THE MATHEMATICS STUDENT

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THE FOURTEENTH CONFERENCE OF THE INDIAN MATHEMATICAL SOCIETY

The Indian Mathematical Society held its fourteenth biennial Conference under the auspices of the Delhi University at their Headquarters on the 21st, 22nd, 23rd and 24th of December 1945

The Conference opened at 3 P. M. on Friday, the 21st December 1945, with an Address of Welcome from Sir Maurice Gwyer, Vice-Chancellor, University of Delhi which, in his unavoidable absence due to indisposition was read out by Dr. Ram Behari, Secretary of the Reception Committee. The Conference was then inaugurated by Sir Benegal Narasinga Rau, who in a neat speech gave the reactions of a succesful Judge and Administrator to the mathematical training at Cambridge he had received in his earlier years.

In the absence of the Secretary, the Report of the Society's activities since the last conference at Annamalainagar was read out by Muhamad Khwaja Mohiuddin of the Osmania University.

This was followed by an Address by the President of the Society Prof. F. W. Levi, in which he discussed the connection between "relations" and "operations" and the fundamental *role* they have played in building up the basic concepts of mathematics.

Dr. Ram Behari then gave a brief account of the "Narasinga Rao Medal for Mathematical Research", and read out the first Prize Problem for which solutions had been invited. The gold medal in the shape of a five pointed star was then presented by Sir B. N. Rau to Dr. V. Ganapathy Iyer (Annamalai University) who had been selected by the committee of Judges as the winner of the award.

Besides the reading of 25 papers, there were two *symposia: the* first on the "Foundations of Projective Geometry" presided over by Dr. R. Vaidyanathaswamy (Madras), and the second on "Stellar

Evolution" under the Chairmanship of Prof. A. C. Banerji (Allahabad). A discussion on the Mathematical Syllabuses of various Indian Universities and the improvement of the teaching of mathematics was held in the open under the sun, but nevertheless conducted within the proper limits of temperature for a healthy discussion.

The programme included three popular lectures on the 21st, 23rd and 24th evenings. The first was a talk by Prof. A. Narasinga Rao (Annamalainagar) on "Mathematical Recreations and Puzzles" and was enlivened with several illustrations and models; the second by Prof. K. S. Krishnan (Allahabad) was entitled "Polydedra in Nature" and was illustrated by a large number of solid models and lantern slides while the last one by Prof. A. C. Banerji (Allahabad) on "A peep into the Wonderland" was devoted to the marvels of astronomical discovery and was illustrated profusely by lantern projections.

A unique feature of the Conference was the "Mathematical Variety entertainment"—the first of its kind—which included mathematical recitations both serious and humourous, a dramatization of the mixed population which constitutes the real number system, and some Demlo Number Magic by the irrepressible Mr. Kaprekar (Devlali).

As at Aligarh the annual meeting of the Benares Mathematical Society was held as a Joint session with the Indian Mathematical Society on Sunday the 23rd December.

The conference was well attended by members of the Society and delegates from all over India.

While the serious side thus received adequate attention, the social side was by no means neglected. The delegates were comfortably lodged in the St. Stephen's College Hostel, where the students kindly vacated some rooms at considerable inconvenience and placed them at the disposal of the guests.

On the 21st December the conference was invited by L. Shankar Lall of the Delhi Cloth and General Mills Limited to a garden party on the university grounds, while the Reception Committee was at Home to the Delegates on the 22nd.

On the 23rd December the delegates were taken on an excursion to Jantar Mantar, Birla Mandir and Upper Air Observatory.

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Sir Maurice Gwyer's Address of Welcome

DELHI,
21st December, 1945

MY DEAR MR. PRESIDENT,

I regret very much that I am laid up with a bad knee and am unable to welcome in person on behalf both of the University and of the Reception Committee the Fourteenth Session of the Mathematical Conference which you are to inaugurate this afternoon.

The University is honoured by having so distinguished a gathering in their midst. It is now some years since the Conference was held in Delhi, and those members who were present on the last occasion will have observed, I hope with pleasure, the very considerable changes and developments which have taken place. We are now more qualified than we were then to receive and entertain men of such eminence in their own sphere as are present here today; and we have learned with the utmost satisfaction of the share in your deliberations which is taken by our Delhi mathematicians.

My science friends tell me that mathematics is the basic science, the foundation upon which all scientific learning is built; and that fact is sufficient to indicate the importance of this assembly. I regret that to myself your science is a closed book. That is my misfortune; but it has not prevented me from appreciating, the immense contribution which mathematicians have made and are still making towards the advance of learning and the solution of the innumerable problems which vex humanity. I was privileged some weeks ago, with many others here present, to listen to a lecture by Dr. Bhabha, on the contribution of mathematics to science, a lecture so clear and perspicuous that I was almost persuaded that I understood what the lecturer was saying; and I remember being much struck by the examples which he gave of solutions to abstract mathematical problems which had been reached long ago and which many generations later afforded the solution to practical problems which have baffled researchers in, I think, both Physics and Chemistry. It must be very agreeable for mathematicians to know that when they are setting forth on the still uncharted seas of their own science they may perhaps, like Columbus, be the instrument of the discovery of new continents or worlds.

I hope that I may be permitted to congratulate the Conference on your acceptance of our invitation to inaugurate these proceedings.

If your distinguished career as a mathematician in your earlier years has been over shadowed since by your labours as an administrator and a Judge and as the impartial arbitrator to whom the Government of India always have recourse for the settlement of issues of the greatest public interest and importance, I am sure that the studies of your Cambridge days are still vivid in your mind, and that you will feel yourself no stranger in a gathering of this kind. In my own country there has indeed always been a close association between the science of mathematics and the science of law ; and I recall that in my very early days at the Bar I witnessed the unique spectacle of three Senior Wranglers sitting together as Lords Justices in the English Court of appeal. Permit me then, Mr. President, to offer you our warmest greetings here today and to request you to inaugurate the Fourteenth Session of the Indian Mathematical Conference, of which this University are on the present occasion proud to be the hosts.

Believe me,

Yours most sincerely,

MAURICE GWYER

Vice-Chancellor.

To

Sir B. N. RAU,

Indian Mathematical Conference, Fourteenth Session,

Delhi University, DELHI.

Opening Address by Sir B. N. Rau

MR. PRESIDENT, LADIES AND GENTLEMEN,

When the Vice-Chancellor, in his usual irresistible way, asked me a few days ago to inaugurate this Conference, I had, of course, no real choice except to agree. But ever since then I have been feeling more and more that the task would have been better discharged by someone in closer touch with recent developments in mathematical theories. Mathematics, like other branches of knowledge, is becoming highly specialized in these days. Many of you have doubtless read the "Recollections and Reflections" of the late Sir J. J. Thomson, a great mathematician and one of the greatest physicists of the century. In the very last paragraph of the book, published, it may be noted, in 1936, the author, writing of Einstein's general theory of relativity, says, "This involves much very abstruse and difficult mathematics and there is much of it I do not profess to understand". If these are the words of a great master, what should a mere amateur feel in addressing an audience some of whom are doubtless specialists in these very recondite subjects? It is many, many years since I tried to do any original work myself; but the mention of Sir J. J. Thomson's name brings back to my mind the history of my first effort. I was then an undergraduate at Cambridge an enthusiastic student of Mathematics—and on one occasion, I thought I had discovered an interesting result in the Mathematical Theory of Electricity. I wrote a longish paper on it and in my youthful presumption sent it to "J, J.", as he was affectionately known in Cambridge, asking him if he thought it worth publishing. He wrote back almost immediately to say that he had read the paper with interest and that it might be worth publishing. Very pleased with the verdict, I sent it to one of the Mathematical Journals and waited; an answer came back from the editor six months later that there was an error in the calculations entirely vitiating the result etc., etc. I have not forgotten the incident, because it shows how even the greatest men in Cambridge try to encourage the humblest workers. But, as I have already said, my efforts in this field ceased long ago and you will not, therefore, expect from me any serious contribution to your labours.

I do not know what papers will be read at this Conference or what their form or contents will be; but I should like to make certain general observations which I hope you will take in the proper spirit. Mathematics is not a subject for the million; it stirs no interest in the press: it inspires no headlines. But it is often made duller than it need be. A journalist who once attended a Mathematical Conference in this very city came back so exasperated at his inability to understand a single word of any of the papers read that he swore he would not attend another. Pondering over the incident, I have been wondering whether you could not clothe your contributions in rather more attractive garb than usual. You have all doubtless heard of Lord Moulton: Senior Wrangler of his year; first Smith's Prizeman; later. a distinguished lawyer and finally a member of the Judicial Committee of the Privy Council. Well, after leaving Cambridge and while studying for the Bar, he wrote two papers on "Intermittence in the Electric Discharge" in collaboration with another distinguished mathematician—papers which secured for him the Fellowship of the Royal Society. So they must have been of outstanding merit; but what was more interesting, there was not, in these two long papers, from beginning to end, a single mathematical symbol. Not that the mere absence of symbols makes a paper attractive; but certainly, the presence of too many frightens the ordinary reader. To avoid all symbols in a really mathematical paper may be deemed a counsel of perfection; there are, however, other devices by which one may relieve the reader's tedium. For example, in the last century, there were a race of men called the "pi-computers", mathematicians who spent years in computing the true value of 'pi', the ratio of the circumference of a circle to its diameter. The culminating point was reached in 1873 when Mr. Shanks published his result correct to 707 places of decimals, which to the layman may seem rather too many. One of these computers, however, contented himself with only 13 places of decimals and adopted the pleasing device of putting the result into the form of a mnemonic couplet:—

How I wish I could recollect of circle round
The exact relation Archimede unwound!

If you write for each word the number of letters it contains, you get the value of "pi" 3·1415926535897. There are other merits about this couplet, besides numerical accuracy: the opening words are exactly those one would say to oneself when one has forgotten something—"How I wish I could recollect etc."—and the closing words give due importance to Archimedes, who was the first to compute "pi" with reasonable accuracy. Another device, which the crossword-puzzle "fan" of to-day would welcome, found favour with the astronomers of the 17th century. Astronomy, though one of the noblest and most exalting of the sciences, does not cure man of all his weaknesses, in particular, of the craving for priority and precedence. So these astronomers, fearing that while they were engaged in verifying an important discovery or in working out its results, some rival might steal the credit for it, would immediately publish it in the form of an anagram, thereby protecting both the secret and their own priority. Thus Galileo announced in a long Latin anagram his discovery that the planet Venus had phases like the moon. These are some of the devices which I commend to your attention if you want your papers to be best-sellers.

In a slightly more serious vein, may I make, in all diffidence, another suggestion of a different kind? In reading a mathematical work, even if it is a short paper, one often feels dissatisfied or rather unsatisfied: the theorem may be of fundamental importance and the proof a model of elegance; but the question occurs to the reader, How did the author come to discover the theorem? Did he see the result first and discover the proof afterwards or *vice versa*? If the former, what process of thought was he engaged on when he saw the result? If the author could explain this, his work would be much more satisfying to the reader. It is said of the ancient Hindu mathematicians that proofs did not interest them; only the results did and some of their results are of remarkable accuracy. Coming to more recent times, we all know how the celebrated Indian Mathematician Ramanujan left a large number of unproved theorems in his note books, some of which have since been proved by others; it is said that he himself had a very vague idea of what constituted a mathematical proof. By what process, then did he discover these theorems to be true or likly to be true? It would be most interesting to know, but we do not know: only the author himself could reveal the secret, and he is no more. In this connection, one of his friends recently told me of a very puzzling fact: it seems that upon returning from Cambridge, after five years' training in the ordinary methods of mathematics, he confessed to this friend that his previous gift or instinct had almost deserted him: he could no longer see results in advance of the proof.

People often ask, what is the use of Mathematics? It is, of course, very good training, particularly for minds apt to be ' woolly" because it compels accuracy and rigour of thought. But apart from that, is it of any use? For example, is it of any use to the student of law? Let me give a few instances some of them trivial. Many years ago, in the interior of Bengal, I had to try a receiver of stolen property, piece-goods in this particular case. Some 50 different varieties, all specifically numbered, were found in the accused's shop. The numbers were exactly those stolen from another establishment. The defence was that the accused had bought all these varieties and no more from different shops in Calcutta. It was proved that there were at least 200 varieties on sale in Calcutta. What was the probability that a man buying 50 varieties at random from among 200 in Calcutta would pick on the very 50 varieties stolen from a local establishment? The probability works out to one in several millions and neither I nor the appellate court had any difficulty in rejecting the defence. To come to another example, the question arose a few years ago whether a tax on the production of electricity was an "excise" and, as such, a federal subject. This turned on the definition of "excise" in the Constitution Act: according to the Act, an excise was a tax on goods produced in the country. The question therefore was whether electricity was "goods". Undoubtedly electricity is akin to "goods" in several respects: it flows from place to place like a material fluid; it can be bought and sold; it can even be stolen; but is it "goods"? A student of modern mathematical theory knows that whether electricity, is "goods" or not, certainly "goods" are electricity, because every atom of matter is just a system of whirling electric charges. Obviously, a lawyer, in touch with mathematical theories, would be at an advantage in arguing this question before a court over one whose knowledge was confined to more orthodox topics. I should perhaps say "*would have been* at an advantage", for the Constitution Act has since been amended so as to make special provision for the electricity tax. The most recent example in which modern mathematical theories were sought to be pressed into service in a court of law is furnished by a House of Lords care decided in June last. It turned on the interpretation of a certain section of the English Law of Property Act which provides that when two or more persons have died in circumstances rendering it uncertain which of them survived the other or others, the younger shall be presumed to have survived the elder. In September 1940, a small house in Chelsea was struck by a high explosive bomb and 5 inmates were killed. Did the section apply to the case.? One side said, No, because the deaths were simultaneous and therefore there was no uncertainty involved; it was as certain could be that not one of them had survived the other or others. The argument on the other side was that simultaneous deaths are impossible and since the order of the deaths was uncertain, the section applied. The House of Lords by a majority of 3 to 2 accepted the latter contention—a triumph for Einstein's analysis of the idea of simultaneity. The Lord Chancellor was in the minority and his judgment makes interesting reading: "The view has been advanced in the course of argument that simultaneous deaths are impossible and this is said to be in some way related to the infinite divisibility of time. The mathematical theory of infinitesimals is a difficult topic which I feel illqualified to expound.............." So here we have a confession from no less an authority than the Lord Chancellor that he would have been better qualified for his task if he had been acquainted with the theory of infinitesimals. It must, however, be confessed that mathematicians themselves have often thrown doubt on the utility of their subject by speaking in riddles. Many of you—particularly those who have had to dive into the mysteries of Non-Euclidean Geometries—doubtless remember the celebrated mathematician Poincare's remark: "There is no sense in asking whether Euclidean Geometry is true or false". Well, if a thing is neither true nor false, is there much use studying it? The doubt is reinforced by the humourous phrase of an English philosopher: "Mathematics is a science in which you never know what you are talking about and never care whether what you are saying be true or false". Another distinguished mathematician, Borel, far from disputing this proposition, finds it literally true. The answer is meant to be equally literal. There is, for example, no sense in asking whether the English language or the French language is true or false; but we find it useful to study both. Again, we habitually talk about electricity; but even Lord Kelvin once confessed that he did not know what it was, much less do we. And so, what is literally true of Mathematics is also equally true of languages and science. But of course these observations are not meant to be taken too seriously. Even pure abstract Mathematics has its uses. The Greek geometers went on with their studies of conic sections without bothering about the ultimate use to be made of them: in fact the Greeks despised all practical applications as "debasing and corrupting the excellence of geometry": twenty centuries later, Kepler applied their results to the planetary motions: then Newton came and explained Kepler's results on the theory of gravitation. The theory may not have been absolutely exact—there was a discrepancy of 43 seconds of arc per century in respect of the perihelion of the planet Mercury—but it was sufficiently accurate to enable Adams and Leverrier to predict the existence and the exact position in the sky of the then-unkown planet Neptune. So mathematics can literally discover new worlds beyond the power of man's unaided eye. Another striking instance of the use of mathematics is furnished by Cauchy's theory of functions of a complex variable. Little did he dream, when he elaborated the theory, that it would one day be useful in the calculations of technical electricity. It is now employed regularly and has been a valuable aid in the theory of wave propagation including the equations of Maxwell and Hertz, with which one must directly associate the invention of wireless telegraphy. More recently still, the absolute differential calculus of Ricci and Levi-Civita has furnished Einstein with a powerful instrument for developing his theories of relativity. And so you cannot say of any branch of Mathematics, however remote from "reality" it may seem at the moment, that it is barren or its pursuit futile.

The student of history has only too much reason to remember Gibbon's desolating description of it as little more than the register of the crimes, follies, and misfortunes of mankind. As a branch of liberal education, Mathematics has this signal merit and advantage over ordinary history: the study of Mathematics and of the history of Mathematics induces a nobler conception of Man and his destiny. Mathematics is nothing if it is not the rule of Law; and every mathematical theory of note is only an extension of the domain of law. The mathematician abhors chaos, even if it is in the third or fourth place of decimals; remember that it was a stellar displacement which, measured in degrees, was significant only in the third place of decimals, that led Bradley to two beautiful theories, the aberration of light and the nutation of the earth. The mathematician cannot rest until he has reduced to order every apparent aberration, and found the law behind every chance—indeed, he has discovered the very laws of chance, which he calls the theory of probability. He pierces into the mysteries of the infinitely vast as well as of the infinitely small; and finds the laws that bind the planets in their motions round the Sun, as well as those that bind the electrons in their motions round the atomic nucleus. In spite of all his conquests, he is very humble: no one living and moving in these lofty regions can be anything else. We have all read of Newton and his gathering of pebbles on the shore while the ocean of truth stretched out before him; also of Kelvin, who, after half a century of indefatigable research and epoch-making discoveries, could still say at his jubilee, "One word characterizes the most strenuous of the efforts for the advancement of science that I have made perseveringly during 55 years and that word is Failure". What is the picture that emerges from all this? Let us think of it for a moment. The earth, considered astronomically, is a small planet in an insignificant solar system—a mere speck in the depths of space. If the entire solar system were to disappear tomorrow, its disappearance would not even be noticed from some of the more distant stars of our own galaxy—let alone other galaxies. On this small planet, Man is one of a myriad forms of life and of comparatively recent origin. His expectation of life, even in the most advanced countries of today is under three score years and most of this period is spent in a fierce fight for existence. And yet, as someone has said, perched precariously on this rotating speck of mud and water that we call the earth, Man, while struggling to live, has wrested some of the profoundest secrets, not only of the visible but also of the invisible universe, and has now created even a miniature Sun out of atomic energy. As we contemplate his victories, the figure that emerges is not of Man the victim of folly and crime and misfortune, but of Man triumphing over Matter, Man the Conqueror—great, and yet humble in his greatness—"The chief handiwork of the Power that hung the stars in the firmament".

Gentlemen, I wish you all success in your labours.

Secretary's Report on the Working of the Society for the years 1944 and 1945

It is my plesant duty, on behalf of the Managing Committee of the Indian Mathematical Society, to extend to you all a hearty welcome to this Conference. We are particularly grateful to the authorities of the Delhi University for inviting us to hold our fourteenth Conference here. We are also thankful to Sir B. N. Rau, for the trouble he has taken in coming and inaugurating our Conference this afternoon.

We met here exactly ten years ago and many of us still carry pleasant memories of that occasion, when the founder of our Society, the late Mr. V. Ramaswami Aiyar, was still in our midst, and was helping us in our deliberations.

Our last three Conferences, viz. those held at Hyderabad, Aligarh and Annamalainagar, had been overshadowed by the war, but it speaks volumes for the courage and devotion of our members that neither the work of the Conference nor the activities of the society were appreciably curtailed by this global catastrophe. We are thankful to the Almighty that this terrible ordeal is over, and we are now meeting again in a peaceful atmosphere. We hope now to be able to expand and enlarge our activities, so that the Society may contribute its quota to the rapid development of the Mathematical Sciences throughout the country.

I am glad to say that the necessary conditions for this development exist. The three Mathematical Societies in the country, the Indian Mathematical Society, the Calcutta Mathematical Society and the Benares Mathematical Society, are sincerely co-operating with each other, and there is absolutely no rivalry between them. The office-bearers of one society are often holding office in the other two societies also. The time is perhaps ripe, therefore, for taking the next step, and establishing one Mathematical Association for the whole country, of which the various mathematical societies may be individual members. If such a step is contemplated, I hope it will not be overlooked that the Indian Mathematical Society, besides being the oldest scientific body in India, is truly of an All-India character.

The activities of the society in the past have been fourfold: (*a*) establishing a Central Library at Poona and circulation of the journals and periodicals to members, (*b*) publication of the 'Journal' (*c*) publication of "The Mathematics Student" (*d*) holding of periodical conferences.

The Library naturally suffered the most during the period of the war. Journals and periodicals from the European continent and from Japan stopped entirely, and probably many of them do not exist any longer. Journals from England and America dropped in at irregular intervals, and many numbers of even these few suffered the casualties of the war. I hope that the post-war period will enable us to make good these deficiencies and to bring our library to modern research standards.

Since the last Conference was held at Annamalainagar, our Librarian, Prof. Shintre has retired, and Prof. Kosambi, now of the Tata Institute of Fundamental Research, Bombay, has been appointed in his place. The Managing Committee has placed on record its deep appreciation of Prof. Shintre's services to the Library. It is hoped that Prof. Kosambi's vigorous efforts will play a large part in the development of the Library.

The period under report was a somewhat critical one in the affairs of the Library. The authorities of the Fergusson College, Poona, were finding it difficult to allot the usual accommodation for the Society's Library. The Managing Committee and the General Body of the Society had therefore decided to shift the Library to the room placed at its disposal by the Osmania University, Hyderabad. Meanwhile, the authorities of the Fergusson College, kindly arranged to place suitable accommodation at our disposal, and therefore it was not found necessary to shift the Library from Poona. The Society's thanks are due to Prof. D. D. Kapadia, a former Secretary of the Society, for his efforts in this connection.

As long as the society does not have a building of its own for the Library and the Office, the situation cannot be considered to be satisfactory.

During the period 1944—45, 39 papers were received for publication in the *Journal*. The total number of papers published in Vol. VIII is 18. Here again, our Editor has been handicapped for want of an adequate press, though every effort has been made to maintain the standard and regularity of the *Journal*. The paper control order did not affect our periodicals appreciably.

because through the co-operation of the authorities of the Department concerned, we were able to secure almost a complete exemption from the ordinance. Dr. Vaidyanathaswamy and Dr. C. N. Srinivasiengar continued as the Editor and Secretary of the Journal respectively.

The Mathematics Student continues to flourish under the able editorship of Prof. A. Narasinga Rao, who is always trying to introduce new features, and to make it as interesting and instructive to the students and the teachers as possible.

I am happy to report that the Rockfeller Foundation has kindly given us, through the National Institute of Sciences, India, a grant of Rs. 450, for the publication of our Journals. It is but fitting that on this occasion we should place on record our gratitude to the authorities of the Rockfeller Foundation for their grant-in-aid. I hope this will be a prelude to more generous donations in future for the purposes of the Society, which has already decided to launch a scheme of publishing monographs. This scheme could not be given effect to because so far we had no funds at our disposal. The Government of His Highness the Maharaja of Travancore has kindly given a grant of Rs. 1,000, towards these publications, but obviously we need many more such grants from the patrons of learning before we can undertake to publish the monographs regularly.

Another new feature of our programme is the award of a Gold medal biennially for the best original thesis on a problem announced in advance. This award has been made possible by a generous donation of Rs. 1,000 by Prof. A. Narasinga Rao. The first medal will be awarded in the course of the present session.

For a number of years now, we have been holding discussions on the teaching of Mathematics in the schools and colleges, but it was felt that these discussions were not having much permanent value. There is still much diversity in the Mathematics curricula of the various universities in the country, and students who migrate from one university to another are much handicapped by the difference in the syllabus of the two universities. The Managing Committee has set up a Sub-Committee, consisting of the representatives of the various Mathematical Societies and of the Science Congress to draw up a comprehensive syllabus for Intermediate, B.A. and B.Sc. (Pass and Honours, Main and Subsidiary) and M.A. and M.Sc., which can be adopted with slight local variations by the various universities. A first draft of such a syllabus will be presented for discussion at the symposium on the Teaching of Mathematics. In the light of the criticisms and suggestions of the general body of members the Committee will draw up a final syllabus and will send it to the Inter-University Board for consideration.

Prof. L. N. Subramaniam retired from the Treasurership last year, and the Managing Committee has placed on record its deep appreciation of the long and devoted services of Prof. Subramaniam. At the request of the Managing Committee, Prof. A. Narasinga Rao who is already rendering valuable services to the Society as the Editor of the Mathematics Student, has consented to act temporarily as the Treasurer.

In conclusion, I have to offer grateful thanks to Sir Maurice Gwyer, and to the authorities of the Delhi University for their kind invitation and hospitality to our Society, I have particularly to thank Prof. Ram Behari, who is now the Local Secretary, but who up to a couple of years ago was the Honorary Secretary of the Society, and who is still a valuable and active member of our Managing Committee. I wish also to thank Members of the Local Committee and the volunteers for the careful arrangements they have made for the comfort and convenience of our members and delegates.

RELATIONS AND OPERATIONS

Presidential Address

delivered at the 14th Indian Mathematical Conference Delhi, December 21st 1945

BY

DR. F. W. LEVI, *Hardinge Professor, Calcutta University*

On the invitation of the University of Delhi, we are gathered in this Imperial city for an exchange of ideas and reports on recent progress in mathematics.—First, our heartiest thanks are due to our hosts. In extending their invitation to the Indian Mathematical Society, they have shown a deep understanding for the needs of science. To promote scientific progress in a country, it does not suffice to encourage research and teaching of only those subjects which attract public attention on account of their obvious and immediate usefulness. As no plant can grow without roots, so science cannot flourish unless the basic topics develop vigorously; because for a lasting growth, their strength is indispensable. Mathematics is a basic subject and not a very popular one. Perhaps it is the claim of absolute truth which makes our science look haughty and unhuman; wrongly it is sometimes taken for a thing out-side normal human activities. Moreover, people wonder how Mathematics can go on developing without outgrowing itself, since each statement once correctly established in the course of centuries, remains a permanent portion of the substance of our science. However, Mathematics has developed and is developing further: it has even completely altered its character during the last two generations. Most of you will remember Dr. Vaidyanathaswamy in his Aligarh address encouraging young Indian Mathematicians to come to a true understanding of to-day's Mathematics and to contribute to its development. It will take some time before his challenge can find a full response, but we are already on the right path. Even a superficial observer will realise that in recent times, Mathematics has shifted from the investigation of "number" and "space" to a general research of *relations* between objects of any kind. One might even call Mathematics the general science of relations, but one cannot proceed from one relation to another without using an interconnecting link. This necessary interconnection is supplied by our power of mathematical *operation* which has already been used from the beginnings of our science in simple forms. In modern times however, the most multifarious of operations are being applied in a free manner corresponding to the great diversity of relations which are to be investigated. "*Relations and Operations*" will be the topic of this address. As a full discussion of the subject would mean a nearly complete survey of Mathematics. I shall restrict myself to a few examples illustrating the complex interconnection of these two notions. Let us start with a simple example, the routine followed for the construction of a triangle out of given data. At first an "Analysis" is made, i.e., one states relations which hold between certain mathematical entities. Say two segments or angles are of equal measure. This is a relation holding between these two entities; moreover three points may be shown to be related by the condition of being collinear, or 4 points by being concyclic; orthogonality of straight lines and congruence of triangles are other instances of relations. The Analysis is followed by "Construction" consisting of a sequence of "operations". To connect 2 points A and B by a straight line s means an operation on A and B, the result being s; similarly the dropping of the perpendicular p from A on a line g is an operation on A and g; again by a different operation on two lines g and h one obtains the point of intersection S. All these operations are based on existing relations, e.g., there is a relation

R (A, B, s) which means that the line s passes through the points A and B. Given A and B, there exists one and only one straight line s which satisfies the relation R (A, B, s) whereas given A and s, the point B is not uniquely determined, it may be any point on s which is different from A. Thus the relation R gives rise to an operation leading from A and B to s, but not to an operation which leads from A and s to B. There are other relations which generate-operations and their inverse operations. Thus denote by P (A,B,C,S) the relation between the vertices A,B,C, of a triangle and its centre of gravity S; then each of the 4 "variables" A,B,C,S is uniquely determined by the 3 other ones. Thus the relations occuring in Mathematics are of different "Operational value" according as one or more or no operation can be derived from them. In the Analysis preceding the construction one looks for such relations only from which operation can be derived, and then one has to bring these operations into a suitable succession. Eventually one obtains the triangle ABC which satisfies the relations, as proposed. Thus operations lead from relations to relations. But the interdependence of relations and operations is in no way restricted to Geometry. Let A and B be any two different numbers, then either $A < B$ or $B < A$; *one of these two relations is bound to hold. If* A is given, than neither of the relations determines uniquely B; thus one cannot derive an operation from these relations. If however we restrict ourselves to the natural numbers 1, 2, 3,......then there exists among all the numbers B satisfying $A<B$ a particular smallest one, which is the *successor* or right hand neighbour of A. The passing from a number to its successor is the simplest as well as the most important operation in Mathematics; in a somewhat amplified form, it is called the method of Mathematical Induction and it applies to all kinds of mathematical investigations. Every natural number—with the only exception of the number 1—is the successor of one and only one natural number; thus we have here an operation which is not always invertible, but if so, then uniquely. To make it invertible, one creates zero and the negative numbers, so obtaining the system of the integral numbers which admits the operations of addition, subtraction and multiplication, all derived from the successor operation.

As the multiplication is not invertible, we have reached a cross-road and we have to decide about our way. Either we take it with some resignation that multiplication cannot be inverted, or we extend the "ring" of the integral numbers to the "field" of the rational numbers by introducing fractions. The first way leads to Arithmetic and Theory of Numbers, the second to Algebra. This distinction reappears in Mathematics at other levels and gives birth to complex and very interesting interconnections. In the field of the rational numbers, addition, subtraction, multiplication and division can freely be applied, but the original operation has got lost. In the natural order of the rational numbers, no number has a successor. The loss of the operation on which mathematical induction is based, is so great, that it became necessary to look for a substitute and two were found. Firstly the passage to a limit. The upper limit A of a bounded set S of numbers without a maximal number can be considered as the successor of the whole set S, because A is the smallest number, greater than every number in S, and that is just what one expects of a successor. However in the field of the rational numbers the passage to limit cannot be performed without restrictions; thus it was extended to the *continuum* of the real numbers which is the playfield of the great portion of modern Mathematics. That the rational numbers appear to be an intermediary entity only seems to be strange. As a second substitute of the successor-operation, one disarranges the natural order of the rational numbers and puts them into some very unnatural order which is of the same character as the natural order of the natural numbers; that means one "enumerates" the rational numbers. In this way the successor-operation is maintained, but it has lost its original meaning and therefore the expedient does not look very helpful at first. Although the enumerability of the rational numbers was discovered 60 years ago, its practical application is of much later date. The operational value of the method became apparent only in connection with the Lebesgue-integration. What I called the two substitutes of the successor-operation are now often applied jointly. We have the continuum of the real numbers in which limit-operations can be performed and inside the continuum there is the field of the rational numbers which is enumerable. These two entities are interconnected by a relation, namely the field is everywhere dense in the continuum, and this relation is just the interconnecting link between the two operations. Wherever a continuous entity and an enumerable one are interconnected by this relation, that combination of methods can be applied; for this reason the methods of Analysis can be generalised to the topology of "separable" spaces. A different kind of combination of the limit and successor-operations may be mentioned only *en passant*—the method of transfinite induction.

By these last remarks I might have exhausted the patience of those of my audience who are not professional mathematicians and I expect some objections like these: "Apparently the mathematicians are developing their ideas in a very arbitrary and fastidious manner following their taste for beautiful relations and their preference for operations which can be performed without inconvenient restrictions. Instead of indulging in these hobbies, they would do better to listen to the voice of nature and follow in their investigations the paths suggested by the laws of the physical universe". It will not suffice to face such objections simply by referring to the great services rendered by the modern mathematical methods to physics A short review like this, which jumps over the developments of centuries may give a wrong impression of the continuous work of generations of mathematicians. It does not consist in inventing arbitrarily new operations and relations according to taste. The most important steps have been done under the pressure of necessity, often very reluctantly and mostly by the great masters of our science. However the advice to listen to nature is certainly a good one; I shall follow it now. As modern Physics is a highly mathematical and not yet a popular subject, I shall look now at classical Mechanics which obviously has not been influenced by the trend of modern mathematical ideas.

The object of classical Mechanics is a system of material points each moving with a particular velocity at a given moment. On these points forces are operating, which result in accelerations of material points. The initial conditions (the situations, velocities and masses) of the material points together with the operating forces determine uniquely the conditions at any later time. The mathematical difficulty of how to establish these uniquely determined conditions, will not be discussed here. But beside the operating forces, the problems actually occurring also show relations. Say two heavy bodies (considered as material points) are connected by a strong rod: in this case the distance of the bodies has to remain constant which implies a relation. Similarly any other kind of machinery imposes some relations on the movement of the material points concerned. Thus the laws of nature as expressed in classical mechanics contain operations as well as relations and they invite mathematicians to study both concepts thoroughly. There is one method where the relations are replaced by forces, whereas in the pure Mechanics of motion, the forces are replaced by relations. One could even speak of a competition between operation and relation in Mechanics: but in general, one uses both operations and relations interconnected by those famous principles of d'Alembert, Lagrange and Hamilton. Lagrange's method uses generalised co-ordinates and generalised forces; in modern terms one might say that the problems are reduced to the notion of a single point in a multidimensional non-Euclidean space moving under non-Newtonian forces. But the mind of the mathematicians of that time —and there were very great masters among them—was very far from general concepts of that kind. Gauss, the greatest of all, held back his knowledge of Non-Euclidean Geometry, afraid of the "clamouring of the Boethians". Time was not ripe for these ideas. There exists an interconnection between the general trend of ideas in any particular period, the discoveries of the scientists and the creations of the mathematicians which has not yet been investigated systematically. Whosoever will dare to undertake this work will get the most interesting clues from the history of the 19th century. It is not the place here to describe how just those portions of Mathematics which were best fitted to explain classical Mechanics and classical Electrodynamics sailed slowly in the backwaters of Mathematics until Minkowski made the striking remark that the most complex looking special theory of relativity afforded a very simple explanation by generalised Geometry. I mention these well-known facts only to show that the development of Mathematics by free use of general relations and operations is the right response to the challenge of Science. That physicists cannot understand mathematical concepts going beyond differential equations, would be considered as an insulting statement now, but not very long ago it was so to speak an axiom.

Whereas in earlier times only the system of numbers and space were objects of mathematical investigations, present-day Mathematics deals with a great variety of systems Each of them has its own laws, just as different countries use different codes, but there are affinities between them and similar methods can often be applied in various cases. A system may be of geometrical, algebraic, analytic, arithmetical or any other nature. It is composed of elements which inside the system itself have no particular significance; they may be compared with pawns on the chessboard. Moreover there are relations which might or might not hold between the elements, and finally there exist operations. If the system is a geometrical space, then the elements a, b, c may be points and the relation R (a, b, c) may mean that those 3 points lie on the same straight line; if the system consists of numbers, then the relation may mean e.g. that $a+b+c=0$. By an operation an element a is transformed into another element say a', similarly b into b', c into c'. Therefore an operation means a mapping of the system under consideration upon itself. If for example the system consists of numbers and the operation consists of doubling every number, then $a+b+c=0$ implies $a'+b'+c'=0$. Thus the operation of doubling is such that elements connected by that particular relation R (a, b, c) will be transformed into elements connected by the same relation. One says that R (a, b, c) is *invariant* for that operation. Given now a system with some relations, then we may search for all those operations for which the given relations remain invariant; they form an aggregate of operations Ω. Now again we may investigate all the relations which remain invariant for every operation out of Ω; these form an aggregate A of relations including the originally given relations. The given set of elements together with the aggregates A of relations and Ω of operations form an object of mathematical investigation: we may call it a space or a field, a group, a ring, a structure, etc., according to its formal character. The distinction of mathematical investigations according to the mathematical systems has at first been clearly elaborated in Geometry, later Algebra followed, then all the more recent branches of Mathematics, like Topology and the theory of linear operations, but even classical portions of Analysis like the theory of functions of a complex variable are treated with advantage from this point of view. Very far from disrupting Mathematics into a heap of incoherent sciences, this principle discloses the interconnection of various threads of ideas in the fabric of Mathematics.

A few words may be added to describe how one mathematical system originates from another one. Suppose we are interested in some mathematical system with an aggregate A of relations and an aggregate Ω of operations. Let α and β be two operations out of Ω, then α maps the elements $a, b, c, \ldots$ on elements $a', b', c' \ldots$ in such a way that every relation holding between them, holds also between the corresponding dashed elements; again β maps the elements a', b', c' on elements $a'', b'', c'' \ldots$ in such a way that every relation holding between the $a, b, c, \ldots$ and therefore between the a', b', c' holds also between the $a'', b'', c'', \ldots$. Hence the transformation mapping $a, b, c, \ldots$ on the corresponding doubly dashed elements leaves all the relations of the aggregate A invariant; hence this mapping is an operation out of Ω; we call it $\beta\alpha$. Thus we can consider the operations of Ω as elements of a new system in which a relation $\gamma = \beta\alpha$ is defined. The properties of this new system are certainly important for the investigation of the original system. It is called a semigroup and it is somewhat arithmetical in its character. Given γ and β, it is not certain that $\gamma = \beta\alpha$ can be satisfied by a suitable α; so questions of divisibility arise and for this reason I said that the system is somewhat arithmetical. However we can derive another new system which is more algebraic in its character, namely the group of all those operations out of Ω for which there exists an inverse operation in Ω. The nature of these two new systems is important for the investigation of the original one; perhaps they are very simple admitting well known methods, otherwise they will give occasion to new research. It may be that they are recognised as difficult problems known from other branches of Mathematics. Sometime one observes common features in very different systems and it seems to be worthwhile to construct a common basis for them by the help of a new, generalised system in which only the common relations are defined.

As an interesting example of this kind of generalisation, I shall mention the structures and partially ordered sets explaining the pith of the matter by the help of a non-mathematical example rather than by exact definitions. Take a map of India; the country appears to be sub-divided into several areas according to its purpose as being a political, historical, hydrographic, ethnographic, orographic, cultural, religious, agricultural, commercial etc. map. Take a political map showing the division into States, foreign possessions, provinces and their repeated sub-division into smaller administrative units. Each unit should be an element of the system which we are just constructing. Now we want to introduce an order-relation between these elements. This would certainly lead to great difficulties, if we insisted on a linear order making any two elements comparable; however let us restrict ourselves to the relation that the whole is greater than any of its parts. So we have the relation India >U.P.> Fyzabad district > Fyzabad city etc. but there is no order relation say between Goa and Santiniketan none of two units being a part of the other one. This system is already a partially ordered set, but it is a simple ramification only. For our present purpose it is too simple and we take a second map of India where the division is made on some other principle, say, geographical zoning. The tropic of cancer cuts India into two zones call them India N and India S and so it subdivides serveral of those administrative units big and small which will also be distinguished by the indices N and S. These portions too are supposed to be elements of our system. Finally we add an element which strictly speaking is not a geographical portion of India but an old invention of Indian mathematicians. Namely "naught". Certainly all the other elements are greater than "naught". So we have e.g. India> India N> Bengal N> Calcutta> naught. On the other hand we have also India> Bengal N etc. Bengal and India N are not in a direct order relation, but there is one element, India, which is greater than both and one element Bengal N which is less than both. These two elements are the nearest common upper and lower elements. One can see easily that to any two or more elements of this system there exists exactly one nearest common upper element which is greater or equal to the given elements and similarly one nearest common lower element. The finding of the nearest common upper or lower element is an operation and these two operations have certain properties like commutativity, associativity, idempotency etc. Obviously the system—we shall call it a structure—reminds us of the lowest common multiple and the highest common factor of integral numbers; in this case 1 stands as the lowest element, being a factor of every integral number and 0 is the highest element, because it is divisible by every integral number. In general, structures are defined by the help of the two operations mentioned above which have to satisfy certain axioms. Every structure is a partially ordered set; on the other hand every partially ordered set can be completed to a structure. Geometry, Topology, Theory of groups, Mathematical logic seem to be very different branches of Mathematics, but a closer investigation done in recent years shows that in each of them there is some portion which deals with structural properties and these can be derived from the general theory of structure. This example shows how and why new mathematical systems are created and that relations and operations are their characteristic features.

Eventually one may ask: What is more important: the relations or the operations — Is it the aim of Mathematics to establish relations, whilst the operations are only a method to get them, or are the operations the really important thing and the relations auxiliary notions only ?—A contemplative philosopher may be inclined to put "relations" first, whereas those who deal with Mathematics to get methods for solving their problem only, will appreciate the importance of operations. One could perhaps classify the mathematicians according to whether their mind works more in the ways of operations or of relations. It will not be easy to decide in actual cases, but this psychological distinction seems to me to be sounder than the physiological classification according to visual or acoustic intuition. But what have Psychology and Physiology to do with Mathematics? Does not our science stand above human qualities and weakness, a part of the absolute, the pleasure of the Gods ?—I am afraid that enthusiastic exuberances of this kind, though they might be full of poetry and beauty, are seriously misleading. I personally believe that Mathematics is an approach towards the Absolute—*one* of the possible approaches and a human approach. The ways, we are going in Mathematics are determined by our qualities and our limitations; the results which we obtain are necessarily imperfect, and every result raises new questions. There can never be a perfect and completed Mathematics as long as there are men who think in the mathematical way. As contemplative human beings we want to see the interconnections between the mathematical entities in their full beauty; as acting individuals we are pushing forward and we create new mathematical objects.

Relations and operations, the fundamentals of Mathematics, are the expression of two basic qualities of human spiritual life. Doing Mathematics is a human activity—an important and necessary one—but human and not a conversation with supernatural beings. As our work is determined by our human qualities and limitations, we have to be courageous; every victory in our research is disappointing for it is necessarily incomplete; every setback should be a stimulus for renewed activity. Even brave men need encouragement; isolation enervates and weakens. Exchange of ideas, personal contact between mathematicians, collaboration and competition give encouragement. Our Society aims to promote every attempt in this direction. The ties binding mathematicians together all over the world have partly broken down by war and other political activities, partly they have been loosened by the difficulties of foreign exchange, restrictions in the traffic of books and other financial and commercial difficulties arising from the present world situation. We shall try our best to have these ties restored, but at present we mathematicians of India must rely to a great extent on our own resources. For this reason our common enterprises, journals, other publications and conferences are of greater importance than at any time before. I hope that the 14th Mathematical Conference which is beginning now, will lead to a successful exchange of ideas, that it will be a source of encouragement to progress in mathematical research to all our members and guests who join in our proceedings and that the generosity of our hosts will be justified by the results of this meeting.

ABSTRACTS OF PAPERS

V. S. Krishnan, Madras.

The Completely Regular Spaces.

A fundamental property of '*the completely regular space*' as defined by A. Tychonoff[1] is its topological imbeddability in a Tore $T\lambda$ (i.e., in the direct product space of a family of spaces—of cardinal λ—each homomorphic to the closed real segment $(0 \leqslant x \leqslant 1)$. Another aspect of this space is brought to light in A. Weil's[2] characterisation of the completely regular space as '*a space of uniform structure*'; namely, that the space embodies the features common to two important special types of spaces—the metric space and the topological group. Using the property of imbeddability in a Tore I have been led to yet another characterisation of these spaces: a completely regular space R, is shown to consist of an abstract set R which has been assigned the strongest among all the topologies that can be assigned to it subject to the single condition that, each one of a given separating[3] family of real valued functions defined over R shall be continuous. This view also leads in a direct and natural manner to some of the results regarding these spaces given by E. Cech[4].

In the above characterisation the effect of the term 'separating', qualifying the family of real-valued functions which define the topology, is to make the space a T_1 space. Its omission leads to what I call '*a quasi uniform space*' which fails to be a space of uniform structure by just the lack of the T_1 separation axiom. And even as a space of uniform structure is a generalisation of the metric space and the topological group, this quasi uniform space is a generalisation of the quasi-metric space and the quasi topological group (which arise from the metric space and the topological group respectively by the omission of the condition '$d(x, y)=0$ implies $x=y$' for the distance function d in the case of the metric space, and the condition '$P\lambda\ V\lambda\ (O)=(O)$ where $V\lambda\ (O)$ is a fundamental

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1. A. Tychonoff: "Über die topologische Erweiterung von Räumen" *Math. Annalen*, Teil 102, (1929).
2. A. Weil: "Sur les espaces des structure uniforme . . . . ." *Actualites Scientifiques et Industr*, 551.
3. For a definition of this term see Lefschetz's book on "*Algebraic Topology*" in the Colloquium Series.
4. E. Cech: "On Bicompact Spaces". *Ann. of Maths* Vol. 38, No. 4 (1937).

system of neighbourhoods of the unit element O in the case of the topological group).

The importance of these non-T_1 generalisations lies in that we are able to show that in '*the lattice of all possible topologies assignable to a fixed set R*' the quasi-uniform topologies can be obtained as lattice products of the quasi-metric topologies, so that the quasi metric topologies form a multiplicative basis for the quasi uniform topologies (and so also for the uniform structure topologies, metric topologies, and—when R is a group, for the quasi group and group-topologies also) whereas it is not true that the uniform structure topologies can be always obtained as lattice products of metric topologies.

V. Narasimhamurti, Guntur.

On certain 2-2-k Tactical Configurations.

This paper deals with inequivalent 2-2-k configurations for the cases $k=5$ and 6. The case $k=6$ is studied here. An "elegant solution" of the 2-2-k configurations when $k=9$ is given in this paper.

S. C. Ghosal, Alwar

A Numerical method of extracting roots of numbers and solving of equations.

D. R. Kaprekar, Devlali.

Demlo Remainders.

The remainders when numbers of the form $(a)_n$ are divided by divisors of the form $(9)_{k-1}\,8\,(0)_{k-1}\,1=(9_k)^2$ are discussed in this paper.

Examples: $(263)_{59}\equiv 958520 \bmod (998001)$ and $(7)_{52}\equiv 67 \bmod (81)$

A. Vindhyachal Prasad, Bhagalpore.

On the lower limit of NF (Nx) when N→∞

The lower limit of NF (Nx) when $N\to\infty$ where F(Nx) denotes the fractional part of Nx where N is integral and x is any positive real number is (i) 0 when x is rational and

$$\text{(ii)}\quad \underline{\lim}_{n\to\infty}\ 1\Big/\left(x_{n+1}+\frac{q_{n-1}}{q_n}\right)(n \text{ odd}) \text{ when } x \text{ is irrational,}$$

where $a_1\ a_2,\ a_3\ \ldots$ are the partial quotients and $x_1,\ x_2,\ x_3\ \ldots$ are the complete quotients and $p_n,\ q_n$ the numerator and denominator of the nth convergent when x is converted into a simple continued fraction.

Some results.

(i) $x=\sqrt{a^2+1}$ $\lim_{N\to\infty}$ NF (Nx) $=\dfrac{1}{2\sqrt{a^2+1}}$

(ii) $x=\sqrt{\dfrac{a}{b}\left(ab+2\right)}$; $\lim_{N\to\infty}$ NF (Nx) $=\dfrac{1}{\sqrt{\dfrac{b}{a}\left(ab+2\right)}}$

(iii) $x=\sqrt{\left(a+\dfrac{1}{b}\right)\left(a+\dfrac{c}{bc+2}\right)}$; $\lim_{N\to\infty}$ NF (Nx) $=\dfrac{x^2-a^2}{2x}$.

This incidentally solves Q. 784 (viii 159) Page 334 (*Collected works* of Ramanujan) which, as far as I know, has not been solved yet.

Again $\lim_{(n \text{ odd})} 1\Big/\left(x_{n+1}+\dfrac{q_{n-1}}{q_n}\right)=\dfrac{1}{2\sqrt{M}}$ where M is a non square integer such that the cycle is odd when $\sqrt{M}$ is converted into a continued fraction.

A. A. Krishnaswami Ayyangar, Mysore.
n-ic Residues as Difference Sets.

The problem of this paper is to find a set of distinct non-zero integers $(X_1, X_2, \ldots, X_t)$ less than a prime v (>2) such that

(i) $X_i \equiv y_i^n$, (mod v), $i \leqslant t$

(ii) $X_i - X_j \equiv m$, (mod v) for a given number λ of pairs i, j m being any non-zero integer.

(iii) $t\ (t-i)=\lambda\ (v-i)$.

The discussion is based on Gauss's and Cayley's results in the theory of cyclotomy. If $v-1=pn$ and g a primitive root of v, we call

$$(g^k, g^{n+k}, g^{2n+k}, \ldots, g^{(p-1)\,n+k}) \text{ reduced (mod } v)$$

the index set of the k-th p-nomial period defined by

$$\eta_k = r^{g^k} + r^{g^{n+k}} + \cdots + r^{g^{(p-1)\,n+k}} \quad \text{where } k=0, 1, 2, \ldots, n-1$$

and r an imaginary v-th root of unity; these η's have the property that every rational symmetric function of the periods with rational coefficients is a rational number.

The following results are proved:

1. The necessary and sufficient conditions for an index set to be a difference set are that p must be odd and therefore n is even and the same number λ of pairs of consecutive integers must occur in each of the index sets corresponding to $\eta_0, \eta_1, \ldots \eta_{\frac{n-2}{2}}$. The index set corresponding to η_0 is the required difference set of n-ic residues, provided the above conditions are satisfied.

Example:—The biquadratic residues of 37 and the non-biquadratic residues of the same character possess the above property and are therefore difference sets.

2. It follows from (1) above that no $(2n+1)$-ic residue can be a difference set; for example, cubic and quintic residues cannot be difference sets.

3. The quadratic residues (or non-residues) of a prime of the form $4m+3$ form a difference set with $\lambda = m$. Incidentally we have the result that in this case there are m pairs of consecutive integers that occur as quadratic residues (or non-residues). Thus (4, 5), (5, 6), (6, 7), (16, 17) are the four pairs of consecutive residues of the prime 19.

4. $\eta_i \, \eta_{i+\frac{n}{2}} = \eta_j \, \eta_{j+\frac{n}{2}}$ for all $i, j > n/2$

$= p - \frac{p-1}{n}$, which shows that n must be an even factor of $(p-1)$.

5. From (4) above, we have when $n=4$ and $v=4p+1$ where p is an odd positive integer,

$$\eta_0 \, \eta_2 = \eta_1 \, \eta_3 = \frac{3p+1}{4}$$

From Cayley's results for the η's (vide Cayley: *Collected Papers* Vol. XI, pp. 92, 93) we get $v = A^2 + B^2$ where A, B are integers,

$$A = -l + 4(m-l),$$

and $\eta_0 \, \eta_2 = (\eta_0 + \eta_2)(l - m) + m$, where l, m are integers, which shows that $l = m$ and therefore $A^2 = 1$ and p an odd square.

A neighbouring result is easily perceived. A set of biquadratic residues mod. v, with the adjunction of 0 can also be a difference set, provided $(v-9)/4$ is an odd square. In this case

$$(1+\eta_0)(1+\eta_2) = (3p+1)/4$$
$$= (1 + l - m)(\eta_0 + \eta_2) + m$$

so that $l-m=-1$, $A=3$, $v=A^2+B^2$ as before, $(4p-8)$ a perfect square i.e., four times an odd square, since p is odd, thus, $v=9+$ four times an odd square.

When $v=109$, the biquadratic difference set is

$$(0, 10^4, 10^8, \ldots, 10^{108}) \text{ mod. } 109 \text{ with } \lambda=7.$$

6. There is one instance of an octic difference set,

$$(1, 2, 4, 8, 16, 32, 37, 55, 64) \text{ mod. } 73 \text{ with } \lambda=1.$$

7. There are no instances of residue sets other than those pointed out here, for values of v less than 100.

With the help of a table of powers of primitive roots one can easily extend the investigation for $v > 100$ utilising the necessary and sufficient conditions mentioned above.

K. Sambasiva Rao, Guntur.

On certain arithmetical functions of square-free integers.

In this paper are given the asymptotic formulae for the sums $\sum_{q \leqslant x} \sigma(q)$, $\sum_{q \leqslant x} d(q)$ and $\sum_{q \leqslant x} \phi(q)$ extended over all square-free integers up to x, as also the probability that a pair of square-free integers chosen at random may be relatively prime.

S. S. Pillai, Calcutta

1. Report on L. G. Sathe's solution of a problem of G. H. Hardy.

Let $\pi_\nu(x)$ denote the number of square-free numbers $\leqslant x$, which are composed of ν prime factors. Hardy's problem is to obtain an asymptotic formula for $\pi_\nu(x)$ when $\nu \sim \log\log x$. Hardy, Erdös and Pillai independently tried this without reaching the final goal. Sathe has succeeded in obtaining the asymptotic formula. In the paper several other interesting results are proved.

2. On a wastage about error terms

Let $a_1 < a_2 < a_3 \ldots\ldots$ be a sequence of integers and

$$(1)^{*} \quad F(x) = \sum_{a_n \leqslant x} 1 = f(x) + E(x),$$

$$(2) \quad S(x) = \sum_{a_n \leqslant x} \frac{1}{a_n} = s(x) + K(x),$$

where $E(x)$ and $K(x)$ are error terms.

Calculate K (x) from (1) and then calculate F (x) from (2) and let the final result be

$$F(x)=f(x)+E_1'(x).$$

The problem is whether E (x) and $E_1(x)$ are of the same order or not.

If $W(x)=E_1(x)/E(x)$, it is shown that

$$W(x)=\left\{\int^x dy \int^y E(x)/x^2\, dx\right\} \div E(x).$$

If $\left[\int^x \phi(x)\,dx\right] \Big/ x\,\phi(x) \leqslant M(x)$ for all integrable $\phi(x)$ and G (x) be the lower bound of M (x), it is shown further that $W(x) \leqslant G(x)$.

It may be that $W(x)=O(1)$ in all cases, but I am not able to prove it.

M. S. Srinivasan, Tanjore.

Application of De Moivre's Theorem to the solution of Congruence equations.

K. G. Ramanathan, Annamalainagar.

1. Congruence properties of σ_a (N).

In this paper congruence properties of $\sigma_a(N)$ of the type $\sigma_a(km+l)\equiv 0 \pmod{k}$, $m \geqslant 0$ for $(k, l)=d>1$ are discussed. They contain as particular cases the results for $d=1$.

2. On the invariants of a finite abelian Group.

It is well-known that a finite abelian group of order $N>0$ is the direct product of cyclic groups of orders $\lambda_1, \lambda_2, \ldots \lambda_r$ where $\lambda_i | \lambda_{i-1}$ $(i=2, 3, \ldots r)$. In this paper binomial equations of the type

$$x^{\frac{1}{2}\lambda}=B$$

where λ belongs to the series $\lambda_1, \lambda_2, \ldots \lambda_r$ and B is an element of the abelian group such that B^2 = identity are studied.

The arithmetical significance of these results is also pointed out.

F. C. Auluck, Delhi.

On the number of partitions of n into k parts.

Some asymptotic properties of $p_k(n)$—the number of partitions of n into exactly k non-zero summands—were discussed by Erdös, Lehmer and others. Their work suggested that $p_k(n)$ possessed a unique maximum. The present paper gives asymptotic expressions for $p_k(n)$

for different intervals of k. Properties of $q_k(n)$, the number of partitions of n into exactly k non-zero summands when they are all different, are also discussed.

B. S. Madhava Rao, Central College, Bangalore.

Pauli's identities in the Dirac Algebra.

It is shown in this paper that by choosing suitable forms for 4×4 matrices as the products of Dirac matrices and matrices of rank unity (*i.e.*, products of 4×1 and 1×4 matrices), and expressing them as linear combinations of the 16 basis elements of the Dirac Algebra, one can derive the generalised identities of Pauli which hold in this Algebra. Generalisations are given for the cases not dealt with by Pauli, and also the use of his B-matrix is avoided. The same method also yields further 'tensor', multilinear, and polynomial indentities. It is shown that, in general, only five types of such products are sufficient to obtain all the identities.

T. Venkatarayudu, Guntur.

On the one dimensional representations of symmetric groups

In this paper we prove that the well-known symmetric and anti-symmetric representations are the only one dimensional representations in symmetric groups.

K. Venkatachaliengar, Bangalore.

The congruent reduction of an orthogonal symmetric pair of matrices.

In this paper the problem of congruent reduction of the matrix pair $\lambda A + \mu G$ where G is symmetric and $AG^{-1} A' = G$ is treated. It is shown that corresponding to the splitting of the minimum polynomial of AG^{-1} (which is reciprocal), the pencil breaks up by a congruent transformation into the direct sum of a number of irreducible pairs of the same type. The criteria for the congruence of two such pairs have been obtained as the equivalence of diagonal quadratic forms in the appropriate algebraic fields over the ground field as is the case with a symmetric-symmetric pencil. As a particular case the criteria for the orthogonal equivalence of two orthogonal matrices over a field of characteristic $\neq 2$ has also been obtained.

K. Chandrasekharan, Madras.

On Bessel Summation of Series.

The following results on J_μ summation are proved.

THEOREM 1. If $\sum a_n$ is summable (c, r), $r > 0$, and

$$\int_{\omega}^{\omega\lambda} \left| \frac{d}{dx} s^r(x) \right| dx = O\left[f(\lambda)\, \omega^{2r}\right],$$

for $\lambda > 1$, then $\sum a_n$ is summable J_μ for $\mu = r + \frac{1}{2}$; in fact, for $\mu >$ max. $(h + \frac{1}{2},\ r - \frac{1}{2})$, where h is the greatest integer less then r.

THEOREM 2. If (i) $\sum_{1}^{\infty} a_n\, \alpha_\mu(nt)$ converges dominatedly for $t > 0$, and tends to zero with t,

(ii) $\int_0^t \left| \phi_\mu(t) \right| dt = O(t)$ as $t \to \infty$,

(iii) $\omega^\delta \int_{1/\omega}^{\infty} \dfrac{\phi_\mu(t) \cos\left(\omega t + \overline{2\mu + 1 - \delta}\, \frac{\pi}{2}\right)}{t^{1-\delta}}\, dt = o(1)$, as $\omega \to \infty$,

then $\sum a_n$ is summable $(c,\ \mu + \frac{1}{2} - \delta)$ for $0 \leqslant \delta < 1$, to the sum zero.

P. D. Shukla, Lucknow.

On the Differentiability of an indefinite integral.

The object of this paper is to investigate the existence of the differential co-efficient at $x = 0$ of the indefinite integral $\int_0^x f(x)\, dx$ where the function $f(x)$ is any bounded and continuous function of x which has a discontinuity of the second kind at $x = 0$. The necessary and sufficient condition has been established, as also three other conditions which are sufficient but not necessary. Suitable examples have been constructed to illustrate the various possibilities which may arise.

A. A. Krishnaswami Ayyangar, Mysore.

Three interesting problems in Maxima and Minima.

1. Given two sets of positive numbers

$$(a_1,\ a_2,\ .\ .\ .\ ,\ a_n),\ (b_1,\ b_2,\ .\ .\ .\ ,\ b_n)$$

to find the maximum and minimum values of $a_i b_j$, $(i \leqslant n,\ j \leqslant n)$, every

a being associated only with one b and every b with one a. This problem has several applications, for example in Spearman's Rank Correlation and in the arrangement of pulleys in mechanics.

2. To express a number in the Hindu place-value system of notation using positive and negative digits, so that the arithmetical sum of the digits may be the *least* possible. A knowledge of this will save labour in working with calculating machines. To multiply a number by 397798 will involve 43 turns of the handle whereas the equivalent $40\bar{2}20\bar{2}$, where the bars indicate negative digits will involve only 10 turns. There are certain criteria for this expression which require some careful formulation.

3. Given n dissimilar surds $\sqrt{a_1}, \sqrt{a_2}, \ldots \sqrt{a_n}$, to find the minimum number of surds possible in $(p_1\sqrt{a_1} + p_2\sqrt{a_2} + \ldots + p_n\sqrt{a_n})^2$ where $p_1, p_2, \ldots, p_n$ are any set of rational numbers. It is well-known that the maximum number is $n(n-1)/2$, but it can be shown that in a class of cases the number can be reduced to n and sometimes even less than n. The least possible number seems to be an unsolved riddle. Some text-book writers without full consciousness of the quality of surds wrongly assert that an expression of the form $a \pm \sqrt{b} \pm \sqrt{c}$ cannot have a square root since it has too few terms to be the square of a trinomial and too many terms to be the square of a binomial. I wish to add, however, that the number can be the square of a quadrinomial. Witness the example,

$$6 - 2\sqrt{2} + \sqrt{3}$$

for which a square root does exist in the form $\sqrt{a} \pm \sqrt{b} \pm \sqrt{c} \pm \sqrt{d}$. The wisdom of the Intermediate Students will be equal to discovering the solution!

N. N. Bose, Lucknow.

A theorem in operational calculus.

In this paper a theorem is established that connects a definite integral with limits 0 to ∞ with an integral with limits 1 to ∞. The theorem is illustrated by means of some examples.

R. S. Varma.

On the generalisation of Laplace's integral.

At the last Science Congress held at Nagpur, I gave a generalisation of Laplace's integral, and mentioned three theorems based thereon. Some further results concerning this transform have since been obtained.

N. D. Tewari, Lucknow

A theorem on generalized Laplace transform.

At the last Science Congress held at Nagpur, Dr. R. S. Varma gave a generalization of Laplace's transform in the form

$$\phi_m^k (p) = p \int_0^\infty (2xp)^{-\frac{1}{4}} W_{k,m} (2xp) f(x)\, dx$$

where $W_{k,m}(z)$ stands for Whittaker function. The object of this paper is to give an integral representation of $\phi_m^k(p)$ when $f(x)$ is self-reciprocal in the Hankel-Transform.

S. K. Bose, Lucknow.

On Meromorphic Functions.

In this paper a definition is given for the maximum modulus and minimum modulus of a meromorphic function. The equivalence of this definition with the existing ones is established. Finally I give an extension of Schwarz's lemma to meromorphic functions.

H. C. Gupta, Cawnpore.

Two theorems on self-reciprocal Functions.

The paper gives two theorems which enable us to derive by the processes of differentiation and integration new self-reciprocal functions from known ones.

R. P. Agarwal, Lucknow.

On the resultant of two functions.

In this paper I have shown that the resultant of two suitably chosen functions, defined by infinite integrals, is a kernel transforming a self-reciprocal function of a given order to another self-reciprocal function of a different order.

C. T. Rajagopal, Tambaram.

A note on periodic integral functions.

This note contains the following two applications of the canonical-product form of a periodic integral function [V. Ganapathy Iyer: *Journal Indian Math., Soc.* (New Series). 5, (1941), 1-17: Theorems 1, 3].

THEOREM 1. *$f(z)$ is an integral function of $z = x + iy$ of period $\lambda > 0$. If the function is* (i) *real* (ii) *non-negative for all real z, then*

$$f(x) = |\phi(X)|^2$$

where $\phi(X)$ is an integral function of $X = \exp(2\pi i x/\lambda)$

THEOREM 2. *Let $f(z)$ be a singly periodic integral function. Then a necessary and sufficient condition for $f(z)$ to be of order not exceeding $\rho > 1$ is that its Fourier co-efficients should satisfy*

$$a_n = O[\exp\{-n^{\rho/(\rho-1)-\varepsilon}\}], \quad b_n = O[\exp\{-n^{\rho/(\rho-1)-\varepsilon}\}]$$

for all positive ε.

When $\rho \leqslant 1$, the theorem can be restated with an obvious modification.

S. M. Shah, Aligarh.

A note on the Minimum Modulus of a class of Integral Functions.

In this paper are considered integral functions having zeros $a_{11}, a_{12}, \ldots$ situated in rings $(R_n - R_n^{\alpha} \leqslant |z| \leqslant R_n)$ where $0 < \alpha < 1$ and R_n is any sequence of positive numbers such that $R_n/R_{n-1} \geqslant \lambda > 1$.

It is shown that

$$\text{If } f(z) = \prod_{n=1}^{\infty} \prod_{s=1}^{pn} \left\{1 - \frac{z^{\mu_n}}{a_{n,s}^{\mu_n}}\right\}$$

be a canonical product of order ρ, then the values of r for which $m(r, f) > C\,M(r, f)$ where $C = C(\lambda, \varepsilon) > 0$ is satisfied form a set of upper density greater than $1 - \frac{1}{\lambda} - \varepsilon$.

V. Ganapathy Iyer, Annamalainagar.

On the translation numbers of Integral Functions.

A number λ is called a translation number of the integral Function $f(z)$ if

$$f(z + \lambda) = g(z)\, f(z) \quad \ldots \quad (1)$$

where $g(z)$ is an integral function. For instance every period of a periodic function is a translation number. The translation number λ is said to be primitive if λ/m is not a translation number for any positive integer $m > 1$. In this paper the properties of integral functions having a translation number are studied. A general expression for such functions is found. It turns out that unlike periods an integral function can have even an infinite number of linearly independent primitive translation numbers.

Apart from the above, the equation (1) is studied as a functional equation for a given $g(z)$. If all the functions considered are meromorphic it is known that (1) has always a meromorphic solution (See: J. M. Whittaker, *Interpolatory Function Theory*, Cambridge Tract No. 33). But this is no longer true if $f(z)$ is required to be an integral function (except, of course, the trivial solution $f(z) \equiv 0$). The condition for the existence of the solution is investigated, and when it exists a general expression for the solution is found.

Afzal Ahmad, Hyderabad (Deccan)

A Note on an infinite system of non-linear integro differential equations.

G. R. Seth, Delhi.

On the differential equation $f^r(x) = f^s(1/x)$

A particular case of the differential equation $f^r(x) = f^s(1/x)$ has been already obtained by P. N. Sharma (*Math. Stud.* 10 (942), 173—74). In this paper we discuss the general case.

Hari Shanker, Delhi.

1. *On integral representation of the product of two parabolic cylinder functions.*

In this paper certain integral-representations for the product of two parabolic cylinder functions have been obtained by the method of operational calculus, e.g.

$$D_n(ix)\, D_{-2m-n-1}(x) = \frac{\Gamma(m+1)\, 2^{m+\frac{1}{2}}}{\Gamma(2m+1)\sqrt{\pi}} \int_0^\infty \frac{e^{-\frac{1}{4}x^2t^2}\, t^{2m}}{(2+t^2)^{m+1}}\, {}_2F_1\left[\begin{matrix} -n, & m+1 \\ 2m+1; & \dfrac{2t^2}{2+t^2} \end{matrix}\right] dt.$$

where n is a positive integer and $R\,(m+\frac{1}{2}) > 0$;

and

$$D_n(x)\, D_{-m-1}(x) = \frac{e^{-\frac{1}{2}x^2}\, x}{2^{\frac{m-n-1}{2}}\, \Gamma\left(\frac{m-n+1}{2}\right)} \int_0^\infty \frac{e^{-t^2/2}\, t^{m-n}}{(x^2+t^2)^{\frac{m-n}{2}+1}}\, {}_2F_1\left[\begin{matrix} -n, & m+1; \\ \dfrac{m-n+1}{2}; & \dfrac{t^2}{2(x^2+t^2)} \end{matrix}\right] dt.$$

where $R\,(m-n+1) > 0$

J. N. Kapur, Delhi.
On the differential equation $\nabla^4 w + k^4 w = 0$

Some solutions of the differential equation $\nabla^4 w + k^4 w = 0$ have been obtained by combining solutions of a set of auxiliary equations.

S. Minakshisundaram, Guntur.
A Note on Fourier expansions.

A. A. Krishnaswami Ayyangar, Mysore.
A Geometrical Construction for $ax^2 + 2hxy + by^2 = 0$.

Without the knowledge of a simple geometrical construction for the pair of lines $ax^2 + 2hxy + by^2 = 0$, when the equation does not split up into rational simple equations, the equipment for tracing conics is incomplete. This note supplies the gap which has not been filled in by any of the multitudinous text-books in Analytical Geometry.

With OX, OY as two perpendicular axes of reference, plot P $(0, a)$ and Q $(-2h, a-b)$, $(a \neq 0)$. Draw the circle on OQ as diameter to cut the parallel through P to OX in L and M which will be possible if $h^2 > ab$. Then OL, OM are the straight lines whose equation is $ax^2 + 2hxy + by^2 = 0$.

If AB parallel to OY be the diameter of the circle LMQ, and it cuts LM in N; then the greater or less of the two segments AN, NB will be the reciprocal of the square of the semitransverse axis of the hyperbola

$$ax^2 + 2hxy + by^2 = 1, \ (h^2 > ab),$$

according as $(a+b)$ is positive or negative. It is also readily seen from the figure that $\tan L\hat{O}M = \pm 2\sqrt{h^2 - ab}/|a+b|$ according as $a^2 + ab$ is greater or less than 0. When $a = 0$, one of the lines OL, OM becomes the tangent at O to the circle LMQ and the other coincides with OX. When $a + b = 0$, $L\hat{O}M = \pi/2$.

Sri Niwas Asthana, Mehgaon.
On The trisection of an angle.

M. Venkataraman, Annamalainagar
1. The De Longchamps—Morley chain.

We define a curve P_n to be a curve of class n touching the line at infinity in $(n-1)$ points in equispaced directions (i.e.) apolar to the circular points at infinity. It is seen that the locus of foci of P_n's touching $n+1$ given lines is a circle C_n; and that the centres of the circles C_{n-1} for any set of n out of the $(n+1)$ lines lies upon the circle C_n. This gives us a new proof of the centre circle chain of theorems of De Longchamps and Morley.

2. Chain theorems in Geometry.

All known chains of theorems giving the same types of elements have been classified into four types – the Cox-Grace type, the centre circle type, the perspectivity type and the centroidal type. The fundamental chains of these types are discussed, and it is shown that all chains of these types must be made up of a number of overlapping fundamental chains. Besides known chains, a number of new chains are also derived.

K. Rangaswami, Annamalainagar.

On a cubic transformation in the geometry of pedal and contact circles.

G. L. Chandratreya, Poona.

Use of abridged notation in finding some results connected with conicoids.

With the help of Joachimsthal's equation, equations are obtained for the tangent planes from a straight line, for the points of contact, for the tangent planes at the points where the line meets the quadric and for the points themselves; for the cone with a given vertex and with its base a given section of the conicoid, and for the plane section etc. Conditions for various relations between points, lines, planes and a quadric are derived. The envelope of all plane sections of a quadric which subtend an orthogonal cone at a given point is found. If the point lies on the quadric, this envelope reduces to two points. Also the locus of a point, the enveloping cone from which has 3 mutually perpendicular tangent planes is found.

Ratan Shanker Mishra, Delhi

The five families of ruled surfaces through a line of a rectilinear congruence

V. Rangachariar, Patna.

On a certain scroll associated with a net of quadrics.

In a paper published in the *Bulletin of the Calcutta Mathematical Society* (1944) the author discussed a scroll generated by lines whose polars with respect to a net of quadrics are concurrent. It was proved that the surface R generated by such lines is of order 9, having the Jacobian Curve Γ of the net as a multiple curve. In this paper some other properties of R are obtained. It is found that R is generated by certain trisectants of Γ which is a multiple curve of order three on R. It is also found that Γ is the only multiple curve on R whose genus is 10. Lines whose polars with regard to four quadrics not belonging to a net are also discussed.

Sahib Ram, Lahore.

Commutative law in 4-dimensional Geometry

In this paper it is shown that

(i) the plane intersecting 4 lines of an associated set intersects the fifth if the commutative law holds;

(ii) the existence of self-polar pentads can be established if the commutative property holds.

S. R. Das, Delhi

Determination of the motions set up when an aeroplane is suddenly struck by a gust of wind.

A sudden gust of wind which quickly subsides disturbs the motion of the aeroplane. It is essential that there must be dynamical stability for the aeroplane otherwise it will either deviate further from the position of equilibrium or oscillations will begin and continually increase. The change of wind velocity will cause an alteration in the condition of the equilibrium and the problem will be to secure for the aeroplane the steady motion consistent with the new conditions.

P. N. Sharma, Delhi.

On some vibrational problems.

It is pointed out that certain vibrational solutions given by D. G. Christopherson (*Quart. Jowl. of Math.*, 11 (1940), 65) are incomplete inasmuch as they contain only one variable parameter, the system possessing two degrees of freedom. The corresponding complete solutions for one set of boundary conditions are given in two recent papers (B. R. Seth, *Proc.*, *Ind.*, *Acad.*, *Sci.* A, 12 (1940), 487; 13, (1941), 390).

For the other set of boundary conditions exact solutions are given for a number of boundaries.

B. R. Seth, Delhi.

Bending of Rectilinear plates

A general method has been developed for determining the bending of rectilinear plates with supported edges. It is shown that when the corresponding torsion solution for the boundary is known the case of a plate bent by uniform pressure can be dealt with. A simple solution of the form $w = \frac{1}{192} \frac{p}{Da} (y-a)(y^2-3x^2)(3x^2+3y^2-4ay)$, which has not been noticed before, has been obtained for an equilateral plate. Complete solutions are also given for an isosceles right-angled triangular plate, both when it is bent by uniform pressure and by pressure varying uniformly over a face. The case when the pressure is a polynomial of any degree is also discussed.

R. Krishnamurti, Hyderabad (Deccan)

Precession or Ayanamsa.

Equinoxes are referred to as "Agni" and "Indra" in the Vedas The movements of Indra and Agni are described. Agni's motion is stated as due to the revolution of the celestial pole about the pole of the ecliptic. The period of precession is given as 28,800 years or 150 revolutions in a period of Mahayuga of 4,320,000 years. The movement of the equinox round the ecliptic causes the oscillation of the Aswini point on the horizon and the number of quarter oscillations are 600 in a period of 4,320,000 years.

S. A. Hamid and R. D. Syal, Lahore.

On the determination of astronomical latitude.

Modification of certain methods depending on correct reading of times of observation, the instrument used being a theodolite. An attempt is made to minimise the effect of instrumental errors. Results reduced to computable forms.

J. P. Jaiswal, Lucknow.

On the electric potential of a single electron in gravitational fields. II.

In an earlier paper I have obtained the electrostatic potential of a single electron for the metric,

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \frac{1}{c^2}\left\{dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta d\phi^2\right\}\left(1 + \frac{2m}{r}\right) \quad \ldots \quad (1)$$

This is the isotropic form of the Schwarzschild's metric which satisfies Einstein's simple gravitational laws

$$G_{\mu\nu=0}$$

and which has been considered by Whittaker.

The object of this paper is to consider Einstein's general law of gravitation, *viz*,

$$G_{\mu\nu} = \alpha g_{\mu\nu}$$

and to obtain the potential for the metric,

$$ds^2 = \left\{1 - \frac{2m}{r} - \frac{1}{3}\alpha r^2\right\} dt^2 - \frac{1}{c^2}\left\{\frac{dr^2}{1 - \frac{2m}{r} - \frac{1}{3}\alpha r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right\} \quad \ldots \quad (2)$$

The differential equations in this case behave in the same way as those in the previous paper; that is to say two of the three equations are solvable in terms of known functions. The third cannot be so solved. However, it possesses as in the previous case a series solution valid everywhere in the neighbourhood of the origin which is a regular singularity of the differential equation.

A. A. Krishnaswami Ayyangar, and A. K. Srinivasan, Mysore.

Digital Bias in 'Pi'

In the *Journal of the Royal Statistical Society* 1938, Kendall and Smith have described four tests for what they call local randomness. They state that the tests are very powerful when taken together and in particular the gap test is very difficult to evade. On application of these tests to the first 707 digits of π (3. 14159265358. . ,) we find that serial and frequency tests to be the most important in this particular case and probably in other cases as well. The next in importance is the poker test, while the gap test seems to be the most unreliable. It is also possible that the gap test will be satisfied by almost all non-artificial sets of numbers like the digits in well known irrational numbers.

Anyhow, it is clear that a number which has an infinite number of random digits in its fractional part must be irrational.

The results of our analysis of the digits of π are as follows:

Test	χ^2	Degrees of freedom	Probability	Conclusion
Frequency	8·78	9	0·3 – 0·5	no bias
Serial	104·74	90	0·3 – 0·4	no bias
Poker	4·47	3	0·2 – 0·3	no bias
Gap	16·42	10	0·05 – 0·1	dubious

A. A. Krishnaswami Ayyangar, Mysore.

On test criteria for statistical hypotheses.

This is a critical review of the criteria for statistical hypotheses as expounded by Neyman, Pearson, Wilks, Fisher, Jeffreys, Henry Scheffe and Wald. Several phases of statistical inference are touched upon briefly, such as prior probability, rejection of observations and randomisation method. The modern ramifications of the subject are lost in the mists of function spaces, distance functions and suitable choice of metrics.

Resolutions passed at the General Body Meeting of the Society at 12-30 p.m. on Saturday the 22nd December 1945

1. A resolution expressing the great loss sustained by the Society in the deaths of the following members of the Society was passed, all the members standing:—

(i) N. Durairajan Esq. Superintending Engineer, Madras;

(ii) K. N. Kandekar Esq., Headmaster, S.S.P. College, Jodhpur.

2. Resolved that the following mathematicians of international reputation be elected as Honorary Members of the Indian Mathematical Society:

(i) Prof. M. H. Stone, Harvard University.

(ii) Prof. Herman Weyl, Institute for Advanced Study, Princeton;

(iii) Prof. J. E. Littlewood, Oxford University;

(iv) Prof. J. Hadamord, New York;

(v) Prof. Elie Cartan, Université de Paris;

(vi) Prof. Max Dehn, Black Mountain College, U.S.A;

(vii) Prof. Shiing-Shen Chern, Institute of Mathematics, Academia Sinica, Shanghai.

(viii) Prof. Vinagradov, Moscow.

(ix) Prof. Kolomogorov, Moscow.

3. Resolved that the Indian Mathematical Society should restore and further strengthen her connections with sister societies all over the world, and should try to increase the number of Journals and publications of other societies obtained by way of exchange with its own publications.

Resolved further that steps be taken to ensure closer collaboration between the mathematical societies in India.

4. The General Body empowers the Managing Committee to examine the question of registration of the Society and the fixing of its Headquarters, and to take the necessary action in the matter.

5. Resolved that the "Syllabus Committee" be requested to draft recommendations regarding the minimum equipment that should be expected of a graduate of an Indian University in Mathematics, and also to outline the list of topics to be included in the M.A. and M.Sc. courses in mathematics.

6. Resolved that steps should be taken to make the lending of books and periodicals from the Society's Library at Poona more prompt, and efficient; that books and periodicals should be properly bound and made accessible. To ensure this result, the society may increase its expenditure on clerical work. The general body is of opinion that copies of *Mathematical Reviews* should be made available for circulation among those who desire it, an additional copy being purchased for the purpose if necessary.

7. Resolutions were passed thanking Sir Maurice Gwyer, Vice-Chancellor of the Delhi University and the other Authorities of the University for their kind invitation and hospitality, and Sir B. N. Rau, Constitutional Adviser to the Government of India, for inaugurating the Conference: the authorities of St. Stephen's College where the guests were accommodated and the Staff and volunteers and the actors in the mathematical entertainment; the members of the reception committee; the authorities of the upper air observatory; those who had taken part in the Symposia and those who delivered the popular evening lectures.

A Meeting of the Managing Committee of the Indian Mathematical Society was held on the evening of Monday the 24th December 1945. The following members of the Managing Committee were present:—

Prof. Levi, Dr. Vaidyanathaswami, Dr. Ram Behari, Dr. A. N. Singh and Dr. A. Narasinga Rao.

At this meeting the action to be taken on the resolutions passed by the General Body were considered.

Regarding the second part of Resolution 3 of the General Body, it was decided to appoint a committee consisting of Prof. Levi (Convener), Prof. Siddiqi and Prof. A. N. Singh to ensure a closer collaboration of the mathematical societies in India.

Regarding resolution 4 of the General Body, the Managing Committee resolved to request Dr. Ram Behari to consult the authorities of the Delhi University and to report on the feasibility of transferring the Headquarters of the Society to Delhi.

The Managing Committee passed a resolution thanking Mrs. Chote Lal and her sons Phul Chand Esq. Junior Controller of Military Accounts Patna, and Pratap Chandra Esq. Advocate, Delhi for their donation of Rs. 501 towards the publication Fund of the Society in memory of the late Chote Lal Esq.

SYMPOSIUM ON "FOUNDATIONS OF PROJECTIVE GEOMETRY"

President: Dr. R. Vaidyanathaswami

Prof. F. W. Levi on "Collineations and Linear Transformations"

In ordinary projective plane geometry one considers the point-point and line-line automorphisms for which the relation of incidence and the values of the cross-ratios remain invariant. These are shown to be generated by linear transformations of the co-ordinates. It can be proved that for every collineation, i e. every incidence—conserving point-point and line-line automorphism, the cross-ratios remain invariant. The proof is based on the property of the real numbers that every function satisfying the conditions

$$f(a+b) = f(a) + (b)$$

$$f(ab) = f(a) \cdot f(b)$$

is necessarily the identity. In the field of the complex number, the corresponding theorem does not hold and therefore in the (ordinary) complex projective geometry, the notion of collineation is more general than that of linear transformation. The speaker gave a sketch of a method used by E. Kamke (Jahresber. D. M. V. Vol. 36) to construct a completely dicontinuous collineation which is not a linear transformation and for which every "real" branch of a complex straight line is completely scattered. The method can be used for every projective geometry over a closed field. On the other hand, in every formally real field which admits the drawing of square roots of its formally positive elements the functional equations given above are satisfied by the indentity only. In the corresponding geometry, every collineation is a linear transformation. The distinction between collineation and linear transformation shows, that the casual use of complex co-ordinates in investigations on real projective geometry involves some logical difficulties.

Mr. B. C. Chatterji on "Axiomatisation of 'mid-point' in Affine Plane"

The speaker reported about the axiomatisation of "mid-point" according to R. Baer (Transaction of American Mathematical Society, Vol. 56 pp 94-107)

In the affine plane we define the notion of middle point in three different ways and study their interconnections:

(1) We define a relation $P \cdot Q = R$, (R is a *mid-point* of PQ) for three different collinear points P, Q, R satisfying the conditions:

(i) $P \cdot Q = Q \cdot P$,

(ii) there exists one and only one mid-point for every pair of different points P, Q;

(iii) the mid-point relation is invariant for parallel projections.

(2) We define a relation $P \odot Q = R$, (R is an M-point of PQ) for three different collinear points P, Q, R if there exist points S, T not lying on PQ such that R, S, T are three different collinear points and such that PS is parallel to QT and PT is parallel to QS.

(3) Thirdly, we introduce the middle point R as the fixed point of a point-reflection interchanging two different points P, Q.

The study of the mutual relationship between the notions of a middle point defined as in (1), (2) and (3) reveal the following:

Every mid-point relation satisfying (i), (ii) and (iii) is also an M-point relation.

For every pair of points P, Q, the M-point is unique if and only if the M-point is independent of the choice of the particular pairs of points S, T in (2) and the Fano's axiom (the diagonals of a parallelogram are not parallel) holds good.

The M-point is unique and is invariant for every parallel projection if and only if there exists to every pair of different points a reflection interchanging them.

Their exists to every pair of different points a reflection interchanging them if and only if the plane is the plane over a Cartesian number system which is right distributive and of characteristic not equal to 2. If the characteristic is 3, the medians of a triangle are parallel, otherwise they are concurrent.

DISCUSSION ON THE TEACHING OF MATHEMATICS IN INDIAN UNIVERSITIES

Delhi 23rd, December 1945.

A meeting to discuss the teaching of mathematics in Indian Universities in connection with the standards to be aimed at in the several courses, the question of uniformity, and means for raising the level of teaching prevailing at present, was held in the open air behind the St. Stephen's College. Prof. F. W. Levi presided and referred to an excellent article which was contributed to the discussion by Prof. W. W. Sawyer of the College of Technology in Leicester, England. This paper (vide pp. 85–91) was then read out by Mr. B. C. Chatterjee (Calcutta University).

Next, a scheme embodying topics suitable for inclusion in the syllabuses for the several courses of study in mathematics prepared by Dr. Siddiqui to serve as a basis for discussion was read out by Dr. A. Narasinga Rao.

During the discussion that followed, DR. RAM BEHARI (Delhi) said that at the B.A., stage one should not keep the requirements of research in view in framing the course, but that in the Honours and M.A. stages this shoud be done. He then gave a list of topics suitable for inclusion in the pass and Honours courses.

DR. R. DHAR MISRA (Lucknow) was against any attempt at adopting uniform courses in different Universities, which move he regarded as undesirable. He said that Universities must be free to develop their own traditions and maintain standards higher than the average as, for example, Cambridge does. He described the courses, ordinary and advanced, which were being taught at his own University and showed that they were much higher than the courses proposed. He, however, agreed that no University should have courses below a certain minimum and suggested that the proposed courses may be taken to indicate the necessary minimum. He further suggested that even in this no attempt should be made to force any University to change its courses and that the discussion that had taken place should be considered enough to give inspiration to the backward Universities.

Prof. HAMID (Lahore) invited the views of those present on the desirability or otherwise of separate degrees for Pure and for Applied Mathematics.

Dr. R. VAIDYANATHASWAMI (Madras) was of the opinion that, without infringing on the rights of the Universities or their expert bodies, it would be relevant for a body like the Indian Mathematical Society to say what, in its opinion, was the level of knowledge that may be expected from a mathematics graduate, and secondly, if there is a postgraduate degree what level it should reach. The development of mathematical thought at the present day had brought out certain more general methods and higher and more unifying ideas, which should find a reflection in our University courses. If they lagged behind, the Society should help in restoring the proper level.

DR. A. N. SINGH (Lucknow) suggested that in view of fact that there are wide differences between the syllabuses followed by different Indian Universities, and some of them being teaching institutions while others were of the affiliating type, a small committee should be appointed which would get into touch with the chairmen of the boards of study of all Indian Universities and evolve a well thought out scheme securing a fair measure of Uniformity in their syllabuses.

MD. KHAJA MOHIUDEEN (Hyderabad), explained the aims which had been kept in view in framing the syllabus for the several courses —Intermediate, B.A., B.Sc., (Main and Subsidiary Mathematics) and M. A., and M. Sc. He remarked that as many of the members assembled there, were themselves Heads of the Department of Mathematics or Chairmen of the Boards of study in one or other of the various Indian Universities, it should not be difficult to ensure by agreement that the 2 or 4 years spent by students in the Universities should be spent in the most profitable manner. He did not consider the proposed syllabus too heavy because it had been successfully followed at the Osmania University.

Annual Meeting of the Benares Mathematical Society

Delhi, 23rd December 1945

1. During the Indian Mathematical Conference, the Benares Mathematical Society also held its Annual General Meeting on Sunday December 23, 1945 at 10 a.m., in the University Buildings. Prof. A. C. Banerjee, Vice President of the Society presided owing to the unavoidable absence of the President Prof. M. R. Siddiqi. 45 persons attended the meeting and these included the members of the Benares Mathematical Society and also members of the Indian Mathematical Society who were present by invitation.

2. The Annual Report was read by the Secretary, Dr. Rama Dhar Misra, and was approved by the house. A summary of the report is given below:

It is my pleasant duty to report that the society has been able to carry on successfully during the unusual and difficult war years. Our membership has increased to 107, the highest since 1918, the year of birth of our Society. The financial position is satisfactory. I take this opportunity of expressing our thanks to the Rockfeller Foundation for their grant-in-aid of Rs. 250 given to us through the National Institute of Sciences, India, to meet partly the expenses of publication of Vol. V of our *Proceedings.*

The last Annual General Meeting was held at Benares and along with the election of the Office bearers, Dr. M. R. Siddiqi was elected the President for the year 1945.

The Society has been publishing annually a journal called "*The Proceedings of the Benares Mathematical Society,*" containing original papers in pure and applied mathematics. Now we propose to bring out two volumes every year.

It is my pleasant duty to welcome you all to our proceedings to-day. We are once again holding a joint session with the sister body, the Indian Mathematical Society. We thank the members of the Indian Mathematical Society for their co-operation and for the consideration we have always received from them. Nearly half of our membership is common. It has given us great pleasure to hold, whenever possible, our General Meetings at the same place where the sister body meets. And we hope that the reciprocal regard and the spirit of accommodation we have for each other will grow with time and we shall co-operate more and more in advancing the cause of mathematical learning in our country.

We are also thankful to the authorities of the Delhi University and also of St. Stephen's College for extending to us also all facilities and hospitality. It has been a real pleasure to us to come to Delhi to hold our Annual Meeting.

3. The following were elected as office bearers for the year 1946 :—

President:—Dr. M. R. Siddiqi,
Vice-Presidents:—Prof. V. V. Narlikar,
Dr. B. N. Prasad,
Dr. B. R. Seth.
Secretary:—Dr. Rama Dhar Misra
Treasurer:—Dr. R. S. Varma
Librarian:—Dr. Brij Mohan
Editor of the Proceedings:—Prof. A. N. Singh
Council Members:—Prof. A. C. Banerji,
Dr. Ram Behari,
Dr. N. G. Shabde,
Dr. Gorakh Prasad,
Mr. N. N. Bose,
Mr. Chandi Prasad.

4. It was resolved to institute a gold medal in memory of the late Dr. Ganesh Prasad, the Founder-President and a benefactor of the Society.

5. The council expressed its appreciation and gratitude for the services of the retiring office-bearers.

6. The following were admitted as members of the Society:—

Messers Shanti Narain, K. S B. Shastri, B. C. Chatterjee, S. P. Kaushik, J. N. Mitra, P. L. Bhatnagar, P. B. Bhattacharjee, Ratan Shankar Mishra, Siva Ram Gupta, Ratan Prakash Agarwala, S. A. Hamid and Mohd. Khwaja Mohiuddin.

7. The following members read papers:—

Dr. Pillai, Mr. J. P. Jaiswal, Mr. R. P. Agarwala, Mr. R. S. Mishra, and Mr. A. Sharma.

The meeting terminated after a vote of thanks to the chair.

Lucknow University

RAMA DHAR MISRA
Secretary.

List of Members and Delegates who attended the 14th Conference of the Indian Mathematical Society at Delhi.

Abdulla Butt,	Aligarh
Abhyankar S. K.,	Gwalior
Afzal Hussain,	Hyderabad (Deccan)
Afzal Ahmed,	Hyderabad (Deccan)
Agarwal, R. P.	Lucknow
Ambikeswar Sharma	Pilani
Asthana, Sri Niwas,	Gwalior
Auluck, F. C.	Delhi
Bagi, B. B	Dharwar
Balagangadharan, K.	Poona
Banerji, A. C.	Allahabad
Bhatnagar, Suryaprakash,	Kotah
Bhatnager, P. L.	Delhi
Bhattachorya, P. B.	Delhi
Bose, N. N.	Lucknow
Chandratreya, M. L.	Poona
Chatterji, Bankim Ch.	Calcutta
Chaturvedi, H. C.	Agra
Chowla, S.	Lahore
Das, S. R.	Delhi
Dayal, Rameshwar	Meerut
Dhawan, K. C.	Lahore
Ganapati Iyer, V.	Annamlainagar
Ghosal, S. C.	Alwar
Gupta, P. D.	Delhi
Gupta, Hansraj	Hoshiarpur
Gupta, H. C.	Cawnpore
Gupta, S. R.	Delhi
Gupta, Ram Lal	Lahore
Hamid, S. A.	Lahore
Hari Shanker	Delhi
Jain, Om Prakash	Alwar
Jaiswal, J. P.	Lucknow
Karmalkar, S. M.	Akola
Kapadia, D. D.	Poona
Kapoor, J. N.	Delhi
Kaprekar, D. R.	Devlali
Kaushik	Bikaner
Krishnamurti, A. M.	Trichinopoly
Krishnamurti, R.	Hyderabad, (Dn.)

Krishnan, V. S.	Madras
Krishnaswami Ayyangar, A. A.	Mysore
Levi, F. W.	Calcutta
Majumdar, R. C.	Delhi
Menon, C. P. S.	Dehra Dun
Minakshisundaram, S.	Guntur
Misra, Rama Dhar	Lucknow
Mohideen, Md. Khwaja	Hyderabad, (Dn.)
Mitra, J. N.	Delhi
Narasinga Rao, A.	Annamalainagar
Pande, K. D.	Etawa
Patni, Gopi Chand.	Jaipur
Pillai, S. S.	Calcutta
Ramamurti, B.	Delhi
Ram Behari	Delhi
Rizvi, Abbas	Hyderabad (Dn.)
Saheb Ram Mandan	Lahore
Sambasiva Rao, K.	Guntur
Sastri, K. S. B.	Hyderabad (Sind)
Saxena, K. N.	Muzaffarnagar
Saxena, H. C.	Delhi
Seth, B. R.	Delhi
Shah, S. M.	Aligarh
Shanti Narain	Lahore
Subramaniam, S.	Delhi
Sushila, Miss	Madras
Shukla, P. D.	Lucknow
Singh, A. N.	Lucknow
Sita Ram	Lahore
Srinivasan, A. K.	Mysore
Sukhatue, P. V.	Delhi
Syal, K. D.	Lahore
Tewari, N. D.	Lucknow
Thakur Das, Miss, V.	Lahore
Varma, R. S.	Lucknow
Venkatrayudu, T.	Guntur
Vindhyachal Prasad, A.	Bhagalpur
Vythynathaswami, R.	Madras
Ziauddin, M.	Lahore

ANNOUNCEMENTS AND NEWS

The following gentlemen have been admitted as members of the Indian Mathematical Society:—

T. Govindarajan Esq. B. Sc. Hons. Assistant Lecturer, Annamalai University, Annamalainagar.

V. S. Krishnamurti Esq. B. A. Hons. Assistant Lecturer, Annamalai University, Annamalainagar.

Dharamraj Lal Srivastava Esq. M. Sc. Lecturer, Govindaram Sakseria College, Basti U. P.

The following gentlemen have become Life Members of the Society by paying the life composition fee of Rs. 150:—

Dr. R. D. Misra Reader, Lucknow University.

S. L. Malurkar, Esq., Meteorologist, Poona.

The Society is glad to announce that it has established exchange relations with the following Societies or periodicals:—

Academie des Sciences, Paris,

Societie Mathematique de France,

National Academy of Sciences, U. S. A.,

American Mathematical Society

Annals of Mathematics

Sri K. Chandrasekharan, and Dr. S. Minakshesundaram left India for the U. S. A. in November 1946 in connection with their membership of the Institute for Advanced Study at Princeton.

Dr. T. Vijayaraghavan formerly at Dacca and Dr. A. Narasinga Rao formerly at Annamalainagar have accepted appointments in the Andhra University, Waltair, as Professor of Mathematics and Gandhian Professor of Mathematical Physics respectively. Sri. G. V. Krishnaswami Ayyangar has been appointed Head of the Mathematics Department at the Annamalai University.

We have received the following information regarding courses of study and Research in Statistics in the Punjab University Research work is done in the Department of Statistics and research students are given guidance. There is a university scholarship of Rs. 100 per month offered for research workers. There is a post graduate course extending over a year at the end of which a certificate is given after an examination. Statistics is a subject for the M. Com. Examination. There is a paper in Statistics for the following examinations.

M. A. Economics M. A. Mathematics

M.Sc. Agriculture, B. A. Hons. and B.A. in Mathematics.

We regret to report the death of Sir James Hopwood Jeans, Member of the Order of Merit, Fellow of the Royal Society, at Dorking (Surrey) on 16-10-46. Sir James Jeans was a great Astronomer, a daring thinker, and had the gift of explaining profound scientific results in a vivid and telling manner to the layman. Students will always remember him through his well known books—"The Mysterious Universe" and "the Mathematical theory of Electricity and Magnetism."

EXTRACTS

Cat righting itself in mid-air

Suppose the cat to start falling vertically with no rotation, back downwards and legs extended at full length perpendicular to the body. Let us regard the cat as made up of a fore part and a hind part, whose movements of inertia A, B are equal when the legs are fully extended. The photographs show that it first contracts its fore legs (thereby making $A < B$) and then turns its fore part around. This action necessitates the hind part being turned in the *opposite direction* (since the total angular momentum about the axis is zero), but to a *less extent* since $B > A$. The animal then contracts its hind legs, extends its fore legs and gives its *hind part* a turn. This necessitates the fore part being turned in the reversed direction, but again to a less extent since A is now greater than B. It will thus be seen that by continued action of this kind the cat can turn itself through any required angle, though at no time has it any angular momentum about its "axis."

From CRABTREE; *Spinning Tops and Gyroscopic Motion*

INDIAN MATHEMATICAL SOCIETY

Narasinga Rao Medal for Mathematical Research

NEXT PRIZE PROBLEM

To make a contribution to the theory of plane projective geometries particularly the following types of non-desarguesian geometries:

(i) The geometries where a single (p,L) Desargues' Theorem holds where L is a line passing through the point p. [The (p,L) Desargues Theorem is the theorem which states that if the three joints of the corresponding vertices of 2 triangles concur at p, and if two pairs of their corresponding sides intersect on L, then the third pair also intersect on L];

(ii) The geometries in which there is a transitive group of translations;

(iii) The geometries in which the theorem of the complete quadrangle (that is, the existence of a unique harmonic conjugate C′ of C with respect to AB for every triad ABC of collinear points) holds;

The required "contribution" is to bear on the following questions:—

1. The distribution of points p and lines L passing through p, such that the (p,L) Desargues theorem is true, the distribution of collinear triads of points A, B, C, such that C has a unique harmonic conjugate with respect to AB.

2. The distribution of quadrangles with three collinear diagonal points.

3. The projective group of the plane and the projective groups of its straight lines.

References

R. MOUFANG: (i) Zur Structur der projectiven Geometrie der Ebene, *Math. Annalen* vol. 105 (1931) pp 536-601.

(ii) Die Einführung der idealen Elemente in die ebene Geometrie mit Hilfe des Satzes vom vollstandigen Vierseit, *Math. Annalen* vol. 105 (1931) pp 759-778.

(iii) Die Schnittpunktsätze des projectiven speziellen Funfecksnetzes in ihrer Abhängigkeit von einander, *Math. Annalen*, vol. 106 (1932) pp 755-795.

(iv) Ein Satz uber die Schnittpunktsätze des allgemeinen Funfecknetzes, *Math. Annalen,* vol. 107 (1932) pp 124-139.

(v) Die Desarguesсhen Sätze vom Rang 10, *Math. Annalen,* vol. 108 (1933) pp 296-310.

(vi) Alternativkörper and der Satz vom vollständigen Vierseit (D_9): *Abh. Math. Seminar Hamburgischen Universität,* vol. 9 (1933) pp 207-222.

(vii) Zur Structur von Alternativkörper *Math. Annalen,* vol. 110 (1934) pp 416-430.

B. H. NEUMANN: On the commutativity of addition, *Jour. Lond. Math. Soc.* vol. 15 (1940) pp 203-208.

W. WAGNER: Über die Grundlagen der projectiven Geometrie und allgemeine Zahlensysteme, *Math. Annalen* vol. 113 (1936-37) pp 528-567.

M. ZORN: (i) Theorie der alternativen Ringe, *Abh. Math. Seminar Hamburg. Universität* vol. 8 (1930) pp 123-147.

(ii) Alternativkörper und quadratische Systeme, *Abh. Math. Seminar Hamburg Univ.* vol. 9 (1933) pp 395-402.

R. BAER: (i) Homogeneity of projective planes - *Amer. Jour. of Maths.* vol. 64 (1942)

(ii) The fundamental theorems of elementary geometry, *Trans. Amer. Math. Soc* vol. 56.

According to the conditions of award of the Narasinga Rao Gold Medal for Mathematical Research, any one of Indian domicile may compete for the prize if he is a member of the Indian Mathematical Society both at the time of competing and at the time of award. The medal shall be awarded for the best solution or contribution towards the solution of a specified problem in mathematics which shall be announced sufficiently in advance of the date fixed for the submission of the theses.

The solutions of the prize problem are to be sent to the President of the Society at the time, and the last date for the submission of the theses is the 1st of July 1948.

A. NARASINGA RAO,
Editor of *The Mathematics Student.*