FOUNDATIONS IN ARITHMETIC

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INTRODUCTION.

It is now very generally recognised that the confusion and inaccuracy which many children exhibit in the arithmetic class are not inherent in the nature of childhood, but are products of the school for which the teacher must be held responsible. In number study no child should be allowed to outrun his understanding, and the teacher of the young child particularly must safeguard his future by resisting all attempts to fix a goal of achievement which necessitates straining the powers of any of her little pupils. Number study is essentially individual work, and the rate at which a given child progresses depends largely on inner factors which are outside of our control.

In this little book, Miss MacKenzie has supplied teachers with a path along which little children will advance with confidence and pleasure, and with a clarity of understanding which will serve them well in all their later work. The use of her material and her method renders it possible in a class of forty children for each one to do number work precisely suited to his stage of development, and renders it possible for the teacher to keep in touch with each one and give the word of encouragement or guidance just when it is needed.

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AUTHOR'S PREFACE.

In sending this little book out upon its travels, the author would like to express indebtedness to those who have done pioneer work in helping to introduce wholesome activities into schools for little children, and to bring these schools into close relationship with the unconscious needs of childhood.

While holding them in no way responsible for the matter contained in the book, she would like in particular to express gratitude for inspiration which came, both through the written and spoken word, from Dr. Maria Montessori, Miss Margaret Drummond, and Mademoiselle Hamaïde, collaborator with Dr. Decroly.

Many and varied are the methods by which little folks seize upon that which they need for their education. At one time they may be absorbed in reproducing by imitation, and on a smaller scale, the activities of the adult life around them. At another time they carry through with absorption games of skill which make the instrument of the body ready for the great game of life, rejoicing in the repetition of exercises in which they have recently acquired the power to succeed.

In ministering in our schools to the needs of youth is it necessary that, in welcoming the message of Montessori, we should neglect the teaching of Decroly, or in acclaiming the

results of the patient, scientific research of Seguin, Itard, Binet, we should set on one side the wisdom of Froebel, gained through his love for little children, his observation of them, and his intuitive genius for seeing beyond the outer expression to the inner needs of childhood ?

The children themselves must be our guide ; we shall learn through their response to our methods ; if our schools become happy places in which learning is associated with the joy which springs from achievement we are not likely to go far wrong.

ANNIE F. MACKENZIE.

Moray House.

CHAPTER I.

NURSERY NUMBER.

Differences in Mental Development.

For the sake of clearness the work outlined in this little book is arranged in accordance with the age groupings common in our schools. It will be understood throughout that no hard and fast scheme of work is desirable for any such age group. We now know, as the result of careful scientific investigation, that a child's mental age may differ greatly from his chronological age, and that it is possible to have, in a so-called five-year-old class, children whose mental ages may range, for example, from three to seven years.

Methods of individual work make it possible to keep children of varying mental ability happily and profitably employed in one class-room. Pre-school activities overflow into the Infant Class, and in their turn Infant room activities overflow into the Junior School. In this way it is possible to help children to build up a connected body of knowledge which is likely to prove a stable foundation for later work.

In the following pages is outlined a scheme which was evolved so that children might work each at his or her own pace, through Nursery, Infant and Junior School, together with methods of recording progress, and descriptions of class or group activities likely to prove enjoyable and profitable at these successive stages.

Home Activities in the Nursery or Nursery School.

A considerable portion of the Nursery or Nursery School day is taken up with ordinary activities of home life, activities which little ones enjoy, and which are a real means of early education. Hands are washed, hair is brushed, overalls are put on, floors are swept, tables are dusted, lunches are set, flowers are arranged, toys are taken out and replaced in cupboards, beds and blankets are set in place for the afternoon sleep, and folded away again when it is over. Very simply and naturally number ideas may be gleaned from such experiences, and very simply and naturally number names may be introduced little by little into ordinary conversation.

We need **four** plates for this table, Kitty. One—two —three—four."

"Bring me your **two** shoes, John, and I will help you to put them on."

"Here are **three** crayons for you, Jimmy—**one** red one, **one** blue one, and **one** yellow one."

"May has brought some pretty daisies. Let's count them, and see how many there are"—and so on.

Informal, incidental use of number names in connection with familiar everyday experience forms a very important introduction to number work. Without the help which number names supply number ideas remain vague. Gradually through frequent use of the names in connection with familiar objects of every-day life the child is helped—when he is ready for it—to form ideas of threeness, twoness, etc., and moreover applies these ideas to a variety of objects in a simple natural way.

The Importance of Accurate Counting.

We have observed that many little ones become eager little counters at about four years of age. Just a few suggestions, such as,

"Let's count to see how many pencils there are in this box," or

"Let's count the steps as we go upstairs,"

may be sufficient stimulus to awaken interest in the number aspect of experience, and many a little one will, independent of further help, prepare himself for future work by counting everything countable in his environment—making his own by frequent repetition the number series of names, and gradually learning to apply each name accurately to a definite quantity.

Ability to count objects accurately one by one is an ability which matures slowly in the little child, and probably indicates a higher level of development than ability to guess at the number in a group and sometimes guess correctly without being able to prove to one's self by accurate counting that the group consists of just so many objects—neither more nor less. We neither can, nor should we try, to force this ability, but children who are mentally ready will seize on the suggestion to count, and will continue the practice of counting until they become skilled in the art.

Adaptability is essential in one who superintends in the Nursery or in the Nursery School, for her function is to minister to the inner needs of the child mind—neither seeking to force mental development, nor to neglect giving the helpful suggestions likely to awaken happy activities for which a child here and there may be ready.

With our fuller knowledge of psychology we are not so afraid to-day as in former years of providing the bright child with the mental food for which he obviously hungers, but our giving must

always be guided—and especially during the critical early years —by the child's response.

Rhyme and Rhythm as Aids to Counting.

Rhythmic repetition of the number rhymes serves more than one purpose. The rhymes themselves please the ear of the little ones ; they enjoy the rhythm ; the number names become familiar, and the rhyme may emphasise the end of a **two, three, four** or **five** group—thus forming an introduction to the counting in twos, threes, etc., which will come at a later stage.

In learning the rhymes the children may clap their hands for each number name, and repeat softly with hands still, the other portions of the rhyme.

One, **two** (clapping),
Buckle my shoe (softly).
Three, **four** (clapping),
Shut the door (softly).
Five, **six** (clapping),
Chop up sticks (softly).
Seven, **eight** (clapping),
Lay them straight (softly).
Nine, **ten** (clapping),
A good fat hen.

Also—

One, two, three, **four** (clapping),
Mary at the cottage door (softly),
Eating cherries off a plate, (softly),
Five, six, seven **eight** (clapping).

And—

One, two, three, four, **five** (clapping),
Once I caught a fish alive (softly) ;
Six, seven, eight, nine, **ten** (clapping),
I had to let him go again (softly).

Games, such as the following, will be found to interest little ones.

Five little children are chosen to be birds. They sit in the centre of a circle, and fly away one by one in response to the words of the song—

"Five little birdies, sitting at the door,
One flew away, and then there were four."

(One rises, and pretending to flap wings, flies round circle and away to settle in far corner of the room, while all sing the chorus) :—

"Birdie, birdie, happy and gay,
Spread out your wings and fly away !
Four little birdies sitting in a tree,
One flew away and then there were three.
(Repeat action and chorus.)

Three little birdies looking at you,
One flew away and then there were two.
(Repeat action and chorus.)

Two little birdies sitting in the sun,
One flew away and then there was one.
(Repeat action and chorus.)

One little birdie left all alone,
He flew away and then there was none.
(Repeat action and chorus.)"

The following rhyme is also enjoyed by little under-fives. While it is being repeated thumbs are held up to represent the

two birdies, and they disappear and re-appear in response to the appropriate words in the rhyme.

"Two little birdies sat upon a hill,
One named Jack and the other named Jill.
Fly away, Jack; fly away, Jill.
Come back, Jack; come back, Jill."

Number connected with Stories and Handwork.

Number ideas and number names very naturally enter into story-telling, story-acting, and the hand-work which children undertake in connection with their stories. They may make, for example, out of clay or plasticine, **three** bowls for the Three Bears' porridge and **three** spoons with which they might sup it. They may choose **three** people to act the part of the three bears, and **one** person to be Goldilocks. Another day teacher may draw **three** houses for the Three Little Pigs, or she may tell the story of Little Gingerbread Boy and the children may model him from clay or plasticine, making his **two** legs, his **two** arms, his **one** round little head, his **two** eyes, his **one** nose, and the **six** curranty buttons down the front of his coat.

If nature conditions are favourable the children may model or draw, for example, the nest with **five** little eggs in it that robin has built in the garden, or the **six** bulbs that they are about to plant in a bowl, or the **seven** tadpoles that swim about in the schoolroom aquarium, and many other things in which they show interest.

Number in connection with Free Occupations.

The free occupations of the children provide many opportunities for incidental number teaching. The room may

contain a rocking-horse, and the little ones will readily learn to share it, if it is decided, for example, that each one shall have ten rocks. Contentedly they chant the number series, coming to understand that at the magic sound of **ten** it is someone else's turn. It will probably be found that here and there a child comes to know, possibly through connection with rhythm, just how many more rocks he or she should have to complete the series.

Some of our little folks were accustomed at one time to be given **ten** chances to bounce a favourite ball. One day we asked four-year-old Isa, when she had bounced the ball seven times, how many more bounces she should have, **"Three"** was the immediate answer. One day later we asked her (from curiosity to know how much she understood), **"How many are seven and three?"** but she could not tell us. She appeared to understand the problem only in connection with the practical situation.

It is something that children should come to appreciate number thus, as a measure for equal sharing. Through use in such concrete and interesting situations they learn to appreciate its value. It is important, however, that **no** attempt should be made to force number ideas upon the children. Those who are ready for the ideas will seize upon them and make them gradually their own.

Nursery children should have varied playthings, picture books, bricks, balls, tea-sets, dollies, spades and pails and a sand-pit in which to use them, and fine big coloured beads for threading. Simply and naturally counting may be introduced in connection with these playthings. A box may, for instance, contain ten, big, coloured, wooden beads. **"Let's see if they're all there,"** suggests the grown-up. "One, two, three . . ." she goes on, the little one joining in as he or she is able, for the essence of good teaching

at this stage consists in urging **not at all,** but in leaving the little one free to join with us as he pleases. He is more than likely to recognise such counting as a pleasant and useful occupation —for of course if the ten beads are not present he gets one or two more to make up the required number !

" Let's see how many baby pigs this mother pig has," studying a picture book, or " Let's see how many baby chickens are running after this big mother hen," etc., are also suggestions which may be found to interest pre-school children who feel the urge to analyse experience in order to make impressions more clear and definite.

By studying impressions from various angles—colour, form, number, and through the binding power of common names little folks are helped gradually to reduce to order the bewildering world in which they find themselves. Number is of real **use** to the little ones in helping them to clarify impressions.

Number and the Montessori Sense-Training Material.

Many Nursery Schools of to-day are provided with the Montessori sense-training material. Much of this material has a direct contribution to make towards the preparation for the understanding of number. If the children are to receive the training which the material was designed to give them they will be shown how to use it for its correct purpose, and how to take care of it.

The cubes of the tower, the prisms of the broad stair, the rods of the long stair, so exactly graded in size in order that they may, in the clearest way possible, train the eye and the hand of the little one nicely to discriminate between shades of dimension, should no more be used, **as a general rule,** to build bridges or

castles, than should so be used the nicely-bound, brightly-coloured picture books, which also are designed for another purpose.

The words "as a general rule" are inserted advisedly, for the "artist" teacher would not be likely to break in on any occupation in which a child appeared to be whole-heartedly and peacefully absorbed. The material has not been designed for free creative activity and when a suitable moment arises the child may be shown the correct method of using the material, or may have his attention drawn to the box of blocks specially designed for free creative work.

Much may be learned by the adult who quietly observes the little ones who use the Montessori material. While she watches, it may be, a three-year-old absorbedly selecting or rejecting the various cubes of the tower in an earnest endeavour to make it symmetrical, or a four-year-old carefully grading the prisms of the broad stair, she will, at one time or another, be impressed by the intentness of the little one, by his absorption, and his pleasure in the completed task.

Wherein lies the attraction for the little ones in what to us might seem an arid occupation? Part of the answer may be that it satisfies a definite need of his growing organism—a need to gain delicate muscular control, to gain precision in observation, and in discrimination, to use independent judgment. The mood in which he works corresponds to the mood in which he spontaneously enters upon numerous other "games of skill," such as balancing on a narrow plank, bouncing a ball, learning to skip, solving puzzles of various kinds—all at their appropriate stages in his development.

It is a mistake to think that these Montessori exercises are easy for the little ones. It has been found that numerous five-

year-old children cannot build the tower. Yet the little ones attack these exercises spontaneously, and, when the mood is upon them, and in an environment with many counter attractions, pursue their task with an absorption of which we hardly believed such youthful students to be capable. Independently, and without fear of intervention from an adult, they observe carefully, discriminate nicely between the blocks, lay them neatly in place, maintaining their balance meanwhile as they bend to do so. The completed task is for them an achievement, and joy springs from the power they gain of controlling this wonderful instrument, the body, and bringing it well under the control of the directing mind.

The tower, the broad stair, the long stair, the three sets of solid insets each have a contribution to make towards the child's growing perception of differences of quantity. The divided stair with its accompanying figures will more rarely be used during the nursery stage, although from time to time it will be found that here and there a little one will master its secret and find joy in the mastery.

It has been found interesting to keep a record of the achievements of Nursery children on a sheet arranged as is shown in illustration. The sheet may be fixed inside a teacher's cupboard door, or on the door of the class-room, and needless to say it is for the teacher's use only. No reference is made in the children's hearing to their achievements or failures, and parents also may be helped to understand the possible dangers of discussing the ability or want of ability of their little ones in their presence.

The supreme lesson for those in charge of pre-school children is to learn to wait on development, to be content to **show** little ones the uses of the things in their environment, and to leave them free to seize upon what they need, and to master for themselves their secrets.

NURSERY SCHOOL
CLASS RECORD SHEET
Session 19___ to 19.....

Name.	Birth date.	Tower.	Broad Stair.	Long Stair.	Divided Stair	Age July 31, 19.....
Mary Reid, Charlie Ross, Isobel Scott, etc.,	17/3/29 etc.,	24/5/32 etc.,				

CHAPTER 2.

NUMBER IN THE FIRST SCHOOL YEAR.

It is important that the transition from home or nursery school to ordinary school life should be as smooth as possible. Sudden changes of environment suit young children as little as they suit young plants. Continuity in training makes for economy in learning, and prevents the mental, physical and emotional jars which sudden changes in treatment give to a young and tender organism. It is to be hoped that soon we may have teachers in infant rooms who understand the mental and physical needs of children under five so that this continuity in treatment and in training may be ensured.

Incidental Teaching in Connection with Everyday Affairs and Projects.

Little children's interests tend to centre round the home and home activities. The room in which they work should be made to look as home-like as possible ; toys and materials which were present in the nursery should be available here also and wherever possible children should be encouraged to engage in home-like activities—such as sweeping, dusting, arranging flowers. It will be found, too, that much valuable teaching may be given through projects which start off from some familiar experience of home life, such as making scones, or a plum pudding, or getting breakfast ready, buying fruit or vegetables or, it may be, milk or cream.

Such projects give scope for learning many things, and among them number has a place. The making of scones, for example, may embrace talks on bakers, millers, farmers, fields of waving grain, cows and milk. The making of the scones involves putting together an exact quantity of flour, of baking powder, of salt, of sugar, perhaps, and of milk. The recipe may be written up in simple form.

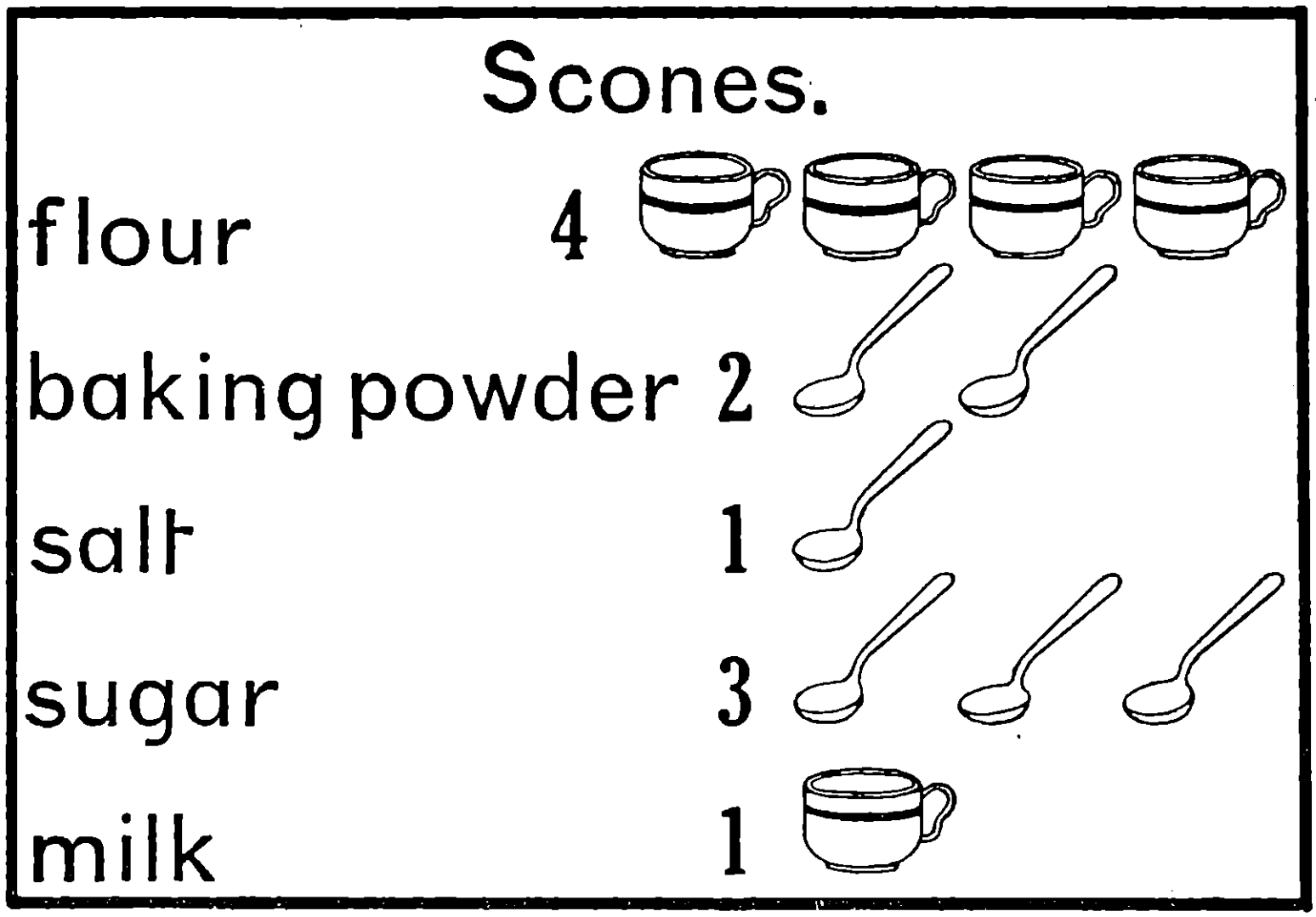


Illustration 1.

Making the tea for breakfast also gives scope for learning a variety of things and when it is decided that tea is to be made for, let us say, six people, a big figure " 6 " may be written up, and children will count while one child carefully measures out six tea-spoonfuls of tea. Buying fruit or vegetables (modelled from clay in handwork lesson) gives scope for gaining familiarity with coins up to sixpence or one shilling, and words such as dozen and half-dozen may be learnt in an interesting and natural context.

Children like to weigh out apples and oranges with pound, or half-pound weights. They may find out how many are needed to make the scales balance. Pint, quart, and gill measures may be used in connection with the exact measuring out of whitened water which represents milk. Little children, in working-class areas at least, often accompany their mothers on shopping expeditions, and they are readily interested in these things which are part of their every-day experience.

They are interested, too, in comparing the weight of, let us say, the pound and half-pound if these weights are laid on the palms of their outstretched hands. They like also to find out how often they can empty the pint into the quart measure, or the little gill—used for the precious cream—into the pint measure.

The drawing or modelling or paper-cutting which children undertake in connection with these projects also provide scope for much incidental number teaching.

The teacher will also seize opportunities for connecting number with all the familiar objects in the school-room. The children may count the windows, the window-panes, the pictures, the doors, the electric-lights, and drawings of these may be made with figures beside them. She will also use number names in connection with the giving out of material, *e.g.*, "Here are **eight** people. We must have **eight** pieces of paper," etc. The teacher may often audibly count material, or the children themselves—to see if all are there, the children joining in as they wish.

Rhymes and Rhythm in Connection with Counting.

To ensure that children learn the correct sequence of number names to ten and to twenty they may from time to time repeat them rhythmically, clapping as they do so. Sometimes this exercise may be varied by getting one child to skip or bounce a ball or walk round the room while the others count. It is useful also to have the figures written large and in sequence where the children can see them. From time to time the teacher may say, for example, "Let us see whether Mary can bounce the ball exactly **seven** times"—underlining 7 in the sequence, or "Let us see whether Jimmy can bring me **nine** pencils"—underlining 9 in the sequence, or "Let's count while Peggy skips up to **ten**"—and so on.

Rhymes of the "**Ten Little Nigger Boys,**" kind may, with advantage, be used in teaching the children to count up to ten and back again. Ten little children take up their places on the floor and one by one they run away as they sing of something that happens to each little boy in turn.

The old folk song, "**Ten Men Went to Mow, Went to Mow a Meadow,**" may be used somewhat in the same way.

As time goes on and as the teacher comes to be sure that the children know the value of the separate figures, rhymes such as the following, which involve simple addition, may be found interesting and helpful :—

"One and one are two—
That's for me and you.

Two and two are four—
That's a couple more.

Three and three are six—
Barley sugar sticks.

Four and four are eight—
Tumblers at the gate.

Five and five are ten—
Bluff sea-faring men."

(*Christina Rosetti.*)

It is important, however, that teachers should realise that before any number analysis—addition or subtraction—is attempted the children should be able to count things accurately, and should understand, for example, that the word "**six**" means an exact number of things. "**More hurry, less speed,**" is a proverb very applicable to early number teaching.

To attempt to teach little folks that "three and two are

five," or even that "one and one are two" before they have had ample and varied opportunities of counting is to court disaster. The words can be for them nothing but a jingle, for much experience is necessary before little ones come to understand what number is all about. To realise the number aspect in connection with any concrete experience the little ones have to eliminate so many other interesting aspects—colour, shape, playability, etc. They must, for example, think of the "twoness" of their counters, separating this quality from their redness, their roundness, their position on the table, their use for rolling about, etc.

Experience shows that children are quite ready—often at four years of age—to become interested in counting, but much counting and arranging is necessary, much time for the assimilation of experience must be allowed, before the mind is ready for number analysis. After all, we cannot teach a child that one and one are two. He either sees it for himself as a result of experience or he does not see it.

Our teaching is successful only in so far as it puts into words for the child something of which he is already aware, helping him thus to more clear, definite knowledge, through the use of language. In other words language must come after, and not precede experience, and furthermore we must be sure that the child has profited by the experience before he is asked to crystallise the experience in words.

Shipwreck in number teaching is certain to result from undue haste during the earliest years, because number from the very beginning, and even when concrete material is used, is an abstract study. Better far is delay in teaching than is a hurry which confuses a little mind not yet ready for dealing with abstractions. The readiness will come in due course through the counting and arranging of material.

Importance of Individual Work Periods.

In a big class it is often difficult to be sure that each separate child is gaining the number experience for which he or she is ready, and it is difficult for the teacher to be aware of just what stage a child has reached. Many have come to realise the great importance in a school day of the " Individual Work " period. During that time each child may work at his or her own rate, along a carefully graded pathway.

Our time-table allows for a considerable portion of time to be spent each day in individual study. Our class-rooms which accommodate forty children, are arranged with tables and chairs in groups of eight. The groups are roughly graded according to the mental ability or maturity of the children. The teacher, for instance, may know that, let us say, Group 1, consisting of the brightest children, will probably go ahead quickly with little help, while Group 5, slower, or less mature, or containing children who have been absent, will go slowly and require helpful assistance.

Our day resolves itself, as a rule, into the following " Daily Programme " :—

9.30–10.—Prayer, Praise, Music.
10–10.55.—Class Lessons.
11.5–12.30.—Individual Work.
12.30–1.30.—Dinner Interval.
1.30–3.—Singing Games, Projects, Handwork, etc.

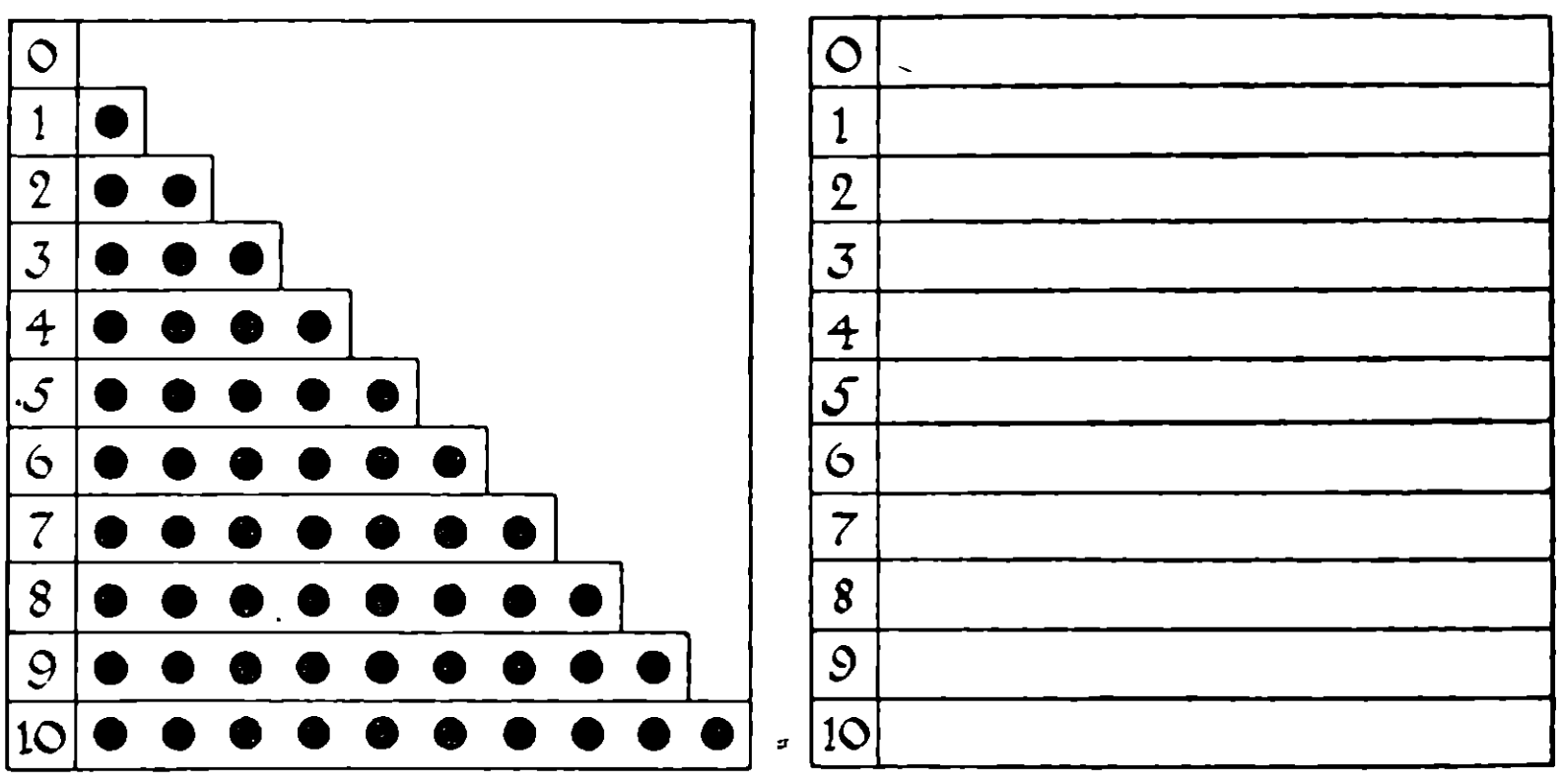
Individual Work.

Class lessons in number centre round aspects of work which children can enjoy together, such as rhythmic counting, problems connected with projects, or other interests of everyday life. The individual work period provides for the building up, by each individual child, of a connected body of knowledge, and ensures that no child is being dragged forward at too speedy a pace.

A simple series of exercises has been evolved to provide a graded pathway to number, and a simple card has been planned on which individual progress can be recorded.

During the early weeks of their first year in school, the teacher will probably find it helpful to provide groups of children with occupations which require the minimum of supervision, such as the threading of big wooden beads, the studying of picture books, drawing with chalk on black-boards, the teasing of wool to stuff cloth toys, the making of patterns with parquetrie boxes, or templates.

Such occupations, can peacefully be enjoyed by many of the children while the teacher trains, perhaps, one or two groups at a time in the correct use of other material, the purpose of which is not at first so obvious to them. The teacher will aim at training the children to take out, use for its correct purpose, and replace material with care, so that they may early be entrusted to carry out their occupations with independence and with the minimum of guidance.

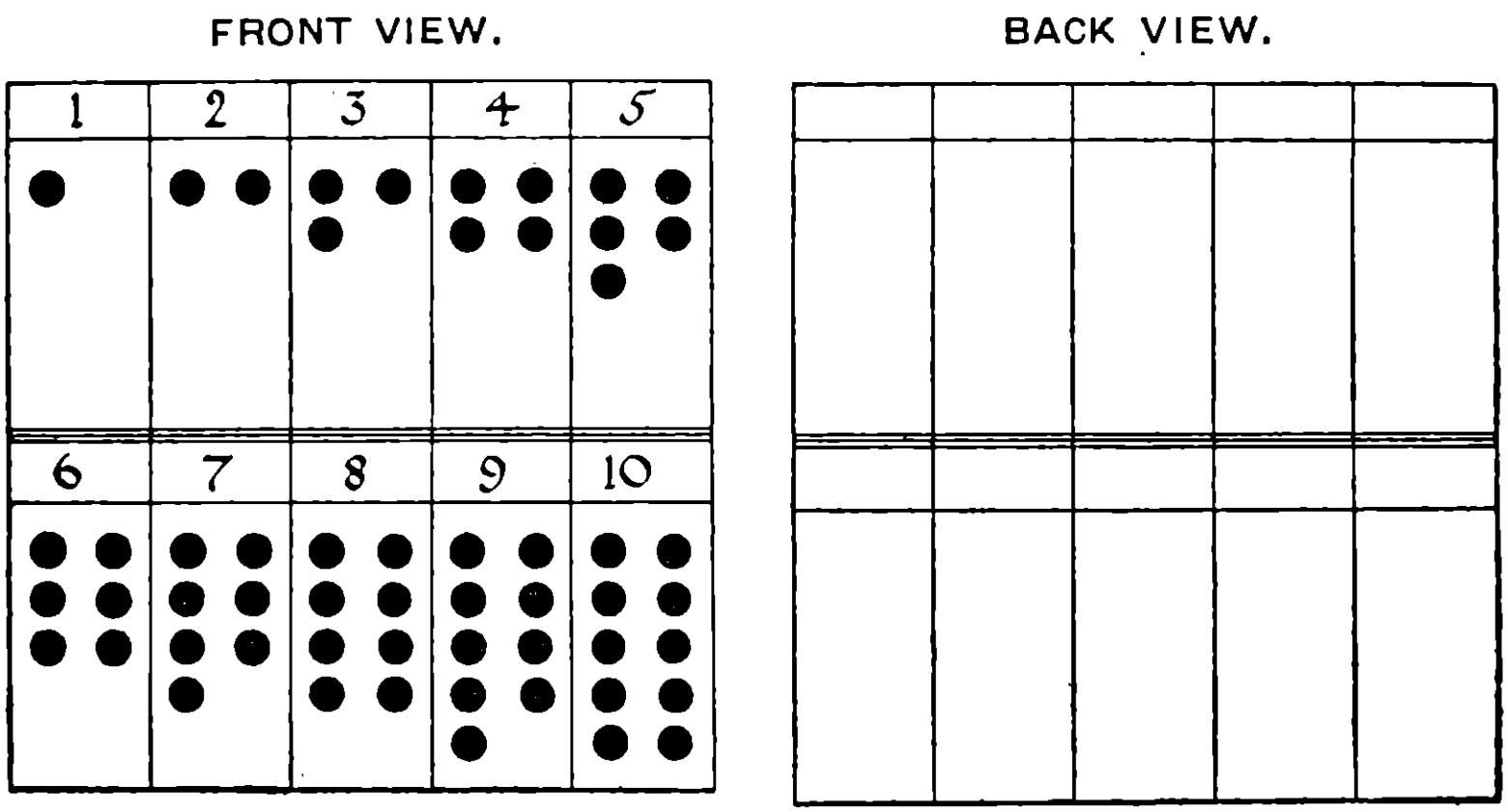


Size $11\frac{3}{4}'' \times 11\frac{1}{2}''$. Illustration 2.

The Stair Board.

This board is designed to make clear to little ones the gradual increase in length of line when counters are arranged opposite the figures from 0 to 10.

On the one side circles are drawn to show exactly where counters are to be placed opposite the figures, and on the back the figures appear, but no circles. Carrying out the exercise on the back of the board thus forms a test of the child's knowledge of the value of each figure. (See illustration above.)



Size $11\frac{1}{4}'' \times 11\frac{1}{4}''$. Illustration 3.

The Odds and Evens Board.

This board is designed to provide another exercise in allotting an exact number of counters to each figure. This board, however, shows another arrangement of counters. The little compartments have to hold counters arranged in pairs and odd ones. The children have their attention drawn to the little odd ones as teacher moves among them. Teacher, too, has often to remind children to fill each little "house" in turn—drawing her finger round the dividing line which separates off one "house" from another. The back of this board, also, is designed to test the knowledge which children have gained through the arranging of counters on the circles drawn on the front of the board. (See illustration above.)

DOMINO CARDS, 11 cards, size 6 ins. × 3 ins.

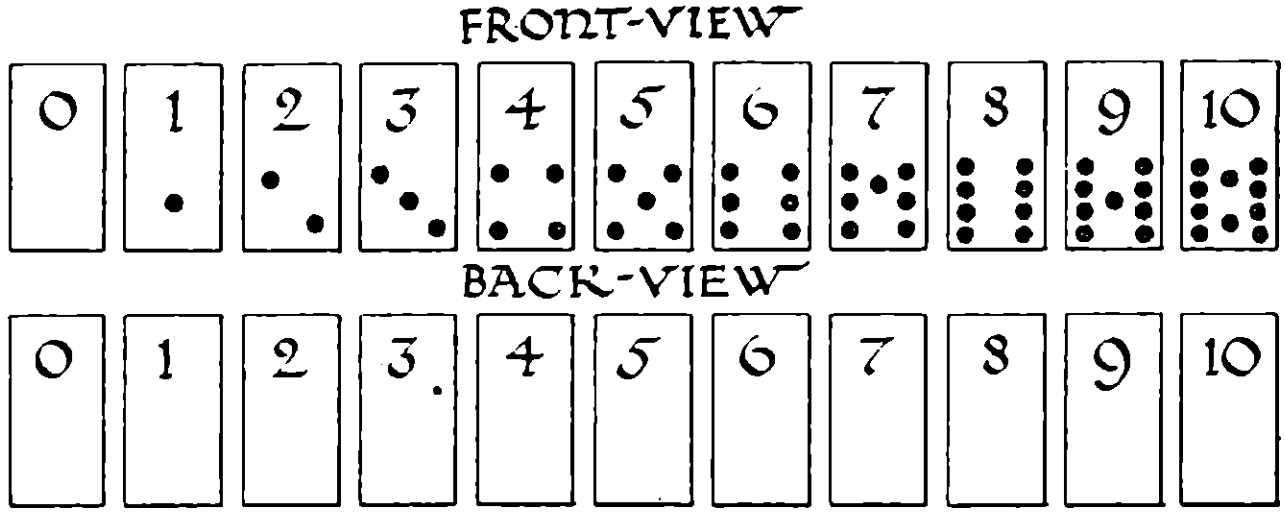
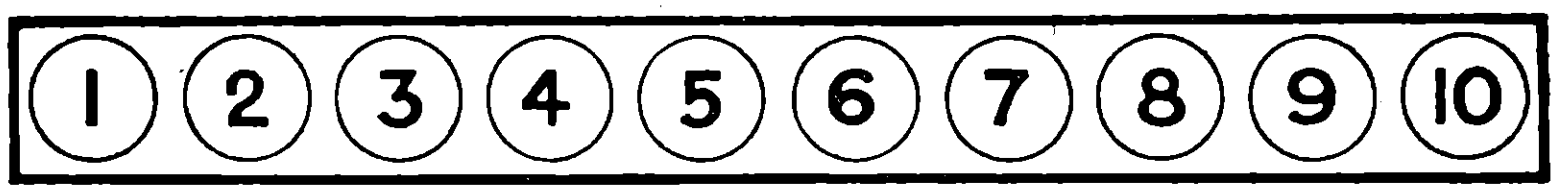


Illustration 4.

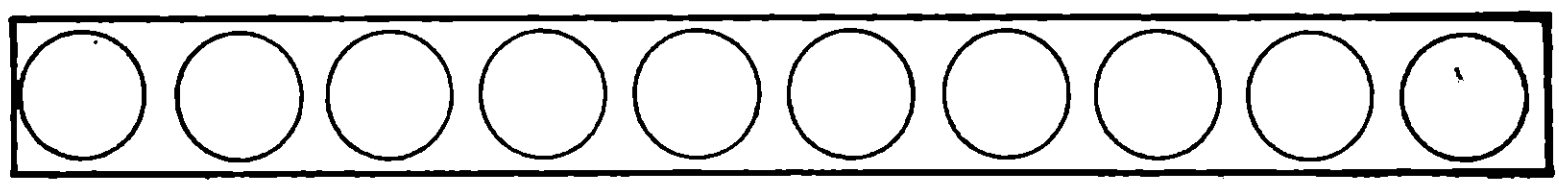
The Domino Boards.

These boards provide for yet another method of counter arrangement. On the front of the boards circles are drawn under the figures in domino arrangement. The children therefore place the counters in domino pattern and thereafter make the same arrangement on the back. In using this material the children have to place the eleven domino boards in sequence. (See illustration above.)

Through the use of the three types of boards the work of this important early stage is varied for the children ; they are helped to see for example that 3 represents just so many things—whether they make any special pattern or no. Each arrangement has some contribution to make towards the child's realisation of number, and by the time all the exercises have been completed the teachers has reason to be assured that the children really know the value of each figure.



Front View.



Back View

Size $11\frac{1}{2}'' \times 2\frac{1}{4}''$. Illustration 5.

The Ten Board. (See illustration above.)

With this little board children can make all possible additions and subtractions up to ten. Ten flat counters coloured red on one side and blue on the other are used with the board.

RED CARDS—Size $4\frac{1}{2}'' \times 3''$.

CARD 1.

3 + 1 =
6 + 1 =
2 + 1 =
4 + 1 =
9 + 1 =
7 + 1 =
8 + 1 =
1 + 1 =
5 + 1 =
6 + 1 =

Illustration 6.

CARD 7.

7 − 1 =
2 − 1 =
10 − 1 =
8 − 1 =
5 − 1 =
9 − 1 =
6 − 1 =
3 − 1 =
4 − 1 =
1 − 1 =

Illustration 6*a*.

Red Cards.

Along with the board are also used twelve red cards, on which are printed sums in their simplest form. On the first six cards are printed exercises in addition, on the following five are printed exercises in subtraction, and on the last card are printed exercises in both, addition and subtraction. (See illustration 6 and illustration 6*a*.)

Illustration 7 shows a convenient method of storing the twelve cards of each series in wall pockets. The wall pockets are made of brown casement cloth and measure 16″ × 18″. The pockets which contain red cards have figures sewn on them in red sylko thread corresponding to the figures on the cards. Blue cards have blue figures on their pockets, and so on.

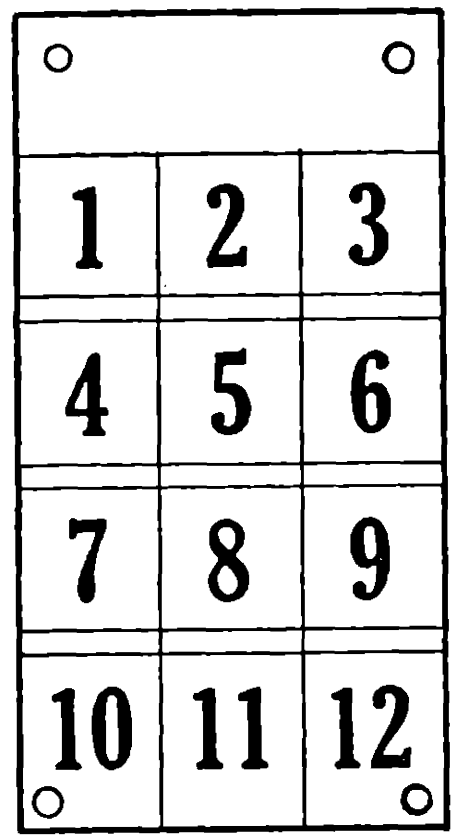


Illustration 7.

The children quickly learn to select their cards in an orderly way and in correct sequence when this method of storing them is provided.

In working an addition sum, *e.g.*, 3 + 4 = 7, the child is taught to lay out three counters with the **red** side uppermost, and four counters with the **blue** side uppermost. In working subtraction sums the whole number is laid out in red, and the child learns to lift away the number indicated by the second figure.

Subtraction is readily understood by young children when a method of lifting away is followed. The close connection between addition and subtraction becomes obvious to children who gradually build up their knowledge from number experience. The method of beginning to teach subtraction by asking, for example, " How many does three need to make seven ? " when working such a sum as 7 — 3 = fails to enable children to grasp the full import of the **minus** sign, and its direct contrast to the **plus** sign.

The last card in the series is planned to make clear to the children that if, for example, $5 + 4 = 9$, $9 - 4$ will equal 5, and $9 - 5$ will equal 4. The children learn such facts, not through explanations, which frequently confuse thought, but through experience in handling and in arranging material.

Wordy explanations are dangerous in number teaching. A child can learn from experience with the concrete; he can memorise what he has learnt; but he is not ready to reason about processes.

Arithmetic is eminently an experimental science, the facts of which, unlike those, for example, of history, may be and ought to be learnt from first-hand experience. Number exercises can be graded so that they bring before the child's attention arithmetical facts in their inter-relations one with another, but it is essential that time should be allowed for the assimilation of knowledge, and that memorisation should follow, and not precede, number experience.

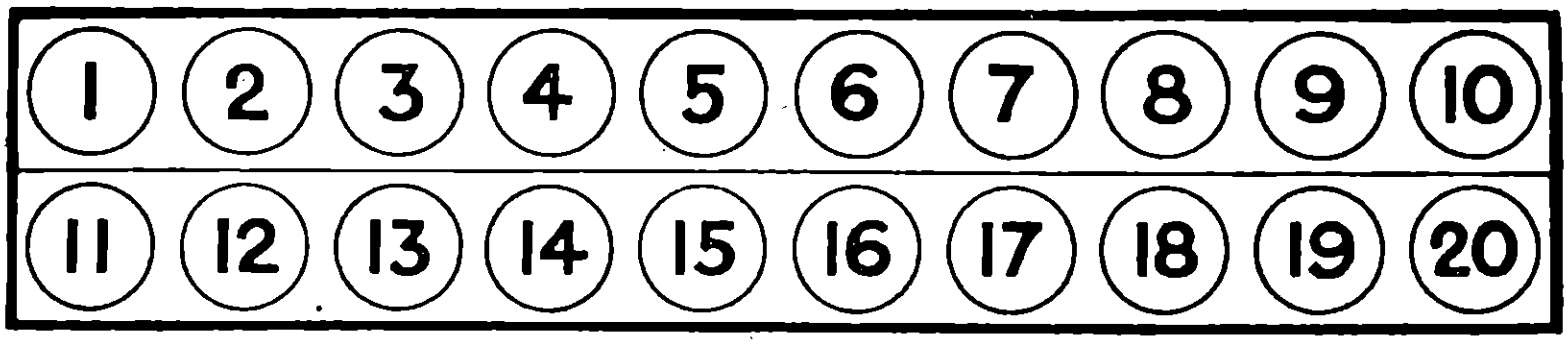
The sums on the cards may be worked first on the front of the board where the figures are a guide to figure writing, then on the back of the board where the circles are unnumbered, and lastly, from memory.

The little board with its ten circles gives the children many pictures of number groups arranged against a background of ten circles. The "ten" is thus constantly before their vision, and each smaller group is seen in relationship to the "ten" whole.

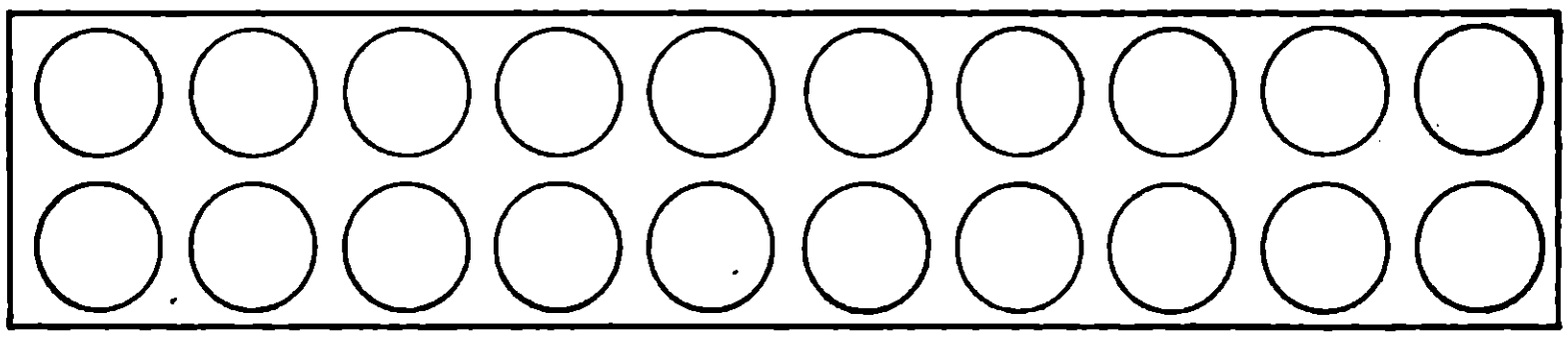
The fact that we have ten fingers and ten toes, that "ten" therefore played an important part in primitive counting, that "tens" are very important units in our decimal system of counting would seem to make "ten" a natural stopping place in our introduction to number analysis.

We need not trouble our little folks with explanations of the "notation" principle underlying the writing of "ten." The writer is inclined to think that at this stage, even when a method of fastening sticks into bundles of "ten" is followed, the children are likely to fail to understand or to appreciate the principle. Little children are practical people. They are content at first to know **how** "ten" is written. They are interested if we tell them that they have learnt to make all the figures they will ever be likely to need to use (*i.e.*, 0—9), and that we use them over again to stand for bigger numbers. The boards which the children use in following this scheme give them much practice in writing figures correctly, and are so arranged that in due course the principle underlying notation will become more and more clear to them.

THE TWENTY BOARD.



Front View.



Back View.

Size $11\frac{1}{2}'' \times 3''$. Illustration 8.

The Twenty Board. (See illustration 8.)

It is unlikely that all the children in a first-year class will be ready to use this board before the end of the session. At the same time it is well to make provision for the children, of whom there are likely to be a fair number, who master the preceding work before the third term and are ready to find "fresh worlds to conquer." The "Individual Work" period allows for a considerable "spread" in attainment, and the writer believes that while it is dangerous to force a child forward beyond his natural pace, it is undesirable to keep the quicker children marking time while the slower people make up, or try to make up, on them.

With this board the child learns to make all possible additions and subtractions to twenty. He learns to write correctly all numbers to twenty, and gains insight into the method by which the symbols he has already learnt are used over again to denote larger quantities. He will come gradually to appreciate the fact that ten denotes one full row and no more, and that eleven denotes one full row and one more, etc. The teacher may draw his attention to this fact, but experience in laying out counters, and writing down the results correctly will enable the child to see this for himself and to make the knowledge his own.

Blue Cards.

Twelve blue cards are provided for use with the twenty board. The children will require little teaching before they carry out the exercises printed on these cards for they are similar in form to those written on the red cards which have already been used.

The first card of this series is arranged to enable the child to realise the import of the first and second figure in the numbers between ten and twenty. In laying out counters to denote, for example :—

10 + 1 =
10 + 2 =
10 + 3 = etc.

the child sees the left-hand figure represented by a **full row** of **ten** red counters. Blue counters represent the right-hand figures in an incomplete row until twenty is reached, when a full row of red counters and a full row of blue counters will lie before him.

The figures which the child makes in writing the answers to his sums are therefore closely connected throughout with their significance.

Other cards in the series are arranged to enable the child to connect his knowledge of the first ten with that of the second, through working out on his board such sums as :—

5 + 4 =
15 + 4 =

Other sums in the series, such as

9 + 3 =
7 + 6 =

give the child practice in bridging the ten.

It may be said that the child who knows number thoroughly up to twenty has the key to number knowledge *ad infinitum*. He knows something of the way in which symbols are used over again to represent ever increasing quantities, he sees the relationship which the numbers of the first ten bear to the numbers of the second ten, and he may gain skill in bridging the ten.

With the use of the hundred board this knowledge will be reinforced. It is not always wise to insist that a child should know this twenty board thoroughly before passing on to the hundred board. The use of the hundred board will throw additional light on the work which has been done with the twenty board, and often it will be found that the work of the earlier stage will best be revised through the additional illumination of the new material.

Correction of Work.

In all exercises absolute accuracy should be expected, for it is important that children should appreciate the inevitable results of adding or subtracting given quantities. If children have acquired the fundamental idea of accurate counting, there is no reason why they should make mistakes when they are working with material. If mistakes arise, the children who make them should be referred back to the material to find out where they have been careless.

Corrections are of little value unless the child is enabled to see his mistakes and correct them himself. It is well that children should find out for themselves why a sum is right or wrong, and not come to depend upon the judgment of one adult. Material is of value in that it may be the unbiased judge of the rightness or wrongness of a child's work. Through its use the child comes to appreciate arithmetic as an exact science, the facts of which are capable of first-hand demonstration.

(See illustrations 9 and 9*a*.)

THE BLUE CARDS—Size $4\frac{1}{2}'' \times 3''$.

CARD 1.

10 + 1 =
10 + 2 =
10 + 3 =
10 + 4 =
10 + 5 =
10 + 6 =
10 + 7 =
10 + 8 =
10 + 9 =
10 + 10 =

Illustration 9.

Card 7.

9 + 8 =
8 + 9 =
7 + 8 =
8 + 7 =
7 + 6 =
6 + 7 =
9 + 7 =
7 + 9 =
8 + 3 =
3 + 8 =

Illustration 9*a*.

FIRST YEAR RECORD CARD.

Name—.....

STAIR BOARD.

ODDS AND EVENS BOARD.

DOMINO CARDS.

RED NUMBER CARDS.

1 2 3 4 5 6 7 8 9 10 11 12

1 2 3 4 5 6 7 8 9 10 11 12

BLUE NUMBER CARDS.

1 2 3 4 5 6 7 8 9 10 11 12

1 2 3 4 5 6 7 8 9 10 11 12

Illustration 10.

Number Record Cards. (See illustration 10.)

Individual Record Cards which lie on the children's tables during the "Individual Work" period have in our experience proved the most practicable means of recording individual progress. These Record Cards have on them only the skeleton or core of work which the children undertake. As will be gathered, their class-room provides for many and varied individual occupations. It would tax the teacher's powers to keep track of all that a child may do during these periods.

The minimum of work outlined on the Record Card forms a clear pathway, towards attainment of skill in dealing with number. This minimum might also be described as a "skeleton" of work, to which "body, substance, and life" is added through other and varied activities. It is a mistake, however, to imagine that these exercises in number skill are dull to the children, or even that "project" and other similar activities are preferred by them. The children like to learn to "do" sums correctly, provided that the sums are graded to meet their needs. They may be as pleased to feel within themselves power accumulating to deal with figures, as they are to learn new and varied ways of bouncing a ball.

In the Record Card (of which an illustration is given) it will be noticed that the figures 1 to 12 appear twice under the headings, "Red Number Cards" and "Blue Number Cards." A red pencil mark through a number in the **first** row of figures indicates that the corresponding Number Card has been done correctly **with** the concrete. A similar mark through a figure in the **second** line of figures indicates that a Number Card has been worked **without** the help of concrete material.

At the end of the first school year the children's Record Cards will be varied in the number attainment which they record. It is probable that the large majority of the class will have completed the exercises with the Red Number Cards; some will have progressed some distance with the Blue Number Cards; a few may have completed these exercises. On the other hand, a few will still have their Red Number Card exercises incomplete.

At the beginning of the new school year the children take their old Record Cards with them to their new class. During the Individual Work period they go on from exactly that point at which they left off—thus preserving continuity in their work.

In number work it has to be remembered that slowness during the early stages does not necessarily mean that the child will continue to be slow. When firm hold has been laid upon simple fundamental arithmetical truths children frequently take strides ahead in their work. As has been said before, it is essential that the assimilation of these truths should be during the early years, unhurried, and that the mind should be undisturbed by wordy explanations, or by the distress which arises from inability to attack difficulties beyond its power.

Supplementary Material.

In our rooms we have a variety of supplementary exercises which the children are free to choose each day during the Individual Work period. As a general rule it has been found convenient to train the children to finish a definite exercise in the series contained in this scheme before they freely choose miscellaneous material. Definite progress, and ease in recording, may thus be secured. On the other hand it is possible from time to time that a child will thrive best if given free choice during the entire period. Here and there we find a child who has come to a stage from which he is unready to progress further, and whose time may more profitably be employed in other ways, for example, in writing, in studying picture books, in drawing. That he does no number for a time does not mean that he is falling behind. His mind is assimilating what has already been done, and he is gathering strength to conquer new worlds.

The following number material is available in addition to that which has been described above :—

The " Montessori " Divided Stair, with sand-paper figures ;

The " County " Number Steps with a set of figures 1 to 10, printed on firm cardboard, 1 inch square ;

The " Croydon " Number Box ;

Boxes of brightly coloured wooden beads, each containing a bootlace knotted at one end on which the beads may be threaded.

CHAPTER 3.

NUMBER IN THE SECOND SCHOOL YEAR.

Number Connected with Everyday Affairs, Projects, etc.

It would make this little work too long were full descriptions given throughout of the diverse and varied ways in which number may naturally interweave itself with the every-day affairs, special projects, hand-work activities, games, etc., which form part of a natural routine of school life for little children. Suffice it to say that the thoughtful teacher will see to it that the skill which children acquire in their individual work periods will be intimately connected with interests of the moment, and that measures of weight, length, time, etc., will be introduced simply and naturally as opportunity arises.

In addition to the **simple weights, a rod of one yard in length, another of one foot, and a little rod of one inch, are useful things to possess.** These measures may be used, for example, in connection with the even spacing of bulbs in a flower-bed, the laying out of a garden in the sand tray, the planning of a class-room frieze. The children will be interested in measuring their room with the yard stick, their desks with the foot stick, their books with the inch stick. They will be interested, too, in comparing their own height, in feet and inches, with that of their class-mates.

Incidental reference may often be made to the passing of time. The room may be made silent while children quietly listen to the clock steadily ticking out the seconds and minutes. Children's attention may be drawn from time to time to the comparative speed with which the long hand travels, and to the

slow progress of the short hand. Reference may also be made, from time to time, to the position of the hands. For instance, "At half-past twelve—when the little hand is between twelve and one and the big hand is here (pointing), half-way round the clock—we shall have to put our things away." Frequent incidental reference to the position of the hands at the more important divisions of the school-day (opening time, lunch time, dinner time, closing time) will give some children sufficient clues to help them to learn to tell the time for themselves.

The beginning of each new day may be celebrated by pulling off the slip for the day which is past from a calendar with large figures. **Weather and nature charts put up anew for each month may be filled up day by day with yellow sun, for sunny days, pencil rain-drops for rainy days, etc.** Autumn, Winter, Spring and Summer Festivals may be held which mark the passing of the seasons and the characteristic features of each.

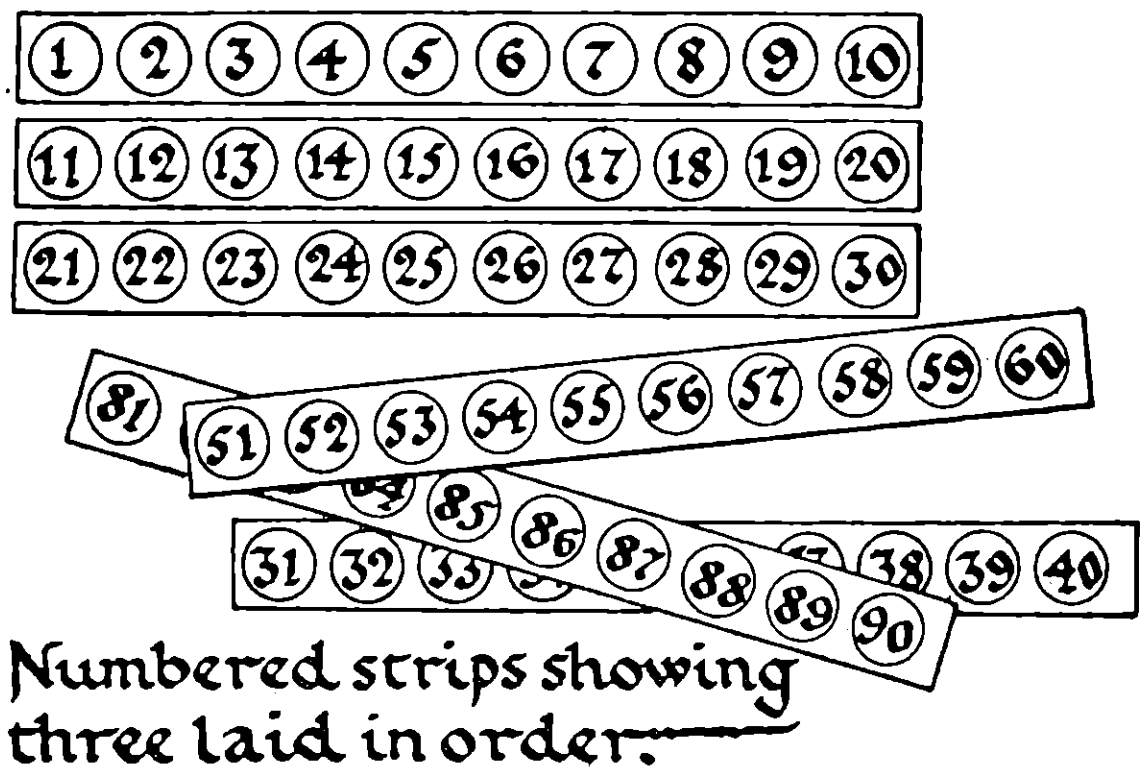
As children approach the age of seven it will often be found that they are ready to become interested in projects connected with primitive life. In connection with such projects they may learn how folk recognised the passing of time in olden days, at first perhaps distinguishing between the cold time and the warm time, the dark time and the light time, and gradually arriving, by means of sun-dial, hour-glass, candles, etc. at complicated clocks, which measure time with accuracy.

Similarly may be gathered from primitive life projects, ideas of early methods of "barter," and the gradual arrival at the standards of measure which are used in the shops of to-day. The standardised yard, foot and inch measures may be connected with the rougher measures used in olden times, and which are still used when absolute accuracy is not necessary. Thus for example, a grown person may roughly measure the length of a room by walking across it, placing one foot immediately in front of the other, or the play-ground may roughly be measured by striding over it, preparatory to guessing its length in yards, and the thumb from the tip to the first joint may be used as a measure of inches. Experienced knitters sometimes measure the foot of a stocking in this way.

Little children may come to appreciate something of the accuracy which underlies fair buying and selling by study of our measures, how they are used, and how they gradually came to be as we know them to-day.

Projects centred round the "Song of Hiawatha," "Robinson Crusoe," "Peter Pan," may also be found to interest children who are approaching the age of seven years. Where a project has thoroughly gripped the interest of a class, number interests may often be interwoven in a simple, natural way. Estimates may be made, for instance, of the number of feathers needed for Indian head-dresses; Crusoe's plans for estimating the passage of time may be discussed; measurements may be considered for the making of a Wendy house. Important as it is that number should connect itself with general interests, it should be unnecessary to make our projects convey all the number knowledge which a little child should gain, and for this end to drag in number problems where they do not naturally belong. Number is found to have its own intrinsic interest for children who work through exercises graded to suit their growing powers.

THE NUMBER STRIPS.



Numbered strips showing three laid in order.

Length 10″. Illustration 11.

INDIVIDUAL WORK.

The Number Strips. (Illustration 11.)

These strips are provided to help children to see for themselves the method by which numbers up to one hundred are written. The strips—each containing ten figures—are laid in consecutive order by the children. Some children will be able to lay the strips in order without guidance. Others will need to be shown by the teacher how to do this. While she lays the strips in order a child or group of children watch, and may repeat with the teacher the numbers on each one as it is laid in place, emphasising the figures, **ten, twenty, thirty,** etc., which come at the end of the strips. When teacher has built the square of ten by laying the strips in order, she mixes them up and invites the children to do the exercise by themselves. The exercise may be further simplified for the slower children by asking them to lay the strips in order on top of the rows of figures on the Hundred Board. The exercise thus becomes a matching one. The Number Strips are pink in colour, and thus form a contrast to the grey Hundred Board.

The Hundred Board and Exercise Cards.

The Hundred Board is similar in structure to the boards which preceded it, and carries a stage further the knowledge which has already been gained by the children. The similarity of the boards enables the child readily to connect the new knowledge with the old.

Four sets of exercise cards with twelve cards in each set provide exercises to be carried out with the Hundred Board, exercises which introduce for the first time ideas of multiplication and division. As each set of cards marks a new stage of work, distinctive colours are used, the first set being yellow, the next green, the next pink, and the last light blue.

The hundred board and the exercise cards which accompany it are likely to occupy the children for some considerable time.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Front View.

Illustration 12.

THE HUNDRED BOARD—Size $11\frac{3}{4}'' \times 11\frac{3}{4}''$.

The reverse side has blank circles for practice.

The Yellow Cards.

On the twelve yellow cards are written exercises which (1) enable the child to gain facility in the correct writing of figures from one to one hundred, and (2) prepare for the learning of multiplication tables by helping the child to count in regular intervals (*e.g.*, 2, 4, 6, or 9, 18, 27, etc.), from one to one hundred.

The exercises on these cards are written in words, but a similar wording is used throughout so that, should reading have lagged behind number work, the child quickly becomes able to deal with the cards himself as the same words are used over and over again.

The exercises on the yellow cards are so arranged that the hundred board is marked off by the child with counters in this order :—(1) in ones, (2) in twos, (3) in fives, (4) in tens, (5) in threes, (6) in fours, (7) in sixes, (8) in sevens, (9) in eights, (10) in nines, (11) in elevens, (12) in twelves.

THE YELLOW CARDS—Size $4\frac{1}{2}'' \times 3''$.

CARD 1.

Put a counter on every number.

Write the numbers in rows of 10.

Illustration 13.

CARD 3.

Put a counter on every 5th number.

Write every 5th number.

Illustration 13*a*.

Pattern Making.

An interesting result of marking off the hundred board in regular intervals is the building up of a pattern on the board. When the board is marked off in twos, fives and tens the counters lie under each other in straight rows, but more or less apart according to the size of the number. When threes, eights, nines are marked off the lines of counters slant across the board. Without explanation the child can see for himself the result of marking off nines, which makes this table an easy one to learn. When fours are marked off it is plainly seen that similar figures recur in every second row, resulting in a pattern.

These exercises, like all others, may be carried out on the unnumbered circles on the back of the board after they have been done on the front. As numbers, when written down, will have to be supplied by the children themselves, this exercise will test their knowledge of the board.

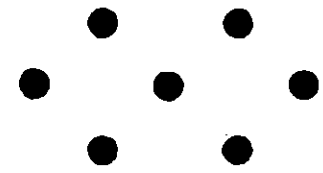


Illustration 14.

The Green Cards.

These Cards introduce for the first time the multiplication sign, and provide the children with exercises in building up for themselves their multiplication tables.

In presenting the first of this new series of cards to the children the teacher would naturally draw their attention to the new " mark " which they probably have not seen before. She might use various devices to make clear to the children its difference in shape from the " plus " sign which they already know, such as asking them to lay flat sticks in the one position and in the other, to make the shapes with their fingers, and to draw the one shape and the other. It is important that the children should **see** clearly the difference between the shapes because clear seeing will lead to clear working.

Without using the hundred board, but in response to exercises written on the blackboard, children might lift two, for example, three times, four times, etc., until the idea of taking a similar group a certain number of times becomes connected with the multiplication sign.

When the meaning of the multiplication sign has become clear, the children may work through the green exercise cards, building up the tables one by one on the hundred board.

In order that the tables may be connected with the work already done on the boards it is suggested that at first the counters should be laid out in rows on the numbered front of the hundred board. In working the table of threes, for example, the child would lay out three red counters, three blue counters, three red counters, covering the figures to nine, and so on.

THE GREEN CARDS—Size $4\frac{1}{2}'' \times 3''$.

CARD 1.

$1 \times 2 =$
$2 \times 2 =$
$3 \times 2 =$
$4 \times 2 =$
$5 \times 2 =$
$6 \times 2 =$
$7 \times 2 =$
$8 \times 2 =$
$9 \times 2 =$
$10 \times 2 =$

Illustration 15.

CARD 10.

$3 \times 2 =$
$2 \times 3 =$
$6 \times 5 =$
$5 \times 6 =$
$7 \times 10 =$
$10 \times 7 =$
$8 \times 3 =$
$3 \times 8 =$
$9 \times 4 =$
$4 \times 9 =$

Illustration 15*a*.

It is an aim of this scheme that the connections between the four rules should be as obvious as possible, and the use of similar material and similar arrangements enables the children all unconsciously to lay hold of the essential similarities and the essential differences between the four rules.

It will be found helpful to let the children work the exercises on each card over again on the back of their boards, using another arrangement, which will help to bring out the connection between such facts as $3 \times 4 = 12$ and $4 \times 3 = 12$, that is, they will lay out the numbers in rows, each consisting of three or four or more counters, according to the table being built up.

It is obvious that such an arrangement may represent 4×3 or 3×4, according to the way we look at it.

The exercises on the green cards follow the same order as those on the yellow cards. Thus they prepare for the learning of the tables in this order :—2, 5, 10, 3, 4, 6, 7, 8, 9.

The three last cards of the series are arranged to enable the children to see connections between the tables.

THE PINK CARDS—Size $4\frac{1}{2}'' \times 3''$.

CARD 1.

$10 \div 2 =$
$8 \div 2 =$
$4 \div 2 =$
$18 \div 2 =$
$2 \div 2 =$
$12 \div 2 =$
$6 \div 2 =$
$20 \div 2 =$
$16 \div 2 =$
$14 \div 2 =$

Illustration 16.

CARD 12.

$2^2 =$
$3^2 =$
$4^2 =$
$5^2 =$
$6^2 =$
$7^2 =$
$8^2 =$
$9^2 =$
$10^2 =$

Illustration 16*a*.

The Pink Cards.

These Cards introduce the division sign, and in presenting the first card the teacher would naturally draw the children's attention to the new "mark" and give them some practice in drawing it. The meaning of the new sign would then be made clear by connecting it with the arrangement of counters on the desk.

We can, of course, approach any example in division in two ways. $12 \div 3$ may be interpreted as "twelve divided into **three,**" or as "twelve divided into **threes.**" The latter interpretation seems best to suit our little learners, and it is well that we should puzzle neither ourselves nor our pupils by over emphasis on the differences between quotition and partition. Children who build up knowledge from experience learn themselves to see connections between the one process and the other.

Examples may be written on the blackboard and worked out by the children with material in order that the meaning of the division sign may become clear. The children will learn for example that in working sums, such as "$12 \div 3 =$", the first figure tells them how many counters to take, and the second figure tells them to arrange the counters in groups of threes.

The children in grouping their counters often like to arrange the groups in patterns, and of course such pattern-making can afterwards be connected up with design, and applied perhaps to some "project" interest.

Several examples may be given until the idea of dividing into similar groups has become clear, and then the pink cards may be worked through one by one.

It is suggested that, in order to connect division with the other rules, these exercises should first be worked out on the numbered front of the hundred board. Thus, for example, in working the exercise $10 \div 2 =$, ten counters may be taken and arranged in alternate red and blue "twos," until ten is reached, and then the number of groups may be counted. Subsequently the same exercises may be carried out on the back of the board, and here the groups may be laid out in rows, each containing two counters, and the number of rows may be counted.

Number and Squares.

On the last of the pink cards are printed exercises in forming number squares. These exercises are intended to be carried out on the unnumbered circles on the back of the hundred board. In showing the child how to carry them out the teacher connects for example, $4^2 =$ with the fact that to find out the answer we must make a square which will measure four across and four down, *i.e.*, it must measure four in **two** directions. Children readily respond to this idea and, when shown how to make one or two squares, will make the others unaided. This early experience in making number squares will not only prove an interesting form of table revision, but will help to illuminate work in higher classes.

THE LIGHT BLUE CARDS—Size $4\frac{1}{2}'' \times 3''$.

CARD 5.

7	+	9	=
37	+	9	=
67	+	9	=
8	+	4	=
28	+	4	=
58	+	4	=
7	+	7	=
17	+	7	=
87	+	7	=
6	+	3	=
46	+	3	=
76	+	3	=

Illustration 17.

CARD 7.

7	6	3
3	2	9
4	5	7
2	4	6
8	6	4
2	1	2
7	9	5
5	3	3

Illustration 17*a*.

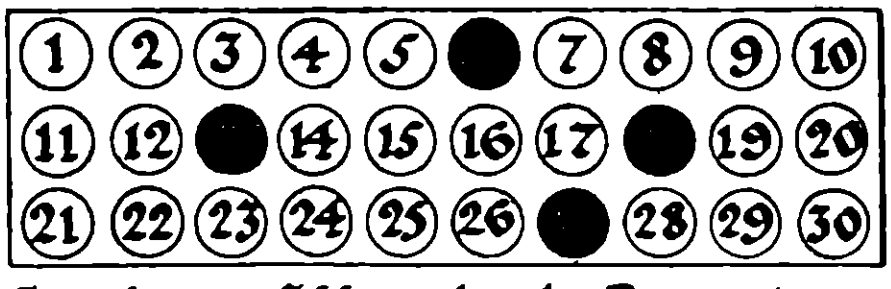
Light Blue Cards.

These cards are arranged in order to give the children practice in associating certain number combinations, such as, 15 + 4 = 19, 35 + 4 = 39, 65 + 4 = 69, etc., and also in adding numbers quickly and in unequal intervals from one to one hundred.

To cover large numbers with counters would be cumbersome, and at this stage unnecessary. The plan is therefore followed of putting a counter at the number first indicated and counting off from there the next number indicated, and placing another counter to mark the combined result.

In working, for example, the following sum (see diagram), the children put **one** counter at 6, the next, when they have counted off 7 more places, at 13, the next counter at 18, and the last at 27.

The children who reach this stage may be encouraged not only to try to work the sums without the material after the material has already been used, but to learn to work them more and more quickly. In order to secure speed in number work it does not appear to be necessary to provide numerous varied examples. Ability to add quickly may be founded upon a few selected examples worked again and again until a measure of speed has been acquired.



Section of Hundred Board showing method of working out exercises.

Illustration 18.

Number Record Cards. (See illustration 19.)

At the beginning of the second school year many children will be finishing exercises with the Twenty Board, and a few may still be using the Ten Board. Teachers cannot too often pause to consider the damage which may be done to a young mind by undue haste. Individual Methods do not necessarily obviate this danger, for children may be hustled through the material at a forced speed, in order that the class may have a semblance of uniformity in achievement, and this speed, unnatural to some children, may prevent them from thoroughly assimilating arithmetical truths. Teachers should be ready to note " danger signals," such as, the giving of quite ridiculous answers, and lack of interest.

Children should be given work **within** their power of achievement, and achievement should be appreciated even though it be the completion of an exercise far behind the average achievement of the class. Care must be taken by the teacher that laziness be not confused with want of ability, for too often in the past have dunces been created in number work through such confusion. Perhaps the greatest incentive to renewed effort is the glow which comes from success in an earlier effort. Failure to succeed has just the opposite effect, and a succession of failures may have serious consequences. The Record Card drawn out for the " Second Year " class is shown in illustration 19, a card which many may complete, and which with others will remain incomplete at the end of the Second School Year.

The second row of figures under " Yellow Cards," " Green Cards," " Pink Cards," may be marked off when each exercise has been completed on the back of the board after having been done on the front of the board.

The second row of figures under " Light Blue Cards " will be marked off when the exercises are done without the help of material.

SECOND YEAR NUMBER RECORD CARD.

Name—.....

NUMBERED STRIPS.

YELLOW CARDS.

1 2 3 4 5 6 7 8 9 10 11 12

1 2 3 4 5 6 7 8 9 10 11 12

GREEN CARDS.

1 2 3 4 5 6 7 8 9 10 11 12

1 2 3 4 5 6 7 8 9 10 11 12

PINK CARDS.

1 2 3 4 5 6 7 8 9 10 11 12

1 2 3 4 5 6 7 8 9 10 11 12

LIGHT BLUE CARDS.

1 2 3 4 5 6 7 8 9 10 11 12

1 2 3 4 5 6 7 8 9 10 11 12

Illustration 19.

Supplementary Material.

Among the supplementary material which children are free to use when they have finished a definite exercise in the scheme are the following :—

(1) A box divided into 6 compartments and containing coloured sticks of 1 in., 2 in., 3 in., 4 in., 5 in. in length, such as are supplied by many publishers. To the bottom of each compartment is pasted a paper with a name, *e.g.*, 2 inches, and the occupation consists in arranging the sticks (about 6 of each length) into their proper compartments.

Inside the lid is pasted a paper with the following instructions :—

Make a line **3** inches long.
Make a line **5** inches long.
Make a line **2** inches long, etc.

(2) A box divided into 5 compartments and containing coloured tablets, preferably of wood, such as are supplied by many educational firms. The occupation consists in fitting the tablets on to the correct pattern. The patterns are drawn on paper which is gummed to the bottom of each compartment and are as follow :—

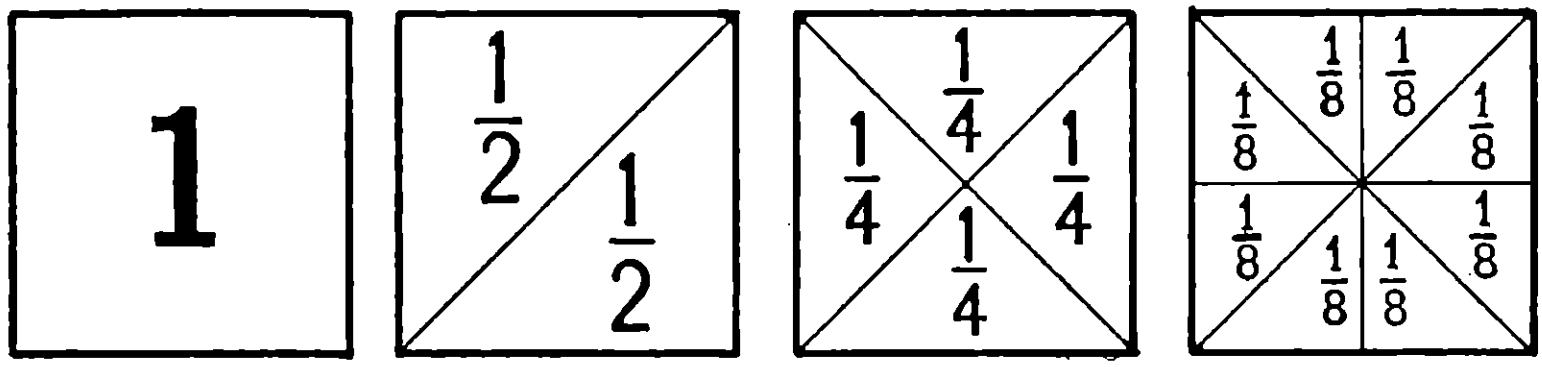


Illustration 20.

Inside the lid is pasted a paper with the following instructions :—

Make **1** square in as many ways as you can.
Draw them.
Make $\frac{1}{2}$ square in as many ways as you can.
Draw them.

Make $\frac{1}{4}$ square in as many ways as you can.
Draw them.
Make some pretty patterns with the tablets.

(It is obvious that the same exercise can be carried out with pieces which divide the square into two oblongs and then into smaller squares, but such pieces are not so easily obtained ready-made.)

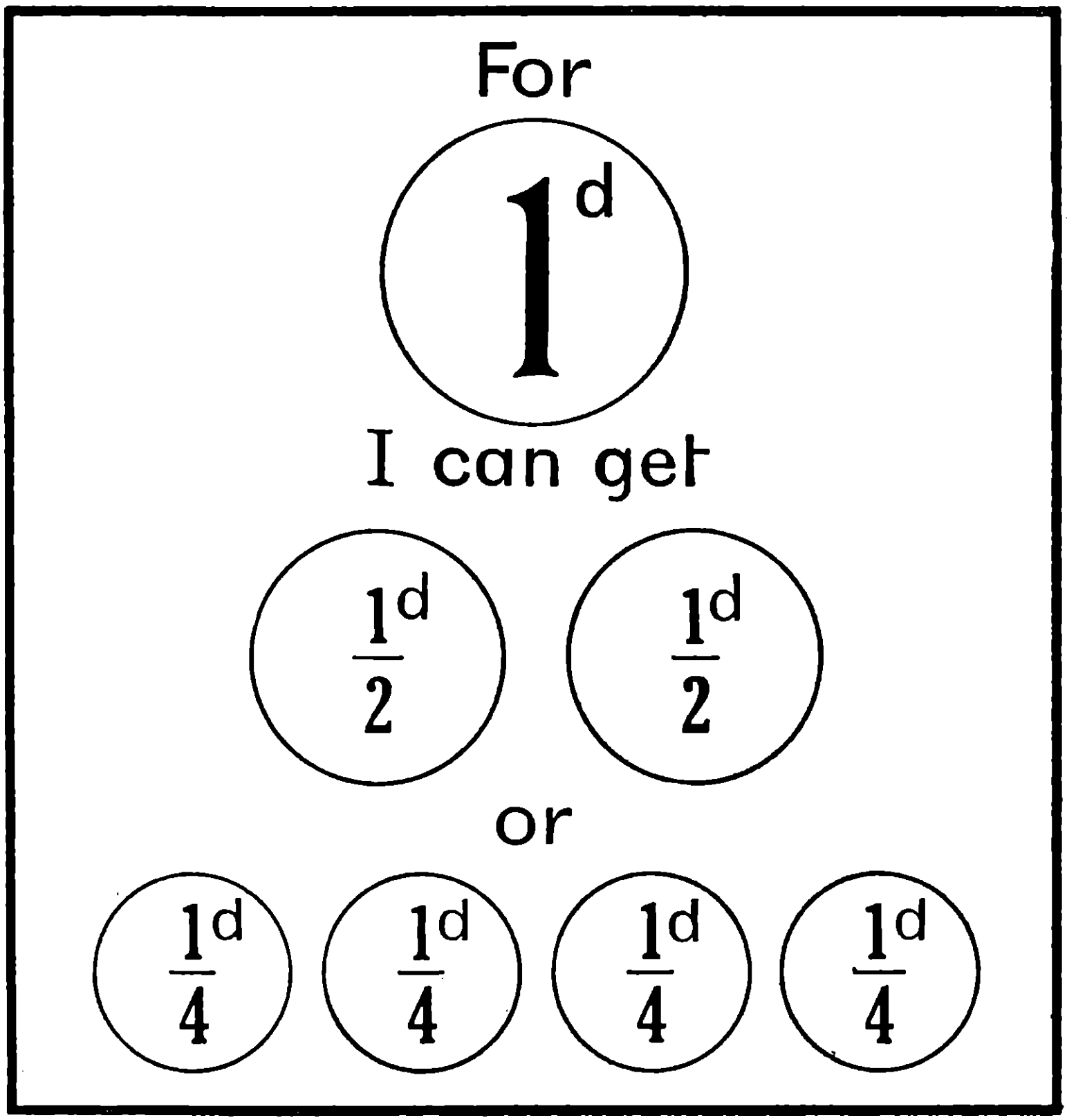


Illustration 21.

Illustration 22.

(3) A box divided into four compartments and containing the following coins :—one penny, two half-pence, four farthings. To the bottom of three of the compartments is fixed a paper on which is drawn a circle to fit one of the coins. In the centre of the circle is the symbol which represents the coin.

The occupation consists in sorting out the coins by placing each one neatly on the circle which fits it. Unsorted coins lie in the fourth compartment provided for them.

When this exercise has been completed the child copies on paper the printed matter pasted inside the lid of the box, drawing round the different coins to get circles of approximately the correct size. (See illustration 21.)

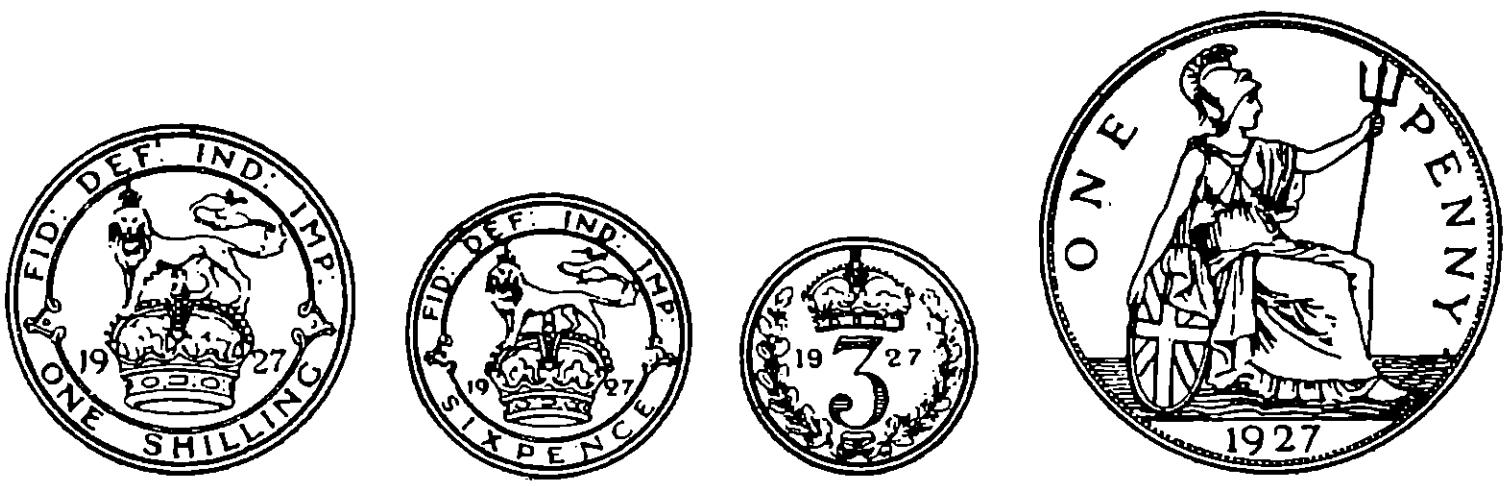


Illustration 23.

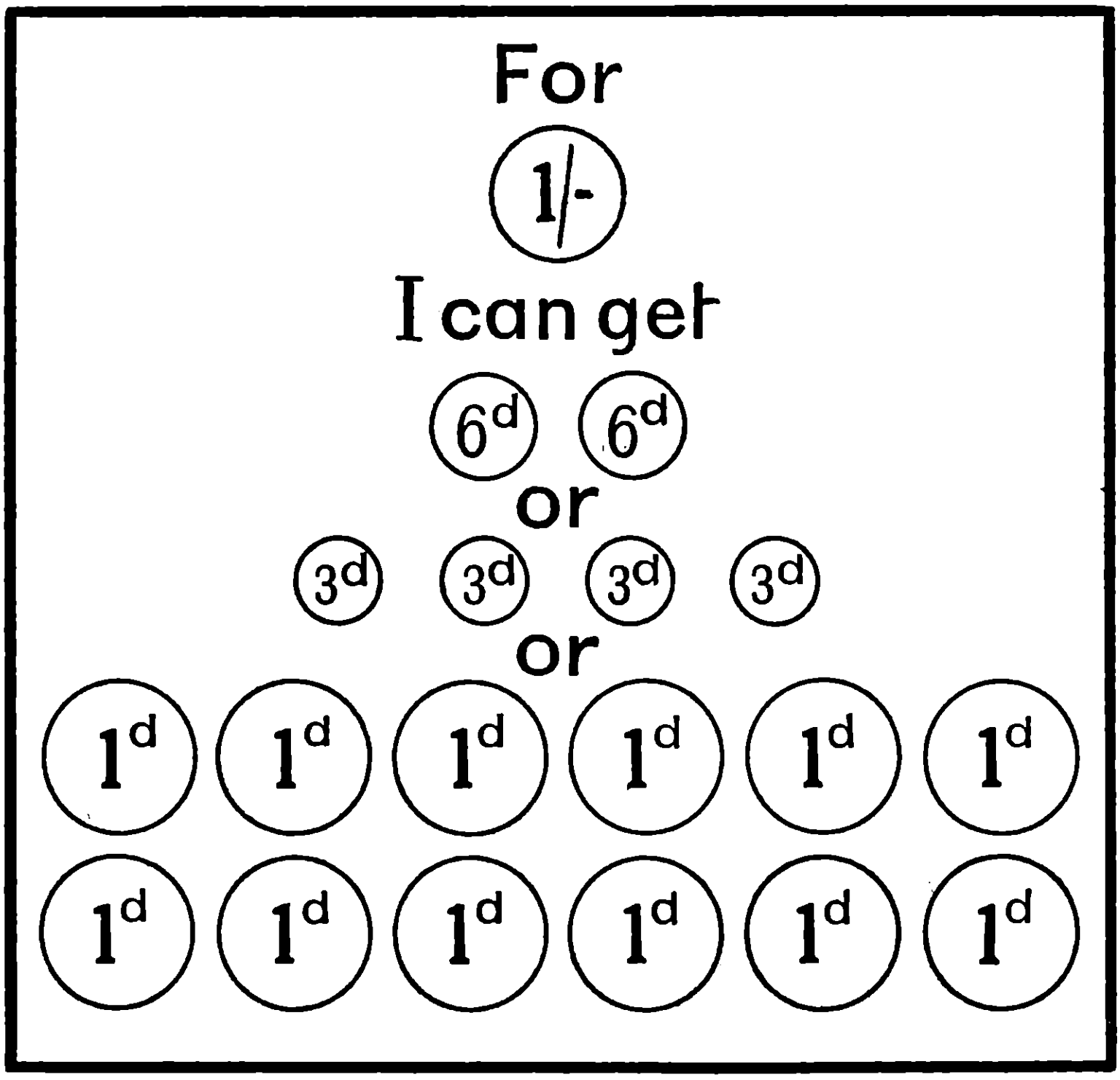


Illustration 24.

(4) A box divided into five compartments is provided for the following coins :—one shilling, two sixpences, four threepennies, and twelve pennies. This box is similarly arranged and is used in a similar manner. Inside the lid is printed the matter shown in illustration.

CHAPTER 4.

NUMBER IN THE THIRD SCHOOL YEAR.

Number Connected with Every-day Affairs, Projects, etc.

DURING the third school year varied experiences of the earlier years become more precisely defined. The children gradually become accustomed to writing the correct date each day in their note-books. At Christmas time when old calendars are nearly finished, or at New Year time when new calendars have been bought, children may learn that the years are counted from about the time when the little Lord Jesus was born, "nineteen hundred and.....years ago." The letters A.D. and B.C. may thus come to be of interest for them. Children like to guess, too, how many days are represented inside the unopened calendar for the new year.

(We always buy a calendar from which a slip for each day can be pulled off. It will be found that in this way children gradually come to know how many days are in each month.)

The children are likely to be interested in learning rhymes, such as :—

Thirty days hath September,
April June and November ;
All the rest have thirty-one,
Excepting February alone,
Which has twenty-eight days clear,
And twenty-nine in each leap year.

Two little poems by Christina Rosetti are also useful, the one beginning :—

"What will you give me for my pound ?"

and the other :—

"How many seconds in a minute ?"

Project work during this year may embrace study of the home environment from many points of view. Measures of length may be used in connection with picture plans of class-room, school, and its immediate environment. The children may come to understand something about "drawing to scale," and will learn to interpret the distance from one place to another on a picture plan of their district. It is important that such studies should be combined with out-door expeditions. The children's first idea of "mile" should spring from some familiar out-door experience of walking from one place to another. These places may later be located on a map of the district.

It will be noted that during Individual Work periods children concentrate on exercises demanding mechanical skill. Such exercises best lend themselves, during the early years, to methods of Individual Work. Sums in problem form may present difficulties to young children of reading, and of setting down, and may send them back in dependence to their teacher. Teachers may, however, supplement the Individual Work exercises by many a fresh, brightly illustrated problem set out on the black-board, *e.g.*,

"Here is Mary's mother's big basket of clothes with twelve towels in it. (Drawing.)
Here is the bag that holds the clothes-pins. (Drawing.)
Mother asks Mary to hang up the towels on this line. (Drawing.)
She says, 'Put **two** pins in each towel.'
How many pins will Mary need ?"

"Peggy went into a lovely green field. She found **nine** golden buttercups. (Draw.)
Her little brother and sister met her when she came home.
She shared her **nine** buttercups among them.
How many would they each get?" (Draw three equal bunches.)

"Here is Bobby's very own bookcase. See it has **three** shelves. (Draw.)
Bobby puts all his books into his book-case—**ten** books on each shelf.
How many books has Bobby got?"

"Tommy had to walk a mile to school every day.
Billy had to take the bus because he lived **twice** as far away.
How far away did Billy live?"

"Mother had a dozen eggs all packed neatly in this box. (Draw.) She boiled **five** eggs for breakfast.
How many were left in the box?"—and so on.

INDIVIDUAL WORK.

The Ten Hundred Squares. (See Illustration 25.)

These squares are planned so that they carry a step further the knowledge which children have gained with the "Hundred Board." Ten squares are now provided, which are numbered

"HUNDRED SQUARES"– HOLYROOD NUMBER SCHEME By Miss A. F. MACKENZIE, L.L.A. NO. 3

201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
291	292	293	294	295	296	297	298	299	300

Size $6\frac{1}{2}'' \times 6''$. Illustration 25.

Square 1, Square 2, etc. On Square 1 are printed the figures from 1 to 100 and on Square 10 are printed the figures from 901 to 1,000. (Illustration 25.)

The Lilac "Notation" Cards.

These cards provide the children with exercises which they carry out on the Ten Hundred Squares, exercises which are planned to give them much practice in writing figures to 1,000, and insight into the way in which with only ten different figures we can represent ever increasing quantities.

Notation Tests.

CARD 1.

Take the Ten Hundred Squares. Lay the squares in order with Square 1 on top. Write every 100th figure.

Illustration 26.

Notation Tests.

CARD 6.

Take Square 3.

Write every 3rd figure.

Illustration 26*a*.

Place Value.

In this number scheme the idea of the place value of figures has been allowed to mature in the mind of the child while counters have been laid out in rows of ten, while the hundred board, consisting of ten rows of ten, has been filled and marked off with counters, and last of all, while ten "hundred squares" are piled one above the other and it is found that the total number of spaces represented is one thousand.

Little children like to count on and on, and being practical people, they like to be able to write down correctly figures representing the numbers they have counted. From counting and the writing of figures in consecutive order they pass to counting and the writing of figures in varied intervals. Gradually they build up very definite ideas of the value of ten, of a hundred, and of a thousand, and this forms a valuable foundation for further exercises in notation which involve the exchange of tens into units, units into tens, tens into hundreds, hundreds into tens, etc.

It is suggested that children should have behind them the number experiences gained through the exercises already described before an attempt is made to make plain to them the principle of relative values underlying notation.

It will be found that when the idea is given to the child **after** he has had adequate number experience he will realise it with greater clearness, and apply it with greater accuracy than if an attempt had been made to explain it, even by graphic methods, at an earlier stage.

We have found two simple pieces of apparatus valuable in making clear to the children the relative values of figures in units, tens, hundreds and thousands columns. These pieces of apparatus we call the Bank and the Place Value Board.

Notation Tests.

CARD 12.

Take the Bank and Place Value Board.

Lay out the numbers below on the board.

After you make each number write the figures in a column under the letters :—

T H H T U

1,436 ; 345 ; 26 ; 9 ;
17 ; 208 ; 1,012 ; 8 ;
619 ; 3,004 ; 32 ;
2,103 ; 400 ; 12.

Illustration 27.

The children who use these pieces of apparatus already understand the four rules ; their purpose is therefore served when they enable the children to extend their knowledge of these rules to the working of sums which involve "exchange." The same apparatus is used for working sums in each of the four rules ; thus the child is enabled to concentrate on processes rather than on apparatus, and to see what there is of similarity and of difference between these processes. Exercises may first be worked with apparatus, and then worked over again without its aid. When errors are made, however, the apparatus will always be available so that the child can work an inaccurate sum

over again with its help. This is a very valuable method of correction.

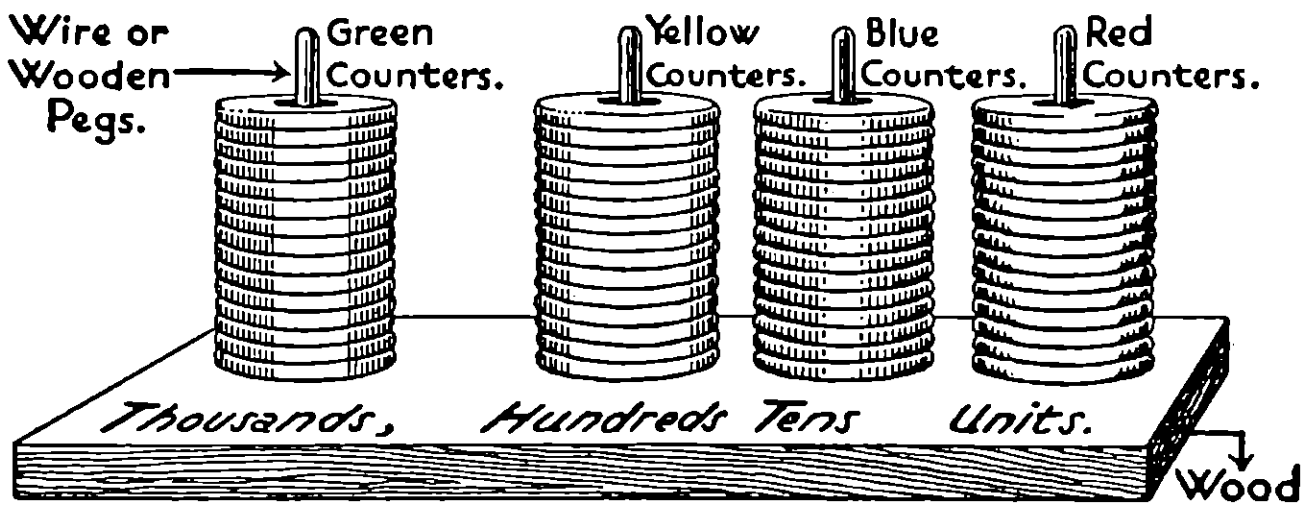


Illustration 28.

THE BANK.

The Bank.

Ring counters of four different colours are provided and "a bank," such as is shown in Fig. 1, may very easily be made from a flat piece of wood and wires cut in four-inch lengths. We find that the name "Bank" is helpful as it suggests the idea of exchange. In presenting it reference may be made to the familiar experience of exchanging shillings for pence, pence for halfpennies, etc.

The teacher might continue :—"We are now going to learn to work sums with big numbers. We cannot manage to use counters for all the big numbers we need. This "Bank" will help us to remember when we are working with figures that stand for tens, hundreds, or thousands of counters. In our bank each of the blue counters is supposed to be worth ten red counters, a yellow counter is worth ten blue ones, and a green counter is worth ten yellow ones."

We then play various games of exchange with our Bank. We say, for example, "What will you give me from your bank for this yellow counter?" "What will you give me for these two blue counters?" "What will you give me in exchange for these ten red counters?" and so on.

The value of the differently coloured counters may be related to the work which the children have previously done with the Hundred Board. For example, the teacher might proceed thus: "This blue counter is worth a whole row on your board." "This yellow counter is worth ten rows full of counters—your Hundred Board covered with counters." "This green counter is worth as many counters as would cover ten hundred boards." "This red counter is just worth one single counter."

The Place Value Board.

This board, used in conjunction with the Bank, gives the child a clear idea of the principle of exchange when the ten circles in any one row are full. It also clarifies the child's ideas of notation. The "Board" is of special value in teaching subtraction and division.

When the Place Value Board is presented to the child his attention is drawn to the ten circles for counters under each number name. We may remark: "There are ten circles for red 'unit' counters. When we have filled the ten places under 'units' with red counters we put the ten counters back in the 'Bank' and take one blue one instead."

Simple exercises in exchange may be given, such as "Let us add nine and three on our Board. We take nine red counters from our Bank and lay them in the units place. We add three

more unit counters. We have a full row and two more. What can we get for the full row of ten ? We put the ten red counters back in the Bank and we get one blue counter instead, which goes in the tens place. Now we have one blue counter in the tens place and two red counters in the units place. How many unit counters would that make altogether ? " etc. Similar simple exercises may be given in adding tens and hundreds.

The idea of repaying to the Bank the exact value of the counter taken from it is an important one for the child to assimilate. Once it has been grasped little difficulty should be found in transferring the idea to sums in the four rules which involve "exchange."

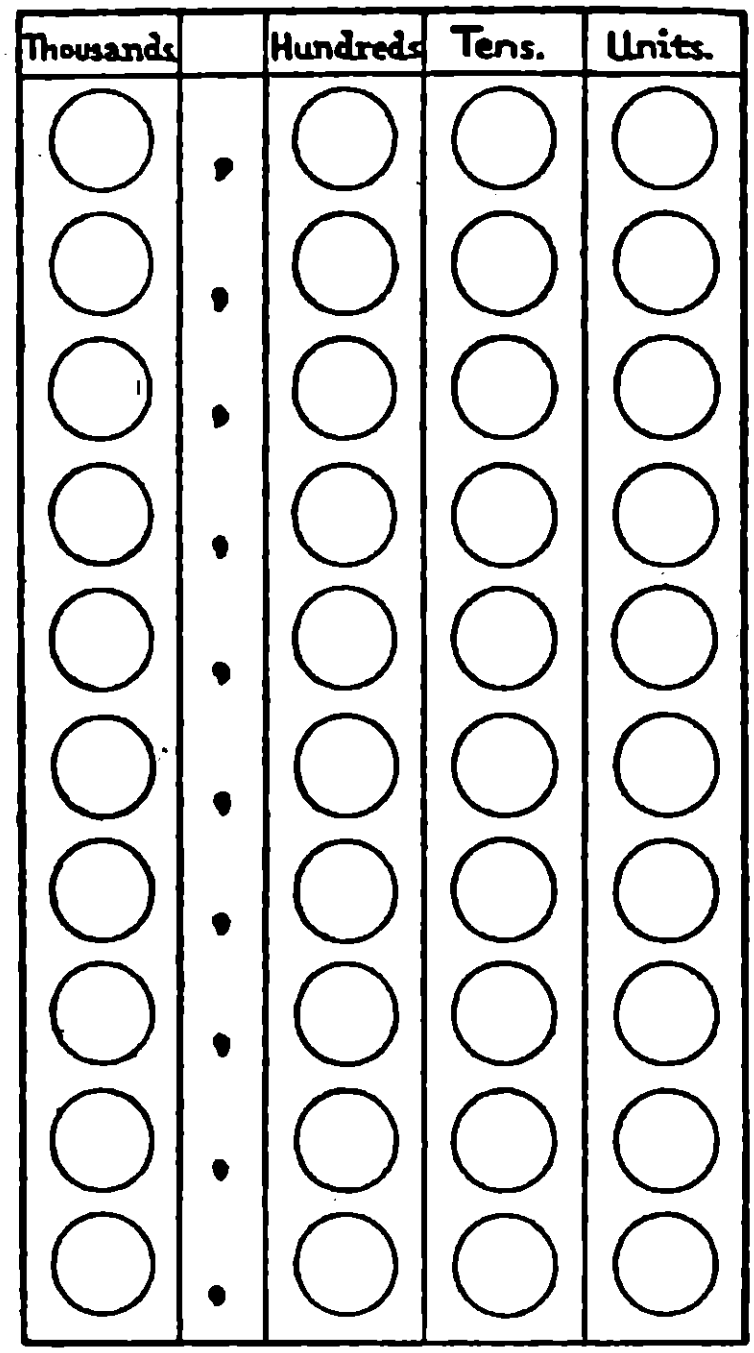


Illustration 29.

PLACE VALUE BOARD.
Size $11\frac{1}{2}'' \times 5\frac{3}{4}''$

Exercises in Notation.

In order to give clearness to ideas of " place value " which the children have gradually been acquiring, they carry out the exercises on Card 17 of the Lilac Notation Series. The coloured counters on the board, and the empty spaces which occur in certain numbers, make graphic pictures for children of the value of each number. (Illustration Card 12.)

SECOND SERIES OF CARDS.

Addition and Subtraction to Thousands.

On these cards are printed exercises in addition and subtraction up to thousands. On the first six cards are printed exercises in addition, and on the last six, exercises in subtraction.

When the child first uses these cards his attention is drawn to the four vertical rows of figures of which the sums consist. It will be found that through connecting these rows with the rows of circles on the Place Value Board, and the counters coloured to signify thousands, hundreds, tens and units in the Bank, the children will readily appreciate the need for keeping these figures in straight vertical rows.

Before working the sums the children may with profit learn to write the name of the process, addition or subtraction, and the number of the card at the top of each page. This will enable them to connect the correct name with each arithmetical process, and to be quite clear about the stage of work which they have reached.

Addition.

The sums on the cards are graded in difficulty so that the child gradually deals with sums which involve more and more exchange.

In teaching the child to work sums in addition he will be shown how to lay out the numbers in the right-hand column with red " unit " counters in the " unit's " circles on the Place Value Board.

Thus, after copying the first sum on Card 1 neatly in his book, he learns to lay out on the " unit " circles on the board eight red counters, then two red counters. His circles will then be full, so he replaces in his Bank the ten red counters and takes out in their stead one blue counter to be added in when he comes to deal with the tens column on the Place Value Board. He then finishes the addition of the units column by laying out five and four red counters, and writes 9 in its proper place in his book. Hé then proceeds to deal with his row of tens in a

similar manner, adding in the **one** extra blue counter taken in exchange for the **ten** red unit counters. In adding this column of his sum he will require to make no exchange, nor will exchange be necessary in adding the hundred, or thousand column. Gradually, however, as he passes from card to card, more exchange will be necessary in working the sums.

It is suggested that one order in adding, such as proceeding from the bottom figure of a column upwards, should be adhered to during the early stages. This tends to make for accuracy, as the child has less opportunity of becoming confused about the numbers with which he has already dealt, and also enables him to deal with varied number combinations.

Subtraction.

The sums on the last six cards will be graded so that the necessity for "exchange" comes less often in the earlier exercises, and is more frequent in the later exercises. In presenting

Second Series.

CARD 1.

ADDITION.

3,324	1,423
125	64
2,312	2,371
208	30
1,320	5,403
2,432	9
3,645	235
201	2,954

Illustration 30.

Second Series.

CARD 7.

SUBTRACTION.

7,249	9,327
3,624	5,194
8,694	6,910
3,257	4,576

Illustration 30*a*.

the child with Card 7, the first of the Subtraction Cards of this series, the name of the process is clearly pronounced and connected with the idea of " taking away." Let the children discover that in each sum the large number is written **above** ; the smaller one below. As they have had previous experience with Bank and Board, it will not be difficult for them to appreciate the fact that it is the left-hand figures which are important in deciding the size of the number. From this the teacher goes on to show by working examples with the children on the Place Value Board that the top figures tell us the numbers we **have,** and the bottom figures the numbers which we must lift away.

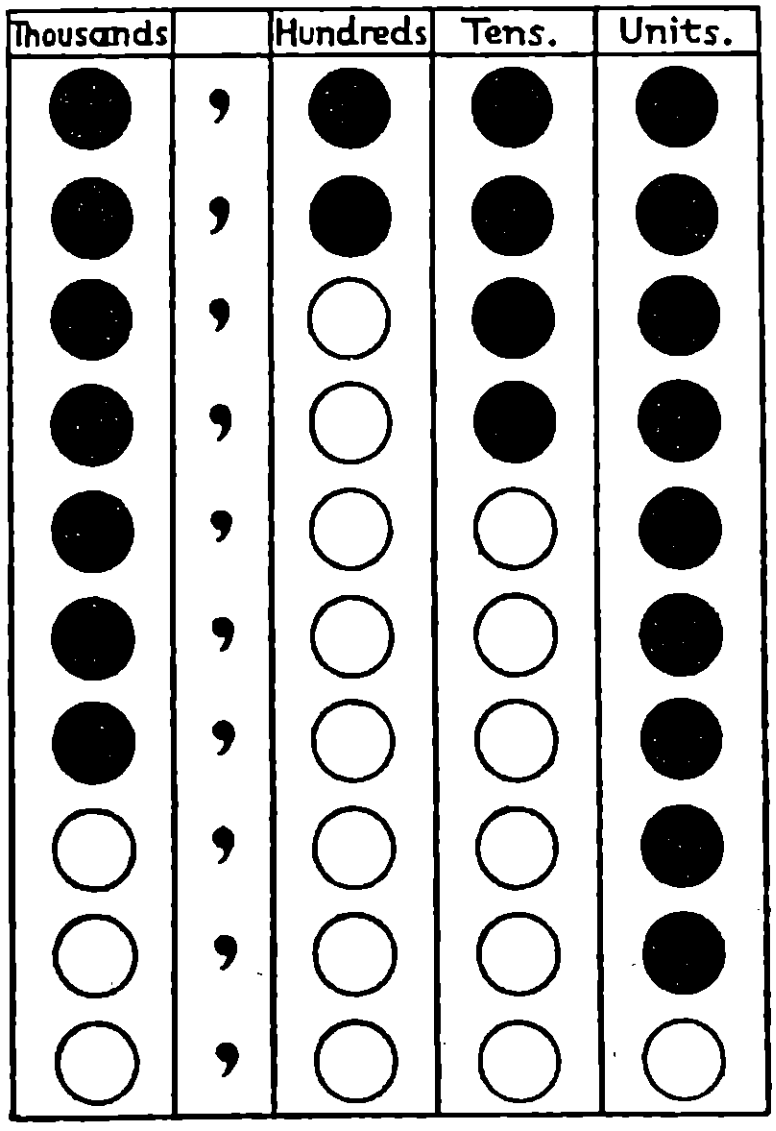


Illustration 31.

Place Value Board Prepared for Example 1—Card 7.

The board is prepared by teacher or child for working the first example. The top number is laid out on the board as shown in diagram, and the child proceeds to lift away from this given number the numbers indicated by the bottom row of figures. The whole principle of exchange in connection with subtraction becomes very much clearer to young children when they are thus helped to realise the top line as indicating the complete number in hand and the second line as merely indicating the number to be lifted away, which therefore has no separate existence apart from the top number.

In working Example 1, after the board has been prepared as shown in diagram, the child would proceed as follows :—

From the nine red counters in his units column he lifts away four, and writes down five in his units place in his exercise book

as indicating the number of red " unit " counters left. Similarly he lifts away two of his four blue " ten " counters, and writes down the result.

He now finds that he must lift away six " hundred " counters. He has only two. But he has seven " thousand " counters. For one of these " green " counters he can get ten " yellow " ones from his bank.

He moves one " green " counter down to the bottom of the board to show that it belongs to the bank, and takes in its stead ten " yellow " counters. He now has twelve yellow counters, lifts away six, and writes down the result.

Now from his seven " green " counters he lifts away four—the one at the bottom which goes back to the bank to pay for the ten " yellow " ones, and three more. He writes down the answer, **three.**

It will be found that children will appreciate the squareness of this " deal " with the bank as a result of previous practice in exchange. From time to time the teacher may, during the early stages, have to remind the child to pay back the extra counter on the left for the ten taken from the bank and added to the number on the right, but gradually the habit will become automatic.

After he has worked through several examples the habit of adding ten to the smaller number at the top and repaying an extra one to the bank from the number on the left should be mechanised. It is essential that no complicated wording should be acquired. The child will readily shed the use of apparatus when it has served its purpose, but cumbrous language clings to the child and hampers thought processes. Children may with advantage be trained to proceed as follows in working such examples as the above. " Four from nine—five. Two from four—two. Six from twelve—six. Four from seven—three."

Methods of Subtraction.

It will be obvious that, in working the above sum, a slightly different method might have been followed. When the ten yellow counters were taken from the bank and added to the two on the board, one green counter might immediately have been lifted from the seven on the board and returned to the bank to pay for them. In this case, instead of taking four from seven in the thousands column, the child would take three from six. In using bank and board either method is intelligible to the child. In the one case he pays for the ten he lifts **at once** with the counter of higher value. In the other the payment is deferred until the next row of counters is being dealt with. This latter method appears to lead to better results both in speed and in accuracy, and therefore it has been followed. When gradually " nothings " are introduced into the top line of the subtraction sums, the child who has learnt automatically to add ten to the top line and takes away an extra one from the figure on its left fares better than he who has to learn to change " nothings " into " nines."

It does not appear to the writer to be necessary to labour the teaching of subtraction sums in which there are " nothings " in the top line with children who, through exchanging counters which symbolise different values, have grasped the principle of exchange. Take, for example, this sum :—

```
5,003
3,769
------
------
```

One of the five " thousand " counters may be moved down to the bottom of its column which allows us to take ten " hundred " counters. One of these moved down to the bottom of its column allows us to take ten " ten " counters. One of these moved to the bottom of its column allows us to take ten " unit " counters.

Debts are repaid to the bank as the sum is worked from right to left. The "three" becomes thirteen, therefore "seven" is taken from "ten" (one counter for the bank). The "nothing" became "ten," therefore "eight" is taken from "ten" (one counter for the bank). Again the "nothing" became "ten," therefore "four" is taken from "five" (one counter for the bank).

The children learn best through "doing," and wordy explanation is to be avoided. The apparatus helps them to see in a practical way what the process means. Thereafter they form the habit of adding "ten" to a smaller number in the top line, and of taking away an extra "one" from the figure on its left. They make that habit automatic, and devoid of superfluous wording.

Multiplication and Division with Bank and Board.

Children who have followed the method outlined in this scheme will have had considerable practice in building up tables, and will thus have been helped to lay a sure foundation for intelligent memorisation. Memorisation will have been encouraged, and will have been facilitated through practice in use of these tables in working the exercises in multiplication and in division already described. These exercises involved no "exchange" or "carrying figures."

Three Sets of Cards.

In order to give the children adequate exercise in use of the multiplication tables, and in applying them to numbers which involve exchange, three sets of cards have been prepared, each set consisting of twelve cards.

Multiplication Cards—Set 1.

On these cards are printed graded exercises in multiplication and in division which involve the use of the tables 2 to 4. Cards 1 and 8 of this series are shown in diagram. Cards 1 and 2 involve multiplication by 2; Cards 3 and 4 multiplication by 3; Cards 5 and 6 multiplication by 4; Cards 7 and 8 division by 2, and so on, the last six cards containing exercises in division as the first six contained exercises in multiplication.

Second Series.

CARD 2.

MULTIPLICATION.

4,231	1,345
2	2
2,364	3,076
2	2

Illustration 32.

Second Series.

CARD 8.

DIVISION.

2) 7,891	2) 9,465
2) 2,670	2) 1,948

Illustration 32*a*.

Multiplication Cards—Set 2.

These cards contain similar graded exercises in the use of the tables 5, 6, 7. At this stage, however, there would appear to be an advantage in using each table first for multiplication and then for division so that the child is helped by a continuous variety of exercises to become thoroughly familiar with the table. Thus Cards 1 and 2 contain exercises in multiplication by 5; Cards 3 and 4 exercises in division by 5, and so on.

In the previous set of cards there would appear, on the other hand, to be an advantage in working continuously six cards of multiplication exercises, followed by six cards of division exercises. At this stage the processes as applied to thousands and involving exchange are new, and are likely to become better known through continuous practice in each process.

Multiplication Cards—Set 3.

On these cards are printed exercises in multiplication and in division involving the use of tables 8, 9 and 10. Here again two cards containing exercises in multiplication are followed by two cards of exercises in division in which the same table is used.

Multiplication with Bank and Board.

To a child who has learnt to exchange the ten for the one of a higher value in connection with addition sums there should be little difficulty in applying the same principle to sums in multiplication. In working, for example, exercise 2 on Card 1, of Set 2, when he takes two counters five times, he finds that he has ten "unit" counters, which he exchanges for one blue counter and writes "nothing" in his units place. The "one" thus comes to be added in when he deals with the "four" in the ten's place.

Division with Bank and Board.

To prepare for working a division sum the number to be divided is laid out on the Place Value Board. The diagram shows the board arranged for the working of exercise 1 on Card 8.

The child's attention is drawn to the new method of indicating that a number is to be divided when that number is a large one. While drawing the line)_____ he may accompany the

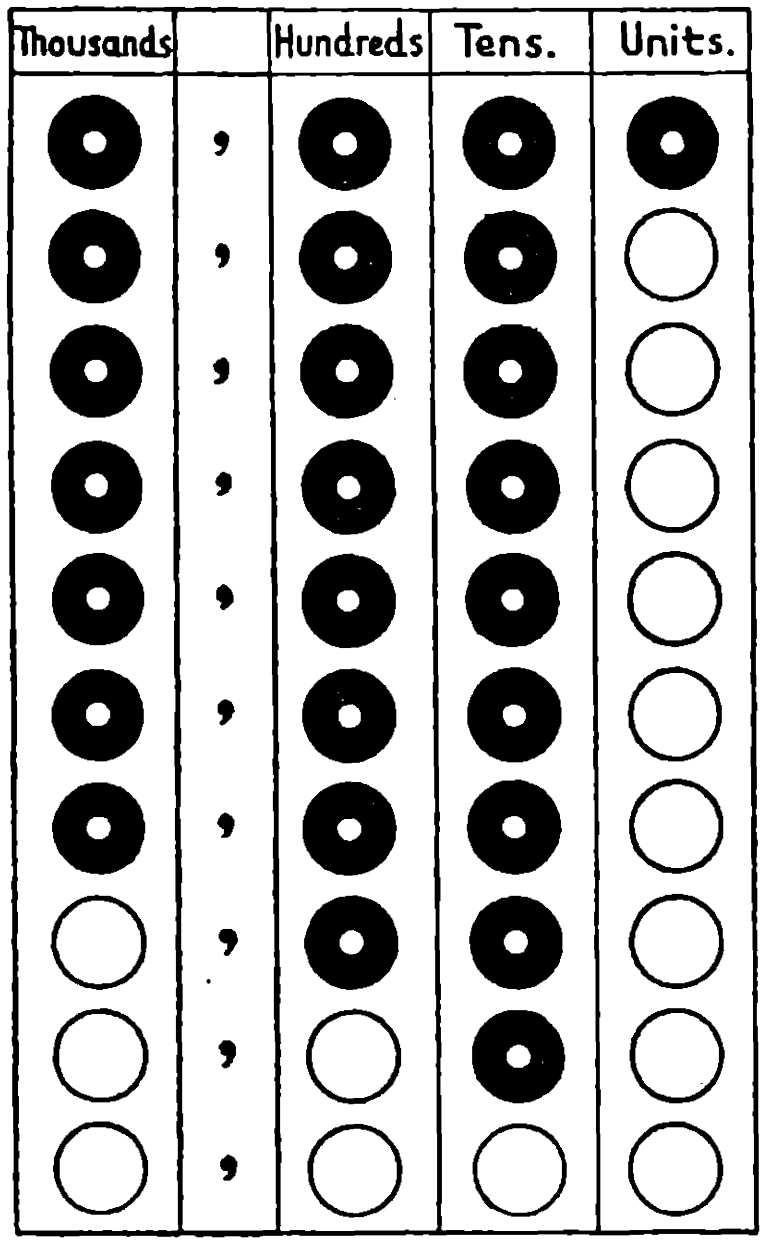


Illustration 33.

Place Value Board arranged for working Exercise 1 on Card 8 of the Light Green Cards.

drawing by saying, "divided into——" and while he writes the figure "2" may complete the phrase by adding "twos."

After arranging the board for the working of the sum, and after neatly copying this sum into his book, the child proceeds to deal with the figures one by one, making the arrangements on his board and recording each in turn in its proper place in his exercise book. The seven green "thousand" counters are arranged in little piles of "twos." He finds there are three twos in his thousand column and records "3" in his book. The one green "thousand" counter which is left over he returns to the bank and takes in its place ten yellow "hundred" counters which are added to the eight already in the "hundreds" column on his board, making a total of eighteen yellow "hundred" counters. These he arranges in little piles of twos, discovering that there are nine twos in eighteen. The result "9" is recorded in its place, and the child proceeds to deal with the nine blue counters in the tens column in a similar way, exchanging in its turn the one blue "ten" counter left over for ten red "unit" counters, making a total of eleven red counters in the units column. These he arranges into five piles of twos, and records the one left over in fraction form, $\frac{1}{2}$, or "one whole divided into twos." This would appear to be the most satisfactory method of recording remainders indicating as it does that the number left over is also expected to be divided up. Supplementary exercises described in this work give the children opportunities of handling fractional parts, and of learning how to write symbols by which they are represented.

Use of the Concrete at this Stage.

The apparatus is intended to make perfectly clear to the child the meaning of each arithmetical process, and the principle of exchange. The use of coloured counters constantly reminds the child of the varying values of the numbers with which he deals so that the principle of notation becomes full of meaning.

Previous experience in number work, and his comparative maturity, enable him to realise "colour" as being merely symbolic of quantity, a useful way of helping us to remember the value of the figures written in different places in a row.

When the apparatus has made perfectly clear to the child the "reasonableness" of his work, it has served its purpose, and at this stage should require to be used for a limited number of examples. Learning of tables should make it unnecessary to use apparatus to any great extent after the Multiplication Cards—Set 1 have been worked through. These exercises will have enabled most children to understand the meaning of the processes, and to apply the principle of exchange. The teacher, however, should be alive to the needs of individuals, who require to handle material more than do others, before a process becomes thoroughly understood.

It will be obvious that when the later tables are reached "exchange" will become a long and laborious process. Nevertheless careless mistakes may profitably be rectified through having recourse to the apparatus. It is important that children should realise the irrevocable results arrived at through arithmetical processes, and no word of mouth is so convincing as the laborious proving to one's self the rightness and wrongness of work.

We wish to train children in self-reliance, in the workmanlike, mental integrity of the man who proves for himself and is sure of his facts. This is one of the main advantages of providing young children with apparatus which may be used as an unbiased judge of their work.

Mechanising Results.

The main concern of an infant department in connection with arithmetic would appear to be the provision of adequate number experience. Children who have had adequate opportunities

of arranging suitable material gain knowledge of number relationships which makes number work intelligible to them, and eliminates the dangers which arise through dependence upon memory alone.

There comes a time, probably during the third school year, when it is well that children should have considerable practice in mechanising results which they have had ample opportunities of proving to be true in their work with material. When the processes of addition, subtraction, multiplication and division have become thoroughly intelligible the children have a background of experience which invests the memorisation of results with meaning and with interest.

The mechanising of results may be entered upon as a game of skill which children thoroughly enjoy, so long as care is taken not to prolong the practice until the children are wearied, or to compare the speed of individual children so that those who are naturally slow in response lose heart and do less well than they otherwise might.

Economy of time and labour would suggest that graded exercises, which would include all possible combinations, should be used when children are ready for speedy mechanisation.

It is to be remembered that regular practice for a few minutes daily is much more effective than lengthy practice taken spasmodically.

It is also to be remembered that the natural reactions of children differ in speed, and that over-anxiety, or over-effort to reach a standard unsuited to the individual child is likely to defeat the end we have in view.

The danger of unwise use of the spirit of competition is connected with the differences of mental ability which the children possess. A child's progress is frequently retarded owing to unwise stimulus being given to him to reach a standard unsuited to one of his mentality, but easy for his class-mate.

When a child feels that he is improving day by day he experiences pleasure in the power developing within him. This pleasure incites to new effort. When he is made too consciously aware, during the earlier stages of school life, of the superior achievements of his class-mates, his enthusiasm is damped and his progress impeded. Older children who have made more of the world's knowledge their own, and have tasted some of the joys of achievement, are more able to withstand the dangers connected with competition.

The "Andrum" Arithmetic Practice Card. (Illustration 34.)

We owe the production of this very simple and helpful device for assisting children to memorise arithmetical results to the combined work of Miss Margaret Drummond, the well-known

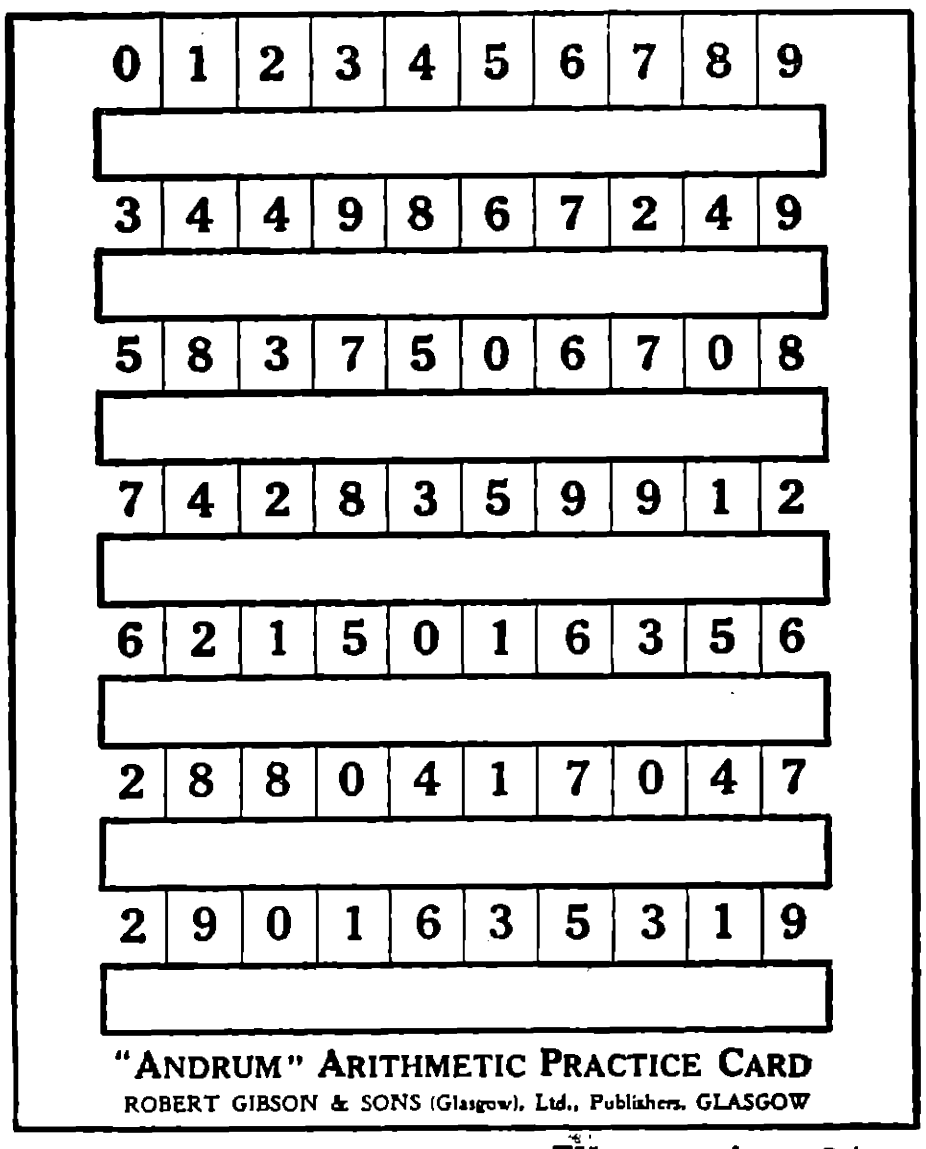


Illustration 34.

Illustration of Card—Size, $7\frac{3}{4}'' \times 6\frac{1}{4}''$.

child psychologist, and Mrs. Anderson, the late headmistress of a large L.C.C. Infant and Junior School, in which its usefulness was proved beyond question.

This Card (illustration) is planned to cover an ordinary exercise book, and can be used for memorising results in any of the four rules. Let us suppose, for example, that we wish to secure speed in addition. The " Andrum " Card is laid on a clean page of the exercise book. A key for our practice is made by adding four, let us say, to each figure in the top line, and writing the results on the exercise book page in the space between the top two bars. When this key has been provided the children proceed to add four to each figure on the card, writing the results in the spaces provided for the purpose. At first the child may have to glance frequently at the key in the top space. This, however, hinders speed and the aim is to finish in record time. If the work is being done by the class as a whole each child may stand when the card is finished and those who finish within record time will be noted as having passed the speed test in adding on fours.

Each figure appears an equal number of times on the card, and nothings are included. Experience goes to prove that children need practice in dealing with nothings.

Children who have completed an addition exercise such as the above can use the rows of figures they have made as a result of the addition of fours for the reverse process. They can subtract four from each of these figures in turn, and by comparing the new results with the rows of figures on the " Andrum " Card they can correct their own work.

The " Andrum " Card may be used, in a similar way, for mechanising the multiplication tables. When the results of multiplying each number on the card, let us say, by seven,

have been written in rows on the exercise book page, each of these results may subsequently be divided by seven, and the results verified by comparing them with the figures on the " Andrum " Card.

It is suggested that the figures under " Speed Tests " on the Individual Record Cards should be marked off when the exercises have been worked through in reasonable time without the help of " key " figures in the top space. Speed Tests would be given in most cases only after considerable individual practice with the card.

THIRD YEAR NUMBER RECORD CARD.

Name—.....

NOTATION.

1 2 3 4 5 6 7 8 9 10 11 12

ADDITION AND SUBTRACTION SPEED TEST.

2 3 4 5 6 7 8 9 10

ADDITION AND SUBTRACTION.

1 2 3 4 5 6 7 8 9 10 11 12
1 2 3 4 5 6 7 8 9 10 11 12

MULTIPLICATION AND DIVISION SPEED TEST.

2 3 4 5 6 7 8 9 10 11 12

MULTIPLICATION AND DIVISION (1).

1 2 3 4 5 6 7 8 9 10 11 12
1 2 3 4 5 6 7 8 9 10 12 11

MULTIPLICATION AND DIVISION (2)

1 2 3 4 5 6 7 8 9 10 11 12
1 2 3 4 5 6 7 8 9 10 11 12

MULTIPLICATION AND DIVISION (3).

1 2 3 4 5 6 7 8 9 10 11 12
1 2 3 4 5 6 7 8 9 10 11 12

Illustration 35.

SUPPLEMENTARY EXERCISES.

The Whole and Its Parts.

The incidental teaching gained in connection with everyday happenings, with projects, handwork and play activities needs to be supplemented by exercises which involve more precision and which are calculated to give definite, clear ideas if children are adequately to appreciate arithmetic as an exact science.

Where a quarter, for example, is measured off to meet the needs of the moment it may seldom be a real quarter.

Important as it is that children should use their number experience in everyday situations, there is also a need to supplement such use by exercises which involve a finer accuracy. Such exercises interest children, and provide them with clear ideas which they transfer to varied life experiences.

CARD 1.

$\frac{1}{2} + \frac{1}{2} =$
$\frac{1}{4} + \frac{1}{4} =$
$\frac{1}{8} + \frac{1}{8} =$
$1 - \frac{1}{2} =$
$1 - \frac{3}{4} =$
$1 - \frac{5}{8} =$

Make the circle in as many ways as you can.
Write the names of the fractions you use to make the circle.

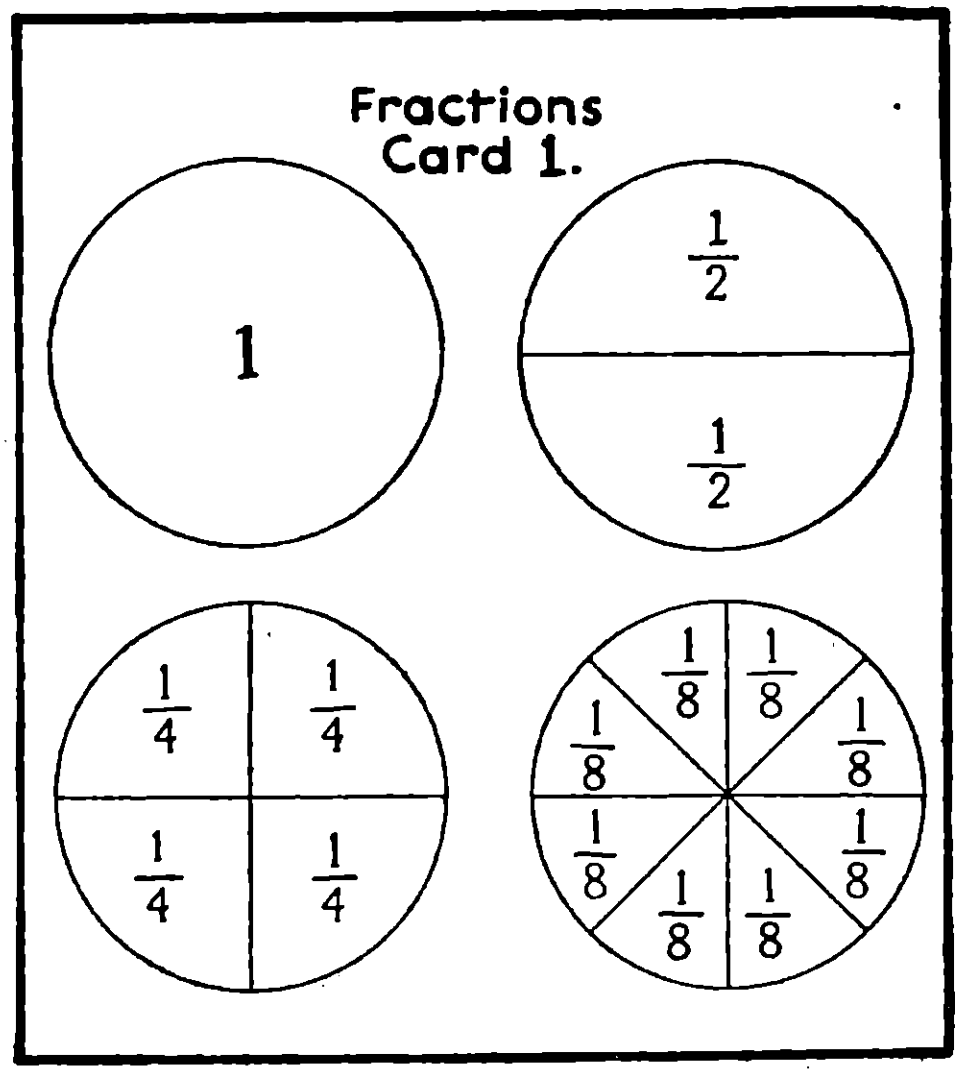


Illustration 36.

The Use of the Cards.

All the cards shown in the diagrams measure 10 ins. by 10 ins., and the circles drawn upon them have a diameter of 4 ins. With each card the necessary fractional parts of the circles should be provided in a little box or bag, of distinctive colours, *e.g.*, red, yellow, purple. These fractional pieces will be used at first for fitting neatly over the divisions marked on the circles on the card.

It will be found helpful to colour the pieces belonging, for instance, to Card 1, red on one side, and blue on the other; those belonging to Card 2, yellow on one side and green on the other; and those belonging to Card 3, purple on one side and orange on the other.

When the child has fitted the fractional parts on all the circles on a card the teacher may mix up all the pieces. Then she may say, "Hold up a half, a quarter, an eighth." Then,

CARD 2.

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$
$\frac{1}{6} + \frac{1}{6} =$
$\frac{1}{12} + \frac{1}{12} =$
$1 - \frac{1}{3} =$
$1 - \frac{2}{6} =$
$1 - \frac{5}{12} =$

Make the Circle in as many ways as you can.

Write the names of the fractions you use to make a circle.

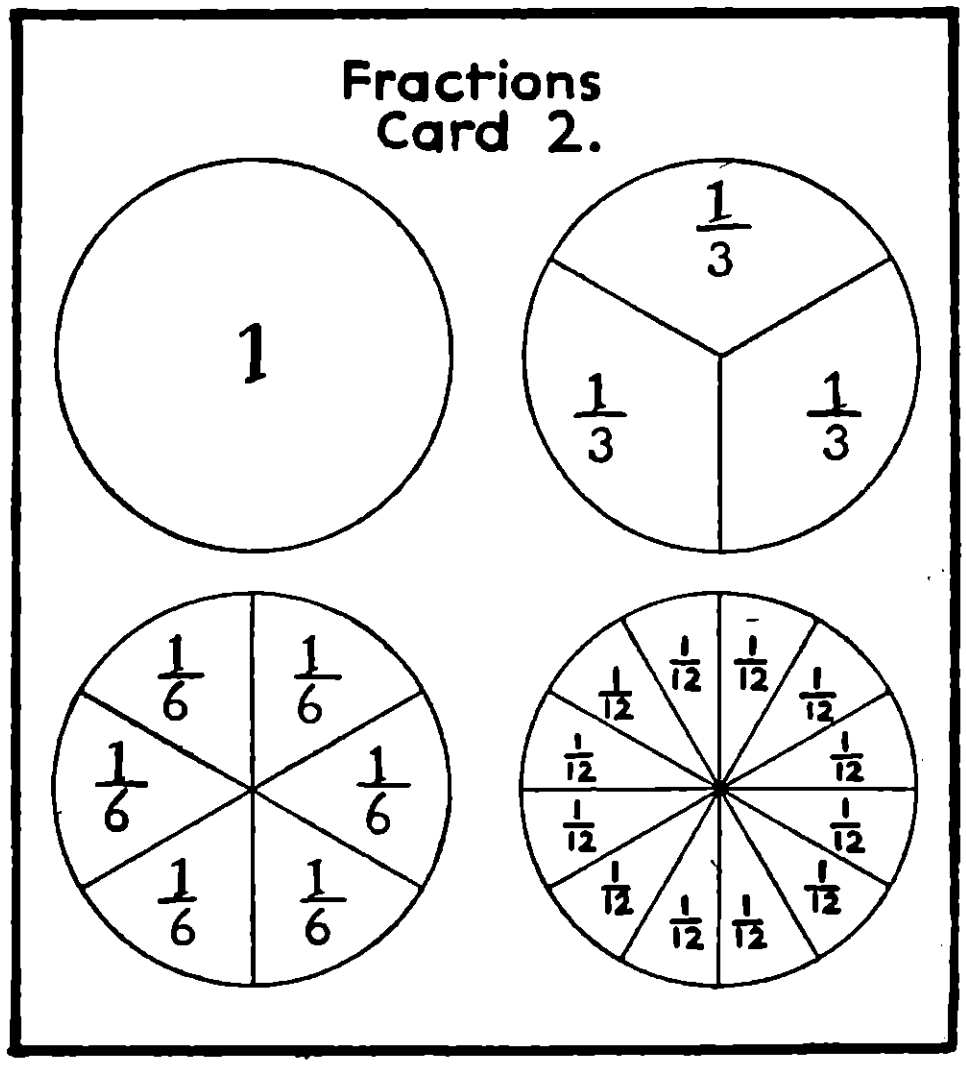


Illustration 37.

it may be, she says : " Hold up three eighths," and shows the child how to write three eighths. By similar methods, and without laborious teaching, she shows the child how to write three quarters, five eighths, etc.

Thereafter the child will, as a rule, be able to work out, with the help of the cards and the loose fractional pieces which accompany them, the exercises printed on the back of each card.

It will be found that the exercise of making an exact circle in as many ways as possible will appeal to the children, and that with a little initial help they will quickly learn to write down their results somewhat as follows :—

$\frac{1}{2} + \frac{2}{4} = \mathbf{1}.$
$\frac{3}{4} + \frac{1}{4} = \mathbf{1}.$
$\frac{1}{8} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} = \mathbf{1}$, etc.

CARD 3.

$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=$
$\frac{1}{10} + \frac{1}{10} =$
$\mathbf{1} - \frac{2}{5} =$
$\mathbf{1} - \frac{3}{10} =$

Make the Circle in as many ways as you can.

Write the names of the fractions you use to make a circle.

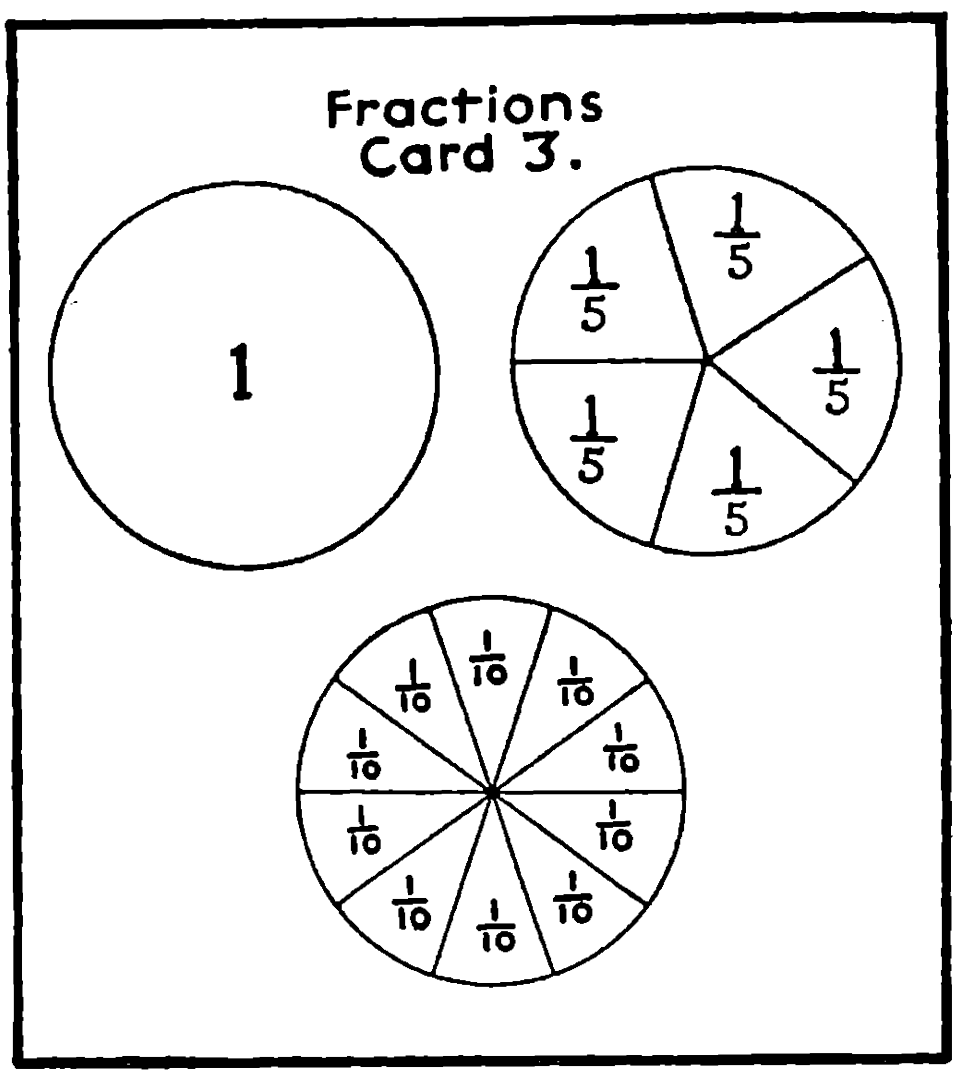


Illustration 38.

A Fraction Game.

When the children have handled and used the fractions for some time, a game may be played with the loose fractional pieces. The teacher may mix up all the fractional parts belonging to all the cards. Then she may hold up any piece she chooses and allow the children to guess its name. The child who first gives the correct name will be given the piece, and in the end the child who had thus acquired the largest number of pieces wins the game. Occasionally the teacher may hold up two or more similar pieces together so that the answer may be, for example, "Two thirds," or "Two fifths."

This game provides excellent eye training for the little folks, and they enjoy it immensely.

Time and Its Divisions.

Provide a clock-face with which children who have had frequent incidental teaching in connection with the school-room clock can find out many things for themselves. Provide also the cards shown in diagram which the children copy into their books, completing them by filling in the missing figures. Interest the children during class-lesson periods in early methods of writing figures, and in, for instance, the Roman V representative of the hand, with its five fingers, and the X representative of two hands, with their ten fingers.

To make this clear a hand may be drawn showing the thumb held apart from the other fingers. which are held close together. From this drawing it may be explained how the V came to represent the hand, and was used to symbolise "five." Similarly the Roman X can be explained as representing two hands (one V on top of the other Λ), and was used as a quick method of representing "ten."

It is well that children should be interested in the way in which such things as figures were gradually evolved.

With the help of a clock-face marked off in Roman figures the children can complete cards 1 and 2.

Card 3 can be completed with the help of a calendar.

1.

Arabic Figures.		Roman Figures.
1	—	
2	—	
3	—	
4	—	
5	—	
6	—	
7	—	
8	—	
9	—	
10	—	
11	—	
12	—	

Illustration 39.

Children will enjoy finding out for themselves by study of the calendar the figures missing in this card. Information which cannot so be obtained may already have been gained during class-lessons, or incidental conversation. If not, the teacher can supply the necessary information when needed.

2.

TIME MEASURES.

1 minute = ...seconds.
1 hour = ...minutes.
$\frac{1}{2}$ hour = ...minutes.
$\frac{1}{4}$ hour = ...minutes.
$\frac{3}{4}$ hour = ...minutes.

Illustration 40.

3.

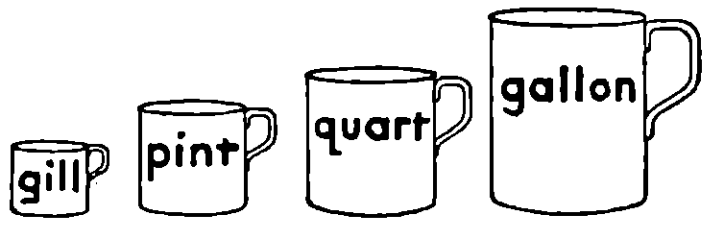
TIME MEASURES.

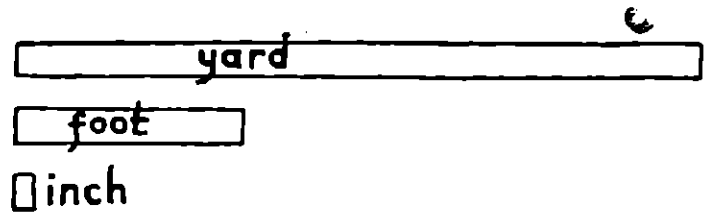
1 day (and night)	=		..... hours.
1 week	=		..... days.
1 fortnight	=		..... weeks.
1 fortnight	=		..... days.
1 month	=	about	..... weeks.
1 month	=	about	..... days.
1 year	=		..... months.
1 year	=		..... weeks.
1 year	=		..... days.
1 leap year	=		..... days.
1 century	=		..... years.

B.C. = Before Christ.
A.D. (ANNO DOMINI) = Year of our Lord.

To-day's date ..... day ..... month ..... year.

Illustration 41.





CARD 1.

LIQUID MEASURE.

1 gallon = ...quarts.

1 quart = ...pints.

1 pint = ...gills.

Illustration 42.

CARD 2.

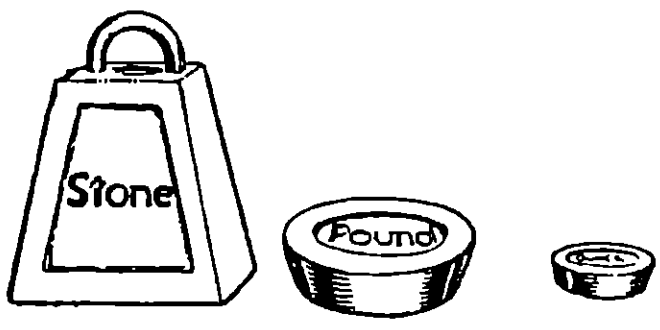
MEASURES OF LENGTH.

1 yard = ...feet.

1 foot = ...inches.

1 yard = ...inches.

Illustration 43.



CARD 3.

MEASURES OF WEIGHT.

1 stone = ...pounds.

1 pound = ...ounces.

Illustration 44.

Weights and Measures. (Illustrations 42, 43, 44.)

The cards shown in illustration can be completed by the children if the necessary weights and measures are present in their class-room.

To complete Card 1 it is necessary that the class-room should contain a gill, pint, quart and gallon measure, and a pail of water. The information should be obtained by actual experiment —the children may find out, for example, how often they can empty the contents of the quart measure into the gallon, and find the capacity in a similar manner of the other measures.

To complete Card 2 by experiment it is necessary to have in the class-room the yard, foot and inch stick, so that each may be measured against the other.

In order that Card 3 may be completed the stone, pound, and ounce weight, and a pair of scales, must be available, and also little bags of sand, or other suitable material which can be weighed, for example :—

14 bags, each containing the weight of one pound ; and

16 little bags, each containing the weight of one ounce.

Another method could be followed if preferred. A box of small pebbles might be available. Pebbles could be placed on the scales until they balanced the stone weight. The stone weight might then be replaced by the pound weight and the children could measure to find out how many times pebbles which balanced the stone weight could be used over again to balance the pound weight. Similarly one pound weight of pebbles might be used over again to find out how many ounce weights they represent.

Squaring and Cubing Numbers.

An exercise which our children thoroughly enjoy is the building up of squares and cubes from numbered rods. From the rods shown in illustration, for instance, the square and cube of 3 may

be made. Our rods are kept in boxes inside the lids of which are pasted the exercises which children are intended to carry out with them. Inside the lid of Box 1, for instance, is written :—

$2^2 =$ $3^2 =$ $4^2 =$ $5^2 =$

$2^3 =$ $3^3 =$ $4^3 =$ $5^3 =$

These exercises help the children to revise their tables in an interesting way, and to gain concrete experience which will form a valuable background for later work in number.

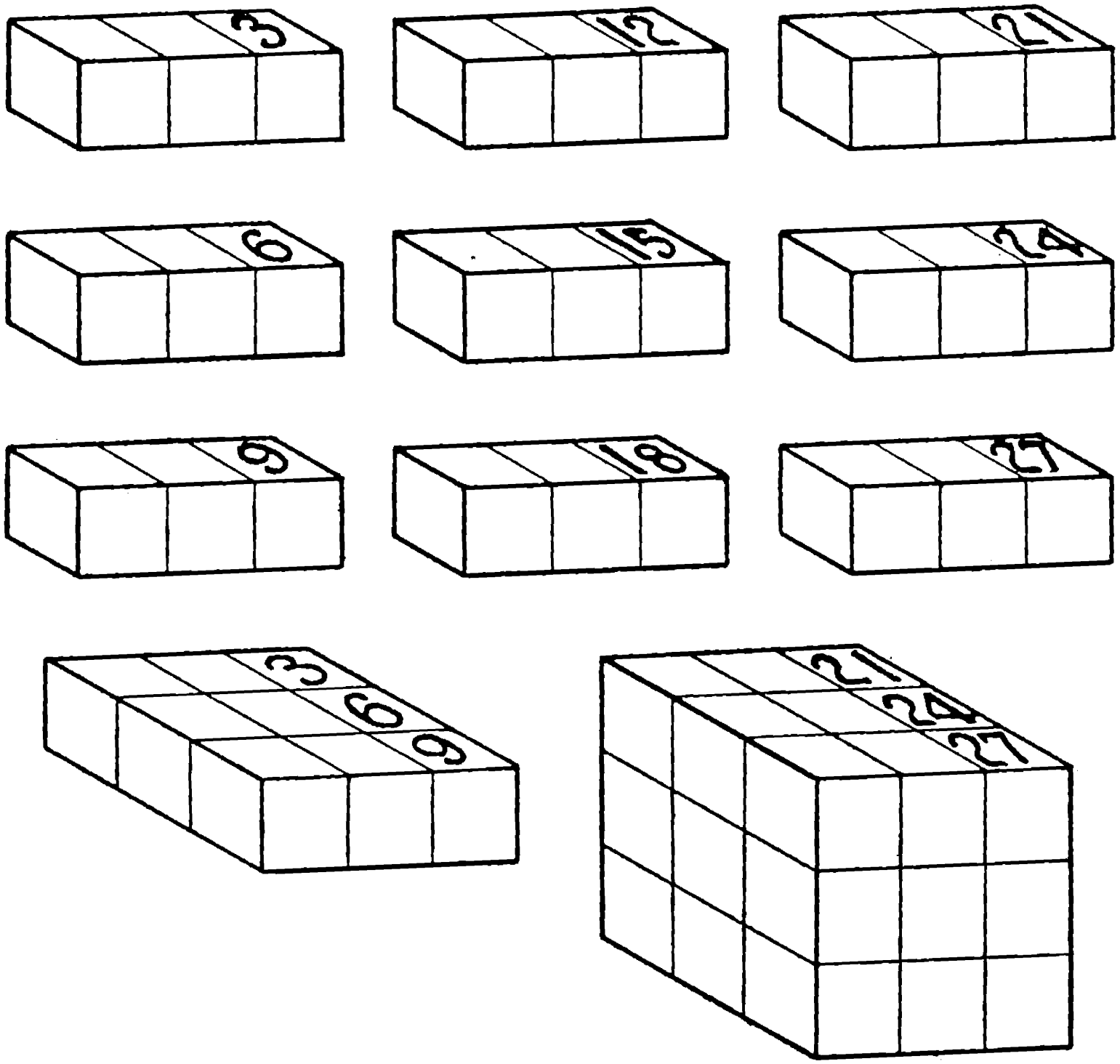


Illustration 45.

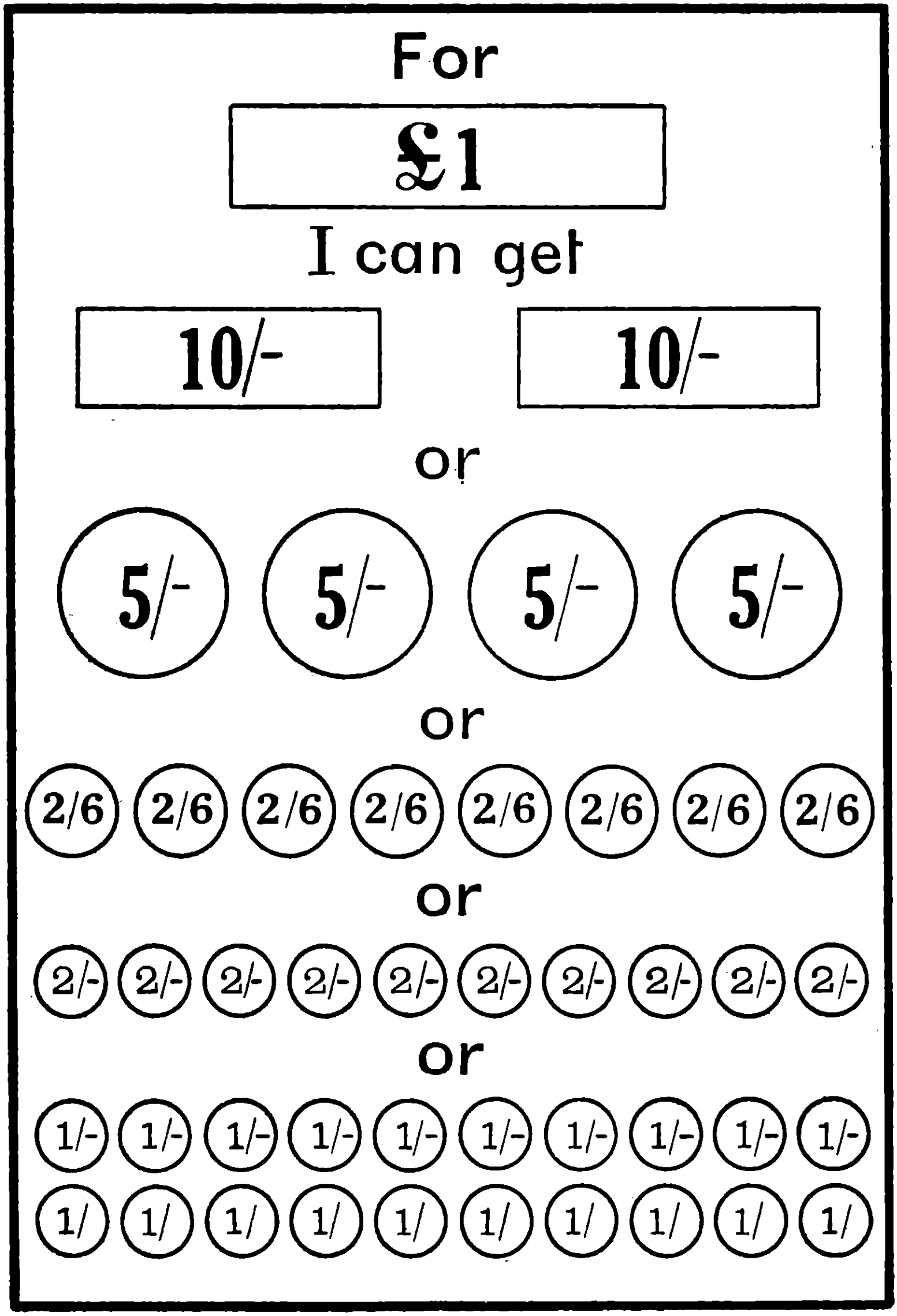


Illustration 46.

Money Values.

A box suitable for this stage might contain seven compartments :—five, arranged each to hold a different type of coin or note, and each having a symbol printed on a slip of paper affixed to the bottom, to represent one type of coin or note. The seventh compartment would hold the unassorted coins preparatory to the carrying out of the sorting exercise.

Inside the lid of this money box would be printed the matter shown in illustration. This is intended to be copied by the child when the exercise of sorting the coins has been completed.

(Illustration 46.)

CHAPTER 5.

SUGGESTED SUBSEQUENT WORK.

Long Division with Bank and Board.

A little experimentation with the Bank and Place Value Board already described will make it clear that Long Division may be demonstrated by their use.

Let us take for example :—

$$23\overline{)7,682}$$

The children are given to understand that this number, 7,682 is to be divided into groups of twenty-threes, and they will be

interested in the method of writing down this long division sum. As in short division the number to be divided is laid out with the different coloured counters from the bank on the Place Value Board.

We proceed to deal first with the first two rows of counters (green and yellow), which the children now recognise as representing thousands and hundreds. The thousands we arrange into three piles of **two,** and the hundreds into two piles of **three.** The one odd green counter in the thousands row is exchanged for ten yellow hundred counters. We make another pile of **three** in the hundreds row to equalise the three piles of **two** in the thousands row ; we have thus three complete " twenty-threes " out of our seventy-six " hundred " counters with **seven** yellow " hundred " counters over.

We record above our 6 in the division sum on our board the figure 3, and lift away (subtract) the three complete groups of twenty-threes on our board and leave the seven yellow counters which are over in the hundreds row. Our proceedings are thus recorded :—

$$\begin{array}{r} 3 \\ \hline 23 \,|\, 7,682 \\ 6,9 \\ \hline 7(8) \end{array}$$

We now proceed to deal with the next pair of figures, writing our eight beside our seven in the sum on our exercise book. The seven is arranged into three piles of **twos** ; the eight is arranged into two piles of **threes** ; the yellow " hundred " counter which is over is exchanged for ten blue " ten " counters and another group of threes is added in the tens row. Thus again we have three complete groups of twenty-threes (counters representing

sixty-nine tens) ; we write 3 in our exercise above the figure 8 ; we lift away (subtract) the three complete groups and find ourselves left with 9 blue counters in the tens row.

Our proceedings are thus recorded :—

```
    33
23 ⟌7,682
   6,9
   ----
    78
    69
    --
     9(2)
```

We now proceed to deal with the last pair of figures, writing our two, beside the nine in our exercise book. We then arrange the nine blue " ten " counters into **twos,** and find that they make four pairs with **one** left over. The one is exchanged for ten red " unit " counters and they are arranged into **threes.** We have thus four complete twenty-threes (counters to the value of ninety-two units on our board). We record our figure four above the two ; lift away the four complete groups of twenty three ; and note that there is no remainder.

```
    334
23 ⟌7,682
   6,9
   ----
    78
    69
    --
     92
     92
     --
```

When expressed in words the proceedings may appear to be complicated. It is suggested that the reader should work out a

few examples with the concrete until the exercise becomes easy, and that teachers should reflect on whether or no the practical working out of a few simple examples with children, might not throw a flash of light on a process which has often been considered unintelligible and irksome in the early stages.

It is to be remembered that the child who approaches the subject for the first time is in a different position from the adult who has perchance built up a conservative attitude towards the method by which he or she has mastered the process.

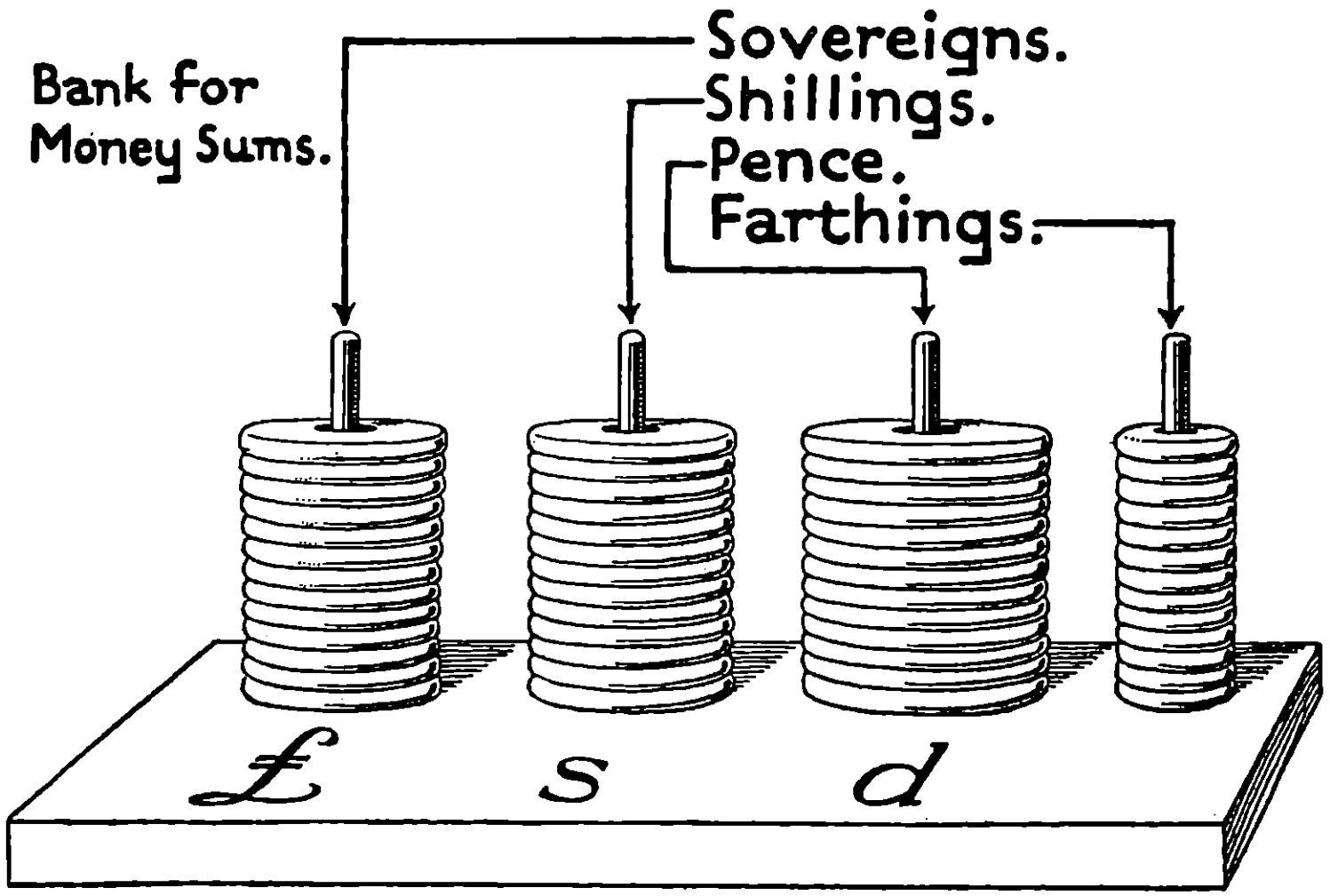
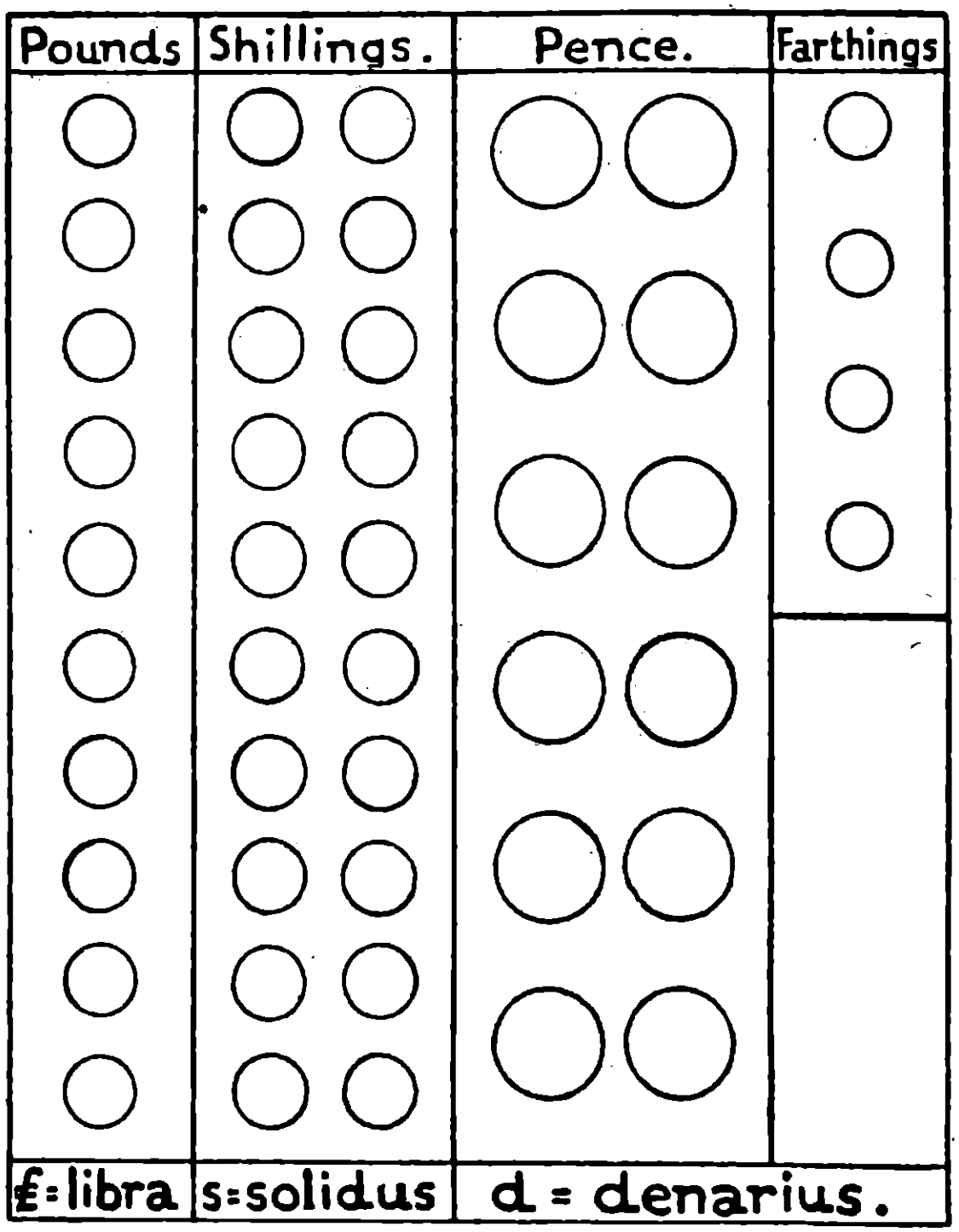


Illustration 47.

The "Money" Bank and Board.

Beyond providing an illustration of the arrangement of a Bank and Board on which money sums can be worked it is unnecessary to go. The Bank and Board are used in a manner similar to that which was used in connection with the Bank and Board which have already been described, the only difference being that they provide for exchange of money instead of exchange of thousands, hundreds, tens or units.



Place Value Board for Money Sums

Illustration 48.

Factors and Prime Numbers.

It is suggested that children be allowed to experiment with beans, shells, or other simple, suitable material, and find out for themselves which numbers (perhaps up to one hundred) are prime numbers and which have factors.

Down the side of their exercise book pages they may write the figures 1 to 100 in consecutive order. They take counters to

represent each figure in turn, and by arranging them into as many equal groups as possible find out which numbers can be so arranged and which numbers will not fit into any system of equal grouping. Results may be recorded as shown in illustration. The exercise will prove an interesting one; the children will have an opportunity of revising multiplication and division tables; and they will have that background of real experience which will make the names "prime" number, and "factors" full of meaning for them.

PRIME NUMBERS AND FACTORS.

1	
2	
3	
4	(2 × 2)
5	
6	(3 × 2), (2 × 3)
7	
8	(4 × 2), (2 × 4)
9	(3 × 3)
10	(5 × 2), (2 × 5)

Illustration 49.