MODERN SCHOOL ARITHMETIC

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and
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WITH ANSWERS

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PREFACE

THIS *Arithmetic* has been arranged to meet the needs of boys and girls who are entering secondary schools, and it assumes some knowledge of the elementary rules. It is intended to provide a full course for School Certificate examinations and to be useful for commercial and technical classes.

Accuracy in arithmetic is essential, and a good foundation is best laid in the early years of school life. Throughout the book the authors have selected those methods which they believe, in the light of experience of teaching and examining, to be most conducive to accuracy.

An introductory chapter provides revision examples on the elementary rules. The work on decimals is divided into two chapters, in the first of which methods of working are established with simple numbers. The second chapter deals with numbers which have no integral part, significant figures, etc. Easy examples on percentages have been placed early because, in addition to their practical importance, they provide useful exercise in fractions and decimals by means of simple, concrete examples. In the chapter on calculation of costs various methods of working an example are compared, one method being new. The chapters on proportion and variation have been illustrated graphically, and the method of equality of ratios replaces the unitary method. Index numbers are introduced as examples on ratio, and interest has been treated practically. The work on mensuration has been divided into three chapters; the early one provides relief from the more mechanical work of decimals, and the second includes exercises on the principle of Archimedes.

Included in the text are exercises marked 'Mental.' The answers to most of these questions should be written down (a figure at a time) as they are worked out, but some can be given orally. They are intended to drive home principles, and are followed by harder exercises for written work.

Many of the examples are original, and several are derived from data given in periodicals relating to trade and finance.

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R. N. H.
E. V. S.

CONTENTS

TABLE OF WEIGHTS AND MEASURES

Two systems are recognized in the United Kingdom: the Imperial, of which the fundamental units are the yard and pound; and the Metric, of which the fundamental units are the metre and Kilogram.

LENGTH

Imperial

12 inches (in.)	=	1 foot (ft.)
3 feet	=	1 yard (yd.)
22 yards	=	1 chain (ch.)
10 chains	=	1 furlong (f.)
8 furlongs	=	1 mile (ml.)
5½ yards	=	1 pole (pl.)
4 poles	=	1 chain
100 links	=	1 chain
1760 yards	=	1 mile

Metric

10 millimetres	=	1 centimetre (cm.)
10 centimetres	=	1 decimetre (dm.)
10 decimetres	=	1 metre (m.)
10 metres	=	1 Decametre (Dm.)
10 Decametres	=	1 Hectometre (Hm.)
10 Hectometres	=	1 Kilometre (Km.)

Sailors' Measurements

1 sea-mile	=	10 cables
1 cable	=	100 fathoms
1 sea-mile	=	6080 feet
1 fathom	=	6·08 feet

WEIGHT

Imperial

16 ounces (oz.)	=	1 pound (lb.)
14 lb.	=	1 stone
28 lb.	=	1 quarter (qr.)
4 qr.	=	1 hundredweight (cwt.)
20 cwt.	=	1 ton
1 cwt.	=	112 lb.
1 ton	=	2240 lb.
1 cental	=	100 lb.

Metric

1000 grammes	=	1 Kilogram (Kg.)
1000 Kg.	=	1 metric tonne
1 quintal	=	100 Kg.

Weights of gold, silver, and other precious metals are now estimated in ounces and decimals of an ounce.

METRIC EQUIVALENTS

				Approximate values	
1 metre	=	39·370113	inches	39·37	in.
1 metre	=	1·093614	yards	1·0936	yd.
1 yard	=	·914399	metres	·9144	metres
1 inch	=	2·5400	centimetres	2·54	cms.
1 sq. in.	=	6·4516	sq. cms.	6·45	sq. cms.
1 c. in.	=	16·387	c.c.'s		
1 lb.	=	0·453592	Kg.	0·4536	Kg.
1 Kg.	=	2·204622	lb.	2·2046	lb.
1 gallon	=	4·545963	litres	4·5460	l.
1 litre	=	1·7598	pints	1·76	pints
1 are	=			119·6	sq. yd.
1 Hectare	=			2·47	acres

A ton is usually taken to be 1016 Kg., but is also taken as 1015 Kg. for freights at some ports.

AREAS

Imperial			Metric		
144 sq. inches	=	1 sq. ft.	100 sq. mms.	=	1 sq. cm.
9 sq. feet	=	1 sq. yard	100 sq. cms.	=	1 sq. dm.
484 sq. yards	=	1 sq. chain	100 sq. dms.	=	1 sq. m.
10 sq. chains	=	1 acre (ac.)	100 sq. ms.	=	1 sq. Dm.
4840 sq. yards	=	1 acre	100 sq. Dms.	=	1 sq. Hm.
640 acres	=	1 sq. mile	100 sq. Hms.	=	1 sq. Km.

Surveyors' Units

$30\frac{1}{4}$ sq. yds.	=	1 sq. pole	1 are	=	100 sq. metres
	=	1 perch (p.)		=	1 sq. Dm.
40 perches	=	1 rood (rd.)	1 Hectare	=	100 ares
4 roods	=	1 acre	100 Hectares	=	1 sq. Km.

VOLUMES

1728 c. ins.	=	1 c. ft.	1 c. decimetre	=	1000 c.c.'s
27 c. ft.	=	1 c. yd.	1 litre	=	1000 c.c.'s[1]
			1 decilitre	=	$\frac{1}{10}$ of 1 litre
			1 Decalitre	=	10 litres
			1 Hectolitre	=	100 litres

[1] To a high degree of accuracy.

For Dry Goods		*For Liquids*	
2 gallons	= 1 peck	4 gills	= 1 pint (pt.)
4 pecks	= 1 bushel	2 pints	= 1 quart (qt.)
8 bushels	= 1 quarter	4 quarts	= 1 gallon (gal.)

The gallon is the standard measure of capacity both for liquids and for dry goods.

CHAPTER I

REVISION OF THE FOUR ELEMENTARY RULES. REDUCTION

Addition and Subtraction

Practise addition both in columns and in lines. Check your work by adding columns both from top to bottom and from bottom to top, and lines from left to right as well as from right to left.

Examples of Subtraction

(1) Subtract 8073 from 10345

10345	Say 3 and *2* make 5
8073	7 and *7* make 14, carry 1
2272	1 and *2* make 3
	8 and *2* make 10

Check by adding the result to 8073.

(2) Subtract £13 17*s.* 10*d.* from £22 5*s.* 6*d.*

£	*s.*	*d.*	
22	5	6	Say 10*d.* and (*2* + *6*)*d.* make 1*s.* 6*d.*, carry 1*s.*
13	17	10	18*s.* and (*2* + *5*)*s.* make £1 5*s.*, carry £1
8	**7**	**8**	£14 and £*8* make £22

Check the result by addition.

This method of subtraction is called 'complementary addition,' or the 'shop method,' since it is used in giving change in shops.

EXERCISES (MENTAL) I

1. Add the following numbers by rows and by columns:

(1)	7	3	19	6	8	24	5	9	12
(2)	15	3	6	21	12	9	11	17	8
(3)	6	31	3	9	5	18	2	22	4
(4)	11	17	19	5	2	8	16	4	13
(5)	5	18	7	19	21	0	13	8	24
(6)	2	30	9	11	13	24	5	16	9
(7)	19	8	27	6	5	4	23	32	11
(8)	22	4	11	33	18	26	9	7	20
	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)

2. Add 523, 159, 462.
3. Add 2*s.* 10*d.*, 6*s.* 5*d.*, 17*s.* 11*d.*, 15*s.* 2*d.*, 14*s.* 10*d.*
4. Add 1 ft. 6 in., 3 ft. 5 in., 5 ft. 11 in., 2 ft. 10 in.
5. Add 2 gal. 3 qt., 3 gal. 2 qt., 5 gal. 3 qt.
6. Add 7 hr. 15 min., 3 hr. 20 min., 6 hr. 50 min.
7. Subtract 267 from 593; 1137 from 3252; 1082 from 2257; 898 from 2367.
8. Subtract 10*d.* from 2*s.* 6*d.*; 3*s.* 3*d.* from 10*s.* 2*d.*; 7*s.* 5*d.* from 16*s.*; 18*s.* 4*d.* from £2 3*s.* 6*d.*
9. Subtract 3 gal. 2 qt. from 6 gal.; 5 gal. 3 qt. from 8 gal. 2 qt.
10. Subtract 2 hr. 50 min. from 7 hr. 10 min.; 16 hr. 25 min. from 24 hr.

Simplify

11. 25 + 97 − 49; 76 + 47 − 89; 23 + 35 − 47 + 88.
12. 3*s.* 10*d.* + 14*s.* 8*d.* − 10*s.* 2*d.*; 15*s.* 3*d.* − 19*s.* 8*d.* + 6*s.* 4*d.*

EXERCISES II

1. Add

(1)	(2) £	s.	d.	(3)	(4) £	s.	d.
60937	371	2	10	54231	6187	13	9
71235	4158	13	2	7093	3215	16	7
8097	29	17	8	16457	982	7	11
273	785	8	5	309	6073	14	10
90972	6230	11	7	89736	157	18	5

2. Add 23 miles 5 f. 3 ch., 7 miles 2 f. 9 ch., 16 miles 7 f. 8 ch., 20 miles 3 ch.

3. Subtract

(1)	(2)	(3) £	s.	d.	(4) £	s.	d.
98073	103259	1037	6	5	6097	17	3
29169	29763	876	17	9	1898	19	6

4. Subtract 27 miles 1093 yd. from 38 miles 540 yd.

5. A motorist kept the following table of the mileage he covered each week-day for 6 weeks:

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	
1st week	86	27	36	52	0	43	(1)
2nd ,,	65	15	47	13	67	59	(2)
3rd ,,	109	0	33	29	17	92	(3)
4th ,,	35	57	18	43	29	37	(4)
5th ,,	77	62	0	49	15	80	(5)
6th ,,	26	0	0	25	30	0	(6)
	(7)	(8)	(9)	(10)	(11)	(12)	(13)

Find the totals (1) to (6) for each week, and the totals (7) to (12).

Add the totals (1) to (6) to get (13), and check by adding the totals (7) to (12).

6. Find the number of hours and minutes between 7.30 A.M. on Monday morning and 12 NOON on the following Saturday.

7. Add together 15 tons 17 cwt. 80 lb., 23 tons 3 cwt. 56 lb., 54 tons 100 lb., and 6 tons 15 cwt. 36 lb.

8. From a stack containing 15 tons of coal, 3 loads of 2 tons 12 cwt., 6 tons 15 cwt., 4 tons 17 cwt. respectively are removed. How much coal is left?

9. Add together 2 qr. 6 bush. 1 pk., 6 qr. 3 bush. 2 pk., 5 bush. 3 pk., and 3 qr. 7 bush.

10. From a ball of 50 yd. of string, pieces of lengths 3 yd. 2 ft. 6 in., 4 yd. 1 ft. 10 in., 2 yd. 1 ft. 9 in., 10 yd. 2ft., and 6 yd. 1 ft. 2 in. were cut. What length of string remains?

Multiplication and Division

Example 1. Multiply £59 13*s.* 2*d.* by 42.

(i) by factors

£	*s.*	*d.*
59	13	2
		6
357	19	0
		7
2505	13	0

Result is **£2505 13s. 0d.**

Rough check £60 × 40 = £2400.

To multiply by 43, we might first multiply by **42** as above, and then add the original quantity taken once.

(ii) by long multiplication

```
    £        s.      d.
   59       13       2
                    42
 ----------------------
 2360      520   12)84
  118       26     ---
   27        7      7s. 0d.
 ----      ---
£2505   20)553
 ----      ---
          £27 13s.
```

Result is **£2505 13s. 0d.**

Rough check as before.

Example 2. Divide 18753 by 35

(i) by factors

```
5|18753
7| 3750  groups of 5, 3 over  }
    535  5 groups of 5 over   }
```

The quotient is **535**, remainder $5 \times 5 + 3 =$ **28.**

(ii) by long division

```
      535
35)18753
    175
    ---
     125
     105
     ---
      203
      175
      ---
       28
```

The quotient is **535**, remainder **28**.

Rough check $18000 \div 30 = 600$.

Example 3. Divide £5625 17*s*. 4*d*. by 73.

	£	*s*.	*d*.
	77	1	4
73)	5625	17	4
	511	80	288
	515	97	292
	511	73	292
	4	24	

The result is **£77 1s. 4d.**

Rough check £5600 ÷ 70 = £80.

Exercises (Mental) III

1. Multiply 2*s*. 8*d*. by 2, 3, 5, 7, 8, 10.
2. Multiply 4*s*. 11*d*. by 2, 3, 5, 7, 8, 10.
3. Multiply 6*s*. 9*d*. by 3, 7, 10, 12.
4. Divide 10*s*. 0*d*. by 3, 4, 6, 8, 12.
5. Divide 17*s*. 6*d*. by 3, 5, 7, 10.
6. Divide £3 6*s*. 8*d*. by 4, 5, 8, 10.
7. Multiply 1 ft. 3 in. by 5, 7, 12.
8. Divide 20 yd. by 3, 6, 12.
9. Multiply 6 hr. 20 min. by 4, 5, 9.
10. Divide 24 hr. by 5, 9, 10.
11. Multiply 2 tons 5 cwt. by 7, 9, 12.
12. Divide 4 cwt. 56 lb. by 2, 7, 9.

Exercises IV

Multiply

1. 3197 by 263.
2. 6193 by 389.
3. 15927 by 47.
4. 68127 by 96.
5. 57813 by 127.
6. 192783 by 573.

Work the following division sums, giving the remainders:

7. 4297 by 23.
8. 6583 by 37.
9. 19297 by 42.
10. 28371 by 64.
11. 617239 by 153.
12. 535973 by 217.

Multiply

13. £2 15*s*. 9*d*. by 36.
14. £17 16*s*. 8*d*. by 55.
15. £65 13*s*. 10*d*. by 42.
16. 3 ml. 5 f. 6 ch. by 15.
17. 13 tons 17 cwt. 2 qr. by 63.
18. 11 cwt. 3 qr. 16 lb. by 32.

Divide

19. £66 12*s.* by 36.
20. £383 15*s.* by 50.
21. £4596 12*s.* by 72.
22. 76 tons 6 cwt. 1 qr. by 35.
23. 248 hr. by 15.
24. 309 ml. 0 f. 110 yd. by 23.
25. Find the value of 28 desks at £3 14*s.* 3*d.* each.
26. Calculate 2701 $\times$ 13 $\times$ 53.
27. In a factory 153 workmen each receive £2 17*s.* 9*d.* per week. What is the total weekly amount paid to the workmen?
28. A man paid a bill of £3 11*s.* 6*d.* for garaging his car for 13 weeks. What was the charge per week?

REDUCTION

Example 1. Reduce 12 tons 17 cwt. 80 lb. to lb.

```
    12 tons 17 cwt. 80 lb.
    20
   ----
   240
    17
   ----
   257 cwt.
   112
 ------
 257
  257
   514
    80
 ------
 28864 lb.
 ------
```

Result is **28864 lb.**

Check 13 $\times$ 2000 = 26000.

Example 2. Express 21583 seconds in hr. min. sec.

```
60|21583 sec.
60|  359 min. 43 sec.
```

5 hr. 59 min. 43 sec.

Check 6 hr. = 6 $\times$ 60 $\times$ 60 sec. = 21600 sec.

EXERCISES (MENTAL) V

Reduce

1. 7*s.* 5*d.* to pence.
2. 113*d.* to shillings and pence.
3. £3 15*s.* to sixpences.
4. £8 7*s.* 6*d.* to half-crowns.
5. 91 florins to pounds and shillings.
6. 71 sixpences to £ *s.* *d.*
7. £1 13*s.* 6*d.* to pence.
8. 75 half-crowns to £ *s.* *d.*
9. £7 17*s.* to shillings.
10. 231 pence to shillings and pence.

Reduce

11. 2 yd. 1 ft. 7 in. to in.
12. 7 ch. 17 yd. to yd.
13. 7 ch. 56 links to links.
14. 79 lb. to qr. and lb.
15. 53 oz. to lb. and oz.
16. 3 tons 19 cwt. to cwt.
17. 317 in. to yd., ft., in.
18. 435 min. to hr. and min.
19. 6 days 15 hr. to hr.
20. 100 yd. to ch. and yd.

Exercises VI

Reduce

1. 15273 pence to £ *s*. *d*.
2. 6573 in. to yards, feet, inches.
3. 81753 lb. to tons, cwt., lb.
4. 16597 sec. to hours, minutes, seconds.
5. £16 15*s*. 7*d*. to pence.
6. 63 ml. 5 f. 7 ch. to chains.
7. 19 tons 3 cwt. 56 lb. to lb.
8. 9 weeks 5 days 15 hr. to hours.
9. 13 ac. 2 rd. 37 sq. pl. to square poles.
10. 7509 sixpences to £ *s*. *d*.
11. £35 17*s*. 6*d*. to half-crowns.
12. 793 oz. to lb. and oz.
13. 37 ml. 6 f. 170 yd. to yards.
14. 72963 ft. to miles and yards.
15. 17897 threepences to £ *s*. *d*.
16. 37189 lb. to tons, cwt., lb.

Exercises VII

1. In the four quarters of last year a motorist covered 1756, 2809, 4385, and 3210 miles respectively. What was his total mileage for the year?

2. In a school's sports a boy threw the cricket-ball a distance of 85 yd. 1 ft. 9 in. By how much did this beat the previous record of 77 yd. 2 ft. 6 in.?

3. A dining-room suite consists of a table costing £7 17*s*. 6*d*., sideboard, £12 19*s*. 6*d*., 2 chairs, £2 13*s*. 6*d*. each, and 4 smaller chairs, £1 16*s*. 6*d*. each. Find the total cost of the suite.

4. How many tins of soup costing 7*d*. each can be bought for 10 shillings, and how much change would there be?

5. The total cost of 5 tons of coal and 2 tons of coke was £14 14*s*. 6*d*. If the coke cost £1 18*s*. 6*d*. per ton, what was the price per ton of the coal?

6. In a motor-race a car covered three laps of the course in 1 hr. 58 min. 36 sec., 2 hr. 3 min. 25 sec., 1 hr. 57 min. 2 sec. respectively. Find its total time for the three laps.

7. Find the change left from £5 when two bills for £2 3*s*. 10*d*. and £1 19*s*. 3*d*. have been paid.

8. A man paid the railway fares for a party of 11 persons with a £1 note. If the fare was 1*s*. 5*d*. for each person, how much change was there?

9. A bookshelf will just hold 67 books, of which 29 are 2 in. thick, the others 1 in. thick. What is the length of the shelf in feet?

10. Using only £1 and 10*s*. notes, half-crowns, florins, shillings, sixpences, and copper coins, how would you make up sums of £2 17*s*. 3*d*., £3 14*s*. 2*d*., and £1 18*s*. 11*d*. so as to use as few notes and coins as possible in each instance?

11. The price of a book is 1*s*. 4*d*. Find the cost of 110 copies, allowing 2*s*. for postage.

12. A woman wished to buy material to make curtains. She required 5 pieces each 84 in. long and 6 pieces 20 in. long. What total length should she buy, and what would be its cost at 2*s*. 11*d*. per yard?

13. A boy who wished to measure the length of his pace found that a distance equal to 20 of his paces measured 15 yd. 1 ft. 8 in. What was the length of his pace in inches? He then measured a distance as 135 paces. What was the distance in yards?

14. Find the cost of taking a party of 32 scouts to camp for 14 days, allowing 1*s*. 9*d*. each per day for cost of food, 5*s*. 4*d*. each for travelling-expenses, and £2 for hire of tents.

15. A man owns 12 houses, and receives a rent of 16*s*. 3*d*. weekly from each of them. Find how much he obtained from them in a year (52 weeks), allowing for repairs, etc., which cost him £117 18*s*. 6*d*.

16. The bill for 65 ft. of hose-pipe, together with connexions which cost 1*s*. 6*d*., amounted to 12*s*. 4*d*. What was the cost per foot of the hose-pipe?

17. A plot of ground of total area 3 ac. 2 rd. was divided into allotments, each of area 10 p. How many allotments were there?

18. Pieces, each 7 ft. 6 in. long, are cut from a 200-ft. length of cord. How many pieces can be cut, and what length of cord will remain over?

19. How many bottles, each holding 1 pint, can be filled from a barrel which contains 65 gal. 2 qt.?

20. Find the total weight of a van which weighs 1 ton 15 cwt. alone, when it is loaded with 160 tins of biscuits each weighing 8 lb. 6 oz.

21. A family takes a daily newspaper, costing 1*d*. on week-

days and 2*d.* on Sundays, and two weekly papers, one costing 2*d.* and the other 3*d.* per week. What is the amount of their bill for these papers after 13 weeks if 1*d.* per week is charged for delivery?

22. A sum of £100 was divided between a number of people. Each person received £2 16*s.* 8*d.*, and 16*s.* 8*d.* was left over. What was the number of people?

23. In aid of a charity some students collected halfpennies. These were placed touching one another in a row on the pavement and made a chain 400 yd. long. If a halfpenny is 1 in. across, find the value in pounds of this collection.

24. Each week a man buys 3 packets of cigarettes at $11\frac{1}{2}d.$ per packet and 3 oz. of tobacco at 1*s.* $1\frac{1}{2}d.$ per oz. How much does he spend in this way in a year (52 weeks)?

25. A man owns a car which travels 25 ml. per gallon of petrol. How much does he pay for petrol per 1000 ml. if the price of petrol is 1*s.* 7*d.* per gallon?

26. Mr Jones had £10 to spend on a holiday and reckoned that it would cost him 14*s.* 6*d* per day. How many days' holiday was he able to have, and how much money would remain?

27. Altogether 685 boys and girls entered for an examination. If there were 117 more boys than girls, how many boys were there?

28. Mr Smith and his friends, Mr Brown and Mr Robinson, went for a tour together, agreeing to share their expenses equally. At the end of the tour Mr Smith had spent a total of £9 10*s.* 6*d.*, Mr Brown £7 13*s.* 10*d.*, and Mr Robinson £6 3*s.* 8*d.* How much should Mr Brown and Mr Robinson each pay Mr Smith?

29. A party of boys had tea together. On receiving the bill they found that if each paid 10*d.*, this would make 9*d.* too little, while if each paid 11*d.*, there would be 6*d.* over. What was the amount of the bill?

30. If light travels at 186,000 ml. a second, how long (in minutes and seconds) does it take to reach the earth from the sun, when the sun is 93,000,000 miles away?

31. The charge for sending packets by letter post is three halfpence for the first 2 oz. plus one halfpenny for each additional 2 oz. or part of 2 oz. What is the total postage for three packets sent separately and weighing 1 oz., 7 oz., and 11 oz. respectively?

32. Find the weight in tons, cwt., lb. of a load of 1000 bricks, each of which weighs 7 lb.

33. The rails for a railway weigh 100 lb. per yard. Find the weight in cwt. and lb. of a rail 120 ft. long.

34. Find the value of 5 tons of butter at 1*s*. 3*d*. per lb.

35. The weight of a fully loaded American air-liner was given as 51,000 lb. Express this in tons, cwt., lb.

36. A motor-car travels 25 ml. per gallon of petrol. Find how many more miles it will go on £2 5*s*. worth of petrol if the price is reduced from 1*s*. 8*d*. to 1*s*. 6*d*. per gallon.

37. For making copies of a manuscript a typist charges 4*s*. 9*d*. for making the stencil of each page of typing and then charges for the paper used at 2*d*. for 12 sheets. How much would she charge for 20 copies of a manuscript which made 75 pages of typing? (Only one side of the paper is used.)

38. If 2 men and 3 boys earn £9 6*s*. 9*d*. in a week, and 5 men and 6 boys £21 18*s*. 9*d*., find the weekly wages of 1 man and of 1 boy respectively.

39. Divide £1 between 2 boys so that one has 2*s*. 6*d*. more than the other.

40. A soldier's marching-step is 2 ft. 6 in.; how many steps does he take in marching a quarter of a mile?

41. A house was to be sold for £600. A man arranged to buy it by paying £40 down, and £3 monthly for 18 years. How much more than £600 did he pay?

42. A shopkeeper bought some socks at 1*s*. 10½*d*. per pair, and sold them at 2*s*. 6*d*. per pair. His total profit was £1 17*s*. 6*d*. How many pairs did he buy?

43. Find the cost of distributing 5000 circulars if the printing costs £2 12*s*. 6*d*. per thousand and there is a half-penny postage on each.

44. A grocer bought a box of 240 oranges for 14*s*. 9*d*. He sold 150 at 5 for 6*d*., 40 at 1*d*. each, and 40 at 4 for 3*d*. The others were spoiled and unsaleable. How much profit did he make?

45. The total cost of 59 small chairs and 19 armchairs was £217 7*s*. 10*d*. If the armchairs cost £6 10*s*. 6*d*. each, what was the price of each small chair?

46. A woman bought some material for curtains at 5*s*. 11*d*. per yard. How many yards did she buy if the bill came to £8 11*s*. 7*d*.?

47. It costs 4¼*d*. to keep a certain kind of electric lamp burning for 16 hr. How much will it cost to keep 6 such lamps burning for 4 hr. a day for 12 weeks?

48. My lawn is 45 yd. long. In mowing it with a small mower I find I push the mower 4 ml. 4 f. How many times up and down the lawn do I go?

CHAPTER II

SIMPLE PRIME FACTORS. H.C.F. AND L.C.M.

If there is no remainder after one number has been divided by another one, the divisor is said to be a **factor** of the original number, and the original number is said to be a **multiple** of that divisor.

A number which has no factors but itself and 1 is said to be a **prime number.**

Numbers which have no common factor except 1 are said to be **prime** to one another.

Tests of Divisibility

A number is divisible by

2, if the last figure is divisible by 2;

4, if the number formed by the last two figures is divisible by 4;

8, if the number formed by the last three figures is divisible by 8;

3 or 9, if the sum of the digits forming the number is divisible by 3 or 9 respectively;

10, if the last figure is 0;

5, if the last figure is 5 or 0.

To test a number for divisibility by 11, add together every other digit, starting from the first one, and then every other digit starting from the second one. Find the difference between these totals. If the difference is 0 or is divisible by 11, 11 is a divisor of the original number.

Reasons. Hundreds are divisible by 4, thousands by 8.

$3456 = 3400 + 56 \quad \therefore$ 3456 is divisible by 4 if 56 is.

$23576 = 23000 + 576 \quad \therefore$ 23576 is divisible by 8 if 576 is.

Hence the tests for divisibility by 4 and by 8.

Consider the number 3465.

$$\begin{aligned} 1000 &= 999 + 1 \quad \therefore \quad 3000 = 3 \text{ times } 999 + 3 \\ 100 &= 99 + 1 \quad \therefore \quad 400 = 4 \text{ times } 99 + 4 \\ 10 &= 9 + 1 \quad \therefore \quad 60 = 6 \text{ times } 9 + 6 \\ &\text{and} \quad 5 = \quad + 5 \end{aligned}$$

Since 999, 99, and 9 are all divisible by 3, 3465 is a multiple of 3 + (3 + 4 + 6 + 5), and is therefore divisible by 3 if (3 + 4 + 6 + 5) is a multiple of 3.

Hence the test for divisibility by 3. The test for 9 is explained similarly from the divisibility by 9 of 9, 99, 999, etc.

Consider the number 54379

$$\begin{aligned} 10000 &= 9999 + 1 \quad \therefore \quad 50000 = 5 \text{ times } 9999 + 5 \\ 1000 &= 1001 - 1 \quad \quad 4000 = 4 \text{ times } 1001 - 4 \\ 100 &= 99 + 1 \quad \quad 300 = 3 \text{ times } 99 + 3 \\ 10 &= 11 - 1 \quad \quad 70 = 7 \text{ times } 11 - 7 \\ & \quad \quad 9 = \quad + 9 \end{aligned}$$

Now the numbers 11, 99, 1001, 9999, 100001, etc. are all divisible by 11; hence 54379 is divisible by 11 if 5 − 4 + 3 − 7 + 9 is divisible by 11.

Exercises (Mental) VIII

Test the following numbers for the factors 2, 4, 8, 3, 9, 5, 11

1. 27444
2. 32730.
3. 31130.
4. 61896.
5. 35706.
6. 57772.
7. 91949.
8. 55555.

Example. Find the prime factors of 1320, 3465, 8712.

10 | 1320	5 | 3465	8 | 8712
4 | 132	9 | 693	9 | 1089
3 | 33	11 | 77	11 | 121
11	7	11

$$\begin{aligned} 1320 &= 10 \times 4 \times 3 \times 11 \\ &= 2 \times 5 \times 2 \times 2 \times 3 \times 11 \\ 3465 &= 5 \times 9 \times 11 \times 7 \\ &= 5 \times 3 \times 3 \times 11 \times 7 \\ 8712 &= 8 \times 9 \times 11 \times 11 \\ &= 2 \times 2 \times 2 \times 3 \times 3 \times 11 \times 11 \end{aligned}$$

Using the index notation, 2^3 stands for three two's multiplied together, 3^2 stands for two three's multiplied together, and generally a^n indicates that n numbers, each a, are to be multiplied together.

$$\left.\begin{array}{l} 1320 = 2^3 \times 3 \times 5 \times 11 \\ 3465 = 3^2 \times 5 \times 7 \times 11 \\ 8712 = 2^3 \times 3^2 \times 11^2 \end{array}\right\}$$

The numbers are now expressed as products of prime factors, the prime factors being arranged in order of magnitude.

Note. It is better to divide by 8 and 9 at once than by a series of two's and three's.

The separate divisions should always be arranged in *vertical* columns.

H.C.F. (Highest Common Factor) sometimes called G.C.M. (Greatest Common Measure)

It will be seen that the three numbers have the factors 3 and 11 in common and no others. 3×11 is said to be the *Highest Common Factor*.

L.C.M. (Least Common Multiple)

If a number is to be divisible by all of the three numbers, it must contain as factors three two's, two three's, one five, one seven and two eleven's.

The product of these, $2^3 \times 3^2 \times 5 \times 7 \times 11^2$, is the *Least Common Multiple* of the three numbers. It is the smallest number into which each of the three numbers divides without remainder.

Note. It is usually unnecessary to multiply these factors out (a long process in the L.C.M.).

Exercises IX

Find the prime factors of

1. 36.	6. 693.	11. 627.	16. 9999.
2. 108.	7. 880.	12. 1001.	17. 78750.
3. 225.	8. 1000.	13. 1638.	18. 37730.
4. 231.	9. 1320.	14. 2205.	19. 11704.
5. 396.	10. 1275.	15. 1911.	20. 43659.

H.C.F. and L.C.M. by Prime Factors

Example. Find the H.C.F. and L.C.M. of 105, 140, 385.

Expressing the numbers in prime factors we get

$105 = 3 \times 5 \times 7$, H.C.F. $= 5 \times 7 =$ **35**.
$140 = 2^2 \times 5 \times 7$, L.C.M. $= 2^2 . 3 . 5 . 7 . 11 =$ **4620**.
$385 = 5 \times 7 \times 11$.

Exercises X

Find, by prime factors, the H.C.F. of

1. 495, 1050.
2. 360, 420, 108.
3. 52, 91, 260.
4. 105, 63, 840.
5. 306, 357, 391.
6. 1001, 616, 429.
7. 1020, 850, 4250.
8. 13923, 25935.

Exercises XI

Find, by prime factors, the L.C.M. of

1. 12, 18, 21.
2. 10, 15, 30.
3. 28, 77, 143.
4. 16, 21, 24, 56.
5. 14, 30, 105, 140.
6. 3003, 16170.
7. 72, 70, 45, 84.
8. 336, 468, 492.

Square Root by Factors

When one number is multiplied by itself, the product is said to be the square of the number. That number which, when multiplied by itself, gives a certain product (say x) is called the square root of x.

Example. $324 = 2^2 . 3^4$.

This is the square of $2 \cdot 3^2$.

$2 \cdot 3^2 = 18$, $\therefore$ **18** is the square root of 324.

Exercises XII

Find, by prime factors, the square roots of

1. 256.
2. 729.
3. 441.
4. 1764.
5. 1521.
6. 5929.
7. 1936.
8. 245025.
9. 78400.
10. 705600.
11. 148225.
12. 9801.
13. Find the smallest multiplier which will make 10008 a perfect square.
14. Find the smallest multiplier which will make 2178 a perfect square.
15. Find the smallest multiplier which will make 49392 a perfect square.

Exercises XIII

1. Find the least number which, when divided by 12, 15, or 18, gives a remainder 1.

2. Find the L.C.M. of 66 × 135 and 66 × 120.

3. What is the smallest number by which 273 must be multiplied to give a multiple of 357?

4. Find the least number by which 1428 must be multiplied so that the product may be divisible by 2898.

5. A sea-mile is 6080 ft. Find, in feet, the greatest length of which both the sea- and the land-mile will be multiples.

6. Find the least sum of money which can be paid in an exact number either of half-crowns or of half-guineas.

7. Make a list of all the integral numbers less than 500 which are divisible by 18 and by 24. Find also the L.C.M. of these numbers.

8. Find the least number which, when divided by 33, leaves 32 as a remainder, divided by 39, leaves 38 as a remainder, and divided by 44, leaves a remainder 43.

H.C.F. and L.C.M. by Long Division

Factors greater than 11 (13, 17, etc.) common to two or more numbers may sometimes be discovered easily by trial, if the numbers are not too big. If not, the work of finding common factors may be completed by a series of divisions.

Example. Find the H.C.F. and L.C.M. of 15283 and 20387.

```
15283)20387(1
      15283
      -----
       5104)15283(2
            10208
            -----
             5075)5104(1
                  5075
                  ----
                    29)5075(175
                       29
                       ---
                       217
                       203
                       ---
                        145
                        145
                        ---
```

The last divisor, **29**, is the H.C.F.

The remainder after each division is used as a new divisor, and is divided into the previous divisor. The principle underlying the method is that every common factor of a divisor and dividend is also a factor of the remainder. (Test this by trial with simple numbers, if you cannot give a formal proof.) Hence any common factor of the first two numbers is a factor of the remainder 5104, and so is common to 15283 and 5104. Also any common factor of these is a factor of 20387, so that the H.C.F. of 20387 and 15283 is also H.C.F. of 15283 and 5104. The H.C.F. of this pair, by the same argument, is also H.C.F. of 5104 and 5075, of 5075 and 29. Since 29 is the H.C.F. of this pair, it is the H.C.F. of the original pair of numbers.

Notes. Before using long division, find all prime factors up to and including 11.

The calculation can frequently be shortened if any factors which clearly do not belong to the H.C.F. can be detected in any remainder.

Thus we might have worked the example above as follows.

```
                          15283)20387(1
                                15283
                                -----
                                 5104
Cut out factor 8                  638
  ,,      ,,   2                  319
  ,,      ,,   11                  29)15283(527
                                      145
                                      ---
                                       78
                                       58
                                      ---
                                      203
                                      203
                                      ---
```

29 is the H.C.F.

To find L.C.M. $20387 = 29 \times 703$
$15283 = 29 \times 527$

703 and 527 are known to have no common factor (since any such factor would belong to the H.C.F.).

$\therefore$ L.C.M. $= \mathbf{29} \times \mathbf{703} \times \mathbf{527}$.

Question. Can you see why the product of the H.C.F. and L.C.M. of two numbers is the same as the product of these numbers?

Exercises XIV

Find the H.C.F. of

1. 16014, 95142.
2. 8987, 14839.
3. 8740, 27531.

Find the L.C.M. of

4. 4158, 10395.
5. 6734, 19943.
6. 3872, 92807.

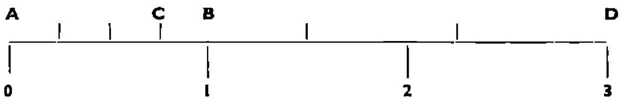
Find the H.C.F. and L.C.M. of the pairs of numbers given in questions 7, 8, 9. Find also (1) the product of the given numbers, and (2) the product of their H.C.F. and L.C.M., and verify that these products are the same.

7. 1537, 1943.
8. 3293, 4181.
9. 6201, 36153.

10. Find the H.C.F. of 7245, 8901, and 11937.

CHAPTER III

VULGAR FRACTIONS



If an inch (AB) be divided into four equal parts, each is a quarter of an inch, written $\frac{1}{4}$ in.; and if three of them be taken, we get a length AC, which is three quarters of an inch and is written $\frac{3}{4}$ in.

Take a pair of dividers, and mark off in succession from A distances equal to AC. When you come to D you will have done this 4 times and covered a distance of 3 inches. Hence we can also get $\frac{3}{4}$ in. by dividing a length of 3 inches into 4 equal parts.

We have here two interpretations of a fraction.

Similarly $\frac{5}{8}$ of any unit is 5-eighths of that unit or 1 eighth of 5 of those units.

5 is called the numerator (Latin, *numerus*, a number), and 8 is called the denominator (Latin, *nomen*, a name) of the fraction $\frac{5}{8}$.

Example. Find $\frac{3}{5}$ of £1.

Dividing £1 by 5, we see that $\frac{1}{5}$ of £1 is 4*s.*

$$\therefore \ \tfrac{3}{5} \text{ of £1 is } 4 \times 3 \text{ shillings, } i.e., \ 12s.$$

Or $\frac{3}{5}$ of £1 equals $\frac{1}{5}$ of £3, *i.e.*, $60 \div 5$ shillings $= 12s.$

Exercises (Mental) XV

1. Find $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{12}$ of £1.
2. Find the same fractions of 10*s.*, 2*s.* 6*d.*, 1*s.* 3*d.*
3. Find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{12}$ of 1 yd.
4. Find in lb. $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{7}$, $\frac{1}{14}$, $\frac{1}{16}$ of 1 cwt.

5. Fill in the gaps in the following tables:

10*s*.	$= \frac{1}{2}$	of £1.
5*s*.	=	
2*s*. 6*d*.	=	
1*s*. 3*d*.	=	
1*s*. 4*d*.	=	
6*s*. 8*d*.	$= \frac{1}{3}$	
3*s*. 4*d*.	=	
1*s*. 8*d*.	=	

112 lb.	$= \frac{1}{20}$	of 1 ton.
56 lb.	=	
28 lb.	=	
14 lb.	=	
7 lb.	=	
8 lb.	=	

6. Find $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{6}$ of £1.
7. Find $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{9}$ of 1 yd. in inches.
8. Find $\frac{3}{4}$, $\frac{4}{7}$, $\frac{5}{8}$ of 1 cwt. in lb.

Exercises XVI

Calculate

1. $\frac{3}{5}$ of £2 10*s*.
2. $\frac{2}{7}$ of £1 8*s*.
3. $\frac{2}{9}$ of £3 12*s*.
4. $\frac{5}{8}$ of £12 8*s*.
5. $\frac{3}{5}$ of £12 8*s*. 4*d*.
6. $\frac{3}{7}$ of £16 3*s*. 2*d*.
7. $\frac{3}{5}$ of 2 tons 2 cwt. 2 qr.
8. $\frac{2}{7}$ of 3 cwt. 2 qr. 14 lb. 7 oz.
9. $\frac{2}{9}$ of 1 ml. 382 yd.
10. $\frac{3}{5}$ of 7 ac. 3 rd. 10 p.
11. Add together $\frac{2}{11}$ of £23 1*s*. 1*d*. and $\frac{5}{6}$ of £17 19*s*. 3*d*.
12. Subtract $\frac{3}{5}$ of £5 7*s*. 1*d*. from $\frac{3}{7}$ of £9 5*s*. 6*d*.
13. Find the difference between $\frac{5}{7}$ of 42 half-crowns and £3 16*s*. 8*d*.
14. Add $\frac{1}{16}$ of £1 to $\frac{11}{12}$ of £5 10*s*.
15. Add together $\frac{5}{7}$ of 1 guinea, $\frac{4}{5}$ of £1 and $\frac{3}{4}$ of 1*s*.

Example. A man owns $\frac{2}{5}$ of a ship, and the value of his share is £5000. What is the value of the whole ship?

	2-fifths of the ship	= £5000
	1-fifth of the ship	= £2500
	5-fifths of the ship	= £2500 × 5
i.e.,	whole ship	= **£12500**

Check by taking $\frac{2}{5}$ of £12500.

This is an example of the *Unitary Method*.

16. If $\frac{3}{7}$ of a man's income is £585, find his income.
17. If $\frac{7}{12}$ of a sum of money is 2 guineas, find the sum.
18. $\frac{3}{10}$ of an estate is 51 ac. How many acres are there in the estate?
19. A man has an income of £500 and spends $\frac{7}{8}$ of it on living-expenses. How much remains?

20. Find the cost of 7 articles at £2 5*s*. per dozen.

21. A shopkeeper pays £66 rent for the ground floor of a house. This is three-fifths of the total rent of the house. What is the total rent?

22. A typist is paid 3 guineas per week. This is $\frac{2}{17}$ of the manager's salary. How many guineas does the manager receive in four weeks?

Equivalent Fractions. Reduction of Vulgar Fractions to their Lowest Terms

If you examine a foot-rule divided into $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ in., you will see that

6-eighths of an inch is the same as 3-fourths of an inch
1-half of an inch is the same as 8-sixteenths of an inch

$$i.e., \frac{6}{8} = \frac{3}{4} \text{ and } \frac{1}{2} = \frac{8}{16}$$

24 in. $= \frac{24}{36}$ of 1 yd., but 24 in. equal 2 ft. and 2 feet are $\frac{2}{3}$ of 1 yd.

$$\therefore \frac{2}{3} \text{ and } \frac{24}{36} \text{ are equivalent fractions.}$$

These are instances of the following important principle. If *both* numerator and denominator of a vulgar fraction are *multiplied* or *divided* by the *same* number, the fraction is *unaltered* in value.

The fraction is said to be in its *lowest* terms when no more common divisors of numerator and denominator can be found.

The process of dividing by common factors is called *cancelling*.

Example. Reduce $\frac{350}{2800}$ to its lowest terms.

$$\frac{350}{2800} = \frac{35}{280} = \frac{7}{56} = \frac{1}{8}$$

by successive divisions of numerator and denominator by 10, 5, 7; or 10 and 35.

Exercises XVII

Reduce the following fractions to their lowest terms

1. $\frac{6}{8}, \frac{15}{30}, \frac{12}{60}, \frac{21}{63}, \frac{33}{77}$.
2. $\frac{32}{48}, \frac{48}{112}, \frac{77}{88}, \frac{50}{120}, \frac{550}{660}$.

3. $\frac{420}{630}$, $\frac{512}{640}$, $\frac{77}{1386}$, $\frac{147}{245}$.

4. $\frac{1024}{1536}$, $\frac{441}{1176}$, $\frac{37}{111}$, $\frac{333}{888}$, $\frac{37}{3737}$.

5. $\frac{111}{111{,}111}$, $\frac{1680}{6300}$, $\frac{2187}{2916}$, $\frac{572}{3003}$.

Mixed Numbers and Improper Fractions

15 farthings $= 3\frac{3}{4}d.$ $\therefore$ $\frac{15}{4}d. = 3\frac{3}{4}d.$, *i.e.*, $\frac{15}{4} = 3\frac{3}{4}$.
$2s.\ 5d. = 29$ pence $= \frac{29}{12}$ shillings.
Also $2s.\ 5d. = 2\frac{5}{12}$ shillings, *i.e.*, $2\frac{5}{12} = \frac{29}{12}$.

Fractions whose numerators are bigger than their denominators are called *improper fractions* ($\frac{15}{4}$, $\frac{29}{12}$, $\frac{22}{7}$, etc.)

$3\frac{3}{4}$, $2\frac{5}{6}$, $3\frac{1}{7}$, *i.e.*, combinations of whole numbers and proper fractions, are called *mixed numbers.*

To change an improper fraction into a mixed number we divide the numerator by the denominator

$$\frac{15}{4} = \frac{12+3}{4} = 3 + \frac{3}{4} = 3\tfrac{3}{4}.$$

To change a mixed number into an improper fraction, we multiply the integral part of the mixed number by the denominator of the fraction and add in the numerator. This gives the numerator of the improper fraction; the denominator is unaltered.

$$2\tfrac{5}{6} = \frac{2 \times 6 + 5}{6} = \frac{\mathbf{17}}{\mathbf{6}}.$$

Exercises XVIII

Express the following mixed numbers as improper fractions:

1. $2\frac{1}{2}$, $3\frac{1}{4}$, $5\frac{2}{7}$.
2. $6\frac{3}{8}$, $7\frac{7}{10}$, $9\frac{1}{14}$.
3. $9\frac{11}{27}$, $10\frac{16}{23}$, $3\frac{1}{33}$.
4. $11\frac{5}{11}$, $13\frac{4}{13}$, $19\frac{5}{10}$.
5. $5\frac{3}{100}$, $8\frac{3}{1000}$, $6\frac{7}{10000}$.

Express as mixed numbers

6. $\frac{15}{7}$, $\frac{19}{4}$, $\frac{31}{11}$.
7. $\frac{55}{14}$, $\frac{83}{20}$, $\frac{117}{100}$.
8. $\frac{22}{7}$, $\frac{77}{14}$, $\frac{355}{113}$.
9. $\frac{95}{15}$, $\frac{100}{85}$, $\frac{48}{32}$.
10. $\frac{99}{63}$, $\frac{112}{100}$, $\frac{1001}{100}$.

Addition and Subtraction of Vulgar Fractions

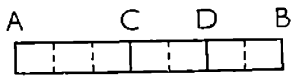
We may read in books on the history of mathematics about the difficulties experienced by ancient races when dealing with fractions. The Egyptians used fractions with numerator 1; other races used fractions with fixed denominators such as 12 or 60. The Egyptians, three or four thousand years ago, wrote $\frac{1}{3}\ \frac{1}{15}$, meaning $\frac{1}{3}$ plus $\frac{1}{15}$, instead of $\frac{2}{5}$, and as late as the time of Archimedes (200–300 B.C.) we find $\frac{1}{6} + \frac{1}{7}$ written when we should use $\frac{13}{42}$.

We can add at once fractions which have the same denominator.

3-sevenths + 2-sevenths = 5-sevenths.

i.e., $$\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}.$$

Illustration



If AB is the unit, AC is 3-sevenths, CD 2-sevenths. The sum of AC and CD is AD, 5-sevenths of the unit.

To add fractions which have different denominators, we first find fractions equivalent to them and which have the same denominator.

Example 1. Add together $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$.

Multiplying all the denominators together, we get $3 \times 6 \times 9 = 162$. This number will have all the denominators as factors.

$$\left.\begin{array}{l}\frac{1}{3} = 54\\ \frac{1}{6} = 27\\ \frac{1}{9} = \underline{18}\end{array}\right\} \text{162th} \qquad \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = 99/\text{162th}$$

$$\text{Add } \overline{99} \qquad\qquad = \frac{99}{162} = \frac{11}{18}.$$

18, the L.C.M. of the denominators, would be a much simpler common denominator.

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{9} = (6 + 3 + 2)\text{18th} = 11/\text{18th} = \frac{11}{18}.$$

Rule. To add fractions. Reduce all the fractions to the same denominator (for convenience, the L.C.M. of the separate denominators) and add the numerators. Integers can be added separately.

$$\text{Thus } 4\tfrac{7}{20} + 3\tfrac{8}{15} + 1\tfrac{5}{12} = (4 + 3 + 1) + \frac{21 + 32 + 25}{60}$$
$$= 8\tfrac{78}{60} = 8 + 1\tfrac{18}{60} = \mathbf{9\tfrac{3}{10}}.$$

Example 2. Subtract $2\frac{3}{4}$ from $3\frac{1}{3}$; *i.e.*, what fraction added to $2\frac{3}{4}$ will give $3\frac{1}{3}$?

$\frac{1}{4}$ added to $2\frac{3}{4}$ makes 3, $\therefore$ answer is $\frac{1}{4} + \frac{1}{3} = \mathbf{\frac{7}{12}}$.

Example 3. $$5\tfrac{1}{3} - 2\tfrac{3}{4} = 3 + \frac{4 - 9}{12} = 2 + \frac{12 + 4 - 9}{12}$$
$$= 2 + \tfrac{7}{12} = \mathbf{2\tfrac{7}{12}}.$$

(3 is changed mentally into 2 + 1, and this 1 is replaced by its equivalent $\frac{12}{12}$.)

Note. Check by addition.

Exercises (Mental) XIX

Add together

1. $\frac{1}{3}$ and $\frac{1}{4}$.
2. $\frac{2}{5}$ and $\frac{7}{10}$.
3. $\frac{1}{6}$ and $\frac{1}{8}$.
4. $\frac{3}{8}$ and $\frac{3}{4}$.
5. $\frac{5}{18}$ and $\frac{7}{36}$.
6. $\frac{8}{21}$ and $\frac{5}{42}$.
7. $\frac{3}{16}$ and $\frac{7}{48}$.
8. $\frac{5}{14}$ and $\frac{9}{21}$.
9. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{6}$.
10. $\frac{3}{5}$, $\frac{1}{2}$, and $\frac{7}{10}$.

Subtract

11. $2\frac{1}{3}$ from 3.
12. $2\frac{3}{7}$ from $3\frac{1}{7}$.
13. $2\frac{5}{6}$ from $3\frac{1}{3}$.
14. $\frac{7}{100}$ from $\frac{3}{10}$.
15. $\frac{1}{5}$ from $\frac{1}{3}$.
16. $2\frac{1}{3}$ from $3\frac{1}{4}$.
17. $\frac{1}{14}$ from $\frac{3}{7}$.
18. $2\frac{5}{8}$ from $4\frac{1}{12}$.
19. $3\frac{1}{6}$ from $4\frac{2}{3}$.
20. $3\frac{2}{3}$ from $5\frac{5}{12}$.

Exercises XX

Express as simple fractions

1. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$.
2. $\frac{3}{7} + \frac{5}{14} + \frac{1}{21}$.
3. $2\frac{1}{3} + 3\frac{1}{6} + \frac{1}{8}$.
4. $\frac{5}{12} + \frac{3}{16} + \frac{7}{48}$.
5. $1\frac{5}{6} + 2\frac{5}{18} + 1\frac{7}{36}$.
6. $1\frac{3}{5} + 2\frac{2}{7} + 3\frac{4}{11}$.
7. $2\frac{69}{100} - 1\frac{7}{10}$.
8. $\frac{19}{100} - \frac{9}{50}$.
9. $\frac{3}{1000} - \frac{19}{100{,}000}$.
10. $3\frac{3}{7} - 1\frac{1}{14}$.
11. $5\frac{1}{3} - 3\frac{1}{5}$.
12. $2\frac{5}{18} - \frac{3}{27}$.

13. $\frac{1}{11} + \frac{3}{22} + \frac{5}{66}$.
14. $3\frac{1}{2} + 5\frac{1}{3} + 2\frac{5}{12}$.
15. $8\frac{11}{36} - 4\frac{5}{18}$.
16. $6\frac{8}{21} - 3\frac{9}{14}$.
17. $3\frac{7}{8} + 2\frac{1}{12} - 4\frac{1}{3}$.
18. $4\frac{1}{7} + 2\frac{1}{3} - 1\frac{3}{5}$.
19. $3\frac{1}{4} + \frac{29}{30} - 2$.
20. $10\frac{1}{2} + 6\frac{1}{4} + \frac{9}{16} - 13\frac{3}{8}$.
21. $6\frac{2}{5} + 4\frac{1}{6} - 8\frac{3}{10}$.
22. $5\frac{2}{5} + \frac{17}{56} + 4\frac{5}{14}$.
23. $7\frac{13}{16} + \frac{17}{72} + 3\frac{2}{9}$.
24. $3\frac{1}{16} + 2\frac{17}{48} + 1\frac{37}{80}$.
25. $5\frac{3}{10} + 4\frac{19}{75} + 1\frac{5}{6} - 6\frac{2}{3}$.
26. $2\frac{3}{4} + 1\frac{5}{8} + 2\frac{3}{10} + 1\frac{8}{15} - 3\frac{5}{24}$.
27. $6\frac{1}{5} - 4\frac{7}{10} + 7\frac{1}{2} - 5\frac{2}{3}$.
28. $12\frac{14}{15} - 3\frac{7}{24} + 16\frac{11}{20} - 10\frac{23}{40}$.
29. $19 - (3\frac{3}{4} + 7\frac{5}{8})$. The sum of the quantities inside the bracket is to be taken from 19.
30. $25\frac{7}{12} - (1\frac{2}{3} + 14\frac{3}{5})$.

To Multiply a Fraction by an Integer

Example 1. Multiply $\frac{5}{32}$ by 8.

$$5/32\text{nds} \times 8 = 40/32\text{nds} = \frac{40}{32} = \frac{5}{4} = \mathbf{1\frac{1}{4}}.$$

Or, since 8/32nds make 1/4th (compare this with 12 pence make 1 shilling)

$$5/32\text{nds} \times 8 = 5/4\text{ths} = \frac{5}{4} = \mathbf{1\frac{1}{4}}.$$

Rule. Either multiply the numerator or divide the denominator of the fraction by the given integer.

Example 2. Multiply $\frac{4}{25}$ by 5.

$\frac{4}{25} \times 5 = \frac{4}{5}$, dividing denominator by 5.

Example 3. Multiply $\frac{3}{14}$ by 6.

$$\frac{3}{14} \times 6 = \frac{18}{14} = \frac{9}{7} = \mathbf{1\frac{2}{7}}.$$

Exercises (Mental) XXI

1. Multiply $\frac{1}{6}$ by 3, $\frac{1}{12}$ by 4, $\frac{1}{36}$ by 6.
2. Multiply $\frac{1}{5}$ by 3, $\frac{1}{12}$ by 7, $\frac{1}{18}$ by 11.
3. Multiply $\frac{3}{7}$ by 2, 5, 6, 10.
4. Multiply $\frac{5}{42}$ by 3, 6, 7.
5. Multiply $\frac{17}{100}$ by 4, 5, 10, 50.
6. Multiply $\frac{4}{15}$ by 3, 4, 5, 20.
7. Multiply $\frac{5}{32}$ by 4, 5, 7, 16.
8. Multiply $\frac{6}{35}$ by 5, 6, 7, 25.
9. Multiply $\frac{9}{40}$ by 3, 5, 6, 8, 10.
10. Multiply $\frac{7}{24}$ by 5, 6, 7, 8, 15.

Division of a Fraction by an Integer

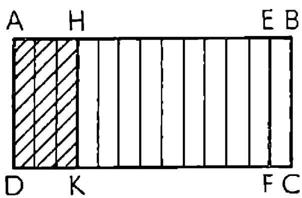
Just as the result of dividing £12 by 4 is £3, so the result of dividing 12-thirteenths by 4 is 3-thirteenths

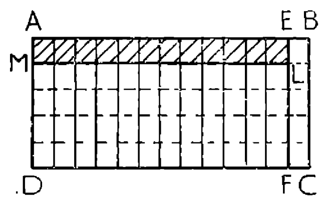
$$\tfrac{12}{13} \div 4 = \tfrac{3}{13}.$$

If we divide £12 by 5, we have to introduce further units, viz., shillings, because 5 is not a factor of 12. In a similar way to divide $\tfrac{12}{13}$ by 5, we may change 12-thirteenths to the equivalent fraction 60 sixty-fifths and then divide by 5, getting $\tfrac{12}{65}$.

$$\tfrac{12}{13} \div 5 = \tfrac{12}{65}.$$

These results are illustrated in these two figures.





ABCD is the unit, *AEFD* $\tfrac{12}{13}$ of it. *AHKD* is one-quarter of *AEFD* and *AELM* one-fifth of *AEFD*.

Hence to divide a fraction by an integer, divide the numerator by the integer if possible, keeping the denominator unchanged. If this cannot be done without leaving a remainder, keep the numerator unchanged and multiply the denominator by the integer, cancelling any common factor.

Thus $$\frac{25}{36} \div 10 = \frac{25}{36 \times 10} = \frac{5}{36 \times 2} = \frac{5}{72}.$$

Exercises XXII

Divide

1. $\tfrac{8}{11}$ by 2 and 4.
2. $\tfrac{3}{7}$ by 2 and 4.
3. $\tfrac{11}{36}$ by 3, 9, 11.
4. $\tfrac{4}{21}$ by 4, 5, 6, 7.
5. $\tfrac{15}{36}$ by 3, 6, 10.
6. $\tfrac{7}{100}$ by 7, 4, 21.
7. $\tfrac{9}{10}$ by 3, 5, 7, 9.
8. $\tfrac{22}{25}$ by 4, 5, 10, 11.
9. $\tfrac{14}{45}$ by 4, 7, 10, 28.
10. $\tfrac{49}{60}$ by 2, 5, 7, 21.

Multiplication and Division of a Mixed Number by an Integer

Multiplication

Example. Multiply $4\frac{2}{9}$ by 3

$$(4 + \tfrac{2}{9}) \times 3 = 12 + \tfrac{6}{9} = \mathbf{12\tfrac{2}{3}}.$$

or $4\frac{2}{9} \times 3 = \frac{38}{9} \times 3 = \frac{38}{3}$ (cancelling) $= \mathbf{12\frac{2}{3}}$.

Division

Example. $4\frac{2}{9} \div 3 = \frac{38}{9} \div \frac{1}{3} = \frac{38}{27} = \mathbf{1\frac{11}{27}}$

or $4\frac{2}{9} \div 3 = (3 + 1\frac{2}{9}) \div 3 = 1 + \frac{11}{9} \div 3$
$= \mathbf{1\frac{11}{27}}$.

General Method. Change the mixed number into an improper fraction and proceed as in the previous sections.

Alternative Method. Multiply or divide the integer and the fraction separately as in the worked examples (this is the better method if you are doing the question mentally).

Exercises XXIII

Multiply

1. $3\frac{1}{3}$ by 2, 3, 5, 6.
2. $2\frac{1}{6}$ by 3, 5, 8, 12.
3. $33\frac{1}{3}$ by 3, 6, 9.
4. $8\frac{5}{9}$ by 3, 5, 9.
5. $16\frac{2}{3}$ by 2, 8, 12.
6. $37\frac{1}{2}$ by 3, 4, 8.

Divide

7. $4\frac{1}{2}$ by 2, 3, 4.
8. $5\frac{1}{7}$ by 2, 3, 5, 7.
9. $3\frac{3}{10}$ by 3, 6, 10.
10. $8\frac{1}{6}$ by 3, 6, 9.
11. $16\frac{3}{7}$ by 3, 5.
12. $12\frac{1}{2}$ by 2, 3, 8.

Multiplication and Division of a Fraction by a Fraction

Multiplication. We are accustomed to such statements as: the cost of 2 yd. of silk at 7 shillings a yard is 7 shillings multiplied by 2. In the same way we would say that the cost of $2\frac{3}{4}$ yd. of this silk is 7 shillings multiplied by $2\frac{3}{4}$, or that the cost of $\frac{3}{4}$ yd. is 7 shillings multiplied by $\frac{3}{4}$. To find the last cost we should multiply 7 shillings by 3 and divide by 4; that is, we take '7 shillings multiplied by $\frac{3}{4}$' to be the same as '$\frac{3}{4}$ of 7 shillings.'

Thus $7 \times \frac{3}{4} = \frac{7 \times 3}{4} = \frac{21}{4} = \mathbf{5\frac{1}{4}}$.

$$\frac{3}{7} \times \frac{4}{5} = \frac{3}{7} \times 4 \div 5 = \frac{3 \times 4}{7} \div 5 = \frac{3 \times 4}{7 \times 5} = \mathbf{\frac{12}{35}}.$$

Method. Multiply the numerators together to form the numerator, and the denominators together to form the denominator, of the product.

To multiply mixed numbers, change the mixed numbers into improper fractions and proceed as above.

$$3\frac{3}{5} \times 4\frac{1}{6} = \frac{18}{5} \times \frac{25}{6}$$

$$= \frac{\overset{3}{\cancel{18}} \times \overset{5}{\cancel{25}}}{\cancel{5} \times \cancel{6}}$$

Cancel the common factors of 18 and 6, of 25 and 5.

$$= \mathbf{15}$$

Division

Example. Divide $\frac{3}{7}$ by $\frac{4}{5}$.

This means 'by what fraction must $\frac{4}{5}$ be multiplied to give $\frac{3}{7}$ as the result?'

$$\frac{4}{5} \times \frac{5}{4} = 1$$

$$\therefore \frac{3}{7} = \frac{3}{7} \times (\frac{5}{4} \times \frac{4}{5})$$

$$= (\frac{3}{7} \times \frac{5}{4}) \times \frac{4}{5}.$$

So that $\frac{3}{7} \times \frac{5}{4}$ is the fraction by which $\frac{4}{5}$ must be multiplied to give $\frac{3}{7}$

$$i.e., \frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4} = \frac{15}{28}.$$

Hence, to divide one fraction by another one, we multiply the first fraction by the second inverted.

Note. Mixed numbers must be changed into improper fractions before this rule is applied.

$$3\frac{1}{5} \div 2\frac{1}{3} = \frac{16}{5} \div \frac{7}{3}$$

$$= \frac{16}{5} \times \frac{3}{7} \text{ (inverted divisor)}$$

$$= \frac{48}{35}$$

$$= \mathbf{1\frac{13}{35}}.$$

Exercises XXIV

Multiply

1. $2\frac{1}{3}$ by $3\frac{1}{5}$.
2. $5\frac{2}{7}$ by $2\frac{3}{37}$.
3. $\frac{14}{25}$ by $\frac{50}{63}$.
4. $1\frac{11}{24}$ by $1\frac{11}{35}$.
5. $\frac{21}{25}$ by $\frac{30}{77}$.
6. $\frac{66}{35}$ by $\frac{14}{11}$.
7. $21\frac{1}{11}$ by $4\frac{16}{20}$.
8. $5\frac{78}{81}$ by $8\frac{37}{46}$.
9. $\frac{5}{6}$ by $2\frac{4}{25}$.
10. $4\frac{8}{15}$ by $1\frac{1}{17}$.

Divide

11. $\frac{3}{5}$ by $\frac{2}{7}$.
12. $\frac{7}{8}$ by $\frac{3}{4}$.
13. $\frac{11}{36}$ by $\frac{2}{9}$.
14. $1\frac{1}{2}$ by $3\frac{3}{4}$.
15. $8\frac{2}{3}$ by $1\frac{4}{9}$.
16. $8\frac{1}{2}$ by $1\frac{1}{4}$.
17. $\frac{50}{63}$ by $\frac{100}{49}$.
18. $10\frac{5}{16}$ by $11\frac{11}{64}$.
19. $3\frac{1}{11}$ by $2\frac{19}{33}$.
20. $3\frac{7}{27}$ by $\frac{16}{81}$.
21. $30\frac{5}{64}$ by $21\frac{7}{128}$.
22. $25\frac{25}{36}$ by $13\frac{7}{8}$.

Simplify

23. $3\frac{1}{3} \times 2\frac{1}{4} \times 1\frac{1}{5}$
24. $3\frac{3}{5} \times 1\frac{7}{8} \times 2\frac{1}{4}$.
25. $\frac{6}{7} \times \frac{2}{13} \times 1\frac{6}{7}$.
26. $\frac{17}{20} \times \frac{5}{42} \div \frac{17}{72}$.
27. $3\frac{4}{7} \times 2\frac{4}{5} \div \frac{27}{35}$.
28. $2\frac{2}{3} \times 3\frac{3}{4} \div 7\frac{1}{7}$.
29. $1\frac{3}{5} \times 2\frac{1}{7} \times 1\frac{5}{16}$.
30. $1\frac{2}{7} \div \frac{5}{21} \times 3\frac{1}{3}$ ($1\frac{2}{7}$ is to be divided by $\frac{5}{21}$, and the result multiplied by $3\frac{1}{3}$).
31. $\frac{15}{29} \div 2\frac{1}{2} \times 6\frac{4}{9}$.
32. $7\frac{9}{13} \div (1\frac{8}{17} \times \frac{10}{13})$ ($7\frac{9}{13}$ is to be divided by the product in the bracket).
33. $3\frac{3}{4} \times \frac{3}{5} \div 9\frac{3}{8}$.
34. $2\frac{2}{3} \times 1\frac{4}{5} \times 4\frac{5}{6}$.
35. $100 \div (3\frac{1}{3} \times 1\frac{1}{4})$.

Complex Fractions

It has been seen that a length of 2 ft. = 24 in. may be expressed as $\frac{24}{36}$ or $\frac{2}{3}$ of a yard. If we wished to express a length of $2\frac{1}{2}$ ft. as a fraction of a length of 2 yd., we could say that since

$$2\tfrac{1}{2} \text{ ft.} = 30 \text{ in.,} \quad 2 \text{ yd.} = 72 \text{ in.}$$

the required fraction is $\frac{30}{72} = \frac{5}{12}$.

We might also say that since 2 yd. = 6 ft., the fraction is $\dfrac{2\frac{1}{2}}{6}$. Hence $\dfrac{2\frac{1}{2}}{6} = \dfrac{5}{12}$. We see that this is in agreement with the rule that multiplication of the numerator and denominator by the same number leaves the value of the fraction unchanged.

Again, to express 2*s*. 6*d*. as a fraction of 3*s*. 4*d*.

$$2s.\ 6d. = 2\tfrac{1}{2}s. = 30d.$$
$$3s.\ 4d. = 3\tfrac{1}{3}s. = 40d.$$

$\therefore$ the required fraction is $\dfrac{2\frac{1}{2}}{3\frac{1}{3}}$ or $\dfrac{30}{40} = \mathbf{\dfrac{3}{4}}$.

To simplify $\dfrac{2\frac{1}{2}}{3\frac{1}{3}}$, multiply numerator and denominator by 6.

$$\text{fraction} = \frac{15}{20} = \mathbf{\frac{3}{4}}.$$

SIMPLE COMPLEX FRACTIONS

Example. $\dfrac{5\frac{1}{35}}{2\frac{1}{5}} = \dfrac{176}{35} \div \dfrac{11}{5}$

$$= \frac{\overset{16}{\cancel{176}}}{\underset{7}{\cancel{35}}} \times \frac{\cancel{5}}{\cancel{11}} = \frac{16}{7} = \mathbf{2\tfrac{2}{7}}.$$

Or multiplying numerator and denominator by 35, the L.C.M. of 35 and 5,

$$\frac{5\frac{1}{35}}{2\frac{1}{5}} = \frac{176}{77} = \frac{16}{7} = \mathbf{2\tfrac{2}{7}}.$$

EXERCISES XXV

Simplify

1. $\dfrac{1\frac{3}{4}}{3\frac{1}{2}}, \dfrac{3\frac{1}{2}}{5}, \dfrac{2\frac{1}{3}}{7}, \dfrac{\frac{4}{5}}{1\frac{2}{3}}$.

2. $\dfrac{12\frac{1}{2}}{100}, \dfrac{16\frac{2}{3}}{100}, \dfrac{8\frac{1}{3}}{10}, \dfrac{22\frac{1}{2}}{100}$.

3. $\dfrac{4\frac{1}{11}}{20}, \dfrac{70}{8\frac{3}{4}}, \dfrac{40}{4\frac{1}{6}}, \dfrac{4\frac{4}{9}}{4}$.

4. $\dfrac{6\frac{9}{11}}{11\frac{3}{22}}, \dfrac{3\frac{1}{14}}{4\frac{2}{21}}, \dfrac{5\frac{1}{3}}{6\frac{1}{2}}, \dfrac{3\frac{1}{6}}{5\frac{1}{15}}$.

TO EXPRESS ONE COMPOUND QUANTITY AS A FRACTION OF ANOTHER ONE

Example. Find what fraction £1 12*s*. 6*d*. is of £2 17*s*. 6*d*.

Taking £1 as the unit,

$$\text{fraction} = \frac{1\frac{5}{8}}{2\frac{7}{8}} = \mathbf{\frac{13}{23}}.$$

Taking 2*s*. 6*d*. as the unit,

$$\text{fraction} = \mathbf{\frac{13}{23}}.$$

Taking 1*s*. as the unit,

$$\text{fraction} = \frac{32\frac{1}{2}}{57\frac{1}{2}} = \frac{65}{115} = \frac{13}{23}.$$

Taking 1*d*. as the unit,

$$\text{fraction} = \frac{390}{690} = \frac{39}{69} = \frac{13}{23}.$$

(The larger units often involve less computation.)

Exercises XXVI

Find what fraction

1. 6*s*. 8*d*. is of £1 13*s*. 4*d*.
2. £1 3*s*. is of £1 6*s*. 10*d*.
3. £1 2*s*. 6*d*. is of £2 2*s*. 6*d*.
4. 3 yd. 1 ft. is of 5 yd. 2 ft.
5. 1 ml. 10 ch. is of 2 ml. 3 f.
6. 8 dollars 50 cents is of 3 dollars 25 cents.
7. 5 gal. 1 qt. is of 3 gal. 2 qt.
8. 2 hr. 15 min. is of 3 hr. 10 min.
9. 2 tons 5 cwt. is of 3 tons 7 cwt. 2 qr.
10. 2 ac. 1 r. 14 p. is of 3 ac. 2 r. 1 p.
11. 2 tons 6 cwt. 1 qr. 15 lb. is of 5 tons 2 cwt. 5 lb.
12. 2 hr. 7 min. is of 5 hr. 30 min. 12 sec.

Exercises XXVII

Example. Simplify $3\frac{1}{2} - 2\frac{1}{3} \div 1\frac{1}{6}$.

When there are no brackets to show which of the operations should be done first, the convention that multiplication and division precede addition and subtraction must be followed.

Expression $= 3\frac{1}{2} - \frac{7}{3} \times \frac{6}{7} = 3\frac{1}{2} - 2 = 1\frac{1}{2}$.

Simplify

1. $3\frac{1}{2} \times 2\frac{1}{3} + 5\frac{1}{6}$.
2. $\frac{2}{3} \times (3\frac{3}{4} + \frac{1}{8})$.
3. $3 \div (2\frac{2}{3} - \frac{5}{9})$.
4. $3\frac{1}{2} - 2\frac{1}{3} \times 1\frac{1}{2}$.
5. $\dfrac{12 - 8\frac{1}{3}}{4\frac{1}{3} + 2\frac{1}{6}}$.
6. $\dfrac{10}{1\frac{1}{4}} + \dfrac{16}{2\frac{2}{3}}$.
7. $\dfrac{14 \times 3\frac{2}{7}}{2\frac{2}{3} + 3\frac{1}{12}}$.
8. $\dfrac{8\frac{1}{2} \div 3\frac{1}{3}}{6 - 2\frac{2}{5}}$.
9. $\dfrac{\frac{11}{28} + \frac{16}{35}}{4\frac{1}{4}}$.
10. $\dfrac{4\frac{3}{4} + 6\frac{1}{5}}{7\frac{3}{10} \div 2\frac{1}{4}}$.

11. $\dfrac{1}{2 + \dfrac{1}{3 + \frac{1}{4}}}$.

12. $\dfrac{2\frac{1}{2} - \dfrac{1}{2\frac{1}{2}}}{5\frac{1}{4}}$.

13. $3\frac{1}{3} + \frac{1}{6} \times 3\frac{1}{3}$.

14. $(3\frac{1}{2} + 2\frac{1}{3})(3\frac{1}{2} - 2\frac{1}{3})$.

15. $\dfrac{2\frac{3}{4} + 5\frac{1}{8} - 3\frac{1}{2}}{3\frac{1}{2} + \frac{1}{3} \text{ of } 2\frac{1}{4}}$.

16. $\dfrac{17}{17 - 4\frac{1}{4}} - \dfrac{15}{15 - 3\frac{3}{4}}$.

17. $\dfrac{\dfrac{2}{2\frac{2}{3}} + \dfrac{3}{3\frac{3}{4}}}{\dfrac{1}{1\frac{1}{9}} - \dfrac{1}{1\frac{1}{4}}}$.

18. $\dfrac{\frac{1}{4} + \frac{2}{3} \text{ of } \frac{7}{8}}{\frac{1}{2} + \frac{1}{3}} \times \dfrac{2 - \frac{3}{4}}{1 + 1\frac{1}{2}}$.

Exercises (Miscellaneous) XXVIII

1. A man does $\frac{7}{10}$ of a journey by rail, $\frac{1}{6}$ by car, and walks the rest. How far did he walk if the total journey was 20 ml. 350 yd.?

2. $\frac{2}{5}$ of $\frac{3}{4}$ of an estate is 150 ac. What is the area of the estate?

3. If $(\frac{1}{3} + \frac{1}{6} + \frac{1}{12})$ of a sum of money is 11 guineas, find this sum.

4. A jug holds $\frac{7}{11}$ of a quart and another one $\frac{5}{9}$ of a quart. By what fraction of a pint do the contents of one jug exceed those of the other?

5. Find a number such that when one-24th of it is taken away the remainder is 253.

6. Smith major's time for the mile is $\frac{6}{7}$ of Smith minor's time; find the latter's time if Smith major takes 5 min. 12 sec.

7. From a roll of silk 40 yd. long 15 pieces each $2\frac{1}{4}$ yd. long are cut off. What length remains?

8. How many glasses each holding $\frac{7}{10}$ of a pint can be filled from a jug containing $\frac{1}{2}$ gal. How much will be left over?

9. Three-quarters of a cake is divided between 12 boys. What fraction of the cake does each receive?

10. The yearly rent of $6\frac{3}{4}$ ac. of land was £33 15*s*. How much is this per acre?

11. Find the product of 183 × $112\frac{1}{2}$ by multiplying 183 by 100 and adding $\frac{1}{8}$ of the result.

12. Find a similar subtraction method for multiplying 936 by $87\frac{1}{2}$.

Find short methods for working out the following products

13. $37 \times 12\frac{1}{2}$.
14. $638 \times 16\frac{2}{3}$.
15. 894×75.
16. $999 \times 66\frac{2}{3}$.
17. $660 \times 26\frac{2}{3}$.
18. 256×225.
19. $660 \times 13\frac{1}{3}$.
20. $486 \times 116\frac{2}{3}$.
21. $894 \times 33\frac{1}{3}$.
22. 999×375.

23. Find the cost of 100 three-halfpenny stamps. (Since $1\frac{1}{2}d$. is one-eighth of 1*s*., the cost is one-eighth of 100*s*.)

24. Use a similar method to find the value of 500 half-crowns.

25. What fraction is 6*s*. 8*d*. of £1? Find the cost of 1000 articles at 6*s*. 8*d*. each.

26. What fraction of £1 is 1*s*. 4*d*.? Find the cost of 75 articles at 1*s*. 4*d*. each.

Harder Examples on Vulgar Fractions

(To be omitted on a first reading)

Example. Simplify $2\frac{3}{4} + \dfrac{3\frac{1}{2} - 1\frac{7}{8}}{\frac{3}{4} \text{ of } 1\frac{3}{10}} - 4\frac{1}{12}$.

Given expression equals

$$2\frac{3}{4} + \frac{1\dfrac{8+4-7}{8}}{\frac{3}{4} \times \frac{13}{10}} - 4\frac{1}{12}$$

$$= 2\frac{3}{4} + \frac{\frac{13}{8}}{\frac{3 \times 13}{4 \times 10}} - 4\frac{1}{12}$$

$$= 2\frac{3}{4} + \frac{\cancel{13}}{\cancel{8}} \times \frac{\cancel{4} \times \overset{5}{\cancel{10}}}{3 \times \cancel{13}} - 4\frac{1}{12}$$

$$= 2\frac{3}{4} + 1\frac{2}{3} - 4\frac{1}{12}$$

$$= \frac{9 + 8 - 13}{12}$$

$$= \frac{4}{12}$$

$$= \frac{1}{3}.$$

It is generally best to write down the whole expression in each line, carrying out the simplification of the various parts one step at a time. It is then possible to arrange the work neatly and clearly.

If parts of the expression are simplified separately, care must be taken that the sign = is properly used.

EXERCISES XXIX

Express as a simple fraction

1. $\frac{1}{12} + \frac{1}{76} + \frac{1}{114}$.

2. $\frac{3}{44} + \frac{4}{77} + \frac{4}{55} - \frac{4}{35}$.

3. $\dfrac{(1 - \frac{1}{3})(\frac{1}{5} - \frac{1}{7})}{1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}}$.

4. $3\frac{1}{7} \times (5\frac{1}{4} - 3\frac{1}{2}) \times (5\frac{1}{4} + 3\frac{1}{2})$.

5. $\dfrac{3\frac{3}{7} - 1\frac{1}{2}}{1\frac{6}{11} + 1\frac{2}{5}} - \frac{1}{12}$.

6. $\dfrac{3\frac{1}{3} \times 2\frac{1}{4}}{3\frac{1}{3} - 2\frac{1}{4}} \div \dfrac{5\frac{1}{7}}{2\frac{3}{5}}$.

7. $\dfrac{1\frac{2}{7} + 2\frac{2}{5}}{6\frac{1}{2} + 3\frac{1}{3}} \times \dfrac{3\frac{1}{2} + 6\frac{1}{3}}{3\frac{4}{5} - 2\frac{2}{7}}$.

8. $1 + \dfrac{1}{4 + \dfrac{1}{3 + \frac{1}{5}}}$.

9. $\dfrac{1}{2 + \dfrac{1}{3 + \dfrac{1}{1 + \frac{1}{4}}}}$.

10. $\dfrac{1}{2 + \dfrac{3}{4 + \dfrac{5}{6 + \frac{7}{8}}}}$.

11. $\dfrac{3}{2 + \dfrac{2}{3 + \dfrac{2}{3 + \frac{2}{3}}}}$.

12. $\dfrac{\frac{1}{4} + \frac{2}{3} \text{ of } \frac{7}{8}}{\frac{1}{2} + \frac{1}{3}} \times \dfrac{2 - \frac{3}{4}}{1 + \frac{3}{2}}$.

13. $\dfrac{2\frac{2}{9} + 3\frac{11}{12} + 4\frac{3}{8} + 2\frac{5}{6}}{3\frac{7}{18} - 2\frac{5}{16}}$.

14. $\dfrac{2}{3 + \frac{2}{3}} \times \dfrac{2}{3 + \dfrac{2}{3 + \frac{2}{3}}}$.

15. $\dfrac{8\frac{7}{12}}{5\frac{11}{18} + 5\frac{5}{6}} - \dfrac{11\frac{3}{8}}{16\frac{11}{12} + 5\frac{5}{6}}$.

16. $\dfrac{3\frac{1}{2} \times 3\frac{1}{2} \times 3\frac{1}{2} - 3}{3\frac{1}{2} \times 3\frac{1}{2} - 3} \div 8\frac{23}{37}$.

17. $2 - \dfrac{17^2 - 15^2}{19^2 - 17^2} - \frac{1}{2} \text{ of } \dfrac{2 - \frac{7}{8}}{4 - 1\frac{3}{4}}$.

18. $\dfrac{\frac{1}{3}(\frac{1}{9} - \frac{1}{24}) + \frac{1}{4}(\frac{1}{16} - \frac{1}{18}) + \frac{1}{6}(\frac{1}{36} - \frac{1}{12})}{\frac{1}{3} + \frac{1}{4} + \frac{1}{6}}$.

CHAPTER IV

METHOD OF PARTS OR SIMPLE PRACTICE

Exercises XXX

1. Find the cost of 36 yd. of cotton at $11\frac{3}{4}d.$ per yd. by estimating the cost at 1*s.* per yd. and subtracting the cost at $\frac{1}{4}d.$ per yd.

In a similar way, find the cost of

2. 800 yd. at 1*s.* $11\frac{3}{4}d.$ per yd.
3. 85 articles at 19*s.* $10\frac{1}{2}d.$ each. (19*s.* $10\frac{1}{2}d.$ = £1 − $1\frac{1}{2}d.$)
4. 29 articles at 7*s.* 11*d.* each. (7*s.* 11*d.* = 8*s.* − 1*d.*)
5. 37 articles at 9*s.* 9*d.* each.

6. 8*s.* 4*d.* = 5*s.* + 3*s.* 4*d.* Find the cost of 360 articles at 8*s.* 4*d.* each by estimating $\frac{1}{4}$ and $\frac{1}{6}$ of £360 and adding the results. (8*s.* 4*d.* is also $\frac{5}{12}$ of £1; check your answer by finding $\frac{5}{12}$ of £360.)

7. £1 14*s.* 4*d.* = £1 + 10*s.* + 4*s.* + 4*d.*; 10*s.* = $\frac{1}{2}$ of £1; 4*s.* = $\frac{1}{5}$ of £1; 4*d* = $\frac{1}{12}$ of 4*s.* Find the cost of 720 articles at 10*s.*, 4*s.*, and 4*d.* each respectively, and hence, by addition, the cost at £1 14*s.* 4*d.* each.

8. Find the cost of 160 articles at 3*s.* 9*d.* each by determining the costs at 2*s.* 6*d.* and 1*s.* 3*d.* each. Check your answer by finding $\frac{3}{16}$ of £160.

The method of multiplication illustrated by Exercises XXX is called the *method of parts*, or *practice*. We now show the method of setting out, and how each line of the working is obtained simply from some previous line.

Example 1. Find the cost of 350 cwt. of metal at £3 13*s.* 9*d.* per cwt.

	£	*s.*	*d.*
Cost at £1	350	0	0
,, ,, £3	1050	0	0
,, ,, 10*s.*, $\frac{1}{2}$ of £1 ($\frac{1}{2}$ top line)	175	0	0
,, ,, 2*s.* 6*d.*, $\frac{1}{4}$ of 10*s.* ($\frac{1}{4}$ previous line)	43	15	0
,, ,, 1*s.* 3*d.*, $\frac{1}{2}$ of 2*s.* 6*d.* ($\frac{1}{2}$ previous line)	21	17	6
Total cost	**£1290**	**12**	**6**

Example 2. Find the cost of 478 articles at 9*s*. $2\frac{1}{2}$*d*. each.

	£	*s.*	*d.*
Cost at 1*s*.	23	18	0
,, ,, 9*s*.	215	2	0
,, ,, 2*d*., $\frac{1}{6}$ of top line,	3	19	8
,, ,, $\frac{1}{2}$*d*., $\frac{1}{4}$ of previous line,		19	11
Total cost	**£220**	**1**	**7**

It is sometimes simpler to use subtraction, as in the following example.

Example 3. Find the cost of 700 quarters of cereals at £1 19*s*. 6*d*. per quarter.

	£	*s.*	*d.*	
Cost at £1	700	0	0	
,, ,, £2	1400	0	0	
,, ,, 6*d*., $\frac{1}{40}$ of £1	17	10	0	(£70)
Subtract. Cost at £1 19*s*. 6*d*.	**£1382**	**10**	**0**	

The cost at 6*d*. is $\frac{1}{40}$ of cost at £1. This may be worked mentally in one step or, if this is found difficult, in two steps by factors 10 and 4. Put the result of the first division in brackets on the right as shown.

Exercises XXXI

Find the cost of

1. 43 lb. of tea at 2*s*. 3*d*. per lb.
2. 360 yd. of silk at 17*s*. 6*d*. per yd.
3. 450 lunches at 1*s*. 2*d*. each.
4. 65 motor-car batteries at £3 12*s*. each.
5. 35 armchairs at £5 16*s*. 3*d*. each.
6. 47 cricket-bats at 17*s*. 9*d*. each.
7. 27 lawn-mowers at £8 5*s*. 6*d*. each.
8. 880 qr. of cereals at £1 19*s*. 9*d*. per quarter.
9. 330 cwt. of rice at £1 3*s*. 9*d*. per cwt.
10. 725 shares in a company at 25*s*. 6*d*. per share.
11. 500 oz. of silver at 2*s*. $1\frac{1}{2}$*d*. per ounce.
12. 550 therms of gas at 1*s*. $1\frac{1}{4}$*d*. per therm.
13. 500 oz. of gold at £6 16*s*. 8*d*. per ounce.
14. 3284 articles at 9*s*. $1\frac{1}{2}$*d*. each.
15. 170 typewriters at £10 15*s*. 9*d*. each.
16. 50,000 cigarettes at 4*s*. 5*d*. per 100.

17. 119 quarterly payments of £4 9*s*. 11*d*.

18. 1000 yd. of carpet at 7*s*. 11*d*. per yard.

19. 175 pairs of shoes at 18*s*. 11*d*. per pair.

20. 85 bicycles at £7 17*s*. 3*d*. each.

21. Find the cost of printing a year's issue of 389 million tube-tickets at 2*s*. 8*d*. per thousand.

CHAPTER V

DECIMALS (I). METRIC SYSTEM

Consider the fraction $\frac{5}{16}$.

Multiplying both numerator and denominator by 10, 100, 1000, etc., we get the following equivalent fractions

$$\frac{5}{16}=\frac{50}{160}=\frac{500}{1600}=\frac{5000}{16000}=\frac{50000}{160000}.$$

By means of a series of divisions it will be found that 50000 is the first of these numerators that can be divided by 16 exactly.

$$\begin{array}{r} 3\ 1\ 2\ 5 \\ 16)\ 5\ 0\ 0\ 0\ 0 \\ 4\ 8 \\ \hline 2\ 0\ 0\ 0 \\ 1\ 6\ 0\ 0 \\ \hline 4\ 0\ 0 \\ 3\ 2\ 0 \\ \hline 8\ 0 \\ 8\ 0 \\ \hline \end{array}$$

$$\therefore \frac{5}{16}=\frac{3125}{10000}$$

$\frac{3125}{10000}$ is an example of a decimal fraction.

A decimal fraction can be defined as a fraction whose denominator is a power of 10, *i.e.*, one of 10, 100, 1000, 10000, etc.

Decimal Notation and Place-value

You will have realized when dealing with whole numbers that the value of each figure varies with its position: in the number 555, each 5 has a different value: 5 hundreds, 5 tens, 5 units.

In the decimal notation one-tenth, one-hundredth, one-thousandth are represented by ·1, ·01, ·001, etc., two-tenths by ·2, three thousandths by ·003, etc.

As a whole number 3125 = three thousand + one hundred + two tens + five units.

$$\frac{3125}{10000} = \frac{3000}{10000} + \frac{100}{10000} + \frac{20}{10000} + \frac{5}{10000}$$

$$= \frac{3}{10} + \frac{1}{100} + \frac{2}{1000} + \frac{5}{10000} \text{ [A]}$$

Using the decimal notation, we write this down as ·3125; each figure has a definite **place-value** according to its position with respect to the point.

53·267 = five tens + three units + two-tenths + six-hundredths + seven-thousandths.

To understand decimals you must realize the importance of place-value. Moving from the point to the left the digits increase in value by ten times; moving to the right they diminish by tenths. The point follows the units figure and shows where the sub-division of the unit begins.

Conversion of a Vulgar Fraction into a Decimal One

From [A] $\frac{5}{16}$ = **·3125**.

To change a vulgar fraction into a decimal one, we divide the numerator by the denominator, adding as many noughts as may be necessary; the number of figures after the decimal point is the same as the number of noughts added.

Example 1. Express $\frac{3}{7}$ as a decimal.

$$7 \,|\, \underline{3{\cdot}000000 \;.\;.\;.}$$
$$\mathbf{{\cdot}428571428571} \;.\;.\;.$$

The division never terminates, but the figures are repeated in a cycle. This kind of decimal is called a *recurring decimal.*

Example 2. Express ·2375 as a vulgar fraction.

$${\cdot}2375 = \frac{2375}{10000} = \frac{\mathbf{19}}{\mathbf{80}} \text{ (cancelling).}$$

Exercises XXXII

1. Add as vulgar fractions

(a) $\frac{3}{10} + \frac{5}{100} + \frac{6}{1000}$.
(b) $\frac{7}{10} + \frac{8}{1000} + \frac{9}{10000}$.
(c) $3 + \frac{5}{100} + \frac{7}{1000}$.
(d) $25 + \frac{3}{10} + \frac{7}{1000}$.

2. Express the answers to question 1 as decimals.

3. Express as decimals $\frac{3}{10}$, $\frac{5}{100}$, $\frac{6}{1000}$, $\frac{23}{100}$, $\frac{203}{1000}$, $\frac{2300}{10000}$, $\frac{350}{100000}$.

4. Add together 35, 65, and 30 centimes and express the answer in francs (1 franc = 100 centimes).

5. Add together 78, 56, 203, and 5 cents, and express the answer in dollars (1 dollar = 100 cents).

6. Subtract $\frac{1}{10}$ from $\frac{17}{100}$ and express the answer as a decimal.

7. Subtract $\frac{3}{100}$ from $\frac{5}{100}$ and express the answer as a decimal.

8. Subtract $\frac{17}{100}$ from $\frac{3}{10}$ and express the answer as a decimal.

9. Subtract 1 franc 15 centimes from 3 francs 20 centimes and express the answer in francs.

10. Subtract 8 dollars 50 cents from 30 dollars 20 cents, expressing the answer in dollars.

11. Express $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$ as decimals, and learn these results.

12. Express as decimals $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{15}{16}$.

13. What decimals of a foot are $1\frac{1}{2}$, 6, $7\frac{1}{2}$, $10\frac{1}{2}$ inches?

14. What decimals of a shilling are 3*d.*, $4\frac{1}{2}$*d.*, 9*d.*, $10\frac{1}{2}$*d.*?

Exercises XXXIII

Express as vulgar fractions in their lowest terms

1. 0·25, 0·5, 0·75.
2. 0·125, 0·25, 0·375.
3. 0·625, 0·875, 0·0625.
4. 0·1875, 0·4375, 0·8225

Express as mixed numbers

5. 5·13, 6·35, 8·48.
6. 9·05, 8·5, 7·205.
7. 3·008, 5·0505, 6·606.

Addition and Subtraction of Decimals

Addition	5·00567	*Subtraction*	8·08233
	8·08092		5·90726
	9·11506	Difference	2·17507
Sum	22·20165		(Check by addition)

Arrange the numbers with their decimal points directly below one another, as indicated by the arrows, so that figures with the same place-value are in the same columns. Then proceed as with whole numbers.

When a decimal is less than 1, zero is often placed before the point; 0·356 has the same value as ·356.

Exercises XXXIV

Add together

1. 3·5, 2·6, 4·07, 5·703.
2. 0·056, 0·56, 0·6.
3. 5·185, 2·9, 0·008.
4. 4·307, 0·076, 15·4, 19·37.
5. 55·78, 4·016, 0·054, 19·7.
6. 0·0056, 0·083, 0·000723.
7. 3·4076, 53·703, 0·8438.
8. 13·009, 4·6572, 1·89, 0·0079.
9. Subtract 0·1, 0·01, and 0·0001 from 1. (Three answers.)
10. Subtract 0·5, 0·005, and 0·00005 from 10. (Three answers.)
11. Which is the greater, 11·3 or 11·295, and by how much?
12. Which is the greater, 0·7 or 0·699, and by how much?

Subtract, and check your answers by addition

13. 0·101 from 1·0101.
14. 115·65 from 200.
15. 0·357 from 35·7.
16. 0·00265 from 0·003.
17. 8·715 from 11·2.
18. 39·9834 from 840·8923.

Add together

19. 1*s.* 11·62*d.*, 1*s.* 10·42*d.*, 1*s.* 9·86*d.*, 1*s.* 7·10*d.*
20. 18*s.* 9·12*d.*, 17*s.* 6·35*d.*, 12*s.* 3·13*d.*, 15*s.* 5·4*d.*
21. £1·50, £2·73, £3·27.
22. 15 francs 35 centimes, 17 francs 82 centimes, 19 francs 75 centimes.
23. 5·37 dollars and 12·13 dollars.
24. 18·68 dollars and 19·12 dollars.
25. Subtract each of the following sums of money from 2*s.*: 1*s.* 9·5*d.*, 1*s.* 10·46*d.*, 1*s.* 11·75*d.*
26. Subtract each of the following sums of money from 3s. 9d.: 2*s.* 1·75*d.*, 2*s.* 2·33*d.*, 3*s.* 7·82*d.*; and check by addition.
27. Subtract 37 dollars 18 cents from 50 dollars 5 cents.
28. Subtract 28·38 dollars from 30·15 dollars.

Multiplication and Division of a Decimal by 10, 100, 1000, etc.

Example 1. Multiply 34·567 by 10, 100, 1000, etc.

Number	=	34·567	or	= 34·567
10 times	=	345·67		= 345·67
100 times	=	3456·7		= 3456·7
1000 times	=	34567·		= 34567

To multiply by 10, 100, 1000, etc., leave the point in its original position and move the figures one, two, three . . . places to the *left* of the point, or move the point one, two, three . . . places to the *right* of its original position.

Example 2. Divide 1256·78 by 10, 100, 1000, etc.

Number	=	1256·78	or	= 1256·78
$\frac{1}{10}$	=	125·678		= 125·678
$\frac{1}{100}$	=	12·5678		= 12·5678
$\frac{1}{1000}$	=	1·25678		= 1·25678

Method. To divide by 10, 100, 1000, etc., leave the point in its original position and move the figures one, two, three . . . places to the *right* of the point, or move the point one, two, three . . . places to the *left* of its original position.

Exercises XXXV

1. Find the cost of 10, 100, 1000 lb. of rubber at 11·25*d*. per lb.

2. If $P = 3{\cdot}7 + 0{\cdot}058W$, find P when $W = 10, 100, 1000$.

3. If $W = 50{\cdot}4P - {\cdot}05$, find W when $P = 0{\cdot}01, 0{\cdot}001$.

4. Find the cost of 1000 lb. of cotton at 7·21*d*. per lb.

5. Find, in dollars, the cost of 100 lb. of spelter at 4·25 cents per lb. and of 1000 lb. of copper at 12·125 cents per lb.

6. 100 lb. of maize cost £0·45, find the price of 1000 lb.

7. If 100,000 lb. of wheat cost £700, what would be the price of 100 lb.?

8. Find the cost of 100,000 bushels of maize at 74·25 dollars per 100 bushels.

9. A chain 22 yd. long is divided into 100 links; find the length of a link in inches.

10. 1000 copies of a book weighed 6 cwt. 21 lb.; find the weight per copy in lb.

The Metric System

This system, which takes its name from the unit of length, was established by law in France at the end of the eighteenth century and is now used in many countries.

The unit of length, the **metre**, was originally intended to be one forty-millionth part of that meridian of the earth which passes through Paris, a difficult measurement to make with accuracy. Later it was defined as the distance at the temperature of melting ice between the centres of two lines traced on a platinum-iridium bar kept in the National Bureau.

The metre corresponds to the British yard, and is approximately 39·37 in.

Multiples of the fundamental units proceed upward by tens, and sub-multiples downward by tenths. The former are named by means of the prefixes '**Deka,**' '**Hecto,**' and '**Kilo**' derived from the Greek for 10, 100, and 1000, and the sub-multiples by means of the prefixes '**deci,**' '**centi,**' and '**milli,**' derived from the Latin for the same numbers.

Upward			*Downward*		
10 metres	=	1 Deca-metre	0·1 metre	=	1 deci-metre
100 metres	=	1 Hecto-metre	0·01 metre	=	1 centi-metre
1000 metres	=	1 Kilo-metre	0·001 metre	=	1 milli-metre

Quantities can be expressed in any other denomination without altering the figures, and this is one of the great advantages of the Metric system.

3567·892 metres	=	3567892·	milli-metres
		356789·2	centi-metres
		35678·92	deci-metres
		3567·892	metres
		356·7892	Deca-metres
		35·67892	Hecto-metres
		3·567892	Kilo-metres

It is easy to pass from one unit to another. It is a mere matter of moving the decimal point to right or left, or moving the figures with respect to the point.

The same prefixes are put before the units of volume and mass.

The unit of volume or capacity was derived from the unit of length. For scientific purposes the unit is the cubic centimetre (c.c.), but for practical purposes the **litre** (= 1000 c.c.'s) is the unit. Thus the litre is a cubic decimetre, and is a little more than a pint and three-quarters. Multiples and sub-multiples of the litre are denoted by the prefixes already given.

The unit of mass was derived from that of volume. The **gramme** was intended to be the mass of 1 cubic centimetre of water at the temperature at which it is heaviest. This unit, though suitable for scientific purposes, is small, and the Kilogram (1000 gramme, the mass of 1 cubic decimetre of water) is used for commercial purposes. The Kilogram is approximately equal to 2·2 lb. avoirdupois, and the demi-Kilo which is much used in trade corresponds fairly closely to our 1 lb.

The units of area are derived from those of length, thus square centimetre, square metre, etc.

For the measurement of land the units used are the **Are** (100 square metres) and the Hectare (100 ares).

The centimetre, gramme, second (of time) are universal units in scientific measurement, and the system is called the C.G.S. system.

The greater accuracy of modern methods of measurement has affected the relations between the various units, *e.g.*, between litre and cubic decimetre, but so slightly that the original relations given above may still be considered correct for all ordinary purposes.

Coinage. In most countries other than Great Britain and India the coinage follows a decimal system. Thus in Canada and the U.S.A. the dollar is divided into 100 cents, in France the franc is divided into 100 centimes, in Germany the mark = 100 pfennige, in Italy the lira = 100 centesimi.

These systems are not, however, entirely decimal. America has quarter and half dollars and the 5-cent piece (nickel); the 20-franc piece and 5-centime (sou) are common coins in France.

Exercises XXXVI

Express

1. 3525 millimetres in metres, Decametres, and Kilometres.
2. 5·356 Kilometres in Hectometres and Decametres.
3. 0·356 metres in deci-, centi-, and millimetres.
4. 23·567 Kilograms in grammes and in deci-, centi-, milli-grams.
5. 385 metres in Kilometres.
6. 535·25 litres in Hectolitres.
7. 357 decilitres in Decalitres.
8. 5385 centigrams in Decagrams.
9. 356 centilitres in Hectolitres.
10. 157 decigrams in Hectograms.
11. 5878 milligrams in Kilograms.

Multiplication and Division of a Decimal by a Simple Integer

Example 1. Multiply 3·264 by 6.

$$\begin{array}{r} 3{\cdot}264 \\ 6 \\ \hline \mathbf{19{\cdot}584} \end{array}$$

Example 2. Divide 3·1527 by 9.

$$\begin{array}{r} 9)3{\cdot}1527 \\ \hline \mathbf{0{\cdot}3503} \end{array}$$

Work just as with whole numbers, putting in the decimal point when you come to it in the multiplicand or dividend.

Exercises (Mental) XXXVII

Multiply (and check by division)

1. 2·23 by 2, 3, 7.
2. 14·67 by 3, 5, 8.
3. 6·25 by 3, 4, 5.
4. 0·1709 by 2, 6, 9.
5. 21·34 by 5, 6, 11.

Divide (and check by multiplication)

6. 17·35 by 5.
7. 21·225 by 3, 5.
8. 14·49 by 7.
9. 3·52 by 2, 11.
10. 2·121 by 3, 7.

11. Taking 1 in. as 2·54 cms., find $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ in. in cm.
12. Find also $\frac{3}{16}$ in. and $\frac{5}{16}$ in. in cm.

Multiplication of Decimals by Whole Numbers

Those readers who have worked out the products of whole numbers by proceeding from left to right of the multiplier will have noticed how the partial products diminish in value by tenths as the work proceeds from one line to the one below.

```
 236
  444
─────
 944   = 94400
  944  =  9440
   944 =   944
──────
104784
```

The 4 on the left of the multiplier represents hundreds, the next one tens, and the next units. This may not have been fully realized at the time, but it is necessary to grasp it if decimals are to be worked correctly.

Example. Taking a Kilogram as 2·204 lb., express 375 Kg. in lb.

We must multiply 2·204 by 375.

Now 375 = 3 hundreds + 7 tens + 5 units. The figure of highest place-value is 3 and represents hundreds. Multiply 2·204 by 100, and proceed as follows.

```
220·4|
    3|75
─────|──
661·2|
154·2|8      Rough check, 2 × 375 = 750.
 11·0|20
─────|──
826·5|00
```

$\therefore$ 375 Kg. equal **826·5** lb.

Multiplication of One Decimal by Another

Example. Multiply 145·62 by 34·56.

The method is the same as in the last example. The figure of highest place-value in the multiplier is 3 and represents tens.

Multiply 145·62 by 10, and proceed as shown

$$\begin{array}{r|l} 1456\cdot2 & \\ 3 & 456 \\ \hline 4368\cdot6 & \\ 582\cdot4 & 8 \\ 72\cdot8 & 10 \\ 8\cdot7 & 372 \\ \hline 5032\cdot6 & 272 \\ \hline \end{array}$$

Rough check, $150 \times 30 = 4500$.

The product is **5032·6272.**

Note. This method of multiplying decimals is equivalent to changing $145\cdot62 \times 34\cdot56$ into $1456\cdot2 \times 3\cdot456$. The multiplier 3·456 has one figure before the decimal point. Numbers such as this, which have one and only one figure before the decimal point, are said to be in **standard form.** One way of multiplying a decimal by another one is to arrange the multiplier in standard form and to compensate by moving the decimal point in the multiplicand the same number of places in the opposite direction.

Alternative Method

Example. Multiply 29·37 by 571·9.

$$\text{Since } 29\cdot37 = \frac{2937}{100}$$

$$\text{and } 571\cdot9 = \frac{5719}{10}$$

$$\text{their product is } \frac{2937 \times 5719}{1000}$$

We therefore multiply 2937 by 5719 and insert the decimal point before the third figure from the right, to allow for division by 1000.

$$\begin{array}{r} 2937 \\ 5719 \\ \hline 14685 \\ 20559 \\ 2937 \\ 26433 \\ \hline 16796703 \\ \hline \end{array}$$

Rough check, $30 \times 600 = 18000$.

The required product is **16796·703.**

Rule. Multiply the two numbers together, disregarding the decimal points; then insert the decimal point in the product by marking off from the right-hand figure a number of decimal places equal to the sum of the numbers of decimal places in the multiplier and multiplicand.

In the example above the total number of decimal places $= 2 + 1 = 3$.

Therefore we put the point before the third figure from the right.

Exercises (Mental) XXXVIII

Multiply

1. 1·3 by 1·2.
2. 3·75 by 1·1.
3. 38·7 by 20.
4. 3·87 by 30.
5. 18·3 by 500.
6. 9·83 by 12.
7. 16·71 by 110.
8. 153·82 by 130.

Exercises XXXIX

Multiply

1. 12·5 by 6·4.
2. 8·75 by 3·2.
3. 16·25 by 1·6.
4. 10·5 by 150.
5. 3·25 by 3·5.
6. 70·4 by 17·5.
7. 342·5 by 2·64.
8. 7·34 by 7·06
9. 53·56 by 20·05.
10. 990 by 9·9.
11. 4·086 by 30·7 and also by 3·07.
12. 3·215 by 7·24 and also by 724.
13. 13·02 by 6·98 and also by 69·8.
14. 183·02 by 2·009 and also by 200·9.
15. 28·395 by 1·4.
16. 170·8 by 3·09.
17. 17·62 by 490·4.
18. 542·875 by 24·8.
19. 12·3 by 51·23.
20. 3·0008 by 37·5.

Find the values of

21. $(2{\cdot}001)^2$.
22. $(1{\cdot}003)^2$.
23. Taking 1 m. as 39·37 in., find the equivalent in inches of 3·7 m.
24. Taking £1 as worth 4·8725 dollars, find the value in dollars of £375.
25. A cubic centimetre of metal weighs 9·73 grammes. Find the weight of 7·8 cubic centimetres of the metal.
26. Some silk costs 29·4 fr. per metre. Find the cost of 6·75 m. in francs and centimes.
27. If a ton is equal to 1016 Kg., find the equivalent in Kilograms of 5·65 tons.
28. A gallon of sea-water weighs 10·26 lb. Find the weight in lb. of 7·35 gal.

Division of One Decimal by Another

Example. Divide 879·27 by 37·1.

Make the divisor a whole number by multiplying by a suitable power of 10, and multiply the dividend by the same power of 10 to compensate.

$$\frac{879{\cdot}27}{37{\cdot}1} = \frac{8792{\cdot}7}{371}$$ multiplying numerator and denominator by 10.

```
        23·7
371)8792·7
    742
    1372
    1113
     2597
     2597
```

Rough check, $\frac{880}{40} = 22$.

The quotient is **23·7.**

Divide as with whole numbers, inserting the point in the quotient before the first figure after the point in the dividend is brought down.

Exercises (Mental) XL

Divide

1. 65 by 2, and by 20.
2. 125 by 4, and by 40.
3. 72·6 by 3, and by 30.
4. 10·8 by 1·2.
5. 187·6 by 70.
6. 197·5 by 25, and by 2·5.
7. 17·03 by 1·3.
8. 38·25 by 1·5.

Exercises XLI

Divide

1. 32·4 by 18.
2. 856·8 by 17.
3. 80·36 by 5·6.
4. 730 by 58·4.
5. 23·67 by 4·5.
6. 36·27 by 11·7.
7. 22·7088 by 3·8.
8. 16·1 by 3·68.
9. 135 by 11·25.
10. 1602·3 by 29·4.
11. 99·994 by 2·89.
12. 28·386 by 1·14.
13. 120 by 3·75.
14. 64·01 by 34·6.
15. 1575·05 by 37·06.
16. 173·667 by 2·19.
17. 2347·936 by 3·07.
18. 578·721 by 20·9.
19. 831·183 by 230·5.
20. 99·9 by 15·984.

21. How many pieces of string each 8·4 cms. long can be cut from a length of 10 m.? What is the length of the remainder?

22. Find the weight per c.c. of a substance 19·3 c.c. of which weigh 49·022 gr.

23. If £1 is worth $4·78, find the value in pounds of $1314·5.

24. A substance is sold at 37·5 fr. per Kilogram. How many Kilograms of the substance can be bought for 1500 fr.?

25. A liner travelled 3219·3 sea-miles in 109·5 hr. How many sea-miles per hour is this?

CHAPTER VI

DECIMALIZATION OF MONEY, WEIGHTS, AND MEASURES

Example 1. Express £5 13*s.* $10\frac{1}{2}$*d.* as a decimal of £1.

$$\text{Since } \tfrac{1}{2} = \cdot 5,\ 10\tfrac{1}{2}d. = 10\cdot 5d.$$
$$= \frac{10\cdot 5}{12}s.$$
$$= \cdot 875s.$$
$$\therefore\ 13s.\ 10\tfrac{1}{2}d. = 13\cdot 875s.$$
$$= £\frac{13\cdot 875}{20}$$
$$= £0\cdot 69375$$
$$\therefore\ £5\ 13s.\ 10\tfrac{1}{2}d. = \mathbf{£5\cdot 69375.}$$

The work is arranged conveniently as follows:

2	1 halfpenny
12	10·5 pence
20	13·875 shillings
	5·69375 pounds.

Begin with the lowest denomination, 1 halfpenny, divide by 2 and prefix the whole pence (10), divide by 12 and prefix the whole shillings (13), divide by 20 and prefix the whole pounds (5).

Example 2. Express 5 tons 3 cwt. 3 qr. 21 lb. as a decimal of 1 ton.

28	7	21 lb.
	4	3
	4	3·75 qr.
	20	3·9375 cwt.
		5·196875 tons.

Example 3. Express 5 miles 7 f. 176 yd. as a decimal of 1 mile.

22 {	2	176 yd.
	11	88
	10	8 chains
	8	7·8 f.
		5·975 miles.

The method is the same for all three examples.

Exercises XLII

Express as decimals of £1

1. £5 13*s*. 9*d*.
2. £4 7*s*. 3*d*.
3. 19*s*. $1\frac{1}{2}$*d*.
4. £3 5*s*. $2\frac{1}{4}$*d*.
5. £4 11*s*. $9\frac{3}{4}$*d*.
6. 9*s*. $5\frac{1}{4}$*d*.
7. 17*s*. $2\frac{1}{4}$*d*.
8. 8*s*. $3\frac{3}{4}$*d*.
9. £1 19*s*. $8\frac{1}{4}$*d*.

Express as decimals of 1 cwt.

10. 8 cwt. 1 qr. 21 lb.
11. 7 cwt. 2 qr. 21 lb.
12. 11 cwt. 1 qr. 14 lb.
13. 15 cwt. 3 qr. 7 lb.

Express as decimals of a ton

14. 4 tons 5 cwt. 2 qr. 14 lb.
15. 3 tons 7 cwt. 2 qr. 7 lb.
16. 9 tons 14 cwt. 2 qr. 21 lb.
17. 4 cwt. $17\frac{1}{2}$ lb.

Express as decimals of an acre

18. 3 ac. 2 r. 8 p.
19. 8 ac. 3 r. 16 p.
20. 3 r. 27 p.

[Land surveys are reliable only to the nearest perch.]

Express as decimals of a mile

21. 1 f. 4 pl.
22. 3 ml. 3 f. 5 ch.
23. 5 ml. 3 f. 77 yd.
24. 5 ml. 330 yd.

Express as decimals of an hour

25. 13 hr. 21 min. 36 sec.
26. 7 hr. 36 min. 45 sec.

Example 1. Express £5·234375 in £ *s. d.*

```
5·234375   pounds
      20
4·6875     shillings
    12                  Multiply decimal parts only.
8·25       pence
   4
1·00       farthings
```

Answer: **£5 4s. $8\frac{1}{4}$d.**

Example 2. Express 6·96 hr. in hours, minutes, and seconds.

```
 6·96   hr.
   60
57·6   min.   Multiply decimal parts only.
   60
36·     sec.
```

Answer: **6 hr. 57 min. 36 sec.**

Example. 3. Express 17·813 cwt. in cwt., qr., lb.

```
17·813 cwt.
      4
 3·252 qr.
      4 ⎫
 1·008  ⎬ 28
      7 ⎭
 7·056 lb.
```

Answer: **17 cwt. 3 qr. 7·056 lb.**

The method is the same in all cases.

Exercises XLIII

Express in shillings, pence, and farthings

1. £0·375.
2. £0·584375.
3. £0·671875.
4. £0·896875.
5. £0·903125.
6. £0·140625.

Express in cwt., qr., lb.

7. 5·625 cwt.
8. 9·4375 cwt.
9. 12·57825 cwt.
10. 0·89675 cwt.

Express in miles, furlongs, chains, yards

11. 0·725 ml.
12. 15·845 ml.
13. 8·09625 ml.

Express in hours, minutes, seconds

14. 7·825 hr.
15. 15·075 hr.

Express in acres, roods, perches

16. 7·65625 ac.
17. 19·83125 ac.

Express in tons, hundredweights, quarters

18. 14·7125 tons.
19. 9·6875 tons.

20. Express in yards, feet, inches 5·723 yd.

CHAPTER VII

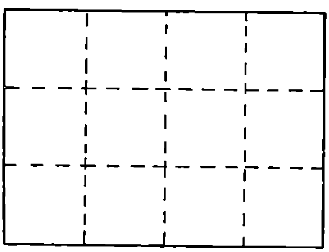
MENSURATION (I). RECTANGULAR AREAS AND VOLUMES

A line is measured by its length, a surface by its area. The unit of measurement of area is a square each of whose sides is one unit of length. Thus we measure areas in square inches, square centimetres, square miles, etc.

A square foot is sometimes called a **foot super** (from *superficies,* a surface).

RECTANGULAR AREAS

If the sides of a rectangle are, for example, 3 and 4 in. long, we can divide it into 3 rows each of 4 squares, each square having an area of 1 sq. in., by drawing lines parallel to the sides, as in the diagram below.



The area of the rectangle is 3 × 4 or 4 × 3, *i.e.*, **12** sq. in.

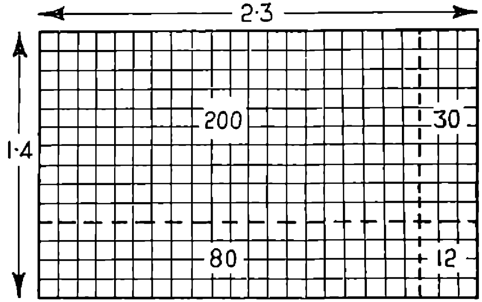
Similarly a rectangle 2·3 in. long and 1·4 in. wide contains 200 + 80 + 30 + 12 or 23 × 14, *i.e.*, 322 small squares, 100 of which make up 1 sq. in. See p. 69.

Hence the area is $\frac{322}{100}$ sq. in.

$$= 3{\cdot}22 \text{ sq. in.}$$

$$i.e., \text{ area} = \frac{23 \times 14}{100} = 2{\cdot}3 \times 1{\cdot}4 = \mathbf{3{\cdot}22} \text{ sq. in.}$$

Hence if we multiply the number of units of length in one side of a rectangle by the number of those units in one of the perpendicular sides, we get the number of units of area in the rectangle, the units of area being those corre-



sponding to the units of length in which the sides are measured.

The sides must be measured in the same units of length; the area is then found in the corresponding units of area. When this is understood you may say, briefly, **area (of rectangle) = length × breadth.**

Exercises (Mental) XLIV

Find the areas of the rectangles whose sides are

1. $2\frac{1}{2}$ in. and 2 in.
2. 13 in. and 5 in.
3. $7\frac{1}{2}$ in. and 6 in.
4. 19 in. and 7 in.
5. 5 ft. and $3\frac{1}{2}$ ft.
6. 15 yd. and 9 yd.
7. 11 ch. and $4\frac{1}{2}$ ch.
8. 1·7 m. and 8 m.
9. 2·6 in. and 1·2 in.
10. 8 ml. and 3·7 ml.

The area and one side of the following rectangles are given; find the other side.

11. 15 sq. in., 2 in.
12. 36 sq. ft., 8 ft.
13. 105 sq. ch., 15 ch.
14. 143 sq. m., 11 m.
15. 256 sq. yd., 16 yd.
16. 196 sq. in., 2 ft. 4 in.
17. 6·5 sq. ml., 5 ml.
18. 7·2 sq. dm., 2·4 dm.

Exercises XLV

1. Draw a figure to show 1 sq. yd. divided into sq. ft. How many sq. ft. are there in a sq. yd.?

Fill in the gaps in the following statements:

2. 1 ft.	= 12 in.	1 sq. ft. =	sq in.
3. 1 pl.	= $5\frac{1}{2}$ yd.	1 sq. pl. =	sq. yd.
4. 1 ch.	= 22 yd.	1 sq. ch. =	sq. yd.
5. 1 ch.	= 100 links	1 sq. ch. =	sq. links
6. 1 m.	= 100 cm.	1 sq. m. =	sq. cm.
7. 1 Km.	= 1000 m.	1 sq. Km. =	sq. m.
8. 1 Dm.	= 100 dm.	1 sq. Dm. =	sq. dm.

9. Area of square = 10,000 sq. m. Side = m.

10. Area of square = 1,000,000 sq. m. Side = Km.

11. The French 'are' is a square of 10 m. side; find the 'are' in sq. m., and also the Hectare.

12. Prove that 100 Hectares = 1 sq. Km.

13. Express 2535 sq. m. in sq. Km.

14. Express 3578 sq. m. in 'ares' and in Hectares.

15. Express 3356 sq. mm. in sq. dm.

16. A map is drawn on the scale of $\frac{1}{10}$ (linear); calculate the areas on the map which represent areas of 100 sq. ft., 52·5 sq. ft., 350 sq. m., 50 ares, and 10 Hectares.

Find the areas of rectangles whose sides are

17. 3·2 in. and 4·6 in.
18. 3·25 ft. and 6·25 ft.
19. 3·56 cm. and 1·2 cm.
20. 5 ch. and 2·5 ch.
21. 3 ft. 6 in. and 5 ft. 6 in.
22. 6 ft. 3 in. and 7 ft. 6 in.
23. 18 ft. 4 in. and 12 ft. 9 in.
24. 8 ft. 6 in. and 5 ft. $4\frac{1}{2}$ in.

25. A railway track is 8·1 m. wide on the average, and 437·2 Km. long. Find its area in square Kilometres.

Find the lengths of the other sides of the following rectangles:

26. Area = 100 sq. ft., one side = 3·125 ft.
27. Area = 24·075 sq. yd., one side = 5·35 yd.
28. Area = 8100 sq. cm., one side = 13·5 cm.
29. Area = 410 ares, one side = 51·25 m.
30. Area = 535 sq. yd., one side = 45 yd.

Find the areas of the following rectangular plots of ground, in acres, sq. chains, and decimals of a square chain:

31. Length of plot = 13 ch. 20 links, breadth 7 ch. 50 links.
32. Length of plot = $7\frac{1}{2}$ ch., breadth $2\frac{3}{4}$ ch.

Find the breadths of the following rectangular plots in chains and links:

33. Area of plot = 5 sq. ch., length of plot 55 yd.
34. Area of plot = 32·25 sq. ch., length of plot 7 ch. 50 links.

35. Find the area of a room 15 ft. by 14 ft., and the cost of covering the floor with carpet at 7*s.* 6*d.* per sq. yd.

36. Find the area of a room 16 ft. 6 in. by 11 ft., and the cost of polishing the floor at 1*s.* 6*d.* per foot super.

37. What would it cost to give a wall 10 ft. by 13 ft. 6 in. two coatings of paint at 1*s.* 9*d.* per yard super?

38. 1344 tiles each 9 in. by $4\frac{1}{2}$ in. are required to pave a courtyard. Find the area of the courtyard in square feet.

39. Find the cost of tarring a road 1 mile long and 8 yd. wide at 3*s.* 9*d.* per square yard.

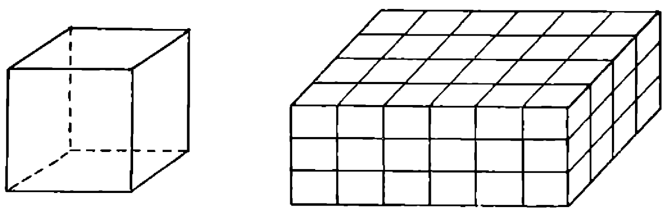
40. How many square tiles of side 8 in. would be required to cover a floor 36 ft. by 27 ft.?

41. What length of carpet 27 in. wide would be required to cover the floor of a room 13 ft. 6 in. by 11 ft. 6 in.?

42. The perimeter of a square field is half a mile. Find its area in acres.

Volumes

The unit of measurement of volume is a cube of edge 1 unit of length; thus cubic inch, cubic centimetre, etc.



Place centimetre cubes in rows on top of one another to form a cuboid (rectangular solid), and verify that the volume of a cuboid is measured by the product of the measures (in the same units) of its length, breadth, and height.

Volume (cubic units) = length × breadth × height (each in linear units)

Since the product of length and breadth determines the area of a cross-section, we have

Volume = base cross-section × height.

In a similar way, we note that

Volume = end cross-section × length.

Exercises (Mental) XLVI

Find the volumes of the rectangular blocks whose lengths, breadths, and depths are

1. 13, 7, 2 in.
2. 5, 4, 2·5 ft.
3. 10, 9, 8 yd.
4. 16, 3, 1·5 m.
5. 6, 5, 2·4 in.
6. $3\frac{1}{2}$, $2\frac{1}{2}$, 1 ft.

Find the depths of the rectangular solids whose volumes and base areas are

7. 120 cu. in., 50 sq. in.
8. 2·7 cu. ft., 1·8 sq. ft.
9. 216 cu. in., 36 sq. in.
10. 343 cu. cm., 49 sq. cm.
11. $6\frac{3}{4}$ cu. ft., $4\frac{1}{2}$ sq. ft.
12. 180 cu. in., 360 sq. in.

Find the depths of the rectangular solids whose volumes, lengths, and breadths are

13. 168 cu. in., 8 in., 7 in.
14. $37\frac{1}{2}$ cu. in., 10 in., $2\frac{1}{2}$ in.
15. 252 cu. ft., 9 ft., 7 ft.

Exercises XLVII

Fill in the gaps in the following statements:

1. 1 ft.	= 12 in.	1 cu. ft. =		cu. in.
2. 1 yd.	= 3 ft.	1 cu. yd. =		cu. ft.
3. 1 m.	= 10 dm.	1 cu. m. =		cu. dm.
4. 1 m.	= 100 cm.	1 cu. m. =		cu. cm.

5. Find the number of litres in 3567 c.c. (1 litre = 1 cu. dm.).
6. Express 35 litres as a decimal of 1 cu. m.
7. Express 3·5 cu. m. in c.c.
8. Find the number of Hectolitres in 5 cu. m.

Find the volumes of the following rectangular solids:

9. Length 8·5 ft., breadth 6 ft., height 3·5 ft.
10. Length 6·8 m., breadth 5 m., height 6·5 m.
11. Length $12\frac{1}{2}$ in., breadth $8\frac{1}{2}$ in., height $5\frac{1}{2}$ in.
12. Length 8 ft. 3 in., breadth 2 ft. 6 in., height 5 ft. 3 in.

Find the volumes of the following solids:

13. Area of base 50 sq. ft., height 8 ft. 5 in. (in cu. ft.).
14. Area of base 300 sq. cm., height 35 mm. (in c.c.).
15. Area of base 3 sq. cm., height 5 dm. (in c.c.).
16. Area of base 5 ares, height 5 cm. (in cu. m.).
17. Area of base 1 acre, height 1 in. (in cu. ft.).
18. The volume of a cuboid is 350 cu. ft. If its height is 10 ft., find the area of its base.

19. The volume of a cuboid is 3750 c.c., and its height 1·2 dm. Find its base-area.

20. The volume of a room is 1980 cu. ft. Its length and breadth are 15 ft. and 12 ft. Find its height.

21. Find the air space in a room 30 ft. by 25 ft. by 18 ft. 6 in.

22. How many bars of chocolate, each 4 in. by $1\frac{1}{2}$ in. by $\frac{1}{3}$ in., can be packed in a box 1 ft. square and 2 in. deep?

23. Find the volume of a stone wall 100 yd. long, 5 ft. high, and $2\frac{1}{2}$ ft. thick. Answer in cubic feet.

24. A trench 6 ft. wide and 50 yd. long is dug 4 ft. 6 in. deep. Find how many cubic yards of earth are removed.

CHAPTER VIII

DECIMALS (II)

Harder Examples on Multiplication and Division

Beginners are apt to find difficulty with the multiplication and division of decimals which have no integral part. The rules already given in Chapter V are, however, sufficient to work any problem of this kind.

Example 1. Multiply ·0652 by ·078.

Method 1. The figure of highest place-value in the multiplier is 7 and represents hundredths. Multiply ·0652 by $\frac{1}{100}$, and proceed as follows:

```
·000652|
      7|8
-------
·004564
    521|6
-------
·005085|6
```

The product is **·0050856.**

Method 2. Multiply 652 by 78, getting 50856, and insert the decimal point by marking off 4 + 3, *i.e.*, 7, places of decimals from the right.

Result is **·0050856.**

To check the result: approximately the product is ·06 × ·08. This may be worked out according to the rules for decimals, but it is a good plan to use vulgar fractions instead, to avoid the possible repetition of an error in the manipulation of decimals.

Thus $\frac{6}{100} \times \frac{8}{100} = \frac{48}{10000}$ = ·0048, and the position of the decimal point is verified.

Example 2. Divide ·0015623 by ·017.

Make the divisor a whole number, 17, and the dividend 1·5623.

```
       ·0919
  17 | 1·5623
       1 53
       ----
         32
         17
         ---
         153
         153
         ---
```

The quotient is **·0919.**

17 does not divide into 1 or into 15; there are no units or tenths in the quotient. Put the point in the quotient above the point in the dividend, and 0 above the 5. 17 divides into 156 9 times, put 9 in the quotient above the 6 of the dividend, and carry on as in ordinary division.

$$\text{Rough check: } \cdot0015 \div \cdot017 = \tfrac{15}{10000} \div \tfrac{17}{1000}$$
$$= \tfrac{15}{10000} \times \tfrac{1000}{17}$$
$$= \tfrac{15}{170}$$

which is about $\frac{1}{11}$, or ·09.

Remainders. Notice that in division of decimals when both divisor and dividend have been multiplied (or divided) by a power of 10, all partial remainders are also multiplied (or divided) by the same power of 10.

Thus in the actual division above the last partial remainder is ·0153, but in the original sum the corresponding remainder would be ·0000153.

i.e., $1{\cdot}5623 = 17 \times {\cdot}091 + {\cdot}0153$

but $\cdot0015623 = \cdot017 \times \cdot091 + \cdot0000153.$

Exercises (Mental) XLVIII

Multiply

1. ·02 by 30, ·3, ·003.
2. ·05 by 20, ·2, ·02.
3. ·3 by 700, ·7, ·007.
4. 1·5 by 0·4.
5. ·008 by ·25.

The answers to the following divisions will be less than 1, and thus there will be no figures before the decimal point in the quotient. State whether or not there will be any noughts following the point in the quotient and, if so, how many. (Do not perform the divisions.)

6. 1·56 ÷ 13.
7. 0·256 ÷ 21.
8. 0·7562 ÷ 36.
9. 135·67 ÷ 150.
10. 0·56 ÷ 49, 57.

Divide

11. ·8 by ·2.
12. ·8 by ·02.
13. 1·6 by 40.
14. 3·5 by ·07.
15. 20 by 50.
16. ·108 by ·3.
17. 80 by ·16.
18. ·072 by ·06.

Exercises XLIX

Multiply

1. 33·7 by ·23.
2. 6·185 by ·37.
3. ·581 by ·059.
4. ·3872 by ·193.
5. ·09837 by 31·2.
6. ·0619 by ·0181.
7. 10·813 by ·0037.
8. 123·4 by ·00579.
9. ·00871 by ·0029.
10. 36·713 by ·0872.

Divide

11. 3·762 by ·57.
12. ·3762 by 5·7.
13. ·24543 by 8·1.
14. ·081875 by ·125.
15. 11·026 by ·037.
16. ·91609 by 2·3 and by ·023.
17. 17·873 by ·0061.
18. 1·87335 by 345 and by ·345.
19. 25·67 by 1·3, and give the remainder.
20. 6·8272 by ·39, and give the remainder.

Note that divisions such as Exercises 19 and 20, which do not terminate, can be carried on to any number of places of decimals by bringing down noughts from the dividend. See Chapter V, p. 52.

To express One Compound Quantity as a Decimal of Another

Example. Express 7*s*. 6*d*. as a decimal of 12*s*.

$$\frac{7s.\ 6d.}{12s.} = \frac{15 \text{ sixpences}}{24 \text{ sixpences}}$$
$$= \tfrac{5}{8}$$
$$= \cdot 625.$$

Hence 7*s*. 6*d*. is **625** of 12*s*.

Exercises L

Express the first of the given quantities as a decimal of the second:

1. 3*s*. 6*d*.; 8*s*.
2. £1 13*s*. 6*d*.; £2.
3. 2 tons 8 cwt.; 3 tons.
4. 1 cwt. 3 qr.; 5 tons.
5. 4 ft. 6 in.; 7 ft. 6 in.
6. 7 f.; $2\frac{1}{2}$ ml.
7. 45 sq. in.; 1 sq. ft.
8. 3 hr.; 1 hr. 20 min.
9. 35 fr.; 12 fr. 50 c.
10. $1\frac{1}{2}$ ac.; 2000 sq. yd.

Accuracy in Measurement

We often think of goods priced in the shops at, *e.g.*, $11\frac{3}{4}$*d*. or 19*s*. 11*d*. as costing us 1*s*. or £1, and a man with an income of £2025 might state it as £2000 roughly.

In measuring a distance with a ruler divided into eighths of an inch we might call it $5\frac{5}{8}$ in. if just over the fifth of the eighth divisions, or $5\frac{3}{4}$ in. if very near the sixth division. Boys weighing something in the science laboratory would probably at first take a result to the nearest centigramme.

No physical measurement can be made with absolute accuracy: a possible error of 1 part in ten millions would be a very high degree of accuracy.

There are two ways in which the degree of accuracy of a number may be stated. We may say the number is corrected to so many places of decimals, or corrected to so many significant figures. These two methods, of which the latter is the more important, are explained in the succeeding paragraphs.

Results corrected to a Given Number of Decimal Places

Consider the number 25·087138 . . . . The figure 1 represents 1 ten-thousandth part of the unit, the figure 3, hundred-thousandth parts, and so on. If for any reason we are to disregard parts smaller than the ten-thousandth part, we take the number as 25·0871, since it lies between this and 25·0872 and is nearer to 25·0871, being less than the number 25·08715, which is half-way between.

We say that the number is 25·0871 *to the nearest fourth place* or *corrected to four places* of decimals.

In the same way we might take the number as 25·087 'to the nearest thousandth' or 'corrected to 3 places of decimals.'

However, if we give the number to the nearest hundredth, or corrected to 2 places of decimals, it will not be 25·08 but 25·09, since it is nearer to the latter, being greater than 25·085, which is half-way between.

You will see that to state a number corrected to a given number of decimal places, you have only to copy the number as far as the given number of decimal places, and if the next figure (the first to be omitted) is a 5 or more you must increase by 1 the last figure that is retained.

A point which sometimes gives difficulty is illustrated by the number 2·05971. If this is to be stated corrected to 3 places of decimals, we must write it as 2·060, writing in the third place of decimals even though it is a nought, to indicate that the value is taken to the nearest thousandth.

Significant Figures

For many purposes measurements or calculations are sufficiently accurate if there is a possible error of about 1 part in a thousand, and only the first 3 or 4 figures of a number need be known. Thus the average distance of the sun from the earth is often given as 92,800,000 miles; the population of a town may be given as 325,800 persons; the average wave-length of light as ·000053 cm. In each of these instances the place-value of the first figure (other than a zero) in the number tells us the scale of magnitude of the quantity being measured; thus 9 in the first instance tells us that the sun's distance is in the neighbourhood of 9 times ten million miles, the 3 in the second instance that the population is in the neighbourhood of 3 hundred thousand people, and the 5 in the last instance that the wavelength is round about 5 hundred thousandths of a centimetre. The succeeding figures in each number then make the number more precise.

We call these first figures (9, 3, 5 respectively) the first **significant figures** in the respective numbers, and count significant figures from them. Thus, in writing down a number corrected to so many significant figures, we begin with the first figure from the left which is not zero, and, from that figure, count all figures whether zero or not.

Thus the number 583·58913 would be written as

583·589,	if corrected to 6	significant figures,
583·59	,, ,, 5	,, ,,
583·6	,, ,, 4	,, ,,
584	,, ,, 3	,, ,,
580	,, ,, 2	,, ,,
600	,, ,, 1	,, ,,

Note that the noughts at the end of 580 and 600 must be put in to give the first figures (5 and 6 respectively) their proper value.

Note also that the number 92,800,000 might represent distance in miles correct to more than 3 significant figures. It would be correct to 4 significant figures if it differed from the true value by less than 5000.

Large and small numbers are often most conveniently expressed as a number in standard form multiplied or divided by a power of 10.

Thus the average distance of the sun is given as $1{\cdot}494 \times 10^{11}$ m., and the wavelength of light in the form $5{\cdot}3 \div 10^5$ (or $5{\cdot}3 \times 10^{-5}$, see Chapter XX).

1·494 and 5·3 are significant figures and the units of measurement are 10^{11} m. and $\frac{1}{10^5}$ cm. respectively.

Exercises (Mental) LI

1. 1 yd. = 0·914399 m. Express a yard in metres, (*a*) to 3 places of decimals, (*b*) to 4 significant figures.

Estimate the following decimals (*a*) to 3 places of decimals, (*b*) to 3 significant figures:

2. 1 m. = 39·370113 in.
3. 1 cm. per sec. = 0·02237 ml. per hr.
4. 1 c.c. = 0·061024 cu. in.
5. 1 pt. = 0·5682 litres.
6. 1 oz. Troy = 31·035 gr.

Exercises LII

Find the values, corrected to 3 decimal places, of

1. 3·175 × 2·71.
2. 3·175 ÷ 2·71.
3. 61·83 × 3·142.
4. 61·83 ÷ 3·142.
5. 0·537 × 25·3.
6. 0·321 ÷ 0·987.

Find the values, corrected to 3 significant figures, of

7. $3{\cdot}15 \times {\cdot}017$.
8. $91{\cdot}7 \div {\cdot}751$.
9. ${\cdot}0719 \times {\cdot}0813$.
10. $21{\cdot}3 \div 819{\cdot}2$.
11. $21{\cdot}3 \times 819{\cdot}2$.
12. ${\cdot}0719 \div {\cdot}0813$.

13. Find to 3 places of decimals the value of $\frac{1}{u} + \frac{1}{v}$ when $u = 3{\cdot}3$, $v = 5{\cdot}6$.

14. Find to 3 significant figures the value of $\frac{1}{2}ft^2$ when $f = 32{\cdot}2$, $t = 0{\cdot}55$.

Find, corrected to 3 significant figures, the values of

15. $\dfrac{7{\cdot}019 + 13{\cdot}56}{8{\cdot}32 - 2{\cdot}195}$.
16. $\dfrac{18{\cdot}92 - 5{\cdot}1}{23{\cdot}5}$.
17. $\dfrac{16{\cdot}71 \times 2{\cdot}59}{0{\cdot}73}$.
18. $\dfrac{148{\cdot}2 \times 0{\cdot}096}{10{\cdot}3}$.
19. $\dfrac{75{\cdot}7 \times 21{\cdot}3}{65{\cdot}1}$.
20. $\dfrac{11{\cdot}9}{2{\cdot}51 \times 3{\cdot}2}$.
21. $\dfrac{{\cdot}0617}{{\cdot}09 \times {\cdot}23}$.
22. $\dfrac{13{\cdot}5 + 8{\cdot}73}{13{\cdot}5 \times 8{\cdot}7}$.
23. $\dfrac{5{\cdot}2\,(6{\cdot}1 - 2{\cdot}47)}{10{\cdot}7}$.
24. $\dfrac{{\cdot}0873}{3{\cdot}142 \times ({\cdot}02)^2}$.

Rough Estimates

When you have had some experience of working decimals, you will find that you can insert the decimal point in the result of a calculation by making a mental estimate of the result.

Thus to evaluate $\dfrac{16{\cdot}71 \times 2{\cdot}59}{0{\cdot}73}$ we might proceed as follows. Multiply 1671 by 259, and divide the product by 73. We obtain the significant figures 5928 . . . . Now we might take the given expression as roughly equal to

$$16 \times 2\tfrac{1}{2} \div \tfrac{7}{10}$$
$$= 40 \times \tfrac{10}{7} = \tfrac{400}{7} = 57 \text{ roughly.}$$

Hence the decimal point must be placed after the second significant figure, and corrected to 3 figures, the value of the given expression is **59·3.**

Example. Find roughly the value of $\dfrac{13\cdot5 + 8\cdot73}{13\cdot5 \times 8\cdot7}$.

The expression is roughly $\dfrac{22}{13 \times 9} = \dfrac{22}{117}$, which is a little less than $\frac{1}{5}$. Hence the value is about **0·2.**

Exercises LIII

Give rough estimates of the values of the following:

1. $\dfrac{7\cdot019 + 13\cdot56}{8\cdot32 - 2\cdot195}$.
2. $\dfrac{148\cdot2 \times \cdot096}{10\cdot3}$.
3. $\dfrac{5\cdot2\,(6\cdot1 - 1\cdot14)}{1\cdot97}$.
4. $\dfrac{8\cdot73}{3\cdot142 \times (\cdot02)^2}$.
5. $\dfrac{(3\cdot142)^2 \times 179}{2178}$.
6. $\dfrac{25 \times 730 \times 300}{760 \times 273}$.

Multiplication of Decimals by Mixed Numbers

Example 1. $15\cdot65 \times 3\frac{1}{3}$.

	15·65
	$3\frac{1}{3}$
3 times top line	46·95
$\frac{1}{3}$,, ,,	5·2166 . . .
	52·1666 . . .

Example 2. $0\cdot1563 \times 3\frac{3}{4}$.

	·1563
	$3\frac{3}{4}$
3 times top line	·4689
$\frac{3}{4}$,, ,,	·117225 (divide previous line by 4)
	0·586125

Example 3. $2\cdot563 \times 3\frac{5}{8}$.

			or	
	2·563			2·563
	$3\frac{5}{8}$			$3\frac{5}{8}$
3 times top line	7·689			7·689
5 ,, ,,	12·815			12·815
$\frac{5}{8}$,, ,,	1·601875	(divide last line)		1·601875
	9·290875			**9·290875**

The intermediate line, which is not to be added into the total, may be written in the column and lightly crossed out, or may be written at the right-hand side.

The method is similar to that of Simple Practice (Chapter IV). It is not necessary to change the mixed numbers into decimals.

Example 4. $15{\cdot}648 \times 4\frac{7}{8}$. $\quad 4\frac{7}{8} = 5 - \frac{1}{8}$.

	15·648
5 times top line	78·240
$\frac{1}{8}$,, ,,	1·956
Subtract	**76·284**

Exercises LIV

1. The perimeter of a circle is $3\frac{1}{7}$ times the diameter (approximately). Find, to 3 significant figures, the perimeters of circles whose diameters are 12·5, 0·25, 3·75 in.

2. The diameter of a penny is 1·2 in.; find its perimeter to 3 significant figures.

3. Multiply 22·05 by $3\frac{3}{4}$, $2\frac{1}{2}$, $3\frac{7}{8}$.

4. Multiply 0·0236 by $2\frac{1}{8}$, $3\frac{3}{8}$, $4\frac{5}{6}$.

The area of a circle is $3\frac{1}{7}$ times the area of the square on its radius (approximately). Find the areas of the following circles, giving the answers to the nearest whole number:

5. Radius 10 ft.
6. Radius 12·5 cm.
7. Radius 35 mm.
8. Diameter 8·4 ft.
9. Diameter 3·8 cm.
10. Diameter $\frac{1}{4}$ ch. (answer in sq. yd.)

11. Find the cost of 100000 articles at $2\frac{1}{7}d.$ each:

(1) By multiplying $2\frac{1}{7}d.$ by 100000.
(2) By multiplying $2{\cdot}14d.$ by 100000.

What is the difference between the two results, and why does it arise?

Values Expressed by Means of a Decimal Fraction Followed by a Simple Vulgar Fraction

The value of the British pound sterling has been quoted as $4{\cdot}91\frac{1}{4}$ dollars in New York or $4{\cdot}91\frac{7}{8}$ dollars in Montreal, that is, to two decimal places and a vulgar fraction. This has been the usual custom.

$$\begin{aligned} 4{\cdot}91\tfrac{7}{8} \text{ dollars} &= 4 \text{ dollars } 91\tfrac{7}{8} \text{ cents} \\ &= 4 \text{ dollars } 91{\cdot}875 \text{ cents} \\ &= \mathbf{4{\cdot}91875} \text{ dollars,} \end{aligned}$$

so that $4{\cdot}91\frac{7}{8} = \mathbf{4{\cdot}91875}$ as a complete decimal.

Similarly at 1016 Kilos to the ton

$$\left.\begin{array}{lcl} 1 \text{ cwt.} & = & 50{\cdot}8 \\ 1 \text{ qr.} & = & 12{\cdot}7 \\ 7 \text{ lb.} & = & 3{\cdot}175 \\ 1 \text{ lb.} & = & 0{\cdot}453\frac{4}{7} \end{array}\right\} \text{Kilos.}$$

Exercises LV

1. Express $1{\cdot}25\frac{1}{2}$, $2{\cdot}63\frac{3}{4}$, $2{\cdot}17\frac{7}{8}$, $1{\cdot}29\frac{9}{16}$, $1{\cdot}45\frac{7}{16}$ as complete decimals.
2. Find, in dollars and cents, the cost of 10, 100, and 1000 bush, of wheat at $1{\cdot}49\frac{3}{4}$\$ per bushel.
3. Taking 1016 Kilos. to the ton, express 1, 2, 3, 4, 5, 6 lb. as decimals of 1 Kilo. to three places and a vulgar fraction.

Reciprocals

The reciprocal of any number = 1 ÷ (that number).

Example 1. Add together $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$, $\frac{1}{11}$, $\frac{1}{16}$, giving the result in decimals to 5 places.

$$\begin{array}{rcl} \frac{1}{5} & = & 0{\cdot}200000 \\ \frac{1}{7} & = & 0{\cdot}142857 \\ \frac{1}{9} & = & 0{\cdot}111111 \\ \frac{1}{11} & = & 0{\cdot}090909 \\ \frac{1}{16} & = & 0{\cdot}062500 \\ \hline & & \mathbf{{\cdot}60738} \end{array}$$

Keep an additional place of of decimals. The sum of the figures in the 6th place is 17. This is nearer 20 than 10, therefore carry 2.

If tables of reciprocals are used the numbers in the difference columns must be subtracted. Why?

Exercises LVI

1. Prove that $\frac{1}{375} = 0{\cdot}002\frac{2}{3}$.
2. Prove that $\frac{1}{240} = 0{\cdot}004\frac{1}{6}$.
3. Find to seven decimal places the difference between $\frac{1}{73}$ and $0{\cdot}0137$.
4. Find to seven decimal places the difference between $\frac{1}{365}$ and ·00274. Express a salary of £375 per annum as a daily wage, by multiplying by ·00274.
5. Find the difference to seven decimal places between $\frac{1}{2240}$ and 0·00045. Express 723 lb. as an approximate decimal of a ton by multiplying by $\cdot0004\frac{1}{2}$.

Add together, to 4 decimal places,

6. $\frac{1}{8}, \frac{1}{11}, \frac{1}{15}, \frac{1}{16}$.

7. $\frac{1}{9}, \frac{1}{90}, \frac{1}{900}, \frac{1}{9000}$.

8. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}$. (Calculate each of these from the preceding one.)

9. Use tables of reciprocals to find how many (whole) shares can be bought for £1000 at the following prices per share: £3 3*s.* 8*d.*, £6 15*s.* 9*d.* (First express the price as a decimal of £1.)

10. An important formula in optics is $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$.

If $u = 32{\cdot}5$, $v = 23{\cdot}6$, find f, to 3 significant figures.

11. If $u = 18{\cdot}9$, $f = 15{\cdot}7$, find v, to 3 significant figures.

12. Given that $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$, find R when $r_1 = 4{\cdot}2$, $r_2 = 3{\cdot}5$, $r_3 = 11$. Give the result to 3 significant figures.

Miscellaneous Exercises on Decimals

1. The output of oil of the British Burmah Oil Co. for six successive months was 9·86, 11·367, 11·629, 10·706, 10·916, 10·150 thousands of tons. Find the total output in tons for the six months.

2. The output of another oil company for the same period was 130·22, 133·46, 137·23, 138·74, 152·56, 167·32 hundreds of tons. Find the total output in tons for the six months.

3. Find the increase in tons in the output of coal which was 4·86 million tons in one week and 5·027 millions in the next week.

4. If $W = \frac{P - 88}{0{\cdot}32}$, find W when $P = 106, 152, 184$.

5. If $P = 0{\cdot}052\ W + 2{\cdot}4$, find P when $W = 150, 200, 300$.

6. Find the value of $12 - 3{\cdot}6x$ when $x = 1, 0{\cdot}1, 2{\cdot}3$.

7. Find the value of $\frac{x}{1{\cdot}2} + \frac{y}{3{\cdot}5}$ when $x = 3{\cdot}75$ and $y = 34{\cdot}3$.

8. Find the value of $a \times a - b \times b$ when $a = 11{\cdot}6$ and $b = 8{\cdot}4$.

9. Show that a speed of 1 m. per second is the same as 3·6 Km. per hour.

10. Taking 1 ml. per hour = 0·447 m. per second, express a speed of 30·5 ml. per hr. in metres per second.

11. Find the weight in Kilogrammes of 850 c.c. of copper if 1 c.c. weighs 7·8 grm.

12. If a cubic foot of ice weighs 57·2 lb., find the weight of 1 cu. yd.

13. Work out the following to two decimal places:

(*a*) $\dfrac{\text{Weight of silver}}{\text{Weight of same bulk of platinum}} = \dfrac{10{\cdot}6}{21{\cdot}37} =$

(*b*) $\dfrac{\text{Weight of mercury}}{\text{Weight of same bulk of cast iron}} = \dfrac{13{\cdot}596}{7{\cdot}2} =$

14. A cup will hold 773·5 c.c. How often must it be filled to obtain 15·75 litres at least?

15. Find the cost of 49 m. of cloth at 15 fr. 35 centimes per metre.

16. A journey from London to Aberdeen of 532·5 ml. was done in 11 hr. 45 min. Find the average speed in miles per hour to three significant figures.

17. Find the cost of a ton of pig iron at $2·15 per 100 lb.

18. The average breaking load of manila rope in lb. is given as $500 \times d \times (d + 1)$ where d is the diameter of the rope in inches. Find the breaking loads for ropes of diameters 2·5 and 3·6 in.

19. The number of yarns in the rope is given as $50 \times d \times (d + {\cdot}04)$ where d is the diameter of the rope in inches. Find the number of yarns in a rope of $3\frac{3}{4}$ in. diameter.

20. In a certain year the Underground Railway carried 404 million passengers; find the average number per day to the nearest thousand.

21. Find the cost in 1935 of 350 lb. of rubber at 5·875*d*. per lb.

22. Find the value of 0·325 oz. of gold when gold was worth £5 10*s*. 6*d*. per ounce. Give the answer to the nearest penny.

23. How many lengths of 0·625 ft. are there in 32·53 ft.? Express the remainder in decimals of an inch.

24. If 22·4 litres of hydrogen weigh 2 grammes, how many litres will there be in 1 Kg. of hydrogen?

25. If 1000 tons of coal are used to generate 580000 units of electricity, find to two decimal places the number of pounds used per unit generated.

26. Taking 1 m. as 39·37 in., find the error in yards made by a cyclist who takes 11 ml. as the equivalent of 18 Km.

27. A clerk, in finding the value of 3·75 of £50, read the number as 0·375: find his error in pounds and shillings.

28. Find to the nearest cwt. the amount of lead that could be bought for £1000 at £10 2*s*. 6*d*. per ton.

29. In 1919 India produced 305 million gal. of petroleum. and its value was 1·8 million pounds. Find the value per gallon in pence to two decimal places.

30. A gold-mine produced 13,772 oz. of gold in one month at a profit of £14,020. A second mine produced 5491 oz. at a

profit of £6903. Determine the profit per ounce in each case to the nearest penny.

31. Taking 1 lb. as equal to 0·4536 Kg., find the weight of 1 Kg. in pounds and ounces to the nearest tenth of an ounce.

32. A train travels from London to Manchester ($188\frac{3}{4}$ ml.) in 3 hr. 30 min. Find its average speed in miles per hour to two decimal places.

33. The mean distance of the sun from the earth is given as $9{\cdot}29 \times 10^7$ miles and also as $14{\cdot}95 \times 10^7$ Km. Assuming these to be equal, find to three significant figures the number of kilometres in 1 ml.

34. Divide 5·05 into two parts, one of which exceeds the other by 1·105.

35. Express $5000 in pounds and shillings to the nearest shilling at $4{\cdot}65\frac{3}{8}$ dollars to the pound.

36. In a certain month five tin-mines reported as follows:

	Yield	Working Costs
1.	15 tons tin ore	£850
2.	36 tons tin ore	£2000
3.	37 tons tin ore	£1980
4.	89 tons tin ore	£2570
5.	19 tons 5 cwt. tin ore	£1785

Find, with the least possible work, which mine was least and which the most expensive (per ton of ore) to work. Find also to the nearest shilling the cost per ton for these two mines.

37. Show that $\dfrac{2{\cdot}25}{0{\cdot}25} + \dfrac{2}{1{\cdot}25} + \dfrac{0{\cdot}09}{0{\cdot}00625} = 25$.

38. Simplify $\dfrac{0{\cdot}0013133}{2{\cdot}3 \times 57{\cdot}1}$.

39. Simplify $\dfrac{230{\cdot}69 \times 0{\cdot}018}{0{\cdot}059}$.

40. Simplify $\dfrac{(0{\cdot}00185)^2}{(1{\cdot}85)^2}$.

41. Simplify $\dfrac{5{\cdot}327 - 4{\cdot}622}{0{\cdot}015}$.

42. Find to three decimal places the value of $\frac{92}{117} + \frac{4}{105} - \frac{5}{17}$.

43. Find the value of 0·64 of £4 10*s*. + 0·0125 of £5 13*s*. 4*d*. + 0·65 of £3 2*s*. 6*d*. to the nearest penny.

44. Express $\frac{5}{6}$ of £3 2*s*. $7\frac{1}{2}d$. as a decimal of £5.

45. The value of $\frac{17 \cdot 67 - 16 \cdot 986}{60 \cdot 8}$ was given as 0·2. Give a reason for saying that this is obviously incorrect.

46. 700 tons of quartz yielded 786 oz. of fine gold. Find, to the nearest penny, the value of the gold per ton of quartz at £6 8*s*. 6*d*. per ounce.

47. If a bicycle wheel has a circumference of 81·68 in., how many revolutions to the nearest hundred would it make over a journey of $18\frac{1}{4}$ ml.?

48. A racing motorist travelled a mile in 17·45 sec. Express his average speed in miles per hour and in Km. per hour, each to one place of decimals (take 1 Km. = 1093·6 yd.).

49. An American bought 17·5 m. of silk in France at 47 fr. 50 c. per metre. Calculate the cost in dollars and cents, to the nearest cent, if 1 dollar was worth 24 fr. 86 c.

CHAPTER IX

THE HUNDRED AS A UNIT. SIMPLE PERCENTAGES

Per cent. is an abbreviation of *per centum* (Latin, *centum*, a hundred).

33 per cent. = 33 out of every hundred = $\frac{33}{100}$ = ·33.

·01 = $\frac{1}{100}$ = 1 out of every hundred = 1 per cent.

$$16\tfrac{2}{3} \text{ per cent.} = \frac{16\frac{2}{3}}{100} = \frac{1}{6}.$$

Hence any percentage can be expressed as a decimal or vulgar fraction, *if we divide the number representing it by* 100.

50 per cent. = ·5, 10 per cent. = ·1.

Conversely, any fraction can be expressed as a percentage if *we multiply that fraction by* 100.

·3 = 30 per cent. 1·25 = 125 per cent.

$1\frac{3}{8}$ = 1·375 = 137·5 per cent.

Example 1. Find 12 per cent. of a population of 53150 people.

12 per cent. of 53150 = 53150 × ·12 = 6378

Answer: **6378** people.

Example 2. Find $66\frac{2}{3}$ per cent. of 552 francs.

$$66\tfrac{2}{3} \text{ per cent. of } 552 = 552 \times \frac{66\frac{2}{3}}{100} = 552 \times \frac{200}{300}.$$
$$= 552 \times \tfrac{2}{3}$$
$$= 368$$

Answer: **368** francs.

Example 3. Express $\frac{1}{7}$ as a percentage.

$\frac{1}{7} = \frac{1}{7} \times 100$ per cent.

$= \mathbf{14\frac{2}{7}}$ per cent.

Example 4. Express 2*s*. 6*d*. as a percentage of £1.

2*s*. 6*d*. is $\frac{1}{8}$ of £1

$= \frac{1}{8} \times 100$ per cent. of £1

$= \mathbf{12\frac{1}{2}}$ per cent. of £1.

EXERCISES LVII

(Many can be done mentally)

1. Express 5, 10, 20, 25, 50, 75 per cent. as decimals and also as vulgar fractions.

2. Express $3\frac{1}{3}$, $6\frac{2}{3}$, $8\frac{1}{3}$, $12\frac{1}{2}$, $33\frac{1}{3}$, $133\frac{1}{3}$ per cent. as vulgar and also as decimal fractions.

3. Express 0·05, 0·2, 0·1, 0·25 as percentages.

4. Express $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{32}$ as percentages.

5. Express the following fractions as percentages and arrange them in order of magnitude: $\frac{1}{3}$, $\frac{3}{5}$, $\frac{2}{7}$, $\frac{4}{15}$.

6. What fractions of any quantity are 75, 100, 125, and 250 per cent. of that quantity?

7. 5 per cent. of £1 = £0·05 = 1*s*.
$2\frac{1}{2}$ per cent. of £1 = £0·025 = 6*d*.
$1\frac{1}{4}$ per cent. of £1 = £0·012$\frac{1}{2}$ = 3*d*.

Use these results to express $6\frac{1}{4}$, $7\frac{1}{2}$, $8\frac{3}{4}$, 10, $11\frac{1}{4}$, and $12\frac{1}{2}$ per cent. of £1 in shillings and pence.

8. What percentage of £1 is 1*d*.?

9. 1 per cent. of £1 = 2·4 pence. Use this result to write down 3, 8, 12, 15, 21 per cent. of £1 in shillings, pence, and decimals of a penny.

10. Prove that $\frac{1}{8}$ per cent. of £1 is 0·3*d*. Hence find $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $1\frac{1}{8}$, $4\frac{7}{8}$, $13\frac{1}{8}$ per cent. of £1 in pence and decimals of a penny.
Hint. $5\frac{3}{8}$ = 43-eighths, 0·3 × 43 pence = 1*s*. 0·9*d*.

Approximately 1 yd. = 1 m., less 10 per cent. of 1 m.
Approximately 2 lb. = 1 Kg., less 10 per cent. of 1 Kg.
Approximately 0·2 gal. = 1 litre, less 10 per cent. of 1 litre.

Use these results to convert

11. 8, 23, 350 yd. into metres.

12. 12, 126, 567·8 lb. into Kg.

13. 16, 46, 48·4 gal. into litres.

14. Prove that £1 = 1000 farthings less 4 per cent. of that amount, and use this fact to find the number of farthings in

15. £12, £35, £37 10*s*.

16. £0·125, £0·675, £5·375.

Example. $13\frac{1}{3}$ per cent. of £60 = £0·6 × $13\frac{1}{3}$ or £60 × ·$13\frac{1}{3}$ = £8.

17. Find 4 per cent. of £75.

18. Find 3 per cent. of £80.

19. Find 6 per cent. of 535 dollars.

20. Find $12\frac{1}{2}$ per cent. of £60.

21. Find $8\frac{1}{2}$ per cent. of 550 fr.

22. Find $26\frac{1}{3}$ per cent. of £30.

23. How many quarters are there in 13 per cent. of 3 tons 15 cwt.?

24. How many litres are there in 32 per cent. of 5 Hectolitres?

Express the first of the given quantities as a percentage of the second:

25. $4\frac{1}{2}d.$, 1*s.*
26. 3 pt., 2 qt.
27. 5 f., 1 ml.
28. $1\frac{1}{4}$ hr., 30 min.
29. 6*s.* 3*d.*, 15*s.* 0*d.*
30. 67 links, 1 chain.
31. 3 fr. 50 c., 5 fr.
32. 1 ton, 12 cwt. 56 lb.

To Find any Percentage of any Quantity

Example. Find $7\frac{3}{8}$ per cent. of £1256 12*s.* 6*d.*

£1256 12*s.* 6*d.* = £1256·625.

1 per cent. of given amount	=	£12·56625	
		$7\frac{3}{8}$	
7 ,, ,,	=	87·96375	
$\frac{3}{8}$,, ,,	=	4·71234	37·69875 / 8
		92·67609	

or by parts

1 per cent. of given amount	=	£12·56625	·67609
			20
7 ,, ,, ,, ,,	=	87·96375	13·5218 *s.*
$\frac{1}{4}$,, ,, ,, ,,	=	3·14156	12
$\frac{1}{8}$,, ,, ,, ,,	=	1·57078	6·2616*d.*
		92·67609	

Answer: **£92 13s. 6d.** to the nearest penny.

Exercises LVIII

The symbol % is often used for 'per cent.'

1. Find 5% of £124 10*s.*
2. Find $6\frac{1}{2}$% of £325 12*s.* 6*d.*
3. Find $32\frac{1}{2}$% of £12 8*s.* 4*d.*
4. Find $15\frac{1}{4}$% of £260 16*s.* to the nearest penny.
5. Find $62\frac{1}{2}$% of 7520 fr.
6. Find $1\frac{1}{2}$% of 675 dollars.
7. Find $2\frac{1}{4}$% of 1 ton.
8. Find 85% of $10\frac{1}{2}$ tons.
9. Find $3\frac{1}{3}$% of 375 tons 7 cwt. 56 lb.
10. Find $17\frac{1}{2}$% of 85 Kg.
11. Find $66\frac{2}{3}$% of 550 fr. 50 c.
12. Find 13·7% of a population of 1,278,000 persons.

PERCENTAGES WITH TIME. SIMPLE INTEREST

Interest is money paid for the use of money. It is usually estimated as an annual percentage on the sum lent or borrowed, which is called the *principal*. In its simplest form the computation of interest involves (1) determination of how much this annual percentage produces in one year, followed by (2) multiplication by the number of years.

Hence simple interest affords useful practice in decimal multiplication by mixed numbers and is put here for that reason.

Example 1. Find the simple interest on £375 for $3\frac{1}{2}$ years at 4 per cent. per annum.

We may use 'practice' as follows.

		£	
	1% of Principal	3·75	
Int. for 1 yr. =	4% ,, ,,	15·00	(Multiply top line by 4)
,, ,, 3 yr.		45·00	(3 times previous line)
,, ,, $\frac{1}{2}$ yr.		7·50	($\frac{1}{2}$ second line)
Interest for $3\frac{1}{2}$ yr. is		£52·5	= **£52 10s.**

Example 2. Find the simple interest on 5376 dollars for 2 years 9 months at $3\frac{3}{4}$ per cent. per annum.

		Dollars.	
	1% of Principal	53·76	
	3% ,, ,,	161·28	
	$\frac{3}{4}$% ,, ,,	40·32	(Divide previous line by 4)
Interest for 1 yr. =	$3\frac{3}{4}$% ,, ,,	201·60	
Interest for 2 yr.		403·2	
,, ,, 6 months		100·8	($\frac{1}{2}$ of 1 year's interest)
,, ,, 3 months		50·4	($\frac{1}{2}$ previous line)
The interest for 2 yr. 9 months	=	**554·4**	dollars.

Note. Ex. 1. $3\frac{1}{2} \times 4 = 14$.

Ex. 2. $3\frac{3}{4} \times 2\frac{3}{4} = \frac{15}{4} \times \frac{11}{4} = \frac{165}{16} = 10\frac{5}{16}$.

The interests might have been found as 14 and $10\frac{5}{16}$ per cent. respectively of the given principals.

Exercises LIX

Find the simple interest on

1. £500 for 2 yr. at 5 per cent. per annum.
2. £375 for 3 yr. at 4 per cent. per annum.
3. £125 for $2\frac{1}{2}$ yr. at 6 per cent. per annum.
4. £450 for $3\frac{3}{4}$ yr. at 4 per cent. per annum.
5. £650 for 1 yr. 6 months at 3 per cent. per annum.
6. £850 for 2 yr. at $3\frac{3}{4}$ per cent. per annum.
7. £950 for 6 months at 6 per cent. per annum.
8. £425 for 9 months at $1\frac{1}{2}$ per cent. per annum.
9. £5760 for $2\frac{1}{2}$ yr. at $3\frac{1}{3}$ per cent. per annum.
10. \$535 for $1\frac{1}{4}$ yr. at $5\frac{1}{2}$ per cent. per annum.
11. 725 fr. for $2\frac{1}{2}$ yr. at $4\frac{1}{2}$ per cent. per annum.
12. 925 lire for $3\frac{1}{3}$ yr. at $2\frac{1}{2}$ per cent. per annum.

Note. Apart from the practice in computation afforded, this type of question has little more than academic interest.

Money is not usually put out at simple interest for periods greater than one year. For example, interest is added to an account in the Post Office Savings bank yearly, and becomes part of the principal on which the next year's interest is calculated. Interest calculated in this way is called *compound* interest.

Commissions

House-agents, auctioneers, commercial travellers, and others are often paid according to the value of the business they transact. They may be paid a fixed percentage of this value; such a percentage is called commission.

Sometimes firms pay a commission to persons introducing business to them.

Insurance against loss of property by fire is charged as a percentage of the value of the goods insured. Many other instances of this custom could be given.

Exercises LX

1. Find the annual cost of insuring £250 worth of furniture against fire at $\frac{1}{4}$ per cent.

2. A house-agent charged 5 per cent. on the annual rental of a house let by him. What did the owner pay if his house was let at a rental of £62 10*s.*?

3. A traveller was paid a fixed salary of £200 a year plus a commission of $2\frac{1}{2}$ per cent. on his sales. If his sales were £1250 for the year, how much did he earn?

4. A man introduced business to the value of £5000 to a firm and was paid $7\frac{1}{2}$ per cent. for his services; how much did he receive?

5. The furniture of a house was sold by an auctioneer for £375 at his sales-room, and he charged $12\frac{1}{2}$ per cent. for his services (including advertisement and other expenses). What was the net sum received by the owner?

6. If a shop-assistant receives a bonus of $\frac{1}{2}$ per cent. on all sales, how much will this bonus be if the sales are £75?

7. A professional received $87\frac{1}{2}$ per cent. of the gate money at his benefit. If this amounted to £825 15*s*., how much did he get?

8. 10 per cent. was added to the bill of a tourist for the services of the staff of an hotel. If his bill was 625 fr. 50 c. before this addition, how much, to the nearest penny, did his stay cost him, assuming that the franc was worth 3*d*.?

9. A broker buys £15000 of goods and is paid a commission of $\frac{1}{4}$ per cent. on his purchases; how much does he receive?

10. An agent sells a farmer's fruit for £380 and charges $1\frac{1}{2}$ per cent. commission; what does the farmer receive?

11. A ship reported missing is re-insured at a charge of 40 guineas per cent. on the sum insured. If the new policy taken out cost £3675, for how much was the ship re-insured?

MARKING OF PRICES

Example 1. Mark articles which cost 15*s*. to gain 12 per cent. on the cost price.

12 per cent. of 15*s*. = 15*s*. × ·12 = 1·80*s*.

= 1*s*. 9·6*d*.

∴ Mark price as **16s. 10d.**

Example 2. Goods which are marked for sale at 18*s*. return the seller a profit of 15 per cent. on the 'selling' price. Find the cost price.

Profit on selling price = 18*s*. × ·15 = 2·70*s*.

∴ cost price = 18*s*. − 2·70*s*.

= 15·3*s*. = **15s. 4d.** to the nearest penny.

Exercises LXI

1. Mark goods which cost 30*s.* to gain 15, 25, 40 per cent. on the cost price.

2. Mark goods which cost 50*s.* to gain $16\frac{2}{3}$, $33\frac{1}{3}$, 35 per cent. on the cost price.

3. Mark goods which cost 17*s.* 6*d.* to gain 5, 10, $12\frac{1}{2}$, 80 per cent. on the cost price.

4. Goods are marked at 12*s.* 6*d.* to give a profit of 15 per cent. of the selling price. Find the cost price.

5. A dealer sold some furniture for £320 and estimated that 18 per cent. of this sum was profit. Find what he paid for the furniture.

CHAPTER X

AVERAGES. WEIGHTED AVERAGES

Example 1. The daily pre-War wages of eight men were 4*s*., 4*s*. 6*d*., 5*s*., 5*s*. 3*d*., 6*s*., 6*s*. 9*d*., 7*s*., and 7*s*. 6*d*. respectively. Find the average daily wage.

Total wages $= 46s.$

$$\text{Average wage} = \frac{\text{Total wages}}{\text{Total number of men}} = \frac{46s.}{8} = \textbf{5s. 9d.}$$

Example 2. The length of a wire is measured four times and found to be 15·258, 15·259, 15·257, 15·258 in.

Find the average (or mean) of these measurements.

The first four figures in each measurement are the same.

Adding the last ones, $8 + 9 + 7 + 8 = 32$. $\frac{32}{4} = 8$.

Mean length = **15·258** in.

Example 3. A traveller's weekly sales amount to £510, £515, £520, £530, £560 for five successive weeks.

Find his average weekly sale.

Take £520 as being very near to the average.

$$\begin{aligned}
£510 &= £520 - £10 \\
£515 &= \quad ,, \quad - £5 \\
£520 &= \quad ,, \quad + £0 \\
£530 &= \quad ,, \quad + £10 \\
£560 &= \quad ,, \quad + £40 \\
\hline
\text{Total sales} &= £520 \times 5 + £35 \\
\therefore \text{ Average} &= £520 + £7 \\
&= \mathbf{£527.}
\end{aligned}$$

Exercises LXII

1. The height of a cone is measured three times and found to be 11·525, 11·530, 11·528 decimetres. Find the average of these measurements.

2. The ages of a class of 5 boys are 11 years 2 months, 11 years 9 months, 11 years 10 months, 12 years 2 months, and 12 years 3 months. Find the average age of the class.

3. Find the mean of the following lengths: 8·92, 8·93, 8·96, and 8·95 cm.

4. The prices of wheat for the six years 1915 to 1920 were 55*s.* 2*d.*, 56*s.* 7*d.*, 78*s.* 4*d.*, 74*s.* 7*d.*, 73*s.* 4*d.*, 85*s.* 10*d.* per quarter. Find the average price per quarter for the whole period.

The price in 1914 was four-sevenths of this average price; find the price in 1914.

5. Rubber prices in 1921 were $9\frac{1}{3}d.$, $9\frac{1}{4}d.$, $9\frac{1}{2}d.$, $10\frac{1}{8}d.$, $10\frac{3}{4}d.$, $10\frac{1}{2}d.$ per lb; find the average price per lb.

At the end of 1934 the price was $6\frac{1}{4}d.$ per lb.; what fraction is this price of the average price in 1921?

6. The average of 6 numbers is 18. The average of the first two is 20 and of the last two, 16; what is the average of the other two?

7. The average temperature for Monday, Tuesday, and Wednesday was 53°. The average for Tuesday, Wednesday, and Thursday was 56°. The temperature for Thursday being 60°, what was the temperature for Monday?

8. The heights of five boys in a class are 5 ft. 3 in., 5 ft. $3\frac{1}{2}$ in., 5 ft. 4 in., 5 ft. 5 in., and 5 ft. $4\frac{1}{2}$ in.; find the average height of the five boys.

9. The average prices of silver per ounce for the years 1917 to 1921 were $40\frac{7}{8}$, $47\frac{9}{16}$, $57\frac{1}{16}$, $61\frac{9}{16}$, and $36\frac{7}{8}$ pence; find the average price for the period.

10. The average prices of silver for the years 1929 to 1934 were $24\frac{7}{16}$, $17\frac{11}{16}$, $14\frac{9}{32}$, $17\frac{27}{32}$, $18\frac{3}{32}$, and $21\frac{7}{32}$ pence per oz.; find the average price for the period.

11. In 1920 six Lancashire cotton-spinning companies declared dividends of 800, 425, 250, 350, 180, and 140 per cent. respectively on their capital; find the average dividend of the six companies.

12. In 1921 the average dividend of the same six companies was $34\frac{2}{3}$ per cent.; find what fraction this is of the average in 1920.

Weighted Averages

Example. 60 labourers earn 7*s.* each, 200 earn 8*s.* each, and 150 earn 9*s.* each per day. Find the average daily wage of these men.

60	men earn	420	shillings
200	,, ,,	1600	,,
150	,, ,,	1350	,,
	Total wages	3370	,,

$$\therefore \text{ Average wage} = \frac{\text{Total wages}}{\text{Total number of men}}$$
$$= \frac{3370}{410} \text{ shillings}$$
$$= \textbf{8s. } \mathbf{2\tfrac{1}{2}d.} \text{ approximately.}$$

It would be wrong to say that the average wage

$$= \frac{7 + 8 + 9}{3} \text{ shillings}$$
$$= 8 \text{ shillings.}$$

If we did we should attach as much importance (or 'weight') to the smaller number of men as to the larger. Each must have its own proper weight.

Exercises LXIII

1. The average height of six men is 5 ft. 5 in., of ten others, 5ft. $8\frac{1}{2}$ in., and of four more, 5 ft. $7\frac{1}{2}$ in. Find the average height of the twenty men.

2. 10 lb. of tea at 1*s.* 6*d.* per lb. are mixed with 8 lb. at 2*s.* 3*d.* per lb. Find the average price of the mixture per lb.

3. If 30 gal. of spirits at 36*s.* per gal. are mixed with 10 gal. of water, find the value per gallon of the mixture.

4. The weekly wage of 5 men was 58*s.* each, and of seven other men, 65*s.* each. Find the average wage per week of the twelve men.

5. Copper weighs 8·9 and zinc 7·1 grm. per c.c. Find the weight per c.c. of brass containing 80 per cent. by volume of copper and 20 per cent. of zinc.

6. The average height of three men was 5 ft. 9 in. The tallest man was 5 ft. $10\frac{1}{2}$ in. What was the average height of the other two?

7. The average mark of a class of 24 boys in an examination was 47. The average mark of the five top boys was 81 and the bottom boy got 21. Find the average mark of the other eighteen boys.

8. A man's cricket average is 35 for his first twelve innings and 32 for the last six; what is his average for the season?

CHAPTER XI

MENTAL PROCESS FOR DECIMALIZATION OF MONEY

$2s. = £\cdot 1$
$1s. = £\cdot 05$
$6d. = £\cdot 025$
$3d. = £\cdot 0125$ or $£\cdot 012\frac{1}{2}$

The vulgar fraction $\frac{1}{2}$ is equivalent to 5 in the fourth place of decimals.

Observe also that we can express shillings as a decimal of £1 if we divide the number of shillings by 2 and prefix the decimal point.

$$14s. = £\cdot 7. \quad 17s. = £\cdot 85.$$

Example 1. Express 15*s*. 9*d*. as a decimal of £1.

$14s. = £\cdot 7$		*or*	$15s. = £\cdot 75$
$1s. = £\cdot 05$			$6d. = £\cdot 025$
$6d. = £\cdot 025$			$3d. = £\cdot 0125$
$3d. = £\cdot 0125$			**£·7875**
$15s.\ 9d. =$ **£·7875**			

Example 2. Express £·775 in shillings and pence.

$£\cdot 7 = 14s$	*or* $£\cdot 75 = 15s.$ (multiplying by 2)
$£\cdot 05 = 1s.$	
$£\cdot 025 = 6d.$	$£\cdot 025 = 6d.$
$\therefore£\cdot 775 =$ **15s. 6d.**	**15s. 6d.**

As there are 20 cwt. in 1 ton, and 4 qr. in 1 cwt., considering cwts. as shillings and qrs. as 3*d*., we have

$$15 \text{ cwt. } 3 \text{ qr.} = \cdot 75 \text{ tons} + \cdot 0375 \text{ tons}$$
$$= \mathbf{\cdot 7875\ tons.}$$

$$\cdot 775 \text{ tons} = \cdot 750 \text{ tons} + \cdot 025 \text{ tons}$$
$$= \mathbf{15\ cwt.\ 2\ qr.}$$

EXERCISES (MENTAL) LXIV

Express as decimals of £1:

1. 2*s*. 6*d*., 5*s*. 6*d*., 8*s*. 6*d*., 12*s*. 6*d*., 14*s*. 6*d*., 17*s*. 6*d*., 18*s*. 6*d*.
2. £1 6*s*., £2 12*s*., £3 13*s*., £1 11*s*. 6*d*., £2 14*s*. 6*d*., £5 19*s*. 6*d*.
3. 1*s*. 3*d*., 2*s*. 3*d*., 3*s*. 9*d*., 17*s*. 3*d*., 10*s*. 9*d*., 4*s*. 3*d*., 18*s*. 9*d*.
4. £1 16*s*. 3*d*., £3 0*s*. 9*d*., £5 13*s*. 6*d*., £4 7*s*. 9*d*., £8 10*s*. 3*d*., £10 1*s*. 9*d*.
5. £9 2*s*. 9*d*., 15*s*. 6*d*., £2 0*s*. 6*d*., £6 19*s*. 3*d*., £5 4*s*. 9d., £1 9*s*. 3*d*.

Express in shillings and pence the following decimals of £1:

6. ·55, ·75, ·125, ·375.
7. ·15, ·85, ·175, ·225.
8. ·875, ·525, ·475, ·675.
9. ·625, ·6375, ·875, ·8875.
10. ·35, ·1625, ·95, ·9625.

For sums less than 3*d*. the decimals will often be non-terminating.

We will use the term *mil* for the thousandth part.

$$6d. = 25 \text{ mils of £1.}$$

$$\therefore\ 24 \text{ farthings} = 25 \text{ mils of £1.}$$

$$\therefore\ \text{each farthing} = 1\frac{1}{24} \text{ mils of £1.}$$

$$13 \text{ farthings} = 13\frac{13}{24} \text{ mils of £1}$$

$$= 13\frac{6{\cdot}5}{12} \text{ mils of £1}$$

$$= 13{\cdot}5416\ldots \text{ mils of £1}$$

$$= £{\cdot}0135416\ldots$$

to as many decimal places as are wanted.

$$5d. = 20 \text{ farthings} = 20\frac{20}{24} \text{ mils of £1}$$

$$= £{\cdot}020\frac{10}{12}$$

$$= £{\cdot}020833\ldots$$

DECIMALIZATION TO 3 PLACES (corrected)

If a sum of money is to be expressed as a decimal of £1, corrected to 3 places, that is, to the nearest mil of £1, we first decimalize the shillings and sixpence, if there is one, according to the rules already given. To deal with the remainder, a sum less than sixpence, we note that, since each farthing equals $1\frac{1}{24}$ mils of £1, any number of farthings less than 12 will equal the same number of mils of £1, to the nearest mil; but we must add 1 to a number of farthings from 12 to 23 ($5\frac{3}{4}d.$) to obtain the equivalent number of mils of £1, to the nearest mil.

Example 1. Express £5 13*s*. $10\frac{1}{4}d.$ as a decimal of £1.

$$13s. = £\cdot65$$
$$6d. = £\cdot025$$
$$4\tfrac{1}{4}d. = 17 \text{ farthings} = £\cdot018 \text{ (to 3 places)}$$
Answer = **£5·693** (corrected to 3 places)

To *more places*, $17\frac{17}{24}$ mils

$$= 17\frac{8\cdot5}{12} \text{ mils}$$
$$= 17\cdot708\ldots \text{ mils} = \cdot017708\ldots$$
£5 13*s*. $10\frac{1}{4}d.$ = **£5·692708**...

Example 2. Express £5·693 in £ *s*. *d*.

$$£\cdot6 = 12s.$$
$$£\cdot05 = 1s.$$
$$£\cdot025 = 6d.$$
$$£\cdot018 = 18 \text{ mils of } £1 = 17 \text{ farthings} = 4\tfrac{1}{4}d.$$
Answer = **£5 13s. 10¼d.** to nearest farthing.

Note. When the rule given above is used to change mils of £1 to farthings, it fails to give the value of 12 mils. £·012 = 11·52 farthings, and so should be taken as 12 farthings.

EXERCISES LXV

You may be able to write down the answers to many of the following examples at once. If you find it hard, try them in two mental steps as follows:

7*s*. $8\frac{1}{2}d.$ = 7*s*. 6*d*. + $2\frac{1}{2}d.$ = £·375 + £·010 = £·385.
4*s*. $11\frac{1}{2}d.$ = £·225 + £·023 = £·248.
£·543 = £·525 + £·018 = 10*s*. 6*d*. + $4\frac{1}{4}d.$ = 10*s*. $10\frac{1}{4}d.$
£15·123 = £15 2*s*. + $5\frac{1}{2}d.$ = £15 2*s*. $5\frac{1}{2}d.$

Express as decimals of £1 corrected to 3 places:

1. 1*d*.
2. $3\frac{3}{4}d$.
3. 8*d*.
4. 2*s*. 1*d*.
5. 4*s*. $3\frac{1}{2}d$.
6. 6*s*. 11*d*.
7. 13*s*. 10*d*.
8. 14*s*. $2\frac{1}{4}d$.
9. 6*s*. $3\frac{1}{2}d$.
10. 17*s*. $8\frac{1}{4}d$.
11. £15 10*s*. 4*d*.
12. £32 3*s*. $10\frac{1}{4}d$.
13. £19 8*s*. $9\frac{3}{4}d$.
14. £86 11*s*. 5*d*.
15. £16 4*s*. $8\frac{1}{2}d$.
16. £63 15*s*. 1*d*.
17. £47 19*s*. $2\frac{1}{2}d$.
18. £18 7*s*. $7\frac{3}{4}d$.
19. £6 2*s*. $11\frac{1}{4}d$.
20. £23 17*s*. $3\frac{1}{2}d$.

Express in pounds, shillings, and pence to the nearest farthing the following decimals of £1:

21. ·007.
22. ·015.
23. ·023.
24. ·033.
25. ·029.
26. ·042.
27. ·066.
28. ·112.
29. ·161.
30. ·321.
31. ·618.
32. ·210.
33. ·345.
34. ·768.
35. 10·282.
36. 86·575.
37. 2·063.
38. 5·357.
39. 9·987.
40. 16·543.

Exercises LXVI

Express as decimals of £1 corrected to five places of decimals:

1. £73 2*s*. 10*d*.
2. £28 16*s*. $3\frac{1}{2}d$.
3. £5 3*s*. 8*d*.
4. £16 17*s*. $2\frac{3}{4}d$.
5. £41 4*s*. 5*d*.
6. £50 15*s*. $11\frac{1}{2}d$.
7. £13 5*s*. $6\frac{1}{4}d$.
8. £86 19*s*. 7*d*.
9. £109 8*s*. $10\frac{3}{4}d$.
10. £69 7*s*. $0\frac{1}{2}d$.

Alternative Method

$$\begin{aligned}\tfrac{1}{4}d. = 0{\cdot}250d. &= {\cdot}240d. + {\cdot}010d.\\ &= £{\cdot}001 + {\cdot}01d. \text{ exactly.}\\ \therefore\ £5\ 13s.\ 10\tfrac{1}{4}d. &= £5\ 13s.\ 6d. + 4\tfrac{1}{4}d.\\ &= £5{\cdot}675 + £{\cdot}017 + {\cdot}17d. \text{ exactly.}\\ &= £5{\cdot}692 + {\cdot}17\text{d. exactly.}\\ \text{Each farthing} &= £{\cdot}001 + {\cdot}01d. \text{ exactly.}\\ \therefore\ \text{each mil of } £1 &= \text{one farthing} - {\cdot}01d. \text{ exactly.}\end{aligned}$$

Express £3·896 in £ *s*. *d*.

$$\begin{aligned}£{\cdot}85 &= 17s.\\ £{\cdot}025 &= 6d.\\ £{\cdot}021 &= 5\tfrac{1}{4} - {\cdot}21d.\\ \hline £3{\cdot}896 &= £3\ 17s.\ 11\tfrac{1}{4}d. - {\cdot}21d.\end{aligned}$$

The answer lies between £3 17*s*. 11*d*. and £3 17*s*. 11¼*d*. and is nearer to the former than to the latter.

This method is often convenient.

Example. Find the cost of 1000 oz. of silver at 2*s*. 2¼*d*. per oz.

$$2s.\ 2\tfrac{1}{4}d. = £\cdot109 + \cdot09d.$$

$$\therefore\ 1000 \text{ oz. cost } £109 + 90d. = \mathbf{£109\ 7s.\ 6d.}$$

The error in taking the decimal to three places is explicitly stated.

See the next chapter for more examples.

Exercises LXVII

Fill in the blank spaces in

1.	3*s*. 1½*d*.	=	£·156	+	*d*.
2.	12*s*. 8*d*.	=	£·633	+	*d*.
3.	15*s*. 10½*d*.	=	£·793	+	*d*.
4.	6*s*. 11½*d*.	=	£	+	·22*d*.
5.	17*s*. 4¼*d*.	=	£	+	·17*d*.
6.	£·136	=	2*s*. 8¾*d*.	−	*d*.
7.	£·528	=	10*s*. 6¾*d*.	−	*d*.
8.	£·343	=	6*s*. 10½*d*.	−	*d*.
9.	£·971	=		−	·21*d*.
10.	£·589	=		−	·14*d*.

Use the method to find

11. The values of £1000 sterling in the following cities on a certain day.

(*a*) Paris, where the pound sterling was worth £2 1*s*. 6*d*.
(*b*) Brussels, where the pound sterling was worth £2 3*s*. 2½*d*.
(*c*) Rome, where the pound sterling was worth £3 17*s*. 6½*d*.
(*d*) Copenhagen, where the pound sterling was worth £1 3*s*. 3*d*.

12. The cost of 560 oz. of silver at 2*s*. 4¼*d*. per oz.

CHAPTER XII

CALCULATION OF COSTS, ETC., BY VARIOUS METHODS

Example 1. Find the cost of 335 shares at £2 13*s*. 7½*d*. each.

Method 1. Long multiplication.

£	*s*.	*d*.
335	335	335
2	13	7½
670	335	2345
228	1005	167½
£898	209	12)2512½
	20)4564	209*s*. 4½*d*.
	228 — 4*s*.	

Answer **£898 4s. 4½d.**

Start on the right with the pence and reduce the total pence to shillings and pence. Move to the left, carrying forward the shillings and so on.

Method 2. By parts.

			£	*s*.	*d*.
Cost at £1			335	0	0
,,	£2		670	0	0
,,	10*s*.	½ of £1	167	10	0
,,	2*s*.	$\frac{1}{5}$ of 10*s*.	33	10	0
,,	1*s*.	½ of 2*s*.	16	15	0
,,	6*d*.	½ of 1*s*.	8	7	6
,,	1½*d*.	¼ of 6*d*.	2	1	10½
			£898	**4**	**4½**

Other possible ways of taking parts are 10*s*., 2*s*. 6*d*. (¼ of 10*s*.), 1*s*. ($\frac{1}{10}$ of 10*s*.), 1½*d*. ($\frac{1}{8}$ of 1*s*.);

or 10*s*., 3*s*. 4*d*. ($\frac{1}{6}$ of £1, or $\frac{1}{3}$ of 10*s*.), 3*d*. ($\frac{1}{40}$ of 10*s*.), ½*d*. ($\frac{1}{6}$ of 3*d*.).

We may work by parts using decimals of £1 as follows.

			£	
Cost at £1			335	
,,	£2		670	Multiply top line by 6, starting 1 place to right.
,,	12*s*.	·6 of £1	201·0	
,,	1*s*.	·05 of £1	16·75	Multiply by 5, starting 2 places to right.
,,	6*d*.	$\frac{1}{2}$ of 1*s*.	8·375	
,,	$1\frac{1}{2}$*d*.	$\frac{1}{4}$ of 6*d*.	2·09375	
			£898·21875	= **£898 4s. $4\frac{1}{2}$d.**

Method 3. £2 13*s*. $7\frac{1}{2}$*d*. = £2·681 + ·06*d*.

£	*d*.
2681	335
335	6
8043	20·10
8043	
13405	
898·135	

Answer £898 2*s*. $8\frac{1}{2}$*d*. — ·10*d*. + 1*s*. 8·10*d*.

= **£898 4s. $4\frac{1}{2}$d.**

In this method the multiplicand is restricted to 3 decimal places.

Example 2. Find the cost of 1255 lb. of rubber at 1*s*. $8\frac{3}{4}$*d*. per lb.

Method 1. By parts

		using £	*s*.	*d*.	using decimals of £1
at £1		1255	0	0	£1255
,, 1*s*.	$\frac{1}{20}$ of £1	62	15	0	62·75
,, 6*d*.	$\frac{1}{2}$ of 1*s*.	31	7	6	31·375
,, 2*d*.	$\frac{1}{3}$ of 6*d*.	10	9	2	10·4583
,, $\frac{3}{4}$*d*.	$\frac{1}{8}$ of 6*d*.	3	18	$5\frac{1}{4}$	3·9219
		£108	10	$1\frac{1}{4}$	£108·5052
					= **£108 10s. $1\frac{1}{4}$d.**

Rough check: 2*s*. × 1200 = £120.

Method 2. 1*s*. 8*d*. is a simple fraction, viz. $\frac{1}{12}$, of £1.

	£	*s*.	*d*.
	12)1255	0	0
Cost at 1*s*. 8*d*.	104	11	8
,, $\frac{3}{4}$*d*.	3	18	$5\frac{1}{4}$
Answer	**£108**	**10**	**$1\frac{1}{4}$**

1255	
3	
4)3765	farthings
12) $941\frac{1}{4}$	pence
78*s*. $5\frac{1}{4}$*d*.	

Method 3. 1*s*. $8\frac{3}{4}$*d*. = £·086 + ·11*d*.

£	*d*.
1255	1255
86	1255
10040	138·05*d*.
7530	
£107·93	

Answer is £107 18*s*. $7\frac{1}{4}$*d*. — ·05*d*. + 11*s*. 6·05*d*.

= **£108 10s. $1\frac{1}{4}$d.**

Example 3. Multiply 4 tons 9 cwt. 3 qr. by 739.

Method 1. Long multiplication.

tons	cwt.	qr.
739	739	739
4	9	3
2956	6651	4)2217
360	554	554 cwt. 1 qr.
3316	20)7205	
	360 tons 5 cwt.	

Answer **3316 tons 5 cwt. 1 qr.**

Method 2. By parts.

	Using	tons	cwt.	qr.	Using decimals of 1 ton
		739	0	0	739
4 tons		2956	0	0	2956
5 cwt.	$\frac{1}{4}$ of 1 ton	184	15	0	184·75
4 cwt.	$\frac{1}{5}$ of 1 ton	147	16	0	147·8
2 qr.	$\frac{1}{8}$ of 4 cwt.	18	9	2	18·475
1 qr.	$\frac{1}{2}$ of 2 qr.	9	4	3	9·2375
		3316	5	1	3316·2625
					20
					5·25
					4
					1·

Answer **3316 tons 5 cwt. 1 qr.**

Example 4. Find the cost of 3 cwt. 3 qr. 21 lb. of metal at £2 1*s.* 6*d.* per cwt., to the nearest penny.

Method 1.

			Using £	*s.*	*d.*	Using decimals of £1 £
Cost of 1 cwt.			2	1	6	2·075
,,	3 cwt.		6	4	6	6·225
,,	2 qr.	$\frac{1}{2}$ of 1 cwt.	1	0	9	1·0375
,,	1 qr.	$\frac{1}{2}$ of 2 qr.		10	4·5	·51875
,,	14 lb.	$\frac{1}{2}$ of 1 qr.		5	2·25	·25938
,,	7 lb.	$\frac{1}{2}$ of 14 lb.		2	7·125	·12969
			£8	3	4·875	£8·17032

Answer **£8 3s. 5d.**

Method 2. 3 cwt. 3 qr. 21 lb. = 4 cwt. — 7 lb.

	£	*s.*	*d.*		£
	2	1	6	or	2·075
Cost of 4 cwt.	8	6	0		8·300
Cost of 7 lb. $\frac{1}{16}$ of 1 cwt.		2	$7\frac{1}{8}$(10*s*.$4\frac{1}{2}$*d*.)		·1297(·5188)
	£8	3	5		£8·1703
					= **£8 3s. 5d.**

To find $\frac{1}{16}$ of £2 1*s*. 6*d*., either divide mentally in one step or divide by 4 and 4 again, the result of the first division being written to the right of the main working.

When sums of money can be decimalized mentally with speed and accuracy, it is often quicker to use decimals of £1 in the method of parts than to use £ *s*. *d*. Also in the use of £ *s*. *d*. mistakes may arise in the frequent change of units from pounds to shillings, shillings to pence, etc. On the other hand errors in calculation may sometimes be more easily noticeable when £ *s*. *d*. are used.

Note that the last example might have been worked as follows:

3 cwt. 3 qr. 21 lb.	= 3·9375 cwt.
£2 1*s*. 6*d*.	= £2·075
∴ Cost	= £2·075 × 3·9375.

This may be evaluated by long multiplication of decimals, but experience shows that more slips are made when examples are worked by this method. For shortened methods of multiplication see Chapter XXXII.

Choose your method and think out the neatest and simplest arrangement of the work before you begin the computation.

Exercises LXVIII

Find the cost of

1. 23 tons of copper at £65 7*s*. 6*d*. per ton.
2. 136 tons of lead at £26 18*s*. 9*d*. per ton.
3. 97 tons of spelter at £27 3*s*. 9*d*. per ton.
4. 1500 cwt. of Danish butter at £5 19*s*. 6*d*. per cwt.
5. 875 Anglo-Iranian shares at 97*s*. 6*d*. per share.
6. 280 rubber shares at 18*s*. $4\frac{1}{2}$*d*. per share.
7. Find the total amount of the rates on a house which is assessed at £75, if the rates are 9*s*. 10*d*. in the £.
8. A bankrupt owes £3450 and pays 13*s*. 9*d*. in the £; find his total assets.
9. Find the dividend on £560 10*s*. at 16*s*. 10*d*. in the £.
10. Find the cost of 584 gal. of petrol at 1*s*. $5\frac{1}{2}$*d*. per gallon.
11. The cost of tyres on a motor-lorry is estimated at $1\frac{1}{4}$*d*. per mile run. Find the cost for a mileage of 7044 ml.
12. Find the cost of importing 8750 lb. of New Zealand mutton at $6\frac{3}{4}$*d*. per lb.
13. Find the value of a ton of rubber at 1*s*. $1\frac{5}{16}$*d*. per lb.

14. Find the value of £150 in Canadian dollars when £1 is worth $4{\cdot}88\frac{5}{8}$ dollars.

15. Find the total cost of 2500 electric-lamp bulbs at 1*s.* $7\frac{1}{2}$*d.* each.

16. Find the cost of supplying 2500 meals at the rate of $7\frac{1}{4}$*d.* per meal.

Find the cost, to the nearest penny, of

17. 3 cwt. 2 qr. 14 lb. of copper at £3 5*s.* 6*d.* per cwt.

18. 3 cwt. 3 qr. 7 lb. of lead at £1 11*s.* 9*d.* per cwt.

19. 12 cwt. 1 qr. 21 lb. of tin at £22 16*s.* per cwt.

20. 6 cwt. 7 lb. of sisal at £1 8*s.* 9*d.* per cwt.

21. 3 tons 5 cwt. 21 lb. of hemp at £13 7*s.* 6*d.* per ton.

22. 3 tons 80 lb. of copper at £72 15*s.* per ton.

23. 5 tons 3 cwt. 60 lb. of metal at £11 18*s.* 9*d.* per ton.

24. 12 ac. 3 r. 25 p. of land at £15 per acre.

25. 5 ac. 2 r. 15 p. of land at £133 10*s.* per acre.

26. The repairs to a road 10 ml. 3 f. 121 yd. long at £87 16*s.* per mile.

27. 15 ac. 2 r. 30 p. of land at £3 17*s.* 6*d.* per acre.

28. The rent of 18 ac. 2 r. 16 p. at £18 16*s.* per acre.

29. The cost of a road 50 ml. 6 f. 8 ch. long at £150 per mile.

30. The cost of 13 tons 455 lb. of Portland cement at £3 18*s.* per ton.

CHAPTER XIII

SQUARE ROOT

The square of any whole number must contain twice as many figures, or one less than twice as many, as the original number. The reader may easily convince himself of the truth of this. The square root of a square number must therefore have half the number of figures, or half of one more than the number of figures, of the square number.

We may use the algebraic identity

$$(a + x)^2 = a^2 + 2ax + x^2 = a^2 + x(2a + x)$$

to find square roots by trial.

1849 is greater than 40^2, but less than 50^2. Assume that it is the square of $(40 + x)$

Then $1849 = (40 + x)^2 = 1600 + 80x + x^2$

Subtracting 1600 from both sides of the equation,

$$249 = 80x + x^2 = x(80 + x)$$

Try $x = 3$

$$x(80 + x) = 3 \times 83 = 249$$

$\therefore$ The square root of 1849 is **43.**

To find the square root of 242064.

The square root will have 3 figures and lies between 400 and 500. Let it be $400 + x$.

Then $242064 = (400 + x)^2 = 160000 + 800x + x^2$.

Subtracting 160000 from each side,

$$82064 = 800x + x^2 = x(800 + x)$$

To get a rough idea of the value of x, divide 80000 by 800, and remembering that x is less than 100, try $x = 90$

$$x(800 + x) \text{ is then } 90 \times 890 = 80100$$

which falls short of 82064 by 1964.

The square of 490 is therefore 1964 less than 242064.

$\therefore$ If $490 + x$ is the square root we must have

$$x(2.490 + x) = 980x + x^2 = 1964$$

To find x we divide 1964 by 980, and so try $x = 2$. We find the equation is satisfied by $x = 2$, so that the square root is **492.**

The principle is the same throughout. Double the part of the root already found to form a trial divisor. Find x by trial and add it to the trial divisor. Multiply the result by x, and subtract.

The above example is now worked out again, to show the method of setting out a square root. Compare the steps in the two solutions. You will see that they are really the same, except that in the formal method the later figures are not written down until it is necessary to do so.

Example. Find the square root of 242064.

Mark off periods of two figures, starting from the units figure. This gives a pair 24 at the extreme left of the number. (With some numbers there will be one figure only.) Find the largest square number not greater than 24, *i.e.*, 16. Place 16 under 24, and its square root (4) above 24 as the first figure in the root. Subtract, and bring down the next pair of figures.

(1)

```
  4
 ________
 24'20'64
 16
 --
  8 20
```

The remaining steps are as follows.

(2)

```
       4  9
      ________
      24'20'64
      16
      --
  89   8 20
       8 01
       ----
         19 64
```

Multiply 4 by 2, getting 8, and write this opposite 820. Divide 820 mentally by 80, or 82 by 8. We cannot have a quotient greater than 9. Try 9 as the next figure in the root. Write 9 after the 8 to make 89 as a divisor into 820, multiply by 9, subtract, and bring down the next pair of figures.

(3)

```
        4  9  2
       ________
       24'20'64
       16
       --
   89   8 20
        8 01
        ----
  982     19 64
          19 64
          -----
```

Multiply 49 by 2, getting 98, and write this opposite 1964. Divide 1964 by 980, or 19 by 9. This gives 2 as the next figure in the root. Put 2 after 98, making the divisor 982, multiply by 2, and subtract. There is no remainder, and **492** is the square root required.

Note. If too large a divisor is chosen at any stage, the product obtained will be bigger than the number from which it is to be subtracted. Try a smaller one, but make certain that the largest possible one is chosen each time.

Square Root of a Decimal

Starting from the decimal point, mark off periods of two figures to the left from the integral part, and periods of two figures to the right from the decimal part. Then proceed as already described.

Example. Find the square root of 1621·6729.

```
           4  0 ·2  7
          ____________
          16'21·67'29
          16
          __
  802        21 67
             16 04
             _____
  8047        5 63 29
              5 63 29
              _______
```

Beginners often find a difficulty here.

When we multiply 4 by 2, getting 8, and think of 80 as a trial divisor, we find this greater than 21, which is to be divided by it. We place a zero in the answer, getting 40, bring down the next two figures (67), multiply 40 by 2, getting 80, and divide 2167 by 800, or 21 by 8, for the next figure in the root, 2, and proceed as before.

Irrational Numbers

If we apply Pythagoras' Theorem (see p. 122) to find the length of the diagonal of a rectangle whose sides are 1 and 2 in. long respectively, we find that the area of the square on the diagonal is 5 sq. in. It is natural to suppose that there is a number which measures the length of the diagonal and whose square therefore is 5. We write the number $\sqrt{5}$.

If we work out the square root of 5 we obtain

```
          2·2  3  6
         ----------
         5·00'00'00
         4
   42    1 00
           84
  443      16 00
           13 29
 4466       2 71 00
            2 67 96
            -------
               3 04
```

and it is found that the process continues indefinitely. It can be shown that $\sqrt{5}$ is not exactly equal to any vulgar fraction or to any terminated or recurring decimal (each of which can be expressed as a vulgar fraction).

The numbers 2, 2·2, 2·23, 2·236, etc., obtained in working out the square root, are successive approximations to $\sqrt{5}$, their squares being

4, 4·84, 4·9729, 4·999696 respectively,

and $\sqrt{5}$ can be replaced in calculations by any one of them which is sufficiently near to it.

If several figures are required, contracted methods may be used with advantage (see Chapter XXXII).

Numbers which cannot be expressed exactly as vulgar fractions are called irrational numbers. Square and other roots which are not exact are examples of irrational numbers, and π (see p. 125) is a further example.

Example. Find the square root of ·8 correct to 3 significant figures.

```
           ·8  9  4 (4)
          ---------
          ·80'00'00
           64
  169      16 00
           15 21
 1784         79 00
              71 36
17884          7 64 00
```

The square root is ·**894** correct to 3 significant figures. Note that the square root of a number which is less than 1 is greater than the number itself.

Example. Find the square root of $18\frac{1}{16}$.

$$18\tfrac{1}{16} = \frac{289}{16}$$

$$\therefore \sqrt{18\tfrac{1}{16}} = \sqrt{\frac{289}{16}}$$

$$= \frac{17}{4}$$

$$= \mathbf{4\tfrac{1}{4}}.$$

Example. Find the square root of $2\frac{1}{7}$.

Express $2\frac{1}{7}$ as a decimal, and proceed according to rule.

In finding the square root of a mixed number, express the mixed number as an improper fraction if the denominator of the fractional part is a perfect square, and take the square roots of denominator and numerator. Otherwise express the fraction as a decimal fraction, and work in decimals.

Exercises LXIX

Find the square roots of

1. 576.	7. 17424.	13. 1267·36.
2. 841.	8. 91809.	14. ·008649.
3. 1681.	9. 97969.	15. 199·6569.
4. 3249.	10. 32·49.	16. 6430·4361.
5. 8649.	11. ·0625.	17. 20107·24.
6. 8281.	12. 2530·09.	18. 281·9041.

Find, to three decimal places, the square roots of

19. 10.	23. 7.	27. 0·9.	31. 3·1416.
20. 40.	24. 8.	28. 2·5.	32. ·0072.
21. 2.	25. 0·2.	29. ·025.	33. ·072.
22. 5.	26. 0·7.	30. 3·6.	34. 6·1416.

Find the square roots of the following vulgar fractions, giving the results in fractions or decimals correct to three places:

35. $1\frac{25}{144}$.
36. $342\frac{1}{4}$.
37. $11\frac{70}{81}$.
38. $4\frac{1}{9}$.
39. $3\frac{1}{7}$.
40. $5\frac{1}{3}$.
41. $36\frac{1}{6}$.
42. $11\frac{37}{40}$.
43. $182\frac{1}{4}$.
44. $81\frac{44}{90}$.

Find the value to 3 places of decimals of

45. $\sqrt{(1{\cdot}2)^2 - (0{\cdot}8)^2}$.

46. $\sqrt{(3{\cdot}875)^2 - (1{\cdot}125)^2}$.

47. $\sqrt{(23{\cdot}73)^2 - (3{\cdot}73)^2}$.

48. If $r^2 = \frac{100}{\pi}$ find r to two decimal places, taking $\pi = 3{\cdot}142$.

49. Show that the square root of 27 differs by less than 0·0001 from $\frac{265}{51}$.

50. Find two numbers differing by ·001, between which the square root of 17·4791 lies.

51. Find the least whole number whose square is greater than 20000.

52. Find the least whole number whose fourth power is greater than 25000.

53. The time in seconds taken for a body to fall any given distance is approximately one-quarter of the square root of the measure in feet of the distance. How many seconds (to the nearest ·01 second) would a stone take to fall to the bottom of a well 250 ft. deep?

CHAPTER XIV

MENSURATION (II)

Area of the Four Walls of a Room or the Four Vertical Sides of a Box (assumed to be Rectangular and of the Same Height)

Height	End Wall	Side Wall	End Wall	Side Wall	Height

The walls can be imagined to be placed end to end to form a rectangle.

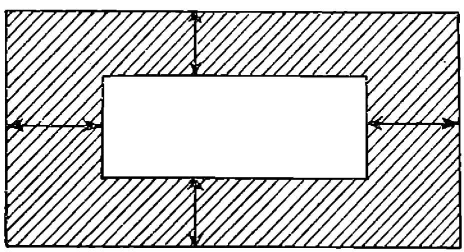
This can be illustrated by cutting out a cardboard model, or by using the tray of a match-box.

Perimeter of room = twice sum of length and breadth.

Area of walls = perimeter × height.

Note. When a formula is put in this concise form it is to be understood that the linear measurements are expressed in terms of the *same* unit. The area will then be expressed in the corresponding square units.

Area of a Border round a Rectangle



The area of the shaded part is best found as the difference between the whole area and the unshaded area.

By how much do the lengths and breadths of the internal and external areas differ?

EXERCISES LXX

Find the areas of the four vertical sides, and also the total internal surface areas of the closed boxes, whose internal measurements are:

1. Length 5 ft. 6 in., breadth 4 ft. 3 in., height 3 ft.
2. Length $8\frac{1}{2}$ in., breadth $5\frac{1}{2}$ in., height 4 in.
3. Length 125 cm., breadth 75 cm., height 1 m.
4. An open box 22 in. by $7\frac{1}{2}$ in. and $4\frac{1}{2}$ in. deep is made of thin cardboard. Find the total area of the sides and bottom of the box.

Find the total areas of the four walls of rooms whose measurements are:

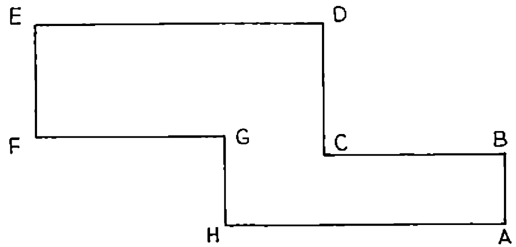
5. Length 10 ft., breadth 8 ft., height 9 ft.
6. Length 15 ft., breadth 12 ft. 6 in., height 10 ft. 6 in.
7. Length 13 ft. 6 in., breadth 12 ft., height 10 ft. 3 in.
8. Length 6 yd., breadth 4 yd. 2 ft., height 12 ft.
9. Find how many (whole) pieces of wall-paper 20 in. wide and 12 yd. long must be bought to repaper the walls of a room 12 ft. 6 in. by 12 ft. and 9 ft. 6 in. high, if 60 sq. ft. are allowed for door, windows, etc.
10. Find how many (whole) pieces of paper 20 in. wide and 12 yd. long are required to repaper a room 20 ft. by 16 ft. 6 in. and 11 ft. 6 in. high, if 120 sq. ft. are allowed for doors, windows, etc., and the cost at 2*s.* 8*d.* per piece.
11. A piece of English wallpaper is supposed to be 12 yd. long by 20 in. wide. Show that each yard length will contain 5 sq. ft. (5 ft. super), and each piece will contain 60 ft. super.

    Hence the number of feet super of surface to be papered, divided by 60, will give the number of pieces of paper required, any remainder being taken as one piece. One piece in seven is allowed for wastage in cutting and fitting.

    Find the cost of covering the walls of a room 15 ft. 6 in. long, 11 ft. 3 in. wide, and 10 ft. high, with paper costing 2*s.* 4*d.* per piece, allowing 120 sq. ft. for door, windows, etc., and one piece in seven for wastage.
12. Find the area of a gravel path 4 ft. wide, which surrounds a rectangular lawn 80 ft. by 110 ft.
13. A photograph measures $9\frac{1}{2}$ in. by $7\frac{1}{2}$ in., and is mounted with a border $2\frac{1}{2}$ in. wide all round. Find the area of this border.
14. A rectangular plate of metal is 27 in. by 33 in. Sixteen rectangular pieces $4\frac{3}{4}$ in. by $3\frac{1}{2}$ in. are stamped out of it. Calculate the area remaining.
15. A swimming bath is 120 ft. long and 35 ft. wide, and is

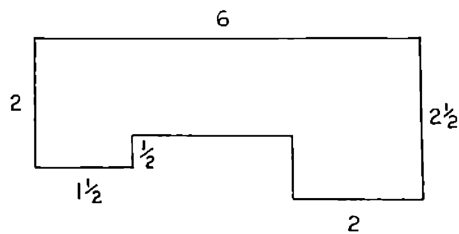
surrounded by a promenade 7 ft. wide. Find the areas of the bath and promenade.

16. Find the area of the garden shown in the figure, all the sides being straight and the angles right-angles.

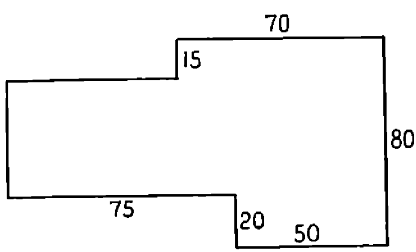


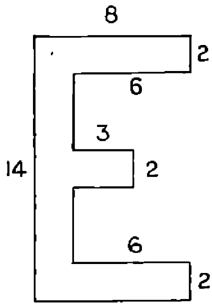
$AB = 11, BC = 21, CD = 18, DE = 35, EF = 15, FG = 22$ yd. (Divide the area into rectangles.)

17. Find the area of the figure, measurements being in inches.



18. The diagram below represents the plan of a factory. The measurements are given in feet. Find the area of the floor space in square feet.



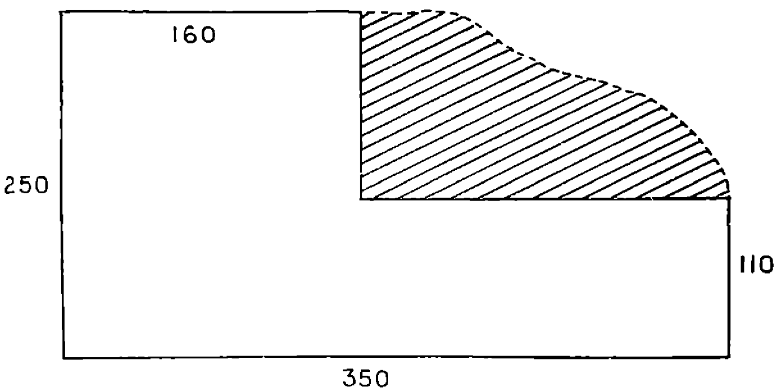


19. Find the area of the letter E, the measurements being given in feet.

20. The area of a square field is 10 ac. Find the length of its side.

21. How long will it take to walk round a square field of 40 ac. at 4 m.p.h.? If it takes the same time to walk round another field which is three times as long as it is broad, find the area of this field.

22. The figure below shows the plan of a farm, the given measurements being in yards. The farmer has just bought the shaded part, and this brings the total area of the farm to 16 acres. Find the area of the shaded part in square yards.



Volumes

Example 1. A sheet of lead is allowed to sink in a tank of water 6 ft. long and 3 ft. 6 in. wide. The level of the water rises 0·2 in.; find the volume of the lead.

Volume of water displaced

$$= \left(6 \times 3{\cdot}5 \times \frac{0{\cdot}2}{12}\right) \text{ cu. ft.}$$

$$= 0{\cdot}35 \text{ cu. ft.}$$

$\therefore$ volume of lead = **0·35** cu. ft.

Example 2. How many tiles $4\frac{1}{2}$ in. long, 4 in. wide, and

$\frac{3}{4}$ in. thick can be placed in a box 2 ft. by 18 in. by 1 ft. (internal measurement).

$$\text{Volume of interior} = (2 \times 1\tfrac{1}{2} \times 1) \text{ cu. ft.}$$
$$= 3 \text{ cu. ft.} = 3 \times 1728 \text{ cu. in.}$$
$$\text{Volume of one tile} = (4\tfrac{1}{2} \times 4 \times \tfrac{3}{4}) \text{ cu. in.}$$
$$= 13\tfrac{1}{2} \text{ cu. in.}$$
$$\therefore \text{ number of tiles} = (3 \times 1728) \div 13\tfrac{1}{2}$$
$$= \cancel{3} \times \overset{192}{\cancel{1728}} \times \frac{2}{\underset{9}{\cancel{27}}}$$
$$= \mathbf{384.}$$

Note. Questions of this kind may be complicated by difficulties of fitting the tiles into the box.

Example 3. A box whose length, breadth, and depth are 27 in., 20 in., and 12 in. (measured externally) is made of wood $\frac{1}{2}$ in. thick and has no lid. Find the volume of the wood.

$$\text{External volume} = 27 \times 20 \times 12$$
$$= 6480 \text{ cu. in.}$$
$$\text{Internal volume} = 26 \times 19 \times 11\tfrac{1}{2} \text{ cu. in.}$$
$$= 5681 \text{ cu. in.}$$
$$\therefore \text{ volume of wood} = 6480 - 5681 \text{ cu. in.}$$
$$= \mathbf{799} \text{ cu. in.}$$

Exercises LXXI

1. How many litres will it take to fill a tank which contains $3\frac{1}{2}$ cu. m.?

2. If a cubic foot of water weighs 62·5 lb. find the weight in tons and lb. of the water that would fill a cistern measuring 6 ft. each way.

3. Find the weight of a rectangular stack of coal 20 yd. long, 8 ft. wide, and 6 ft. 6 in. high, reckoning 44 cu. ft. to the ton.

4. Find, to the nearest ounce, the weight of water that can be contained in a vessel 2 ft. 6 in. by 1 ft. $4\frac{1}{2}$ in. by $7\frac{3}{4}$ in., taking 1 cu. ft. of water to weigh 1000 oz.

5. What is the weight in Kg. of an iron girder which is 5·4 m. long and whose cross-section is 85 sq. cm.? Take 1 c.c. of iron to weigh 7·76 grammes.

6. The area of the section of an excavator is 1325 sq. ft., and the machine is driven forward 4 ft. a day. How many cubic yards of earth are excavated in a day?

7. An English standard of deal planks contains 120 planks, each 12 ft. long, 9 in. wide, and 3 in. thick. How many cu. ft. of wood does it contain?

8. If a cubic foot of water weighs 1000 oz., what is the weight of water (in cwt., quarters, etc.) in a cistern 6 ft. long and $3\frac{1}{2}$ ft. wide filled to a depth of 10 in.?

9. A beam, having a square section, is 9 ft. long and weighs $3\frac{1}{2}$ cwt. One cubic foot of the substance of the beam weighs 32 lb. What is the thickness of the beam?

10. A cistern which is 9 ft. 4 in. long and 7 ft. 6 in. wide contains 6 tons 5 cwt. of water; if a cubic foot of water weighs 1000 oz., what is the depth of water in the cistern?

11. A pane of glass measures 140 cm. by 90 cm., and is 0·3 cm. thick. The material of which it is made weighs 2·6 grm. per c.c. Find the weight of the pane.

12. Find the number of litres of water in a rectangular trough whose length is 35·3 m. and width 56 cm., the depth being 74·8 mm. Give the answer to the nearest litre.

13. A cubic foot of water weighs 62·5 lb. Express a rainfall of 1 in. in tons of water per acre to the nearest ton.

14. How many gallons of tar per mile are required to give a road 40 ft. wide a coating of tar $\frac{1}{10}$ in. thick on the average? (Assume that a cubic foot contains $6\frac{1}{4}$ gal.)

15. A closed box whose external measurements are 3 ft., 18 in., 14 in. is made of wood 1 in. thick. Find the volume of the wood.

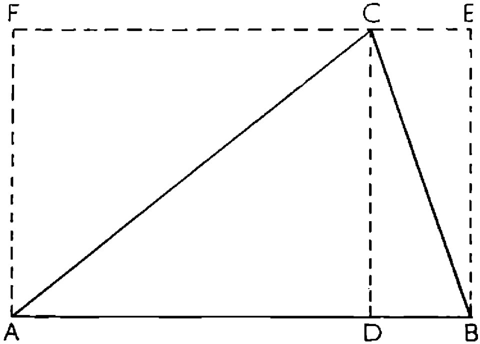
16. An open box whose inside measurements are 1 ft. $10\frac{1}{2}$ in., $13\frac{1}{2}$ in., and $10\frac{1}{2}$ in. (depth) is made of wood $\frac{3}{4}$ in. thick. Find the external measurements of the box and the volume of wood.

17. A closed box whose external measurements are 3 ft. 6 in., 2 ft. 6 in., and 1 ft. 6 in. is made from wood 1 in. thick. Find the volume of wood, and, adding 10 per cent. to your result to allow for waste in making the box, calculate its cost at $4\frac{1}{2}d.$ per square foot to the nearest penny.

18. A gravel path 3 ft. wide surrounds a rectangular lawn 60 ft. by 84 ft. Find the number of cubic yards of gravel necessary to cover the path to a depth of 2 in.

Area of Triangle, Parallelogram, Trapezium

The diagram shows that any acute-angled triangle ABC has half the area of the rectangle whose sides are equal to the base and perpendicular height of the triangle. The triangle ABC is divided into two parts, ACD, BCD, which are halves of the rectangles $AFCD$, $BECD$ respectively.



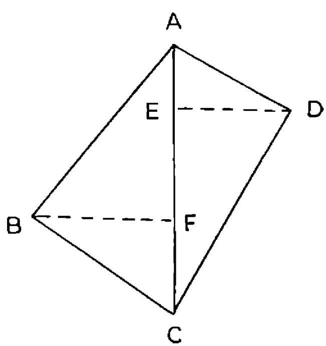
EXERCISES LXXII

1. Draw a figure to show that, when the angle B is greater than a right angle, the area of the triangle ABC is equal to half the area of a rectangle whose sides are equal to AB and the perpendicular distance of C from AB.

2. Draw a parallelogram on squared paper, and show that it is equal in area to a rectangle drawn on the same base, and with the same height.

3. Find the distance between the opposite sides of a parallelogram if they are 6·3 cm. long, and the area of the figure is 31·5 sq. cm.

4. Construct a triangle whose sides are 3 in., 2·5 in., and 4 in. respectively. Drop perpendiculars from each vertex to the opposite side, measure them, and find in three ways the area of the triangle.



5. In the quadrilateral $ABCD$ the length of the diagonal AC is 13 in., offsets BF and DE perpendicular to AC are 4·2 in. and 3·6 in. respectively. Find the area of the quadrilateral.

6. The diagonals of a quadrilateral are at right angles to each other, and their lengths

are 3·6 in. and 2·4 in. respectively; find the area of the quadrilateral.

7. A trapezium is a quadrilateral with a pair of opposite sides parallel. By dividing a trapezium into two triangles, show that its area is equal to half the sum of the parallel sides, multiplied by their perpendicular distance apart.

Find the areas of the following trapeziums:

8. Parallel sides 12 ft. and 5 ft., distance apart 10 ft.

9. Parallel sides 3·5 cm. and 5·6 cm., distance apart 2·5 cm.

10. Parallel sides 12 ft. 6 in. and 15 ft. 3 in., distance apart 6 ft. 6 in.

Diagonal of a Rectangle. Hypotenuse of Right-angled Triangle

Pythagoras' Theorem. It is shown in books on geometry that the squares on the two sides of a right-angled triangle which contain the right-angle are together equal in area to the square on the third side, the hypotenuse.

Example. Find the length of the diagonal of a rectangle whose sides are 5 cm. and 6 cm.

$$\begin{aligned} \text{The square on the diagonal} &= 5^2 + 6^2 \text{ sq. cm.} \\ &= 25 + 36 \text{ sq. cm.} \\ &= 61 \text{ sq. cm.} \\ \therefore \text{ The diagonal} &= \sqrt{61} \text{ cm.} \\ &= \mathbf{7{\cdot}81} \text{ cm. to nearest } {\cdot}01 \text{ cm.} \end{aligned}$$

Exercises LXXIII

1. Find the length of the hypotenuse of a right-angled triangle, if the lengths of the other two sides are 6 and 2·5 cm. respectively.

2. Find the length of the third side of a right-angled triangle, if the lengths of the hypotenuse and one other side are 6·25 and 5·25 ft. respectively. Give the answer to 2 decimal places.

3. The sides of a rectangle are 2·3 in. and 3·7 in. Find the length of its diagonal to the nearest tenth of an inch.

4. The area of a square lawn is 15625 sq. ft. Find the length of its diagonal to the nearest foot.

5. The area of a square field is 5 ac. Find the length of a diagonal to the nearest chain.

6. The edges of a cuboid are 6, 7, and 8 cm. Find the length of a diagonal to the nearest mm.

7. The edges of a box being 1·1, 1·5, and 2·5 ft., find the length of the diagonal to the nearest tenth of a foot.

8. A flagstaff stands at the centre of a square of side 10 ft. Wire stays are fixed to the ground at the corners of the square and to the flagstaff at a height of 20 ft. above the ground. Find the length of each stay to the nearest tenth of a foot.

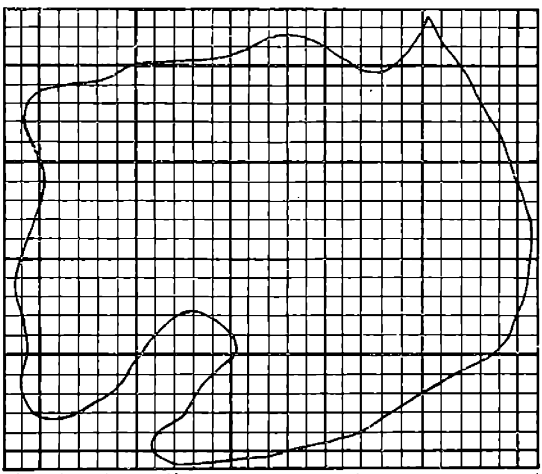
9. A ladder leans against a wall. It is 25 ft. long, and its lower end is 7 ft. from the wall. How far up the wall does the ladder reach?

Determination of Areas by means of Squared Paper

The area of a figure can be found approximately by drawing its outline on squared paper, and counting the squares included. A good method of counting the squares is as follows. First count the large squares and note the result, then count the small ones left over. Count a small square which is only partly inside the figure as one small square, if it appears that half or more of it is inside, otherwise neglect it. The totals of large and small squares are then expressed in terms of the same unit, and added together.

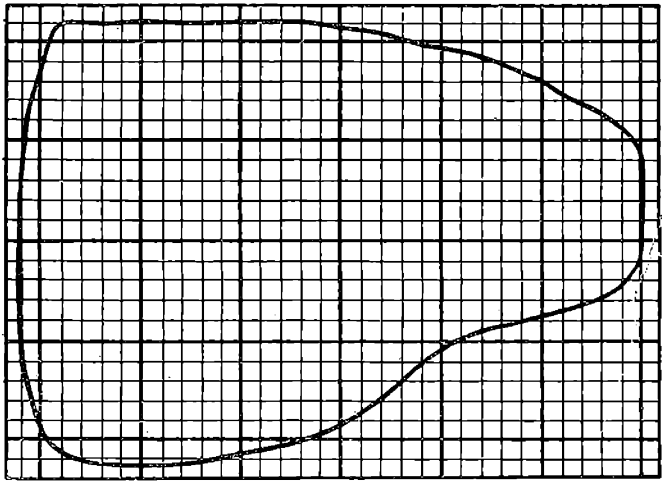
Exercises LXXIV

1. The figure shows the map of an island. Find its area, giving the answer to the nearest 10 sq. ml.



Scale: 1 small square to the square mile

2. The diagram below shows the plan of a reservoir on a scale of the side of one large square to 20 yd. Find the area to the nearest square chain.



3. Compare the area of a circle with the area of the square on its radius by counting squares. Divide the area of the circle by the area of the square on its radius.

Areas can also be investigated by weighing.

Exercises LXXV

1. Weigh two rectangles cut from the same kind of cardboard. Divide (1) the weight of the first by the weight of the other, and (2) the area of the first by the area of the other. Compare the results.

2. Cut out from the same kind of cardboard, a circle and a square whose side is equal to the radius of the circle. Divide the weight of the circle by the weight of the square. Compare your result with that of question 3 of the last set of examples.

Calculate the area of the square, and deduce the area of the circle.

Circle

The perimeter of a circle can be determined experimentally by stepping off from it equal small distances in succession with a pair of dividers opened at a small angle.

This will give an approximate result only, because the lengths are small arcs and not straight lines. It can also be found by rolling a penny on a sheet of paper and measuring the track, or by putting stretched thread round it and measuring the length of the thread. See p. 124 for experimental determination of the area of a circle.

It can be shown that the fraction

$$\frac{\text{circumference of a circle}}{\text{diameter of the circle}}$$

is a fixed number, independent of the size of the circle. The Greek letter, π, is used to denote this number, which is an irrational number (see p. 111); so that, if C denotes the circumference, d the diameter, and r the radius of any circle,

$$C = \pi d = 2\pi r.$$

To eight places of decimals $\pi = 3{\cdot}14159265\ldots$

Approximate values are $3\frac{1}{7}$, $\frac{355}{113}$, $3{\cdot}1416$, etc.

$$3\tfrac{1}{7} = 3{\cdot}14285714$$

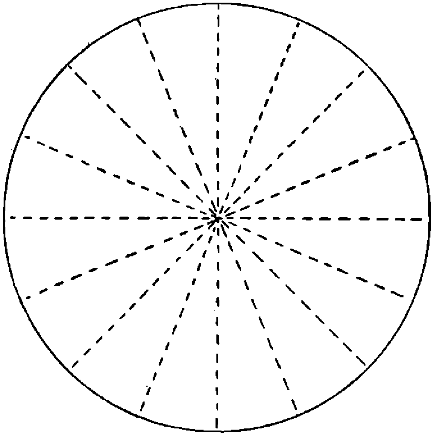
$$\pi = 3{\cdot}14159265$$

$$3\tfrac{1}{7} - \pi = {\cdot}00126449 = {\cdot}00125 + {\cdot}00001449$$

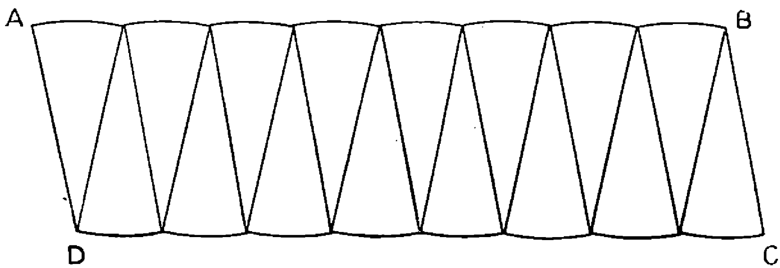
So that $\quad 3\frac{1}{7} - \pi$ is approximately $\frac{1}{800}$

or $\pi = 3\frac{1}{7} - \frac{1}{800}$ approximately.

$3{\cdot}1416$ is a closer approximation.



If a circle is divided by diameters into a number of equal sectors, and these are cut out, they may be rearranged to make the figure *ABCD*, in which the sides *AB* and *CD* are made up of parts of the circumference. The greater the number of sectors, the more nearly will the figure *ABCD* approach a rectangle whose sides *AD*, *BC* equal the radius



of the circle and whose sides *AB*, *CD* are each half of its circumference. The area of such a rectangle would be

$$\pi r \times r = \pi r^2$$

so that the area of a circle is given by this formula.

Area $= \pi$ times the square on the radius.

We may obtain this result in a different way as follows: Consider a polygon *ABCDE* . . . described about a circle. *Oa*, *Ob*, *Oc*, *Od*, etc., are radii drawn to the points of contact of the sides of the polygon.

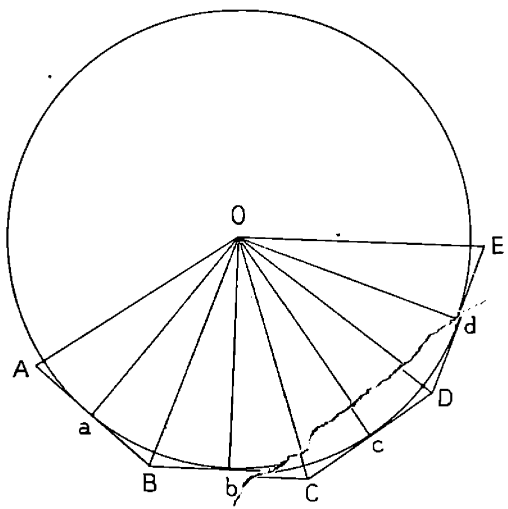
The figure shows a portion of the polygon on an enlarged scale.

Area *OABCDE* . . .

$= \frac{1}{2}(AB + BC + CD + DE + \ldots) \times$ radius

$= \frac{1}{2}$ perimeter of circumscribing polygon $\times$ radius of circle.

The smaller the lengths *AB*, *BC*, *CD* . . . etc., the nearer the area of the polygon will approximate to the area of the circle and the perimeter of the polygon to the circumference of the circle.



From this we deduce area of circle

$$= \tfrac{1}{2} \text{ circumference} \times r$$

$$= \tfrac{1}{2} \times 2\pi r \times r = \pi r^2 \text{ or } \pi\frac{d^2}{4}.$$

From the same argument

Area of sector of circle $= \frac{1}{2}$ arc $\times$ radius.

EXERCISES LXXVI

In the following examples, take π to be $3\frac{1}{7}$.

Find the circumference and area of the circles whose radii are

1. $3\frac{1}{2}$ in.
2. 1 ft. 2 in.
3. 3 cm.
4. 6 ft.
5. 25 ft.
6. 77 ft.
7. A circle has a circumference of 11 ft.; find its radius.
8. A circle has a circumference of 55 ft.; find its area.
9. A running-track in the form of a circle has a perimeter of a quarter of a mile; find its radius.

10. A target consists of a black circle of diameter 1 in., surrounded by concentric circles of diameters 2 and 3 in. Find the areas between the circles.

11. A circle has an area of 154 sq. ft.; find its circumference.

12. A circular flower bed of diameter 10 ft. 6 in. is made in the centre of a rectangular lawn 25 ft. by 30 ft. Find the area of the remainder of the lawn.

Example. The diameter of a circle is 12·45 cm.; find its circumference, taking π as $3\frac{1}{7} - \frac{1}{800}$ and also as 3·1416.

	12·45	12·45
	$3\frac{1}{7}$	3·1416
	37·35	37·35
	1·7786	1·245
	39·1286	·4980
$\frac{1}{800}$th =	·0156	·01245
	39·1130	·007470
		39·11292

To the nearest third decimal place both answers agree. Since the measurement of the diameter will often be only approximate to the second decimal place, and multiplication will increase the error that there may be, the result should not be given to more than two decimal places.

Work some of the previous exercises (LXXVI) by these methods and compare the results.

DETERMINATION OF VOLUMES BY WEIGHING

Archimedes' Principle. When a body is immersed in a liquid, the liquid exerts a vertical upward pressure on the body and this pressure or upthrust is equal to the weight of liquid displaced by the body.

(Consult a treatise on physics for experimental verification of this principle.)

Suspend a body from an accurate spring balance and read the weight of the body from the balance. Then place a beaker of water below the balance so that the body hangs totally immersed in the water, and read the balance again. This reading will be less than the first by an amount equal to the upthrust of the water, which, by Archimedes' principle, is equal to the weight of water displaced. As the weight of unit volume of water is known, we can at once find the volume of water displaced, *i.e.*, the volume of the body.

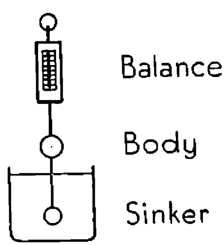
If the body is lighter than water, it sinks into the water till it has displaced a volume of water whose weight is equal to the weight of the body, and then floats.

Example 1. To find (1) the weight per cubic centimetre of one body and (2) the weight per cubic foot of another body, both of which sink in water.

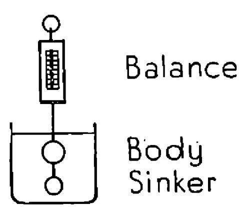
	Reading of balance
Solid in air . .	56·5 grm.
Solid in water . .	33·5 grm.
Weight of water displaced	= 23·0 grm.
But 1 c.c. of water weighs . .	1 grm.
∴ Vol. of water displaced . .	= 23·0 c.c.
i.e., Vol. of body	= 23·0 c.c.
∴ 1 c.c. of body weighs	$\frac{56\cdot5}{23\cdot0}$ grm.
	= **2·45** grm.

	Reading of balance
	23·5 oz.
	21·0 oz.
	2·5 oz.
1 cu. ft of water weighs	1000 oz.
Vol. of water	= ·0025 cu. ft.
Vol. of body	= ·0025 cu. ft.
1 cu. ft. of body weighs	$\frac{23\cdot5}{\cdot0025}$ oz.
	= 9400 oz.
	= **587½** lb.

Example 2. To find the weight per cubic centimetre of a body which floats in water. A 'sinker' is needed for the complete immersion of the body, and three readings are required.



	Reading of balance
Body in air	20·0 grm.
Sinker in water attached to body in air	125·0 grm.
Sinker and body both in water	99·5 grm.



The difference between the last two readings gives the upthrust due to displacement of water by body, *i.e.*, is the weight of the water displaced by the body.

$\therefore$ weight of water displaced by body = 25·5 grm.
$\therefore$ volume of water displaced by body = 25·5 cub. cm.
i.e., volume of body = 25·5 cub. cm.

$$\therefore \text{ weight per cub. cm. is } \frac{20}{25\cdot 5} \text{ grm.}$$

= **·78** grm. approximately.

THE WEIGHT PER CUBIC CENTIMETRE OF A LIQUID

The volume of liquid may be measured directly by means of burette, pipette, or other apparatus and the weight by the balance; or Archimedes' principle may be used as follows.

Take a glass stopper or other solid body and weigh it in air. Then weigh it again (1) suspended in water and (2) suspended in the liquid. From the results the volume of stopper and the weight of an equal volume of liquid can be deduced.

	Reading of balance
Stopper not immersed	25·7 grm.
Stopper immersed in liquid	18·3 grm.
Stopper immersed in water	19·6 grm.

Weight of water displaced = 25·7 − 19·6 grm. = 6·1 grm.
$\therefore$ Volume of water displaced = 6·1 cub. cm.

Weight of liquid displaced = 25·7 − 18·3 = 7·4 grm., but volume of liquid displaced = volume of water displaced = 6·1 cub. cm.

$$\therefore \text{ Weight per cub. cm. of liquid} = \frac{7\cdot 4}{6\cdot 1} \text{ grm.}$$

= **1·2** grm. approximately.

In the preceeding examples the phrase 'weight per c.c.' has been used. At some stage or other according to the view of the teacher, the more technical terms, mass, density, specific gravity will be explained and used. For other methods of finding density, specific gravity, etc., see text-books on physics.

EXERCISES LXXVII

1. The weight of a glass vessel is 163·7 grm. 25 c.c. of a liquid are placed in the vessel and the total weight is found to be 197·3 grm. Find the weight of the liquid in grm. per c.c.

2. A piece of metal weighed 189·2 grm. When the metal was immersed completely in water in a graduated vessel the water level rose from 150 c.c. to 173·2 c.c. Find the weight in grm. per c.c. of the metal to 3 significant figures.

3. A small bottle weighed 105·9 grm. When filled with water it weighed 123·5 grm., and when filled with another liquid, 129·3 grm. Find the weight per c.c. of this liquid to 3 significant figures.

4. A body weighs 98·7 grm., and its volume is 13·5 c.c. Find its apparent weight when suspended (1) in water, (2) in a liquid of weight ·86 grm. per c.c.

5. A piece of lead weighs 627 grm. What would be its apparent weight when suspended in a liquid of weight 1·25 grm. per c.c., the weight of lead being 11·4 grm. per c.c.?

6. A piece of wood floats in water with $\frac{3}{5}$ of its volume below the surface. What is the weight of the wood in grm. per c.c.? (When a body floats its weight is just balanced by the upthrust equal to the weight of fluid displaced.)

7. Weight of a body in air is 85·65 grm. Apparent weight in water is 73·28 grm. Find the weight per c.c. of the body to 3 significant figures.

8. The weight of a piece of wood in air = 50·28 grm. Apparent weight of body in air with sinker in water = 264·62 grm. Apparent weight of body and sinker, both in water = 189·40 grm. Find the weight per c.c. of the body.

9. If a body weighs 23·5 oz. in air, and appears to weigh 21·0 oz. in water, find the weight in lb. per cubic foot of the body, assuming that a cubic foot of water weighs 1000 oz.

10. When a body is weighed in air with a sinker attached in water the balance reads 125·00 grm; when both sinker and body are in water the reading is 99·50 grm. If the body weighs 0·77 grm. per c.c., find its total weight in air.

11. A glass stopper when immersed in a liquid appears to weigh 19·3 grm, and when immersed in water, to weigh 18·6 grm. If the weight of the stopper in air is 25·7 grm., find the weight per c.c. of the liquid to two decimal places.

12. A body appears to weigh 152·3 grm. when totally immersed in water and 158·6 grm. when totally immersed in a liquid (X). If the body weighs 200 grm. in air, find the weight per c.c. of X to two decimal places.

13. A boat of weight 50 tons floats in fresh water; being

given that a cubic foot of fresh water weighs 62·5 lb., find the volume of water displaced. (Condition for flotation is: weight of body = weight of water displaced by the body.)

14. The same boat floats in sea water; find the volume of sea water displaced if 1 cu. ft. of sea water weighs 64 lb.

15. A man, whose weight is 140 lb., finds that he can just float in fresh water when he is, practically, wholly immersed; find his volume in cubic feet.

16. A ship and its cargo weigh 3341 tons, and the section of the ship by the plane of the water is 9120 sq. ft., the sides being vertical in the neighbourhood of the water line. Find (to the nearest inch) the distance by which the water line is lowered as the ship sails from fresh water (specific gravity 1) to salt water (specific gravity 1·03). Take the weight of fresh water as 62·3 lb. per cubic foot. (A substance has specific gravity 'x' when it weighs 'x' times as much as an equal volume of water.)

17. A hollow box, with a lid, floats with one quarter of its volume out of the water. When 10 lb. of lead are placed in it and the lid is closed water-tight the box just sinks; find the volume of the box.

18. The dimensions of a rectangular block of wood are 29 cm., 11 cm., 9·5 cm., and its weight is 2800 grm. If it floats in water, find what percentage (to two figures) of its volume will be above the water.

CHAPTER XV

GRAPHS

GRAPHS OF STATISTICS

A cricketer's scores in ten successive innings were 14, 7, 38, 43, 6, 11, 25, 56, 33, 8. This may be represented in a diagram as shown on p. 134.

The lines OX, OY, upon which the scales are marked, are called the *axes*. OX is always drawn from left to right parallel to the top and bottom edges of the paper, and OY at right angles to it, up the page.

To represent that the score in the third innings was 38, we measure from the point 3 on OX (which represents the third innings) a distance representing 38 runs according to the scale marked, parallel to OY. If squared paper is used, geometrical instruments are not needed for the marking (or plotting) of such points.

When all the points have been plotted, the figure is called the graph of the given statistics, but it is customary to join the points up in order by segments of straight lines, so that the eye grasps more readily their relative positions and the information given by the graph is made more vivid.

EXERCISES LXXVIII

1. The figure on p. 135 shows the numbers of school certificates gained by candidates from a certain school in successive years.

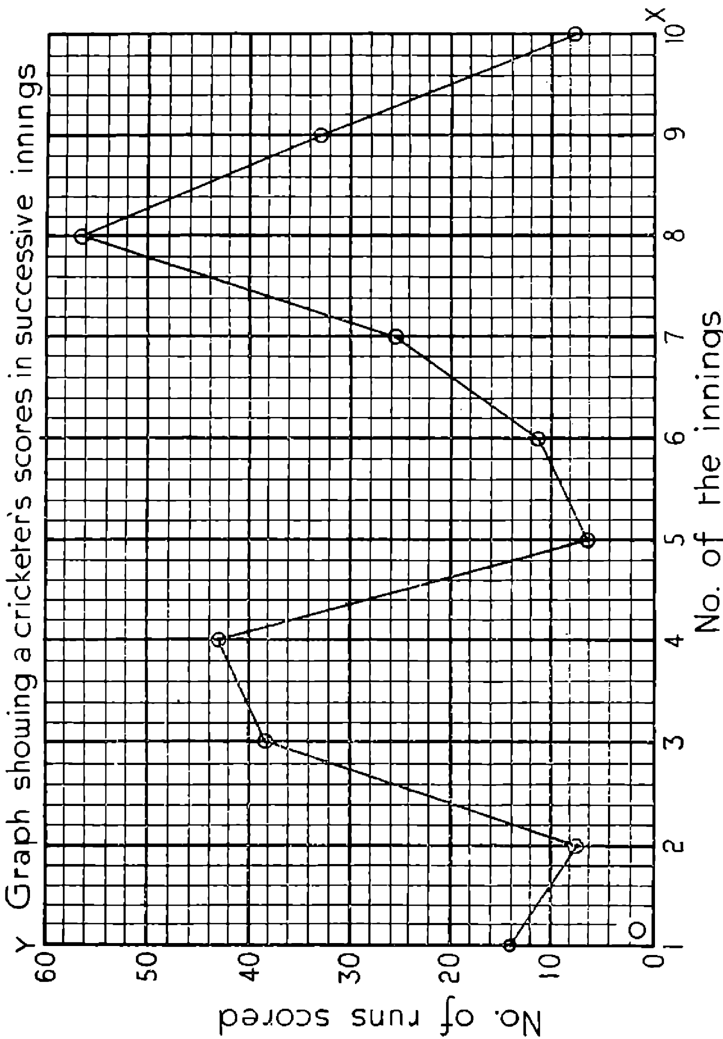
Notice that to save space, and to show their variation as well as possible, the numbers of certificates allowed for by the scale range from 50 to 65, not from 0 to 65.

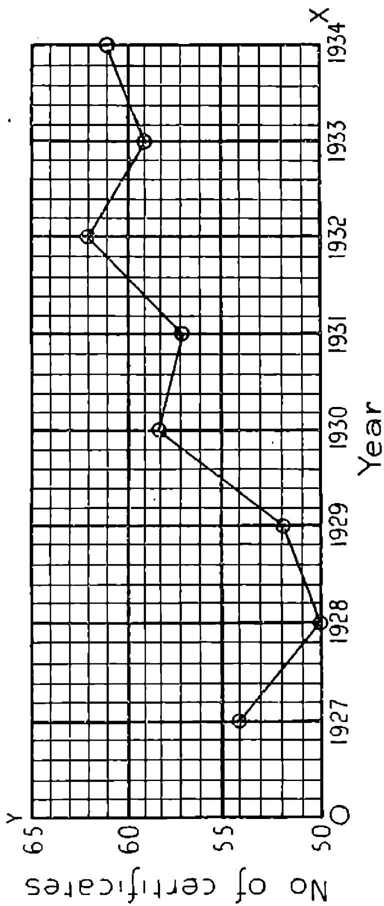
Read from the graph:

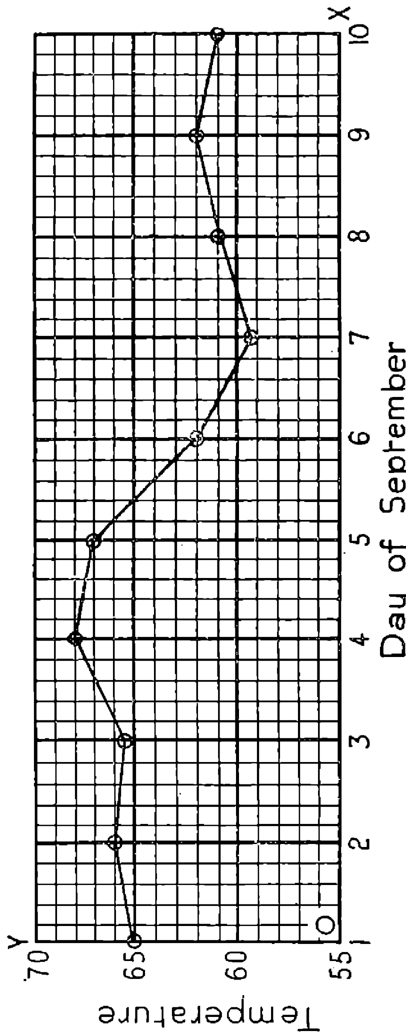
(*a*) How many certificates were gained in 1927, 1929, 1931, 1933?

(*b*) In what year was the greatest number of certificates gained?

(*c*) In what years were more than 60 certificates gained?


Graph showing a cricketer's scores in successive innings
No. of the innings
No. of runs scored
X
Y
O
0
10
20
30
40
50
60
1
2
3
4
5
6
7
8
9
10


X
Y
O
1927
1928
1929
1930
1931
1932
1933
1934
Year
65
60
55
50
No of certificates


X
Y
O
70
65
60
55
1
2
3
4
5
6
7
8
9
10
Day of September
Temperature

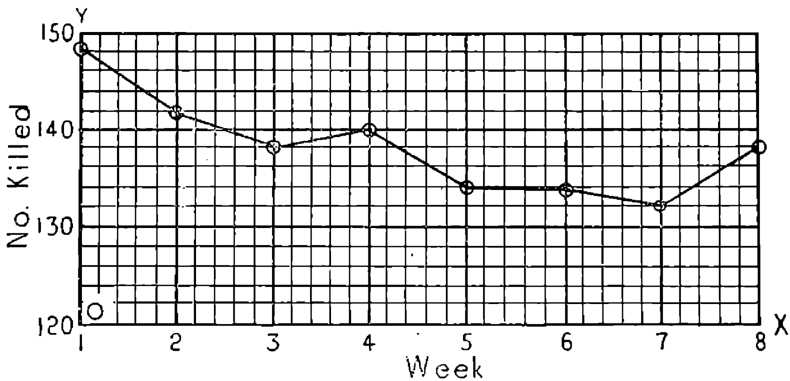
2. The maximum shade-temperatures recorded by a thermometer, on ten consecutive days from the beginning of September, are shown in the graph on p. 136.

(*a*) What were the readings for September 5 and September 6?

(*b*) What were the highest and lowest readings, and on what dates were they taken?

(*c*) Give a general account of the facts represented by the graph, *i.e.*, how the maximum shade-temperature varied throughout the period.

3. The figure below shows the number of persons killed on the roads in eight successive weeks.



(*a*) In which weeks was the number killed less than 140?

(*b*) What were the numbers in the second and seventh weeks?

(*c*) Describe the general tendency of the weekly numbers of persons killed.

Note that in the examples given above the quantity or number which is calculated or observed is represented by a length parallel to OY.

4. The following table shows the output of a gold-mine for seven successive months.

Month No.	1	2	3	4	5	6	7
Output in fine oz.	3180	3050	3075	3150	3100	3210	3160

Construct a graph to represent these figures.

Take points at equal intervals along an axis OX to represent successive months, and represent amounts of gold by lengths parallel to OY, beginning at 3000 oz. and taking 1 in. to represent 100 oz.

5. The numbers of boys who left a certain school during certain years are given in this table:

Year	1925	1926	1927	1928	1929	1930	1931	1932
No. of boys leaving	102	96	90	105	88	95	107	101

Represent these figures in a graph.

Choose scales such that readings from your graph are quickly and easily made, and so that you may obtain a figure of reasonable size, without waste of paper.

6. The rate of exchange of dollars to the pound sterling at the close of business each day for a week was as follows:

Day	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
Dollars to £1	$4{\cdot}98\frac{1}{2}$	$4{\cdot}98\frac{7}{8}$	$4{\cdot}98\frac{3}{4}$	$4{\cdot}99\frac{1}{8}$	$4{\cdot}98\frac{5}{8}$	$4{\cdot}98\frac{1}{2}$

Draw a graph to represent this.

Continuous Graphs

Observations were taken of the height of the tide at a point on the coast over a period of 18 hours, and the results are shown in the graph opposite. The plotted points are joined by a smooth curve, rather than by straight lines, because additional observations might have been made at any other times during the period and the information have been added to the graph. It is probable that the values obtained would lie on, or very close to, the smooth curve drawn.

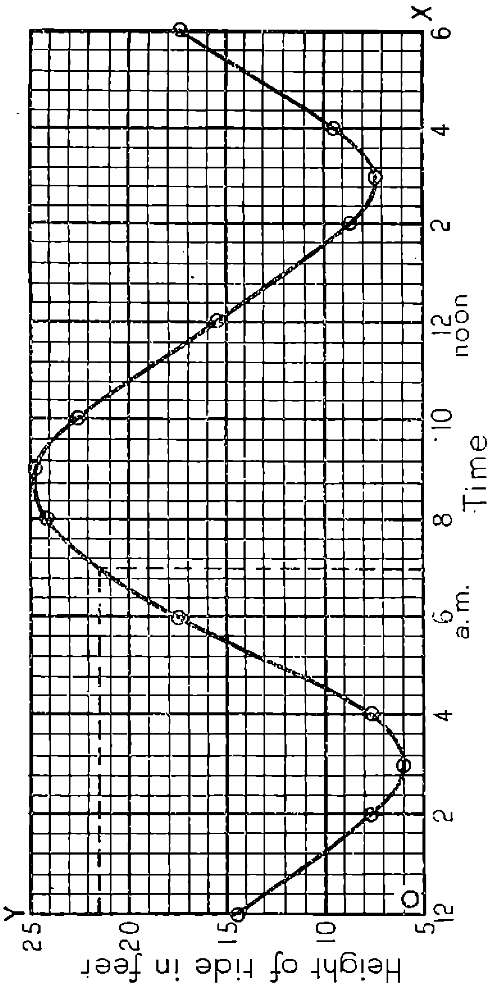
In the previous examples such intermediate values have not been possible.

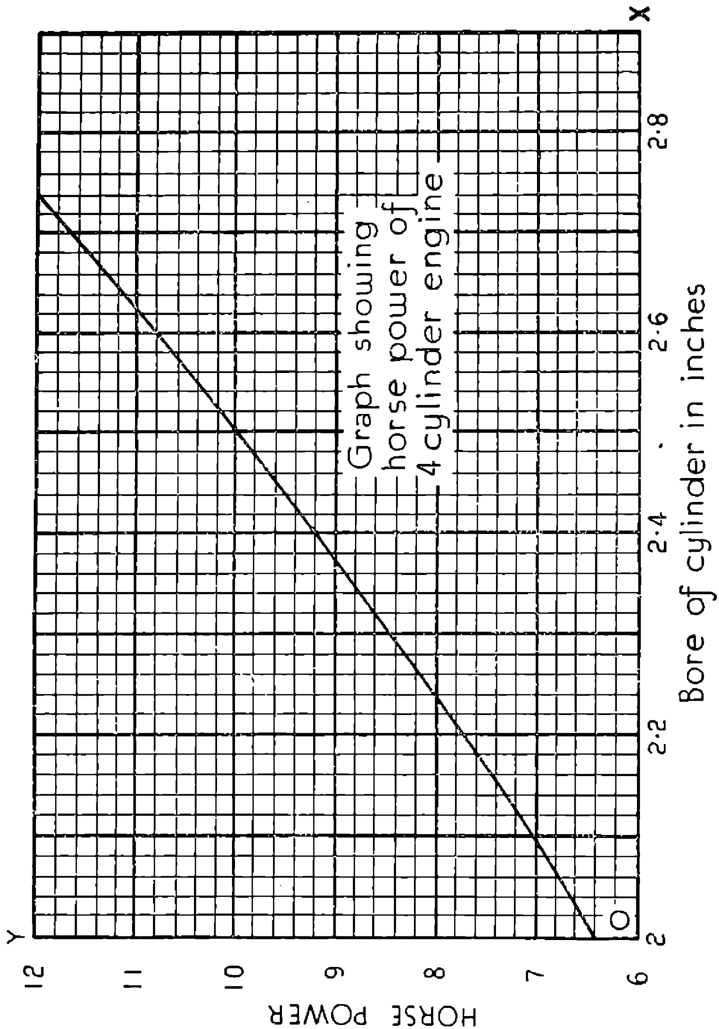
The times are marked along the axis OX since they were selected, the results of the observations are represented parallel to OY.

From this graph we can estimate the height of the tide at any time. Thus at 7 A.M., for example, the height was approximately 21·5 ft. (see the dotted lines in the figure).

Exercises LXXIX

1. The figure on p. 140 shows the horse-power obtained from a four-cylinder engine, according to the formula used for purposes of taxation.


Height of tide in feet
Time
a.m.
noon
Y
X
O
25
20
15
10
5
12
2
4
6
8
10
12
2
4
6


Graph showing
horse power of
4 cylinder engine
Bore of cylinder in inches
HORSE POWER
X
Y
O
2
2·2
2·4
2·6
2·8
6
7
8
9
10
11
12

Read from the graph the power obtained from cylinders of bore 2·55 in. and the bore required to give a horse-power of 8.

2. The volumes and radii of some spheres are given in the following table:

Radius in inches	0·5	1	1·5	2
Volume in cubic inches	0·52	4·19	14·14	33·52

Draw a graph to represent these figures. Use a length of 2 in. along OX to represent a radius of 1 in. and a length of $\frac{1}{2}$ in. along OY to represent a volume of 5 cu. in.

From the graph estimate the volume of a sphere of radius 1·25 in. and the radius of a sphere of volume 25 cu. in.

3. The connexion between the volume and depth of the water in a vase is given by this table:

Depth in cm.	1	2	4	6	8	9	10
Vol. in c.c.	12	28	68	98	115	123	135

Sketch a graph, and estimate the depth when the volume is 80 c.c.

4. Calculate the times taken to cover a journey of 150 ml. at the following average speeds: 10, 20, 30, 50, 60 miles per hour. Express the results graphically, and thus estimate the average speeds necessary to cover the distance in $4\frac{1}{2}$ and $9\frac{1}{2}$ hr.

5. On a certain wireless set the readings of a condenser scale required to receive particular wave-lengths are as follows:

Wave-length in metres	296	342	373	390	452
Condenser reading	$22\frac{1}{2}$	32	39	44	67

Find the wave-length of a station which is received when the condenser reads 56.

6. The annual payments required by a certain insurance company for a sum of £1000 at the age of 55 are as follows:

Beginning at age	20	25	30	35	40
Yearly payment	£18 10*s*.	£23	£29	£39	£55

From a graph estimate the yearly payments for such insurances begun at the ages of 28, 32, 37.

Straight-line Graphs

The upper figure on p. 142 shows the cost of eggs at 2*s*. per score. The points are plotted from calculation and lie in a straight line through the origin, *i.e.*, the point which corresponds to zero on each scale. The lower figure on

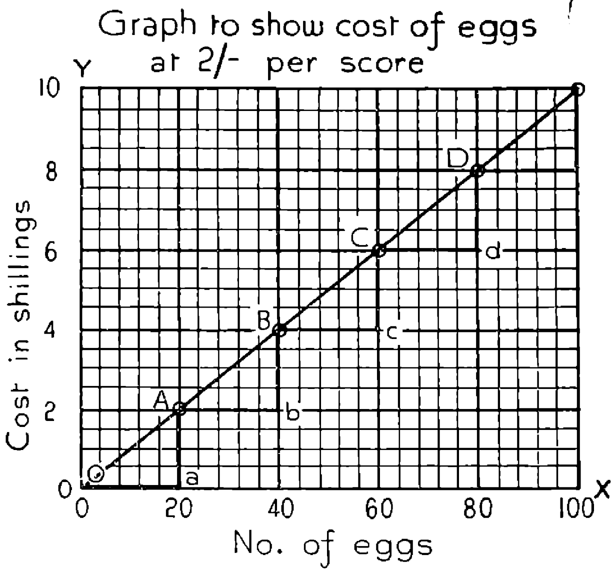

Graph to show cost of eggs
at 2/- per score
Y
10
8
6
4
2
0
Cost in shillings
A
B
C
D
a
b
c
d
0
20
40
60
80
100
X
No. of eggs

FIG. 1

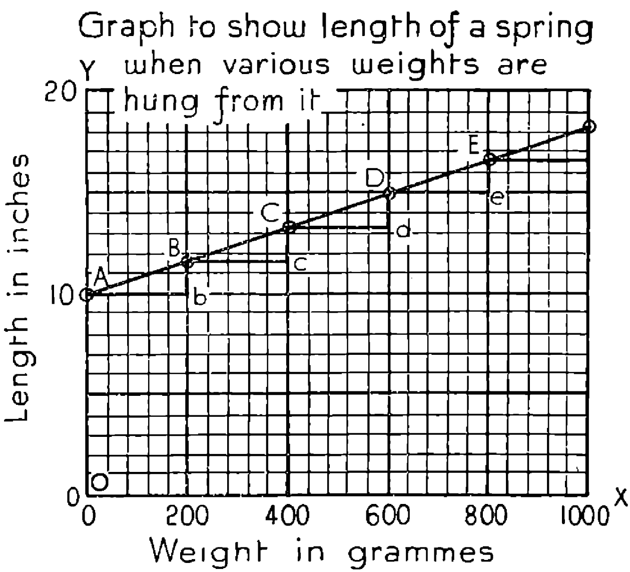

Graph to show length of a spring
when various weights are
hung from it
Y
20
10
0
Length in inches
A
B
C
D
E
b
c
d
e
O
0
200
400
600
800
1000
X
Weight in grammes

FIG. 2

p. 142 shows the length of a spring which is stretched by having weights hung from it. The lengths corresponding to different weights are found by experiment, and the results are plotted. The points are found to lie on a straight line which does not pass through the origin.

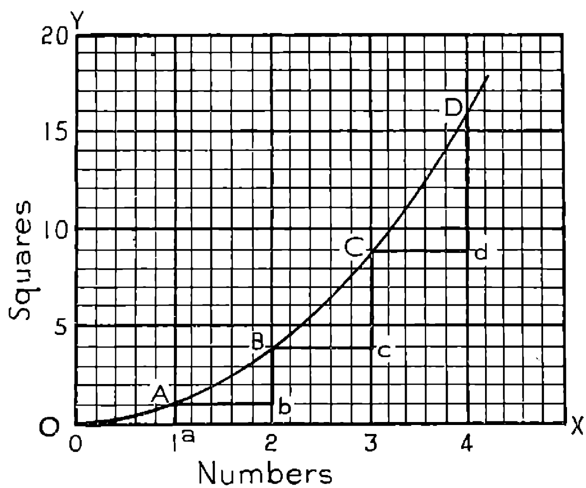
Imagine a pencil point to start from *O* and to trace out the zig-zag path *OaAbBcCdD* in Fig. 1, or to start from *A* and trace the path *AbBcCdDeE* in Fig. 2, so that after moving each distance parallel to *OX* it moves a distance parallel to *OY*.

Then, in Fig. 1, *Oa*, *Ab*, *Bc*, *Cd* are equal steps parallel to *OX*, and *aA*, *bB*, *cC*, *dD* are equal steps parallel to *OY* but not necessarily equal to the horizontal steps *Oa*, etc.

A similar result is true for Fig. 2, or for any straight-line graph, including graphs which slope downward from left to right instead of upward as in Figs. 1 and 2; and the reader will see that when any set of equal steps parallel to *OX* corresponds to a set of equal steps parallel to *OY* as in these figures, the graph must be a straight line.

The equal steps *Ab*, *Bc*, etc., correspond in Fig. 1 to equal changes in the number of eggs, in Fig. 2 to equal changes

GRAPH TO SHOW SQUARES OF NUMBERS



in the weight supported, while the equal steps bB, cC, etc., correspond to equal changes in the cost and length of spring respectively.

We may say then that when equal changes in one quantity correspond to equal changes in a second quantity the graph which shows the connexion between the quantities is a straight line.

In both figures it will be seen that

$$\frac{Bb}{Ab} = \frac{Cc}{Bc} = \frac{Dd}{Cd} = \ldots$$

The common value of these fractions is called the *slope* of the line. The slope is the same for all parts of the straight line.

A graph showing squares of numbers is on p. 143.

It will be seen that in this graph equal changes in the number do not give rise to equal changes in the squares.

$Oa = Ab = Bc = Cd$, etc., but aA, bB, cC, dD are not equal to one another.

Exercises LXXX

1. The upper diagram on p. 145 shows the estimated cost of giving a tennis club dance according to the number of guests. Read from the graph the costs when there are 50, 80, 100 guests respectively.

The total cost is formed from a fixed charge for hire of room, band, etc., and a certain cost per person for refreshments. What is the fixed charge for hire of band, etc.?

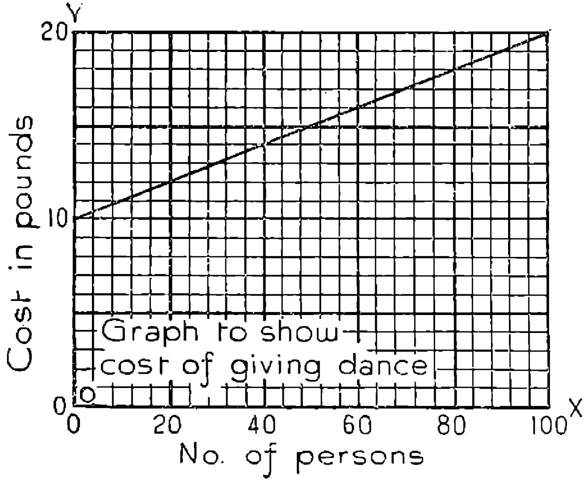
2. The lower diagram shows the number of gallons of petrol left in the tank of a light car during a journey.

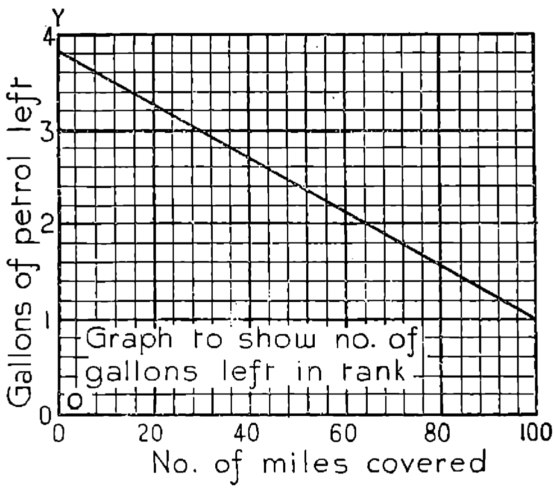
How many gallons were left in the tank (i) at the start, (ii) after 50 ml., (iii) after 100 ml.?

How many miles does the car travel per gallon of petrol?

3. The Centigrade and Fahrenheit temperature scales are such that freezing-point is 0° C. or 32° F., while the boiling-point of water is 100° C. or 212° F. Equal rises on one scale correspond to equal rises on the other, hence the relation between them can be expressed by a straight-line graph. Draw the graph and find the Fahrenheit temperatures corresponding to 35° C. and 65° C. and the Centigrade temperatures corresponding to 40° F. and 180° F.

4. Draw a graph to show the duty on quantities of spirit


Y
20
10
0
Cost in pounds
Graph to show
cost of giving dance
O
0
20
40
60
80
100
X
No. of persons


Y
4
3
2
1
0
Gallons of petrol left
Graph to show no. of
gallons left in tank
O
0
20
40
60
80
100
No. of miles covered

up to $\frac{1}{2}$ gal., taking the duty as 72*s*. 6*d*. per gallon. Read off from it the duty on ·15, ·3, ·35, and ·45 gal.

5. Taking 1 m. as 39·37 in., draw a graph which will convert lengths up to 1 m. given in cm. to inches and vice versa. Find the equivalents of 13, 29, 65 cm. and 12, 20, 30 in.

6. Draw a graph which will enable numbers up to 50 to be multiplied by $3\frac{1}{7}$. From it read off the values of $6 \times 3\frac{1}{7}$, $15{\cdot}5 \times 3\frac{1}{7}$, $26 \times 3\frac{1}{7}$, $33{\cdot}5 \times 3\frac{1}{7}$.

7. A man began a holiday with £20, and his total expenditure amounted to 12*s*. 6*d*. a day. Draw a graph to show how much money was left up to 30 days.

His friend began at the same time with £25, and his expenditure was 17*s*. 6*d*. a day. Draw in the same figure a graph to show how much of his money remained, and find after how long each man had the same amount left.

Travel-graphs

Position along a definite route at different times may be represented graphically.

Example. A cyclist leaves a town by a main road at noon, travelling steadily at 12 m.p.h. Draw a graph to represent his distance from the town during the first hour.

Represent times along OX beginning at 12 noon, and distance from town along OY, choosing large convenient scales.

Insert in the same figure a graph representing the progress of a pedestrian starting from the same place at noon, and walking along the same road at 4 m.p.h.

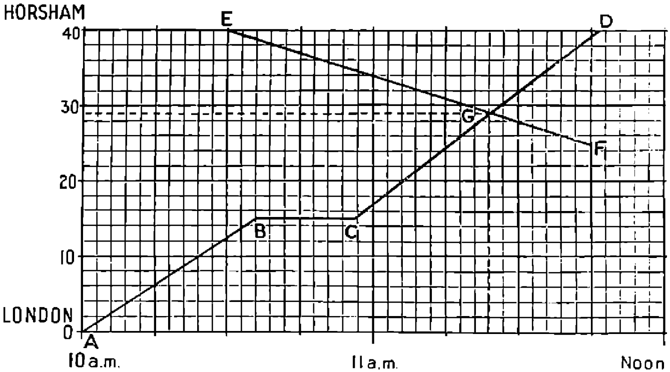
How does the greater speed of the cyclist show itself in the graph?

How would change of speed during a journey show itself in a travel-graph?

Approximate answers may be obtained to problems on the travel of trains, cars, etc., by treating them as if the different parts of the journeys were carried out at uniform speeds. This may be done by calculation, but more simply by means of a graph.

Example. A motorist left London at 10 A.M. and proceeded towards Horsham (40 miles from London) at an average speed of 25 m.p.h. He stopped to transact business at a place 15 miles out from London, taking 20 min. over it, and then went on at an average of 30 m.p.h. Find where and when he met a cyclist who left Horsham at 10.30 and cycled towards London at 12 m.p.h.

The travel-graphs of the two are shown in this figure, *ABCD* for the motorist; *EF* for the cyclist.



The point *G* in the graph represents that the two are simultaneously at the same spot on the road; hence they meet at 11.24 A.M. approximately **11 miles** from Horsham (or 29 from London).

Exercises LXXXI

1. A train leaving London at noon arrived at a station 73 ml. away at 1.25 P.M. Assuming it to travel uniformly, draw its travel-graph. How many miles from London was it at 1 P.M.? What was its average speed in m.p.h.?

2. A train left Birmingham at 10.45 A.M. and travelled at an average speed of 28 m.p.h. towards London (110 ml.). Draw its travel-graph by marking points to represent its position at 10.45 A.M., 11.45 A.M., 12.45 P.M. These should be in a straight line. Draw the line and produce it.

From your graph find at what time the train passed through Rugby (30 ml. from Birmingham) and at what time it should have reached London.

3. In the same figure as for Question 2, put in a graph for a train leaving Birmingham at 11.15 A.M. and going towards London at an average of 55 m.p.h. How does the faster speed

show itself in the graph? At what time and how far from London would the faster train pass the slower.

4. Find graphically which of the following journeys from London is the fastest on the average:

Brighton	(51 ml.)	63 min.
Swindon	(77 ml.)	83 min.
Crewe	(158 ml.)	182 min.

5. The following is an extract from a railway time-table. Draw a graph and so find approximately where and at what time the trains pass one another.

London			9.15	Bath	9.17
Reading	36 ml.	arr.	10.3		
		dep.	10.7		
Swindon	77 ml.	arr.	10.58		
		dep.	11.3		
Bath	107 ml.		11.46	London	11.5

6. A and B are two towns 70 ml. apart. A motorist left A at 10.15 A.M., and reached B at 12.40 P.M. Another motorist left B at 10.45 A.M. and travelled to A by the same road, reaching A at 1.30 P.M. Assuming that each travelled uniformly, find approximately when and where they passed each other.

7. The upper diagram on p. 149 shows a portion of the travel-graph of an electric train. Explain the meaning of its upward curvature.

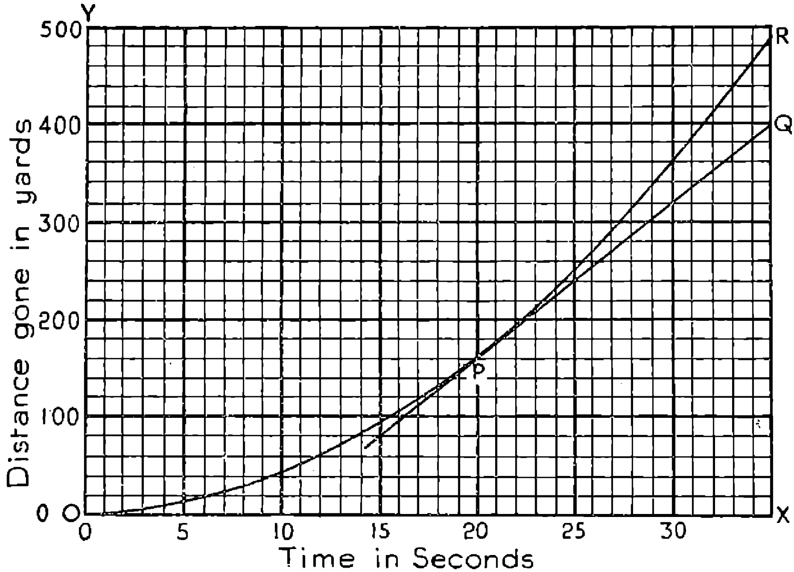
If after 20 sec. the train had continued to travel with the speed it had at the end of the twentieth second, its travel-graph would have continued along the tangent line PQ instead of along the curve PR. In this case how many yards would it have travelled in the interval from time 20 to time 30 sec.? What speed in yards per minute does this represent?

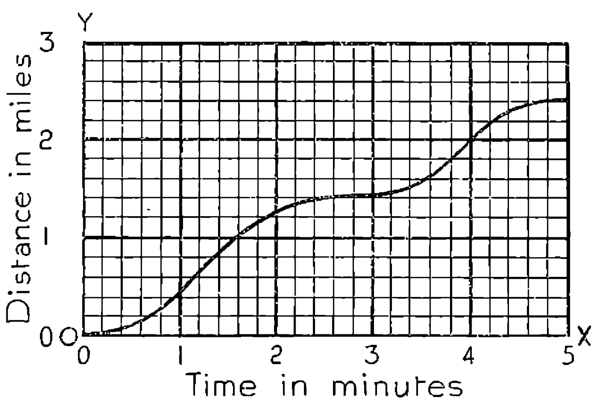
8. Describe in words the motion of the car whose travel-graph is shown on the lower diagram on p. 149.

9. Sketch curves to show the shapes of the travel-graphs of two trains A and B, if

(i) A passed a station at top speed, slowed down through a tunnel, and stopped at a station beyond it.

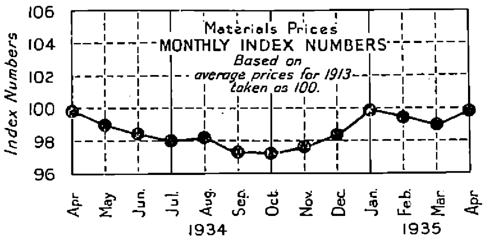
(ii) B travelled steadily, stopped quickly at a signal, waited a few minutes, and then got up speed again.


Y
500
400
300
200
100
0 O
R
Q
P
X
0
5
10
15
20
25
30
Distance gone in yards
Time in Seconds

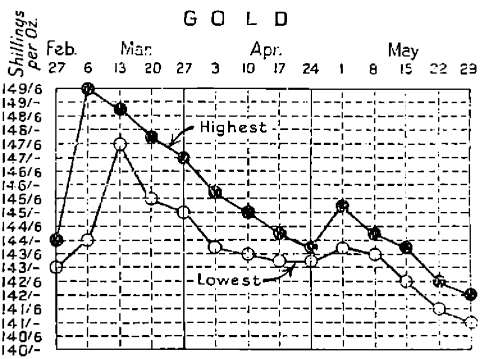

Y
3
2
1
0 O
X
0
1
2
3
4
5
Distance in miles
Time in minutes

Exercises LXXXII

1. The diagram below shows the **Index** Numbers for the price of materials for 12 months.



In which months were the Index Numbers (*a*) highest, (*b*) lowest? Describe in general terms the variation of prices during the year.



2. The diagram above shows the highest and lowest prices for gold for 3 months in 1935.

When was the difference between highest and lowest prices (*a*) greatest, (*b*) least?

Describe in general terms how prices changed during the period.

3. The diagram on p. 151 shows highest and lowest prices quoted for tin during the same 3 months.

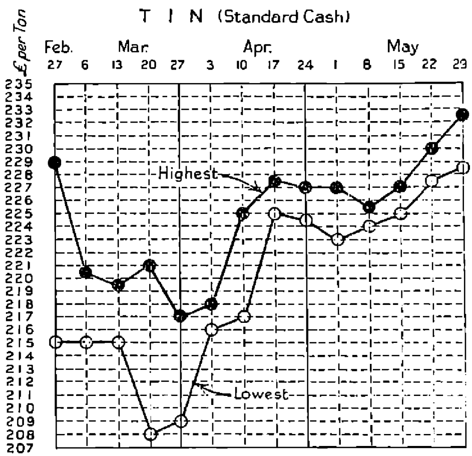
Answer the same questions about this graph as in Question 2.

(These graphs are taken from *The Times* "*Trade and Engineering*," which is issued monthly.)

4. The table shows the imports into Jamaica of artificial silk goods from Japan during the first six months of 1934.

Month	Jan.	Feb.	Mar.	April	May	June
Thousands of sq. yd.	246	263	440	509	564	193

Plot a graph to illustrate these figures.



5. A salesman kept the following record of the distances he travelled on business during eight successive weeks:

Week	1	2	3	4	5	6	7	8
Miles travelled	260	340	290	420	190	280	470	300

Plot a graph to show this information.

6. Calculate the cubes of 0, 1, 2, 3, 4, 5, and draw a graph to show the results. Use the graph to find approximately the cubes of $2\frac{1}{2}$ and $3\frac{3}{4}$, and the numbers whose cubes are 50 and 100.

7. A certain kind of glass, $\frac{3}{16}$ in. thick, is used for windows. The following table shows the largest square area of this glass which will withstand a given wind pressure. ($11\frac{1}{2}$ lb. per square foot corresponds to a 60 m.p.h. gale, 26 lb. to a 90 m.p.h. gale.)

Wind pressure in lb. per sq. ft.	$11\frac{1}{2}$	14	18	22	26
Area of square window in sq. ft.	50	40·8	31·6	25	21·8

Plot these figures in a graph, and find approximately:

(i) What wind pressure a window 6 ft. square could withstand.

(ii) The largest square area which would withstand a pressure of 20 lb. per square foot.

8. The connexion between the load raised by a screw-jack and the force applied is given in this table:

Load in lb.	100	500	1000	1500	2000
Force in lb. wt.	$\frac{3}{4}$	3	$6\frac{1}{4}$	10	$14\frac{1}{4}$

Draw a graph to represent these facts, and read off from it the loads raised by forces of 5 and 12 lb. weight respectively and the force which must be applied to raise a weight of 1200 lb.

9. The angles of slope for different road gradients are given in this table:

Gradient of 1 in	3	4	5	8	10	15	20	30	50
Angle of slope in degrees	$19\frac{1}{2}$	$14\frac{1}{2}$	$11\frac{1}{2}$	$7\frac{1}{4}$	$5\frac{3}{4}$	$3\frac{3}{4}$	3	2	$1\frac{1}{8}$

Plot these figures in a graph and determine:

(*a*) the gradient corresponding to a slope of 10°;

(*b*) the angle of slope for gradients of 1 in 6, and 1 in 12.

10. *A* and *B* are two villages 20 ml. apart. Find roughly, by means of a graph, when and where a cyclist leaving *A* for *B* at 8.30 A.M., at an average of 12 m.p.h., would meet a car which left *B* for *A* at 9 A.M. at an average of 25 m.p.h.

11. *X* and *Y* are two stations on a railway 75 ml. apart. A train left *X* at 10 A.M. and reached *Y* without stopping at 11.10 A.M. At 10.55 it passed another train which had left *Y* at 10.30. Assuming both trains to travel uniformly find approximately at what time the second train would reach a station 35 ml. from *Y*.

12. Taking £1 as equivalent to $5·20, draw a graph to change sums of money up to £1 into dollars. From the graph find the dollar equivalents of 11*s*. 9*d*. and 16*s*. 6*d*., and the equivalent in English money of $3·50.

CHAPTER XVI

RATIO. INDEX NUMBERS

We can compare two quantities by estimating how much more or less than the other one of them is; *e.g.*, $4\frac{1}{2}$ tons is $2\frac{1}{2}$ tons heavier than 2 tons, 2*s*. 6*d*. is 2*s*. less than 4*s*. 6*d*.

We can also find what part or fraction $2\frac{1}{2}$ tons is of $4\frac{1}{2}$ tons, or 2*s*. 6*d*. of 4*s*. 6*d*., but certainly not what part 2*s*. 6*d*. is of $4\frac{1}{2}$ tons, because they are quantities of entirely different kinds.

Every concrete quantity must have

(i) A unit (lb., ton, ft., etc.).

(ii) A measure, *i.e.*, a number (integral or fractional) which tells us how many of that particular unit there are in the quantity.

The **ratio** of one quantity to another of the *same* kind is got by dividing the measure of the first by that of the second, both quantities being expressed in terms of the *same* unit.

This ratio tells us how many times one quantity is contained in the other one or what multiple or part one is of the other.

If the two quantities are expressed in terms of more than one unit (£ *s*. *d*., tons, cwt. lb., etc.), they must be reduced to one and the same unit when we are finding their ratio.

Example 1. £3 2*s*. 6*d*. = £3·125 = 62·5*s*.
= 125 sixpences.
£10 = £10·0 = 200*s*.
= 400 sixpences.

$$\text{Ratio of £3 2}s.\ 6d.\text{ to £10} = \frac{3\cdot125}{10} \text{ or } \frac{62\cdot5}{200} \text{ or } \frac{125}{400}$$

$$= \frac{0\cdot3125}{1} \text{ or } \frac{5}{16}.$$

$$\text{Ratio of £10 to £3 2}s.\ 6d. = \frac{3\cdot2}{1} \text{ or } \frac{16}{5}.$$

£3 2*s*. 6*d*. is 0·3125 or $\frac{5}{16}$ of £10.

£10 is 3·2 or $3\frac{1}{5}$ times £3 2*s*. 6*d*.

We can also say that £3 2*s*. 6*d*. is to £10 in the ratio of 5 to 16 or that £10 bears to £3 2*s*. 6*d*. the ratio of 16 to 5.

Example 2. A map is drawn on the scale of 6 in. to the mile. What fraction of the real distance is any distance on the map?

$$1 \text{ mile} = 1760 \times 36 \text{ in.}$$

$$\text{Ratio of 6 in. to 1 mile} = \frac{6}{1760 \times 36} = \frac{1}{1760 \times 6}$$

$$= \frac{\mathbf{1}}{\mathbf{10560}}$$

Any distance on the map is $\frac{1}{10560}$ of the actual distance. This fraction is called the **representative fraction** (*R.F.*) of the map.

Ratio as a Percentage

Any ratio may be expressed as a percentage if the fraction that represents that ratio is multiplied by 100. (See Chapter IX.)

Example. 3 cwt. 2 qr. 14 lb. $= 14\frac{1}{2}$ qr.

5 cwt. 3 qr. 14 lb. $= 23\frac{1}{2}$ qr.

$$\text{Ratio of first to second} = \frac{14\frac{1}{2}}{23\frac{1}{2}} = \frac{29}{47}$$

$$= 0{\cdot}617 \text{ (approx.)}$$

$$\text{Check } \frac{4 \text{ cwt.}}{6 \text{ cwt.}} = \frac{2}{3} = \cdot 6 \ldots$$

∴ the first quantity is **61·7** per cent. of the second one.

Exercises (Mental) LXXXIII

Express the following ratios as vulgar fractions, decimal fractions, and also as percentages.

1. 2*s*. to 16*s*.; 2*s*. 6*d*. to 17*s*. 6*d*.; 12*s*. 6*d*. to £2; 17*s*. 6*d*. to £7.
2. 14 lb. to 1 cwt.; 3 cwt. to 1 ton; 5 cwt. 2 qr. to 2 tons.
3. 10 grm. to 1 Kg.; 35 grm. to 7 Kg.; 3 litres to 5 Hectolitres.
4. 7 cm. to 5 mm.; 3 Dm. to 4 dm.; 3 sq. m. to 5 sq. mm.
5. 1 sq. ft. to 1 sq. yd.; 9 sq. in. to 1 sq. ft.; 1 c.dm. to 1 c.cm.

6. £1 12*s.* 6*d.* to £2 12*s.* 6*d.*; 3*s.* 4*d.* to £2; £5 to 16*s.* 8*d.*

7. 10 per cent. to $12\frac{1}{2}$ per cent.; 8 per cent. to 25 per cent.; 25 per cent. to $37\frac{1}{2}$ per cent.

8. 3 ch. to 5 f.; 44 sq. yd. to 1 ac.; 132 yd. to 1 f.

9. A map is drawn on the scale of $\frac{1}{1000}$ (*R.F.*). What lengths on the map represent 1 ch., 1 ml., 5 m., and 8 Km. 5 m. respectively?

Exercises LXXXIV

1. Express the ratio of 33 yd. to 2 f. as a vulgar and also as a decimal fraction.
2. Find the ratio of 15*s.* 9*d.* to £8 15*s.*
3. Find the ratio of 510 gr. to 17 Kg.
4. Find the ratio of 3 cwt. 2 qr. to 1 ton 10 cwt.

Express as a decimal corrected to three significant figures the ratio of:

5. A guinea (21*s.*) to a £1.
6. £1 to 1 guinea.
7. 3 f. 70 yd. to 5 f.
8. 3 ac. 2 r. 10 p. to 5 ac. 1 r. 20 p.
9. 3·78 m. to 6·25 Km.
10. Express 6 f. 5 ch. as a percentage of 2 ml.
11. Express a short ton (2000 lb.) as a percentage of a long ton (2240 lb.).
12. An arc of a circle is the same percentage of the circumference as the angle it subtends at the centre is of 360°. If the subtended angles are 20°, 60°, 70°, 90°, 125°, 150°, 180°, what percentages of the circumference are the arcs?

The Ordnance Survey issue the following maps:

13. Scale $\frac{1}{500}$ for towns. How many inches to the mile is this?
14. Scale $\frac{1}{1250}$ for suburban areas. How many inches to the mile is this?
15. Scale 3 inches to the mile. What is the *R.F.*?
16. Scale 2, 4, and 10 ml. to the inch. What are the respective *R.F.*'s?
17. At one time the world's production of fixed nitrogen was estimated at 661700 tons. Of that total Germany produced 424000 tons and the British Empire 12800 tons. Express Germany's production as a percentage of the British.
18. 130000 bales of wool were sold in London and 83000 of these were bought for the Continent. What percentage (to 3 significant figures) went to the Continent?

Find the ratio, expressed as a decimal to three significant figures, of the wholesale price in 1934 to the price in 1913 for each of the following commodities:

	1934	1913
19. Flour (280 lb.)	23*s*. 6*d*.	27*s*. 6*d*.
20. Potatoes (per ton)	£6 0*s*. 0*d*.	£3 15*s*. 0*d*.
21. Rubber (per lb.)	6¼*d*.	3*s*. 1*d*.
22. Bacon (per cwt.)	89*s*. 0*d*.	77*s*. 0*d*.

23–26. Express the increase or reduction in price in 1934 as a percentage of the price in 1913, to 3 significant figures.

Index Numbers

The price of pig-iron was 50*s*. per ton in 1913 and 140*s*. per ton in 1921. It was, therefore, 2·8 times as dear in 1921 as in 1913.

If the price in 1913 is taken as 100, then the price in 1921 is 280.

100 and 280 are said to be **Index Numbers.**

For purposes of comparison the price in a certain year is taken as 100. This is the standard, and the prices for other years are estimated in terms of this standard.

Example. The wholesale price of beef in 1913 was 4*s*. 2*d*. per 8 lb. and in 1921 was 9*s*. per 8 lb.

Ratio of 1921 to 1913 price $= \frac{108}{50} = \frac{216}{100}$.

Hence the 1921 price was 216 per cent. of the 1913 price. If the index number for the cost of beef in 1913 is taken as 100 the index number for 1921 must be **216.**

It will be seen that the estimation of the Index Number for any year is the same as the determination of the percentage that the price in that year is of the price in the year that is taken as the standard.

Exercises LXXXV

Taking the price in 1913 as the standard (Index Number 100), find (to the nearest integer) the index numbers for the years 1921 and 1934. All prices are wholesale.

	1913	1921	1934
1. Imported mutton (per 8 lb.)	3*s*. 3*d*.	5*s*. 4*d*.	3*s*. 8*d*.
2. Eggs (per 120)	12*s*.	30*s*.	13*s*. 6*d*.
3. Suit of clothes	5 guin.	9½ guin.	6 guin.
4. Copper (per ton)	£71 15*s*.	£92 2*s*. 6*d*.	£31 10*s*.
5. Sugar (per cwt.)	18*s*. 3*d*.	51*s*. 6*d*.	21*s*. 9*d*.

Many more examples could be obtained from *The Times*, *Economist*, and other papers.

CHAPTER XVII

PROPORTION AND VARIATION, DIRECT AND INVERSE

THE 'UNITARY' METHOD

Example 1. If 16 cub. ft. of water weigh 1000 lb., what will be the weight of 56 cub. ft.?

$$16 \text{ cub. ft. weigh } 1000 \text{ lb.}$$

$$1 \text{ cub. ft. weighs } \frac{1000}{16} \text{ lb.}$$

$$\therefore 56 \text{ cub. ft. weigh } \frac{1000}{16} \times 56 \text{ lb.}$$

$$= \frac{1000 \times 7}{2} \text{ lb.}$$

$$= \mathbf{3500} \text{ lb.}$$

Example 2. If a train moving at the rate of 30 miles per hour completes a journey in 3 hr., how long would it take at 36 miles per hour?

At 30 m.p.h. the journey is done in 3 hr.

$\therefore$ At 1 ,, ,, ,, ,, 3×30 hr.

$\therefore$ At 36 $\frac{3 \times 30}{36}$ hr.

$= \mathbf{2\frac{1}{2}}$ hr.

Ex. 1 is an instance of **direct variation** and Ex. 2 an instance of **inverse variation.**

These kinds of variation are discussed in the succeeding pages.

The examples at the end of the chapter can be worked by the unitary method.

PROPORTION AND VARIATION

Consider the following statements of the cost of a number of articles:

Number of articles bought	10,	50,	100,	150,	200.
Cost in shillings	5,	25,	50,	75,	100.

It will be seen that the ratio of any two costs is equal to the ratio of the corresponding numbers of articles.

The interests on a certain sum of money for different numbers of days are given in this table:

Interest in pounds	1,	2,	3,	4,	10.
Number of days	9,	18,	27,	36,	90.

The ratio of any two interests is equal to the ratio of the corresponding numbers of days.

In these examples, costs are said to be 'directly proportional to' or to 'vary directly as' the number of articles bought; interests are 'directly proportional to' or 'vary directly as' the number of days.

Observe that

$$\frac{\text{the measure of any cost}}{\text{corresponding number of articles bought}}$$

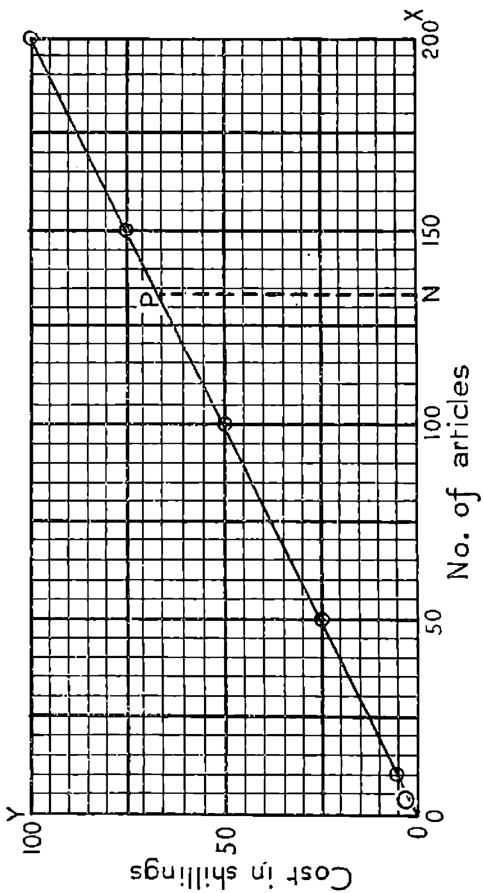
gives always the same fraction $\frac{1}{2}$, and

$$\frac{\text{the measure of any interest}}{\text{corresponding number of days}}$$

gives always the fraction $\frac{1}{9}$, *i.e.*, if a and b are the corresponding measures of the two quantities, $\frac{a}{b}$ is a constant number (often written k).

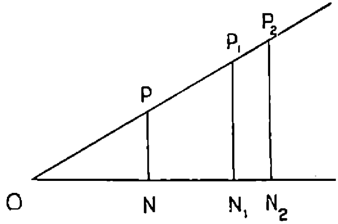
GRAPHICAL ILLUSTRATION

In the figure on p. 159 the costs and numbers of articles in the first example are plotted. The graph is a straight line through the origin (the point which represents zero on both scales simultaneously).


No. of articles
0
50
100
N
150
200
X
Cost in shillings
0
50
100
Y
P

Consult a text-book on geometry for a proof of the following theorem.

If PN, P_1N_1, P_2N_2 are parallel, $\frac{PN}{ON} = \frac{P_1N_1}{ON_1} = \frac{P_2N_2}{ON_2}$ wherever P, P_1, P_2 may be on the straight line $OP\ P_1\ P_2$.



This will explain the nature of the graph above, where PN represents a cost, ON the corresponding number of articles, and it has been seen that $\frac{\text{measure of cost}}{\text{number of articles}}$ is always the same.

A straight line through the origin is characteristic of direct variation.

INVERSE PROPORTION

Consider a distance of 60 miles travelled at various rates. The table shows the times taken.

Rate in miles per hour	4,	8,	10,	12,	30.
Time taken in hours	15,	$7\frac{1}{2}$,	6,	5,	2.

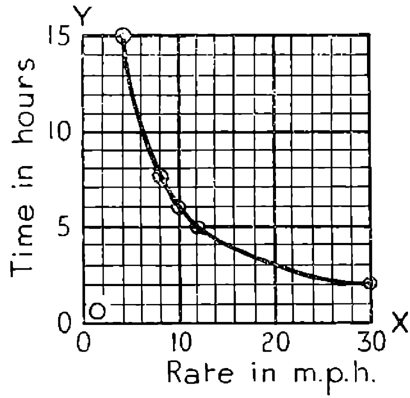
In this example the ratio of any two times, *e.g.*, $\frac{15}{7\frac{1}{2}}$, is not equal to the ratio of the corresponding rates, $\frac{4}{8}$, but to this ratio inverted, $\frac{8}{4}$.

That is, the ratio of any two times is equal to the ratio of the corresponding rates inverted.

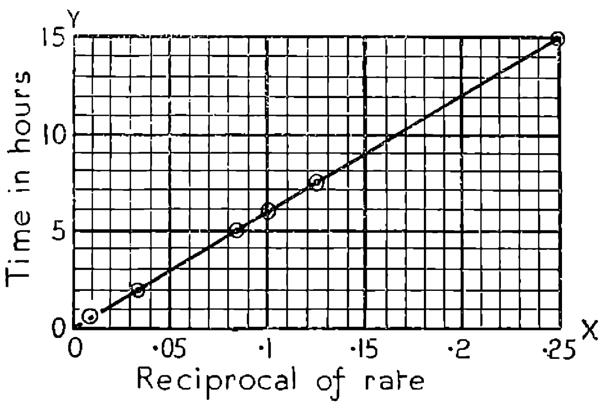
The time is said to be 'inversely proportional' to the rate or to 'vary inversely' as the rate.

Note that here 'the measure of any rate' × 'the measure of the corresponding time' is a constant number, viz., 60, *i.e.*, if a and b are the corresponding measures of the two quantities, ab is a constant number.

Graphical Illustration. The times of the last example are plotted first against the rates and secondly against the reciprocals of the rates. $\left(\text{Reciprocal of rate} = \frac{1}{\text{rate}}.\right)$



The curve in Fig. 1 is called an hyperbola, and is characteristic of inverse variation. It is not, however, so easy



to tell whether a curve is a true hyperbola as to see that a number of points lie in a straight line.

The second graph gives a straight line which, when produced, passes through the origin. This is also characteristic of inverse variation when one quantity is plotted against the reciprocal of the other; for, from what has been said above, if a straight line through the origin results from the plotting of y against $\frac{1}{x}$, then $\frac{y}{\frac{1}{x}}$ is a constant, *i.e.*, xy is a constant, and y varies inversely as x.

Before passing on to the solution of problems on direct and inverse proportion, be sure that you can recognize whether or not there is variation of either kind between two quantities a and b in a given example. The simplest test is this: if a is doubled, what happens to b? If it is also doubled then there is direct variation, if it is halved then there is inverse variation.

Exercises LXXXVI

Answer the following, if they can be answered:

1. A toy costs 6*d*. How much will 2 of these toys cost?
2. A boy can stand on one leg for 5 min. How long can he stand on both legs?
3. If you double the side of a square, do you double the area?
4. A car is travelling at half a mile a minute. How far will it go in 5 min.?
5. It takes 20 min. to boil a potato. How long will it take to boil 6 potatoes?
6. The railway fare on a certain journey for 2 persons was 30*s*. What would it be for 4 persons?
7. A boy would take 3 hr. to dig a certain plot of ground. If another boy works at the same rate how long would they take to dig the plot working together?
8. A man shot 10 rabbits in half an hour. How long would he take to shoot 100 rabbits?
9. A cistern is fed by two exactly similar taps. Either turned on alone would fill the cistern in an hour. How long will it take for the cistern to fill when both are turned on?
10. A boy aged 10 eats 3 meals in one day. How many meals will a man aged 40 eat in a day?

Example 1. 36 articles cost £1 4*s*. 6*d*. Find the price of 50 if there is no discount for increased quantities.

Ratio of costs = ratio of numbers bought.

$$\therefore \frac{\text{cost of } 50}{£1\ 4s.\ 6d.} = \frac{50}{36}$$

$$\therefore \text{cost of } 50 = £1\ 4s.\ 6d. \times \tfrac{50}{36}$$

$$= £1{\cdot}225 \times \tfrac{50}{36}$$

$$= £1{\cdot}701$$

$$= \mathbf{£1\ 14s.}\ \text{approximately.}$$

Example 2. 35 cub. cm. of a gas are collected at a pressure of 758 mm. of mercury. What would be the volume at 760 mm. pressure, temperature being kept constant (by Boyle's Law the volume is inversely proportional to the pressure)?

$$\frac{\text{Volume at 760 mm.}}{\text{Volume at 758 mm.}} = \frac{758}{760}.$$ Note the inversion.

$$\therefore \text{Volume at 760 mm.} = 35 \times \frac{758}{760} =$$

34·9 cub. cm. approximately.

Example 3. Of what sum is £35 6 per cent.?

$$\frac{\text{Required Sum}}{£35} = \frac{100}{6}$$

$$\therefore \text{Answer is } £35 \times \frac{100}{6}$$

$$= £\frac{3500}{6}$$

$$= \mathbf{£583\ 6s.\ 8d.}$$

EXERCISES (MENTAL) LXXXVII

1. If 10 cigarettes cost 6*d*., find the cost of 25 cigarettes.

2. A child has enough money to buy a pound of toffee at 6*d*. per ¼ lb. How much toffee could he buy at 8*d*. per ¼ lb.?

3. A car covered 24 ml. in an hour. How long at this speed would it take to cover 40 ml.?

4. A watch was put right at 9 P.M. on Monday. At 6 P.M. on Tuesday it was 3½ min. fast. How much fast would it be at 9 A.M. on Wednesday?

5. If a pot of paint is just sufficient to give an area of 400 sq. ft. 2 coats, will it be sufficient to give an area of 250 sq. ft. 3 coats?

6. A man has a piece of work to do, over which he will take 6 days alone. How long will it take if he has the help of a boy who works half as fast as the man?

7. A halfpenny dropped from a height would fall 16 ft. in the first second. How far would a penny dropped at the same time fall in the first second?

8. A pile of note-paper contains 480 sheets. Another pile of the same height contains sheets two-thirds the thickness of the sheets of the first pile. How many sheets does the second pile contain?

Exercises LXXXVIII

1. If 20 articles cost 15*s.* 6*d.*, find the cost of 75.

2. If 5 sovereigns weigh 41 gr., how many sovereigns will weigh 2 Kg. 50 gr.?

3. The simple interest on a certain sum of money for 15 days was £5 2*s.* 6*d.* Find the interest for 33 days.

4. If 3 tons 5 cwt. of copper cost £220, how much could be bought for £704?

5. A man cycles a certain distance at 12 ml. per hour in $2\frac{2}{3}$ hr. How long would he take at 16 ml. per hour?

6. 15 per cent. of a certain sum of money is £50. What would $37\frac{1}{2}$ per cent. of the same sum be?

7. Find the price of 448 lb. of wheat at 5*s.* $2\frac{1}{2}$*d.* per 100 lb.

8. 70 c.c. of gas are collected at a pressure of 755 mm. of mercury; what would be the volume in c.c. to 1 place of decimals at 760 mm. of mercury?

9. A gas has a volume of 125 litres at a pressure of 75·8 cm. of mercury. What pressure to the nearest mm. will reduce the volume to 121 litres?

10. If 360 men do a piece of work in 15 days, in how many days should 800 men be able to do it?

11. If 8000 lb. of wire cost £37 10*s.*, what would be the cost of 8 cwt. of the same wire?

12. A man is paid at the rate of £4 2*s.* 6*d.* per week of 48 hr. What would he receive for 40 hours' work?

13. A man is paid at the rate of £3 17*s.* per working week of 42 hr. and every additional hour of overtime is reckoned as an hour and a half. What does he receive for a week of 45 hr.?

14. Three-eighths of a ship is worth £15000. What is the value of $\frac{9}{16}$ of the same ship?

15. If 360 men have enough food to last for 27 days, how long should the food last if the number of men is reduced by 10 per cent.?

16. Find the volume of 336 lb. of fresh water if 12 cu. ft. weigh 750 lb.

17. Of what amount is £5 10*s*. 6*d*. $37\frac{1}{2}$ per cent.?

18. Of what amount is 25 fr. 50 c. 15 per cent.?

19. If 5 ac. 3 r. of land cost £36, find the cost of 12 ac. 2 r. (nearest shilling).

20. If 10 Hectares of land cost 50000 fr., what would the price of 8 Hectares 25 ares of this land be?

21. The cost of oats per cental (100 lb.) was 12*s*. 6*d*. Find the cost of a trade quarter of 336 lb. at that time.

22. If a sack of flour (280 lb.) was bought for 63*s*. 9*d*., find the cost per cwt.

23. 1000 Kg. = 1 metric ton. 1 English ton = 1016 Kg. Find the cost to the nearest shilling of 8 English tons of copper at £30 per metric ton.

24. An Axminster carpet 12 ft. by 12 ft. cost £16 16*s*.; find the cost of a similar carpet 10 ft. 6 in. by 9 ft.

25. If 216 grm. of oxide of mercury yield 16 grm. of oxygen when heated, how many grm., to the nearest hundredth, of oxygen would be obtained from 100 grm. of the oxide?

26. If 63 grm. of nitric acid contain 14 grm. of nitrogen, how much nitrogen (to the nearest grm.) would there be in 1 Kg. of nitric acid?

27. If 22·4 litres of nitrogen weigh 28 grm., find the weight of 5250 c.c. of nitrogen, to the nearest centigramme.

28. If 2 grm. of hydrogen are obtained when 65 grm. of zinc are dissolved in acid, what weight of hydrogen, in grm. to the nearest hundredth, will be liberated if $\frac{1}{4}$ Kg. of zinc is dissolved?

29. 73 grm. of hydrochloric acid are required to neutralize 106 grm. of sodium carbonate. Find the weight of acid required to neutralize 100 grm. of carbonate, to the nearest decigramme.

30. The electrical resistance of a wire of constant cross-section varies directly as the length. The resistance of 100 yd. of wire being 75 ohms, what length of this wire would have a resistance of 100 ohms?

31. The electrical resistances of wires of the same material and equal lengths vary inversely as their cross-sections. If the resistance of a wire of cross-section 2 sq. mm. is 12·5 ohms, find the resistance of a wire of the same material of cross-section 5 sq. mm, (1) of equal length, (2) 5 times as long.

32. If the volume of a gas is kept constant the pressure varies as its absolute temperature (temp. in degrees centigrade + 273). If the pressure is 748 mm. of mercury when the temperature is 19° C., find the pressure, to the nearest mm., if the temperature is reduced to 0° C.

Examples for graphical treatment:

33. In an experiment with a spring it was found that different weights hung from the end produced different extensions of the spring according to the following table:

Wt. hung (in grm.)	100	200	500	1000
Increase in length (in cm.)	2·8	5·6	13·9	27·8

Plot these values in a graph and verify that approximately the increase in length varies as the weight supported.

Calculate, on this assumption, the increase in length when the weight is 700 grm., and verify the result by reference to the graph.

34. A uniform metre stick was supported on a pivot at its middle point, and a weight was hung at a fixed point of the stick. This was balanced successively by different weights hung one at a time at suitable points on the other side of the pivot. The weights and their distances from the pivot were noted as follows:

Weight (in grm.)	100	200	500	1000
Distance from pivot (in cm.)	47·0	23·5	9·4	4·7

Plot these results in a graph. What kind of variation is suggested by it? Test by means of a suitable graph.

CHAPTER XVIII

DIVISION INTO PROPORTIONAL PARTS

Since the value of a fraction is unaltered when we multiply or divide *both* numerator and denominator by the same number, it follows that a ratio will be unaltered in value when both terms of it are multiplied or divided by the same number.

Ratio of 30 to 12 = 5 to 2, dividing by 6.
Ratio of 35 to 84 = 5 to 12, dividing by 7.

Example 1. A substance contains 23 parts by weight of sodium, 14 of nitrogen, and 48 of oxygen. What percentage of each does it contain (by weight)?

The weights are in the ratio of 23 to 14 to 48.

$$23 + 14 + 48 = 85$$

Hence, of every 85 parts by weight of the substance 23 are sodium, 14 nitrogen, and 48 oxygen, therefore

per cent. of sodium $= \frac{23}{85} \times 100 =$ **27·1** to nearest tenth
,, ,, nitrogen $= \frac{14}{85} \times 100 =$ **16·5**
,, ,, oxygen $= \frac{48}{85} \times 100 =$ **56·5**
100·1.

Example 2. A's share of a relative's will is to B's as 3 to 5 and B's share is to C's as 8 to 9. If the total legacies amount to £5450 how much does each receive?

We have to get a standard of comparison through B.

3 to 5 = 24 to 40 } B's number is made the same in
8 to 9 = 40 to 45 } each ratio.

The shares are as 24 to 40 to 45.

$$24 + 40 + 45 = 109$$

A's share = $\frac{24}{109}$ of £5450 = £50 × 24 = **£1200**
B's share = $\frac{40}{109}$ of £5450 = £50 × 40 = **£2000**
C's share = $\frac{45}{109}$ of £5450 = £50 × 45 = **£2250**
£5450

Exercises (Mental) LXXXIX

1. Divide £100 into two parts in the ratio of 3 to 2.

2. *A* and *B* were paid for their work at the same rate per hour. *A* received £3 6*s.* 8*d.* for 40 hr. What should *B* receive for 48 hr.?

3. Divide £70 between *A* and *B*, so that *A* has twice as much as *B*.

4. The cost of 2 lb. of butter and 1 lb. of tea is 5*s.* 0*d.* 1 lb. of tea costs twice as much as 1 lb. of butter. Find the cost of 1 lb. of each.

5. The fare by bus from *X* to *Y* is to the fare by tram in the ratio of 7 to 4. To go from *X* to *Y* by bus and return by tram costs $5\frac{1}{2}d.$ What is the bus fare?

6. The salaries of a father and his two sons are in the ratios of 7 to 4 to 3. Their total annual salaries amount to £1680. What does each receive?

7. The costs of 3 cars, *P*, *Q*, and *R*, are such that the cost of *P* is to the cost of *Q* as 5 is to 3, while the cost of *Q* is to the cost of *R* as 5 to 4. If the cost of *R* is £120, what is the cost of *P*?

Exercises XC

1. Divide £56 between *A*, *B*, *C*, and *D* in the ratio of the numbers 3, 5, 7, and 9.

2. *A*, *B*, and *C* are partners in a business in which *A* invests £4000, *B* £3000, and *C* £2000. If the total profits for a year are £1530, what should each partner receive?

3. A body of 4270 troops is quartered in three villages, the number assigned to each village being proportional to the number of its inhabitants. How many does each village receive if their populations are 685, 1125, and 1240?

4. Divide a weekly wage of £7 12*s.* 6*d.* between a man and a youth, the man's share being to the youth's in the ratio of 3 to 2.

5. The constituents of an alloy are copper 80, tin 15, and zinc 5 parts by weight. How much of each metal is there in 1 cwt. of the alloy?

6. An alloy contains 85 per cent. of copper, $12\frac{1}{2}$ per cent. of tin, and $2\frac{1}{2}$ per cent. of zinc. How much of each metal is there in 56 lb. of the alloy?

7. Two men subscribe £11050 and £18850 to an enterprise and agree to divide the profits between them in the ratio of their investments. How much should each receive out of a total profit of £1840?

8. A sovereign weighs 123·274 grains and $\frac{11}{12}$ of it is pure gold. Find the weight of pure gold, to the nearest grain, in 50 sovereigns.

9. A sum of money is divided between four children in the ratio of their ages, which are 9, 12, 15, 18 years. If the youngest child gets 3*s.* 6*d.*, how much is distributed and what do each of the other children get?

10. A sum of £37 19*s.* is made up of equal numbers of sovereigns, shillings, and pence. How many of each were there?

11. Two partners agree that the profits of their business shall be divided so that one-third goes to each and the remaining third is divided between them in proportion to the numbers of days each has worked. If the total profits are £1018 and they have worked for 257 and 314 days respectively, find, to the nearest pound, the amount each should receive.

12. A man leaves £32818 to be divided between his three sons in the ratio of the fractions $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$. Find the share of each to the nearest pound. (*Hint.* Multiply each fraction by 60.)

13. The proportions by weight of zinc and oxygen in a powder are 65 and 16. Find the amount of each, to the nearest hundredth of a gramme, in 50 grm. of the powder.

14. The proportions by weight of potassium, oxygen, and hydrogen in caustic potash are 39, 16, 1. Find, to the nearest hundredth of a lb., the weight of each constituent in 100 lb. of caustic potash.

15. If £15 be divided among 6 men, 12 women, and 17 boys so that 2 men may receive as much as 5 boys and 2 women as much as 3 boys, how much will each man, woman, and boy receive?

16. How much of each metal is there in 2 tons 2 cwt. 2 qr. of an alloy which contains 65 per cent. of copper, $22\frac{1}{2}$ per cent. of zinc, and $12\frac{1}{2}$ per cent. by weight of tin?

17. Find the weight in pounds of each constituent in 1 ton of cordite which is composed of 57 per cent. of nitro-glycerine, 38 per cent. of nitro-cellulose, and 5 per cent. of vaseline.

18. On a committee of 20 persons the proportion of gentlemen to ladies is 3 to 1. How many ladies must be added to the committee so that the proportion may be 3 to 2?

19. In a certain business *A* has £20000, *B* £30000, and *C* £20000 invested. In any year half the profits are divided equally between *B* and *C* for managing the business and the

other half is divided between A, B, and C in proportion to their capital invested. In a year in which C receives £880, what will each of the others receive?

20. Divide £1000 between A, B, C, D so that C's share is $\frac{2}{3}$ of A's share, B's share is $\frac{3}{4}$ of C's share, and D's share is equal to the sum of B's and C's shares.

21. The weight of 1 c.c. of an alloy of three metals in the proportion 3 to 4 to 5 by weight and 13 to 17 to 26 by volume is 7·28 grm. Find the weight of 1 c.c. of each of these metals, to the nearest hundredth of a grm.

22. An alloy consists of 48 parts by weight of gold to 52 of silver, the volume of the alloy being equal to the sum of the volumes of the gold and silver. If a c.c. of gold weighs 19·6 grm. and of silver 10·2 grm., what is the volume of 13·5 grm. of the alloy to the nearest ·01 c.c.?

23. An alloy is made by mixing three metals in the proportion 7 to 9 to 12 by volume, and the weights per unit volume of the first two of these metals and of the alloy itself are in the proportion 3 to 5 to 6. What is the ratio of the weight per unit volume of the third metal to that of the alloy?

CHAPTER XIX

NEGATIVE NUMBERS

Some writers believe that the signs + and — were invented by Italian Customs Officers who marked packages + if they contained more than the stated quantity and — if they contained less. $\overset{+}{2}$ meant 2 too many, $\bar{2}$ meant 2 too few.

It is often convenient to be able to associate with numbers an idea of direction, of up or down, of right or left, etc., reckoning from some fixed standard. With this extended idea of numbers, the ordinary numbers 2, 5, etc., are replaced by $\overset{+}{2}$, $\overset{+}{5}$, and the negative numbers $\bar{1}$, $\bar{2}$, $\bar{3}$, . . . continue the scale below zero. Thus we refer to a temperature of 10 degrees below freezing point on the Centigrade scale as a temperature of $\bar{10}$ degrees, and, when we are considering heights above sea-level a height of $\bar{100}$ ft. would mean 100 ft. below sea-level.

$\overset{+}{2}$ is to be read 'plus 2' and $\bar{2}$, or (— 2) as it is often written, 'minus 2.'

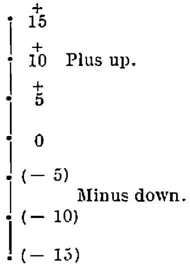
Addition of Negative Numbers

Using the idea of the first paragraph above

$$\left.\begin{array}{l} \bar{2} + \bar{4} \text{ is evidently } \bar{6} \\ \bar{2} + \overset{+}{4} \quad ,, \qquad ,, \quad \overset{+}{2} \\ \overset{+}{2} + \bar{4} \quad ,, \qquad ,, \quad \bar{2} \end{array}\right\} \text{and so on.}$$

If mixed positive and negative numbers are to be added together, add the positive ones and add the negative ones; subtract one result from the other, and attach the sign + if the positive number are in excess, — if the negative numbers are in excess.

These results are also plain from the diagram:



Start from 0

$\overset{+}{5}$ added to $\overset{+}{10}$, 5 up and 10 more up, $= \overset{+}{15}$

$\bar{5}$,, ,, $\overline{10}$, 5 down and 10 down, $= \overline{15}$

$\bar{5}$,, ,, $\overset{+}{10}$, 5 down and 10 up, $= \overset{+}{5}$

$\overset{+}{5}$,, ,, $\overline{10}$, 5 up and 10 down, $= \bar{5}$

Subtraction of Negative Numbers

$a - b$ is the number which must be added to b to make a.

Take $\overset{+}{8}$ from $\overset{+}{10}$, $\overset{+}{10}$ is 2 up from $\overset{+}{8}$, ∴ answer $\overset{+}{2}$

$\overset{+}{10}$ from $\overset{+}{8}$, $\overset{+}{8}$ is 2 down from $\overset{+}{10}$, ∴ ,, $\bar{2}$

,, $\bar{5}$ from $\bar{7}$, $\bar{7}$ is 2 down from $\bar{5}$, ∴ ,, $\bar{2}$

,, $\overset{+}{5}$ from $\bar{7}$, $\bar{7}$ is 12 down from $\overset{+}{5}$, ∴ ,, $\overline{12}$

,, $\bar{5}$ from $\overset{+}{8}$, $\overset{+}{8}$ is 13 up from $\bar{5}$, ∴ ,, $\overset{+}{13}$

It is convenient to omit the + sign from positive numbers, retaining only the − sign to distinguish negative numbers.

Exercises (Mental) XCI

Add together

1. $\bar{3}$, $\bar{5}$, $\bar{7}$.
2. $\bar{4}$, $\bar{6}$, $\bar{8}$.
3. $\bar{3}$, 3.
4. $\bar{5}$, 2.
5. $\bar{3}$, $\bar{5}$, 8.
6. $\bar{5}$, $\bar{7}$, 15.
7. $\bar{8}$, $\overline{10}$, 16.
8. $\bar{4}$, $\bar{8}$, 5, 7.

Subtract

9. $\bar{5}$ from $\bar{8}$.
10. $\bar{8}$ from $\bar{5}$.
11. 5 from $\overline{10}$.
12. $\bar{5}$ from 10.
13. $\bar{6}$ from 9.
14. 7 from $\bar{8}$.
15. $\bar{3}$ from $\bar{4}$.
16. $\bar{3}$ from $\bar{2}$.

17. The Centigrade thermometer registers 2 degrees below zero. What will it register if the temperature falls 3, 6, 8 degrees?

18. What will the same thermometer register if the temperature rises 1, 2, 5 degrees?

19. A man walks 5 ml. East and returns 3 ml. due *W*. How far from his starting-point is he?

20. If a man walks 5 ml. *S* and returns 7 ml. *N*, how far and in what direction is he from his starting-point?

Simple Calculations Preliminary to the Use of Logarithms

As will be explained in the next chapter a logarithm is a number and is usually expressed in the form of a positive decimal fraction together with an integral part which may be positive or negative. Thus $\bar{2}\cdot456$ means that the 2 is negative, but the decimal part $\cdot456$ is positive; $2\cdot456$ is all positive.

It will be necessary to add and subtract such numbers and to multiply and divide them by positive integers.

Addition

$$\begin{array}{r} \bar{2}\cdot456 \\ 3\cdot567 \\ \bar{4}\cdot892 \\ \hline \mathbf{\bar{2}\cdot915} \end{array}$$

The integral parts add up to $\bar{3}$, and 1 is to be carried from the decimal part.

Subtraction

$$\begin{array}{r} 4\cdot289 \\ \bar{2}\cdot567 \\ \hline \mathbf{5\cdot722} \end{array}$$

1 to be carried from decimal part. Adding this to $\bar{2}$ gives $\bar{1}$. Subtracting $\bar{1}$ from 4 we get 5.

$$\begin{array}{r} \bar{4}\cdot289 \\ \bar{2}\cdot567 \\ \hline \mathbf{\bar{3}\cdot722} \end{array}$$

As in the last example, except subtract $\bar{1}$ from $\bar{4}$.

Check subtractions by addition.

Multiplication

$\bar{2}\cdot6 \times 7 = \mathbf{\overline{10}\cdot2}$ — 4 is carried from the decimal part, and is added to $\overline{14}$, to make $\overline{10}$.

$\bar{3}\cdot85 \times 8 = \mathbf{\overline{18}\cdot8}$ — Carry 6 to add to $\overline{24}$. This gives $\overline{18}$.

Division. (1) Divide $\bar{3}\cdot6$ by 3.

Divide negative and positive parts separately.

The result is $\mathbf{\bar{1}\cdot2}$.

(2) Divide $\bar{3}\cdot6$ by 2.

The negative part does not divide exactly by 2, and it is desired to have the result in the same form, *i.e.*, as a combination of positive decimal with a negative integer.

Now $\bar{3}\cdot6 = \bar{3} + \cdot6 = \bar{4} + 1\cdot6 = \bar{5} + 2\cdot6 = \bar{6} + 3\cdot6$ $= \ldots$ and so on.

We use the first of these forms which has a negative part exactly divisible by the given divisor 2, *i.e.*, $\bar{4} + 1\cdot6$.

On dividing by 2 we obtain $\bar{2} + \cdot8 = \mathbf{\bar{2}\cdot8}$.

(3) Divide $\bar{5}\cdot821$ by 7.

$$\bar{5}\cdot821 = \bar{7} + 2\cdot821$$

dividing by 7, result is $\bar{1} + \cdot403 = \mathbf{\bar{1}\cdot403}$.

With a little practice the work can be done mentally thus

$$6)\underline{\bar{4}\cdot88} \qquad 6 \text{ into } \bar{6} \text{ gives } \bar{1}$$
$$\mathbf{\bar{1}\cdot48} \qquad 6 \text{ into } 2\cdot88 \text{ gives } \cdot48.$$

(4) Multiply $\bar{3}\cdot76$ by $2\frac{1}{4}$

$$\left.\begin{array}{l} \bar{3}\cdot76 \times 2 = \bar{5}\cdot52 \\ \bar{3}\cdot76 \div 4 = \bar{1}\cdot44 \end{array}\right\} \text{add.}$$
$$\bar{6}\cdot96$$

Result is $\mathbf{\bar{6}\cdot96}$.

Exercises XCII

Multiply

1. $\bar{1}\cdot2$ by 6.
2. $\bar{2}\cdot35$ by 3.
3. $\bar{4}\cdot62$ by 4.
4. $\bar{2}\cdot45$ by 5.
5. $\bar{3}\cdot67$ by 7.
6. $\bar{1}\cdot16$ by 8.

Divide

7. $\bar{1}\cdot5$ by 2.
8. $\bar{2}\cdot5$ by 2.
9. $\bar{3}\cdot5$ by 4.
10. $\bar{2}\cdot32$ by 7.
11. $\bar{6}\cdot822$ by 6.
12. $\bar{1}\cdot43$ by 10.

Multiply

13. $\bar{2}\cdot56$ by $1\cdot25$.
14. $\bar{4}\cdot68$ by $3\cdot5$.
15. $\bar{2}\cdot14$ by $3\frac{1}{8}$.
16. $\bar{3}\cdot4$ by $3\frac{3}{4}$.

Add together

17. $2\cdot35$, $\bar{1}\cdot78$.
18. $\bar{2}\cdot56$, $\bar{1}\cdot42$.
19. $\bar{2}\cdot43$, $4\cdot67$.
20. $\bar{2}\cdot37$. $\bar{3}\cdot48$.

Subtract

21. $\bar{2}\cdot47$ from $\bar{3}\cdot58$.
22. $2\cdot93$ from $\bar{2}\cdot89$.
23. $\bar{3}\cdot78$ from $\bar{1}\cdot52$.
24. $2\cdot56$ from $1\cdot93$.
25. $0\cdot357$ from 0.

CHAPTER XX

LOGARITHMS

We use the notation 10^3 to mean $10 \times 10 \times 10$, the *index* 3 denoting that 3 factors each 10 are to be multiplied together. Using this notation we have $10^3 \times 10^2 = 5$ factors each 10 multiplied together $= 10^5$, and in general $10^m \times 10^n = 10^{m+n}$, m and n being positive integers.

$$10^5 \div 10^2 = 5 \text{ factors each } 10 \div 2 \text{ factors each } 10$$
$$= 3 \text{ factors each } 10 = 10^3$$

and in general, $10^m \div 10^n = 10^{m-n}$, m and n being positive integers and m greater than n.

Also $(10^4)^3 =$ the product of 3 numbers each of which is the product of 4 tens
$=$ the product of 12 factors each 10
$= 10^{12}$

and in general $(10^m)^n = 10^{mn}$.

Thus if m and n are positive integers we have the following results:

$$10^m \times 10^n = 10^{m+n} \quad . \quad . \quad (1)$$
$$10^m \div 10^n = 10^{m-n},\ m > n \quad . \quad (2)$$
$$(10^m)^n = 10^{mn} \quad . \quad (3)$$

EXTENSION TO FRACTIONAL AND NEGATIVE INDICES

It is convenient to assign to such expressions as 10^0, $10^{\frac{1}{2}}$, 10^{-2}, etc., for which the original meaning of an index fails, such meanings that the expressions still obey the laws (1), (2) and (3) above.

By law (2), removing the restrictions on m and n,

$$10^m \div 10^m = 10^{m-m} = 10^0$$

but as $10^m \div 10^m = 1$, we therefore interpret 10^0 as equal to 1. It is easy to verify that this interpretation is consistent with laws (1) and (3).

Hence $10^0 = 1$ . . . . . . (4)

Assuming law (1) to hold for all indices

$$10^2 \times 10^{-2} = 10^{2+(-2)} = 10^0 = 1$$

$$\therefore\ 10^{-2} = \frac{1}{10^2}$$

Similarly $10^{-3} = \dfrac{1}{10^3}$

and in general $10^{-m} = \dfrac{1}{10^m}$. (5)

Again $10^{\frac{2}{3}} \times 10^{\frac{2}{3}} \times 10^{\frac{2}{3}} = 10^{\frac{2}{3}+\frac{2}{3}+\frac{2}{3}} = 10^2$

$$\therefore\ 10^{\frac{2}{3}} \text{ is the cube root of } 10^2 = \sqrt[3]{10^2}$$

and in general $10^{\frac{m}{n}} = \sqrt[n]{10^m}$ . . . . . . (6)

It is easy to verify that the results (5) and (6) are consistent with laws (2) and (3).

Logarithms to base 10

When a number is expressed as a power of 10, the index is called the **logarithm** of the number to the base 10. (If the number were expressed as a power of 8, then the index would be its logarithm to base 8, and so on.) In the following we deal only with logarithms to base 10; common logarithms, as they are called.

Since $10^1 = 10$, the logarithm of 10 is 1.
,, $10^0 = 1$, the logarithm of 1 is 0.
,, $10^{-2} = \frac{1}{100} = \cdot01$, the logarithm of ·01 is -2, and so on.

Exercises (Mental) XCIII

Find the logarithms of

1. 100
2. 1000
3. 100000
4. $\dfrac{1}{10}$
5. $\dfrac{1}{1000}$
6. $\dfrac{1}{1000000}$
7. ·01
8. ·0001
9. $\sqrt{10}$
10. $\sqrt[3]{10}$
11. $\dfrac{1}{\sqrt{10}}$
12. $\dfrac{1}{\sqrt[3]{10}}$.

The word 'logarithm' is usually abbreviated to 'log.' Since log 1 = 0 and log 10 = 1, the log of a number which lies between 1 and 10 will lie between 0 and 1; that is, it will be a proper fraction, or decimal with no integral part.

Numbers which are between 1 and 10, and thus have one figure before the decimal point, are said to be in the standard form.

Any number can be expressed as a number in the standard form multiplied by a positive or negative integral power of 10.

E.g., $478{\cdot}9 = 4{\cdot}789 \times 10^2$
$\cdot 004789 = 4{\cdot}789 \times 10^{-3}$.

To express 4·789 as a power of 10 is not easy, and we rely in practice upon tables of logarithms.

Since $\sqrt{10} = 3{\cdot}162$ approximately, we have

$$\left.\begin{array}{ll} 3{\cdot}162 = \sqrt{10} = 10^{\frac{1}{2}} = 10^{\cdot 5} & \therefore \log 3{\cdot}162 = \cdot 5 \\ 31{\cdot}62 = 10^{\frac{1}{2}} \times 10 = 10^{1\cdot 5} & \therefore \log 31{\cdot}62 = 1{\cdot}5 \\ 316{\cdot}2 = 10^{\frac{1}{2}} \times 10^2 = 10^{2\cdot 5} & \therefore \log 316{\cdot}2 = 2{\cdot}5 \\ \cdot 3162 = 10^{\frac{1}{2}} \div 10 = 10^{\frac{1}{2}} \times 10^{-1} & \therefore \log \cdot 3162 = \cdot 5 - 1 \end{array}\right\} A$$

Using the notation of the last chapter we write ·5 − 1 as $\bar{1}{\cdot}5$, and read it 'bar 1, point 5.'

$$\cdot 003162 = 10^{\frac{1}{2}} \div 10^3 = 10^{\frac{1}{2}} \times 10^{-3} = 10^{\bar{3}\cdot 5}$$
$$\therefore \log \cdot 003162 = \bar{3}{\cdot}5$$

Note that the numbers in A have the same significant figures, and their logs have the same decimal part. This decimal part is called the **mantissa** of the logarithm. The position of the decimal point in the original number determines the positive or negative integral part of the logarithm. This part is called the **characteristic** of the logarithm.

Thus the logarithm of any number consists of a decimal part (mantissa) which is determined by the significant figures of the number, and a characteristic which is determined by the position of the decimal point.

Logarithm tables give the mantissa of the logarithm. It is unwise to trust to rules for the determination of the characteristic. Observe how it is found.

Example. Given log 4·789 = ·6802, find the logs of 478·9, 47890, ·4789, and ·004789.

$478{\cdot}9 = 4{\cdot}789 \times 10^2 = 10^{\cdot 6802} \times 10^2 = 10^{2\cdot 6802}$

$\therefore \log 478{\cdot}9 = \mathbf{2{\cdot}6802}$

$47890 = 4{\cdot}789 \times 10^4 = 10^{4\cdot 6802}$

$\therefore \log 47890 = \mathbf{4{\cdot}6802}$

$\cdot 4789 = 4{\cdot}789 \times 10^{-1} = 10^{\bar{1}\cdot 6802}$

$\therefore \log \cdot 4789 = \mathbf{\bar{1}{\cdot}6802}$

$\cdot 004789 = 4{\cdot}789 \times 10^{-3} = 10^{\bar{3}\cdot 6802}$

$\therefore \log \cdot 004789 = \mathbf{\bar{3}{\cdot}6802}$

Example. Given log 2 = ·3010, find the numbers whose logs are 2·3010, 5·3010, $\bar{2}$·3010, $\bar{4}$·3010.

Since log 2 = ·3010, $10^{\cdot 3010} = 2$.

$\therefore 10^{2\cdot 3010} = 10^{\cdot 3010} \times 10^2 = 2 \times 100 = \mathbf{200}$

$10^{5\cdot 3010} = 10^{\cdot 3010} \times 10^5 = 2 \times 100000 = \mathbf{200000}$

$10^{\bar{2}\cdot 3010} = 10^{\cdot 3010} \div 10^2 = 2 \div 100 = \mathbf{\cdot 02}$

$10^{\bar{4}\cdot 3010} = 10^{\cdot 3010} \div 10^4 = 2 \div 10000 = \mathbf{\cdot 0002}$

Exercises (Mental) XCIV

Given that log 3·5 = ·5441, find the logs of

1. 35.
2. 3500.
3. ·035.
4. 350.
5. ·35.
6. ·0035.

Given that log 6·35 = ·8028, find the logs of

7. 63·5.
8. ·0635.
9. ·000635.
10. 6350.

Given that log 8 = ·9031, find the numbers whose logs are

11. 1·9031.
12. $\bar{1}$·9031.
13. 3·9031.
14. $\bar{2}$·9031.

Given that log 5·45 = ·7364, find the numbers whose logs are

15. $\bar{3}$·7364.
16. $\bar{1}$·7364.
17. 5·7364.
18. $\bar{6}$·7364.

Graphical Determination of Logarithms

Approximate values of the logarithms of numbers from 1 to 10 may readily be obtained from a graph.

Using the fact that $10^{\frac{1}{2}} = \sqrt{10} = 3{\cdot}162$, $10^{\frac{1}{4}} = \sqrt{3{\cdot}162} = 1{\cdot}779$, $10^{\frac{1}{8}} = \sqrt{1{\cdot}779} = 1{\cdot}333$, $10^{\frac{3}{8}} = 10^{\frac{1}{4}} \times 10^{\frac{1}{8}}$, $10^{\frac{5}{8}} = 10^{\frac{1}{2}} \times 10^{\frac{1}{8}}$, etc., we get the table

Nos.	1	1·333	1·779	2·371	3·162	4·22	5·62	7·50	10
Logs	0	·125	·25	·375	·5	·625	·75	·875	1

from which we can plot the preceding graph.

From this graph logarithms can be obtained to two figures, and numbers can be found from their logarithms. Thus log 7·4 is approximately ·87; the number whose log is ·58 is approximately 3·8.

It is worth while to construct such a graph, and use it for the first few simple calculations with logarithms. As soon as principles are mastered, the more accurate tables must be used.

To find a Logarithm from the Tables

Suppose the log of ·07583 is required. To begin with, concentrate on the figures 7583. Look in the first column of the tables for the first pair of figures 75; the row containing them and the headings of the columns are reproduced here:

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5

Look across the row to the column headed by the third figure, 8; in this row and column the figures 8797 are found. (This is the mantissa of the log for the figures 7580.) To this must be added the figure found in the same row in that one of the last nine columns (the 'difference' columns) which is headed by the fourth figure, 3.

$$8797 + 2 = 8799.$$

This means that the log of 7·583 (no. in standard form)

$$= {\cdot}8799 \text{ (pure decimal)}.$$

The log of the required number ·07583 is then $\bar{2}{\cdot}8799$, since ·07583 is $7{\cdot}583 \div 10^2$, *i.e.*, $7{\cdot}583 \times 10^{-2}$.

Exercises XCV

Find from the tables the logarithms of

1. 6·541	4. 1·070	7. 31·42	10. 987·3
2. 5·078	5. ·6379	8. 31420	11. ·8975
3. 1·893	6. ·01342	9. ·004615	12. ·07633

To find a Number from its Logarithm

When the logarithm is known, the number (antilogarithm) can be found either by using the logarithm table backward, or by using the antilogarithm table in the same manner as the logarithm table. Look up the mantissa of the logarithm in the tables to obtain the significant figures, and use the characteristic to fix the decimal point.

Example. Find the number whose logarithm is 3·5841. The figures of the number are found by looking up the number 5841 (the decimal part of the log) in the tables. We find the figures 3838. This means that ·5841 is the log of 3·838.

Hence 3·5841 is the log of $3{\cdot}838 \times 10^3 =$ **3838.**

Similarly the number whose log is $\bar{3}{\cdot}5841$ is $3{\cdot}838 \times 10^{-3}$.

$=$ **·003838.**

Exercises XCVI

Find the numbers whose logarithms are

1. ·6121	4. 1·5189	7. $\bar{2}{\cdot}6187$	10. $\bar{3}{\cdot}7913$
2. ·3789	5. $\bar{1}{\cdot}5189$	8. 3·1113	11. 4·4971
3. ·5189	6. 2·6187	9. 1·0192	12. $\bar{2}{\cdot}6990$

Calculations using Logarithms

Multiplication

Example 1. Find the value of $107{\cdot}9 \times 21{\cdot}58 \times {\cdot}02795$.
Using the tables,

$$\log 107{\cdot}9 = 2{\cdot}0330 \quad i.e., \ 107{\cdot}9 = 10^{2{\cdot}0330}$$
$$\log 21{\cdot}58 = 1{\cdot}3340 \quad 21{\cdot}58 = 10^{1{\cdot}3340}$$
$$\log {\cdot}02795 = \bar{2}{\cdot}4464 \quad {\cdot}02795 = 10^{\bar{2}{\cdot}4464}$$

$$\therefore \text{ the required product } = 10^{2{\cdot}0330 + 1{\cdot}3340 + \bar{2}{\cdot}4464}$$
$$= 10^{1{\cdot}8134}$$

remembering that $\bar{2} = -2$.

∴ the logarithm of the product is 1·8134.
∴ using the tables, the product is

$6{\cdot}507 \times 10^1 = 65{\cdot}07$
$= 65{\cdot}1$ corrected to 3 significant figures.

From this example it will be clear that to multiply numbers, all that is necessary is to find their logs, add these together to form the log of the product, and so find the product. The work can be conveniently arranged as follows:

	Numbers	Logs	
107·9 × 21·58 × ·02795	107·9	2·0330	R.C. 100 × 20 × ·03 = 60
	21·58	1·3340	
	·02795	$\bar{2}$·4464	
The product =	$6{\cdot}507 \times 10^1$ ←	1·8134 (adding).	

Therefore the product is 6·507 × 10 = **65·07**.

Division

Since to divide one power of 10 by another we subtract the index of the divisor from the index of the dividend to obtain the index of the quotient, we do the same with logarithms.

Example 2. Find the value of

$$\frac{21{\cdot}65 \times {\cdot}08923}{7 \times {\cdot}3789}.$$

The log of expression
= log of numerator − log of denominator
= (log 21·65 + log ·08923) − (log 7 + log ·3789).

	Numbers	Logs	
	21·65	1·3355	
	·08923	$\bar{2}$·9505	
		·2860	log of numerator.
	7	·8451	Add these and subtract from log numerator in one step.
	·3789	$\bar{1}$·5785	
Result	$7{\cdot}285 \times 10^{-1}$ ←	$\bar{1}$·8624	

Therefore the value is **·7285.**

Rough check $\frac{20 \times \cdot 09}{7 \times \cdot 4} = \frac{20 \times 9}{7 \times 40} = \frac{9}{14} = \cdot 7. \ . \ . \ .$

Note. Since tables are correct to 4 figures, and errors may accumulate, the fourth figure in the result is unreliable. In this example we cannot say whether the value is ·728 or ·729 to 3 significant figures.

Observe the tabular arrangement. It should always be used.

Example 3. Find the value of $\sqrt[3]{158 \cdot 7}$.

$$\log 158 \cdot 7 = 2 \cdot 2006, \textit{ i.e.}, 158 \cdot 7 = 10^{2 \cdot 2006}$$

$$\therefore \sqrt[3]{158 \cdot 7} = (10^{2 \cdot 2006})^{\frac{1}{3}}$$
$$= 10^{2 \cdot 2006 \times \frac{1}{3}}$$
$$= 10^{\cdot 7335}$$
$$= 5 \cdot 414$$

$$\therefore \sqrt[3]{158 \cdot 7} = 5 \cdot 41 \text{ to 3 significant figures.}$$

In tabular form

Numbers	Logs	
158·7	2·2006	divide by 3.
5·414←	—·7335	

Result is **5·41.**

Check. $5^3 = 125$, $6^3 = 216$. Therefore result is between 5 and 6.

Example 4. Find the value of $\sqrt{\frac{1 \cdot 876}{3 \cdot 142}}$.

	Numbers	Logs	
	1·876	·2732	
	3·142	·4972	subtract
Exp. under sq. rt.		$\bar{1} \cdot 7760$	divide by 2, $-2 + 1 \cdot 7760$
	$7 \cdot 727 \times 10^{-1}$	$\bar{1} \cdot 8880$	

The value is ·7727, or **·773** correct to 3 significant figures.

Check. $\sqrt{\frac{2}{3}} = \sqrt{\cdot 7} = \cdot 8. \ . \ . \ .$

For division of negative characteristic refer to the last chapter.

Example 5. Find the value of $\dfrac{74{\cdot}56 \times \sqrt[3]{{\cdot}0002476}}{\sqrt{0{\cdot}2594 \times 647{\cdot}6}}$

Numbers	Logs	
74·56	1·8726	
$\sqrt[3]{{\cdot}0002476}$	$\bar{2}{\cdot}7979$	$\dfrac{\bar{4}{\cdot}3938}{3} = \dfrac{-6 + 2{\cdot}3938}{3}$
Numerator	·6705	
0·2594	$\bar{1}{\cdot}4140$	
647·6	2·8113	
	2·2253	divide by 2.
Denominator	1·1126	subtract from log numerator
$3{\cdot}613 \times 10^{-1}$ ←	$\bar{1}{\cdot}5579$.	

The result is ·3613 or **·361** to 3 significant figures.

$$\text{Rough check}\quad \frac{70 \times \sqrt[3]{\dfrac{250}{10^6}}}{\sqrt{160}} \simeq \frac{70 \times \dfrac{6}{10^2}}{13}$$

$$= \frac{42}{130} = {\cdot}3\ \ldots$$

EXERCISES XCVII

Find the values, to 3 significant figures, of

1. $5{\cdot}356 \times 4{\cdot}892$
2. $15{\cdot}78 \times {\cdot}4956$
3. $0{\cdot}1562 \times 0{\cdot}3425$
4. $153{\cdot}7 \times 0{\cdot}00982$
5. $1{\cdot}094 \times 0{\cdot}1914$
6. $9{\cdot}567 \div 8{\cdot}593$
7. $153{\cdot}9 \div 0{\cdot}5678$
8. $2{\cdot}205 \div 0{\cdot}4536$
9. $0{\cdot}07276 \div 278{\cdot}3$
10. $0{\cdot}0156 \div 0{\cdot}00792$

Estimate to three significant figures

11. $\sqrt{25{\cdot}6}$
12. $\sqrt[3]{256}$
13. $\sqrt[4]{25{\cdot}6}$
14. $(1{\cdot}663)^3$
15. $3{\cdot}1416^2$
16. $3{\cdot}1416^{-2}$
17. $1{\cdot}333^{\frac{2}{5}}$
18. $\frac{355}{113}$
19. $100 \times \frac{758}{760} \times \frac{273}{288}$
20. $144 \times {\cdot}06871 \times 3{\cdot}217$

Exercises XCVIII

Find the values, to 3 significant figures, of

1. $3{\cdot}142 \times 6{\cdot}843^2$
2. $\dfrac{4 \times 3{\cdot}142 \times 5{\cdot}12^3}{8 \times 640}$
3. $\dfrac{1275 \times 4{\cdot}5 \times 107}{100 \times 365}$
4. $\dfrac{121{\cdot}5 \times 78{\cdot}64 \times 13{\cdot}6}{9 \times 4840}$
5. $\dfrac{1}{{\cdot}05871}$

6. $3{\cdot}142\ (10{\cdot}87^2 - 4{\cdot}52^2)$. Begin by expressing the difference of squares as a product.

7. $\dfrac{3{\cdot}142\ ({\cdot}817^2 - {\cdot}658^2) \times 960}{13{\cdot}6}$
8. $\dfrac{5{\cdot}128 \times ({\cdot}01234)^3}{({\cdot}0781)^2}$
9. $(8{\cdot}173 \times {\cdot}8735)^{\frac{1}{2}}$
10. $(15{\cdot}61 \times {\cdot}0783)^{\frac{1}{3}}$
11. $(25{\cdot}89)^{1{\cdot}2}$
12. $(58{\cdot}29)^{-2}$
13. $3{\cdot}146 \times (0{\cdot}00812)^2$
14. $\dfrac{15{\cdot}9 \times 75{\cdot}76 \times 273}{76 \times 291{\cdot}8}$
15. $\sqrt[3]{\dfrac{3 \times 517{\cdot}5}{4 \times 3{\cdot}142}}$

16. Find the number of digits in 19^{33}.

17. The earth's equatorial radius is $6{\cdot}378 \times 10^6$ m., and its polar radius $6{\cdot}357 \times 10^6$ m. Express the difference between the lengths of the two radii as a percentage of the equatorial radius.

18. Find the value of $p.r^u$ when $p = 93{\cdot}75$, $r = 1{\cdot}03$, and $u = 4$.

19. Find the value of $\pi\sqrt{\dfrac{l}{g}}$ if $\pi = 3{\cdot}142$, $l = 85$, and $g = 980{\cdot}5$.

20. Find, by using logarithm tables, the weight in pounds of $\frac{1}{2}$ pt. of mercury, if 1 c.c. of mercury weighs 13·6 grm., 1 lb. = 453·6 grm., and 1000 c.c. = 1·76 pt.

CHAPTER XXI

HARDER EXAMPLES ON PERCENTAGES

PROFIT AND LOSS

Example. A man bought 4 tons 11 cwt. of goods for £6 10*s*. and sold them at £2 per ton. What was his percentage profit?

		£
Selling price	$= £2 \times 4{\cdot}55 =$	9·1
Cost price	$=$	6·5
Total profit	$=$	£2·6.

Profit per cent. on *cost* profit

$$= \frac{2{\cdot}6}{6{\cdot}5} \times 100 = \frac{2}{5} \times 100 = \mathbf{40} \text{ per cent.}$$

Profit per cent. on *selling* price

$$= \frac{2{\cdot}6}{9{\cdot}1} \times 100 = \frac{2}{7} \times 100 = \mathbf{28{\cdot}6} \text{ per cent.}$$

Method. Find what fraction of the cost or selling price the total profit is, and multiply by 100.

Traders, wholesale and retail, often estimate their profits as a percentage of the *selling price.* They find it convenient for many reasons (sound and legitimate). As this custom does not seem to be well known, some of the following examples are worked in the two ways. (If the price on which the percentage is to be computed is not stated in any example, take it as the cost price.)

Alternative Method for the previous example.

$$100 \text{ per cent. of cost price} = £6{\cdot}5$$
$$x(?) \text{ per cent. of cost price} = £9{\cdot}1$$

$$\therefore \frac{x}{100} = \frac{9{\cdot}1}{6{\cdot}5} = \frac{7}{5}$$

$$x = \frac{700}{5}$$

i.e., selling price = 140 per cent. of cost price.

Therefore profit = **40** per cent. of cost price.

100 per cent. of selling price $= £9{\cdot}1$
$x(?)$ per cent. of selling price $= £6{\cdot}5$

$$\therefore \frac{x}{100} = \frac{6{\cdot}5}{9{\cdot}1} = \frac{5}{7}$$

$$x = \frac{500}{7}$$

i.e., cost price = 71·4 per cent. of selling price.
Therefore profit = **28·6** per cent. of selling price.

Marking of Prices to give a Certain Percentage of Profit

If the price on which the profit is based be *given* the question is a simple calculation of percentages as on p. 93.

If unknown, take the price on which the profit is based as 100 units.

Example 1. Mark an article which costs £2 10*s*. to give a profit of 12 per cent. on the selling price.

Selling price = 100 units. Cost price = 88 units.

$$\frac{\text{Selling price}}{\text{Cost price}} = \frac{100}{88}$$

$$\therefore \text{Selling price} = £2{\cdot}5 \times \frac{100}{88}$$
$$= £\frac{250}{88}$$
$$= £2{\cdot}841$$

= **£2 16s. 10d.** to the nearest penny.

Example 2. An article is listed at £3 6*s*. Find the cost price if 10 per cent. profit is made on the cost price.

Cost price = 100 units. Selling price = 110 units.

$$\therefore \frac{\text{Cost price}}{\text{Selling price}} = \frac{100}{110}$$

$$\therefore \text{Cost price} = £3{\cdot}3 \times \frac{100}{110}$$

= **£3.**

TWO COMMON TYPES OF EXAMINATION QUESTION

1. A merchant makes a profit of 15 per cent. on the cost price by selling an article for £3 16*s*. 8*d*. At what price should he have sold it to gain 18 per cent. on the cost price?

Take cost price as 100 units, then the selling prices are 115 and 118 units respectively.

$$\therefore \frac{\text{New selling price}}{£3\ 16s.\ 8d.} = \frac{118}{115}$$

$$\text{New selling price} = £3\ 16s.\ 8d. \times \frac{118}{115}$$

$$= £3\tfrac{5}{6} \times \frac{118}{115}$$

$$= £\frac{\overset{}{\cancel{23}}}{\underset{3}{\cancel{6}}} \times \frac{\overset{59}{\cancel{118}}}{\underset{5}{\cancel{115}}}$$

$$= £\frac{59}{15}$$

$$= £3\tfrac{14}{15}$$

$$= \mathbf{£3\ 18s.\ 8d.}$$

2. By selling a plot of ground for £121 10*s*. the owner made 8 per cent. profit on what it cost him. What would have been his profit per cent. on his outlay if he had sold for £126?

Take cost price = 100 units.

First selling price = 108, second = 100 + x units.

$$\therefore \frac{100 + x}{108} = \frac{£126}{£121\ 10s.}$$

$$100 + x = \frac{126}{121{\cdot}5} \times 108 = \frac{\overset{28}{\cancel{84}}\!\!\!\!\!\!\!\!\!\!\ \cancel{252}}{\underset{27}{\cancel{81}}\!\!\!\!\!\!\!\!\!\!\ \cancel{243}} \times \overset{4}{\cancel{108}}$$

$$= 112.$$

$\therefore$ His profit would have been **12** per cent.

Example 3. Mark an article which cost 5*s.* to gain 20 per cent. on the cost price after allowing a discount of 5 per cent. on the list price.

95 per cent. of listed price = 120 per cent. of cost price
= 6*s.*

$\therefore$ 100 per cent. of listed price $= 6s. \times \frac{100}{95}$
$= \frac{120}{19}s.$
= **6s. 4d.** to the nearest penny.

Example 4. Mark the same article to return a profit of 20 per cent. on the selling price after allowing the same discount.

Cost price = 80 per cent. of money received
= 80 per cent. of 95 per cent. of list price
= list price × ·80 × ·95.

$\therefore$ List price $= \frac{5}{\cdot80 \times \cdot95}s.$
$= \frac{1}{\cdot152}s.$
= 6·58*s.*
= **6s. 7d.** to the nearest penny.

Exercises (Mental) XCIX

Of how much is

1. £15, 30 per cent.
2. £45, 33⅓ per cent.
3. £50, 12½ per cent.
4. 80 dollars, 75 per cent.
5. 70 fr., 25 per cent.
6. 36 yd., 18 per cent.?

Find the gain or loss per cent. on the cost price if

7. Cost price is £40, selling price is £50.
8. Cost price is £32, selling price is £30.
9. Cost price is $50 25 cents, selling price is $60 30 cents.
10. Cost price is 12 fr. 50 c., selling price is 10 fr.

Find the cost price of articles sold for

11. £80 at a loss of 20 per cent. on cost price.
12. £150 at a loss of 25 per cent. on cost price.

13. 33 fr. at a profit of 32 per cent. on cost price.

14. $37 80 cents at a profit of $12\frac{1}{2}$ per cent. on cost price.

15. 20 per cent. on cost price is gained by selling for £3; what must you sell for to gain 40 per cent.?

16. 15 per cent. is lost by selling for £8 10*s.*; what must you sell for to gain 18 per cent.?

17. 5 per cent. is gained by selling for 21*s.*; what must you sell for to gain $3\frac{1}{3}$ per cent.?

18. 10 per cent. is lost by selling for £9; find the percentage profit when sold for £11.

19. 25 per cent. is gained by selling for £6 5*s.*; find the percentage profit when sold for £7.

20. $33\frac{1}{3}$ per cent. is gained by selling for $16; find the percentage profit when sold for $20.

Exercises C

Find the gain or loss per cent. on the following sales:

1. Bought at £5 and sold for £5 10*s.*
2. Bought at £7 10*s.* and sold for £9 5*s.*
3. Bought at £133 6*s.* 8*d.* and sold for £170.
4. Bought at 15 fr. and sold for 13 fr.
5. Bought at 135 lire and sold for 200 lire.
6. Bought at $4 80 cents and sold for $3 60 cents.
7. Calculate the percentage profits on the selling prices of any of the previous six sales that you care to do.
8. When the price of National Savings Certificates was raised from 15*s.* 6*d.* to 16*s.*, what was the percentage increase?
9. Turmeric test papers are bought (wholesale) at 2*s.* 3*d.* per doz. books and retailed at 5*d.* per book; find the percentage profit.
10. Some pills are bought (wholesale) at 10*s.* 6*d.* per thousand and retailed in boxes of 24 at 9*d.* per box; find the percentage profit.

Price the following articles to make the given percentage profit on the cost price:

	Cost price	Percentage profit	
11.	£10	$12\frac{1}{2}$	
12.	£5 2*s.* 6*d.*	15	
13.	£6 17*s.* 6*d.*	$22\frac{1}{2}$	
14.	$15 25 cents	$37\frac{1}{2}$	Give the answer to the nearest cent.

Try one or two of Exercises 11–14, assuming that the profit is based on the selling price.

Find the cost price in each of the following examples:

	Sale price	Percentage profit on the cost price
15.	£5 1*s.* 9*d.*	10
16.	£12 15*s.*	25
17.	£250 10*s.*	$33\frac{1}{3}$
18.	£8 8*s.* 9*d.*	$12\frac{1}{2}$
19.	$98	40
20.	90 fr. 75 c.	$37\frac{1}{2}$

21. Prove that a profit of x per cent. on the cost price is the same as a profit of $\frac{100x}{100 + x}$ per cent. on the selling price.

22. 10, 15, 20, 25, $33\frac{1}{3}$ are profits made on the cost prices. Use the formula of Exercise 21 to find the corresponding profits per cent. on the selling prices.

Find the catalogue or list prices of the following articles if the stated percentage gains on the cost price are made after allowing the discounts for cash:

	Cost price	Gain per cent.	Cash discount per cent.
23.	£18	$33\frac{1}{3}$	10
24.	£18 10*s.*	25	$7\frac{1}{2}$
25.	£9 10*s.*	30	5
26.	78 fr.	15	$2\frac{1}{2}$
27.	$48 50 cents.	$22\frac{1}{2}$	3

28. A gold watch is sold for £20 8*s.* at a loss of 15 per cent. At what price should it have been sold to gain 20 per cent.?

29. If a grocer gained $12\frac{1}{2}$ per cent. by selling tea at 12 guineas per cwt., what would he have gained per cent. by selling at 2*s.* 8*d.* per lb.?

30. Goods in a shop are marked at a price 20 per cent. above cost price. A customer receives a cash discount of 1*s.* in the £ from the marked price. Find the cost price of goods sold for 57 guineas.

31. Certain goods passed from manufacturer to consumer through an agent. The manufacturer's profit was 25 per cent. and the agent's 20 per cent. on what he paid to the manufacturer. If the consumer paid £5, what was the cost to the manufacturer?

Example 1. The population of a town has decreased 5 per cent. in the last 10 years and is now 31160. What was the population 10 years ago.

For every 100 people 10 years ago there are now 95.

$$\therefore \frac{\text{Population 10 years ago}}{\text{Present population}} = \frac{100}{95}$$

$$\therefore \text{Population 10 years ago} = \frac{100 \times 31160}{95}$$

$$= \mathbf{32800.}$$

Example 2. A man used to be able to save 10 per cent. of his income. If his income has increased 20 per cent., and his expenditure by 25 per cent., what percentage of his income can he save now?

If his old income is taken as 100 units his expenditure used to be 90 units.

His new income is 120 units, and his new expenditure is

$$90 + \tfrac{90}{4} \text{ units, } i.e., 112\tfrac{1}{2} \text{ units.}$$

$\therefore$ He can save $7\frac{1}{2}$ units out of 120.

$$\therefore \text{He can save } \frac{7{\cdot}5 \times 100}{120} \text{ per cent. of his income.}$$

i.e., **6¼** per cent.

Exercises CI

[Give answers which do not work out exactly to a reasonable degree of accuracy, *e.g.*, to the nearest penny or to 3 significant figures, unless definite instructions are given.]

1. A traveller's sales amount to £750, and his expenses are £17 15*s.*; express his expenses as a percentage of the sales.

2. The cost of coal for the power house of an electric railway was 14*s.* 9*d.* per ton in 1914 and 50*s.* per ton in 1921; express the increase in price as a percentage of the price in 1914.

3. Crops are grown on 80 per cent. of the area of a farm; 95 per cent. of the remaining area is used for grazing, and buildings which cover 2 roods complete the total area. Find this area.

4. There were 520,000 unemployed in November, 1920, and 1,800,000 in November, 1921; express the increase in unemployment as a percentage of the number of unemployed in 1920.

5. If the birth-rate of a district was 14·8 per 1000 living people, find the number of births if the population was 527500.

6. The population of a country was 42 million people, and the total number of births was 676200 in 1934; estimate the birth-rate per thousand people.

7. A man bought 450 articles at £2 6*s*. 8*d*. each and 100 at £2 14*s*. each. He sold all of them at £2 17*s*. 6*d*. each; find his percentage profit.

8. A man bought certain goods of which he sold one-third at a profit of 14 per cent., three-fifths at a profit of $17\frac{1}{2}$ per cent., and the remainder at a profit of 20 per cent. on the cost price. What was his percentage profit on the whole transaction?

9. If the manufacturer makes a profit of 25 per cent., the middleman 5 per cent., and the retailer 30 per cent., what is the cost to the manufacturer of an article that is sold in the shops for 13 guineas?

10. A man used to spend five-sevenths of his income on necessaries and two-sevenths on luxuries. If the cost of necessaries rises by 32 per cent., by how much per cent. will the money available for luxuries be reduced?

11. A manufacturer sells a piano to a middleman at a profit of 25 per cent. on the cost of manufacture, and the middleman sells it to a shopkeeper at a profit of 5 per cent. If the shopkeeper makes a profit of 30 per cent. by selling the piano for 52 guineas, what was the cost of manufacture?

12. A man sells two horses for £30 each, thereby making a profit of 20 per cent. on one of them and a loss of 20 per cent. on the other; what percentage of profit or loss does he make on the sale of the two horses taken together?

13. A's wages are half as much again as B's. If A's wages are now increased by 20 per cent. and B's by 10 per cent., by how much per cent. is the total payment to them increased?

14. A manufacturer marks his goods at an advance of 80 per cent. on the cost price, but he allows 15 articles to the dozen and also gives 10 per cent. discount for cash. What rate of profit on his outlay does he get from a customer who pays cash?

15. A profit of 20 per cent. on the cost price is made when a discount of 10 per cent. is given on the list price. What profit per cent. will be made when a discount of 20 per cent. on the list price is given?

16. A manufacturer used to make a profit of 20 per cent. by selling a bicycle for £10. What percentage of profit will he make if he reduces the price to £7 15*s*. when the cost of manufacture decreases by 25 per cent.?

17. An article could be sold at 2*s*. 6*d*. to give a profit of 20 per cent. on the cost price. If the cost price is increased by 10 per cent., what would be the gain per cent. by selling for 2*s*. 9*d*.?

18. A tradesman has sold three-quarters of his stock of a certain article at a profit of 35 per cent. At what percentage

below cost can he sell off the remainder to make a profit of 25 per cent. on the whole stock?

19. A tradesman used to make a profit of 20 per cent. by selling an article for 6*d.* The cost price has been increased by $1\frac{1}{4}d.$, and he has raised his selling price to $7\frac{1}{2}d.$ What percentage of profit is he making now?

20. If the cost of food had increased by 140 per cent. and the consumption had decreased by 25 per cent., for how many days could 4 persons have been fed with the money that used to feed 6 persons for 30 days?

21. A tradesman's rate of profit is 35 per cent. on his outlay. If he now lowers his selling price 2*d.* in the shilling, in what ratio must his sales increase if his whole profit increases in the ratio of 9 to 7?

22. In one year 56000 tons of ore were taken from a gold-mine and yielded 14000 oz. of gold. If, in the next year, the output of ore is increased by 20 per cent. and the yield of gold per ton is raised by 2 per cent., how many ounces of gold to the nearest thousand will be obtained?

23. A sold goods to B at a profit of 20 per cent. B became bankrupt and paid 13*s.* 6*d.* in the £. What was A's percentage loss?

24. Through the bankruptcy of a retailer who pays 12*s.* 6*d.* in the £ a merchant makes a loss of 25 per cent. on the cost price of goods sold to the dealer. What would have been his percentage gain had the dealer paid in full?

25. A manufacturer makes a new kind of box (cuboid) which is 10 per cent. longer and 8 per cent. wider than the old one; by what percentage of the old depth must the depth of the new box be decreased if the volume is to be unchanged?

26. The clerical work (which was practically uniform) in an office was done by two senior men, A and B, and a youth, each working 6 days a week. A did 50 per cent. of the work and B did 40 per cent. In a certain week the work was suddenly increased by one half. A increased his by 50 per cent. for 4 days and then went on sick leave. The youth did no more than he did before; by how much per cent. did B have to increase his work to get the whole job done?

27. An alloy of brass contains 80 per cent. of copper and 20 per cent. of zinc by weight. If copper weighs 8·5 and zinc 7·1 grm. per c.c., find the proportions by volume of copper and zinc in the alloy.

28. The population of a town was 85700 and increased in three successive years by 4, 3·2, 3·5 per cent. of the population at the beginning of the year. Find the population at the end of the period to the nearest hundred.

29. The expenditure on Health Insurance was £36·7 × 10^6 in one year, and the number of people who benefited by it was 18·5 × 10^6; find to three significant figures the expenditure per thousand benefited.

30. Taking the index number for 1914 as 100, the index numbers for February, 1937, were: Food 135, Rent 159, Fuel and Light 175. Find how much a householder would pay in 1937 for each £ spent on the three items in 1914.

31. The number of unemployed was 2,660,000 in March, 1932, and 1,625,000 in February, 1937; express the reduction in unemployment as a percentage of the number unemployed in 1932.

32. If £42·4 × 10^6 is spent on Old-age Pensions in one year and the number of pensioners is 1·8 × 10^6 people, find the average cost of the pensions per thousand pensioners to the nearest thousand pounds.

CHAPTER XXII

SIMPLE INTEREST AND BANKER'S DISCOUNT

If r is the rate per cent. per annum and t the number of years, the simple interest on £1 for t years is $£\frac{r}{100} \times t$, and the interest on a principal of £P would be

$$£P \times \frac{r}{100} \times t.$$

Hence $I = \frac{P \times r \times t}{100}$ where I stands for the total interest and is measured in the same units as P.

Computations of simple interest can be made by means of this formula.

Example. Find the simple interest on £1256 5*s*. for $3\frac{1}{2}$ years at $1\frac{5}{8}$ per cent. per annum.

$$I = \frac{P.r.t}{100}$$

$$= £\frac{1256\frac{1}{4} \times 1\frac{5}{8} \times 3\frac{1}{2}}{100}$$

$$= £\frac{5025 \times 13 \times 7}{4 \times 8 \times 2 \times 100}$$

$$= £\frac{201 \times 13 \times 7}{4 \times 8 \times 2 \times 4} \text{ (cancelling by 25)}$$

$$= £\frac{18291}{256}$$

= **£71 9s.** to the nearest penny.

INTEREST FOR DAYS

Find the simple interest on £3156 for 191 days at $3\frac{1}{2}$ per cent. per annum.

Counting 365 days to the year, 191 days $= \frac{191}{365}$ years.

$$I = \frac{P.r.t}{100} = £3156 \times 3\tfrac{1}{2} \times \frac{191}{365} \times \frac{1}{100}$$

$$= £\frac{3156 \times 7 \times 191}{73000}$$

$$= £\frac{22{\cdot}092 \times 191}{73}$$

```
      22092
     198828
       22092
73)4219·572(57·8023
   365
    569
    511
     585
     584
       172
       146
        260
```

$= £57{\cdot}8023$

$=$ **£57 16s. 1d.** to the nearest penny.

Note. In these calculations 365 should be changed into 730, the numerator being doubled to compensate, unless there is a factor 2 in the denominator, as in this example.

COMPUTATION ON THE BASIS OF 360 DAYS TO THE YEAR

Banks in America and some other countries reckon 360 days to the year.

This makes computation easier, as 360 has so many factors. Since $\frac{360}{365} = \frac{72}{73}$, the interest computed on our 365-day basis would be $\frac{72}{73}$ of that computed on 360 days to the year. Hence it would be necessary to subtract $\frac{1}{73}$ of the interest—computed in this way—to get the interest on our 365-day year, but the simpler divisor 72 would often give the answer as nearly as would be wanted.

The deduction might often be got mentally; it is 1*d.* on every 6*s.* 1*d.* or 6*s.* It would also be a simple matter to make a table of deductions for general use. More than 100 years ago Augustus De Morgan pointed out the application of parts, *i.e.*, practice, to this method.

Taking the previous example

		£	
		31·56	
		3·5	
Interest for 360 days	=	110·46	
,, ,, 180 ,,	=	55·23	$\frac{1}{2}$ of previous line
,, ,, 9 ,,	=	2·7615	$\frac{1}{20}$,, ,,
,, ,, 1 ,,	=	·30683	$\frac{1}{9}$,, ,,
,, ,, 1 ,,	=	·30683	
		58·6052	
$\frac{1}{72}$	=	·8139	$\frac{1}{73}$ = ·803
Subtract		57·7913	
	=	£57 15*s*. 10*d*.	

making a difference of 3d.

INTEREST ON DEPOSIT ACCOUNT

Example. A man placed on deposit with his bank £375 on March 12, £380 on March 18, and £550 on March 21; find the total interest due at the end of the month at 2 per cent. per annum.

March 12 to March 18 = 6 days
March 18 to March 21 = 3 days
March 21 to March 31 = 10 days
} Do not count both first and last days.

Hence £375 is drawing interest for 6 days.

£755 = £375 + £380 is drawing interest for 3 days.

£1305 = £755 + £550 is drawing interest for 10 days

£375 × 6	=	£2250	for one day
380			
755 × 3	=	£2265	
550			
1305 × 10	=	£13050	
Add		£17565	

Therefore interest on the account = interest on £17565 for one day at 2 per cent. per annum.

$$= £\frac{175\cdot65 \times 2}{365} = £\frac{351\cdot3}{365}$$

$$= £\frac{702\cdot6}{730}$$

$$= £\cdot962$$

$$= \mathbf{19s.\ 2d.}$$[1]

```
       ·962
73)7026
   657
    456
    438
     180
```

Note. Since £1 10*s.* = 360*d.*, the interest on £100 at $1\frac{1}{2}$ per cent. per annum is 1*d.* per day, on a 360-day-year basis.

This provides a simpler method for the calculation.

Interest = 175·65*d.* at $1\frac{1}{2}$ per cent.
58·55*d.* at $\frac{1}{2}$ per cent., dividing by 3

∴ Interest = 234·2*d.* at 2 per cent.
= 19*s.* 6*d.*

Subtract 1*d.* for every 6*s.* 1*d.* = 4*d.*

19*s.* 2*d.*

Exercises CII

Find, to the nearest penny, the simple interest on

1. £350 for 3 months at $4\frac{1}{3}$ per cent. per annum
2. £950 for 9 months at $5\frac{1}{3}$ per cent. per annum
3. £5000 for 9 months at $3\frac{3}{8}$ per cent. per annum
4. £300 for 73 days at $3\frac{1}{2}$ per cent. per annum.
5. £750 for 219 days at 4 per cent. per annum.
6. £375 for 57 days at $1\frac{3}{4}$ per cent. per annum.
7. £950 for 183 days at $1\frac{1}{2}$ per cent. per annum.
8. £240 for 216 days at 2 per cent. per annum.
9. £575 for 324 days at $2\frac{1}{2}$ per cent. per annum.

(Questions 1–3: Take 12 months to the year.)

Find the interests at the end of the corresponding half-years (June 30 or December 31) on the following deposits in a bank:

10. £535 deposited on March 1 at $1\frac{1}{2}$ per cent. per annum.
11. £380 deposited on April 17 at 2 per cent. per annum.

[1] Banks do not count fractions of a penny.

12. £3250 deposited on September 30 at 3 per cent. per annum.

13. A man puts on deposit with his bank £135 12*s.* 6*d.* on March 31, £335 16*s.* 0*d.* on April 30, £225 3*s.* 6*d.* on May 31. Find the total interest due to him at the end of the half-year, June 30, if the rate of interest is $2\frac{1}{2}$ per cent. per annum. (When computing the interests, reckon on the nearest £.)

14. The interest on £400 for 30 days at 5 per cent. per annum is £1 12*s.* 10*d.* Calculate the interest for 103, 176, and 322 days with the least possible work. (It can be done mentally.)

Banker's Discount

Trade is commonly conducted on **credit** terms by means of **Bills of Exchange.** Extract from the Bills of Exchange Act, 1882: "A **Bill of Exchange** is an unconditional order in writing addressed by one person to another requiring the person to whom it is addressed to pay on demand or at fixed or determinable future time a sum certain in money to or to the order of a specified person or to bearer."

Note. A cheque is a bill of exchange drawn on a banker, payable on demand.

Bills are usually drawn for 7, 18, 36, 60 days, or 3 or 6 months.

Bills for more than 6 months are much rarer.

This method of payment by *Bills* is convenient for both buyer and seller.

The buyer gets credit for a certain period and the seller can get his price at any time, should he wish to do so, by selling the Bill to a banker or to one of the numerous bill-brokers established for this purpose.

Naturally, a commission must be paid for the services of the broker, because he has paid in advance and will have to wait some time for the return of this money advanced.

This commission is called **Banker's discount** and is deducted from the amount of the Bill. It is computed in the same way as Interest, the rate of discount being estimated as a percentage per annum on the Bill.

Days of Grace. Payment cannot be enforced legally until *three* days after the date on which the Bill becomes due. These are called 'Days of Grace' and must be added to the number of days for which the Bill has been drawn.

EXERCISES CIII

Find, to the nearest penny, the banker's discount on the following bills (counting 3 days of grace):

1. £125 drawn at 30 days at $1\frac{1}{2}$ per cent. per annum.
2. £375 drawn at 60 days at $2\frac{1}{4}$ per cent. per annum.
3. £500 drawn at 120 days at $1\frac{7}{8}$ per cent. per annum.
4. £850 drawn at 93 days at $3\frac{1}{8}$ per cent. per annum.
5. £9500 drawn at 18 days at $5\frac{1}{16}$ per cent. per annum.
6. £725 drawn at 72 days at $3\frac{3}{4}$ per cent. per annum.
7. £300 drawn at 60 days on July 3 and discounted on July 26 at $2\frac{3}{8}$ per cent. per annum.
8. £525 drawn on August 10 at 30 days and discounted on August 17 at $3\frac{1}{2}$ per cent. per annum.

ANOTHER METHOD OF CALCULATION OF INTEREST

Computed on a 360-day year, the interest on £100 at $1\frac{1}{2}$ per cent. per annum is 1*d*. per day. Hence the interest on £100 for any number of days is as many pence as there are days.

Example. The interest on £100 at $1\frac{1}{2}$ per cent. per annum for 193 days = 193 pence = 16*s*. 1*d*.

∴ Interest on £325 for this time

= 16*s*. 1*d*. × 3·25
= 48*s*. 3*d*. + 4*s*. $0\frac{1}{4}$*d*.
= 52*s*. $3\frac{1}{4}$*d*. (neglect the farthing).

Correction for 365-day year—deduct 1*d*. for every 6*s*. 1*d*.
= 9*d*.

∴ Interest = **51s. 6d.**

Interest at other rates on £100 for a number of days, on 360-day year: at 3 per cent. it is twice as many pence as there are days; at $4\frac{1}{2}$ per cent., three times as many pence as there are days; at 6 per cent., four times as many pence as there are days; and so on.

For other percentages, use such facts as

2 per cent. = $1\frac{1}{2}$ per cent. + $\frac{1}{3}$ of $1\frac{1}{2}$ per cent.
$2\frac{1}{2}$ per cent. = 3 per cent. − $\frac{1}{6}$ of 3 per cent.
5 per cent. = 6 per cent. − $\frac{1}{6}$ of 6 per cent.

and so on.

Example. Find the interest on £736 for 294 days at 6 per cent. per annum.

The interest on £100 at 6 per cent. is 4*d*. per day or 1*s*. every 3 days (on a 360-day year).

∴ Interest on £100 at 6 per cent. for 294 days

$$= \tfrac{294}{3}s.$$
$$= 98s.$$

∴ Interest on £736 at 6 per cent. for 294 days

736	$= 98 \times 7{\cdot}36s.$
14·72	$= 721{\cdot}28s.$
721·28	$= £36\ 1s.\ 3d.$

Subtract 1*d*. for every 6*s*. 1*d*. = 9*s*. 11*d*. (or $\frac{1}{73}$)

Interest is **£35 11s. 4d.**

Try some of Exercises CII by this method.

Note. All the computations that are required in this method could be got from a good Ready Reckoner, and expensive tables would not be necessary.

CHAPTER XXIII

INVERSE QUESTIONS ON SIMPLE INTEREST

The four quantities that enter into questions on Interest are **Principal** (P), **Rate per cent.** (r), **Time** (t), and **Interest** (I) or **Amount** ($A = P + I$). Simple interest is directly proportional to the time and also to the rate per cent., but the amount is not. *E.g.*, if we double the time or the rate we double the interest, but we do not double the amount.

To find the Principal

Example 1. What sum of money will amount to £350 in 3 years at 5 per cent. per annum, simple interest?

Interest on £100 for 3 years at 5 per cent. per annum = £15.

Amount of £100 for 3 years at 5 per cent. per annum = £115.

Ratio of Amounts = Ratio of Principals

$$\therefore \frac{\pounds P}{\pounds 100} = \frac{\pounds 350}{\pounds 115}$$

$$\therefore \pounds P = \pounds 100 \times \frac{350}{115}$$

$$= \pounds\frac{7000}{23}$$

$$= \pounds 304{\cdot}348$$

$\therefore$ Principal = **£304 7s.**

Example 2. If the interest was £350.

Ratio of Principals = Ratio of Interests

$$\therefore \frac{P}{100} = \frac{350}{15} = \frac{70}{3}$$

$$P = \frac{7000}{3}$$

$\therefore$ Principal = **£2333 6s. 8d.**

Note. Make sure whether it is interest or amount that is given before you start the computation.

Definition. The **Present Worth** (or Present Value) of a given sum of money due at the end of a given time is that principal which, if put out to interest at the present time, would amount to the given sum at the end of the given time.

Find the present worth of £1000 due in 3 years at 5 per cent. per annum simple interest.

Take a present worth of £100.

Amount of £100 after 3 years = £115

$$\therefore \frac{P.W.}{£100} = \frac{£1000}{£115} = \frac{200}{23}$$

$$\therefore P.W. = \frac{£20000}{23}$$

∴ Present worth = **£869 11s. 4d.** to the nearest penny.

Note. Present worth is of much more importance in compound interest. In banking circles, the present worth of £1 or £100 is taken from tables.

To find the Time

Example. In how many years will £300 amount to £420 at 5 per cent. per annum, simple interest?

Amount = £420
Principal = £300
Total Interest = £120

Interest on £300 for year = £15

Ratio of Interests = Ratio of Times

∴ if n be the number of years

$$\frac{n}{1} = \frac{120}{15} = 8$$

∴ Time = **8 years.**

To find the Rate

Example. The simple interest on £4500 for $1\frac{1}{4}$ years is £350. What is the rate per cent.?

Ratio of interests = Ratio of rates per cent.

Interest at 1 per cent. = £45 × 1·25

∴ if r be the rate per cent.,

$$\frac{r}{1} = \frac{350}{45 \times 1{\cdot}25}$$

$$= \frac{70}{9 \times 1{\cdot}25} \quad \text{or by vulgar fractions}$$

$$= \frac{14}{9 \times {\cdot}25}$$

$$= \frac{14}{2{\cdot}25}$$

∴ Rate per cent. = **6·22.**

Note. In both time and rate questions, if the amount is given the first step is to determine the total interest.

Exercises (Mental) CIV

Principal = P. Interest = I. Amount = A. No. of years = n. Rate per cent. = r.

Find the principal at simple interest when:

1. I = £50, $r = 5$, $n = 2$.
2. I = £60, $r = 4$, $n = 3$.
3. I = £150, $r = 6$, $n = 2\frac{1}{2}$.
4. A = £360, $r = 5$, $n = 4$.
5. A = £550, $r = 4$, $n = 2\frac{1}{2}$.
6. A = £625, $r = 3\frac{1}{8}$, $n = 8$.

Find the time when

7. P = £300, $r = 5$, I = £60.
8. P = £375, $r = 6$, I = £90.
9. P = £5000, $r = 4\frac{1}{2}$, I = £675.
10. P = £300, $r = 3\frac{1}{3}$, A = £360.

Find the rate when

11. P = £500, $n = 2\frac{1}{2}$, I = £37 10*s*.
12. P = £1000, $n = 3\frac{1}{4}$, I = £130.
13. P = £300, $n = \frac{1}{2}$, I = £2.
14. P = £100, $n = 3$, A = £130.
15. P = £250, $n = 4$, A = £300.

EXERCISES CV

1. What sum of money will amount to £810 in $2\frac{1}{2}$ years at 5 per cent. per annum?

2. On what principal will the simple interest amount to £51 in 3 years at 4 per cent. per annum?

3. Find the present worth of £35 14*s.* due at the end of 6 months at 4 per cent. per annum.

4. How much money must be lent for 1 year 3 months at 5 per cent. per annum to produce £37 10*s.* interest?

5. What sum of money will amount to £974 14*s.* in 2 years at 4 per cent. per annum?

6. Find the sum of money on which the interest for 146 days at $5\frac{1}{2}$ per cent. per annum will be £1 2*s.*

Find to the nearest shilling the present values of

7. £100 due in 2 years at 4 per cent. per annum.

8. £550 due in $3\frac{1}{2}$ years at $2\frac{1}{2}$ per cent. per annum.

9. £878 due in 3 years at 5 per cent. per annum.

10. £10000 due in 5 years at 4 per cent. per annum.

11. In what time will £480 produce an interest of £42 at $3\frac{1}{3}$ per cent. per annum?

12. In what time will £150 amount to £171 at 4 per cent. per annum?

13. At what rate per cent. will £360 amount to £414 in 3 years?

14. At what rate per cent. will a sum of money double itself in 18 years?

15. At what rate per cent. will the interest on £325 be £53 12*s.* 6*d.* at the end of 3 years?

16. In what time will the interest on £1500 amount to £312 10*s.* at $3\frac{1}{3}$ per cent. per annum?

17. In what time will a sum of money double itself at $7\frac{1}{2}$ per cent. per annum?

18. At what rate per cent. will £1275 amount to £1301 15*s.* 6*d.* in 219 days?

19. A sum of money trebles itself in 40 years. What is the rate per cent.?

20. At what rate per cent. will 1000 fr. amount to 1125 fr. in 4 years?

21. In what time will 265 dollars amount to 302 dollars 10 cents at $3\frac{1}{2}$ per cent. per annum?

22. With simple interest 2360 fr. amounts to 2628 fr. 45 c. in $3\frac{1}{2}$ years. What is the rate per cent.?

CHAPTER XXIV

COMPOUND PERCENTAGES

Compound Interest, Depreciation, Compound Discounts

In compound interest, the interest at the end of each year is added to the principal at the beginning of that year, and the interest for each succeeding year is computed on the new principal for that year.

Example 1. Find the compound interest on £250 placed in the Post Office Savings Bank for 3 years at $2\frac{1}{2}$ per cent. per annum.

	£	
	250·	1st principal.
$2\frac{1}{2}$ per cent. $= \frac{1}{40}$	6·25	1st year's interest.
	256·25	2nd principal or amount at end of 1st year.
$\frac{1}{40} =$	6·40625	2nd year's interest.
	262·65625	3rd principal or amount at end of 2nd year.
$\frac{1}{40} =$	6·56640 | 625	3rd year's interest.
	269·2227	Amount at end of 3rd year.
Original principal =	250	
Compound int. =	19·2227	
	= **£19 4s. 5d.**	

It is unnecessary to keep in any figures to the right of the marked vertical line because the answer is only required to a penny.

Note. Keep the decimal points directly below one another.

Example 2. Find the compound interest on £387 10*s*. for 4 years at $3\frac{1}{2}$ per cent. per annum.

	£
	387·5
Int. 1st yr. at 3 %	11·625
½ %	1·9375
	401·0625
Int. 2nd yr. at 3 %	12·03187
½ %	2·00531
	415·09968
Int. 3rd yr. at 3 %	12·45299
½ %	2·07549
	429·62816
Int. 4th yr. at 3 %	12·88884
½ %	2·14814
	444·66514
Original principal	387·5
Compound interest	57·16514

To calculate 3 per cent., multiply by 3, writing the result two places to the right. Omit figures to right of the line, but correct the fifth decimal figure to allow for them. To calculate ½ per cent., divide principal by 2, writing the result two places to the right, or divide previous interest by 6.

= £57 3s. 4d.

Exercises CVI

Find, to the nearest penny, the compound interest on

1. £750 for 2 yr. at 5 per cent. per annum.
2. £680 for 2 yr. at 2½ per cent. per annum.
3. £3780 for 3 yr. at 4 per cent. per annum.
4. £215 for 3 yr. at 5½ per cent. per annum.
5. £590 for 3 yr. at 3¾ per cent. per annum.
6. £875 10*s*. for 3 yr. at 4¼ per cent. per annum.
7. £375 for 4 yr. at 3⅓ per cent. per annum.
8. £67 12*s*. 6*d*. for 4 yr. at 6½ per cent. per annum.
9. £333 6*s*. 8*d*. for 3 yr. at 8⅔ per cent. per annum.
10. £625 10*s*. for 4 yr. at 12½ per cent. per annum.

Find the compound interest on

11. 1425 fr. for 3 yr. at 4¾ per cent. per annum.
12. $5875 for 2 yr. 6 mo. at 4½ per cent. per annum.
13. 350 lire for 3 yr. 4 mo. at 5 per cent. per annum.
14. £333 13*s*. 4*d*. for 2 yr. 9 mo. at 4¾ per cent. per annum.

If the interest be added half-yearly, find, to the nearest penny, the amount at compound interest of

15. £1000 in 2 yr. at 4½ per cent. per annum.
16. £2500 in 2½ yr. at 7½ per cent. per annum.

17. £3760 in $1\frac{1}{2}$ yr. at $3\frac{3}{4}$ per cent. per annum.

18. In consequence of migration into towns the population of a rural district is diminished each year by 10 per cent. of its number at the beginning of that year. To what number, to the nearest hundred, is a population of 15136 reduced at the end of 5 years?

19. £387 10*s.* amounts to £500 9*s.* 6*d.* in 5 years at $5\frac{1}{4}$ per cent. compound interest. What will be the amount to the nearest shilling at the end of an additional 5 years?

20–23. Prepare tables which will give the compound interest on £1 to five decimal places at $3\frac{1}{3}$, $4\frac{1}{2}$, 6, and $2\frac{1}{4}$ per cent. respectively for 4 years, and use them to find the amount at compound interest of £375 at each of these rates (to the nearest penny).

Depreciations are examples of percentages *subtracted.* The value of machinery, etc., in use becomes less each year, and allowance must be made for this.

Example. The original value of a chemical plant was £20000. The Income Tax Commissioners allowed depreciation at the rate of $7\frac{1}{2}$ per cent. per annum. Find the estimated value of the plant at the end of 3 years.

	£	
	20000·	
5 per cent.	1000·	First year's depreciation.
$2\frac{1}{2}$ per cent. (half of 5 per cent.)	500·	
Subtract	18500·	Value of plant at end of first year.
5 per cent.	925·	Second year's depreciation.
$2\frac{1}{2}$ per cent.	462·5	
Subtract	17112·5	Value at end of second year.
5 per cent.	855·625	Third year's depreciation.
$2\frac{1}{2}$ per cent.	427·8125	
Subtract	15829·0625	Value at end of third year.

Answer: **£15829 1s. 3d.**

The subtractions should be done by complementary addition and the results checked by adding.

Compound Discounts

In wholesale catalogue lists, prices are quoted subject to various discounts. The first is usually a large one called **Trade Discount,** and the others, which may be one or two, are smaller and given for various commercial reasons.

Example. Catalogued price is £15 12*s*. 6*d*., subject to successive discounts of $33\frac{1}{3}$, 10, and $2\frac{1}{2}$ per cent. Find the price paid by the retailer.

	£
	15·625
$33\frac{1}{3}$ per cent. = $\frac{1}{3}$rd	5·20833
Subtract	10·41667
10 per cent. = ·1	1·04167
Subtract	9·37500
$2\frac{1}{2}$ per cent. = $\frac{1}{40}$th	·234375
Subtract	9·140625

Retailer pays **£9 2s. 10d.**

Exercises CVII

1. A motor-lorry depreciates in value at the end of each year by 20 per cent. of its value at the beginning of the year. Find, to the nearest shilling, its value at the end of 4 years if the original cost was £1000.

Find the value of the following machinery to the nearest ten shillings at the end of the given times. The allowances for depreciation are the percentages accepted by the Income Tax Commissioners:

	Cost £	Depreciation per annum	Value required at end of
2. Railway wagons	20000	5 per cent.	3 yr.
3. Hosiery machinery	20000	$7\frac{1}{2}$ per cent.	4 yr.
4. Flour-milling machinery	5000	7 per cent.	3 yr. 6 mo.

Calculate the single equivalent discounts on a catalogued price of £100 subject to successive discounts of

5. $33\frac{1}{3}$ per cent., 10 per cent.
6. 30 per cent., 5 per cent.
7. 25 per cent., 4 per cent.
8. 20 per cent., 5 per cent., $1\frac{1}{2}$ per cent.
9. Find what the retailer pays the manufacturer for goods catalogued at £275, and subject to successive discounts of $33\frac{1}{3}$ per cent. and $16\frac{2}{3}$ per cent.
10. The catalogued price of an article is £350, and successive discounts are 25 per cent. and $8\frac{1}{3}$ per cent. Find what the retailer pays to the wholesaler.

COMPOUND INTEREST. LOGARITHMIC METHOD

Let P be the principal, r the rate per cent. per annum, n the number of years, and A the amount.

Every unit in P amounts to $\left(1 + \frac{r}{100}\right)$ units at the end of a year.

Call the amount of £1 after 1 year, £R; then

$$R = 1 + \frac{r}{100}.$$

At the end of 1 year £1 amounts to £R.

,, ,, ,, £R amounts to £$R \times R$ = £R^2,

i.e., at the end of 2 years £1 amounts to £R^2.

At the end of 1 year £R^2 amounts to £$R^2 \times R$ = £R^3,

i.e., at the end of 3 years £1 amounts to £R^3, and so on.

Hence, at the end of n years £1 amounts to £R^n, and £P amounts to £$P.R^n$.

$$\therefore\ A = P.R^n = P\left(1 + \frac{r}{100}\right)^n$$

$$\text{and } \log A = \log P + n \log R$$

$$= \log P + n \log\left(1 + \frac{r}{100}\right).$$

From these equations we can find the fourth quantity, if we are given three of them.

Example 1. Find the amount of £100 in 3 years at 5 per cent. Compound Interest.

$$A = P.R^n$$

$$= 100\ (1{\cdot}05)^3$$

$$\log A = \log 100 + 3 \log 1{\cdot}05.$$

Using four-figure tables

$$\log A = 2 + {\cdot}0212 \times 3$$

$$= 2{\cdot}0636.$$

$$A = £115{\cdot}8$$

$$\therefore\ \text{Amount} = £115\ 16s.$$

By direct calculation without logs, A = £115 15s. 3d.

The four-figure tables cannot give the answer to the nearest penny, and for larger principals the error would be larger. Seven-figure tables would be necessary for practical calculation of interests by this method.

If the time is not a whole number of years the formula must be modified to be in agreement with the practical method of calculation. Thus to find the amount of £100 in 3 years 6 months at 5 per cent. per annum we should have to calculate £100. $(1{\cdot}05)^3 . (1{\cdot}025)$, allowing $2\frac{1}{2}$ per cent. for the final six months. The formula £100 $\times$ $(1{\cdot}05)^{3{\cdot}5}$ would give less than $2\frac{1}{2}$ per cent., about 2·47 per cent., for the final half-year.

Example 2. In how many years will a sum of money double itself at 5 per cent. compound interest?

$$\begin{aligned} A &= 2P \\ \therefore\ 2P &= P(1{\cdot}05)^n \\ 2 &= 1{\cdot}05^n \\ \log 2 &= n \log 1{\cdot}05. \\ \therefore\ n &= \frac{0{\cdot}3010}{0{\cdot}0212} \\ &= \mathbf{14{\cdot}2} \end{aligned}$$

i.e., in 14 years the sum will be not quite doubled; more than doubled in 15 years.

Example 3. At what rate per cent. per annum will £250 amount to £325 in 6 years at compound interest?

$$325 = 250\left\{1 + \frac{r}{100}\right\}^6$$

$$\therefore \log 325 = \log 250 + 6 \log\left(1 + \frac{r}{100}\right)$$

$$\therefore 6 \log\left(1 + \frac{r}{100}\right) = \log 325 - \log 250$$

$$\begin{aligned} \log 325 &= 2{\cdot}5119 \\ \log 250 &= 2{\cdot}3979 \\ 6)\overline{0{\cdot}1140} \\ &\ \ 0{\cdot}0190 \end{aligned}$$

$$\therefore \log\left(1 + \frac{r}{100}\right) = 0{\cdot}0190$$

$$\therefore 1 + \frac{r}{100} = 1{\cdot}045$$

$$\therefore \frac{r}{100} = {\cdot}045$$

$$r = 4{\cdot}5.$$

Rate per cent. = **4·5**

Exercises CVIII

(To be worked as accurately as four-figure tables permit.)

Find the amounts at compound interest of

1. £250 after 5 years at $3\frac{1}{2}$ per cent. per annum.
2. £180 after 7 years at $2\frac{1}{2}$ per cent. per annum.
3. £425 after 10 years at $3\frac{1}{4}$ per cent. per annum.

Find what principals will amount at compound interest to £1000

4. In 5 years at 5 per cent. per annum.
5. In 15 years at $2\frac{1}{2}$ per cent. per annum.
6. In 10 years at $3\frac{1}{2}$ per cent. per annum.

7. In how many years will £100 amount to £150 at $4\frac{1}{2}$ per cent. compound interest?

8. In how many years will a sum of money double itself at $3\frac{1}{2}$ per cent. per annum compound interest?

9. At what rate per cent. must a sum of money be invested at compound interest to double itself in 10 years?

10. At what rate per cent. compound interest was a sum of money invested if it had increased by 50 per cent. in 15 years?

11. The original War Savings Certificates rose in value from 15*s*. 6*d*. to £1 in 5 years. What is the equivalent rate of compound interest per annum?

12. Later certificates increased in value from 16*s*. to 24*s*. in 10 years. To what annual rate of compound interest is this equivalent?

13. The population of a town decreased by the same percentage for 3 successive years from 15650 to 13880. What was the annual percentage decrease?

14. The value of a motor-car originally costing £180 was £80 after 3 years. Assuming its value to depreciate by equal annual percentages, calculate the annual percentage depreciation.

CHAPTER XXV

STOCKS AND SHARES

Shares

When money is wanted for some enterprise, the public may be invited to lend it and thus become shareholders. Companies offer their shares in definite amounts ranging from £100 down to 2*s*. or 1*s*. shares. If the company prospers, the net profits, after part has been set aside for reserve funds, etc., are divided among the shareholders. These distributed profits are called **dividends** and may be considered as interest on the money lent. Dividends are usually estimated as percentages of the **nominal** value of the shares, occasionally as so much (in shillings and pence) per share. It is important to distinguish between the nominal value of a share and its cash value. The nominal value is fixed when the shares are first issued and remains unchanged; the cash value varies from day to day.

If a shareholder wants his money back, he can get it only by selling his shares to some other person. The cash he will receive for the shares will not usually be the same as their nominal value. If the dividend paid on the shares is higher than the general level of dividends and seems likely to continue so, competition of buyers to acquire such shares will have caused their price to rise above the nominal value. On the other hand if the shares are paying small dividends, their price will be lower, probably, than their nominal value.

Stock

The name **stock** is applied to shares in Government (National or Municipal) loans, railways, or other big national undertakings. The nominal value of a stock is always taken as £100, and is not mentioned when the price or dividend of the stock is quoted. Thus the phrase "$2\frac{1}{2}$ per cent. Consols at $86\frac{1}{8}$" means Government Consolidated stock, £100 stock (nominal value) costing £$86\frac{1}{8}$ (cash value), and paying $2\frac{1}{2}$ per cent. (of the nominal value) dividend per annum.

It is possible to buy odd amounts of stock, *e.g.*, £325 12*s.* 6*d.* (nominal value) of stock, but it is not possible to buy fractions of shares.

When the cash price exceeds the nominal value, stocks or shares are said to be at a **premium.** When the cash price is lower than the nominal value, they are at a **discount**, and when cash price and nominal value are equal, at **par.**

Shares and stocks are of various kinds—Debentures, Preference, Ordinary, etc. Debentures and Preference stocks or shares carry fixed dividends and have the first call on any profits. Dividends on ordinary shares are paid from the profits remaining after the dividends on debentures and preference shares have been paid.

Brokerage

The actual buying or selling of stocks and shares is done through a bank or a stockbroker. These make a charge for their services, and there are other small charges, *e.g.*, stamp duties, to be paid as well. The minimum charge for buying or selling most British, Indian, and Colonial Government securities, county and Corporation stock is $\frac{1}{4}$ per cent. of the stock bought. For Consols the charge is $\frac{3}{16}$ per cent. On railway stock and foreign Government bonds the brokerage varies from $\frac{1}{4}$ to $\frac{1}{2}$ per cent. of the amount of stock, but in many cases the charge is made on the cash value of the stock at 10*s.* per £100 of price. The brokerage on shares varies from $\frac{1}{2}d.$ to 3*d.* per share for shares of value between 1*s.* and 30*s.*, with proportional charges on shares of higher value.

To find the Cost of a given Amount of Stock

Example. How much will be paid for £5350 of Consols at $86\frac{1}{8}$? £$86\frac{1}{8}$ will buy £100 nominal value of stock.
Ratio of prices paid = Ratio of amounts of stock bought.

$$\therefore \frac{\text{Price paid}}{£86\frac{1}{8}} = \frac{5350}{100}$$

$$\therefore \text{Price paid} = £86\tfrac{1}{8} \times 53{\cdot}5.$$

$$\begin{array}{r} 53{\cdot}5 \\ 86\tfrac{1}{8} \\ \hline 4280{\cdot} \\ 321{\cdot} \\ 6{\cdot}6875 \\ \hline 4607{\cdot}6875 \end{array}$$

$\therefore$ Price = **£4607 13s. 9d.**

The broker will charge $\frac{3}{16}$ per cent. as commission on every £100 of stock. This is usually made out separately.

It could also be added to the price, making it $86\frac{5}{16}$, or deducted from it if the holder is selling. The money he received would then be $85\frac{15}{16}$ per £100 stock.

To find how much Stock a given Sum of Money will buy

Example. How much railway stock at 66 can be bought for £1254?

£66 cash will buy £100 nominal value of stock.

Ratio of amounts of stock bought = Ratio of moneys paid.

$\therefore$ Amount bought = £100 × $\frac{1254}{66}$
= £100 × 19
= **£1900** stock.

Broker's commission on the sale or purchase of railway stocks is 10*s*. per cent. on the *money paid*.

10*s*. per cent. on £1254 = £6·27
= **£6 5s. 5d.**

To find the Income derived from a given Holding of Stock

Example. What income would be got from £5350 of 5 per cent. stock?

Income on £100 stock = £5.

$\therefore$ Income on £5350 = £5 × $\frac{5350}{100}$
= £53·5 × 5
= **£267 10s.**

To find the Income from an Investment in Stocks

Example. What will be the income from £2000 invested in $2\frac{1}{2}$ per cent. Consols at 80?

£80 cash invested gives an income of £$2\frac{1}{2}$.

Incomes vary directly as the sums of money invested.

$$\therefore \frac{\text{Required Income}}{£2\frac{1}{2}} = \frac{2000}{80}$$

$$\therefore \text{Income} = £2\frac{1}{2} \times 25$$
$$= \mathbf{£62\ 10s.}$$

To find the Return (or Yield) per cent. on an Investment

Example. Find the percentage yield on an investment in $4\frac{1}{2}$ per cent. stock at 86.

£86 invested brings an income of £$4\frac{1}{2}$.

Interests are proportional to the sums invested.

$$\therefore \frac{\text{Interest on £100 invested}}{\text{Interest on £86 invested}} = \frac{100}{86}$$

$$\therefore \text{Interest on £100 invested} = £4{\cdot}5 \times \tfrac{100}{86}$$
$$= £\tfrac{450}{86}$$
$$= £5{\cdot}232.$$

$\therefore$ Yield per cent. = **£5 4s. 8d.** approximately.

Note. Beginners seem to have more difficulty with this type than with the preceding ones.

To compare the Total Yields of Two Stocks

Example. Which gives the better return: a 5 per cent. stock at $101\frac{1}{4}$ or a 4 per cent. stock at $87\frac{1}{2}$?

Method 1. Invest in each company the product of the prices of the two stocks.

Income on £$101\frac{1}{4} \times 87\frac{1}{2}$ invested in 5 per cent. is $87\frac{1}{2} \times 5$.

Income on £$101\frac{1}{4} \times 87\frac{1}{2}$ invested in 4 per cent. is $101\frac{1}{4} \times 4$.

First income = £$437\frac{1}{2}$.
Second income = £405.

$\therefore$ the **5 per cent. stock** is the better investment.

Method 2. Invest in each company the price of one of the two stocks.

Method 3. Invest £100 cash in each company.

Try all three methods, and say which you think is the most convenient.

Yield with Redemption

Government stock is issued redeemable at the end of a certain period, *e.g.*, a 4 per cent. loan issued in 1930 at 75 and redeemable at par 20 years later.

This means that anyone who invests £75 in this loan can get the nominal value £100 returned to him in 1950. Hence he will benefit by a sum of £100 — £75, in addition to the interest he receives throughout the 20 years.

We can work out the total profit on £100 invested in such a way, and, dividing by the number of years, obtain an average yield per cent. per annum, but for more accurate purposes, the different times of the payments of interest and the final repayment must be allowed for, and the present value of the different payments might form a basis for comparison.

Exercises (Mental) CIX

Find the cost of

1. £5000 stock at 90.
2. £4500 stock at 80.
3. 1000 shares at 70*s*. per share.
4. 200 shares at 6*s*. 8*d*. per share.
5. £2500 stock at 75.

Find the income from

6. £2000 3½ per cent. stock.
7. £7500 2 per cent. stock.
8. £2150 6 per cent. stock.
9. 1200 shares, dividend 6*s*. per share.
10. 500 shares, dividend 2*s*. 3*d*. per share.

How much stock can be bought with

11. £2000; stock at 80.
12. £5000; stock at 125.
13. £7000; stock at 70.
14. £100; stock at 60.
15. £525; stock at 105?

How much stock bearing the given dividends is required to produce an income of £300?

16. 5 per cent.
17. 3 per cent.
18. $2\frac{1}{2}$ per cent.
19. 15 per cent.
20. $7\frac{1}{2}$ per cent.

What is the income from an investment of

21. £180 in $2\frac{1}{2}$ per cent. stock at 90.
22. £200 in 3 per cent. stock at 80.
23. £500 in 4 per cent. stock at 125.
24. £351 in $3\frac{1}{2}$ per cent. stock at 117.
25. £1700 in 3 per cent. stock at 85?

Exercises CX

(Neglect brokerage, etc.)

Find the cost of the following stocks in December, 1934:

1. £525 of 4 per cent. G.W.R. Debentures at 114.
2. £3750 of L.M.S. 4 per cent. Debentures at 108.
3. £32360 of L.N.E. 3 per cent. Debentures at 84.
4. £575 of $2\frac{1}{2}$ per cent. Consols at $92\frac{1}{2}$.
5. £850 of S.Rly. 5 per cent. Preference at $117\frac{1}{2}$.
6. £3550 of Calcutta 6 per cent. at $104\frac{3}{4}$.

How much of the following stocks would a buyer have got if he had invested at 1921 price?

7. £1070 in London County Stock at $53\frac{1}{2}$.
8. £2632 10*s*. in Caledonian 4 per cent. at 65.
9. £7060 in 5 per cent. War Loan at $88\frac{1}{4}$.
10. £5000 in G.W.Rly. 5 per cent. at 80.
11. \$7100 in 4 per cent. Grand Trunks at $88\frac{3}{4}$.
12. 3000 fr. in 5 per cent. French Rentes at $46\frac{7}{8}$.

Find the incomes derived from the following stocks:

13. £1250 of $3\frac{1}{2}$ per cent. Conversion Loan.
14. £325 of 4 per cent. Victory Bonds.
15. £275 of G.W.Rly. 4 per cent.
16. £225 of Belfast $5\frac{1}{2}$ per cent.
17. £475 of Japanese $4\frac{1}{2}$ per cent.
18. £8750 of Rhodesian 4 per cent. Debentures.

Find the incomes derived from investing the following sums of money:

19. £2000 in $2\frac{1}{2}$ per cent. Consols at 56 (1921).

20. £3000 in $2\frac{1}{2}$ per cent. Consols at $87\frac{1}{2}$ (1935).

21. £256 10*s.* in 5 per cent. War Loan at the original issue price of 95.

22. £2299 10*s.* in 7 per cent. Belgian stock at $109\frac{1}{2}$ (1934).

23. £728 in Czechoslovakia 8 per cents. at 105 (1934).

24. £237 10*s.* in Bulgarian $7\frac{1}{2}$ per cents. at 19 (1934).

What return per cent. (to 3 decimal places and also in £ *s. d.*) would there be on investments in the following stocks?

25. $3\frac{1}{2}$ per cent. Conversion Loan at 106.

26. 7 per cent. Belgian at $109\frac{1}{2}$.

27. 5 per cent. Chinese at 86.

28. $5\frac{1}{2}$ per cent. Japanese at 87.

29. 4 per cent. G.W.R. Debentures at 110.

30. $2\frac{1}{2}$ per cent. Consols at $86\frac{3}{32}$.

Which investment would give the better return?

31. Sudan $4\frac{1}{2}$ per cent. stock at 93 or Victoria 5 per cent. stock at $99\frac{1}{2}$.

32. South Australia 5 per cent. stock at 99 or Jamaica $4\frac{1}{2}$ per cent. stock at 89.

Find the brokerage on the sale of

33. £5350 Nigeria stock at $\frac{1}{4}$ per cent. of the amount of stock.

34. £560 Consols at $\frac{3}{16}$ per cent. of the amount of stock.

35. 200 motor shares at 2*d.* per share.

36. £4500 railway stock at 72, brokerage at 10*s.* per cent. of the cash price.

Exercises CXI

Prices of some industrial, catering, and other companies shares in 1935, £1 being the nominal value.

Selfridge's 6 per cent. preference, 29*s.*
J. Lyons 8 per cent. preference, 37*s.* 9*d.*
General Electric $6\frac{1}{2}$ per cent. preference, 32*s.* $1\frac{1}{2}$*d.*
Gaumont British Pictures $5\frac{1}{2}$ per cent. preference, 17*s.* $11\frac{1}{2}$*d.*

1. Find the cost of 100 shares in each company.
2. How many shares in each company can you buy for £100?
3. Which of the shares gives the best return on an investment?

4. The 10 per cent. £1 preference shares of a certain company are quoted at 13*s.* 3*d.* Find the yield per cent., and suggest reasons for or against buying these shares.

5. How much $4\frac{1}{2}$ per cent. stock must I sell at $78\frac{3}{4}$ in order to buy £2000 of 5 per cent. War Loan at $94\frac{1}{2}$ with the proceeds? What will be the change in my income?

6. How much 3 per cent. stock does a man hold if, by selling out at 80 and investing the proceeds in a 5 per cent. stock at 85, he increases his income by £203 per annum?

7. A man had £1250 of a certain 4 per cent. stock which he sold at 90, and with the proceeds he purchased $3\frac{1}{2}$ per cent. stock at 75. Find the change in his yearly dividend.

8. A man has £4500 of 3 per cent. stock. He sells it at 74 and invests the proceeds in a 4 per cent. stock, and thereby increases his income by £50. Find the price of the latter stock.

9. What annual income is gained by investing £1900 in $3\frac{1}{2}$ per cent. stock at 76? Find, to the nearest penny, the change in income if the stock be sold at 70 and the proceeds are invested in 5 per cent. stock at 95.

10. If a 3 per cent. stock be at 75, what sum must I invest to secure a yearly income of £500 after paying an income tax of 5*s.* in the £?

11. A man invests £600 in 3 per cents. at 75; how much must he invest in 6 per cents. at 108 so that the rate of interest on his whole outlay may be 5 per cent.?

12. A man's net income from a 5 per cent. stock after payment of income tax of 3*s.* 4*d.* in the pound was £600. He sold his stock at 90 and invested in a 6 per cent. stock at par. Find the change in his income after allowing for income tax.

13. What is the lowest price at which a man must sell a 4 per cent. stock, so as to be able to invest the proceeds in a $4\frac{1}{2}$ per cent. stock at $97\frac{7}{8}$ without loss of income? If £1000 of the 4 per cent. stock is sold, the proceeds are invested in the $4\frac{1}{2}$ per cents. at $97\frac{7}{8}$, and a gain of £5 in income is thus secured, at what price must the original stock have been sold?

14. Find, to the nearest penny, what sum of money must be invested in $2\frac{1}{2}$ per cent. Consols at 45 in order that after 6*s.* in the £ income tax has been deducted, the net income may be £100 per annum.

15. Two persons invest equal sums of money, one in a $3\frac{1}{2}$ and the other in a 4 per cent. stock, and receive the same amount in dividends. If the $3\frac{1}{2}$ per cents. were at $75\frac{1}{4}$, find the price of the 4 per cents.

16. Find the average yield per annum, allowing for redemption, of Belfast $3\frac{1}{2}$ per cent. loan bought in 1921 at 66, and redeemed in 1935 at par. Give the answer to the nearest penny.

17. Find the average yield per cent. per annum, allowing for redemption, of Cardiff 3 per cent. stock bought in 1922 at 58 and redeemed in 1954 at par. Give the answer to the nearest penny.

18. A man holds £4600 of $3\frac{1}{2}$ per cent. stock, and also a certain amount of $4\frac{1}{2}$ per cent. stock which he bought for £5148. If his income from both sources amounts to £377, find at what price the latter stock was bought.

19. A man invests £2425 in 5 per cent. Exchequer bonds at 97, the principal being repayable at par at the end of 5 years. Find (*a*) the total profit received by him during the five years, including the final repayment, (*b*) the average yield per cent. per annum on the sum invested. Give the answer to the nearest penny.

20. At what price must a stock which pays a dividend of $4\frac{3}{4}$ per cent. be bought in order to yield a return of 5 per cent. on the money invested?

21. A man has an annual income of £567 derived from an investment in $4\frac{1}{2}$ per cent. stock. In one year he spends £800 and provides for the excess of his expenditure above his income by selling out sufficient stock at $87\frac{3}{8}$ to produce the amount of this deficiency. What will be his income in the following year?

CHAPTER XXVI

MENSURATION (III)

AREA OF TRIANGLE OF GIVEN SIDES

If a, b, c be the lengths of the sides of a triangle, and $2s = a + b + c$ its perimeter, *i.e.*, s is its semi-perimeter, it can be proved that the area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}.$$

(See any text-book on trigonometry for a proof.)

Let the sides of a triangle be 11·3, 12·2, 13·1 cm. respectively; to find the area.

$$\begin{array}{rcl|l} s &=& 18{\cdot}3 \leftarrow & \\ \hline a &=& 11{\cdot}3 & \\ b &=& 12{\cdot}2 & \\ c &=& 13{\cdot}1 & \\ \hline 2s &=& 36{\cdot}6 & \\ \hline s &=& 18{\cdot}3 & \end{array} \qquad \begin{array}{lcr} s-a &=& 7{\cdot}0 \\ s-b &=& 6{\cdot}1 \\ s-c &=& 5{\cdot}2 \\ \hline \text{Check} \quad s &=& 18{\cdot}3 \end{array}$$

$$\therefore \text{ Area} = \sqrt{18{\cdot}3 \times 7{\cdot}0 \times 6{\cdot}1 \times 5{\cdot}2} \text{ sq. cm.}$$

Nos.	Logs.	
18·3	1·2625	
7·0	·8451	
6·1	·7853	
5·2	·7160	
	3·6089	divide by 2
6·376 × 10	1·8044(5)	

The area is **63·76** sq. cm.

$$\text{Rough check: area} \simeq \sqrt{18 \times 7 \times 6 \times 5}$$

$$= \sqrt{36 \times 105} \simeq 6 \times 10 = 60$$

EXERCISES CXII

Find, to 3 significant figures, the areas of the triangles whose sides are

1. 4, 5, and 6 in.
2. 5·9, 7·2, 8·7 in.
3. 6·1, 7·3, 11·8 ft.
4. 52·7, 120·3, 135·5 m.
5. 6·35, 2·18, 5·79 ch.
6. Find the area of the quadrilateral *ABCD* whose sides *AB*, *BC*, *CD*, *DA* are 4·78, 7·15, 3·89, and 8·64 ch. respectively, and whose diagonal *AC* is 9·56 ch.

CYLINDERS AND PRISMS

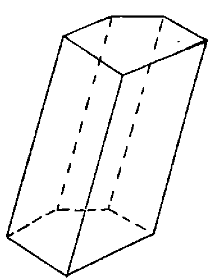
If parallel lines are drawn in any direction through all points of the boundary of a closed plane figure to meet a parallel plane (*as in the diagrams*), the solid figure so formed is called a prism or cylinder according as the plane ends (*which are congruent figures*) are polygons or closed curves.

The parallel lines are called generators.

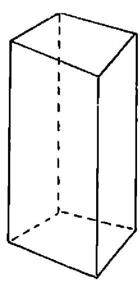
If the generators are perpendicular to the plane ends the solid is called a *right* prism or cylinder.

THE RIGHT PRISM

The ends are polygons, and the other faces rectangles. The surface area may therefore be found from the rules



OBLIQUE PRISM



RIGHT PRISM

already given. Since each unit of area of cross-section gives one unit of volume for each unit of length of the prism, we can find the volume by multiplying the area of cross-section by the length (or height) of the prism, thus

$$\text{Volume} = \text{Area of cross-section} \times \text{length.}$$

THE RIGHT CIRCULAR CYLINDER

This is a right cylinder whose cross-section is a circle. The circumference and area of the ends of the cylinder are therefore known. The curved surface of the cylinder could evidently be just covered by bending a sheet of paper cut in the form of a rectangle, whose length and breadth are equal to the length and circumference of the cylinder. If l be the length, r the radius, and $d = 2r$ the diameter of the cylinder measured in the same units, then

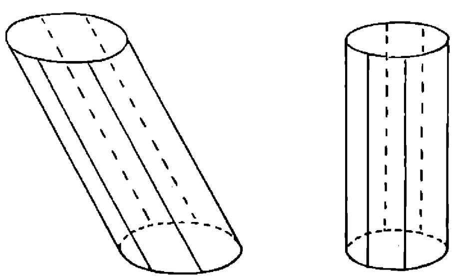
$$\text{Area of curved surface} = 2\pi r \times l$$
$$= 2\pi rl = \pi dl \text{ sq. units.}$$

It will be clear that the volume can be calculated in the same way as for a prism, so that, since the cross-sectional area is $\pi r^2 = \frac{1}{4}\pi d^2$ sq. units, the

$$\text{Volume of cylinder} = \pi r^2 \times l$$
$$= \pi r^2 l = \frac{1}{4}\pi d^2 l \text{ cub. units.}$$

THE RIGHT CYLINDER

In a similar way, the curved surface of any right cylinder is given by the product of the length of the cylinder and the perimeter of its cross-section, while the volume is



OBLIQUE CYLINDER RIGHT CYLINDER

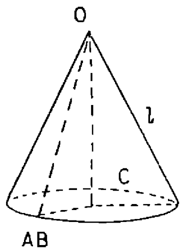
given by the product of the length of the cylinder and the area of its cross-section.

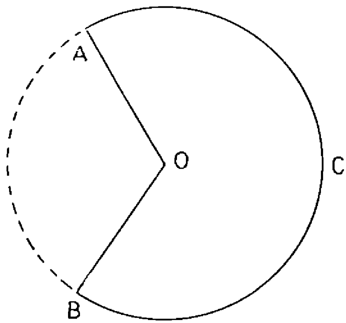
CONES AND PYRAMIDS

If all points of the boundary of a closed plane curve or polygon are joined by straight lines to a point (called the vertex) not in the same plane, the solids formed are called cone and pyramid respectively.

RIGHT CIRCULAR CONE

This solid is formed when a right-angled triangle is rotated about either of the sides which contain the right





angle, or when a sector is cut from a circular piece of paper, and the two bounding radii are drawn together.

The area of the curved surface of the cone is therefore equal to that of a sector of a circle whose radius is the slant height, l, of the cone, and whose arc is the circumference of the circular base of the cone. Hence if r is the radius of the base and $d = 2r$ its diameter,

$$\therefore \frac{\text{Area of curved surface of cone}}{\text{Area of circle radius } l}$$

$$= \frac{\text{circumference of base of cone}}{\text{circumference of circle radius } l} = \frac{2\pi r}{2\pi l} = \frac{r}{l}.$$

$\therefore$ Area of curved surface of cone

$$= \pi l^2 \times \frac{r}{l} = \pi r l \text{ square units}$$

$$= \frac{\pi d l}{2} \text{ square units.}$$

It can be shown that the volume of the right circular cone is given by $\frac{1}{3}$ of the product of the base-area and the perpendicular height, h, thus

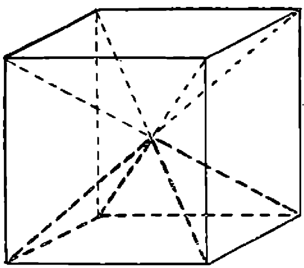
$$\text{Volume of cone} = \tfrac{1}{3} \times \pi r^2 \times h$$
$$= \tfrac{1}{3}\pi r^2 h \text{ or } \tfrac{1}{12}\pi d^2 h.$$

Note also that by applying Pythagoras' Theorem to the right-angled triangle whose rotation generates the cone we have

$$l^2 = r^2 + h^2 = \tfrac{1}{4}d^2 + h^2.$$

THE PYRAMID

The surface is composed of triangles, and the base is a polygon; its area may therefore be found from rules already given. It may be shown that the volume is given by the formula



$$V = \tfrac{1}{3}A \cdot h,$$

where A is the base area and h the perpendicular distance of the vertex from the base.

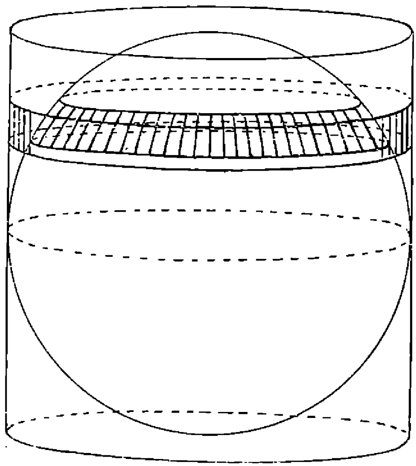
This formula may be verified in the case of a right pyramid on a square base, and having a perpendicular height equal to one half the length of a side of the base. Such a pyramid is one-sixth of the cube on the same base.

THE SPHERE

This is the solid which is formed when a semicircle is rotated about the diameter. Its surface differs from the curved surfaces of cylinder and right circular cone in that it cannot be reproduced by cutting and bending a plane sheet of paper. It can be shown that if r be the radius, $d = 2r$ the diameter of a sphere, then

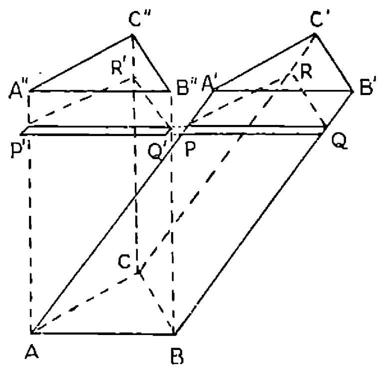
Surface area of sphere $= 4\pi r^2 = \pi d^2$ square units
and Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$ cubic units.

It can be also be shown that if a cylinder be drawn just to enclose the sphere (see p. 227), then planes drawn at right angles to the axis of the cylinder cut off zones of equal area on the surfaces of the sphere and the cylinder. From this result we may deduce that the area of the curved



surface of part of a sphere cut off by two parallel planes at distance h apart is $2\pi rh$.

In particular taking one of these planes as a tangent plane we find that the area of a 'spherical cap' of radius r and height h is $2\pi rh$.



Oblique Cylinder and Prism

If the generators are not perpendicular to the plane ends no simple rule can be given for the calculation of the surface area of a cylinder or prism, but it can be shown that the volume is given by the product of the area of either end and the perpendicular distance between the ends.

Suppose $ABCA'B'C'$ to be an oblique prism, and imagine it divided into thin slices, such as PQR, by planes parallel to the ends ABC and $A'B'C'$. These slices may be imagined to slide over one another till the solid takes the form $ABC\ A''B''C''$, in which AA'', BB'', CC'' are perpendicular to the plane ends. There will be no change in the distance between the planes ABC and $A'B'C'$ during the sliding, and the small irregularities in the faces of the new solid $ABC\ A''B''C''$ will be made smaller if the slices are made thinner, and can be made as small as we please in this way. The volume, therefore, of the solid $ABC\ A''B''C''$ is is equal to that of $ABC\ A'B'C'$, and it is given by the product of the area ABC and the distance between the planes ABC and $A'B'C'\ A''B''C''$.

Oblique Cone and Pyramid

A similar argument might be applied to show that cones or pyramids on the same base, and having the same perpendicular height, are equal in volume.

It can be shown that the formula already given for the volume of a right cone or pyramid holds in all cases. Thus

Volume of any cone or pyramid $= \frac{1}{3}$ of the product of the base-area and perpendicular height.

For convenience we collect together here the results that have been given in this chapter.

Area of triangle:

$$\tfrac{1}{2}\text{ base} \times \text{height} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$\text{where } 2s = a + b + c.$$

Volume of cylinder or prism:

Area of cross-section × length measured at right angles to cross-section.

Volume of pyramid or cone:

$\frac{1}{3}$ area of base × height measured at right angles to base.

Right circular cylinder:

$$\text{Curved surface} = 2\pi rl \text{ or } \pi dl.$$

$$\text{Volume} = \pi r^2 l \text{ or } \frac{\pi}{4}\cdot d^2 l.$$

Right circular cone:

Curved surface $= \pi r l$.
Volume $= \frac{1}{3}\pi r^2 h$.

Sphere:

Curved surface $= 4\,\pi r^2$ or πd^2.

Volume $= \frac{4}{3}\pi r^3$ or $\frac{\pi d^3}{6}$.

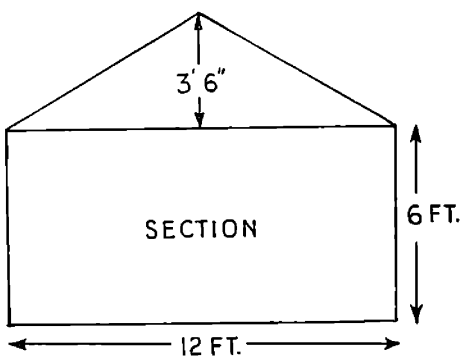
Example 1. Find the area of the cross-section of a cylindrical tube whose outer and inner radii are 4·45 and 4·10 cm. respectively.

$$\text{Area} = \pi(4{\cdot}45^2 - 4{\cdot}10^2)$$
$$= \pi(4{\cdot}45 + 4{\cdot}10)(4{\cdot}45 - 4{\cdot}10) \text{ sq. cm.}$$
$$= \pi \times 8{\cdot}55 \times 0{\cdot}35 \text{ sq. cm.}$$
$$= 2{\cdot}9925\,\pi \text{ sq. cm.}$$
$$= \mathbf{9{\cdot}40} \text{ sq. cm.}$$

Rough check $3 \times 9 \times \frac{1}{3} = 9$.

855
35
2565
4275
2·9925
$3\frac{1}{7}$
8·9775
·4275
9·4050
$\frac{1}{800}$th = ·0037
9·40
or use logs.

Example 2. A tent is in the form of a cone resting on a cylinder 6 ft. high and 12 ft. in diameter. The vertex of



the cone is $9\frac{1}{2}$ ft. from the ground. Find the internal volume of the tent.

Volume of cone

$= \frac{1}{3}$ area of base $\times$ height

$= \frac{1}{3} \times \frac{\pi}{4} \times 12^2 \times 3{\cdot}5$ cub. ft.

$= 42\pi$ cub. ft.

Volume of cylinder

$= \frac{\pi}{4} \times 12^2 \times 6$ cub. ft.

$= 216\pi$ cub. ft.

$\therefore$ Total internal volume

$= 258\pi$ cub. ft.

Taking π as $3\frac{1}{7}$,

Volume = 810·86 cub. ft.

Taking π as $3\frac{1}{7} - \frac{1}{800}$,

Volume = 810·53 cub. ft.

Taking π as 3·1416,

Volume = **810·53** cub. ft.

(1)

```
            258
             3 1/7
            ------
            774
             36·857
            -------
            810·857
1/800th =     ·3225
            -------
            810·534
```

or (2)

```
            3·1416
               258
            -------
            628·32
            157·080
             25·1328
            --------
            810·5328
```

or (3)

Nos.	Logs
258	2·4116
π	·4971
810·4	2·9087

Example 3. Find the radius of a circle of area 100 sq. in.

Since $\pi \times [\text{radius}]^2 = 100$

$\therefore [\text{radius}]^2 = \frac{100}{\pi}$ sq. in.

radius $= \sqrt{\frac{100}{\pi}}$ in.

$=$ **5·64** in.

Nos.	Logs
100	2
π	·4971
	1·5029 ÷ 2
5·642	·7514(5)

Alternatively, $\frac{100}{\pi}$ may be worked out or found from reciprocal tables, and its square root either worked or found from tables.

Exercises CXIII

(Unless otherwise directed take π = 3·1416, or log π = ·4971. Give answers to a reasonable number of figures.)

Areas

1. Find the area of a circle radius 12·5 cm., taking π = 3·1416.
2. Find the area of cross-section of a hollow cylindrical tube

whose internal and external radii are 3·4 and 3·6 cm. respectively. (Take $\pi = 3\frac{1}{7} - \frac{1}{800}$.)

3. A circle of radius $2\frac{1}{2}$ ft. lies within another circle of radius $3\frac{1}{2}$ ft. Find the area between them. Take $\pi = 3{\cdot}142$.

4. Find the area of surface of a sphere of diameter 5 ft. 6 in.

5. A (right-angled) triangle has sides of length 3, 4, 5, ft. Find the area of the curved surface of the cone formed by rotating the triangle about its shortest side.

6. A square of side 3·5 cm. is rotated about one side. Find the total surface of the cylinder formed.

7. The slant edges of a pyramid are each 10 ft. long, and the base is a square of side 12 ft.; find the total area of the four sloping sides of the pyramid.

8. Find the area of the curved surface of a right circular cone whose base-radius is 6 ft. and slant height 12 ft.

9. Find the total area of surface of a right circular cone whose base-radius is 8 ft. and perpendicular height 6 ft.

10. Find the radius of a circle whose area is equal to that of an equilateral triangle of side 10 in.

Exercises CXIV

Volumes

(Unless otherwise directed take $\pi = 3{\cdot}1416$, or $\log \pi = {\cdot}4971$. Give answers to a reasonable number of figures.)

1. Find the volume of a sphere of radius 2 ft. 6 in.

2. Find the volume of a sphere of diameter 0·3 m.

3. Find the volume of a cone of base-radius 3·6 cm. and vertical height 12 cm.

4. Find the volume of a cone of base-radius 1·2 m., slant height 1·3 m.

5. The volume of a cylinder is 1000 cu. ft. and the area of its base 80·5 sq. ft. Find its height.

6. How many complete cylinders each 2 in. long and of $1\frac{1}{2}$ in. radius could be cast from a cubic foot of lead?

7. The volume of a cone is 5000 c.c.'s, and its perpendicular height is 80 cm.; find the radius of its base.

8. Find the volume of a hollow cylinder 10 ft. long, and having internal and external diameters 2 ft. 8 in. and 3 ft. 0 in.

9. Find the radius of a sphere, whose volume is 1 litre.

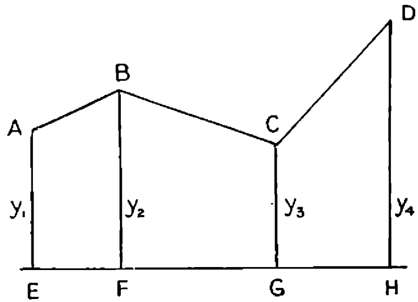
10. Find the ratio (using the formulæ given) of the volumes of a sphere and a circular cylinder, if the length and diameter of the cylinder are each equal to the diameter of the sphere.

11. A pyramid stands on a square base of side 6 in., and its sloping edges are each 9 in. long. Find its volume.

12. Find the volume of a regular tetrahedron of side 5 in. (*i.e.*, of a pyramid on a triangular base, each edge being 5 in.). In this solid, the perpendicular from the vertex to the base, which is an equilateral triangle, meets the base at its 'centre.' This is a point of trisection of the altitudes of the triangle.

Area enclosed by a Curve

This figure, in which *AE*, *BF*, *CG*, *DH* are drawn perpendicular to a common base *EFGH*, is called a trapezoid.



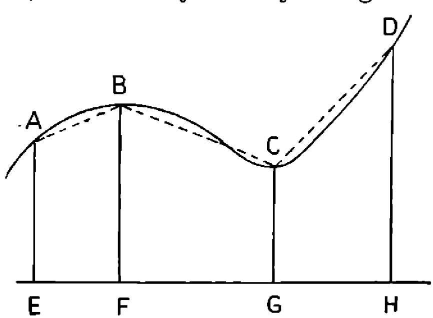
Since it is made up of three parts, each of which is a trapezium, its total area is

$$\tfrac{1}{2}(y_1 + y_2)EF + \tfrac{1}{2}(y_2 + y_3)FG + \tfrac{1}{2}(y_3 + y_4)GH \quad . \quad (1)$$

If $EF = FG = GH = x$, the area is

$$(\tfrac{1}{2}y_1 + y_2 + y_3 + \tfrac{1}{2}y_4)x.$$

If *A*, *B*, *C*, *D* are not joined by straight lines, but are



points on a curve, an approximation to the area enclosed by the curve *AE*, *EH*, and *HD* might be got by replacing the arcs of the curve by the chords *AB*, *BC*, *CD*, an excess in one part of the area being set off against a defect in another part. Thus expression (1) above is an approximation to this area.

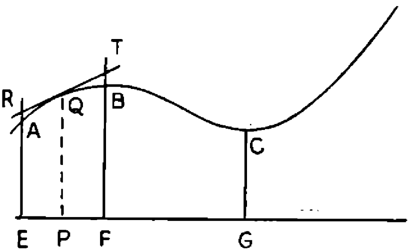
AE, BF, etc., are called **ordinates,** and the area $AEHD$ is called the area under the curve $ABCD$.

By taking a larger number of strips of equal width we may express the area under the curve approximately by the formula (known as the **trapezoidal rule**),

Common width of strips $\times$ ($\frac{1}{2}$ sum of end ordinates plus sum of intermediate ordinates)

The Mid-ordinate Rule

Another method is as follows. Bisect EF at P, and draw the ordinate PQ to meet the curve at Q. Draw RQT,



the tangent at Q, meeting AE at R and BF at T. Then the area of trapezium $ERTF = EF \times PQ$, since $PQ = \frac{1}{2}(ER + FT)$. If we take this area as an approximation to the area $AQBFE$, and proceed similarly for other strips, we can obtain by addition an approximate value for the area. In general this method gives a rather closer approximation to the area than the trapezoidal rule.

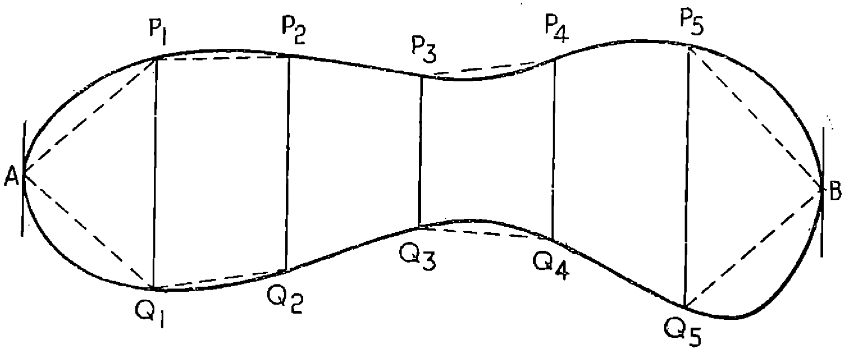
Simpson's Rule

This gives generally a still closer approximation. The area must be divided into an even number of strips of equal width. This requires an odd number of ordinates. The area is then approximately

$\frac{1}{3}$ (sum of end ordinates, plus twice sum of other odd ordinates, plus four times sum of even ordinates) multiplied by the common width.

Each of the above formulae may be applied to find the area contained within a closed curve.

For example, this figure is divided by ordinates P_1Q_1, P_2Q_2, etc., parallel to one another and to the tangents at A and B, into strips of equal width. By replacing the arcs AP_1, P_1P_2, etc., by the corresponding chords and using



the trapezoidal rule, we have as an approximate result for the area

(Sum of P_1Q_1, . . . P_5Q_5) $\times$ common width of the strips. The approximation is fairly close except when the curve is bending sharply, as at the two ends. For such parts of the curve, closer approximations to the area may be obtained by taking ordinates closer together.

VOLUMES

Simpson's rule for an area reduces when there are three equidistant ordinates to the form

$$\frac{h}{3}(y_1 + 4y_2 + y_3)$$

where h is the distance between ordinates.

A similar approximate formula holds for volumes. If y_1 and y_3 are the areas of the end sections and y_2 the area of the section half-way between, then the volume may be found approximately from the formula

$$V = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

where $2h$ is the distance between the end sections.

From this prismoidal formula (as it is called) the formulæ for the volumes of cylinder, pyramid, and sphere may be deduced.

For a cylinder $y_1 = y_2 = y_3$.

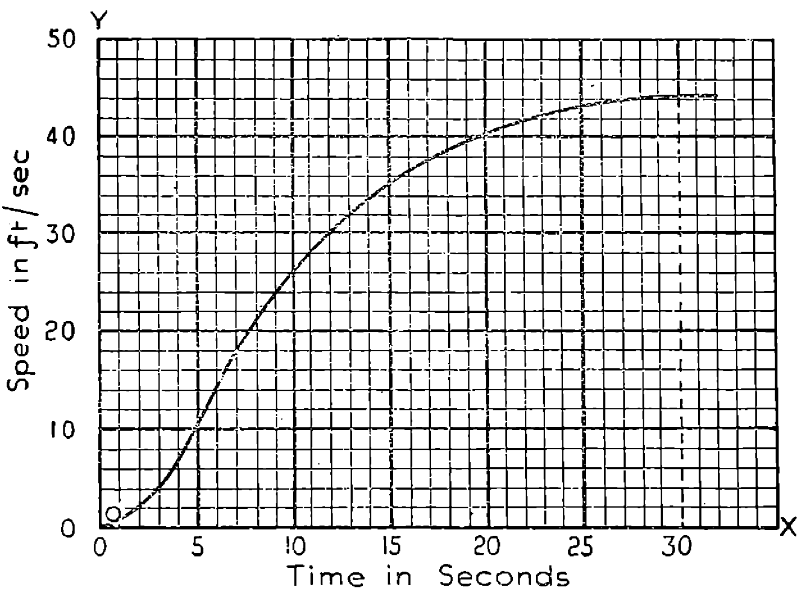
For a pyramid or cone $y_1 = 0$, $y_2 = \frac{y_3}{4}$.

For a sphere $y_1 = y_3 = 0$.

Exercises CXV

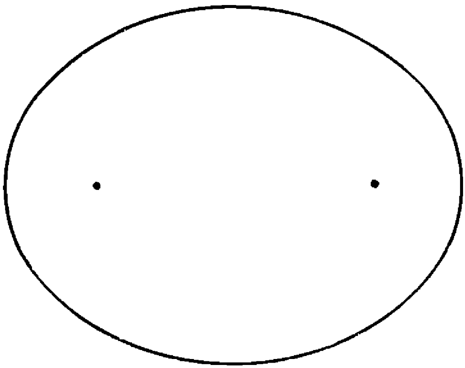
1. Draw a circle of radius 3 in., and find its area by using in turn each of the formulæ given above. Compare your results with the value found by calculation, taking $\pi = 3 \cdot 142$.

2. The figure shows the connexion between the time and the speed of a motor-car. It can be shown that 'the area under the graph,' that is, the area enclosed by the curve, the axis OX, and the ordinates at any two points on OX, represents the distance travelled during the corresponding interval of



time, on a scale 1 large square to 10×5 ft. Find approximately the distance travelled in 30 seconds.

3. The figure shows an ellipse. Find its area.



4. Water flows in a horizontal drain-pipe of diameter 1 ft. to a depth of 8 in. Find the area of the cross-section of the water by means of a scale drawing on squared paper.

5. Use the prismoidal formula to find approximately the volume of a water-butt, whose depth is 6 ft., whose ends are circles of radius 2 ft, and whose central section is a circle of radius 2 ft. 6 in.

EXERCISES CXVI

1. A map is drawn on the scale of 10 yd. to an inch. What area on the map will represent a lake of area 20 ac.?

2. A square room is covered with carpet 27 in. wide, costing 3*s*. $4\frac{1}{2}$*d*. per yard. If the total cost is £15 12*s*. 6*d*., find the length of the room.

3. The floor of a corridor 6 ft. wide, 18 ft. long has a strip of boarding 2 ft. wide down the middle, and the remainder is filled in with tiles, each of which measures 4 in. by 9 in. How many tiles are used?

4. If a square beam is cut from a cylindrical tree, find roughly what percentage of wood is cut away.

5. Find in yards the length of wire $\frac{3}{8}$ in. in diameter which can be drawn from 11 cwt. of copper. (Take 1 cu. ft. of copper to weigh 525 lb.)

6. The area of the base of a tank which can hold 150 gal. is 1 sq. yd.; find the depth of the tank. (1 gal. = 277·24 cu. in.)

7. The area of a circular running-track of uniform width is 2926 sq. yd., and the distance round the outside boundary is one quarter of a mile; find the width of the track. (Take $\pi = \frac{22}{7}$.)

8. A rectangular tank 8 ft. long and 4 ft. wide is filled with water to a depth of 2 ft. 9 in. If it is emptied in 15 min. through a pipe whose area of cross-section is 3·2 sq. in., find (in m.p.h.) the average speed at which water flows through the pipe.

9. A hollow circular cylinder has its internal and external diameters 4 and 5 ft. respectively. If it weighs as much as a solid ball of the same material and of diameter $4\frac{1}{2}$ ft., find the height of the cylinder.

10. Calculate the volume of a pyramid, 70 ft. high, which stands on a square base of side 60 ft. Calculate also the area of each side face.

11. If a sphere and a cube have equal surfaces, find the ratio of their volumes.

12. The slant side of a cone is 25 ft. and the area of its curved surface is 550 sq. ft. Find its volume ($\pi = \frac{22}{7}$).

13. A section of a railway cutting 250 yd. long is in the form of a trapezium with parallel horizontal sides. The top of the cutting is 56 ft. wide, the bottom 36 ft. wide, and the vertical depth 25 ft. How many cubic yards of earth to the nearest hundred were removed in making it? (Assume the top was originally flat.)

14. A rail weighs 84 lb. per yard length. Find the cross-sectional area of the rail if a cubic foot of the metal weighs 470 lb.

15. The capacity of an oil-can may be calculated approximately by regarding the lower part as a cylinder and the upper part as a cone. The total height is $14\frac{1}{2}$ in., the height of the cylindrical part 10 in., and the circumference 27 in. How many gallons will the can hold? (Take $\pi = 3\frac{1}{7}$, 1 gal. = 277 cu. in.)

16. Calculate to the nearest thousand cubic feet the volume of the envelope of an airship, regarding it as equivalent to that of a cylinder 500 ft. long and 48 ft. in diameter.

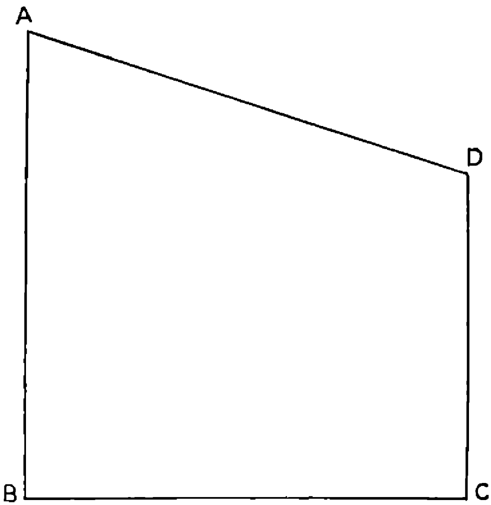
17. A hollow sphere 1 ft. in external diameter and 2 in. thick is made of metal weighing 504 lb. per cubic foot. Find its weight to the nearest pound.

18. A hollow iron sphere weighs 72 lb., and its external diameter is 9 in. If iron weighs 480 lb. per cubic foot, find the volume of the cavity.

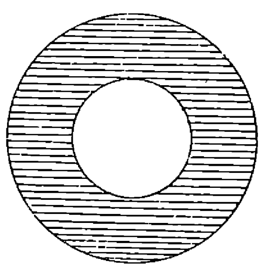
19. 200 grm. of metal weighing 7·93 grm. per c.c. are melted

and cast into the form of a solid cylinder 2·5 cm. high. Calculate the diameter of the cylinder.

20. A graduated glass cylinder is 2 in. in internal diameter and contains water. A dozen equal glass marbles are lowered into the water and are entirely immersed. If the level of the water rises 0·9 in., find the diameter of one of the marbles.

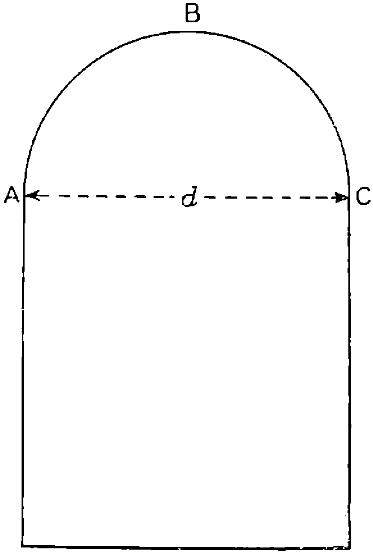


21. This is the side view of a corn-bin on a square base. *AD* is the lid. If the figure is on a scale 1 cm. to the foot, find the volume in cubic feet.



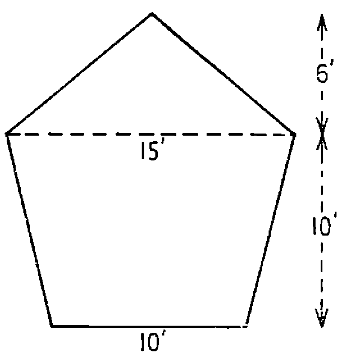
22. The diagram is the pattern of a washer full size. The thickness is $\frac{1}{24}$ in. Find the number of washers to the pound of metal, if the metal weighs 448 lb. per cubic foot.

23. This figure shows the section of an arched passage-way with vertical sides and semicircular roof. Scale $\frac{1}{50}$. The



passage is 10 yd. long. Find the cost to the nearest 5*s.* of painting the walls and roof at 4*d.* per square yard. (Do not include the ends of the passage.)

24. This figure shows the cross-section of a haystack. Find the volume per yard length in cubic feet.



25. A circular steel washer of thickness t in. is of external diameter D in. and internal diameter d in. Write down a formula giving the weight W of the washer in ounces, supposing that a cubic foot of steel weighs 490 lb. Show that the formula may be expressed thus: $W = n(D + d)(D - d)t$, and find n to three significant figures.

26. An exercise ground is in the form of an irregular polygon, A, B, C being consecutive vertices and ABC a re-entering angle; $AB = 121$ yd., $BC = 706$ yd., $CA = 725$ yd. If the fences AB, BC are removed, and one is made from A to C, by how many acres will the area of the ground be increased?

CHAPTER XXVII

JOINT VARIATION. VARIATION WITH THE SQUARE, CUBE, ETC.

JOINT VARIATION

The cost of carriage of goods depends on the weight of the goods and the distance they are carried. More precisely, the cost varies for a given distance directly as the weight carried, and for a given weight directly as the distance.

Example 1. The cost of carriage of 5 tons for 30 miles is £2. What will be the cost of carrying 25 tons for 15 miles?

The ratio of the distances is $\frac{15}{30}$.

$\therefore$ The cost of carriage of 5 tons for 15 miles would be

$$£2 \times \tfrac{15}{30}.$$

The ratio of the weights is $\frac{25}{5}$.

$\therefore$ The cost of carriage of 25 tons for 15 miles is

$$£2 \times \tfrac{15}{30} \times \tfrac{25}{5}.$$
$$= \mathbf{£5.}$$

Notice that the cost is multiplied in succession by the ratios of the distances and of the weights.

The time taken to complete some building work will vary inversely with the number of men put on, and inversely with the length in hours of the working week.

Example 2. If 10 men, working 48 hr. a week, would complete some work in 21 days, how long would 12 men, working 42 hr. a week, take to do the same work?

Ratio of new number of men to the old $= \frac{12}{10}$.

Ratio of new number of hours worked to the old $= \frac{42}{48}$.

$$\therefore \frac{\text{Time required}}{21 \text{ days}} = \tfrac{10}{12} \times \tfrac{48}{42} \quad \text{(The ratios are inverted)}$$

$$\therefore \text{Time required} = 21 \times \tfrac{10}{12} \times \tfrac{48}{42} \text{ days}$$
$$= \mathbf{20} \text{ days.}$$

These are examples of joint variation. A further example is afforded by the area of a rectangle, which varies directly with the length and directly with the breadth. If A_1 is the area of a rectangle of length a_1 and breadth b_1, and A_2 the area of a rectangle of length a_2 and breadth b_2

$$\frac{A_1}{A_2} = \frac{a_1 b_1}{a_2 b_2} = \frac{a_1}{a_2} \times \frac{b_1}{b_2}$$

$$\text{and } \frac{A_1}{a_1 b_1} = \frac{A_2}{a_2 b_2} = 1.$$

Similarly, the area of a triangle varies jointly with the base and height.

Exercises CXVII

1. If the simple interest on a certain sum of money is £3 for 146 days at $3\frac{1}{2}$ per cent. per annum, what would it be for 219 days at $2\frac{1}{3}$ per cent. per annum?

2. If 18 men make a road 100 yd. long in 5 days, how many men would make one 150 yd. long in 3 days?

3. If the meat ration for 10000 men is 1 lb. per head per day and the supplies would last for 35 days, to what should the daily ration be reduced if the supplies have to last 15000 men for 33 days?

4. If 3 tons 5 cwt. can be carried 50 ml. for £2 2*s*. 6*d*., what would be the cost of carrying 4 tons 17 cwt. 2 qr. for 75 ml. at the same rate?

5. The number of planks 8 ft. long and 6 in. wide required to make a floor was 150. How many planks 6 ft. long and 5 in. wide would have been required for this floor?

6. The volume of a gas varies inversely as the pressure and directly as the absolute temperature (273° + Centigrade temperature). If a quantity of gas has a volume of 200 c.c. at temperature 15° C. and pressure 755 mm., what would be its volume at a pressure of 760 mm. and temperature 31° C.?

7. The cost of a gas-fire varies directly with the number of burners, and directly with the time for which it is alight. If the cost when seven burners are alight was 3*s*. 6*d*. for 40 hr., what would be the cost for nine burners for 64 hr.?

8. The cost of carpet for a given room varies directly as the price per yard and inversely as the width of carpet used. If carpet 27 in. wide at 7*s*. 6*d*. per yard is used, the cost is £25. What would be the cost if carpet 30 in. wide at 5*s*. 6*d*. per yard were used?

9. The weight of a rectangular sheet of metal of uniform thickness measuring 60 in. by 45 in. was $10\frac{1}{2}$ lb. A rectangular piece 25 in. by 36 in. was cut away. Find the weight of the remainder.

10. If the wages of 130 men amounted to £2145 in 6 weeks, what would the total wages of 175 men amount to in 5 weeks, if all the men earned the same weekly wage?

VARIATION WITH THE SQUARE, CUBE, ETC.

From the formulæ given in Chapter XIV, the area, A, of a circle is given in terms of the radius, r, by the formula

$$A = \pi r^2.$$

Consider circles of different radii, and divide the area of one by the area of another. We see that, since the factor π cancels out in the division, the ratio of the areas is equal to the ratio of the squares of the radii.

We see also that the formula may be written $\frac{A}{r^2}$ = constant number π. We say therefore that the area of a circle varies directly as the square of the radius.

Similarly, if V denotes the volume of a sphere of radius r, $V = \frac{4}{3}\pi r^3$; we may write this

$$\frac{V}{r^3} = \text{a constant number } \frac{4\pi}{3}$$

and we may say that the volume of a sphere varies directly as the cube of the radius.

The volume of a circular cylinder is given by the formula $V = \pi r^2 h$ where r is the radius, h the height. Hence

$$\frac{V}{r^2 h} = \text{a constant number } \pi$$

and we say that the volume of a cylinder varies jointly as the square of the radius, and the height.

Example. If the pressure of the wind on the front of an engine is 0·4 lb. per sq. ft. when the train is moving at 10 miles an hour, what will the pressure per square foot be when the speed is 35 miles per hour, assuming that the pressure varies with the square of the speed?

Let P = pressure in lb. per sq. ft.

$$\therefore \frac{P}{0 \cdot 4} = \frac{35^2}{10^2}.$$

$$P = 0 \cdot 4 \times \frac{1225}{100} = 4 \cdot 9.$$

The required pressure is **4·9** lb. per sq. ft.

Exercises CXVIII

1. The radii of two circles are as 3 to 2. Find the ratio of their areas.

2. A circle whose radius is 3 ft. has an area of 28·27 sq. ft. Find the areas of circles whose radii are 6, 15, $4\frac{1}{2}$ ft.

3. The volumes of two spheres are 5 and 625 cu. ft. Find the ratio of their diameters.

4. The radii of two spheres are 1·25 and 2·5 cm. Find the ratio of the areas of their surfaces.

5. The edges of two cubes are in the ratio of 1·25 to 1. Find the ratio of their volumes.

6. The heights of two cones are the same and their volumes are 15 and 45 c.c. Find the ratio of the radii of their bases.

7. Two similar models of the same statue are made of the same material. One is 4 in. high and weighs 16 oz. If the other is 3 in. high, what will it weigh?

8. If a four-oared boat costs £80, what will a six-oar boat cost, assuming that the cost varies directly with the square of the number of oars?

9. If a pipe of 9 in. bore discharges a certain quantity of water in 6 hr., how long would four pipes of 6 in. bore take to discharge three times the quantity, if the water passes along all the pipes at the same speed?

10. If the rate at which a ship moves is proportional both to the total area of sails set and to the velocity of the wind, and if it travels at 8 knots with a wind of 15 knots and 400 sq.ft. of canvas set, at what rate will it move if the wind freshens to 20 knots and 50 sq. ft. of canvas be taken in?

11. The weights of leaden spheres vary as the cubes of their diameters; in what ratio is the weight increased if the radius is increased by 50 per cent.?

12. A boat is 450 ft. long and a model of it is 10 in. long. Find the displacement of the boat in tons if the model displaces 3 oz. of water. (Take displacements to vary as cubes of linear measurements.)

13. The time of oscillation of a pendulum varies as the square root of its length. If a pendulum 39 in. long swings once per second, find, to the nearest hundredth of a second, the time of swing of a pendulum 2 ft. long.

Examples for graphical treatment

14. The greatest weight which can be hung on a rope of diameter d in. is $2{\cdot}63\, d^2$ tons. Find the greatest weights which can be hung on ropes of diameters 0·5 in., 1·5 in., 2·5 in., 3·5 in., 4·5 in.

Plot graphs (1) of weights against diameters;
(2) of weights against squares of diameters.

15. In an experiment with a pendulum, consisting of a small heavy bob at the end of a fine string, the times of a swing corresponding to given lengths of pendulum were noted as follows:

Length in inches (l)	5	10	15	20	25
Time of swing in seconds (t)	·36	·51	·62	·71	·79

Plot a graph of the lengths against the squares of the times. What is your conclusion from the resulting graph? Read off the value of $\frac{l}{t^2}$ from the graph.

16. The following table gives the electrical resistance of copper wires:

Diameter of wire (in inches)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$
Resistance per 1000 yd. (in ohms)	1·95	·487	·216	·122

Plot a graph of the resistance against the reciprocal of the square of the diameter. What is your conclusion? Find a formula (approximately) to give the resistance R ohms of 1000 yd. of wire of diameter d in.

CHAPTER XXVIII

PROBLEMS ON WORK, CLOCKS, TRAINS, ETC. MIXTURES

Example 1. If A takes 10 days to do a certain piece of work and B takes 12 days, how long would they take working together?

In 1 day A does $\frac{1}{10}$ of the work.

In 1 day B does $\frac{1}{12}$ of the work.

$\therefore$ In 1 day A and B together do $\frac{1}{10} + \frac{1}{12}$ of the work

$$= \frac{6 + 5}{60} \; i.e., \; \frac{11}{60} \text{ of the work.}$$

$\therefore$ They would take $\frac{60}{11}$ days to complete the work,

i.e., $\mathbf{5\frac{5}{11}}$ days.

Example 2. A cyclist left a town travelling at an average speed of 12 miles per hour. An hour later a car averaging 30 miles per hour left the town by the same road and followed the cyclist. After how long and at what distance from the town should the car overtake the cyclist?

The car gains on the cyclist at the rate of $30 - 12$ miles per hour, *i.e.*, 18 miles per hour.

The cyclist had a start of 12 miles.

$\therefore$ The car overtakes him in $\frac{12}{18}$ hr., *i.e.*, 40 min.

In this time the car has travelled $30 \times \frac{2}{3} = 20$ miles.

$\therefore$ Car overtakes cyclist after **40** min., **20** miles from town.

Example 3. A train 80 yd. long travelling at 40 miles per hour passes another train 52 yd. long travelling at 20 miles per hour in the opposite direction. How long do the two trains take to pass?

The motion of one train past the other is the same as if one train were stationary and the second were to pass at a rate of $40 + 20 = 60$ miles per hour.

In passing, the second train travels a distance equal to the sum of the lengths of the trains, *i.e.*, $80 + 52 = 132$ yd.

∴ Time taken to pass is the time taken to travel 132 yd. at 60 miles per hour.

$$= \frac{132}{1760} \text{ min.}$$

$$= \frac{132 \times 60}{1760} \text{ sec.}$$

$$= 4\tfrac{1}{2} \text{ sec.}$$

Example 4. At what times between 9 and 10 A.M. will the hands of a watch be (1) coincident, (2) at right angles, (3) opposite?

At 9 A.M. the minute-hand is 45 minute-divisions behind the hour-hand.

∴ (1) it will have to gain 45 minute-divisions.
(2) ,, ,, ,, 30 ,, ,,
(3) ,, ,, ,, 15 ,, ,,

The minute-hand moves over 60 minute-divisions while the hour-hand moves over 5.

∴ It gains 55 minute-divisions in 60 min.

∴ Answers are (1) $60 \times \frac{45}{55}$, (2) $60 \times \frac{30}{55}$, (3) $60 \times \frac{15}{55}$ min.

i.e., (1) $\mathbf{49\frac{1}{11}}$, (2) $\mathbf{32\frac{8}{11}}$, (3) $\mathbf{16\frac{4}{11}}$ min. after 9 A.M.

Exercises CXIX

(Many can be done graphically.)

1. A man takes 8 days to do a certain piece of work while a boy takes 12 days. How long would they take working together?

2. *A* takes 10 days to do a piece of work, but if *B* helps him they can finish the work in 6 days. How long would *B* take alone?

3. *A* and *B* together would take 18 days to do a piece of work, while *A* and *C* would take 20 days, and *B* and *C* together 24 days. Find how long *A* would take by himself. Answer to the nearest day.

4. A bath would be filled by its cold-water tap in 5 min. or by its hot-water tap in 15 min. In how long would it be filled if both were running together?

5. Two men started one day at noon from places 12 ml. apart and walked towards each other, one at $3\frac{1}{2}$ m.p.h. and the other at 4 m.p.h. Where and at what time would they meet?

6. Two motorists drive in the same direction on an oval racing-track 2 ml. round, one at 60 and one at 70 m.p.h. At a certain moment they are 1 ml. apart. After how many laps will the faster motorist overtake the slower?

7. How long does the minute-hand of a clock take to gain 20 min. divisions on the hour-hand?

At what time after 1 P.M. will the minute-hand first be at right angles to the hour-hand?

8. At what time between 8 and 9 o'clock are the two hands of a clock in a straight line?

9. Two trains, each 60 yd. long, are running on parallel rails, one at 30 and one at 45 m.p.h. How long do they take to pass (1) if they are moving in opposite directions, and (2) if they are moving in the same direction?

10. A train travelling at 45 m.p.h. passed a signal in 3 sec. It then passed a goods train travelling in the same direction on a parallel line at 15 m.p.h. in 12 sec. Find the length of the goods train.

11. 3 men and 5 boys can do a piece of work in 45 hr.; 2 men and 9 boys can do the same in 42 hr. Compare the rates of working of a man and a boy.

12. In a hundred-yards race A can beat B by 10 yd. and B can beat C by 5 yd. By how many yards can A beat C?

13. A can give B 50 yd. start in a quarter-mile race. B could give C 33 yd. start in a similar race. How many yards should A give C?

14. A boat's crew can row a four-mile course on a tidal river in 20 min. in still water and in 16 min. with the tide. How long would the crew take to row the course against the tide?

15. A man walks from A to B uphill at $3\frac{3}{4}$ m.p.h. and back again at $4\frac{1}{2}$ m.p.h. If he takes 2 hr. 40 min. for the whole journey, find the distance from A to B.

16. At what times between 9 and 10 A.M. will the hands of a watch be (1) coincident, (2) at right angles, (3) opposite, (4) 5 divisions apart?

17. A man walking along a road at $3\frac{1}{2}$ m.p.h. is overtaken by a motor-car. From the moment it passes him to the time of its disappearance round a corner he takes 27 steps and, walking on, reaches the corner in 135 steps more. Find the speed of the car.

18. A motorist whose average speed is 20 m.p.h. overtakes a cyclist whose average speed is 10 m.p.h. Twenty-five miles farther on the motorist stops for 10 min. and returns at the same speed. How soon after he has started to return will he meet the cyclist?

MIXTURES

Example 1. Mix golds 9 carat fine and 21 carat fine to make an alloy 18 carat fine.

$$21 = 18 + \mathbf{3}$$
$$9 = 18 - \mathbf{9}.$$

Weight the components so that the excess of one component may be balanced by the defect in the other.

Since $3 \times \mathbf{3} - 1 \times \mathbf{9} = 0$, we must take **three** parts of 21 carat to **one** part of 9 carat.

Example 2. Mix teas at 3*s.* 6*d.*, 2*s.* 10*d.*, 2*s.* 6*d.* and 2*s.* 3*d.* per lb. so that the cost price of the mixture may be 2*s.* 9*d.* per lb.

$$\text{Above the mean}\begin{cases} 3s.\ 6d. = 2s.\ 9d. + \mathbf{9d.} \\ 2s.\ 10d. = 2s.\ 9d. + \mathbf{1d.} \end{cases}$$

$$\text{Below the mean}\begin{cases} 2s.\ 3d. = 2s.\ 9d. - \mathbf{6d.} \\ 2s.\ 6d. = 2s.\ 9d. - \mathbf{3d.} \end{cases}$$

We have to weight the various components so that the mean price may be 2*s.* 9*d.* The problem is to find numbers a, b, c, d such that

$$(9 \times a + 1 \times b) - (6 \times c + 3 \times d) = 0.$$

There are many answers. Three possible ones are **1, 0, 1, 1 ; 1, 6, 1, 3 ;** and **3, 6, 5, 1.**

EXERCISES CXX

1. Mix golds 15 carat and 21 carat fine to make an alloy 18 carat fine.

2. Mix spirits at 45*s.* per gallon with water so that the mixture may be valued at 36*s.* per gallon.

3. Mix tea at 2*s.* per lb. with tea at 1*s.* 8*d.* per lb. so that the cost price of the blend may be 1*s.* 10½*d.* per lb.

4. In what proportion must a merchant mix tea at 3*s.* per lb. with tea at 1*s.* 6*d.* per lb. in order to make a profit of 25 per cent. on the cost price by selling the tea at 2*s.* 6*d.* per lb. (*Hint.* Start by finding the cost price of the mixture.)

5. A grocer has two kinds of tea which cost him 2*s.* 4*d.* and 3*s.* 6*d.* per lb. How must he mix them to make a profit of 7½ per cent. on his outlay if he sells the mixture at 3*s.* 7*d.* per lb.?

6. A man buys spirits at 35*s.* per gallon, adds water, and sells

the mixture at 36*s.* a gallon, thereby making a profit of 20 per cent. on his outlay. How much water did he add?

7. One flask of nitric acid contains 25 per cent. of acid, and a second flask contains 37·5 per cent. of acid. How much of the solution must be taken from each flask to get a mixture containing $35\frac{5}{7}$ per cent. of acid?

8. How should teas which cost 1*s.* 4*d.* and 1*s.* 8*d.* per lb. be mixed so that the mixture can be retailed at 1*s.* $9\frac{1}{2}d.$ per lb. and yield a profit of 25 per cent. on the outlay?

9. A man bought 1000 oranges for £2 15*s.* He sold some at 5 for 4*d.* and the remainder at 3 for 2*d.*, and made a profit of 20 per cent. on his original outlay. How many did he sell at the lower price?

10. How much tea at 1*s.* 1*d.* per lb. must be mixed with 1 cwt. which cost £4 18*s.* in order that 25 per cent. may be gained on the outlay by selling the mixture at 1*s.* 3*d.* per lb.?

11. A grocer pays 1*s.* 3*d.*, 1*s.* 6*d.*, 1*s.* 9*d.* per lb. for three kinds of teas. If he mixes them together in the proportion of 2 to 3 to 5 by weight, at what price per lb. must he sell the mixture in order to gain $33\frac{1}{3}$ per cent. on the outlay?

12. What weight of alloy must be melted down with 10 sovereigns of 22 carat gold each weighing 122·5 grains to make an alloy of 18 carat gold?

13. A mixture contains 25 per cent. of water. Find how much water must be added to 10 gal. 2 qt. in order that the resulting mixture shall contain 30 per cent. of water.

14. A merchant bought 136 litres of wine at 2 fr. per litre; after adding water he reduced the price to 170 c. per litre and yet gained 68 fr. How much water did he add?

15. A certain kind of brass contains 80 per cent. by weight of copper, and 20 per cent. of zinc. Copper weighs 8·95 grm. per c.c. and zinc 7·1 gr. Find to the nearest tenth how many c.c.'s of copper there are in 100 c.c.'s of the brass.

16. A man added 10 gal. of water to 60 gal. of wine which he bought at £1 1*s.* per gal. At what price per gal. must he sell the mixture to gain 50 per cent. on his outlay?

17. A dealer bought 100 lb. of tea at 1*s.* 6*d.* per lb.; he mixed it with 25 lb. of another sort and sold the mixture at 2*s.* 2*d.* per lb., making 30 per cent. profit on his outlay. How much per lb. did he pay for the second sort of tea?

18. A grocer pays 1*s*, 1*s.* 3*d.*, and 1*s.* 8*d.* per lb. for three kinds of tea. If he mixes them together in the proportion of 7, 8, and 9 by weight, at what price per lb. must he sell the mixture in order to gain $12\frac{1}{2}$ per cent. on his outlay?

CHAPTER XXIX
CONVERSION OF UNITS

Example 1. Express price of 9*s*. 6*d*. per pound in francs per Kg., taking £1 as 45·65 francs. (1 lb. = 0·4536 Kg.)

$$\text{Cost} = £0{\cdot}475 \text{ per lb.}$$
$$= 0{\cdot}475 \times 45{\cdot}65 \text{ francs per } 0{\cdot}4536 \text{ Kg.}$$
$$= \frac{0{\cdot}475 \times 45{\cdot}65}{0{\cdot}4536} \text{ francs per Kg.}$$
$$= \mathbf{47{\cdot}80} \text{ francs per Kg.}$$

Example 2. A litre of water weighs 1000 grm. A pint of water weighs 20 oz., and 1 lb. is 453·6 grm.

Express a pint in terms of a litre.

$$\text{Weight of 1 pt.} = \tfrac{20}{16} \text{ lb.}$$
$$= \tfrac{20}{16} \times 453{\cdot}6 \text{ grm.}$$
$$= 453{\cdot}6 \times \tfrac{5}{4} \text{ grm.}$$
$$= 567 \text{ grm.}$$
$$= {\cdot}567 \text{ Kg.}$$
$$= \text{weight of } 0{\cdot}567 \text{ litres.}$$
$$\therefore \text{ 1 pt.} = \mathbf{0{\cdot}567} \text{ litres.}$$

Exercises CXXI

1. Taking a rupee as 1*s*. $4\frac{1}{2}$*d*. and 25·50 fr. as the equivalent of £1, express 32760 rupees in francs, to the nearest franc.

2. Rubber was sold in Paris at 4 fr. 30 c. per kilo, when the exchange was 50·25 fr. to the £. Taking a ton as 1016 kilos, find the cost of 1 ton in British money to the nearest pound.

3. If wheat was sold in Liverpool at 11*s*. per cental, find its price in francs per quintal when the exchange was £1 = 48 fr. (1 cental = 100 lb. 1 quintal = 100 Kg. 1 Kg. = 2·2046 lb.)

4. Find, to two decimal places, the weight in pounds of a cubic inch of mercury, being given that 1 litre of

mercury weighs 13·596 Kg. (Take 1 in. as 2·5400 cm. and 1 lb. as 0·4536 Kg.)

5. The rainfall in a certain district is 5·76 cm. How many litres per are is this? Find also the equivalent number of cubic feet per acre, taking 40 ares as 1 acre, and 1 litre as ·03531 cu. ft. Give the answer to 3 significant figures.

6. Taking 1 m. as 39·37 in. and 1 gal. as 277·27 cu. in., find (to two decimal places) the number of pints in a litre.

7. Express in pounds per square inch a pressure of 10·5 Kg. per square centimetre, taking 1 Kg. as 2·2 lb., and 1 Km. as 0·621 ml. Give the answer to 3 significant figures.

Verify the following statements:

8. Pounds per lineal foot × 1·4880 = Kg. per lineal metre.

9. Tons per lineal foot × 3333·3 = Kg. per lineal metre.

10. Pounds per square foot × 4·882 = Kg. per square metre.

11. Pounds per cubic foot × 16·018 = Kg. per cubic metre.

12. Find the cost of coal at 28*s*. per ton in francs per metric tonne when the exchange was £1 = 49·50 fr. (1 metric tonne = 1000 Kg. 1 Kg. = 2·2046 lb.)

13. The German Ruhr production of coal in 1921 was (approximately) 1,835,000 metric tons per week. Mines in the United Kingdom produced 4,900,000 tons per week. Express the German weekly output as a percentage of the British to three significant figures.

14. When the exchange was £1 = 49·50 fr. luggage was charged at 5 fr. per Kg. on a French aeroplane, while an English one allowed 30 lb. free and charged 1*s*. for every pound in excess of 30 lb. What was the charge for 93 lb. on each line in shillings?

15. A new railway 110 Km. long in Holland cost 6,441,000 florins. Find the cost per mile in English money to the nearest thousand pounds.

(Take 1 Km. as $\frac{5}{8}$ ml. and £1 as 12 fl. 11 c.)

16. When the exchange was 25 fr. to the £ a wine merchant bought a cask of claret in Bordeaux, containing 208 litres of wine, at 75 c. per litre. The cask was charged at 12 fr., freight to London was £1, and duty 1*s*. 3*d*. per gallon. He bottled the wine at a cost of 2*s*. per dozen bottles and sold it at 18*s*. per dozen bottles. What percentage of this price was profit?

(6 bottles = 1 gal. 1 litre = 0·22 gal. Neglect waste in bottling, and give your answer to the nearest integer.)

CHAPTER XXX

APPROXIMATE EVALUATION OF SERIES

Example. Find to 6 decimal places the value of

$$1-\frac{1}{2}+\frac{1}{1\cdot 2}\cdot\left(\frac{1}{2}\right)^2-\frac{1}{1\cdot 2\cdot 3}\cdot\left(\frac{1}{2}\right)^3+\frac{1}{1\cdot 2\cdot 3\cdot 4}\left(\frac{1}{2}\right)^4-\ldots \text{ etc.}$$

In examples of this kind tabulate the values of the terms, positive terms in one column, negative ones in another.

		+	−
1		1	
$\frac{1}{2}$			·5
$\frac{1}{1.2}\left(\frac{1}{2}\right)^2$	$\frac{1}{4}$ last term	·125	
$\frac{1}{1.2.3}\left(\frac{1}{2}\right)^3$	$\frac{1}{6}$ last term		·0208333
$\frac{1}{1.2.3.4}\left(\frac{1}{2}\right)^4$	$\frac{1}{8}$ last term	·0026042	
$\frac{1}{1.2.3.4.5}\left(\frac{1}{2}\right)^5$	$\frac{1}{10}$ last term		·0002604
$\frac{1}{1\ldots 6}\left(\frac{1}{2}\right)^6$	$\frac{1}{12}$ last term	·0000217	
$\frac{1}{1\ldots 7}\left(\frac{1}{2}\right)^7$	$\frac{1}{14}$ last term		·0000015
$\frac{1}{1\ldots 8}\left(\frac{1}{2}\right)^8$	$\frac{1}{16}$ last term	·0000001	No more terms required for six figures in answer
		1·1276260 −	·5210952
		·5210952	
		·606531	Answer

EXERCISES CXXII

Express as a decimal to six places

1. $\frac{1}{5} + \frac{1}{5^3} + \frac{1}{5^5} + \ldots.$

2. $\frac{1}{7} + \frac{1}{3.7^3} + \frac{1}{5.7^5} + \ldots.$

3. $\frac{1}{6} - \frac{1}{6^2} + \frac{1}{6^3} - \frac{1}{6^4} + \ldots.$

4. $1 - \frac{1}{3} + \frac{1}{1.2}\left(\frac{1}{3}\right)^2 - \frac{1}{1.2.3}\left(\frac{1}{3}\right)^3 + \frac{1}{1.2.3.4}\left(\frac{1}{3}\right)^4 - \ldots.$

CHAPTER XXXI

ERROR. PERCENTAGE ERROR. LIMITS OF ACCURACY

Measurements are rarely exact. There is, usually, some difference between the estimated value of a quantity and its true value, and this difference is called the error or *absolute error* in the measurement. The ratio of this error to the true value is called the *relative error* or, when the ratio is expressed as a percentage, the *percentage error*. This relative or percentage error is of more importance in calculations than the absolute error.

An error (absolute) of 1 in. in measuring a yard of silk is of more importance than the same error in measuring the side of a field; while an error of 50 miles is large if the distance being measured is that between two towns but small if it is that between the earth and the moon.

When a length is given as 7·53 in. to the nearest ·01 in., this means that the true value lies between 7·525 and 7·535 in., and in taking 7·53 in. as the value the maximum error made is ·005 in., and the maximum percentage error

$$\frac{\cdot 005}{7\cdot 53} \times 100 \text{ per cent.}$$

$$= \cdot 066 \text{ per cent. approximately.}$$

Since the error ·005 is a maximum possible error and is not an exact estimate of the error, there is no object in working out the percentage to more than one or two significant figures. All we need is a rough idea of the possible percentage error.

Consider the area of a rectangle whose length and breadth are given as 5·63 and 8·75 in., each to the nearest hundredth of an inch. Then by multiplying 5·63 by 8·75, we obtain the area as 49·2625 sq. in.

The area really lies somewhere between

$$5\cdot 625 \times 8\cdot 745 = 49\cdot 190625$$

and

$$5\cdot 635 \times 8\cdot 755 = 49\cdot 334425,$$

so that only the first two figures of the first answer are to be relied upon.

Now it can be shown (see Exercises CXXIII, Nos. 10–13) that when numbers are multiplied or divided, the percentage error in the product or quotient is the sum of the percentage errors in the numbers separately. When a number is squared the percentage error is doubled, and when its square root is taken the percentage error is halved, and so on; hence in the above example we might have calculated the possible error of the result 49·2625 as follows:

$$\text{Percentage error in side } 5{\cdot}63 \quad \frac{{\cdot}005 \times 100}{5{\cdot}63} = \frac{50}{563} \simeq {\cdot}09.$$

$$\text{,,} \qquad\qquad \text{,,} \quad 8{\cdot}75 \quad \frac{50}{875} = \frac{2}{35} \simeq {\cdot}06.$$

$\therefore$ Percentage error in area $\simeq {\cdot}09 + {\cdot}06 = {\cdot}15$.

$\therefore$ Error $\simeq {\cdot}15$ per cent. of 49·2625
$\simeq {\cdot}074$ sq. in.

$$\begin{array}{r} {\cdot}04926 \\ {\cdot}02463 \\ \hline {\cdot}074 \end{array}$$

and the answer may be given appropriately as

49·26 $\pm$ ·08 sq. in.

As a general rule the result of a calculation should not be given to more figures than are given in the data used, and even then the last figure is unreliable.

Exercises CXXIII

1. Find, to one decimal place, the percentage error in taking half a Kilogramme as 1 lb. (1 Kg. = 2·2046 lb.)

2. Find the percentage error in taking 12 m. as equal to 13 yd. when converting metres to yards; answer to two significant figures. (1 m. = 39·37 in.)

3. The length and breadth of a rectangle are measured and recorded as 11·3 and 15·7 in., each to the nearest tenth of an inch. Find, to two significant figures, the possible percentage errors in these estimates of length and breadth.

4. Find the area of the rectangle in Question 3, by taking it

(i) as 11·3 × 15·7;
(ii) as (not greater than) 11·35 × 15·75;
(iii) as (not less than) 11·25 × 15·65.

Find the percentage errors in taking the value given by (i) instead of the values given by (ii) and (iii) respectively. Compare with the sum of the percentage errors obtained in Question 3.

5. A rectangular block of stone measures (to the nearest inch) 18 in. by 42 in. by 147 in. Estimate to two figures the percentage errors in length, breadth, and depth, and (by adding these) in the volume. Hence, taking the volume as $18 \times 42 \times 147$ cu. in., find the possible error in this estimate, and express the volume in the form $V \pm v$ where v is the possible error.

6. The radius of a circle is given as 15 ft. 4 in. to the nearest inch. Find its area in square feet in the form $A \pm a$, where a is the possible error. Take $\pi = 3{\cdot}142$.

7. The volume of a sphere is measured experimentally as 165 c.c. with a possible error of 1 c.c. Find the relative error.

8. If the radius of the sphere in Question 7 is calculated from the given data, the relative error in the calculated value will be $\frac{1}{3}$ the relative error in the observed volume. Why is this?

Calculate the radius, and the possible error in the value obtained.

9. A motor-car was timed to cover a mile in 13·47 sec. to the nearest ·01 sec., and the average speed was given as 267·26 m.p.h. Criticize the latter statement, and give the average speed in the most appropriate form $V \pm v$ m.p.h. where v is the possible error.

10. a, b are the relative errors in two quantities whose values are measured as A, B. Show that the first quantity lies between $A\ (1 - a)$ and $A\ (1 + a)$.

Write down values between which the product AB must lie.

11. Show that $(1 + a)(1 + b) \bumpeq 1 + (a + b)$ if a and b are small, and that $(1 - a)(1 - b) \bumpeq 1 - (a + b)$ if a and b are small. Hence show that the product AB in Question 10 lies between $AB\,[1 + (a + b)]$ and $AB\,[1 - (a + b)]$. What is the relative error in the product AB?

12. Hence show that if a, b, c are the relative errors in the measurements of three quantities, the relative error in the product is $a + b + c$.

13. By division show that $\dfrac{1}{1 + b} \bumpeq 1 - b$, if b is small, and that $\dfrac{1}{1 - b} \bumpeq 1 + b$.

If a, b are the relative errors in two quantities A and B, between what limits must $\dfrac{A}{B}$ lie?

Hence show that the relative error in $\dfrac{A}{B}$ is $a + b$.

CHAPTER XXXII
CONTRACTED METHODS

MULTIPLICATION AND DIVISION

Example 1. Multiply 521·6229 by 1·232893, giving the answer to five significant figures.

A rough estimate of the answer is $500 \times 1\frac{1}{4} = 625$.

```
√√√√√√
5216229 × 1232893
-------
5216229
1043246
 156487
  10432
   4173
    469
     16
-------
6431052     Answer 643·11.
```

Multiply in the usual way until two more figures than are required in the answer are obtained. This occurs above in the first line of the working. To obtain the second line, cut off the last figure (9) from the multiplicand (placing a √ over it) and multiply by 2, but allow for the 9 by carrying 2 ($2 \times 9 = 18$, nearer 20 than 10). For the next line place a tick over the 2 on the left of the 9 in the multiplicand, and multiply by 3, carrying 1 ($3 \times 2 = 6$, nearer 10 than 0), and so on.

It will be seen that the last figure in each line is sometimes too big, sometimes too small; on the whole it is very probable that the errors will balance out sufficiently to give the answer correct to the required number of figures, but this cannot be relied upon.

Example 2. Divide 643·10522 by 1·232893, giving the answer to one decimal place.

Rough estimate of the answer: $650 \div 1\frac{1}{4} = 520$; four significant figures are required.

Keep six figures (two more) in the divisor, the last figure being 'allowed for' at the first division.

```
  ✓✓✓✓
123289)64310522(5216
       61645
        2665
        2466
         199
         123
          76   Answer 521·6.
```

The last 9 of the divisor is ticked and allowed for at the first division, 61645 being $12328 \times 5 + 5$ (since $9 \ldots \times 5 = 45 \ldots$, nearer 50 than 40); for the next division, the 8 in the divisor is ticked, and the same process repeated, no figures being brought down from the dividend. The last figure in the quotient comes from the division of 76 by 12(3); this is evidently nearest to 6.

There is a possibility of ambiguity arising as to the last figure, but this happens very rarely, and contracted division is very convenient and expeditious.

In more complicated cases which involve both multiplication and division, it is advisable first to determine how many significant figures are required in the result, and then to work throughout keeping two more figures than this.

Square Root

Contracted method may also be used if it is desired to work out a square root to several figures. As a general rule, if more than half the number of figures required have been found by the ordinary method, the remaining figures may be found by contracted division.

Example. Find the square root of 5 to 6 places of decimals.

As seven significant figures are required, the first four must be found by the ordinary method.

```
         2·236068
         ________
         5·
         4
  42     100
          84
  443    1600
         1329
  ✓✓✓
  4466    27100
          26796
            304
            268
             36
```

Carry on contracted division by 4466. The saving of labour is obvious.

If eight significant figures were required, it would be necessary to obtain the first five before completing the divison by contracted method.

Exercises CXXIV

Multiply

1. 0·93969 by 77. Answer to 3 significant figures.
2. 0·70711 by 8·5. Answer to 3 significant figures.
3. 0·86603 by 77·8. Answer to 3 significant figures.
4. 19·68 by 0·17365. Answer to 3 significant figures.

Divide

5. 0·9369 by 0·3402. Answer to 4 significant figures.
6. 0·5 by 0·8603. Answer to 4 significant figures.
7. 0·258824 by 0·96593. Answer to 4 significant figures.
8. 0·9848176 by 0·173657. Answer to 4 significant figures.

Multiply

9. 123·45678 by 65·4321. Answer to nearest whole number.
10. 26607 by 149372. Answer to nearest million.
11. 29·3506 by 13·56813. Answer to 1 decimal place.
12. 10·07276 by 782·345. Answer to 2 decimal places.

Divide

13. 0·044672 by 72·568. Answer to 5 decimal places.
14. 0·0727643 by 278·289. Answer to 3 significant figures.
15. 782·345 by 10·07276. Answer to 4 significant figures.

Evaluate to 3 significant figures

16. $(3{\cdot}14159)^2$.

17. $(0{\cdot}071432)^2$.

18. $\dfrac{1}{0{\cdot}26607}$.

19. $\dfrac{1}{7{\cdot}8956}$.

20. $\dfrac{1000}{23{\cdot}567}$.

21. 800 tons of coal are used to generate an output of 535670 units of electricity. Find (to two decimal places) the number of pounds of coal used per unit of output of electricity.

22. £35,390,609 of gold was produced in 1919 and £35,758,316 in 1918. Express the 1918 output as a percentage of the 1919 one. Answer to 4 significant figures.

Find the value, to 4 significant figures, of

23. $(1{\cdot}045)^3$.

24. $100/(1{\cdot}045)^3$.

25. $\dfrac{3 \times 28{\cdot}73}{4 \times 3{\cdot}1416}$.

26. $180 \times (1{\cdot}00034)^3$.

27. Find, to six significant figures, $1 \div (1{\cdot}025)^4$.

28. Find, to the nearest shilling, the sum which will amount at 3 per cent. per annum compound interest paid yearly to £1000 in 5 years.

29. Find the square root of 10 to 6 places of decimals.

30. Find the square root of 1·05 to 6 places of decimals.

PROBLEM PAPERS (MENTAL)

These papers are intended for practice in rapid calculation and for revision of principles. The designation 'Mental' need not be interpreted strictly.

I

1. Add 35, 79, 68.
2. Subtract 15*s.* 7*d.* from £1 8*s.* 3*d.*
3. Multiply 87 by 13.
4. Add $3\frac{3}{4}$, $4\frac{1}{3}$.
5. Divide 1853 by 8, and give the remainder.
6. Find the cost of 2 gal. of vinegar at 7*d.* a pint.
7. How many pence are there in 17*s.* 5*d.*?
8. Multiply 21·4 by ·6.

II

1. Subtract 978 from 1205.
2. Add 12*s.* 5*d.*, 3*s.* 8*d.*, 4*s.* 11*d.*
3. Divide 7631 by 13.
4. From $10\frac{3}{5}$ take $8\frac{3}{4}$.
5. Divide 21·7 by ·07.
6. Find the cost of a gross of eggs at 1*s.* 7*d.* a dozen.
7. How many $1\frac{1}{2}d.$ stamps can be bought for 4*s.*?
8. Find the cost of 60 oranges at 5 for 6*d.*?

III

1. Simplify $\frac{3+5}{5+7}$.
2. Multiply $2\frac{2}{3}$ by $\frac{3}{4}$.
3. How many half-crowns make £5 12*s.* 6*d.*?
4. Divide ·0126 by ·06.
5. Find the cost of 20 articles at 1*s.* 5*d.* each.
6. Find the area of a rectangle 1·7 in. by 2·4 in.
7. Express $\frac{5}{8}$ as a decimal.
8. Find the value in £ *s.* *d.* of 1000 pence.

IV

1. Divide 16 by $2\frac{2}{3}$.
2. Reduce $\frac{12}{32}$ to its lowest terms.
3. Find the cost of $2\frac{1}{2}$ lb. of butter at 1*s.* 4*d.* per lb.

4. Add 14·72, 2·16, 16·8.

5. Find the length of a rectangle whose area is 14·4 sq. yd. and width 2·4 yd.

6. Find 15 per cent. of £1.

7. Divide £176 10*s*. by 12.

8. Express 3*s*. 4*d*. as a fraction of 7*s*. 6*d*.

V

1. Find the cost of $3\frac{3}{4}$ lb. of ham at 3*s*. 6*d*. per lb.

2. If 5 copies of a book cost 37*s*. 6*d*., find the cost of 8 copies.

3. Express as a single fraction $12\frac{2}{3} + 2\frac{3}{4} + 5\frac{5}{6}$.

4. Simplify $(0{\cdot}9)^2 \div 30$.

5. Express 2*s*. 4*d*. as a decimal fraction of 5*s*. 10*d*.

6. How many pints are there in $59\frac{1}{2}$ gal.?

7. Find the volume of a cuboid 11 in. by $5\frac{1}{2}$ in. by 3 in.

8. On a map 6 in. represent a mile. What length represents 13 f.?

VI

1. Fill in the blank space in this addition sum:

```
 2189
 8753

15027
```

2. Find the value of $({\cdot}02)^3$.

3. Find the cost of 15 boxes of cigarettes at 4*s*. 3*d*. per box.

4. Add together the multiples of 5 from 15 to 40 inclusive.

5. What fraction (in its lowest terms) is 11*s*. 8*d*. of £1?

6. A girl has ·75 of £1; she spends ·75 of it. How much in shillings and pence has she left?

7. What decimal of 1 yd. 2 ft. is 9 in.?

8. If $2\frac{1}{2}$ lb. of chocolate cost 6*s*. 8*d*., what is the cost per lb.?

VII

1. Multiply 63 by 19.

2. Subtract $16\frac{1}{12}$ from $20\frac{3}{4}$.

3. How many eggs at 1*s*. 6*d*. per dozen can you buy for £1?

4. Reduce 5 tons 17 cwt. 2 qr. to quarters.

5. If 3 pennies weigh 1 oz., find the weight of £5 worth of pennies.

6. Find the prime factors of 108.

7. Fill in the blank space in this addition sum:

£	s.	d.
37	2	$8\frac{1}{2}$
28	19	$3\frac{1}{4}$
£110	18	$4\frac{1}{2}$

8. How many days are there from April 27 to July 27? (Count only one of the given dates.)

VIII

1. Divide 1732 by 17, giving quotient and remainder.
2. Express as a decimal 60 ÷ 2500.
3. The area of a rectangle is 6·23 sq. in. and the length 3·5 in.; find the breadth.
4. If 8 Km. equal 5 ml., how many Km. equal a length of $17\frac{1}{2}$ ml.?
5. Find $\frac{3}{5}$ of 6 ml. 2 f.
6. Express 253 grm. in Hectograms.
7. A man's travelling expenses are 1*s.* 5*d.* per day, six days each week. How much does this amount to in 13 weeks?
8. Tom is 5 years younger than Dick, who was 13 in 1936. In what year will Tom be 19?

IX

1. Multiply 8·5 by ·012.
2. Find the cost of $3\frac{1}{4}$ lb. of tea at 2*s.* 6*d.* per lb.
3. Reduce 7 ac. 3 r. 20 p. to square poles.
4. A matchstick is $\frac{1}{10}$ in. by $\frac{1}{10}$ in. by $1\frac{3}{4}$ in. Write down its volume as a decimal of a cubic inch.
5. Find 35 per cent. of 1 ml. in chains.
6. If January 1 is a Tuesday, what day of the week is February 1?
7. A tennis court measures 78 ft. by 36 ft. What is its area in square yards?
8. A monthly railway ticket costs 1*d.* per mile. What is the fare to and from a place 228 ml. away?

X

1. Multiply 76 by 21.
2. Simplify $3\frac{1}{3} \div 2\frac{2}{3}$.
3. Find how many minutes it is from 10·45 A.M. to 2·20 P.M.
4. How many square yards are there in $\frac{1}{5}$ ac.?
5. What percentage of £1 is 3*s.* 6*d.*?

6. Find the L.C.M. of 36 and 84.

7. Express 25700 litres in Kilolitres.

8. 100 lb. of wheat produce 70 lb. of flour. How many pounds of flour will a ton of wheat produce?

XI

1. Find the value of $(50)^2 \div (0{\cdot}5)$.

2. How many rose-trees at 10*s.* 6*d.* per dozen can be bought for £21?

3. The volume of a rectangular room is 1710 cu. ft. and the floor area 180 sq. ft. Find the height.

4. Convert a price of $7\frac{1}{2}$*d.* per lb. into £ *s.* per cwt.

5. Find the value of 1000 eggs at 9*s.* per 120.

6. Express 2*s.* 4*d.* as a percentage of 5*s.*

7. A sheet of paper 6 in. by $8\frac{1}{2}$ in. is divided by lines parallel to its edges into $\frac{1}{2}$ in. squares. How many squares are there?

8. Tom and Tim had £1 17*s.* 6*d.* between them. How much had Tom if he had 3*s.* 6*d.* more than Tim?

XII

1. Reduce £6 1*s.* 9*d.* to threepences.

2. Find 45 per cent. of 2 tons.

3. Find the cost of 240 oranges at 7 for 6*d.*, odd ones 1*d.* each.

4. Find the volume of a cuboid 3·5 in. by 2·5 in. by 2 in.

5. A hundredweight costs 14*s.* How much per lb. is this?

6. If January 1 is a Friday, and a man earns 12*s.* every day except on Sundays, how much will he earn during the month?

7. A bottle and stopper together weighed 5 oz. The bottle weighed 3 times as much as the stopper. What did the bottle weigh?

8. Give a rough estimate of the value of $\sqrt{15 \times 4840}$.

XIII

1. Simplify $0{\cdot}6 \div 0{\cdot}012$.

2. Multiply 873 by 25.

3. What percentage of 1 ton is 7 cwt. 1 qr.?

4. A common unit of length in medieval times was 13·2 in. What percentage of a foot is this?

5. How many yards are there in 7 ch. 17 yd.?

6. A father and his two sons are 49, 17, and 15 yr. old respectively. What is the average of their ages?

7. Find the simple interest on £200 at $3\frac{1}{2}$ per cent. per annum for 3 years.

8. A man making a wool rug found that each row took 25 min. to do. How long in hours will he take to make the whole rug, which has 240 rows?

XIV

1. Divide $4\frac{2}{7}$ by $3\frac{3}{4}$.

2. Find the cost of $3\frac{1}{2}$ yd. of silk at 4*s*. 11*d*. per yard.

3. Fertilizer is to be applied at a rate of 4 oz. to the square yard to a rectangular plot of ground 90 ft. by 140 ft. How much fertilizer (in cwt. and lb.) will be required?

4. Posts are set in the ground at intervals of 25 ft. How far is it from the first post to the tenth?

5. Subtract 173 tons 9 cwt. 3 qr. from 241 tons 5 cwt.

6. An agent receives a commission of 5 per cent. on his sales. What commission would he receive on sales totalling £325?

7. Fill in the blank space in this division sum:

$$\underline{\quad)18123}$$
$$1647 \text{ Remainder: } 6.$$

8. The houses on one side of a street bear the odd numbers from 1 to 57 inclusive; the houses on the other side bear the even numbers from 2 to 54. By how much does the sum of the numbers of the houses on one side exceed the sum of the numbers on the other?

XV

1. Reduce 500 in. to yards, feet, and inches.

2. Divide 98375 by 25.

3. How many tins of sardines at $5\frac{1}{2}d$. each could be bought for 10*s*., and how much change would there be?

4. Express 3*s*. $2\frac{1}{2}d$. as a decimal of £1 to 3 places.

5. Find the perimeter of a room 18 ft. 3 in. by 13 ft. 6 in.

6. Find the cost of 103 tins of polish at $6\frac{1}{2}d$. per tin.

7. When it is noon in London it is 7 A.M. in New York; what is the London time at 8.30 P.M. in New York?

8. Find the average of 30·3, 30·28, 30·27, 30·31.

XVI

1. How many steps of 2 ft. 6 in. must be taken to go $\frac{1}{2}$ ml.?

2. If $2\frac{1}{2}$ tons of coal cost £6 5*s*., find the cost of 3 tons.

3. What common multiples of 6, 8, 10 lie between 800 and 1000?

4. A bag contains £5 worth of shillings and half-crowns. The value of the half-crowns alone is half as much again as that of the shillings alone. How many coins are there in all?

5. If a gallon of water weighs 10 lb., how many ounces does a quarter-pint weigh?

6. Find the cost of 1·7 m. at 3 fr. 50 c. per metre.

7. A sheet of paper is $9\frac{1}{2}$ in. from the top to the bottom edge and has 23 lines ruled at equal intervals parallel to these edges. The space below the bottom line equals that between any two consecutive lines, and that above the top line is twice this distance. What is the space between lines?

8. Express 17*s*. $10\frac{1}{4}$*d*. as a decimal of £1 to 3 places.

XVII

1. Multiply 583 by 31.

2. A cyclist travelled 36 ml. in 2 hr. 15 min. What was his average speed in miles per hour?

3. If 4 men can dig a trench in 6 days, how long should 3 men take to do the same?

4. Express £·817 in shillings and pence to the nearest farthing.

5. How many hours and minutes is it from 7·35 P.M. on Thursday to 6·20 A.M. on Saturday?

6. Find the simple interest on £325 at $4\frac{1}{2}$ per cent. per annum for 4 years.

7. Find the cost of £3000 stock at $87\frac{3}{8}$.

8. Divide £350 between *A* and *B* in the ratio of 5 to 3.

XVIII

1. Find the change from £5 after a bill for $4\frac{1}{2}$ yd. of velvet at 18*s*. 11*d*. per yard has been paid.

2. A house bought for £960 was sold at a profit of $7\frac{1}{2}$ per cent. What was the selling price?

3. A shopkeeper found that he had taken £55 during one day and estimated that 15 per cent. of this sum was profit. What was this profit in pounds and shillings?

4. How many hours, minutes, seconds are there in 10,000 sec.?

5. The simple interest on £450 for 2 yr. was £27. What was the rate?

6. Find the income from £650 $3\frac{1}{2}$ per cent. stock.

7. A mixture of tea contains 11 parts by weight of cheap tea to 5 parts of a better tea. How much of the better tea is there in 1 cwt. of the mixture?

8. A rectangular tank 6 ft. long, 4 ft. wide contains 600 gal. of water. How deep is the water if 1 cu. ft. = $6\frac{1}{4}$ gal.?

XIX

1. A car was sold for £189 at a loss of 10 per cent. on the cost price. What was the cost price?

2. In what time would the simple interest on £425 at 4 per cent. p.a. amount to £42 10*s.*?

3. How much stock at 88 can be bought for £11000?

4. The value of $\frac{10{\cdot}93 \times {\cdot}00712}{{\cdot}518}$ has as significant figures 150. Insert the decimal point.

5. How much $2\frac{1}{2}$ per cent. stock at 86 must be bought to give an income of £300?

6. Divide 196,875 by 125.

7. Find the area of a border 2 in. wide outside a rectangle 8 in. by $10\frac{1}{2}$ in.

8. In a certain school boys either learn French or German or do both. Altogether 85 per cent. learn French and 55 per cent. learn German. What percentage of the boys learn both languages?

XX

1. Divide 28752 by 31, and give the remainder.

2. Find the simple interest on £160 at $3\frac{1}{2}$ per cent. per annum for 6 months.

3. Find the cost of £5500 stock at $93\frac{1}{4}$.

4. Trams running at an average speed of 12 m.p.h. pass a certain point at intervals of $1\frac{1}{2}$ min. What is the distance between successive trams?

5. A car travelling at 30 m.p.h. is catching up a cyclist who is travelling at 10 m.p.h. In how long will the car overtake the cyclist if the latter is $\frac{1}{4}$ ml. in front of the car?

6. The value of $\frac{({\cdot}391)^2 \times 2751}{(6{\cdot}81)^3}$ is given as 133, but the decimal point has been omitted. Where should it be?

7. Find the income from £550 $4\frac{1}{4}$ per cent. stock.

8. How many tiles 4 in. square would be required for a passage-way 3 ft. 8 in. by 15 ft.?

PROBLEM PAPERS

Papers I–X,	not	beyond	Ch.	X.
XI–XX,	,,	,,	,,	XVIII.
XXI–XXX,	,,	,,	,,	XXIV.
XXXI–XL,	,,	,,	,,	XXVI.
XLI—L,	,,	,,	,,	XXIX.
LI–LX, on the whole book.				

Give all answers which do not work out exactly to a reasonable degree of approximation, unless otherwise instructed.

I

1. 325,000 tons of sulphate of ammonia are produced annually from coke-ovens. Find its value at £9 7*s.* 6*d.* per ton.

2. The weekly wages of certain workers were reduced in 1931 from £4 2*s.* 11*d.* to £3 14*s.* 2*d.* Find the difference made by this fall in the weekly wage-bill of a company employing 1250 of these workers.

3. Add together 1*s.* 11·56*d.*, 2*s.* 1·32*d.*, 1*s.* 8·37*d.*, and 2*s.* $9\frac{3}{4}$*d.*, and subtract the total from 10*s.*

4. The amount of grease in Australian merino fleeces is 48 per cent. of the weight. Find the weight of 1 ton 2 cwt. 11 lb. of wool, after it has been cleansed of grease.

5. (1) Simplify $\dfrac{4\frac{1}{4} - 2\frac{1}{2}}{1\frac{1}{3} - 1\frac{1}{4}}$. (2) Divide 6·2747 by 0·085.

6. Find the prime factors of 532 and 342, and their H.C.F.

II

1. Cape crayfish are canned and sold at 2 guineas per 48 lb.; find the cost of 6 cwt.

2. The Burma yield of rubber is 243 lb. per acre. Find the yield from 9500 acres in tons.

3. The English foot in the twelfth century was 13·22 in. long. How many of these feet, to the nearest whole number, would there be in the mile of to-day?

4. Ironstone in the raw state contains 24·28 per cent. by weight of iron. Find the amount of iron in a year's output of 12,025,000 tons of ironstone.

5. (1) Find the square of 0·582. (2) Divide 93 by 0·205 to 2 places of decimals.

6. How many Swiss francs will £5 8*s*. buy, when the exchange is £1 = 25·15 fr.?

III

1. A pigeon won a race of 68 ml. by flying at an average rate of 1530 yd. a minute. How long did it take?

2. In the course of a year a County Council supplied 4,867,000 free dinners to school-children at an average cost of £1 16*s*. 6*d*. per hundred dinners. Find the total amount that the Council paid for the dinners.

3. Find the L.C.M. of 11011 and 8281.

4. What number, added to the sum of 13·0009, 4·5672, 1·89, and 0·0079, will make a total of 20?

5. Find $3\frac{3}{4}$ per cent. of £255.

6. A farm labourer started work on the morning of Thursday, July 18, and worked every day until the evening of the last day of the next month. If he was paid 6*s*. 6*d*. for every week-day and half as much again on Sundays, find his total wages.

IV

1. South Wales pits can put their coal on board ship at a cost of 2*s*. 8*d*. per ton, while South Yorkshire pits pay 7*s*. 6*d*. per ton for carriage to Hull. Find how much more carriage costs the Yorkshire than it would the Welsh pits for an export of 7,500,000 tons.

2. The Aire and Calder Canal is 85 ml. long and carries $3\frac{1}{2}$ million tons per annum. Find, to the nearest 100 tons, the number of tons carried on the average per mile of canal.

3. A man who takes 7 steps in 3 sec. walks $6\frac{1}{2}$ Km. in an hour. Find the length of his stride to the nearest cm.

4. Calculate the value of 56·29 × 43·7 to 3 significant figures.

5. Find the value of $12{\cdot}8 \times 4{\cdot}25 - 5{\cdot}5 \times 3\frac{1}{11}$.

6. Find the cost of 1 cwt. of rubber at $6\frac{1}{4}$*d*. per lb.

V

1. Wire ropes 11 ml. long and weighing 31 tons have been made in Sheffield. Find the weight per foot in lb. to two decimal places.

2. Middlesbrough was rated at £535902 and South Shields at £495913. How much more would a rate of 2*d*. in the £ have produced in Middlesbrough than in South Shields?

3. Find the cost of carting 5 tons 8 cwt. of goods for 10 ml. at 2*s.* $7\frac{1}{2}$*d.* per ton per mile.

4. Express £5 12*s.* 6*d.* in francs when £1 is worth 75·52 fr.

5. Simplify (1) $\dfrac{2 + \frac{1}{5}}{3 - \frac{1}{10}}$, (2) 0·56 of £5 7*s.* 6*d.*

6. Express the weight of a gallon of nitric acid, 14 lb. $3\frac{1}{4}$ oz., as a decimal of 1 lb.

VI

1. Find the cost of making 350 tons of steel at £6 18*s.* 9*d.* per ton.

2. A rate of 2*s.* $7\frac{1}{2}$*d.* in the £ on a parish produced £745 10*s.* What was the rateable value of the parish?

3. Telegraph posts are placed along a certain road at distances of 60 yd. apart. A motor-car is timed to travel from the first to the fifth post in 27 seconds. Find the average speed of the car in miles per hour.

4. (1) Find the value of 0·325 of £3 5*s.* (2) Express 1 ton 3 cwt. $17\frac{1}{2}$ lb. as the decimal of a ton.

5. Simplify $3\frac{5}{51} - 2\frac{1}{34} + 1\frac{5}{68}$.

6. Find the prime factors of 6776, and deduce the smallest multiplier which will make it a perfect square.

VII

1. Find the cost of carriage of 2 tons 5 cwt. 2 qr. of goods for 24 ml. at 1*s.* $11\frac{1}{2}$*d.* per ton per mile.

2. Express 0·567 of £1 in shillings and pence to the nearest farthing.

3. The total area of rubber-plantations in the Indian Empire was given as 204663 ac. If 45 per cent. of these were in Burma and 32 per cent. in Travancore, find the number of acres in each of these provinces to the nearest 100.

4. Find the value of $\dfrac{0{\cdot}66 \times 1{\cdot}69}{{\cdot}078}$.

5. If $\frac{3}{8}$ of $\frac{5}{7}$ of a man's income is £591 7*s.* 6*d.*, find his total income.

6. If a goods train is 176 yd. long and takes 30 sec. to pass a signal, at what rate is it travelling?

VIII

1. Out of a total year's output of 7,894,000 tons of steel ingots and castings, 1,740,000 were made in the Cleveland district. Express the Cleveland output as a percentage of the whole to 3 figures.

2. What is the smallest number which, when multiplied by 21, gives a product of which each digit is 1?

3. A householder pays rates at 7*s.* 6*d.* in the £1 on five-sixths of the rent of his house. If the rates amount to £17 10*s.*, what is the rent of his house?

4. Find by prime factors the L.C.M. of 1404 and 1836.

5. Multiply (1) 50·17$\frac{3}{4}$ by 36, (2) 5 Hg. 3 grm. by 101.

6. Find the cost of 1200 articles at £1 1*s.* 3*d.* each.

IX

1. If £1 6*s.* 11*d.* is divided between 8 boys and 5 girls so that a boy has half as much again as a girl, what does each receive?

2. Some property is insured against fire for 85 per cent. of its value; how much will the owner receive if the whole property is destroyed, and its value is £2350?

3. 35 wardrobes of equal value were bought for £465 10*s.* How many more of another kind costing £1 1*s.* each less could have been bought for the same sum?

4. Simplify $\frac{1·96 \times 2·25}{0·14 \times 15}$.

5. Estimate 87$\frac{1}{2}$ per cent. of £225 4*s.* 8*d.*

6. Calculate the simple interest on £380 for 2$\frac{1}{2}$ yr. at 5 per cent. per annum.

X

1. Taking a rupee as 1*s.* 4$\frac{1}{2}$*d.*, find the value in English money of 1750 rupees.

2. A bachelor with an income of £400 pays 7·5 per cent. of it in income tax, but in 1912 he would have been charged only 2·25 per cent. of the same income. Find the difference between the two amounts of tax.

3. In 20 years 560 passengers were killed in railway accidents in the United Kingdom. The total number of passengers carried was estimated at 25,000 million passengers. Find the number killed per hundred thousand passengers.

4. A man is walking at 3$\frac{1}{2}$ ml. per hour. How many steps each 2 ft. 9 in. does he take in a minute?

5. Find the simple interest on £550 for 3 yr. at 2$\frac{7}{8}$ per cent. per annum.

6. Find the area in square feet of the floor of a room 13 ft. 4 in. by 16 ft. 6 in.

XI

1. A mining-company sold 1753 tons of lead in one month at £23 7*s*. 1*d*. per ton; how much did it receive?

2. Find the square root of 480249.

3. Find the simple interest on £550 10*s*. for 3 yr. at $3\frac{1}{3}$ per cent. per annum.

4. Divide 253·6 by 15·7 to two places of decimals, and give the exact remainder.

5. 3*s*. $2\frac{1}{4}$*d*. = £0·159 + ·09*d*.; find the cost of 550 oz. at 3*s*. $2\frac{1}{4}$*d*. per oz.

6. A tank measuring 3 ft. each way is filled with water to a depth of 1·6 ft. How many solid cubes each of volume 0·8 cu. ft. can be placed in the tank before the water begins to overflow?

XII

1. The terms at an hotel are 27·50 fr. per day, and a 10 per cent. tax is added to this for service. How many days can a visitor stay for 350 fr., and how much will he have left?

2. If 2·237 miles per hour = 1 metre per sec., find the equivalent of 60 miles per hour in metres per sec.

3. Find to four significant figures the value of $\frac{1}{32{\cdot}5} + \frac{1}{23{\cdot}6}$.

4. An engineering workshop is 550 ft. long, 365 ft. wide. Find its area in perches to the nearest perch.

5. Convert the sum of $4{\cdot}83\frac{3}{4}$, $5{\cdot}72\frac{7}{8}$, and $3{\cdot}25\frac{1}{2}$ dollars into English money at 4·75 dollars to the £.

6. If there are 32 oz. of sulphur in 98 oz. of sulphuric acid, find the weight of sulphur in 1 cwt. of acid.

XIII

1. Find the number of £1 shares in a company that could be bought for £50 at 18*s*. 6*d*. per share. How much money would be left over?

2. Find the cost of 488 oz. of silver at 2*s*. $5\frac{7}{8}$*d*. per oz.

3. Find (1) $67\frac{1}{2}$ per cent. of £520 10*s*., (2) $113\frac{1}{3}$ per cent. of 3 tons 3 cwt. 3 qr.

4. A room is 18 ft. long and 13 ft. 6 in. wide. How many yards of carpet 27 in. wide would be required to cover the floor completely? What would be its cost at 5*s*. 11*d*. per yard?

5. Find the cost of printing a year's issue of 389 million tube tickets at an average price of 2*s*. 8*d*. per thousand.

6. Divide the sum of £1170 6*s.* 6*d.* between three partners in the ratio 6 to 4 to 3.

XIV

1. A gallon of castor oil weighs 9 lb. $2\frac{1}{2}$ oz. Find the weight of 144 gal.

2. Out of a population of 42 millions, there were 450,000 children attending secondary schools. Find the number per thousand of the population to 3 significant figures.

3. If a Kilogram equals 2·204 lb., find the number of Kilograms in 1 lb. Answer to 3 decimal places.

4. Find the value of 0·375 of £3 6*s.* 8*d.* + 3·75 of 3*s.* 4*d.*

5. If 24 men can do a piece of work in 20 days, how long would 15 men take to do the work?

6. Find the volume of a wooden block which measures $10\frac{1}{2}$ in. by $4\frac{2}{3}$ in. by $3\frac{1}{4}$ in.

XV

1. A gallon of sulphuric acid weighs 18 lb. 7 oz. Find to 1 decimal place the number of gallons per ton of acid.

2. Find the square of 691, and hence write down the squares of 69·1 and 0·0691.

3. Divide a sum of £850 between two persons so that one gets half as much again as the other.

4. Simplify $1\frac{3}{4} \times (10\frac{2}{3} - 6\frac{5}{7})$.

5. Find the simple interest on £830 for 2 months at 3 per cent. per annum.

6. A room is 20 ft. long, 15 ft. wide, and 14 ft. high. Find the area of walls to be distempered, allowing 70 sq. ft. for doors, windows, and fireplace.

XVI

1. In a certain year the taxation per head in Great Britain was £22 0*s.* 6*d.*, while in the United States it was £10 1*s.* 3*d.* Express the taxation in the United States as a percentage of that in Great Britain to the nearest integer.

2. A map is drawn on a scale of 2·5 in. to the mile. Give the ratio of the distance on the map to the distance it represents (1) as a decimal to 2 significant figures, (2) as a fraction whose numerator is 1.

3. The length and breadth of a tennis court are 78 ft. and 36 ft. respectively. Calculate the length of the diagonal to the nearest $\frac{1}{2}$ ft.

4. If 12 cu. ft. of bathstone cost 38*s.* 9*d.*, find the cost of 135 cu. ft. to the nearest shilling.

5. Prove that a fall of $\frac{1}{10}$ of an inch of rain represents about 10 tons to the acre. (1 cu. ft. of rain-water weighs 62·3 lb.)

6. Black copper oxide contains copper and oxygen in the proportion by weight of 63 to 16. Find to the nearest gramme the weight of each constituent in 1 Kg. of the oxide.

XVII

1. Spanish orange-marmalade pulp was sold at 22*s.* 6*d.* per 100 lb. Find the price per ton.

2. A gravel walk 6 ft. wide runs round a grass plot 60 ft. long and 40 ft. wide. If gravel costs 3*s.* per cu. yd., find the cost of laying gravel to a depth of 3 in.

3. Simplify $12\frac{3}{4} - 1\frac{3}{5}$ of $3\frac{1}{3}$.

4. If 5 men and 2 boys do a piece of work in 6 days, how long would 3 men and 4 boys take, assuming that 1 man does as much work as 2 boys?

5. The population of the United Kingdom at the beginning of a year was 43,660,000 people. In the course of the year, for every 10,000 inhabitants at the beginning of the year there were 269 births and 155 deaths. In addition 205,000 persons entered the country, and 459,000 emigrated from it. Find the population at the end of the year to the nearest ten thousand.

6. Of what sum is £45 15*s.* $7\frac{1}{2}$ per cent.?

XVIII

1. On a map whose scale is $\dfrac{1}{633{,}600}$ the distance between Oxford and Cambridge is $6\frac{3}{8}$ in. Find the actual distance between them in miles.

2. Find the cost of 13500 bricks at £5 1*s.* 6*d.* per thousand.

3. By converting them into decimal fractions, arrange in ascending order of magnitude $\frac{25}{91}$, $\frac{102}{365}$, $\frac{407}{1459}$.

4. Divide 85 ml. into two parts so that $\frac{2}{3}$ of one part equals $\frac{3}{4}$ of the other.

5. A mowing-machine cuts a lawn, advancing at 4 miles per hour. How wide is its cutter if it mows 10,560 sq. yd. in 72 min.? (Neglect overlapping.)

6. Find V from the formula $V = 19{\cdot}74\, r^2x$, when $r = 2{\cdot}5$, $x = 6{\cdot}1$.

XIX

1. Find the cost of 3 cwt. 2 qr. 7 lb. of cobalt at £81 7*s*. 6*d*. per cwt.

2. A train travels at 35 miles per hour for half an hour and then at 55 miles for three-quarters of an hour. Find the average rate for the whole time.

3. What length of carpet 30 in. wide will be required for a room 17 ft. 4 in. long, 13 ft. 9 in. wide. If the carpet costs £5 19*s*. 2*d*., find the price per yard.

4. Simplify $\frac{3}{4}\{1\frac{3}{5} + 2\frac{2}{3}\} \div \{\frac{1}{3} - \frac{1}{5}\}$.

5. The number of passengers who travelled on a country bus route was 484 in one week. This represented an increase of 10 per cent. on the number for the previous week. Find that number.

6. Find the cost of 488 bush. at $0{\cdot}85\frac{5}{8}$ dollars per bushel.

XX

1. A certain stock of potatoes is sufficient to supply 11 men for 6 days and leave 201 lb. over, or to supply 21 men for 8 days and leave 48 lb. How many pounds are there in the stock?

2. Taking the cost of freight of imported beef as $1\frac{3}{4}$*d*. per lb. plus 5 per cent., find the total cost of freight for an import of 25 tons 5 cwt.

3. 12379 new books and editions were published in 1913, 7716 in 1921, and 9547 in 1934. Express as percentages to 2 significant figures the increase in 1934 compared with 1921, and the decrease in 1934 compared with 1913.

4. Find the simple interest on £365 10*s*. 6*d*. for 7 months at $3\frac{1}{2}$ per cent. per annum (to nearest 1*d*.).

5. If $R = 2{\cdot}5 + \dfrac{V^2}{50{\cdot}8 + 0{\cdot}0278\,L}$, find R when $V = 60$, $L = 300$.

6. The cost of making steel before the introduction of the Bessemer process was 715 per cent. above the market price afterwards. If the latter price was £6 18*s*. 9*d*. per ton, what, to the nearest shilling, was the previous cost?

XXI

1. A night watchman was paid at the rate of 1*s*. 5*d*. per hour up to 12 midnight and after that at a time and a quarter rate, but on Sundays each hour after midnight was paid at time and a half rate. What did he earn in a week of 7 days, starting each night at 8·30 P.M. and finishing at 5.30 A.M.?

2. Find to the nearest franc the cost of lining an open tank 4·6 m. long, 2·25 m. wide, and 1·5 m. deep at 4 fr. 75 c. per square metre.

3. Calcium carbonate is bought wholesale at 42*s.* per cwt. and retailed at 7*d.* per lb. Find the total profit per cwt., and express it as a percentage (*a*) of the cost price, (*b*) of the selling price.

4. Divide the sum of £951 15*s.* between 3 people in the ratio of the fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.

5. Find, to four places of decimals, the value of

$$\frac{4{\cdot}7 \times 1{\cdot}25 - 0{\cdot}26 \times 1{\cdot}99}{0{\cdot}072 \times 5{\cdot}8}.$$

XXII

1. A diamond-mining company reports that the average selling price per carat was £6 5*s.* and the working costs 20*s.* per carat. The Government taxation is 66 per cent. of the value of the diamond less 70 per cent. of working cost. Find how much the Government receives per carat, and the company's profit.

2. Thirty-six loads of gravel are laid evenly on a path 5 ft. wide and $\frac{1}{4}$ ml. long. If each load contains 25 cu. ft., find to the nearest tenth of an inch the thickness of the layer.

3. Explain why the following statement is obviously wrong:

$$\frac{27^2 \times 7 \times 23}{8 \times 22 \times 277} = 24{\cdot}07.$$

4. If 1 ch. = 20·1168 m., express 1 m. in chains to 3 significant figures.

5. In a cricket match between two schools, *A* and *B*, *A* made 85 runs more than *B* in the first innings, but in the second innings *B* made 64 per cent. more runs than *A* and won by 43 runs. What were the second innings scores?

XXIII

1. A wheel of 7 in. diameter is rotating at 3000 revolutions per minute. Find the speed of a point on the rim of the wheel in feet per second. (Take $\pi = 3\frac{1}{7}$.)

2. If the price of gas is given as 7·79*d.* per therm, equivalent to 3*s.* 1*d.* per 1000 cu. ft., find a decimal multiplier which will reduce a price in pence per 1000 cu. ft. to pence per therm.

Use it to find the price per therm equivalent to 2*s*. 6*d*. per 1000 cu. ft.

3. Why are the following statements obviously wrong? (1) $(0{\cdot}339^2) = 1{\cdot}151$. (2) $\sqrt{0{\cdot}4816} = 0{\cdot}2194$.

4. A man bought an estate and subsequently sold it for £625 less than he paid for it, losing $1\frac{1}{4}$ per cent. on his outlay. At what price did he buy?

5. A gas-meter indicated that 89300 cu. ft. of gas had been consumed. A test showed that it registered 1030·9 cu. ft. for every 1000 cu. ft. actually consumed. Find the total amount actually consumed and the cost at 2*s*. 9*d*. per 1000 cu. ft.

XXIV

1. A man bought 5000 articles at $2\frac{1}{4}d$. each, and sold 4250 at $3\frac{3}{4}d$. each, and the remainder at 2*d*. each. Find his total profit.

2. The cost of producing a photogravure of a certain kind is £30 for the plate and 3*s*. 6*d*. for each copy. What is the least number of copies that must be sold at 15*s*. each to ensure a profit?

3. The area of a square field is 4·9 ac. How long will a man take to walk round the edge of it at $3\frac{1}{2}$ ml. per hour?

4. 15 per cent. of a certain sum of money is £77 17*s*. 6*d*. Find $37\frac{1}{2}$ per cent. of the same sum.

5. A legacy of £2420 was left to three persons so that after each had paid a duty of 15 per cent. the first had twice as much as the second and the second three times as much as the third. What were their legacies before payments of duty?

XXV

1. Find the cost of 3 ac. 2 r. 30 p. of land at £75 an acre.

2. The hold of a barge is 30 ft. long, 5 ft. 6 in. wide, and 7 ft. deep. Find in tons (to the nearest integer) what weight of coal it would carry, the top surface being level, allowing 40 cu. ft. to the ton.

3. A man rode a bicycle from *A* to *B* (54 ml.) at an average speed of 8 ml. an hour; another man started from *A* half an hour after the first and arrived at *B* 15 min. before him. Find the average speed of the second man.

4. Find the compound interest on £350 at $3\frac{1}{2}$ per cent. paid yearly for 3 years.

5. Express as decimals to 4 places $\frac{9}{5} + \sqrt{\frac{9}{5}}$, $\frac{355}{113}$, $\frac{19\sqrt{7}}{16}$.

XXVI

1. A shopkeeper sold a number of eggs one week when the price was $2\frac{1}{4}d.$ each, and the next week when the price had fallen to $2d.$ each, he sold 10 dozen more than the first week. Altogether he received £9 10*s.* for them. How many eggs did he sell altogether in the two weeks?

2. Ferrous sulphide contains iron and sulphur in the proportion by weight of 7 to 4. Find the weight of iron in 1 Kg. of sulphide to the nearest gramme.

3. What is the value to the nearest pound of a rectangular stack of coal 20 yd. long, 8 ft. wide, and 6 ft. high, worth 34*s.* 6*d.* per ton, reckoning 44 cu. ft. to the ton?

4. The old golf-ball had a diameter of 1·75 in., the new one has a diameter of 1·62 in. Find the difference between their circumferences (taking π as 3·142) to 3 significant figures.

5. One rupee = 16 annas = 64 pice. Express 85 rupees 13 annas 2 pice as a decimal of a rupee, and find the value in English money when the rupee is worth 1*s.* $7\frac{1}{2}d.$

XXVII

1. The area of the total surface of a cube is 18 sq. ft. 54 sq. in. Find the length of an edge of the cube.

2. The coal-bunkers of a steamer, when filled with a certain kind of coal, hold 375 tons. How many tons would they hold of a lighter coal which requires more space per ton than the first kind in the ratio $11\frac{1}{4}$ to 10?

3. Nitrate of soda is sold at £11 5*s.* per ton, this price representing the value of the nitrogen in it. If it contains $15\frac{1}{2}$ per cent. of nitrogen, find the value of an agricultural unit of nitrogen, *i.e.*, 1 per cent. of 1 ton.

4. The interest on a sum of money at $5\frac{1}{2}$ per cent. per annum amounted to £83 1*s.* in 6 months. What was the sum?

5. A, B, and C contributed £3000, £4500, £3500 of capital to start a business. After a year profits amounted to 16 per cent. of the total capital. A received £440 for managing the business, and the remainder of the profits was shared between them in the ratio of their capitals. How much did each receive?

XXVIII

1. How many postage-stamps, each 0·93 in. long and 0·77 in. wide, are there in a sheet of these stamps just over $21\frac{1}{4}$ in. long, and just under $15\frac{1}{2}$ in. wide, the length and breadth of sheet and stamps being measured the same way?

2. Find the value of £350 at New York when the exchange was $4{\cdot}68\frac{7}{16}$ dollars to the pound, and find the value in English money of 350 dollars.

3. Find to the nearest penny the simple interest on £300 for 91 days at 2 per cent. per annum.

4. If a cubic foot of rain-water weighs 63 lb., find to the nearest ton the weight of rain falling on a field of 7 ac. during a rainfall of ·08 in.

5. A man's income averaged £471 5*s.* for the two years 1932–1933, £499 for the two years 1933–1934, and £486 for the three years 1932–1934. What was his income in each year?

XXIX

1. A unit in the valuation of fertilizers is 1 per cent. of 1 ton. Peruvian Guano contains 12 per cent. of nitrogen valued at 14*s.* 6*d.* per unit, 5 per cent. of soluble phosphates valued at 2*s.* per unit, 15 per cent. of insoluble phosphates valued at 1*s.* 2*d.* per unit, and 2 per cent. of potash valued at 3*s.* $7\frac{1}{2}$*d.* per unit. The remainder is considered of no value. What should the price of a ton of the Guano be?

2. Calculate to 3 significant figures

$$\frac{1470 \times 0{\cdot}0589}{12{\cdot}5 \times 7\frac{1}{2} + 2{\cdot}75 \times 3\frac{1}{2}}.$$

3. Find the square root of 61 correct to 3 significant figures.

4. The value of a motor-car sold after 1 year had decreased by 15 per cent. of its cost. If it was sold for £153, how much did it cost?

5. A circular metal plate has a diameter of 11 in. and has 3 circular pieces stamped out of it, each of diameter 2 in. Find the area of the remainder, taking $\pi = 3\frac{1}{7}$.

XXX

1. On a map drawn to a scale of 25 in. to the mile, find what area is occupied by a farm of 100 ac. Answer in square inches to 1 place of decimals.

2. If 5 litres of a liquid weighing 1·13 times as much as water are mixed with 2 litres of a liquid which weighs 0·92 times as much as water, find how many times as heavy as water the mixture will be.

3. Simplify $\dfrac{3\frac{1}{3} \times 2\frac{1}{4}}{3\frac{1}{3} - 2\frac{1}{4}} \times \dfrac{2\frac{3}{5}}{5\frac{1}{7}}$.

4. Find the rate per cent. per annum at which the simple interest for 3 months on £570 would amount to £5 6*s*. $10\frac{1}{2}$*d*.

5. A contractor undertook to finish 4 ml. 960 yd. of road in 144 days. After employing 104 men for 112 days he found that only 2 ml. 640 yd. was completed. How many additional men should he engage just to fulfil his contract?

XXXI

1. Taking an inch as 2·54 cm., find the internal diameters of a 75-mm. field-gun and of a 42-cm. howitzer in inches to the nearest tenth.

2. Raspberry pulp is sold for preserving at 116*s*. per 100 lb. Find the price per ton.

3. A room is 20 ft. 6 in. long, 17 ft. 4 in. broad, and 12 ft. high, and the area of windows, doors, and fireplace altogether is 162 sq. ft. How much, to the nearest shilling, will it cost to paint the walls and ceiling at 2*s*. per square yard?

4. The bisector of the vertical angle of a triangle divides the base in the ratio of the other two sides. If these sides are 7 and 9 in. long and the base is 12 in., find the lengths of the parts of the base which are cut off by the bisector of the vertical angle.

5. A man invested £100 in German marks, and sold them at a profit of 20 per cent. when they stood at 22 to the pound; at what price did he buy them?

XXXII

1. In an examination a candidate must get 35 per cent. of the total marks in order to pass. There are 3 papers; on the first his marks are 56 out of 100, on the second 23 out of 100. How many must he get out of 200 marks on the last paper, if he is to pass?

2. Some Sheffield wire ropes can bear a load of 90 tons per square inch of cross-section. What load can a wire of $\frac{3}{16}$ in. diameter support?

3. Which is the greater rate of interest, $\frac{1}{4}$*d*. a week for every pound lent or 5 per cent. per annum? (Take 52 weeks = 1 year.)

4. In 1935 income tax for a married man with two children was calculated as follows. From the gross income one-fifth was subtracted, and then a further allowance was made of £270. On the remainder tax was 1*s*. 6*d*. per £ up to £135, and on any

further income, 4*s.* 6*d.* in the £. Find the tax paid by such a man whose income was £630.

5. Calculate the income from an investment of £2000 in a 3 per cent. stock at $62\frac{1}{2}$, neglecting brokerage.

XXXIII

1. What sum of money will produce an interest of 351 fr. in 2 yr. at $2\frac{1}{4}$ per cent. simple interest?

2. A strip of building land on the side of a straight road has a depth of 100 yd. It is sold at £7 10*s.* per foot of road frontage. How much is this per acre?

3. A dairyman obtained 41 lb. of butter from 100 gal. of milk, and a gallon of milk weighs 10·3 lb. What percentage of the total weight of milk was the weight of the butter produced?

4. A penny measures 1·23 in. in diameter and a halfpenny 1 in. If the thicknesses were in the same ratio, what would be the ratio of their weights? (Answer to 3 significant figures.)

5. A man who runs at 10 ml. per hour gives a boy 44 yd. start, and overtakes him after running a furlong. At what rate does the boy run?

XXXIV

1. The simple interest on £242 13*s.* 4*d.* for 5 years amounted to £30 6*s.* 8*d.* What was the rate per cent.?

2. An estate is represented by an area of 3·7 sq. in. on a map which is drawn on the scale of 10 in. to the mile. Find the area of the estate in acres.

3. The following were the average costs to a certain railway:

	April, 1913	April, 1935
Wages per man	£1 12*s.* 0*d.*	£4 6*s.* 8*d.*
Coal per ton	14*s.* 9*d.*	33*s.* 11*d.*

Taking the index numbers for 1913 as 100, find the numbers for 1935.

4. A debtor owes £1537 3*s.* 4*d.*, but his assets are only worth £840; find, to the nearest penny, how much in the £ he can pay.

5. A set-square 5 mm. thick has its two shorter sides 22 cm. and 38 cm. respectively. Find its volume in c.c.

XXXV

1. A man is due at a certain place at a certain time. If he walks at the rate of 3 ml. an hour he will be 15 min. late; if he walks at 4 ml. per hour he will be 15 min. early. Find the distance he has to walk.

2. A ball has a diameter of 1·62 in. Find the area of its surface.

3. Find, to the nearest penny, the simple interest on £693 10*s*. for 122 days at $3\frac{3}{4}$ per cent. per annum.

4. A salesman's salary consists of a fixed sum per week plus a percentage of the value of his orders. In two weeks when he obtained orders to the values of £30 and £42 respectively, his salaries were £4 5*s*. and £5 3*s*. What is his salary when he takes orders worth £20?

5. A man sells £5000 $3\frac{1}{2}$ per cent. stock at 95 and invests the proceeds in £1 shares at 38*s*. If a dividend of 1*s*. $10\frac{1}{2}$*d*. per share is paid on the shares, what will be the change in his income? (Neglect brokerage.)

XXXVI

1. An insurance company charges 5*s*. per cent. per annum for insurance of the contents of a house against damage by fire, etc. What would be the annual premium for an insurance of the contents of a house if they are valued at £1250?

2. A graduated jar is in the form of a cylinder of circular cross-section. It is 2·4 in. in diameter (internal) and contains water. An irregular piece of metal is lowered into it, and the level of water rises from 6·7 in. to 7·6 in. Find the volume of the metal.

3. A runner took 4 min. 13 sec. to run a mile. Find his average speed in miles per hour to the nearest $\frac{1}{4}$ ml.

4. The price of a machine is given in a catalogue as £1000. Successive discounts of 15, 5, and $2\frac{1}{2}$ per cent. are allowed from this price. What does the buyer pay?

5. Find the value of $\sqrt{\dfrac{4840}{3{\cdot}142}}$.

XXXVII

1. A rectangular plot intended for a tennis court 78 ft. by 36 ft. was measured with a wooden yard measure which had half an inch broken from the end, and so measured only $35\frac{1}{2}$ in. The error was not noticed. Find the resulting deficiency in the area of the court.

2. Find the cost of 3 tons of wheat at 11*s*. 6*d*. per 100 lb.

3. If a cubic foot of water equals $6\frac{1}{4}$ gal., find, to the nearest

100, the number of gallons discharged in an hour from a pipe 6 in. in diameter through which water is flowing at $3\frac{1}{2}$ ft. per second. (Take $\pi = 3\frac{1}{7}$.)

4. It was required in a parish of rateable value £45,360 to raise a sum of £3500. What rate in the pound would be necessary? Answer in pence to 2 decimal places.

5. Find which is the better investment, $2\frac{1}{2}$ per cent. stock at 78 or $3\frac{1}{2}$ per cent. at 102.

XXXVIII

1. Find the difference between $1{\cdot}623 \times 950$ and $1{\cdot}578 \times 950$.

2. The largest mill in England can produce 1500 tons per week of steel wire rods $\frac{3}{16}$ in. in diameter. If the wire weighs 480 lb. per cubic foot, find the total length in miles of a full week's output.

3. A sum of money put out to simple interest at 4 per cent. per annum amounts to £878 2*s*. in a number of years, but at 3 per cent. it would have amounted to £841 10*s*. 3*d*. Find the sum and the number of years.

4. Find the amount of £100 in 3 yr. at 4 per cent. per annum compound interest. Find also the sum which at the same rate would amount to £100 in 3 yr.

5. Alternative rates of payment for electricity are as follows: (*a*) an initial charge of £2 10*s*. and a further charge of $\frac{1}{2}d.$ per unit consumed, or (*b*) a charge of $4\frac{1}{2}d.$ each for the first 50 units, 3*d*. each for the next 100 units, and $1\frac{1}{2}d.$ each for any further units consumed. Find how many units must be used if (*a*) is to be more economical than (*b*).

XXXIX

1. If $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$, find R when $r_1 = 3{\cdot}5$, $r_2 = 5$, $r_3 = 8$.

2. A sum of £450 was lent at simple interest, and was repaid after nine months together with the interest. Find the rate per cent. if the payment was £466 17*s*. 6*d*.

3. How much rubber can be bought for £10 when the price is 7·78*d*. per lb.?

4. A swimming bath is 132 ft. long, 44 ft. wide, and the bottom slopes so that the depth of water increases steadily from 3 ft. at one end to 8 ft. at the other. Find the number of gallons of water required to fill it, taking 1 cu. ft. = $6\frac{1}{4}$ gal.

5. Find the cost price of £8450 3 per cent. stock at 92 and the income from it.

XL

1. Find the cost of 17 cwt. 3 qr. 21 lb. at £21 per ton.

2. A sum of money less than £50 was shared among 385 boys. Each of them received an exact number of pence, and 70 pence were left over. The same sum was similarly shared between 286 girls, and again there were 70 pence over. What was the sum?

3. A builder spent £1450 in building a house, and sold it to an agent at a profit of 15 per cent. The agent resold the house for £1800. What was the agent's profit per cent. on his outlay?

4. In what ratio should teas costing 2*s.* 8*d.* and 1*s.* 8*d.* per lb. respectively be mixed, if the mixture is to be sold at 2*s.* 6*d.* per lb., and a profit of 20 per cent. made on the outlay.

5. A small metal ball of diameter 1·32 cm. weighed 7·83 grm. What would be the weight of a similar ball of diameter 2·5 cm.

XLI

1. Find the cost of 1792 oz. of silver at $29\frac{7}{16}$ pence per oz.

2. An estate of 6950 sq. yd. was offered for sale for £400,000. How much is this per acre to the nearest £100?

3. Find in its simplest form the value of

$$(12\tfrac{3}{8} - 9\tfrac{2}{3}) \div (3\tfrac{5}{6} - \tfrac{3}{4} \times 2\tfrac{2}{5}).$$

4. Two trains, one 60 yd. and the other 85 yd. long, passed when travelling in opposite directions on a railway. The shorter train was moving at 32 ml. per hour and the other at 55 ml. per hour. How long did they take to pass?

5. A man measures the length and breadth of a rectangular field by pacing, and taking his pace as 30 in., calculates the area as $2\frac{3}{4}$ ac. What is the true area if his pace is really 32 in.?

XLII

1. Of what amount is 500·31 fr. 27 per cent.?

2. Two clocks are set right at 12 noon on March 1; one loses 17 sec. and the other 8 sec. in 12 hr. If they are kept going but are not reset, when will one be a quarter of an hour before the other, and what time will it show?

3. Half an inch of rain falls on a horizontal roof of area 500 sq. ft., and runs off into a cylindrical tank 5 ft. in diameter. Find the depth of water in the tank due to this rainfall to the nearest half-inch.

4. Find, to the nearest penny, the compound interest on £2250 for 3 yr. at $2\frac{1}{4}$ per cent. per annum.

5. A tobacconist mixed 7 lb. of tobacco costing 11*s.* a pound with $2\frac{1}{2}$ lb. costing 13*s.* 6*d.* a pound. What percentage of profit on his outlay did he make by selling the mixture at 1*s.* per ounce?

XLIII

1. A map is drawn on the scale 2 cm. to 1 Km.; how many inches on the map represent a mile?

2. What principal will amount to £500 in 3 yr. at $4\frac{1}{8}$ per cent. per annum simple interest?

3. The old type of golf-ball was 1·75 in. in diameter and weighed 1·4 oz.; find its weight per cubic inch.

4. The area of the Trent basin is given as 4082 sq. ml., and its annual rainfall as 31 in. The river discharges 220,000 cu. ft. per minute. What percentage of the rainfall to the nearest integer finds its way to the river?

5. The cost of a new 5-ton steam-wagon was £1025. Depreciation is reckoned at $17\frac{1}{2}$ per cent. per annum. What will be its value to the nearest 10*s.* after 3 yr.?

XLIV

1. Two cogged wheels with 15 and 24 teeth respectively work into each other. If the first makes 16 revolutions in 10 sec., how many revolutions per minute does the second make?

2. A quart of water weighs $2\frac{1}{2}$ lb., and a kilogram equals 2·2046 lb. Find, to 2 significant figures, the percentage error in taking 8 litres as equal to 7 quarts.

3. A cube of metal, each edge of which is $\frac{5}{8}$ in. long, weighs 0·625 lb. What would be the length of an edge of a cube of this metal which weighed 40 lb.?

4. Find in litres the volume of a cylindrical cask 98 cm. high and 60 cm. in diameter.

5. Find the present value of £3000 due at the end of 5 yr. at $3\frac{3}{4}$ per cent. simple interest.

XLV

1. Six men and 2 boys can do a piece of work in 6 days; how long would 3 men and 4 boys take to do the same work if a man can do in 3 hr. what a boy would do in 5 hr.?

2. The Derby was won in the record time of 2 min. $34\frac{1}{5}$ sec. If the distance is 1 ml. 909 yd., what was the average speed in miles per hour to the nearest tenth?

3. A man changed £20 into francs at 74·50 fr. to the £1 and stayed 8 days in France. His hotel expenses were 65 fr. per day, plus 10 per cent. for service, and travelling and other expenses amounted to 450 fr. The remainder of his money he changed back into English money at 74·75 fr. to the £. How much was left from the £20?

4. How much water should be added to half a gallon of mixture which contains 60 per cent. of alcohol and 40 per cent. of water, so that the final mixture may contain 10 per cent. of alcohol?

5. Find the banker's discount on a bill of £75 drawn on March 30 at three months, and discounted on April 23 at $3\frac{1}{2}$ per cent. per annum. (Allow 3 days of grace.)

XLVI

1. A cyclist with a 28-in. wheel (diameter) says that he can find his speed in miles per hour by counting the number of clicks made by his cyclometer in 5 sec., each click indicating a revolution of the wheel. Test this assertion.

2. The assets of a bankrupt are worth £9000, and this sum would pay his creditors 7*s*. 6*d*. in the £. If, however, the claim of one creditor were disallowed, the others could be paid 9*s*. in the £. What was the claim of this creditor?

3. Find the number of square feet of canvas required to make a conical tent of height 9 ft. and base-radius 6 ft. Allow 5 per cent. more to cover waste in making it.

4. The following sums were placed on deposit in a bank; £535 10*s* on March 28 and £145 18*s*. on May 5. Find the total interest due on June 30 at $1\frac{1}{2}$ per cent. per annum.

5. From the formula $T = \dfrac{(L - \frac{3}{8}B) \times B^2}{188}$, calculate T when $L = 320$, $B = 55\frac{1}{2}$.

XLVII

1. Taking the weights of 1 gal. of glycerine and 1 gal. of sulphuric acid as 12 lb. $10\frac{1}{2}$ oz. and 18 lb. 7 oz. respectively, find how many gallons of glycerine weigh as much as 16 gal. of the acid.

2. At simple interest a sum of money would amount to £7848 in two years, and in six years to £9144; find the sum and the rate per cent. per annum.

3. All fractions except $\frac{2}{3}$ used by the Egyptians had the numerator 1. Express $\frac{3}{4}$ as the sum of two fractions with numerator 1, and $\frac{2}{5}$ as the sum of three such fractions.

4. A cylindrical pipe has a diameter of 20 in., and water is flowing through it at the rate of 10 ft. per second, the area of the cross-section of the stream in the pipe being $\frac{3}{4}$ of that of the pipe. Taking $\pi = \frac{22}{7}$ and a cubic foot = $6\frac{1}{4}$ gal., find, to 3 significant figures, how many gallons are discharged per minute.

5. A man sold out £5750 of 5 per cent. War Loan at 95 and invested half of the proceeds in a $3\frac{1}{2}$ per cent. stock at 76, and half in a 6 per cent. stock at $103\frac{1}{2}$. Find the change in his income.

XLVIII

1. A rectangular plot of ground measures 80 ft. by 35 ft. 6 in. A trench 3 ft. 6 in. wide is dug round and outside it, and the soil from the trench when distributed evenly over the plot forms a layer 3 in. deep. Find the depth of the trench.

2. What was the average speed in knots (sea-miles per hour) of the *Mauretania* when she made a passage of 3166 sea-miles in 4 days 18 hr. 35 min.?

3. By selling articles at 9*s*. a hundred a dealer gains 20 per cent. on the cost price; what would he gain per cent. if he sold 150 for 13*s*.?

4. A rod in the form of a cylinder $\frac{1}{2}$ in. in diameter and 10 in. long weighs 1 lb. $13\frac{1}{2}$ oz. Find the weight of the metal in ounces per cubic inch.

5. A man receives £450 annually in rents from his property, and spends on an average 18 per cent. of this in repairs, etc. What capital sum invested in $4\frac{1}{2}$ per cent. stock at 95 would bring him the same net income?

XLIX

1. Fourteen Jersey pounds are equivalent to 15 lb. 2 oz. avoirdupois. What, to the nearest halfpenny, should be the price per lb. avoirdupois of tea sold at 2*s*. 6*d*. per Jersey lb.?

2. A cistern can be filled by a pipe *A* in 40 min. or by a pipe *B* in 50 min. When the cistern is $\frac{3}{10}$ full the pipe *A* is turned on, and pipe *B* 10 min. later. In how many more minutes would the cistern be full?

3. A railway engine running at 47 ml. per hour picks up water from a trough $\frac{1}{4}$ ml. long between the rails at the rate of 177 gal. a minute. Find the number of pounds of water picked up in the $\frac{1}{4}$ ml. if 1 gal. weighs 10 lb.

4. Find the cost of 12·6 thousand cu. ft. of gas at 7·37*d*. per therm, if 1 thousand cu. ft. are reckoned as equal to 4·75 therms. Answer to the nearest penny.

5. Find the present value at compound interest of £300 due in 2 yr. at $3\frac{3}{4}$ per cent. per annum.

L

1. A box contained 12 dozen golf-balls, some of which were worth 1*s*. 3*d*. and the others 2*s*. each. They were sold without gain or loss at a uniform price of 1*s*. 7*d*. each. How many were of each kind?

2. If $r^2 = A/4\pi$, find r when $A = 3560$ and $\pi = 3{\cdot}142$.

3. *A*, *B*, and *C* enter into partnership with capitals of £2500, £3500, £4000 respectively, and after 3 months *A* subscribes a further £1500 of capital. How should the first year's profit of £1780 be divided between them?

4. A man invested a sum of money in $4\frac{1}{2}$ per cent. stock at $87\frac{1}{2}$ and, after receiving 2 years' dividends, sold out at $89\frac{3}{4}$; find his total profit per cent. on the original investment.

5. An estate of 18·43 ac. is intersected by a straight road 18 yd. wide and 1350 yd. long. Find, to the nearest ·01 ac., how much is available for building, and the greatest number of houses each occupying with garden 550 sq. yd. that could be built on the estate.

LI

1. Find the rate of exchange between London and New York when 2352 dollars 66 cents are received for £520 10*s*. (Answer in dollars to the £.)

2. A room is 15 ft. long by 13 ft. 6 in. wide, and its floor is to be covered with linoleum of which a piece 4 ft. 6 in. wide costs 8*s*. 3*d*. a yard. If there is no waste in cutting the linoleum, find the cost.

3. The depreciation in value of a machine is reckoned as 16 per cent. per annum of its value at the beginning of the year. Three years after it was bought the machine was sold for half its original cost. What percentage of its estimated value at the time did the seller lose?

4. Find, correct to 3 significant figures, the values of

$$(1)\ \sqrt{17{\cdot}39 \times 4840}; \qquad (2)\ \sqrt[3]{\frac{3 \times 9{\cdot}735}{4 \times 3{\cdot}142}}.$$

5. If a body falls 400 ft. from rest in 5 sec., how many seconds, to the nearest tenth, will it take to fall 800 ft? (The distance fallen from rest varies as the square of the time taken.)

LII

1. A man left $\frac{3}{7}$ of his estate to his wife, $\frac{2}{9}$ of the remainder to each of his two younger sons, and the rest to his eldest son. What is the eldest son's share if the estate is worth £12285?

2. If $V = \frac{h}{3}\left\{A + B + \sqrt{AB}\right\}$, calculate V when $A = 28{\cdot}5$, $B = 78{\cdot}6$ and $h = 4{\cdot}5$.

3. Find the banker's discount on a bill of £375 due on September 1 and discounted at $3\frac{1}{2}$ per cent. per annum on July 18.

4. A man bought £1 shares at 18*s.* and, after receiving dividends of $10\frac{1}{2}d.$ and $7\frac{1}{2}d.$ per share in successive years, sold the shares when they had fallen to 16*s.* 1*d.* Allowing for the dividends, he lost altogether £25. How many shares did he buy?

5. Find the value, to 5 places of decimals, of

$$1 + \frac{1}{3} + \frac{1}{1.2}\left(\frac{1}{3}\right)^2 + \frac{1}{1.2.3}\left(\frac{1}{3}\right)^3 + \frac{1}{1.2.3.4}\left(\frac{1}{3}\right)^4 + \dots\,.$$

LIII

1. A rod of zinc 2 yd. long at a temperature of 0° Centigrade was heated to 100° Centigrade, and its length was found to have increased by ·178 in. Find the expansion per unit length per degree Centigrade.

2. The slant side of a cone is 25 ft. and its curved surface 550 sq. ft. Find its volume, taking $\pi = 3\frac{1}{7}$.

3. When are the hands of a clock at right angles between 5 and 6 o'clock?

4. If a company earns a profit of 24 per cent. on its capital, and out of this pays a dividend of 7 per cent. on the capital, places £1600 to reserve, and carries forward £1375 to the next year's account, find its capital.

5. Convert the price of 35 fr. per litre to shillings and pence per gallon, given that £1 = 74·5 fr., 1 litre = 1·760 pt.

LIV

1. Four cogged wheels work each into the next. The numbers of teeth are 35, 30, 25, 20 respectively. If the first turns at 100 revolutions a minute, at what rate will the last turn? At what intervals will all four wheels be in the same relative positions?

2. A tradesman allows 4 per cent. off the marked price of his goods, and in selling at cash price, makes a profit of 24 per cent. on his outlay. What did goods marked £31 cost him?

3. *A* starts from his home for Epsom, distant 14 ml., walking at $3\frac{1}{2}$ ml. per hour. *B* starts from the same house $1\frac{1}{2}$ hr. later and cycles along the same road at $10\frac{1}{2}$ ml. per hour. At what distance from Epsom will *B* overtake *A*?

4. Simplify the expression $\dfrac{6\frac{1}{2} - 1\frac{5}{8} + \frac{3}{4} \text{ of } (1\frac{3}{4} - \frac{2}{3})}{11 - \frac{1}{6}}$.

5. How much must a man invest in $2\frac{1}{2}$ per cent. Consols at $77\frac{3}{4}$ to produce a net income of £200 per annum after the deduction of income tax at 4*s*. 6*d*. in the £? (Answer to the nearest £.)

LV

1. *A* runs 100 yd. in $10\frac{1}{4}$ sec. and *B* in $10\frac{2}{5}$ sec. What start must be given to *B* to secure a dead heat?

2. An oil-tank is in the form of a horizontal cylinder 30 ft. long and 5 ft. in diameter, closed by a hemisphere at each end. Find the capacity of the tank in gallons, if 1 cu. ft. = $6\frac{1}{4}$ gal.

3. The length and breadth of a piece of cardboard are given as 10·4 in. and 4·7 in., each to the nearest tenth of an inch. Find the possible percentage errors in length, breadth, and area, and find the area in the form $A \pm e$, where e is the possible error.

4. The population of a town is increasing at the rate of 2 per cent. each year of its population at the beginning of that year. In December, 1934, it was 589700. What is it likely to be in December, 1936, and what was it in December, 1933? Answer to the nearest 100 in each case.

5. The volume of a quantity of gas at 19° C. and 768 mm. pressure was 237·5 c.c.; find its volume at 0° C. and pressure 760 mm. (The volume varies inversely as the pressure and directly as the 'absolute temperature,' *i.e.*, temp. in degrees Centigrade + 273.)

LVI

1. *A*'s wages are half as much again as *B*'s. If *A*'s wages are now increased by 10 per cent., and *B*'s by 15 per cent., by how much per cent. is the total paid to them increased?

2. Find the cost of insurance of goods worth £15,000 at £2 17*s*. 6*d*. per cent. (*i.e.*, per £100 value).

3. A sum is invested at compound interest payable yearly. The interest for two successive years was £112 10*s.* and £115 6*s.* 3*d.* Find the rate per cent. and the total amount with interest at the end of these two years.

4. Calculate the volume of a pyramid whose height is 70 ft., and whose base is a square of side 60 ft. Find also the area of each side face.

5. Use logarithms to find the rate per cent. compound interest paid yearly at which 16*s.* would amount to £1 in six years.

LVII

1. A litre of water weighs 1 Kg., and a litre of another liquid weighs 1·340 Kg. A mixture of the two weighs 1·255 Kg. per litre. Find how much of each component there is in a litre of the mixture.

2. Find the value of $\pi l (r_1^2 - r_2^2)$ when $l = 120$, $r_1 = 3{\cdot}6$, $r_2 = 3{\cdot}3$.

3. Find the number of cubic yards required to build earth walls, 18 in. thick throughout, for a shed 20 ft. long, 15 ft. wide, and 8 ft. high, allowing for a door-opening 6 ft. high and 4 ft. wide, and two windows each 3 ft. by 2 ft. Take the measurements as internal, and assume that 6 cu. yd. of earth must be dug to build 5 cu. yd. of wall.

4. Find the average yield per cent. per annum on 5 per cent. Bonds bought at 102 in 1930 and sold after 4 yr. at 103.

5. The wavelength of a wireless station of frequency 877 Kc. is 342·1 m. Find the wavelength corresponding to a frequency of 1474 Kc. (The wavelength is inversely proportional to the frequency.)

LVIII

1. A cask contains 15 gal. of spirit, which contains 65 per cent. of alcohol. Find how much water must be added to the spirit to reduce the strength to 50 per cent. of alcohol.

2. The compound interest on a certain sum of money for 2 yr. at $3\frac{1}{2}$ per cent. is £5 2*s.* 1*d.* more than the simple interest would have been. Find the sum.

3. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$. Find, to 2 significant figures, the percentage error in taking the volume to be $\frac{11}{21} \times (\text{diameter})^3$. ($\pi = 3{\cdot}14159$.)

4. If a fall of £2 6*s.* 9*d.* in the selling price of a certain article converts the seller's profit of $5\frac{1}{2}$ per cent. into a loss of $5\frac{1}{2}d.$

in the pound on the cost price, find the original selling price of the article.

5. Find, to 5 places of decimals, the value of

$$1 + \tfrac{1}{2} \cdot (\tfrac{1}{5}) + \frac{\frac{1}{2} \cdot \frac{3}{2}}{1.2}(\tfrac{1}{5})^2 + \frac{\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2}}{1.2.3}(\tfrac{1}{5})^3 + \frac{\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2} \cdot \frac{7}{2}}{1.2.3.4}(\tfrac{1}{5})^4 + \dots .$$

LIX

1. A certain kind of coal, when converted into coke, loses one-third of its weight but increases one-tenth in bulk. Find, to the nearest cwt., how much of this coke could be stored in a cellar which would hold 16 tons of the coal.

2. A man spends equal sums of money in buying two kinds of stock at 93 and 95 respectively. He sells when their prices have changed to 90 and 99 respectively. Find whether he gains or loses, and by what percentage of his original outlay.

3. A workshop is 120 ft. long and 50 ft. wide. The roof slopes from a height of 20 ft. at each side to 35 ft. in the centre. Find the volume of the workshop.

4. A manufacturer marks his goods at an advance of 80 per cent. on the cost price, but he allows 15 articles to the dozen and gives 10 per cent. discount for cash. What rate of profit does the manufacturer get from cash customers?

5. The 'expectation of life' for men is given in the following table (*i.e.*, number of years a man who has attained the given age is likely to live):

Age	1	10	30	50	70	90
Expectation of life	52	50	33	19	8	$2\frac{1}{2}$

Represent these figures by means of a graph, and from it estimate the expectation of life for a man at the age of twenty, and the age a man of 60 may expect to reach.

LX

1. From the formula $A = \dfrac{2Q}{0{\cdot}7 \cdot T\sqrt{2gh}}$, calculate A when $Q = 300000$, $h = 26$, $T = 300$, and $g = 32{\cdot}1$.

2. A man invests equal sums in a 5 per cent. stock at 92 and in a 6 per cent. stock at 105. If the difference between the incomes obtained is £3 7*s.* 6*d.*, find the amount invested in each.

3. A frustum of a pyramid is contained between two parallel planes distant 9 ft. apart, and the areas of the parallel faces are 5 sq. ft. and 20 sq. ft. Determine the height of the pyramid and the volume of the frustum.

4. Taking the earth as a sphere of radius 3960 ml. find the distance in miles to 3 significant figures per minute of arc along the equator, *i.e.*, the distance on the earth between two points on the equator whose longitudes differ by 1 min. Take $\pi = 3{\cdot}142$.

5. Different amounts of acid are added to 50 gal. of water. and in each case the percentage of acid in the mixture is calculated. Complete the following table:

No. of gallons of acid added	0	5	10	20	25
Percentage of acid	0				

Express the result by means of a graph, and so estimate (*a*) the percentage of acid when 15 gal. are added, and (*b*) the number of gallons to be added to make a 20 per cent. solution.

LOGARITHMS AND ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969					3	7	11	14	18	21	25	28	32
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	28
					1584	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903					3	6	9	11	14	17	20	23	26
							1931	1959	1987	2014	3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2405	2430					3	5	8	10	13	15	18	20	23
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16

25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6996	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

LOGARITHMS—*continued*

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	9096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5

75	8751	8756	8762	8768	8774	8779	8785	8791	8707	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1536	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4

·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12

·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8270	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

ANSWERS

ANSWERS

Answers to Mental Examples are not included.

EXERCISES II

1. (1) 231514; (2) £11575 13*s.* 8*d.*; (3) 167826; (4) £16617 11*s.* 6*d.* **2.** 68 ml. 3 ch. **3.** (1) 68904; (2) 73496; (3) £160 8*s.* 8*d.*; (4) £4198 17*s.* 9*d.* **4.** 10 ml. 1207 yd. **5.** (1) 244; (2) 266; (3) 280; (4) 219; (5) 283; (6) 81; (7) 398; (8) 161; (9) 134; (10) 211; (11) 158; (12) 311; (13) 1373. **6.** 124 hr. 30 min. **7.** 99 tons 17 cwt. 48 lb. **8.** 16 cwt. **9.** 13 qr. 6 bush. 2 pk. **10.** 21 yd. 2 ft. 9 in.

EXERCISES IV

1. 840811. **2.** 2,409,077. **3.** 748569. **4.** 6,540,192. **5.** 7,342,251. **6.** 110,464,659. **7.** 186, 19. **8.** 177, 34. **9.** 459, 19. **10.** 443, 19. **11.** 4034. 37. **12.** 2469, 200. **13.** £100 7*s.* **14.** £980 16*s.* 8*d.* **15.** £2759 1*s.* **16.** 55 ml. 4 f. **17.** 874 tons 2 cwt. 2 qr. **18.** 19 tons 2 qr. 8 lb. **19.** £1 17*s.* **20.** £7 13*s.* 6*d.* **21.** £63 16*s.* 10*d.* **22.** 2 tons 3 cwt. 2 qr. 12 lb. **23.** 16 hr. 32 min. **24.** 13 ml. 3 f. 110 yd. **25.** £103 19*s.* **26.** 1860989. **27.** £441 15*s.* 9*d.* **28.** 5*s.* 6*d.*

EXERCISES VI

1. £63 12*s.* 9*d.* **2.** 182 yd. 1 ft. 9 in. **3.** 36 tons 9 cwt. 105 lb. **4.** 4 hr. 36 min. 37 sec. **5.** 4027. **6.** 5097. **7.** 42952. **8.** 1647. **9.** 2197. **10.** £187 14*s.* 6*d.* **11.** 287. **12.** 49 lb. 9 oz. **13.** 66610. **14.** 13 ml. 1441 yd. **15.** £223 14*s.* 3*d.* **16.** 16 tons 12 cwt. 5 lb.

EXERCISES VII

1. 12160. **2.** 7 yd. 2 ft. 3 in. **3.** £33 10*s.* **4.** 17 tins, 1*d.* **5.** £2 3*s.* 6*d.* **6.** 5 hr. 59 min. 3 sec. **7.** 16*s.* 11*d.* **8.** 4*s.* 5*d.* **9.** 8 ft. **10.** £2 + 10*s.* + 2 h.c. + 1 fl. + 3*d.*, £3 + 10*s.* + 2 fl. + 2*d.*, £1 + 10*s.* + 3 h.c. + 1*s.* + 5*d.* **11.** £7 8*s.* 8*d.* **12.** 15 yd., £2 3*s.* 9*d.* **13.** 28 in., 105 yd. **14.** £49 14*s.* 8*d.* **15.** £389 1*s.* 6*d.* **16.** 2*d.* **17.** 56. **18.** 26, 5 ft. **19.** 524. **20.** 2 tons 6 cwt. 108 lb. **21.** 15*s.* 2*d.* **22.** 35. **23.** £30. **24.** £16 5*s.* **25.** £3 3*s.* 4*d.* **26.** 13 days, 11*s.* 6*d.* **27.** 401. **28.** 2*s.* 2*d.*, £1 12*s.* 4*d.* **29.** 13*s.* 3*d.* **30.** 8 min. 20 sec. **31.** 8½*d.* **32.** 3 tons 2 cwt. 56 lb. **33.** 35 cwt. 80 lb. **34.** £700. **35.** 22 tons 15 cwt. 40 lb. **36.** 75. **37.** £18 17*s.* 1*d.* **38.** £3 5*s.* 3*d.*, 18*s.* 9*d.* **39.** 11*s.* 3*d.*, 8*s.* 9*d.* **40.** 528. **41.** £88. **42.** 60. **43.** £23 10*s.* 10*d.*, **44.** 6*s.* 1*d.* **45.** £1 11*s.* 8*d.* **46.** 29. **47.** £2 4*s.* 7½*d.* **48.** 176.

Exercises IX

1. $2^2, 3^2$. **2.** $2^2, 3^3$. **3.** $3^2, 5^2$. **4.** 3, 7, 11. **5.** $2^2, 3^2$, 11. **6.** 3^2, 7, 11. **7.** 2^4, 5, 11. **8.** $2^3, 5^3$. **9.** 2^3, 3, 5, 13. **10.** 3, 5^2, 17. **11.** 3, 11, 19. **12.** 7, 11, 13. **13.** 2, 3^2, 7, 13. **14.** 3^2, 5, 7^2. **15.** 3, 7^2, 13. **16.** 3^2, 11, 101. **17.** 2, 3^2, 5^4, 7. **18.** 2, 5, 7^3, 11. **19.** 2^3, 7, 11, 19. **20.** 3^4, 7^2, 11.

Exercises X

1. 15. **2.** 12. **3.** 13. **4.** 21. **5.** 17. **6.** 11. **7.** 170. **8.** 273.

Exercises XI

1. 252. **2.** 30. **3.** 4004. **4.** 336. **5.** 420. **6.** 210210. **7.** 2520. **8.** 537264.

Exercises XII

1. 16. **2.** 27. **3.** 21. **4.** 42. **5.** 39. **6.** 77. **7.** 44. **8.** 495. **9.** 280. **10.** 840. **11.** 385. **12.** 99. **13.** 278. **14.** 2. **15.** 7.

Exercises XIII

1. 181. **2.** 71280. **3.** 17. **4.** 69. **5.** 160 ft. **6.** £2 12*s.* 6*d.* **7.** 72, 144, 216, 288, 360, 432, 4320. **8.** 1715.

Exercises XIV

1. 942. **2.** 209. **3.** 437. **4.** 20790. **5.** 518518. **6.** 2969824. **7.** 29; 102979. **8.** 37; 372109. **9.** 117; 1916109. **10.** 69.

Exercises XVI

1. £1 10*s.* **2.** 8*s.* **3.** 16*s.* **4.** £7 15*s.* **5.** £7 9*s.* **6.** £6 18*s.* 6*d.* **7.** 1 ton 5 cwt. 2 qr. **8.** 1 cwt. 4 lb. 2 oz. **9.** 476 yd. **10.** 4 ac. 2 r. 30 p. **11.** £14 3*s.* 5*d.* **12.** 15*s.* 3*d.* **13.** 1*s.* 8*d.* **14.** £5 4*s.* 7*d.* **15.** £1 11*s.* 9*d.* **16.** £1365. **17.** £3 12*s.* **18.** 170 ac. **19.** £62 10*s.* **20.** £1 6*s.* 3*d.* **21.** £110. **22.** 102 guineas.

Exercises XVII

1. $\frac{3}{4}, \frac{1}{2}, \frac{1}{5}, \frac{1}{3}, \frac{3}{7}$. **2.** $\frac{2}{3}, \frac{3}{7}, \frac{7}{8}, \frac{5}{12}, \frac{5}{6}$. **3.** $\frac{2}{3}, \frac{4}{5}, \frac{1}{18}, \frac{3}{5}$. **4.** $\frac{2}{3}, \frac{3}{8}, \frac{1}{3}, \frac{3}{8}, \frac{1}{101}$. **5.** $\frac{1}{1001}, \frac{4}{15}, \frac{3}{4}, \frac{4}{21}$.

Exercises XVIII

1. $\frac{5}{2}, \frac{13}{4}, \frac{37}{7}$. **2.** $\frac{51}{8}, \frac{77}{10}, \frac{127}{14}$. **3.** $\frac{254}{27}, \frac{246}{23}, \frac{100}{33}$. **4.** $\frac{126}{11}, \frac{173}{13}, \frac{365}{19}$. **5.** $\frac{503}{100}, \frac{6003}{1000}, \frac{60007}{10000}$. **6.** $2\frac{1}{7}, 4\frac{3}{4}, 2\frac{9}{11}$. **7.** $3\frac{13}{14}, 4\frac{3}{20}, 1\frac{17}{100}$. **8.** $3\frac{1}{7}, 5\frac{1}{2}, 3\frac{16}{113}$. **9.** $6\frac{1}{3}, 1\frac{3}{17}, 1\frac{1}{2}$. **10.** $1\frac{4}{7}, 1\frac{3}{25}, 10\frac{1}{100}$.

Exercises XX

1. $1\frac{1}{12}$. **2.** $\frac{5}{6}$. **3.** $5\frac{5}{8}$. **4.** $\frac{3}{4}$. **5.** $5\frac{13}{36}$. **6.** $7\frac{96}{385}$. **7.** $\frac{99}{100}$. **8.** $\frac{1}{100}$. **9.** $\frac{261}{100000}$. **10.** $2\frac{5}{14}$. **11.** $2\frac{2}{15}$. **12.** $2\frac{1}{8}$. **13.** $\frac{10}{33}$. **14.** $11\frac{1}{4}$. **15.** $4\frac{1}{36}$. **16.** $2\frac{11}{12}$. **17.** $1\frac{5}{8}$. **18.** $1\frac{92}{105}$. **19.** $2\frac{13}{60}$. **20.** $3\frac{15}{16}$. **21.** $2\frac{4}{15}$. **22.** $10\frac{17}{280}$. **23.** $11\frac{13}{48}$. **24.** $6\frac{211}{240}$. **25.** $4\frac{16}{25}$. **26.** 5. **27.** $3\frac{1}{3}$. **28.** $15\frac{37}{60}$. **29.** $7\frac{5}{8}$. **30.** $9\frac{19}{60}$.

Exercises XXII

1. $\frac{4}{11}$, $\frac{2}{11}$. **2.** $\frac{3}{14}$, $\frac{3}{28}$. **3.** $\frac{11}{108}$, $\frac{11}{324}$, $\frac{1}{36}$. **4.** $\frac{1}{21}$, $\frac{4}{105}$, $\frac{2}{63}$, $\frac{4}{147}$. **5.** $\frac{5}{36}$, $\frac{5}{72}$, $\frac{1}{24}$. **6.** $\frac{1}{100}$, $\frac{7}{400}$, $\frac{1}{300}$. **7.** $\frac{3}{16}$, $\frac{9}{80}$, $\frac{9}{112}$, $\frac{1}{16}$. **8.** $\frac{11}{50}$, $\frac{22}{125}$, $\frac{11}{120}$, $\frac{2}{25}$. **9.** $\frac{7}{90}$, $\frac{2}{45}$, $\frac{7}{225}$, $\frac{1}{90}$. **10.** $\frac{49}{120}$, $\frac{49}{300}$, $\frac{7}{60}$, $\frac{7}{180}$.

Exercises XXIII

1. $6\frac{2}{3}$, 10, $16\frac{2}{3}$, 20. **2.** $6\frac{1}{2}$, $10\frac{5}{6}$, $17\frac{1}{3}$, 26. **3.** 100, 200, 300. **4.** $25\frac{2}{3}$, $42\frac{7}{9}$, 77. **5.** $33\frac{1}{3}$, $133\frac{1}{3}$, 200. **6.** $112\frac{1}{2}$, 150, 300. **7.** $2\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{1}{8}$. **8.** $2\frac{4}{7}$, $1\frac{5}{7}$, $1\frac{1}{35}$, $\frac{36}{49}$. **9.** $1\frac{1}{10}$, $\frac{11}{20}$, $\frac{33}{100}$. **10.** $2\frac{19}{27}$, $1\frac{19}{54}$, $\frac{73}{81}$. **11.** $5\frac{10}{21}$, $3\frac{2}{7}$. **12.** $6\frac{1}{4}$, $4\frac{1}{6}$, $1\frac{9}{16}$.

Exercises XXIV

1. $7\frac{7}{15}$. **2.** 11. **3.** $\frac{4}{9}$. **4.** $2\frac{1}{10}$. **5.** $\frac{18}{55}$. **6.** $2\frac{2}{5}$. **7.** 96. **8.** $52\frac{1}{2}$. **9.** $1\frac{4}{5}$. **10.** $4\frac{4}{5}$. **11.** $2\frac{1}{10}$. **12.** $1\frac{1}{6}$. **13.** $1\frac{3}{8}$. **14.** $\frac{2}{5}$. **15.** 6. **16.** $6\frac{1}{5}$. **17.** $\frac{7}{18}$. **18.** $\frac{12}{13}$. **19.** $1\frac{1}{5}$. **20.** $16\frac{1}{2}$. **21.** $1\frac{3}{7}$. **22.** $1\frac{23}{27}$. **23.** 9. **24.** $15\frac{3}{16}$. **25.** $\frac{12}{49}$. **26.** $\frac{3}{7}$. **27.** $12\frac{36}{37}$. **28.** $1\frac{2}{5}$. **29.** $4\frac{1}{2}$. **30.** 18. **31.** $1\frac{1}{3}$. **32.** $6\frac{4}{5}$. **33.** $\frac{6}{25}$. **34.** $23\frac{1}{3}$. **35.** 24.

Exercises XXV

1. $\frac{1}{2}$, $\frac{7}{10}$, $\frac{1}{3}$, $\frac{12}{25}$. **2.** $\frac{1}{8}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{9}{40}$. **3.** $\frac{9}{44}$, 8, $9\frac{3}{5}$, $1\frac{1}{9}$. **4.** $\frac{30}{49}$, $\frac{3}{4}$, $\frac{32}{39}$, $\frac{5}{8}$.

Exercises XXVI

1. $\frac{1}{5}$. **2.** $\frac{6}{7}$. **3.** $\frac{9}{17}$. **4.** $\frac{10}{17}$. **5.** $\frac{9}{19}$. **6.** $2\frac{6}{13}$. **7.** $1\frac{1}{2}$. **8.** $\frac{27}{35}$. **9.** $\frac{2}{3}$. **10.** $\frac{2}{3}$. **11.** $\frac{5}{11}$. **12.** $\frac{5}{13}$.

Exercises XXVII

1. $13\frac{1}{3}$. **2.** $2\frac{7}{12}$. **3.** $1\frac{11}{16}$. **4.** 0. **5.** $\frac{23}{30}$. **6.** 14. **7.** 8. **8.** $\frac{17}{24}$. **9.** $\frac{1}{5}$. **10.** $3\frac{3}{8}$. **11.** $\frac{13}{30}$. **12.** $\frac{2}{5}$. **13.** 4. **14.** $6\frac{29}{36}$. **15.** $1\frac{1}{34}$. **16.** 0. **17.** $15\frac{1}{2}$. **18.** $\frac{1}{2}$.

Exercises XXVIII

1. 2 ml. 35 yd. **2.** 500 ac. **3.** £19 16*s*. **4.** $\frac{16}{99}$. **5.** 264. **6.** 6 min. 4 sec. **7.** $6\frac{1}{4}$ yd. **8.** 9, $\frac{1}{17}$ pt. **9.** $\frac{1}{16}$. **10.** £5. **11.** $20587\frac{1}{2}$. **12.** 81900. **13.** $462\frac{1}{2}$. **14.** $10633\frac{1}{3}$. **15.** 67050. **16.** 66600. **17.** 17600. **18.** 57600. **19.** 8800. **20.** 56700. **21.** 29800. **22.** 374625. **23.** 12*s*. 6*d*. **24.** £62 10*s*. **25.** $\frac{1}{3}$, £333 6*s*. 8*d*. **26.** $\frac{1}{15}$, £5.

Exercises XXIX

1. $\frac{2}{19}$. **2.** $\frac{11}{140}$. **3.** $\frac{1}{19}$. **4.** $48\frac{1}{8}$. **5.** $\frac{4}{7}$. **6.** $3\frac{1}{2}$. **7.** $2\frac{23}{53}$. **8.** $1\frac{16}{69}$. **9.** $\frac{19}{43}$. **10.** $\frac{52}{137}$. **11.** $1\frac{17}{100}$. **12.** $\frac{1}{2}$. **13.** $12\frac{2}{5}$. **14.** $\frac{4}{19}$. **15.** $\frac{1}{4}$. **16.** $\frac{1}{2}$. **17.** $\frac{31}{36}$. **18.** $\frac{1}{48}$.

Exercises XXX

1. £1 15*s*. 3*d*. **2.** £79 3*s*. 4*d*. **3.** £84 9*s*. $4\frac{1}{2}$*d*. **4.** £11 9*s*. 7*d*. **5.** £18 0*s*. 9*d*. **6.** £150. **7.** £1236. **8.** £30.

Exercises XXXI

1. £4 16*s.* 9*d.* **2.** £315. **3.** £26 5*s.* **4.** £234. **5.** £203 8*s.* 9*d.* **6.** £41 14*s.* 3*d.* **7.** £223 8*s.* 6*d.* **8.** £1749. **9.** £391 17*s.* 6*d.* **10.** £924 7*s.* 6*d.* **11.** £53 2*s.* 6*d.* **12.** £30 7*s.* 3½*d.* **13.** £3416 13*s.* 4*d.* **14.** £1498 6*s.* 6*d.* **15.** £1833 17*s.* 6*d.* **16.** £110 8*s.* 4*d.* **17.** £535 0*s.* 1*d.* **18.** £395 16*s.* 8*d.* **19.** £165 10*s.* 5*d.* **20.** £668 6*s.* 3*d.* **21.** £51866 13*s.* 4*d.*

Exercises XXXII

1. (*a*) $\frac{356}{1000}$; (*b*) $\frac{7089}{1000}$; (*c*) $3\frac{57}{1000}$; (*d*) $25\frac{307}{1000}$. **2.** 0·356, 0·7089, 3·057, 25·307. **3.** ·3, ·05, ·006, ·23, ·203, ·23, ·0035. **4.** 1·30. **5.** 3·42. **6.** ·07. **7.** ·02. **8.** ·13. **9.** 2·05. **10.** 21·70. **11.** ·125, ·25, ·375, ·5, ·625, ·75, ·875. **12.** ·0625, ·1875, ·3125, ·4375, ·5625, ·9375. **13.** ·125, ·5, ·625, ·875. **14.** ·25, ·375, ·75, ·875.

Exercises XXXIII

1. $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$. **2.** $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$. **3.** $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{16}$. **4.** $\frac{3}{16}$, $\frac{7}{16}$, $\frac{329}{400}$. **5.** $5\frac{9}{50}$, $6\frac{7}{20}$, $8\frac{12}{25}$. **6.** $9\frac{1}{20}$, $8\frac{1}{2}$, $7\frac{41}{200}$. **7.** $3\frac{1}{125}$, $5\frac{101}{2000}$, $6\frac{303}{500}$.

Exercises XXXIV

1. 15·873. **2.** 1·216. **3.** 8·093. **4.** 39·153. **5.** 79·55. **6.** ·089323. **7.** 57·9544. **8.** 19·5641. **9.** ·9, ·99, ·9999. **10.** 9·5, 9·995, 9·99995. **11.** 11·3 by ·005. **12.** ·7 by ·001. **13.** ·9091. **14.** 84·35. **15.** 35·343. **16.** ·00035. **17.** 2·485. **18.** 800·9089. **19.** 7*s.* 3*d.* **20.** £3 4*s.* **21.** £7·5. **22.** 52 fr. 92 c. **23.** \$17 50 cents. **24.** \$37 80 cents. **25.** 2·5*d.*, 1·54*d.*, ·25*d.* **26.** 1*s.* 7·25*d.*, 1*s.* 6·67*d.*, 1·18*d.* **27.** \$12 87 cents. **28.** \$1 77 cents.

Exercises XXXV

1. 9*s.* 4½*d.*, £4 13*s.* 9*d.*, £46 17*s.* 6*d.* **2.** 4·28, 9·5, 61·7. **3.** ·454, ·0004. **4.** £30 0*s.* 10*d.* **5.** \$4 25 cents, \$121 25 cents. **6.** £4 10*s.* **7.** 14*s.* **8.** \$74250. **9.** 7·92 in. **10.** ·693 lb.

Exercises XXXVI

1. 3·525, ·3525, ·003525. **2.** 53·56, 535·6. **3.** 3·56, 35·6, 356. **4.** 23567, 235670, 2356700, 23567000. **5.** ·385. **6.** 5·3525. **7.** 3·57. **8.** 5·385. **9.** ·0356. **10.** ·157. **11.** ·005878.

Exercises XXXIX

1. 80. **2.** 28. **3.** 26. **4.** 1575. **5.** 11·375. **6.** 1232. **7.** 904·2. **8.** 51·8204. **9.** 1073·878. **10.** 9801. **11.** 125·4402, 12·54402. **12.** 23·2766, 2327·66. **13.** 90·8796, 908·796. **14.** 367·68718, 36768·718. **15.** 39·753. **16.** 527·772. **17.** 8640·848. **18.** 13463·3. **19.** 630·129. **20.** 112·53. **21.** 4·004001. **22.** 1·006009. **23.** 145·669. **24.** 1827·1875. **25.** 75·894 grm. **26.** 198 fr. 45 c. **27.** 5740·4. **28.** 75·411.

Exercises XLI

1. 1·8. **2.** 50·4. **3.** 14·35. **4.** 12·5. **5.** 5·26. **6.** 3·1. **7.** 5·976. **8.** 4·375. **9.** 12. **10.** 54·5. **11.** 34·6. **12.** 24·9. **13.** 32. **14.** 1·85. **15.** 42·5. **16.** 79·3. **17.** 764·8. **18.** 27·69. **19.** 3·606. **20.** 6·25. **21.** 119, 4 mm. **22.** 2·54 grm. **23.** £275. **24.** 40. **25.** 29·4.

Exercises XLII

1. £5·6875. **2.** £4·3625. **3.** £0·95625. **4.** £3·259375. **5.** £4·590625. **6.** £0·471875. **7.** £0·859375. **8.** £0·415625. **9.** £1·984375. **10.** 8·4375 cwt. **11.** 7·6875 cwt. **12.** 11·375 cwt. **13.** 15·8125 cwt. **14.** 4·28125 tons. **15.** 3·378125 tons. **16.** 9·734375 tons. **17.** 0·2078125 tons. **18.** 3·55 ac. **19.** 8·85 ac. **20.** 0·91875 ac. **21.** 0·1375 m. **22.** 3·4375 m. **23.** 5·41875 m. **24.** 5·1875 m. **25.** 13·36 h. **26.** 7·6125 h.

Exercises LXIII

1. 7*s.* 6*d.* **2.** 11*s.* $8\frac{1}{4}$*d.* **3.** 13*s.* $5\frac{1}{4}$*d.* **4.** 17*s.* $11\frac{1}{4}$*d.* **5.** 18*s.* $0\frac{3}{4}$*d.* **6.** 2*s.* $9\frac{3}{4}$*d.* **7.** 5 cwt. 2 qr. 14 lb. **8.** 9 cwt. 1 qr. 21 lb. **9.** 12 cwt. 2 qr. 8·764 lb. **10.** 3 qr. 16·436 lb. **11.** 5 f. 8 ch. **12.** 15 ml. 6 f. 7 ch. 13·2 yd. **13.** 8 ml. 7 ch. 15·4 yd. **14.** 7 hr. 49 min. 30 sec. **15.** 15 hr. 4 min. 30 sec. **16.** 7 ac. 2 r. 25 p. **17.** 19 ac. 3 r. 13 p. **18.** 14 tons 14 cwt. 1 qr. **19.** 9 tons 13 cwt. 3 qr. **20.** 5 yd. 2 ft. 2·028 in.

Exercises XLV

17. 14·72 sq. in. **18.** 20·3125 sq. ft. **19.** 4·272 cm. **20.** 12·5 sq. ch. **21.** 19 sq. ft. 36 sq. in. **22.** 46 sq. ft. 126 sq. in. **23.** 233 sq. ft. 108 sq. in. **24.** 45 sq. ft. 99 sq. in. **25.** 3·54132 sq. Km. **26.** 32 ft. **27.** 4·5 yd. **28.** 600 cm. **29.** 800 m. **30.** $11\frac{5}{9}$ yd. **31.** 9 ac. 9 sq. ch. **32.** 2 ac. 0·625 sq. ch. **33.** 2 ch. **34.** 4 ch. 30 links. **35.** 210 sq. ft., £8 15*s.* **36.** $181\frac{1}{2}$ sq. ft., £13 12*s.* 3*d.* **37.** £2 12*s.* 6*d.* **38.** 378 sq. ft. **39.** £2640. **40.** 2187. **41.** 23 yd. **42.** 10 ac.

Exercises XLVII

9. $178\frac{1}{2}$ cu. ft. **10.** 221 cu. in. **11.** $584\frac{3}{8}$ cu. in. **12.** $108\frac{9}{32}$ cu. ft. **13.** $420\frac{5}{6}$ cu. ft. **14.** 1050 c.c. **15.** 150 c.c.s. **16.** 25 cu. m. **17.** 3630 cu. ft. **18.** 35 sq. ft. **19.** $312\frac{1}{2}$ sq. cm. **20.** 11 ft. **21.** 13875 cu. ft. **22.** 12 dozen. **23.** 3750 cu. ft. **24.** 150 cu. yd.

Exercises XLIX

1. 7·751. **2.** 2·28845. **3.** ·034279. **4.** ·0747296. **5.** 3·069144. **6.** ·00112039. **7.** ·0400081. **8.** ·714486. **9.** ·000025259. **10.** 3·2013736. **11.** 6·6. **12.** ·066. **13.** ·0303. **14.** ·655. **15.** 298. **16.** ·3983, 39·83. **17.** 2930. **18.** ·00543, 5·43. **19.** 19·7, ·06. **20.** 17·50, ·0022.

Exercises L

1. ·4375. **2.** ·8375. **3.** ·8. **4.** ·0175. **5.** ·6. **6.** ·35. **7.** ·3125. **8.** 2·25. **9.** 2·8. **10.** 3·63.

Exercises LII

1. 8·604. **2.** 1·172. **3.** 194·270. **4.** 19·679. **5.** 13·586. **6.** 0·325. **7.** 0·0536. **8.** 122. **9.** 0·00585. **10.** 0·0260. **11.** 17400. **12.** 0·884. **13.** 0·482. **14.** 4·87. **15.** 3·36. **16.** 0·588. **17.** 59·3. **18.** 1·38. **19.** 24·8. **20.** 1·48. **21.** 2·98. **22.** 0·189. **23.** 1·76. **24.** 69·5.

Exercises LIII

1. 3. **2.** $1\frac{1}{2}$. **3.** 12. **4.** 7000. **5.** ·8. **6.** 25.

Exercises LIV

1. 39·3, 0·786, 11·8. **2.** 3·77 in. **3.** 82·6875, 55·125, 85·44375. **4.** 0·05015, 0·07965, 0·114066 . . **5.** 314 sq. ft. **6.** 491 sq. cm. **7.** 3850 sq. mm. **8.** 55 sq. ft. **9.** 1134 sq. cm. **10.** 24. sq. yd. **11.** £892 17*s.* $1\frac{5}{7}$*d.*, £891 13*s.* 4*d.*, £1 3*s.* $9\frac{5}{7}$*d.* Difference between $2\frac{1}{7}$*d.* and 2·14*d.* multiplied by 100000.

Exercises LV

1. 1·255, 2·6375, 2·17875, 1·295625, 1·454375. **2.** \$14 $97\frac{1}{2}$ cents, \$149 75 cents, \$1497 50 cents. **3.** $\cdot453\frac{4}{7}$, $\cdot907\frac{1}{7}$, $1\cdot360\frac{5}{7}$, $1\cdot814\frac{2}{7}$, $2\cdot267\frac{6}{7}$, $2\cdot721\frac{3}{7}$.

Exercises LVI

3. ·0000014. **4.** ·0000003, £1 0*s.* 6·6*d.* **5.** ·0000036, ·32535. **6.** ·3451 **7.** ·1234. **8.** ·4979. **9.** 314, 147. **10.** 13·7. **11.** 92·7. **12.** 1·63.

Miscellaneous Exercises on Decimals

1. 64628. **2.** 85953. **3.** 167000. **4.** 56·25, 200, 300. **5.** 10·2, 12·8, 18. **6.** 8·4, 11·64, 3·72. **7.** 12·925, **8.** 64. **10.** 13·6335. **11.** 6·63. **12.** 1544·4 lb. **13.** 0·50, 1·89. **14.** 21. **15.** 752 fr. 15 c. **16.** 45·3 m.p.h. **17.** \$48·16. **18.** 4375, 8280 lb. **19.** 710. **20.** 1107 thousand. **21.** £8 11*s.* $4\frac{1}{4}$*d.* **22.** £1 15*s.* 11*d.* **23.** 52, ·36 in. **24.** 11200. **25.** 3·86. **26.** 325. **27.** £168 15*s.* **28.** 98 tons 15 cwt. **29.** 1·42*d.* **30.** £1 0*s.* 4*d.* £1 5*s.* 2*d.* **31.** 2 lb. 3·3 oz. **32.** 53·93 m.p.h. **33.** 1·61 Km. **34.** 1·9725, 3·0775. **35.** £1074 8*s.* **36.** No. 4 cheapest, £28 18*s.*; No. 5 dearest, £92 15*s.* **38.** ·00001. **39.** 70·38. **40.** ·000001. **41.** 47. **42.** ·530. **43.** £4 19*s.* 8*d.* **44.** ·521875. **46.** £7 4*s.* 3*d.* **47.** 14200. **48.** 206·3, 332·0. **49.** \$33 44 cents.

Exercises LVII

1. ·05, ·1, ·2, ·25, ·5, ·75; $\frac{1}{20}$, $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$. **2.** $\frac{1}{30}$, $\frac{1}{15}$, $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{3}$, $1\frac{1}{3}$; ·033 . ., ·066 . ., ·0833 . ., ·125, ·33 . ., 1·33 . . **3.** 5, 20, 10, 25. **4.** $33\frac{1}{3}$, 25, $12\frac{1}{2}$, $16\frac{2}{3}$, $3\frac{1}{8}$. **5.** $33\frac{1}{3}$ (2), 60 (1), $28\frac{4}{7}$ (3), $26\frac{2}{3}$ (4). **6.** $\frac{3}{4}$, 1, $1\frac{1}{4}$, $2\frac{1}{2}$. **7.** 1*s.* 3*d.*, 1*s.* 6*d.*, 1*s.* 9*d.*, 2*s.*, 2*s.* 3*d.*, 2*s.* 6*d.* **8.** $\frac{5}{12}$. **9.** 7·2*d.*, 1*s.* 7·2*d.*, 2*s.* 4·8*d.*, 3*s.* 0*d.*, 4*s.* 2·4*d.* **10.** 0·6, 1·2, 1·8, 2·7, 11·7*d.*, 2*s.* 7·5*d.* **11.** 7·2, 20·7, 315 m. **12.** 5·4, 56·7, 255·51 Kg. **13.** 72, 207, 217·8. **15.** 11520, 33600, 36000 f. **16.** 120, 648, 5160 f. **17.** £3. **18.** £2 8*s.* **19.** \$32 10 cents. **20.** £7 10*s.* **21.** 46 fr. 75 c. **22.** £7 18*s.* **23.** 39 qr. **24.** 160. **25.** $37\frac{1}{2}$. **26.** 75. **27.** $62\frac{1}{2}$. **28.** 250. **29.** $41\frac{2}{3}$. **30.** 67. **31.** 70. **32.** 160.

Exercises LVIII

1. £6 4*s*. 6*d*. **2.** £21 3*s*. $3\frac{3}{4}$*d*. **3.** £4 0*s*. $8\frac{1}{2}$*d*. **4.** £39 15*s*. 5*d*. **5.** 4700 fr. **6.** \$10 $12\frac{1}{2}$ cents. **7.** 50·4 lb. **8.** 8 tons 18 cwt. 2 qr. **9.** 12 tons 10 cwt. 1 qr. **10.** 14 Kg. 875 grm. **11.** 367 f. **12.** 175086.

Exercises LIX

1. £50. **2.** £45. **3.** £18 15*s*. **4.** £67 10*s*. **5.** £29 5*s*. **6.** £63 15*s*. **7.** £28 10*s*. **8.** £4 15*s*. $7\frac{1}{2}$*d*. **9.** £480. **10.** \$36 $78\frac{1}{8}$ cents. **11.** 81 fr. $56\frac{1}{4}$ c. **12.** $77\frac{1}{12}$ lire.

Exercises LX

1. 12*s*. 6*d*. **2.** £3 2*s*. 6*d*. **3.** £231 5*s*. **4.** £375. **5.** £328 2*s*. 6*d*. **6.** 7*s*. 6*d*. **7.** £722 10*s*. $7\frac{1}{2}$*d*. **8.** £8 12*s*. **9.** £37 10*s*. **10.** £374 6*s*. **11.** £8750.

Exercises LXI

1. 34*s*. 6*d*., 37*s*. 6*d*., 42*s*. **2.** 58*s*. 4*d*., 66*s*. 8*d*., 67*s*. 6*d*. **3.** 18*s*. $4\frac{1}{2}$*d*., 19*s*. 3*d*., 19*s*. $8\frac{1}{4}$*d*., 31*s*. 6*d*. **4.** 10*s*. $7\frac{1}{2}$*d*. **5.** £262 8*s*.

Exercises LXII

1. $11{\cdot}527\frac{2}{3}$. **2.** 11 yrs. 10 months. **3.** 8·94. **4.** 70*s*. $7\frac{2}{3}$*d*., 40*s*. $4\frac{8}{21}$*d*. **5.** $9\frac{7}{8}$*d*., $\frac{59}{70}$. **6.** 18. **7.** 51°. **8.** 5 ft. 4 in. **9.** $48\frac{63}{80}$*d*. **10.** $18\frac{89}{96}$. **11.** $357\frac{1}{2}$ per cent. **12.** $\frac{16}{165}$.

Exercises LXIII

1. 5 ft. $7\frac{1}{4}$ in. **2.** 1*s*. 10*d*. **3.** 27*s*. **4.** 62*s*. 1*d*. **5.** 8·54. **6.** 5 ft. $8\frac{1}{4}$ in. **7.** 39. **8.** 34.

Exercises LXV

1. ·004. **2.** ·016. **3.** ·033. **4.** ·104. **5.** ·215. **6.** ·346. **7.** ·692. **8.** ·709. **9.** ·315. **10.** ·884. **11.** 15·517. **12.** 32·193. **13.** 19·441. **14.** 86·571. **15.** 16·235. **16.** 63·754. **17.** 47·960.. **18.** 18·382. **19.** 6·147. **20.** 23·865. **21.** $1\frac{3}{4}$*d*. **22.** $3\frac{1}{2}$*d*. **23.** $5\frac{1}{2}$*d*. **24.** 8*d*. **25.** 7*d*. **26.** 10*d*. **27.** 1*s*. $3\frac{3}{4}$*d*. **28.** 2*s*. 3*d*. **29.** 3*s*. $2\frac{3}{4}$*d*. **30.** 6*s*. 5*d*. **31.** 12*s*. $4\frac{1}{4}$*d*. **32.** 4*s*. $2\frac{1}{2}$*d*. **33.** 6*s*. $10\frac{3}{4}$*d*. **34.** 15*s*. $4\frac{1}{4}$*d*. **35.** £10 5*s*. $7\frac{3}{4}$*d*. **36.** £86 11*s*. 6*d*. **37.** £2 1*s*. 3*d*. **38.** £5 7*s*. $1\frac{3}{4}$*d*. **39.** £9 19*s*. 9*d*. **40.** £16 10*s*. $10\frac{1}{4}$*d*.

Exercises LXVI

1. £73·14167. **2.** £28·81458. **3.** £5·18333. **4.** £16·86146. **5.** 41·22083. **6.** £50·79792. **7.** £13·27604. **8.** £86·97917. **9.** £109·44479. **10.** £69·35208.

Exercises LXVII

11. £2075, £2160 8*s*. 4*d*., £3877 1*s*. 8*d*., £1162 10*s*. **12.** £65 18*s*. 4*d*.

Exercises LXVIII

1. £1503 12*s.* 6*d.* **2.** £3663 10*s.* **3.** £2637 3*s.* 9*d.* **4.** £8962 10*s.* **5.** £4265 12*s.* 6*d.* **6.** £257 5*s.* **7.** £36 17*s.* 6*d.* **8.** £2371 17*s.* 6*d.* **9.** £471 15*s.* 1*d.* **10.** £42 11*s.* 8*d.* **11.** £36 13*s.* 9*d.* **12.** £246 1*s.* $10\frac{1}{2}$*d.* **13.** £124 5*s.* **14.** $732 $93\frac{3}{4}$ cents. **15.** £203 2*s.* 6*d.* **16.** £75 10*s.* 5*d.* **17.** £11 17*s.* 5*d.* **18.** £6 1*s.* 1*d.* **19.** £283 11*s.* 6*d.* **20.** £8 14*s.* 4*d.* **21.** £43 11*s.* 11*d.* **22.** £220 17*s.* **23.** £61 15*s.* 11*d.* **24.** £193 11*s.* $10\frac{1}{2}$*d.* **25.** £746 15*s.* 4*d.* **26.** £916 19*s.* 3*d.* **27.** £60 15*s.* 9*d.* **28.** £349 13*s.* 7*d.* **29.** £7627 10*s.* **30.** £51 9*s.* 10*d.*

Exercises LXIX

1. 24. **2.** 29. **3.** 41. **4.** 57. **5.** 93. **6.** 91. **7.** 132. **8.** 303. **9.** 313. **10.** 5·7. **11.** ·25. **12.** 50·3. **13.** 35·6. **14.** ·093. **15.** 14·13. **16.** 80·19. **17.** 141·8. **18.** 16·79. **19.** 3·162. **20.** 6·325. **21.** 1·414. **22·** 2·236. **23.** 2·646. **24.** 2·828. **25.** ·447. **26.** ·837. **27.** ·949. **28.** 1·581. **29.** ·158. **30.** 1·897. **31.** 1·772. **32.** ·085. **33.** ·268. **34.** 2·478. **35.** $1\frac{1}{12}$. **36.** $18\frac{1}{2}$. **37.** $3\frac{4}{9}$. **38.** 2·028. **39.** 1·773. **40.** 2·309. **41.** 6·037. **42.** $3\frac{3}{7}$. **43.** $13\frac{1}{2}$. **44.** 9·025. **45.** 0·894. **46.** 3·708. **47.** 23·435. **48.** 5·64. **50.** 4·180, 4·181. **51.** 142. **52.** 13. **53.** 3·95.

Exercises LXX

1. $58\frac{1}{2}$, $105\frac{1}{4}$ sq. ft. **2.** 112, $205\frac{1}{2}$ sq. in. **3.** 40000, 58750 sq. cm. **4.** $430\frac{1}{2}$ sq. in. **5.** 324 sq. ft. **6.** $577\frac{1}{2}$ sq. ft. **7.** $522\frac{3}{4}$ sq. ft. **8.** 85 sq. yd. 3 sq. ft. **9.** 7. **10.** 12, £1 12*s.* **11.** 18*s.* 8*d.* **12.** 1584 sq. ft. **13.** 110 sq. in. **14.** 625 sq. in. **15.** 4200 sq. ft., 2366 sq. ft. **16.** 938 sq. yd. **17.** $11\frac{3}{4}$ sq. in. **18.** 7675 sq. ft. **19.** 58 sq. ft. **20.** 220 yd. **21.** 15 min., 30 ac. **22.** 16540 sq. yd.

Exercises LXXI

1. 3500. **2.** 6 tons 60 lb. **3.** $70\frac{10}{11}$ tons. **4.** 138 lb. 12 oz. **5.** 356·184 Kg. **6.** $196\frac{8}{27}$ cu. yd. **7.** 270. **8.** 9 cwt. 3 qr. 1 lb. 12 oz. **9.** 1 ft. 2 in. **10.** 3 ft. 2·4 in. **11.** 9·828 Kg. **12.** 1479 litres. **13.** 101 tons. **14.** 11000 gal. **15.** 1 cu. ft. 816 cu. in. **16.** $860\frac{5}{8}$ cu. in. **17.** $2\frac{163}{216}$ cu. ft., 13*s.* 8*d.* **18.** $5\frac{5}{9}$.

Exercises LXXII

3. 5. **5.** 50·7 sq. in. **6.** 4·32 sq. in. **8.** 85 sq. ft. **9.** 11·375 sq. cm. **10.** $90\frac{3}{16}$ sq. ft.

Exercises LXXIII

1. 6·5 cm. **2.** 3·39 ft. **3.** 4·4 in. **4.** 177 ft. **5.** 10 ch. **6.** 12·2 cm. **7.** 3·1 ft. **8.** 21·2 ft. **9.** 24 ft.

Exercises LXXIV

1. About 440. **2.** 18.

Exercises LXXVI

1. 22 in., $38\frac{1}{2}$ sq. in. **2.** 7 ft. 4 in., $4\frac{5}{18}$ sq. ft. **3.** $18\frac{6}{7}$ cm., $28\frac{2}{7}$ sq. cm. **4.** $37\frac{5}{7}$ ft., $113\frac{1}{7}$ sq. ft. **5.** $157\frac{1}{7}$ ft., $1964\frac{2}{7}$ sq. ft. **6.** 484 ft., 18634 sq. ft. **7.** 1 ft. 9 in. **8.** $240\frac{5}{8}$ sq. ft. **9.** 70 yd. **10.** $2\frac{5}{14}$, $3\frac{13}{14}$ sq. in. **11.** 44 ft. **12.** $663\frac{3}{8}$ sq. ft.

Exercises LXXVII

1. 1·344 grm. **2.** 8·16 grm. **3.** 1·33 grm. **4.** 85·2, 87·09 grm. **5.** 558·25 grm. **6.** 0·6. **7.** 6·90 grm. **8.** ·668 grm. **9.** 587·5 lb. **10.** 19·635 grm. **11.** 0·90 grm. **12.** ·87 grm. **13.** 1792 cu. ft. **14.** 1750 cu. ft. **15.** 2·24 cu. ft. **16.** 4·6 in. **17.** $\frac{16}{75}$ cu. ft. **18.** 7·6 per cent.

Exercises LXXIX

1. 10·4, 2·24. **2.** 8·2, 1·8. **3.** 4·7 cm. **4.** 33·3, 15·8 m.p.h. **5.** 425 m. **6.** £26½, £32½, £44½.

Exercises LXXX

3. 95, 149; 4½, 82. **7.** 20 days.

Exercises LXXXI

1. 51½. **2.** 11·49 A.M., 2·41 P.M. **3.** 11·46 A.M., 81½ m. **5.** 10·19 A.M., 46 m. **6.** 11·47 A.M., 46 m. from A.

Exercises LXXXII

6. 15·6, 52·4; 3·68, 4·64. **7.** 15·8, 27·8. 8. 820, 1740; 7¾. **9.** 1 in 5·8; 9½°, 4¾°. **10.** 9·22 A.M., 10½ m. from A. **11.** 11·25 A.M. **12.** 3·06, 4·29; 13*s.* 6*d.*

Exercises LXXXIV

1. $\frac{3}{40}$, ·075. **2.** ·09. **3.** ·03. **4.** $\frac{7}{60}$. **5.** 1·05. **6.** ·952. **7.** ·664. **8.** ·66$\dot{3}$. **9.** ·000605. **10.** 40⅛. **11.** $89\frac{2}{7}$. **12.** $5\frac{5}{9}$, $16\frac{2}{3}$, $19\frac{1}{6}$, 25, $34\frac{13}{15}$, $41\frac{2}{3}$, 50. **13.** 126·72. **14.** 50·688. **15.** $\frac{1}{21120}$. **16.** $\frac{1}{126720}$; $\frac{1}{253440}$; $\frac{1}{633600}$. **17.** 3312½. **18.** 63·8. **19.** ·855. **20.** 1·6. **21.** ·169. **22.** 1·16. **23.** 14·5. **24.** 60. **25.** 83·1. **26.** 15·6.

Exercises LXXXV

1. 164, 113. **2.** 250, 112½. **3.** 190, 120. **4.** 128, 44. **5.** 282, 119.

Exercises LXXXVIII

1. £2 18*s.* 1½*d.* **2.** 250. **3.** £11 5*s.* 6*d.* **4.** 10 tons 8 cwt. **5.** 2 hr. **6.** £125. **7.** £1 3*s.* 4*d.* **8.** 69·5. **9.** 78·3 cm. **10.** 6¾ days. **11.** £4 4*s.* **12.** £3 8*s.* 9*d.* **13.** £4 5*s.* 3*d.* **14.** £22500. **15.** 30 days. **16.** 5·376 cu. ft. **17.** £14 14*s.* 8*d.* **18.** 170 f. **19.** £78 5*s.* **20.** 41250 f. **21.** £2 2*s.* **22.** £1 5*s.* 6*d.* **23.** £243 17*s.* **24.** £11 0*s.* 6*d.* **25.** 7·41 grm. **26.** 222 grm. **27.** 6·56 grm. **28.** 7·69 grm. **29.** 68·9 grm. **30.** 133⅓ yd. **31.** 5, 25 ohm. **32.** 699 mm.

EXERCISES XC

1. £7, £11 13*s.* 4*d.*, £16 6*s.* 8*d.*, £21. **2.** £680, £510, £340. **3.** 959, 1575, 1736. **4.** £4 11*s.* 6*d.*, £3 1*s.* **5.** 89·6, 16·8, 5·6 lb. **6.** 47·6, 7·0, 1·4 lb. **7.** £680, £1160. **8.** 5650. **9.** £1 1*s.*, 4*s.* 8*d.*, 5*s.* 10*d.*, 7*s.* **10.** 36. **11.** £492, £526. **12.** £10327, £11016, £11475. **13.** 40·12, 9·88. **14.** 69·64, 28·57, 1·79. **15.** 15*s.*, 9*s.*, 6*s.* **16.** 1 ton 7 cwt. 2 qr. 14 lb.; 9 cwt. 2 qr. 7 lb.; 5 cwt. 1 qr. 7 lb. **17.** 1276·8, 851·2, 112 lb. **18.** 5. **19.** *A* £320, *B* £1040. **20.** *A* £300, *B* £150, *C* £200, *D* £350. **21.** 7·84, 7·99, 6·53. **22.** 1·02. **23.** 17 to 12.

EXERCISES XCII

1. $\bar{5}$·2. **2.** $\bar{5}$·05. **3.** $\overline{14}$·48. **4.** $\bar{8}$·25. **5.** $\overline{17}$·69. **6.** $\bar{7}$·28. **7.** $\bar{1}$·75. **8.** $\bar{1}$·25. **9.** $\bar{1}$·375. **10.** $\bar{1}$·76. **11.** $\bar{1}$·137. **12.** $\bar{1}$·943. **13.** $\bar{2}$·2. **14.** $\overline{12}$·38. **15.** $\bar{6}$·1875. **16.** $\overline{10}$·25. **17.** 2·13. **18.** $\bar{3}$·98. **19.** 3·1. **20.** $\bar{5}$·85. **21.** $\bar{1}$·11. **22.** $\bar{5}$·96. **23.** 1·74. **24.** $\bar{1}$·37. **25.** $\bar{1}$·643.

EXERCISES XCV

1. ·8157. **2.** ·7057. **3.** ·2772. **4.** ·0294. **5.** $\bar{1}$·8047. **6.** $\bar{2}$·1278. **7.** 1·4972. **8.** 4·4972. **9.** $\bar{3}$·6642. **10.** 2·9944. **11.** $\bar{1}$·9530. **12.** $\bar{2}$·8827.

EXERCISES XCVI

1. 4·094 **2.** 2·393. **3.** 3·303. **4.** 33·03. **5.** ·3303. **6.** 415·7. **7.** ·04157. **8.** 1292. **9.** 10·45 **10.** ·006184. **11.** 31420. **12** ·05000.

EXERCISES XCVII

1. 26·2. **2.** 7·82. **3.** 0·0535. **4.** 1·51. **5.** 0·209. **6.** 1·11. **7.** 271. **8.** 4·86. **9.** ·000261. **10.** 1·97. **11.** 5·06. **12.** 6·35. **13.** 2·25. **14.** 4·60 **15.** 9·87. **16.** ·101. **17.** 1·12. **18.** 3·14. **19.** 94·6. **20.** 31·8.

EXERCISES XCVIII

1. 147. **2.** ·330. **3.** 16·8. **4.** 2·98. **5.** 17·0. **6.** 307. **7.** 52·0. **8.** ·00158. **9.** 2·67. **10.** 1·04. **11.** 49·6. **12.** ·000294. **13.** ·000207. **14.** 14·8. **15.** 4·98. **16.** 43. **17.** ·329. **18.** 105. **19.** ·925. **20.** 8·52.

EXERCISES C

1. 10. **2.** $23\frac{1}{3}$. **3.** $27\frac{1}{2}$. **4.** $-13\frac{1}{3}$. **5.** $48\frac{4}{27}$. **6.** -25. **8.** $3\frac{7}{31}$. **9.** $122\frac{2}{3}$. **10.** $197\frac{13}{21}$. **11.** £11 5*s.* **12.** £5 17*s.* $10\frac{1}{2}$*d.* **13.** £8 8*s.* $5\frac{1}{4}$*d.* **14.** $20·97. **15.** £4 12*s.* 6*d.* **16.** £10 4*s.* **17.** £187 17*s.* 6*d.* **18.** £7 10*s.* **19.** $70. **20.** 66 f. **22.** $9\frac{1}{11}$, $13\frac{1}{33}$, $16\frac{2}{3}$, 20, 25. **23.** £26 13*s.* 4*d.* **24.** £25. **25.** £13. **26.** 92 f. **27.** $61·25. **28.** £28 16*s.* **29.** $33\frac{1}{3}$. **30.** 50 guineas. **31.** £3 6*s.* 8*d.*

EXERCISES CI

1. $2\frac{11}{30}$. **2.** 239. **3.** 50 ac. **4.** $246\frac{2}{13}$. **5.** 7807. **6.** 16·1. **7.** 19·8. **8.** $16\frac{1}{2}$. **9.** £8. **10.** 80. **11.** £32. **12.** -4. **13.** 16. **14.** 29·6. **15.** $6\frac{2}{3}$. **16.** 24. **17.** 20. **18.** 5. **19.** 20. **20.** 25. **21.** 18 : 5. **22.** 17000. **23.** 19. **24.** 20, **25.** 15·8. **26.** 125. **27.** 284 : 85. **28.** 95200. **29.** £1980. **30.** £1 7*s.*, £1 11*s.* 10*d.*, £1 17*s.* 6*d.* **31.** 38·9. **32.** £24000.

EXERCISES CII

1. £3 12*s.* 2*d.* **2.** £36 10*s.* 4*d.* **3.** £126 11*s.* 3*d.* **4.** £2 2*s.* **5.** £18. **6.** £1 0*s.* 6*d.* **7.** £7 2*s.* 11*d.* **8.** £2 16*s.* 10*d.* **9.** £12 15*s.* 2*d.* **10.** £2 13*s.* 2*d.* **11.** £1 10*s.* 10*d.* **12.** £24 11*s.* 6*d.* **13.** £2 14*s.* 3*d.* **14.** Add £4, £8, £16 to the given interest.

EXERCISES CIII

1. 3*s.* 5*d.* **2.** £1 9*s.* 2*d.* **3.** £3 3*s.* 2*d.* **4.** £6 19*s.* 9*d.* **5.** £27 13*s.* 5*d.* **6.** £5 11*s.* 9*d.* **7.** 15*s.* 7*d.* **8.** £1 6*s.* 2*d.*

EXERCISES CV

1. £720. **2.** £425. **3.** £35. **4.** £600. **5.** £902 10*s.* **6.** £50. **7.** £92 12*s.* **8.** £505 15*s.* **9.** £763 10*s.* **10.** £8333 7*s.* **11.** $2\frac{1}{2}$ yr. **12.** $3\frac{1}{2}$ yr. **13.** 5 per cent. **14.** $5\frac{5}{9}$ per cent. **15.** $5\frac{1}{2}$ per cent. **16.** $6\frac{1}{4}$ yr. **17.** $13\frac{1}{3}$ yr. **18.** $3\frac{1}{2}$ per cent. **19.** 5 per cent. **20.** $3\frac{1}{8}$ per cent. **21.** 4 yr. **22.** $3\frac{1}{4}$ per cent.

EXERCISES CVI

1. £76 17*s.* 6*d.* **2.** £34 8*s.* 6*d.* **3.** £471 19*s.* 9*d.* **4.** £37 9*s.* 3*d.* **5.** £68 17*s.* 11*d.* **6.** £116 8*s.* 9*d.* **7.** £52 11*s.* 1*d.* **8.** £19 7*s.* 5*d.* **9.** £94 7*s.* 11*d.* **10.** £376 8*s.* 7*d.* **11.** 212·86 fr. **12.** $685·00. **13.** 61·92 l. **14.** £45 9*s.* 10*d.* **15.** £1093 1*s.* 8*d.* **16.** £3005 5*s.* **17.** £3975 9*s.* 10*d.* **18.** 8900. **19.** £646 8*s.* **20.** £0·14015, £427 11*s.* 1*d.* **21.** £0·19252, £447 3*s.* 11*d.* **22.** £·26248, £473 8*s.* 7*d.* **23.** £·09308, £409 18*s.* 1*d.*

EXERCISES CVII

1. £409 12*s.* **2.** £17147 10*s.* **3.** £14642. **4.** £3881. **5.** 40 per cent. **6.** $33\frac{1}{3}$ per cent. **7.** 28 per cent. **8.** 25·14 per cent. **9.** £152 15*s.* 7*d.* **10.** £240 12*s.* 6*d.*

EXERCISES CVIII

1. £296 16*s.* **2.** £213 18*s.* **3.** £583 18*s.* **4.** £783 8*s.* **5.** £691. **6.** £709 12*s.* **7.** 9·22 yr. **8.** 20·2 yr. **9.** 7·2 per cent. **10.** 2·8 per cent. **11.** 5·2 per cent. **12.** 4·1 per cent. **13.** 3·93 per cent. **14.** 23·7 per cent.

EXERCISES CX

1. £598 10*s.* **2.** £4050. **3.** £27182 8*s.* **4.** £531 17*s.* 6*d.* **5.** £998 15*s.* **6.** £3718 12*s.* 6*d.* **7.** £2000. **8.** £4050. **9.** £8000. **10.** £6250. **11.** $8000. **12.** 6400 fr. **13.** £43 15*s.* **14.** £13. **15.** £11. **16.** £12 7*s.* 6*d.* **17.** £21 7*s.* 6*d.* **18.** £350. **19.** £89 5*s.* 9*d.* **20.** £85 14*s.* 3*d.* **21.** £13 10*s.* **22.** £147. **23.** £55 9*s.* 4*d.* **24.** £93 15*s.* **25.** 3·302, £3 6*s.* **26.** 6·393, £6 7*s.* 10*d.* **27.** 5·814, £5 16*s.* 3*d.* **28.** 6·322, £6 6*s.* 5*d.* **29.** 3·636, £3 12*s.* 9*d.* **30.** 2·881, £2 17*s.* 7*d.* **31.** 5 per cent. **32.** $4\frac{1}{2}$ per cent. **33.** £13 7*s.* 6*d.* **34.** £1 1*s.* **35.** £1 13*s.* 4*d.* **36.** £16 4*s.*

Exercises CXI

1. £145, £188 15*s*., £160 12*s*. 6*d*. £89 15*s*. 10*d*. **2.** 68, 52, 62, 111. **3.** $5\frac{1}{2}$ per cent. **4.** 15·1 per cent. **5.** £2400, £8 dec. **6.** £11900. **7.** £2 10*s*. inc. **8.** 72. **9.** £87 10*s*., £4 12*s*. 1*d*. inc. **10.** £16666 13*s*. 4*d*. **11.** £1080. **12.** £48 inc. **13.** 87, $97\frac{7}{8}$. **14.** £2571 8*s*. 7*d*. **15.** 86. **16.** £8 19*s*. 8*d*. **17.** £7 8*s*. 8*d*. **18.** $107\frac{1}{4}$. **19.** 5·773, £5 15*s*. 6*d*. **20.** 95. **21.** £555.

Exercises CXII

1. 9·92 sq. in. **2.** 21·1 sq. in. **3.** 18·6 sq. ft. **4.** 3160 sq. m. **5.** 6·29 sq.chs. **6.** 33·3 sq. ch.

Exercises CXIII

1. 490·9 sq. cm. **2.** 4·39825 sq. cm. **3.** 18·85 sq. ft. **4.** 95·03 sq. ft. **5.** 62·83 sq. ft. **6.** 153·9 sq. cm. **7.** 192 sq. ft. **8.** 226·2 sq. ft. **9.** 452·4 sq. ft. **10.** 3·71 in.

Exercises CXIV

1. 65·45 cu. ft. **2.** ·0141 cu. in. **3.** 163 c.c.s. **4.** ·754 cu. m. **5.** 12·42 ft. **6.** 122. **7.** 7·73 cm. **8.** 14·8 cu. ft. **9.** 6·20 cm. **10.** 2 : 3. **11.** 95·2 cu. in. **12.** 14·7 cu. in.

Exercises CXV

2. Nearly 300 yd. **3.** 3·2 sq. in. **4.** 80 sq. in. **5.** 104 cu. ft.

Exercises CXVI

1. 968 sq. in. **2.** 25 ft. **3.** 288. **4.** 36 per cent. **5.** 1020. **6.** 32·1 in. **7.** 7 yd. **8.** 3 m.p.h. **9.** $6\frac{3}{4}$ ft. **10.** 84000 cu. ft. 2285 sq. ft. **11.** 1·38 : 1. **12.** 1232 cu. ft. **13.** 31900. **14.** 8·58 sq. in. **15.** 2·41. **16.** 905000. **17.** 186. **18.** 122·5 cu. in. **19.** 3·58 cm. **20.** ·766 in. **21.** 160 cu. ft. **22.** 98. **23.** £1 10*s*. **24.** 510. **25.** 3·56. **26.** 8·80.

Exercises CXVII

1. £3. **2.** 45 men. **3.** 11·3 oz. **4.** £4 15*s*. $7\frac{1}{2}$*d*. **5.** 240 lb. **6.** 209·7 c.c.s. **7.** 7*s*. 2*d*. **8.** £16 10*s*. **9.** 7 lb. **10.** £2406 5*s*.

Exercises CXVIII

1. 9 to 4. **2.** 113·08, $706\frac{3}{4}$, 63·6075. **3.** 1 : 5. **4.** 1 : 4. **5.** 125 to 64. **6.** 1 to $\sqrt{3}$ (1·732). **7.** $6\frac{3}{4}$ oz. **8.** £180. **9.** 2 hr. **10.** $9\frac{1}{3}$ knots. **11.** 27 to 8. **12.** 13180 tons. **13.** 0·78 sec. **15.** 39. **16.** $\cdot 0305 \div d^2$.

Exercises CXIX

1. $4\frac{4}{5}$ days. **2.** 15 days. **3.** 31 days. **4.** $3\frac{3}{4}$ min. **5.** 1·36 P.M. **6.** $3\frac{1}{2}$. **7.** $21\frac{9}{11}'$, $21\frac{9}{11}'$ past 1. **8.** $10\frac{10}{11}'$ past 8. **9.** $3\frac{3}{11}$ sec., $16\frac{4}{11}$ sec. **10.** 330 ft. **11.** man = 3 boys. **12.** $14\frac{1}{2}$ yd. **13.** $79\frac{1}{4}$ yd. **14.** $26\frac{2}{3}$ min. **15.** $5\frac{5}{11}$ ml. **16.** $49\frac{1}{11}$, $32\frac{8}{11}$, $16\frac{4}{11}$, $43\frac{7}{11}$ and $54\frac{6}{11}$ min. after 9. **17.** 21 m.p.h. **18.** $21\frac{2}{3}$ min.

Exercises CXX

1. Equal parts. **2.** 4 to 1. **3.** 5 to 3. **4.** 1 to 2. **5.** 1 to 6. **6.** 1 to 6. **7.** 1 to 6. **8.** 7 to 3. **9.** 60. **10.** $1\frac{1}{2}$ cwt. **11.** 2*s.* $1\frac{1}{5}$*d.* **12.** $272\frac{3}{8}$ gr. **13.** 3 qt. **14.** 64 l. **15.** 76·0 c.c.s. **16.** £1 7*s.* **17.** 2*s.* 4*d.* **18.** 1*s.* 6*d.*

Exercises CXXI

1. 57432 fr. **2.** £87. **3.** 58 fr. 20 c. **4.** 0·49 lb. **5.** 5760, 8135. **6.** 1·76. **7.** 149. **12.** 68 fr. 20c. **13.** 36·9. **14.** 85*s.*, 63*s.* **15.** £8000. **16.** 38 per cent.

Exercises CXXII

1. ·208333. **2.** ·143841. **3.** ·142857. **4.** ·716531.

Exercises CXXIII

1. 9·3. **2.** ·94. **3.** ·44, ·32. **4.** 177·41, 178·7625, 176·0625, ·76. **5.** 2·8, 1·2, ·34, 4·4, 111100 ± 4900 cu. in. **6.** 738·7 ± 4·1 sq. ft. **7.** ·0061. **8.** 3·402, ·007. **9.** 267·26 ± ·10.

Exercises CXXIV

1. 72·4. **2.** 6·01. **3.** 67·4. **4.** 3·42. **5.** 2·754. **6.** ·5812. **7.** ·2680. **8.** 5·671. **9.** 8078. **10.** 3,974,000,000. **11.** 398·2. **12.** 7880·37. **13.** ·00062. **14.** ·000261. **15.** 77·67. **16.** 9·87. **17.** ·00510. **18.** 3·76. **19.** ·127. **20.** 42·4. **21.** 3·35. **22.** 101·0. **23.** 1·141. **24.** 87·63. **25.** 6·859. **26.** 180·2. **27.** ·905951. **28.** £862 12*s.* **29.** 3·162278. **30.** 1·024695.

PROBLEM PAPERS: MENTAL

I

1. 182. **2.** 12*s.* 8*d.* **3.** 1131. **4.** $8\frac{1}{12}$. **5.** 231, 5. **6.** 9*s.* 4*d.* **7.** 209. **8.** 12·84.

II

1. 227. **2.** 21*s.* **3.** 587. **4.** $1\frac{17}{20}$. **5.** 310. **6.** 19*s.* **7.** 32. **8.** 6*s.*

III

1. $\frac{2}{3}$. **2.** 2. **3.** 45. **4.** ·21. **5.** £1 8*s.* 4*d.* **6.** 4·08 sq. in. **7.** ·625. **8.** £4 3*s.* 4*d.*

IV

1. 6. **2.** $\frac{3}{8}$. **3.** 3*s.* 4*d.* **4.** 33·68. **5.** 6 yd. **6.** 3*s.* **7.** £14 14*s.* 2*d.* **8.** $\frac{4}{9}$.

V

1. 13*s.* $1\frac{1}{2}$*d.* **2.** £3. **3.** $21\frac{1}{4}$. **4.** ·027. **5.** ·4. **6.** 476. **7.** $181\frac{1}{3}$ cu. in. **8.** $9\frac{3}{4}$ in.

VI

1. 4085. **2.** ·000008. **3.** £3 3*s.* 9*d.* **4.** 165. **5.** $\frac{7}{12}$. **6.** 3*s.* 9*d.* **7.** ·15. **8.** 2*s.* 8*d.*

VII

1. 1197. **2.** $3\frac{5}{6}$. **3.** 160. **4.** 470. **5.** 25 lb. **6.** 2^2. 3^3. **7.** £44 16*s.* $4\frac{3}{4}$*d.* **8.** 91.

VIII

1. 101, 15. **2.** ·024. **3.** 1·78 in. **4.** 28. **5.** 3 ml. 6 f. **6.** 2·53 **7.** £5 10*s.* 6*d.* **8.** 1947.

IX

1. ·102. **2.** 8*s.* $1\frac{1}{2}$*d.* **3.** 1260. **4.** ·0175. **5.** 28. **6.** Friday. **7.** 312 sq. yd. **8.** £1 18*s.*

X

1. 1596. **2.** $1\frac{1}{5}$. **3.** 215. **4.** 968. **5.** $17\frac{1}{2}$. **6.** 252. **7.** 25·7. **8.** 1568.

XI

1. 5000. **2.** 480. **3.** 9 ft. 6 in. **4.** £3 10*s.* **5.** £3 15*s.* **6.** $46\frac{2}{3}$. **7.** 204. **8.** £1 0*s.* 6*d.*

XII

1. 487. **2.** 18 cwt. **3.** 17*s.* 2*d.* **4.** 17·5 cu. in. **5.** $1\frac{1}{2}$*d.* **6.** £15 12*s.* **7.** $3\frac{3}{4}$ oz. **8.** 250–280.

XIII

1. 50. **2.** 21825. **3.** $36\frac{1}{4}$. **4.** 110. **5.** 171. **6.** 27. **7.** £21. **8.** 100 hr.

XIV

1. $1\frac{1}{7}$. **2.** 17*s.* 2½*d.* **3.** 3 cwt. 14 lb. **4.** 75 yd. **5.** 67 tons 15 cwt. 1 qr. **6.** £16 5*s.* **7.** 11. **8.** 85.

XV

1. 13 yd. 2 ft. 8 in. **2.** 3935. **3.** 21, 4½*d.* **4.** ·160. **5.** 63 ft. 6 in. **6.** £2 15*s.* 9½*d.* **7.** 1.30 A.M. **8.** 30·29.

XVI

1. 1056. **2.** £7 10*s.* **3.** 840, 960. **4.** 64. **5.** 5. **6.** 5 fr. 95 c. **7.** ·38 in. **8.** ·893.

XVII

1. 18073. **2.** 16 m.p.h. **3.** 8 days. **4.** 16*s.* 4*d.* **5.** 34 hr. 45 min. **6.** £58 10*s.* **7.** £2621 5*s.* **8.** £218 15*s.*, £131 5*s.*

XVIII

1. 14*s.* 10½*d.* **2.** £1032. **3.** £8 5*s.* **4.** 2 hr. 46 min. 40 sec. **5.** 3 per cent. **6.** £22 15*s.* **7.** 35 lb. **8.** 4 ft.

XIX

1. £210. **2.** 2½ y. **3.** £12500. **4.** ·150. **5.** £12000. **6.** 1575. **7.** 89 sq. in. **8.** 40 per cent.

XX

1. 927, 15. **2.** £2 16*s.* **3.** £5128 15*s.* **4.** 528 yd. **5.** 45 sec. **6.** 1·33. **7.** £23 7*s.* 6*d.* **8.** 495.

PROBLEM PAPERS

I

1. £3,046,875. **2.** £546 17*s*. 6*d*. **3.** 1*s*. 5*d*. **4.** 11 cwt. 1 qr. 27 lb. **5.** (1) 21; (2) 73·82. **6.** $2^2.7.19$, $2.3^2.19$, 38.

II

1. £29 8*s*. **2.** 1030·5. **3.** 4793. **4.** 2,919,670 tons. **5.** (1) 0·338724; (2) 453·66. **6.** 135·81.

III

1. $78\frac{2}{9}$ min. **2.** £88822 15*s*. **3.** 1,002,001. **4.** 0·534. **5.** £9 11*s*. 3*d*. **6.** £15 12*s*.

IV

1. £1,812,500. **2.** 41200. **3.** 77 cm. **4.** 2460. **5.** 37·4. **6.** £2 18*s*. 4*d*.

V

1. 1·20. **2.** £333 4*s*. 10*d*. **3.** £7 1*s*. 9*d*. **4.** 424·80 fr. **5.** (1) $\frac{23}{25}$; (2) £3 0*s*. 2·4*d*. **6.** 14·203125.

VI

1. £2428 2*s*. 6*d*. **2.** £5680. **3.** $18\frac{2}{11}$. **4.** (1) £1 1*s*. $1\frac{1}{2}$*d*.; (2) 1·1578125. **5.** $2\frac{29}{204}$. **6.** $2^3.7.11^2$; 14.

VII

1. £5 6*s*. 11*d*. **2.** 11*s*. 4*d*. **3.** 92100, 65500. **4.** 14·3. **5.** £1379 17*s*. 6*d*. **6.** 12 m.p.h.

VIII

1. 22·0 per cent. **2.** 5291. **3.** £56. **4.** 23868. **5.** (1) 1806·39; (2) 508 Hg. 3 grm. **6.** £1275.

IX

1. Boy 2*s*. $4\frac{1}{2}$*d*., girl 1*s*. 7*d*. **2.** £1997 10*s*. **3.** 3. **4.** 2·1. **5.** £197 1*s*. 7*d*. **6.** £47 10*s*.

X

1. £120 6*s*. 3*d*. **2.** £21. **3.** ·00224. **4.** 112. **5.** £47 8*s*. 9*d*. **6.** 220 sq. ft.

XI

1. £40939 17*s*. 1*d*. **2.** 693. **3.** £55 1*s*. **4.** 16·15, remainder ·045. **5.** £87 13*s*. $1\frac{1}{2}$*d*. **6.** 15.

XII

1. 11 days, 17·25 fr. **2.** 26·82. **3.** 0·07314. **4.** 737 p. **5.** £2 18*s*. 2*d*. **6.** 36 lb. $9\frac{1}{7}$ oz.

XIII

1. 54, 1*s*. **2.** £60 14*s*. 11*d*. **3** (1) £351 6*s*. 9*d*; (2) 3 tons 12 cwt. 1 qr. **4.** 36, £10 13*s*. **5.** £51866 13*s*. 4*d*. **6.** £540 3*s*., £360 2*s*., £270 1*s*. 6*d*.

XIV

1. 11 cwt. 3 qr. $2\frac{1}{2}$ lb. **2.** 10·7. **3.** ·454. **4.** £1 17*s*. 6*d*. **5.** 32. **6.** $159\frac{1}{4}$ cu. in.

XV

1. 121·5. **2.** 477481, 4774·81, 0·00477481. **3.** £510, £340. **4.** $6\frac{1}{12}$. **5.** £4 3*s*. **6.** 910 sq. ft.

XVI

1. 46. **2.** 0·000039, $\frac{1}{25344}$. **3.** 86 ft. **4.** £21 16*s*. **6.** 797, 203 grm.

XVII

1. £25 4*s*. **2.** £1 17*s*. 4*d*. **3.** $7\frac{5}{12}$. **4.** $7\frac{1}{5}$ days. **5.** 43,900,000. **6.** £610.

XVIII

1. $63\frac{3}{4}$ ml. **2.** £68 10*s*. 3*d*. **3.** $\frac{25}{91}$ (·274 . .), $\frac{407}{1459}$ (·278 . .) $\frac{102}{365}$ (·279 . .). **4.** 45 ml., 40 ml. **5.** 45 in. **6.** 752·6.

XIX

1. £289 18*s*. **2.** 47 m.p.h. **3.** 31 yd. 2 ft. 4 in., 3*s*. 9*d* **4.** 24. **5.** 440. **6.** $417·85.

XX

1. 300 lb. **2.** £433 0*s*. 9*d*. **3.** 24, 23. **4.** £7 9*s*. 3*d*. **5.** 63·4. **6.** £56 11*s*.

XXI

1. £5 4*s*. 10*d*. **2.** 147 fr. **3.** £1 3*s*. 4*d*., $55\frac{5}{9}$ per cent., $35\frac{5}{7}$ per cent. **4.** £405, £303 15*s*., £243. **5.** 12·8295.

XXII

1. £3 8*s*. 6*d*., £1 16*s*. 6*d*. **2.** 1·6 in. **4.** ·0497. **5.** *A* 200, *B* 328.

XXIII

1. 91·7. **2.** 0·2105, 6·32*d*. **4.** £50000. **5.** 86623, £11 18*s*. 3*d*.

XXIV

1. £25 15*s*. $7\frac{1}{2}$*d*. **2.** 53. **3.** 6 min. **4.** £194 13*s*. 9*d*. **5.** £1452, £726, £242.

XXV

1. £276 11*s*. 3*d*. **2.** 29. **3.** 9 m.p.h. **4.** £38 1*s*. **5.** 3·1416, 3·1416, 3·1418.

XXVI

1. 1080. **2.** 636. **3.** £113. **4.** 0·408 in. **5.** 85·84375., £6 19*s*. 6*d*.

XXVII

1. 21 in. **2.** 333⅓. **3.** 14*s*. 6*d*. **4.** £3020. **5.** *A* £800, *B* £540, *C* £420.

XXVIII

1. 460. **2.** $1639·53, £74 14*s*. 4*d*. **3.** £1 9*s*. 11*d*. **4.** 57. **5.** £460, £482 10*s*., £515 10*s*.

XXIX

1. £10 8*s*. 9*d*. **2.** 0·838. **3.** 7·81. **4.** £180. **5.** 85·6 sq. in.

XXX

1. 97·7 sq. in. **2.** 1·07. **3.** 3½. **4.** 3¾. **5.** 232.

XXXI

1. 3·0, 16·5 in. **2.** £129 18*s*. 5*d*. **3.** £12 5*s*. **4.** 5¼, 6¾ in. **5.** 26·40.

XXXII

1. 61. **2.** 46·7 tons. **3.** ¼*d*. a week. **4.** £32 8*s*. **5.** £96.

XXXIII

1. 7800 fr. **2.** £1089. **3.** 3·98. **4.** 1·86 : 1. **5.** 8 m.p.h.

XXXIV

1. 2½. **2.** 23·68. **3.** 271, 230. **4.** 10*s*. 11*d*. **5.** 209.

XXXV

1. 6 ml. **2.** 8·24 sq. in. **3.** £8 13*s*. 10*d*. **4.** £3 10*s*. **5.** £59 7*s*. 6*d*. increase.

XXXVI

1. £3 2*s*. 6*d*. **2.** 4·07 cu. in. **3.** 14¼. **4.** £787 6*s*. 3*d*. **5.** 39·2.

XXXVII

1. 77$\frac{11}{24}$ sq. ft. **2.** £38 12*s*. 10*d*. **3.** 15500. **4.** 18·52. **5.** 3½ per cent.

XXXVIII

1. 42·75. **2.** 6910. **3.** £731 15*s*., 5 yr. **4.** £112 9*s*. 9*d*., £88 18*s*. **5.** More than 300.

XXXIX

1. $1\frac{109}{171} \doteq 1{\cdot}64$. **2.** 5. **3.** 308 lb. **4.** 199650. **5.** £7774, £253 10*s*.

XL

1. £18 16*s*. 8¼*d*. **2.** £42. **3.** 7·95. **4.** 1 : 2. **5.** 53·2 grm.

XLI

1. £219 16*s*. **2.** £278600. **3.** $1\frac{81}{241}$. **4.** 3·41 sec. **5.** 3·13 ac.

XLII

1. 1853 fr. **2.** Noon April 20th. 11 hr. 46 min. 40 sec. **3.** 12½ in. **4.** £155 6*s*. 4*d*. **5.** 37·2.

XLIII

1. 1·2672. **2.** £444 18*s*. 9*d*. **3.** ·499 oz. **4.** 39. **5.** £575 10*s*.

XLIV

1. 60. **2.** 0·78. **3.** 2·5 in. **4.** 277 l. **5.** £2526 6*s*. 4*d*.

XLV

1. 8 days. **2.** 35·4. **3.** £6 5*s*. 3*d*. **4.** 2½ gal. **5.** 10*s*. 3*d*.

XLVI

2. £4000. **3.** 214. **4.** £2 8*s*. 1*d*. **5.** 4700.

XLVII

1. 23·3. **2.** £7200. 4½. **3.** $\frac{1}{2} + \frac{1}{4}$, $\frac{1}{4} + \frac{1}{10} + \frac{1}{20}$. **4.** 6140. **5.** £3 7*s*. 8½*d*. decrease.

XLVIII

1. 9·9 in. **2.** 27·63. **3.** $15\frac{5}{9}$. **4.** 15·0. **5.** £7790.

XLIX

1. 2*s*. 4*d*. **2.** 10. **3.** 565. **4.** £1 16*s*. 9*d*. **5.** £278 14*s*. 1*d*.

L

1. 80, 64. **2.** 16·8. **3.** £580, £560, £640. **4.** $12\frac{6}{7}$. **5.** 13·41 ac., 118.

LI

1. 4·52. **2.** £6 3*s*. 9*d*. **3.** 17·6. **4.** 290, 1·32. **5.** 7·1.

LII

1. £3900. **2.** 231·6. **3.** £1 14*s*. 6*d*. **4.** 1200. **5.** 1·39561.

LIII

1. ·0000247. **2.** 1232 cu. ft. **3.** 5·10$\frac{10}{11}$, 5·43$\frac{7}{11}$. **4.** £17500. **5.** 42*s*. 8½*d*.

LIV

1. 175 r.p.m., 36 sec. **2.** £24. **3.** 6⅛ ml. **4.** $\frac{21}{40}$. **5.** £8026.

LV

1. 52 in. **2.** 4090. **3.** ·48, 1·06, 1·55, 48·88 ± ·76. **4.** 613500, 578100. **5.** 224·4.

LVI

1. 12. **2.** £431 5*s*. **3.** 2½, £4727 16*s*. 3*d*. **4.** 84000 cu. ft., 2285 sq. ft. **5.** 3·8.

LVII

1. ¼, ¾. **2.** 780. **3.** 38·1. **4.** 5·147 (£5 2*s*. 11*d*.). **5.** 203·5 m.

LVIII

1. 4½ gal. **2.** £4166 13*s*. 4*d*. **3.** 0·27. **4.** £31 13*s*. **5** 1·11803.

LIX

1. 9 tons 14 cwt. **2.** Gains 0·49 per cent. **3.** 165000 cu. ft. **4.** 29·6. **5.** 41⅓, 73.

LX

1. 69·9 **2.** £1207 10*s*. **3.** 18 ft. 105 cu. ft. **4.** 1·15. **5.** 23, 12½.