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VII Standard

Untouchability
Inhuman - Crime

Department of School Education

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Preface

The great book of nature lies ever open before our eyes
And the true philosophy is written in it ... But we cannot
read it unless we have first learned the language
and the characters in which it is written ... It is written
in mathematical language....

- Galileo Galilei

This book is prepared based upon the common syllabus-2010 proposed by the Government of Tamilnadu. This book is designed as per the guidelines given in NCF 2005 and the need of Matriculation students as well as Anglo Indian students is taken care of.

This book is written not only for Government school students but also for Matriculation school students and Anglo Indian school students. So the writing team has exhaustively referred to so many standard reference books in various libraries.

Each chapter begins with the aim of motivating the young minds. Basic facts studied in the lower classes are recalled in a brief introduction to facilitate easy reference and continuity of study. A unique feature of this text book is to enrich the students knowledge. So the writing team has introduced the following steps **"Do you know?"**, **"Try these"** and **"Think it"** in each and every chapter for easy understanding and enrichment.

Further the important results of **each** chapter are pulled out and presented as **"Points to remember"**. Multiple Choice Question pattern has been introduced in almost all the exercises. Theoretical concepts have been presented with clear explanations. Many examples have been fully worked out for better understanding. An extensive collection of problems is given at the end of each section. We hope that the students would enrich their mathematical knowledge by using this book.

Despite all our efforts, the book may still contain some errors. The writing team will be grateful if the reader points out the errors. We also welcome suggestions for effective improvement of the book in future.

N.Varadarajan
Chairperson

Contents

1 REAL NUMBER SYSTEM

No World without Water

No Mathematics without Numbers

1.1 Introduction

In the development of science, we should know about the properties and operations on numbers which are very important in our daily life. In the earlier classes we have studied about the whole numbers and the fundamental operations on them. Now, we extend our study to the integers, rationals, decimals, fractions and powers in this chapter.

Numbers

In real life, we use Hindu Arabic numerals - a system which consists of the symbols 0 to 9. This system of reading and writing numerals is called, “Base ten system” or “Decimal number system”.

1.2 Revision

In VI standard, we have studied about Natural numbers, Whole numbers, Fractions and Decimals. We also studied two fundamental operations addition and subtraction on them. We shall revise them here.

Natural Numbers

Counting numbers are called natural numbers. These numbers start with smallest number 1 and go without end. The set of all natural numbers is denoted by the symbol ‘N’.

$N = \{1, 2, 3, 4, 5, ...\}$ is the set of all natural numbers.

Whole numbers

Natural numbers together with zero (0) are called whole numbers. These numbers start with smallest number 0 and go without end. The set of all whole numbers is denoted by the symbol ‘W’.

$W = \{0, 1, 2, 3, 4, 5, ...\}$ is the set of all whole numbers.

Integers

The whole numbers and negative numbers together are called integers. The set of all integers is denoted by Z.

$Z = \{... -2, -1, 0, 1, 2, ...\}$ is the set of all integers (or) $Z = \{0, \pm 1, \pm 2, ...\}$ is the set of all Integers.

Do you know?

Ramanujam, the greatest Mathematician was born at Erode in Tamil Nadu.

1.3 Four Fundamental Operations on Integers

(i) Addition of Integers

Sum of two integers is again an integer.

For example,

i) $10 + (-4) = 10 - 4 = 6$

ii) $8 + 4 = 12$

iii) $6 + 0 = 6$

iv) $6 + 5 = 11$

v) $4 + 0 = 4$

(ii) Subtraction of integers

To subtract an integer from another integer, add the additive inverse of the second number to the first number.

For example,

i) $5 - 3 = 5 +$ addditive inverse of $3 = 5 + (-3) = 2$.

ii) $6 - (-2) = 6 +$ addditive inverse of $(-2) = 6 + 2 = 8$.

iii) $(-8) - (5) = (-8) + (-5) = -13$.

iv) $(-20) - (-6) = -20 + 6 = -14$.

(iii) Multiplication of integers

In the previous class, we have learnt that multiplication is repeated addition in the set of whole numbers. Let us learn about it now in the set of integers.

Rules :

1. The product of two positive integers is a positive integer.
2. The product of two negative integers is a positive integer.
3. The product of a positive integer and a negative integer is a negative integer.

Example

i) $5 \times 8 = 40$

ii) $(-5) \times (-9) = 45$

iii) $(-15) \times 3 = -(15 \times 3) = -45$

iv) $12 \times (-4) = -(12 \times 4) = -48$

Try these

1) $0 \times (-10) =$
2) $9 \times (-7) =$
3) $-5 \times (-10) =$
4) $-11 \times 6 =$

Activity

Draw a straight line on the ground. Mark the middle point of the line as '0' (Zero). Stand on the zero. Now jump one step to the right on the line. Mark it as + 1. From there jump one more step in the same direction and mark it as + 2. Continue jumping one step at a time and mark each step (as + 3, + 4, + 5, ...). Now come back to zero position on the line. Move one step to the left of '0' and mark it as – 1. Continue jumping one step at a time in the same direction and mark the steps as – 2, – 3, – 4, and so on. The number line is ready. Play the game of numbers as indicated below.

i) Stand on the zero of the number line facing right side of 0. Jumping two steps at a time. If you continue jumping like this 3 times, how far are you from '0' on number line?

ii) Stand on the zero of number line facing left side of 0. Jump 3 steps at a time. If you continue jumping like this 3 times, how far are you from '0' on the number line?

Activity

×	4	– 6	– 3	2	7	8
– 6	– 24					
– 5			15			– 40
3					21	

Example 1.1

Multiply (– 11) and (– 10).

Solution

$$-11 \times (-10) = (11 \times 10) = 110$$

Example 1.2

Multiply (– 14) and 9.

Solution

$$(-14) \times 9 = -(14 \times 9) = -126$$

Example 1.3

Find the value of 15×18.

Solution

$$15 \times 18 = 270$$

Example 1.4

The cost of a television set is ₹5200. Find the cost of 25 television sets.

Solution

The cost one television set = ₹5200

∴ the cost of 25 television set = 5200×25

= ₹130000

Exercise 1.1

1. Choose the best answer:

i) The value of multiplying a zero with any other integer is

(A) positive integer (B) negative integer

(C) 1 (D) 0

ii) -15^2 is equal to

(A) 225 (B) – 225 (C) 325 (D) 425

iii) $-15 \times (-9) \times 0$ is equal to

(A) – 15 (B) – 9 (C) 0 (D) 7

iv) The product of any two negative integers is a

(A) negative integer (B) positive integer

(C) natural number (D) whole number

2. Fill in the blanks:

i) The product of a negative integer and zero is __________.

ii) __________ $\times (-14) = 70$

iii) $(-72) \times$ __________ $= -360$

iv) $0 \times (-17) =$ __________.

3. Evaluate:

i) $3 \times (-2)$

ii) $(-1) \times 25$

iii) $(-21) \times (-31)$

iv) $(-316) \times 1$

v) $(-16) \times 0 \times (-18)$

vi) $(-12) \times (-11) \times 10$

vii) $(-5) \times (-5)$

viii) 5×5

ix) $(-3) \times (-7) \times (-2) \times (-1)$

x) $(-1) \times (-2) \times (-3) \times 4$

xi) $7 \times (-5) \times (9) \times (-6)$

xii) $7 \times 9 \times 6 \times (-5)$

xiii) $10 \times 16 \times (-9)$

xiv) $16 \times (-8) \times (-2)$

xv) $(-20) \times (-12) \times 25$

xvi) $9 \times 6 \times (-10) \times (-20)$

4. Multiply

i) (-9) and 15

ii) (-4) and (-4)

iii) 13 and 14

iv) (-25) with 32

v) (-1) with (-1)

vi) (-100) with 0

5. The cost of one pen is ₹15. What is the cost of 43 pens?

6. A question paper contains 20 questions and each question carries 5 marks. If a student answered 15 questions correctly, find his mark?

7. Revathi earns ₹ 150 every day. How much money will she have in 10 days?

8. The cost of one apple is ₹20. Find the cost of 12 apples?

(iv) Division of integers

We know that division is the inverse operation of multiplication.

We can state the rules of division as follows:

$$\frac{\text{Positive integer}}{\text{Positive integer}} = \text{Positive number}$$

$$\frac{\text{Negative integer}}{\text{Negative integer}} = \text{Positive number}$$

$$\frac{\text{Positive integer}}{\text{Negative integer}} = \text{Negative number}$$

$$\frac{\text{Negative integer}}{\text{Positive integer}} = \text{Negative number}$$

Try these

a) $\frac{0}{10} =$ b) $\frac{9}{-3} =$

c) $\frac{-3}{-3} =$ d) $\frac{-10}{2} =$

Division by zero

Division of any number by zero (except 0) is meaningless because division by zero is not defined.

Example 1.5

Divide 250 by 50.

Solution

Divide 250 by 50 is $\frac{250}{50} = 5$.

Example 1.6

Divide (– 144) by 12.

Solution

Divide (– 144) by 12 is $\frac{-144}{12}$ = – 12.

Example 1.7

Find the value $\frac{15\times(-30)\times(-60)}{2\times10}$.

Solution

$$\frac{15\times(-30)\times(-60)}{2\times10} = \frac{27000}{20} = 1350.$$

Example 1.8

A bus covers 200 km in 5 hours. What is the distance covered in 1 hour?

Solution

Distance covered in 5 hours = 200 km.

∴ Distance covered in 1 hour = $\frac{200}{5}$ = 40 km.

Exercise 1.2

1. Choose the best answer:

i) Division of integers is inverse operation of

(A) addition (B) subtraction (C) multiplication (D) division

ii) 369 ÷ .... = 369.

(A) 1 (B) 2 (C) 369 (D) 769

iii) – 206 ÷ .... = 1.

(A) 1 (B) 206 (C) – 206 (D) 7

iv) – 75 ÷ .... = – 1.

(A) 75 (B) – 1 (C) – 75 (D) 10

2. Evaluate

i) $(-30)\div6$ ii) $50 \div 5$ iii) $(-36)\div(-9)$

iv) $(-49)\div49$ v) $12 \div[(-3)+1]$ vi) $[(-36)\div6]-3$

vii) $[(-6)+7]\div[(-3)+2]$ viii) $[(-7)+(-19)]\div[(-10)+(-3)]$

ix) $[7+13]\div[2+8]$ x) $[7+23]\div[2+3]$

3. Evaluate

i) $\frac{(-1)\times(-5)\times(-4)\times(-6)}{2\times3}$ ii) $\frac{8\times5\times4\times3\times10}{4\times5\times6\times2}$ iii) $\frac{40\times(-20)\times(-12)}{4\times(-6)}$

4. The product of two numbers is 105. One of the number is (– 21). What is the other number?

1.5 Properties of Addition of integers

(i) Closure Property

Observe the following examples:

1. $19 + 23 = 42$
2. $-10 + 4 = -6$
3. $18 + (-47) = -29$

In general, for any two integers a and b, $a + b$ is an integer.

Therefore the set of integers is closed under addition.

(ii) Commutative Property

Two integers can be added in any order. In other words, addition is commutative for integers.

We have $8 + (-3) = 5$ and $(-3) + 8 = 5$

So, $8 + (-3) = (-3) + 8$

In general, for any two integers a and b we can say, $a + b = b + a$

Therefore addition of integers is commutative.

Try these

Are the following equal?

i) $(5) + (-12)$ and $(-12) + (5)$

ii) $(-20) + 72$ and $72 + (-20)$

(iii) Associative Property

Try these

Are the following pairs of expressions equal?

i) $7 + (5 + 4), (7 + 5) + 4$

ii) $(-5) + [(-2) + (-4)]$, $[(-5) + (-2)] + (-4)$

Observe the following example:

Consider the integers 5, – 4 and 7.

Look at $5 + [(-4) + 7] = 5 + 3 = 8$ and

$[5 + (-4)] + 7 = 1 + 7 = 8$

Therefore, $5 + [(-4) + 7] = [5 + (-4)] + 7$

In general, for any integers a, b and c, we can say, $a + (b + c) = (a + b) + c$.

Therefore addition of integers is associative.

(iv) Additive identity

When we add zero to any integer, we get the same integer.

Observe the example: $5 + 0 = 5$.

In general, for any integer a, $a + 0 = a$.

Therefore, zero is the additive identity for integers.

Try these

i) $17 + \_\_\_ = 17$

ii) $0 + \_\_\_ = 20$

iii) $-53 + \_\_\_ = -53$

Properties of subtraction of integers.

(i) Closure Property

Observe the following examples:

i) $5 - 12 = -7$

ii) $(-18) - (-13) = -5$

From the above examples it is clear that subtraction of any two integers is again an integer. In general, for any two integers a and b, $a - b$ is an integer.

Therefore, the set of integers is closed under subtraction.

(ii) Commutative Property

Consider the integers 7 and 4. We see that

$7 - 4 = 3$

$4 - 7 = -3$

$\therefore\ 7 - 4 \neq 4 - 7$

In general, for any two integers a and b

$a - b \neq b - a$

Therefore, we conclude that subtraction is not commutative for integers.

(iii) Associative Property

Consider the integers 7, 4 and 2

$7 - (4 - 2) = 7 - 2 = 5$

$(7 - 4) - 2 = 3 - 2 = 1$

$\therefore\ 7 - (4 - 2) \neq (7 - 4) - 2$

In general, for any three integers a , b and c

$a - (b - c) \neq (a - b) - c.$

Therefore, subtraction of integers is not associative.

Properties of multiplication of integers

(i) Closure property

Observe the following:

$$-10 \times (-5) = 50$$
$$40 \times (-15) = -600$$

In general, $a \times b$ is an integer, for all integers a and b.

Therefore, integers are closed under multiplication.

(ii) Commutative property

Observe the following:

$$5 \times (-6) = -30 \quad \text{and} \quad (-6) \times 5 = -30$$
$$5 \times (-6) = (-6) \times 5$$

Therefore, multiplication is commutative for integers.

In general, for any two integers a and b, $a \times b = b \times a$.

Try these

Are the following pairs equal?

i) $5 \times (-7)$, $(-7) \times 5$

ii) $9 \times (-10)$, $(-10) \times 9$

(iii) Multiplication by Zero

The product of any nonzero integer with zero is zero.

Observe the following:

$$5 \times 0 = 0$$
$$-8 \times 0 = 0$$

In general, for any nonzero integer a

$$a \times 0 = 0 \times a = 0$$

Try these

i) $0 \times 0 =$ _____

ii) $-100 \times 0 =$ _____

iii) $0 \times x =$ _____

(iv) Multiplicative identity

Observe the following:

$$5 \times 1 = 5$$
$$1 \times (-7) = -7$$

This shows that '1' is the multiplicative identity for integers.

In general, for any integer a we have

$$a \times 1 = 1 \times a = a$$

Try these

i) $(-10) \times 1 =$ ___

ii) $(-7) \times$ ___ $= -7$

iii) ___ $\times 9 = 9$

(v) Associative property for Multiplication

Consider the integers 2, – 5, 6.

Look at

$$[2\times(-5)]\times 6 = -10\times 6$$
$$= -60 \text{ and}$$
$$2\times[(-5)\times 6] = 2\times(-30)$$
$$= -60$$

Thus $[2\times(-5)]\times 6 = 2\times[(-5)\times 6]$

So we can say that integers are associative under multiplication.

In general, for any integers a, b, c, $(a \times b) \times c = a \times (b \times c)$.

(vi) Distributive property

Consider the integers 12, 9, 7.

Look at

$$12\times(9+7) = 12\times 16 = 192$$
$$(12\times 9)+(12\times 7) = 108+84 = 192$$

Thus $12\times(9+7) = (12\times 9)+(12\times 7)$

In general, for any integers a, b, c.

$$a\times(b+c) = (a\times b)+(a\times c).$$

Therefore, integers are distributive under multiplication.

Try these

Are the following equal?

1. $4\times(5+6)$ and $(4\times 5)+(4\times 6)$
2. $3\times(7-8)$ and $(3\times 7)+(3\times -8)$
3. $4\times(-5)$ and $(-5)\times 4$

Properties of division of integers

(i) Closure property

Observe the following examples:

(i) $15 \div 5 = 3$

(ii) $(-3)\div 9 = \frac{-3}{9} = \frac{-1}{3}$

(iii) $7\div 4 = \frac{7}{4}$

From the above examples we observe that ***integers are not closed under division.***

(ii) Commutative Property

Observe the following example:

$8 \div 4 = 2$ and

$4 \div 8 = \frac{1}{2}$

$\therefore\ 8 \div 4 \neq 4 \div 8$

We observe that ***integers are not commutative under division.***

(iii) Associative Property

Observe the following example:

$12 \div (6 \div 2) = 12 \div 3 = 4$

$(12 \div 6) \div 2 = 2 \div 2 = 1$

$\therefore\ 12 \div (6 \div 2) \neq (12 \div 6) \div 2$

From the above example we observe that ***integers are not associative under division.***

1.6 Fractions

Introduction

In the early classes we have learnt about fractions which included proper, improper and mixed fractions as well as their addition and subtraction. Now let us see multiplication and division of fractions.

Recall :

Proper fraction: A fraction is called a proper fraction if its

Denominator > Numerator.

Example: $\frac{3}{4}, \frac{1}{2}, \frac{9}{10}, \frac{5}{6}$

Improper fraction: A fraction is called an improper fraction if its

Numerator > Denominator.

Example : $\frac{5}{4}, \frac{6}{5}, \frac{41}{30}, \frac{51}{25}$

Mixed fraction : A fraction consisting of a natural number and a proper fraction is called a mixed fractions.

Example: $2\frac{3}{4},\ 1\frac{4}{5},\ 5\frac{1}{7}$

Think it : Mixed fraction = Natural number+ Proper fraction

Discuss : How many numbers are there from 0 to 1.

Do you know?

All whole numbers are fractional numbers with 1 as the denominator.

Recall : Addition and subtraction of fractions.

Example (i)

Simplify: $\frac{2}{5}+\frac{3}{5}$

Solution

$$\frac{2}{5}+\frac{3}{5}=\frac{2+3}{5}=\frac{5}{5}=1$$

Example (ii)

Simplify: $\frac{2}{3}+\frac{5}{12}+\frac{7}{24}$

Solution

$$\begin{aligned}\frac{2}{3}+\frac{5}{12}+\frac{7}{24} &= \frac{2\times 8+5\times 2+7\times 1}{24}\\ &= \frac{16+10+7}{24}\\ &= \frac{33}{24}=1\frac{3}{8}\end{aligned}$$

Example (iii)

Simplify: $5\frac{1}{4}+4\frac{3}{4}+7\frac{5}{8}$

Solution

$$\begin{aligned}5\frac{1}{4}+4\frac{3}{4}+7\frac{5}{8} &= \frac{21}{4}+\frac{19}{4}+\frac{61}{8}\\ &= \frac{42+38+61}{8}=\frac{141}{8}\\ &= 17\frac{5}{8}\end{aligned}$$

Example (iv)

Simplify: $\frac{5}{7}-\frac{2}{7}$

Solution

$$\frac{5}{7}-\frac{2}{7}=\frac{5-2}{7}=\frac{3}{7}.$$

Example (v)

Simplify: $2\frac{2}{3}-3\frac{1}{6}+6\frac{3}{4}$

Solution

$$2\frac{2}{3}-3\frac{1}{6}+6\frac{3}{4}=\frac{8}{3}-\frac{19}{6}+\frac{27}{4}$$

$$= \frac{32 - 38 + 81}{12}$$

$$= \frac{75}{12} = 6\frac{1}{4}$$

(i) Multiplication of a fractions by a whole number

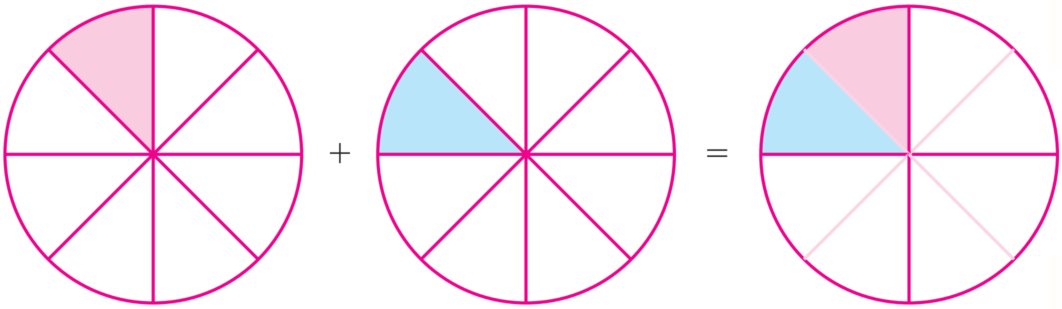


Fig. 1.1

Observe the pictures at the (fig.1.1) . Each shaded part is $\frac{1}{8}$ part of a circle. How much will the two shaded parts represent together?

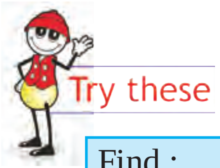
They will represent $\frac{1}{8} + \frac{1}{8} = 2 \times \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$

To multiply a proper or improper fraction with the whole number:

we first multiply the whole number with the numerator of the fraction, keeping the denominator same. If the product is an improper fraction, convert it as a mixed fraction.

To multiply a mixed fraction by a whole number, first convert the mixed fraction to an improper fraction and then multiply.

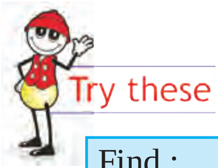
Therefore, $4 \times 3\frac{4}{7} = 4 \times \frac{25}{7} = \frac{100}{7} = 14\frac{2}{7}$



Find :

i) $\frac{2}{5} \times 4$ ii) $\frac{8}{5} \times 4$

iii) $4 \times \frac{1}{5}$ iv) $\frac{13}{11} \times 6$



Find :

i) $6 \times 7\frac{2}{3}$

ii) $3\frac{2}{9} \times 7$

(ii) Fraction as an operator 'of'

From the figure (fig. 1.2) each shaded portion represents $\frac{1}{3}$ of 1. All the three shaded portions together will represent $\frac{1}{3}$ of 3.

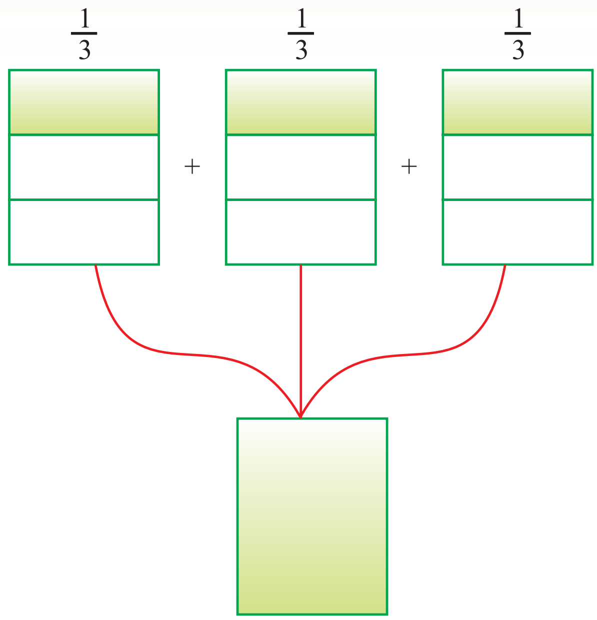


Fig. 1.2

Combining the 3 shaded portions we get 1.

Thus, one-third of 3 = $\frac{1}{3} \times 3 = 1$.

We can observe that 'of' represents a multiplication.

Prema has 15 chocolates. Sheela has $\frac{1}{3}$rd of the number of chocolates what Prema has. How many chocolates Sheela has? As, 'of' indicates multiplication,

Sheela has $\frac{1}{3} \times 15 = 5$ chocolates.

Example 1.9

Find : $\frac{1}{4}$ of $2\frac{1}{5}$

Solution

$$\begin{aligned} \frac{1}{4} \text{ of } 2\frac{1}{5} &= \frac{1}{4} \times 2\frac{1}{5} \\ &= \frac{1}{4} \times \frac{11}{5} \\ &= \frac{11}{20} \end{aligned}$$

Example 1.10

In a group of 60 students $\frac{3}{10}$ of the total number of students like to study Science, $\frac{3}{5}$ of the total number like to study Social Science.

(i) How many students like to study Science?

(ii) How many students like to study Social Science?

Solution

Total number of students in the class $= 60$

(i) Out of 60 students, $\frac{3}{10}$ of the students like to study Science.

Thus, the number of students who like to study Science $= \frac{3}{10}$ of 60

$$= \frac{3}{10} \times 60 = 18.$$

(ii) Out of 60 students, $\frac{3}{5}$ of the students like to study Social Science.

Thus, the number of students who like to study Social Science

$$= \frac{3}{5} \text{ of } 60$$

$$= \frac{3}{5} \times 60 = 36.$$

Exercise 1.3

1. Multiply :

i) $6 \times \frac{4}{5}$ ii) $3 \times \frac{3}{7}$ iii) $4 \times \frac{4}{8}$ iv) $15 \times \frac{2}{10}$

v) $\frac{2}{3} \times 7$ vi) $\frac{5}{2} \times 8$ vii) $\frac{11}{4} \times 7$ viii) $\frac{5}{6} \times 12$

ix) $\frac{4}{7} \times 14$ x) $18 \times \frac{4}{3}$

2. Find :

i) $\frac{1}{2}$ of 28 ii) $\frac{7}{3}$ of 27 iii) $\frac{1}{4}$ of 64 iv) $\frac{1}{5}$ of 125

v) $\frac{8}{6}$ of 216 vi) $\frac{4}{8}$ of 32 vii) $\frac{3}{9}$ of 27 viii) $\frac{7}{10}$ of 100

ix) $\frac{5}{7}$ of 35 x) $\frac{1}{2}$ of 100

3. Multiply and express as a mixed fraction :

i) $5 \times 5\frac{1}{4}$ ii) $3 \times 6\frac{3}{5}$ iii) $8 \times 1\frac{1}{5}$ iv) $6 \times 10\frac{5}{7}$

v) $7 \times 7\frac{1}{2}$ vi) $9 \times 9\frac{1}{2}$

4. Vasu and Visu went for a picnic. Their mother gave them a baggage of 10 one litre water bottles. Vasu consumed $\frac{2}{5}$ of the water Visu consumed the remaining water. How much water did Vasu drink?

(iii) Multiplication of a fraction by a fraction

Example 1.11

Find $\frac{1}{5}$ of $\frac{3}{8}$.

Solution

$$\frac{1}{5} \text{ of } \frac{3}{8} = \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$$

Example 1.12

Find $\frac{2}{9} \times \frac{3}{2}$.

Solution

$$\frac{2}{9} \times \frac{3}{2} = \frac{1}{3}$$

Example 1.13

Leela reads $\frac{1}{4}^{th}$ of a book in 1 hour. How much of the book will she read in $3\frac{1}{2}$ hours?

Solution

The part of the book read by leela in 1 hour $= \frac{1}{4}$

So, the part of the book read by her in $3\frac{1}{2}$ hour $= 3\frac{1}{2} \times \frac{1}{4}$

$$= \frac{7}{2} \times \frac{1}{4}$$

$$= \frac{7 \times 1}{4 \times 2}$$

$$= \frac{7}{8}$$

$\therefore$ Leela reads $\frac{7}{8}$ part of a book in $3\frac{1}{2}$ hours.

Try these

Find

i) $\frac{1}{3} \times \frac{7}{5}$

ii) $\frac{2}{3} \times \frac{8}{9}$

Exercise 1.4

1. Find :

i) $\frac{10}{5}$ of $\frac{5}{10}$ ii) $\frac{2}{3}$ of $\frac{7}{8}$ iii) $\frac{1}{3}$ of $\frac{7}{4}$ iv) $\frac{4}{8}$ of $\frac{7}{9}$

v) $\frac{4}{9}$ of $\frac{9}{4}$ vi) $\frac{1}{7}$ of $\frac{2}{9}$

2. Multiply and reduce to lowest form :

i) $\frac{2}{9} \times 3\frac{2}{3}$ ii) $\frac{2}{9} \times \frac{9}{10}$ iii) $\frac{3}{8} \times \frac{6}{9}$ iv) $\frac{7}{8} \times \frac{9}{14}$

v) $\frac{9}{2} \times \frac{3}{3}$ vi) $\frac{4}{5} \times \frac{12}{7}$

3. Simplify the following fractions :

i) $\frac{2}{5} \times 5\frac{2}{3}$ ii) $6\frac{3}{4} \times \frac{7}{10}$ iii) $7\frac{1}{2} \times 1$

iv) $5\frac{3}{4} \times 3\frac{1}{2}$ v) $7\frac{1}{4} \times 8\frac{1}{4}$

4. A car runs 20 km. using 1 litre of petrol. How much distance will it cover using $2\frac{3}{4}$ litres of petrol.

5. Everyday Gopal read book for $1\frac{3}{4}$ hours. He reads the entire book in 7 days. How many hours in all were required by him to read the book?

The reciprocal of a fraction

If the product of two non-zero numbers is equal to one then each number is called the reciprocal of the other. So reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$, the reciprocal of $\frac{5}{3}$ is $\frac{3}{5}$.

Note: Reciprocal of 1 is 1 itself. 0 does not have a reciprocal.

(iv) Division of a whole number by a fraction

To divide a whole number by any fraction, multiply that whole number by the reciprocal of that fraction.

Example 1.14

Find (i) $6 \div \frac{2}{5}$ (ii) $8 \div \frac{7}{9}$

Solution

(i) $6 \div \frac{2}{5} = 6 \times \frac{5}{2} = 15$

(ii) $8 \div \frac{7}{9} = 8 \times \frac{9}{7} = \frac{72}{7}$

While dividing a whole number by a mixed fraction, first convert the mixed fraction into improper fraction and then solve it.

Try these

Find:

i) $6 \div 5\frac{2}{3}$ ii) $9 \div 3\frac{3}{7}$

Example 1.15

Find $6 \div 3\frac{4}{5}$

Solution

$6 \div 3\frac{4}{5} = 6 \div \frac{19}{5} = 6 \times \frac{5}{19} = \frac{30}{19} = 1\frac{11}{19}$

(v) Division of a fraction by another fraction

To divide a fraction by another fraction, multiply the first fraction by the reciprocal of the second fraction.

We can now find $\frac{1}{5} \div \frac{3}{7}$

$$\frac{1}{5} \div \frac{3}{7} = \frac{1}{5} \times \text{reciprocal of } \frac{3}{7}.$$

$$= \frac{1}{5} \times \frac{7}{3} = \frac{7}{15}$$

Try these

Find:

i) $\frac{3}{7} \div \frac{4}{5}$, ii) $\frac{1}{2} \div \frac{4}{5}$, iii) $2\frac{3}{4} \div \frac{7}{2}$

Exercise 1.5

1. Find the reciprocal of each of the following fractions:

   i) $\frac{5}{7}$ ii) $\frac{4}{9}$ iii) $\frac{10}{7}$ iv) $\frac{9}{4}$

   v) $\frac{33}{2}$ vi) $\frac{1}{9}$ vii) $\frac{1}{13}$ viii) $\frac{7}{5}$

2. Find :

   i) $\frac{5}{3} \div 25$ ii) $\frac{6}{9} \div 36$ iii) $\frac{7}{3} \div 14$ iv) $1\frac{1}{4} \div 15$

3. Find :

   (i) $\frac{2}{5} \div \frac{1}{4}$ (ii) $\frac{5}{6} \div \frac{6}{7}$ (iii) $2\frac{3}{4} \div \frac{3}{5}$ (iv) $3\frac{3}{2} \div \frac{8}{3}$

4. How many uniforms can be stitched from $47\frac{1}{4}$ metres of cloth if each scout requires $2\frac{1}{4}$ metres for one uniform?

5. The distance between two places is $47\frac{1}{2}$ km. If it takes $1\frac{3}{16}$ hours to cover the distance by a van, what is the speed of the van?

1.7 Introduction to Rational Numbers

A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Here p is the numerator and q is the denominator.

For example $\frac{7}{3}, \frac{-5}{7}, \frac{2}{9}, \frac{11}{-7}, \frac{-3}{11}$ are the rational numbers

A rational number is said to be in *standard form* if its denominator is positive and the numerator and denominator have no common factor other than 1.

If a rational number is not in the standard form, then it can be reduced to the standard form.

Example 1.16

Reduce $\frac{72}{54}$ to the standard form.

Solution

We have, $\frac{72}{54} = \frac{72 \div 2}{54 \div 2}$

$$= \frac{36}{27} = \frac{36 \div 3}{27 \div 3}$$

$$= \frac{12}{9} = \frac{12 \div 3}{9 \div 3}$$

$$= \frac{4}{3}$$

Aliter: $\frac{72}{54} = \frac{72 \div 18}{54 \div 18} = \frac{4}{3}$

In this example, note that 18 is the highest common factor (H.C.F.) of 72 and 54.

To reduce the rational number to its standard form, we divide its numerator and denominator by their H.C.F. ignoring the negative sign if any.

If there is negative sign in the denominator divide by " – H.C.F.".

Example 1.17

Reduce to the standard form.

(i) $\frac{18}{-12}$ (ii) $\frac{-4}{-16}$

Try these

Write in standard form.

i) $\frac{-18}{51}$, ii) $\frac{-12}{28}$, iii) $\frac{7}{35}$

Solution

(i) The H.C.F. of 18 and 12 is 6

Thus, its standard form would be obtained by dividing by – 6.

$$\frac{18}{-12} = \frac{18 \div (-6)}{-12 \div (-6)} = \frac{-3}{2}$$

(ii) The H.C.F. of 4 and 16 is 4.

Thus, its standard form would be obtained by dividing by – 4

$$\frac{-4}{-16} = \frac{-4 \div (-4)}{-16 \div (-4)} = \frac{1}{4}$$

1.8 Representation of Rational numbers on the Number line.

You know how to represent integers on the number line. Let us draw one such number line.

The points to the right of 0 are positive integers. The points to left of 0 are negative integers.

Let us see how the rational numbers can be represented on a number line.

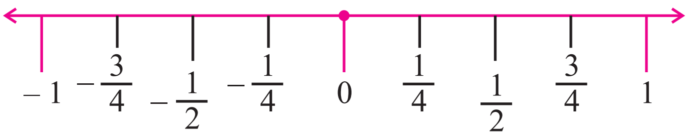


Fig. 1.3

Let us represent the number $-\frac{1}{4}$ on the number line.

As done in the case of positive integers, the positive rational numbers would be marked on the right of 0 and the negative rational numbers would be marked on the left of 0.

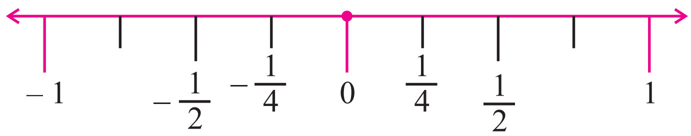


Fig. 1.4

To which side of 0, will you mark $-\frac{1}{4}$? Being a negative rational number, it would be marked to the left of 0.

You know that while marking integers on the number line, successive integers are marked at equal intervals. Also, from 0, the pair 1 and – 1 is equidistant .

In the same way, the rational numbers $\frac{1}{4}$ and $-\frac{1}{4}$ would be at equal distance from 0. How to mark the rational number $\frac{1}{4}$? It is marked at a point which is one fourth of the distance from 0 to 1. So, $-\frac{1}{4}$ would be marked at a point which is one fourth of the distance from 0 to – 1.

We know how to mark $\frac{3}{2}$ on the number line. It is marked on the right of 0 and lies halfway between 1 and 2. Let us now mark $\frac{-3}{2}$ on the number line. It lies on the left of 0 and is at the same distance as $\frac{3}{2}$ from 0.

Similarly $-\frac{1}{2}$ is to the left of zero and at the same distance from zero as $\frac{1}{2}$ is to the right. So as done above, $-\frac{1}{2}$ can be represented on the number line. All other rational numbers can be represented in a similar way.

Rational numbers between two rational numbers

Raju wants to count the whole numbers between 4 and 12. He knew there would be exactly 7 whole numbers between 4 and 12.

Are there any integers between 5 and 6 ?

There is no integer between 5 and 6.

$\therefore$ Number of integers between any two integers is finite.

Now let us see what will happen in the case of rational numbers ?

Raju wants to count the rational numbers between $\frac{3}{7}$ and $\frac{2}{3}$.

For that he converted them to rational numbers with same denominators.

So $\frac{3}{7} = \frac{9}{21}$ and $\frac{2}{3} = \frac{14}{21}$

Now he has, $\frac{9}{21} < \frac{10}{21} < \frac{11}{21} < \frac{12}{21} < \frac{13}{21} < \frac{14}{21}$

So $\frac{10}{21}, \frac{11}{21}, \frac{12}{21}, \frac{13}{21}$ are the rational numbers in between $\frac{9}{21}$ and $\frac{14}{21}$.

Now we can try to find some more rational numbers in between $\frac{3}{7}$ and $\frac{2}{3}$.

we have $\frac{3}{7} = \frac{18}{42}$ and $\frac{2}{3} = \frac{28}{42}$

So, $\frac{18}{42} < \frac{19}{42} < \frac{20}{42} < \cdots < \frac{28}{42}$. Therefore $\frac{3}{7} < \frac{19}{42} < \frac{20}{42} < \frac{21}{42} < \cdots < \frac{2}{3}$.

Hence we can find some more rational numbers in between $\frac{3}{7}$ and $\frac{2}{3}$.

We can find unlimited (infinite) number of rational numbers between any two rational numbers.

Example 1.18

List five rational numbers between $\frac{2}{5}$ and $\frac{4}{7}$.

Solution

Let us first write the given rational numbers with the same denominators.

Now, $\frac{2}{5} = \frac{2\times7}{5\times7} = \frac{14}{35}$ and $\frac{4}{7} = \frac{4\times5}{7\times5} = \frac{20}{35}$

So, we have $\frac{14}{35} < \frac{15}{35} < \frac{16}{35} < \frac{17}{35} < \frac{18}{35} < \frac{19}{35} < \frac{20}{35}$

$\frac{15}{35}, \frac{16}{35}, \frac{17}{35}, \frac{18}{35}, \frac{19}{35}$ are the five required rational numbers.

Example 1.19

Find seven rational numbers between $-\frac{5}{3}$ and $-\frac{8}{7}$

Solution

Let us first write the given rational numbers with the same denominators.

Now, $-\frac{5}{3} = -\frac{5\times7}{3\times7} = -\frac{35}{21}$ and $-\frac{8}{7} = -\frac{8\times3}{7\times3} = -\frac{24}{21}$

So, we have $\frac{-35}{21} < \frac{-34}{21} < \frac{-33}{21} < \frac{-32}{21} < \frac{-31}{21} < \frac{-30}{21}$

$$< -\frac{29}{21} < -\frac{28}{21} < -\frac{27}{21} < -\frac{26}{21} < -\frac{25}{21} < -\frac{24}{21}$$

$\therefore$ The seven rational numbers are $-\frac{34}{21}, -\frac{33}{21}, -\frac{32}{21}, -\frac{31}{21}, -\frac{30}{21}, -\frac{29}{21}, -\frac{28}{21}$.

(We can take any seven rational numbers)

Exercise 1.6

1. Choose the best answer.

i) $\frac{3}{8}$ is called a

(A) positive rational number (B) negative rational number
(C) whole number (D) positive integer

ii) The proper negative rational number is

(A) $\frac{4}{3}$ (B) $\frac{-7}{-5}$ (C) $-\frac{10}{9}$ (D) $\frac{10}{9}$

iii) Which is in the standard form?

(A) $-\frac{4}{12}$ (B) $-\frac{1}{12}$ (C) $\frac{1}{-12}$ (D) $\frac{-7}{14}$

iv) A fraction is a

(A) whole number (B) natural number (C) odd number (D) rational number

2. List four rational numbers between:

i) $-\frac{7}{5}$ and $-\frac{2}{3}$ ii) $\frac{1}{2}$ and $\frac{4}{3}$ iii) $\frac{7}{4}$ and $\frac{8}{7}$

3. Reduce to the standard form:

i) $\frac{-12}{16}$ ii) $\frac{-18}{48}$ iii) $\frac{21}{-35}$
iv) $\frac{-70}{42}$ v) $\frac{-4}{8}$

4. Draw a number line and represent the following rational numbers on it.

i) $\frac{3}{4}$ ii) $\frac{-5}{8}$ iii) $\frac{-8}{3}$
iv) $\frac{6}{5}$ v) $-\frac{7}{10}$

5. Which of the following are in the standard form:

i) $\frac{2}{3}$ ii) $\frac{4}{16}$ iii) $\frac{9}{6}$
iv) $\frac{-1}{7}$ v) $\frac{-4}{7}$

1.9 Four Basic Operations on Rational numbers

You know how to add, subtract, multiply and divide on integers. Let us now study these four basic operations on rational numbers.

(i) Addition of rational numbers

Let us add two rational numbers with same denominator.

Example 1.20

Add $\frac{9}{5}$ and $\frac{7}{5}$.

Solution

$$\frac{9}{5}+\frac{7}{5} = \frac{9+7}{5} = \frac{16}{5}.$$

Let us add two rational numbers with different denominators.

Example 1.21

Simplify: $\frac{7}{3}+\left(\frac{-5}{4}\right)$

Solution

$$\frac{7}{3}+\left(\frac{-5}{4}\right)$$

$$= \frac{28-15}{12} \quad \text{(L.C.M. of 3 and 4 is 12)}$$

$$= \frac{13}{12}$$

Example 1.22

Simplify $\frac{-3}{4}+\frac{1}{2}-\frac{5}{6}$.

Solution

$$\frac{-3}{4}+\frac{1}{2}-\frac{5}{6} = \frac{(-3\times 3)+(1\times 6)-(5\times 2)}{12} \quad \text{(L.C.M. of 4,2 and 6 is 12)}$$

$$= \frac{-9+6-10}{12}$$

$$= \frac{-19+6}{12} = \frac{-13}{12}$$

(ii) Subtraction of rational numbers

Example 1.23

Subtract $\frac{8}{7}$ from $\frac{10}{3}$.

Solution:

$$\frac{10}{3}-\frac{8}{7} = \frac{70-24}{21} = \frac{46}{21}$$

Example 1.24

Simplify $\frac{6}{35}-\left(\frac{-10}{35}\right)$.

Solution:

$$\frac{6}{35}-\left(\frac{-10}{35}\right) = \frac{6+10}{35} = \frac{16}{35}$$

Example 1.25

Simplify $\left(-2\frac{7}{35}\right)-\left(3\frac{6}{35}\right)$.

Solution

$$\left(-2\frac{7}{35}\right)-\left(3\frac{6}{35}\right) = \frac{-77}{35}-\frac{111}{35}$$

$$= \frac{-77-111}{35} = \frac{-188}{35} = -5\frac{13}{35}$$

Example 1.26

The sum of two rational numbers is 1. If one of the numbers is $\frac{5}{20}$, find the other.

Solution

Sum of two rational numbers = 1

Given number + Required number = 1

$\frac{5}{20}$ + Required number = 1

Required number $= 1-\frac{5}{20}$

$= \frac{20-5}{20}$

$= \frac{15}{20} = \frac{3}{4}$

$\therefore$ Required number is $\frac{3}{4}$.

Try these

i) $\frac{7}{35}-\frac{5}{35}$, ii) $\frac{5}{6}-\frac{7}{12}$,

iii) $\frac{7}{3}-\frac{3}{4}$, iv) $\left(3\frac{3}{4}\right)-\left(2\frac{1}{4}\right)$,

v) $\left(4\frac{5}{7}\right)-\left(6\frac{1}{4}\right)$

Exercise1.7

1. Choose the best answer.

i) $\frac{1}{3}+\frac{2}{3}$ is equal to

(A) 2 (B) 3 (C) 1 (D) 4

ii) $\frac{4}{5}-\frac{9}{5}$ is equal to

(A) 1 (B) 3 (C) – 1 (D) 7

iii) $5\frac{1}{11}+1\frac{10}{11}$ is equal to

(A) 4 (B) 3 (C) – 5 (D) 7

iv) The sum of two rational numbers is 1. If one of the numbers is $\frac{1}{2}$, the other number is

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{-3}{4}$ (D) $\frac{1}{2}$

2. Add :

i) $\frac{12}{5}$ and $\frac{6}{5}$ ii) $\frac{7}{13}$ and $\frac{17}{13}$ iii) $\frac{8}{7}$ and $\frac{6}{7}$

iv) $-\frac{7}{13}$ and $-\frac{5}{13}$ v) $\frac{7}{3}$ and $\frac{8}{4}$ vi) $-\frac{5}{7}$ and $\frac{7}{6}$

vii) $\frac{9}{7}$ and $-\frac{10}{3}$ viii) $\frac{3}{6}$ and $-\frac{7}{2}$ ix) $\frac{9}{4}, \frac{8}{7}$ and $\frac{1}{28}$

x) $\frac{4}{5}, -\frac{7}{10}$ and $-\frac{8}{15}$

3. Find the sum of the following :

i) $-\frac{3}{4}+\frac{7}{4}$ ii) $\frac{9}{6}+\frac{15}{6}$ iii) $-\frac{3}{4}+\frac{6}{11}$

iv) $-\frac{7}{8}+\frac{9}{16}$ v) $\frac{4}{5}+\frac{7}{20}$ vi) $\left(-\frac{6}{13}\right)+\left(-\frac{14}{26}\right)$

vii) $\frac{11}{13}+\left(-\frac{7}{2}\right)$ viii) $\left(-\frac{2}{5}\right)+\frac{5}{12}+\left(-\frac{7}{10}\right)$

ix) $\frac{7}{9}+\left(-\frac{10}{18}\right)+\left(-\frac{7}{27}\right)$ x) $\frac{6}{3}+\left(-\frac{7}{6}\right)+\left(-\frac{9}{12}\right)$

4. Simplify :

i) $\frac{7}{35}-\frac{5}{35}$ ii) $\frac{5}{6}-\frac{7}{12}$ iii) $\frac{7}{3}-\frac{3}{4}$

iv) $\left(3\frac{3}{4}\right)-\left(2\frac{1}{4}\right)$ v) $\left(4\frac{5}{7}\right)-\left(6\frac{1}{4}\right)$

5. Simplify :

i) $\left(1\frac{2}{11}\right)+\left(3\frac{5}{11}\right)$ ii) $\left(3\frac{4}{5}\right)-\left(7\frac{3}{10}\right)$

iii) $\left(-1\frac{2}{11}\right)+\left(-3\frac{5}{11}\right)+\left(6\frac{3}{11}\right)$ iv) $\left(-3\frac{9}{10}\right)+\left(3\frac{2}{5}\right)+\left(6\frac{5}{20}\right)$

v) $\left(-3\frac{4}{5}\right)+\left(2\frac{3}{8}\right)$ vi) $\left(-1\frac{5}{12}\right)+\left(-2\frac{7}{11}\right)$

vii) $\left(9\frac{6}{7}\right)+\left(-11\frac{2}{3}\right)+\left(-5\frac{7}{42}\right)$ viii) $\left(7\frac{3}{10}\right)+\left(-10\frac{7}{21}\right)$

6. The sum of two rational numbers is $\frac{17}{4}$. If one of the numbers is $\frac{5}{2}$, find the other number.

7. What number should be added to $\frac{5}{6}$ so as to get $\frac{49}{30}$.

8. A shopkeeper sold $7\frac{3}{4}$kg, $2\frac{1}{2}$kg and $3\frac{3}{5}$kg of sugar to three consumers in a day. Find the total weight of sugar sold on that day.

9. Raja bought 25 kg of Rice and he used $1\frac{3}{4}$ kg on the first day, $4\frac{1}{2}$ kg on the second day. Find the remaining quantity of rice left.

10. Ram bought 10 kg apples and he gave $3\frac{4}{5}$kg to his sister and $2\frac{3}{10}$ kg to his friend. How many kilograms of apples are left?

(iii) Multiplication of Rational numbers

To find the multiplication of two rational numbers, multiply the numerators and multiply the denominators separately and put them as new rational number. Simplify the new rational number into its lowest form.

Example 1.27

Find the product of $\left(\frac{4}{-11}\right)$ and $\left(\frac{-22}{8}\right)$.

Solution

$$\left(\frac{4}{-11}\right)\times\left(\frac{-22}{8}\right)$$

$$=\left(\frac{-4}{11}\right)\times\left(\frac{-22}{8}\right) \quad = \frac{88}{88}$$

$$= 1$$

Example 1.28

Find the product of $\left(-2\frac{4}{15}\right)$ and $\left(-3\frac{2}{49}\right)$.

Solution

$$\left(-2\frac{4}{15}\right)\times\left(-3\frac{2}{49}\right) \quad = \quad \left(\frac{-34}{15}\right)\times\left(\frac{-149}{49}\right)$$

$$= \frac{5066}{735} = 6\frac{656}{735}$$

Example 1.29

The product of two rational numbers is $\frac{2}{9}$. If one of the numbers is $\frac{1}{2}$, find the other rational number.

Solution

The product of two rational numbers $= \frac{2}{9}$

One rational number $= \frac{1}{2}$

$\therefore$ Given number $\times$ required number $= \frac{2}{9}$

$\frac{1}{2}\times$ required number $= \frac{2}{9}$

required number $= \frac{2}{9}\times\frac{2}{1} = \frac{4}{9}$

$\therefore$ Required rational number is $\frac{4}{9}$.

Multiplicative inverse (or reciprocal) of a rational number

If the product of two rational numbers is equal to 1, then one number is called the multiplicative inverse of other.

i) $\frac{7}{23} \times \frac{23}{7} = 1$

$\therefore$ The multiplicative inverse of $\frac{7}{23}$ is $\frac{23}{7}$.

Similarly the multiplicative inverse of $\frac{23}{7}$ is $\frac{7}{23}$.

ii) $\left(\frac{-8}{12}\right) \times \left(\frac{12}{-8}\right) = 1$

$\therefore$ The multiplicative inverse of $\left(\frac{-8}{12}\right)$ is $\left(\frac{12}{-8}\right)$.

Try these

Find

1) $\frac{7}{8} \times \frac{9}{12}$, 2) $\frac{11}{12} \times \frac{24}{33}$

3) $\left(-1\frac{1}{4}\right) \times \left(-7\frac{2}{3}\right)$

(iv) Division of rational numbers

To divide one rational number by another rational number, multiply the first rational number with the multiplicative inverse of the second rational number.

Example 1.30

Find $\left(\frac{2}{3}\right) \div \left(\frac{-5}{10}\right)$.

Solution

$$\left(\frac{2}{3}\right) \div \left(\frac{-5}{10}\right) = \frac{2}{3} \div \left(\frac{-1}{2}\right)$$
$$= \frac{2}{3} \times (-2) = \frac{-4}{3}$$

Example 1.31

Find $4\frac{3}{7} \div 2\frac{3}{8}$.

Solution

$$4\frac{3}{7} \div 2\frac{3}{8} = \frac{31}{7} \div \frac{19}{8}$$
$$= \frac{31}{7} \times \frac{8}{19} = \frac{248}{133}$$
$$= 1\frac{115}{133}$$

Exercise 1.8

1. Choose the best answer.

i) $\frac{7}{13} \times \frac{13}{7}$ is equal to

(A) 7 (B) 13 (C) 1 (D) – 1

ii) The multiplicative inverse of $\frac{7}{8}$ is

(A) $\frac{7}{8}$ (B) $\frac{8}{7}$ (C) $\frac{-7}{8}$ (D) $\frac{-8}{7}$

iii) $\frac{4}{-11} \times \left(\frac{-22}{8}\right)$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

iv) $-\frac{4}{9} \div \frac{9}{36}$ is equal to

(A) $\frac{-16}{9}$ (B) 4 (C) 5 (D) 7

2. Multiply :

i) $\frac{-12}{5}$ and $\frac{6}{5}$ ii) $\frac{-7}{13}$ and $\frac{5}{13}$

iii) $\frac{-3}{9}$ and $\frac{7}{8}$ iv) $\frac{-6}{11}$ and $\frac{44}{22}$

v) $\frac{-50}{7}$ and $\frac{28}{10}$ vi) $\frac{-5}{6}$ and $\frac{-4}{15}$

3. Find the value of the following :

i) $\frac{9}{5} \times \frac{-10}{4} \times \frac{15}{18}$ ii) $\frac{-8}{4} \times \frac{-5}{6} \times \frac{-30}{10}$

iii) $1\frac{1}{5} \times 2\frac{2}{5} \times 9\frac{3}{10}$ iv) $-3\frac{4}{15} \times -2\frac{1}{5} \times 9\frac{1}{5}$ v) $\frac{3}{6} \times \frac{9}{7} \times \frac{10}{4}$

4. Find the value of the following :

i) $\frac{-4}{9} \div \frac{9}{-4}$ ii) $\frac{3}{5} \div \left(\frac{-4}{10}\right)$

iii) $\left(\frac{-8}{35}\right) \div \frac{7}{35}$ iv) $-9\frac{3}{4} \div 1\frac{3}{40}$

5. The product of two rational numbers is 6. If one of the number is $\frac{14}{3}$, find the other number.

6. What number should be multiply $\frac{7}{2}$ to get $\frac{21}{4}$?

1.10 Decimal numbers

(i) Represent Rational Numbers as Decimal numbers

You have learnt about decimal numbers in the earlier classes. Let us briefly recall them here.

All rational numbers can be converted into decimal numbers.

For Example

(i) $\frac{1}{8} = 1 \div 8$

$\therefore \frac{1}{8} = 0.125$

(ii) $\frac{3}{4} = 3 \div 4$

$\therefore \frac{3}{4} = 0.75$

(iii) $3\frac{1}{5} = \frac{16}{5} = 3.2$

(iv) $\frac{2}{3} = 0.6666\cdots$ Here 6 is recurring without end.

Decimal Numbers

Addition and Subtraction of decimals:

Example 1.32

Add 120.4, 2.563, 18.964

Solution

$$\begin{array}{r} 120.4 \\ 2.563 \\ 18.964 \\ \hline 141.927 \\ \hline \end{array}$$

Example 1.33

Subtract 43.508 from 63.7

Solution

$$\begin{array}{rr} & 63.700 \\ (-) & 43.508 \\ \hline & 20.192 \\ \hline \end{array}$$

Example 1.34

Find the value of 27.69 – 14.04 + 35.072 – 10.12.

Solution

$$\begin{array}{r} 27.690 \\ 35.072 \\ \hline 62.762 \\ \hline \end{array} \qquad \begin{array}{r} -14.04 \\ -10.12 \\ \hline -24.16 \\ \hline \end{array} \qquad \begin{array}{r} 62.762 \\ -24.16 \\ \hline 38.602 \\ \hline \end{array}$$

The value is 38.602.

Examples 1.35

Deepa bought a pen for ₹177.50. a pencil for ₹4.75 and a notebook for ₹20.60. What is her total expenditure?

Solution

Cost of one pen = ₹177.50

Cost of one pencil = ₹4.75

Cost of one notebook = ₹20.60

∴ Deepa's total expenditure = ₹202.85

1.11 Multiplication of Decimal Numbers

Rani purchased 2.5 kg fruits at the rate of ₹23.50 per kg. How much money should she pay? Certainly it would be ₹(2.5 × 23.50). Both 2.5 and 23.5 are decimal numbers. Now, we have come across a situation where we need to know how to multiply two decimals. So we now learn the multiplication of two decimal numbers.

Let us now find 1.5 × 4.3

Multiplying 15 and 43. We get 645. Both, in 1.5 and 4.3, there is 1 digit to the right of the decimal point. So, count 2 digits from the right and put a decimal point. (since 1 + 1 = 2)

While multiplying 1.43 and 2.1, you will first multiply 143 and 21. For placing the decimal in the product obtained, you will count 2 + 1 = 3 digits starting from the right most digit. Thus 1.43 × 2.1 = 3.003.

Try these

i) 2.9 × 5

ii) 1.9 × 1.3

iii) 2.2 × 4.05

Example 1.36

The side of a square is 3.2 cm. Find its perimeter.

Do you know?

Perimeter of a square = 4 × side

Solution

All the sides of a square are equal.

Length of each side = 3.2 cm.

Perimeter of a square = 4 × side

Thus, perimeter = 4 × 3.2 = 12.8 cm.

Example 1.37

The length of a rectangle is 6.3 cm and its breath is 3.2 cm. What is the area of the rectangle?

Solution:

Length of the rectangle = 6.3 cm

Breadth of the rectangle = 3.2 cm.

Area of the rectangle = (length) × (breath)

= 6.3 × 3.2 = 20.16 cm^2

Multiplication of Decimal number by 10, 100 and 1000

Rani observed that $3.7 = \frac{37}{10}$, $3.72 = \frac{372}{100}$ and $3.723 = \frac{3723}{1000}$ Thus, she found that depending on the position of the decimal point the decimal number can be converted to a fraction with denominator 10 , 100 or 1000. Now let us see what would happen if a decimal number is multiplied by 10 or 100 or 1000.

For example,

$$3.23 \times 10 = \frac{323}{100} \times 10 = 32.3$$

Decimal point shifted to the right by one place since 10 has one zero over one.

$$3.23 \times 100 = \frac{323}{100} \times 100 = 323$$

Decimal point shifted to the right by two places since 100 has two zeros over two.

$$3.23 \times 1000 = \frac{323}{100} \times 1000 = 3230$$

Try these

i) 0.7×10

ii) 1.3×100

iii) 76.3×1000

Exercise 1.9

1. Choose the best answer.

i) 0.1×0.1 is equal to

(A) 0.1 (B) 0.11 (C) 0.01 (D) 0.0001

ii) $5 \div 100$ is equal to

(A) 0.5 (B) 0.005 (C) 0.05 (D) 0.0005

iii) $\frac{1}{10} \times \frac{1}{10}$ is equal to

(A) 0.01 (B) 0.001 (C) 0.0001 (D) 0.1

iv) 0.4×5 is equal to

(A) 1 (B) 0.4 (C) 2 (D) 3

2. Find :

(i) 0.3×7 (ii) 9×4.5 (iii) 2.85×6 (iv) 20.7×4

(v) 0.05×9 (vi) 212.03×5 (vii) 3×0.86 (viii) 3.5×0.3

(ix) 0.2×51.7 (x) 0.3×3.47 (xi) 1.4×3.2 (xii) 0.5×0.0025

(xiii) 12.4×0.17 (xiv) 1.04×0.03

3. Find :

(i) 1.4×10 (ii) 4.68×10 (iii) 456.7×10 (iv) 269.08×10

(v) 32.3×100 (vi) 171.4×100 (vii) 4.78×100

4. Find the area of rectangle whose length is 10.3 cm and breath is 5 cm.

5. A two-wheeler covers a distance of 75.6 km in one litre of petrol. How much distance will it cover in 10 litres of petrol?

(ii) Division of Decimal Numbers

Jasmine was preparing a design to decorate her classroom. She needed a few colourd strips of paper of length 1.8 cm each. She had a strip of coloured paper of length 7.2 cm. How many pieces of the required length will she get out of this strip? She thought it would be $\frac{7.2}{1.8}$ cm. Is she correct?

Both 7.2 and 1.8 are decimal numbers. So we need to know the division of decimal numbers .

For example,

$$141.5 \div 10 = 14.15$$
$$141.5 \div 100 = 1.415$$
$$141.5 \div 1000 = 0.1415$$

To get the quotient we shift the digits in the decimal number to the left by as many places as there are zeros over 1.

Try these

Find:

i) $432.5 \div 10$

ii) $432.5 \div 100$

iii) $432.5 \div 1000$

Example 1.38

Find $4.2 \div 3$.

Solution

$$\begin{aligned} 4.2 \div 3 &= \frac{42}{10} \div 3 = \frac{42}{10} \times \frac{1}{3} \\ &= \frac{42 \times 1}{10 \times 3} = \frac{1 \times 42}{10 \times 3} \\ &= \frac{1}{10} \times \frac{42}{3} = \frac{1}{10} \times 14 \\ &= \frac{14}{10} = 1.4 \end{aligned}$$

Try these

Find:

i) $85.8 \div 3$

ii) $25.5 \div 5$

Example 1.39

Find $18.5 \div 5$.

Solution

First find $185 \div 5$. We get 37.

There is one digit to the right of the decimal point in 18.5. Place the decimal point in 37 such that there would be one digit to its right. We will get 3.7.

Try these

Find:

i) $73.12 \div 4$

ii) $34.55 \div 7$

Division of a Decimal Number by another Decimal number

Example 1.40

Find $\frac{17.6}{0.4}$.

Solution

We have $17.6 \div 0.4 = \frac{176}{10} \div \frac{4}{10}$

$$= \frac{176}{10} \times \frac{10}{4} = 44.$$

Try these

> Find :
>
> i) $\frac{9.25}{0.5}$
>
> ii) $\frac{36}{0.04}$
>
> iii) $\frac{6.5}{1.3}$

Example 1.41

A car covers a distance of 129.92 km in 3.2 hours. What is the distance covered by it in 1 hour?

Solution

Distance covered by the car = 129.92 km.

Time required to cover this distance = 3.2 hours.

So, distance covered by it in 1 hour $= \frac{129.92}{3.2} = \frac{1299.2}{32} = 40.6$km.

Exercise 1.10

1. Choose the best answer.

i) $0.1 \div 0.1$ is equal to

(A) 1 (B) 0.1 (C) 0.01 (D) 2

ii) $\frac{1}{1000}$ is equal to

(A) 0.01 (B) 0.001 (C) 1.001 (D) 1.01

iii) How many apples can be bought for ₹50 if the cost of one apple is ₹12.50?

(A) 2 (B) 3 (C) 4 (D) 7

iv) $\frac{12.5}{2.5}$ is equal to

(A) 4 (B) 5 (C) 7 (D) 10

2. Find :

(i) $0.6 \div 2$ (ii) $0.45 \div 5$ (iii) $3.48 \div 3$

(iv) $64.8 \div 6$ (v) $785.2 \div 4$ (vi) $21.28 \div 7$

3. Find :

(i) $6.8 \div 10$ (ii) $43.5 \div 10$ (iii) $0.9 \div 10$

(iv) $44.3 \div 10$ (v) $373.48 \div 10$ (vi) $0.79 \div 10$

4. Find :

(i) $5.6 \div 100$ (ii) $0.7 \div 100$ (iii) $0.69 \div 100$

(iv) $743.6 \div 100$ (v) $43.7 \div 100$ (vi) $78.73 \div 100$

5. Find :

(i) $8.9 \div 1000$ (ii) $73.3 \div 1000$ (iii) $48.73 \div 1000$

(iv) $178.9 \div 1000$ (v) $0.9 \div 1000$ (vi) $0.09 \div 1000$

6. Find :

(i) $9 \div 4.5$ (ii) $48 \div 0.3$ (iii) $6.25 \div 0.5$

(iv) $40.95 \div 5$ (v) $0.7 \div 0.35$ (vi) $8.75 \div 0.25$

7. A vehicle covers a distance of 55.2 km in 2.4 litres of petrol. How much distance will it cover in one litre of petrol?

8. If the total weight of 11 similar bags is 115.5 kg, what is the weight of 1 bag?

9. How many books can be bought for ₹362.25, if the cost of one book is ₹40.25?

10. A motorist covers a distance of 135.04 km in 3.2 hours. Find his speed?

11. The product of two numbers is 45.36. One of them is 3.15. Find the other number?

1.12 Powers

Introduction

Teacher asked Ramu, "Can you read this number 2560000000000000?"

He replies, "It is very difficult to read sir".

"The distance between sun and saturn is 1,433,500,000,000 m. Raja can you able to read this number?" asked teacher.

He replies, "Sir, it is also very difficult to read".

Now, we are going to see how to read the difficult numbers in the examples given above.

Exponents

We can write the large numbers in a shortest form by using the following methods.

$$10 = 10^1$$

$$100 = 10^1 \times 10^1 = = 10^2$$

$$1000 = 10^1 \times 10^1 \times 10^1 = 10^3$$

Similarly,

$$2^1 \times 2^1 = 2^2$$

$$2^1 \times 2^1 \times 2^1 = 2^3$$

$$2^1 \times 2^1 \times 2^1 \times 2^1 = 2^4$$

$a \times a = a^2$ [read as 'a' squared or 'a' raised to the power 2]

$a \times a \times a = a^3$ [read as 'a' cubed or 'a' raised to the power 3]

$a \times a \times a \times a = a^4$ [read as 'a' raised to the power 4 or the 4^{th} power of 'a']

…………………… ……………………

$a \times a \times ...\ m$ times $= a^m$ [read as 'a' raised to the power m or m^{th} power of 'a']

Here 'a' is called the base, 'm' is called the exponent (or) power.

Note: Only a^2 and a^3 have the special names "a squared' and "a cubed".

$\therefore$ we can write large numbers in a shorter form using exponents.

Example 1.42

Express 512 as a power .

Solution

We have $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

So we can say that $512 = 2^9$

Example: 1.43

Which one is greater 2^5 , 5^2 ?

Solution

We have $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

and $5^2 = 5 \times 5 = 25$

Since $32 > 25$.

Therefore 2^5 is greater than 5^2.

Example: 1.44

Express the number 144 as a product of powers of prime factors.

Solution

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$= 2^4 \times 3^2$$
$$\text{Thus, } 144 = 2^4 \times 3^2$$

Example 1.45

Find the value of (i) 4^5 (ii) $(-4)^5$

Solution

(i) 4^5

$$4^5 = 4 \times 4 \times 4 \times 4 \times 4$$
$$= 1024.$$

(ii) $(-4)^5$

$$(-4)^5 = (-4) \times (-4) \times (-4) \times (-4) \times (-4)$$
$$= -1024.$$

Excercise 1.11

1. Choose the best answer.

i) -10^2 is equal to

(A) – 100 (B) 100 (C) – 10 (D) 10

ii) $(-10)^2$ is equal to

(A) 100 (B) – 100 (C) 10 (D) – 10

iii) $a \times a \times a \times \ldots\ldots n$ times is equal to

(A) a^m (B) a^{-n} (C) a^n (D) a^{m+n}

iv) $103^3 \times 0$ is equal to

(A) 103 (B) 9 (C) 0 (D) 3

2. Find the value of the following :

(i) 2^8 (ii) 3^3 (iii) 11^3

(iv) 12^3 (v) 13^4 (vi) 0^{10}

3. Express the following in exponential form :

(i) $7 \times 7 \times 7 \times 7 \times 7 \times 7$ (ii) $1 \times 1 \times 1 \times 1 \times 1$

(iii) $10 \times 10 \times 10 \times 10 \times 10 \times 10$ (iv) $b \times b \times b \times b \times b$

(v) $2 \times 2 \times a \times a \times a \times a$ (vi) $1003 \times 1003 \times 1003$

4. Express each of the follwing numbers using exponential notation. (with smallest base)

(i) 216	(ii) 243	(iii) 625
(iv) 1024	(v) 3125	(vi) 100000

5. Identify the greater number in each of the following :

(i) 4^5 , 5^4	(ii) 2^5 , 5^2	(iii) 3^2 , 2^3
(iv) 5^6 , 6^5	(v) 7^2 , 2^7	(vi) 4^7 , 7^4

6. Express each of the following as product of powers of their prime factors :

(i) 100	(ii) 384	(iii) 798
(iv) 678	(v) 948	(vi) 640

7. Simplify :

(i) 2×10^5	(ii) 0×10^4	(iii) $5^2 \times 3^4$
(iv) $2^4 \times 3^4$	(v) $3^2 \times 10^9$	(vi) $10^3 \times 0$

8. Simplify :

(i) $(-5)^3$	(ii) $(-1)^{10}$	(iii) $(-3)^2 \times (-2)^3$
(iv) $(-4)^2 \times (-5)^3$	(v) $(6)^3 \times (7)^2$	(vi) $(-2)^7 \times (-2)^{10}$

Laws of exponents

Multiplying powers with same base

1) $$\begin{aligned} 3^2 \times 3^4 &= (3 \times 3) \times (3 \times 3 \times 3 \times 3) \\ &= 3^1 \times 3^1 \times 3^1 \times 3^1 \times 3^1 \times 3^1 \\ &= 3^6 \end{aligned}$$

2) $$\begin{aligned} (-5)^2 \times (-5)^3 &= [(-5) \times (-5)] \times [(-5) \times (-5) \times (-5)] \\ &= (-5)^1 \times (-5)^1 \times (-5)^1 \times (-5)^1 \times (-5)^1 \\ &= (-5)^5 \end{aligned}$$

3) $$\begin{aligned} a^2 \times a^5 &= (a \times a) \times (a \times a \times a \times a \times a) \\ &= a^1 \times a^1 \times a^1 \times a^1 \times a^1 \times a^1 \times a^1 \\ &= a^7 \end{aligned}$$

From this we can generalise that for any non-zero integer a, where m and n are whole numbers $a^m \times a^n = a^{m+n}$

Try these

i) $2^5 \times 2^7$	ii) $4^3 \times 4^4$
iii) $p^3 \times p^5$	iv) $(-4)^{100} \times (-4)^{10}$

Dividing powers with the same base

We observe the following examples:

i) $2^7 \div 2^5 = \dfrac{2^7}{2^5}$

$= \dfrac{2\times2\times2\times2\times2\times2\times2}{2\times2\times2\times2\times2}$

$= 2^2$

ii) $(-5)^4 \div (-5)^3 = \dfrac{(-5)^4}{(-5)^3}$

$= \dfrac{(-5)\times(-5)\times(-5)\times(-5)}{(-5)\times(-5)\times(-5)}$

$= -5$

From these examples, we observe: In general, for any non-zero integer 'a',

$a^m \div a^n = a^{m-n}$ where m and n are whole numbers and $m > n$.

Power of a power

Consider the following:

(i) $(3^3)^2 = 3^3 \times 3^3$

$= 3^{3+3} = 3^6$

(ii) $(2^2)^3 = 2^2 \times 2^2 \times 2^2$

$= 2^{2+2+2}$

$= 2^6$

From this we can generalise for any non-zero integer 'a'

$(a^m)^n = a^{mn}$, where m and n are whole numbers.

Example: 1.46

Write the exponential form for $9 \times 9 \times 9 \times 9$ by taking base as 3.

Solution

We have $9 \times 9 \times 9 \times 9 = 9^4$

We know that $9 = 3 \times 3$

Therefore $9^4 = (3^2)^4$

$= 3^8$

Exercise 1.12

1. Choose the best answer.

i) $a^m \times a^x$ is equal to

(A) a^{mx} (B) a^{m+x} (C) a^{m-x} (D) a^{m^x}

ii) $10^{12} \div 10^{10}$ is equal to

(A) 10^2 (B) 1 (C) 0 (D) 10^{10}

iii) $10^{10} \times 10^2$ is equal to

(A) 10^5 (B) 10^8 (C) 10^{12} (D) 10^{20}

iv) $(2^2)^{10}$ is equal to

(A) 2^5 (B) 2^{12} (C) 2^{20} (D) 2^{10}

Using laws of exponents, simplify in the exponential form.

2. i) $3^5 \times 3^3 \times 3^4$

ii) $a^3 \times a^2 \times a^7$

iii) $7^x \times 7^2 \times 7^3$

iv) $10^0 \times 10^2 \times 10^5$

v) $5^6 \times 5^2 \times 5^1$

3. i) $5^{10} \div 5^6$

ii) $a^6 \div a^2$

iii) $10^{10} \div 10^0$

iv) $4^6 \div 4^4$

v) $3^3 \div 3^3$

4. i) $(3^4)^3$

ii) $(2^5)^4$

iii) $(4^5)^2$

iv) $(4^0)^{10}$

v) $(5^2)^{10}$

Points to Remember

1. Natural numbrs N = {1, 2, 3, ...}
2. Whole numbers W = {0, 1, 2, ...}
3. Integers Z = {..., – 3, – 2, – 1, 0, 1, 2, 3, ...}
4. The product of two positive integers is a positive integer.
5. The product of two negative integers is a positive integer.
6. The product of a positive integer and a negative integer is a negative integer.
7. The division of two integers need not be an integer.
8. Fraction is a part of whole.
9. If the product of two non-zero numbers is 1 then the numbers are called the reciprocal of each other.
10. $a \times a \times a \times \ldots$ m times = a^m
    (read as ‘a’ raised to the power m (or) the m^{th} power of ‘a’)
11. For any two non-zero integers a and b and whole numbers m and n,
    i) $a^m a^n = a^{m+n}$
    ii) $\frac{a^m}{a^n} = a^{m-n}$, where m > n
    iii) $(a^m)^n = a^{mn}$
    iv) $(-1)^n = 1$, when n is an even number
    $(-1)^n = -1$, when n is an odd number

2 ALGEBRA

2.1 ALGEBRAIC EXPRESSIONS

(i) Introduction

In class VI, we have already come across simple algebraic expressions like $x + 10$, $y - 9$, $3m + 4$, $2y - 8$ and so on.

Expression is a main concept in algebra. In this chapter you are going to learn about algebraic expressions, how they are formed, how they can be combined, how to find their values, and how to frame and solve simple equations.

(ii) Variables, Constants and Coefficients

Variable

A quantity which can take various numerical values is known as a **variable** (or a **literal**).

Variables can be denoted by using the letters a, b, c, x, y, z, etc.

Constant

A quantity which has a fixed numerical value is called a **constant**.

For example, 3, -25, $\frac{12}{13}$ and 8.9 are constants.

Numerical expression

A number or a combination of numbers formed by using the arithmetic operations is called a **numerical expression** or **an arithmetic expression**.

For example, $3 + (4 \times 5)$, $5 - (4 \times 2)$, $(7 \times 9) \div 5$ and $(3 \times 4) - (4 \times 5 - 7)$ are numerical expressions.

Algebraic Expression

An algebraic expression is a combination of variables and constants connected by arithmetic operations.

Example 2.1

	Statement	Expressions
(i)	5 added to y	$y + 5$
(ii)	8 subtracted from n	$n - 8$
(iii)	12 multiplied by x	$12\,x$
(iv)	p divided by 3	$\frac{p}{3}$

Term

A term is a constant or a variable or a product of a constant and one or more variables.

$3x^2$, $6x$ and -5 are called the terms of the expression $3x^2 + 6x - 5$.

A term could be

(i) a constant
(ii) a variable
(iii) a product of constant and a variable (or variables)
(iv) a product of two or more variables

In the expression $4a^2 + 7a + 3$, the terms are $4a^2$, $7a$ and 3. The number of terms is 3.

In the expression $-6p^2 + 18pq + 9q^2 - 7$, the terms are $-6p^2$, $18pq$, $9q^2$ and -7. The number of terms is 4.

Find the number of terms.

(i) $8b$
(ii) $3p - 2q$
(iii) $a^2 + 4a - 5$
(iv) $7x^2y - 4y + 8x - 9$
(v) $4m^2n + 3mn^2$

Coefficient

The coefficient of a given variable or factor in a term is another factor whose product with the given variable or factor is the term itself.

Do you know?

In the term $6xy$, the factors are 6, x and y.

If the coefficient is a constant, it is called a constant coefficient or a numerical coefficient.

Example 2.2

In the term $5xy$,

coefficient of xy is 5 (numerical coefficient),

coefficient of $5x$ is y,

coefficient of $5y$ is x.

Find the numerical coefficient in

(i) $3z$ (ii) $8ax$ (iii) ab

(iv) $-pq$ (v) $\frac{1}{2}mn$ (vi) $-\frac{4}{7}yz$

Example 2.3

In the term $-mn^2$,

coefficient of mn^2 is -1,

coefficient of $-n^2$ is m,

coefficient of m is $-n^2$.

S.No.	Expression	Term which contains y	Coefficient of y
1	$10 - 2y$		
2	$11 + yz$	yz	z
3	$yn^2 + 10$		
4	$-3m^2y + n$		

Exercise 2.1

1. Choose the correct answer:

(i) The numerical coefficient in $-7xy$ is

(A) -7 (B) x (C) y (D) xy

(ii) The numerical coefficient in $-q$ is

(A) q (B) $-q$ (C) 1 (D) -1

(iii) 12 subtracted from z is

(A) $12 + z$ (B) $12z$ (C) $12 - z$ (D) $z - 12$

(iv) n multiplied by -7 is

(A) $7n$ (B) $-7n$ (C) $\frac{7}{n}$ (D) $\frac{-7}{n}$

(v) Three times p increased by 7 is

(A) 21p (B) $3p - 7$ (C) $3p + 7$ (D) $7 - 3p$

2. Identify the constants and variables from the following:
a, 5, $-xy$, p, -9.5

3. Rewrite each of the following as an algebraic expression
   (i) 6 more than x
   (ii) 7 subtracted from $-m$
   (iii) 11 added to $3q$
   (iv) 10 more than 3 times x
   (v) 8 less than 5 times y

4. Write the numerical coefficient of each term of the expression $3y^2 - 4yx + 9x^2$.

5. Identify the term which contains x and find the coefficient of x

(i) $y^2x + y$ (ii) $3 + x + 3x^2y$

(iii) $5 + z + zx$ (iv) $2x^2y - 5xy^2 + 7y^2$

6. Identify the term which contains y^2 and find the coefficient of y^2

(i) $3 - my^2$ (ii) $6y^2 + 8x$ (iii) $2x^2y - 9xy^2 + 5x^2$

(iii) Power

If a variable a is multiplied five times by itself then it is written as $a \times a \times a \times a \times a = a^5$(read as a to the power 5). Similarly, $b \times b \times b = b^3$ (b to the power 3) and $c \times c \times c \times c = c^4$($c$ to the power 4). Here a, b, c are called the base and 5, 3, 4 are called the exponent or power.

Example 2.4

(i) In the term $-8a^2$, the power of the variable a is 2

(ii) In the term m, the power of the variable m is 1.

(iv) Like terms and Unlike terms

Terms having the same variable or product of variables with same powers are called **Like terms**. Terms having different variable or product of variables with different powers are called **Unlike terms**.

Example 2.5

(i) x, $-5x$, $9x$ are like terms as they have the same variable x

(ii) $4x^2y$, $-7yx^2$ are like terms as they have the same variable x^2y

Example 2.6

(i) $6x$, $6y$ are unlike terms

(ii) $3xy^2$, $5xy$, $8x$, $-10y$ are unlike terms.

Identify the like terms and unlike terms:

(i) $13x$ and $5x$

(ii) $-7m$ and $-3n$

(iii) $4x^2z$ and $-10zx^2$

(iv) $36mn$ and $-5nm$

(v) $-8p^2q$ and $3pq^2$

(v) Degree of an Algebraic expression

Consider the expression $8x^2 - 6x + 7$. It has 3 terms $8x^2$, $-6x$ and 7.

In the term $8x^2$, the power of the variable x is 2.

In the term $-6x$, the power of the variable x is 1.

The term 7 is called as a constant term or an independent term.

The term 7 is $7 \times 1 = 7x^0$ in which the power of the variable x is 0.

In the above expression the term $8x^2$ has the highest power 2. So the degree of the expression $8x^2 - 6x + 7$ is 2.

Consider the expression $6x^2y + 2xy + 3y^2$.

In the term $6x^2y$, the power of variable is 3.

(Adding the powers of x and y we get 3 (*i.e.*) $2 + 1 = 3$).

In term $2xy$, the power of the variable is 2.

In term $3y^2$, the power of the variable is 2.

So, in the expression $6x^2y + 2xy + 3y^2$, the term $6x^2y$ has the highest power 3. So the degree of this expression is 3.

Hence, the degree of an expression of one variable is the highest value of the exponent of the variable. The degree of an expression of more than one variable is the highest value of the sum of the exponents of the variables in different terms.

Note: The degree of a constant is 0.

Example 2.7

The degree of the expression: (i) $5a^2 - 6a + 10$ is 2

(ii) $3x^2 + 7 + 6xy^2$ is 3

(iii) $m^2n^2 + 3mn + 8$ is 4

(vi) Value of an Algebraic expression

We know that an algebraic expression has variables and a variable can take any value. Thus, when each variable takes a value, the expression gives some value.

For example, if the cost of a book is ₹ x and if you are buying 5 books, you should pay ₹ $5x$. The value of this algebraic expression $5x$ depends upon the value of x which can take any value.

If $x = 4$, then $5x = 5 \times 4 = 20$.

If $x = 30$, then $5x = 5 \times 30 = 150$.

So to find the value of an expression, we substitute the given value of x in the expression.

Example 2.8

Find the value of the following expressions when $x = 2$.

(i) $x + 5$ (ii) $7x - 3$ (iii) $20 - 5x^2$

Solution Substituting $x = 2$ in

(i) $x + 5 = 2 + 5 = 7$

(ii) $7x - 3 = 7(2) - 3$

$= 14 - 3 = 11$

(iii) $20 - 5x^2 = 20 - 5(2)^2$

$= 20 - 5(4)$

$= 20 - 20 = 0$

Example 2.9

Find the value of the following expression when $a = -3$ and $b = 2$.

(i) $a + b$ (ii) $9a - 5b$ (iii) $a^2 + 2ab + b^2$

Solution Substituting $a = -3$ and $b = 2$ in

(i) $a + b = -3 + 2 = -1$

(ii) $9a - 5b = 9(-3) - 5(2)$

$= -27 - 10 = -37$

(iii) $a^2 + 2ab + b^2 = (-3)^2 + 2(-3)(2) + 22$

$= 9 - 12 + 4 = 1$

Try these

1. Find the value of the following expressions when $p = -3$

   (i) $6p - 3$ (ii) $2p^2 - 3p + 2$

2. Evaluate the expression for the given values

x	3	5	6	10
$x - 3$				

3. Find the values for the variable

x				
$2x$	6	14	28	42

Exercise 2.2

1. Choose the correct answer

(i) The degree of the expression $5m^2 + 25mn + 4n^2$ is

(A) 1 (B) 2 (C) 3 (D) 4

(ii) If $p = 40$ and $q = 20$, then the value of the expression $(p - q) + 8$ is

(A) 60 (B) 20 (C) 68 (D) 28

(iii) The degree of the expression $x^2y + x^2y^2 + y$ is

(A) 1 (B) 2 (C) 3 (D) 4

(iv) If $m = -4$, then the value of the expression $3m + 4$ is

(A) 16 (B) 8 (C) -12 (D) -8

(v) If $p = 2$ and $q = 3$, then the value of the expression $(p+q)-(p-q)$ is

(A) 6 (B) 5 (C) 4 (D) 3

2. Identify the like terms in each of the following:

(i) $4x, 6y, 7x$

(ii) $2a, 7b, -3b$

(iii) $xy, 3x^2y, -3y^2, -8yx^2$

(iv) $ab, a^2b, a^2b^2, 7a^2b$

(v) $5pq, -4p, 3q, p^2q^2, 10p, -4p^2, 25pq, 70q, 14p^2q^2$

3. State the degree in each of the following expression:

(i) $x^2 + yz$ (ii) $15y^2 - 3$ (iii) $6x^2y + xy$

(iv) $a^2b^2 - 7ab$ (v) $1 - 3t + 7t^2$

4. If $x = -1$, evaluate the following:

(i) $3x - 7$ (ii) $-x + 9$ (iii) $3x^2 - x + 7$

5. If $a = 5$ and $b = -3$, evaluate the following:

(i) $3a - 2b$ (ii) $a^2 + b^2$ (iii) $4a^2 + 5b - 3$

2.2 Addition and subtraction of expressions

Adding and subtracting like terms

Already we have learnt about like terms and unlike terms.

The basic principle of addition is that we can add only like terms.

To find the sum of two or more like terms, we add the numerical coefficient of the like terms. Similarly, to find the difference between two like terms, we find the difference between the numerical coefficients of the like terms.

There are two methods in finding the sum or difference between the like terms namely,

(i) Horizontal method

(ii) Vertical method

(i) Horizontal method: In this method, we arrange all the terms in a horizontal line and then add or subtract by combining the like terms.

Example 2.10

Add $2x$ and $5x$.

Solution:

$$2x + 5x = (2+5) \times x$$
$$= 7 \times x$$
$$= 7x$$

(ii) Vertical method: In this method, we should write the like terms vertically and then add or subtract.

Example 2.11

Add $4a$ and $7a$.

Solution:

$$\begin{array}{r} 4\,a \\ +\quad 7\,a \\ \hline 11\,a \\ \hline \end{array}$$

Example 2.12

Add $7pq,\ -4pq$ and $2pq$.

Solution: **Horizontal method**

$7pq - 4pq + 2pq$

$= (7-4+2)\times pq$

$=5\,pq$

Vertical method

$$\begin{array}{r} 7\,pq \\ -4\,pq \\ +\,2\,pq \\ \hline 5\,pq \\ \hline \end{array}$$

Example 2.13

Find the sum of $5x^2y,\ 7x^2y,\ -3x^2y,\ 4x^2y$.

Solution: **Horizontal method**

$5x^2y + 7x^2y - 3x^2y + 4x^2y$

$=(5+7-3+4)x^2y$

$= 13x^2y$

Vertical method

$$\begin{array}{r} 5x^2y \\ +7x^2y \\ -3x^2y \\ +\ 4x^2y \\ \hline 13x^2y \\ \hline \end{array}$$

Example 2.14

Subtract $3a$ from $7a$.

Solution: **Horizontal method**

$7a - 3a = (7-3)a$

$= 4\,a$

Vertical method

$$\begin{array}{rl} 7\,a & \\ +\,3\,a & \\ (-) & \text{(Change of sign)} \\ \hline 4\,a & \\ \hline \end{array}$$

Do you know?

When we subtract a number from another number, we add the additive inverse to the earlier number. i.e., in subtracting 4 from 6 we change the sign of 4 to negative (additive inverse) and write as $6 - 4 = 2$.

Note: Subtracting a term is the same as adding its inverse. For example subtracting $+3a$ is the same as adding $-3a$.

Example 2.15

(i) Subtract $-2xy$ from $9xy$.

Solution:

$$\begin{array}{rl} 9\,xy & \\ -2\,xy & \\ (+) & \text{(change of sign)} \\ \hline 11\,xy & \\ \hline \end{array}$$

(ii) Subtract

$8p^2q^2$ from $-6p^2q^2$

Solution:

$$\begin{array}{r} -6p^2q^2 \\ +8p^2q^2 \\ (-) \\ \hline -14p^2q^2 \\ \hline \end{array}$$

Unlike terms cannot be added or subtracted the way like terms are added or subtracted.

For example when 7 is added to x we write it as $x + 7$ in which both the terms 7 and x are retained.

Similarly, if we add the unlike terms $4xy$ and 5, the sum is $4xy + 5$. If we subtract 6 from $5pq$ the result is $5pq - 6$.

Example 2.16

Add $6a + 3$ and $4a - 2$.

Solution:

Like terms

$6a + 3 + 4a - 2$

Like terms

$= 6a + 4a + 3 - 2$ (grouping like terms)

$= 10a + 1$

Example 2.17

Simplify $6t + 5 + t + 1$.

Solution:

Like terms

$6t + 5 + t + 1$

Like terms

$= 6t + t + 5 + 1$ (grouping like terms)

$= 7t + 6$

Example 2.18

Add $5y + 8 + 3z$ and $4y - 5$

Solution:

$5y + 8 + 3z + 4y - 5$

$= 5y + 4y + 8 - 5 + 3z$ (grouping like terms)

$= 9y + 3 + 3z$ (The term 3z will remain as it is.)

Example 2.19

Simplify the expression $15n^2 - 10n + 6n - 6n^2 - 3n + 5$

Solution:

Grouping like terms we have

$15n^2 - 6n^2 - 10n + 6n - 3n + 5$

$= (15 - 6)n^2 + (-10 + 6 - 3)n + 5$

$= 9n^2 + (-7)n + 5$

$= 9n^2 - 7n + 5$

Example 2.20

Add $10x^2 - 5xy + 2y^2$, $-4x^2 + 4xy + 5y^2$ and $3x^2 - 2xy - 6y^2$.

Solution:

$$
\begin{array}{r}
10x^2 - 5xy + 2y^2 \\
-4x^2 + 4xy + 5y^2 \\
+3x^2 - 2xy - 6y^2 \\
\hline
9x^2 - 3xy + y^2 \\
\hline
\end{array}
$$

Try these

Add:

(i) $8m - 7n,\ 3n - 4m + 5$

(ii) $a + b,\ -a + b$

(iii) $4a^2,\ -5a^2,\ -3a^2,\ 7a^2$

Example 2.21

Subtract $6a - 3b$ from $-8a + 9b$.

Solution:

$$\begin{array}{rr} -8a & +\ 9b \\ +6a & -\ 3b \\ (-) & (+) \\ \hline -14a & +\ 12b \\ \hline \end{array}$$

Example 2.22

Subtract $2(p - q)$ from $3(5p - q + 3)$

Solution: $3(5p - q + 3) - 2(p - q)$

$= 15p - 3q + 9 - 2p + 2q$

$= 15p - 2p - 3q + 2q + 9$

$= 13p - q + 9$

> **Do you know?**
>
> Just as
>
> $-(8 - 5) = -8 + 5,$
>
> $-2(m - n) = -2m + 2n$
>
> the signs of algebraic terms are handled in the same way as signs of numbers.

Example 2.23

Subtract $a^2 + b^2 - 3ab$ from $a^2 - b^2 - 3ab$.

Solution:

Horizontal method

$(a^2 - b^2 - 3ab) - (a^2 + b^2 - 3ab)$

$= a^2 - b^2 - 3ab - a^2 - b^2 + 3ab$

$= -b^2 - b^2$

$= -2b^2$

Vertical method

$$\begin{array}{rrr} a^2 & -\ b^2 & -\ 3ab \\ a^2 & +\ b^2 & -\ 3ab \\ (-) & (-) & (+) \\ \hline & -\ 2b^2 & \\ \hline \end{array}$$

Example 2.24

If $A = 5x^2 + 7x + 8$, $B = 4x^2 - 7x + 3$, find $2A - B$.

Solution: $2A = 2(5x^2 + 7x + 8)$

$= 10x^2 + 14x + 16$

Now $2A - B = (10x^2 + 14x + 16) - (4x^2 - 7x + 3)$

$= 10x^2 + 14x + 16 - 4x^2 + 7x - 3$

$= 6x^2 + 21x + 13$

Subtract:

(i) $(a-b)$ from $(a+b)$

(ii) $(5x-3y)$ from $(-2x+8y)$

Example 2.25

What should be subtracted from $14b^2$ to obtain $6b^2$?

Solution:

$$\begin{array}{r} 14b^2 \\ 6b^2 \\ (-) \\ \hline 8b^2 \\ \hline \end{array}$$

Example 2.26

What should be subtracted from $3a^2 - 4b^2 + 5ab$ to obtain $-a^2 - b^2 + 6ab$.

Solution:

$$\begin{array}{rrr} 3a^2 & -4b^2 & +5ab \\ -a^2 & -b^2 & +6ab \\ (+) & (+) & (-) \\ \hline 4a^2 & -3b^2 & -ab \\ \hline \end{array}$$

Exercise 2.3

1. Choose the correct answer:

(i) Sum of $4x$, $-8x$ and $7x$ is

(A) $5x$ (B) $4x$ (C) $3x$ (D) $19x$

(ii) Sum of 2ab, 4ab, $-8ab$ is

(A) 14 ab (B) $-2ab$ (C) $2ab$ (D) $-14ab$

(iii) $5ab + bc - 3ab$ is

(A) $2ab + bc$ (B) $8ab + bc$ (C) $9ab$ (D) $3ab$

(iv) $5y - 3y^2 - 4y + y^2$ is

(A) $9y + 4y^2$ (B) $9y - 4y^2$ (C) $y + 2y^2$ (D) $y - 2y^2$

(v) If $A = 3x + 2$ and $B = 6x - 5$, then $A - B$ is

(A) $-3x + 7$ (B) $3x - 7$ (C) $7x - 3$ (D) $9x + 7$

2. Simplify :

(i) $6a - 3b + 7a + 5b$

(ii) $8l - 5l^2 - 3l + l^2$

(iii) $-z^2 + 10z^2 - 2z + 7z^2 - 14z$

(iv) $p - (p - q) - q - (q - p)$

(v) $3mn - 3m^2 + 4nm - 5n^2 - 3m^2 + 2n^2$

(vi) $(4x^2 - 5xy + 3y^2) - (3x^2 - 2xy - 4y^2)$

3. Add :

(i) $7ab,\ 8ab,\ -10ab,\ -3ab$

(ii) $s + t,\ 2s - t,\ -s + t$

(iii) $3a - 2b,\ 2p + 3q$

(iv) $2a + 5b + 7,\ 8a - 3b + 3,\ -5a - 7b - 6$

(v) $6x + 7y + 3, -8x - y - 7, 4x - 4y + 2$

(vi) $6c - c^2 + 3,\ -3c - 9,\ c^2 + 4c + 10$

(vii) $6m^2n + 4mn - 2n^2 + 5,\ n^2 - nm^2 + 3,\ mn - 3n^2 - 2m^2n - 4$

4. Subtract :

(i) $6a$ from $14a$

(ii) $-a^2b$ from $6a^2b$

(iii) $7x^2y^2$ from $-4x^2y^2$

(iv) $3xy - 4$ from $xy + 12$

(v) $m(n - 3)$ from $n(5 - m)$

(vi) $9p^2 - 5p$ from $-10p - 6p^2$

(vii) $-3m^2 + 6m + 3$ from $5m^2 - 9$

(viii) $-s^2 + 12s - 6$ from $6s - 10$

(ix) $5m^2 + 6mn - 3n^2$ from $6n^2 - 4mn - 4m^2$

5. (i) What should be added to $3x^2 + xy + 3y^2$ to obtain $4x^2 + 6xy$?

(ii) What should be subtracted from $4p + 6q + 14$ to get $-5p + 8q + 20$?

(iii) If $A = 8x - 3y + 9$, $B = -y - 9$ and $C = 4x - y - 9$ find $A + B - C$.

6. Three sides of a triangle are $3a + 4b - 2$, $a - 7$ and $2a - 4b + 3$. What is its perimeter?

7. The sides of a rectangle are $3x + 2$ and $5x + 4$. Find its perimeter.

8. Ram spends $4a+3$ rupees for a shirt and $8a - 5$ rupees for a book. How much does he spend in all?

9. A wire is $10x - 3$ metres long. A length of $3x + 5$ metres is cut out of it for use. How much wire is left out?

10. If $A = p^2 + 3p + 5$ and $B = 2p^2 - 5p - 7$, then find

(i) $2A + 3B$ (ii) $A - B$

11. Find the value of $P - Q + 8$ if $P = m^2 + 8m$ and $Q = -m^2 + 3m - 2$.

2.3 Simple expressions with two variables

We have learnt about rectangle. Its area is $l \times b$ in which the letters 'l' and 'b' are variables.

Variables follow the rules of four fundamental operations of numbers.

Let us now translate a few verbal phrases into expressions using variables.

Operation	Verbal phrase	Algebraic Expression
Addition	Sum of x and y	$x + y$
Subtraction	Difference between a and b	$a - b$ (if $a > b$) (or) $b - a$ (if $b > a$)
Multiplication	product of x and y	$x \times y$ (or) xy
Division	p divided by q	$p \div q$ (or) $\frac{p}{q}$

The following table will help us to learn some of the words (phrases) that can be used to indicate mathematical operations:

Addition	Subtraction	Multiplication	Division
The sum of	the difference of	the product of	the quotient of
increased by	decreased by	multiplied by	divided by
plus	minus	times	the ratio of
added to	subtracted from		
more than	less than		

Example 2.27

Write the algebraic expressions for the following:

1) Twice the sum of m and n.
2) b decreased by twice a.
3) Numbers x and y both squared and added.
4) Product of p and q added to 7.
5) Two times the product of a and b divided by 5.
6) x more than two-third of y.

7) Half a number x decreased by 3.

8) Sum of numbers m and n decreased by their product .

9) 4 times x less than sum of y and 6.

10) Double the sum of one third of a and m.

11) Quotient of y by 5 added to x.

Solution:

1) $2(m+n)$ 2) $b-2a$

3) x^2+y^2 4) $7+pq$

5) $\frac{2ab}{5}$ 6) $\frac{2}{3}y+x$

7) $\frac{x}{2}-3$ 8) $(m+n)-mn$

9) $(y+6)-4x$ 10) $2\left(\frac{1}{3}a+m\right)$

11) $\frac{y}{5}+x$

Try these

Express each of the following as an algebraic expression

(i) a times b.

(ii) 5 multiplied by the sum of a and b.

(iii) Twice m decreased by n .

(iv) Four times x divided by y .

(v) Five times p multiplied by 3 times q.

Exercise 2.4

1. Choose the correct answer:

(i) The sum of 5 times x, 3 times y and 7

(A) $5(x+3y+7)$ (B) $5x+3y+7$

(C) $5x+3(y+7)$ (D) $5x+3(7y)$

(ii) One half of the sum of numbers a and b

(A) $\frac{1}{2}(a+b)$ (B) $\frac{1}{2}a+b$ (C) $\frac{1}{2}(a-b)$ (D) $\frac{1}{2}+a+b$

(iii) Three times the difference of x and y

(A) $3x-y$ (B) $3-x-y$ (C) $xy-3$ (D) $3(y-x)$

(iv) 2 less than the product of y and z

(A) $2 - yz$ (B) $2 + yz$ (C) $yz - 2$ (D) $2y - z$

(v) Half of p added to the product of 6 and q

(A) $\frac{p}{2} + 6q$ (B) $p + \frac{6q}{2}$ (C) $\frac{1}{2}(p + 6q)$ (D) $\frac{1}{2}(6p + q)$

2. Write the algebraic expressions for the following using variables, constants and arithmetic operations:

(i) Sum of x and twice y.
(ii) Subtraction of z from y.
(iii) Product of x and y increased by 4
(iv) The difference between 3 times x and 4 times y.
(v) The sum of 10, x and y.
(vi) Product of p and q decreased by 5.
(vii) Product of numbers m and n subtracted from 12.
(viii) Sum of numbers a and b subtracted from their product.
(ix) Number 6 added to 3 times the product of numbers c and d.
(x) Four times the product of x and y divided by 3.

2.4 Simple Linear Equations

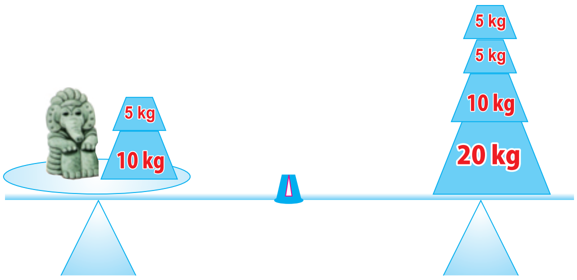
Malar's uncle presented her a statue. She wants to know the weight of that statue. She used a weighing balance to measure its weight. She knows her weight is 40kg . She finds that the statue and potatoes balance her weight.

i.e.,

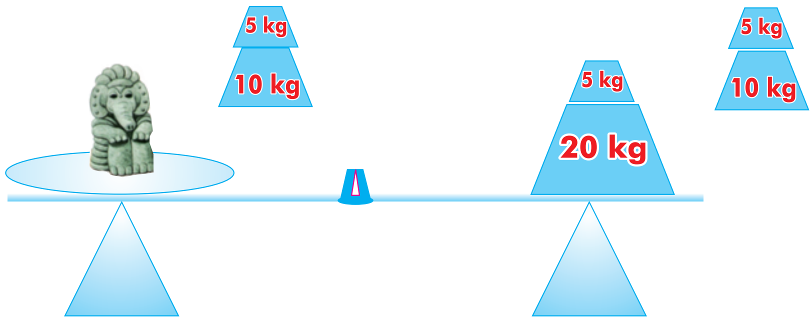
Weight of statue	Plus	Weight of potatoes	Equal	Malar's weight
s	+	15	=	40

Table 2.1

Now we will think about a balance to find the value of s.



Take away 15 from both sides.



Now the balance shows the weight of the statue.

$$\begin{aligned} s + 15 &= 40 \text{ (from Table 2.1)} \\ s + 15 - 15 &= 40 - 15 \text{ (Taking away 15 from both the sides)} \\ s &= 25 \end{aligned}$$

So the statue weighs 25 kg.

The statement $s + 15 = 40$ is an equation. i.e., a statement in which two mathematical expressions are equal is called an equation.

In a balance, if we take away some weight from one side, to balance it we must take away the same weight from the other side also.

If we add some weight to one side of the balance, to balance it we must add the same weight on the other side also.

Similarly, an equation is like a weighing balance having equal weights on each side. In an equation there is always an equality sign. This equality sign shows that value of the expression on the left hand side (LHS) is equal to the value of the expression on the right hand side (RHS).

★ Consider the equation $x + 7 = 15$

Here LHS is $x + 7$

RHS is 15

We shall subtract 7 from both sides of the equation

$$x + 7 - 7 = 15 - 7 \quad \text{(Subtracting 7 reduces the LHS to } x\text{)}$$

$$x = 8 \quad \text{(variable } x \text{ is separated)}$$

★ Consider the equation $n - 3 = 10$

LHS is $n - 3$

RHS is 10

Adding 3 to both sides, we get

$$n - 3 + 3 = 10 + 3$$

$$n = 13 \qquad \text{(variable } n \text{ is separated)}$$

★ Consider the equation $4m = 28$

Divide both sides by 4

$$\frac{4m}{4} = \frac{28}{4}$$

$$m = 7$$

★ Consider the equation $\frac{y}{2} = 6$

Multiply both sides by 2

$$\frac{y}{2} \times 2 = 6 \times 2$$

$$y = 12$$

So, if we add (or subtract) any number on one side of an equation, we have to add (or subtract) the same number the other side of the equation also to keep the equation balanced. Similarly, if we multiply (or divide) both sides by the same non-zero number, the equation is balanced. Hence to solve an equation, one has to perform the arithmetical operations according to the given equations to separate the variable from the equation.

Example 2.28

Solve $3p + 4 = 25$

Solution: $3p + 4 - 4 = 25 - 4$ (Subtracting 4 from both sides of the equation)

$$3p = 21$$

$$\frac{3p}{3} = \frac{21}{3} \qquad \text{(Dividing both sides by 3)}$$

$$p = 7$$

Example 2.29

Solve $7m - 5 = 30$

Solution: $7m - 5 + 5 = 30 + 5$ (adding 5 on both sides)

$$7m = 35$$

$$\frac{7m}{7} = \frac{35}{7} \quad \text{(Dividing both sides by 7)}$$

$$m = 5$$

While solving equations, the commonly used operation is adding or subtracting the same number on both sides of the equation. Instead of adding or subtracting a number on both sides of the equation, we can transpose the number.

Transposing a number (i.e., changing the side of the number) is the same as adding or subtracting the number from both sides. While transposing a number we should change its sign. Let us see some examples of transposing.

Example 2.30

Solve $2a - 12 = 14$

Solution:

Adding or subtracting on both sides	Transposing
$2a - 12 = 14$ $2a - 12 + 12 = 14 + 12$ (adding 12 on both sides) $2a = 26$ $\frac{2a}{2} = \frac{26}{2}$ (dividing both sides by 2) $a = 13$	$2a - 12 = 14$ Transpose (– 12) from LHS to RHS $2a = 14 + 12$ (on transposing -12 becomes $+12$) $2a = 26$ $\frac{2a}{2} = \frac{26}{2}$ (Dividing both sides by 2) $a = 13$

Example 2.31

Solve $5x + 3 = 18$

Solution: Transposing +3 from LHS to RHS

$$5x = 18 - 3 \quad \text{(on Transposing } +3 \text{ becomes } -3\text{)}$$

$$5x = 15$$

$$\frac{5x}{5} = \frac{15}{5} \quad \text{(Dividing both sides by 5)}$$

$$x = 3$$

Example 2.32

Solve $2(x+4)=12$

Solution: Divide both sides by 2 to remove the brackets in the LHS.

$$\frac{2(x+4)}{2}=\frac{12}{2}$$

$$x+4=6$$

$$x=6-4 \qquad \text{(transposing +4 to RHS)}$$

$$x=2$$

Example 2.33

Solve $-3(m-2)=18$

Solution: Divide both sides by (-3) to remove the brackets in the LHS.

$$\frac{-3(m-2)}{-3}=\frac{18}{-3}$$

$$m-2=-6$$

$$m=-6+2 \qquad \text{(transposing } -2 \text{ to RHS)}$$

$$m=-4$$

Example 2.34

Solve $(3x+1)-7=12$

Solution:

$$\begin{aligned}(3x+1)-7 &= 12\\ 3x+1-7 &= 12\\ 3x-6 &= 12\\ 3x &= 12+6\\ \frac{3x}{3} &= \frac{18}{3}\\ x &= 6\end{aligned}$$

Example 2.35

Solve $5x+3=17-2x$

Solution:

$$5x+3=17-2x$$

$$5x + 2x = 17 - 3 \quad \text{(transposing + 3 to RHS and } - 2x \text{ to LHS)}$$

$$7x = 14$$

$$\frac{7x}{7} = \frac{14}{7}$$

$$x = 2$$

Example 2.36

Sum of three consecutive integers is 45. Find the integers.

Solution: Let the first integer be x.

$\Rightarrow$ second integer $= x + 1$

Third integer $= x + 1 + 1 = x + 2$

Their sum $= x + (x + 1) + (x + 2) = 45$

$$3x + 3 = 45$$

$$3x = 42$$

$$x = 14$$

Hence, the integers are $x = 14$

$$x + 1 = 15$$

$$x + 2 = 16$$

Example 2.37

A number when added to 60 gives 75. What is the number?

Solution: Let the number be x.

The equation is $60 + x = 75$

$$x = 75 - 60$$

$$x = 15$$

Example 2.38

20 less than a number is 80. What is the number?

Solution: Let the number be x.

The equation is $x - 20 = 80$

$$x = 80 + 20$$

$$x = 100$$

Example 2.39

$\frac{1}{10}$ of a number is 63. What is the number?

Solution: Let the number be x.

The equation is $\frac{1}{10}(x) = 63$

$$\frac{1}{10}(x) \times 10 = 63 \times 10$$

$$x = 630$$

Example 2.40

A number divided by 4 and increased by 6 gives 10. Find the number.

Solution: Let the number be x.

The equation is $\frac{x}{4} + 6 = 10$

$$\frac{x}{4} = 10 - 6$$

$$\frac{x}{4} = 4$$

$$\frac{x}{4} \times 4 = 4 \times 4$$

$\therefore$ the number is 16.

Example 2.41

Thendral's age is 3 less than that of Revathi. If Thendral's age is 18, what is Revathi' s age?

Solution: Let Revathi's age be x

$\Rightarrow$ Thendral's age $= x - 3$

Given, Thendral's age is 18 years

$$\Rightarrow \quad x - 3 = 18$$

$$x = 18 + 3$$

$$x = 21$$

Hence Revathi's age is 21 years.

Exercise - 2.5

1. Choose the correct answer.

(i) If $p+3=9$, then p is

(A) 12 (B) 6 (C) 3 (D) 27

(ii) If $12-x=8$, then x is

(A) 4 (B) 20 (C) -4 (D) -20

(iii) If $\frac{q}{6}=7$, then q is

(A) 13 (B) $\frac{1}{42}$ (C) 42 (D) $\frac{7}{6}$

(iv) If $7(x-9)=35$, then x is

(A) 5 (B) -4 (C) 14 (D) 37

(v) Three times a number is 60. Then the number is

(A) 63 (B) 57 (C) 180 (D) 20

2. Solve :

(i) $x-5=7$ (ii) $a+3=10$ (iii) $4+y=-2$

(iv) $b-3=-5$ (v) $-x=5$ (vi) $-x=-7$

(vii) $3-x=8$ (viii) $14-n=10$ (ix) $7-m=-4$

(x) $20-y=-7$

3. Solve :

(i) $2x=100$ (ii) $3l=42$ (iii) $36=9x$

(iv) $51=17a$ (v) $5x=-45$ (vi) $5t=-20$

(vii) $-7x=42$ (viii) $-10m=-30$ (ix) $-2x=1$

(x) $-3x=-18$

4. Solve :

(i) $\frac{1}{2}x=7$ (ii) $\frac{a}{6}=5$ (iii) $\frac{n}{3}=-8$

(iv) $\frac{p}{-7}=8$ (v) $\frac{-x}{5}=2$ (vi) $\frac{-m}{3}=-4$

5. Solve :

(i) $3x+1=10$ (ii) $11+2x=-19$ (iii) $4z-3=17$

(iv) $4a-5=-41$ (v) $3(x+2)=15$ (vi) $-4(2-x)=12$

(vii) $\frac{y+3}{5}=14$ (viii) $\frac{x}{3}+5=7$ (ix) $6y=21-y$

(x) $11m=42+4m$ (xi) $-3x=-5x+22$ (xii) $6m-1=2m+1$

(xiii) $3x-14=x-8$ (xiv) $5x-2x+7=x+1$ (xv) $5t-3=3t-5$

6. The sum of two numbers is 33. If one number is 18, what is the other number?
7. A number increased by 12 gives 25. Find the number.
8. If 60 is subtracted from a number, the result is 48. Find the number.
9. 5 times a number is 60. Find the number.
10. 3 times a number decreased by 6 gives 18. Find the number.
11. The sum of 2 consecutive integers is 75. Find the numbers.
12. Ram's father gave him 70 rupees. Now he has 130 rupees. How much money did Ram have in the beginning?
13. 8 years ago, I was 27 years old. How old am I now?

Solve:

(i) $y + 18 = -70$ (ii) $-300 + x = 100$

(iii) $\frac{t}{3} - 5 = -6$ (iv) $2x + 9 = 19$

(v) $3x + 4 = 2x + 11$

Fun game

Ram asked his friends Arun, Saranya and Ravi to think of a number and told them to add 50 to it. Then he asked them to double it. Next he asked them to add 48 to the answer. Then he told them to divide it by 2 and subtract the number that they had thought of. Ram said that the number could now be 74 for all of them. Check it out if Arun had thought of 16, Saranya had thought of 20 and Ravi had thought of 7.

		Arun	Saranya	Ravi
Think of a number	x	16	20	7
Add 50	$x+50$			
Double it	$2x + 100$			
Add 48	$2x + 148$			
Divide by 2	$x + 74$			
Take away the number you thought of	74			

Points to Remember

1. Algebra is a branch of Mathematics that involves alphabet, numbers and mathematical operations.
2. A variable or a literal is a quantity which can take various numerical values.
3. A quantity which has a fixed numerical value is a constant.
4. An algebraic expression is a combination of variables and constants connected by the arithmetic operations.
5. Expressions are made up of terms.
6. Terms having the same variable or product of variables with same powers are called Like terms. Terms having different variable or product of variables with different powers are called Unlike terms.
7. The degree of an expression of one variable is the highest value of the exponent of the variable. The degree of an expression of more than one variable is the highest value of the sum of the exponents of the variables in different terms
8. A statement in which two expressions are equal is called an equation.
9. An equation remains the same if the LHS and RHS are interchanged.
10. The value of the variable for which the equation is satisfied is called the solution of the equation.

3 LIFE MATHEMATICS

3.1 Introduction

In most of our daily activities like following a recipe or decorating our home or calculating our daily expenses we are unknowingly using mathematical principles. People have been using these principles for thousands of years, across countries and continents. Whether you're sailing a boat off the coast of Chennai or building a house in Ooty, you are using mathematics to get things done.

How can mathematics be so universal? First human beings did not invent mathematical concepts, we discovered them. Also the language of mathematics is numbers, not English or German or Russian. If we are well versed in this language of numbers, it can help us make important decisions and perform everyday tasks. Mathematics can help us shop wisely, remodel a house within a budget, understand population growth, invest properly and save happily.

Let us learn some basic mathematical concepts that are used in real life situations.

3.2 Revision - Ratio and Proportion

Try and recollect the definitions and facts on Ratio and Proportion and complete the following statements using the help box:

1. The comparison of two quantities of the same kind by means of division is termed as ___________.
2. The two quantities to be compared are called the _________ of the ratio.
3. The first term of the ratio is called the __________ and the second term is called the ________.
4. In a ratio, only two quantities of the ___________ unit can be compared.
5. If the terms of the ratio have common factors, we can reduce it to its lowest terms by cancelling the ______.
6. When both the terms of a ratio are multiplied or divided by the same number (other than zero) the ratio remains __________ .The obtained ratios are called___________.

7. In a ratio the order of the terms is very important. (Say True or False)

8. Ratios are mere numbers. Hence units are not needed. (Say True or False)

9. Equality of two ratios is called a ___________. If $a, b; c, d$ are in proportion, then $a:b::c:d$.

10. In a proportion, the product of extremes =____________

Help Box:

1) Ratio	2) terms	3) antecedent, consequent
4) same	5) common terms	6) unchanged, equivalent ratios
7) True	8) True	9) proportion
10) product of means		

Example 3.1:

Find 5 equivalent ratios of 2:7

Solution: 2 : 7 can be written as $\frac{2}{7}$.

Multiplying the numerator and the denominator of $\frac{2}{7}$ by 2, 3, 4, 5, 6

we get $\quad \frac{2\times 2}{7\times 2} = \frac{4}{14}, \frac{2\times 3}{7\times 3} = \frac{6}{21}, \frac{2\times 4}{7\times 4} = \frac{8}{28}$

$$\frac{2\times 5}{7\times 5} = \frac{10}{35}, \quad \frac{2\times 6}{7\times 6} = \frac{12}{42}$$

4 : 14, 6 : 21, 8 : 28, 10 : 35, 12 : 42 are equivalent ratios of 2 : 7.

Example 3.2:

Reduce 270 : 378 to its lowest term.

Solution:

$$270:378 = \frac{270}{378}$$

Dividing both the numerator and the denominator by 2, we get

$$\frac{270 \div 2}{378 \div 2} = \frac{135}{189}$$

Aliter:

Factorizing 270,378 we get

$$\frac{270}{378} = \frac{2\times 3\times 3\times 3\times 5}{2\times 3\times 3\times 3\times 7}$$

$$= \frac{5}{7}$$

by 3, we get

$$\frac{135 \div 3}{189 \div 3} = \frac{45}{63}$$

by 9, we get

$$\frac{45 \div 9}{63 \div 9} = \frac{5}{7}$$

270 : 378 is reduced to 5 : 7

Example 3.3

Find the ratio of 9 months to 1 year

Solution: 1 year = 12 months

Ratio of 9 months to 12 months = 9 : 12

9 : 12 can be written as $\frac{9}{12}$

$= \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$

= 3 : 4

> Quantities of the same units only can be compared in the form of a ratio. So convert year to months.

Example 3.4

If a class has 60 students and the ratio of boys to girls is 2:1, find the number of boys and girls.

Solution:

Number of students = 60

Ratio of boys to girls = 2 : 1

Total parts = 2 + 1 = 3

Number of boys = $\frac{2}{3}$ of 60

$= \frac{2}{3} \times 60 = 40$

Number of boys = 40

Number of girls = Total Number of students – Number of boys

= 60 – 40

= 20

Number of girls = 20

[OR]

> Number of girls
> $= \frac{1}{3}$ of $60 = \frac{1}{3} \times 60$
> = 20

Example 3.5

A ribbon is cut into 3 pieces in the ratio 3: 2: 7. If the total length of the ribbon is 24 m, find the length of each piece.

Solution:

Length of the ribbon $=$ 24m

Ratio of the 3 pieces $= 3 : 2 : 7$

Total parts $= 3 + 2 + 7 = 12$

Length of the first piece of ribbon $= \frac{3}{12}$ of 24

$= \frac{3}{12} \times 24 = 6$ m

Length of the second piece of ribbon $= \frac{2}{12}$ of 24

$= \frac{2}{12} \times 24 = 4$ m

Length of the last piece of ribbon $= \frac{7}{12}$ of 24

$= \frac{7}{12} \times 24 = 14$ m

So, the length of the three pieces of ribbon are 6 m, 4 m, 14 m respectively.

Example 3.6

The ratio of boys to girls in a class is 4 : 5. If the number of boys is 20, find the number of girls.

Solution: Ratio of boys to girls $= 4 : 5$

Number of boys $= 20$

Let the number of girls be x

The ratio of the number of boys to the number of girls is $20 : x$

$4 : 5$ and $20 : x$ are in proportion, as both the ratios represent the number of boys and girls.

(i.e.) $4 : 5 :: 20 : x$

Product of extremes $= 4 \times x$

Product of means $= 5 \times 20$

In a proportion, product of extremes = product of means

$$4 \times x = 5 \times 20$$

$$x = \frac{5 \times 20}{4} = 25$$

$$\text{Number of girls} = 25$$

Example 3.7

If A : B = 4 : 6, B : C = 18 : 5, find the ratio of A : B : C.

Solution:

A : B = 4 : 6

B : C = 18 : 5

L.C.M. of 6, 18 = 18

A : B = 12 : 18

B : C = 18 : 5

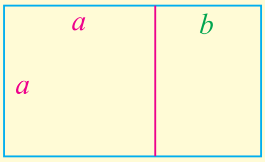
A : B : C =12 : 18 : 5

HINT

To compare 3 ratios as given in the example, the consequent (2nd term) of the 1st ratio and the antecedent (1st term) of the 2nd ratio must be made equal.

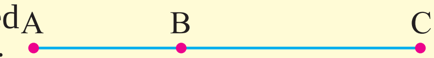
Do you Know?

Golden Ratio: Golden Ratio is a special number approximately equal to 1.6180339887498948482⋯. We use the Greek letter Phi (Φ) to refer to this ratio. Like Phi the digits of the Golden Ratio go on forever without repeating.



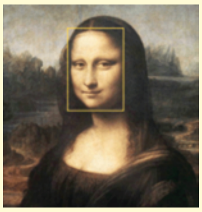
Golden Rectangle: A Golden Rectangle is a rectangle in which the ratio of the length to the width is the Golden Ratio. If width of the Golden Rectangle is 2 ft long, the other side is approximately = 2 (1.62) = 3.24 ft

Golden segment: It is a line segment divided into 2 parts. The ratio of the length of the 2 parts of this segment is the Golden Ratio



$$\frac{AB}{BC} = \frac{BC}{AC}$$

Applications of Golden Ratio:



Think!

1. Use the digits 1 to 9 to write as many proportions as possible. Each digit can be used only once in a proportion. The numbers that make up the proportion should be a single digit number.

   Eg: $\frac{1}{2} = \frac{3}{6}$

2. Suppose the ratio of zinc to copper in an alloy is 4 : 9, is there more zinc or more copper in the alloy?

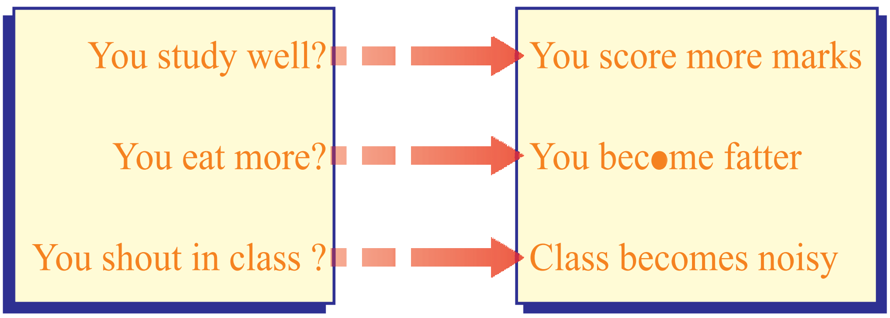
3. A bronze statue is made of copper, tin and lead metals. It has $\frac{1}{10}$ of tin, $\frac{1}{4}$ of lead and the rest copper. Find the part of copper in the bronze statue.

3.2 Variation

What do the above said statements indicate?

These are some changes.

What happens when......



In the above cases, a change in one factor brings about a change in the related factor. These changes are also termed as variations.

Now, **try and match the answers to the given questions:**

What happens when............

You buy more pens?	More number of teachers
Number of students are more?	Costs you more
You travel less distance?	Weight of bag is less
Number of books are reduced?	Time taken is less

The above examples are interdependent quantities that change numerically.

We observe that, an increase $(\uparrow)$ in one quantity brings about an increase $(\uparrow)$ in the other quantity and similarly a decrease $(\downarrow)$ in one quantity brings about a decrease $(\downarrow)$ in the other quantity .

Now, look at the following tables:

Cost of 1 pen (₹)	Cost of 10 pens (₹)
5	$10 \times 5 = 50$
20	$10 \times 20 = 200$
30	$10 \times 30 = 300$

As the number of pens increases, the cost also increases correspondingly.

Cost of 5 shirts (₹)	Cost of 1 shirt (₹)
3000	$\frac{3000}{5} = 600$
1000	$\frac{1000}{5} = 200$

As the number of shirts decreases, the cost also decreases correspondingly.

Thus we can say, if an increase (↑) [decrease (↓)] in one quantity produces a proportionate increase (↑) [decrease (↓)] in another quantity, then the two quantities are said to be in **direct variation**.

Now, let us look at some more examples:

i) When the speed of the car increases, do you think that the time taken to reach the destination will increase or decrease?

ii) When the number of students in a hostel decreases, will the provisions to prepare food for the students last longer or not?

We know that as the speed of the car increases, the time taken to reach the given destination definitely decreases.

Similarly, if the number of students decreases, the provisions last for some more number of days.

Thus, we find that if an increase (↑) [decrease (↓)] in one quantity produces a proportionate decrease (↓) [increase (↑)] in another quantity, then we say that the two quantities are in **inverse variation**.

Try these

Identify the direct and inverse variations from the given examples.

1. Number of pencils and their cost
2. The height of poles and the length of their shadows at a given time
3. Speed and time taken to cover a distance
4. Radii of circles and their areas
5. Number of labourers and the number of days taken to complete a job
6. Number of soldiers in a camp and weekly expenses
7. Principal and Interest
8. Number of lines per page and number of pages in a book

Look at the table given below:

Number of pens	x	2	4	7	10	20
Cost of pens (₹)	y	100	200	350	500	1000

We see that as 'x' increases (↑) 'y' also increases (↑).

We shall find the ratio of number of pens to cost of pens

$$\frac{\text{Number of pens}}{\text{Cost of pens}} = \frac{x}{y}, \text{ to be } \frac{2}{100}, \frac{4}{200}, \frac{7}{350}, \frac{10}{500}, \frac{20}{1000}$$

and we see that each ratio = $\frac{1}{50}$ = Constant.

Ratio of number of pens to cost of pens is a constant.

$$\therefore \frac{x}{y} = \text{constant}$$

It can be said that **when two quantities vary directly the ratio of the two given quantities is always a constant**.

Now, look at the example given below:

Time taken (Hrs)	$x_1 = 2$	$x_2 = 10$
Distance travelled (km)	$y_1 = 10$	$y_2 = 50$

We see that as time taken increases (↑), distance travelled also increases (↑).

$$X = \frac{x_1}{x_2} = \frac{2}{10} = \frac{1}{5}$$

$$Y = \frac{y_1}{y_2} = \frac{10}{50} = \frac{1}{5}$$

$$X = Y = \frac{1}{5}$$

From the above example, it is clear that in **direct variation, when a given quantity is changed in some ratio then the other quantity is also changed in the same ratio**.

Now, study the relation between the given variables and find *a* and *b*.

Time taken (hrs)	x	2	5	6	8	10	12
Distance travelled (Km)	y	120	300	a	480	600	b

Here again, we find that the ratio of the time taken to the distance travelled is a constant.

$$\frac{\text{Time taken}}{\text{Distance travelled}} = \frac{2}{120} = \frac{5}{300} = \frac{10}{600} = \frac{8}{480} = \frac{1}{60} = \text{Constant}$$

(i.e.) $\frac{x}{y} = \frac{1}{60}$. Now, we try to find the unknown

$$\frac{1}{60} = \frac{6}{a}$$

$$\frac{1 \times \boxed{6}}{60 \times \boxed{6}} = \frac{6}{360}$$

$$a = 360$$

$$\frac{1}{60} = \frac{12}{b}$$

$$\frac{1 \times \boxed{12}}{60 \times \boxed{12}} = \frac{12}{720}$$

$$b = 720$$

Look at the table given below:

Speed (Km / hr)	x	40	48	60	80	120
Time taken (hrs)	y	12	10	8	6	4

Here, we find that as x increases (↓) y decreases (↑)

$$xy = 40 \times 12 = 480$$

$$= 48 \times 10 = 60 \times 8 = 80 \times 6 = 120 \times 4 = 480$$

$$\therefore xy = \text{constant}$$

It can be stated that **if two quantities vary inversely, their product is a constant**.

Look at the example below:

Speed (Km/hr)	$x_1 = 120$	$x_2 = 60$
Time taken (hrs)	$y_1 = 4$	$y_2 = 8$

As speed increases (↑), time taken decreases (↓).

$$X = \frac{x_1}{x_2} = \frac{120}{60} = 2$$

$$Y = \frac{y_1}{y_2} = \frac{4}{8} = \frac{1}{2} \quad 1/Y = 2$$

$$X = \frac{1}{Y}$$

Thus, it is clear that in inverse variation, when a given quantity is changed in some ratio the other quantity is changed in inverse ratio.

Now, study the relation between the variables and find a and b.

No of men	x	15	5	6	b	60
No of days	y	4	12	a	20	1

We see that, $xy = 15 \times 4 = 5 \times 12 = 60 = \text{constant}$

$$xy = 60$$

$$6 \times a = 60$$

$$6 \times 10 = 60$$

$$a = 10$$

$$xy = 60$$
$$b \times 20 = 60$$
$$3 \times 20 = 60$$
$$b = 3$$

Try these

1. If x varies directly as y, complete the given tables:

(i)

x	1	3			9	15
y	2		10	16		

(ii)

x	2	4	5		
y	6			18	21

2. If x varies inversely as y, complete the given tables:

(i)

x	20	10	40	50	
y			50		250

(ii)

x		200	8	4	16
y	10		50		

Example 3.8

If the cost of 16 pencils is ₹48, find the cost of 4 pencils.

Solution:

Let the cost of four pencils be represented as 'a'.

Number of pencils	**Cost (₹)**
x	y
16	48
4	a

As the number of pencils decreases (↓), the cost also decreases (↓). Hence the two quantities are in **direct variation**.

We know that, in direct variation, $\frac{x}{y}$ = constant

$$\frac{16}{48} = \frac{4}{a}$$

$$16 \times a = 48 \times 4$$

$$a = \frac{48 \times 4}{16} = 12$$

Cost of four pencils = ₹12

Aliter:

Let the cost of four pencils be represented as 'a' .

Number of pencils	Cost (₹)
x	y
16	48
4	a

As number of pencils decreases (↓), cost also decreases (↓), **direct variation (Same ratio)**.

$$\frac{16}{4} = \frac{48}{a}$$

$$16 \times a = 4 \times 48$$

$$a = \frac{4 \times 48}{16} = 12$$

Cost of four pencils = ₹12.

Example 3.9

A car travels 360 km in 4 hrs. Find the distance it covers in 6 hours 30 mins at the same speed.

Solution:

Let the distance travelled in $6\frac{1}{2}$ hrs be a

Time taken (hrs)	Distance travelled (km)
x	y
4	360
$6\frac{1}{2}$	a

$30 \text{ mins} = \frac{30}{60}\text{hrs}$

$= \frac{1}{2}\text{of an hr}$

$6 \text{ hrs } 30 \text{ mins} = 6\frac{1}{2} \text{ hrs}$

As time taken increases (↑), distance travelled also increases (↑), **direct variation.**

In direct variation, $\frac{x}{y}$ = constant

$$\frac{4}{360} = \frac{6\frac{1}{2}}{a}$$

$$4 \times a = 360 \times 6\frac{1}{2}$$

$$4 \times a = 360 \times \frac{13}{2}$$

$$a = \frac{360 \times 13}{4 \times 2} = 585$$

Distance travelled in $6\frac{1}{2}$ hrs = 585 km

Aliter: Let the distance travelled in $6\frac{1}{2}$ hrs be a

Time taken (hrs)	Distance travelled (km)
4	360
$6\frac{1}{2}$	a

As time taken increases (↑), distance travelled also increases (↑), **direct variation (same ratio).**

$$\frac{4}{6\frac{1}{2}} = \frac{360}{a}$$

$$4 \times a = 360 \times 6\tfrac{1}{2}$$

$$4 \times a = 360 \times \frac{13}{2}$$

$$a = \frac{360}{4} \times \frac{13}{2} = 585$$

Distance travelled in $6\frac{1}{2}$ hrs = 585 km.

Example 3.10

7 men can complete a work in 52 days. In how many days will 13 men finish the same work?

Solution: Let the number of unknown days be a.

Number of men	Number of days
x	y
7	52
13	a

As the number of men increases (↑), number of days decreases (↓), **inverse variation**

In inverse variation, xy = constant

$$7 \times 52 = 13 \times a$$

$$13 \times a = 7 \times 52$$

$$a = \frac{7 \times 52}{13} = 28$$

13 men can complete the work in 28 days.

Aliter:

Let the number of unknown days be a.

Number of men	Number of days
7	52
13	a

As number of men increases (↑), number of days decreases (↓), **inverse variation (inverse ratio).**

$$\frac{7}{13} = \frac{a}{52}$$

$$7 \times 52 = 13 \times a$$

$$13 \times a = 7 \times 52$$

$$a = \frac{7 \times 52}{13} = 28$$

13 men can complete the work in 28 days

Example 3.11

A book contains 120 pages. Each page has 35 lines . How many pages will the book contain if every page has 24 lines per page?

Solution: Let the number of pages be a.

Number of lines per page	Number of pages
35	120
24	a

As the number of lines per page decreases (↓) number of pages increases (↑) it is in **inverse variation (inverse ratio)**.

$$\frac{35}{24} = \frac{a}{120}$$

$$35 \times 120 = a \times 24$$

$$a \times 24 = 35 \times 120$$

$$a = \frac{35 \times 120}{24}$$

$$a = 35 \times 5 = 175$$

If there are 24 lines in one page, then the number of pages in the book = 175

Exercise 3.1

1. Choose the correct answer

i) If the cost of 8 kgs of rice is ₹160, then the cost of 18 kgs of rice is

(A) ₹480 (B) ₹180 (C) ₹360 (D) ₹1280

ii) If the cost of 7 mangoes is ₹35, then the cost of 15 mangoes is

(A) ₹75 (B) ₹25 (C) ₹35 (D) ₹50

iii) A train covers a distance of 195 km in 3 hrs. At the same speed, the distance travelled in 5 hours is

(A) 195 km. (B) 325 km. (C) 390 km. (D) 975 km.

iv) If 8 workers can complete a work in 24 days, then 24 workers can complete the same work in

(A) 8 days (B) 16 days (C) 12 days (D) 24 days

v) If 18 men can do a work in 20 days, then 24 men can do this work in

(A) 20 days (B) 22 days (C) 21 days (D) 15 days

2. A marriage party of 300 people require 60 kg of vegetables. What is the requirement if 500 people turn up for the marriage?
3. 90 teachers are required for a school with a strength 1500 students. How many teachers are required for a school of 2000 students?
4. A car travels 60 km in 45 minutes. At the same rate, how many kilo metres will it travel in one hour?
5. A man whitewashes 96 sq.m of a compound wall in 8 days. How many sq.m will be white washed in 18 days?
6. 7 boxes weigh 36.4 kg. How much will 5 such boxes weigh?
7. A car takes 5 hours to cover a particular distance at a uniform speed of 60 km / hr. How long will it take to cover the same distance at a uniform speed of 40 km / hr?
8. 150 men can finish a piece of work in 12 days. How many days will 120 men take to finish the same work?
9. A troop has provisions for 276 soldiers for 20 days. How many soldiers leave the troop so that the provisions may last for 46 days?
10. A book has 70 pages with 30 lines of printed matter on each page. If each page is to have only 20 lines of printed matter, how many pages will the book have?
11. There are 800 soldiers in an army camp. There is enough provisions for them for 6o days. If 400 more soldiers join the camp, for how many days will the provisions last?

Just for fun

If an owl builds a nest in 1 second , then what time will it take if there were 200 owls?
Owls don't build their own nests. They simply move into an old hawk's nest or rest in ready made cavities.

Try these

Read the questions. Recollect the different methods that you have learnt earlier. Try all the different methods possible and solve them.

1. A wheel makes 48 revolutions in 3 seconds. How many revolutions does it make in 30 seconds?
2. A film processor can develop 100 negatives in 5 minutes. How many minutes will it take to develop 1200 negatives?
3. There are 36 players in 2 teams. How many players are there in 5 teams?

3.3 Percent

In the banners put up in the shops what do you understand by 25%, 20% ?

Ramu's mother refers to his report card to analyze his performance in Mathematics in standard VI.

His marks in Maths as given in his report card are

17 / 25 , 36 / 50 , 75 / 100 , 80 / 100 , 22 / 25, 45 / 50

Name: Ramu K. Class & Sec.: VI 'A'

SUBJECTS	Unit Test-I	Mid Term I	Quarterly Exam.	Half Yearly Exam.	Unit Test-II	Mid Term II
Max. Marks.	25	50	100	100	25	50
ENGLISH	23	41	75	80	22	40
II LANGUAGE	20	35	85	80	21	41
MATHEMATICS	17	36	75	80	22	45
SCIENCE	23	39	92	90	21	42
SOCIAL SCIENCE	18	42	86	92	24	42
Sign. of the Teacher						
Sign. of the H.M.						
Sign. of the Parent						

She is unable to find his best mark and his least mark by just looking at the marks.

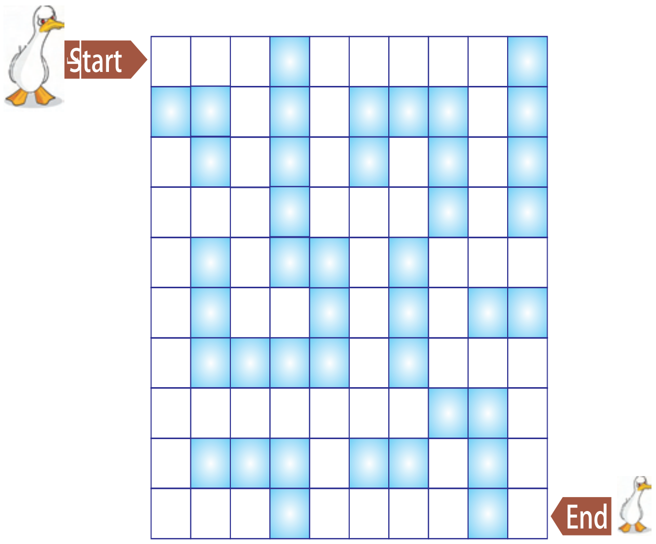
So, she converts all the given marks for a maximum of 100 (equivalent fractions with denominator 100) as given below:

Unit Test 1	Monthly Test 1	Quarterly Exam	Half - yearly Exam	Unit Test 2	Monthly Test 2
$\frac{68}{100}$	$\frac{72}{100}$	$\frac{75}{100}$	$\frac{80}{100}$	$\frac{88}{100}$	$\frac{90}{100}$

Now, all his marks are out of 100. So, she is able to compare his marks easily and is happy that Ramu has improved consistently in Mathematics in standard VI.

Now let us learn about these special fractions.

Try and help the duck to trace the path through the maze from ‘Start’ to ‘End’. Is there more than one path?



No, there is only one path that can be traced from ‘Start’ to ‘End’.

Total number of the smallest squares = 100

Number of shaded squares = 41

Number of unshaded squares = 59

Number of squares traced by the path = ____

Now, look at the table below and fill in the blanks:

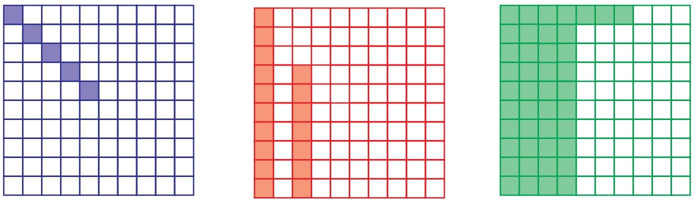
		Ratio	Fraction	Percent
Shaded Portion	41 out of 100	41 : 100	$\frac{41}{100}$	41%
Unshaded Portion	59 out of 100	59 : 100	$\frac{59}{100}$	59%
Portion traced by the path	____ out of 100	____ : 100	$\frac{\quad}{100}$	____ %

The fraction with its denominator 100 is called a Percent.

- The word ‘Percent’ is derived from the Latin word ‘Percentum’, which means ‘per hundred’ or ‘hundredth’ or ‘out of 100’.
- Percentage also means ‘percent’.
- Symbol used for percent is %
- Any ratio $x : y$, where $y = 100$ is called ‘Percent’.

To Express Percent in Different Forms:

Pictorial Representation



Shaded portion represented in the form of :

Ratio	5 : 100	17 : 100	43 : 100
Fraction	$\frac{5}{100}$	$\frac{17}{100}$	$\frac{43}{100}$
Percent	5%	17%	43%

Exercise 3.2

1) Write the following as a percent:

   (i) 20:100 (ii) $\frac{93}{100}$ (iii) 11 divided by 100 (iv) $\frac{1}{100}$ (v) $\frac{100}{100}$

2) Write the following percent as a ratio:

   (i) 43% (ii) 75% (iii) 5% (iv) $17\frac{1}{2}\%$ (v) $33\frac{1}{3}\%$

3) Write the following percent as a fraction:

   (i) 25% (ii) $12\frac{1}{2}\%$ (iii) 33% (iv) 70% (v) 82%

Think!

Shop - I

Shop - II

Find the selling price in percentage when 25% discount is given, in the first shop.

What is the reduction in percent given in the second shop?

Which shop offers better price?

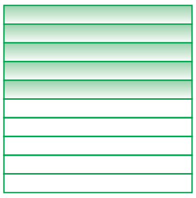
I. To Express a Fraction and a Decimal as a Percent

We know that $\frac{5}{100} = 5\%$, $\frac{1.2}{100} = 1.2\%$, $\frac{175}{100} = 175\%$.

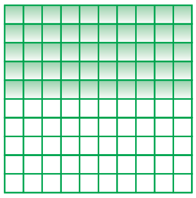
To convert $\frac{5}{10}$ to a percent

$\frac{5}{10}$ represented pictorially can be converted to a percent as shown below:

$\frac{5}{10}$



$\frac{50}{100}$



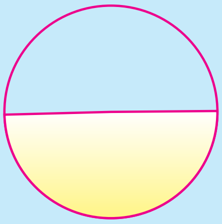
Multiply the numerator and denominator by 10 to make the denominator 100

$$\frac{5\times 10}{10\times 10} = \frac{50}{100} = 50\%$$

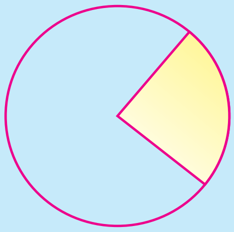
This can also be done by multiplying $\frac{5}{10}$ by 100%

$$\left(\frac{5}{10}\times 100\right)\% = 50\%$$

Try these



50% of the circle is shaded.



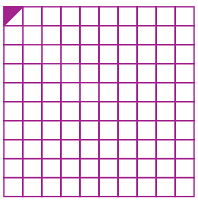
25% of the circle is shaded.

Try drawing circles with (i) 50%, (ii) 25% portion shaded in different ways.

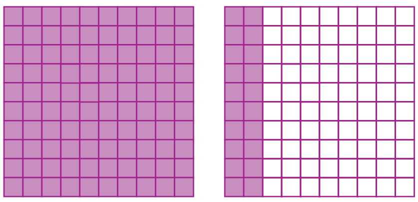
Do you know?

Less than 1 and more than 100 can also be represented as a percent.

$\frac{1}{2}\%$



120%



(i) Fractions with denominators that can be converted to 100

Try these

1) $2 \times$ ____ $= 100$

2) $5 \times 20 =$ ______

3) $4 \times 25 =$ ______

4) $10 \times$ ______ $= 100$

5) $1 \times$ ______ $= 100$

Example 3.12

Express $\frac{3}{5}$ as a percent

Solution:

5 multiplied by 20 gives 100

$$\frac{3 \times 20}{5 \times 20} = \frac{60}{100} = 60\%$$

$$\frac{3}{5} = 60\%$$

Example 3.13

Express $6\frac{1}{4}$ as a percent

Solution:

$$6\frac{1}{4} = \frac{25}{4}$$

4 multiplied by 25 gives 100

$$\frac{25 \times 25}{4 \times 25} = \frac{625}{100} = 625\%$$

(ii) Fractions with denominators that cannot be converted to 100

Example 3.14

Express $\frac{4}{7}$ as a percent

Solution: Multiply by 100%

$$\left(\frac{4}{7} \times 100\right)\% = \frac{400}{7}\%$$

$$= 57\frac{1}{7}\% = 57.14\%$$

Example 3.15

Express $\frac{1}{3}$ as a percent

Solution: Multiply by 100%

$$\left(\frac{1}{3} \times 100\right)\% = \left(\frac{100}{3}\right)\%$$

$$= 33{}^{1}\!/_{3}\% \text{ (or) } 33.33\%$$

Example 3.16

There are 250 students in a school. 55 students like basketball, 75 students like football, 63 students like throw ball, while the remaining like cricket. What percent of students like (a) basket ball? (b) throw ball?

Solution:

Total number of students = 250

(a) Number of students who like basket ball = 55

55 out of 250 like basket ball which can be represented as $\frac{55}{250}$

Percentage of students who like basket ball = $\left(\frac{55}{250}\times 100\right)\%$

$= 22\%$

(b) Number of students who like throw ball = 63

63 out of 250 like throw ball and that can be represented as $\frac{63}{250}$

Percentage of students who like throw ball $= \left(\frac{63}{250}\times 100\right)\%$

$= \frac{126}{5}\% = 25.2\%$

22% like basket ball, 25.2% like throw ball.

(iii) To convert decimals to percents

Example 3.17

Express 0.07 as a percent

Solution:

Multiply by 100%

$(0.07\times 100)\% = 7\%$

Aliter:

$$0.07 = \frac{7}{100} = 7\%$$

Example 3.18

Express 0.567 as a percent

Solution:

Multiply by 100%

$(0.567\times 100)\% = 56.7\%$

Aliter: $0.567 = \frac{567}{1000} = \frac{567}{10\times 100}$

$= \frac{56.7}{100} = 56.7\%$

Note: To convert a fraction or a decimal to a percent, multiply by 100%.

Think!

1. $\frac{9}{10}$ of your blood is water. What % of your blood is not water.
2. $\frac{2}{5}$ of your body weight is muscle. What % of body is muscle?

   About $\frac{2}{3}$ of your body weight is water. Is muscle weight plus water weight more or less than 100 %? What does that tell about your muscles?

Exercise 3.3

1. Choose the correct answer:

(i) 6.25 =

(A) 62.5% (B) 6250% (C) 625% (D) 6.25%

(ii) 0.0003 =

(A) 3% (B) 0.3% (C) 0.03% (D) 0.0003%

(iii) $\frac{5}{20}$ =

(A) 25% (B) $\frac{1}{4}$% (C) 0.25% (D) 5%

(iv) The percent of 20 minutes to 1 hour is

(A) $33\frac{1}{3}$ (B) 33 (C) $33\frac{2}{3}$ (D) none of these

(v) The percent of 50 paise to Re. 1 is

(A) 500 (B) $\frac{1}{2}$ (C) 50 (D) 20

2. Convert the given fractions to percents

i) $\frac{20}{20}$ ii) $\frac{9}{50}$ iii) $5\frac{1}{4}$ iv) $\frac{2}{3}$ v) $\frac{5}{11}$

3. Convert the given decimals to percents

i) 0.36 ii) 0.03 iii) 0.071 iv) 3.05 v) 0.75

4. In a class of 35 students, 7 students were absent on a particular day. What percentage of the students were absent?
5. Ram bought 36 mangoes. 5 mangoes were rotten. What percentage of the mangoes were rotten?
6. In a class of 50, 23 were girls and the rest were boys. What is the percentage of girls and the percentage of boys?
7. Ravi got 66 marks out of 75 in Mathematics and 72 out of 80 in Science. In which subject did he score more?
8. Shyam’s monthly income is ₹12,000. He saves ₹1,200 Find the percent of his savings and his expenditure.

II. To Express a Percent as a Fraction (or) a Decimal

i) A percent is a fraction with its denominator 100. While expressing it as a fraction, reduce the fraction to its lowest term.

Example 3.19

Express 12% as a fraction.

Solution:

$$12\% = \frac{12}{100} \text{(reduce the fraction to its lowest terms)}$$
$$= \frac{3}{25}$$

Example 3.20

Express $233\frac{1}{3}\%$ as a fraction.

Solution:

$$233\tfrac{1}{3}\% = \frac{700}{3}\%$$
$$= \frac{700}{3 \times 100} = \frac{7}{3}$$
$$= 2\tfrac{1}{3}$$

Percents that have easy fractions

$$50\% = \frac{1}{2}$$
$$25\% = \frac{1}{4}$$
$$33\frac{1}{3}\% = \frac{1}{3}$$

Find more of this kind

Example 3.21

Express $\frac{1}{4}\%$ as a fraction

Solution:

$$\frac{1}{4}\% = \frac{1}{4 \times 100} = \frac{1}{400}$$

(ii) A percent is a fraction with its denominator 100. To convert this fraction to a decimal, take the numerator and move the decimal point to its left by 2 digits.

Example 3.22

Express 15% as a decimal.

Solution:

$$15\% = \frac{15}{100} = 0.15$$

Example 3.23

Express 25.7% as a decimal.

Solution:

$$25.7\% = \frac{25.7}{100}$$
$$= 0.257$$

Math game - To make a triplet (3 Matching cards)

This game can be played by 2 or 3 people.

Write an equivalent ratio and decimal for each of the given percent in different cards as shown.

5%	1 : 20	0.05
$33\frac{1}{3}\%$	1 : 3	0.33

Make a deck of 48 cards (16 such sets of cards) - 3 cards to represent one particular value - in the form of %, ratio and decimal.

Shuffle the cards and deal the entire deck to all the players.

Players have to pick out the three cards that represent the same value of percent, ratio and decimal and place them face up on the table.

The remaining cards are held by the players and the game begins.

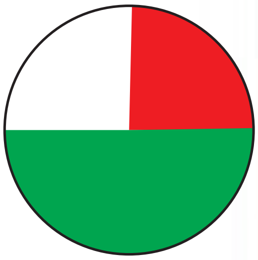
One player chooses a single unknown card from the player on his left. If this card completes a triple (3 matching cards) the 3 cards are placed face up on the table. If triplet cannot be made, the card is added to the player's hand. Play proceeds to the left.

Players take turns to choose the cards until all triplets have been made.

The player with the most number of triplets is the winner.

TO FIND THE VALUES OF PERCENTS

Colour 50% of the circle green and 25% of the circle red.

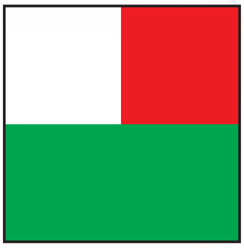


$50\% = \frac{50}{100} = \frac{1}{2}$ of the circle is to be coloured green.

Similarly, $25\% = \frac{25}{100} = \frac{25}{100} = \frac{1}{4}$

$\frac{1}{4}$ of the circle is to be coloured red.

Now, try colouring $\frac{1}{2}$ of the square, green and $\frac{1}{4}$ of the square, red.



Do you think that the green coloured regions are equal in both the figures?

No, 50% of the circle is not equal to 50% of the square.

Similarly the red coloured regions, 25% of the circle is not equal to 25% of the square.

Now, let's find the value of 50% of ₹100 and 50% of ₹10.

What is 50% of ₹100?

$$50\% = \frac{50}{100} = \frac{1}{2}$$

So, $\frac{1}{2}$ of $100 = \frac{1}{2} \times 100 = 50$

50% of ₹100 = ₹50

What is 50% of ₹10?

$$50\% = \frac{50}{100} = \frac{1}{2}$$

$\frac{1}{2}$ of $10 = \frac{1}{2} \times 10 = 5$

50% of ₹10 = ₹5

Example 3.24

Find the value of 20% of 1000 kg.

Solution:

$$\begin{aligned} 20\% \text{ of } 1000 &= \frac{20}{100} \text{ of } 1000 \\ &= \frac{20}{100} \times 1000 \\ 20\% \text{ of } 1000 \text{ kg} &= 200 \text{ kg.} \end{aligned}$$

Example 3.25

Find the value of $\frac{1}{2}\%$ of 200.

Solution:

$$= \frac{\frac{1}{2}}{100} \text{ of } 200$$

$$= \frac{1}{2 \times 100} \times 200$$

$$\frac{1}{200} \times 200 = 1$$

$$\frac{1}{2}\% \text{ of } 200 = 1$$

Example 3.26

Find the value of 0.75% of 40 kg.

Solution:

$$0.75\% = \frac{0.75}{100}$$

$$0.75\% \text{ of } 40 = \frac{0.75}{100} \times 40$$

$$= \frac{3}{10} = 0.3$$

$$0.75\% \text{ of } 40\text{kg} = 0.3\text{kg}.$$

Example 3.27

In a class of 70, 60% are boys. Find the number of boys and girls.

Solution:

Total number of students = 70

Number of boys = 60% of 70

$$= \frac{60}{100} \times 70$$

= 42

Number of boys = 42

Number of girls = Total students – Number of boys

= 70 – 42

= 28

Number of girls = 28

Example 3.28

In 2010, the population of a town is 1,50,000. If it is increased by 10% in the next year, find the population in 2011.

Solution:

Population in 2010 = 1,50,000

Increase in population $= \frac{10}{100} \times 1,50,000$

= 15,000

Population in 2011 = 150000 + 15000

= 1,65,000

Exercise 3.4

1. Choose the correct answer:

(i) The common fraction of 30 % is

(A) $\frac{1}{10}$ (B) $\frac{7}{10}$ (C) $\frac{3}{100}$ (D) $\frac{3}{10}$

(ii) The common fraction of $\frac{1}{2}\%$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{200}$ (C) $\frac{200}{100}$ (D) 100

(iii) The decimal equivalent of 25% is

(A) 0.25 (B) 25 (C) 0.0025 (D) 2.5

(iv) 10% of ₹300 is

(A) ₹10 (B) ₹20 (C) ₹30 (D) ₹300

(v) 5% of ₹150 is

(A) ₹7 (B) ₹7.50 (C) ₹5 (D) ₹100

2. Convert the given percents to fractions:

i) 9% ii) 75% iii) $\frac{1}{4}\%$ iv) 2.5% v) $66\frac{2}{3}\%$

3. Convert the given percents to decimals:

i) 7% ii) 64% iii) 375% iv) 0.03% v) 0.5%

4. Find the value of:

i) 75% of 24 ii) $33\frac{1}{3}\%$ of ₹72 iii) 45% of 80m

iv) 72% of 150 v) 7.5% of 50kg

5. Ram spent 25% of his income on rent. Find the amount spent on rent, if his income is ₹25,000.

6. A team played 25 matches in a season and won 36% of them. Find the number of matches won by the team.

7. The population of a village is 32,000. 40% of them are men. 25% of them are women and the rest are children. Find the number of men and children.

8. The value of an old car is ₹45,000. If the price decreases by 15%, find its new price.

9. The percentage of literacy in a village is 47%. Find the number of illiterates in the village, if the population is 7,500.

Think!

1) Is it true?

   20% of 25 is same as 25% of 20.

2) The tax in a restaurant is 1.5% of your total bill.

   a) Write the tax % as a decimal.

   b) A family of 6 members paid a bill of ₹ 750. What is the tax for their bill amount?

   c) What is the total amount that they should pay at the restaurant?

3.4 Profit and Loss

Ram & Co. makes a profit of ₹1,50,000 in 2008.

Ram & Co. makes a loss of ₹25,000 in 2009.

Is it possible for Ram & Co. to make a profit in the first year and a loss in the subsequent year?

Different stages of a leather product - bag are shown below:

Factory

Wholesale Dealer

Retailer

Where are the bags produced?

Do the manufactures sell the products directly?

Whom does the products reach finally?

PRICE LIST

Mango ₹10 each

Apple ₹6 each

Banana ₹3 each

Orange ₹5 each

Wholesale Market

Fruit Stall

PRICE LIST

Mango ₹15 each

Apple ₹8 each

Banana ₹2 each

Orange ₹5 each

Raja, the fruit stall owner buys fruits from the wholesale market and sells it in his shop.

On a particular day, he buys apples, mangoes and bananas.

Each fruit has two prices, one at each shop, as shown in the price list.

The price at which Raja buys the fruit at the market is called the Cost Price (C.P.). The price at which he sells the fruit in his stall is called the Selling Price (S.P.).

From the price list we can say that the selling price of the apples and the mangoes in the shop are greater than their respective cost price in the whole sale market. (i.e.) the shopkeeper gets some amount in addition to the cost price. This additional amount is called the **profit**.

Selling Price of mango = Cost Price of mango + Profit

Selling price – Cost price = Profit

Profit = Selling Price – Cost Price

= 15 – 10

Profit = ₹5

i.e., Profit = Selling Price – Cost Price

In case of the apples,

Selling price of apple > Cost price of apple, there is a profit.

Profit = S.P. – C.P.

= 8 – 6

Profit = ₹2

As we know, bananas get rotten fast, the shop keeper wanted to sell them without wasting them. So, he sells the bananas at a lower price (less than the cost price). The amount by which the cost is reduced from the cost price is called **Loss**.

In case of bananas,

Cost price of banana > selling price of banana, there is a loss.

S.P. of the banana = C.P. of the banana – Reduced amount

S.P. = C.P. – Loss

Loss = C.P. – S.P.

Loss = 3 – 2

Loss = ₹1

So, we can say that

- When the selling price of an article is greater than its cost price, then there is a profit.

  Profit = Selling Price - Cost Price

- When the cost price of an article is greater than its selling price, then there is a loss.

  Loss = Cost Price – Selling Price

- S.P = C.P + Profit

- S.P = C.P - Loss.

To find Profit / Loss %

Rakesh buys articles for ₹10,000 and sells them for ₹11,000 and makes a profit of ₹1,000, while Ramesh buys articles for ₹1,00,000 and sells them for ₹1,01,000 and makes a profit of ₹1,000.

Both of them have made the same amount of profit. Can you say both of them are benefited equally? No.

To find who has gained more, we need to compare their profit based on their investment.

Try these

1) Any fraction with its denominator 100 is called ___________
2) $\frac{1}{2}$ = __________%
3) 35% = __________ (in fraction)
4) 0.05 = __________%
5) $\frac{1}{4}$ = __ %

We know that comparison becomes easier when numbers are expressed in percent. So, let us find the profit %

Rakesh makes a profit of ₹1,000, when he invests ₹10,000.

Profit of ₹1,000 out of ₹10,000

For each 1 rupee, he makes a profit of $\frac{1000}{10000}$

Therefore for ₹100, profit = $\frac{1000}{10,000} \times 100$

Profit % = 10

Ramesh makes a profit of ₹1000, when he invests ₹1,00,000.

$$\text{Profit of 1000 out of 1,00,000} = \frac{1000}{100000}$$

$$\text{Profit \%} = \frac{1000}{100000} \times 100 = 1$$

So, from the above we can say that Rakesh is benefited more than Ramesh.

So, $\boxed{\text{Profit\%} = \frac{\text{Profit}}{\text{C.P}} \times 100}$

Loss % is also calculated in the same way.

$$\boxed{\text{Loss\%} = \frac{\text{Loss}}{\text{C.P.}} \times 100}$$

Profit % or Loss % is always calculated on the cost price of the article.

Example 3.29

A dealer bought a television set for ₹10,000 and sold it for ₹12,000. Find the profit / loss made by him for 1 television set. If he had sold 5 television sets, find the total profit/loss

Solution:

Selling Price of the television set = ₹12,000

Cost Price of the television set = ₹10,000

S.P. > C.P, there is a profit

Profit = S.P. – C. P.

= 12000 – 10000

Profit = ₹2,000

Profit on 1 television set = ₹2,000

Profit on 5 television sets = 2000 × 5

Profit on 5 television sets = ₹10,000

Example 3.30

Sanjay bought a bicycle for ₹5,000. He sold it for ₹600 less after two years. Find the selling price and the loss percent.

Solution:

Cost Price of the bicycle = ₹5000

Loss = ₹600

Selling Price = Cost Price – Loss

= 5000 – 600

Selling Price of the bicycle = ₹4400

Loss % = $\frac{\text{Loss}}{\text{C.P.}} \times 100$

= $\frac{600}{5000} \times 100$

= 12

Loss % = 12

Example 3.31

A man bought an old bicycle for ₹1,250. He spent ₹250 on its repairs. He then sold it for ₹1400. Find his gain or loss %

Solution:

Cost Price of the bicycle = ₹1,250

Repair Charges = ₹250

Total Cost Price = 1250 + 250 = ₹1,500

Selling Price = ₹1,400

C.P. > S.P., there is a Loss

Loss = Cost Price – Selling Price

= 1500 – 1400

= 100

Loss = ₹100

Loss % = $\frac{\text{Loss}}{\text{C.P.}} \times 100$

= $\frac{100}{1500} \times 100$

= $\frac{20}{3}$

= $6\frac{2}{3}$ (or) 6.67

Loss % = 6.67

Example 3.32

A fruit seller bought 8 boxes of grapes at ₹150 each. One box was damaged. He sold the remaining boxes at ₹190 each. Find the profit / loss percent.

Solution:

Cost Price of 1 box of grapes = ₹150

Cost Price of 8 boxes of grapes = 150×8

= ₹1200

Number of boxes damaged = 1

Number of boxes sold = 8 – 1

= 7

Selling Price of 1 box of grapes = ₹190

Selling Price of 7 boxes of grapes = 190 × 7

= ₹1330

S.P. > C.P, there is a Profit.

Profit = Selling Price – Cost Price

= 1330 – 1200

= 130

Profit = ₹130

$$\text{Profit }\% = \frac{\text{Profit}}{\text{C.P}} \times 100$$

$$= \frac{130}{1200} \times 100$$

= 10.83

Profit % = 10.83

Example 3.33

Ram, the shopkeeper bought a pen for ₹50 and then sold it at a loss of ₹5. Find his selling price.

Solution:

Cost price of the pen = ₹50

Loss = ₹5

S.P. = C.P. – Loss

= 50 – 5

= 45

Selling price of the pen = ₹45.

Example 3.34

Sara baked cakes for the school festival. The cost of one cake was ₹55. She sold 25 cakes and made a profit of ₹11 on each cake. Find the selling price of the cakes and the profit percent.

Solution:

Cost price of 1 cake = ₹55

Number of cakes sold = 25

Cost price of 25 cakes = 55 × 25 = ₹1375

Profit on 1 cake = ₹11

Profit on 25 cakes = 11 × 25 = ₹275

S.P. = C.P. + Profit

= 1375 + 275

= 1,650

= ₹1,650

$$\text{Profit \%} = \frac{\text{Profit}}{\text{C. P}} \times 100$$

$$= \frac{275}{1375} \times 100$$

= 20

Profit % = 20

Exercise 3.5

1. Choose the correct answer:

i) If the cost price of a bag is ₹575 and the selling price is ₹625, then there is a profit of ₹

(A) 50 (B) 575 (C) 625 (D) none of these

ii) If the cost price of the box is ₹155 and the selling price is ₹140, then there is a loss of ₹

(A) 155 (B) 140 (C) 15 (D) none of these

iii) If the selling price of a bag is ₹235 and the cost price is ₹200, then there is a

(A) profit of ₹235 (B) loss of ₹3

(C) profit of ₹35 (D) loss of ₹200

iv) Gain or loss percent is always calculated on

(A) cost price (B) selling price (C) gain (D) loss

v) If a man makes a profit of ₹25 on a purchase of ₹250, then profit% is

(A) 25 (B) 10 (C) 250 (D) 225

2. Complete the table by filling in the appropriate column:

C.P. ₹	S.P. ₹	Profit ₹	Loss ₹
144	168		
59	38		
600	635.45		
26599	23237		
107.50	100		

3. Find the selling price when cost price and profit / loss are given.
   i) Cost Price = ₹450 Profit = ₹80
   ii) Cost Price = ₹760 Loss = ₹140
   iii) Cost Price = ₹980 Profit = ₹47.50
   iv) Cost Price = ₹430 Loss = ₹93.25
   v) Cost Price = ₹999.75 Loss = ₹56.25

4. Vinoth purchased a house for ₹27, 50,000. He spent ₹2,50,000 on repairs and painting. If he sells the house for ₹33,00,000 what is his profit or loss % ?

5. A shop keeper bought 10 bananas for ₹100. 2 bananas were rotten. He sold the remaining bananas at the rate of ₹11 per banana. Find his gain or loss %

6. A shop keeper purchased 100 ball pens for ₹250. He sold each pen for ₹4. Find the profit percent.

7. A vegetable vendor bought 40 kg of onions for ₹360. He sold 36 kg at ₹11 per kg. The rest were sold at ₹4.50 per kg as they were not very good. Find his profit / loss percent.

Choose one product and find out the different stages it crosses from the time it is produced in the factory to the time it reaches the customer.

Think!

Do you think direct selling by the manufacturer himself is more beneficial for the costumers? Discuss.

Do it yourself

1. A trader mixes two kinds of oil, one costing ₹100 per Kg. and the other costing ₹80 per Kg. in the ratio 3: 2 and sells the mixture at ₹101.20 per Kg. Find his profit or loss percent.
2. Sathish sold a camera to Rajesh at a profit of 10 %. Rajesh sold it to John at a loss of 12 %. If John paid ₹4,840, at what price did Sathish buy the camera?
3. The profit earned by a book seller by selling a book at a profit of 5% is ₹15 more than when he sells it at a loss of 5%. Find the Cost Price of the book.

4.5 Simple Interest

Deposit ₹10,000 now. Get ₹20,000 at the end of 7 years.

Deposit ₹10,000 now. Get ₹20,000 at the end of 6 years.

Is it possible? What is the reason for these differences?

Lokesh received a prize amount of ₹5,000 which he deposited in a bank in June 2008. After one year he got back ₹5,400.

Why does he get more money? How much more does he get?

If ₹5,000 is left with him in his purse, will he gain ₹400?

Lokesh deposited ₹5,000 for 1 year and received ₹5,400 at the end of the first year.

When we borrow (or lend) money we pay (or receive) some additional amount in addition to the original amount. This additional amount that we receive is termed as Interest (I).

As we have seen in the above case, money can be borrowed deposited in banks to get Interest.

In the above case, Lokesh received an interest of ₹400.

The amount borrowed / lent is called the Principal (P). In this case, the amount deposited - ₹5,000 is termed as Principal (P).

The Principal added to the Interest is called the Amount (A).

In the above case, Amount = Principal +Interest

= ₹5000 + ₹400 = ₹5,400.

Will this Interest remain the same always?

Definitely not. Now, look at the following cases

(i) If the Principal deposited is increased from ₹5,000 to ₹10,000, then will the interest increase?

(ii) Similarly, if ₹5,000 is deposited for more number of years, then will the interest increase?

Yes in both the above said cases, interest will definitely increase.

From the above, we can say that interest depends on principal and duration of time. But it also depends on one more factor called the rate of interest.

Rate of interest is the amount calculated annually for ₹100

(i.e.) if rate of interest is 10% per annum, then interest is ₹10 for ₹100 for 1 year.

So, Interest depends on:

Amount deposited or borrowed – Principal (P)

Period of time - mostly expressed in years (n)

Rate of Interest (r)

This Interest is termed as Simple Interest because it is always calculated on the initial amount (ie) Principal.

Calculation of Interest

If ‘r’ is the rate of interest, principal is ₹100, then Interest

for 1 year $= 100 \times 1 \times \frac{r}{100}$

for 2 years $= 100 \times 2 \times \frac{r}{100}$

for 3 years $= 100 \times 3 \times \frac{r}{100}$

for n years $= 100 \times n \times \frac{r}{100}$

So,

$$I = \frac{Pnr}{100}$$

$$A = P + I$$

$$A = P + \frac{Pnr}{100}$$

$$A = P\left(1 + \frac{nr}{100}\right)$$

$$\text{Interest} = \text{Amount} - \text{Principal}$$

$$I = A - P$$

The other formulae derived from

$I = \frac{Pnr}{100}$ are

$$r = \frac{100I}{Pn}$$

$$n = \frac{100I}{Pr}$$

$$P = \frac{100I}{rn}$$

Note: 'n' is always calculated in years. When 'n' is given in months \ days, convert it into years.

Try these

Fill in the blanks

Principal ₹	Interest ₹	Amount ₹
5,000	500	
12,500		17,500
	6,000	25,000
8,450	750	
12,000		15,600

Example 3.35

Kamal invested ₹3,000 for 1 year at 7 % per annum. Find the simple interest and the amount received by him at the end of one year.

Solution:

Principal (P) = ₹3,000

Number of years (n) = 1

Rate of interest (r) = 7 %

$$\text{Interest (I)} = \frac{Pnr}{100}$$

$$= \frac{3000 \times 1 \times 7}{100}$$

$$I = ₹210$$

$$A = P + I$$

$$= 3000 + 210$$

$$A = ₹3,210$$

Example 3.36

Radhika invested ₹5,000 for 2 years at 11 % per annum. Find the simple interest and the amount received by him at the end of 2 years.

Solution:

Principal (P) = ₹5,000

Number of years (n) = 2 years

Rate of interest (r) = 11 %

$$I = \frac{Pnr}{100}$$

$$= \frac{5000 \times 11 \times 2}{100}$$

$$= 1100$$

$$I = ₹1,100$$

Amount (A) = P + I

= 5000 + 1100

A = ₹6,100

Example 3.37

Find the simple interest and the amount due on ₹7,500 at 8 % per annum for 1 year 6 months.

Solution:

P = ₹7,500

n = 1 yr 6 months

$$= 1\frac{6}{12} \, yrs$$

$$= 1\frac{1}{2} = \frac{3}{2} \, yrs$$

r = 8 %

Know this

12 months = 1 years

$$6 \text{ months} = \frac{6}{12} \text{ year}$$

$$= \frac{1}{2} \text{ year}$$

$$3 \text{ months} = \frac{3}{12} \text{ year}$$

$$= \frac{1}{4} \text{ year}$$

$$I = \frac{Pnr}{100}$$

$$= \frac{7500 \times \frac{3}{2} \times 8}{100}$$

$$= \frac{7500 \times 3 \times 8}{2 \times 100}$$

$$= 900$$

$$I = ₹900$$

$$A = P + I$$

$$= 7500 + 900$$

$$= ₹8,400$$

Interest = ₹900, Amount = ₹8,400

Aliter:

$$P = ₹7,500$$

$$n = \frac{3}{2} \text{ years}$$

$$r = 8\ \%$$

$$A = P\left(1 + \frac{nr}{100}\right)$$

$$= 7500\left(1 + \frac{\frac{3}{2} \times 8}{100}\right)$$

$$= 7500\left(1 + \frac{3 \times 8}{2 \times 100}\right)$$

$$= 7500\left(\frac{28}{25}\right)$$

$$= 300 \times 28$$

$$= 8400$$

$$A = ₹8400$$

$$I = A - P$$

$$= 8400 - 7500$$

$$= 900$$

$$I = ₹900$$

Interest = ₹900

Amount = ₹8,400

Example 3.38

Find the simple interest and the amount due on ₹6,750 for 219 days at 10 % per annum.

Solution:

$$P = ₹6,750$$

$$n = 219 \text{ days}$$

$$= \frac{219}{365} \text{ year} = \frac{3}{5} \text{ year}$$

$$r = 10\ \%$$

$$I = \frac{Pnr}{100}$$

$$I = \frac{6750 \times 3 \times 10}{5 \times 100}$$

$$= 405$$

$$I = ₹405$$

$$A = P + I$$

$$= 6750 + 405$$

$$= 7,155$$

$$A = ₹7,155$$

$$\text{Interest} = ₹405, \text{ Amount} = ₹7,155$$

Know this

$$365 \text{ days} = 1 \text{ year}$$

$$219 \text{ days} = \frac{219}{365} \text{ year}$$

$$= \frac{3}{5} \text{ year}$$

$$73 \text{ days} = \frac{73}{365} \text{ year}$$

$$= \frac{1}{5} \text{ year}$$

Example 3.39

Rahul borrowed ₹4,000 on 7th of June 2006 and returned it on 19th August2006. Find the amount he paid, if the interest is calculated at 5 % per annum.

Solution:

$$P = ₹4,000$$

$$r = 5\ \%$$

Number of days, June = 24 (30 – 6)

July = 31

August = 18

Total number of days = 73

$$n = 73 \text{ days}$$

Know this

Thirty days hath September, April, June and November. All the rest have thirty - one except February.

$$= \frac{73}{365} \text{ year}$$

$$= \frac{1}{5} \text{ year}$$

$$A = P\left(1 + \frac{nr}{100}\right)$$

$$= 4000\left(1 + \frac{1 \times 5}{5 \times 100}\right)$$

$$= 4000\left(1 + \frac{1}{100}\right)$$

$$= 4000\left(\frac{101}{100}\right)$$

$$= 4,040$$

$$\text{Amount} = ₹4,040$$

Example 3.40

Find the rate percent per annum when a principal of ₹7,000 earns a S.I. of ₹1,680 in 16 months.

Solution:

$$P = ₹7,000$$

$$n = 16 \text{ months}$$

$$= \frac{16}{12} yr = \frac{4}{3} yr$$

$$I = ₹1,680$$

$$r = ?$$

$$r = \frac{100I}{Pn}$$

$$= \frac{100 \times 1680}{7000 \times \frac{4}{3}}$$

$$= \frac{100 \times 1680 \times 3}{7000 \times 4}$$

$$= 18$$

$$r = 18\ \%$$

Example 3.41

Vijay invested ₹10,000 at the rate of 5 % simple interest per annum. He received ₹11,000 after some years. Find the number of years.

Solution:

$$A = ₹11,000$$

$$P = ₹10,000$$

$$r = 5\ \%$$
$$n = ?$$
$$\text{I} = \text{A} - \text{P}$$
$$= 11{,}000 - 10{,}000$$
$$= 1{,}000$$
$$\text{I} = ₹1000$$
$$n = \frac{100\,\text{I}}{\text{P}r}$$
$$= \frac{100 \times 1000}{10000 \times 5}$$
$$n = 2 \text{ years.}$$

Aliter:

$$\text{A} = \text{P}\left(1 + \frac{nr}{100}\right)$$
$$11000 = 10000\left(1 + \frac{n \times 5}{100}\right)$$
$$\frac{11000}{10000} = 1 + \frac{n}{20}$$
$$\frac{11}{10} = \frac{20 + n}{20}$$
$$\frac{11}{10} \times 20 = 20 + n$$
$$22 = 20 + n$$
$$22 - 20 = n$$
$$n = 2 \text{ years}$$

Example 3.42

A sum of money triples itself at 8 % per annum over a certain time. Find the number of years.

Solution:

Let Principal be ₹P.

$$\text{Amount} = \text{triple the principal}$$
$$= ₹3\text{ P}$$
$$r = 8\ \%$$
$$n = ?$$

$$
\begin{aligned}
I &= A - P \\
&= 3P - P \\
&= 2P \\
I &= ₹2\ P \\
n &= \frac{100I}{Pr} \\
&= \frac{100 \times 2P}{P \times 8} \\
n &= 25 \text{ years} \\
\text{Number of years} &= 25
\end{aligned}
$$

Aliter:

Let Principal be ₹100

$$
\begin{aligned}
\text{Amount} &= 3 \times 100 \\
&= ₹300 \\
I &= A - P \\
&= 300 - 100 \\
I &= ₹200. \\
n &= \frac{100I}{Pr} = \frac{100 \times 200}{100 \times 8} \\
n &= \frac{200}{8} = 25 \\
\text{Number of years} &= 25.
\end{aligned}
$$

Example 3.43

A certain sum of money amounts to ₹10,080 in 5 years at 8 % . Find the principal.

Solution:

$$
\begin{aligned}
A &= ₹10{,}080 \\
n &= 5 \text{ years} \\
r &= 8\ \% \\
P &= ? \\
A &= P\left(1 + \frac{nr}{100}\right) \\
10080 &= P\left(1 + \frac{5 \times 8}{100}\right)
\end{aligned}
$$

$$10080 = P\left(\frac{7}{5}\right)$$

$$10080 \times \frac{5}{7} = P$$

$$7{,}200 = P$$

Principal = ₹7,200

Example 3.44

A certain sum of money amounts to ₹8,880 in 6 years and ₹7,920 in 4 years respectively. Find the principal and rate percent.

Solution:

Amount at the end of 6 years = Principal + interest for 6 years

$= P + I_6 = 8880$

Amount at the end of 4 years = Principal + Interest for 4 years

$= P + I_4 = 7920$

$I_2 = 8880 - 7920$

$= 960$

Interest at the end of 2 years = ₹960

Interest at the end of 1st year $= \frac{960}{2}$

$= 480$

Interest at the end of 4 years $= 480 \times 4$

$= 1{,}920$

$P + I_4 = 7920$

$P + 1920 = 7920$

$P = 7920 - 1920$

$P = 6{,}000$

Principal = ₹6,000

$$r = \frac{100I}{pn}$$

$$= \frac{100 \times 1920}{6000 \times 4}$$

$$r = 8\ \%$$

Exercise 3.6

1. Choose the correct answer:

i) Simple Interest on ₹1000 at 10 % per annum for 2 years is

(A) ₹1000 (B) ₹200 (C) ₹100 (D) ₹2000

ii) If Amount = ₹11,500, Principal = ₹11,000, Interest is

(A) ₹500 (B) ₹22,500 (C) ₹11,000 (D) ₹11,000

iii) 6 months =

(A) $\frac{1}{2}$ yr (B) $\frac{1}{4}$ yr (C) $\frac{3}{4}$ yr (D) 1 yr

iv) 292 days =

(A) $\frac{1}{5}$ yr (B) $\frac{3}{5}$ yr (C) $\frac{4}{5}$ yr (D) $\frac{2}{5}$ yr

v) If P = ₹14000 I = ₹1000, A is

(A) ₹15000 (B) ₹13000 (C) ₹14000 (D) ₹1000

2. Find the S.I. and the amount on ₹5,000 at 10 % per annum for 5 years.

3. Find the S.I and the amount on ₹1,200 at $12\frac{1}{2}$% per annum for 3 years.

4. Lokesh invested ₹10,000 in a bank that pays an interest of 10 % per annum. He withdraws the amount after 2 years and 3 months. Find the interest, he receives.

5. Find the amount when ₹2,500 is invested for 146 days at 13 % per annum.

6. Find the S.I and amount on ₹12,000 from May 21^{st} 1999 to August 2^{nd} 1999 at 9 % per annum.

7. Sathya deposited ₹6,000 in a bank and received ₹7500 at the end of 5 years. Find the rate of interest.

8. Find the principal that earns ₹250 as S.I. in $2\frac{1}{2}$ years at 10 % per annum.

9. In how many years will a sum of ₹5,000 amount to ₹5,800 at the rate of 8 % per annum.

10. A sum of money doubles itself in 10 years. Find the rate of interest.

11. A sum of money doubles itself at $12\frac{1}{2}$% per annum over a certain period of time. Find the number of years.

12. A certain sum of money amounts to ₹6,372 in 3 years at 6 % Find the principal.

13. A certain sum of money amounts to ₹6,500 in 3 years and ₹5,750 in $1\frac{1}{2}$ years respectively . Find the principal and the rate percent?

Think!

1) Find the rate per cent at which, a sum of money becomes $\frac{9}{4}$ times in 2 years.
2) If Ram needs ₹6,00,000 after 10 years, how much should he invest now in a bank if the bank pays 20 % interest p.a.

Points to Remember

1. Two quantities are said to be in direct variation if the increase (decrease) in one quantity results in a proportionate increase (decrease) in the other quantity.
2. Two quantities are said to be in inverse variation if the increase (decrease) in one quantity results in a proportionate decrease (increase) in the other quantity.
3. In direct proportion, the ratio of one quantity is equal to the ratio of the second quantity.
4. In indirect proportion, the ratio of one quantity is equal to the inverse ratio of the second quantity.
5. A fraction whose denominator is 100 or a ratio whose second term is 100 is termed as a percent.
6. Percent means per hundred, denoted by %
7. To convert a fraction or a decimal to a percent, multiply by 100.
8. The price at which an article is bought is called the cost price of an article.
9. The price at which an article is sold is called the selling price of an article.
10. If the selling price of an article is more than the cost price, there is a profit.

11. If the cost price of an article is more than the selling price, there is a loss.

12. Total cost price = Cost Price + Repair Charges / Transportation charges.

13. Profit or loss is always calculated for the same number of articles or same units.

14. Profit = Selling Price – Cost Price

15. Loss = Cost Price – Selling Price

16. Profit% = $\frac{\text{Profit}}{\text{C.P.}} \times 100$

17. Loss% = $\frac{\text{Loss}}{\text{C.P.}} \times 100$

18. Selling Price = Cost Price + Profit

19. Selling Price = Cost Price - Loss

20. The formula to calculate interest is $\mathrm{I} = \frac{\mathrm{Pnr}}{100}$

21. $$\begin{aligned} A &= P + I \\ &= \mathrm{P} + \frac{\mathrm{P}nr}{100} \\ &= P\left(1 + \frac{nr}{100}\right) \end{aligned}$$

22. $\mathrm{I} = A - P$

23. $P = \frac{100I}{nr}$

24. $r = \frac{100\mathrm{I}}{\mathrm{P}n}$

25. $n = \frac{100I}{\mathrm{Pr}}$

4 MEASUREMENTS

In class VI, we have learnt about the concepts and formulae for finding the perimeter and area of simple closed figures like rectangle , square and right triangle. In this chapter, we will learn about the area of some more closed figures such as triangle, quadrilateral, parallelogram, rhombus, trapezium and circle.

4.1 Revision

Let us recall what we have learnt about the area and perimeter of rectangle, square and right triangle.

Perimeter

When we go around the boundary of the closed figure, the distance covered by us is called the perimeter.

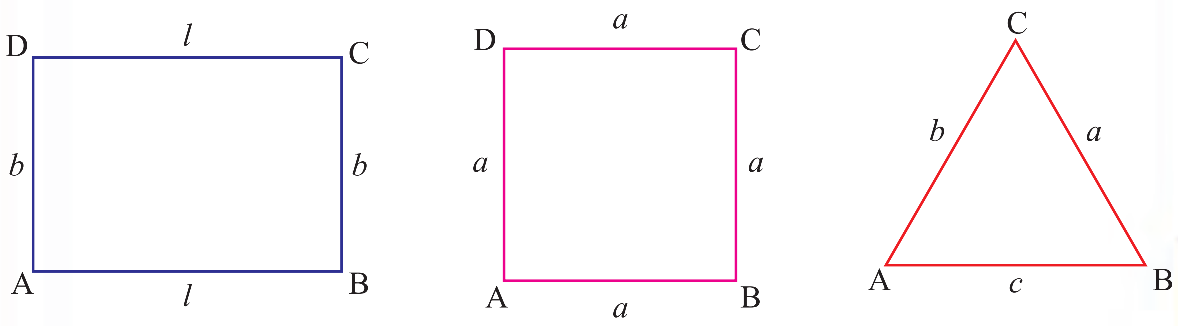


Fig. 4.1

Perimeter of the rectangle = $2 \times$ (length) + $2 \times$ (breadth)

= 2 [length + breadth]

Perimeter of the rectangle = $2\,(l + b)$ units where l = length, b = breadth

Perimeter of the square = $4 \times$ length of its side

= $4 \times$ side

Perimeter of the square = $4\,a$ units where a = side

Perimeter of the triangle = Sum of the sides of the triangle

Perimeter of the triangle = $(a + b + c)$ units

where a, b, c are the sides of the triangle

Area

The surface enclosed by a closed figure is called its area.

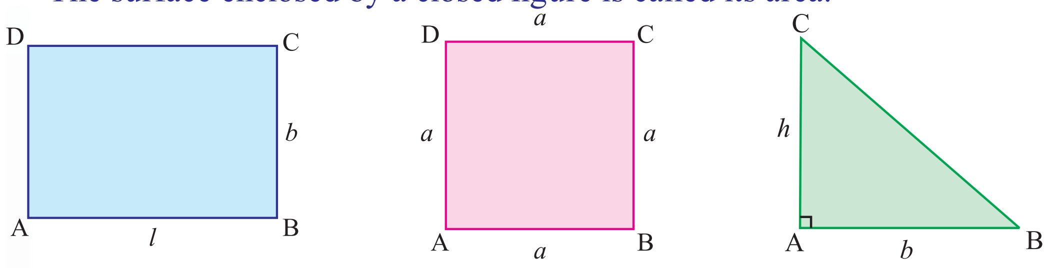


Fig. 4.2

$$\text{Area of the rectangle} = \text{length} \times \text{breadth}$$

$$\text{Area of the rectangle} = l \times b \text{ sq. units}$$

$$\text{Area of the square} = \text{side} \times \text{side}$$

$$\text{Area of the square} = a \times a \text{ sq. units}$$

$$\text{Area of the right triangle} = \frac{1}{2} \times \text{product of the sides containing } 90^\circ$$

$$\text{Area of the right triangle} = \frac{1}{2} \times (b \times h) \text{ sq. units}$$

where b and h are adjacent sides of the right angle.

Try these

- Find the area and perimeter of your class room blackboard, table and windows.
- Take a sheet of paper, cut the sheet into different measures of rectangles, squares and right triangles. Place them on a table and find the perimeter and area of each figure.

Example 4.1

Find the area and the perimeter of a rectangular field of length 15 m and breadth 10 m.

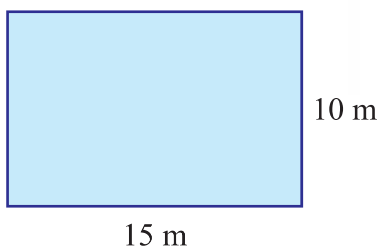


Fig.4.3

Solution

Given: length = 15 m and breadth = 10 m

$$\begin{aligned} \text{Area of the rectangle} &= \text{length} \times \text{breadth} \\ &= 15 \text{ m} \times 10 \text{ m} \\ &= 150 \text{ m}^2 \end{aligned}$$

$$\text{Perimeter of the rectangle} = 2\,[\text{length} + \text{breadth}]$$
$$= 2\,[15 + 10] = 50 \text{ m}$$
$$\therefore \text{Area of the rectangle} = 150 \text{ m}^2$$
$$\text{Perimeter of the rectangle} = 50 \text{ m}$$

Example 4.2

The area of a rectangular garden 80m long is 3200sq.m. Find the width of the garden.

Solution

Given: length = 80 m, Area = 3200 sq.m

$$\text{Area of the rectangle} = \text{length} \times \text{breadth}$$
$$\text{breadth} = \frac{\text{area}}{\text{length}}$$
$$= \frac{3200}{80} = 40 \text{ m}$$
$$\therefore \text{Width of the garden} = 40 \text{ m}$$

Example 4.3

Find the area and perimeter of a square plot of length 40 m.

Solution

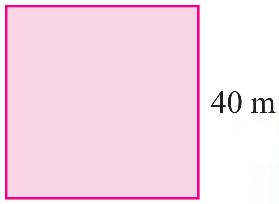


Fig. 4.4

Given the side of the square plot = 40 m

$$\text{Area of the square} = \text{side} \times \text{side}$$
$$= 40 \text{ m} \times 40 \text{ m}$$
$$= 1600 \text{ sq.m}$$
$$\text{Perimeter of the square} = 4 \times \text{side}$$
$$= 4 \times 40 = 160 \text{ m}$$
$$\therefore \text{Area of the square} = 1600 \text{ sq.m}$$
$$\text{Perimeter of the square} = 160 \text{ m}$$

Example 4.4

Find the cost of fencing a square flower garden of side 50 m at the rate of ₹10 per metre.

Solution

Given the side of the flower garden = 50 m

For finding the cost of fencing, we need to find the total length of the boundary (perimeter) and then multiply it by the rate of fencing.

Perimeter of the square flower garden = 4 × side

= 4 × 50

= 200 m

cost of fencing 1m = ₹10 (given)

∴ cost of fencing 200m = ₹10 × 200

= ₹2000

Example 4.5

Find the cost of levelling a square park of side 60 m at ₹2 per sq.m.

Solution

Given the side of the square park = 60 m

For finding the cost of levelling, we need to find the area and then multiply it by the rate for levelling.

Area of the square park = side × side

= 60 × 60

= 3600 sq.m

cost of levelling 1 sq.m = ₹2

∴ cost of levelling 3600 sq.m = ₹2 × 3600

= ₹7200

Example 4.6

In a right triangular ground, the sides adjacent to the right angle are 50 m and 80 m. Find the cost of cementing the ground at ₹5 per sq.m

Solution

For finding the cost of cementing, we need to find the area and then multiply it by the rate for cementing.

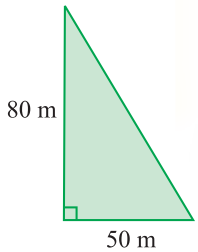


Fig. 4.5

Area of right triangular ground = $\frac{1}{2} \times b \times h$

where b and h are adjacent sides of the right anlges.

$= \frac{1}{2} \times (50\,m \times 80\,m)$

$= 2000\text{ m}^2$

cost of cementing one sq.m = ₹5

∴ cost of cementing 2000 sq.m = ₹5 × 2000

= ₹10000

Do you know?

1 are = 100 m^2

1 hectare = 100 are (or)

= 10000 m^2

4.2 Area of Combined Plane Figures

In this section we will learn about the area of combined plane figures such as rectangle, square and right triangle taken two at a time.

A villager owns two pieces of land adjacent to each other as shown in the Fig.4.6. He did not know the area of land he owns. One land is in the form of rectangle of dimension 50 m × 20 m and the other land is in the form of a square of side 30m. Can you guide the villager to find the total area he owns?

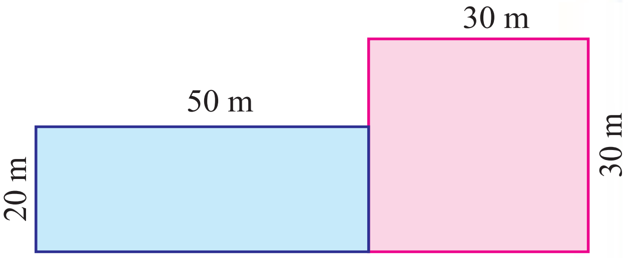


Fig. 4.6

Now, Valarmathi and Malarkodi are the leaders of Mathematics club in the school. They decorated the walls with pictures. First, Valarmathi made a rectangular picture of length 2m and width 1.5m. While Malarkodi made a picture in the shape of a right triangle as in Fig. 4.7. The adjacent sides that make the right angle are 1.5m and 2m. Can we find the total decorated area?

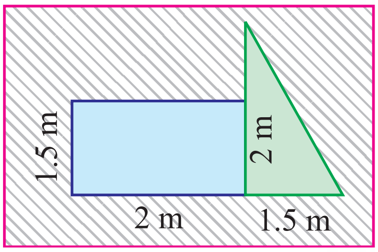


Fig. 4.7

Now, let us see some examples for combined figures

Example 4.7

Find the area of the adjacent figure:

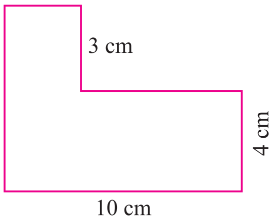


Fig. 4.8

Solution

Area of square (1) $=$ $3\text{ cm} \times 3\text{ cm} = 9\text{ cm}^2$

Area of rectangle (2) $=$ $10\text{ cm} \times 4\text{ cm} = 40\text{ cm}^2$

$\therefore$ Total area of the figure (Fig. 4.9) $= (9 + 40)\text{ cm}^2$

$= 49\text{ cm}^2$

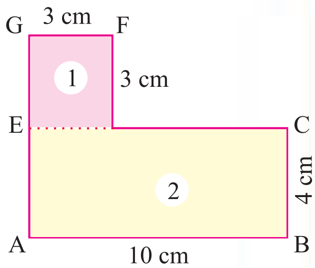


Fig. 4.9

Aliter:

Area of rectangle (1) $=$ $7\text{ cm} \times 3\text{ cm} = 21\text{ cm}^2$

Area of rectangle (2) $=$ $7\text{ cm} \times 4\text{ cm} = 28\text{ cm}^2$

$\therefore$ Total area of the figure (Fig. 4.10) $= (21 + 28)\text{ cm}^2$

$= 49\text{ cm}^2$

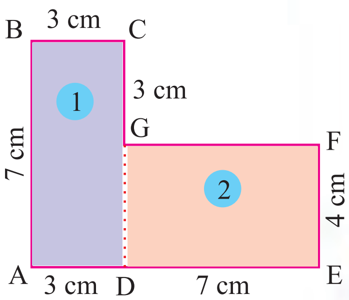


Fig. 4.10

Example 4.8

Find the area of the following figure:

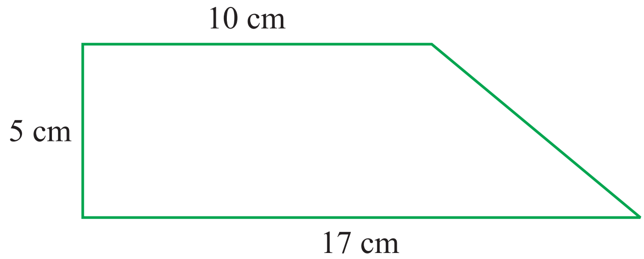


Fig. 4.11

Solution

The figure contains a rectangle and a right triangle

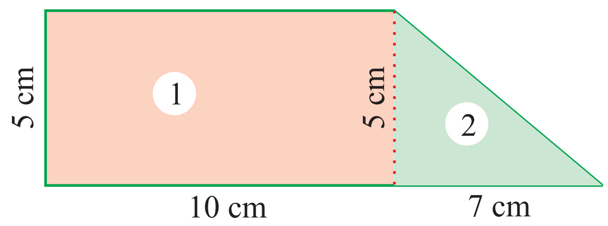


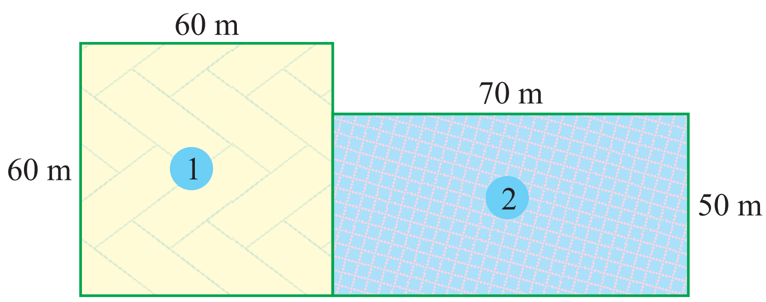
Fig. 4.12

$$
\begin{aligned}
\text{Area of the rectangle (1)} &= 5\text{ cm} \times 10\text{ cm} \\
&= 50\text{ cm}^2 \\
\text{Area of the right triangle (2)} &= \frac{1}{2} \times (7\text{ cm} \times 5\text{ cm}) \\
&= \frac{35}{2}\text{ cm}^2 = 17.5\text{ cm}^2 \\
\therefore \text{Total area of the figure} &= (50 + 17.5)\text{ cm}^2 \\
&= 67.5\text{ cm}^2 \\
\text{Total area} &= 67.5\text{ cm}^2
\end{aligned}
$$

Example 4.9

Arivu bought a square plot of side 60 m. Adjacent to this Anbu bought a rectangular plot of dimension 70 m $\times$ 50 m. Both paid the same amount. Who is benefited ?

Solution



Fi.g 4.13

Area of the square plot of Arivu (1) $= 60 \text{ m} \times 60 \text{ m} = 3600 \text{ m}^2$

Area of the rectangular plot of Anbu (2) $= 70 \text{ m} \times 50 \text{ m} = 3500 \text{ m}^2$

The area of the square plot is more than the rectangular plot.

So, Arivu is benefited.

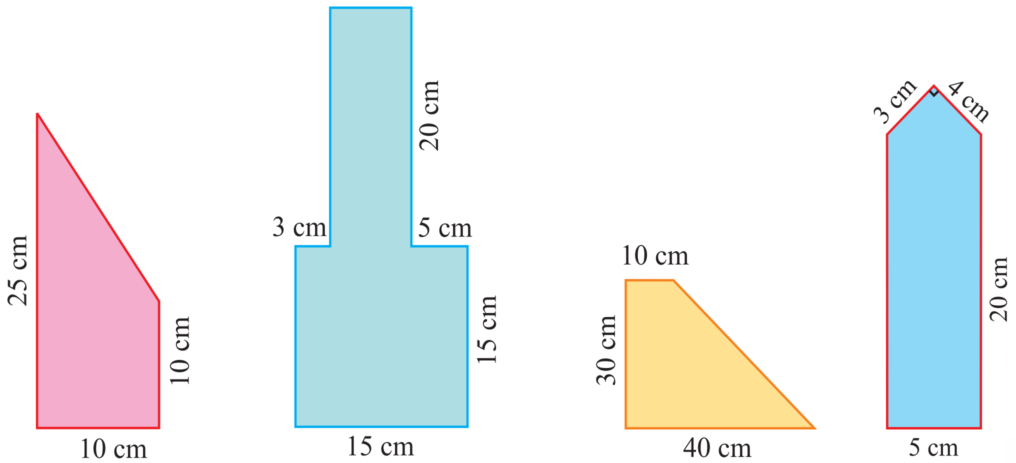
Try these

Take two square sheets of same area. Cut one square sheet along the diagonal. How many right triangles do you have? What can you say about their area? Place them on the other square sheet. Observe and discuss.

Now, take two rectangular sheets of same dimensions. Cut one rectangular sheet along the diagonal. How many right triangles do you have? What can you say about their area? Place them on the other sheet. What is the relationship between the right triangle and the rectangle?

Exercise 4.1

1. Find the area of the following figures:



(i) (ii) (iii) (iv)

2. Sibi wants to cover the floor of a room 5 m long and width 4 m by square tiles. If area of each square tiles is $\frac{1}{2} m^2$, then find the number of tiles required to cover the floor of a room.
3. The cost of a right triangular land and the cost of a rectangular land are equal. Both the lands are adjacent to each other. In a right triangular land the adjacent sides of the right angles are 30 m and 40 m. The dimensions of the rectangular land are 20 m and 15 m. Which is best to purchase?
4. Mani bought a square plot of side 50 m. Adjacent to this Ravi bought a rectangular plot of length 60 m and breadth 40 m for the same price. Find out who is benefited and how many sq. m. are more for him?
5. Which has larger area? A right triangle with the length of the sides containing the right angle being 80 cm and 60 cm or a square of length 50 cm.

4.3 Area of Triangle

The area of a right triangle is half the area of the rectangle that contains it.

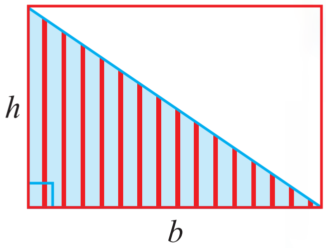


Fig. 4.14

The area of the right triangle

$= \frac{1}{2}$(Product of the sides containing 90^0)

(or) $= \frac{1}{2} b\, h$ sq.units

where b and h are adjacent sides of right angle

In this section we will learn to find the area of triangles.

To find the area of a triangle

Take a rectangular piece of paper. Name the vertices as A, B, C and D. Mark any point E on DC. Join AE and BE. We get a triangle ABE inscribed in the rectangle ABCD as shown in the Fig. 4.15 (i)

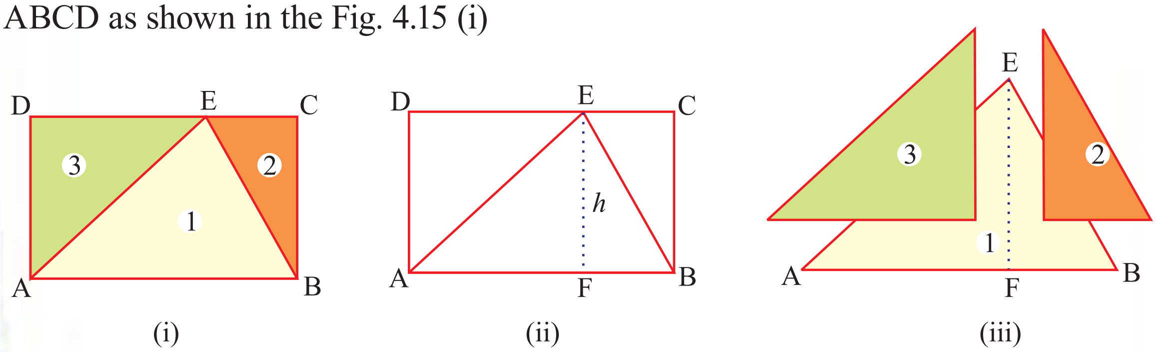


Fig. 4.15

Now mark a point F on AB such that DE = AF. join EF. We observe that EF = BC. We call EF as h and AB as b.

Now cut along the lines AE and BE and superpose two triangles (2) and (3) on ABE as shown in the Fig. 4.15 (iii).

$\therefore$ Area of ΔABE = Area of ΔADE + Area of ΔBCE ..... (1)

Area of Rectangle ABCD = Area of ΔABE + (Area of ΔADE + Area of ΔBCE)

= Area of ΔABE + Area of Δ ABE (By using (1))

= 2 Area of ΔABE

(i.e.) 2 Area of ΔABE = Area of the rectangle ABCD

$\therefore$ Area of the triangle ABE

$$= \frac{1}{2}(\text{area of rectangle ABCD})$$

$$= \frac{1}{2}(\text{length} \times \text{breadth})$$

$$= \frac{1}{2}\ bh \text{ sq.units}$$

$\therefore$ Area of any triangle = $\frac{1}{2}\ bh$ sq.units

Where b is the base and h is the height of the triangle.

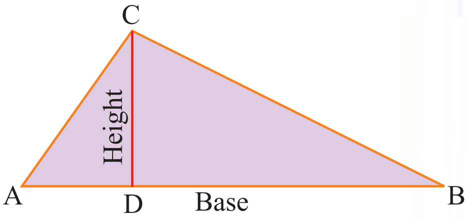


Fig. 4.16

Think it!

Consider an obtuse angled triangle ABC. The perpendicular drawn from C meets the base BA produced at D.

What is the area of the triangle?

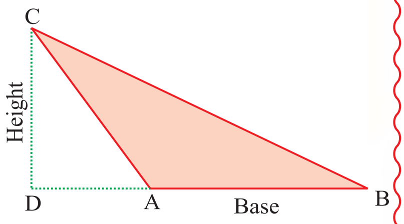


Fig. 4.17

Try these

Paper folding method

Take a triangular piece of paper. Name the vertices as A, B and C. Consider the base AB as b and altitude by h.

Find the midpoint of AC and BC, say D and E respectively. Join D and E and draw a perpendicular line from C to AB. It meets at F on DE and G on AB. We observe that CF = FG.

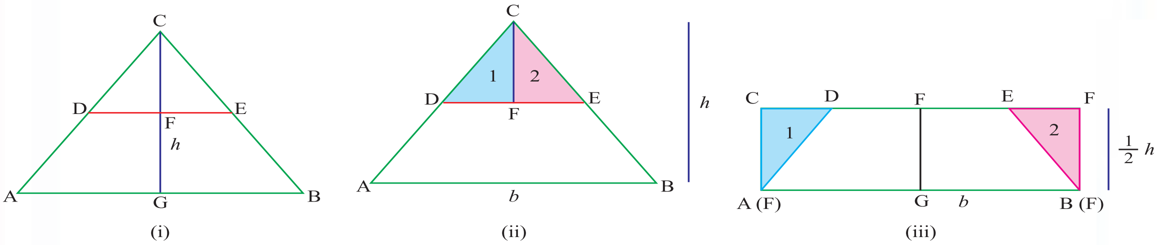


Fig. 4.18

Cut along DE and again cut it along CF to get two right triangles. Now, place the two right triangles beside the quarilateral ABED as shown in the Fig. 4.18 (iii).

Area of figure (i) = Area of figure (iii)

(i.e.) Area of the triangle = Area of the rectangle

$$= b \times (\frac{1}{2}h) \text{ sq. units} \qquad [CF + FG = h]$$

$$= \frac{1}{2} b\, h \text{ sq. units.}$$

Example 4.10

Find the area of the following figures:

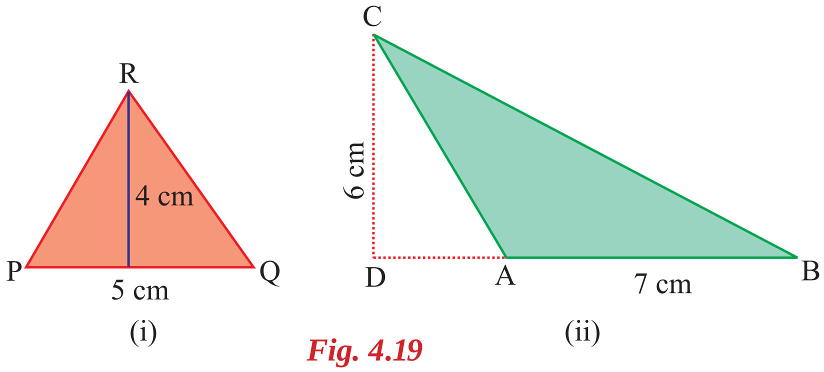


Fig. 4.19

Solution

(i) Given: Base $= 5$ cm, Height $= 4$ cm

$$\begin{aligned}\text{Area of the triangle PQR} &= \frac{1}{2} b\, h \\ &= \frac{1}{2} \times 5 \text{ cm} \times 4 \text{ cm} \\ &= 10 \text{ sq.cm (or) cm}^2\end{aligned}$$

(ii) Given: Base $= 7$cm, Height $= 6$cm

$$\begin{aligned}\text{Area of the triangle ABC} &= \frac{1}{2} b\, h \\ &= \frac{1}{2} \times 7\text{cm} \times 6\text{cm} \\ &= 21 \text{ sq.cm (or) cm}^2\end{aligned}$$

Example 4.11

Area of a triangular garden is 800 sq.m. The height of the garden is 40 m. Find the base length of the garden.

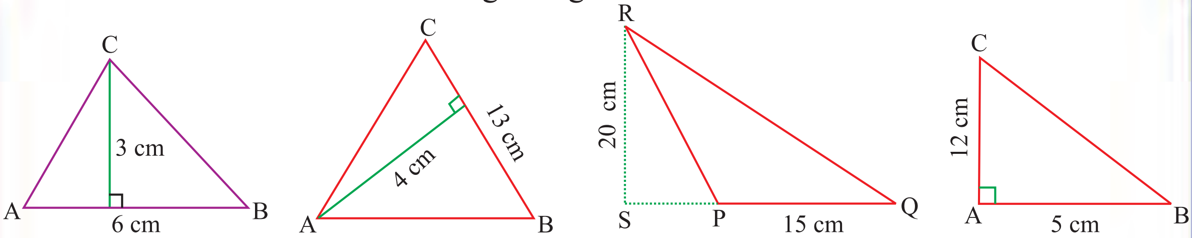
Solution

$$\begin{aligned}\text{Area of the triangular garden} &= 800 \text{ sq.m. (given)} \\ \frac{1}{2} b\, h &= 800 \\ \frac{1}{2} \times b \times 40 &= 800 \quad (\text{since } h = 40) \\ 20\, b &= 800 \\ b &= 40 \text{ m}\end{aligned}$$

$\therefore$ Base of the garden is 40 m.

Exercise 4.2

1. Find the area of the following triangles:



(i) (ii) (iii) (iv)

2. Find the area of the triangle for the following measurements:
   (i) base = 6 cm, height = 8 cm
   (ii) base = 3 m, height = 2 m
   (iii) base = 4.2 m , height = 5 m

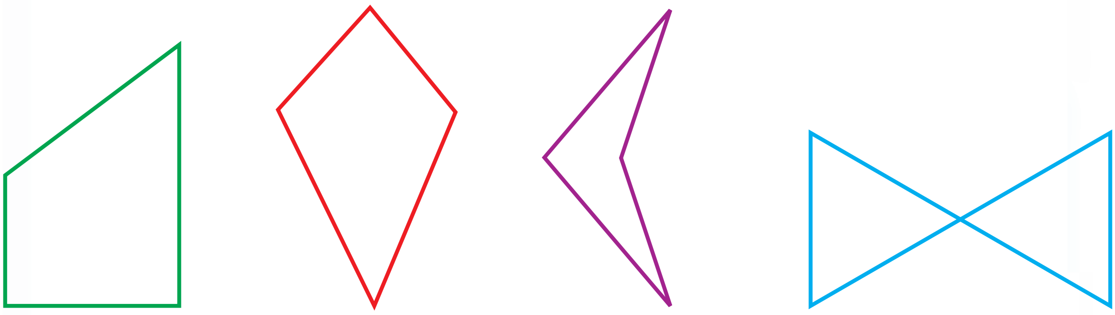
3. Find the base of the triangle whose area and height are given below:
   (i) area = 40 m^2 , height = 8 m
   (ii) area = 210 cm^2 , height = 21 cm
   (iii) area = 82.5 m^2 , height = 10 m

4. Find the height of the triangle whose area and the base are given below:
   (i) area = 180 m^2 , base = 20 m
   (ii) area = 62.5 m^2 , base = 25 m
   (iii) area = 20 cm^2 , base = 5 cm

5. A garden is in the form of a triangle. Its base is 26 m and height is 28 m. Find the cost of levelling the garden at ₹5 per m^2.

4.4 Area of the Quadrilateral

A quadrilateral is a closed figure bounded by four line segments such that no two line segments cross each other.



(i) (ii) (iii) (iv)

Fig. 4.20

In the above figure
fig (i), (ii), (iii) are quadrilaterals.
fig (iv) is not a quadrilateral.

Types of quadrilateral

The figure given below shows the different types of quadrilateral.

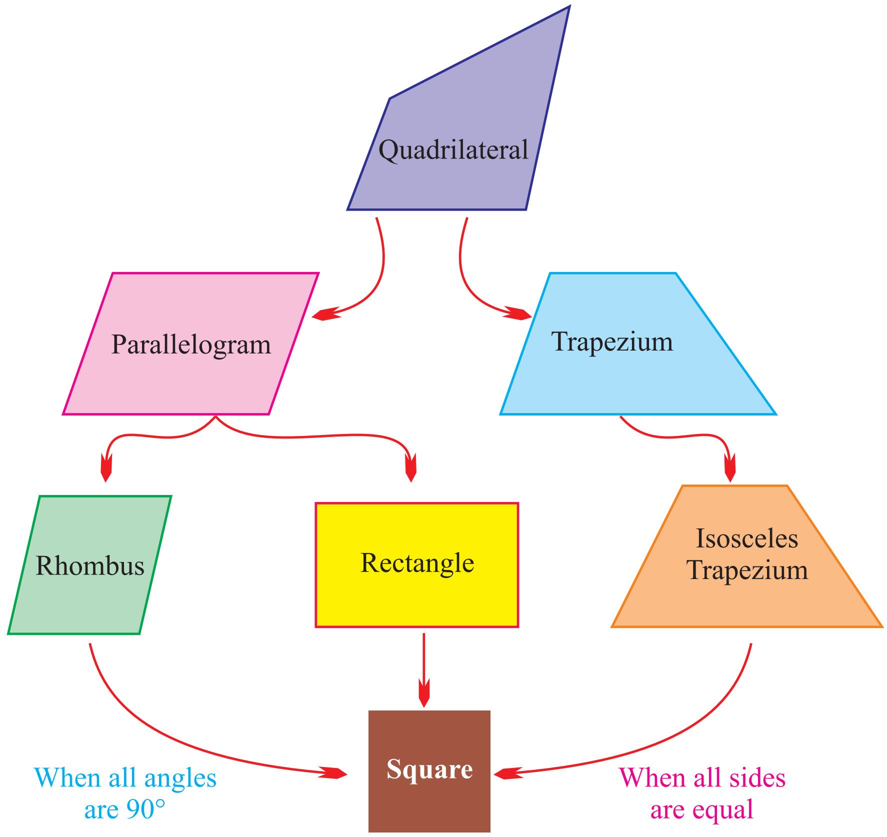


Fig. 4.21

Area of the quadrilateral

In a quadrilateral ABCD, draw the diagonal AC. It divides the quadrilateral into two triangles ABC and ADC. Draw altitudes BE and DF to the common base AC.

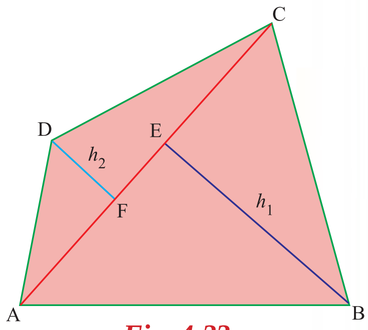


Fig. 4.22

Area of the quadrilateral ABCD

$$= \text{Area of } \triangle ABC + \text{Area of } \triangle ADC$$

$$= [\frac{1}{2} \times AC \times h_1] + [\frac{1}{2} \times AC \times h_2]$$

$$= \frac{1}{2} \times AC \times (h_1 + h_2)$$

$$= \frac{1}{2} \times d \times (h_1 + h_2) \text{ sq. units}$$

where d is the length of the diagonal AC and h_1 and h_2 are perpendiculars drawn to the diagonal from the opposite vertices.

$\therefore$ Area of the quadrilateral $= \frac{1}{2} \times d \times (h_1 + h_2)$ sq.units.

Example 4.12

Calculate the area of a quadrilateral PQRS shown in the figure

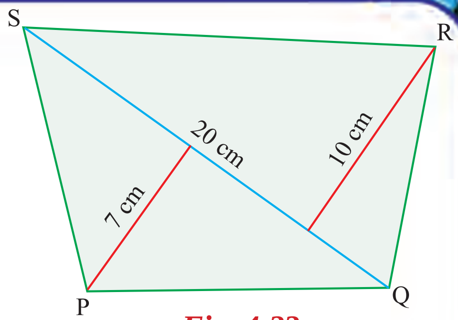


Fig. 4.23

Solution

Given: $d = 20\text{cm}$,$h_1 = 7\text{cm}$, $h_2 = 10\text{cm}$.

Area of a quadrilateral PQRS

$$= \frac{1}{2} \times d \times (h_1 + h_2)$$

$$= \frac{1}{2} \times 20 \times (7 + 10)$$

$$= 10 \times 17$$

$$= 170 \text{ cm}^2$$

$\therefore$ Area of the quadrilateral PQRS = 170 cm^2.

Example 4.13

A plot of land is in the form of a quadrilateral, where one of its diagonals is 200 m long. The two vertices on either side of this diagonals are 60 m and 50 m away from the diagonal. What is the area of the plot of land ?

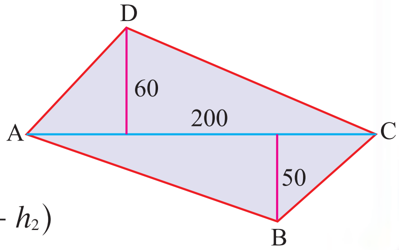


Fig. 4.24

Solution

Given: $d = 200$ m, $h_1 = 50$ m, $h_2 = 60$ m

Area of the quadrilateral ABCD $= \frac{1}{2} \times d \times (h_1 + h_2)$

$$= \frac{1}{2} \times 200 \times (50 + 60)$$

$$= 100 \times 110$$

$\therefore$ Area of the quadrilateral $= 11000 \text{ m}^2$

Example 4.14

The area of a quadrilateral is 525 sq. m. The perpendiculars from two vertices to the diagonal are 15 m and 20 m. What is the length of this diagonal ?

Solution

Given: Area = 525 sq. m, $h_1 = 15$ m, $h_2 = 20$ m.

Now, we have

Area of the quadrilateral = 525 sq.m.

$$\frac{1}{2} \times d \times (h_1 + h_2) = 525$$

$$\frac{1}{2} \times d \times (15 + 20) = 525$$

$$\frac{1}{2} \times d \times 35 = 525$$

$$d = \frac{525 \times 2}{35} = \frac{1050}{35} = 30 \text{ m}$$

$\therefore$ The length of the diagonal = 30 m.

Example 4.15

The area of a quadrilateral PQRS is 400 cm^2. Find the length of the perpendicular drawn from S to PR, if PR = 25 cm and the length of the perpendicular from Q to PR is 15 cm.

Solution

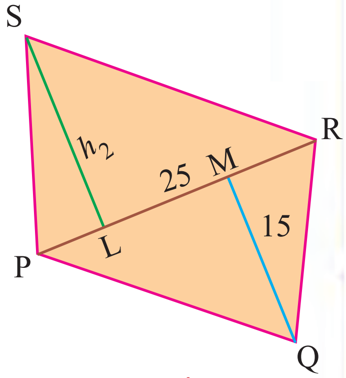


Fig. 4.25

Given: $d = 25$ cm, $h_1 = 15$ cm, Area = 400 cm^2

Area of a quadrilateral PQRS= 400 cm^2

$$\frac{1}{2} \times d \times (SL + QM) = 400 \text{ where } SL = h_1, QM = h_2$$

(i.e.) $$\frac{1}{2} \times d \times (h_1 + h_2) = 400$$

$$\frac{1}{2} \times 25 \times (15 + h_2) = 400$$

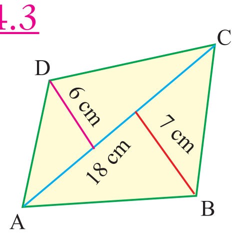
$$15 + h_2 = \frac{400 \times 2}{25} = 16 \times 2 = 32$$

$$h_2 = 32 - 15 = 17$$

$\therefore$ The length of the perpendicular from S to PR is 17 cm.

Excercise 4.3

1. From the figure, find the area of the quadrilateral ABCD.



2. Find the area of the quadrilateral whose diagonal and heights are:
   (i) $d = 15$ cm, $h_1 = 5$ cm, $h_2 = 4$ cm
   (ii) $d = 10$ cm, $h_1 = 8.4$ cm, $h_2 = 6.2$ cm
   (iii) $d = 7.2$ cm, $h_1 = 6$ cm, $h_2 = 8$ cm
3. A diagonal of a quadrilateral is 25 cm, and perpendicular on it from the opposite vertices are 5 cm and 7 cm. Find the area of the quadrilateral.
4. The area of a quadrilateral is 54 cm^2. The perpendicualrs from two opposite vertices to the diagonal are 4 cm and 5 cm. What is the length of this diagonal?
5. A plot of land is in the form of a quadrilateral, where one of its diagonals is 250 m long. The two vertices on either side of the diagonal are 70 m and 80 m away. What is the area of the plot of the land?

4.5 Area of a Parallelogram

In our daily life, we have seen many plane figures other than square, rectangle and triangle. Do you know the other plane figures?

Parallelogram is one of the other plane figures.

In this section we will discuss about the parallelogram and further we are going to discuss the following:

How to find the area of a land which is a parallelogram in shape ?

Can a parallelogram be converted into rectangle of equal area ?

Can a parallelogram be converted into two triangles of equal area ?

Definition of Parallelogram

Take four broom sticks. Using cycle valve tube rubber, join them and form a rectangle (see Fig. 4.26 (i))

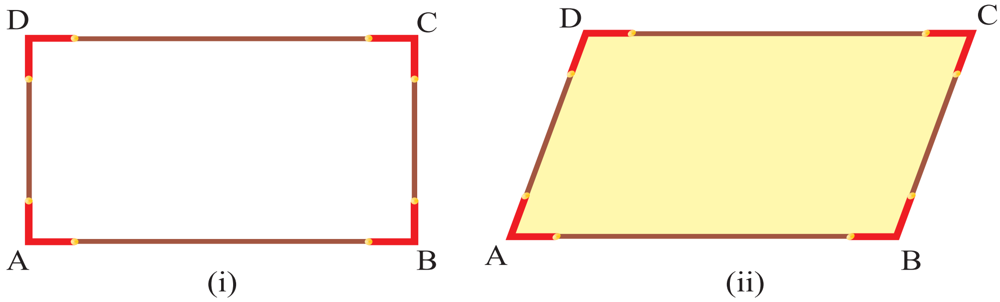


Fig. 4.26

Keeping the base AB fixed and slightly push the corner D to its right, you will get the shape as shown in Fig. 4.26 (ii).

Now answer the following:

Do the shape has parallel sides ? Which are the sides parallel to each other?

Here the sides AB and DC are parallel and AD and BC are parallel. We use the symbol '||' which denotes "is parallel to" i.e., AB || DC and AD || BC. (Read it as AB is parallel to DC and AD is parallel to BC).

So, in a quadrilateral, if both the pair of opposite sides are parallel then it is called a parallelogram. Fig.4.27.

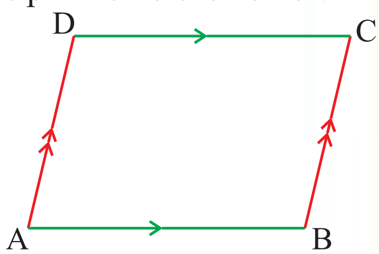


Fig. 4.27

Area of the parallelogram

Draw a parallelogram on a graph paper as shown in Fig. 4.28 (i)

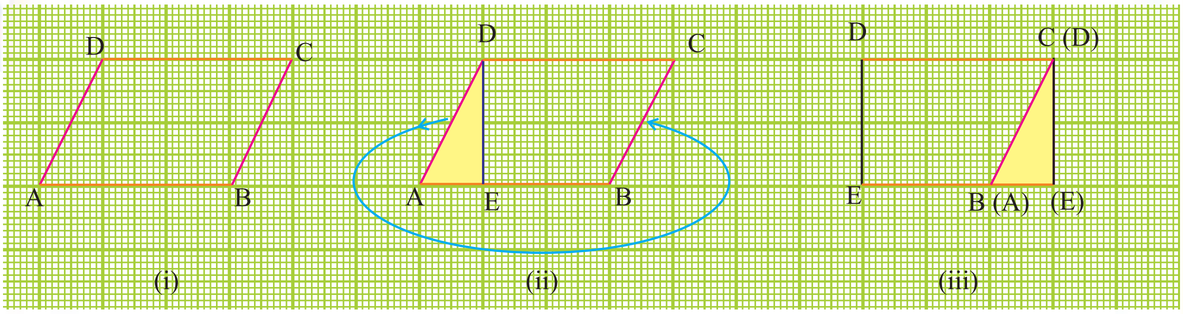


Fig. 4.28

Draw a perpendicular line from the vertex D to meet the base AB at E.

Now, cut out the triangle AED. Place the triangle AED to the other side as shown in Fig. 4.28 (iii)

What shape do you get? Is it a rectangle?

Is the area of the parallelogram equal to the area of the rectangle formed?

Yes, Area of the parallelogram = Area of the rectangle formed

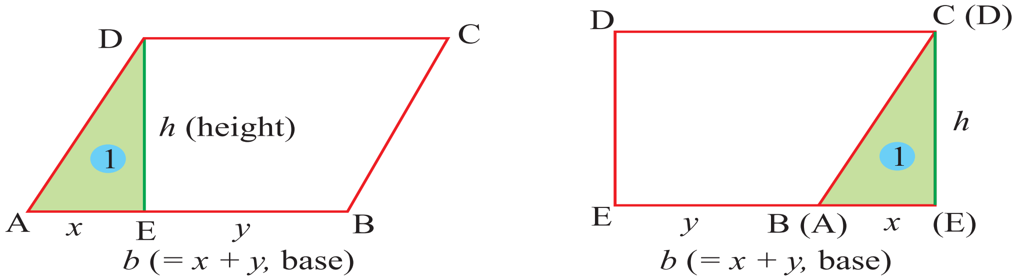


Fig. 4.29

We find that the length of rectangle formed is equal to the base of the parallelogram and breadth of rectangle is equal to the height of the parallelogram. (see Fig. 4.29)

$\therefore$ Area of parallelogram = Area of rectangle

= (length × breadth) sq. Units

= (base × height) sq. Units

Area of parallelogram = bh sq. Units

Where b is the base and h is the height of the parallelogram.

$\therefore$ area of the parallelogram is the product of the base (b) and its corresponding height (h).

Note: Any side of a parallelogram can be chosen as base of the parallelogram. The perpendicular dropped on that side from the opposite vertex is known as height (altitude).

Do you know?

In a parallelogram

- the opposite sides are parallel.
- the opposite angles are equal.
- the opposite sides are equal.
- the diagonals are not equal.
- the diagonals bisect each other.

Example 4.16

Using the data given in the figure,

(i) find the area of the parallelogram with base AB.

(ii) find the area of the parallelogram with base AD.

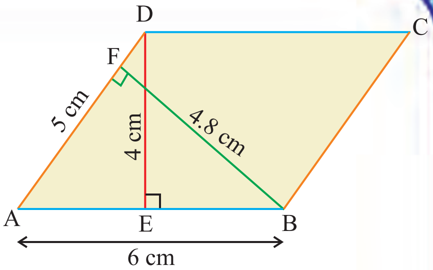


Fig. 4.30

Solution

The area of the parallelogram = base × height

(i) Area of parallelogram with base AB = base AB × height DE

$= 6 \text{ cm} \times 4 \text{ cm}$

$= 24 \text{ cm}^2$

(ii) Area of parallelogram with base AD = base AD × height FB

$= 5 \text{ cm} \times 4.8 \text{ cm}$

$= 24 \text{ cm}^2$

Note: Here, area of parallelogram with base AB is equal to the area of parallelogram with base AD.

∴ we conclude that the area of a parallelogram can be found choosing any of the side as its base with its corresponding height.

Try these

Find the relationship between the area of the parallelogram and the triangles using Fig. 4.31.

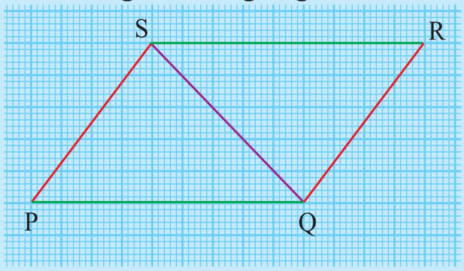


Fig. 4.31

Example 4.17

Find the area of a parallelogram whose base is 9 cm and the altitude (height) is 5 cm.

Solution

Given: $b = 9$ cm, $h = 5$ cm

Area of the parallelogram $= b \times h$

$= 9 \text{ cm} \times 5 \text{ cm}$

∴ Area of the parallelogram $= 45 \text{ cm}^2$

Example 4.18

Find the height of a parallelogram whose area is 480 cm^2 and base is 24 cm.

Solution

Given: Area = 480 cm^2, base b = 24 cm

Area of the parallelogram = 480

$$b \times h = 480$$

$$24 \times h = 480$$

$$h = \frac{480}{24} = 20 \text{ cm}$$

$\therefore$ height of a parallelogram = 20 cm.

Example 4.19

The area of the parallelogram is 56 cm^2. Find the base if its height is 7 cm.

Solution

Given: Area = 56 cm^2, height h = 7 cm

Area of the parallelogram = 56

$$b \times h = 56$$

$$b \times 7 = 56$$

$$b = \frac{56}{7} = 8 \text{ cm.}$$

$\therefore$ base of a parallelogram = 8 cm.

Example 4.20

Two sides of the parallelogram PQRS are 9 cm and 5 cm. The height corresponding to the base PQ is 4 cm (see figure). Find

(i) area of the parallelogram

(ii) the height corresponding to the base PS

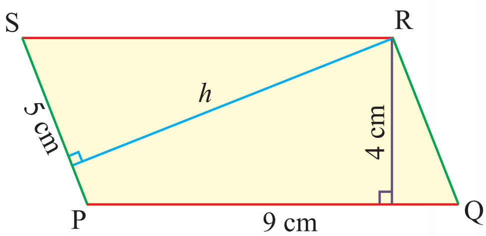


Fig. 4.32

Solution

(i) Area of the parallelogram $= b \times h$

$= 9 \text{ cm} \times 4 \text{ cm}$

$= 36 \text{ cm}^2$

(ii) If the base PS (b) = 5 cm, then

$$\text{Area} = 36$$

$$b \times h = 36$$

$$5 \times h = 36$$

$$h = \frac{36}{5} = 7.2 \text{ cm.}$$

$\therefore$ height corresponding to the base PS is 7.2 cm.

Think and Discuss:

- Draw different parallelograms with equal perimeters.
- Can you say that they have same area?

Excercise 4.4

1. Choose the correct answer.

i) The height of a parallelogram whose area is 300 cm^2 and base 15 cm is

(A) 10 cm (B) 15 cm (C) 20 cm (D) 30 cm

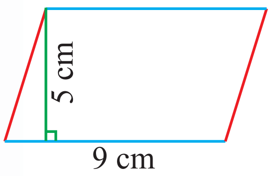
ii) The base of a parallelogram whose area is 800 cm^2 and the height 20 cm is

(A) 20 cm (B) 30 cm (C) 40 cm (D) 50 cm

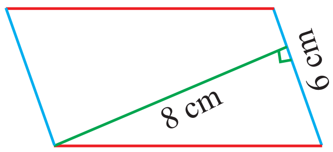
iii) The area of a parallelogram whose base is 20 cm and height is 30 cm is

(A) 300 cm^2 (B) 400 cm^2 (C) 500 cm^2 (D) 600 cm^2

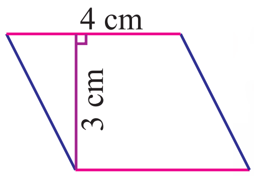
2. Find the area of each of the following parallelograms:



(i)



(ii)



(iii)

3. Find the area of the parallelogram whose base and height are :
   (i) $b = 14$ cm, $h = 18$ cm
   (ii) $b = 15$ cm, $h = 12$ cm
   (iii) $b = 23$ cm, $h = 10.5$ cm
   (iv) $b = 8.3$ cm, $h = 7$ cm
4. One of the sides and the corresponding height of a parallelogram are 14 cm and 8 cm respectively. Find the area of the parallelogram.
5. A ground is in the form of a parallelogram. Its base is 324 m and its height is 75 m. Find the area of the ground.
6. Find the height of the parallelogram which has an area of 324 sq. cm. and a base of 27 cm.

4.6 Rhombus

In a parallelogram if all the sides are equal then it is called rhombus.

Let the base of the rhombus be b units and its corresponding height be h units.

Since a rhombus is also a parallelogram we can use the same formula to find the area of the rhombus.

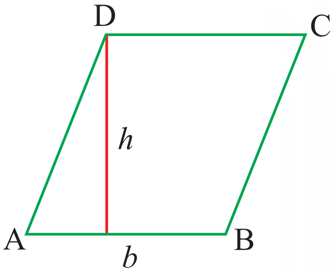


Fig. 4.33

$\therefore$ The area of the rhombus = $b \times h$ sq. units.

Do you know?

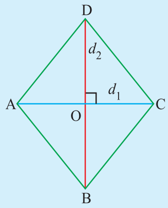
In a rhombus,

(i) all the sides are equal

(ii) opposite sides are parallel

(iii) diagonal divides the rhombus into two triangles of equal area.

(iv) the diagonal bisect each other at right angles.



Area of the rhombus in terms of its diagonals

In a rhombus ABCD , AB ∥ DC and BC ∥ AD

Also, AB = BC = CD = DA

Let the diagonals be d_1 (AC) and d_2 (BD)

Since, the diagonals bisect each other at right angles

AC ⊥ BD and BD ⊥ AC

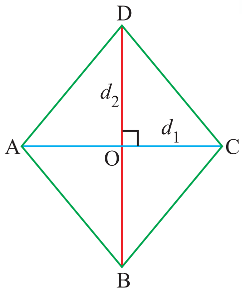


Fig. 4.34

Area of the rhombus ABCD

$$= \text{Area of } \Delta\ ABC + \text{Area of } \Delta\, ADC$$

$$= \left[\frac{1}{2} \times AC \times OB\right] + \left[\frac{1}{2} \times AC \times OD\right]$$

$$= \frac{1}{2} \times AC \times (OB + OD)$$

$$= \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times d_1 \times d_2 \text{ sq. units}$$

$\therefore$ Area of the rhombus $= \frac{1}{2}[d_1 \times d_2]$ sq. units

$= \frac{1}{2} \times$ (product of diagonals) sq. units

Think and Discuss

Square is a rhombus but a rhombus is not a square.

Example 4.21

Find the area of a rhombus whose side is 15 cm and the altitude (height) is 10cm.

Solution

Given: base = 15 cm, height = 10 cm

$$\text{Area of the rhombus} = \text{base} \times \text{height}$$
$$= 15 \text{ cm} \times 10 \text{ cm}$$
$$\therefore \text{Area of the rhombus} = 150 \text{ cm}^2$$

Example 4.22

A flower garden is in the shape of a rhombus. The length of its diagonals are 18 m and 25 m. Find the area of the flower garden.

Solution

Given: $d_1 = 18$ m, $d_2 = 25$ m

$$\text{Area of the rhombus} = \frac{1}{2} \times d_1 \times d_2$$
$$= \frac{1}{2} \times 18 \times 25$$
$$\therefore \text{Area of the flower garden} = 225 \text{ m}^2$$

Example 4.23

Area of a rhombus is 150 sq. cm. One of its diagonal is 20 cm. Find the length of the other diagonal.

Solution

Given: Area = 150 sq. cm, diagonal $d_1 = 20$ cm

$$\text{Area of the rhombus} = 150$$
$$\frac{1}{2} \times d_1 \times d_2 = 150$$
$$\frac{1}{2} \times 20 \times d_2 = 150$$
$$10 \times d_2 = 150$$
$$d_2 = 15 \text{ cm}$$

$\therefore$ The length of the other diagonal = 15 cm.

Example 4.24

A field is in the form of a rhombus. The diagonals of the fields are 50 m and 60 m. Find the cost of levelling it at the rate of ₹2 per sq. m.

Solution

Given: $d_1 = 50$ m, $d_2 = 60$ m

$$\text{Area} = \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 50 \times 60 \text{ sq. m}$$

$$= 1500 \text{ sq. m}$$

Cost of levelling 1 sq. m = ₹2

∴ cost of levelling 1500 sq. m = ₹2 × 1500

= ₹3000

Try these

Take a rectangular sheet. Mark the midpoints of the sides and join them as shown in the Fig. 4.35.

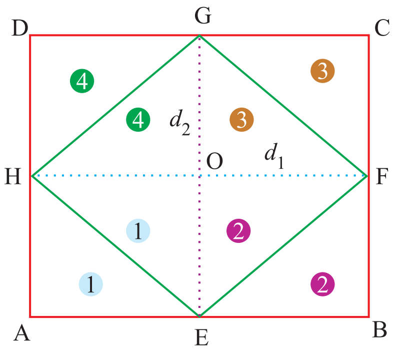


Fig. 4.35

The shaded figure EFGH is a rhombus. Cut the light shaded triangles and join them to form a rhombus. The new rhombus is identical to the original rhombus EFGH see Fig.4.36.

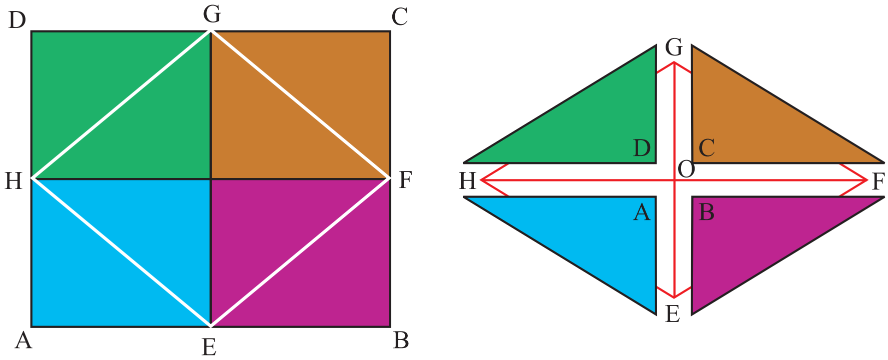


Fig. 4.36

∴ The area of rectangle = Twice the area of rhombus

Area of a rhombus $= \frac{1}{2}$ [area of rectangle]

$$= \frac{1}{2}[AB \times BC]$$

$$= \frac{1}{2}[HF \times EG] \quad [\text{ see Fig. 4.35 }]$$

Area of a rhombus $= \frac{1}{2}(d_1 \times d_2)$ sq. units.

Exercise 4.5

1. Choose the correct answer.

i) The area of a rhombus

(A) $d_1 \times d_2$ (B) $\frac{3}{4}(d_1 \times d_2)$ (C) $\frac{1}{2}(d_1 \times d_2)$ (D) $\frac{1}{4}(d_1 \times d_2)$

ii) The diagonals of a rhombus bisect each other at

(A) 30° (B) 45° (C) 60° (D) 90°

iii) The area of a rhombus whose diagonals are 10 cm and 12 cm is

(A) 30 cm^2 (B) 60 cm^2 (C) 120 cm^2 (D) 240 cm^2

2. Find the area of rhombus whose diagonals are

i) 15 cm, 12 cm ii) 13 cm, 18.2 cm
iii) 74 cm, 14.5 cm iv) 20 cm, 12 cm

3. One side of a rhombus is 8 cm and the altitude (height) is 12 cm. Find the area of the rhombus.

4. Area of a rhombus is 4000 sq. m. The length of one diagonal is100 m. Find the other diagonal.

5. A field is in the form of a rhombus. The diagonals of the field are 70 m and 80 m. Find the cost of levelling it at the rate of ₹3 per sq. m.

4.7 Trapezium

A trapezium is a quadrilateral with one pair of opposite sides are parallel.

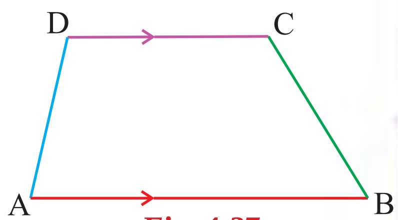


Fig. 4.37

The distance between the parallel sides is the height of the trapezium. Here the sides AD and BC are not parallel, but AB || DC.

If the non - parallel sides of a trapezium are equal (AD = BC), then it is known as an isosceles trapezium.

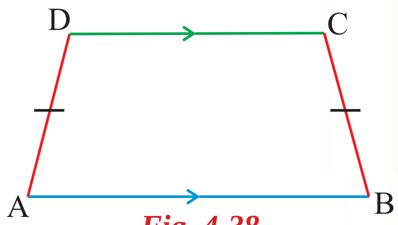


Fig. 4.38

Here $\angle A = \angle B$; $\angle C = \angle D$

$AC = BD$

$\angle A + \angle D = 180°$; $\angle B + \angle C = 180°$

Area of a trapezium

ABCD is a trapezium with parallel sides AB and DC measuring '*a*' and '*b*'. Let the distance between the two parallel sides be '*h*'. The diagonal BD divides the trapezium into two triangles ABD and BCD.

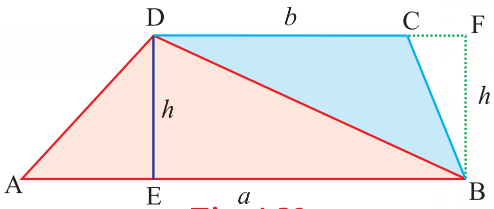


Fig. 4.39

Area of the trapezium

$= \text{area of } \Delta ABD + \text{area of } \Delta BCD$

$= \frac{1}{2} \times AB \times h + \frac{1}{2} \times DC \times h$

$= \frac{1}{2} \times h[AB + DC]$

$= \frac{1}{2} \times h[a + b]$ sq. units

$\therefore$ Area of a trapezium = $\frac{1}{2} \times$ height $\times$ (sum of the parallel sides) sq. units

Example 4.25

Find the area of the trapezium whose height is 10 cm and the parallel sides are 12 cm and 8 cm of length.

Solution

Given: h = 10 cm, a = 12 cm, b = 8 cm

$$\text{Area of a trapezium} = \frac{1}{2} \times h(a+b)$$
$$= \frac{1}{2} \times 10 \times (12 + 8) = 5 \times (20)$$
$$\therefore \text{Area of the trapezium} = 100 \text{ sq. cm}^2$$

Example 4.26

The length of the two parallel sides of a trapezium are 15 cm and 10 cm. If its area is 100 sq. cm. Find the distance between the parallel sides.

Solution

Given: a = 15 cm, b = 10 cm, Area = 100 sq. cm.

$$\text{Area of the trapezium} = 100$$
$$\frac{1}{2}h(a+b) = 100$$
$$\frac{1}{2} \times h \times (15 + 10) = 100$$
$$h \times 25 = 200$$
$$h = \frac{200}{25} = 8$$

$\therefore$ the distance between the parallel sides = 8 cm.

Example 4.27

The area of a trapezium is 102 sq. cm and its height is 12 cm. If one of its parallel sides is 8 cm. Find the length of the other side.

Solution

Given: Area = 102 cm^2, h = 12 cm, a = 8 cm.

$$\text{Area of a trapezium} = 102$$

$$\frac{1}{2}h(a+b) = 102$$

$$\frac{1}{2}\times 12\times(8+b) = 102$$

$$6\,(8+b) = 102$$

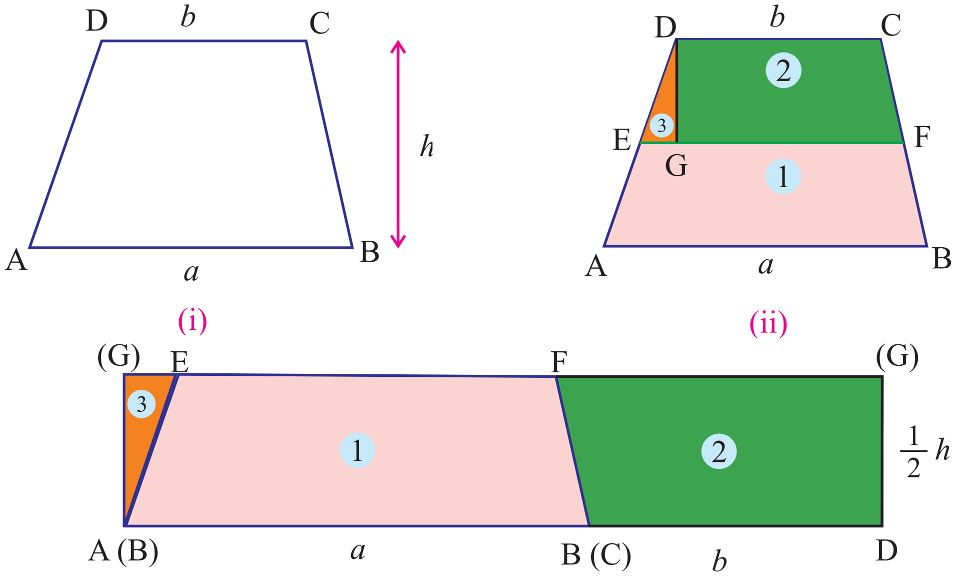
$$8+b = 17 \quad\Rightarrow\quad b = 17-8 = 9$$

$$\therefore \text{ length of the other side} = 9 \text{ cm}$$

Try these

By paper folding method:

In a chart paper draw a trapezium ABCD of any measure. Cut and take the trapezium separately. Fold the trapezium in such a way that DC lies on AB and crease it on the middle to get EF.



(iii)

Fig. 4.40

EF divides the trapezium in to two parts as shown in the Fig. 4.40 (ii)

From D draw DG $\perp$ EF. Cut the three parts separately.

Arrange three parts as shown in the Fig. 4.40 (iii)

The figure obtained is a rectangle whose length is AB + CD = $a + b$

and breadth is $\frac{1}{2}$(height of trapezium) = $\frac{1}{2}h$

$$\therefore \text{ Area of trapezium} = \text{area of rectangle as shown in Fig. 4.40 (iii)}$$

$$= \text{length} \times \text{breadth}$$

$$= (a+b)\left(\frac{1}{2}h\right)$$

$$= \frac{1}{2}h(a+b) \text{ sq. units}$$

Excercise 4.6

1. Choose the correct answer.

i) The area of trapezium is ____________ sq. units

(A) $h(a+b)$ (B) $\frac{1}{2}$ h (a + b) (C) $h(a-b)$ (D) $\frac{1}{2}$ h $(a-b)$

ii) In an isosceles trapezium

(A) non parallel sides are equal (B) parallel sides are equal

(C) height = base (D) parallel side = non parallel side

iii) The sum of parallel sides of a trapezium is 18 cm and height is 15 cm. Then its area is

(A) 105 cm^2 (B) 115 cm^2 (C) 125 cm^2 (D) 135 cm^2

iv) The height of a trapezium whose sum of parallel sides is 20 cm and the area 80 cm^2 is

(A) 2 cm (B) 4 cm (C) 6 cm (D) 8 cm

2. Find the area of a trapezium whose altitudes and parallel sides are given below:
   i) altitude = 10 cm, parallel sides = 4 cm and 6 cm
   ii) altitude = 11 cm, parallel sides = 7.5 cm and 4.5 cm
   iii) altitude = 14 cm, parallel sides = 8 cm and 3.5 cm

3. The area of a trapezium is 88 cm^2 and its height is 8 cm. If one of its parallel side is 10 cm. Find the length of the other side.

4. A garden is in the form of a trapezium. The parallel sides are 40 m and 30 m. The perpendicular distance between the parallel side is 25 m. Find the area of the garden.

5. Area of a trapezium is 960 cm^2. The parallel sides are 40 cm and 60 cm. Find the distance between the parallel sides.

4.8 Circle

In our daily life, we come across a number of objects like wheels, coins, rings, bangles,giant wheel, compact disc (C.D.)

What is the shape of the above said objects?

'round', 'round', 'round'

Yes, it is round. In Mathematics it is called a circle. Now, let us try to draw a circle.

Take a thread of any length and fix one end tightly at a point O as shown in the figure. Tie a pencil (or a chalk) to the other end and stretch the thread completely to a point A.

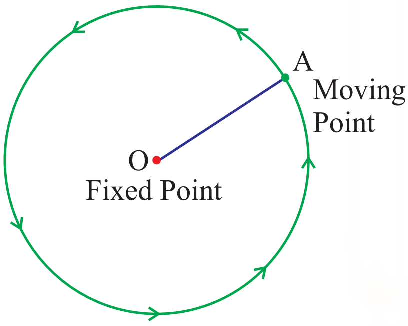


Fig. 4.41

Holding the thread stretched tightly, move the pencil. Stop it when the pencil again reaches the point A. Now see the path traced by the pencil.

Is the path traced by the pencil a circle or a straight line?

'Circle'

Yes, the path traced by the point, which moves at a constant distance from a fixed point on a given plane surface is called a circle.

Parts of a Circle

The fixed point is called the centre of the circle.

The constant distance between the fixed point and the moving point is called the radius of the circle.

i.e. The radius is a line segment with one end point at the centre and the other end on the circle. It is denoted by 'r'.

A line segment joining any two points on the circle is called a chord.

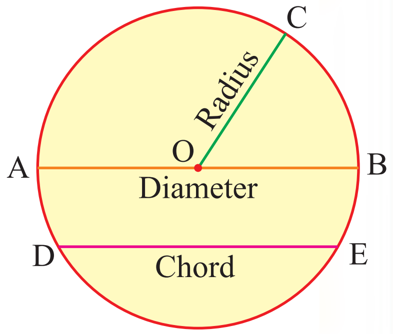


Fig. 4.42

Diameter is a chord passing through the centre of the circle. It is denoted by 'd'.

The diameter is the longest chord. It is twice the radius.(i.e. $d = 2r$)

The diameter divides the circle into two equal parts. Each equal part is a semicircle.

> **Think it:**
>
> How many diameters can a circle have ?

> **Do you know?**
>
> The plural of radius is "radii".
> All the radii of a circle are equal.

Circumference of a circle:

Can you find the distance covered by an athlete if he takes two rounds on a circular track.

Fig. 4.43

Since it is a circular track, we cannot use the ruler to find out the distance.

So, what can we do ?

Take a one rupee coin.Place it on a paper and draw its outline. Remove the coin. Mark a point A on the outline as shown in the Fig. 4.44

Take a thread and fix one end at A. Now place the thread in such a way that the thread coincides exactly with the outline. Cut the other end of the thread when it reaches the point A.

Length of the thread is nothing but the circumference of the coin.

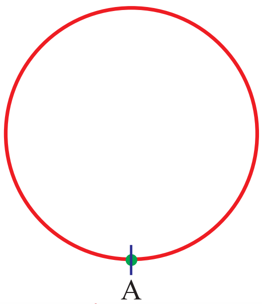


Fig. 4.44

So,

the distance around a circle is called the circumference of the circle, which is denoted by 'C'. i.e., The perimeter of a circle is known as its circumference.

Take a bottle cap or a bangle or any other circular objects and find the circumference. If possible find the relation between the circumference and the diameter of the circular objects.

Relation between diameter and circumference of the circle

Draw four circles with radii 3.5 cm, 7 cm, 5 cm, 10.5 cm in your note book. Measure their circumferences using a thread and the diameter using a ruler as shown in the Fig. 4.45 given below.

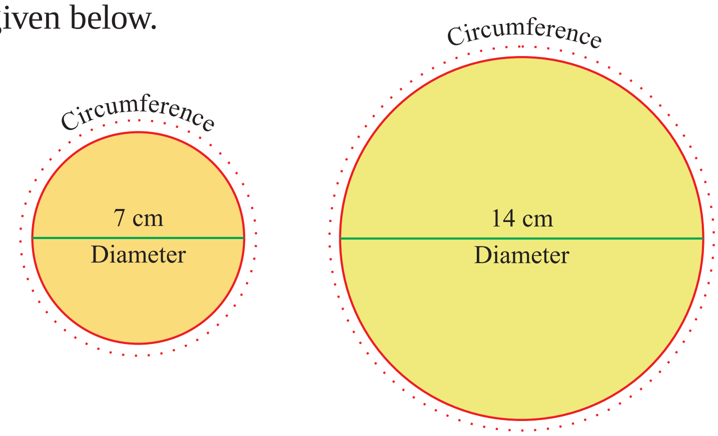


Fig. 4.45

Fill up the missing values in table 4.1 and find the ratio of the circumference to the diameter.

Circle	Radius	Diameter (d)	Circumference (C)	Ratio $\left(\frac{C}{d}\right)$
1	3.5 cm	7 cm	22 cm	$\frac{22}{7} = 3.14$
2	7 cm	14 cm	44 cm	$\frac{44}{14} = \frac{22}{7} = 3.14$
3	5 cm	10 cm	----	----
4	10.5 cm	21 cm	----	----

Table 4.1

What do you infer from the above table?. Is this ratio $\left(\frac{C}{d}\right)$ approximately the same?

Yes !

$$\frac{C}{d} = 3.14 \quad \Rightarrow \quad C = (3.14)d$$

So, can you say that the circumference of a circle is always more than 3 times its diameter ?

Yes !

In all the cases, the radio $\frac{C}{d}$ is a constant and is denoted by the Greek letter π (read as 'pi'). Its approximate value is $\frac{22}{7}$ or 3.14.

so, $\frac{C}{d} = \pi \quad \Rightarrow \quad C = \pi d$ units where d is the diameter of a circle.

We know that the diameter of a circle is twice the radius r. i.e., $d = 2r$.

from the above formula, $C = \pi d = \pi(2r) \Rightarrow$ $C = 2\pi r$ units.

Do you know?

The value of π is calculated by many mathematicians.

Babylonians : $\pi = 3$

Greeks : $\pi = \frac{22}{7}$ or 3.14

Archemides : $3\frac{1}{7} < \pi < 3\frac{10}{71}$

Aryabhata : $\pi = \frac{62838}{2000}$ (or) 3.1416

Now, we use $\pi = \frac{22}{7}$ or 3.14

Example 4.28

Find the circumference of a circle whose diameter is 21 cm.

Solution

$$\begin{aligned} \text{Circumference of a circle} &= \pi d \\ &= \frac{22}{7} \times 21 \qquad \text{Here } \pi = \frac{22}{7} \\ &= 66 \text{ cm.} \end{aligned}$$

Example 4.29

Find the circumference of a circle whose radius is 3.5 m.

Solution

$$\begin{aligned} \text{Circumference of a circle} &= 2\pi r \\ &= 2 \times \frac{22}{7} \times 3.5 \\ &= 2 \times 22 \times 0.5 \\ &= 22 \text{ m} \end{aligned}$$

Example 4.30

A wire of length 88 cm is bent as a circle. What is the radius of the circle.

Solution

$$\text{Length of the wire} = 88 \text{ cm}$$
$$\text{Circumference of the circle} = \text{Length of the wire}$$
$$2\pi r = 88$$
$$2 \times \frac{22}{7} \times r = 88$$
$$r = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

$\therefore$ radius of a circle is 14 cm.

Example 4.31

The diameter of a bicycle wheel is 63 cm. How much distance will it cover in 20 revolutions?

Solution

When a wheel makes one complete revolutions,

$$\text{Distance covered in one rotation} = \text{Circumference of wheel}$$
$$\therefore \text{circumference of the wheel} = \pi d \text{ units}$$
$$= \frac{22}{7} \times 63 \text{ cm}$$
$$= 198 \text{ cm}$$

For one revolution, the distance covered = 198 cm

$\therefore$ for 20 revolutions, the distance covered = 20 × 198 cm

= 3960 cm

= 39 m 60 cm [100 cm = 1 m]

Example 4.32

A scooter wheel makes 50 revolutions to cover a distance of 8800 cm. Find the radius of the wheel.

Solution

$$\text{Distance travelled} = \text{Number of revolutions} \times \text{Circumference}$$
$$\text{Circumference} = \frac{\text{Distance travelled}}{\text{Number of revolutions}}$$
$$2\pi r = \frac{8800}{50}$$
$$\text{i.e., } 2\pi r = 176$$

$$2 \times \frac{22}{7} \times r = 176$$

$$r = \frac{176 \times 7}{2 \times 22}$$

$$r = 28 \text{ cm}$$

$\therefore$ radius of the wheel = 28 cm.

Example 4.33

The radius of a cart wheel is 70 cm. How many revolution does it make in travelling a distance of 132 m.

Solution

Given: r = 70 cm, Distance travelled = 132 m.

$\therefore$ Circumference of a cart wheel = $2\pi r$

$$= 2 \times \frac{22}{7} \times 70$$

$$= 440 \text{ cm}$$

Distance travelled = Number of revolutions $\times$ Circumference

$$\therefore \text{Number of revolutions} = \frac{\text{Distance travelled}}{\text{Circumference}}$$

$$= \frac{132 \text{ m}}{440 \text{ cm}}$$

$$= \frac{13200 \text{ cm}}{440 \text{ cm}} \quad (1 \text{ m} = 100 \text{ cm}, 132 \text{ m} = 13200 \text{ cm})$$

$$= 30$$

$\therefore$ Number of revolutions = 30.

Example 4.34

The circumference of a circular field is 44 m. A cow is tethered to a peg at the centre of the field. If the cow can graze the entire field, find the length of the rope used to tie the cow.

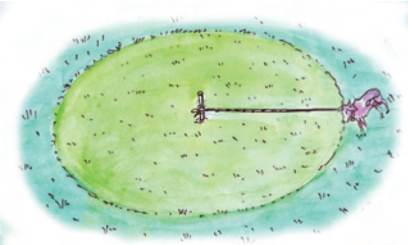


Fig. 4.46

Solution

Length of the rope = Radius of the circle

Circumference = 44 m (given)

i.e., $2\pi r = 44$

$$2 \times \frac{22}{7} \times r = 44$$

$$\therefore r = \frac{44 \times 7}{2 \times 22} = 7 \text{ m}$$

$\therefore$ The length of the rope used to tie the cow is 7 m.

Example 4.35

The radius of a circular flower garden is 56 m. What is the cost of fencing it at ₹10 a metre ?

Solution

Length to be fenced = Circumference of the circular flower garden

Circumference of the flower garden = $2\pi r$

$$= 2 \times \frac{22}{7} \times 56 = 352 \text{ m}$$

$\therefore$ Length of the fence = 352 m

Cost of fencing per metre = ₹10

$\therefore$ cost of fencing 352 m = ₹10 $\times$ 352

= ₹3520

$\therefore$ Total cost of fencing is ₹3520.

Example 4.36

The cost of fencing a circular park at the rate of ₹5 per metre is ₹1100. What is the radius of the park.

Solution

Cost of fencing = Circumference $\times$ Rate

$$\therefore \text{Circumference} = \frac{\text{Cost of fencing}}{\text{Rate}}$$

$$\text{i.e., } 2\pi r = \frac{1100}{5}$$

$$2\pi r = 220$$

$$\therefore 2 \times \frac{22}{7} \times r = 220$$

$$r = \frac{220 \times 7}{2 \times 22}$$

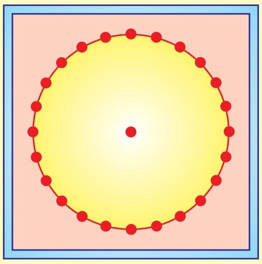
$$= 35 \text{ m}$$

$\therefore$ Radius of the park = 35 m.

Activity - Circular Geoboard

Take a square Board and draw a circle.

Fix nails on the circumference of the circle. (See fig)



Using rubber band, form various diameters, chords, radii and compare.

Excercise 4.7

1. Choose the correct answer:

i) The line segment that joins the centre of a circle to any point on the circle is called

(A) Diameter (B) Radius (C) Chord (D) None

ii) A line segment joining any two points on the circle is called

(A) Diameter (B) Radius (C) Chord (D) None

iii) A chord passing through the centre is called

(A) Diameter (B) Radius (C) Chord (D) None

iv) The diameter of a circle is 1 m then its radius is

(A) 100 cm (B) 50 cm (C) 20 cm (D) 10 cm

v) The circumference of a circle whose radius is 14 cm is

(A) 22 cm (B) 44 cm (C) 66 cm (D) 88 cm

2. Fill up the unknown in the following table:

	radius (r)	diameter (d)	circumference (c)
(i)	35 cm	-----	-----
(ii)	-----	56 cm	-----
(iii)	-----	-----	30.8 cm

3. Find the circumference of a circle whose diametre is given below:

(i) 35 cm (ii) 84 cm (iii) 119 cm (iv) 147 cm

4. Find the circumference of a circle whose radius is given below:

(i) 12.6 cm (ii) 63 cm (iii) 1.4 m (iv) 4.2 m

5. Find the radius of a circle whose circumference is given below:

(i) 110 cm (ii) 132 cm (iii) 4.4 m (iv) 11 m

6. The diameter of a cart wheel is 2.1 m. Find the distance travelled when it complets 100 revolutions.

7. The diameter of a circular park is 98 m. Find the cost of fencing it at ₹4 per metre.

8. A wheel makes 20 revolutions to cover a distance of 66 m. Find the diameter of the wheel.

9. The radius of a cycle wheel is 35 cm. How many revolutions does it make to cover a distance of 81.40 m?

Area of a circle

Consider the following

A farmer levels a circular field of radius 70 m. What will be the cost of levelling?

What will be the cost of polishing a circular table-top of radius 1.5 m ?

How will you find the cost ?

To find the cost what do you need to find actually?

Area or perimeter ?

Area, area, area

Do you know?

The region enclosed by the circumference of a circle is a circular region.

Yes. In such cases we need to find the area of the circular region.

So far, you have learnt to find the area of triangles and quadrilaterals that made up of straight lines. But, a cirlce is a plane figure made up of curved line different from other plane figures.

So, we have to find a new approach which will make the circle turn into a figure with straight lines.

Take a chart paper and draw a circle. Cut the circle and take it separately. Shade one half of the circular region. Now fold the entire circle into eight parts and cut along the folds (see Fig. 4.48).

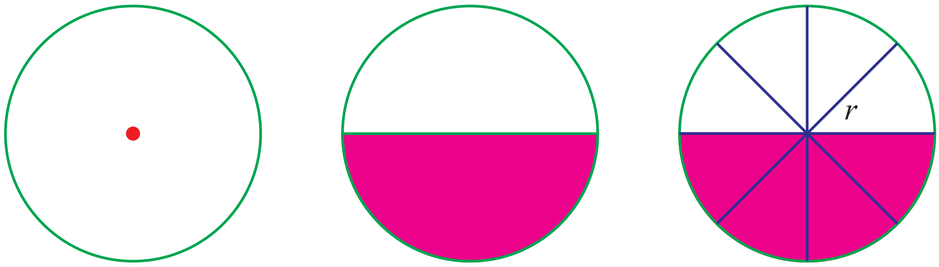


Fig. 4.48

Arrange the pieces as shown below.

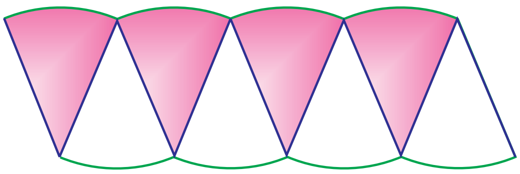
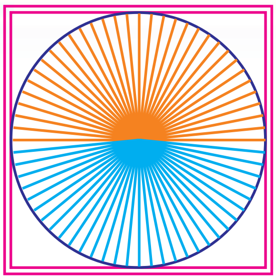


Fig. 4.49

What is the figure obtained?

These eight pieces roughly form a parallelogram.

Similarly, if we divide the circle into 64 equal parts and arrange these, it gives nearly a rectangle. (see Fig. 4.50)



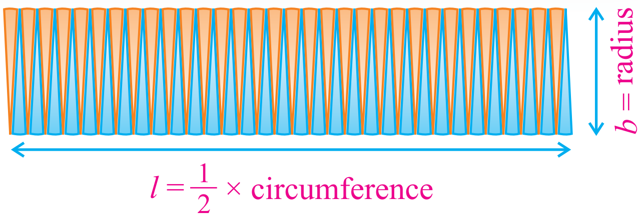


Fig. 4.50

What is the breadth of this rectangle?

The breadth of the rectangle is the radius of the circle.

i.e., breadth $b = r$ ..... (1)

What is the length of this rectangle ?

As the whole circle is divided into 64 equal parts and on each side we have 32 equal parts. Therefore, the length of the rectangle is the length of 32 equal parts, which is half of the circumference of a circle.

$$\therefore \text{length } l = \frac{1}{2}[\text{circumference of the circle}]$$
$$= \frac{1}{2}[2\pi r] = \pi r$$
$$\therefore l = \pi r \quad \text{..... (2)}$$

$$\text{Area of the circle} = \text{Area of the rectangle (from the Fig. 4.50)}$$
$$= l \times b$$
$$= (\pi r) \times r \quad \text{(from (1) and (2))}$$
$$= \pi r^2 \text{ sq. units.}$$

$$\therefore \text{Area of the circle} = \pi r^2 \text{ sq. units.}$$

Example 4.37

Find the area of a circle whose diameter is 14 cm

Solution

$$\text{Diameter } d = 14 \text{ cm}$$

So, $$\text{radius } r = \frac{d}{2} = \frac{14}{2} = 7 \text{ cm}$$

$$\text{Area of circle} = \pi r^2$$
$$= \frac{22}{7} \times 7 \times 7$$
$$= 154 \text{ sq. cm}$$

$$\therefore \text{Area of circle} = 154 \text{ sq. cm}$$

Example 4.38

A goat is tethered by a rope 3.5 m long. Find the maximum area that the goat can graze.

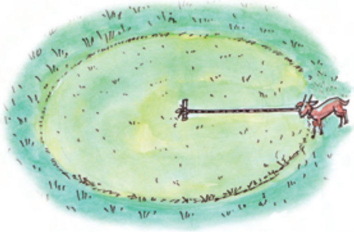


Fig. 4.51

Solution

Radius of the circle = Length of the rope

$\therefore$ radius $r = 3.5\text{ m} = \frac{7}{2}$ m

maximum area grazed by the goat = πr^2 sq. units.

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{77}{2} = 38.5 \text{ sq. m}$$

$\therefore$ maximum area grazed by the goat is 38.5 sq. m.

Example 4.39

The circumference of a circular park is 176 m. Find the area of the park.

Solution

$$\begin{aligned}
\text{Circumference} &= 176 \text{ m} \quad \text{(given)} \\
2\pi r &= 176 \\
2 \times \frac{22}{7} \times r &= 176 \\
r &= \frac{176 \times 7}{44} \\
\therefore r &= 28 \text{ m} \\
\text{Area of the park} &= \pi r^2 \\
&= \frac{22}{7} \times 28 \times 28 \\
&= 22 \times 4 \times 28 \\
&= 2464 \text{ sq. m.}
\end{aligned}$$

Example 4.40

A silver wire when bent in the form of a square encloses an area of 121 sq. cm. If the same wire is bent in the form of a circle. Find the area of the circle.

Solution

Let a be the side of the square

Area of the square = 121 sq. cm. (given)

$$a^2 = 121 \Rightarrow a = 11 \text{ cm} \qquad (11 \times 11 = 121)$$

$$\text{Perimeter of the square} = 4a \text{ units}$$
$$= 4 \times 11 \text{ cm}$$
$$= 44 \text{ cm}$$
$$\text{Length of the wire} = \text{Perimeter of the square}$$
$$= 44 \text{ cm}$$

The wire is bent in the form of a circle

The circumference of the circle = Length of the wire

$\therefore$ circumference of a circle = 44 cm

$$2\pi r = 44$$
$$\therefore 2 \times \frac{22}{7} \times r = 44$$
$$r = \frac{44 \times 7}{44}$$
$$r = 7 \text{ cm}$$
$$\therefore \text{Area of the circle} = \pi r^2$$
$$= \frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm}$$
$$\text{Area of the circle} = 154 \text{ cm}^2.$$

Example 4.41

When a man runs around circular plot of land 10 times, the distance covered by him is 352 m. Find the area of the plot.

Solution

$$\text{Distance covered in 10 times} = 352 \text{ m}$$
$$\text{Distance covered in one time} = \frac{352}{10} \text{ m} = 35.2 \text{ m}$$
$$\text{The circumference of the circular plot} = \text{Distance covered in one time}$$
$$\therefore \text{circumference} = 35.2 \text{ m}$$
$$2\pi r = 35.2$$
$$2 \times \frac{22}{7} \times r = 35.2$$
$$r = \frac{35.2 \times 7}{44}$$
$$= 0.8 \times 7$$
$$= 5.6 \text{ m}$$
$$\text{Area of the circular plot} = \pi r^2$$
$$= \frac{22}{7} \times 5.6 \times 5.6$$
$$= 22 \times 0.8 \times 5.6$$
$$= 98.56 \text{ m}^2$$
$$\therefore \text{Area of circular plot} = 98.56 \text{ m}^2$$

Example 4.42

A wire in the shape of a rectangle of length 37 cm and width 29 cm is reshaped in the form of a circle. Find the radius and area of the circle.

Solution

Length of the wire = perimeter of the rectangle
= 2 [length + breadth]
= 2 [37 cm + 29 cm] = 2 × 66 cm
= 132 cm.

Since wire is bent in the form of a circle,

The circumference of the circle = The length of the wire

$\therefore$ Circumference of a circle = 132

$$2\pi r = 132$$

$$2 \times \frac{22}{7} \times r = 132$$

$$r = \frac{132 \times 7}{44} = 21$$

$\therefore$ radius of the circle = 21 cm

$$\text{Area of the circle} = \pi r^2$$

$$= \frac{22}{7} \times 21 \times 21 = 22 \times 3 \times 21$$

$\therefore$ Area of the circle = 1386 sq. cm.

Exercise 4.8

1. Find the area of the circles whose diameters are given below:

   (i) 7 cm (ii) 10.5 cm (iii) 4.9 m (iv) 6.3 m (take $\pi = \frac{22}{7}$)
2. Find the area of the circles whose radii are given below:

   (i) 1.2 cm (ii) 14 cm (iii) 4.2 m (iv) 5.6 m (take $\pi = \frac{22}{7}$)
3. The diameter of a circular plot of ground is 28 m. Find the cost of levelling the ground at the rate of ₹3 per sq. m.
4. A goat is tied to a peg on a grass land with a rope 7 m long. Find the maximum area of the land it can graze.
5. A circle and a square each have a perimeter of 88 cm. Which has a larger area?
6. A wheel goes a distance of 2200 m in 100 revolutions. Find the area of the wheel.
7. A wire is in the form of a circle of radius 28 cm. Find the area that will enclose, if it is bent in the form of a square having its perimeter equal to the circumference of the cirlce.
8. The area of circular plot is 3850 m^2. Find the radius of the plot. Find the cost of fencing the plot at ₹10 per metre.

4.9 Area of the path way

In our day - to - day life we go for a walk in a park, or in a play ground or even around a swimming pool.

Can you represent the path way of a park diagrammatically ?

Have you ever wondered if it is possible to find the area of such paths ?

Can the path around the rectangular pool be related to the mount around the photo in a photo frame ?

Can you think of some more examples?

In this section we will learn to find

- Area of rectangular pathway
- Area of circular pathway

Area of rectangular pathway

(a) Area of uniform pathway outside the rectangle

Consider a rectangular building. A uniform flower garden is to be laid outside the building. How do we find the area of the flower garden?

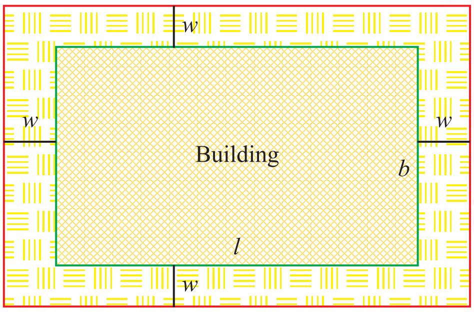


Fig. 4.52

The uniform flower garden including the building is also a rectangle in shape. Let us call it as outer rectangle. We call the building as inner rectangle.

Let l and b be the length and breadth of the building.

$\therefore$ Area of the inner rectangle = $l\,b$ sq. units.

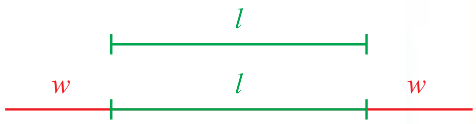


Fig. 4.53

Let w be the width of the flower garden.

What is the length and breadth of the outer rectangle ?

$$\text{The length of the outer rectangle (L)} = w + l + w = (l + 2w) \text{ units}$$

$$\text{The breadth of the outer rectangle (B)} = w + b + w = (b + 2w) \text{ units}$$

$$\therefore \text{ area of the outer rectangle} = \text{L} \times \text{B}$$

$$= (l + 2w)(b + 2w) \text{ sq. units}$$

Now, what is the area of the flower garden ?

Actually, the area of the flower garden is the pathway bounded between two rectangles.

$\therefore$ Area of the flower garden = (Area of building and flower garden) – (Area of building)

Generally,

Area of the pathway = (Area of outer rectangle) – (Area of inner rectangle)

i.e. Area of the pathway = $(l + 2w)(b + 2w) - lb$.

Example 4.43

The area of outer rectangle is 360 m^2. The area of inner rectangle is 280 m^2. The two rectangles have uniform pathway between them. What is the area of the pathway?

Solution

Area of the pathway = (Area of outer rectangle) – (Area of inner rectangle)

= $(360 - 280)\ m^2$

= $80\ m^2$

$\therefore$ Area of the pathway = $80\ m^2$

Example 4.44

The length of a building is 20 m and its breadth is 10 m. A path of width 1 m is made all around the building outside. Find the area of the path.

Solution

Inner rectangle (given)	Outer rectangle
l = 20 m b = 10 m Area = $l \times b$ Area = 20 m × 10 m = 200 m^2	width, w = 1 m L = $l + 2w$ = 20 + 2 = 22 m B = $b + 2w$ = 10 + 2 = 12 m Area = $(l + 2w)(b + 2w)$ Area = 22 m × 12 m = 264 m^2

Area of the path = (Area of outer rectangle) – (Area of inner rectangle)

= $(264 - 200)\ m^2 = 64\ m^2$

$\therefore$ Area of the path = $64\ m^2$

Example 4.45

A school auditorium is 45 m long and 27 m wide. This auditorium is surrounded by a varandha of width 3 m on its outside. Find the area of the varandha. Also, find the cost of laying the tiles in the varandha at the rate of ₹100 per sq. m.

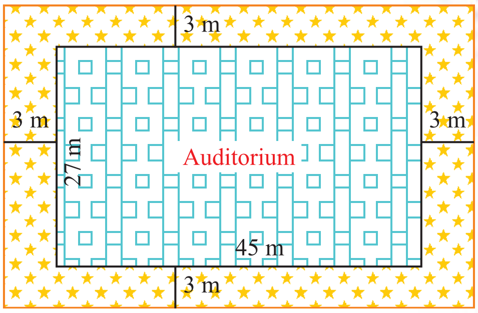


Fig. 4.54

Solution

Inner (given) rectangle	Outer rectangle
l = 45 m b = 27 m Area = 45m × 27 m = 1215 m^2	Width, w = 3 m L = $l + 2w$ = 45 + 6 = 51 m B = $b + 2w$ = 27 + 6 = 33 m Area = 51m × 33 m = 1683 m^2

(i) Area of the verandha = (Area of outer rectangle) – (Area of inner rectangle)

= (1683 – 1215) m^2

= 468 m^2

∴ Area of the verandha = 468 m^2 (or) 468 sq. m.

(ii) Cost of laying tiles for 1 sq. m = ₹100

Cost of laying tiles for 468 sq. m = ₹100 × 468

= ₹46,800

∴Cost of laying tiles in the verandha = ₹46,800

(b) Area of uniform pathway inside a rectangle

A swimming pool is built in the middle of a rectangular ground leaving an uniform width all around it to maintain the lawn.

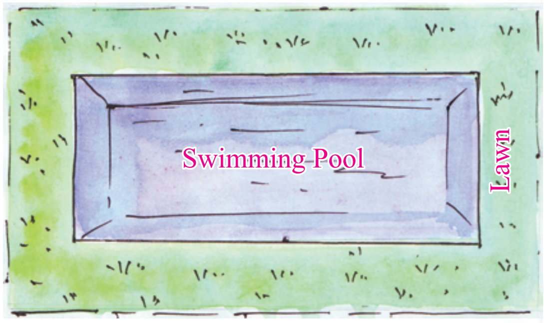


Fig. 4.55

If the pathway outside the pool is to be grassed, how can you find its cost ?

If the area of the pathway and cost of grassing per sq. unit is known, then the cost of grassing the pathway can be found.

Here, the rectangular ground is the outer rectangle where l and b are length and breadth.

$\therefore$ Area of the ground (outer rectangle) = $l\,b$ sq. units

If w be the width of the pathway (lawn), what will be the length and breath of the swimming pool ?

The length of the swimming pool $= l - w - w$

$= l - 2w$

The breadth of the swimming pool $= b - w - w$

$= b - 2w$

$\therefore$ Area of the swimming pool (inner rectangle) = $(l - 2w)(b - 2w)$ Sq. units

Area of the lawn = Area of the ground – Area of the swimming pool.

Generally,

Area of the pathway = (Area of outer rectangle) – (Area of inner rectangle)

$= lb - (l - 2w)(b - 2w)$

Example 4.46

The length and breadth of a room are 8 m and 5 m respectively. A red colour border of uniform width of 0.5 m has been painted all around on its inside. Find the area of the border.

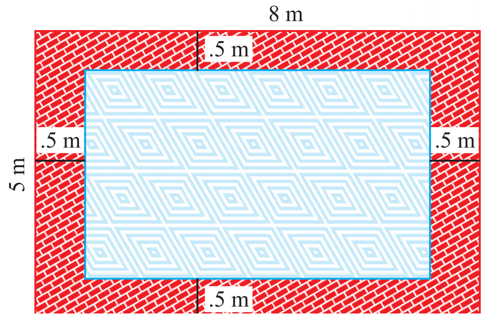


Fig. 4.56

Solution

Outer (given)rectangle	Inner rectangle
l = 8 m b = 5 m Area = 8m × 5 m = 40 m^2	width, w = 0.5 m L = $l - 2w$ = (8 – 1) m = 7 m B = $b - 2w$ = (5 – 1) m = 4 m Area = 7m × 4 m = 28 m^2

Area of the path = (Area of outer rectangle) – (Area of inner rectangle)

$= (40 - 28)\ m^2$

$= 12\ m^2$

$\therefore$ Area of the border painted with red colour = 12 m^2

Example 4.47

A carpet measures 3 m $\times$ 2 m. A strip of 0.25 m wide is cut off from it on all sides. Find the area of the remaining carpet and also find the area of strip cut out.

Solution

Outer rectangle carpet before cutting the strip	Inner rectangle carpet after cutting the strip
l = 3 m b = 2 m Area = 3m $\times$ 2m = 6 m^2	width, w = 0.25 m L = $l - 2w$ = (3 – 0.5) m = 2.5 m B = $b - 2w$ = (2 – 0.5) m = 1.5 m Area = 2.5m $\times$ 1.5m = 3.75 m^2

The area of the carpet after cutting the strip = 3.75 m^2

Area of the strip cut out = (Area of the carpet) – (Area of the remaining part)

= (6 – 3.75) m^2

= 2.25 m^2

$\therefore$ Area of the strip cut out = 2.25 m^2

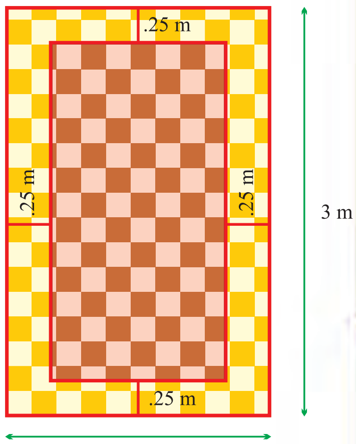


Fig. 4.57

Note: If the length and breadth of the inner rectangle is given, then the length and breadth of the outer rectangle is $l + 2w$, $b + 2w$ respectively where w is the width of the path way.

Suppose the length and breadth of the outer rectangle is given, then the length and breadth of the inner rectangle is $l - 2w$, $b - 2w$ respectively.

Exercise 4.9

1. A play ground 60 m $\times$ 40 m is extended on all sides by 3 m. What is the extended area.

2. A school play ground is rectangular in shape with length 80 m and breadth 60 m. A cemented pathway running all around it on its outside of width 2 m is built. Find the cost of cementing if the rate of cementing 1 sq. m is ₹20.

3. A garden is in the form of a rectangle of dimension 30 m $\times$ 20 m. A path of width 1.5 m is laid all around the garden on the outside at the rate of ₹10 per sq. m. What is the total expense.

4. A picture is painted on a card board 50 cm long and 30 cm wide such that there is a margin of 2.5 cm along each of its sides. Find the total area of the margin.
5. A rectangular hall has 10 m long and 7 m broad. A carpet is spread in the centre leaving a margin of 1 m near the walls. Find the area of the carpet. Also find the area of the un covered floor.
6. The outer length and breadth of a photo frame is 80 cm , 50 cm. If the width of the frame is 3 cm all around the photo. What is the area of the picture that will be visible?

Circular pathway

Concentric circles

Circles drawn in a plane with a common centre and different radii are called concentric circles.

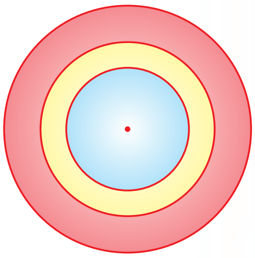


Fig. 4.58

Circular pathway

A track of uniform width is laid around a circular park for walking purpose.

Can you find the area of this track ?

Yes. Area of the track is the area bounded between two concentric circles. In Fig. 4.59, O is the common centre of the two circles.Let the radius of the outer circle be R and inner circle be *r*.

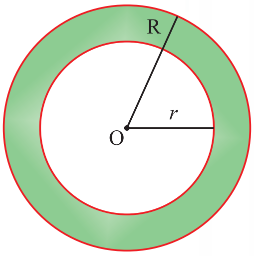


Fig. 4.59

The shaded portion is known as the circular ring or the circular pathway. i.e. a circular pathway is the portion bounded between two concentric circles.

width of the pathway, $w = R - r$ units

i.e., $w = R - r \Rightarrow R = w + r$ units

$r = R - w$ units.

The area of the circular path = (area of the outer circle) – (area of the inner circle)

$= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$ sq. units

$\therefore$ The area of the circular path $= \pi(R^2 - r^2)$ sq. units

$= \pi(R + r)(R - r)$ sq. units

Example 4.48

The adjoining figure shows two concentric circles. The radius of the larger circle is 14 cm and the smaller circle is 7 cm. Find

(i) The area of the larger circle.

(ii) The area of the smaller circle.

(iii) The area of the shaded region between two circles.

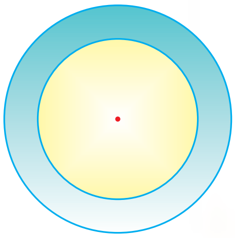


Fig. 4.60

Solution

i) Larger circle	ii) Smaller circle
$R = 14$	$r = 7$
area $= \pi R^2$	area $= \pi r^2$
$= \frac{22}{7} \times 14 \times 14$	$= \frac{22}{7} \times 7 \times 7$
$= 22 \times 28$	$= 22 \times 7$
$= 616 \text{ cm}^2$	$= 154 \text{ cm}^2$

iii) The area of the shaded region

= (Area of larger circle) – (Area of smaller circle)

$= (616 - 154) \text{ cm}^2$

$= 462 \text{ cm}^2$

Example 4.49

From a circular sheet of radius 5 cm, a concentric circle of radius 3 cm is removed. Find the area of the remaining sheet ? (Take $\pi = 3.14$)

Solution

Given: R = 5 cm, r = 3 cm

Area of the remaining sheet $= \pi(R^2 - r^2)$

$= 3.14\,(5^2 - 3^2)$

$= 3.14\,(25 - 9)$

$= 3.14 \times 16$

$= 50.24 \text{ cm}^2$

Aliter:

Outer circle	Inner circle
R $= 5$ cm	r $= 3$ cm
Area $= \pi R^2$ sq. units	Area $= \pi r^2$ sq. units
$= 3.14 \times 5 \times 5$	$= 3.14 \times 3 \times 3$
$= 3.14 \times 25$	$= 3.14 \times 9$
$= 78.5 \text{ cm}^2$	$= 28.26 \text{ cm}^2$

Area of the remaining sheet = (Area of outer circle) – (Area of inner circle)

$= (78.5 - 28.26) \text{ cm}^2$

$= 50.24 \text{ cm}^2$

$\therefore$ Area of the remaining sheet $= 50.24 \text{ cm}^2$

Example 4.50

A circular flower garden has an area 500 m^2. A sprinkler at the centre of the garden can cover an area that has a radius of 12 m. will the sprinkler water the entire garden (Take π = 3.14)

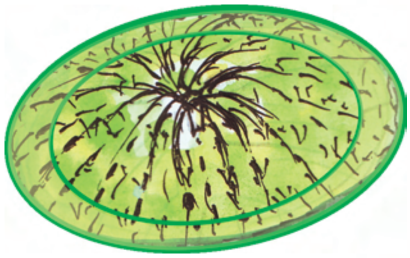


Fig. 4.60

Solution

Given, area of the garden $= 500 \text{ m}^2$

Area covered by a sprinkler $= \pi r^2$

$= 3.14 \times 12 \times 12$

$= 3.14 \times 144$

$= 452.16 \text{ m}^2$

Since, the area covered by a sprinkler is less than the area of the circular flower garden, the sprinkler cannot water the entire garden.

Example 4.51

A uniform circular path of width 2 m is laid out side a circular park of radius 50 m. Find the cost of levelling the path at the rate of ₹5 per m^2 (Take π = 3.14)

Solution

Given: r = 50 m, w = 2 m, R = $r + w$ = 50 + 2 = 52 m

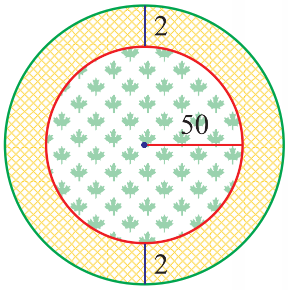


Fig. 4.61

Area of the circular path $= \pi(R+r)(R-r)$

$= 3.14 \times (52+50)(52-50)$

$= 3.14 \times 102 \times 2$

$= 3.14 \times 204$

$= 640.56 \text{ m}^2$

The cost of levelling the path of area 1 sq m = ₹5

The cost of levelling the path of 640.56 m^2 = ₹5 × 640.56

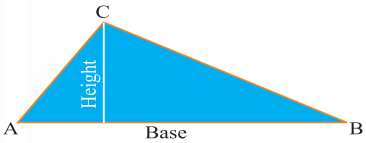
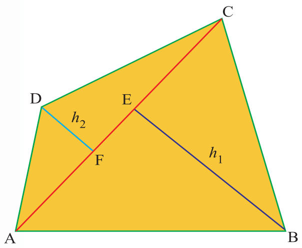
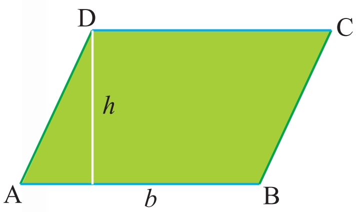
= ₹3202.80

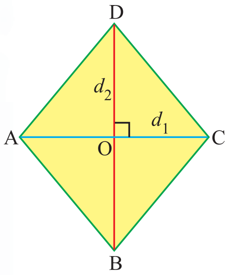
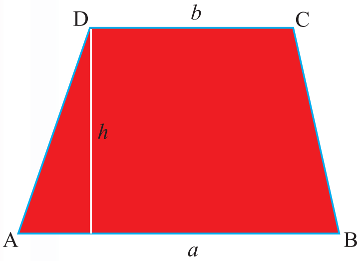
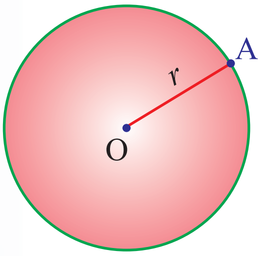
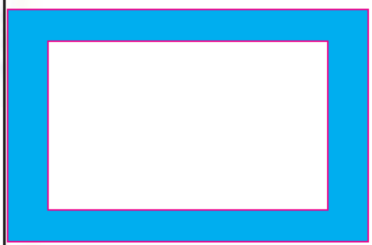
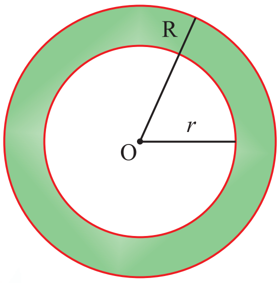
$\therefore$ the cost of levelling the path = ₹3202.80

Exercise 4.10

1. A circus tent has a base radius of 50 m. The ring at the centre for the performance by an artists is 20 m in radius. Find the area left for the audience. (Take $\pi = 3.14$)
2. A circular field of radius 30 m has a circular path of width 3 m inside its boundary. Find the area of the path (Take $\pi = 3.14$)
3. A ring shape metal plate has an internal radius of 7 cm and an external radius of 10.5 cm. If the cost of material is ₹5 per sq. cm, find the cost of 25 rings.
4. A circular well has radius 3 m. If a platform of uniform width of 1.5 m is laid around it, find the area of the platform . (Take $\pi = 3.14$)
5. A uniform circular path of width 2.5 m is laid outside a circular park of radius 56m. Find the cost of levelling the path at the rate of ₹5 per m^2 (Take $\pi = 3.14$)

Points to Remember

Figure	Area	Forumula
 Triangle	$\frac{1}{2} \times$ base $\times$ height	$\frac{1}{2} \times b \times h$ sq. units.
 Quadrilateral	$\frac{1}{2} \times$ diagonal $\times$ (sum of the perpendicular distances drawn to the diagonal from the opposite vertices)	$\frac{1}{2} \times d \times (h_1 + h_2)$ sq. units
 Parallelogram	base $\times$ corresponding altitude	bh sq. units

 **Rhombus**	$\frac{1}{2}$ × product of diagonals	$\frac{1}{2} \times d_1 \times d_2$ sq. units
 **Trapezium**	$\frac{1}{2}$ × height × sum of parallel sides	$\frac{1}{2} \times h \times (a + b)$ sq. units
 Circle	Perimeter of the circle = $2 \times \pi \times$ radius	$2\pi r$ units
	Area of the circle = π × radius × radius	πr^2 sq. units
	**Area of the pathway** i) area of the rectangular pathway	Area of outer rectangle – Area of inner rectangle
 **Circular Pathway**	ii) area of the circular pathway	Area of outer circle – Area of inner circle $= \pi\ (R^2 - r^2)$ sq. units $= \pi\ (R + r)\ (R - r)$ sq. units

5

GEOMETRY

Geometry is a branch of Mathematics that deals with the properties of various geometrical shapes and figures. In Greek the word "Geometry" means "Earth Measurement". Geometry deals with the shape, size, position and other geometrical properties of various objects. Geometry is useful in studying space, architecture, design and engineering.

5.1. Revision

Basic Geometrical concepts:

In earlier classes you have studied about some geometrical concepts. Let us recall them.

Point

A fine dot made with a sharp pencil may be taken as roughly representing a point. A point has a position but it has no length, breadth or thickness. It is denoted by a capital letters. In the figure A, B, C, D are points.

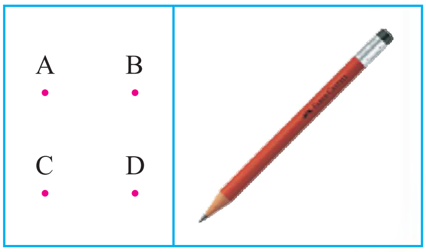


Fig. 5.1

Line

A line is traced out by a moving point. If the point of a pencil is moved over a sheet of paper, the trace left represents a line. A line has length, but it has no breadth. A line has no end points. A line AB is written as $\overleftrightarrow{AB}$. A line may be named with small letters l, m, n, etc. we read them as line l, line m, line n etc. A line has no end points as it goes on endlessly in both directions.

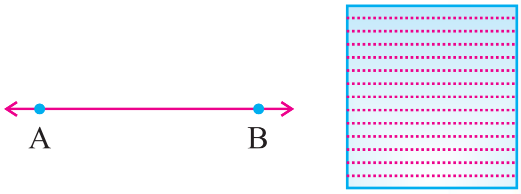


Fig. 5.2

Ray

A ray has a starting point but has no end point. The starting point is called the initial point.

Fig. 5.3

Here OA is called the ray and it is written as $\overrightarrow{OA}$. That is the ray starts from O and passes through A.

Line Segment

Let $\overleftrightarrow{AB}$ be a straight line.

Two points C and D are taken on it. CD is a part of AB. CD is called a line segment, and is written as $\overline{CD}$. A line segment has two end points.

Fig. 5.4

Plane

A plane is a flat surface which extends indefinitely in all directions. The upper surface of a table, the blackboard, the walls are some examples of planes.

5.2. Symmetry

Symmetry is an important geometrical concept commonly seen in nature and is used in every field of our life. Artists, manufacturers, designers, architects and others make use of the idea of symmetry. The beehives, flowers, tree leaves, hand kerchief, utensils have symmetrical design.

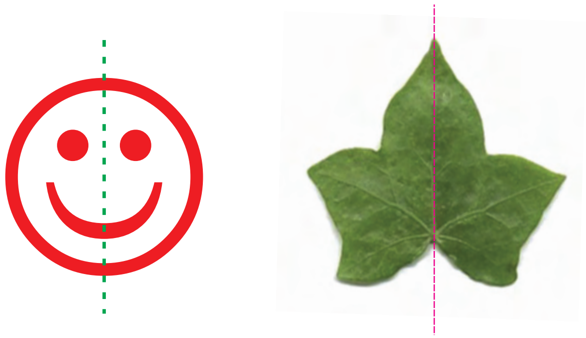


Fig. 5.5

Symmetry refers to the exact match in shape and size between two halves of an object. If we fold a picture in half and both the halves-left half and right half - match exactly then we say that the picture is symmetrical.

For example, if we cut an apple into two equal halves, we observe that two parts are in symmetry.

Fig. 5.6

Do you know?

Tajmahal in Agra is a symmetrical monument.

A butterfly is also an example of a symmetrical form. If a line is drawn down the centre of the butterfly's body, each half of the butterfly looks the same.

Fig. 5.7

Symmetry is of different types. Here we discuss about

1. Line of symmetry or axis of symmetry
2. Mirror symmetry
3. Rotational symmetry

1. Line of symmetry

In the Fig 5.8 the dotted lines divide the figure into two identical parts. If figure is folded along the line, one half of the figure will coincide exactly with the other half. This dotted line is known as line of symmetry.

When a line divides a given figure into two equal halves such that the left and right halves matches exactly then we say that the figure is symmetrical about the line. This line is called the line of symmetry or axis of symmetry.

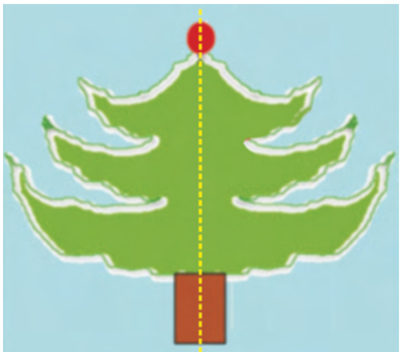


Fig. 5.8

Activity 1:

Take a rectangular sheet of paper. Fold it once lengthwise, so that one half fits exactly over the other half and crease the edges. Now open it, and again fold it once along its width.

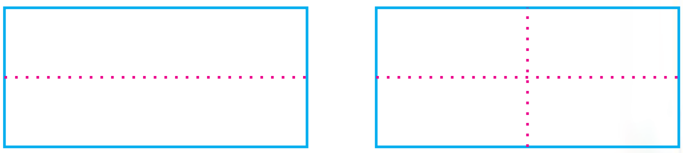


Fig. 5.9

In this paper folding,

You observe that a rectangle has two lines of symmetry.

Discuss: Does a parallelogram have a line of symmetry?

Activity 2:

One of the two set squares in your geometry box has angle of measure $30^{0}, 60^{0}, 90^{0}$. Take two such identical set squares. Place them side by side to form a 'kite' as shown in the Fig. 5.10.

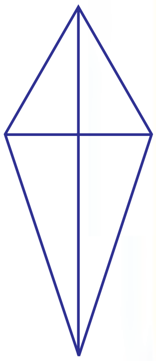


Fig. 5.10

How many lines of symmetry does the shape have?

You observe that this kite shape figure has one line of symmetry about its vertical diagonal.

Activity 3:

For the given regular polygons find the lines of symmetry by using paper folding method and also draw the lines of symmetry by dotted lines.

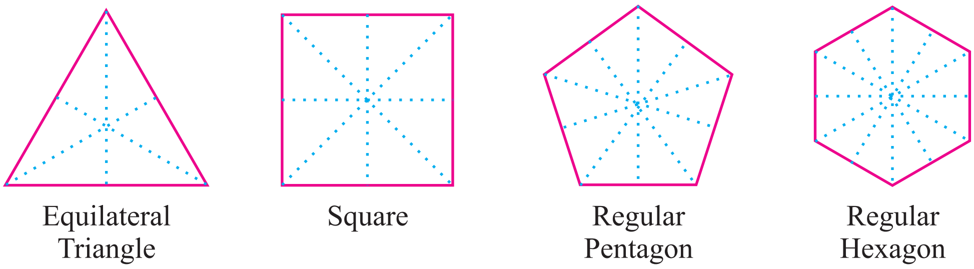


Fig. 5.11

In the above paper folding, you observe that

(i) An equilateral triangle has three lines of symmetry.

(ii) A square has four lines of symmetry

(iii) A regular pentagon has five lines of symmetry.

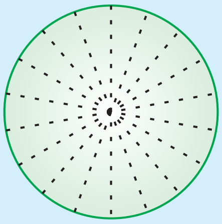
(iv) A regular hexagon has six lines of symmetry.

Do you know?

A polygon is said to be regular if all its sides are of equal length and all its angles are of equal measure.

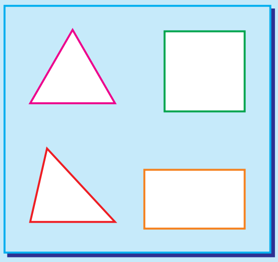
Each regular polygon has as many lines of symmetry as it has sides.

Do you know?



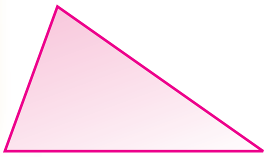
A circle has many lines of symmetry.

Try these

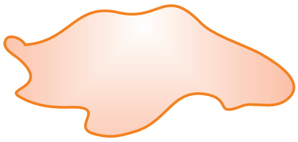


Identify the regular polygon

Some objects and figures have no line of symmetry.



Scalene triangle



Irregular shape

Fig. 5.12

Try these

Make a list of English alphabets which have no line of symmetry

The above figures have no line of symmetry; because these figures are not symmetrical. We can say that these figures are asymmetrical.

Do you know? To reflect an object means to produce its mirror image.

Mirror line symmetry

When we look into a mirror we see our image is behind the mirror. This image is due to reflection in the mirror. We know that the image is formed as far behind the mirror as the object is in front of it.

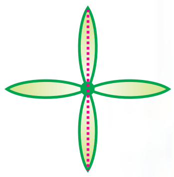


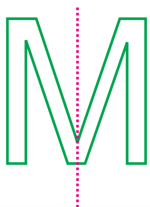
Fig. 5.13

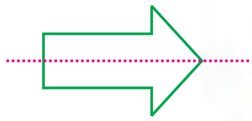
In the above figure if a mirror is placed along the line at the middle, the half part of the figure reflects through the mirror creating the remaining identical half. In other words, the line were the mirror is placed divides the figure into two identical parts in Fig. 5.13. They are of the same size and one side of the line will have its reflection exactly at the same distance on the other side. Thus it is also known as mirror line symmetry.

While dealing with mirror reflection, we notice that the left-right changes as seen in the figure.

Example 5.1

The figure shows the reflection of the mirror lines.





Exercise 5.1

1. Choose the correct answer.

i) An isosceles triangle has

(A) no lines of symmetry (B) one line of symmetry

(C) three lines of symmetry (D) many lines of symmetry

ii) A parallelogram has

(A) two lines of symmetry (B) four lines of symmetry

(C) no lines of symmetry (D) many lines of symmetry

iii) A rectangle has

(A) two lines of symmetry (B) no lines of symmetry

(C) four lines of symmetry (D) many lines of symmetry

iv) A rhombus has

(A) no lines os symmetry (B) four lines of symmetry

(C) two lines of symmetry (D) six lines of symmetry

v) A scalene triangle has

(A) no lines of symmetry (B) three lines of symmetry

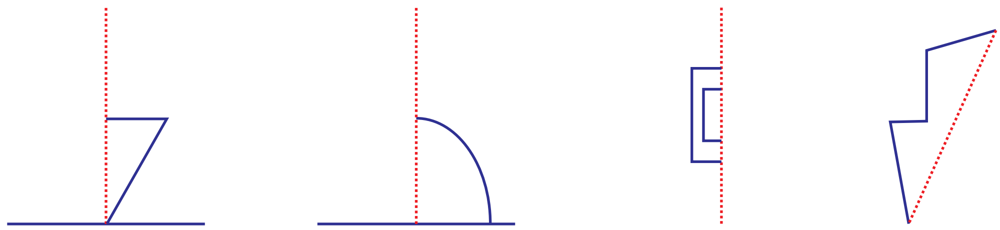
(C) one line of symmetry (D) many lines of symmetry

2. Which of the following have lines of symmetry?

(i) (ii) (iii) (iv)

How many lines of symmetry does each have?

3. In the following figures, the mirror line (i.e. the line of symmetry) is given as dotted line. Complete each figure performing reflection in the dotted (mirror) line.



(i) (ii) (iii) (iv)

4. Complete the following table:

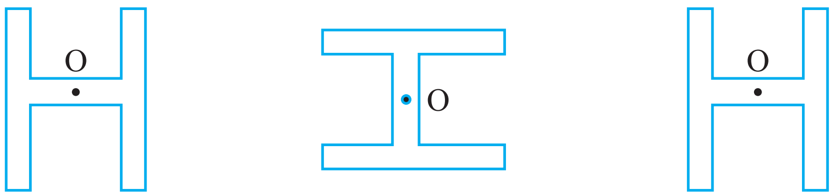
Shape	Rough figure	Number of lines of symmetry
Equilateral triangle		
Square		
Rectangle		
Isosceles triangle		
Rhombus		

5. Name a triangle which has
   (i) exactly one line of symmetry.
   (ii) exactly three lines of symmetry.
   (iii) no lines of symmetry.

6. Make a list of the capital letters of English alphabets which
   (i) have only one line of symmetry about a vertical line.
   (ii) have only one line of symmetry about a horizontal line.
   (iii) have two lines of symmetry about both horizontal and vertical line of symmetry.

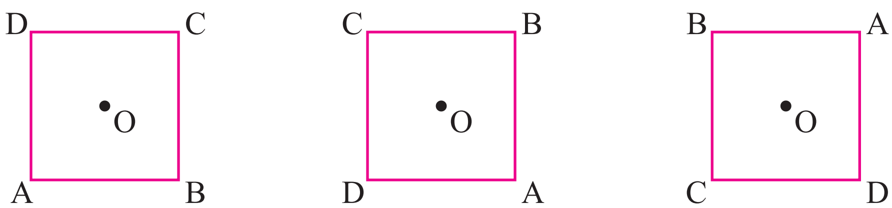
5.3 Rotational Symmetry

Look at the following figures showing the shapes that we get, when we rotate about its centre 'O' by an angle of 90^0 or 180^0



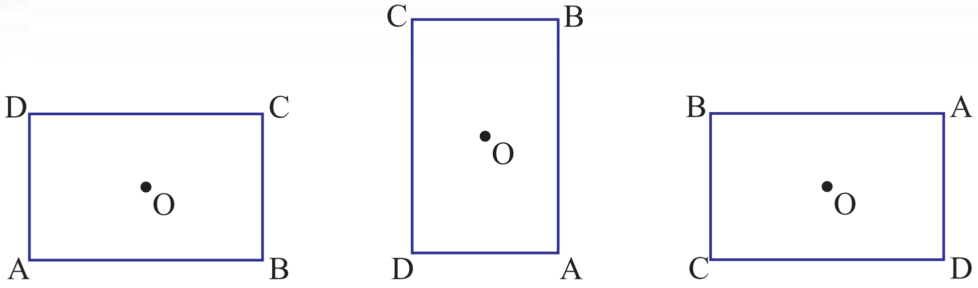
Letter H | 90° rotation | 180° rotation

Fig. 5.14



Square | 90° rotation | 180° rotation

Fig. 5.15



Rectangle　　90° rotation　　180° rotation

Fig. 5.16

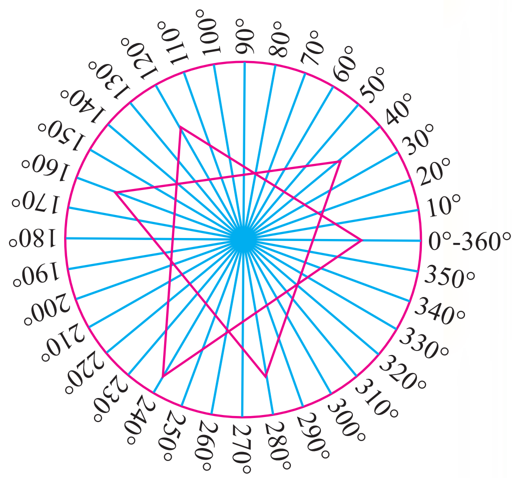
In the case of a square, we get exactly the same shape after it is rotated by 90^0 while in the case of a rectangle, we get exactly the same shape after it is rotated by 180° such figures which can be rotated through an angle less than 360° to get the same shape are said to have rotational symmetry.

Angle of Rotation

The minimum angle through which the figure has to be rotated to get the original figure is called the angle of rotation and the point about which the figure is rotation is known as centre of rotation.

Activity 4:

Take two card board sheets and cut off one equilateral triangle in each sheet such that both the triangles are identical. Prepare a circle on a card board and mark the degrees from 0 to 360 degree in the anticlockwise direction. Now palce one triangle exactly over the other and put a pin through the centres of the figures. Rotate the top figure until it matches with the lower figure.



You observe that the triangle has been rotated through an angle 120°.

Again rotate the top figure until it matches with the lower figure for the second time. Now you observe that the top of figure has been rotated through an angle 240° from the original position.

Rotate the top figure for the third time to match with the lower figure. Now the top triangle has reached its original position after a complete rotation of 360° From the above activity you observe that an equilateral triangle has angle of rotation 120°.

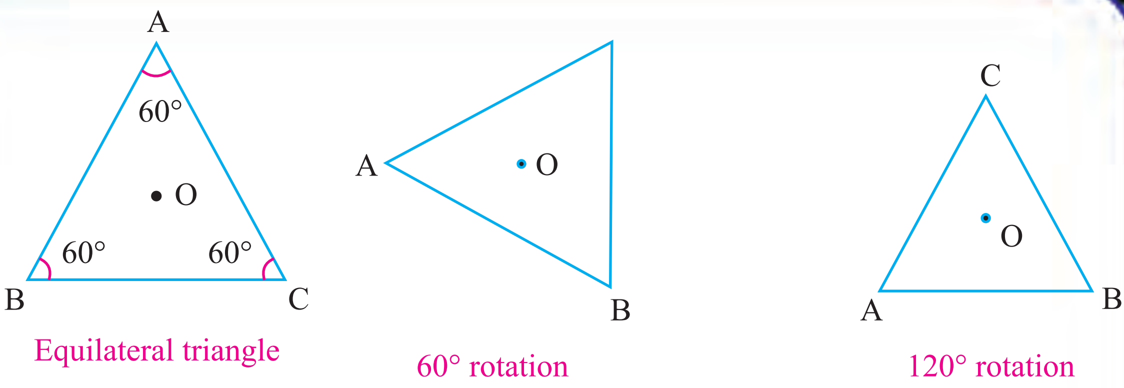


Fig. 5.17

Angle of rotation of a hexagon

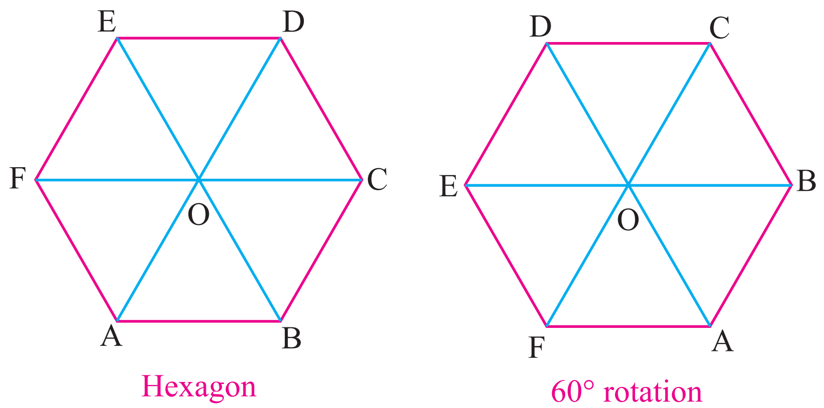


Fig. 5.18

In the above Fig. 5.15 to 5.18.

We get exactly the same shape of square, rectangle, equilateral triangle and hexagon after it is rotated by $90^0, 180^0, 120^0, 60^0$ respectively.

Thus the angle of rotation of

(i) a square is 90^0

(ii) a rectangle is 180^0

(iii) an equilateral triangle is 120^0

(iv) a hexagon is 60^0

Order of rotational symmetry

The order of rotational symmetry is the number that tell us how many times a figure looks exactly the same while it takes one complete rotation about the centre.

Thus if the angle of rotation of an object is x^0

It's order of rotational symmetry symmetry $= \dfrac{360}{x^0}$

In Fig. 5.15 to 5.18.

The order of rotational symmetry of

(i) a square is $\frac{360^0}{90^0} = 4$

(ii) a rectangle is $\frac{360^0}{180^0} = 2$

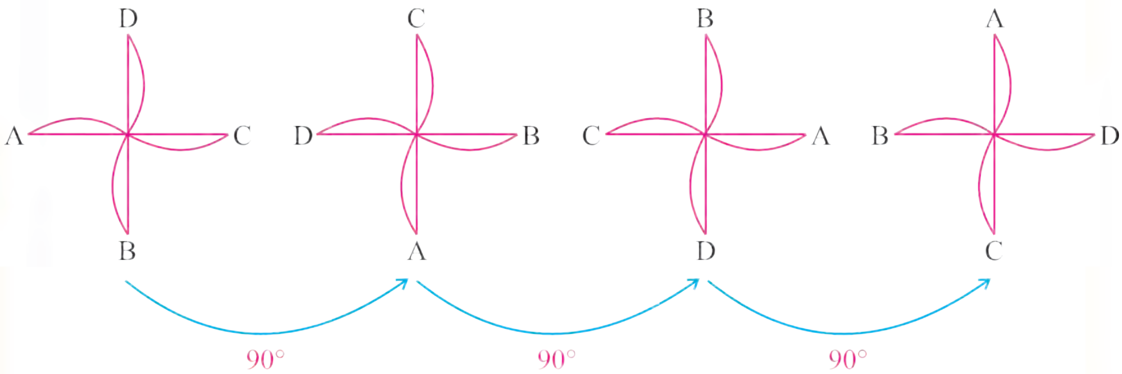
(iii) an equilateral triangle is $\frac{360^0}{120} = 3$

(iv) a hexagon is $\frac{360^0}{60^0} = 6.$

Example 5.2

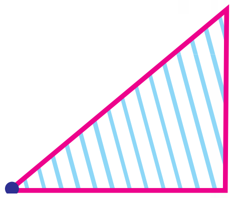
The objects having no line of symmetry can have rotational symmetry.

Have you ever made a paper wind mill? The paper wind mill in the picture looks symmetrical. But you do not find any line of symmetry. No folding can help you to have coincident halves. However if you rotate it by 90° about the the centre, the windmill will look exactly the same. We say the wind mill has a rotational symmetry.



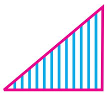
In a full turn, there are four positions (on rotation through the angles $90^0, 180^0 270^0$ and 360^0) in which the wind mill looks exactly the same. Because of this, we say it has a rotational symmetry of order 4.

Activity 5:

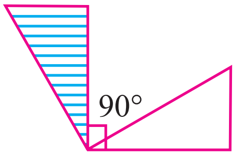


As shown in figure cut out a card board or paper triangle. Place it on a board and fix it with a drawing pin at one of its vertices. Now rotate the triangle about this vertex, by 90^0 at a time till it comes to its original position.

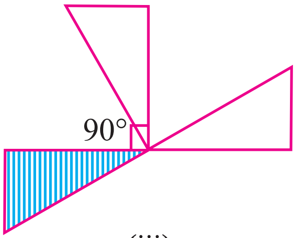
You observe that, for every 90^0 you have the following figures (ii to v).



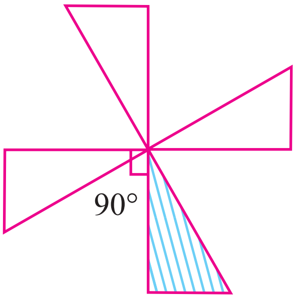
(i)



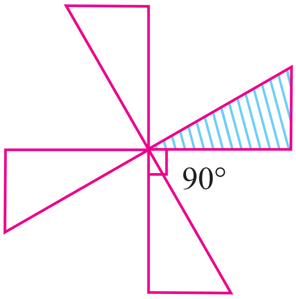
(ii)



(iii)



(iv)



(v)

The triangle comes back to its original position at position (v) after rotating through 360^0 Thus the angle of rotation of this triangle is 360^0 and the order of rotational symmetry of this triangle is $\frac{360^0}{360^0} = 1$.

Exercise 5.2

1. Choose the correct answer:

i) The angle of rotation of an equilateral triangle is

(A) 60^0 (B) 90^0 (C) 120^0 (D) 180^0

ii) The order of rotational symmetry of square is

(A) 2 (B) 4 (C) 6 (D) 1.

iii) The angle of rotation of an object is 72^0 then its order of rotational symmetry is

(A) 1 (B) 3 (C) 4 (D) 5

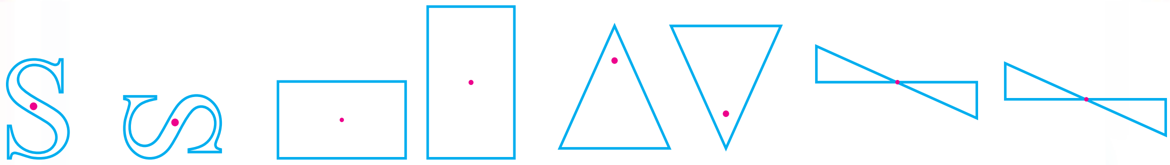
iv) The angle of rotation of the letter 'S' is

(A) 90^0 (B) 180^0 (C) 270^0 (D) 360^0

v) the order of rotational symmetry of the letter 'V' is one then its angle of rotation is

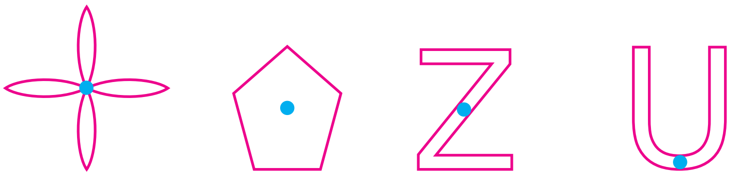
(A) 60^0 (B) 90^0 (C) 180^0 (D) 360^0

2. The following figures make a rotation to come to the new position about a given centre of rotation. Examine the angle through which the figure is rotated.



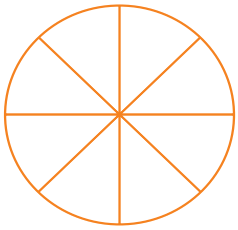
(i) (ii) (iii) (iv)

3. Find the angle of rotation and the order of rotational symmetry for the following figures given that the centre of rotation is ‘0’.



(i) (ii) (iii) (iv)

4. A circular wheel has eight spokes.



What is the angle of rotation and the order of rotation?

5.3 Angle

Two rays starting from a common point form an angle. In $\angle AOB$, O is the vertex, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the two arms.

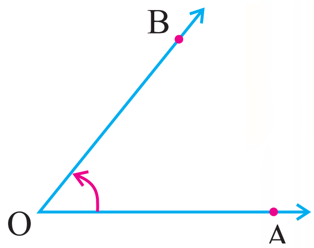


Fig. 5.19

Types of angles

(i) Acute angle:

An angle whose measure is greater than 0° but less than 90^0 is called an acute angle.

Example: $15^0, 30^0, 60^0, 75^0$, In Fig. 5.20 $\angle AOB = 30^0$ is an acute angle.

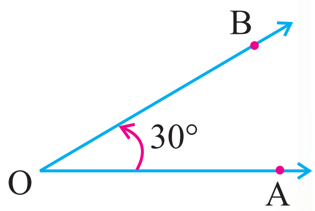


Fig. 5.20

(ii) Right angle

An angle whose measure is 90^0 is called a right angle.

In Fig. 5.21 $\angle AOB = 90^0$ is a right angle.

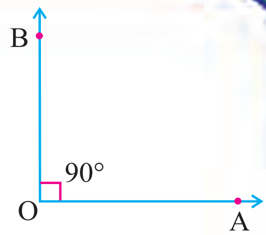


Fig. 5.21

(iii) Obtuse angle

An angle whose measure is greater than 90^0 and less than 180^0 is called an obtuse angle.

Example: $100^0, 110^0, 120^0, 140^0$

In Fig. 5.22 $\angle$ AOB $= 110^0$ is an obtuse angle.

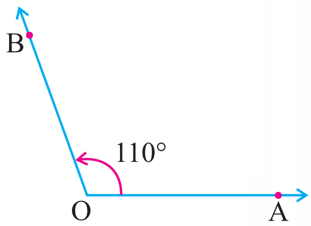


Fig. 5.22

(iv) Straight angle

When the arms of an angle, are opposite rays forming a straight line. The angle thus formed is a straight angle and whose measure is 180^0 In Fig. (5.23) $\angle$ AOB $= 180^0$ is a straight angle.

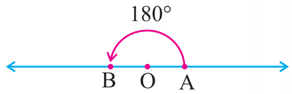


Fig. 5.23

(v) Reflex angle

An angle whose measure is more than 180^0 but less than 360^0 is called a reflex angle. In Fig. 5.24 $\angle AOB = 220°$ is a reflex angle.

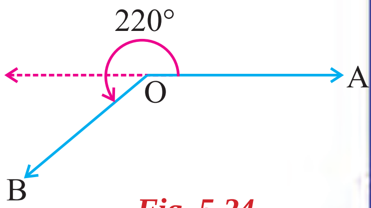


Fig. 5.24

(vi) Complete angle

In Fig. 5.25

The angle formed by $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ is one complete circle, that is 360^0.Such an angle is called a complete angle

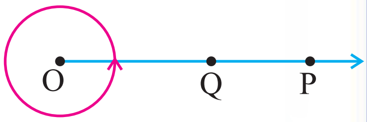


Fig. 5.25

Related Angles

(i) Complementary angles

If the sum of the measures of two angle is 90^0, then the two angles are called complementary angles. Here each angle is the complement of the other.

The complement of 30^0 is 60^0 and the complement of 60^0 *is* 30^0

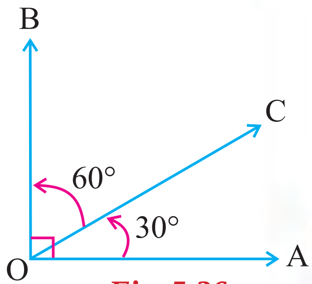


Fig. 5.26

(ii) Supplementary angles

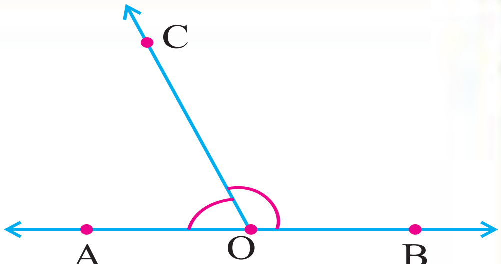


Fig. 5.27

If the sum of the measures of two angle is 180°, then the two angles are called supplementary angles. Here each angle is the supplement of the other.

The supplement of 120° is 60° and 60° is the supplement of 120°

Try these

Identify the following pairs of angles are complementary or supplementary

(a) 80° and 10°_____

(b) 70° and 110°_____

(c) 40° and 50°______

(d) 95° and 85°_______

(e) 65° and 115°______

Try these

Fill in the blanks.

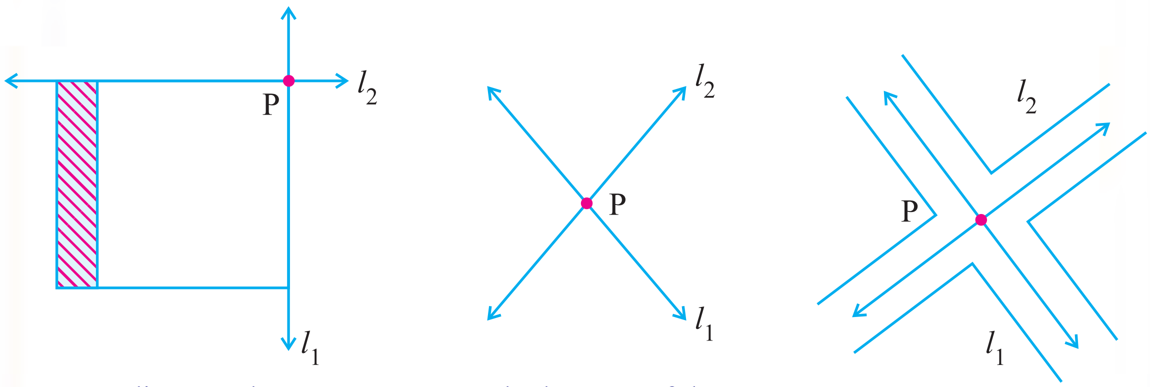
(a) Complement of 85° *is*____

(b) Complement of 30° is ___

(c) Supplement of 60° is ____

(d) Supplement of 90° *is*_____

Intersecting lines



Two adjacent edges of your book

The letter X of the English alphabet

Crossing Roads

Fig. 5.28

Look at the Fig. 5.28. Two lines l_1 and l_2 are shown. Both the lines pass through a point P. We say l_1 and l_2 intersect at P. If two lines have one common point, they are called intersecting lines. The common point 'P' is their point of intersection.

Angles in intersecting lines

When two lines intersect at a point angles are formed.

In Fig. 5.29 the two lines AB and CD intersect at a point ‘O’, $\angle$COA, $\angle$AOD, $\angle$DOB, $\angle$BOC are formed. Among the four angles two angles are acute and the other two angles are obtuse.

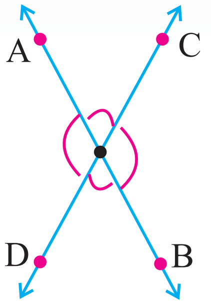


Fig. 5.29

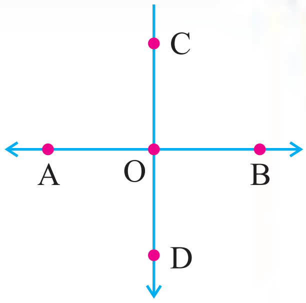


Fig. 5.30

But in figure 5.30 if the two intersecting lines are perpendicular to each other then the four angles are at right angles.

Adjacent angles

If two angles have the same vertex and a common arm, then the angles are called adjacent angles.

In Fig. 5.31 $\angle$BAC and $\angle$CAD are adjacent angles (i.e . $\angle x$ and $\angle y$) as they have a common arm $\overrightarrow{AC}$, a common vertex A and both the angle $\angle$BAC and $\angle$CAD are on either side of the common arm $\overrightarrow{AC}$.

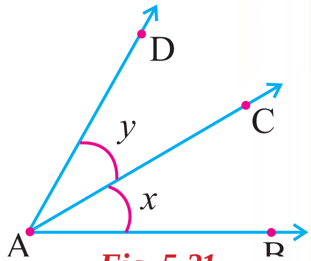
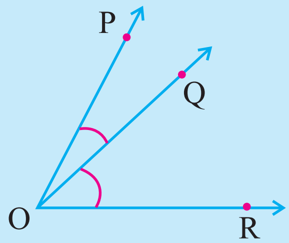


Fig. 5.31

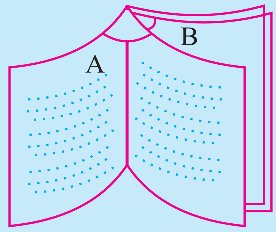
Try these

$\angle$ ROP and $\angle$QOP are not adjacent angle. Why?



Try these

Look at the following figure



Open a book looks like the above figure. Is the pair of angles are adjacent angles?

(i) Adjacent angles on a line.

When a ray stands on a straight line two angles are formed. They are called linear adjacent angles on the line.

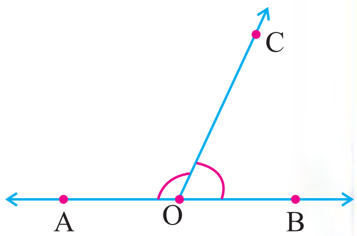


Fig. 5.32

In Fig. 5.32 the ray OC stands on the line AB. $\angle BOC$ and $\angle COA$ are the two adjacent angles formed on the line AB. Here 'O' is called the common vertex, $\overrightarrow{OC}$ is called the common arm. The arms OA and OB lie on the opposite sides of the common arm OC.

Two angles are said to be linear adjacent angles on a line if they have a common vertex, a common arm and the other two arms are on the opposite sides of the common arm.

(ii) The sum of the adjacent angles on a line is 180°

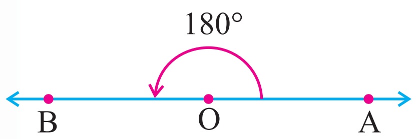


Fig. 5.33

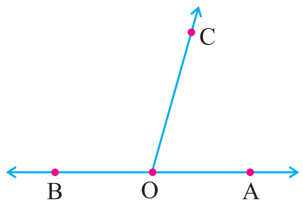


Fig.5.34

In Fig. 5.33 $\angle AOB = 180^0$ is a straight angle.

In Fig. 5.34 The ray OC stands on the line AB. $\angle AOC$ and $\angle COB$ are adjacent angles. Since $\angle$ AOB is a straight angle whose measure is 180^0

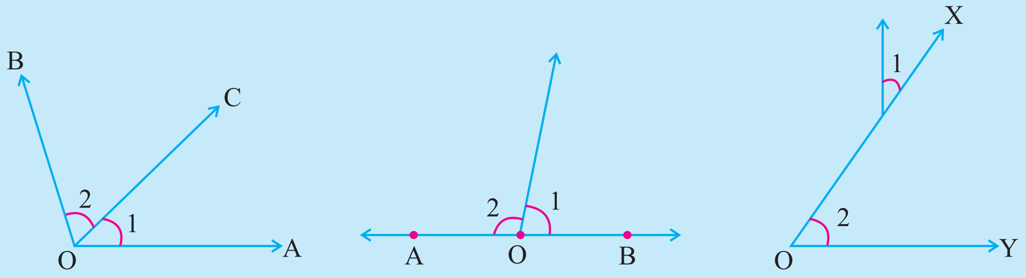
$\angle AOC + \angle COB = 180^0$

From this we conclude that the sum of the adjacent angles on a line is 180^0

Note 1: A pair of adjacent angles whose non common arms are opposite rays.

Note 2: Two adjacent supplementary angles form a straight angle.

Try these



(*i*) (*ii*) (*iii*)

Are the angles marked 1 and 2 adjacent?

If they are not adjacent, Justify your answer.

Do you know?

A vegetable chopping board

A pen stand

The chopping blade makes a linear pair of angles with the board.

The pen makes a linear pair of angles with the stand.

Discuss:

(i) Can two adjacent acute angles form a liner pair?

(ii) Can two adjacent obtuse angles form a linear pair?

(iii) Can two adjacent right angles form a linear pair?

(iv) Can an acute and obtuse adjacent angles form a linear pair?

(iii) Angle at a point

In Fig. 5.35, four angles are formed at the point 'O'. The sum of the four angles formed is 360^0.

(i.e) $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^0$

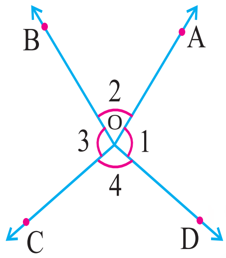


Fig. 5.35

(iv) Vertically opposite angles

If two straight lines AB and CD intersect at a point 'O'. Then $\angle AOC$ and $\angle BOD$ form one pair of vertically opposite angles and $\angle DOA$ and $\angle COB$ form another pair of vertically opposite angles.

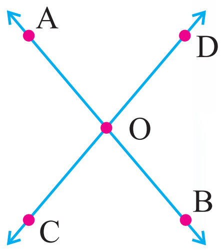


Fig. 5.36

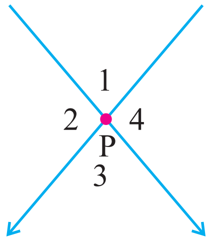
Do you know?

The following are some real life example for vertically Opposite angles

Activity 6: Draw two lines 'l' and 'm', intersecting at a point 'P' mark $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ as in the Fig. 5.37.

Take a trace copy of the figure on a transparent sheet. Place the copy on the original such that $\angle$ 1 matches with its copy, $\angle 2$, matches with its copy.. etc...

Fix a pin at the point of intersection of two lines 'l' and 'm' at P. Rotate the copy by 180^0. Do the lines coincide again?



Can be rotated to get

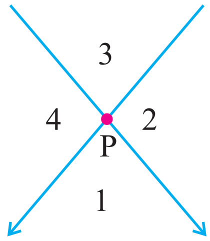


Fig. 5.37

You find that $\angle 1$ and $\angle 3$ have interchanged their positions and so have $\angle 2$ and $\angle 4$. (This has been done without disturbing the position of the lines).

Thus $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$.

From this we conclude that when two lines intersect, the vertically opposite angles are equal.

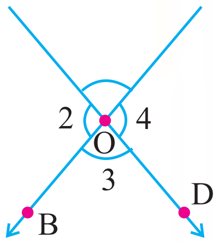


Fig. 5.38

Now let us try to prove this using Geometrical idea.

Let the lines AB and CD intersect at 'O' making angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

Now $\angle 1 = 180^0 - \angle 2 \rightarrow$ (i)

(Since sum of the adjacent angle on a line 180^0)

$\angle 3 = 180^0 - \angle 2 \rightarrow$ (ii)

(Since sum of the adjacent angle on a line 180^0).

From (i) and (ii)

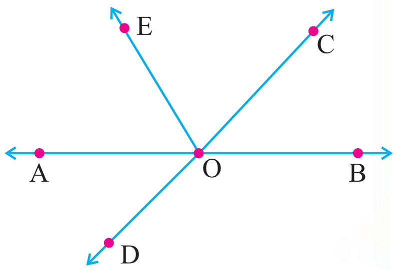
$\angle 1 = \angle 3$ and similarly we prove that $\angle 2 = \angle 4$.

Example 5.3

In the given figure identify

(a) Two pairs of adjacent angles.

(b) Two pairs of vertically opposite angles.



Solution:

(a) Two pairs of adjacent angles are

(i) $\angle EOA$, $\angle COE$ since OE is common to $\angle EOA$ and $\angle COE$

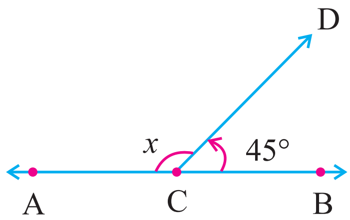
(ii) $\angle COA$, $\angle BOC$ since OC is common to $\angle COA$ and $\angle BOC$

(b) Two pairs of vertically opposite angles are i) $\angle BOC$, $\angle AOD$

ii) $\angle COA$, $\angle DOB$.

Example 5.4

Find the value of x in the given figure.



Solution:

$\angle BCD + \angle DCA = 180^0$

(Since $\angle BCA = 180^0$ is a straight angle)

$45° + x = 180°$

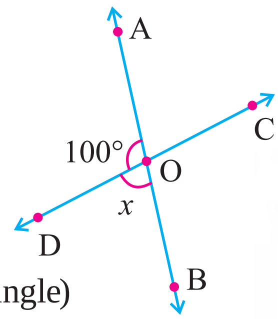
$x = 180° - 45°$

$= 135°$

$\therefore$ The value of x is 135^0.

Example 5.5

Find the value of x in the given figure.



Solution:

$\angle AOD + \angle DOB = 180^0$

(Since $\angle AOB = 180^0$ is a straight angle)

$100^0 + x = 180^0$

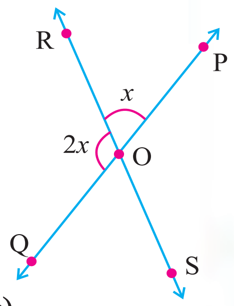
$x = 180^0 - 100^0$

$= 80^0$

$\therefore$ The value of x is 80^0.

Example 5.6

Find the value of x in the given figure.



Solution:

$\angle POR + \angle ROQ = 180^0$

(Since $\angle POQ = 180^0$ is a straight angle)

$$x + 2x = 180^{0}$$

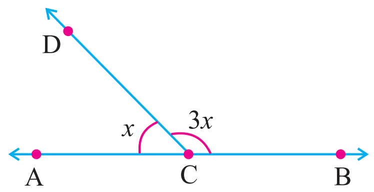
$$3x = 180^{0}$$

$$x = \frac{180^{0}}{3}$$

$$= 60^{0}$$

$\therefore$ The value of x is 60^{0}

Example 5.7



Find the value of x in the given figure.

Solution:

$\angle BCD + \angle DCA = 180^{0}$

(Since $\angle BCA = 180^{0}$ is a straights angle)

$$3x + x = 180^{0}$$

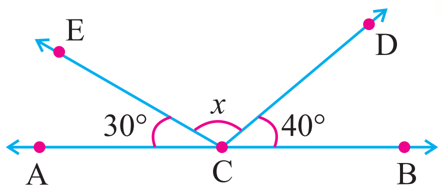
$$4x = 180^{0}$$

$$x = \frac{180^{0}}{4}$$

$$= 45^{0}$$

$\therefore$ The value of x is 45^{0}

Example 5.8



Find the value of x in the given figure.

Solution:

$\angle BCD + \angle DCE + \angle ECA = 180^{0}$

(Since $\angle BCA = 180^{0}$ is a straight angle)

$$40^{0} + x + 30^{0} = 180^{0}$$

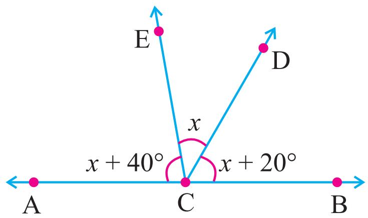
$$x + 70^{0} = 180^{0}$$

$$x = 180^{0} - 70^{0}$$

$$= 110^{0}$$

$\therefore$ The value of x is 110^{0}

Example 5.9



Find the value of x in the given figure.

Solution:

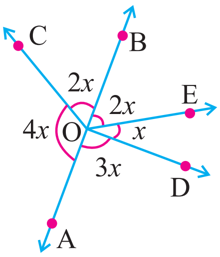
$\angle BCD + \angle DCE + \angle ECA = 180^{0}$ (Since $\angle BCA = 180^{0}$ straight angle).

$$x + 20^0 + x + x + 40^0 = 180^0$$
$$3x + 60^0 = 180^0$$
$$3x = 180^0 - 60^0$$
$$3x = 120^0$$
$$x = \frac{120}{3} = 40^0$$

$\therefore$ The value of x is 40^0

Example 5.10

Find the value of x in the given figure.



Solution:

$$\angle BOC + \angle COA + \angle AOD + \angle DOE + \angle EOB = 360^0$$

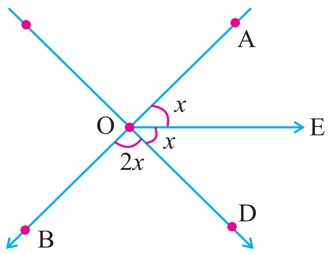
(Since angle at a point is 360^0)

$$2x + 4x + 3x + x + 2x = 360^0$$
$$12x = 360^0$$
$$x = \frac{360^0}{12}$$
$$= 30^0$$

$\therefore$ The value of x is 30^0

Example 5.11

Find the value of x the given figure.



Solution:

$$\angle BOD + \angle DOE + \angle EOA = 180^0$$

(Since $\angle$ AOB = 180^0 is straight angles)

$$2x + x + x = 180^0$$
$$4x = 180^0$$
$$x = \frac{180^0}{4}$$
$$= 45^0$$

$\therefore$ The value of x is 45^0

Exercise: 5.3

1. Choose the correct answer:

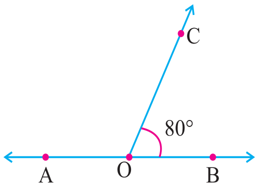
i) The number of points common to two intersecting line is

(A) one (B) Two (C) three (D) four

ii) The sum of the adjacent angles on a line is

(A) 90^0 (B) 180^0 (C) 270^0 (D) 360^0

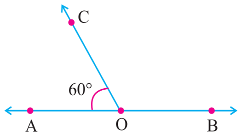
iii) In the figure $\angle COA$ will be



(A) 80^0 (B) 90^0

(C) 100^0 (D) 95^0

iv) In the figure $\angle BOC$ will be

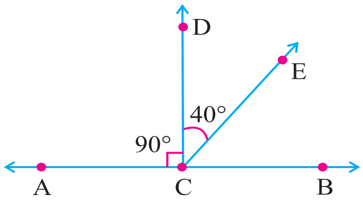


(A) 80^0 (B) 90^0

(C) 100^0 (D) 120^0

v) In the figure CD is perpendicular to AB.

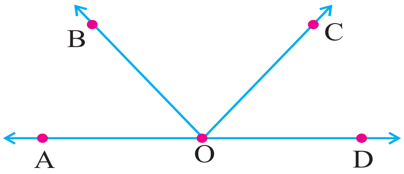
Then the value of $\angle BCE$ will be



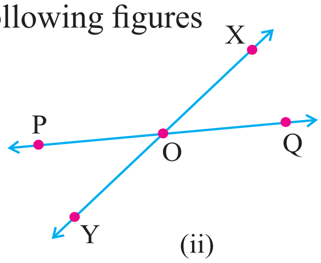
(A) 45^0 (B) 35^0

(C) 40^0 (D) 50^0

2. Name the adjacent angles in the following figures

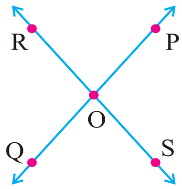


(i)



(ii)

3. Identity the vertically opposite angles in the figure.



4. Find $\angle B$ if $\angle A$ measures?

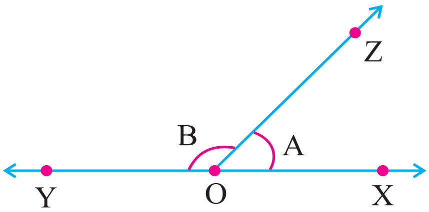
(i) 30^0

(ii) 80^0

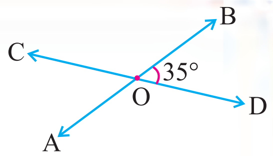
(iii) 70°

(iv) 60°

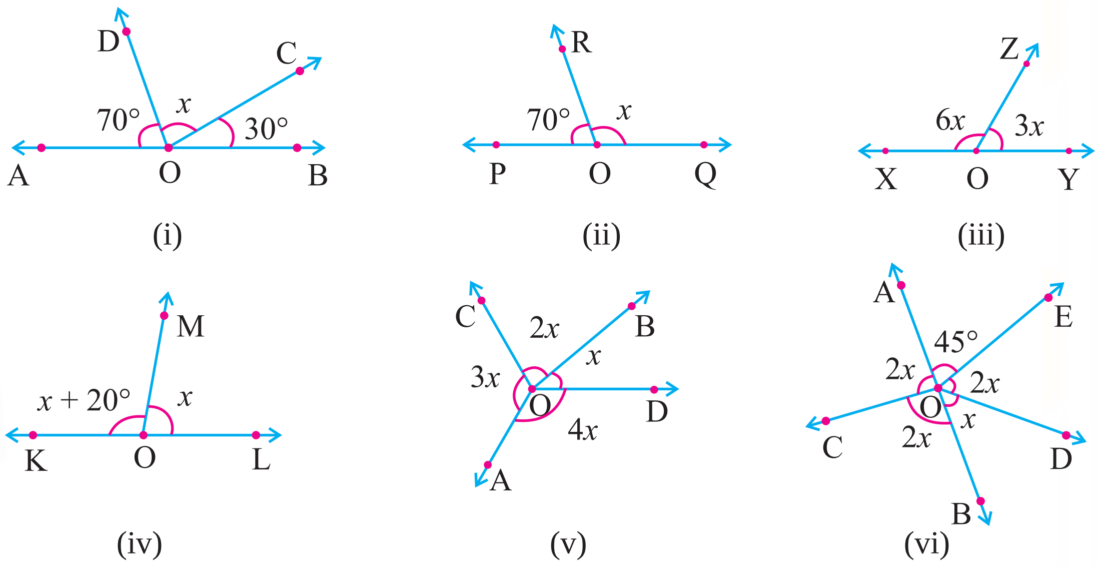
(v) 45^0



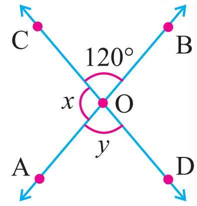
5. In figure AB and CD be the intersecting lines if $\angle DOB = 35^0$ find the measure of the other angles.



6. Find the value of x in the following figures.



7. In the following figure two lines AB and CD intersect at the point O. Find the value of x and y.



8. Two linear adjacent angles on a line are $4x$ and $(3x + 5)$ Find the value of x.

Parallel Lines

Look at the table.

The top of the table ABCD is a flat surface. Are you able to see some points and line segment on the top? Yes.

The line segment AB and BC intersects at B. which line segment intersects at A, C and D? Do the line segment AD and CD intersect? Do the line segment AD and BC intersect?

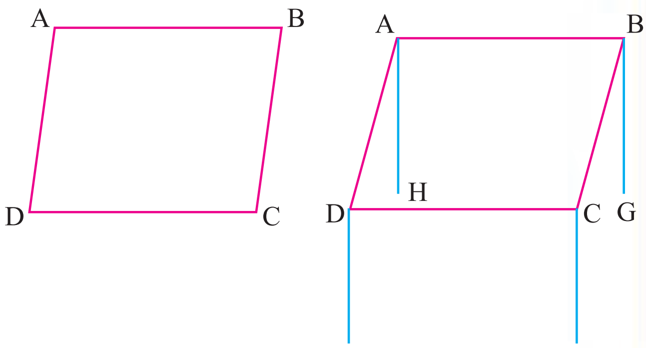


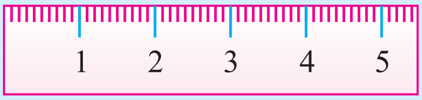
Fig. 5.39

The line segment AB and CD will not meet however they are extended such lines are called parallel lines. AD and BC form one such pair. AB and CD form another pair.

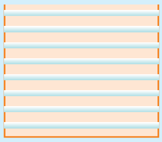
If the two lines AB and CD are parallel. We write AB || CD.

Do you know?

The following are the examples of parallel lines

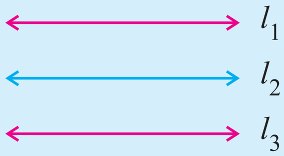


The opposite edges of ruler



The cross bars of this window

Do you know?



l_3 is parallel to l_3

Two straight lines are said to be parallel to each other if they do not intersect at any point.

In the given figure, the perpendicular distance between the two parallel lines is the same everwhere.

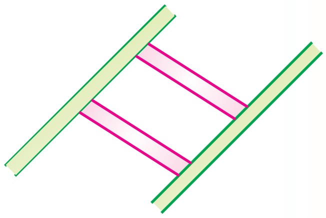


Fig. 5.40

Transversal

A straight line intersects two or more given lines at distinct points is called a transversal to the given lines. The given lines may or may not be parallel.

Names of angles formed by a transversal.

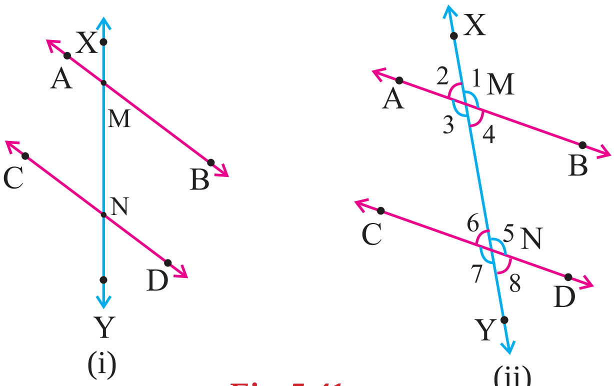
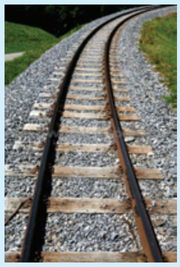


Fig. 5.41

Do you know?



The above figure give an idea of a transversal. You have seen a railway line crossing several lines.

In Fig. 5.41 (i), a pair of lines AB and CD, are cut by a transversal XY, intersecting the two lines at points M and N respectively. The points M and N are called points of intersection.

Fig. 5.41 (ii) when a transversal intersects two lines the eight angles marked 1 to 8 have their special names. Let us see what those angles are

1. Interior angles

All the angles which have the line segment MN as one arm in Fig. 5.41 (ii) are known as interior angles as they lie between the two lines AB and CD. In Fig. 5.41 (ii), $\angle$ 3, $\angle$ 4, $\angle$ 5, $\angle$ 6 are interior angles.

2. Interior alternate angles

When a transversal intersects two lines four interior angles are formed. Of the interior angles, the angles that are on opposite sides of the transversal and lie in separate linear pairs are known as interior alternate angles. $\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$ are interior alternate angles in Fig. 5.41 (ii).

3. Exterior angles

All the angles which do not have the line segment MN as one arm, are known as exterior angles. $\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$ are exterior angles in Fig. 5.41 (ii).

4. Exterior alternate angles

When a transversal intersects two lines four exterior angles are formed. Of the exterior angles, the angles that are on opposite sides of the transversal and lie in separate linear pairs are known as exterior alternate angles.

In Fig. 5.41 (ii), $\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$ are exterior alternate angles.

5. Corresponding angles

The pair of angles on one side of the transversal, one of which is an exterior angle while the other is an interior angle but together do not form a linear pair, are known as corresponding angles.

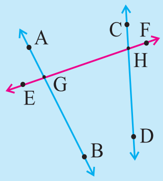
The pairs of corresponding angles in Fig. 5.41 (ii) are $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$.

Notice that although both $\angle 6$ and $\angle 7$ lie on the same side of the transversal and $\angle 6$ is an interior angle while $\angle 7$ is an exterior angle but $\angle 6$ and $\angle 7$ are not corresponding angles as together they form a linear pair. Now we tabulate the angles.

a	Interior angles	$\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$
b	Exterior angles	$\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$
c	Pairs of corresponding angles	$\angle 1$ and $\angle 5$; $\angle 2$ and $\angle 6$ $\angle 3$ and $\angle 7$; $\angle 4$ and $\angle 8$
d	Pairs of alternate interior angles	$\angle 3$ and $\angle 5$; $\angle 4$ and $\angle 6$
e	Pairs of alternate exterior angles	$\angle 1$ and $\angle 7$; $\angle 2$ and $\angle 8$
f	Pairs of interior angles on the same side of the transversal.	$\angle 3$ and $\angle 6$; $\angle 4$ and $\angle 5$

Try these

Name the following angles:

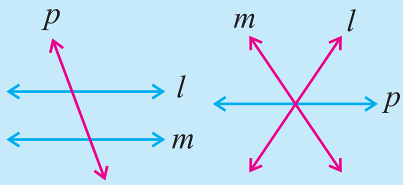


a) Any two interior angles ____ and _____

b) Any two exterior angles ____ and _____

c) A pair of interior angles _____and _____

d) A pair of corresponding _____and _____ angles.



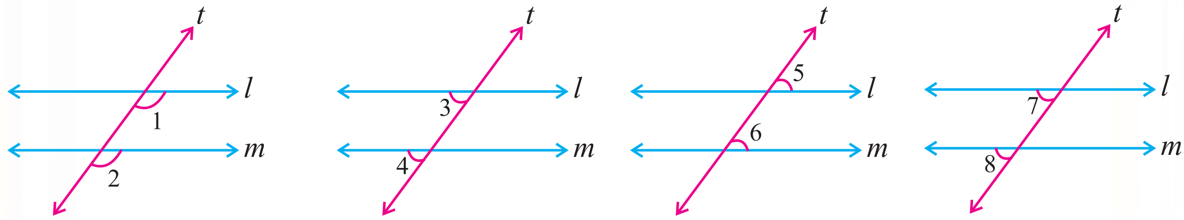
(i) (ii)

In Fig. (i) p is a transversal to the lines l and m. In Fig. (ii) the line p is not a transversal, although it cuts two lines l and 'm' can you say why?

Properties of parallel lines cut by a transversal

Activity 7:

Take a sheet of white paper. Draw (in thick colour) two parallel lines 'l' and 'm'. Draw a transversal 't' to the lines 'l' and 'm' . Label $\angle 1$ and $\angle 2$ as shown in Fig 5.42.



(i) (ii) (iii) (iv)

Fig. 5.42

Place a trace paper over the figure drawn. Trace the lines 'l', 'm' and 't'. Slide the trace paper along 't' until 'l' coincides with 'm'.

You find that $\angle 1$ on the traced figure coincides with $\angle 2$ of the original figure. In fact, you can see all the following results by similar tracing and sliding activity.

(i) $\angle 1 = \angle 2$ (ii) $\angle 3 = \angle 4$ (iii) $\angle 5 = \angle 6$ (iv) $\angle 7 = \angle 8$

From this you observe that.

When two parallel lines are cut by a transversal,

(a) each pair of corresponding angles are equal

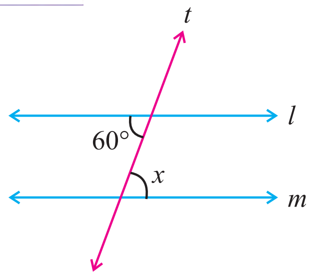
(b) each pair of alternate angles are equal

(c) each pair of interior angles on the same side of the transversal are supplementary (i.e 180^0)

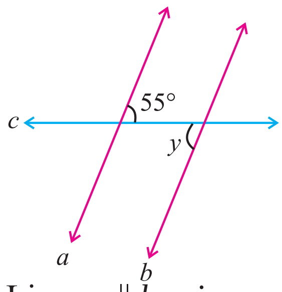
Try these

Draw parallel lines cut by a transversal. Verify the above three statements by actually measuring the angles.

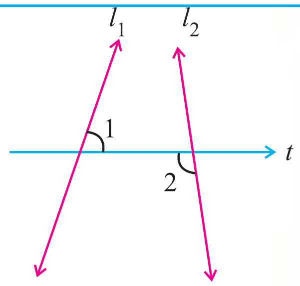
Try these



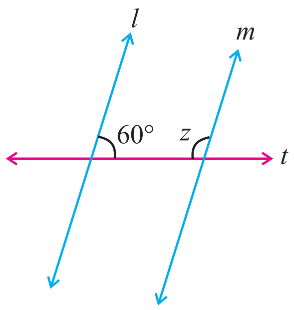
Lines $l \parallel m$, t is a transversal, $\angle x = ?$



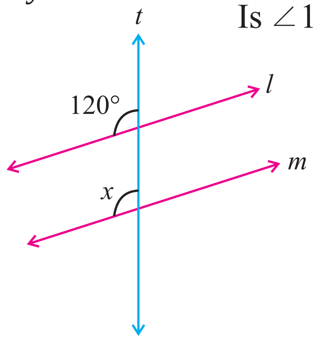
Lines $a \parallel b$, c is a transversal, $\angle y = ?$



l_1, l_2 be two lines and t is a transversal. Is $\angle 1 = \angle 2$?



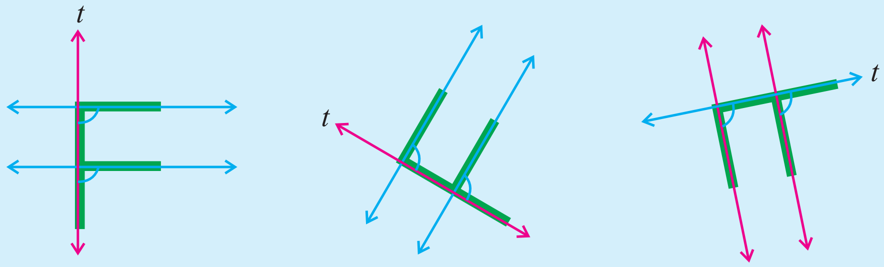
Lines $l \parallel m$, t is a transversal, $\angle z = ?$



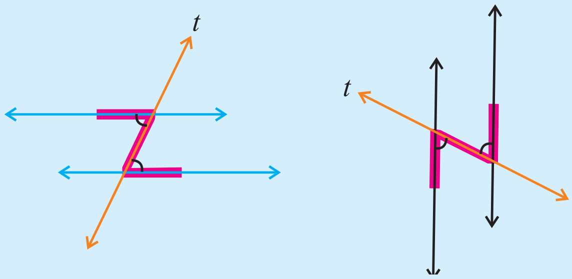
Lines $l \parallel m$, t is a transversal, $\angle x = ?$

Do you know?

The F - shape stands for corresponding angles.



The Z - shape stands for alternate angles.

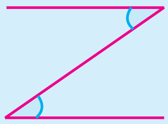


Try these

Fold a sheet of paper so as to get a pair of parallel lines. Again fold the paper across such that a transversal is obtained. Press the edges of folded paper and open it. You will see a pair of parallel lines with the transversal. Measure the angles and verify the properties of parallel lines when cut by a transversal.

Do you know?

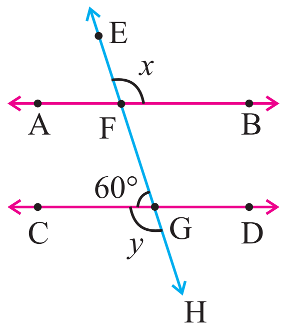
Checking for Parallel Lines:
Look at the letter z.



The horizontal segments are parallel, because the alternate angles are equal.

Example 5.12

In the figure, find $\angle CGH$ and $\angle BFE$.



Solution

In the figure, AB || CD and EH is a transversal.

$\angle FGC = 60°$ (given)

$y = \angle CGH = 180° - \angle FGC$ (Since $\angle CGH$ and $\angle FGC$ are adjacent angles on a line)

$= 180° - 60°$

$= 120°$

$\angle EFA = 60°$ (Since $\angle EFA$ and $\angle FGC$ are corresponding angles)

$\angle EFA + \angle BFE = 180°$ (Since sum of the adjacent angles on a line is 180°)

$60° + x = 180°$

$x = 180° - 60°$

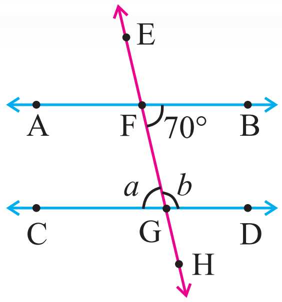
$= 120°$

$\therefore \quad x = \angle BFE = 120°$

$y = \angle CGH = 120°$

Example 5.13

In the given figure, find $\angle CGF$ and $\angle DGF$.



Solution

In the figure AB || CD and EH is a transversal.

$\angle GFB = 70°$ (given)

$\angle FGC = a = 70°$ (Since alternate interior angles $\angle GFB$ and $\angle CGF$ are equal)

$\angle CGF + \angle DGF = 180°$ (Since sum of the adjacent angle on a line is 180°)

$a + b = 180°$

$70 + b = 180°$

$b = 180° - 70°$

$= 110°$

$\angle CGF = a = 70°$

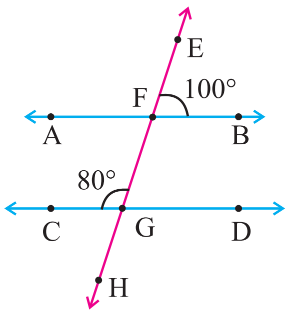
$\angle DGF = b = 110°$

Example 5.14

In the given figure, $\angle BFE = 100°$ and $\angle CGF = 80°$.

Find i) $\angle EFA$, ii) $\angle DGF$,

iii) $\angle GFB$, iv) $\angle AFG$, v) $\angle HGD$.



Solution

$\angle BFE = 100°$ and $\angle CGF = 80°$ (given)

i) $\angle EFA = \angle 80°$ (Corresponding angles)

ii) $\angle DGF = 100°$ (Since corresponding angles are equal)

iii) $\angle GFB = 80°$ (Since alternate interior angles are equal)

iv) $\angle AFG = 100°$ (Since corresponding angles $\angle CGH$ and $\angle AFG$ are equal)

v) $\angle HGD = 80°$ (Since corresponding angles are equal)

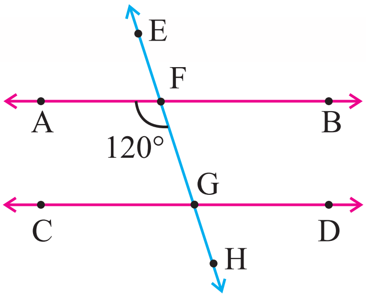
Example 5.15

In the figure, AB || CD, ∠AFG = 120° Find

(i) ∠DGF

(ii) ∠GFB

(iii) ∠CGF



Solution

In the figure, AB || CD and EH is a transversal

(i) ∠AFG = 120° (Given)

∠AFG = ∠DGF = 120° (Since alternate interior angles are equal)

∴ ∠DGF = 120°

(ii) ∠AFG + ∠GFB = 180° (Since sum of the adjacent angle on a line is 180°)

120° + ∠GFB = 180°

∠GFB = 180° – 120°

= 60°

(iii) ∠AFG + ∠CGF = 180°

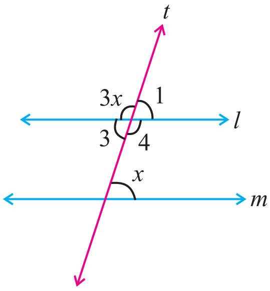
120° + ∠CGF = 180° (Since sum of the adjacent angles on a line is 180°)

∠CGF = 180° – 120°

= 60°

Example 5.16

Find the measure of x in the figure, given $l \parallel m$.



Solution

In the figure, $l \parallel m$

$\angle 3 = x$ (Since alternate interior angles are equal)

$3x + x = 180°$ (Since sum of the adjacent angles on a line is 180°)

$4x = 180°$

$x = \frac{180^0}{4}$

$= 45°$

Exercise 5.4

1. Choose the correct answer

i) If a transversal intersect two lines, the number of angles formed are

(A) 4 (B) 6 (C) 8 (D) 12

ii) If a transversal intersect any two lines the two lines

(A) are parallel (B) are not parallel

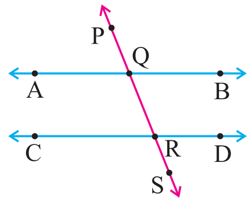
(C) may or may not be parallel (D) are perpendicular

iii) When two parallel lines are cut by a transversal, the sum of the interior angles on the same side of the transversal is

(A) 90° (B) 180° (C) 270° (D) 360°

iv) In the given figure

$\angle$BQR and $\angle$QRC are a pair of



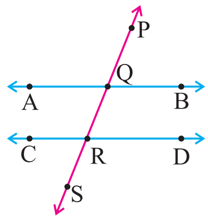
(A) vertically apposite angles

(B) exterior angles

(C) alternate interior angles

(D) corresponding angles

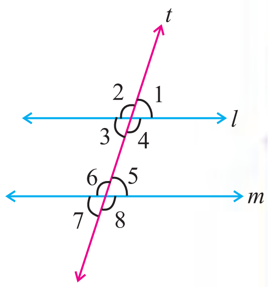
v) In the given figure $\angle$SRD = 110° then the value of $\angle$BQP will be



(A) 110° (B) 100°

(C) 80° (D) 70°

2. In the given figure, state the property that is used in each of the following statement.

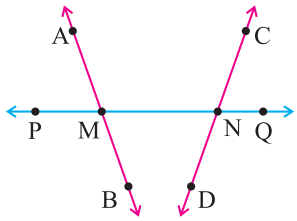


(i) If $l \parallel m$ then $\angle 1 = \angle 5$.

(ii) If $\angle 4 = \angle 6$ then $l \parallel m$.

(iii) If $\angle 4 + \angle 5 = 180°$ then $l \parallel m$.

3. Name the required angles in the figure.



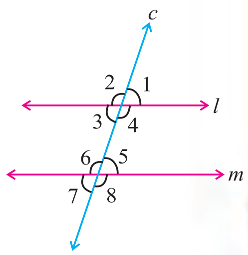
(i) The angle vertically opposite to $\angle$AMN

(ii) The angle alternate to $\angle$CNQ

(iii) The angle corresponding to $\angle$ BMP

(iv) The angle corresponding to $\angle$ BMN

4. In the given figure identify



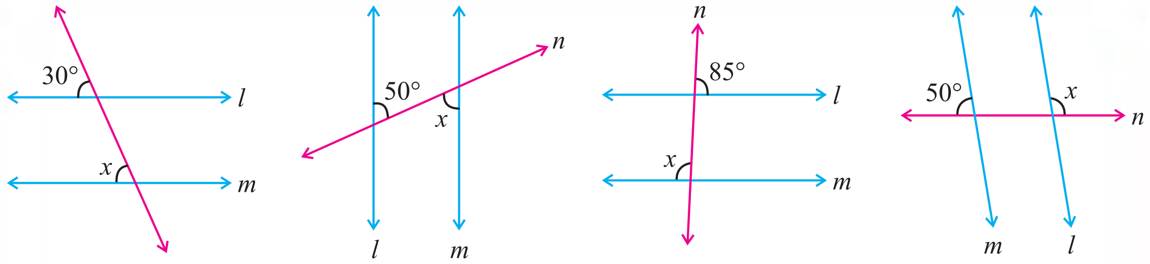
(i) Pairs of corresponding angles

(ii) Pairs of alternate interior angles.

(iii) Pairs of interior angles on the same side of the transversal

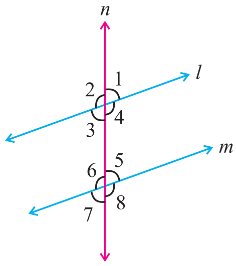
(iv) Vertically opposite angles.

5. Given $l \parallel m$, find the measure of x in the following figures

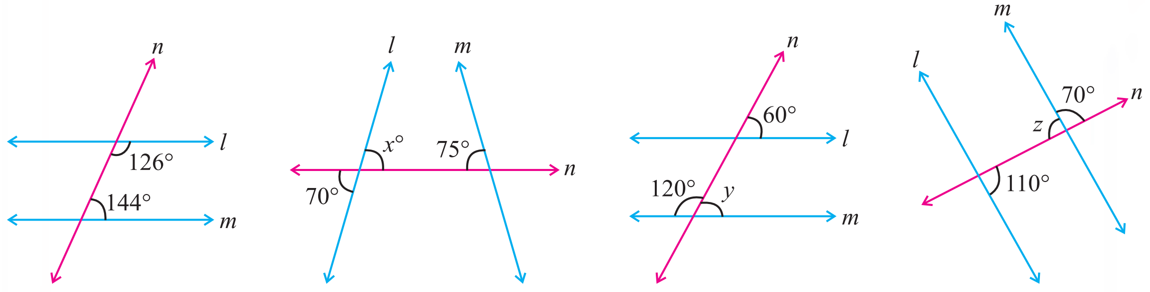


(i) (ii) (iii) (iv)

6. Given $l \parallel m$ and $\angle 1 = 70°$, find the measure of $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$, $\angle 7$ and $\angle 8$.

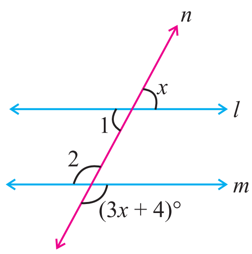


7. In the given figures below, decide whether $l \parallel m$? Give reasons.



(i) (ii) (iii) (iv)

8. Given $l \parallel m$, find the measure of $\angle 1$ and $\angle 2$ in the figure shown.



Triangle: Revision

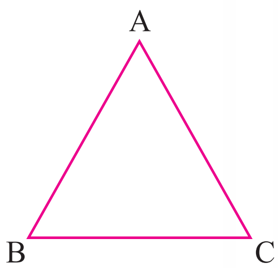


Fig. 5.43

A triangle is a closed plane figure made of three line segments.

In Fig. 5.43 the line segments AB, BC and CA form a closed figure. This is a triangle and is denoted by ΔABC. This triangle may be named as ΔABC or ΔBCA or ΔCAB.

The line segments forming a triangle are the three sides of the triangle. In Fig.5.43 $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are the three sides of the triangle.

The point where any two of the three line segments of a triangle intersect is called the vertex of the triangle. In Fig. 5.43 A,B and C are the three vertices of the ΔABC.

When two line segments intersect, they form an angle at that point. In the triangle in Fig. 5.43 $\overline{AB}$ and $\overline{BC}$ intersect at B and form an angle at that vertex. This angle at B is read as angle B or $\angle$B or $\angle$ABC. Thus a triangle has three angles $\angle$A, $\angle$B and $\angle$C.

In Fig. 5.43 ΔABC has

Sides : $\overline{AB}, \overline{BC}, \overline{CA}$

Angles : $\angle$CAB, $\angle$ABC, $\angle$BCA

Vertices : A, B, C

The side opposite to the vertices A, B, C are BC, AC and AB respectively. The angle opposite to the side BC, CA and AB is $\angle$A, $\angle$B and $\angle$C respectively.

A triangle is a closed figure made of three line segments. It has three vertices, three sides and three angles.

Types of Triangles

Based on sides

A triangle is said to be

Equilateral, when all its sides are equal.

Isosceles, when two of its sides are equal.

Scalene, when its sides are all unequal.

Based on angles

A triangle is said to be

Right angled, when one of its angle is a right angle and the other two angles are acute.

Obtuse - angled, when one of its angle is obtuse and the other two angles are acute.

Acute - angled, when all the three of its angles are acute.

The sum of the lengths of any two sides of a triangle is always greater than the length of the third side.

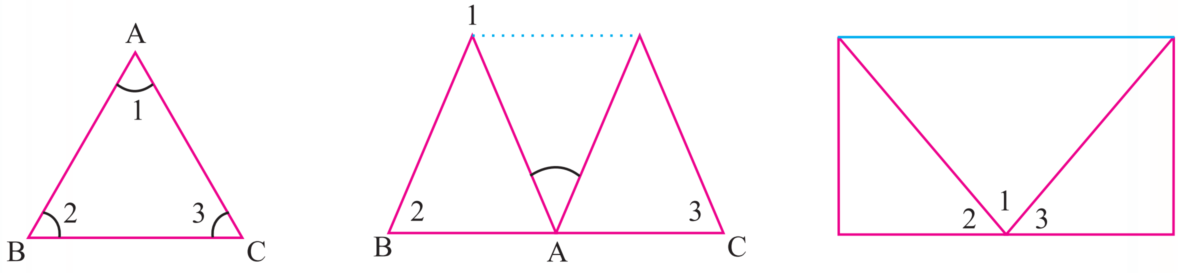
Angle sum property of a triangle:

Try these

Is it possible to form a triangle whose sides are 7cm, 5cm and 13cm?

Activity 8

Draw any triangle ABC on a sheet of paper and mark the angles 1, 2 and 3 on both sides of the paper as shown in Fig. 5.44 (i).



(i) (ii) (iii)

Fig. 5.44

Cut the triangle ABC. Fold the vertex A to touch the side BC as shown in the Fig. 5.44 (ii) Fold the vertices B and C to get a rectangle as shown in the Fig. 5.44 (iii) Now you see that $\angle 1$, $\angle 2$ and $\angle 3$ make a straight line.

From this you observe that

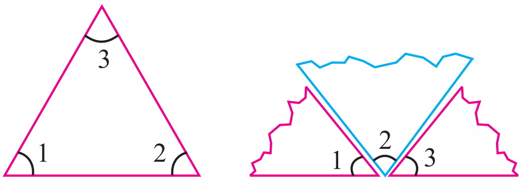
$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

$$\angle A + \angle B + \angle C = 180^\circ$$

The sum of the three angles of a triangle is 180°

Activity 9

Draw a triangle. Cut on the three angles. Re arrange them as shown in Fig. 5.45 (ii). You observe that the three angles now constitute one angle. This angle is a straight angle and so has measure 180°



(i) (ii)

Fig. 5.45

The sum of the three angles of a triangle is 180°

Think it.

1. Can you have a triangle with the three angles less than 60°?

2. Can you have a triangle with two right angles?

Exterior angle of a triangle and its property

Activity 10

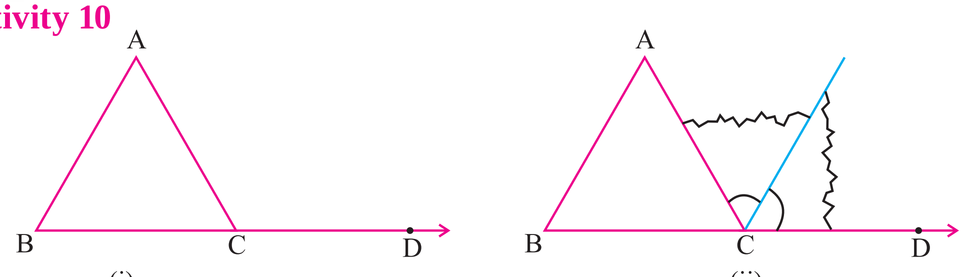


Fig. 5.46

Draw a triangle ABC and produce one of its sides, say BC as shown in Fig. 5.46 (i) observe the angles ACD formed at the point C. This angle lies in the exterior of ΔABC formed at vertex C.

$\angle BCA$ is an adjacent angle to $\angle ACD$. The remaining two angles of the triangle namely $\angle A$ nd $\angle$ B are called the two interior opposite angles.

Now cut out (or make trace copies of) $\angle A$ and $\angle B$ and place them adjacent to each other as shown in Fig. 5.46 (ii)

You observe that these two pieces together entirely cover $\angle$ ACD.

From this we conclude that the exterior angle of a triangle is equal to the sum of the two interior opposite angles.

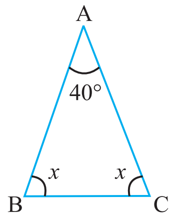
The relation between an exterior angle and its two interior angles is referred to as the exterior angle property of a triangle.

Try these

Draw a triangle ABC and produce one of its sides BC as shown in Fig. 5.46 (i) $\angle ACD$ formed at the point C. Now take a protractor and measure $\angle$ ACD, $\angle A$ and $\angle B$.
Find the sum $\angle$ A + $\angle$ B. and compare it with the measure of $\angle ACD$. Do you observe that $\angle ACD = \angle A + B$?

Example 5.17

In the given figure find the value of *x*.



Solution

$$\angle CAB + \angle ABC + \angle BCA = 180^\circ$$ (Since sum of the three angles of a triangle is 180°)

$$40^\circ + x + x = 180^\circ$$

$$40^\circ + 2x = 180^\circ$$

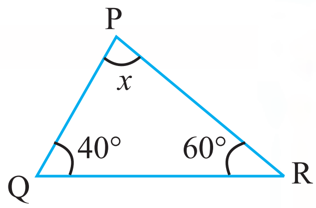
$$2x = 180^\circ - 40^\circ$$

$$2x = 140^\circ$$

$$x = \frac{140^\circ}{2} = 70^\circ$$

The value of $x = 70^\circ$.

Example 5.18



Two angles of a triangle are 40° and 60°. Find the third angle.

Solution

$$\angle RPQ + \angle PQR + \angle QRP = 180^\circ \quad \text{(Since sum of the three angles of a triangle is } 180^\circ\text{)}$$

$$x + 40^\circ + 60^\circ = 180^\circ$$

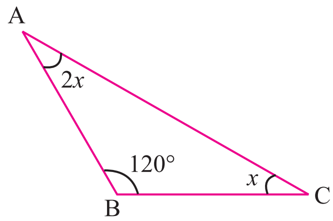
$$x + 100^\circ = 180^\circ$$

$$x = 180^\circ - 100^\circ$$

$$= 80^\circ$$

$$\therefore \text{ The third angle } x = 80^\circ$$

Example 5.19



In the given figure, find the measure of $\angle A$.

Solution

$$\angle CAB + \angle ABC + \angle BCA = 180^\circ \quad \text{(Since sum of the three angles of a triangle is } 180^\circ\text{)}$$

$$2x + 120^\circ + x = 180^\circ$$

$$3x + 120^\circ = 180^\circ$$

$$3x = 180^\circ - 120^\circ$$

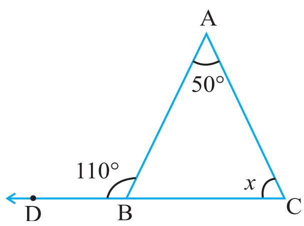
$$3x = 60^\circ$$

$$x = \frac{60^\circ}{3}$$

$$= 20^\circ$$

$$\therefore \angle A = 2x = 2 \times 20^\circ = 40^\circ$$

Example 5.20



In the given figure. Find the value of x.

Solution

In the figure exterior angle = $\angle ABD = 110^\circ$.

Sum of the two interior opposite angle = $\angle BCA + \angle CAB$

$$= x + 50^\circ$$

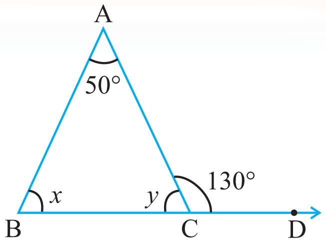
$$x + 50^\circ = 110^\circ \quad \text{(Since the sum of the two interior opposite angle is equal to the exterior angle)}$$

$$x = 110^\circ - 50^\circ$$

$$= 60^\circ$$

$\therefore$ The value of x is 60°.

Example 5.21



In the given figure find the values of x and y.

Solution

In the give figure,

$$\begin{aligned} \text{Exterior angle} &= \angle DCA = 130^\circ \\ 50^\circ + x &= 130^\circ \qquad \text{(Since sum of the two interior opposite} \\ x &= 130^\circ - 50^\circ \ \text{angle is equal to the exterior angle)} \\ &= 80^\circ \end{aligned}$$

In ΔABC,

$$\begin{aligned} \angle A + \angle B + \angle C &= 180^\circ \ \text{(Since sum of three angles of a triangle is } 180^\circ\text{)} \\ 50^\circ + x + y &= 180^\circ \\ 50^\circ + 80^\circ + y &= 180^\circ \\ 130^\circ + y &= 180^\circ \\ y &= 180^\circ - 130^\circ \\ &= 50^\circ \end{aligned}$$

$\therefore$ The values of $x = 80^\circ$ and $y = 50^\circ$.

Aliter:

$$\begin{aligned} \angle ACB + \angle DCA &= 180^\circ \ \text{(Since sum of the adjacent angles on a line is } 180^\circ\text{)} \\ y + 130^\circ &= 180^\circ \\ y &= 180^\circ - 130^\circ \\ &= 50^\circ \end{aligned}$$

In Δ ABC,

$$\begin{aligned} \angle A + \angle B + \angle C &= 180^\circ \ \text{(Since sum of the three angles of a triangle is } 180^\circ\text{)} \\ 50^\circ + x + y &= 180^\circ \\ 50^\circ + x + 50^\circ &= 180^\circ \\ 100^\circ + x &= 180^\circ \\ x &= 180^\circ - 100^\circ \\ &= 80^\circ \end{aligned}$$

Example 5.22

Three angles of a triangle are $3x + 5°$, $x + 20°$, $x + 25°$. Find the measure of each angle.

Solution

$$\begin{aligned}
\text{Sum of the three angles of a triangle} &= 180° \\
3x + 5° + x + 20° + x + 25° &= 180° \\
5x + 50° &= 180° \\
5x &= 180° - 50° \\
5x &= 130° \\
x &= \frac{130°}{5} \\
&= 26° \\
3x + 5° &= (3 \times 26°) + 5° = 78° + 5° = 83° \\
x + 20° &= 26° + 20° = 46° \\
x + 25° &= 26° + 25° = 51°
\end{aligned}$$

$\therefore$ The three angles of a triangle are 83°, 46° and 51°.

Exercise 5.5

1. Choose the correct answer.

i) The sum of the three angles of a triangle is

(A) 90° (B) 180° (C) 270° (D) 360°

ii) In a triangle, all the three angles are equal, then the measure of each angle is

(A) 30^0 (B) 45^0 (C) 60^0 (D) 90^0

iii) Which of the following can be angles of a triangle?

(A) $50^0, 30^0, 105^0$ (B) $36^0, 44^0, 90^0$ (C) $70^0, 30^0, 80^0$ (D) $45^0, 45^0 80^0$

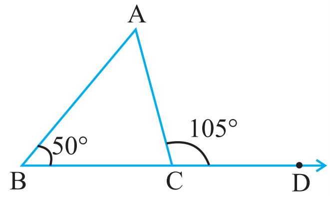
iv) Two angles of a triangle are 40° and 60°, then the third angle is

(A) 20^0 (B) 40^0 (C) 60^0 (D) 80^0

v) In ΔABC, BC is produced to D and $\angle ABC = 50^0$, $\angle ACD = 105^0$, then $\angle$ BAC will be equal to

(A) 75° (B) 15°

(C) 40° (D) 55°



2. State which of the following are triangles.

   (i) $\angle A = 25^0$ $\quad \angle B = 35^0$ $\quad \angle C = 120^0$

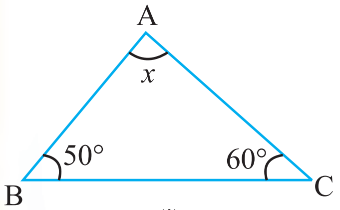
   (ii) $\angle P = 90^0$ $\quad \angle Q = 30^0$ $\quad \angle R = 50^0$

   (iii) $\angle X = 40^0$ $\quad \angle Y = 70^0$ $\quad \angle Z = 80^0$

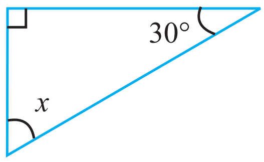
3. Two angles of a triangle is given, find the third angle.

   (i) $75^0, 45^0$ $\quad$ (ii) $80^0, 30^0$ $\quad$ (iii) $40^0, 90^0$ $\quad$ (iv) $45^0, 85^0$

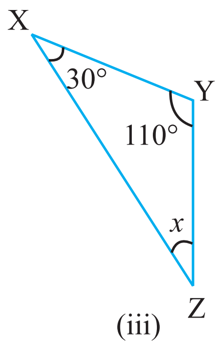
4. Find the value of the unknown x in the following diagrams:



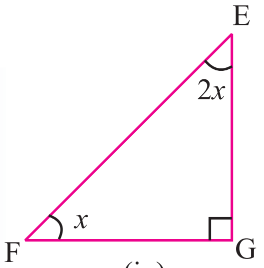
(i)



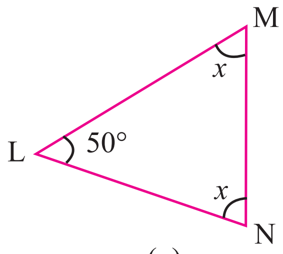
(ii)



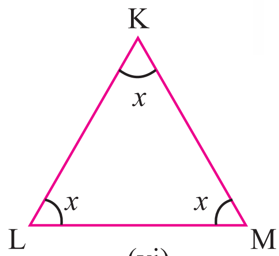
(iii)



(iv)

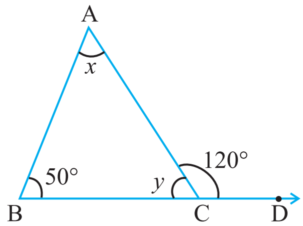


(v)

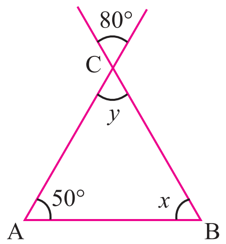


(vi)

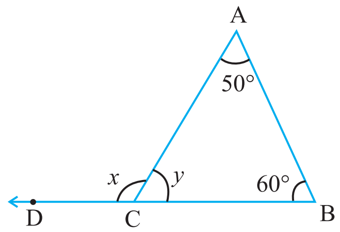
5. Find the values of the unknown x and y in the following diagrams:



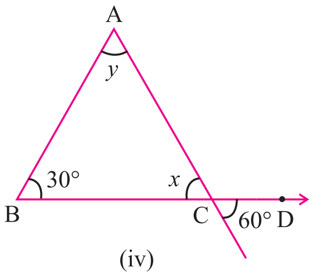
(i)



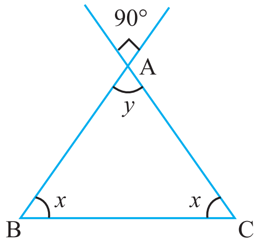
(ii)



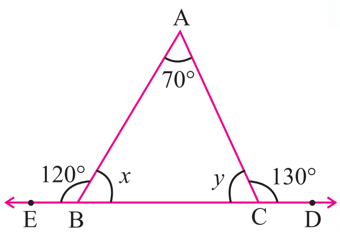
(iii)



(iv)



(v)



(vi)

6. Three angles of a triangle are $x + 5°$, $x + 10°$ and $x + 15°$ find x.

Points to Remember

1. Symmetry refers to the exact match in shape and size between two halves of an object.
2. When a line divides a given figure into two equal halves such that the left and right halves matches exactly then we say that the figure is symmetrical about the line. This line is called the line of symmetry or axis of symmetry.
3. Each regular polygon has as many lines of symmetry as it has sides.
4. Some objects and figures have no lines of symmetry.
5. Figures which can be rotated through an angle less than 360° to get the same shape are said to have rotational symmetry.
6. The order of rotational symmetry is the number that tell us how many times a figure looks exactly the same while it takes one complete rotation about the centre.
7. The objects having no line of symmetry can have rotational symmetry.
8. If two angles have the same vertex and a common arm, then the angles are called adjacent angles.
9. The sum of the adjacent angles on a line is 180°.
10. When two lines interset, the vertically opposite angles are equal.
11. Angle at a point is 360°.
12. Two straight lines are said to be parallel to each other if they do not intersect at any point.
13. A straight line intersects two or more lines at distinct points is called a transversal to the given line.
14. When two parallel lines are cut by a transversal,

    (a) each pair of corresponding angles are equal.

    (b) each pair of alternate angles are equal.

    (c) each pair of interior angles on the same side of the transversal are supplementary.
15. The sum of the three angles of a triangle is 180°.
16. In a triangle an exterior angle is equal to the sum of the two interior opposite angles.

6 PRACTICAL GEOMETRY

6.1 Introduction

This chapter helps the students to understand and confirm the concepts they have learnt already in theoretical geometry. This also helps them to acquire some basic knowledge in geometry which they are going to prove in their later classes. No doubt, all the students will do the constructions actively and learn the concepts easily.

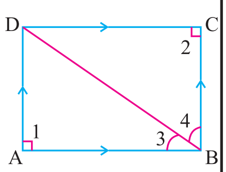
In the previous class we have learnt to draw a line segment, the parallel lines, the perpendicular lines and also how to construct an angle.

Here we are going to learn about the construction of perpendicular bisector of a line segment, angle bisector, some angles using scale and compass and the construction of triangles.

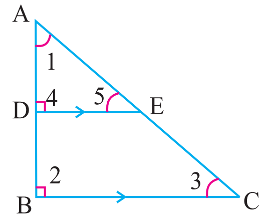
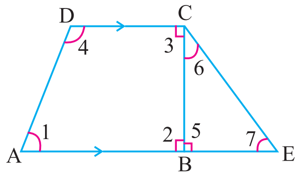
Review

To recall the concept of angles, parallel lines and perpendicular lines from the given figure.

We shall identify the points, the line segments, the angles, the parallel lines and the perpendicular lines from the figures given below in the table.

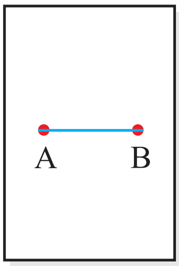
	Figures	Points identified	Lines identified	Angles identified	Parallel lines	Perpendicular lines
1		A, B, C and D	AB, BC, CD, AD, and BD	1 - $\angle BAD$ ($\angle A$) 2 - $\angle DCB$ ($\angle C$) 3 - $\angle DBA$ 4 - $\angle CBD$	AB $\parallel$ DC BC $\parallel$ AD	AB $\perp$ AD AB $\perp$ BC BC $\perp$ CD CD $\perp$ AD

Try these

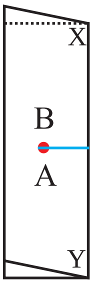
No.	Figures	Points identified	Lines identified	Angles identified	Parallel lines	Perpendicular lines
2						
3						

6.2 Perpendicular bisector of the given line segment

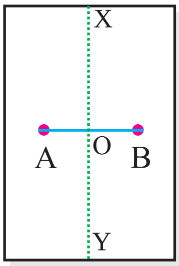
(i) Activity : Paper folding



- Draw a line segment AB on a sheet of paper.



- Fold the paper so that the end point B lies on A. Make a crease XY on the paper.



- Unfold the paper. Mark the point O where the line of crease XY intersects the line AB.

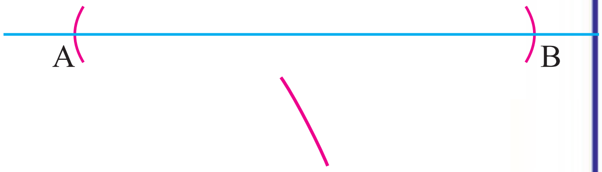
- By actual measurement we can see that OA = OB and the line of crease XY is perpendicular to the line AB.

The line of crease XY is the perpendicular bisector of the line AB.

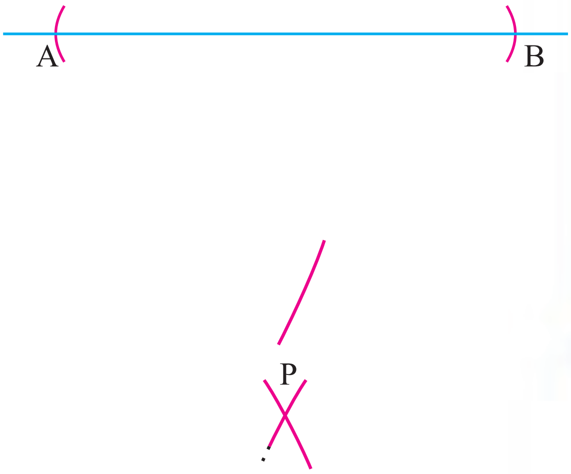
> The perpendicular bisector of a line segment is a perpendicular line drawn at its midpoint.

(ii) To construct a perpendicular bisector to a given line segment.

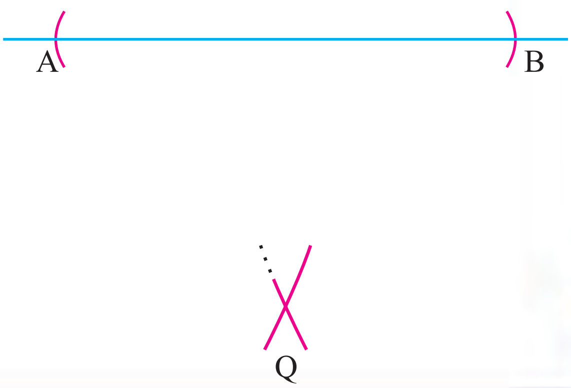
Step 1 : Draw a line segment AB of the given measurement.



Step 2 : With 'A' as centre draw arcs of radius more than half of AB, above and below the line AB.

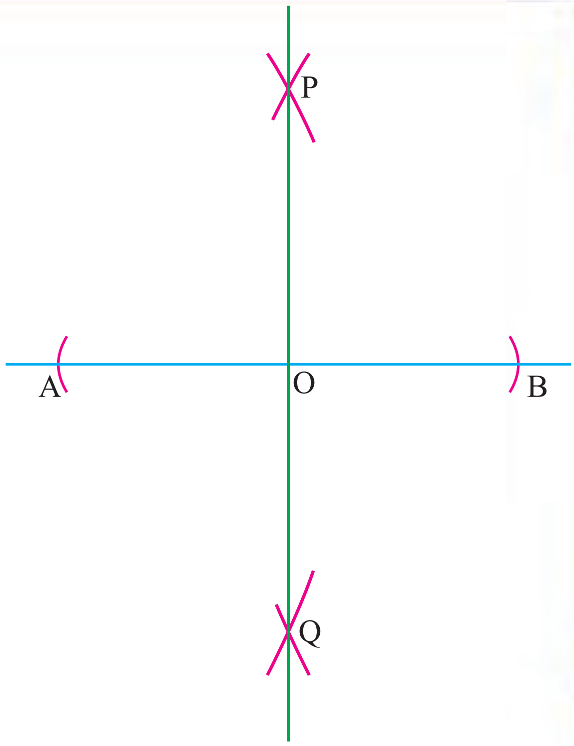


Step 3 : With 'B' as centre and with the same radius draw two arcs. These arcs cut the previous arcs at P and Q.



Step 4 : Join PQ. Let PQ intersect AB at 'O'.

PQ is a perpendicular bisector of AB.



Try these

Mark any point on the perpendicular bisector PQ. Verify that it is equidistant from both A and B.

Do you know?

The perpendicular bisector of a line segment is the axis of symmetry for the line segment.

Think !

Can there be more than one perpendicular bisector for the given line segment?

Example 6.1

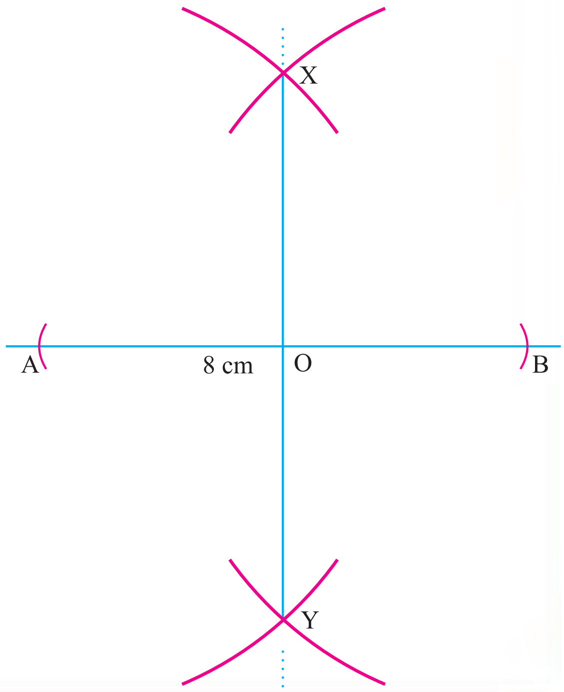
Draw a perpendicular bisector to the line segment AB = 8 cm.

Solution

Step 1 : Draw the line segment AB = 8cm.

Step 2 : With 'A' as centre draw arcs of radius more than half of AB above and below the line AB.

Step 3 : With 'B' as centre draw the arcs of same radius to cut the previous arcs at X and Y.



Step 4 : Join XY to intersect the line AB at O.

XY is the perpendicular bisector of AB.

Try these

1. With PQ = 6.5 cm as diameter draw a circle.
2. Draw a line segment of length 12 cm. Using compass divide it into four equal parts. Verify it by actual measurement.
3. Draw a perpendicular bisector to a given line segment AC. Let the bisector intersect the line at 'O'. Mark the points B and D on the bisector at equal distances from O. Join the points A, B, C and D in order. Verify whether all lines joined are of equal length.

Think!

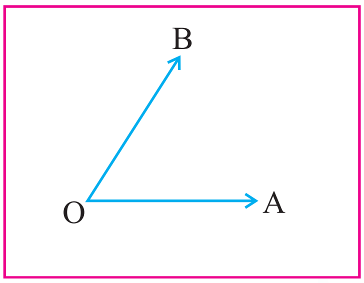
In the above construction mark the points B and D on the bisector, such that OA = OB = OC = OD. Join the points A, B, C and D in order. Then

1. Do the lines joined are of equal length?
2. Do the angles at the vertices are right angles?
3. Can you identify the figure?

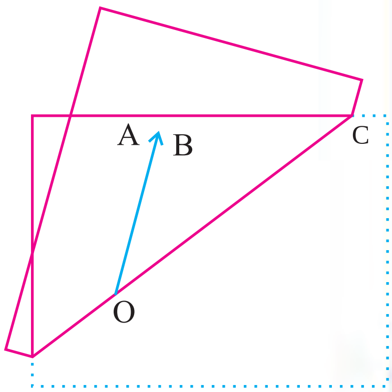
6.3 Angle Bisector

(ii) Activity : Paper folding

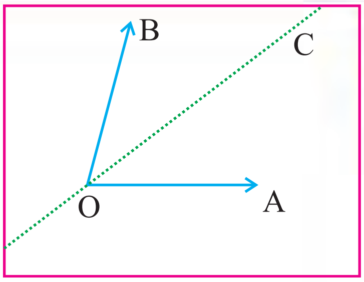
- Take a sheet of paper and mark a point O on it. With O as initial point draw two rays OA and OB to make $\angle AOB$.



- Fold the sheet through 'O' such that the rays OA and OB coincide with each other and make a crease on the paper.



- Let OC be the line of crease on the paper after unfold. By actual measurement, ∠AOC and ∠BOC are equal.
- So the line of crease OC divides the given angle into two equal parts.

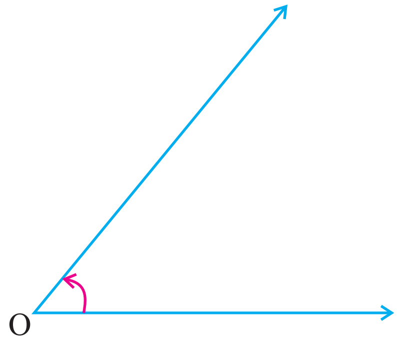


- This line of crease is the line of symmetry for ∠AOB.
- This line of symmetry for ∠AOB is called the angle bisector.

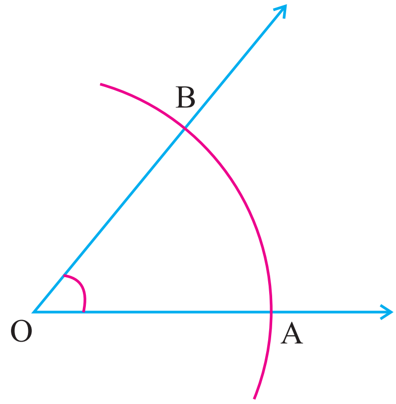
The angle bisector of a given angle is the line of symmetry which divides the angle into two equal parts.

(ii) To construct an angle bisector of the given angle using scale and compass

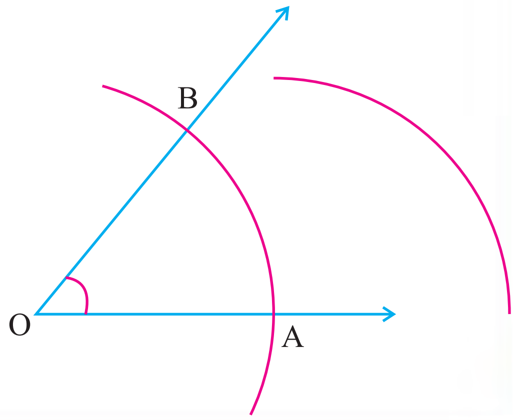
Step 1 : Construct an angle of given measure at O.



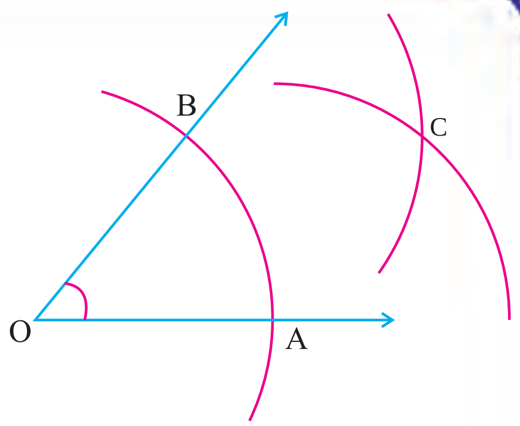
Step 2 : With 'O ' as centre draw an arc of any radius to cut the rays of the angle at A and B.



Step 3 : With 'A' as centre draw an arc of radius more than half of AB, in the interior of the given angle.

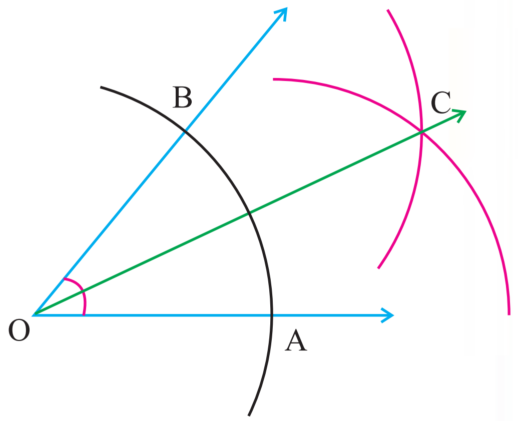


Step 4 : With 'B' as centre draw an arc of same radius to cut the previous arc at 'C'.



Step 5 : Join OC.

OC is the angle bisector of the given angle.



Try these

Mark any point on the angle bisector OC. Verify that it is equidistant from the rays OA and OB.

Example 6.2

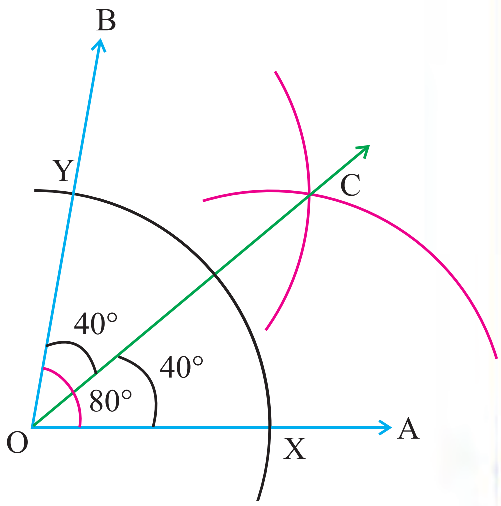
Construct $\angle AOB = 80°$ and draw its angle bisector.

Solution

Step 1 : Construct $\angle AOB = 80°$ angle at the point 'O' using protractor.

Step 2 : With 'O' as centre draw an arc of any radius to cut the rays OA and OB at the points X and Y respectively.

Step 3 : With 'X' as centre draw an arc of radius more than half of XY in the interior of the angle.



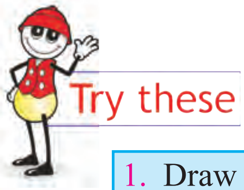
Step 4 : With 'Y' as centre draw an arc of the same radius to cut the previous arc at C. Join OC.

OC is the angle bisector of the given angle 80°.

Draw an angle of measure 120° and divide into four equal parts.

Exercise 6.1

1. Draw the line segment AB = 7cm and construct its perpendicular bisector.
2. Draw a line segment XY = 8.5 cm and find its axis of symmetry.
3. Draw a perpendicular bisector of the line segment AB = 10 cm.
4. Draw an angle measuring 70° and construct its bisector.
5. Draw an angle measuring 110° and construct its bisector.
6. Construct a right angle and bisect it using scale and compass.



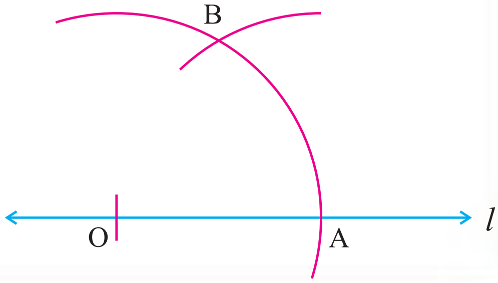
1. Draw a circle with centre 'C' and radius 4 cm. Draw any chord AB. Construct perpendicular bisector to AB and examine whether it passes through the centre of the circle.
2. Draw perpendicular bisectors to any two chords of equal length in a circle. (i) Where do they meet? (ii) Verify whether the chords are at a same distance from the centre.
3. Plot three points not on a straight line. Find a point equidistant from them.

Hint: Join all the points in order. You get a triangle. Draw perpendicular bisectors to each side. They meet at a point which is equidistant from the points you have plotted. This point is called circumcentre.

6.4 To construct angles 60°, 30°, 120°, 90° using scale and compass.

(i) Construction of 60° angle

Step 1 : Draw a line '*l*' and mark a point 'O' on it.

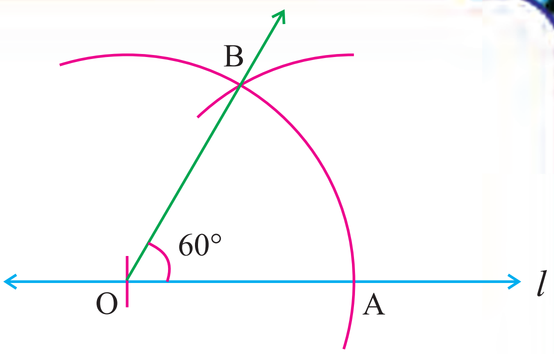


Step 2 : With 'O' as centre draw an arc of any radius to cut the line at A.

Step 3 : With the same radius and A as centre draw an arc to cut the previous arc at B.

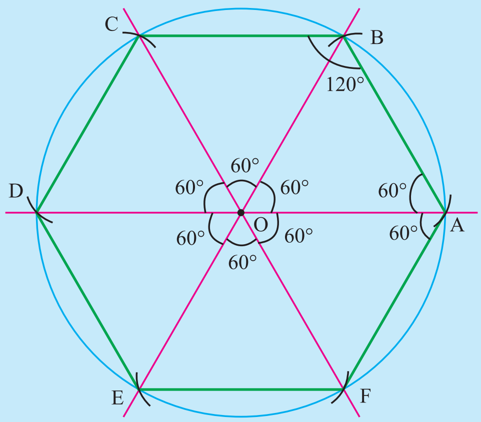
Step 4 : Join OB.

$\angle AOB = 60°$.



Try these

Draw a circle of any radius with centre 'O'. Take any point 'A' on the circumference. With 'A' as centre and OA as radius draw an arc to cut the circle at 'B'. Again with 'B' as centre draw the arc of same radius to cut the circle at 'C'. Proceed so on. The final arc will pass through the point 'A'. Join all such points A, B, C, D, E and F in order. ABCDEF is a regular Hexagon.



From the above figure we came to know

(i) The circumference of the circle is divided into six equal arc length subtending 60° each at the centre. In any circle a chord of length equal to its radius subtends 60° angle at the centre.

(ii) Total angle measuring around a point is 360°.

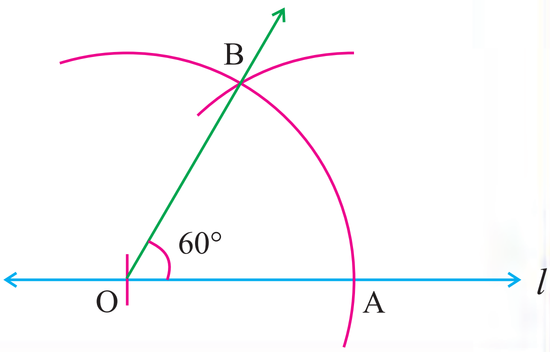
(iii) It consists of six equilateral triangles.

(ii) Construction of 30° angle

First you construct 60° angle and then bisect it to get 30° angle.

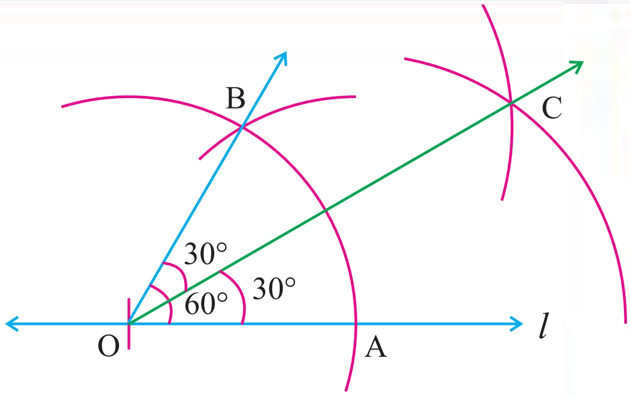
Step 1 : Construct 60° (as shown in the above construction (i))

Step 2 : With 'A' as centre, draw an arc of radius more than half of AB in the interior of $\angle AOB$.



Step 3 : With the same radius and with B as centre draw an arc to cut the previous one at C. Join OC.

$\angle AOC$ is 30°.



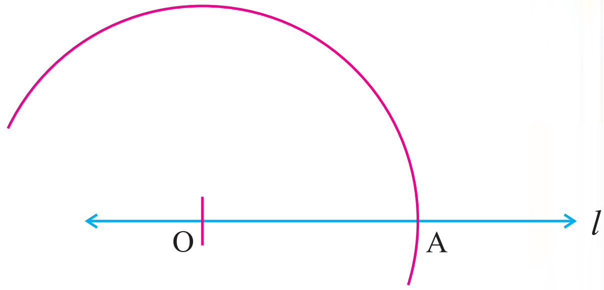
Try these

How will you construct 15° angle.

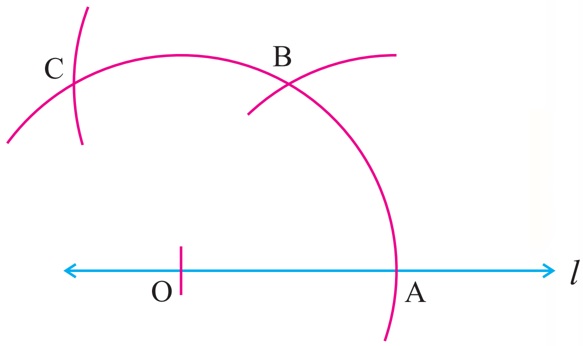
(iii) Construction of 120° angle

Step 1 : Mark a point 'O' on a line 'l'.

Step 2 : With 'O' as centre draw an arc of any radius to cut the line l at A.



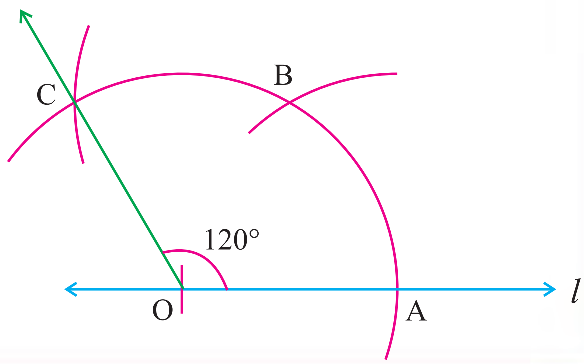
Step 3 : With same radius and with 'A' as centre draw another arc to cut the previous arc at 'B'.



Step 4 : With 'B' as centre draw another arc of same radius to cut the first arc at 'C'.

Step 5 : Join OC.

$\angle AOC$ is 120°.

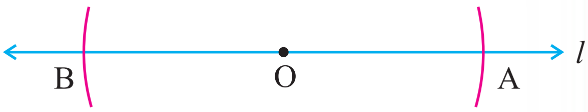


(iv) Construction of 90° angle

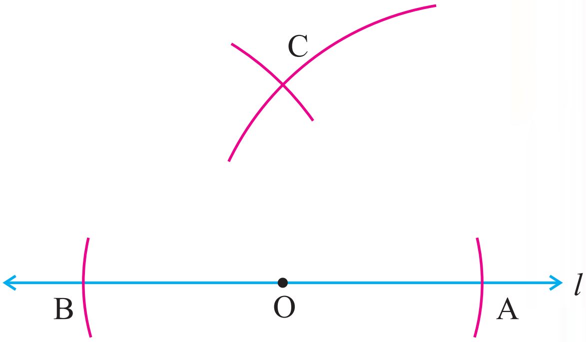
To construct 90° angle, we are going to bisect the straight angle 180°.

Step 1 : Mark a point 'O' on a straight line 'l'.

Step 2 : With 'O' as centre draw arcs of any radius to cut the line l at A and B. Now $\angle AOB = 180°$.

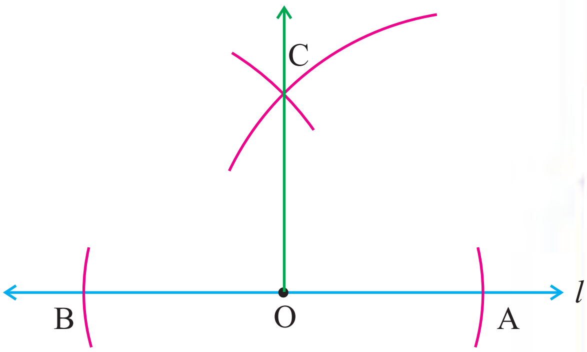


Step 3 : With A and B as centres and with the radius more than half of AB draw arcs above AB to instersect each other at 'C'.



Step 4 : Join OC.

$\angle AOC = 90°$.



Try these

1. Construct an angle of measure 60° and find the angle bisector of its complementary angle.
2. Trisect the right angle.
3. Construct the angles of following measures: 22½°, 75°, 105°, 135°, 150°

Do you know?

To construct a perpendicular for a given line at any point on it, you can adopt this method for the set-square method, as an alternate.

Exercise 6.2

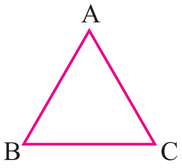
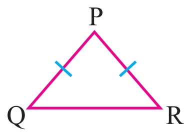
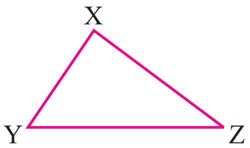
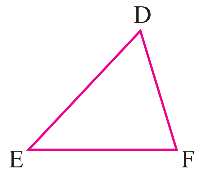
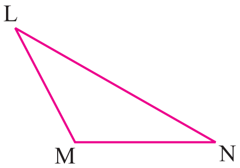
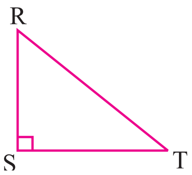
1. Construct the angles of following measures with ruler and compass.

   (i) 60° (ii) 30° (iii) 120° (iv) 90°

6.5 Construction of triangles

In the previous class, we have learnt the various types of triangles on the basis of using their sides and angles. Now let us recall the different types of triangles and some properties of triangle.

Classification of triangles

CLASSIFICATION OF TRIANGLES		No.	Name of Triangle	Figure	Note
	on the basis SIDES	1	Equilateral triangle		Three sides are equal
		2	Isosceles triangle		Any two sides are equal
		3	Scalene triangle		Sides are unequal
	on the basis ANGLES	4	Acute angled triangle		All the three angles are acute (less than 90°)
		5	Obtuse angled triangle		Any one of the angles is obtuse (more than 90°)
		6	Right angled triangle		Any one of the angles is right angle (90°)

Some properties of triangle

1. The sum of the lengths of any two sides of a triangle is greater than the third side.
2. The sum of all the three angles of a triangle is 180°.

To construct a triangle we need three measurements in which at least the length of one side must be given. Let us construct the following types of triangles with the given measurements.

(i) Three sides (SSS).

(ii) Two sides and included angle between them (SAS).

(iii) Two angles and included side between them (ASA).

(i) To construct a triangle when three sides are given (SSS Criterion)

Example 6.3

Construct a triangle ABC given that AB = 4cm, BC = 6 cm and AC = 5 cm.

Solution

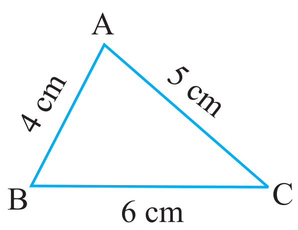
Given measurements

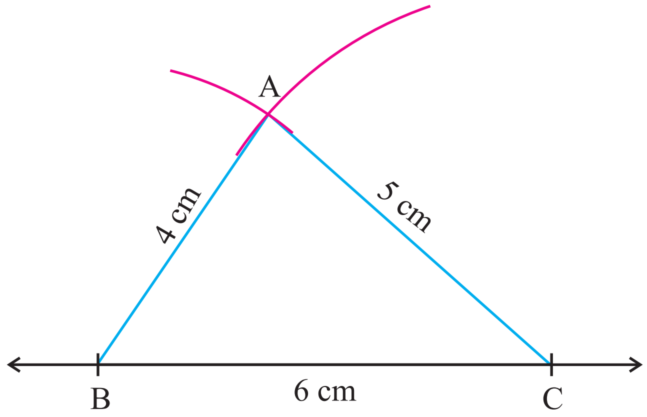
AB = 4cm

BC = 6 cm

AC = 5 cm.

Rough Diagram





Steps for construction

Step 1 : Draw a line segment BC = 6cm

Step 2 : With ‘B’ as centre, draw an arc of radius 4 cm above the line BC.

Step 3 : With ‘C’ as centre, draw an arc of 5 cm to intersect the previous arc at ‘A’

Step 4 : Join AB and AC.

Now ABC is the required triangle.

Try these

By using protector measure all the angles of a triangle. Find the sum of all the three angles of a triangle.

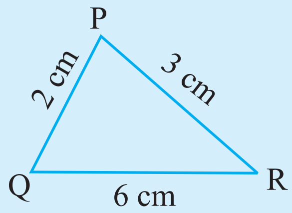
Do you know?

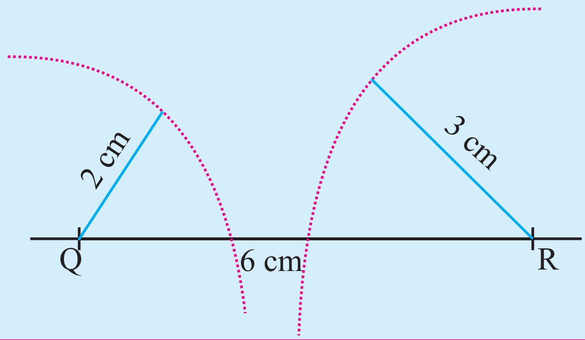
1. A student attempted to draw a triangle with given measurements PQ = 2cm, QR = 6cm, PR = 3 cm. (as in the rough figure). First he drew QR = 6cm. Then he drew an arc of 2cm with P as centre and he drew an arc of radius 3 cm with R as centre. They could not intersect each to get P.

   (i) What is the reason?

   (ii) What is the triangle property in connection with this?

Rough Figure





The sum of any two sides of a triangle is always greater than the third side.

Try these

Draw the bisectors of the three angles of a triangle. Check whether all of them pass through a same point. This point is incentre.

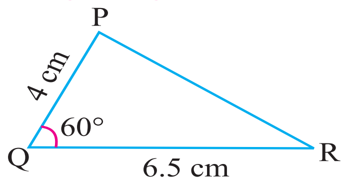
(ii) To construct a triangle when Two sides and an angle included between them are given. (SAS Criterion)

Example 6.4

Construct a triangle PQR given that PQ = 4 cm, QR = 6.5 cm and $\angle PQR = 60°$.

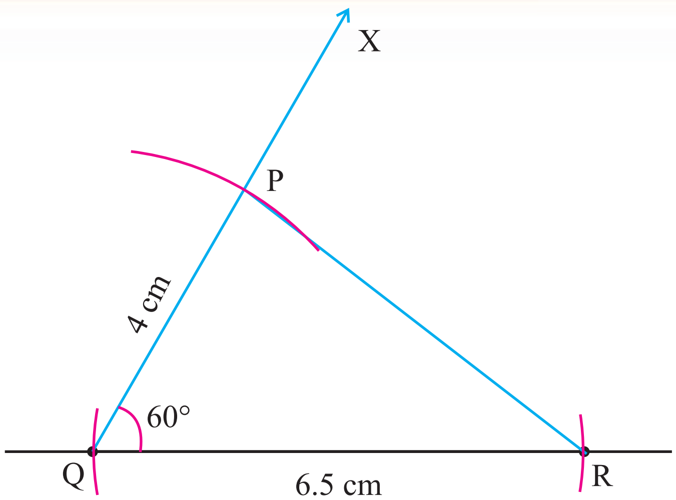
Solution

Rough Diagram



Given measurements

PQ	=	4 cm
QR	=	6.5 cm
$\angle PQR$	=	60°



Steps for construction

Step 1 : Draw the line segment QR = 6.5 cm.

Step 2 : At Q, draw a line QX making an angle of 60^0 with QR.

Step 3 : With Q as centre, draw an arc of radius 4 cm to cut the line (QX) at P.

Step 4 : Join PR.

PQR is the required triangle.

Try these

Construct a triangle with the given measurements XY = 6cm, YZ = 6cm and ∠XYZ = 70°. Measure the angles of the triangle opposite to the equal sides. What do you observe?

(iii) To construct a triangle when two of its angles and a side included between them are given. (ASA criterion)

Example 6.5

Construct a triangle XYZ given that XY = 6 cm, ∠ZXY = 30° and ∠XYZ = 100°. Examine whether the third angle measures 50°.

Solution

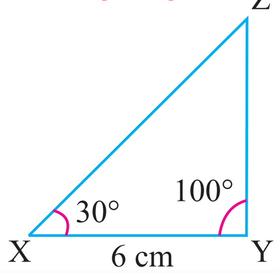
Given measurements

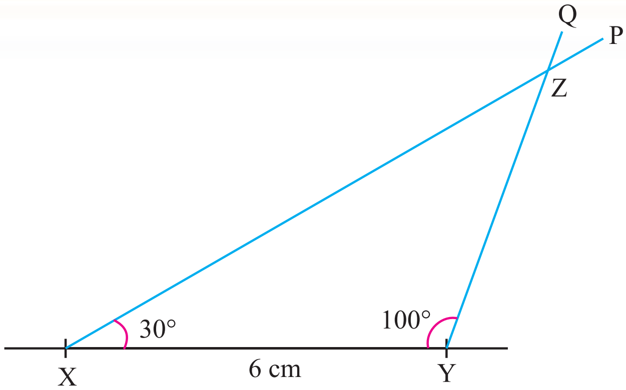
XY = 6 cm

∠ZXY = 30°

∠XYZ = 100°

Rough Figure





Step 1 : Draw the line segment XY = 6cm.

Step 2 : At X, draw a ray XP making an angle of 30° with XY.

Step 3 : At Y, draw another ray YQ making an angle of 100° with XY. The rays XP and YQ intersect at Z.

Step 4 : The third angle measures 50° i.e $\angle Z = 50°$.

Construct a triangle PQR given that PQ = 7 cm, $\angle Q = 70°$, $\angle R = 40°$.

Hint: Use the Angle Sum Property of a triangle.

Exercise : 6.3

I. Construct the triangles for the following given measurements.

1. Construct $\triangle PQR$, given that PQ = 6cm, QR = 7cm, PR = 5cm.
2. Construct an equilateral triangle with the side 7cm. Using protector measure each angle of the triangle. Are they equal?
3. Draw a triangle DEF such that DE = 4.5cm, EF = 5.5cm and DF = 4.5cm. Can you indentify the type of the triangle? Write the name of it.

II. Construct the triangles for the following given measurements.

4. Construct $\triangle XYZ$, given that YZ = 7cm, ZX = 5cm, $\angle Z = 50°$.
5. Construct $\triangle PQR$ when PQ = 6cm, PR = 9cm and $\angle P = 100°$.
6. Construct $\triangle ABC$ given that AB = 6 cm, BC = 8 cm and $\angle B = 90°$ measure length of AC.

III. Construct the triangles for the following given measurements.

7. Construct $\triangle XYZ$, when X = 50°, Y = 70° and XY = 5cm.
8. Construct $\triangle ABC$ when A = 120°, B = 30° and AB = 7cm.
9. Construct $\triangle LMN$, given that $\angle L = 40°$, $\angle M = 40°$ and LM = 6cm. Measure and write the length of sides opposite to the $\angle L$ and $\angle M$. Are they equal? What type of Triangle is this?

7 DATA HANDLING

Introduction

Data Handling is a part of statistics. The word statistics is derived from the Latin word " Status". Like Mathematics, Statistics is also a science of numbers. The numbers referred to here are data expressed in numerical form like,

(i) Marks of students in a class

(ii) Weight of children of particular age in a village

(iii) The amount of rainfall in a region over a period of years.

Statistics deals with the methods of collection, classification, analysis and interpretation of such data.

Any collection of information in the form of numerical figures giving the required information is called data.

Raw data

The marks obtained in Mathematics test by the students of a class is a collection of observations gathered initially. The information which is collected initially and presented randomly is called a raw data.

The raw data is an unprocessed and unclassified data.

Grouped data

Some times the collected raw data may be huge in number and it gives us no information as such. Whenever the data is large, we have to group them meaningfully and then analyse.

The data which is arranged in groups or classes is called a grouped data.

Collection of data

The initial step of investigation is the collection of data. The collected data must be relevant to the need.

Primary data

Try these

Find the relevant data for the students from tribal villages are good visual learners.

For example, Mr. Vinoth, the class teacher of standard VII plans to take his students for an excursion. He asks the students to give their choice for

(i) particular location they would like to go

(ii) the game they would like to play

(iii) the food they would like to have on their trip

For all these, he is getting the information directly from the students. This type of collection of data is known as primary data.

7.1 Collection and Organizing of Continues Data

Secondary data

Mr. Vinoth, the class teacher of standard VII is collecting the information about weather for their trip. He may collects the information from the internet, news papers, magazines, television and other sources. These external sources are called secondary data.

Variable

As for as statistics is concerned the word variable means by measurable quantity which takes any numerical values within certain limits.

Few etxamples are (i) age, (ii) income, (iii) height and (iv) weight.

Frequency

Suppose we measure the height of students in a school. It is possible that a particular value of height say 140 cm gets repeated. We then count the number of times the value occurs. This number is called the frequency of 140 cm.

The number of times a particular value repeats itself is called its frequency.

Range

The difference between the highest value and the lowest value of a particular data is called the range.

Example 7.1

Let the heights (in cm) of 20 students in a class be as follows.

120, 122, 127, 112, 129, 118, 130, 132, 120, 115

124, 128, 120, 134, 126, 110, 132, 121, 127, 118.

Here the least value is 110 cm and the highest value is 134 cm.

Range = Highest value - Lowest value

$= 134 - 110 = 24$

Class and Class Interval

The above example we take 5 classes say 110 - 115, 115, -120, 120 - 125, 125 - 130, 130 - 135 and each class is known as class interval. The class interval must be of equal size. The number of classes is neither too big nor too small. i.e The optimum number of classes is between 5 and 10.

Class limits

In class 110 - 115, 110 is called the lower limit of the class and 115 is the upper limit.

Width (or size) of the class interval:

The difference between the upper and lower limit is called the width of the class interval. In the above example the width of the class interval is 115 - 110 = 5. By increasing the class interval, we can reduce the number of classes.

There are two types of class intervals. They are (i) inclusive form and (ii) Exclusive form.

(i) Inclusive form

In this form, the lower limit as well as upper limit will be included in that class interval. For example in the first class interval 110 - 114 the heights 110 as well as 114 are included. In the second class interval 115 - 119 both the heights 115 and 119 are included and so on.

(ii) Exclusive form:

In the above example 7.1, in the first class interval 110 - 115, 110 cm is included and 115 cm is excluded. In the second class interval 115 is included and 120 is excluded and so on. Since the two class intervals contain 115 cm. It is customary to include 115 cm in the class interval 115 cm - 120 cm, which is the lower limit of the class interval.

Tally marks

In the above example 7.1, the height 110 cm, 112 cm belongs to in the class interval 110 - 115. We enter | | tally marks. Count the tally marks and enter 2 as the frequency in the frequency column.

If five tally marks are to be made we mark four tally marks first and the fifth one one is marked across, so that $\cancel{||||}$ represents a cluster of five tally marks.

To represent seven, we use a cluster of five tally marks and then add two more tally marks as shown $\cancel{||||}$ ||.

Frequency Table

A table which represents the data in the form of three columns, first column showing the variable (Number) and the second column showing the values of the variable (Tally mark) and the third column showing their frequencies is called a **frequency table** (Refer table 7.3).

If the values of the variable are given using different classes and the frequencies are marked against the respective classes, we get a frequency distribution. All the frequencies are added and the number is written as the total frequency for the entire intervals. This must match the total number of data given. The above process of forming a frequency table is called tabulation of data.

Now we have the following table for the above data. (Example 7.1)

Inclusive form

<table>
<tr><th>Class Interval</th><th>Tally Marks</th><th>Frequency</th></tr>
<tr><td>110 - 114</td><td>||</td><td>2</td></tr>
<tr><td>115 - 119</td><td>|||</td><td>3</td></tr>
<tr><td>120 - 124</td><td>卌 |</td><td>6</td></tr>
<tr><td>125 - 129</td><td>卌</td><td>5</td></tr>
<tr><td>130 - 134</td><td>||||</td><td>4</td></tr>
<tr><td></td><td>Total</td><td>20</td></tr>
</table>

Table 7.1

Exclusive form

<table>
<tr><th>Class Interval</th><th>Tally Marks</th><th>Frequency</th></tr>
<tr><td>110 - 115</td><td>||</td><td>2</td></tr>
<tr><td>115 - 120</td><td>|||</td><td>3</td></tr>
<tr><td>120 - 125</td><td>卌 |</td><td>6</td></tr>
<tr><td>125 - 130</td><td>卌</td><td>5</td></tr>
<tr><td>130 - 135</td><td>||||</td><td>4</td></tr>
<tr><td></td><td>Total</td><td>20</td></tr>
</table>

Table 7.2

Frequency table for an ungrouped data

Example 7.2

Construct a frequency table for the following data.

5, 1, 3, 4, 2, 1, 3, 5, 4, 2

1, 5, 1, 3, 2, 1, 5, 3, 3, 2.

Solution:

From the data, we observe the numbers 1, 2, 3, 4 and 5 are repeated. Hence under the number column, write the five numbers 1, 2, 3, 4, and 5 one below the other.

Now read the number and put the tally mark in the tally mark column against the number. In the same way put the tally mark till the last number. Add the tally marks against the numbers 1, 2, 3, 4 and 5 and write the total in the corresponding frequency column. Now, add all the numbers under the frequency column and write it against the total.

Number	Tally Marks	Frequency
1	卌	5
2	∣∣∣∣	4
3	卌	5
4	∣∣	2
5	∣∣∣∣	4
	Total	20

Table 7.3

In the formation of Frequency distribution for the given data values, we should

(i) select a suitable number of classes, not very small and also not very large.

(ii) take a suitable class - interval (or class width) and

(iii) present the classes with increasing values without any gaps between classes.

Frequency table for a grouped data

Example 7.3

The following data relate to mathematics marks obtained by 30 students in standard VII. Prepare a frequency table for the data.

25, 67, 78, 43, 21, 17, 49, 54, 76, 92, 20, 45, 86, 37, 35

60, 71, 49, 75, 49, 32, 67, 15, 82, 95, 76, 41, 36, 71, 62

Solution:

The minimum marks obtained is 15.

The maximum marks obtained is 95.

Range $=$ Maximum value – Minimum value

$= 95 - 15$

$= 80$

Choose 9 classes with a class interval of 10. as 10 - 20, 20 - 30,$\cdots$,90 - 100. The following is the frequency table.

Class Interval (Marks)	Tally Marks	Frequency
10 - 20	||	2
20 - 30	|||	3
30 - 40	||||	4
40 - 50	~~||||~~	5
50 - 60	||	2
60 - 70	||||	4
70 - 80	~~||||~~ |	6
80 - 90	||	2
90 - 100	||	2
	Total	30

Table 7.4

7.2 Continuous grouped Frequency distribution Table

To find the class limits in continuous grouped frequency distribution.

Steps to do

(i) Find the difference between the upper limit of the first class and lower limit of the second class.

(ii) Divide the difference by 2. Let the answer be x.

(iii) Subtract 'x' from lower limits of all the class intervals.

(iv) Add 'x' to all the upper limits of all the class intervals. Now the new limits will be true class limits.

Example 7.4

Form the frequency distribution table for the following data which gives the ages of persons who watched a particular channel on T.V.

Class Interval (Age)	10 -19	20 -29	30 - 39	40 - 49	50 - 59	60 - 69
Number of persons	45	60	87	52	25	12

Solution:

In this table, the classes given here have gaps. Hence we rewrite the classes using the exclusive method.

Difference between upper limits of first class and lower limits of second class

$$= 20 - 19 = 1$$

Divide the difference by 2 then,

$$x = \frac{1}{2} = 0.5$$

Now subtract 0.5 from lower limits and add 0.5 to the upper limits. Now we get continuous frequency distribution table with true class limits.

Class Interval (Age)	Frequency (Number of persons)
9.5 - 19.5	45
19.5 - 29.5	60
29.5 - 39.5	87
39.5 - 49.5	52
49.5 - 59.5	25
59.5 - 69.5	12

Table 7.5

Exercise 7.1

1. Choose the correct answer.

i) The difference between the highest and lowest value of the variable in the given data. is called.

(A) Frequency (B) Class limit (C) Class interval (D) Range

ii) The marks scored by a set of students in a test are 65, 97, 78, 49, 23, 48, 59, 98. The range for this data is

(A) 90 (B) 74 (C) 73 (D) 75

iii) The range of the first 20 natural numbers is

(A) 18 (B) 19 (C) 20 (D) 21

iv) The lower limit of the class interval 20 - 30 is

(A) 30 (B) 20 (C) 25 (D) 10

v) The upper of the class interval 50 - 60 is

(A) 50 (B) 60 (C) 10 (D) 55

2. Construct a frequency table for each of the following data:

   10, 15, 13, 12, 14, 11, 11, 12, 13, 15

   11, 13, 12, 15, 13, 12, 14, 14, 15, 11

3. In the town there were 26 patients in a hospital.

   The number of tablets given to them is given below. Draw a frequency table for the data.

   2, 4, 3, 1, 2, 2, 2, 4, 3, 5, 2, 1, 1, 2

   4, 5, 1, 2, 5, 4, 3, 3, 2, 1, 5, 4.

4. The number of savings book accounts opened in a bank during 25 weeks are given as below. Find a frequency distribution for the data:

   15, 25, 22, 20, 18, 15, 23, 17, 19, 12, 21, 26, 30

   19, 17, 14, 20, 21, 24, 21, 16, 22, 20, 17, 14

5. The weight (in kg) 20 persons are given below.

   42, 45, 51, 55, 49, 62, 41, 52, 48, 64

   52, 42, 49, 50, 47, 53, 59, 60, 46, 54

   Form the frequency table by taking class intervals 40 - 45, 45 - 50, 50 - 55, 55 - 60 and 60 - 65.

6. The marks obtained by 30 students of a class in a mathematics test are given below.

   45, 35, 60, 41, 8, 28, 31, 39, 55, 72, 22, 75, 57, 33, 51

   76, 30, 49, 19, 13, 40, 88, 95, 62, 17, 67, 50, 66, 73, 70

   Form the grouped frequency table:

7. Form a continuous frequency distribution table from the given data.

Class Interval (weight in kg.)	21 - 23	24 - 26	27 - 29	30 - 32	33 - 35	36 - 38
Frequency (Number of children)	2	6	10	14	7	3

8. The following data gives the heights of trees in a grove. Form a continuous frequency distribution table.

Class Interval (Height in metres)	2 - 4	5 - 7	8 - 10	11 - 13	14 - 16
Frequency (Number of trees)	29	41	36	27	12

7.3 Mean Median, Mode of ungrouped data

Arithmetic mean

We use the word 'average' in our day to day life.

Poovini spends on an average of about 5 hours daily for her studies.

In the month of May the average temperature at Chennai is 40 degree celsius.

What do the above statement tell us?

Poovini usually studies for 5 hours. On some days, she may study for less number of hours and on the other day she may study longer.

The average temperature of 40 degree celsius, means that, the temperature at the month of May in chennai is 40 degree celsius. Some times it may be less than 40 degree celsius and at other time it may be more than 40 degree celsius.

Average lies between the highest and the lowest value of the given data.

Rohit gets the following marks in different subjects in an examination.

62, 84, 92, 98, 74

In order to get the average marks scored by him in the examination, we first add up all the marks obtained by him in different subjects.

$62 + 84 + 92 + 98 + 74 = 410.$

and then divide the sum by the total number of subjects. (i.e. 5)

The average marks scored by Rohit = $\frac{410}{5} = 82.$

This number helps us to understand the general level of his academic achievement and is referred to as mean.

$\therefore$ **The average or arithmetic mean or mean is defined as follows.**

$$\textbf{Mean} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$$

Example 7.5

Gayathri studies for 4 hours, 5 hours and 3 hours respectively on 3 consecutive days. How many hours did she study daily on an average?

Solution:

$$\text{Average study time} = \frac{\text{Total number of study hours}}{\text{Number of days for which she studied.}}$$

$$= \frac{4+5+3}{3} \text{ hours}$$

$$= \frac{12}{3}$$

$$= 4 \text{ hours per day.}$$

Thus we can say that Gayathri studies for 4 hours daily on an average.

Example 7.6

The monthly income of 6 families are ₹ 3500, ₹ 2700, ₹ 3000, ₹ 2800, ₹ 3900 and ₹ 2100. Find the mean income.

Solution:

$$\text{Average monthly income} = \frac{\text{Total income of 6 familes}}{\text{Number of families}}$$

$$= \frac{₹\ 3500+2700+3000+2800+3900+2100}{6}$$

$$= ₹\ \frac{18000}{6}$$

$$= ₹\ 3{,}000.$$

Example 7.7

The mean price of 5 pens is ₹ 75. What is the total cost of 5 pens?

Solution:

$$\text{Mean} = \frac{\text{Total cost of 5 pens}}{\text{Number of pens}}$$

$$\text{Total cost of 5 pens} = \text{Mean} \times \text{Number of pens}$$

$$= ₹\ 75 \times 5$$

$$= ₹\ 375$$

Median

Consider a group of 11 students with the following height (in cm)

106, 110, 123, 125, 115, 120, 112, 115, 110, 120, 115.

The Physical EducationTeacher Mr. Gowtham wants to divide the students into two groups so that each group has equal number of students. One group has height lesser than a particular height and the other group has student with height greater than the particular height.

Now, Mr. Gowtham arranged the students according to their height in ascending order.

106, 110, 110, 112, 115, 115,115, 120, 120, 123, 125

The middle value in the data is 115 because this value divides the students into two equal groups of 5 students each. This values is called as median. Median refers to the value 115 which lies in the middle of the data.Mr. Gowtham decides to keep the middle student as a referee in the game.

Median is defined as the middle value of the data when the data is arranged in ascending or descending order.

Find the median of the following:

40, 50, 30, 60, 80, 70

Arrange the given data in ascending order.

30, 40, 50, 60, 70, 80.

Try these

Find the actual distance between your school and house. Find the median of the place.

Here the number of terms is 6 which is even. So the third and fourth terms are middle terms. The average value of the two terms is the median.

(i.e) Median $= \frac{50+60}{2} = \frac{110}{2} = 55.$

(i) **When the number of observations is odd, the middle number is the median.**

(ii) **When the number of observations is even, the median is the average of the two middle numbers.**

Do you know?

In highways, the yellow line represents the median.

Example 7.8

Find the median of the following data.

3, 4, 5, 3, 6, 7, 2.

Solution:

Arrange the data in ascending order.

2, 3, 3, 4, 5, 6, 7

The number of observation is 7 which is odd.

$\therefore$The middle value 4 the median.

Example 7.9

Find the median of the data

12, 14, 25, 23, 18, 17, 24, 20.

Solution:

Arrange the data in ascending order

12, 14, 17, 18, 20, 23, 24, 25.

The number of observation is 8 which is even.

$\therefore$ Median is the average of the two middle terms 18 and 20.

$$\text{Median} = \frac{18+20}{2} = \frac{38}{2} = 19$$

Example 7.10

Find the median of the first 5 prime numbers.

Solution:

The first five prime numbers are 2, 3, 5, 7, 11.

The number of observation is 5 which is odd.

$\therefore$ The middle value 5 is the median.

Mode

Look at the following example,

Mr. Raghavan, the owner of a ready made dress shop says that the most popular size of shirts he sells is of size 40 cm.

Observe that here also, the owner is concerned about the number of shirts of different sizes sold. He is looking at the shirt size that is sold, the most. The highest occurring event is the sale of size 40 cm. This value is called the mode of the data.

Mode is the variable which occurs most frequently in the given data.

Mode of Large data

Putting the same observation together and counting them is not easy if the number of observation is large. In such cases we tabulate the data.

Example 7.11

Following are the margin of victory in the foot ball matches of a league.

1, 3, 2, 5, 1, 4, 6, 2, 5, 2, 2, 2, 4, 1, 2, 3, 2, 3, 2,

1, 1, 2, 3, 2, 6, 4, 3, 2, 1, 1, 4, 2, 1, 5, 3, 4, 2, 1, 2. Find the mode of this data.

Solution:

Margin of victory	Tally Marks	Number of Matches
1	卌 ||||	9
2	卌 卌 ||||	14
3	卌 ||	7
4	卌	5
5	||	3
6	||	2
	Total	40

Table 7.6

Now we quickly say that '2' is the mode. Since 2 has occurred the more number of times, then the most of the matches have been won with a victory margin of 2 goals.

Example 7.12

Find the mode of the following data.

3, 4, 5, 3, 6, 7

Try these

Find the mode of the transport in your place.

Solution:

3 occurs the most number of times.

$\therefore$ Mode of the data is 3.

Example 7.13

Find the mode of the following data.

2, 2, 2, 3, 3, 4, 5,5, 5, 6,6, 8

Solution:

2 and 5 occur 3 times.

$\therefore$ Mode of the data is 2 and 5.

Try these

Find the mode of the flower.

Example 7.14

Find the mode of the following data

90, 40, 68, 94, 50, 60.

Solution:

Here there are no frequently occurring values. Hence this data has no mode.

Example 7.15

The number of children in 20 families are 1, 2, 2, 1, 2, 1, 3, 1, 1, 3

1, 3, 1, 1, 1, 2, 1, 1, 2, 1. Find the mode.

Solution:

Number of Children	Tally Marks	Number of Families
1	$\cancel{||||}$ $\cancel{||||}$ $||$	12
2	$\cancel{||||}$	5
3	$|||$	3
	Total	20

Table 7.7

12 families have 1 child only, so the mode of the data is 1.

Exercise: 7.2

1. Choose the correct answer:

i) The arithmetic mean of 1, 3, 5, 7 and 9 is

(A) 5 (B) 7 (C) 3 (D) 9

ii) The average marks of 5 children is 40 then their total mark is

(A) 20 (B) 200 (C) 8 (D) 4

iii) The median of 30,50, 40, 10, 20 is

(A) 40 (B) 20 (C) 30 (D) 10

iv) The median of 2, 4, 6, 8, 10, 12 is

(A) 6 (B) 8 (C) 7 (D) 14

v) The mode of 3, 4, 7, 4, 3, 2, 4 is

(A) 3 (B) 4 (C) 7 (D) 2

2. The marks in mathematics of 10 students are

 56, 48, 58, 60, 54, 76, 84, 92, 82, 98.

 Find the range and arithmetic mean

3. The weights of 5 people are

 72 kg, 48 kg, 51 kg, 69 kg, 67 kg.

 Find the mean of their weights.

4. Two vessels contain 30 litres and 50 litres of milk separately. What is the capacity of the vessels if both share the milk equally?

5. The maximum temperature in a city on 7 days of a certain week was 34.8°C, 38.5°C, 33.4°C, 34.7°C, 35.8°C, 32.8°C, 34.3°C. Find the mean temperature for the week.

6. The mean weight of 10 boys in a cricket team is 65.5 kg. What is the total weight of 10 boys.

7. Find the median of the following data.

 6, 14, 5, 13, 11, 7, 8

8. The weight of 7 chocolate bars in grams are

 131, 132, 125, 127, 130, 129, 133. Find the median.

9. The runs scored by a batsman in 5 innings are

 60, 100, 78, 54, 49. Find the median.

10. Find the median of the first seven natural numbers.

11. Pocket money received by 7 students is given below.

 ₹ 42, ₹ 22, ₹ 40, ₹ 28, ₹ 23, ₹ 26, ₹ 43. Find the median.

12. Find the mode of the given data.

 3, 4, 3, 5, 3, 6, 3, 8, 4.

13. Twelve eggs collected in a farm have the following weights.

    32 gm,40 gm, 27 gm, 32 gm, 38 gm, 45 gm,

    40 gm, 32 gm, 39 gm, 40 gm, 30 gm, 31 gm,

    Find the mode of the above data.

14. Find the mode of the following data.

    4, 6, 8, 10, 12, 14

15. Find the mode of the following data.

    12, 14, 12, 16, 15, 13, 14, 18, 19, 12, 14, 15, 16, 15, 16, 16,

    15, 17, 13, 16, 16, 15, 13, 15, 17, 15, 14, 15, 13, 15, 14.

Points to Remember

1. Any collection of information in the form of numerical figures giving the required information is called data.
2. The raw data is an unprocessed and unclassified data.
3. The data which is arranged in groups (or classes) is called a grouped data.
4. The number of times a particular value repeats itself is called its frequency.
5. Range = Highest value – Lowest value.
6. The difference between the upper and the lower limit is called the width of the class interval.
7. Average lies between the highest and the lowest value of the given data.
8. Mean = $\dfrac{\text{sum of all the observations}}{\text{total number of observations}}$
9. Median is defined as the middle value of the data, when the data is arranged in ascending or descending order.
10. Mode is the variable which occurs most frequently in the given data.

ANSWERS

Unit 1

Exercise 1.1

1. i) D ii) B iii) C iv) B
2. i) 0 ii) – 5 iii) 5 iv) 0
3. i) – 6 ii) – 25 iii) 651 iv) – 316 v) 0 vi) 1320
   vii) 25 viii) 25 ix) 42 x) – 24 xi) 1890 xii) – 1890
   xiii) –1440 xiv) 256 xv) 6000 xvi) 10800
4. i) – 135 ii) 16 iii) 182 iv) – 800 v) 1 vi) 0
5. ₹ 645 6. 75 marks 7. ₹1500 8. ₹240

Exercise 1.2

1. i) C ii) A iii) C iv) A
2. i) – 5 ii) 10 iii) 4 iv) – 1 v) – 6 vi) – 9
   vii) – 1 viii) 2 ix) 2 x) 6
3. i) 20 ii) 20 iii) – 400
4. – 5

Exercise 1.3

1. i) $\frac{24}{5}$ ii) $\frac{9}{7}$ iii) 2 iv) 3 v) $\frac{14}{3}$ vi) 20
   vii) $\frac{77}{4}$ viii) 10 ix) 8 x) 24
2. i) 14 ii) 63 iii) 16 iv) 25 v) 288 vi) 16
   vii) 9 viii) 70 ix) 25 x) 50
3. i) $26\frac{1}{4}$ ii) $19\frac{4}{5}$ iii) $9\frac{3}{5}$ iv) $64\frac{2}{7}$ v) $52\frac{1}{2}$ vi) $85\frac{1}{2}$
4. Vasu drank 4 litres.

Exercise 1.4

1. i) 1 ii) $\frac{7}{12}$ iii) $\frac{7}{12}$ iv) $\frac{7}{18}$ v) 1 vi) $\frac{2}{63}$
2. i) $\frac{22}{27}$ ii) $\frac{1}{5}$ iii) $\frac{1}{4}$ iv) $\frac{9}{16}$ v) $\frac{9}{2}$ vi) $\frac{48}{35}$
3. i) $2\frac{4}{15}$ ii) $4\frac{29}{40}$ iii) $7\frac{1}{2}$ iv) $20\frac{1}{8}$ v) $59\frac{13}{16}$
4. 55 km 5. $12\frac{1}{4}$ hrs

Exercise 1.5

1. i) $\frac{7}{5}$ ii) $\frac{9}{4}$ iii) $\frac{7}{10}$ iv) $\frac{4}{9}$ v) $\frac{2}{33}$ vi) 9
   vii) 13 viii) $\frac{5}{7}$
2. i) $\frac{1}{15}$ ii) $\frac{1}{54}$ iii) $\frac{1}{6}$ iv) $\frac{1}{12}$
3. i) $\frac{8}{5}$ ii) $\frac{35}{36}$ iii) $4\frac{7}{12}$ iv) $1\frac{11}{16}$
4. 21 uniforms
5. 40 km/hour

Exercise 1.6

1. i) A ii) C iiii) B iv) D
2. i) $\frac{-20}{15}, \frac{-19}{15}, \frac{-18}{15}, \frac{-17}{15}$ ii) $\frac{7}{6}, \frac{6}{6}, \frac{5}{6}, \frac{4}{6}$
   iii) $\frac{48}{28}, \frac{47}{28}, \frac{46}{28}, \frac{45}{28}$
3. i) $\frac{-3}{4}$ ii) $\frac{-3}{8}$ iii) $\frac{-3}{5}$ iv) $\frac{-5}{3}$ v) $\frac{-1}{2}$
5. i, iv, v

Exercise 1.7

1. i) C ii) C iii) D iv) D
2. i) $\frac{18}{5}$ ii) $\frac{24}{13}$ iii) 2 iv) $\frac{-12}{13}$ v) $\frac{13}{3}$ vi) $\frac{19}{42}$
   vii) $\frac{-43}{21}$ viii) -3 ix) $\frac{24}{7}$ x) $\frac{-13}{30}$
3. i) 1 ii) 4 iii) $\frac{-9}{44}$ iv) $\frac{-5}{16}$ v) $\frac{23}{20}$ vi) -1
   vii) $\frac{-69}{26}$ viii) $\frac{-41}{60}$ ix) $\frac{-1}{27}$ x) $\frac{1}{12}$
4. i) $\frac{2}{35}$ ii) $\frac{1}{4}$ iii) $\frac{19}{12}$ iv) $\frac{3}{2}$ v) $\frac{-43}{28}$
5. i) $4\frac{7}{11}$ ii) $-3\frac{1}{2}$ iii) $1\frac{7}{11}$ iv) $5\frac{3}{4}$ v) $-1\frac{17}{40}$ vi) $-4\frac{7}{132}$
   vii) $-6\frac{41}{42}$ viii) $-3\frac{7}{210}$
6. $\frac{7}{4}$
7. $\frac{4}{5}$
8. $13\frac{17}{20}$ kg.
9. $18\frac{3}{4}$ kg.
10. $3\frac{9}{10}$ kg.

Exercise 1.8

1. i) C ii) B iii) A iv) A
2. i) $\frac{-72}{25}$ ii) $\frac{-35}{169}$ iii) $\frac{-7}{24}$ iv) $\frac{-12}{11}$ v) -20 vi) $\frac{2}{9}$
3. i) $\frac{-15}{4}$ ii) -5 iii) $26\frac{98}{125}$ iv) $66\frac{44}{375}$ v) $\frac{45}{28}$
4. i) $\frac{16}{81}$ ii) $\frac{-3}{2}$ iii) $\frac{-8}{7}$ iv) $-9\frac{3}{43}$
5. $\frac{9}{7}$
6. $\frac{3}{2}$

Exercise 1.9

1. i) C ii) C iii) A iv) C
2. i) 2.1 ii) 40.5 iii) 17.1 iv) 82.8 v) 0.45 vi) 1060.15
   vii) 2.48 viii) 1.05 ix) 10.34 x) 1.041 xi) 4.48 xii) 0.00125
   xiii) 2.108 xiv) 0.0312
3. i) 14 ii) 468 iii) 4567 iv) 2960.8 v) 3230 vi) 17140
   vi) 478
4. 51.5 cm^2
5. 756 km.

Exercise 1.10

1. i) A ii) B iii) C iv) B
2. i) 0.3 ii) 0.09 iii) 1.16 iv) 10.8 v) 196.3 vi) 3.04
3. i) 0.68 ii) 4.35 iii) 0.09 iv) 4.43 v) 37.348 vi) 0.079
4. i) 0.056 ii) 0.007 iii) 0.0069 iv) 7.436 v) 0.437 vi) 0.7873
5. i) 0.0089 ii) 0.0733 iii) 0.04873
   iv) 0.1789 v) 0.0009 vi) 0.00009
6. i) 2 ii) 160 iii) 12.5 iv) 8.19 v) 2 vi) 35
7. 23 km
8. 10.5 kg
9. ₹ 9
10. 42.2 km/hour
11. 14.4

Exercise 1.11

1. i) A ii) A iii) C iv) C
2. i) 256 ii) 27 iii) 1331 iv) 1728 v) 28561 vi) 0
3. i) 7^6 ii) 1^5 iii) 10^6 iv) b^5 v) 2^2a^4 vi) $(1003)^3$
4. i) $2^3 \times 3^3$ ii) 3^5 iii) 5^4 iv) 2^{10} v) 5^5 vi) 10^5
5. i) 4^5 ii) 2^5 iii) 3^2 iv) 5^6 v) 2^7 vi) 4^7

6. i) $5^2 \times 2^2$ ii) $2^7 \times 3^1$ iii) $2^1 \times 3^1 \times 133^1$ iv) $2^1 \times 3^1 \times 113^1$
   v) $2^2 \times 3 \times 79$ vi) $2^7 \times 5^1$
7. i) 200000 ii) 0 iii) 2025 iv) 1296
   v) 9000000000 vi) 0
8. i) – 125 ii) 1 iii) 72 iv) – 2000 v) 10584 vi) – 131072

Exercise 1.12

1. i) A ii) A iii) C iv) C
2. i) 3^{12} ii) a^{12} iii) 7^{5+x} iv) 10^7 v) 5^9
3. i) 5^4 ii) a^4 iii) 10^{10} iv) 4^2 v) $3^0 = 1$
4. i) 3^{12} ii) 2^{20} iii) 2^{20} iv) 1 v) 5^{20}

Unit - 2

Exercise 2.1

1. (i) A (ii) D (iii) D (iv) B (v) C
2. Constants: 5, – 9.5; Variables: a, $-xy$, p.
3. (i) $x + 6$ (ii) $-m - 7$ (iii) $3q + 11$ (iv) $3x + 10$ (v) $5y - 8$
4. 3, – 4, 9
5. (i) $y^2 x$, coefficient = y^2. (ii) x, coefficient = 1.
   (iii) $3x$, coefficient = 3. (iv) $-5xy^2$, coefficient = $-5y^2$.
6. (i) $-my^2$, coefficient = $-m$. (ii) $6y^2$, coefficient = 6.
   (iii) $-9xy^2$, coefficient = $-9x$.

Exercise 2.2

1. (i) B (ii) D (iii) D (iv) D (v) A
2. (i) $4x, 7x$ (ii) $7b, -3b$ (iii) $3x^2y, -8yx^2$ (iv) $a^2b, 7a^2b$
   (v) $5pq, 25pq$; $3q, 70q$; $p^2q^2, 14\,p^2\,q^2$
3. (i) 2 (ii) 2 (iii) 3 (iv) 4 (v) 2
4. (i) – 10 (ii) 10 (iii) 11
5. (i) 21 (ii) 34 (iii) 82

Exercise 2.3

1. (i) C (ii) B (iii) A (iv) D (v) A
2. (i) $13a + 2b$ (ii) $5l - 4l^2$ (iii) $16z^2 - 16z$
   (iv) $p - q$ (v) $7m^2n - 4m^2 - 6n^2 + 4mn^2$ (vi) $x^2 - 3xy + 7y^2$

3. (i) $2ab$ (ii) $2s + t$ (iii) $3a - 2b + 2p + 3q$
   (iv) $5a - 5b + 4$(v) $2x + 2y - 2$
   (vi) $7c + 4$ (vii) $3m^2n + 5mn - 4n^2 + 4$
4. (i) $8a$ (ii) $7a^2b$ (iii) $-11x^2y^2$ (iv) $-2xy + 16$
   (v) $5n - 2mn + 3m$ (vi) $-5p - 15p^2$ (vii) $8m^2 - 6m - 12$
   (viii) $s^2 - 6s - 4$ (ix) $9n^2 - 10mn - 9m^2$
5. (i) $x^2 + 5xy - 3y^2$ (ii) $9p - 2q - 6$ (iii) $4x - 3y + 9$
6. $6a - 6$
7. $16x + 12$
8. $12a - 2$ rupees
9. $7x - 8$ metres
10. (i) $8p^2 - 9p - 11$ (ii) $-p^2 + 8p + 12$
11. $2m^2 + 5m + 10$

Exercise 2.4

1. (i) B (ii) A (iii) D (iv) C (v) A
2. (i) $x + 2y$ (ii) $y - z$ (iii) $xy + 4$
   (iv) $3x - 4y$ (if $3x > 4y$) or $4y - 3x$ (if $4y > 3x$)
   (v) $10 + x + y$ (vi) $pq - 5$ (vii) $12 - mn$
   (viii) $ab - (a + b)$ (ix) $3cd + 6$ (x) $\frac{4xy}{3}$

Exercise 2.5

1. (i) B (ii) A (iii) C (iv) C (v) D
2. (i) $x = 12$ (ii) $a = 7$ (iii) $y = -6$ (iv) $b = -2$ (v) $x = -5$
   (vi) $x = 7$ (vii) $x = -5$ (viii) $n = 4$ (ix) $m = 11$ (x) $y = 27$
3. (i) $x = 50$ (ii) $l = 14$ (iii) $x = 4$ (iv) $a = 3$ (v) $x = -9$
   (vi) $t = -4$ (vii) $x = -6$ (viii) $m = 3$ (ix) $x = \frac{-1}{2}$ (x) $x = 6$
4. (i) $x = 14$ (ii) $a = 30$ (iii) $n = -24$(iv) $p = -56$ (v) $x = -10$
   (vi) $m = 12$
5. (i) $x = 3$ (ii) $x = -15$ (iii) $z = 5$ (iv) $a = -9$ (v) $x = 3$
   (vi) $x = 5$ (vii) $y = 67$ (viii) $x = 6$ (ix) $y = 3$ (x) $m = 6$
   (xi) $x = 11$ (xii) $m = \frac{1}{2}$ (xiii) $x = 3$ (xiv) $x = -3$ (xv) $t = -1$
6. 15 7. 13 8. 108 9. 12 10. 8
11. 37 12. 60 13. 35

Unit - 3

Exercise 3.1

1. (i) ₹ 360 (ii) ₹ 75 (iii) 325 km (iv) 8 (v) 15
2. 100 kg
3. 120 teachers
4. 80 km
5. 216 sq.m.
6. 26 kg
7. 7.5 hours
8. 15 days
9. 156 soldiers
10. 105 pages
11. 40 days

Exercise 3.2

1. (i) 20% (ii) 93% (iii) 11% (iv) 1% (v) 100%
2. (i) 43 : 100 (ii) 75 : 100 (iii) 5 : 100 (iv) 35:200 (v) 100 : 300
3. (i) $\frac{25}{100}$ (ii) $\frac{25}{200}$ (iii) $\frac{33}{100}$ (iv) $\frac{70}{100}$ (v) $\frac{82}{100}$

Exercise 3.3

1. (i) 625% (ii) 0.03% (iii) 25% (iv) $33\frac{1}{3}$ (v) 50
2. (i) 100% (ii) 18% (iii) 5.25% (iv) 66.67% (v) 45.45%
3. (i) 36% (ii) 3% (iii) 7.1% (iv) 305% (v) 75%
4. 20%
5. 13.89%
6. Girls 46%; Boys 54
7. He got more marks in Science.
8. Savings 10%; Expenditure 90%

Exercise 3.4

1. (i) $\frac{3}{10}$ (ii) $\frac{1}{200}$ (iii) 0.25 (iv) ₹ 30 (v) ₹ 7.50
2. (i) $\frac{9}{100}$ (ii) $\frac{3}{4}$ (iii) $\frac{1}{400}$ (iv) $\frac{1}{40}$ (v) $\frac{2}{3}$
3. (i) 0.07 (ii) 0.64 (iii) 3.75 (iv) 0.0003 (v) 0.005
4. (i) 18 (ii) ₹ 24 (iii) 36 m (iv) 108 (v) 3.75 kg
5. ₹ 6250
6. 9 matches
7. 12,800 men; 11,200 children
8. ₹ 38250
9. 3975 illiterates

Exercise 3.5

1. (i) 50 (ii) 15 (iii) Profit of ₹ 35

   (iv) cost price (v) 10

2. Profit = ₹ 24, Loss = ₹ 21;

   Profit = ₹ 35.45, Loss = ₹ 3362, Loss = ₹ 7.50

3. (i) ₹ 530 (ii) ₹ 620 (iii) ₹ 1027.50

   (iv) ₹ 336.75 (v) ₹ 943.50

4. 10% 5. 12% 6. 60% 7. 15%

Exercise 3.6

1. (i) ₹ 200 (ii) ₹ 500 (iii) ½ year

   (iv) $\frac{4}{5}$ year (v) ₹ 15,000

2. ₹ 2,500; ₹ 7,500 3. ₹ 450; ₹ 1,650 4. ₹ 2,250

5. ₹ 2,630 6. ₹ 216; ₹ 12,216 7. 5% 8. ₹ 1,000

9. 2 years 10. 10% 11. 8 years

12. ₹ 5,400 13. ₹ 5,000; 10%

Unit - 4

Exercise 4.1

1. (i) 175 cm^2 (ii) 365 cm^2 (iii) 750 cm^2 (iv) 106 cm^2
2. 40 tiles
3. triangular land
4. Mani benefited more.
5. Square has larger area.

Exercise 4.2

1. (i) 9 cm^2 (ii) 26 cm^2 (iii) 150 cm^2 (iv) 30 cm^2
2. (i) 24 cm^2 (ii) 3 m^2 (iii) 10.5 m^2
3. (i) 10 m (ii) 20 cm (iii) 16.5 m
4. (i) 18 m (ii) 5 m (iii) 8 cm
5. Cost ₹ 1,820

Exercise 4.3

1. 117 cm^2

2. (i) 67.5 cm^2 (ii) 73 cm^2 (iii) 50.4 cm^2

3. 150 cm^2 4. 12 cm 5. 18750 cm^2

Exercise 4.4

1. (i) C (ii) C (iii) D

2. (i) 45 cm^2 (ii) 48 cm^2 (iii) 12 cm^2

3. (i) 252 cm^2 (ii) 180 cm^2 (iii) 241.5 cm^2 (iv) 58.1 cm^2

4. 112 cm^2 5. 24300 m^2 6. 12 cm

Exercise 4.5

1. (i) C (ii) D (iii) B

2. (i) 90 cm^2 (ii) 118.3 cm^2 (iii) 536.5 cm^2 (iv) 120 cm^2

3. 96 cm^2 4. 80 cm 5. ₹ 8400

Exercise 4.6

1. (i) B (ii) A (iii) D (iv) D

2. (i) 50 cm^2 (ii) 66 cm^2 (iii) 80.5 cm^2

3. 12 cm 4. 875 m^2 5. 19.2 cm

Exercise 4.7

1. (i) B (ii) C (iii) A (iv) D (v) D

2. (i) $d = 70$ cm, $c = 220$ cm

   (ii) $r = 28$ cm, $c = 176$ cm

   (iii) $r = 4.9$ cm, $d = 9.8$ cm

3. (i) 110 cm (ii) 264 cm (iii) 374 cm (iv) 462 cm

4. (i) 79.2 cm (ii) 396 cm (iii) 8.8 m (iv) 26.4 m

5. (i) 17.5 cm (ii) 21 cm (iii) 0.7 m (iv) 1.75 m

6. 660 m 7. ₹ 1232 8. 1.05 m 9. 37

Exercise 4.8

1. (i) 38.5 cm^2 (ii) 86.625 cm^2
   (iii) 18.865 m^2 (iv) 124.74 m^2
2. (i) 4.525 cm^2 (ii) 616 cm^2
   (iii) 55.44 m^2 (iv) 98.56 m^2
3. ₹ 1848
4. 154 m^2
5. circle has larger area
6. 38.5 m^2
7. 1936 cm^2
8. $r = 35$, ₹ 2200

Exercise 4.9

1. 636 m^2
2. ₹ 1152
3. ₹ 1590
4. 375 cm^2
5. 40 m^2, 30 m^2
6. 3256 cm^2

Exercise 4.10

1. 6594 m^2
2. 536.94 m^2
3. ₹ 24,050
4. 21.195 m^2
5. ₹ 4494

Unit - 5

Exercise 5.1

1. (i) B (ii) C (iii) A (iv) B (v) A
2. Equilateral triangle - 3 lines of symmetry; Rhombus - 4 lines of symmetry
5. (i) isosceles triangle (ii) equilateral triangle (iii) scalene triangle

Exercise 5.2

1. (i) C (ii) B (iii) D (iv) B (v) D
2. (i) 90° (ii) 90° (iii) 180° (iv) 180°
3. (i) 90°, 4 (ii) 72°, 5 (iii) 180°, 2 (iv) 360°, 1
4. 45°, 8

Exercise 5.3

1. (i) A (ii) B (iii) C (iv) D (v) D
2. (i) $\angle$DOC, $\angle$COB; $\angle$COB, $\angle$BOA
   (ii) $\angle$QOX, $\angle$XOP; $\angle$POY, $\angle$YOQ; $\angle$YOQ, $\angle$QOX; $\angle$XOP, $\angle$POY

3. $\angle$POR, $\angle$QOS; $\angle$SOP, $\angle$ROQ

4. (i) 150° (ii) 100°

   (iii) 110° (iv) 120° (v) 135°

5. $\angle BOC = 145°$; $\angle AOD = 145°$; $\angle COA = 35°$.

6. (i) 80° (ii) 110°

   (iii) 20° (iv) 80°

   (v) 36° (vi) 45°

7. $y = 120°$; $x = 60°$ 8. $x = 25°$

Exercise 5.4

1. (i) C (ii) C (iii) B (iv) C (v) D

2. (i) corresponding angles (ii) alternate interior angle

   (iii) sum of the interior angles on the same side of the transversal.

3. (i) $\angle$PMB (ii) $\angle$PMB (iii) $\angle$DNM (iv) $\angle$DNQ

4. (i) $\angle 1, \angle 5; \angle 4, \angle 8; \angle 2, \angle 6; \angle 3, \angle 7$ (ii) $\angle 4, \angle 6; \angle 3, \angle 5$

   (iii) $\angle 3, \angle 6; \angle 4, \angle 5$ (iv) $\angle 1, \angle 3; \angle 2, \angle 4; \angle 5, \angle 7; \angle 6, \angle 8$

5. (i) 30° (ii) 50° (iii) 95° (iv) 130°

6. $\angle 1 = 70°, \angle 2 = 110°, \angle 3 = 70°, \angle 4 = 110°$

   $\angle 5 = 70°, \angle 6 = 110°, \angle 7 = 70°, \angle 8 = 110°$

7. (i) l is not parallel to m. (sum of the interior angles on the same side of the transversal is not 180°).

   (ii) l is not parallel to m. ($x = 75°$. Sum of the interior angles on the same side of the transversal is not 180°).

   (iii) l is parallel to m. ($y = 60°$. Corresponding angles are equal).

   (iv) l is parallel to m. ($z = 110°$. Alternate angles are equal).

8. $\angle 1 = 44°$, $\angle 2 = 136°$

Exercise 5.5

1. (i) B (ii) C (iii) C (iv) D (v) D

2. (i) $\angle A = 25°$, $\angle B = 35°$, $\angle C = 120°$

3. (i) 60° (ii) 70° (iii) 50° (iv) 50°

4. (i) 70° (ii) 60° (iii) 40° (iv) 30°

   (v) 65°, 65° (vi) 60°, 60°, 60°

5. (i) $y = 60°, x = 70°$ (ii) $y = 80°, x = 50°$ (iii) $y = 70°, x = 110°$

   (iv) $x = 60°, y = 90°$ (v) $y = 90°, x = 45°$ (vi) $x = 60°, y = 50°$

6. $x = 50°$.

Unit - 7

Exercise 7.1

1. (i) D (ii) D (iii) B (iv) B (v) B

Exercise 7.2

1. (i) A (ii) B (iii) C (iv) C (v) B

2. Range is 50; A.M. = 70.8

3. 61.4 kg. 4. 40 litres 5. 34.9°C

6. 655.0 kg 7. 8 8. 130 gram 9. 60 10. 4 11. ₹ 28

12. 3 13. 32 gm and 40 gm 14. no mode 15. 15