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EXERCISES IN ARITHMETIC

FOR MIDDLE AND UPPER FORMS

BY

ROBERT WALKER M.A.

THE TRAINING CENTRE JORDANHILL, GLASGOW
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PREFACE

THE present volume is designed as a class-book for middle and upper forms. It presupposes a knowledge of notation and of the four fundamental rules employed with integral numbers. It will be found to meet the requirements for the School Certificate and for the University Matriculation Examinations; and particular attention has been paid also to the special requirements of pupils following commercial and industrial courses.

The reception given to *The Essentials of Arithmetic: a Manual for Teachers*, published in 1929, has led to requests from many quarters for an amplified class-book of exercises for pupils, and it is in response to these that the new book is published. Its aim is identical with that of its predecessor—viz., to insist on broad fundamental principles, and to indicate how a grasp of these principles will equip the student for solving intelligently, and with the minimum of drudgery, the problems that arise daily in counting-house, workshop, and laboratory. In general a chapter has been devoted to each subject; and each set of formal exercises is preceded by (i) hints on methods, (ii) model examples worked out in detail, and (iii) a set of easy introductory questions to be answered orally.

The methods recommended are those advocated in *The Essentials* as being natural, practical, and labour-saving.

The successive examples are graded in degree of difficulty, although from time to time easy, straightforward cases are allowed to occur with the view of encouraging pupils to realize their gradually increasing power.

The tables of logarithms and antilogarithms given in the Appendix are reproduced by kind permission of the Controller of His Majesty's Stationery Office from *Mathematical Tables for the Use of Students*, published by that Office.

R. W.

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EXERCISES IN ARITHMETIC

CHAPTER I ELEMENTARY REVISORY EXERCISES

EXERCISES I

(Involving revision of the fundamental operations.)

1. A man earns £70 per calendar month. If he spends £12 10s. per week, what does he save in a year? (Take a year as 52 weeks.)
2. Find the cost of 214 fowls at 5s. 9d. each.
3. If a man's salary is £620 yearly, what does he receive per day?
4. The hotel expenses for 7 visitors amount to £3 13s. 2d. per head. If a banknote for £100 is offered in paying the total bill, what change will be received?
5. Two men are paid at the same rate per hour. One is employed on a piece of work for 9 hours, and the other takes 13 hours to finish it. If £5 13s. 8d. is paid for the whole work, what should each receive?
6. A lady buys 2 dozen cakes at 3d. each, 4 lb. of biscuits at 2s. 4d. per lb., and 3 loaves at 1s. 4½d. each. What change should she get out of a £5 note?
7. I buy 18 pens and 25 pencils for £3 9s. 3d. If the pencils cost 3d. each, what is the price of a pen?
8. Find the total sum spent in the following purchases: 6 tablespoons at 1s. 9d. each, 9 dessert spoons at 2s. 3½d. each, and 15 teaspoons at 2s. 7d. each.
9. Notebooks sell at 7 for 1s. How many will £3 14s. buy?
10. A lady buys 2 lb. of tea at 3s. 8d. per lb., 7 lb. of sugar at 5½d. per lb., and 4 dozen biscuits at 1½d. each. Find the total cost.

11. If hyacinth bulbs cost $6d.$ each, how many can I buy for $\pounds 1\ 6s.$?

12. If tulips cost $9d.$ each, how many can I buy for $\pounds 1\ 9s.\ 3d.$?

13. A boy wishes to buy 50 crocuses, which cost $1\frac{1}{2}d.$ each. He has saved up only $3s.\ 6d.$ How much more must he save before he can carry out his wish?

14. The bus fare from one town to another is $1s.\ 3d.$ If the conductor takes $\pounds 1\ 15s.$, how many people have paid?

15. For a picnic 27 people subscribe $3s.\ 6d.$ each. The bus fare costs them $2s.\ 1d.$ each. The driver receives $5s.$ for himself. Tea costs the subscribers $9d.$ each. What sum is left over?

16. A householder has 3 gas-fires burning for 7 hours. Each fire burns per hour gas worth $3\frac{1}{2}d.$ How much will he have to pay?

17. The bus fare from one town to another is $1s.\ 9d.$ single and $3s.\ 1d.$ return. A passenger makes the double journey daily for the month of August. What will he save by taking return tickets?

18. Find the cost of 87 golf-bags at $\pounds 1\ 7s.\ 9d.$ each.

19. How many wireless valves at $13s.\ 4d.$ each can I buy for $\pounds 14$?

20. A lady sends 25 poor children to the coast for a day's holiday. The railway fare amounts to $3s.\ 4d.$ per head. The meals cost her altogether $\pounds 2\ 2s.\ 6d.$ What will the whole entertainment cost her?

21. I buy in a shop 3 dozen collars at $9s.\ 6d.$ per dozen, 18 handkerchiefs at $1s.\ 1\frac{1}{2}d.$ each, 5 ties at $4s.\ 6d.$ each, and 4 pairs of gloves at $8s.\ 9d.$ per pair. What will the shopkeeper charge me altogether?

22. A boy resolves to save $\pounds 13$ in the course of a year by regular weekly savings. How much must he set aside per week?

23. How many articles at $\pounds 2\ 11s.\ 4d.$ each can be bought with $\pounds 79\ 8s.\ 4d.$, and what sum (if any) will be left over?

24. Find the total cost of 27 yds. at $3s.\ 7\frac{1}{2}d.$ per yard, 15 yds. at $4s.\ 2d.$ per yard, and 31 yds. at $11s.\ 7\frac{1}{2}d.$ per yard.

ELEMENTARY REVISORY EXERCISES 11

25. A woman buys a gross of eggs for 12s. She loses $1\frac{1}{2}$ dozen by breakage. At what rate per dozen must she sell the remainder in order to make 3s. 9d. profit?

26. Add:

<i>Hours</i>	<i>Minutes</i>	<i>Seconds</i>
7	29	41
3	16	38
14	44	26
5	34	50

27. Add:

<i>Acres</i>	<i>Roods</i>	<i>Sq. Yds.</i>
37	1	180
62	3	345
11	2	850

28. From 10 gal. 2 qts. take 4 gal. 2 qts. 1 pt.

29. Multiply 7 cwt. 1 qr. 12 lb. by 32.

30. Divide 38 miles 5 fur. 3 ch. by 24. (Obtain the quotient to within a yard.)

31. Find the difference between 37 tons 11 cwt. 1 qr. 11 lb. and 19 tons 16 cwt. 2 qrs. 20 lb.

32. Multiply 5 miles 3 fur. 4 ch. by 23.

33. If a ton of potatoes be divided equally among 37 families, what will be the share of each? (Calculate to within 1 lb.)

34. Multiply 7 cwt. 2 qrs. 11 lb. by 5, and divide the result by 7.

35. Multiply 36 acres 2 rds. 500 sq. yds. by 12, and divide the result by 17.

36. How many 2-lb. packets of tea can be made from a chest containing half a hundredweight?

37. From 2 tons 13 cwt. 1 qr. of metal 48 articles are made, each weighing 3 qrs. 12 lb. What weight of metal is left over?

38. A cycle-wheel measures 9 ft. 2 in. in circumference. How many revolutions should it make in running a distance of 4 miles?

39. From 80 yds. of wire how many pieces 2 ft. 3 in. long can be cut, and what length (if any) will remain?

40. A railway van weighing 5 tons 3 cwt. 1 qr. is loaded with 24 boxes each weighing 1 qr. 20 lb., 100 barrels each weighing 1 qr. 11 lb., and 47 parcels each weighing 33 lb. Find the total weight of the loaded truck.

41. Find the cost of 1 cwt. 6 st. 7 lb. at 2s. 3d. per lb.

42. How many times can a watering-can which holds a gallon and a half be filled from a tank which holds 16 gal. 2 qts.?

43. The side of a square field is 1 fur. 4 ch. 5 yds. 2 ft. A boy wishes to time himself for a 2-mile race by running round the field. How many complete laps must he make, and what extra distance?

44. How many packets each containing 1 lb. 4 oz. can be made from 3 cwt. 1 qr. 14 lb., and what weight (if any) will be left over?

45. Multiply 5 acres 3 rds. 420 sq. yds. by 11, and divide the result by 13.

46. A teacher sends a Christmas card to each of his pupils. The cards cost $4\frac{1}{2}d.$ each; the envelopes cost 2d. per dozen, and each requires a three-halfpenny stamp. If there are 30 pupils in the class, what will be the total cost to the teacher?

47. A field containing 3 acres is to be divided up into plots, each containing 240 sq. yds. How many complete plots can be marked out, and how many square yards of land will be left over?

48. In a railway train the engine weighs $48\frac{1}{2}$ tons and the tender $25\frac{1}{2}$ tons. There are 11 carriages, each weighing 8 tons 6 cwt., and 2 brake-vans, each weighing 8 tons 15 cwt. Find the total weight of the train.

49. A dealer buys a coil of wire 360 ft. in length for 2s. 11d. He cuts it up into lengths of 15 ft., and sells these for $3\frac{1}{2}d.$ each. What profit does he make?

50. Find the total cost of 6 lb. at 4s. 8d. per stone, $5\frac{1}{2}$ yds. at 1s. 10d. per yard, and 78 articles at 3s. 2d. per dozen.

CHAPTER II

MULTIPLES AND FACTORS

(N.B. *In this chapter where the word 'number' is used it refers to whole numbers only.*)

If we multiply, say, 7 by any number we obtain a multiple of 7. Thus 14, 21, 28, etc., are multiples of 7, and 7 is a factor of 14, 21, etc. Similarly, any number of 5's is a multiple of 5—*e.g.*, 35, 40, 75, and 5 is a factor of 35, 40, etc.

A number which has no factor except itself and unity is called a prime number—*e.g.*, 17. Other numbers are called composite numbers because they are made up, or composed, of factors—*e.g.*, 21, which is composed of the factors 3 and 7.

The student should notice carefully that it is convenient sometimes to resolve a number into its *prime factors*—*e.g.*, $225 = 3 \times 3 \times 5 \times 5$ —and sometimes into two or more factors which are not necessarily prime.

Consider

$$\begin{aligned} 225 &= 3 \times 75 \\ &\text{or } 5 \times 45 \\ &\text{or } 9 \times 25 \\ &\text{or } 15 \times 15 \end{aligned}$$

When, as in the last case, we can resolve a number into two *equal* factors we are said to have found its square root. Thus 15 is called the square root of 225, and 225 is called the square of 15.

When we resolve a number into two factors it is clear that if one is less than the square root the other must be greater. It follows that if a number has no factor less than its square root it can have none greater. This fact will shorten our work in finding whether a given number is prime or not.

Tests of Divisibility. The following short tests are worth remembering:

(a) Division by 2 or 5. The remainder, if any, is found by dividing the **last digit**.

Thus the remainder from $71839 \div 2$ and from $71839 \div 5$ is found from $9 \div 2$ and $9 \div 5$ respectively.

(b) Division by 4. The remainder is found by testing the number expressed by the **last two digits**.

Thus the remainder from $68453 \div 4$ is found from $53 \div 4$.

(c) Division by 8. The remainder is found by testing the number denoted by the **last three digits**.

Thus the remainder from $6183416 \div 8$ is found from $416 \div 8$.

(d) Division by 3 (or by 9). Test the **sum of the digits**.

Thus the remainder from $618347 \div 3$ is found from $29 \div 3$ (since 29 is the sum of the digits). Similarly, the remainder from $618347 \div 9$ is found from $29 \div 9$.

Proofs of these tests will be readily grasped by the student at a later stage.

MODEL EXAMPLES

(a) Find whether (i) 221 and (ii) 503 are prime.

(i) Partly by observation and partly by trial we find that 221 is not exactly divisible by 2 or 3 or 5 or 7 or 11. It is, however, exactly divisible by 13, and is therefore not prime.

(ii) 503 is not divisible by 2 or 3 or 5 or 7 or 11 or 13 or 17 or 19 or 23. Now, we know that 23 is beyond the square root of 503, since the quotient from $503 \div 23$ is less than the divisor 23. Hence, since 503 has no divisor less than its square root, it can have none greater, and is therefore prime.

(The student should ask himself how we *know* that 37, for example, cannot be a factor of 503.)

(b) Resolve 396 and 1176 into prime factors.

$$\begin{aligned}
 396 &= 9 \times 44 \text{ (evident from the test for 9)} \\
 &= 3 \times 3 \times 4 \times 11 \\
 &= 2 \times 2 \times 3 \times 3 \times 11. \\
 1176 &= 8 \times 147 \text{ (evident from the test for 8)} \\
 &= 8 \times 7 \times 21 \\
 &= 2 \times 2 \times 2 \times 3 \times 7 \times 7.
 \end{aligned}$$

(c) By what must (i) 396 and (ii) 1176 be multiplied to obtain a complete square?

(i) From above we see that $396 = 2 \times 2 \times 3 \times 3 \times 11$. Clearly we need introduce only an extra factor 11.

Then $396 \times 11 = (2 \times 3 \times 11) \times (2 \times 3 \times 11)$, which is the square of $2 \times 3 \times 11$.

(ii) $1176 = 2 \times 2 \times 2 \times 3 \times 7 \times 7$.

If we introduce two extra factors, a 2 and a 3, we have $1176 \times 2 \times 3 = (2 \times 2 \times 3 \times 7) \times (2 \times 2 \times 3 \times 7)$, which is the square of $2 \times 2 \times 3 \times 7$.

(d) Find the greatest common divisor of 336 and 1260.

$$\begin{aligned} 336 &= 3 \times 112 \\ &= 3 \times 8 \times 14 \\ &= 2 \times 2 \times 2 \times 2 \times 3 \times 7. \\ 1260 &= 10 \times 126 \\ &= 10 \times 6 \times 21 \\ &= 2 \times 2 \times 3 \times 3 \times 5 \times 7. \end{aligned}$$

The common *prime* factors are 2, 2, 3, 7.

We are seeking only the greatest common divisor, which must be $2 \times 2 \times 3 \times 7$ —i.e., 84.

(e) Find the lowest common multiple of 9 and 12.

$$\begin{aligned} 9 &= 3 \times 3. \\ 12 &= 2 \times 2 \times 3. \end{aligned}$$

Our required number must contain 3×3 if it is to contain 9. To make it contain 12 also we must include 2×2 .

Hence, the required lowest common multiple is $3 \times 3 \times 2 \times 2$ —i.e., 36.

(f) Find the least number that is a common multiple of 54, 36, and 48.

$$\begin{aligned} 54 &= 2 \times 3 \times 3 \times 3. \\ 36 &= 2 \times 2 \times 3 \times 3. \\ 48 &= 2 \times 2 \times 2 \times 2 \times 3. \end{aligned}$$

We take all the prime factors of 54. An extra 2 is needed for 36. An extra 2×2 is needed for 48.

Hence, the required number is $54 \times 2 \times 2 \times 2$ —i.e., 432.

ORAL EXERCISES

1. Resolve each of the following numbers into its prime factors: 8, 12, 27, 18, 30, 34, 52, 45, 60, 28, 98, 36, 70, 100.
2. Give all the possible divisors of each of the following: 8, 12, 27, 18, 30, 34, 45, 28, 70.
3. Give the first five multiples of each of the following numbers: 7, 9, 11, 12.
4. Give the prime numbers between 20 and 30, also those between 70 and 90.
5. The number 53 gives a remainder of 5 on being divided by 8. Write down the next six numbers which yield remainders of 5 on division by 8.
6. Give the first five multiples of 8 and the first five multiples of 6. Among these do you notice a *common* multiple of 8 and 6?
7. Give the first four multiples of 6 and the first four multiples of 9. Find a *common* multiple of 6 and 9.
8. Name all the *common* divisors of 12 and 30.
9. Name the prime numbers between 50 and 70.
10. The number 112 is exactly divisible by 7. Write down the next three numbers that are exactly divisible by 7. Hence give three numbers which when divided by 7 will give a remainder of 5.
11. What is the lowest common multiple of 4 and 6? Give the next three numbers that are common multiples of 4 and 6.
12. Can you tell which is the lowest number that is exactly divisible by 2, by 3, and by 5? Give the next three numbers that are common multiples of 2, 3, and 5.
13. What are the prime factors of 12? What extra factor would make a perfect square?
14. By what must 50 be multiplied to make a perfect square?
15. By what must 63 be multiplied to produce a perfect square?

EXERCISES II

1. Examine the following numbers, and classify each as prime or composite: 151, 177, 309, 347, 2481.

2. Resolve each of the following numbers into its prime factors: 120, 339, 375, 308.
3. Write down the first ten prime numbers.
4. 1097 contains none of the primes up to 31. Is it a prime number?
5. Resolve into prime factors 264, 135, 847, 945.
6. Resolve into prime factors 392, 5544, 2808.
7. The number 7172 is a multiple of 11. Write down the next six multiples of 11.
8. Resolve into prime factors 1584, 2835, 1902.
9. Find the remainder on dividing 6372 by 13. What is the least number that must be added to 6372 to make a multiple of 13? Write down the next six multiples of 13.
10. What must be added to 1000 to make a multiple of 19? Write down all the multiples of 19 between 1000 and 1100.
11. Resolve into prime factors 4536, 1397, 3240.
12. Resolve into prime factors 4068, 2425, 2412.
13. By what must 180 be multiplied to make a perfect square?
14. By what must 1260 be multiplied to make a perfect square?
15. Resolve into prime factors 3430 and 25,410.
16. Is 2001 a multiple of 13? Find all the numbers from 2001 to 2101 that are multiples of 13.
17. Resolve into prime factors 1040, 7480.
18. Find the prime numbers between 400 and 409.
19. By what must 3380 be multiplied to give a perfect square?
20. The number 7,413,791 is a multiple of 7. Write down the next six multiples of 7.
21. Here is a list of numbers: 437, 2222, 6190, 4328, 54,540, 444, 20,799, 13,579.
 - (a) Pick out those which divide exactly by 2.
 - (b) Find the multiples of 3.
 - (c) Find the multiples of 5.
 - (d) Find the remainders on dividing by 8.
 - (e) Find the remainders on dividing by 9.

22. Here is a number whose second digit has been rubbed out: 4-613. If the complete number were divided by 9 the remainder would be 4. What must the missing digit be? Test your result.

23. If the complete number is to be exactly divisible by 4, fill in the missing figure in the following number: 31-6. Test your result (or results).

24. Here is a big number of whose digits the last but two is missing: 64077-36. It is known that the complete number is exactly divisible by 8. Find all possible values of the missing digit.

25. If it is known also that the number in Exercise 24 is exactly divisible by 9 as well as by 8, what must the number be?

26. Find all the numbers from 1241 up to 1261 that are exactly divisible by 9. Then write down all those which will give a remainder of 5 when divided by 9.

27. A certain number having four digits is exactly divisible by 9. The first digit is 7 and the last is 5. Can you find the second and third digits? Give as many solutions as you can.

28. Find all the numbers that will divide 385 exactly.

29. Find (*a*) all the *prime* factors of 105, and (*b*) all the divisors.

30. Find all the common divisors of 105 and 120.

31. Find the greatest common divisor of 84 and 140.

32. Find (*a*) the greatest common divisor of 1050 and 4710, and (*b*) the next greatest common divisor.

33. Find the greatest number that is a factor of 252 and also of 945.

34. What is the least number that is exactly divisible by either 42 or 30?

35. Find the least common multiple of 90 and 135.

36. Find the lowest number that is a common multiple of 336 and 420.

37. Find the greatest common divisor of 1320, 3780, and 390. Find also their least common multiple.

38. Find the lowest common multiple, and also the highest common divisor, of the following:

- (a) 306, 714, 1071.
- (b) 195, 380, 110.
- (c) 126, 360, 162.
- (d) 360, 240, 144.
- (e) 784, 1176, 336.
- (f) 378, 945, 630, 567.

39. The front wheel of a bicycle measures 108 in. in circumference, and the back wheel measures 120 in. How far must the bicycle travel before both wheels have made an exact number of complete revolutions?

40. Three men whose paces measure respectively 30 in., 33 in., and 36 in., set out walking. If they are in step at first, and if they remain abreast, how far will they walk before being again in step? How many paces will each man have taken?

41. A rectangular floor 192 in. long and 150 in. broad is to be laid with square tiles. What is the size of the largest possible tile if whole tiles only may be used?

42. Find the greatest number which will divide the numbers 841, 211, 295.

43. Find the lowest common multiple of

- (a) 14, 24, 36, 42.
- (b) 34, 51, 68, 85, 102, 119.
- (c) 42, 64, 70, 112.

44. The front wheel of a carriage is 168 in. in circumference, the rear wheel 132 in. What is the shortest distance the carriage must travel in order that each wheel may make an integral number of complete revolutions?

45. A florist has 120 red roses, 180 white, 225 yellow. He wishes to make up as many uniform bouquets as possible containing all three colours. If he uses all the blooms, what will be the composition of each bouquet?

46. On the roll of a school there are 450 pupils. An examiner tests every fifth pupil in writing, every sixth in arithmetic, every seventh in composition. How many pupils are tested in all three subjects?

47. Street lamps, 40 yds. apart, are erected along a road.

Every sixth lamp is made a tram stopping-place, and every ninth lamp is made a bus stopping-place. How many stopping-places serving both bus and tram will there be in the first mile?

48. Find the smallest number that will give a remainder of 2 when divided by either 6, 7, or 8.

49. Find the greatest number that will divide 145, 421, and 397 giving a remainder of 1 in each case.

50. Find the lowest number that is exactly divisible by each of the numbers 2, 3, 4, 5, 6, 7, 8, 9.

CHAPTER III

VULGAR FRACTIONS

TAKE a strip of paper 1 foot long. If we divide this into two equal parts each is called one-half of a foot. We write this $\frac{1}{2}$ foot. If we divide the whole strip into three equal parts each is called one-third of a foot, and this is written $\frac{1}{3}$ foot. Similarly, we could obtain $\frac{1}{4}$ foot, $\frac{1}{5}$ foot, $\frac{1}{6}$ foot, and so on. The student is already familiar with $\frac{1}{4}d.$, $\frac{1}{2}d.$, and $\frac{3}{4}d.$ in dealing with farthings and halfpennies.

In this way we express 'bits' or fractions, and fractions expressed in this way are called **vulgar fractions**. The word 'vulgar' means *common*. For example, if we divide a pound of, say, butter into seven equal parts each part would be $\frac{1}{7}$ lb., and if we take five such parts this weight would be written $\frac{5}{7}$ lb. The lower number is called the **denominator**—*i.e.*, the name of the part—and the upper number is the **numerator**—*i.e.*, the number of parts taken.

Examine a foot-rule which is divided into inches. Notice that $\frac{1}{2}$ foot = $\frac{2}{4}$ foot = $\frac{3}{6}$ foot, and that $\frac{1}{3}$ foot = $\frac{2}{6}$ foot.

Find what $\frac{1}{8}$ foot is, and then notice that $\frac{3}{4}$ foot = $\frac{6}{8}$ foot.

It is not difficult, then, to understand that if we multiply both the numerator and the denominator of a fraction by the same factor we do not change the *value* of the fraction, although we change its appearance.

Thus, $\frac{5}{6}$ of an inch is the same as $\frac{15}{18}$ of an inch, and $\frac{1}{24}$ of a foot is the same as $\frac{4}{96}$ foot or $\frac{2}{48}$ foot.

ADDITION AND SUBTRACTION

MODEL EXAMPLES

- (a) Which is the greater length, $\frac{5}{12}$ yd. or $\frac{7}{16}$ yd.?
The lowest common denominator is 48.

Now,
and

$$\frac{5}{12} = \frac{20}{48}$$

$$\frac{7}{16} = \frac{21}{48}.$$

Hence the latter fraction is greater than the other by $\frac{1}{48}$ yd.

(b) Add $\frac{1}{8}$, $\frac{5}{12}$, $\frac{2}{9}$.

$$8 = 2 \times 2 \times 2.$$

$$12 = 2 \times 2 \times 3.$$

$$9 = 3 \times 3.$$

The least common denominator, therefore, is $8 \times 3 \times 3$ —*i.e.*, 72.

$$\frac{1}{8} + \frac{5}{12} + \frac{2}{9} = \frac{9}{72} + \frac{30}{72} + \frac{16}{72}.$$

$$= \frac{55}{72}.$$

(c) Add $4\frac{7}{10}$, $2\frac{2}{15}$, $5\frac{5}{6}$.

$$10 = 2 \times 5.$$

$$15 = 3 \times 5.$$

$$6 = 2 \times 3.$$

The least common denominator is 30.

$$4\frac{7}{10} + 2\frac{2}{15} + 5\frac{5}{6} = 11\frac{21}{30} + \frac{4}{30} + \frac{25}{30}$$

$$= 11\frac{50}{30}$$

$$= 12\frac{2}{3}.$$

(d) From $\frac{7}{15}$ subtract $\frac{3}{10}$.

$$\frac{7}{15} - \frac{3}{10} = \frac{14}{30} -$$

$$= \frac{5}{30}$$

$$= \frac{1}{6}.$$

(e) From $7\frac{5}{18}$ take $2\frac{1}{2}\frac{3}{4}$.

$$7\frac{5}{18} - 2\frac{1}{2}\frac{3}{4} = 5\frac{20}{36} - \frac{39}{36}$$

$$= 4\frac{9}{36} - \frac{39}{36}$$

$$= 4\frac{5}{12}.$$

(f) By another useful method:

$$7\frac{5}{18} - 2\frac{1}{2}\frac{3}{4} = 5\frac{20}{36} - \frac{39}{36}.$$

If from $5\frac{20}{36}$ we take away $\frac{39}{36}$ we shall still require to subtract $\frac{19}{36}$.

Hence

$$5\frac{20}{72} - \frac{39}{72} = 5 - \frac{19}{72} \\ = 4\frac{53}{72}.$$

$$(g) \quad 6\frac{11}{24} - 2\frac{19}{20} = 4\frac{55}{120} - \frac{114}{120} \\ = 4 - \frac{59}{120} \\ = 3\frac{61}{120}.$$

(h) Evaluate $7\frac{3}{10} + 4\frac{4}{15} - 2\frac{5}{12} + 1\frac{5}{9} - 4\frac{7}{8}$.

Note. The two fractions which are to be subtracted may be taken away one after the other, or they may be added together and their sum subtracted.

$$\begin{aligned} & (7\frac{3}{10} + 4\frac{4}{15} + 1\frac{5}{9}) - (2\frac{5}{12} + 4\frac{7}{8}) \\ &= (12 + \frac{27}{90} + \frac{24}{90} + \frac{50}{90}) - (6\frac{10}{24} + \frac{21}{24}) \\ &= 13\frac{11}{90} - 7\frac{7}{24} \\ &= 6\frac{44}{360} - \frac{105}{360} \\ &= 6 - \frac{61}{360} \\ &= 5\frac{299}{360}. \end{aligned} \quad \left| \begin{array}{l} 12\frac{101}{90} = 13\frac{11}{90} \\ 6\frac{31}{24} = 7\frac{7}{24} \end{array} \right.$$

(i) When the steps are thoroughly mastered the work may be set down more briefly. For example,

$$\begin{aligned} & 7\frac{5}{18} - 3\frac{7}{12} + 4\frac{4}{15} - 1\frac{9}{10} \\ &= (7\frac{5}{18} + 4\frac{4}{15}) - (3\frac{7}{12} + 1\frac{9}{10}) \\ &= 11\frac{\frac{25+24}{90}}{90} - 4\frac{\frac{35+9}{60}}{60} \\ &= 11\frac{49}{90} - 5\frac{44}{60} \\ &= 6\frac{\frac{98-88}{180}}{180} \\ &= 6\frac{10}{180}. \end{aligned} \quad \left| \begin{array}{l} 4\frac{89}{90} = 5\frac{44}{60} \end{array} \right.$$

ORAL EXERCISES

1. How many eighths are in $\frac{3}{4}$? How many in $\frac{1}{4}$?
2. How many twelfths are in $\frac{2}{3}$? How many in $\frac{5}{6}$? How many in $\frac{3}{4}$?
3. How many tenths are in $\frac{1}{2}$? How many in $\frac{1}{3}$? How many in $\frac{1}{2} + \frac{1}{3}$?
4. What is the simplest way of expressing $\frac{15}{20}$ of a yard?
5. From a yard of ribbon I cut off a piece $\frac{1}{4}$ yd. long. How much is left?
6. How much is left if from a yard of ribbon I cut $\frac{1}{8}$ yd.? How much if I cut $\frac{3}{8}$ yd.?

7. How many eighths are in $\frac{1}{2}$? If from half a yard of ribbon I cut $\frac{1}{8}$ yd. how much is left?

8. How many sixths are in $\frac{1}{2} + \frac{1}{3}$?

9. From $\frac{1}{2}$ yd. of wire I cut $\frac{1}{5}$ yd. How much is left?

10. How many twelfths are in $\frac{3}{4}$? How many in $\frac{5}{6}$? Can you tell now whether $\frac{3}{4}$ yd. or $\frac{5}{6}$ yd. is greater?

11. Express $\frac{5}{6}$ and $\frac{7}{9}$ each in eighteenths. Now tell which fraction is the greater.

12. A ribbon is 10 in. long. If I cut off 3 in., how much is left? If instead I had cut off $3\frac{1}{4}$ in., how much would have been left?

EXERCISES IIIA

1. Add $\frac{3}{8}$ and $\frac{1}{12}$.

2. Add $\frac{1}{6}$, $\frac{3}{8}$, $\frac{2}{9}$.

3. Find the sum of $\frac{3}{4}$, $\frac{3}{10}$, $\frac{1}{8}$.

4. Add $2\frac{2}{15}$, $3\frac{3}{10}$, $4\frac{1}{6}$.

5. Add $7\frac{5}{12}$, $4\frac{11}{15}$, $3\frac{3}{10}$.

6. Add $3\frac{11}{14}$, $5\frac{4}{11}$, $3\frac{5}{6}$.

7. Find the value of $4\frac{5}{18} + 3\frac{7}{12} + 4\frac{3}{8}$.

8. Add $4\frac{3}{8}$, $\frac{5}{6}$, $3\frac{2}{5}$.

9. Add $\frac{7}{16}$, $\frac{11}{12}$, $\frac{13}{15}$.

10. Add $6\frac{4}{9}$, $5\frac{17}{30}$, $7\frac{31}{60}$, $2\frac{7}{12}$, $1\frac{13}{45}$.

11. Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$. Can you find a short way of obtaining the answer?

12. Find the sum of 4s. $3\frac{1}{2}d.$, 2s. $11\frac{3}{8}d.$, 14s. $7\frac{1}{5}d.$, 1s. $10\frac{5}{6}d.$

13. Add 7 lb. $10\frac{1}{4}$ oz., 11 lb. $9\frac{7}{8}$ oz., 12 lb. $14\frac{7}{12}$ oz.

14. Add 5 ft. $2\frac{3}{4}$ in., 12 ft. $6\frac{5}{12}$ in., 7 ft. $11\frac{5}{8}$ in.

15. Evaluate $3\frac{14}{15} + 7\frac{11}{10} + 39\frac{23}{5}$.

16. Evaluate £3 7s. $9\frac{3}{8}d.$, + £6 18s. $10\frac{11}{12}d.$

17. Evaluate $1\frac{1}{2} + 2\frac{2}{3} + 3\frac{3}{4} + 4\frac{4}{5}$.

18. Write down $\frac{2}{13}$ six times and find the sum. Can you now see how to multiply $\frac{2}{13}$ by 6?

19. Add $13\frac{17}{20}$, $9\frac{11}{15}$, $18\frac{13}{18}$, $17\frac{11}{30}$.

20. For the specified purposes the following are the rates of taxation per pound of rental in a certain county: police, $5\frac{1}{5}d.$; county general, $\frac{4}{5}d.$; roads and bridges, 1s. $9\frac{3}{8}d.$; public health, 1s. $1\frac{2}{3}d.$; water, 1s. $2\frac{1}{4}d.$; lighting, $4\frac{3}{5}d.$; draining,

$11\frac{5}{8}d.$; scavenging, $5\frac{4}{5}d.$; poor relief, $9\frac{1}{12}d.$; education, $3s. 6\frac{1}{2}d.$; lunacy, $3\frac{1}{4}d.$ Calculate the 'cumulo'—*i.e.*, the total rate per pound.

21. Find the combined weight of three parcels, one of which weighs $37\frac{3}{4}$ lb., another 3 st. $11\frac{1}{8}$ lb., and the third 1 st. $6\frac{1}{2}$ lb.

22. Add $\frac{1}{3}$ of $\pounds 8\ 7s. 4d.$, $\frac{1}{6}$ of $\pounds 5\ 11s. 7d.$, and $\frac{1}{8}$ of $\pounds 9\ 11s. 10d.$ Do not introduce farthings into your working.

23. I have three coils of wire, each containing 28 yds. 2 ft. I cut off $\frac{1}{5}$ of the first, $\frac{1}{10}$ of the second, $\frac{1}{6}$ of the third. Calculate (in two ways) the total length of wire cut off.

24. One workman is paid at the rate of $2s. 7\frac{5}{8}d.$ per hour, another at $1s. 10\frac{5}{12}d.$ per hour, and a third at $1s. 8\frac{5}{8}d.$ per hour. What will it cost me to employ all three for one hour?

25. Add 8 lb. $5\frac{5}{12}$ oz., 11 lb. $10\frac{5}{6}$ oz., 17 lb. $9\frac{3}{8}$ oz.

26. Add 8 miles $6\frac{2}{3}$ fur., 5 miles $5\frac{5}{8}$ fur., 3 miles $5\frac{7}{10}$ fur.

27. Add $6\frac{4}{15}$, $3\frac{1}{20}$, $4\frac{1}{5}$, $7\frac{1}{4}$.

28. Add $7\frac{2}{3}$, $5\frac{3}{10}$, $9\frac{1}{2}$.

29. Calculate to a fraction of a second the seventh part of an hour, the eleventh part of a day, the ninth part of a minute. Add your three results.

30. Add $1s. 4\frac{5}{12}d.$, $2s. 7\frac{4}{9}d.$, $1s. 11\frac{7}{10}d.$, $1s. 9\frac{3}{8}d.$

EXERCISES III_B

1. Subtract $4\frac{1}{3}$ from $7\frac{5}{9}$.

2. Find the value of $7\frac{4}{15} - 2\frac{1}{12}$.

3. Take $3\frac{5}{8}$ from $7\frac{1}{6}$.

4. From $11\frac{4}{9}$ take $3\frac{5}{6}$, and from the result take $1\frac{1}{3}$.

5. What must be added to $4\frac{5}{12}$ to make $7\frac{2}{9}$?

6. What must be taken from $11\frac{9}{14}$ to leave $6\frac{1}{2}$?

7. From 20 take $11\frac{3}{14}$, and from the result take $2\frac{1}{2}$.

8. Subtract $13\frac{1}{5}$ from $19\frac{7}{5}$.

9. From 3 cwt. subtract 1 cwt. $67\frac{7}{12}$ lb.

10. From a wire $22\frac{5}{8}$ ft. long I cut off $7\frac{1}{12}$ ft., and then $3\frac{3}{4}$ ft. How much is left? (Find the result in two ways.)

11. From $2s. 4\frac{1}{6}d.$ subtract $1s. 9\frac{1}{12}d.$

12. Evaluate $237\frac{1}{18} - 179\frac{1}{20}$.

13. Evaluate $7\frac{1}{9} - 2\frac{1}{2}\frac{1}{4}$.
 14. From $16\frac{3}{4}$ take $5\frac{1}{3}\frac{3}{5}$, and from the result take $2\frac{1}{4}\frac{9}{2}$.
 15. Subtract 2 qrs. $11\frac{7}{2}$ lb. from $237\frac{1}{5}\frac{1}{5}$ lb.
 16. From 17s. $10\frac{1}{5}d$. take 3s. $10\frac{7}{8}d$.
 17. Evaluate $23\frac{1}{2}\frac{3}{4} - 11\frac{1}{3}\frac{9}{10}$.
 18. From $164\frac{1}{6}\frac{1}{6}$ take $97\frac{1}{6}$, and from the result take $12\frac{1}{2}\frac{3}{4}$.
 19. By how much is $19\frac{1}{9}$ greater than $11\frac{4}{15}$?
 20. Subtract $17\frac{7}{8}$ from $31\frac{5}{12}$.
 21. Find the difference between 3s. $7\frac{4}{15}d$. and 2s. $9\frac{1}{2}\frac{1}{2}d$.
 22. Find a third of 17s. 8d., and from the result take 2s. $9\frac{3}{8}d$.
- Do not introduce farthings.
23. From a fifth of £3 7s. 9d. take a ninth of £4 11s.
 24. Subtract a sixth part of 13 ft. 5 in. from a ninth part of 20 ft. 8 in.
 25. Calculate the difference between an eighth part of 18s. 7d. and a sixth part of 9s. 5d.
 26. Evaluate $147\frac{1}{2}\frac{3}{10} - 98\frac{1}{4}\frac{7}{5}$.
 27. Subtract the seventh part of a day from 5 hrs. 13 min. $11\frac{1}{3}$ sec.
 28. Subtract $11\frac{1}{5}\frac{4}{5}$ from $23\frac{5}{18}$, and from the result take $7\frac{3}{4}$.
 29. Evaluate $(19\frac{1}{4}\frac{7}{5} - 3\frac{7}{3}\frac{7}{10}) - 2\frac{5}{18}$.
 30. From the ninth part of 3 cwt. 1 qr. 15 lb. subtract the twelfth part of 3 cwt. 3 qrs. 5 lb.

EXERCISES IIIc

1. Simplify $6\frac{3}{10} + 2\frac{1}{6} - 3\frac{4}{15}$.
2. Simplify $13\frac{5}{8} - 1\frac{1}{2}\frac{1}{2} - 4\frac{1}{18}$.
3. Simplify $4\frac{1}{8}\frac{3}{8} + 3\frac{1}{2}\frac{1}{2} - 2\frac{3}{8}$.
4. Find the value of $5\frac{2}{15} - 3\frac{7}{12} + 1\frac{1}{6}$.
5. Evaluate $4\frac{5}{14} - 2\frac{1}{2}\frac{1}{1} + 5\frac{5}{28} - 4\frac{5}{6}$.
6. Evaluate $4\frac{4}{9} - \frac{1}{8} - 2\frac{5}{12} + 3$.
7. What must be taken from $3\frac{7}{12} + 1\frac{1}{8}$ to leave $2\frac{1}{2}\frac{9}{4}$?
8. Simplify $5\frac{1}{15} - 1\frac{7}{10} - \frac{2}{9} - \frac{4}{45}$.
9. Simplify $9\frac{5}{7} - 8\frac{4}{5} - 9\frac{1}{2}\frac{1}{1} + 10\frac{4}{15} + \frac{3}{35}$.
10. Simplify $2\frac{2}{5} - 3\frac{3}{10} - 4\frac{7}{15} - 1\frac{4}{25} + 11\frac{1}{30}$.
11. Simplify $5\frac{5}{12} + 3\frac{7}{18} - 2\frac{4}{9} - 1\frac{1}{2}\frac{7}{4}$.
12. Simplify $8\frac{1}{2}\frac{5}{2} + \frac{1}{15}\frac{4}{5} - 3\frac{4}{5}\frac{9}{5} - 2\frac{2}{3}\frac{3}{3}$.

13. Simplify $4\frac{1}{3} - 2\frac{5}{6} - \frac{1}{2}\frac{1}{6} - \frac{4}{3}\frac{1}{9}$.

14. Add $2\frac{5}{8}$, $3\frac{1}{12}$, $5\frac{5}{6}$, $4\frac{1}{24}$.

15. Simplify $9\frac{7}{16} + 2\frac{1}{80} - 1\frac{1}{2} - 1\frac{1}{20}$.

16. Simplify $4\frac{1}{9} - 1\frac{5}{18} + 3\frac{5}{8} + 4\frac{9}{16} - \frac{1}{2}\frac{1}{7}$.

17. Simplify $14\frac{5}{7} - 1\frac{2}{3} - 2\frac{5}{24} - 2\frac{5}{8}$.

18. Simplify $7\frac{4}{15} - \frac{3}{10} - \frac{4}{25} - \frac{1}{5}$.

19. The following are the rates of taxation for the different branches of expenditure in a certain county: police, $5\frac{3}{8}d.$; roads, $1s. 10\frac{1}{2}d.$; health, $1s. 1\frac{5}{12}d.$; water, $1s. 2\frac{5}{6}d.$; lighting and cleansing, $11\frac{7}{16}d.$; general purposes, $2\frac{3}{4}d.$ Find the total rate.

20. Complete the following table:

TAX	RATE, Nov. 1927	RATE, Nov. 1926	INCREASE	DECREASE
Police	$5\frac{3}{8}d.$	$5\frac{1}{6}d.$		
Roads	$1s. 10\frac{1}{2}d.$	$1s. 8\frac{5}{8}d.$		
Health	$1s. 1\frac{5}{12}d.$	$1s. 2\frac{1}{6}d.$		
Water	$1s. 2\frac{5}{6}d.$	$1s. 3\frac{3}{4}d.$		
Lighting and cleansing	$11\frac{7}{16}d.$	$10\frac{1}{2}d.$		
General	$2\frac{3}{4}d.$	$1\frac{5}{6}d.$		
Totals				

Check by comparing the difference between the totals of the first and second columns of figures with that between the totals of the third and fourth columns.

21. From the sum of $7s. 9\frac{5}{8}d.$, $3s. 4\frac{1}{6}d.$, and $2s. 11\frac{3}{8}d.$ take the sum of $5s. 3\frac{7}{10}d.$ and $1s. 6\frac{4}{15}d.$

22. Evaluate $9 - 3\frac{1}{15} - 2\frac{1}{10} - 1\frac{5}{12}$.

23. Evaluate $6\frac{4}{21} - 3\frac{1}{14} - 1\frac{5}{6} + 3\frac{1}{2}\frac{1}{8}$.

24. Simplify $(7\frac{7}{9} - 3\frac{5}{12}) - (2\frac{4}{15} - 1\frac{1}{20})$.

25. Simplify $7\frac{1}{3} - (2\frac{4}{9} - 1\frac{1}{3} + \frac{5}{6})$.

26. Simplify $(3\frac{1}{4} - 2\frac{7}{10}) + (1\frac{4}{5} - \frac{5}{6})$.

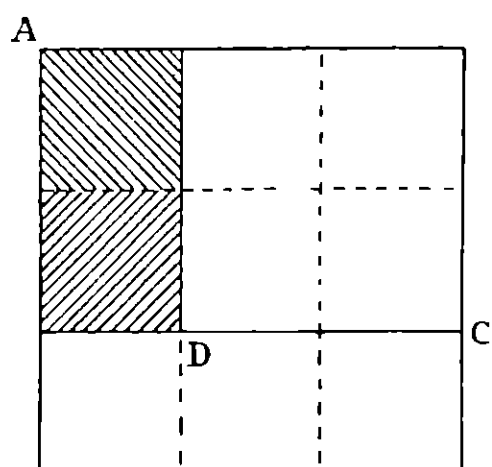
27. The price of a certain article is $7s. 5\frac{5}{8}d.$ at the opening of the market. The price rises $\frac{3}{16}d.$, then falls $\frac{1}{12}d.$, then falls again $\frac{1}{6}d.$ Calculate the price after these changes.

28. Evaluate $(6\frac{7}{18} - 2\frac{1}{20}) - (8\frac{1}{15} - 6\frac{1}{12})$.

29. Simplify $7\frac{5}{9} - 1\frac{4}{5} - 2\frac{1}{15} + 3\frac{1}{18} - (4\frac{1}{7} - 1\frac{9}{14})$.

30. Simplify $(6\frac{1}{3} - 2\frac{4}{5}) - (3\frac{5}{9} - 1\frac{1}{6}) + (2\frac{5}{8} - 1\frac{1}{12})$.

MULTIPLICATION



Example (a): Find $\frac{1}{3}$ of $\frac{2}{3}$.

Let AB represent the unit—say, 1 sq. ft.

Then $AC = \frac{2}{3}$ of the unit
and $AD = \frac{1}{3}$ of AC
 $= \frac{1}{3}$ of $\frac{2}{3}$.

But clearly AB is divided into 9 ninths and AD is 2 ninths.

$$\frac{1}{3} \text{ of } \frac{2}{3} =$$

Students should try to deduce the rule for obtaining the answer without drawing a diagram.

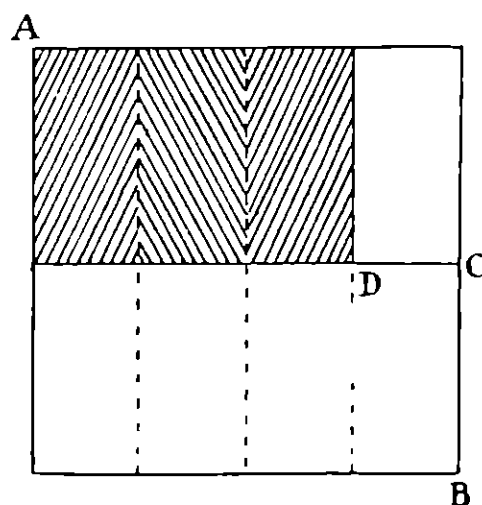
Example (b): Find $\frac{3}{4}$ of $\frac{1}{2}$.

If AB represents the unit,

then $AC = \frac{1}{2}$ of the unit
and $AD = \frac{3}{4}$ of AC
 $= \frac{3}{4}$ of $\frac{1}{2}$.

But AD is clearly $\frac{3}{8}$ of the unit.

$$\frac{3}{4} \text{ of } \frac{1}{2} = \frac{3}{8}.$$



Again the students should deduce the rule for obtaining the answer—viz., multiply the numerators and multiply the denominators.

Thus,

$$\frac{3}{4} \text{ of } \frac{1}{2} = \frac{1 \cdot 3}{2 \cdot 4} \text{ (according to the rule just evolved)} \\ = \frac{3}{8}.$$

The student should now ask whether the result, $\frac{3}{8}$, could not have been found without having to obtain $\frac{1 \cdot 3}{2 \cdot 4}$. Obviously we may cancel out a 3 in the numerator with a 3 in the denominator, likewise a 2 above and below.

Thus,

$$\frac{\overset{1}{3}}{\underset{1}{4}} \text{ of } \frac{\overset{1}{3}}{\underset{2}{4}} = \frac{1}{2}.$$

Similarly,

$$\frac{\overset{9}{18}}{\underset{5}{25}} \text{ of } \frac{\overset{3}{15}}{\underset{8}{16}} = \frac{27}{40}.$$

The student should now observe that the form $\frac{36}{70} \times \frac{14}{27}$ implies $\frac{14}{27}$ of $\frac{36}{70}$.

Obviously, however, the answer can be found without rewriting:

$$\frac{\overset{4}{36}}{\underset{5}{70}} \times \frac{\overset{1}{14}}{\underset{3}{27}} = \frac{4}{15}.$$

DIVISION

The rule for division by a fraction may be elicited by considering some obvious examples.

Students should note that if we diminish the divisor the quotient is increased.

Thus,

$$\begin{aligned} 12 \div 12 &= 1. \\ 12 \div 6 &= 2. \\ 12 \div 4 &= 3. \\ 12 \div 3 &= 4. \\ 12 \div 2 &= 6. \\ 12 \div 1 &= 12. \\ 12 \div \frac{1}{2} &= 24. \text{ (This is obvious.)} \\ 12 \div \frac{1}{4} &= 48. \end{aligned}$$

From this we may suggest that

$$\begin{aligned} 12 \div \frac{1}{2} &= 12 \times \frac{2}{1}, \\ \text{and that} \quad 12 \div \frac{1}{4} &= 12 \times \frac{4}{1}, \end{aligned}$$

and from these examples deduce a rule—namely, *invert* the divisor and multiply.

$$\begin{aligned} \text{Thus,} \quad \frac{14}{25} \div \frac{21}{40} &= \frac{14}{25} \times \frac{40}{21} \\ &= \frac{16}{15}, \text{ or } 1\frac{1}{15}. \end{aligned}$$

MULTIPLICATION AND DIVISION

MODEL EXAMPLES

(a) Evaluate $2\frac{3}{4} \times \frac{3}{7}$

$$\frac{11}{4} \times \frac{3}{7} = \frac{33}{28}$$

$$= 1\frac{5}{4}.$$

(b) Find the value of $\frac{2}{9} \times \frac{1}{8} \times 1\frac{1}{3}$.

$$\frac{2}{9} \times \frac{1}{8} \times \frac{4}{3} = \frac{2}{9}.$$

(c) Divide $3\frac{1}{7}$ by $2\frac{3}{4}$.

$$\frac{22}{7} \div \frac{11}{4} = \frac{22}{7} \times \frac{4}{11}$$

$$= \frac{8}{7}$$

$$= 1\frac{1}{7}.$$

(d) Divide $\frac{7}{15}$ of $\frac{5}{8}$ by $3\frac{1}{2}$.

$$(\frac{7}{15} \text{ of } \frac{5}{8}) \div \frac{7}{2} = \frac{7}{15} \text{ of } \frac{5}{8} \times \frac{2}{7}$$

$$= \frac{1}{12}.$$

(e) Divide $\frac{7}{12}$ by $\frac{3}{8}$ of $2\frac{1}{3}$.

$$\frac{7}{12} \div \left(\frac{3}{8} \text{ of } \frac{7}{3}\right) = \frac{7}{12} \div \frac{7}{8}$$

$$= \frac{7}{12} \times \frac{8}{7} = \frac{2}{3}.$$

Or, more briefly, $\frac{7}{12} \div (\frac{3}{8} \text{ of } \frac{7}{3})$

$$= \frac{7}{12} \times \frac{8}{3} \times \frac{3}{7} = \frac{2}{3}.$$

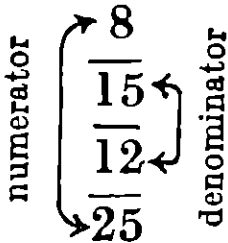
(f) Simplify $\frac{\frac{15}{19}}{\frac{25}{25}}$.

This may be written $\frac{8}{15} \div \frac{12}{25}$.

$$\begin{aligned} &= \frac{2}{8} \times \frac{5}{25} \\ &= \frac{3}{12} \times \frac{3}{25} \\ &= \frac{10}{9}, \text{ or } 1\frac{1}{9}. \end{aligned}$$

It is useful, however, to treat the fraction in its given form. Note that the two outside numbers are the factors of the numerator and that the two inside are those of the denominator; thus:

Therefore we may cancel any outside factor with an inside factor.



$$\begin{aligned} &\frac{8}{15} \div \frac{12}{25} = \frac{10}{9} \\ &= 1\frac{1}{9}. \end{aligned}$$

(g) Simplify $\frac{8\frac{2}{5} \times \frac{5}{24} \div \frac{35}{36}}{2\frac{2}{5} - \frac{5}{24} - \frac{35}{36}}$.

The given expression

$$\begin{aligned} &= \frac{\frac{42}{5}}{1\frac{79}{360}} \\ &= \frac{9}{51} \\ &= \frac{439}{360} \\ &= \frac{648}{439} \\ &= 1\frac{209}{439} \end{aligned}$$

$$\begin{aligned} &\frac{1}{8} \times \frac{1}{24} \times \frac{9}{35} = \frac{9}{5} \\ &= 2\frac{144-75-350}{360} \\ &= 2\frac{144-425}{360} \\ &= 2 - \frac{281}{360} \\ &= 1\frac{79}{360}. \end{aligned}$$

ORAL EXERCISES

1. Multiply $\frac{1}{4}$ yd. by 3. Multiply $\frac{1}{5}$ yd. by 2. Multiply $\frac{1}{5}$ yd. by 2. Multiply $\frac{3}{14}$ yd. by 3.
2. What is $\frac{1}{2}$ of $\frac{1}{2}$, $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{3}$ of $\frac{1}{4}$, $\frac{1}{3}$ of $\frac{1}{6}$?
3. What is $\frac{1}{2}$ of $\frac{6}{7}$, $\frac{1}{2}$ of $\frac{8}{9}$, $\frac{1}{2}$ of $\frac{1}{7}$, $\frac{1}{2}$ of $\frac{3}{7}$?
4. What is $\frac{1}{3}$ of $\frac{1}{7}$, $\frac{2}{3}$ of $\frac{1}{7}$, $\frac{2}{3}$ of $\frac{5}{7}$?
5. Multiply $3\frac{1}{4}$ by 2, $4\frac{1}{5}$ by 3, $3\frac{2}{15}$ by 4.
6. How many half-yards are there in 3 yards?
7. Divide 3 by $\frac{1}{2}$. Is not this the same problem as No. 6?
8. Divide 7 by $\frac{1}{3}$. Divide 7 by $\frac{1}{5}$.
9. Multiply $3\frac{1}{5}$ yds. by 3. Multiply $4\frac{1}{2}$ yds. by 3.
10. Divide $\frac{1}{2}$ by 7. Divide 7 by $\frac{1}{2}$.

Preliminary Note to Exercises III_D. In setting any example involving the employment of several signs we use *brackets* wherever there would otherwise be a doubt as to the order in which the various operations are to be carried out. For example, $\frac{5}{8} \div \frac{3}{4} \times \frac{2}{3}$ should be set as $(\frac{5}{8} \div \frac{3}{4}) \times \frac{2}{3}$, which is its legitimate meaning.

As a guide, however, it may be stated that 'of' is regarded as the most closely binding sign. Thus, $\frac{5}{8} \div \frac{3}{4}$ of $\frac{2}{3}$ is to be treated as if it were written $\frac{5}{8} \div (\frac{3}{4} \text{ of } \frac{2}{3})$. Wherever the omission of brackets has left room for doubt the student should give precedence first to operations indicated by 'of,' next to the signs $\times$ and $\div$ in the order in which they occur, and finally to the signs $+$ and $-$ in their order.

Thus the example $\frac{2}{3} + \frac{4}{5} \div \frac{3}{7}$ of $\frac{1}{2} - \frac{3}{8} \times \frac{2}{7}$ is really badly set. The student should treat it as if it were written

$$\frac{2}{3} + \left\{ \frac{4}{5} \div \left(\frac{3}{7} \text{ of } \frac{1}{2} \right) \right\} - \left(\frac{3}{8} \times \frac{2}{7} \right).$$

EXERCISES III_D

Evaluate:

1. $\frac{4}{15}$ of $\frac{25}{32}$.
2. $\frac{14}{33}$ of $\frac{22}{35}$.
3. $\frac{4}{9}$ of $\frac{27}{28}$ of $\frac{14}{45}$.
4. $\frac{3}{16} \times \frac{12}{25} \times \frac{10}{27}$.
5. $\frac{4}{15} \times \frac{25}{48} \times \frac{9}{10}$.

6. $\frac{1\frac{3}{2}}{2} \times \frac{7}{26} \times \frac{3\frac{3}{3}}{5}$.
7. $\frac{5}{18} \times \frac{27}{35} \times \frac{14}{51} \times$
8. $3\frac{1}{9} \times \frac{27}{40} \times 1\frac{1}{14}$.
9. $\frac{25}{27} \times 3\frac{3}{5} \times 1\frac{1}{15}$.
10. $3\frac{1}{15} \times 3\frac{1}{18} \times 1\frac{1}{23}$.
11. $\frac{14}{50} \times 1\frac{1}{15} \times \frac{20}{21} \times 1\frac{1}{39} \times 6\frac{1}{4}$.
12. $2\frac{5}{14} \times 1\frac{4}{21} \times 2\frac{9}{5} \times 1\frac{5}{44}$.
13. $\frac{34}{57} \times 4\frac{7}{22} \times \frac{11}{85}$.
14. $(\frac{3}{8} + \frac{5}{12}) \times \frac{8}{9}$.
15. $\frac{15}{16} \times (\frac{2}{3} + \frac{9}{5})$.
16. $(\frac{8}{9} - \frac{2}{3}) \times (\frac{1}{2} + \frac{1}{4})$.
17. $\frac{7}{9}$ of $(\frac{13}{16} - \frac{1}{4})$.
18. $(1\frac{5}{12} - \frac{1}{2}) \times (1\frac{5}{22} - \frac{6}{11})$.
19. $1\frac{1}{6} \times 1\frac{4}{11} \times 1\frac{5}{28} \times \frac{9}{10}$.
20. $3\frac{1}{15} \times 1\frac{13}{22} \times \frac{44}{69} \times 1\frac{9}{21}$.
21. $(\frac{5}{12} + 3\frac{4}{9} + \frac{7}{8}) \times \frac{9}{11}$.
22. $(9\frac{1}{7} \times 1\frac{13}{15} \times 3\frac{1}{8}) - 31\frac{7}{9}$.
23. $(1\frac{7}{12} - \frac{5}{8}) \times (2\frac{7}{10} - 1\frac{1}{15}) \times \frac{60}{161}$.
24. $\frac{5}{11} \times \frac{4}{7} \times \frac{1}{3} \times \frac{2}{3}$.
25. $(2\frac{2}{3} + 2\frac{2}{15}) \times (2\frac{1}{39} - \frac{1}{3}) \times (1\frac{0}{33} + \frac{2}{3})$.
26. $\frac{11}{15}$ of $(2\frac{1}{3} - 1\frac{6}{7})$ of $\frac{14}{55}$.
27. $\frac{39}{380} \times 1\frac{41}{91} \times 2\frac{4}{17} \times \frac{34}{99}$.
28. $(\frac{10}{39}$ of $1\frac{1}{25}) + 1\frac{8}{15} - (\frac{14}{54}$ of $\frac{24}{35})$.
29. $\frac{29}{105} + \frac{8}{35}$ of $\frac{7}{8} - \frac{7}{22}$ of $7\frac{1}{3} \times \frac{6}{49}$.
30. $2\frac{31}{52} \times \frac{23}{33} \times 4\frac{1}{3} \times \frac{11}{115}$.

EXERCISES III_E

Evaluate:

1. $\frac{7}{9} \div \frac{3}{5}$.
2. $\frac{5}{11} \div \frac{6}{7}$.
3. $8\frac{1}{3} \div \frac{15}{16}$.
4. $\frac{29}{30} \div \frac{3}{25}$.
5. $7\frac{4}{15} \div \frac{11}{30}$.
6. $(\frac{4}{5} + \frac{3}{10}) \div \frac{1}{2}$.
7. $\frac{8}{9} \div (\frac{2}{3} + \frac{1}{2})$.
8. $(\frac{4}{5}$ of $\frac{21}{40}) \div \frac{1}{25}$. (*N.B.* One step is sufficient.)
9. $(\frac{11}{30} \div \frac{14}{16}) \times (\frac{2}{33}$ of $\frac{7}{8})$. (Can this be done in one step?)

10. $\frac{27}{91} \div \frac{9}{26}$.
11. $(\frac{3}{5} + \frac{1}{2}) \div (\frac{2}{3} - \frac{1}{10})$.
12. $(\frac{11}{20} \div \frac{22}{35}) + (\frac{6}{7} \div \frac{4}{21})$.
13. $1\frac{29}{40} \div \frac{46}{55}$.
14. $3\frac{1}{3} \div (2\frac{1}{5} - 1\frac{1}{3})$.
15. $11\frac{3}{7} \div \frac{40}{49}$.
16. $\frac{2}{7} \div (\frac{5}{14} \times \frac{35}{36})$. (Can this be done in one step?)
17. $18\frac{1}{3} \div \frac{22}{27}$.
18. $1\frac{9}{20} \div 2\frac{3}{5}$.
19. $8\frac{5}{6} \div \frac{4}{9}$.
20. $6\frac{9}{10} \div 28\frac{3}{4}$.
21. $\frac{\frac{4}{15}}{\frac{10}{21}}$.
22. $\frac{\frac{3}{5} + \frac{1}{10}}{\frac{8}{15} - \frac{3}{10}}$.
23. $\frac{(1\frac{1}{3} \times 2\frac{5}{6}) - 3\frac{1}{2}}{1\frac{5}{6} + \frac{2}{3}}$.
24. $\frac{6\frac{1}{2} - 3\frac{1}{4}}{7\frac{1}{3} + 1\frac{1}{7}} \times \frac{2\frac{3}{7} \text{ of } 3\frac{2}{3}}{4\frac{1}{3} \text{ of } 1\frac{1}{4}}$. (This can be done in *three* steps.)
25. $(\frac{8}{9} - \frac{7}{8}) \div (\frac{2}{2\frac{1}{4}} \text{ of } \frac{1\frac{3}{4}}{2})$.
26. $\frac{(\frac{1}{12} + \frac{1}{16}) \text{ of } \frac{5}{14}}{(\frac{1}{12} - \frac{1}{16}) \text{ of } \frac{4}{21}}$.
27. $\frac{\frac{3}{8} + (\frac{3}{5} \text{ of } \frac{1}{4})}{\frac{3}{8} - (\frac{3}{5} \text{ of } \frac{1}{4})}$.
28. $(7\frac{5}{12} + 8\frac{7}{8} - 13\frac{5}{6} + 10\frac{2}{3}) \div \frac{5}{24}$.
29. $\frac{4\frac{1}{4} \div 2\frac{1}{2}}{1\frac{1}{3} \text{ of } 1\frac{1}{4}} + \frac{4\frac{1}{4} \text{ of } 2\frac{1}{2}}{1\frac{1}{3} \div 1\frac{1}{4}}$.
30. $(4\frac{3}{4} + 5\frac{1}{20} + 3\frac{1}{9} - 11\frac{2}{5}) \div (\frac{9}{20} - \frac{4}{9})$.

EXERCISES III_F

1. By what must $8\frac{1}{3}$ be divided to give $2\frac{1}{2}$?
2. What must be added to $\frac{5}{14} \div \frac{4}{21}$ to make 2?
3. Find the difference between $\frac{2}{9} \div \frac{5}{6}$ and $\frac{4}{9}$ of $\frac{3}{10}$.
4. From a piece of wire 40 ft. long how many pieces $6\frac{3}{4}$ in. can be cut off, and what length, if any, will remain?

5. From an iron rod $\frac{3}{7}$ of the length is cut off; then $\frac{3}{7}$ of the remainder is taken. What fraction of the rod is left? (Find two ways of getting the answer.)

6. For a particular wireless set 6 pieces of wire each $5\frac{1}{2}$ in. long and 7 pieces each $4\frac{5}{8}$ in. long are required. How many sets can be wired from 50 ft. of wire, and what length, if any, will be left?

7. Calculate how much wire will be required to make 36 sets similar to that mentioned in the preceding question.

8. Evaluate $\frac{3}{10}$ of $\frac{5}{6}$ of £7 18s.

9. Add $\frac{2}{3}$ of £3 17s., $\frac{1}{4}$ of £8 2s., $\frac{3}{5}$ of £6 2s. 6d.

10. Add $\frac{3}{4}$ of 37s. 8d., $\frac{2}{9}$ of 48s. $4\frac{1}{2}$ d., and $\frac{7}{10}$ of 53s. 4d.

11. Find the difference between $\frac{6}{7}$ of £4 11s. and $\frac{5}{6}$ of £4 1s. 6d.

12. A man earns £4 10s. weekly. If he gives away a tenth in charity, and puts a fifth in the bank, what sum has he left each week for other purposes?

13. A load of 7 tons is to be removed by two motor-cars and a motor-lorry. If each car takes a fifth of the load, what weight is left to be taken by the lorry?

14. Three-eighths of a garden is planted with roses, one-sixth with sweet peas, and the remainder is divided equally between chrysanthemums and carnations. If the whole garden is exactly 3 acres, find how many square yards of space are given to carnations.

15. A boy spends half of his money on fares, and a third of the remainder on food. What fraction of his money remains?

16. Find the value of $\frac{14\frac{1}{4}}{12\frac{2}{3}}$ of 1 ton 11 cwt. 7 lb.

17. (a) What fraction of 4s. 3d. is 1d.?

(b) What fraction of 4s. 3d. is 2d.?

(c) What fraction of 4s. 3d. is 7d.?

(d) What fraction of 4s. 3d. is 2s. 1d.?

18. What fraction of 3 cwt. 11 lb. is 1 cwt. 10 lb.?

19. Find the difference between $\frac{5}{8}$ of 3 miles 2 fur. and $\frac{7}{12}$ of 2 miles 1 fur. 4 yds.

20. Add $\frac{3}{8}$ of £4 12s., $\frac{3}{8}$ of £5 9s., and $\frac{3}{8}$ of £7 11s. (Be sure you take the *easy* way of getting the answer.)

21. Divide $20\frac{6}{7}$ by $\left\{ \left(\frac{3}{7} \text{ of } 5\frac{1}{4} \right) + \left(1\frac{5}{9} \text{ of } 10\frac{2}{7} \right) \right\}$.
22. Simplify $\frac{\frac{3}{4} \text{ of } \frac{7}{8} \text{ of } \frac{5}{8}}{\frac{1}{24} \text{ of } \frac{3}{14} \text{ of } \frac{2}{5}}$.
23. Simplify $\frac{3\frac{1}{2} - 1\frac{7}{8}}{3\frac{1}{4} \text{ of } 2\frac{1}{2}} \div \left(\frac{1}{5} \text{ of } 2\frac{3}{4} \right)$.
24. Simplify $\frac{17\frac{5}{12} - 9\frac{3}{4} + 4\frac{5}{7}}{\frac{5}{9} \times 9\frac{2}{7}} \div \frac{5}{12}$.
25. Simplify $\frac{6\frac{2}{3} - 4\frac{4}{5}}{5\frac{2}{5} + 3\frac{1}{3}} \div \frac{7}{9}$.
26. Divide $15\frac{1}{2} + \left(3\frac{1}{3} \text{ of } 5\frac{2}{5} \right)$ by $\left(3\frac{1}{18} \text{ of } \frac{2}{40} \text{ of } 1\frac{1}{3} \right) - \frac{1}{6}$.
27. Simplify $\left(3\frac{4}{15} + 2\frac{7}{12} - 1\frac{5}{6} \right) \div \left(8\frac{8}{9} + 1\frac{3}{4}\frac{7}{5} \right)$.
28. How many times is $1\frac{3}{8}$ cwt. contained in $4\frac{2}{5}$ tons?
29. Add $\frac{1}{\frac{2}{3} - \frac{1}{7}}$ of 8s. 3d. and $\frac{\frac{5}{6}}{1\frac{3}{10} - \frac{1}{15}}$ of 2s. 9d.
30. Add $\frac{3}{5}$ of 17s. 2d., $\frac{7}{10}$ of 9s. 4d., $\frac{1}{15}$ of 7s. 6d.

CHAPTER IV

DECIMAL FRACTIONS

THE following steps may serve to present the subject.

(a) Consider the digits of the number 1111. Note that the value of the first digit is a thousand; that of the next a tenth of a thousand—*i.e.*, a hundred; that of the next a tenth of a hundred—*i.e.*, a ten; that of the next a tenth of a ten—*i.e.*, a unit.

(b) Naturally, then, what should be the value of a 1 written after the units place? Clearly a tenth of a unit.

(c) Thus we may write 111^u miles (where u marks the units digit), which means 10 miles, 1 mile, and a tenth of a mile.

Obviously this length could not be written simply 111 miles, which would mean 100 miles, 10 miles, and 1 mile. Hence the necessity (when the last digit is *not* units) of showing which

digit is the units digit—*e.g.*, 6732^u miles, meaning 673 miles and 2 tenths of a mile, or, as it is usually written, 673·2 miles.

(d) Similarly, in 111111^u yds. the final 1 (written after the 1 tenth) would mean a tenth of a tenth, which is a hundredth. The usual form is 1111·11.

Note. The u may with advantage be employed instead of the 'point' throughout addition and subtraction, and also in the introductory examples in multiplication.

In the exercises in this chapter the units digit is sometimes indicated by the letter u written above it, and sometimes by a point written after it.

$$\text{Example (i) } 56 \text{ yds., } \frac{4}{10} \text{ yd., } \frac{3}{100} \text{ yd.} \\ = 56.43 \text{ yds.}$$

$$\text{(ii) } 314 \text{ lb., } \frac{7}{100} \text{ lb.} \\ = 314.07 \text{ lb.}$$

ADDITION AND SUBTRACTION

MODEL EXAMPLES

One example of each is sufficient. There is nothing to explain.

(a) Addition:

$$\begin{array}{r} u \\ 62347 \\ 5708 \\ 643 \\ 28079 \\ \hline 667656 \end{array}$$

(b) Subtraction:

$$\begin{array}{r} u \\ 40137 \\ 286984 \\ \hline 114386 \end{array}$$

ORAL EXERCISES

1. Add 7, 5, 0.3.
2. Add 0.7, 0.4.
3. Add 307, 0.2.
4. Add 3.6, 0.7.
5. Add 0.8, 0.06.
6. Add 0.8, 0.16.
7. Add 0.04, 0.03.
8. Add 7, 0.6, 0.05.
9. Add 3, 0.02, 0.01.
10. Add 20, 0.7, 0.1.
11. From 0.8 take 0.5.
12. From 1 take 0.3.
13. From 1 take 0.07.
14. From 20 take 3.
15. From 20 take 0.03.
16. From 100 take 0.1.
17. Add 0.7 to 5 and take 3 from the result.
18. From 1 take 0.99.
19. From 7 take 6.99.
20. Add 0.5, 0.04, 0.003, and take 0.1 from the result.

EXERCISES IV_A

Note. The student should be required to state the value of each digit in his answer.

1. Add:

$$\begin{array}{r} u \\ 64309 \\ 2104 \\ \underline{3978} \end{array}$$

2. Add:

$$\begin{array}{r} u \\ 037 \\ 6824 \\ \underline{079} \end{array}$$

3. Subtract:

$$\begin{array}{r} u \\ 47239 \\ \underline{186478} \end{array}$$

4. Subtract:

$$\begin{array}{r} u \\ 39218 \\ \underline{0687} \end{array}$$

5. Add:

$$\begin{array}{r} u \\ 43 \\ 1682 \\ 07 \\ \underline{4098} \end{array}$$

6. Add:

$$\begin{array}{r} 320 \\ 71.86 \\ 0.435 \\ \underline{7.2} \end{array}$$

7. Subtract:

$$\begin{array}{r} 370 \\ \underline{1.986} \end{array}$$

8. Subtract:

$$\begin{array}{r} 4612.3 \\ \underline{370} \end{array}$$

EXERCISES IN ARITHMETIC

9. Add:

$$\begin{array}{r}
 4.13 \\
 17 \\
 2.9 \\
 604 \\
 \hline
 3.879
 \end{array}$$

10. Subtract:

$$\begin{array}{r}
 317.28 \\
 96.9 \\
 \hline
 \end{array}$$

11. Add 73.9, 6.08, 0.41, 297.53.
12. Add 0.086, 0.77, 1.9, 39.28.
13. Add 7.68, 0.9, 44.827, 600.09.
14. From 907.86 take 48.777.
15. From 0.209 subtract 0.0864.
16. From the sum of 9.73, 18.096, and 0.379 take 11.4968.
17. Subtract the sum of 0.083, 0.729, and 0.6667 from 10.
18. Take the sum of 8.62, 4.937, and 0.9024 from the sum of 11.09, 13.7, and 6.073.
19. Simplify $4.817 + 2.639 - 1.08 + 7.42$.
20. Simplify $1.1 - 0.01 - 0.001 + 0.101$.
21. Add 0.047, 3.26, 11, 0.4318.
22. Add 16.44, 0.732, 4.068, 0.7704.
23. From 4 take $0.814 + 0.3774 + 0.018 + 1.047 + 0.3109$.
24. Add 7.1, 0.0806, 1.333, 24.2974.
25. Simplify $(0.374 + 0.888 + 4.0962) - (1.73 - 0.084)$.
26. Simplify $0.61 - 0.279 - 1.43 + 4.97 + 3$.
27. Simplify $(8 - 0.64) + (4 - 0.09) - (0.237 + 0.6 - 0.084)$.
28. Add 14.734, 0.6988, 2.0974, 3.7909.
29. Simplify $7.03 - 2.86 + 1.39 - 0.62$.
30. Subtract the sum of 7.18 in. and 1.69 in. from 1 ft.

MULTIPLICATION

MODEL EXAMPLES

First Method. Note the following principle!

$$\begin{aligned}
 400 \times 30 &= 40 \times 300 \\
 &= 4 \times 3000 \\
 &= 4000 \times 3.
 \end{aligned}$$

That is to say, when two numbers are to be multiplied together we may multiply the one by any factor and divide the other by the same factor without affecting the required product. Examples are given below.

$$(a) \quad 713.26 \times 0.4 = 71.326 \times 4.$$

$$\begin{array}{r} 71.326 \\ 4 \\ \hline 285.304 \end{array}$$

$$(b) \quad 643.972 \times 0.008 = 0.643972 \times 8.$$

$$\begin{array}{r} 0.643972 \\ 8 \\ \hline 5.151776 \end{array}$$

$$(c) \quad 43.268 \times 0.402 = 4.3268 \times 4.02.$$

$$\begin{array}{r} 4.3268 \\ 4.02 \\ \hline 17.30720 \\ 86536 \\ \hline 17.393736 \end{array}$$

$$(d) \quad 56.29 \times 0.032 = 0.5629 \times 3.2.$$

$$\begin{array}{r} 0.5629 \\ 3.2 \\ \hline 1.6887 \\ .11258 \\ \hline 1.80128 \end{array}$$

This method of ‘standardizing’ the multiplier—*i.e.*, making the multiplier begin with units—eliminates the difficulty of placing rightly the first partial product.

The student should note that the number of decimal figures in the final product equals the sum of the numbers of decimal figures in the original multiplicand and multiplier. For instance, in Example (d) above there are five decimal figures in the final product, 1.80128. In the original multiplicand, 56.29, there were two, and in the original multiplier, .032,

there were three. This affords a mechanical test of the accuracy of the result.

Second Method. (The use of squared paper is strongly recommended.)

$$\begin{array}{r}
 (e) \qquad \qquad u \\
 \qquad \qquad 74329 \\
 \qquad \qquad \underline{6} \\
 \qquad \qquad 445974
 \end{array}$$

$$\begin{array}{r}
 (f) \qquad \qquad u \\
 \qquad \qquad 83964 \\
 \qquad \qquad \underline{10} \\
 \qquad \qquad 83964
 \end{array}$$

Simply promote each digit one place to the left.

$$\begin{array}{r}
 (g) \qquad \qquad u \\
 \qquad \qquad 83964 \\
 \qquad \qquad \underline{70} \\
 \qquad \qquad 587748
 \end{array}$$

Multiply by 7 and promote one place.

$$\begin{array}{r}
 (h) \qquad \qquad u \\
 \qquad \qquad 83964 \\
 \qquad \qquad \underline{300} \\
 \qquad \qquad 251892
 \end{array}$$

Promote two places.

$$\begin{array}{r}
 (i) \qquad \qquad u \\
 \qquad \qquad 83964 \\
 \qquad \qquad \underline{23} \\
 \qquad \qquad 167928 \\
 \qquad \qquad \underline{251892} \\
 \qquad \qquad 1931172
 \end{array}$$

The first multiplier, 2, is one place to the left of the units. Therefore, move every digit in this partial product one place to the left.

$$\begin{array}{r}
 (j) \qquad \qquad u \\
 \qquad \qquad 81726 \\
 \qquad \qquad \underline{432} \\
 \qquad 326904 \\
 \qquad \quad 245178 \\
 \qquad \quad \underline{163452} \\
 \qquad 35305632
 \end{array}$$

The first multiplier, 4, is two places to the left of the units. Therefore move every digit in this partial product two places to the left.

$$\begin{array}{r}
 (k) \qquad \qquad u \\
 \qquad \qquad 96437 \\
 \qquad \qquad \underline{304} \\
 \qquad 2893110 \\
 \qquad \quad 385748 \\
 \qquad \quad \underline{000} \\
 \qquad 29316848
 \end{array}$$

The first multiplier, 3, is two places to the left of the units. Therefore move the digits accordingly.

The next multiplier is 0. Simply write 0 after the partial product already found. Then multiply by 4.

$$\begin{array}{r}
 (l) \qquad \qquad u \\
 \qquad \qquad 71326 \\
 \qquad \qquad \underline{04} \\
 \qquad 285304
 \end{array}$$

The multiplier is one place to the *right* of the units. Therefore move every digit in the product one place to the right.

$$\begin{array}{r}
 (m) \qquad \qquad u \\
 \qquad \qquad 643972 \\
 \qquad \qquad \underline{008} \\
 \qquad 5151776
 \end{array}$$

The multiplier is two places to the right of the units. Therefore move every digit in the product two places to the right.

Note. In the remaining examples the units digit is marked

by a point written *after* it instead of by the letter *u* written above it.

$$\begin{array}{r}
 (n) \qquad 43\cdot268 \\
 \quad \quad 0\cdot402 \\
 \hline
 \quad \quad 17\cdot30720 \\
 \quad \quad \quad 86536 \\
 \hline
 \quad \quad 17\cdot393736
 \end{array}$$

$$\begin{array}{r}
 (o) \qquad 56\cdot29 \\
 \quad \quad 0\cdot032 \\
 \hline
 \quad \quad 1\cdot6887 \\
 \quad \quad \quad 11258 \\
 \hline
 \quad \quad 1\cdot80128
 \end{array}$$

If working on plain paper the pupil should begin Example (o) thus:

$$\begin{array}{r}
 56\cdot29 \\
 0\cdot032 \\
 \hline
 \quad \quad - - 7
 \end{array}$$

(p) Similarly:

$$\begin{array}{r}
 142\cdot37 \\
 0\cdot003 \\
 \hline
 \quad \quad - - - 1
 \end{array}$$

ORAL EXERCISES

1. $0\cdot3 \times 2$; $0\cdot34 \times 2$; $0\cdot312 \times 3$.
2. $5\cdot4 \times 2$; $3\cdot1 \times 4$.
3. 7×10 ; 7×1 ; $7 \times 0\cdot1$.
4. 72×10 ; 72×1 ; $72 \times 0\cdot1$.
5. 31×20 ; 31×2 ; $31 \times 0\cdot2$.
6. Observe that $873 \times 0\cdot2 = 87\cdot3 \times 2$, and that $643\cdot75 \times 0\cdot3 = 64\cdot375 \times 3$.

Change in the same way:

$$\begin{array}{rcl}
 43\cdot6 & \times & 0\cdot3. \\
 5\cdot72 & \times & 0\cdot4. \\
 0\cdot816 & \times & 0\cdot7.
 \end{array}$$

7. Observe that

$$\begin{aligned} 174.36 \times 0.31 &= 17.436 \times 3.1. \\ 13.617 \times 0.471 &= 1.3617 \times 4.71. \\ 0.925 \times 0.86 &= 0.0925 \times 8.6. \end{aligned}$$

Change in the same way—*i.e.*, standardize the multiplier:

$$\begin{aligned} 29.86 \times 0.43. \\ 103.2 \times 0.516. \\ 0.71 \times 0.32. \end{aligned}$$

8. 0.2×0.2 ; $0.2 \times 0.2 \times 0.2$; 0.3×0.7 .
 9. 0.12×0.2 ; 0.14×0.1 ; 0.14×0.01 .
 10. 0.03×0.1 ; 0.03×0.01 ; 0.03×0.04 .

EXERCISES IV_B

Note. Successful work is almost guaranteed by the use of squared paper throughout these exercises. ($\frac{1}{4}$ -inch squares, or squares very slightly larger, are suitable.)

1. Multiply ["]42376 by 4, by 7, by 9, by 8, by 6, by 5, by 3.
2. Multiply ["]31925 by 2, by 4, by 7, by 8, by 9.
3. Multiply ["]21371 by 10, by 100, by 20, by 300, by 40, by 600, by 700, by 80, by 900.
4. Multiply ["]41937 by 10, by 41, by 86, by 91, by 307, by 29, by 403.
5. Multiply 2741 by 42 (by factors), by 10, by 100; similarly by 56, by 35, by 28, by 72.
6. Multiply ["]2137 by 3 tenths, by 7 tenths, by 5 tenths, by 9 tenths.
7. Multiply 413.27 by 0.6, by 10, by 0.8, by 0.2, by 0.5, by 0.9.
8. Multiply 61.38 by 3.4, by 2.8, by 19, by 23, by 2.3, by 4.6, by 6.7.
9. Multiply 42.37 by 0.41, by 0.26, by 3.7, by 0.37, by 0.42, by 100.

10. Multiply 423.67 by 0.04, by 0.07, by 0.041, by 0.41, by 4.1, by 0.08, by 0.084, by 100.

11. Multiply 0.2307 by 0.023, by 4.1, by 0.78, by 0.061, by 200.

12. Multiply 3.812 by 6.04, by 4.03, by 0.209, by 40.7, by 300.

13. Multiply 17.016 by 3.82, by 0.0382, by 38.2, by 0.206, by 10.

14. Multiply 81.074 by 2.004, by 30.06, by 0.0304, by 40.07, by 400.

15. Multiply 6.823 by 0.042, by 320, by 600, by 0.084, by 100, by 30.

16. Multiply 0.4307 by 0.206, by 0.024, by 30.8, by 200, by 10, by 40.

17. Multiply 26.068 by 8.2, by 2.03, by 0.04, by 15.1.

18. Multiply 39.102 by 25.004, by 17.083, by 111.011.

19. Evaluate $6.28 \times 10.2 \times 10.2$.

20. Evaluate $15.2 \times 2.5 \times 1.3 \times 2.1$.

21. Evaluate $0.302 \times 0.8 \times 2.3 \times 10$.

22. Evaluate $31.7 \times 3.02 \times 4.1$.

23. Evaluate $0.07 \times 0.4 \times 4.1 \times 20$.

24. Evaluate $3.71 \times 0.23 \times 4.1 \times 40$.

25. Evaluate $17.3 \times 0.06 \times 3.2 \times 310$.

26. Evaluate $1.7 \times 0.17 \times 0.017$.

27. Evaluate $0.413 \times 30 \times 0.3$.

28. Evaluate $16.4172 \times 110 \times 4.1$.

29. Evaluate $17.03 \times 0.42 \times 8.01 \times 30$.

30. Evaluate $0.723 \times 0.07 \times 4.1 \times 200$.

DIVISION

MODEL EXAMPLES

$$(a) \quad \begin{array}{r} 5 \overline{)56.12} \\ 11.224 \end{array}$$

$$(b) \quad \begin{array}{r} 8 \overline{)0.713} \\ 0.089-- \end{array}$$

$$(c) \quad \begin{array}{r} 4 \overline{)0.0261} \\ 0.0065 \text{ --} \end{array}$$

$$(d) \quad \begin{array}{r} 5.41 \text{ --} \\ 23 \overline{)124.61} \\ 115 \\ \hline 96 \\ 92 \\ \hline 41 \\ 23 \\ \hline 18 \end{array}$$

$$(e) \quad \begin{array}{r} 0.062 \text{ --} \\ 37 \overline{)2.3178} \\ 222 \\ \hline 97 \end{array}$$

Clearly there is no difficulty in dividing by a *whole number*. In all other cases we modify the problem so as to make the divisor a whole number.

$$(f) \quad 1.318 \div 0.4 = 13.18 \div 4.$$

$$\begin{array}{r} 4 \overline{)13.18} \\ 3.29 \text{ --} \end{array}$$

$$(g) \quad 0.26813 \div 0.05 = 26.813 \div 5.$$

$$\begin{array}{r} 5 \overline{)26.813} \\ 5.362 \text{ --} \end{array}$$

$$(h) \quad 4.7183 \div 6.01 = 471.83 \div 601.$$

$$\begin{array}{r} 0.78 \text{ --} \\ 601 \overline{)471.83} \\ 4207 \\ \hline 5113 \end{array}$$

The student should verify examples such as

$$\begin{aligned} &60 \div 12 \\ &= 30 \div 6 \\ &= 10 \div 2 \\ &= 5 \div 1. \end{aligned}$$

Note particularly the next two examples.

$$(i) \quad 1.2379 \div 20 = 0.12379 \div 2.$$

$$\begin{array}{r} 2 \overline{)0.12379} \\ 0.06189 \text{ --} \end{array}$$

(j) $312 \cdot 314 \div 73000 = 0 \cdot 312314 \div 73.$

$$\begin{array}{r} 0 \cdot 0042 - - \\ 73 \overline{) 0 \cdot 312314} \\ \underline{292} \\ 203 \\ \underline{146} \\ 571 \end{array}$$

(k) Make a rough estimate of the value of $21 \cdot 637 \div 0 \cdot 619$.

Convert to $216 \cdot 37 \div 6 \cdot 19$.

Now, $216 \div 6 = 36$ approximately.

(l) Estimate $0 \cdot 01628 \div 0 \cdot 00347$.

Convert to $16 \cdot 28 \div 3 \cdot 47$.

Then $16 \div 3 = 5$ approximately.

(m) Estimate $6 \div 0 \cdot 00028$.

Convert to $60,000 \div 2 \cdot 8$.

Then $60,000 \div 3$ (rather than 2) = 20,000 approximately.

(n) Evaluate (i) $\frac{1}{3 \cdot 142}$ and (ii) $\frac{1}{(3 \cdot 142)^2}$ (in both cases to four decimal places).

(i) $1 \div 3 \cdot 142 = 1000 \div 3142.$

$$\begin{array}{r} 0 \cdot 3182 \\ 3142 \overline{) 1000 \cdot 0} \\ \underline{9426} \\ 5740 \\ \underline{3142} \\ 25980 \\ \underline{25136} \\ 8440 \end{array}$$

Required result = $0 \cdot 3182$.

(ii) We take the result already found and divide again.

$$0.3182 \div 3.142 = 318.2 \div 3142.$$

$$\begin{array}{r} 0.1012 \\ 3142 \overline{)318.2} \\ \underline{314 } \\ 4 \\ \underline{3 } \\ 8580 \end{array}$$

Required result = 0.1012.

N.B. In finding $\frac{1}{(3.142)^2}$ it would be laborious to obtain $(3.142)^2$, and then divide 1 by the result.

(o) Evaluate $\frac{7.2 \times 0.65 \times 0.049}{3.5 \times 10.4 \times 0.63}.$

The expression = $\frac{72 \times 65 \times 49}{35 \times 104 \times 63 \times 100}.$

(This is the result of multiplying numerator and denominator by $10 \times 10 \times 10 \times 10 \times 10 \times 10$ —i.e., by 10^6).

Proceeding, we get

$$\begin{array}{ccc} \text{I} & \text{I} & \text{I} \\ 9 & 13 & 7 \\ 72 \times 65 \times 49 \\ \hline 35 \times 104 \times 63 \times 100 \\ 7 & 13 & 9 \\ \text{I} & \text{I} & \text{I} \\ = \frac{\text{I}}{100} \\ = 0.01. \end{array}$$

(p) Evaluate $\frac{6.5 \times 0.7 \times 0.99}{0.26 \times 2.2}.$

$$\begin{aligned}
 \text{The expression} &= \frac{\overset{1}{\cancel{6}5} \times 7 \times \overset{9}{\cancel{9}\cancel{9}}}{\underset{2}{\cancel{2}\cancel{6}} \times \underset{2}{\cancel{2}\cancel{2}} \times \underset{2}{\cancel{1}\cancel{0}}} \\
 &= \frac{63}{8} \\
 &= 7.875.
 \end{aligned}$$

ORAL EXERCISES

1. $0.68 \div 2$; $0.048 \div 3$; $0.018 \div 3$.
2. $0.132 \div 4$; $0.024 \div 6$; $0.18 \div 3$.
3. In the following give the quotient to four decimal places at most: $1.7316 \div 4$; $0.066 \div 4$; $0.108 \div 12$; $0.317 \div 8$.
4. Observe the following:

$$\begin{aligned}
 61.37 \div 0.3 &= 613.7 \div 3. \\
 3.29 \div 0.3 &= 32.9 \div 3. \\
 4.657 \div 0.12 &= 465.7 \div 12. \\
 17 \div 0.4 &= 170 \div 4.
 \end{aligned}$$

Change the following in the same way—*i.e.*, so as to make the divisor a whole number:

$$\begin{aligned}
 6.84 \div 0.7 \\
 0.317 \div 0.8 \\
 0.047 \div 0.5 \\
 0.816 \div 0.12.
 \end{aligned}$$

5. In the following examples make the divisor a whole number, and give the quotient to three decimal places at most:

$$\begin{aligned}
 33 \div 0.2 \\
 0.177 \div 0.2 \\
 6.44 \div 0.02 \\
 0.046 \div 0.4.
 \end{aligned}$$

6. Observe the following:

$$\begin{aligned}
 73.29 \div 20 &= 7.329 \div 2. \\
 41.618 \div 300 &= 0.41618 \div 3. \\
 0.184 \div 120 &= 0.0184 \div 12.
 \end{aligned}$$

Change the following in the same way—*i.e.*, making the divisor a whole number without any final ‘noughts’:

$$\begin{array}{r} 1.673 \div 30 \\ 4.089 \div 400 \\ 7.29 \div 1100. \end{array}$$

7. Give the quotient to four decimal places in each of the following: $0.8645 \div 0.2$, $0.017 \div 0.5$, $0.7313 \div 0.4$.

8. Give the quotient to four decimal places in each of the following: $3.721 \div 20$, $0.816 \div 30$, $14.297 \div 500$.

9. Give the quotient to three decimal places in each of the following: $0.029 \div 1.1$, $0.47 \div 0.11$, $0.306 \div 0.12$.

10. Give the quotient to four decimal places in each of the following: $617.3 \div 120$, $48.6 \div 1200$, $0.087 \div 400$.

11. In the following cases restate each problem, making the divisor a whole number: $61.387 \div 0.47$, $9.0387 \div 7.3$, $0.93768 \div 1.41$, $0.87136 \div 0.472$, $0.71962 \div 0.097$, $183 \div 4.7$.

12. See Model Examples (i) and (j), pages 47 and 48, and restate correspondingly the following: $41987 \div 7300$, $6843.97 \div 1700$, $9.2186 \div 3700$, $516.27 \div 610000$.

EXERCISES IVc

1. Divide 0.6182 by 3, by 4, by 7, by 8, by 9 (quotients to four decimal places).

2. Divide 4.8173 by 3, by 8, by 6, by 2, by 5, to four decimal places.

3. Divide 4.8173 by 10, by 100, by 1000.

4. Divide 5.372 by 31, by 41, by 51, by 61, to three decimal places.

5. Divide 5.372 by 42 (by factors); similarly by 56, by 63, by 72. (All to three decimal places.)

6. Divide 14.379 by 29, by 217, by 34, to three decimal places.

7. Divide 219.386 by 0.4, by 0.7, by 0.8, by 0.9, by 0.6, by 0.5, by 0.3, to three decimal places.

8. Divide 23.6187 by 3.1, by 4.7, by 2.9, by 6.1, by 8.3, to two decimal places.

9. Divide 4.816 by 40.7, by 6.13, by 214, to three decimal places.

10. Divide 61.372 by 400, by 80, by 300, by 70, by 700, by 90, by 20, by 200, by 2000. (*Short* division to three decimal places.)

11. Divide, without unnecessary figures, 1237.816 by 7300, by 3650, by 830, by 41000, by 73000, to three decimal places.

12. Divide 0.8163 by 0.7, by 0.73, by 0.711, to three decimal places.

13. Divide 0.0624 by 0.9, by 0.091, by 92, to four decimal places.

14. Divide 14.7 by 14.8, by 23.6, by 321, to four decimal places.

15. Divide 6 by 0.07, by 8.3, by 70, to four decimal places.

16. Divide 318.74 by 21.3, by 40.1, by 417.3, to three decimal places.

17. Divide 4.73 by 31, by 48.2, by 0.9, to four decimal places.

18. Divide 17.17 by 0.8, by 1.1, by 23.1, to three decimal places.

19. Divide 0.0932 by 0.8, by 0.07, by 0.009, to three decimal places.

20. Divide 173.6 by 70, by 710, by 7900, to four decimal places.

EXERCISES IV_D

1. Evaluate $\frac{0.36 \times 1.5 \times 14}{3.5 \times 0.63}$.

2. Evaluate $\frac{(2.4)^2 \times (1.6)^2 \times 125}{(4.8)^2 \times 80 \times 0.25}$.

(Note that $2.4^2 = 2.4 \times 2.4$.)

3. Evaluate $\frac{43.2 \times 1.78 \times (0.06)^2}{3.56 \times (1.2)^2 \times 0.004}$.

4. Evaluate $\frac{15.3 \times 0.26^2 \times 0.087}{2.9^2 \times 0.39 \times 0.034}$, to three decimal places.

5. Evaluate $\frac{0.012 \times 3.3 \times 10.5 \times 1.53}{5.1 \times 0.055 \times 0.28}$.

6. Find, to four places of decimals, $0.37046 \div 0.914$.

7. Find $0.0974 \div 0.683$, to four decimal places.

8. Find $7.1 \div 0.8306$, to three decimal places.

9. Find $\frac{1}{(4.37)^2}$, to four decimal places.

(Note. Find $\frac{1}{4.37}$ to five places, and divide the result by 4.37.)

10. Find $\frac{1}{0.603^3}$, to three decimal places.

(Note. Divide 1 by 0.603, the result by 0.603 again, and the last result by 0.603 again.)

11. Find $\frac{1}{47.3 \times 684}$, to four places of decimals.

12. Divide 6.31723 by 0.0809, to three places of decimals.

13. Evaluate $0.3714 \times 0.862 \div 4.39$.

14. Simplify $\frac{0.36 \times 0.025 \times 7.7}{3.3 \times 1.5 \times 1.2}$, to four places of decimals.

15. Divide 143.704 by 7300, to four places of decimals.

16. Divide 68.297 by 73000, to six decimal places.

17. Divide 208.3 by 3650, to four places of decimals.

18. Divide 123.7 by 4100, to four places of decimals.

19. Divide 1479.28 by 73000, to four places of decimals.

20. A car runs 23.5 miles to the gallon of petrol. What distance should it run on 7.4 gal.? Find to within a tenth of a gallon the consumption on a journey of 637 miles.

21. If the scale of a map is 0.4 in. to 1 mile, find to within a tenth of a mile the distance between two points which are 15.73 in. apart on the map.

22. How many pence are in £1? How many in £7.475?

23. The circumference of a cycle-wheel is 10.35 ft. How many turns will it make in travelling half a mile?

24. To how many people can I give 18.75 francs each out of a sum of 254.89 francs? State the remainder. (Be careful to give a sensible answer.)

25. The plan of a town is traced on squared paper, and the area of the plan is found to be 42.38 sq. in. If on the map 16 sq. in. represents 1 sq. mile, find the actual area of the town to within 0.01 sq. mile.

26. A watch is set exactly right at 12 o'clock on the last night of the year. If at noon on January 27 it shows 12 hours 10 min. 43 sec., how many seconds has it gained? Find to a tenth of a second the gain per hour. (Assume the total gain to be less than one round of the clock—*i.e.*, less than 12 hours.)

27. A coil of wire measures 68.45 ft. in length. How many pieces each 3.55 ft. long can be cut from the coil, and what will be the length of the remainder?

28. Evaluate $\frac{(3.7 \times 0.013) + (0.91 \times 0.74)}{0.83}$, to four decimal places.

29. How many people can receive 13.95 francs each out of a sum of 346.5 francs, and what will be the value of the remainder?

30. Evaluate $\frac{(0.37 \times 0.23) + (0.081 \times 0.7) + (13.2 \times 0.08)}{7.23}$, to four decimal places.

CONVERSION OF DECIMAL FRACTIONS TO VULGAR FRACTIONS

MODEL EXAMPLES

$$\begin{aligned} (a) \quad 0.4 &= 4 \text{ tenths} \\ &= \frac{4}{10}. \end{aligned}$$

$$\begin{aligned} (b) \quad 0.39 &= 3 \text{ tenths} + 9 \text{ hundredths} \\ &= \frac{3}{10} + \frac{9}{100} \\ &= \frac{39}{100}. \end{aligned}$$

$$(c) \text{ Similarly, } 0.867 = \frac{867}{1000}.$$

CONVERSION OF VULGAR FRACTIONS TO DECIMAL FRACTIONS

MODEL EXAMPLES

(a)	$\frac{2}{5} = \frac{1}{5} \text{ of } 2$ $= 0.4$	$\begin{array}{r} 5 \overline{)2.} \\ \underline{4} \\ 0 \end{array}$
(b)	$\frac{3}{8} = \frac{1}{8} \text{ of } 3$ $= 0.375$	$\begin{array}{r} 8 \overline{)3.} \\ \underline{3} \\ 75 \\ \underline{64} \\ 11 \\ \underline{96} \\ 15 \end{array}$
(c)	$\frac{2}{3} = \frac{1}{3} \text{ of } 2$	$\begin{array}{r} 3 \overline{)2.} \\ \underline{6} \\ 66 \\ \underline{66} \\ 00 \end{array}$

We cannot express $\frac{2}{3}$ as a terminating decimal, but we can write the decimal to as many places as we wish. Thus $\frac{2}{3} = 0.6667$ correct to four decimal places.

Really

$\frac{2}{3}$ lies between 0.6 and 0.7,
or between 0.66 and 0.67,
or between 0.666 and 0.667,

and so on.

Also we may write $\frac{2}{3} = 0.\dot{6}$. (This is called a *repeating* or *recurring* decimal.)

Again,

$$\begin{aligned} \frac{3}{7} &= \frac{1}{7} \text{ of } 3 \\ &= 0.428571428571428 \dots \\ &= 0.\dot{4}28571. \end{aligned}$$

This is called a *circulating* decimal. In practical work, of course, we take just as many of these figures as we require for accuracy in our desired result.

Thus,

$$\begin{aligned} \frac{3}{7} &= 0.43 \text{ approximately} \\ &\text{or } 0.429 \text{ approximately,} \end{aligned}$$

and so on.

Every vulgar fraction may be expressed either as a terminating decimal or as a repeating or circulating decimal.

E.g., In dividing by 13 there are only twelve possible different remainders; so that, unless the division terminates, one of these must sooner or later occur *a second time*.

EXERCISES IV_E

1. Express as vulgar fractions, in their lowest terms:
 $0.5; 0.4; 0.08; 0.25; 0.36; 0.125; 0.045.$
2. Similarly express:
 $0.006; 0.888; 0.505; 0.375; 0.0075.$
3. Decimalize the following:
 $\frac{1}{4}; \frac{3}{5}; \frac{3}{4}; \frac{1}{8}; \frac{5}{8}.$
4. Decimalize to five places, without unnecessary work:
 $\frac{1}{3}; \frac{5}{6}; \frac{3}{7}; \frac{5}{12}; \frac{3}{8}.$
5. Decimalize to five places:
 $\frac{1}{17}; \frac{3}{401}; \frac{13}{504}.$
6. Express as vulgar fractions in their lowest terms:
 $0.36; 0.088; 0.128; 0.96; 0.028.$
7. Decimalize to four places:
 $\frac{7}{11}; \frac{3}{14};$
8. Decimalize to four places:
 $\frac{316}{401}; \frac{397}{1210}; \frac{555}{4840}.$
9. Express as vulgar fractions in their lowest terms:
 $0.888; 0.072; 0.666.$
10. Decimalize to four places:
 $\frac{12}{112}; \frac{17}{105}; \frac{1}{111}; \frac{131}{140}.$

CHAPTER V

CONTRACTED MULTIPLICATION AND DIVISION FOR ADVANCED PUPILS

CONTRACTED MULTIPLICATION

MODEL EXAMPLES

(a) 56.3768×0.9 , correct to two decimal places.

Rewrite as

$$\begin{array}{r} 5.637\bar{6}8 \\ 9 \\ \hline 50.738 \\ \text{I.e., } 50.74. \end{array}$$

Work for three decimal places. Take the carriage from the fourth place—*i.e.*, from 9 times 6. Mark the 6 thus: $\bar{6}$.

(b) 56.3768×0.94 , correct to two decimal places.

Rewrite thus:

$$\begin{array}{r} 5.637\bar{6}8 \\ 9.4 \\ \hline 50.738 \\ 2.255 \\ \hline 52.993 \\ \text{I.e., } 52.99. \end{array}$$

Obtain first partial product as in Example (a).

In multiplying by 4 take the carriage from 4 times 7—*i.e.*, carry 3, since 28 is nearer to 30 than to 20. Mark the 7.

(c) 56.3768×0.946 , correct to two decimal places.

$$\begin{array}{r} 5.637\bar{6}8 \\ 9.46 \\ \hline 50.738 \\ 2.255 \\ .338 \\ \hline 53.331 \\ \text{I.e., } 53.33. \end{array}$$

Obtain first and second partial products as above.

In multiplying by 6 take the carriage from 6 times 3—*i.e.*, carry 2.

(d) 56.3768×0.904 , correct to two decimal places.

$$\begin{array}{r}
 5.6\bar{3}\bar{7}\bar{6}8 \\
 9.04 \overline{) } \\
 \hline
 50.738 \\
 225 \\
 \hline
 50.963 \\
 \text{I.e., } 50.96.
 \end{array}$$

Multiply by 9, as in the previous example, marking the carriage figure, $\bar{6}$.

The next multiplier is 0; therefore simply mark the carriage figure $\bar{7}$, and pass on to the next multiplier, 4.

The carriage is from 4 times 3.

(e) 0.316729^2 , correct to three decimal places.

$$\begin{array}{r}
 0.0\bar{3}\bar{1}\bar{6}\bar{7}29 \\
 3.16729 \overline{) } \\
 \hline
 0.0950 \\
 32 \\
 19 \\
 2 \\
 \hline
 0.1003 \\
 \text{I.e., } .100.
 \end{array}$$

Note that in multiplying by 7 the carriage is from 7 times 3. It is necessary to write in the 2.

(f) 41873^2 , correct to within a million.

$$\begin{array}{r}
 4\bar{1}\bar{8}\bar{7}\bar{3}0000 \\
 4.1873 \overline{) } \\
 \hline
 16749 \\
 419 \\
 334 \\
 29 \\
 1 \\
 \hline
 17532
 \end{array}$$

Result = 1753 millions, or 1,753,000,000.

EXERCISES VA

1. 73.42×4.7383 , correct to two decimal places.
2. 4.63793×214.543 , correct to three decimal places.
3. 0.417386^2 , correct to four decimal places.
4. $63.0274 \times 0.41735 \times 4.623$, correct to three decimal places. (Work the first multiplication to five decimal places and the second to four places.)

5. 72.0348×30.6142 , correct to three decimal places.
6. 15.3174×0.06372 , correct to three decimal places.
7. $64,037 \times 2148$, correct to a million.
8. 31794^2 , correct to a million.
9. 8137.46×3179.58 , correct to within one hundred.
10. $44,326^2$, correct to a million.
11. $14,739 \times 613$, correct to a thousand.

CONTRACTED DIVISION

MODEL EXAMPLES

(a) $51.3982 \div 438.62$, correct to three decimal places.

We work to four places.

Rewrite as $0.513982 \div 4.3862$.

From this we forecast that the first figure of our required quotient will be in the tenths place. The problem is to fill in the blanks in the skeleton quotient shown below

$$43862)513982(0. \text{---}$$

The decimal point may now be omitted in the working.

Contraction consists in discarding a figure from the divisor instead of 'taking down' a fresh figure from the dividend.

Contraction may be begun as soon as the number of digits in the divisor exceeds the number of digits *still to be found* in the quotient.

In the present case we have four quotient figures to be found; hence we discard the final 2 from the divisor so as to leave only four figures.

$$\begin{array}{r} 4'3'8'6'2)513982(1171 \\ \underline{4386} \\ 753 \\ \underline{439} \\ 314 \\ \underline{307} \\ 7 \end{array}$$

Having reached the first remainder, 753, discard the 6 from the divisor instead of 'taking down' a figure from the dividend. Begin each multiplication by taking the carriage from the figure just discarded.

Required quotient = 0.117.

(b) $0.074136 \div 0.934$, correct to four decimal places.

We work to five places.

Rewriting as $0.74136 \div 9.34$ gives the skeleton quotient
 0.0 ---

$$\begin{array}{r}
 9'3'4)74136(7937 \\
 \underline{6538} \\
 8756 \\
 \underline{8406} \\
 350 \\
 \underline{280} \\
 70
 \end{array}$$

Required quotient = 0.0794 .

There are four figures to be found in the quotient. There are three figures altogether in the divisor; therefore no contraction is made until two figures are found in the quotient (in this example the 7 and the 9).

EXERCISES V_B

Note. In every case make a forecast of the result.

1. $41,637 \div 317.2$, correct to the first decimal place.

2. $0.83572 \div 0.77129$, correct to two decimal places.

3. $43.716 \div 812.48$, correct to three decimal places.

4. $\frac{1}{7.173}$, correct to three decimal places.

5. $\frac{0.4183}{0.6129 \times 0.7348}$, correct to three decimal places.

(Work $0.4183 \div 0.6129$ to five places, and divide the result by 0.7348 , to four places.)

6. $0.08374 \div 0.62413$, correct to four decimal places.

7. $91437.6 \div 413.8$, correct to a unit.

8. $7.2916 \div 42.378$, correct to three decimal places.

9. $\frac{1}{3.14159}$, correct to four decimal places.

10. $\frac{1}{(3.14159)^2}$, correct to three decimal places.

(In working Question 10 take the result of Question 9 to five places, and divide by $3\cdot14159$, to four places.)

11. $\frac{1}{(3\cdot14159)^3}$, correct to two decimal places.

(Take the result of Question 10 to four places, and divide by $3\cdot14159$, to three places.)

12. $\frac{1}{(2\cdot39)^3}$, correct to three decimal places.

13. The National Debt at the end of 1929 amounted to £7,595,508,154, and the population of the United Kingdom was estimated at 45,843,630. Calculate to within a pound the debt per head of the population.

14. Evaluate $373\cdot49 \div 3\cdot14159$ to two decimal places.

15. The number of passengers carried by a certain railway for a year is 2,436,814, and the total fares amount to £687,342. Calculate to the third decimal of a pound the average fare per head.

16. Evaluate correct to one decimal place $63\cdot0785 \times 0\cdot7463 \times 0\cdot9863$.

(Work the first multiplication to three decimal places, and the second to two places.)

17. In a savings bank the total sum deposited in a certain year amounted to £137,684, the number of depositors being 11,436. Calculate to three decimal places the average amount in pounds deposited per head.

18. Multiply $81\cdot3974$ by $0\cdot6857$, correct to five significant figures.

19. How many times is 179·6 francs contained in 23,974·65 francs? (Give your answer to within a unit.)

20. Multiply 89,736 by 6048·9, correct to within a million.

21. Find to within 0·01 the difference between $3\cdot8647 \times 0\cdot208$ and $0\cdot8795 \times 0\cdot6437$.

22. Evaluate $\frac{17\cdot936 \times 8\cdot296}{6\cdot439}$, correct to within 0·1.

23. How many books costing £1 13s. 4d. each can be bought with £17 11s. 5d.?

24. In a certain bank the sum of £16,438,297 lies to the

credit of 673,429 depositors. Calculate to within a hundredth of a pound the average sum per head.

25. Evaluate correct to three decimal places $0.7306 \times 0.0968 \times 0.864$.

(Find the first product to five places, and the second to four places.)

26. In a certain country the total import of sugar for a particular year was 936,217 cwt., the value amounting to £748,138. Calculate to the thousandth of a pound the average value per cwt.

27. Evaluate $0.837916 \div 0.096347$, correct to four decimal places.

28. Evaluate $3.14159 \times (0.7306)^2$, correct to four decimal places.

(Multiply 0.314159 by 7.306 , then rearrange the product in order again to multiply by 7.306 .)

HARDER EXAMPLES

29. Evaluate $\frac{6873 \times 2709}{86,134}$, correct to two decimal places.

(Adjust so as to have a standardized denominator. *E.g.*, $\frac{687.3 \times 2.709}{8.6134}$. This will enable you to see how many decimal places are required in the numerator product.)

30. Evaluate $\frac{0.4768 \times 0.0936}{0.7134}$, correct to four decimal places.

31. Evaluate $3.8697 \times 0.0465 \times 93.68$, correct to three decimal places.

32. Evaluate $\frac{73.819}{7.3964 \times 13.297}$, correct to four decimal places.

(*N.B.*—This is an important type. Re-arrange thus:

$$\frac{7.3819}{7.3964} \div 1.3297.$$

Find the first quotient to five places, then likewise the final quotient.)

33. Evaluate $\frac{0.7839 \times 0.6728}{4.738 \times 0.6584}$, correct to four decimal places.

(Readjust thus: $\frac{0.7839 \times 6.728}{4.738} \div 6.584$. Since you are to have two successive divisors, each beginning with units, it is obvious that it will be sufficient to work out the multiplication in the numerator to five decimal places, and likewise each of the two division operations.)

34. Evaluate $\frac{13.897 \times 17.85}{11.29 \times 8.736}$, correct to two decimal places.

35. Evaluate $\frac{0.0973 \times 0.6894}{0.7186 \times 0.346 \times 0.7794}$, correct to four decimal places.

36. Evaluate $18,963 \times 74.6 \times 38.64$, correct to within a million.

37. Evaluate $(683.579)^3$, correct to within a million.

38. Evaluate $\frac{84.397 \times 610.75 \times 0.888}{(371.49)^2}$, correct to four decimal places.

(Make some such adjustment as

$$\frac{0.084397 \times 6.1075 \times 8.88}{3.7149} \div 3.7149.$$

Then consider your procedure.)

39. Complete the following table showing the average population per square mile:

COUNTRY	AREA IN SQUARE MILES	TOTAL POPULATION	POPULATION PER SQUARE MILE
England and Wales	50,874	39,947,931	
Scotland	30,406	4,882,497	
France	212,659	40,745,874	
Germany	185,889	62,500,000	

40. Evaluate to £.001

$$£47.758 \times \frac{37.296}{268.374}.$$

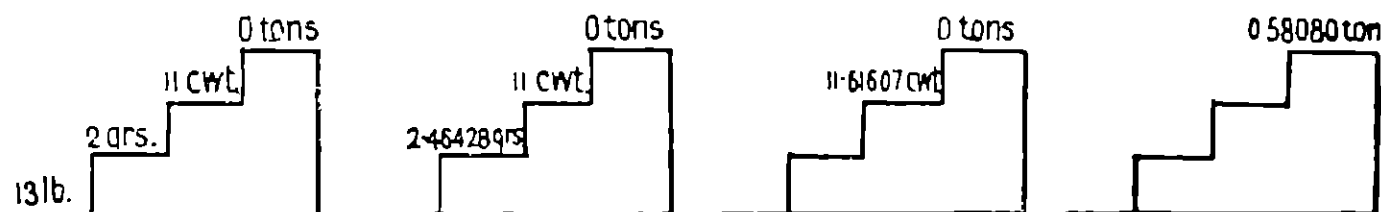
CHAPTER VI

DECIMALIZATION OF COMPOUND QUANTITIES

MODEL EXAMPLES

(a) Express 11 cwt. 2 qrs. 13 lb. as the decimal of a ton, to five decimal places.

This involves simply a series of steps in reduction. The process is analogous to that of picking up a number of weights from successive steps of a stair and conveying them to a higher stage.



$$\begin{array}{r}
 4 \overline{) 13 \text{ lb.}} \\
 7 \overline{) 3 \cdot 25} \\
 4 \overline{) 2 \cdot 46428 \text{ qrs.}} \\
 20 \overline{) 11 \cdot 61607 \text{ cwt.}} \\
 \hline
 0 \cdot 580803 \text{ ton}
 \end{array}$$

Result = 0.58080 ton.

(b) Express $10\frac{3}{4}d.$ as the decimal of 1s., to five decimal places.

$$\begin{array}{r}
 12 \overline{) 10 \cdot 75d.} \\
 \hline
 0 \cdot 89583s.
 \end{array}$$

(c) Express 13s. $4\frac{1}{2}d.$ as the decimal of a pound.

$$\begin{array}{r}
 12 \overline{) 4 \cdot 5d.} \\
 20 \overline{) 13 \cdot 375s.} \\
 \hline
 £0 \cdot 66875
 \end{array}$$

Result = £0.66875.

(d) Express 3 fur. 7 ch. 14 yds. as the decimal of 1 mile, to five decimal places.

$$\begin{array}{r|l} 2 & 14 \text{ yds.} \\ 11 & 7 \\ 10 & 7.636363 \text{ ch.} \\ 8 & 3.763636 \text{ fur.} \\ \hline & 0.470454 \text{ mile} \end{array}$$

Result = 0.47045 mile.

(e) Express 2 rds. 780 sq. yds. as the decimal of 1 acre, to five decimal places.

$$\begin{array}{r|l} 1210 \left\{ \begin{array}{l} 110 \\ 11 \\ 4 \end{array} \right. & \begin{array}{l} 780 \text{ sq. yds.} \\ 7.090909 \\ 2.644628 \text{ rds.} \\ 0.661157 \text{ acre} \end{array} \end{array}$$

Result = 0.66116 acre.

(f) Express 70 acres 2 rds. 1000 sq. yds. as the decimal of 1 sq. mile, to five decimal places.

$$\begin{array}{r|l} 1210 \left\{ \begin{array}{l} 110 \\ 11 \\ 4 \end{array} \right. & \begin{array}{l} 1000 \text{ sq. yds.} \\ 9.090909 \\ 2.826446 \text{ rds.} \end{array} \\ 640 \left\{ \begin{array}{l} 80 \\ 8 \end{array} \right. & \begin{array}{l} 70.706611 \text{ acres} \\ 0.883832 \\ 0.110479 \text{ sq. mile} \end{array} \end{array}$$

Result = 0.11048 sq. mile.

(g) Evaluate 0.66116 acre to within 1 sq. yd.

$$\begin{array}{r} 0.66116 \text{ acre} \\ \frac{4}{2.64464} \text{ rds.} \\ \frac{11}{7.09104} \\ \frac{110}{780.0144} \text{ sq. yds.} \end{array}$$

Result = 2 rds. 780 sq. yds.

(h) Evaluate 0·47045 mile, to within a yd.

$$\begin{array}{r}
 0\cdot47045 \text{ mile} \\
 8 \\
 \hline
 3\cdot7636 \text{ fur.} \\
 10 \\
 \hline
 7\cdot636 \text{ ch.} \\
 2 \\
 \hline
 1\cdot272 \\
 11 \\
 \hline
 13\cdot992 \text{ yds.}
 \end{array}$$

Result = 3 fur. 7 ch. 14 yds.

(i) Evaluate £0·6687, to within a penny.

$$\begin{array}{r}
 £0\cdot6687 \\
 20 \\
 \hline
 13\cdot374 \text{ s.} \\
 12 \\
 \hline
 4\cdot488 \text{ d.}
 \end{array}$$

Result = 13s. 4d.

(j) Evaluate 0·6342 ton + 0·786 cwt. + 1·397 qrs. to within 1 lb.

$$\begin{array}{r}
 0\cdot6342 \text{ ton} \\
 20 \\
 \hline
 12\cdot684 \text{ cwt.} \\
 \text{Add } 0\cdot786 \\
 \hline
 13\cdot470 \\
 4 \\
 \hline
 1\cdot880 \text{ qrs.} \\
 \text{Add } 1\cdot397 \\
 \hline
 3\cdot277 \\
 4 \\
 \hline
 1\cdot108 \\
 7 \\
 \hline
 7\cdot756 \text{ lb.}
 \end{array}$$

Result = 13 cwt. 3 qrs. 8 lb.

(k) Decimalize at sight 14s. 7½d.

Theory:

$$14s. = 7 \text{ florins} = £0.7.$$

$$7\frac{1}{2}d. = 30 \text{ farthings}$$

$$= £ \frac{30}{960}$$

$$= £ \frac{31}{1000} \text{ approx.}$$

$$= £0.031.$$

$$\text{I.e., } 14s. 7\frac{1}{2}d. = £0.731.$$

Practice:

$$14s. 7\frac{1}{2}d. = £(0.730 + 0.001)$$

$$= £0.731. \text{ (This should be done mentally.)}$$

Note. When the pence and farthings to be decimalized exceed (a) 3d. add correction £.001, (b) 9d. add correction £.002.

(l) Decimalize at sight 13s. 7½d.

Theory:

$$13s. = £0.65.$$

$$7\frac{1}{2}d. = £0.031.$$

$$13s. 7\frac{1}{2}d. = £0.681.$$

In practice find first £0.65. Write the £0.6 and retain the 5 in mind.

Next find 7½d., which = £0.031, and then add the 5 in the second decimal place—*i.e.*, to the 3.

(m) More examples in decimalizing at sight:

$$\begin{aligned} 11s. 4\frac{1}{4}d. &= £0.5(5) \\ &+ \underline{£0.018} \text{ (adding £0.001)} \\ &= £0.568 \end{aligned}$$

$$\begin{aligned} 19s. 2\frac{3}{4}d. &= £0.9(5) \\ &+ \underline{£0.011} \\ &= £0.961 \end{aligned}$$

$$\begin{aligned} 7s. 10d. &= £0.35 \\ &+ \underline{£0.042} \\ &= £0.392 \end{aligned}$$

(n) The pupil should verify the following examples:

$$\begin{aligned} 17s. 3d. &= £0.862. \\ 11s. 9\frac{3}{4}d. &= £0.591. \\ 3s. 1d. &= £0.154. \\ 8s. 2d. &= £0.408. \\ 13s. 11d. &= £0.696. \\ 7s. 2\frac{1}{2}d. &= £0.360. \\ 16s. 1\frac{1}{2}d. &= £0.806. \end{aligned}$$

(o) Evaluate at sight £0.983.

Theory:

$$\begin{aligned} £0.95 &= 19s. \\ £0.033 &= 8\frac{1}{4}d., \text{ less } \frac{1}{4}d. \end{aligned}$$

I.e., $£0.983 = 19s. 8d.$

Note. Subtract $\frac{1}{4}d.$ if the resulting pence and farthings do not thereby fall below $3d.$

Subtract $\frac{1}{2}d.$ if the resulting pence and farthings do not thereby fall below $9d.$

(p) Evaluate at sight £0.778.

Theory:

$$\begin{aligned} £0.75 &= 15s. \\ £0.028 &= 7d., \text{ less } \frac{1}{4}d. \end{aligned}$$

I.e., $£0.778 = 15s. 6\frac{3}{4}d.$

(q) Verify:

$$\begin{aligned} £0.139 &= 2s. 9\frac{1}{4}d. \\ £0.078 &= 1s. 6\frac{3}{4}d. \\ £0.333 &= 6s. 8d. \\ £0.897 &= 17s. 11\frac{1}{4}d. \end{aligned}$$

EXERCISES VI

1. Express 11 cwt. 1 qr. 12 lb. as a decimal of 1 ton, to four places.

2. Express 3 qrs. 18 lb. 10 oz. as a decimal of 1 cwt., to four places.

3. Express 17s. $10\frac{1}{4}d.$ as a decimal of £1, to four places.

4. Express 9s. $7\frac{1}{2}d.$ as a decimal of £1, to four places.

5. Express 3 fur. 5 ch. 18 yds. as a decimal of 1 mile, to four places.

6. Express 2 fur. 48 yds. as a decimal of 1 mile, to four places.

7. Express 3 qts. 1 pt. 2 gills as a decimal of 1 gal., to four places.

8. Express 3 rds. 850 sq. yds. as a decimal of 1 acre, to four places.

9. Express 104 acres 3 rds. 700 sq. yds. as a decimal of 1 sq. mile, to five places.

10. Express 13s. 8½d. as a decimal of £1, to four places.

11. Evaluate £0.794, to within a farthing.

12. Evaluate £0.687, to within a farthing.

13. Evaluate £0.444, to within a farthing.

14. Evaluate 0.8394 ton, to within 1 lb.

15. Evaluate 0.7908 mile, to within 1 yd.

16. Evaluate 0.793 gal., to within 1 gill.

17. Evaluate 0.6683 sq. mile, to within 1 rd.

18. Evaluate 0.7433 acre, to within 1 sq. yd.

19. Evaluate £0.9037, to within 1 penny.

20. Evaluate 0.4073 lb. (Troy), to within 1 dwt.

21. Find to within 1 lb. the value of 0.6437 ton + 0.886 rwt. + 0.397 qr.

22. Express 3 lb. 11 oz. 15 drs. as a decimal of 1 qr., to five places.

23. Express 37 fur. 97 yds. as a decimal of 1 mile, to the fourth decimal place.

24. Express 2 rds. 647 sq. yds. as a decimal of 1 acre, to four places.

25. Express 13 cwt. 2 qrs. 20 lb. as a decimal of 1 ton, to four places.

26. Evaluate 0.3978 ton to within 1 lb.

27. Evaluate 0.6387 mile + 3.924 fur. + 100.96 yds., giving the sum in furlongs, yards, and feet.

28. Evaluate £0.973 + 47.39s. + 103.7d., to within a penny.

29. Evaluate £0.8179 + 216.98s. + 307.9d., to within a penny.

30. Evaluate 0.793 ton $+$ 48.79 cwt. $+$ 9.86 qrs., to within 1 lb.

31. Evaluate 0.839276 ton to within 1 lb.

32. Decimalize at sight:

- (a) $6s. 4\frac{1}{4}d.$; $13s. 4\frac{1}{4}d.$; $7s. 2d.$; $15s. 9\frac{1}{4}d.$; $17s. 8d.$
- (b) $3s. 11d.$; $16s. 5\frac{3}{4}d.$; $11s. 7\frac{1}{2}d.$; $14s. 2d.$; $5s. 11d.$
- (c) $18s. 4d.$; $17s. 1\frac{1}{4}d.$; $19s. 5\frac{3}{4}d.$; $16s. 1d.$; $13s. 9d.$
- (d) $11s. 11d.$; $13s. 10d.$; $17s. 4\frac{3}{4}d.$; $8s. 9\frac{3}{4}d.$; $12s. 7\frac{1}{2}d.$

33. Evaluate at sight:

- (a) £0.437; £0.926; £0.678; £0.777; £0.094.
- (b) £0.691; £0.888; £0.071; £0.386; £0.556.
- (c) £0.333; £0.168; £0.713; £0.124; £0.246.
- (d) £0.515; £0.097; £0.186; £0.808; £0.588.

34. Decimalize at sight:

- (a) $12s. 7d.$; $1s. 1\frac{1}{2}d.$; $15s. 5d.$; $18s. 10d.$; $3s. 11\frac{3}{4}d.$
- (b) $14s. 8\frac{1}{4}d.$; $2s. 7\frac{3}{4}d.$; $1s. 11d.$; $16s. 4d.$; $5s. 10d.$
- (c) $1s. 1\frac{1}{4}d.$; $1s. 6\frac{1}{2}d.$; $12s. 9d.$; $7s. 4\frac{1}{4}d.$; $16s. 10\frac{1}{2}d.$
- (d) $2s. 3\frac{3}{4}d.$; $1s. 9d.$; $17s. 2d.$; $8s. 11\frac{3}{4}d.$; $14s. 3d.$

35. Evaluate at sight:

- (a) £0.077; £0.395; £0.666; £0.908; £0.173.
- (b) £0.481; £0.765; £0.019; £0.282; £0.590.
- (c) £0.881; £0.909; £0.479; £0.707; £0.009.
- (d) £0.439; £0.770; £0.178; £0.523; £0.975.

36. Find to within a tenth of a yard the sum of $.37$ fur. $+$ 4.396 ch. $+$ 11.72 yds.

37. Find to within a farthing $£0.7439 + 0.887s. + 0.49d.$

38. Express each of the following as the decimal of 1 mile, to five decimal places, and add the results together: 3 fur. 4 ch. 19 yds., 1 fur. 7 ch. 5 yds., 13 ch. 19 yds.

39. Add the three items given in the preceding question. Then decimalize the result, and compare with your answer to Question 38.

40. Evaluate to within 1 sq. yd. 4.37964 acres $+$ 1.0829 rds.

CHAPTER VII

THE METRIC SYSTEM

THIS system of measures and weights is in general use by most of the civilized nations except the English-speaking peoples, who reserve it for scientific purposes. It is so called because the standard unit of length is named the *metre* (very nearly equivalent to 39.37 in.). The other units of length are obtained either by multiplying the metre by 10, 100, 1000, or by dividing it by 10, 100, 1000.

Similarly, the unit of weight is the *gramme*, and the other units of weight are related to the gramme by the decimal law.

The unit of capacity is the *litre*, the other units being decimal multiples or sub-multiples—*i.e.*, tenths, hundredths, thousandths—of the litre.

The use of this system eliminates our antiquated and laborious compound system; and it is indisputable that by its use the French schoolboy, for example, has an advantage over the British boy of from one to two school years in his arithmetical study. Correspondingly, it is not too much to say that by the decimal system Continental commercial calculations require less than half the labour of corresponding calculations in our country.

One can only wonder how much longer the criminal waste of time will be tolerated by our legislators.

The following are the easily remembered tables of length, area, volume, weight, and capacity.¹

Length

1 metre	=	about 39.37 in.	(m.)
$\frac{1}{10}$ metre,	called 1 decimetre		(dm.)
$\frac{1}{100}$ metre,	called 1 centimetre		(cm.)

¹ The contractions employed here are in accordance with the French decree of 1919.

EXERCISES IN ARITHMETIC

$\frac{1}{1000}$ metre, called 1 millimetre	(mm.)
10 metres, called 1 decametre	(dam.)
100 metres, called 1 hectometre	(hm.)
1000 metres, called 1 kilometre	(km.)

Area

1 square metre (written m. ²)	
1 square decimetre	(dm. ²)
1 square centimetre	(cm. ²)
1 square millimetre	(mm. ²)

The fundamental unit of land measure is the *are* (a.), which equals the square on a length of 1 decametre, and therefore contains 100 square metres. Hence also the *hectare* (ha.), which contains 100 ares. Also we have the square kilometre (km.²).

Note. $\frac{1}{100}$ are, which, of course, is 1 square metre (m.²), is sometimes called a centiare (ca.).

Volume

1 cubic metre (somewhat larger than our cubic yard)	(m. ³)
1 cubic decimetre (a cube whose edge is 1 decimetre)	(dm. ³)
1 cubic centimetre (a cube whose edge is 1 centimetre)	(cm. ³)
1 cubic millimetre (a cube whose edge is 1 millimetre)	(mm. ³)

Hence we have

1000 cubic millimetres = 1 cubic centimetre
1000 cubic centimetres = 1 cubic decimetre
1000 cubic decimetres = 1 cubic metre

Weight

1 gramme (which is the weight of 1 cu. cm. of water at 4° Centigrade), written 1 g.

$\frac{1}{10}$ gramme, called 1 decigramme	(dg.)
$\frac{1}{100}$ gramme, called 1 centigramme	(cg.)
$\frac{1}{1000}$ gramme, called 1 milligramme	(mg.)

(All four of these are employed for very small quantities—*e.g.*, in goldsmiths' and chemists' operations.)

Also

10 grammes, called 1 decagramme	(dag.)
100 grammes, called 1 hectogramme	(hg.)
1000 grammes, called 1 kilogramme	(kg.)

(The kilogramme is the weight most commonly used in the purchase of goods.)

Capacity

1 litre (which equals exactly 1 cubic decimetre) (l.)

Also its decimal multiples and sub-multiples (dal., hl., and dl., cl., ml.)

Useful Relations to be committed to Memory

1 metre = 39.37 in. approximately (a little more than a yard)

1 decimetre = nearly 4 in.

1 kilometre = fully 1000 yds., or, more exactly, 1095 yds. (A good approximation is 5 furlongs.)

1 inch = $2\frac{1}{2}$ centimetres approximately

1 hectare = $2\frac{1}{2}$ acres approximately

1 kilogramme = $2\frac{1}{5}$ lb. avoirdupois (nearly), or, more exactly, 2.205 lb.

1 lb. avoirdupois = 450 grammes approximately, or, more exactly, 454 grammes

1 litre = $1\frac{3}{4}$ pts. approximately

The names of certain of the sub-multiples in the metric system are rarely used. Thus we speak of 0.75 gramme rather than of 7.5 decigrammes or 75 centigrammes, and of 0.75 litre rather than of 75 centilitres.

ORAL EXERCISES

1. Express in metres 6374 centimetres, 4239 decimetres, 865 millimetres, 293 centimetres, 5836 millimetres, 298 decimetres, 57 centimetres.

2. Express in centimetres 483 millimetres, 29 metres, 29.6 metres, 473 decimetres, 7.38 metres, 8.296 metres, 48.6 millimetres.

3. Express in metres 43 kilometres, 43.72 kilometres, 3.7 hectometres, 0.83 kilometre, 5.73 decametres.

4. Express in square metres 3 ares, 3.7 ares, 0.847 are, 17.298 ares.

5. Express in grammes 4.38 kg., 0.897 kg., 13.06 kg., 0.83 hg.

6. Express in kilogrammes 73 hg., 897 hg., 14 dag., 7 g., 293.8 g.

7. Express in litres 39.7 hl., 0.793 hl., 43.78 dal.

8. What is the total weight of 12 buttons, each weighing 10.4 g.?

9. How many pieces of wire each weighing 25 g. can be obtained from a piece weighing 7.5 kg.?

10. From a length of 2·5 kilometres I cut off 50 metres. What length remains?

11. A cyclist travels at a speed of 300 metres per minute. Express this in kilometres per hour.

12. A road 5·8 kilometres long is to be lighted from end to end by lamps placed uniformly at intervals of 1 hectometre. How many lamps will be required?

13. How many decilitre bottles can be filled from 1 decalitre?

14. How many cubic centimetres are there in a cube whose edge measures 1 decimetre?

15. A square court, the side of which measures 12 metres, is to be laid with square wooden blocks, the sides of which measure 50 centimetres. How many blocks will be required?

EXERCISES VII

1. Assuming that gold costs 16 f. 50—*i.e.*, 16 francs 50 centimes—per gramme, calculate in francs the cost of 3·6 grammes of gold.

2. The wheel of a bicycle is 3 m. 30—*i.e.*, 3 metres 30 centimetres—in circumference. How many complete turns will it make in travelling a distance of (a) 3·4 hm. and (b) 3·75 km.?

3. To measure the length of a street two men, A and B, traverse it from end to end, counting the number of paces taken. A takes 12 paces more than B. If A's pace measures 0 m. 77, and he takes altogether 250 paces, calculate (a) the length of the street and (b) the length of B's pace.

4. Find the cost of 1·5 kl. at the rate of 5 f. 40 per litre.

5. Assuming that 1 kg. = 2·205 lb., calculate the number of grammes equivalent to 1 lb.

6. Calculate the cost of 1·5 hl. if the *double decalitre* costs 28·5 francs.

7. Express in metres, correct to 1 centimetre, a length of 4 miles 2 fur. (Take 1 km. as 1095 yds.)

8. Find the cost of half a litre if 1 hectolitre costs 190 f.

9. How many pieces of wire 3·5 cm. long can be cut from a coil of wire 10 m. 50 long?

10. A man wishes to purchase $7\frac{1}{2}$ lb. of paint from a French firm. The selling price is 8 f. 30 per kg. Show (*a*) how the man should order in the metric system and (*b*) what price in francs he will have to pay for his order. (Take 1 lb. as 450 g.)

For harder problems in metric system see Chapter XIII page 138.

CHAPTER VIII

MENSURATION OF AREA

THE RECTANGLE

To measure the area of a surface is to find the number of unit squares in it—*i.e.*, the number of square inches, or square yards, or acres, etc.

Fig. 1 represents a rectangle 6 in. long and 4 in. broad.

We first cut off a strip ADCB 1 in. broad. Now the whole rectangle contains 4 strips like ADCB. It is easy to see that

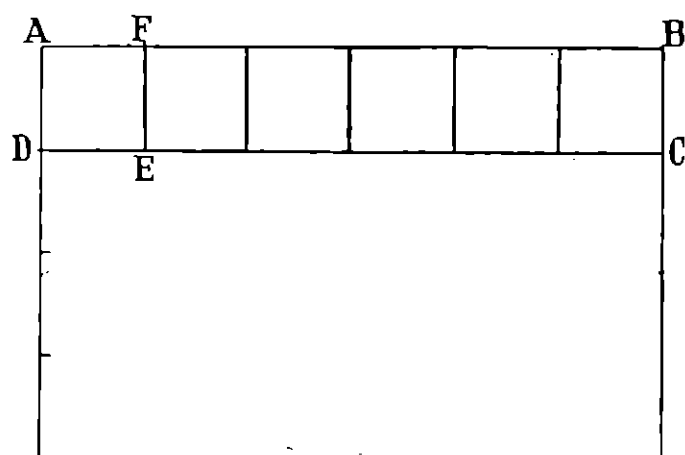


FIG. 1

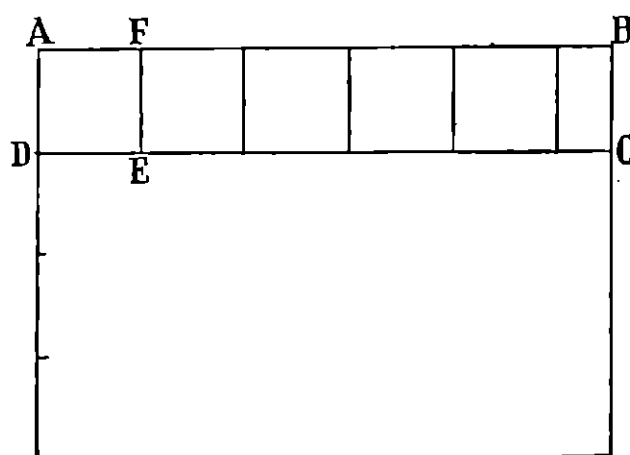


FIG. 2

this strip contains 6 unit squares—*i.e.*, inch squares like ADEF. Thus the whole rectangle contains (6×4) sq. in.

Fig. 2 represents a rectangle $5\frac{1}{2}$ in. long and 4 in. broad.

As in the previous case, we have divided the whole rectangle into 4 strips like ADCB, 1 in. broad. But in this case ADCB contains $5\frac{1}{2}$ inch squares. Thus the whole rectangle contains $(5\frac{1}{2} \times 4)$ sq. in.

Thus it can be shown that the area of any rectangle may be found by multiplying the number of units in the length by the number of units in the breadth.

Although not strictly mathematical it is convenient to remember this rule as follows:

$$\text{Area (of any rectangle)} = \text{length} \times \text{breadth.}$$

It follows that

$$\text{length} = \text{area} \div \text{breadth}$$

and

$$\text{breadth} = \text{area} \div \text{length}.$$

MODEL EXAMPLES

(a) A rectangle is 12 ft. 3 in. long and 9 ft. 8 in. broad. Find its area.

$$\begin{array}{l|l} \text{Area} = 12\frac{1}{4} \times 9\frac{2}{3} \text{ sq. ft.} & 9\frac{2}{3} \times 12 = 116 \\ = 118\frac{5}{12} \text{ sq. ft.} & 9\frac{2}{3} \times \frac{1}{4} = 2\frac{1}{4} + \frac{1}{6} = \frac{2\frac{5}{12}}{118\frac{5}{12}} \\ = 118 \text{ sq. ft. } 60 \text{ sq. in.} & \text{Or, if preferred, take } \frac{49}{4} \times \frac{29}{3}. \end{array}$$

(b) A room is 15 ft. long, 14 ft. broad, and 11 ft. high. A fireplace measures 5 ft. 8 in. by 6 ft. A doorway measures 8 ft. 6 in. by 4 ft., and two windows each 7 ft. by 2 $\frac{1}{4}$ ft. Find the cost of lining the walls with oak lining at 7s. 6d. per square yard.

$$\begin{aligned} \text{Perimeter of room} &= 2(15 + 14) \text{ ft.} \\ &= 58 \text{ ft.} \end{aligned}$$

$$\begin{aligned} \text{area of four walls} &= 58 \times 11 \text{ sq. ft.} \\ &= 638 \text{ sq. ft.} \end{aligned}$$

Deduct

$$\begin{array}{rcl} \text{Fireplace, } 5\frac{2}{3} \times 6 & = & 34 \text{ sq. ft.} \\ \text{Door, } 8\frac{1}{2} \times 4 & = & 34 \\ \text{Windows, } 2 (7 \times 2\frac{1}{4}) & = & 31\frac{1}{2} \\ \text{Total} & & 99\frac{1}{2} \end{array}$$

$$\begin{array}{rcl} \text{Lining required is} & 538\frac{1}{2} & \\ \text{Cost at } 10d. \text{ per sq. ft.} & = 5385d. & \\ & = \text{£}22 \text{ } 8s. \text{ } 9d. & \left| \begin{array}{l} 12 \overline{) 538 \text{ } 5d.} \\ 210 \overline{) 44 \text{ } 8s. \text{ } 9d.} \\ \hline \text{£}22 \text{ } 8s. \end{array} \right. \end{array}$$

Note. The short form of stating dimensions—*i.e.*, length, breadth, and height—of a room, as in Example (b) above, is: room 15' $\times$ 14' $\times$ 11', doorway 8' 6" $\times$ 4', two windows 7' $\times$ 2' 3"

ORAL EXERCISES

1. A sheet of notepaper is 7 in. long and 4 in. broad. How many square inches does the page contain?

A page of a book is 7 in. long and $4\frac{1}{2}$ in. broad. Find the area of the page.

3. A rectangular visiting-card is 3 in. long and $\frac{3}{4}$ in. broad. What is its area?

4. A farmer wishes to enclose a piece of ground in the form of a rectangle containing 400 sq. yds. If the length is to be 40 yds., what must be the breadth?

5. A rectangular photo-print is $2\frac{1}{2}$ in. long and $1\frac{1}{2}$ in. broad. Calculate the area.

6. Find the area of 20 yds. of ribbon $1\frac{1}{2}$ in. broad. (Caution: Will you think of the ribbon as divided into yard squares or inch squares?)

7. How many square yards of carpet will be required to cover the floor of a rectangular room 15 ft. long and 12 ft. broad?

8. I have a rectangular sheet of paper 10 in. long and $8\frac{1}{2}$ in. broad. If I cut it into strips 1 in. broad, and place these strips end to end in a straight line, what will be the length of the combined strips?

9. I have a sheet of 60 foreign stamps, each of which is 1 in. long and $\frac{3}{4}$ in. broad. What is the area of the sheet?

10. A roller $3\frac{1}{2}$ ft. long is pushed along a lawn for a distance of 20 ft. What is the area of the part of lawn rolled?

EXERCISES VIII

(*N.B.* It is often helpful in working these problems to draw a diagram.)

1. Find the area of the floor of a rectangular room in the following cases:

- | | | |
|-----|----------------------|---------------------|
| (a) | Length 17 ft. 6 in., | breadth 13 ft. |
| (b) | 17 ft. 9 in., | 14 ft. |
| (c) | 16 ft. 10 in., | $12\frac{1}{2}$ ft. |
| (d) | 22 ft. 4 in., | 18 ft. |
| (e) | 18 ft. 7 in., | 17 ft. 6 in. |
| (f) | 31 ft. 8 in., | 24 ft. 9 in. |
| (g) | 25 ft. 10 in., | 21 ft. 6 in. |

2. Find the area of a rectangular court in the following cases :

- | | | |
|-----|-----------------------|----------------------|
| (a) | Length 120 ft. 6 in., | breadth 63 ft. 9 in. |
| (b) | 43 yds., | 37 ft. 6 in. |
| (c) | 174·3 metres, | 96 metres. |
| (d) | 7 ch. 11 yds., | 80 yds. |

3. Find in acres the area covered by a straight mile of railway having a uniform width of 32 yds.

4. A man requires 192 sq. ft. of wood. What length must he take from planking 16 in. broad?

5. A rectangular court 120 ft. long and 75 ft. broad is surrounded by netting. A path is made round the court just outside the netting. If the path is to be 10 ft. 6 in. broad calculate the area of the path.

6. How many acres are contained in a rectangular playing-field $\frac{1}{4}$ mile long and 250 yds. broad?

7. A room is 16 ft. long, 12 ft. 6 in. wide, and 9 ft. 3 in. high. Find (a) the area of the floor and (b) the area of the four walls, making no allowance for doors, windows, etc.

8. Make an allowance for a doorway 7 ft. $\times$ 3 ft., two windows 6 ft. $\times$ $3\frac{1}{2}$ ft., and a fireplace 5 ft. $\times$ 4 ft. in Question 7 above. Calculate the cost of painting the walls at 2d. per square foot.

9. The dimensions of a room are 22 ft. $\times$ 15 ft. 6 in. $\times$ 9 ft. Allowing 100 sq. ft. for openings—*i.e.*, doors, windows, etc.—calculate (a) the area of paper required to cover the walls, (b) the number of 'pieces' required, assuming that the standard size of a piece of wallpaper is 12 yds. long and 21 in. broad.

10. In connexion with the preceding question, calculate the cost of painting the ceiling at $7\frac{1}{2}$ d. per square foot.

11. Calculate in each of the following cases (i) the area of the floor, (ii) the total wall area, less openings, and (iii) the cost of painting ceiling and walls :

- (a) Room 18 ft. $\times$ 15 ft. 6 in. $\times$ 10 ft., door 7 ft. 6 in. $\times$ 4 ft., three windows each 7 ft. $\times$ 3 ft.; cost of painting 2s. 3d. per square yard.

- (b) Room 27 ft. $\times$ 20 ft. $\times$ 12 ft., two doors each 8 ft. $\times$ 4 ft. 6 in., four windows each 8 ft. 6 in. $\times$ 4 ft.; cost of painting $2\frac{1}{2}d.$ per square foot.

12. It is required to cover entirely with carpet the floor of a room 17 ft. 6 in. $\times$ 15 ft. If the carpet bought is 4 ft. 3 in. wide, what length will be required?

13. If in the case of the preceding question it had been decided to leave uncovered a margin 2 ft. broad round the room, calculate the cost of staining the margin at $1\frac{1}{2}d.$ per square foot.

14. How many tiles 16 in. $\times$ 12 in. will be required to cover a rectangular court 120 ft. $\times$ 90 ft.?

15. A carpet 12 ft. 9 in. $\times$ 10 ft. 6 in. is laid in the middle of a room 14 ft. $\times$ 12 ft. Calculate the cost of staining the uncovered margin at $2d.$ per square foot.

16. Calculate the area of a path 4 ft. 6 in. broad made immediately round the outside of a rectangular lawn $12\frac{1}{2}$ yds. $\times$ 10 yds.

17. The dimensions of a room are 11.2 metres, 8.7 metres, and 9 metres. Allowing 72 square metres for openings, calculate (a) the cost of painting the ceiling at 10 francs per square metre, (b) the cost of painting the walls at 7.5 francs per square metre.

18. It is desired to lay out a rectangular garden to contain 250 sq. yds. If the breadth is to be 15 yds., what is the necessary length? Give your answer to within a foot.

19. A schoolroom measures 25 ft. $\times$ 20 ft. If the floor space to be allowed per pupil is 15 sq. ft., what is the maximum number of pupils that can be accommodated?

20. A rectangular box 7 ft. $\times$ 5 ft. 9 in. $\times$ 4 ft. is to be covered with tin foil. Calculate the area of tin foil required.

21. What length of carpet 24 in. wide must I buy to cover entirely a floor 33 ft. $\times$ 20 ft. 6 in.?

22. An orchard 300 yds. long and 150 yds. broad is planted with trees. The trees are 15 ft. apart in rows, and the rows are 15 ft. apart. The outer rows are 15 ft. from the boundary walls. How many trees are there in all?

23. Calculate the area of a path $4\frac{1}{2}$ ft. wide made round a rectangular enclosure 12 yds. $\times$ $7\frac{1}{2}$ yds.

24. A straight path 7 ft. 6 in. wide is to be covered with gravel, allowing $\frac{1}{4}$ st. to the square foot. If a workman begins from one end of the path, what length of the path will he cover when he has used 3 tons of gravel?

25. Find the cost of carpeting a floor 42 ft. $\times$ 15 ft. with carpet 4 ft. 6 in. wide at 3s. 6d. per yard—*i.e.*, at the rate of 3s. 6d. for a piece of carpet 1 yd. long and 4 ft. 6 in. wide.

26. A house-owner is charged £17 1s. 8d. by a painter for having the walls of a rectangular store-room varnished. The dimensions of the room are 22 ft. 6 in. $\times$ 18 ft. 3 in. $\times$ 12 ft. 6 in. Without making allowance for openings, calculate the charge per square yard made by the painter. Give your result to within a penny.

27. If the wallpaper required for covering entirely the walls of a room 18 ft. $\times$ 17 ft. $\times$ 11 ft. (neglecting openings) costs 46s. 8d., calculate the cost per linear yard of the paper, which is $\frac{1}{12}$ yd. wide.

28. The length of a rectangular field is $11\frac{1}{2}$ ch. and the breadth $7\frac{1}{2}$ ch. Calculate the area of the field in acres, roods, and square yards.

29. A rectangular field is 205 metres long. If the area is 3.5 hectares, calculate the breadth to within a metre.

30. A rectangular court measures 150 metres $\times$ 120 metres. Calculate the area of a path 4.5 metres wide made round the outside of the court. Calculate also the cost of covering the path with gravel at the rate of 9.25 francs per square metre.

CHAPTER IX

COST PROBLEMS

(THE important method shown below is known as '*practice*.')

The pupil should be made familiar with the following aliquot parts of £1

$$\begin{array}{lll}
 6s. \ 8d. = \frac{1}{3} \text{ of } £1 & 1s. \ 8d. = \frac{1}{12} \text{ of } £1 & 4d. = \frac{1}{60} \text{ of } £1 \\
 3s. \ 4d. = \frac{1}{6} & 8d. = \frac{1}{30} & 3d. = \frac{1}{80} \\
 2s. \ 6d. = \frac{1}{8} & 6d. = \frac{1}{40} & 2d. = \frac{1}{120}
 \end{array}$$

Note also that any *even* number of shillings may be expressed as tenths of £1. This frequently affords a short method in '*practice*.'

TYPE I

MODEL EXAMPLES

- (a) Find the cost of 683 tons at £1 15s. per ton.

$10s. = \frac{1}{2} \text{ of } £1$ $5s. = \frac{1}{2} \text{ of } 10s.$	$\begin{array}{r} £ \\ 683 \\ 341.5 \\ 170.75 \\ \hline 1195.25 \end{array}$	Cost of 683 tons at £1 per ton. 10s. 5s.
---	--	---

Required cost = £1195 5s.

- (b) Find the cost of 297 loads at £2 14s. 8d. each.

$10s. = \frac{1}{2} \text{ of } £1$ $4s. = \frac{1}{5} \text{ of } £1$ $8d. = \frac{1}{6} \text{ of } 4s.$	$\begin{array}{r} £ \\ 297 \\ 2 \\ \hline 594 \\ 148.5 \\ 59.4 \\ 9.9 \\ \hline 811.8 \end{array}$	Cost at £1 per load. £2 10s. 4s. 8d.
--	--	---

Required cost = £811 16s.

Note. The work would be lessened by taking $14s. = \frac{7}{10}$ of £1 and $8d. = \frac{1}{30}$ of £1; or by taking $8s. = \frac{4}{10}$ of £1 and $6s. 8d. = \frac{1}{3}$ of £1.

(c) Find (without unnecessary figures) the cost of 517 barrels at £3 18s. 5d. each.

	£
	517
	3

	1551
	258·5
	129·25
	86·1666
	2·1541

	2027·071

$10s. = \frac{1}{2} \text{ of } £1$
 $5s. = \frac{1}{2} \text{ of } 10s.$
 $3s. 4d. = \frac{1}{3} \text{ of } 10s.$
 $1d. = \frac{1}{40} \text{ of } 3s. 4d.$

Required cost = £2027 1s. 5d.

Notes. (i) Take care to divide the *appropriate* line. For example, in Example (c) above 3s. 4d. is $\frac{1}{3}$ of 10s., not $\frac{1}{3}$ of 5s.

(ii) Carry out the division to the *fourth* decimal place at most.

(iii) In adding, do not write down the fourth decimal figure, but consider it for carriage. For instance, in Example (c) above we have 6 and 1 make 7. Discard this, but carry 1 to the addition of the next column.

N.B. A shorter method in this example would be to take $18s. = \frac{9}{10}$ of £1, $4d. = \frac{1}{60}$ of £1, and $1d. = \frac{1}{4}$ of 4d.

(d) Find the cost of 7 tons 3 cwt. 1 qr. 12 lb. at £4 11s. 7d. per cwt.

Note that the rate is quoted per *cwt.* Therefore express the quantity as cwt., with whatever decimal is necessary.

4	12 lb.
7	3
4	-----
	1·42857 qrs.

	0·35714 cwt.

Add 7 tons 3 cwt.—i.e.,	142	cwt.
Total	143·35714	cwt.

(The fifth decimal figure has been kept for the sake of the carriage when we come to multiply by 4.)

	£
	143·3571(4)
	4
	573·4286
10s. = $\frac{1}{2}$ of £1	71·6785
1s. = $\frac{1}{10}$ of 10s.	7·1678
6d. = $\frac{1}{2}$ of 1s.	3·5839
1d. = $\frac{1}{6}$ of 6d.	·5973
	656·456

Required cost = £656 9s. 1($\frac{1}{2}$)d.

N.B. A shorter form would be to take 8s. = $\frac{4}{10}$ of £1, 3s. 4d. = $\frac{1}{6}$ of £1, and 3d. = $\frac{1}{80}$ of £1.

(e) Find the cost of 12 acres 3 rds. 430 sq. yds. at £9 16s. 10d. per acre.

110	430 sq. yds.
11	3·90909
4	3·35537 rds.
	12·83884 acres.

	£
	12·8388(4)
	9
	115·5496
10s. = $\frac{1}{2}$ of £1	6·4194
5s. = $\frac{1}{2}$ of 10s.	3·2097
1s. = $\frac{1}{10}$ of 10s.	·6419
10d. = $\frac{1}{6}$ of 5s.	·5349
	126·356

Required cost = £126 7s. 1($\frac{1}{2}$)d.

Note. Simplify the work wherever possible. For example:
 (i) We may take 1s. as $\frac{1}{5}$ of 5s. in the above example, but 1s. taken as $\frac{1}{10}$ of 10s. makes easier division. (ii) A thoughtless pupil might break 10d. into 6d. and 4d., thereby making two steps when one would do, as shown above.

N.B. A shorter form would be to take $16s. = \frac{8}{10}$ of £1, $6d. = \frac{1}{10}$ of £1, and $4d. = \frac{1}{60}$ of £1.

(f) Find the cost of 637 articles at £3 4s. 11d. each.

$$\begin{array}{r}
 \text{£} \\
 637 \\
 3 \\
 \hline
 1911 \\
 159 \cdot 25 \\
 \hline
 2070 \cdot 25 \\
 2 \cdot 654 \\
 \hline
 2067 \cdot 596
 \end{array}$$

$5s. = \frac{1}{4}$ of £1
 $1d. = \frac{1}{60}$ of 5s.

Required cost = £2067 11s. 11d.

This introduction of *subtraction* frequently saves labour. Pupils should take advantage of any such opportunity.

(g) Find the cost of 12 acres 3 rds. 430 sq. yds. at £2 11s. 11d. per acre.

$$\begin{array}{r}
 110 \overline{) 430 \text{ sq. yds.}} \\
 11 \overline{) 390909} \\
 4 \overline{) 335537 \text{ rds.}} \\
 \hline
 12 \cdot 83884 \text{ acres}
 \end{array}$$

(Divide by 11 and move digits one place to right.)

$$\begin{array}{r}
 \text{£} \\
 12 \cdot 8388(4) \\
 2 \\
 \hline
 25 \cdot 6777 \\
 6 \cdot 4194 \\
 1 \cdot 2838 \\
 \hline
 33 \cdot 3809 \\
 0 \cdot 535 \\
 \hline
 33 \cdot 3274
 \end{array}$$

$10s. = \frac{1}{2}$ of £1
 $2s. = \frac{1}{10}$ of £1
 $1d. = \frac{1}{120}$ of 10s.

Required cost = £33 6s. 6½d.

Note that 2s. is better taken as $\frac{1}{10}$ of £1 than as $\frac{1}{5}$ of 10s. Also that 1d. is better taken as $\frac{1}{120}$ of 10s. than as $\frac{1}{24}$ of 2s.

Alternatively:

£

12·8388(4)

2

25·6777

6·4194

·6419

·5349

·0534

33·327

10s. = $\frac{1}{2}$ of £1

1s. = $\frac{1}{10}$ of 10s.

10d. = $\frac{1}{12}$ of 10s.

1d. = $\frac{1}{10}$ of 10d.

I.e., £33 6s. 6½d.

(h) The short cut suggested in the notes appended to Examples (b), (c), (d), and (e) is illustrated in the following:

Find the cost of 12 acres 3 rds. 430 sq. yds. at £2 18s. 8d. per acre. We obtain, as already shown, 12·83884 acres.

Then

£

12·8388(4)

2

25·6777

11·5549

etc.

18s. = £ $\frac{9}{10}$

8d. = £ $\frac{1}{30}$

(Multiply 12·83884 by 9, and move digits one place to right.)

This device should be adopted wherever convenient.

(i) **Alternative method**, useful in a limited number of cases, but laborious in the general case.

Find the cost of 2 tons 11 cwt. 1 qr. 8 lb. at £11 17s. 2d. per ton.

£

11·858

2

23·716

5·929

·5929

·1482

·0370

·0053

30·428

10 cwt. = $\frac{1}{2}$ ton

1 cwt. = $\frac{1}{10}$ of 10 cwt.

1 qr. = $\frac{1}{4}$ of 1 cwt.

7 lb. = $\frac{1}{4}$ of 1 qr.

1 lb. = $\frac{1}{7}$ of 7 lb.

2 tons

10 cwt.

1 cwt.

1 qr.

7 lb.

1 lb.

I.e., required cost = £30 8s. 7d., to within a penny.

ORAL EXERCISES

1. 624 tons at 10s. per ton.
2. 738 tons at 5s. per ton.
3. 818 tons at 2s. 6d. per ton.
4. 520 tons at 6s. 8d. per ton.
5. 327 tons at 4s. per ton.
6. 483 tons at 6d. per ton.
7. 319 tons at 8d. per ton.
8. 296 tons at 4d. per ton.
9. 774 tons at 2d. per ton.
10. 417 tons at 2s. per ton.
11. 586 tons at 16s. per ton. (*N.B.* Take 16s. as £ $\frac{8}{10}$.)
12. 371 tons at 14s. per ton.
13. 703 tons at 18s. per ton.
14. 409 tons at 12s. per ton.

EXERCISES IXA

Find the cost of

1. 739 articles at 10s. each.
2. 437 articles at 5s. each.
3. 619 articles at £1 10s. each.
4. 737 articles at £2 10s. each.
5. 198 articles at £1 5s. each.
6. 427 articles at £3 5s. each.
7. 984 articles at £2 15s. each.
8. 713 articles at 2s. 6d. each.
9. 437 articles at £4 2s. 6d. each.
10. 1137 articles at £7 12s. 6d. each.
11. 439 articles at £1 7s. 6d. each.
12. 615 articles at 17s. 6d. each.
13. 237 articles at £2 17s. 6d. each.
14. 169 articles at 4s. each.
15. 217 articles at £1 14s. each.
16. 7931 articles at 6s. each.
17. 448 articles at 13s. each.
18. 293 articles at £1 16s. each.

19. 572 articles at 4s. 6d. each.
20. 931 articles at £2 11s. 6d. each.
21. 407 articles at £3 16s. 6d. each.
22. 966 articles at £2 18s. 4d. each.
23. 378 articles at 9s. 2d. each.
24. 1351 articles at £1 7s. 2d. each.
25. 1315 lb. at 1s. 7½d. per lb.
26. 76 tons at £1 18s. 9d. per ton.
27. 327 articles at £9 15s. 9d. each.
28. 725 tons at £2 7s. 5d. per ton.
29. 43 gallons at £7 7s. 9d. per gallon.
30. 94 tons at £43 6s. 10d. per ton.
31. 61 articles at £1 7s. 5¾d. each.
32. 100 articles at £4 19s. 6½d. each.
33. 117 articles at £3 15s. 3¼d. each.
34. 155 articles at £1 13s. 6d. each.
35. 156 articles at £2 11s. 5¼d. each.
36. 97 lb. at 3s. 4½d. per lb.
37. 89 trees at 16s. 9d. each.
38. 105 articles at £1 10s. 6¼d. each.
39. 365 loads at £1 1s. 11d. per load.
40. 437 cwt. at £2 17s. 10d. per cwt.
41. 648 articles at £1 14s. 7d. each.
42. 239 tons at £3 11s. 4d. per ton.
43. 174 articles at 17s. 8½d. each.
44. 349 loads at £1 13s. 10½d. per load.
45. 563 tons at £13 9s. 7d. per ton.

Find to within a penny the cost in the following cases:

46. 13 tons 10 cwt. at £4 7s. 6d. per ton.
47. 12 tons 5 cwt. at £1 11s. 6d. per ton.
48. 7 miles 6 fur. at £2 12s. 6d. per mile.
49. 28 tons 1 cwt. 2 qrs. at £2 15s. per ton.
50. 317 miles 1 fur. at £7 17s. 6d. per mile.
51. 47 acres 1 rd. at £2 11s. per acre.
52. 11 cwt. 1 qr. 7 lb. at £1 6s. per cwt.
53. 6 tons 7 cwt. 2 qrs. at £3 2s. 9d. per ton.
54. 25 tons 15 cwt. 5 st. at £2 4s. 8d. per ton.

55. 3 tons 15 cwt. 2 qrs. 15 lb. at £2 3s. 8d. per ton.
56. 16 tons 11 cwt. 3 qrs. 1 st. at £8 14s. 6d. per ton.
57. 5 miles 3 fur. 80 yds. at £2 8s. 6d. per mile.
58. 15 tons 3 cwt. 2 qrs. 17 lb. at £3 17s. 3d. per ton.
59. 7 ch. 15 yds. at 25s. 6d. per chain.
60. 4 acres 3 rds. 25 sq. poles at 1s. 3d. per square pole.
61. 14 miles 7 fur. 24 yds. at £2 5s. 9d. per mile.
62. $462\frac{3}{4}$ yds. at 15s. 6d. per yd.
63. 63,500 cu. ft. at 7s. 4d. per 1000 cu. ft.
64. 69 miles 4 fur. 100 yds. at £5 15s. 8d. per mile.
65. 3 miles 6 fur. 120 yds. at £4 12s. 7d. per mile.
66. 273 tons 11 cwt. 3 qrs. at £4 11s. 8d. per ton.
67. 817,396 cu. ft. at 6s. 10d. per 1000 cu. ft.
68. 13 acres 3 rds. 19 sq. poles at £2 15s. 8d. per acre.
69. 3 acres 1 rd. 700 sq. yds. at £9 5s. per acre.
70. 15 acres 3 rds. 550 sq. yds. at £13 13s. per acre.

EXERCISES IX_B

(Additional exercises involving the method of 'practice'.)

1. Find to within a penny the tax on £786 14s. at the rate of 4s. $2\frac{1}{2}$ d. per £1.
2. Find the tax on a rental of £576 10s. at 11s. $4\frac{1}{2}$ d. in the £1.
3. Find the total assessment on a rental of £938 at the rate of 9s. $7\frac{1}{2}$ d. in the pound.
4. A man's 'taxable income' is £793. If income tax is chargeable at 2s. 6d. in the £1 on the first £175 of income, and at 5s. in the pound on the rest, calculate the man's liability.
5. A bankrupt can pay only 7s. 10d. in the pound. Calculate the payments to be made to creditors who have respectively the following claims:

- (a) £1467.
- (b) £933 14s.
- (c) £689 15s.
- (d) £437 11s.

6. Calculate the payment to be made to a creditor having a claim of £11,437 17s. on a bankrupt who can pay only 16s. 8d. in the pound.

7. If income tax is chargeable as set forth in Question 4 above, calculate the tax due by a man whose 'taxable income' amounts to £679.

8. Calculate the tax on a rental of £1055 at the rate of 8s. 10 $\frac{7}{8}$ d. per £1. (Consider the 'parts' 8s., 8d., 2d., $\frac{4}{8}$ d., $\frac{2}{8}$ d., $\frac{1}{8}$ d.; or 10s. less (1s. + 1d. + $\frac{1}{8}$ d.).

9. Calculate the tax on a rental of £675 15s. at the rate of 13s. 7 $\frac{7}{16}$ d. per £1. (Consider the 'parts' 6s. 8d., 6s. 8d., 3d., $\frac{8}{16}$ d., less $\frac{1}{16}$ d.)

10. See Exercises IIIA, Question 20. Assuming the rate of assessment given in that example, calculate the tax on a rental of £480 10s.

TYPE II

(Problems of this type seldom occur in practical experience.)

MODEL EXAMPLES

N.B. It is particularly useful in these examples to make an intelligent forecast of the required result.

(a) If 716 tons cost £1193 6s. 8d., find the price per ton.

$$\begin{aligned}\text{Price per ton} &= \text{£} \frac{1193.333}{716} \\ &= \text{£} 1.666 \\ &= \text{£} 1.667 \\ &= \text{£} 1 \text{ } 13\text{s. } 4\text{d.}\end{aligned}$$

$$\begin{array}{r} 1.6666 \\ 716 \overline{) 1193.333} \\ \underline{716} \\ 4773 \\ \underline{4296} \\ 4773 \\ \underline{4296} \\ 4773 \\ \underline{4296} \\ 477 \end{array}$$

(Advanced pupils should employ contracted division.)

COST PROBLEMS

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(b) If 69 miles 4 fur. 100 yds. cost £402 4s. 9d., find the cost per mile.

$$\begin{aligned}\text{Cost per mile} &= \text{£} \frac{402.237}{69.556} \\ &= \text{£} 5.783 \\ &= \text{£} 5 \text{ } 15\text{s. } 8\text{d.}\end{aligned}$$

$$\begin{array}{r} \left. \begin{array}{l} 20 \\ 11 \\ 8 \end{array} \right\} \begin{array}{l} 100 \text{ yds.} \\ 5 \\ 4.454 \text{ fur.} \\ .556 \text{ mile} \end{array} \\ \hline 5.7829 \\ 69556 \overline{) 402237} \\ \underline{347780} \\ 544570 \\ \underline{486892} \\ 576780 \\ \underline{556448} \\ 203320 \\ \underline{139112} \\ 642080 \end{array}$$

Taking advantage of all possible contraction, we have

$$\begin{array}{r} 6'9'5'5'6 \overline{) 402237(57829} \\ \underline{54457} \\ 5768 \\ \underline{204} \\ 65 \end{array}$$

(c) If 74 acres 2 rds. 880 sq. yds. cost £6437 13s. 9d., find to within a penny the cost per acre.

$$\begin{aligned}\text{Cost per acre} &= \text{£} \frac{6437.687}{74.2015} \\ &= \text{£} 86.2015 \\ &= \text{£} 86 \text{ } 4\text{s.}\end{aligned}$$

Answer is approximately £90, and hence is of the form 90. ---; i.e., we have 6 digits in the quotient.

$$\begin{array}{r} 110 \overline{) 880 \text{ sq. yds.}} \\ 11 \overline{) 8} \\ 4 \overline{) 2.72727 \text{ rds.}} \\ \hline .68181 \text{ acre} \\ 7'4'6'8'1'8'1 \overline{) 6437687(862015} \\ \underline{463142} \\ 15051 \\ \underline{115} \\ 40 \end{array}$$

EXERCISES IXc

(N.B. Remember in each case to make an approximate forecast of the required result.)

1. If 13 tons 10 cwt. cost £59 1s. 3d., calculate the cost per ton.

2. If 12 tons 5 cwt. cost £19 5s. 10½d., calculate the cost per ton.

3. Find the cost per mile if 7 miles 6 fur. cost £20 6s. 10½d.

4. Calculate the cost per ton if 28 tons 1 cwt. 2 qrs. cost £77 4s. 1d.

5. If 317 miles 1 fur. cost £2497 7s. 2d., calculate the cost per mile.

6. If the feu duty on 47 acres 1 rd. amounts to £120 9s. 9d., what is the duty per acre?

7. Find the cost per cwt. if 11 cwt. 1 qr. 7 lb. cost £14 14s. 1d.

8. Find the price per pound (Troy) if 11 lb. 5 oz. 10 dwt. cost £47 18s. 10d.

9. Find the cost per stone if 1 qr. 12 lb. cost £1 19s. 6d.

10. Find the price per thousand cubic feet if 71,500 cu. ft. cost £13 11s. 2d.

11. If 16 tons 11 cwt. 3 qrs. 1 st. cost £144 15s. 7d., find the cost per ton.

12. Find the cost per decagramme if 37.265 kg. cost 49,387.50 francs.

13. Calculate the price per yard if 38 yds. 2 ft. 9 in. cost £3 16s. 4d.

14. Find the cost per gallon if 17 gal. 1 qt. 1 pt. cost £14 15s. 8d.

15. Find the cost per square pole if 4 acres 3 rds. 25 sq. poles cost £49 1s. 3d.

16. Find the cost per square yard if 31 sq. yds. 5 sq. ft. cost £4 16s. 2d.

17. If $462\frac{3}{4}$ yds. cost £358 12s. 7d., find the cost per yard.

18. Calculate the charge per furlong if 3 miles 320 yds. cost £27 13s. 6d.

COST PROBLEMS

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19. Find the cost per mile if 69 miles 4 fur. 100 yds. cost £402 5s. 5d.
20. Find the cost per mile if 3 miles 6 fur. 120 yds. cost £17 13s. 6d.
21. If 273 tons 11 cwt. 3 qrs. cost £1253 18s. 10d., find the cost per ton.
22. If 817,396 cu. ft. cost £279 5s. 6d., find the cost per 1000 cu. ft.
23. Find the price per hundredweight if 1 ton 16 cwt. 42 lb. cost £13 4s. 9d.
24. Find the cost per acre if 3 acres 1 rd. 450 sq. yds. cost £19 11s. 7d.
25. Calculate the cost per hour of running a machine if it takes £2 14s. 3d. to run it for 17 hrs. 32 min.
26. Find the cost per ton if 6 tons 7 cwt. 2 qrs. cost £20.
27. Find the cost per ton if 25 tons 15 cwt. 5 st. cost £57 11s. 7d.
28. Find the cost per ton if 3 tons 15 cwt. 2 qrs. 15 lb. cost £8 5s. 1d.
29. Find the cost per mile if 5 miles 3 fur. 80 yds. cost £13 2s. 11d.
30. Find the cost per ton if 15 tons 3 cwt. 2 qrs. 17 lb. cost £58 12s. 10d.
31. Find the cost per chain if 7 ch. 15 yds. cost £9 15s. 10d.
32. Find the cost per mile if 14 miles 7 fur. 24 yds. cost £34 1s. 2d.
33. Find the cost per 1000 cu. ft. if 63,500 cu. ft. cost £23 5s. 8d.
34. Find the cost per acre if 13 acres 3 rds. 19 sq. poles cost £38 12s.
35. Find the cost per acre if 3 acres 1 rd. 700 sq. yds. cost £31 8s.
36. Find the cost per acre if 15 acres 3 rds. 550 sq. yds. cost £216 10s. 9d.

CHAPTER X

THE EQUATION—CHANGING THE SUBJECT OF A FORMULA

THE EQUATION

It is now generally recognized that the simple form of the equation presents little difficulty to the young pupil, but, on the contrary, is readily accepted by him as a natural 'short-hand' way of writing an arithmetical relation which the mind has already grasped.

It is useful to introduce the form at this stage. If we say that 6 times a certain number equals 132 we have an equation. It saves time in writing (and in subsequent reading) to abbreviate this into

$$6n = 132.$$

Note. The student must never forget that $6n$ means 6 times n —i.e., 6 times the number—whereas the notational form 64 means 6 tens + 4, and not 6 times 4.

Now

$$\begin{aligned} 6n &= 132. \\ n &= 22. \end{aligned}$$

I.e., the required number is 22.

MODEL EXAMPLES

(a) Suppose we know that a man receives a uniform wage weekly, and that at the end of 13 weeks he has received altogether £71 10s. What is his weekly wage?

We may write

$$13 \text{ times the weekly wage} = \text{£}71 \text{ } 10\text{s.}$$

or, briefly,

$$13w = 71\frac{1}{2},$$

understanding that the money is in *pounds*.

$$\begin{aligned} w &= \frac{71\frac{1}{2}}{13} \\ &= 5\frac{1}{2}. \end{aligned}$$

I.e., the weekly wage is £5 10s.

After some practice the student will easily recognize that the equation form may be compared to a pair of scales, in which, if the balance is to be preserved, we must treat both sides (or scale-pans) alike. If we add to the one side we must add an equal amount to the other. If we divide one side by any number—say, 13—we must divide the other side by 13 if we are to be allowed to continue to use the sign of equality.

(b) Given $\frac{n}{2} = 37$, it is required to find the value of n .

Multiplying both sides by 2, we get

$$n = 74.$$

(c) Given $n - 3 = 10$, find the value of n .

Adding 3 to each side, we have

$$\begin{aligned} n &= 10 + 3 \\ &= 13. \end{aligned}$$

(d) Given $7n - 3 = 25$, find n .

Add 3 to both sides. Then

$$\begin{aligned} 7n &= 28. \\ n &= 4. \end{aligned}$$

(e) Given $\frac{6n + 2}{7} = 8$, find n .

Multiply both sides by 7.

Then $6n + 2 = 56$.

Take 2 away from both sides.

Then $6n = 54$.
 $n = 9$.

(f) A boy chooses a number, doubles it, then adds 17. His result is 81. What number did he choose?

Begin with a number.

Double it. This gives 2 times the number, or, using 'short-hand,' we have $2n$.

Add 17.

We have now

$$2n + 17 = 81. \quad (\text{This resembles a pair of scales exactly balanced.})$$

$$2n = 64. \quad (\text{We have taken 17 from each scale-pan.})$$

$$\therefore n = 32.$$

(Verify this result.)

(g) A boy spends half of his money, then receives 3s. He now has 11s. What had he at first?

Suppose we call his money m shillings.

$$\begin{aligned} \text{Then} \quad \frac{1}{2} \text{ of } m + 3 &= 11. \\ \frac{1}{2} \text{ of } m &= 8. \\ m &= 16. \end{aligned}$$

I.e., he had 16s. at first.

(Verify this.)

(h) Three-fifths of a coil of rope is cut off. The remaining piece measures 15 ft. How long was the coil originally?

Suppose the original length was l feet.

$$\text{Then} \quad \text{of } l = 15.$$

We shall get l alone on the left-hand side of our equation if we multiply both sides by $\frac{5}{2}$.

$$\begin{aligned} \text{Then} \quad l &= 15 \times \frac{5}{2} \\ &= 37\frac{1}{2}. \end{aligned}$$

I.e., the coil measured originally $37\frac{1}{2}$ ft.

(Verify this.)

(i) Three-hundredths of a sum of money amounts to £10 5s. 6d. What was the original sum?

If we call the sum s pounds we have

$$\begin{aligned}\frac{3}{100} \text{ of } s &= 10.275. \\ s &= 10.275 \times \frac{100}{3} \\ &= \frac{1027.5}{3} \\ &= 342.5.\end{aligned}$$

I.e., the original sum was £342 10s. (Verify this.)

(j) A man spends $\frac{3}{5}$ of his money. Then he gives away £11, and finds he has just £26 left. What had he at first?

Suppose he had m pounds at first.

He spends $\frac{3}{5}$ of m , and consequently he must have of m left.

Then $(\frac{2}{5} \text{ of } m) - 11 = 26.$

Add 11 to both sides.

Then $\frac{2}{5} \text{ of } m = 37.$

Multiply both sides by $\frac{5}{2}$.

Then $m = 37 \times \frac{5}{2}$
 $= 92.5.$

I.e., he had £92 10s. at first. (Verify this.)

(k) In the following equation the numerator of the fraction has been omitted:

$$\frac{\quad}{100} \text{ of } 625 = 37\frac{1}{2}.$$

Can you supply the missing numerator?

Suppose it was n .

Then $\frac{n}{100} \text{ of } 625 = 37.5.$

Multiply both sides of the equation by $\frac{100}{625}$ in order to get n alone on the left-hand side.

Then $n = 37.5 \times \frac{100}{625}$
 $= \frac{3750}{625}$
 $= 6.$

Thus the missing numerator was 6. (Verify.)

(l) A vat contains 42 gal. of solution, of which 30 gal. is
G

acid and the rest water. It is required to raise the strength so that the acid will constitute $\frac{1}{6}$ of the total mixture. How much acid must be added?

Obviously the 12 gal. of water (which remains constant) is to be $\frac{1}{6}$ of the new total.

Then if the total be raised to $42 + n$ gal. by adding n gal. of acid we have

$$\begin{aligned} 12 \text{ gal. of water} &= \frac{1}{6} (42 + n). \\ 72 \text{ gal.} &= 42 + n. \\ \therefore n &= 30. \end{aligned}$$

I.e., 30 gal. acid must be added.

(Verify.)

Note. The student may suggest a very short method here.

EXERCISES X_A

(*Note.* Verify your answer in every case.)

1. If you double a certain number and then add 23 the result is 97. What is the number?

2. If you multiply a certain number by 7 and then add 61 the result is 621. Find the number.

3. I cut a piece of wire into halves and use one half. From the other half I cut off 10 in., and find that the remaining part measures 43 in. What length of wire had I at first?

4. Four-sevenths of a sum of money is £11 16s. What is the sum?

5. Three-hundredths of a sum is £28 13s. What is the sum?

6. A man puts a sum of money into the bank. The interest amounts to £5 5s. If this equals $\frac{7}{100}$ of the sum he put into the bank, what was the sum?

7. A bank promises to pay as interest four-hundredths of any sum deposited for a year. One depositor receives £22 10s. as interest. What sum did he lodge in the bank?

8. From five-eighths of a certain number 37 is subtracted. The remainder is 138. What was the original number?

9. A number is multiplied by 7, and the result is divided

by 8. Then 43 is added. If the final result is 393, what was the original number?

10. Two-thirds of a certain road was repaired last month. Five miles more was repaired this month. This leaves still $8\frac{1}{2}$ miles to do. What is the length of the road?

11. A man finds that it takes $\frac{5}{9}$ of his income every year to pay for rent and food. Clothes cost him £40 a year. This leaves him with £70 for other purposes. What is his yearly income?

12. Supply the missing numerator in the following equation:

$$\frac{?}{100} \text{ of } 27.5 = 2.2.$$

13. Supply the missing number in the following:

$$\frac{5}{100} \text{ of } ? = 37.$$

14. Supply the missing sum of money in the following:

$$\frac{4\frac{1}{2}}{100} \text{ of } ? = \text{£}47 \text{ 10s. } 3d.$$

15. Supply the missing numerator in

$$\frac{?}{200} \text{ of } 64.4 = 8.05.$$

16. If £10 be subtracted from $\frac{3}{7}$ of a certain sum, the remainder is £47 10s. Find the original sum.

17. $\frac{3}{4}$ of $\frac{5}{6}$ of a sum = £27 10s. Find the sum.

18. £2 13s. 4d. = $\frac{8}{21}$ of what sum?

19. £3 14s. 9d. = $\frac{299}{400}$ of what sum?

20. $\frac{3}{8}$ of a certain number exceeds $\frac{2}{9}$ of the same number by 33. Find the number.

21. Here is an amusement sometimes given at a children's tea-party. A boy is told to "think of a number, double it, add 12 to it (or any number the speaker cares to choose), halve the result." If now he tells the final answer, can you tell the number he first thought of? If his final answer was 23, what number did he think of at first?

22. A lady spends $\frac{3}{5}$ of her money in one shop, and $\frac{1}{2}$ of the remainder in another shop. She then finds that she has 4s. 10d. left. What sum had she at first?

23. A boy chooses a number, trebles it, takes away 8 from

the result, and has 28 left. What number did he choose? Can you see from this how to find the required number in any case like this?

24. $\frac{4}{9}$ of a sum equals 14s. 2d. What is the sum?

25. If a certain weight is multiplied by 7 and the result divided by 40 the final result is 3 qrs. 21 lb. Find the original weight.

26. $\frac{9}{10}$ of a certain sum amounts to 11s. 3d. What is the sum?

27. If $\frac{3}{5}$ of a certain sum is added to the sum itself the result is £42 1s. 4d. Find the sum.

28. The difference between two numbers is 15; their sum is 135. Find the two numbers.

29. A boy has a sum of money. He spends of it, earns an additional 4s. 3d., and then has altogether £2 7s. 7d. What had he at first?

30. A man plants a quarter of his garden with roses, a sixth part with vegetables, and has still 145 sq. yds. unplanted. How many square yards are in his garden altogether?

31. If $\frac{4}{100}$ of a certain number is multiplied by $\frac{80}{85}$ the result is 3.84. Find the number.

32. A certain sum is increased by $\frac{2}{5}$ of itself. The sum thus increased is multiplied by $\frac{7\frac{1}{2}}{100}$, and the result is £6 6s. Find the original sum.

33. A number is squared, and the resulting number is multiplied by $\frac{22}{7}$. If the final result is 154, find the original number.

34. If a certain number is diminished by 13 and the result multiplied by $\frac{22}{7}$ the product is 242. Calculate the original number.

35. From a certain length 3 fur. 4 ch. is subtracted, and the remainder is divided by 8. The result is $12\frac{1}{2}$ ch. Find the original length.

36. Two consecutive numbers are added, and the sum is multiplied by $\frac{6}{5}$. If the result is 288, find the numbers.

37. Three consecutive numbers are added, and the sum is reduced by 100. If the result is 503, find the numbers.

38. Of two consecutive numbers 3 times the smaller plus

5 times the greater amounts to 365. Calculate the two numbers.

39. £120 is multiplied by the fraction $\frac{r}{100}$, and the result is then multiplied by $\frac{3}{5}$. If the final result amounts to £18, find the value of r .

40. $\frac{4}{5}$ of a sum of money is taken and multiplied by $\frac{123}{100}$. To the resulting sum £8 is then added. If the final result amounts to £377, calculate the original sum.

41. A boy receives a sum of money as a prize. He puts it into the bank, and keeps it there until the interest amounts to a quarter of the original sum. He then has altogether in the bank £90 12s. 6d. What was the value of the prize?

42. A rectangular field is 40 yds. longer than it is broad. A boy motors round it 11 times, and finds that the total distance is 3 miles. Find the length and breadth of the field.

43. Supply the missing numerator in the following:

$$\left(\frac{?}{63} \times \frac{2}{51}\right) + 1\frac{85}{189} = 2.$$

44. A bank pays as interest three-hundredths of any sum invested for a year. What sum must I invest in order to have altogether £68 13s. 4d.?

45. The breadth of a rectangular field is 48 yds. less than the length. To travel $5\frac{1}{2}$ times round the field is to travel 3 miles. Find the length and breadth of the field.

46. If $\frac{7}{100}$ of a certain sum is multiplied by $\frac{34}{965}$ the result is £119. Calculate the original sum.

47. If a certain number is subtracted from 1827 the remainder is equal to 6 times the original number. Find the number.

48. Calculate the missing denominator in the following equation:

$$\frac{51}{?} \times 6 = \frac{2}{9} \text{ of } 17.$$

49. If three particular consecutive numbers are added and the sum divided by 15 the result is 7. Calculate the numbers.

50. If $\frac{3}{5}$ of a certain number is added to $17\frac{3}{10}$ and the result multiplied by 30 the result is 3021. Find the original number.

CHANGING THE SUBJECT OF A FORMULA

In arithmetic a *formula* is really an equation which expresses some general rule. For example, we have already seen that the *area* of a rectangle is found by multiplying the number of units in the length by the number in the breadth. This rule is expressed in the equation

$$A = l \times b,$$

and so we speak of having found a *formula* for the area.

In the above equation A is the *subject*.

It is easy to change the subject.

Thus, $l \times b = A$.

Dividing both sides by b , we obtain $l = \frac{A}{b}$, where l is the subject.

Again, from $l \times b = A$, dividing by l , we get $b = \frac{A}{l}$, where b is the subject.

Thus, by memorizing the formula $A = l \times b$, we are able at any time to find the area, or the length, or the breadth, as required.

The student will readily see that he can change the *subject* in similar equations, even although he may not know what the letters represent. *E.g.*,

Given the equation $v = V + gt$, where v is the subject, change the subject (i) to V , (ii) to t .

(i) We have $V + gt = v$.

Subtracting gt from both sides, then

$$V = v - gt.$$

(ii) We have $V + gt = v$.

Subtracting V from both sides, then

$$gt = v - V.$$

Dividing along by g , then

$$t = \frac{v - V}{g}.$$

CHANGING THE SUBJECT OF A FORMULA 103

The following exercises contain easy examples which the student may work at this stage. As he proceeds farther on in his study he will meet with important formulæ in which it is extremely useful to be able to change the subject.

EXERCISES X_B

1. Rewrite each of the following equations, making x the subject:

$$(a) \ y = 3x.$$

$$(b) \ y = \frac{x}{4}.$$

$$(c) \ y = x + 2.$$

$$(d) \ y = 3x + 2.$$

$$(e) \ y = \frac{x}{2} + 5.$$

$$(f) \ y = x - 3.$$

$$(g) \ y = 4x - 1.$$

2. The circumference of a circle can always be calculated from the following equation if we know the length of the diameter:

$$c = \pi d.$$

(The symbol π is a Greek letter used to denote a particular number.)

Find the formula for the diameter—*i.e.*, rewrite the equation, making d the subject.

3. The formula for the circumference of a circle may be stated in the form

$$c = 2\pi r.$$

(r denotes the radius of the circle.)

Rewrite the equation, making r the subject.

4. If p denotes the perimeter of a rectangle, l the length, and b the breadth, we have the equation $p = 2l + 2b$. Examine and verify this.

Rewrite the equation, making l the subject; and again rewrite the equation, making b the subject.

5. If the length, breadth, and depth of a rectangular box be denoted by l , b , and d respectively, then the volume of the box—*i.e.*, the space taken up by the box—is found from the following equation:

$$v = l \times b \times d,$$

or, more briefly,

$$v = lbd.$$

Rewrite the equation, making the subject (i) l , (ii) b , (iii) d .

Practical applications of this principle will be found in the chapters on Percentages (Model Examples (g) , (h) , (i) , (j) , (k)); Interest (Model Example (f) and the preceding note on the formula); Profit and Loss (Model Examples (a) , (c) , (e) , (f) , (g)); Bills of Exchange and Discount (Model Example (c)); Stocks (Model Example (c)); Logarithms (Model Example (h) and the notes immediately preceding and following).

CHAPTER XI

PROBLEMS INVOLVING THE PRINCIPLE OF PROPORTIONALITY

MODEL EXAMPLES

(a) If 6 books cost 10s. 6d., find the cost of 5.

(i) **Unitary Method.** Price of 6 books = 10s. 6d.

$$1 \text{ book} = \frac{10s. 6d.}{6}.$$

$$\begin{aligned} 5 \text{ books} &= \frac{10s. 6d.}{6} \times 5 \\ &= 8s. 9d. \end{aligned}$$

This method is of practical advantage in only a limited number of problems, but in most cases it is tedious, and the principle is frequently awkward to express.

(ii) **Fractional Method** (also known as the *ratio method*).

This is undoubtedly the shortest and the most powerful method, and is applicable in all cases.

The student should state the problem in skeleton form thus:

$$\begin{array}{cc} 6 \text{ books} & 10s. 6d. \\ 5 & ? \end{array}$$

He should then make a rough forecast of the result thus:

5 books will cost *less* than 6.

The reasoning is completed thus:

The number of books in *question* is $\frac{5}{6}$ of the number in the given case.

$$\begin{aligned} \text{Hence required cost} &= 10s. 6d. \times \frac{5}{6} \\ &= 8s. 9d. \end{aligned}$$

This fractional method will be employed in the model examples that follow. The pupil should aim at the briefest possible solution.

(b) If 10 men mow a field in 8 days, how long should 20 men take?

$$\begin{array}{cc} 10 \text{ men} & 8 \text{ days} \\ 20 & ? \end{array}$$

20 men should require *less* time than 10.

$$\begin{aligned} \text{Hence required time} &= 8 \text{ days} \times \frac{10}{20} \\ &= 4 \text{ days.} \end{aligned}$$

Important Note. The student should realize the fundamental difference between problems (a) and (b).

In (a) the *cost* increases as the *number of books* increases. This is a case of direct proportion.

In (b) the *time* decreases as the *number of men* increases. This is a case of inverse proportion.

The student should learn to use these terms freely; and in each problem attempted he should decide (from his own common sense) whether the case is one of direct or of indirect proportion.

(c) A man's wages for 18 hours amount to 37s. 6d. What should he earn in 30 hours?

$$\begin{array}{cc} 18 \text{ hours} & 37s. 6d. \\ 30 & ? \end{array}$$

The wages for 30 hours should be *more* than those for 18—*i.e.*, this is a case of direct proportion.

$$\begin{aligned} \text{Required sum} &= \begin{array}{c} 12s. 6d. \\ 37s. 6d. \end{array} \times \frac{\begin{array}{c} 5 \\ 30 \\ 18 \\ 3 \\ 1 \end{array}}{18} \\ &= £3 \ 2s. 6d. \end{aligned}$$

$$\begin{aligned} \text{Or} \quad \text{required sum} &= \begin{array}{c} 25 \\ 75 \end{array} \text{ sixpences} \times \frac{\begin{array}{c} 5 \\ 30 \\ 18 \\ 3 \\ 1 \end{array}}{18} \\ &= 125 \text{ sixpences} \\ &= £3 \ 2s. 6d. \end{aligned}$$

(d) A wheel of 24 ft. circumference makes 1728 revolutions in travelling a certain distance. How many revolutions will a wheel of 16 ft. circumference make in travelling the same distance?

$$\begin{array}{cc} 24 \text{ ft.} & 1728 \text{ revs.} \\ 16 & ? \end{array}$$

The smaller wheel should make *more* turns—i.e., this is a case of *inverse* proportion.

$$\begin{aligned} \text{Required number} &= 1728 \times \frac{24}{16} \\ &= 2592. \end{aligned}$$

(e) If a 4-lb. loaf costs one shilling when wheat sells at 45s. per quarter, what should the cost be when wheat sells at 60s. per quarter? (Assume that other factors, such as cost of labour, are constant.)

Note that the weight of the loaf, 4 lb., does not vary, and therefore does not enter into the problem.

$$\begin{array}{cc} 1s. & 45s. \\ ? & 60s. \end{array}$$

With dearer wheat we expect a dearer loaf. This is *direct* proportion.

Hence required cost

$$\begin{aligned} &= 1s. \times \frac{4}{3} \\ &= 1s. 4d. \end{aligned}$$

(f) If a ninepenny loaf weighs $5\frac{1}{2}$ lb. when wheat is 52s. 6d. per quarter, what should it weigh when wheat is 57s. 9d. per quarter, assuming that the other factors are constant?

The price (ninepence) does not vary, and therefore does not enter into the solution.

$$\begin{array}{ll} 5\frac{1}{2} \text{ lb.} & 52s. \ 6d. \\ ? & 57s. \ 9d. \end{array}$$

The required weight should be less, since wheat is dearer. This is *inverse* proportion.

Required weight

$$= 5\frac{1}{2} \text{ lb.} \times \frac{52s. \ 6d.}{57s. \ 9d.}$$

$$= \frac{11}{2} \text{ lb.} \times \frac{\begin{array}{c} 10 \\ 210 \\ 231 \\ 21 \\ 1 \end{array}}$$

$$= 5 \text{ lb.}$$

$\frac{52s. \ 6d.}{57s. \ 9d.}$ is to be simplified into a simple fraction by bringing its terms to the same name—say, three-pences.

(g) A motorist running at an average rate of 25 miles an hour goes from Glasgow to Perth in 2 hrs. 42 min. At what rate should he travel to do the same journey in 3 hrs.?

Notes. (i) The journey does not vary. (ii) The speed will be less, since more time is allowed. Thus we have *inverse* proportion.

$$\begin{array}{ll} 25 \text{ m.p.h.} & 2 \text{ hrs. } 42 \text{ min.} \\ ? & 3 \text{ hrs.} \end{array}$$

$$\text{Required speed} = 25 \text{ miles per hour} \times \frac{2 \text{ hrs. } 42 \text{ min.}}{3 \text{ hrs.}}$$

$$\begin{aligned} &= \frac{5}{25} \times \frac{\begin{array}{c} 18 \\ 162 \\ 180 \\ 36 \\ 4 \end{array}}{\text{miles per hour}} \\ &= \frac{90}{4} \text{ miles per hour} \\ &= 22\frac{1}{2} \text{ miles per hour.} \end{aligned}$$

ORAL EXERCISES

1. If 10 exercise-books cost 1s. 3d., what would 1 cost? What would 14 cost?
2. If a man can mow 4 acres in 6 days, how long will he take to mow 1 acre? How long will he take to mow 10 acres?
3. If the rent of $\frac{1}{2}$ acre is £3 5s., what is the rent per acre? What should be the rent of 10 acres?
4. 10 men can mow a certain field in 6 days. How long should 1 man take to mow the same field? How long should 4 men take?
5. A train running at uniform speed travels 18 miles in 40 minutes. How far will it travel in 20 minutes? How far in 1 hour?
6. If 7 articles cost 10½d., what should 1 similar article cost? How many can I buy for 1s. 6d.?
7. 3 lb. of butter cost 5s. What will 4½ lb. cost?
8. 12 articles cost 18s. 6d. What should 1 cost? What should 5 cost?
9. A train running uniformly at 40 miles per hour takes 5½ hrs. to make a certain journey. How long will it take if the speed is halved? How long if the speed were 1 mile per hour? How long if the speed were 50 miles per hour?
10. If oranges are sold at 1s. 9d. per dozen, what should be the price of 20?

EXERCISES XIA

(*N.B.* The fractional or ratio method is advised in all cases.)

1. If 72 bricks weigh 180 lb., find the weight of 125 bricks.
2. Find the cost of 375 rose-bushes at 17s. 6d. per 100.
3. If it takes 12 days to mow 14 acres, how long should it take to mow 21 acres?
4. If 9 men can mow a field in 6 days, how long should 12 men take?
5. A bar of iron of uniform thickness is 12 ft. 9 in. long and weighs 300 lb. If a part 2 ft. 10 in. is cut off, what will be the weight of this part?

6. A motorist is travelling at 18 miles per hour. How long will he take to travel 39 miles?

7. A workman earns £1 14s. in 16 hours. How many hours must he work to earn £2 2s. 6d. at the same rate?

8. The bank interest on a certain sum for 52 days is £3 8s. What would the interest on the same sum amount to in 130 days?

9. When a train is travelling at 60 miles per hour how many yards does it travel per second?

10. A man can finish a job in 21 days by working 4 hrs. per day. How long will he take if he works 7 hrs. daily?

11. If 6 cwt. 2 qrs. is carried 48 miles by rail for a certain sum, how far should 9 cwt. 3 qrs. be carried for the same sum?

12. If 3 oz. of gold is worth £11.025, what is the value of 12.125 lb. of gold? (Troy weight is used here: 12 oz. = 1 lb.)

13. If 2 tons 3 cwt. 3 qrs. cost £61 5s., what should be the cost of 5 cwt. 1 qr. 16 lb. of the same material?

14. If $\frac{1}{8}$ of a sum of money amounts to £808 os. 8d., what fraction will amount to £3131 2s. 7d.?

15. What is the value of 3 bush. if 35 qrs. 2 bush. of the same substance is worth £66 1s. 10½d.?

16. On a certain map a distance of 1½ in. represents 440 yds. What distance should 3.75 in. represent?

17. If 6.6 g. of gold can be drawn into a wire 198 yds. in length, what length of similar wire could be drawn from 5.2 g.?

18. The front wheel of a cycle has a circumference of 132 in., the rear wheel a circumference of 154 in. If the former makes 7560 revolutions in the course of a journey, how many should the rear wheel make in the same journey?

19. If a packing-case weighing 6 cwt. 3 qrs. is carried by rail for 115 miles for a certain sum, how far should 4 cwt. 2 qrs. be carried for the same sum?

20. A train travelling at 30 miles per hour covers a certain distance in 2 hrs. 20 min. How long should it take to do the same journey if its speed were 42 miles per hour?

THE PRINCIPLE OF PROPORTIONALITY 111

21. A reservoir is filled in 54 hrs. by a pump delivering 40 gal. per minute. What must be the rate of delivery of a pump that fills the reservoir in 48 hrs.?

22. If 25 gal. of water fills 4 cu. ft. of space, how many gallons would fill 3 cu. yds.?

23. If on an average the combined weight of 16 men amounts to 1 ton, calculate the total weight of a bus carrying 22 men inside and 24 men outside, if the bus itself weighs 2 tons 12 cwt.

24. If 18 men can mow a field in 10 days, how many men would be required if 15 days are allowed?

25. If the rent of $22\frac{1}{2}$ acres is £23 12s. 6d., what should be the rent of $12\frac{1}{2}$ acres?

26. What should be the cost of 132 sheets of paper at the rate of 7s. 6d. per ream? (A ream contains 480 sheets.)

27. If I can exchange £1 for 122 francs, what should I pay for 793 francs?

28. If the income tax on £280 amounts to £63, what should be the tax on £100?

29. A train runs a distance of 792 yds. in 54 sec. How far will it travel in 10 min. if its rate remains constant?

30. What weight of gravel will be required to cover an acre of ground at the rate of 10 cwt. for 144 sq. yds.?

31. If I can exchange 153 German marks for £7 10s., what should I receive in exchange for 561 marks?

32. Calculate the cost of 7925 cu. ft. of gas at the rate of 5s. 6d. per 1000 cu. ft.

33. If $\frac{5}{9}$ of the value of an estate amounts to £675, what should be the value of $\frac{7}{15}$ of the estate?

34. Find the cost of painting 1200 sq. ft. at the rate of 8d. per square yard.

35. Calculate to within a penny the cost of 1 qr. 12 lb. at the rate of 84s. per cwt.

36. The cost of electric light when 18 uniform lamps are used for a certain period is £6 1s. $4\frac{1}{2}$ d. What would have been the cost if only 15 lamps had been used?

37. A newspaper printer can turn out his required number of copies in 1 hr. 15 min. when he runs 15 machines. How

long would it have taken had he used only 10 of the machines? (Assume that the machines are uniform.)

38. A library pays 17·85 dollars for 21 copies of a book. What would have been the cost if 35 copies had been ordered?

39. In a speed test it is found that a motor-car takes 1 min. 24 sec. to cover a distance of 3000 yds. Calculate the average speed (*a*) in miles per hour and (*b*) in yards per minute.

40. A train travelling at an average speed of 42 miles per hour takes $47\frac{1}{2}$ min. to perform a certain journey. Calculate the necessary speed if the time is to be reduced by 9 min. 30 sec.

41. Find the cost of 6 gal. 3 qts. of a certain mixture if 2 gal. 2 qts. cost £3 12s. 1d.

42. The total weight of a basket containing 10 loaves is 42 lb. If 3 loaves are taken out the weight of the basket and the remaining loaves is $31\frac{1}{2}$ lb. Find (*a*) the weight of a loaf, (*b*) the weight of the basket, and (*c*) the total weight of the basket and 15 loaves.

43. If the total wages of 15 men and 6 apprentices amount to £12 18s. 6d., calculate the total sum earned in the same time by 20 men and 4 apprentices, assuming that the wage of an apprentice is a fourth of a man's wage, and that the men are all paid uniformly.

44. The railway time-table gives the distances London to Glasgow 440 miles, York to Glasgow 252 miles. The return fare for the first journey is given as 167s. 4d. Calculate what the return fare for the second journey should be if the rates of charge were uniform.

45. If in a Paris shop I buy 27 metres of ribbon for 94·5 francs, how much ribbon should I get for 140 francs?

46. The front wheel of a car is 84 in. in circumference, and the rear wheel 90 in. If the latter makes 3210 turns in a certain journey, how many will the front wheel make?

47. A ship's cargo fills 172 trucks, each truck holding 4 tons 8 cwt. How many trucks would be required if each held 5 tons 19 cwt.?

48. Sugar is sold at 32s. 8d. per cwt. How many pounds can be bought for 3s. $2\frac{1}{2}$ d.?

49. 77 lb. of sugar is bought for 22s. $5\frac{1}{2}d.$ What will be the price of 3 st.?

50. Into a square tank containing water 192 bricks are dropped, causing the level of the water to rise 18 in. How many bricks will be required to raise the level 2 ft. 6 in.?

51. A 4-lb. loaf costs 1s. $1\frac{1}{2}d.$ when wheat sells at 7s. $10\frac{1}{2}d.$ per bushel. What should it cost when wheat is at 5s. $10d.$ per bushel, assuming that other factors are constant?

52. A bankrupt owes to one creditor £168 and to another £210. If he can pay to the first only £18 13s. $4d.$, what should he pay to the other?

53. A bankrupt owes to one creditor £138 12s. and to another £51 6s. $8d.$ If at the settlement the first receives £101 5s., what should the second receive?

54. A rectangular field is 350 yds. long and 200 yds. broad. Another is 430 yds. long and 340 yds. broad. If it costs £20 2s. $6d.$ to put a fence round the first, what should it cost to put a similar fence round the second?

55. A shilling loaf weighs $3\frac{3}{4}$ lb. when wheat is at 78s. per quarter. What should it weigh when wheat falls to 65s.?

56. A boy wishes to know the height of a tower. He fixes a walking-stick vertically with its end in the ground. The height of the stick is 3 ft. 2 in. above the ground. The shadow of the tower on the ground is found to measure 141 ft., and at the same time the shadow of the stick is 2 ft. 10 in. Can you find the height of the tower?

57. The scale of a map is 6 in. to 1 mile. Find to within a chain the actual distance between two towns which are 17.39 in. apart on the map.

58. Find the income tax on an income of £475 at the rate of 3s. $9d.$ in the pound.

59. A bankrupt's total debts amount to £794 11s. and his assets to £322 14s. What should a creditor receive whose claim is £49 15s.?

60. If 3 cwt. 1 qr. 27 lb. is carried 48 miles 7 fur. for £3 10s., how far should 2 tons 9 cwt. 1 qr. 11 lb. be carried for the same money? Give result to a hundredth of a mile.

61. If the rent of 21 acres 115 sq. yds. be £45 18s. 9d., what should be the rent of 11 acres 2 rds.?

62. When the rate of exchange with the United States is 4·81 dollars to £1, what is the value of a dollar? (Give the result to a tenth of a penny.)

63. If 2 acres 1 rd. 300 sq. yds. is let at a rental of £19 10s., what should be the rental of 15 acres 2 rds. 725 sq. yds. at the same rate?

64. If 27 men mow 65 acres in 5 days, how long should 18 men take to do the same work?

65. A motorist finds that when travelling at 24 miles per hour he can travel in 4 hrs. 15 min. the same distance a train does in 2 hrs. 33 min. Calculate the speed of the train.

66. A wheel 12 ft. 9 in. in circumference makes 432 revolutions in rolling a certain distance. How many revolutions should another wheel, 11 ft. 4 in. in circumference, make in the same distance?

67. The interest on £475 for a certain time amounts to £17. What should be the interest on £850 for the same time? (Give the result to within a penny.)

68. A squad of men can finish a piece of work in 66 days working 8 hrs. per day. How much longer per day must they work to do the work in 60 days?

69. It takes 8 machines to produce 20,000 bricks in a day. How many extra machines must be installed to raise the output to 32,500 bricks per day?

70. A house is let at the rent of £58 10s. per annum. If the householder resolves to set aside weekly the right proportion of the rent, how much will he have set aside at the end of 20 weeks? (Take a year as (a) 52 weeks, (b) 365 days.)

71. A supply of coal can be conveyed in a train of 35 wagons, each holding 4 tons 15 cwt. How many wagons would be required if each carried 5 tons 14 cwt.?

72. If it costs £10 10s. to feed 45 men for 4 days, what would it cost to feed 32 men for the same time?

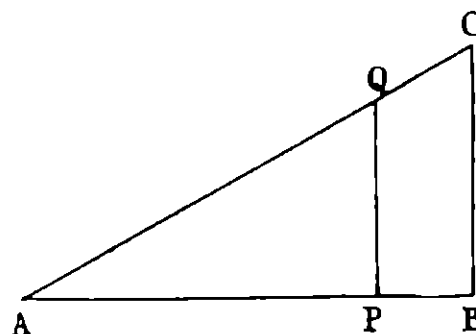
73. The rental of a field of 42 acres 2 rds. is £93 10s. What reduction should be made on the rental if the tenant gives up a field of $8\frac{1}{2}$ acres?

PROBLEMS INVOLVING PROPORTION IN GEOMETRY
AND DRAWING TO SCALE

MODEL EXAMPLES

(a) In the triangle ABC $AB = 3.2$ in. and $AC = 3.6$ in. If PQ is parallel to BC, and $AP = 2.8$ in., find the length of AQ.

Note that because PQ is parallel to BC the figure APQ is exactly the same *shape* as the figure ABC. Hence AP is the same fraction of AB as AQ is of AC, and as PQ is of BC.



Or we may state it thus:

$$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}.$$

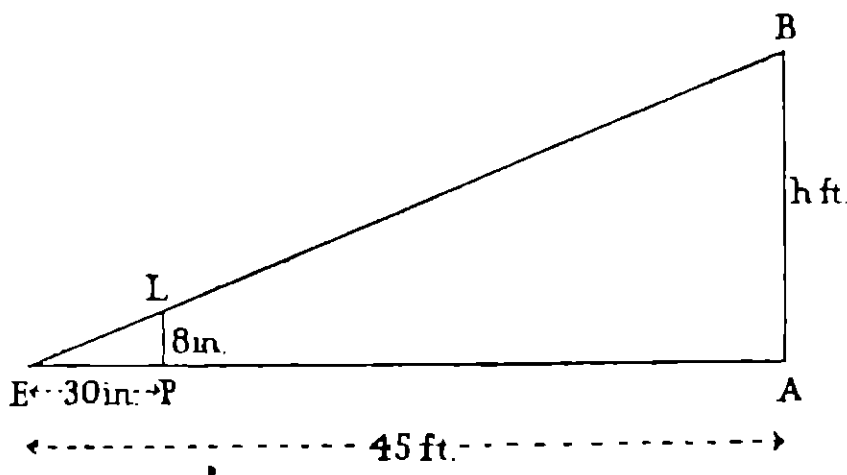
Now, if AQ is n in.,

$$\begin{array}{l|l} \frac{n}{3.6} = \frac{2.8}{3.2} & \frac{28}{32} \times 3.6 \\ & = 3.15. \end{array}$$

I.e.,

$$AQ = 3.15 \text{ in.}$$

(b) A boy, lying on the ground at a distance of 45 ft. from a vertical flag-pole, finds that his pencil, when held parallel to



the pole, and distant 30 in. from the eye, exactly covers the pole. If the pencil is 8 in. long, find the height of the pole.

E denotes the position of the eye.

If the required height be h ft., it is easily seen that the fraction

$$\frac{h \text{ ft.}}{8 \text{ in.}} = \frac{45 \text{ ft.}}{30 \text{ in.}}$$

$$\frac{h \times 12}{8} = \frac{45 \times 12}{30}.$$

Whence

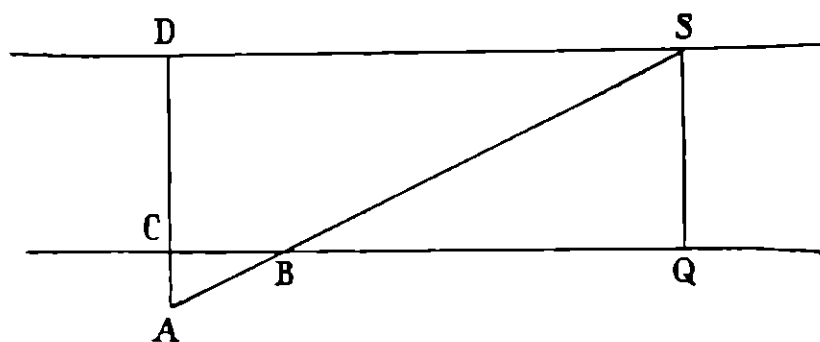
$$h = \frac{45 \times 12}{30} \times \frac{8}{12}$$

$$= 12.$$

I.e., height is 12 ft.

Note. At a later stage the pupil may see why multiplication by 12 in the above solution might be dispensed with.

(c) To find the breadth of a river by the use of a set-square (or, better still, by using a large right-angled triangle cut in wood or cardboard).



A stake is driven into the near bank at some point, Q, directly opposite to, say, a tree or post, S, on the other side. Walk along the

bank from Q until a point C is reached, such that if the set-square ACB is laid down in the manner shown in the diagram CB lies parallel to the water-edge and AB *points exactly to* S.

Measure the distance CQ, and also the sides of the set-square. Suppose $CQ = 30$ ft., $AC = 9$ in., and $CB = 12$ in.

Suppose $AD = n$ feet. Figures ACB and ADS are similar.

Then, since fraction $\frac{AD}{AC} =$ fraction $\frac{DS}{CB}$, we have $\frac{n \text{ feet}}{9 \text{ in.}} = \frac{30 \text{ ft.}}{12 \text{ in.}}$. Whence we easily find AD, and then CD.

ORAL EXERCISES

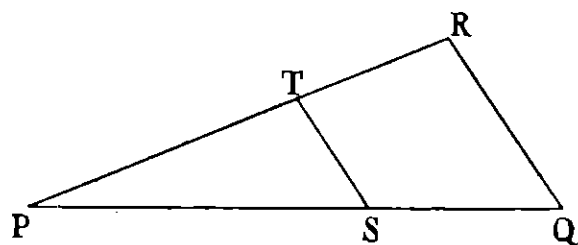
In the triangle PQR $PQ = 15$ units, $PR = 12$ units, and $QR = 7$ units. A point S is chosen in PQ, and ST is drawn parallel to QR.

1. If PS is 10 units, what fraction is PS of PQ? What fraction must PT be of PR? Calculate the length of PT.

What fraction must ST be of QR? Calculate length of ST.

2. If S is chosen so that PS is 3 units, find the length of PT and of ST.

3. If PS is 4 units, find the length of PT and of ST.



4. If SQ is 10 units, find TR and ST.

5. If SQ is 6 units, find TR and ST.

6. If SQ is 1 unit, find TR and ST.

7. If PS is $2\frac{1}{2}$ units, find PT, TR, ST.

8. If SQ is $2\frac{1}{2}$ units, find PT, TR, ST.

9. If ST is $3\frac{1}{2}$ units, find PS and PT.

10. If ST is 1 unit, find PS and PT.

EXERCISES XI_B

1. In a triangle ABC, $AB = 7.2$ in., $AC = 8.1$ in., $BC = 9$ in. P is a point in AC, and PQ is drawn parallel to CB, so that point Q is in AB. Calculate the length of AQ in the following cases:

(a) When $AP = 6$ in.

(c) When $AP = 4.8$ in.

(b) When $AP = 5.4$ in.

(d) When $AP = 3.2$ in.

Give your answers to $\frac{1}{100}$ in.

Verify your results by drawing and measurement.

2. Draw a straight line, AB, 4.93 in. long. From A draw at any angle a straight line, AC, $3\frac{1}{2}$ in. long, and mark the half-inches in the latter. Make a construction to find $\frac{1}{7}$ of AB. Check your results.

3. Use the method of the preceding question to enable you to divide any given straight line into (a) 7 equal parts, (b) 5 equal parts.

4. Draw a straight line, AB, 5.4 in. long. Draw two other straight lines, one 2.9 in. and the other 4.3 in. Find a method of dividing AB into two parts such that the one part will be the same fraction of the other as 2.9 is of 4.3. Check your result.

5. It is required to copy the triangle mentioned in Question 1 on a small scale such that the side AB will be represented by $\frac{1}{2}$ in. Calculate the lengths of the other sides to within a hundredth of an inch.

6. A plan of a golf-course is drawn on the scale of 1 in. to 150 yds. Calculate to within a hundredth of an inch the length on the plan of (a) a bunker whose actual length is 54 ft. and (b) the club-house, which measures actually 200 ft. in length.

7. A spherical captive balloon is known to be 70 ft. in diameter. A boy looking on finds that a halfpenny held at 30 in. from the eye exactly covers the balloon. Calculate the distance of the balloon from the boy, assuming that a halfpenny is 1 in. in diameter.

8. Find a method of calculating the height of a flag-pole by setting up a stake in the ground between the eye and the pole. If in the correct position a 6-ft. stake is 18 ft. distant from the eye, and the flag-pole is 162 ft. from the eye, what is the height of the pole? (Assume the eye to be on the level of the ground.)

9. Find a method of calculating the height of a flag-pole by measuring the length of its shadow cast by the sun, and measuring at the same time the shadow cast by a vertical pole which you have set up.

If your pole is 9 ft. high, and casts a shadow of 14 ft., find to a tenth of a foot the height of a tower whose shadow at the same time is 135 ft.

10. Find to one decimal place, by drawing and measurement, a number which is the same fraction of 37 as 9 is of 21. Check your result by calculation.

11. The height of a cone is 18 in., the diameter of the base 15 in. Find the diameter of a section parallel to the base and 10 in. from it.

12. The base of a triangle is 7.5 in., the altitude is 6.3 in. A straight line is drawn parallel to the base and at a distance of 2.8 in. from the base. If this line cuts the other sides of the triangle, calculate the distance between these two points of section.

PROBLEMS INVOLVING COMPOUND PROPORTION

MODEL EXAMPLES

(a) If I pay £4 for 14 railway-tickets for a 36 miles' journey, what should I pay for 24 tickets for a 21 miles' journey?

Tabulate the data thus:

£4	14 tickets	36 miles' journey
?	24	21

Cost of 14 tickets for 36 miles is £4.

24	„	more, £4 × $\frac{24}{14}$.
	21	less, £4 × $\frac{24}{14}$ × $\frac{21}{36}$.

$$\begin{array}{r} \text{Hence required cost} = \text{£4} \times \frac{24}{14} \times \frac{21}{36} \\ \qquad \qquad \qquad \frac{2}{1} \times \frac{3}{1} \\ = \text{£4.} \end{array}$$

(b) If 7 men take 8 days to mow 42 acres, how long should 10 men take to mow 30 acres?

7 men	8 days	42 acres
10 „	?	30

The given time is 8 days.

Considering the men alone, we get the multiplier $\frac{7}{10}$.
 „ acres „ $\frac{30}{42}$.

$$\begin{array}{l} \text{Hence required time} = 8 \text{ days} \times \frac{7}{10} \times \frac{30}{42} \\ = 4 \text{ days.} \end{array}$$

EXERCISES XIc

1. If the wages of 14 men for 24 days amount to £210, what should 18 men earn in 7 days at the same rate?

2. If the carriage of 10 cwt. 2 qrs. 14 lb. for 253 miles amounts to 19s. 10d., how far should 1 cwt. 3 qrs. 14 lb. be carried for 38s. 6d.?

3. If 18 men mow 45 acres in 3 days, how long should 12 men take to mow 60 acres?

4. A rectangular sheet of metal 34 ft. long and 14 ft. broad weighs 16 cwt. 2 qrs. What should be the weight of a sheet of the same thickness 21 ft. long and 17 ft. broad?

5. If 15 men can unload a boat's cargo of 259 tons in 14 hours, how many men (approximately) should be required to unload 111 tons in 8 hrs. at the same rate?

6. If a shilling loaf weighs 3 lb. when wheat sells at 56s. per quarter, what should be the price of a 4-lb. loaf when wheat is at 63s. per quarter?

7. If a shilling loaf weighs $3\frac{1}{2}$ lb. when wheat is 56s. per quarter, what should be the weight of a ninepenny loaf when wheat is 49s. per quarter?

8. If it costs £86.12s. 6d. to stable and feed 21 horses for 33 days, calculate the bill for 55 horses for 28 days at the same rate.

9. A rectangular skating-pond 150 ft. long and 100 ft. broad is flooded in $18\frac{1}{2}$ hrs. by a pump delivering 40 gal. of water per minute. How long should it take to flood to the same depth a pond 180 ft. long and 120 ft. broad, using a pump which delivers 84 gal. per minute?

10. A garrison of 240 men has sufficient bread to last for 6 days if each man is allowed 10 oz. per day. If 60 men be sent away, how long should the supply last if each man is now allowed 12 oz. per day?

11. It takes 36 men 120 days to lay a road 3 miles 3 fur. long and 50 ft. broad. How long should it take 45 men to lay a road 3 miles 6 fur. long and 30 ft. broad at the same rate?

12. A man walking with an average pace of 33 in. travels $4\frac{1}{2}$ miles in 1 hr. 20 min. If he increases his pace to 36 in. how long will he take to do the next mile, assuming that he does not alter his rate of stepping?

13. When petrol costs 1s. 2d. per gal. a motor-car which runs 24 miles to the gallon uses petrol costing £6 2s. 6d. on a certain journey. Another car, which runs 30 miles to the gallon, does the same journey when petrol has fallen to 1s. 1d.

per gal. What should the journey cost the second motorist for petrol?

14. If 14 men working 10 hrs. per day build a wall in 15 days, how many hours a day should 10 men work to finish the same undertaking in $17\frac{1}{2}$ days?

15. If it takes 12 men 30 days to build a wall 180 yds. long and 6 ft. high, how long should 15 men take to build a similar wall 150 yds. long and 8 ft. high?

16. If a shilling loaf weighs 3 lb. when wheat sells at 6s. per quarter, what would you expect the price of wheat to be when a 4-lb. loaf costs 1s. 2d.?

17. If the carriage of $71\frac{1}{2}$ cwt. for 150 miles costs £3 15s., what should it cost to send 45 cwt. for 180 miles?

18. The cross-section of a metal wire is a square of side 0.25 in. That of another wire is a rectangle of sides 0.2 in. and 0.35 in. If 50 ft. of the first wire weigh 12 lb., calculate to within 1 oz. the weight of 60 ft. of the second wire.

19. A square skating-pond of side 150 ft. can be flooded to a depth of 9 in. in $4\frac{1}{2}$ hrs. To what depth could a similar pond of side 200 ft. be flooded in 10 hrs. at the same rate of inflow of water?

20. If the carriage of 11 cwt. 1 qr. for 42 miles costs £1 16s. 4d., how far should 9 cwt. 1 qr. 14 lb. be carried for £3 15s.?

21. A square foot of metal plate $\frac{3}{4}$ in. in thickness weighs 30 lb. It is required to employ a plate of the same material 405 sq. in. in area and weighing 35 lb. Calculate the necessary thickness to within $\frac{1}{1000}$ in.

22. Metal plates $9\frac{1}{3}$ sq. ft. in area and $\frac{1}{8}$ in. thick are quoted as weighing 430 to the ton. What should be the number of plates to the ton if the area is 11 sq. ft. and the thickness $\frac{1}{12}$ in.?

23. The material for a brick wall 36 yds. long and 5 ft. high costs £11 5s. What height of wall 50 yds. long and of the same thickness as the other, can be provided for by an expenditure of £10 10s.?

24. A certain charitable fund is sufficient to support 280 people for 135 days when the rate is 2s. 9d. per person per

day. How long will the fund last if the number of people is reduced to 252 and the rate is increased to 3s. $1\frac{1}{2}d.$?

25. A rectangular sheet of metal 18 in. long and 16 in. broad costs 15s. Find the cost of a sheet of the same metal 12 in. long and 20 in. broad and of equal thickness.

26. Two rectangular reservoirs are alike in length and breadth, but one is three-fourths as deep as the other. If the shallower is filled in 54 hrs. by a pump delivering 360 cu. ft. per minute, what must be the rate of delivery of a pump required to fill the deeper reservoir in 48 hrs.?

27. It costs £9 10s. for coal to run an engine for 84 hrs. when coal costs 28s. per ton. What should it cost to run the engine at the same rate for 60 hrs. when coal costs 24s. 6d. per ton?

28. When petrol costs 1s. 5d. per gal. a certain car which runs 25 miles to the gallon uses petrol costing 8s. 6d. on a particular journey. If petrol rises to 1s. 6d. per gallon calculate the petrol bill for the same journey made by another car, which runs 27 miles to the gallon.

29. A great inventor calculated that the work done by the average horse in 1 minute is sufficient to raise a weight of 330 lb. to a height of 100 ft. How many horses would be required to raise in 1 minute 1000 lb. to a height of 825 ft.?

30. If it costs £21 13s. 4d. to supply 35 workmen with bread for 21 days, allowing each man 2 lb. per day, how long should 28 men be supplied for the sum of £26 if the ration is reduced to $1\frac{3}{4}$ lb. per man per day?

31. If the hotel expenses of 7 men for 4 days amount to £42, for how many days should 8 men be accommodated for £50 at the same rate?

32. If 2 men take 18 hrs. to paint a surface of 504 sq. yds., how long should 5 men take to paint 173 sq. yds., assuming that the men work at a uniform rate?

33. If £660 in the bank earns 5s. 8d. interest in 26 days, in what time will £390 earn 14s. 2d. interest at the same rate?

34. If the carriage of 1 cwt. $4\frac{1}{2}$ st. for 45 miles costs 5s. 9d., how far should 4 cwt. 1 qr. be carried for 10s. 6d. at the same rate?

THE PRINCIPLE OF PROPORTIONALITY 123

35. If £14 6s. earns 2s. 3d. interest in 65 days, what sum will earn 15s. 9d. interest in 169 days at the same rate?

36. A metal plate measuring 243 sq. in. and $\frac{3}{8}$ in. thick weighs 24 lb. What must be the thickness of a plate of the same metal measuring 297 sq. in. and weighing 33 lb.?

37. If it costs £201 10s. to feed 120 soldiers for 13 days, how many could be fed for 27 days for £279?

38. A wall 3380 ft. long is to be built in 30 days, and it is found that by employing 7 workmen only 980 ft. has been built at the end of 14 days. How many additional men must be employed if the wall is to be finished within the specified time?

39. The wages of 45 men and 12 boys for 15 days amount to £240. Calculate the total wages of 23 men and 8 boys for 17 days, assuming that a boy's wage is $\frac{3}{8}$ of a man's.

40. The weight of 270 rectangular metal plates 12 in. long, 7 in. broad, and $\frac{3}{8}$ in. thick is 1800 lb. Calculate to within a pound the weight of 150 rectangular plates of the same metal, each 10 in. long, 6 in. broad, and $\frac{2}{3}$ in. thick.

PROPORTIONAL DIVISION

MODEL EXAMPLES

(a) £2 9s. 6d. is paid for a certain piece of work, on which A is engaged for 14 hrs. and B for 19 hrs. If A and B are paid at the same rate, how much should each receive?

A's time is 14 hrs. out of the total 33 hrs.

I.e., he works $\frac{14}{33}$ of the total time.

Hence he receives $\frac{14}{33}$ of the total sum.

Then
$$\begin{aligned} \text{A's share} &= \frac{14}{33} \text{ of } £2 \text{ 9s. 6d.} \\ &= \frac{14}{33} \text{ of } \frac{99}{2} \text{s.} \\ &= 21\text{s.} \end{aligned}$$

$$\begin{aligned} \text{B's share} &= \frac{19}{33} \text{ of } \frac{99}{2} \text{s.} \\ &= 28\text{s. 6d.} \end{aligned}$$

(b) A's capital in a business is £3200, B's £4000, C's £1200.

What share should each receive out of a year's profit amounting to £546?

Total capital = £8400.

A holds $\frac{3200}{8400}$ —i.e., $\frac{8}{21}$ of the total.

A receives $\frac{8}{21}$ of £546—i.e., £208.

B receives $\frac{10}{21}$ of £546—i.e., £260.

C receives $\frac{3}{21}$ of £546—i.e., £78.

$$\begin{array}{r} 26 \\ 8 \quad 78 \\ 21 \text{ of } 546 \\ 3 \\ 1 \end{array}$$

(c) A and B rent a grazing-field for £24. A puts in 42 cattle for 20 days, B 36 cattle for 30 days. What proportion of rent should each pay?

Reckon 20 days' grazing for 42 cattle as 42×20 days' grazing for 1 cow.

Then A should pay for 840 days' grazing
and B 1080

Hence A pays $\frac{840}{1920}$ of £24—i.e., £10 10s.

B pays $\frac{1080}{1920}$ of £24—i.e., £13 10s.

$$\frac{84}{192} \text{ of } 24 = 10.5$$

(d) The same problem as in Example (c) above, except that the rental is £31 14s. 8d.

A pays $\frac{840}{1920}$ of £31.733.

I.e., he pays £13 17s. 8d.

B pays £17 17s.

$$\frac{84}{192} \text{ of } 31.733$$

$$\begin{array}{r} 31.733 \\ 7 \\ 4 \overline{) 222.131} \\ 4 \overline{) 55.533} \\ 13.883 \end{array}$$

$$\frac{9}{16} \text{ of } 31.733$$

$$\begin{array}{r} 31.733 \\ 9 \\ 4 \overline{) 285.597} \\ 4 \overline{) 71.399} \\ 17.849 + \end{array}$$

EXERCISES XI_D

1. Divide 15*s.* between A and B in the ratio of 5 to 4.
2. Divide £4 18*s.* 10½*d.* among A, B, and C in the ratio of 5, 7, 9.
3. Two workmen are paid at the same rate per hour. One works for 15 hrs., and the other for 21 hrs. at the same job. If the total payment is £3 3*s.*, what should each receive?
4. A railway-ticket is issued for 11*s.* 4½*d.* The journey consists of 42 miles on one railway and 36 miles on another. What sum is due to each railway?
5. A opens a business with a capital of £1250. Three months later he admits to partnership B, who brings a capital of £1500. At the end of the year the profits amount to £1092 10*s.* What should each receive?
6. A works at a particular job for 11 hrs., B for 3½ hrs., C for 12 hrs. How much should each receive out of a total payment of £3 12*s.* 10½*d.* if the rate of payment is uniform?
7. Three men rent a pasturage for £28 2*s.* 6*d.* A grazes 18 horses for 20 days, B 15 horses for 16 days, C 20 horses for 15 days. What sum should each pay?
8. In a bankruptcy case after the expenses of administration are paid there is left a sum of £960 to be shared by the five creditors. The debt to A is £630, to B £540, to C £480, to D £420, to E £330. What should each receive?
9. The same problem as in the preceding question. Find what the bankrupt can pay in the pound, and hence obtain the results required in Question 8.
10. An alloy consists of lead, antimony, and bismuth in the proportion of 9, 2, 1. What weight of lead is required to make 1½ cwt. of the alloy?
11. The gas bill of a school is paid partly by the day school and partly by the evening school. If the former burns 36 lights for 90 hrs., and the latter 24 lights for 150 hrs., what proportion of the bill should each pay?
12. In a public park there are three square plots, the sides of which are respectively 12 yds., 15 yds., and 21 yds. The sum of £315 is to be expended on the cultivation of these,

being distributed in proportion to the areas. What sum will be spent in each case?

13. A offers to subscribe half a crown for every florin subscribed by B, and C promises to give as much as A and B together. In order to raise a total subscription of 70 guineas, how much must each contribute?

14. A sovereign consists of gold and copper in the ratio of 22 to 2 by weight, and weighs 123.274 grains. Calculate in Troy measure the weight of gold in 1000 sovereigns.

15. Divide 6 tons 12 cwt. in the ratio $2.8 : 3.7 : 5.5$.

16. A bell is composed of copper, tin, and zinc in the ratio $15 : 3 : 1$. If the total weight is 15 cwt. 1 qr. 2 lb., calculate the weight of each metal used in the bell.

17. A day school and an evening school use the same building. During the session the day school uses all the 160 lights for 70 hrs.; the evening school uses 64 lights for 250 hrs. If the total bill for lighting is £14 3s. 4d., what proportion should be paid (*a*) by the day school and (*b*) by the evening school, assuming the lights to be uniform?

18. The total hotel bill for three men A, B, and C amounts to £21. A has stayed for 6 days, B for 8 days, and C for 10 days. What amount should each pay respectively, assuming a uniform rate?

19. One partner in a business holds £7500 of the capital, and the other holds £6500. If the profit amounts to £2926, and is to be shared in proportion to the capital, calculate the share of each partner.

20. There are three partners in a business. A holds £4800 of the capital, B £3900, and C £2900. If the total profit amounts to £4350, and is to be shared according to the capital, calculate the share of each partner.

21. A opens a business, having put into it a capital of £5500. After 3 months B joins him, bringing a capital of £3600. When the business has been carried on for a year there is £1950 to be divided between the partners in proportion to their investment and the time during which they have been in the business. What should be the share of each?

22. Two men rent a grazing-field, the rental being £46.

A puts in 70 cattle for 5 months, B 85 cattle for 4 months. How much of the rent should each pay?

23. The composition of gunpowder is 33 parts of nitre, 7 of charcoal, and 5 of sulphur. What weight of each of these components is contained in $4\frac{1}{2}$ tons of gunpowder?

24. Divide £72 among A, B, and C so that B may receive twice as much as A, and C half as much as A and B together.

25. Divide £2790 into three parts in the ratio $\frac{3}{4} : \frac{14}{15} : \frac{5}{9}$.

26. A tax of £72,000 is to be raised from three towns, each of which will pay in proportion to its assessed rental. The respective rentals of the towns are £107,418, £335,697, and £516,885. Calculate the sum to be paid by each town.

27. Pure chalk is composed of calcium, carbon, and oxygen in the ratio of 10 : 3 : 12. Calculate the weight of each element in $3\frac{3}{4}$ tons of chalk.

28. Divide £371 18s. 6d. into three parts, having the ratio $3 : 11\frac{3}{4} : 14\frac{1}{4}$.

29. Divide £48 into three parts, having the ratio $\cdot 42 : \cdot 31 : \cdot 23$.

30. A, B, and C do a piece of work jointly. All three begin at 12 noon. A works till 5.20 P.M., B till 4.15 P.M., and C till 6.15 P.M. If the sum of £4 15s. is paid for the whole undertaking, what amount should each receive, assuming a uniform rate?

CHAPTER XII

MENSURATION OF AREA

(Continued from Chapter VIII)

Area

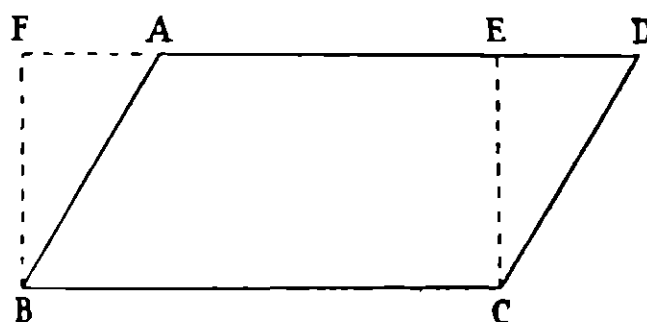
The student is already familiar with the formula for the area of a *rectangle*—viz.,

$$\text{area} = \text{length} \times \text{breadth.}$$

He should now observe that in order to find the area of any other figure we find first the *equivalent rectangle*, and then employ the formula as given above.

(i) **The Parallelogram.** Consider the following figure, ABCD. Cut off the right-angled triangle CED and apply it in the position BFA.

We have now the rectangle FBCE, whose area is found by multiplying the number of units in the *base*, BC, by the



number of units in the *altitude*, BF.

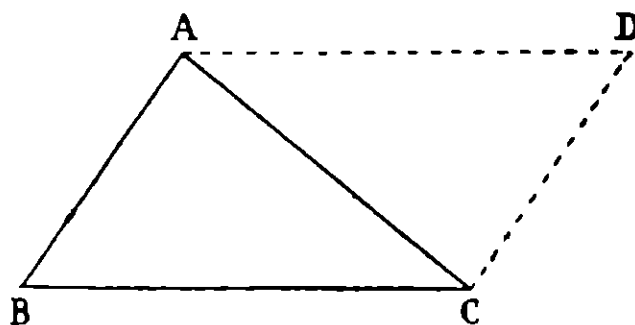
I.e., $\text{area} = \text{base} \times \text{altitude.}$

(*Note.* Beginners must be warned that the altitude is *not* BA.)

(ii) **The Triangle.** The triangle ABC is clearly half of the parallelogram ABCD, which has the same base as the triangle and the same altitude. Hence

$$\text{area} = \frac{1}{2} \text{ base} \times \text{altitude.}$$

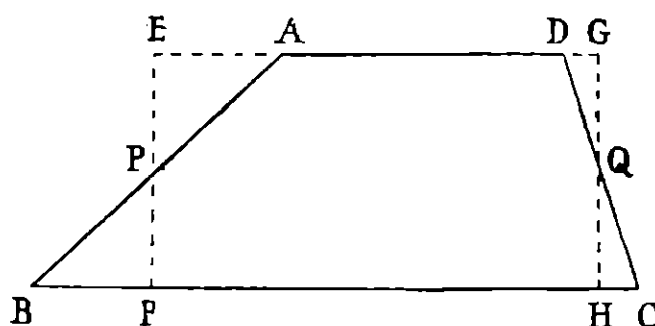
(Note that in the case of the *right-angled* triangle one side is the base and another is the height.)



(iii) **The Trapezium.** This is a quadrilateral with one pair of parallel sides. It deserves special attention because of its use in field measurement.

Let the given figure be ABCD.

Bisect AB at P, and draw EPF at right angles to the parallel sides.



The triangle PBF may be cut off and applied exactly in the position PAE.

Proceeding similarly, the triangle QCH may be cut off and applied in the position QDG.

We thus obtain the equivalent rectangle EFHG.

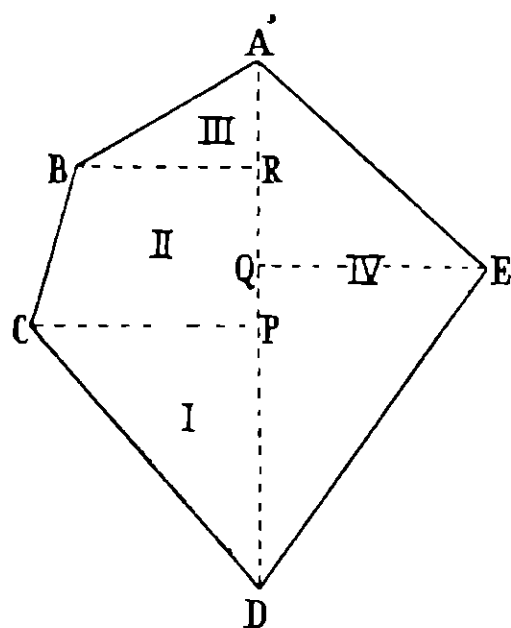
$$\begin{aligned} \text{Now, } FH + EG &= FH + (EA + AD + DG) \\ &= (FH + EA + DG) + AD \\ &= (FH + BF + HC) + AD \\ &= BC + AD. \end{aligned}$$

$$\text{Hence } FH = \frac{1}{2} (BC + AD).$$

Hence the formula for finding the area of a trapezium:

$$\text{area} = \frac{1}{2} \text{ sum of parallel sides } \times \text{ altitude.}$$

This formula, of course, is found at once if we draw a diagonal and take the areas of the two resulting triangles.



(iv) **Irregular Rectilinear Figures.**

The method consists in taking a long diagonal, and then offsets from this to the other vertices, thus dividing the figure into right-angled triangles and trapezia, whose areas are readily calculated.

Let the distances of P, Q, R, and A from the starting-point, D, be respectively 2.7 in., 3.3 in., 4.3 in., and 5.2 in. (Note that 3.3 in. is the length DQ, and not PQ.)

Let PC, QE, RB, be 2.3 in., 2.2 in., and 1.8 in. respectively.

Then a most convenient and lucid form in which to set down all these measurements is the following:

	To A	
	5.2	
1.8	4.3	
	3.3	2.2
2.3	2.7	
	0	
	From D	

This is the form of the field-book. In practice the entries are made from below upward—*i.e.*, from D in the given example.

A sensible way of arranging the results is the following:

AREA	DOUBLE AREAS IN SQUARE INCHES	
I	2.3 × 2.7	6.21
II	4.1 × 1.6	6.56
III	1.8 × .9	1.62
IV	2.2 × 5.2	11.44
		25.83

Then the area of whole figure = 12.91 sq. in.

The foregoing is actually the way in which field measurement is carried out. The long diagonal is measured with the chain; and the offsets with chain, tape-measure, or survey-pole, as may be required.

THE CIRCLE

If the circumference of any circle be measured carefully by using a thread or a fine wire, and the result be divided by the length of the diameter, the resulting quotient is constant, and is approximately 3.1416, or, more roughly, $\frac{22}{7}$. With the utmost care in measuring, the result could hardly be relied on beyond the third or fourth decimal figure.

Students should be required to find the value of this constant ratio for themselves, each student taking three or four

circles of different radii, and calculating the mean result. The work should be tabulated thus :

RADIUS GIVEN		CIRCUMFERENCE BY MEASUREMENT	RATIO $\frac{\text{CIRCUMFERENCE}}{\text{DIAMETER}}$
1			
2			
3			
4			
MEAN VALUE OF RATIOS			

This constant value of the ratio $\frac{\text{circumference}}{\text{diameter}}$ for all circles is usually denoted by the Greek letter π . The numerical value of π can be calculated by mathematics to any degree of accuracy desired.

The value, correct to eight decimal places, is 3.14159265.

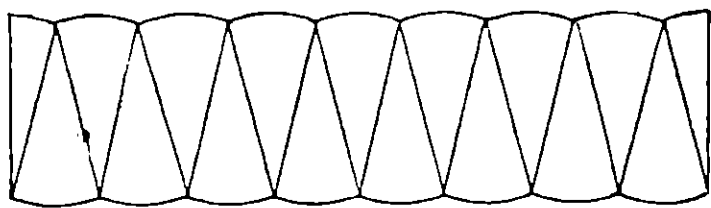
Area of a Circle. The equivalent rectangle can be obtained approximately by the following useful exercise.

Draw on coloured paper two circles of the same radius—say, 2 in.

Cut out one, and paste it on the upper half of a page of an exercise-book.

Divide the other into, say, sixteen (certainly not less than twelve) equal sectors.

On the lower half of the page of the exercise-book draw in pencil two parallel lines 2 in. apart. Between these arrange the sectors (all but one). Bisect the remaining sector, and place the halves one at each end to complete the approximate rectangle as shown below.



Obviously the area of the approximate rectangle is identical with that of the circle.

Now, the length is just half the circumference—*i.e.*, πr —and the breadth is r .

Hence area = $\pi r \times r$
= πr^2 .

MODEL EXAMPLES

(a) Find the area of a trapezium whose parallel sides measure 137 ft. and 68 ft., and whose altitude—*i.e.*, distance between parallel sides—is 87 ft.

Required area
= $\left(\frac{137 + 68}{2} \times 87\right)$ sq. ft.
= 990 sq. yds. $7\frac{1}{2}$ sq. ft.

$$\begin{array}{r} 137 \\ 68 \\ \hline 205 \\ 87 \\ \hline 1640 \\ 1435 \\ \hline 2 \overline{) 17835} \\ 9 \overline{) 8917\frac{1}{2}} \end{array}$$

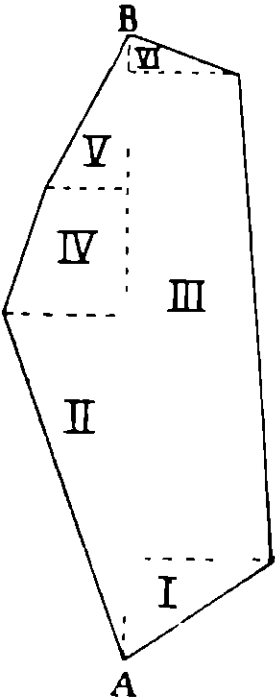
sq. ft.
990 sq. yds. $7\frac{1}{2}$ sq. ft.

(b) From the following entry in a field-book draw a plan of the field in question, and calculate the area :

	To B	
	635	
	585	115
80	475	
120	350	
	100	135
	0	
	From A	

The measurements are in feet.

FIGURE	DOUBLE AREA IN SQ. FT.	
I	100 × 135	13,500
II	350 × 120	42,000
III	485 × 250	121,250
IV	125 × 200	25,000
V	160 × 80	12,800
VI	50 × 115	5,750
		220,300



2

220 300 sq. ft.

9

110 150

11'0

12 23'8 sq. yds. 8 sq. ft.

11

1 11, 28

4

10 rds. 138 sq. yds.

2 acres 2 rds.

Required area = 2 acres 2 rds. 138 sq. yds. 8 sq. ft.

(c) Calculate the circumference of a circle whose diameter is 12 ft. (Take $\pi = \frac{22}{7}$.)

$C = 2\pi r$ or πd

$= 37\frac{1}{7}$ ft., or 37 ft. $8\frac{1}{7}$ in.

$3\frac{1}{7} \times 12$

$= 37\frac{5}{7}$.

(d) Find the area of a circle of which the radius is 4.26 in.

$\text{Area} = \pi r^2$

$= 57.0$ sq. in.

4.26

4.26

17.04

.852

.2556

18.1476

3.1416

54.443

1.815

.726

18

11

57.01

(N.B. The student should not write this result as 57 sq. in., which would imply either (i) that the result was exactly 57, or (ii) that the work had not been carried beyond the units.)

EXERCISES XII

1. Find the area of a rectangular court 24 ft. 6 in. long and 18 ft. 9 in. broad.
2. A rectangular grass plot 32 ft. 8 in. long and 24 ft. broad has a path 6 ft. 6 in. wide round it. Find the area of the path.
3. Find the cost of paper for the walls of a room 16 ft. 6 in. long, 14 ft. broad, and 10 ft. 6 in. high, allowing for a doorway 8 ft. by $4\frac{1}{2}$ ft., a fireplace 7 ft. by 5 ft., and window space 10 ft. by 8 ft. The paper costs 5s. 6d. per piece. (The standard size of a 'piece' of wallpaper is 12 yds. by 21 in.)
4. A rectangular court 80 yds. long and 60 yds. broad is

to be enclosed by wire-netting 10 ft. 6 in. high. Calculate the cost of netting at 1s. 9d. per sq. yd.

5. A circular plot is 15 ft. 6 in. in diameter. Find the area, and then calculate the cost of turfing at 2s. 3d. per sq. yd.

6. In connexion with the preceding exercise, find the cost of surrounding the plot with wire-netting 6 ft. 6 in. high at 1s. 9d. per sq. yd.

7. Round the plot specified in Question 5 a path 6 ft. 6 in. broad is made. Calculate the area of the path.

8. A gardener makes a circular plot 11 ft. in diameter instead of 10 ft., as ordered. How much extra ground has he taken in?

9. Find the cost of material in a metal frame 4 in. broad, surrounding a rectangular parchment 15 in. long, 9 in. broad, at $7\frac{1}{2}d.$ per sq. in.

10. A path 12 ft. broad surrounds a circular enclosure 70 ft. in diameter. Calculate the cost of covering the path with gravel at 15s. per ton, allowing $\frac{1}{2}$ cwt. of gravel per sq. yd.

11. The circumference of a tree-trunk is measured with a string and found to be 11 ft. 7 in. Assuming the section of the trunk to be a circle, calculate its diameter.

12. From the field-book entries given below draw a plan, and calculate the area in each case.

(a) To B		(b) To B	
	630		704
	515	101	601
120	387		451
	147	60	307
	0		0
	From A		From A

The measurements are in feet.

13. What length of carpet 2 ft. 6 in. wide will be required to cover the floor of a room 14 ft. long and 12 ft. 6 in. broad?

14. The area of a district is found from a map (by tracing on squared paper, and then counting squares) to be 43.6 sq. in. If the scale of the map is $2\frac{1}{2}$ in. to 1 mile, calculate the area of the district.

15. If in Question 14 the scale had been $\frac{1}{4}$ in. to 1 mile, what would the area of the district have been?

16. In a square metal plate of 16-in. side a circle of 7 in. diameter is cut. Find the cost of the remainder if the metal costs 1s. 8d. per sq. in.

17. A closed rectangular box 8 ft. 6 in. $\times$ 4 ft. 3 in. $\times$ 3 ft. 10 in. (external dimensions) is made of wood 1 in. thick. Find the area of lead foil required to line the interior completely.

18. A pendulum 30 in. long swings through an angle of 15 degrees. Calculate the area traced out.

19. It is required to make a circular racing-track. Calculate the diameter, allowing

- (a) 3 laps—*i.e.*, circuits—to the mile.
- (b) 2 „
- (c) 1 lap

20. A closed cylindrical tank is 7 ft. high, and stands on a base 3 ft. 6 in. in diameter. Calculate the cost of painting the surface at $7\frac{1}{2}$ d. per sq. yd.

21. ABCD is a parallelogram. AB, the base, is 6.5 in.; AC, a diagonal, is 9.7 in.; and BC, a side, is 5 in. Draw the figure carefully; draw the altitude, and measure it to within .01 in. Calculate the area of the figure.

22. From the field-book entries given below draw a plan, and calculate the area in each case:

	(a) To B			(b) To B	
	990			1800	
240	800			1350	800
	710	130	600	940	
	460	350		580	400
120	250		200	300	
	0			0	
	From A			From A	

The measurements are in feet.

23. A pendulum 36 in. long swings through an angle of 18 degrees. Calculate the area traced out.

24. Calculate the cost of wallpaper for a rectangular room

18 ft. long and 15 ft. broad. The paper is to extend from the 10-in.-deep skirting, which runs round the room, to the picture-rail, which is 8 ft. 6 in. above the floor. Allowance is to be made for two doorways, 8 ft. 6 in. high by $4\frac{1}{2}$ ft. broad, a fireplace, 7 ft. by $4\frac{1}{2}$ ft., and window space, which is 10 ft. by $10\frac{1}{2}$ ft. The paper costs 8s. 9d. per piece. (See Question 3.)

25. A cylinder is 8 ft. 6 in. high and 3 ft. 9 in. in diameter. Calculate the cost of painting the surface at $8\frac{1}{2}d.$ per sq. yd.

26. Calculate to within 1 sq. in. the area of a circular plot having a diameter of 22 ft. 6 in.

27. A grass plot consists of a rectangle 17 yds. $\times$ 15 yds., with a circular pond 30 ft. in diameter in the middle. Calculate the cost of turfing the lawn at $9\frac{1}{2}d.$ per sq. ft.

28. In connexion with Question 17 above calculate the cost of painting the outside of the box at $8\frac{1}{2}d.$ per sq. yd.

29. Find the cost of turfing a circular plot 24 ft. 9 in. in diameter if the turf costs 2s. 9d. per sq. yd.

30. From the following field-book entries draw a plan in each case, and calculate the area :

(a) To B		(b) To B	
	840		1140
	615	230	830
205	480		650
110	320		390
	180	145	170
	0		0
	From A		From A

The measurements are in feet.

31. A closed rectangular box is made of wood $\frac{3}{4}$ in. thick. The external measurements are: length 10 ft. 3 in., breadth 4 ft. 6 in., depth 4 ft. 2 in. Calculate the area of the wood required for its construction.

32. Find the area of a circular track 30 ft. broad, if the diameter of the enclosed circle is 250 ft.

33. Calculate the total surface area of a cylinder 8 ft. 9 in. high having a diameter of 5 ft. 4 in.

34. The circumference of a cylinder is measured with a thread and found to be 8 ft. 8 in. Calculate the area of the cross-section. (Avoid unnecessary calculation.)

35. A path 15 ft. broad surrounds a circular bandstand 25 ft. in diameter. Calculate the cost of covering the path with gravel at the rate of $6\frac{1}{2}d.$ per sq. yd.

36. In connexion with the preceding question calculate the cost of erecting an 8-ft.-high network fence round the entire enclosure, including the path. The rate for fencing is $2s. 4d.$ per sq. yd.

37. Find the area of an equilateral triangle having sides 60 ft. long. (Find the altitude from a carefully drawn figure. See page 115, Example (a).)

38. A pendulum 48 in. long swings through an angle of 26 degrees. Calculate the area traced out.

39. In a circle of 10 in. radius two diameters are drawn at right angles to each other, and the ends of these diameters are joined. Calculate the area of the square thus formed.

40. Calculate the area of a triangular enclosure each side of which measures 32 ft. 6 in. (Make a careful drawing, and find the altitude of the prescribed enclosure by the method of Example (a), page 115.)

CHAPTER XIII

THE METRIC SYSTEM

(Continued from Chapter VII)

HARDER PROBLEMS

(*Note.* In these exercises money is expressed mostly in francs and centimes. The centime is the hundredth part of a franc.)

MODEL EXAMPLES

(a) Find the cost of 84 kilogrammes at 93 centimes per kilogramme.

$$\begin{array}{l} \text{Required cost} = 93 \text{ centimes} \times 84 \\ \quad \quad \quad = 7812 \text{ centimes} \\ \quad \quad \quad = 78.12 \text{ francs, or } 78 \text{ f. } 12. \end{array} \quad \begin{array}{r} 93 \\ 12 \\ \hline 1116 \\ 7 \\ \hline 7812 \end{array}$$

(b) Express a price of 17.42 francs per kilogramme in British money per lb. (Assume 120 francs to £1 and 2.2 lb. to 1 kilogramme.)

$$\begin{array}{l} \text{Rate} = 17.42 \text{ francs per kilo.} \\ \quad \quad \quad = \pounds \frac{17.42}{120} \text{ per } 2.2 \text{ lb.} \\ \quad \quad \quad = \pounds \left(\frac{17.42}{120} \times \frac{1}{2.2} \right) \text{ per lb.} \\ \quad \quad \quad = \pounds 0.0659 \text{ per lb.} \\ \quad \quad \quad = 15. \text{ } 3\frac{3}{4}d. \text{ per lb.} \end{array} \quad \begin{array}{r} 871 \\ \cancel{1742} \\ 120 \times 220 \\ \hline 110 \\ 12 \overline{) 871} \\ 11 \overline{) 871} \\ \hline 7258 \\ \hline 0659 \end{array}$$

(See also the model examples under "The Chain Rule," page 202.)

EXERCISES XIII

1. Find the cost of 17.5 kilogrammes at 73 centimes per kilogramme.

2. Butter is sold at 3·65 francs per kilogramme. What weight in kilogrammes can I buy for £1 if the rate of exchange is 120 francs to the pound sterling?

3. The side of a square field measures 760 metres. Find the cost of fencing the field at 8750 francs per kilometre.

4. The cylinder of a French car is advertised as 81·5 mm. in diameter and as 114 mm. stroke. Express these measurements in inches to within a hundredth of an inch.

5. I require to order 500 yds. of wire from a French maker. How shall I order it in the metric system? If the price is 18·75 francs per kilometre, what sum shall I pay for my order? (Give the answer in British coinage, taking the exchange as 120.)

6. A French maker advertises paint at 4·75 francs per kilogramme. Express this in British money per lb. (Exchange, 124 francs = £1.)

7. Which is the lower price, 7·35 francs per litre or $7\frac{1}{2}d.$ per pint? (Assume that 120 francs = £1 and 1 litre = 1·76 pts.)

8. I require 364 pieces of wire, each 79·4 cm. long. Find the cost at 15·5 francs per hectometre.

9. Find the size of the largest square tile that can be used in paving the floor of a rectangular court 23·10 metres long, 11·22 metres broad, if *whole* tiles only are to be used.

10. If it costs 115·5 francs to send goods 43·4 kilometres by rail, what should it cost to send the same goods 39·2 kilometres?

11. A merchant decides to purchase goods in France at 6·6 francs per litre instead of buying in England at 8d. per pint. Calculate whether this means an advantage or a disadvantage. (Assume value of franc and litre as in Question 7 above.)

12. A bicycle wheel is 3·735 metres in circumference. If every stroke of a pedal produces two and a half revolutions of the wheel, how many strokes of the pedal will the rider make in covering 1 kilometre, assuming that he pedals continuously?

13. Find (in francs) the cost of 3·79 kilometres of wire at the rate of 3 metres for 2 francs.

14. A skein of silk contains 1 hectometre. How many pieces,

each 3·1 metres long, can be cut from the skein? What will be the length of the remaining fragment, if there is any?

15. Express a speed of 64·2 kilometres per hour in metres per second.

16. Find the cost of 1·625 kilogrammes at 1·50 francs per kilogramme.

17. A merchant buys 125 kilogrammes of goods at 175 francs per demi-quintal. (A demi-quintal is 50 kilogrammes.) He sells in packets of $2\frac{1}{2}$ kilogrammes, at 10·5 francs per packet. Calculate the total gain.

18. The petrol consumption of a motor-car is 1 litre to 8 kilometres. If petrol costs 12·50 francs per demi-decalitre, calculate the cost of running 250 kilometres.

19. A merchant buys goods at 135 francs per hectolitre. What will he gain or lose by selling a demi-quintal for 94·5 francs? Assume that 1 hectolitre of the goods weighs 75 kilogrammes. (See Question 17.)

20. If a litre of milk yields ·12 litre of cream, and 1 litre of cream yields 240 grammes of butter, what weight of butter should be obtained from 250 litres of milk?

21. A vessel full of water weighs 3·75 kilogrammes. When empty the vessel weighs 1·15 kilogrammes. Give in litres the capacity of the vessel, knowing that 1 cubic centimetre of water weighs 1 gramme.

22. A rectangular area 200 metres long and 110 metres broad is to be planted with trees in rows parallel to the sides. The outermost trees all round are to be 10 metres back from the boundary. If a uniform distance of $4\frac{1}{2}$ metres is to be observed between the rows, and also between trees in a row, calculate (a) the number of trees required, (b) the total cost at 11·25 francs per tree.

23. Two men set out to walk a distance of 9 kilometres. Their paces measure 72 cm. and 80 cm. respectively. Find (a) the number of paces taken by each in the whole journey, (b) the number of times the men will be exactly in step, assuming that they start off in step, and that they walk abreast all the way.

24. The front wheels of a motor-car are 2·80 metres in

circumference, while the back wheels are 3.50 metres. If in a certain journey a back wheel makes 1080 revolutions, how many should a front wheel make?

25. It is calculated that pure milk contains 15 per cent. of its own weight of cream, and that cream produces butter to the extent of four-fifths of its own weight. If milk weighs 1.1 kilogrammes per litre, find what weight of butter may be obtained from 30 litres of milk. Find also the cost of the butter at 9.9 francs per half-kilogramme.

26. A rectangular classroom is 12.4 metres long, 10.8 metres broad, 4.6 metres high. If it is to accommodate 42 pupils, find (a) the amount of floor space allowed per pupil, (b) the cubic space per pupil.

27. If 20 metal plates .62 metre long, .27 metre broad, and .02 metre thick weigh 80 kilogrammes, what should be the weight of 35 plates of the same material .465 metre long, .32 metre broad, and .018 metre thick?

28. Goods are bought at 225 francs per quintal. Calculate the loss or gain incurred by selling 7 kilogrammes for 25.5 francs.

(*N.B.* 1 quintal = 100 kilogrammes.)

29. Goods are bought in France at 225 francs per quintal. Calculate the loss or gain incurred by selling 1 cwt. in London at 6*d.* per lb., assuming that the freight increases the cost by a tenth. (See Question 28. Take 120 francs = £1 and 1 kilogramme = 2.20 lb.)

30. If the carriage of 560 kilogrammes for 450 kilometres amounts to 90 francs, how far should 50 kilogrammes be carried for 40 francs?

CHAPTER XIV

PERCENTAGES

A PERCENTAGE of any quantity is so many *hundredths* of that quantity. *E.g.*, 1 per cent. of £10 means $\frac{1}{100}$ of £10, which amounts to 2s. 7 per cent. of £60 means $\frac{7}{100}$ of £60, which amounts to £4 4s. So also a 35 per cent. solution of acid means a mixture $\frac{35}{100}$ of which is acid and the rest—viz., $\frac{65}{100}$ —water.

MODEL EXAMPLES

Type I

(*Note that the method of 'practice' is generally advantageous.*)

(a) Find 3 per cent. of 4 tons 10 cwt.

$\frac{3}{100} \text{ of } 4\cdot5 \text{ tons} = \frac{13\cdot5}{100} \text{ tons}$ $= \cdot135 \text{ ton}$ $= 2 \text{ cwt. } 2 \text{ qrs. } 22\cdot4 \text{ lb.}$	<table style="border-collapse: collapse; margin: 0 auto;"> <tr><td style="text-align: right;">·135 ton</td></tr> <tr><td style="text-align: right; border-top: 1px solid black;">20</td></tr> <tr><td style="text-align: right;">2·70 cwt.</td></tr> <tr><td style="text-align: right; border-top: 1px solid black;">4</td></tr> <tr><td style="text-align: right;">2·80 qrs.</td></tr> <tr><td style="text-align: right; border-top: 1px solid black;">28</td></tr> <tr><td style="text-align: right;">22·40 lb.</td></tr> </table>	·135 ton	20	2·70 cwt.	4	2·80 qrs.	28	22·40 lb.
·135 ton								
20								
2·70 cwt.								
4								
2·80 qrs.								
28								
22·40 lb.								

Important. (b) Find 3 per cent. of £29 7s. 8d.

$\begin{array}{r} \text{£} \\ 3 \text{ per cent. } \overline{) 29\cdot383} \\ \underline{0\cdot8815} \end{array}$ <i>I.e.</i>, 17s. 8d., to within a penny.	<i>Method.</i> Decimalize the sum at sight. Multiply by 3, and write the figures of the product two places to the right. Keep four decimal figures only. Evaluate the result at sight.
--	--

Note. It is impossible to over-estimate the importance of this process. The step is of constant occurrence in commercial calculations.

(c) Find $4\frac{1}{2}$ per cent. of £29 7s. 8d.

$$\begin{array}{r} \text{£} \\ 29.383 \\ \hline 3 \text{ per cent.} \quad .8815 \\ 1\frac{1}{2} \text{ per cent.} \quad .4407 \\ \hline 1.322 \end{array}$$

I.e., £1 6s. 5d.

$1\frac{1}{2}$ per cent. is found by taking half of 3 per cent.

(d) Find $13\frac{1}{2}$ per cent. of £617 11s. 5d.

$$\begin{array}{r} \text{£} \\ 617.571 \\ \hline 9 \text{ per cent.} \quad 55.5814 \\ 4\frac{1}{2} \text{ per cent.} \quad 27.7907 \\ \hline 83.372 \end{array}$$

I.e., £83 7s. 5d.

Note. We might have taken $12\frac{1}{2}$ per cent. as $\frac{1}{8}$, and then 1 per cent. as $\frac{1}{100}$ of the original sum.

The alternative method is longer and less reliable.

Required result = £617.571 $\times$ 0.135.

$$\begin{array}{r} 61.7571 \\ 1.35 \\ \hline 61.7571 \\ 18.52713 \\ 3.087855 \\ \hline 83.372 \end{array}$$

(e) Find 14 per cent. of 6 tons 11 cwt. 3 qrs.

$$\begin{array}{r} \text{Tons} \\ 6.5875 \\ \hline 10 \text{ per cent.} \quad .6587 \\ 4 \text{ per cent.} \quad .2635 \\ \hline 0.9222 \end{array}$$

I.e., 18 cwt. 1 qr. 22 lb. approx.

$$\begin{array}{r} 20 \overline{) 11.75} \text{ cwt.} \\ \underline{5875} \text{ ton} \\ \\ 9222 \text{ ton} \\ 20 \overline{) 18.444} \text{ cwt.} \\ \underline{4} \\ 1.776 \text{ qrs.} \\ \underline{4} \\ 3.104 \\ \underline{7} \\ 21.7 \text{ lb.} \end{array}$$

Alternatively:

<i>Tons</i>	<i>Cwt.</i>	<i>Qrs.</i>
6	11	3
		2
13	3	2
		7
100)92	4	2
20		
18 ¹ 44	cwt.	
4		
1 ¹ 78	qrs.	
4		
3 ¹ 12		
7		
21 ¹ 84	lb.	

I.e., 18 cwt. 1 qr. 21·8 lb.

Note. With measures other than the money table either method may be employed. The essential thing is to avoid unnecessary figures.

(*f*) Find the difference between $4\frac{1}{4}$ per cent. of £86 3*s.* 7*d.* and $6\frac{1}{2}$ per cent. of £59 14*s.* 2*d.*

	£	
	86·179	
3 per cent.	2·5854	
1 per cent.	·8618	(Find 1 per cent. as $\frac{1}{100}$ of original
$\frac{1}{4}$ per cent.	·2154	sum.)
	3·6626	

Note. We might have taken $1\frac{1}{4}$ per cent. as $\frac{1}{80}$ of the original sum.

	£	
	59·708	
4 per cent.	2·3883	
$2\frac{1}{2}$ per cent.	1·4927	(Find $2\frac{1}{2}$ per cent. as $\frac{2\frac{1}{2}}{100}$ — <i>i.e.</i> , $\frac{1}{40}$ of
	3·8810	original sum. Divide by 4 and
		move digits down one place.)

The latter sum is greater by £0·218—*i.e.*, by 4*s.* $4\frac{1}{4}$ *d.*

Type II

Two methods of solution: (i) the equational method, for those who have mastered Chapter VIII; (ii) the purely arithmetical method, based on proportion.

(g) What percentage of £29 7s. 8d. is 17s. 8d.?

(i) *Equational Method*

If r per cent. of £29.383 = £.883,
then

$$\frac{r}{100} \text{ of } 29.383 = .883.$$

$$r = \frac{.883 \times 100}{29.383}$$

$$= \frac{88.3}{29.383}$$

$$= 3.0.$$

The quotient obviously begins with units, and one decimal place is enough.

Hence keep no more than three figures in the divisor.

$$\begin{array}{r} 293(8) \overline{)883(3.00} \\ \underline{881} \\ 200 \end{array}$$

(ii) *Arithmetical Method*

Expressing 17s. 8d. as a fraction of £29 7s. 8d., we have

$$\frac{.883}{29.383}.$$

Taking this fraction of 100 will give us the required percentage.

$$\text{Hence required percentage} = \frac{.883}{29.383} \text{ of } 100$$

$$= \text{etc., as in Method (i).}$$

Type III

(h) 17 per cent. of a certain sum amounts to £12 11s. Find the sum to within a penny.

(i) *Equational Method*

We have $\frac{17}{100}$ of $S = £12.55$.

$$S = £12.55 \times \frac{100}{17}$$

$$= £73.823$$

$$= £73 \text{ 16s. } 5d.$$

$$\begin{array}{r} 73.823 \\ 17 \overline{)1255} \\ \underline{119} \\ 65 \\ \underline{51} \\ 140 \\ \underline{136} \\ 40 \\ \underline{34} \\ 60 \end{array}$$

(ii) *Arithmetical Method*

17 hundredths of the sum = £12.55.

100 „ „ = ?

Required sum = £12.55 $\times \frac{100}{17}$
 = ., etc., as in Method (i).

General Problems

(i) 7 gal. of a 40 per cent. acid solution are mixed with 6 gal. of a 50 per cent. solution. Find the percentage strength of the final mixture.

Total amount of final mixture = (7 + 6) gal.
 = 13 gal.

acid in same = ($\frac{40}{100}$ of 7 + $\frac{50}{100}$ of 6) gal.
 = (2.8 + 3) gal.
 = 5.8 gal.

Then, by the equational method, if the final mixture contains r per cent. of acid, we have

$$\begin{aligned}\frac{r}{100} \text{ of } 13 \text{ gal.} &= 5.8 \text{ gal.} \\ r &= \frac{5.8 \times 100}{13} \\ &= 44.61.\end{aligned}$$

I.e., the final mixture contains 44.6 per cent. acid.

$$\begin{array}{r} 44.61 \\ 13 \overline{) 580} \\ \underline{52} \\ 60 \\ \underline{52} \\ 80 \\ \underline{78} \\ 20 \end{array}$$

Or, by the arithmetical method, required percentage

$$\begin{aligned}&= \frac{5.8}{13} \text{ of } 100 \\ &= \quad ., \text{ etc., etc.}\end{aligned}$$

(j) A manufacturer has 60 gal. of acid solution which he knows to be of 72 per cent. strength. How many gallons of acid must he add to raise the strength to 78 per cent.?

(*Note.* The easy way is to consider the relation of the *constant* quantity—*i.e.*, the water.)

Amount of original mixture = 60 gal.

water = 28 per cent. of same

= $\frac{28}{100}$ of 60 gal. = 16.8 gal.

(i) *Equational Method.* (Not recommended here.)

If we add n gal. acid to the mixture the total is $60 + n$ gal.
The water is to constitute 22 per cent. of the total.

Then 16.8 gal. water

$$= 22 \text{ per cent. of } (60 + n) \text{ gal.}$$

$$\text{I.e., } 16.8 = \frac{22}{100} \text{ of } (60 + n).$$

$$16.8 \times \frac{100}{22} = 60 + n.$$

$$\text{I.e., } n + 60 = 76\frac{4}{11}.$$

$$\text{Whence } n = 16\frac{4}{11}.$$

(Verify.)

$$\begin{array}{r} 2 \overline{) 1680} \\ 11 \overline{) 840} \\ \underline{76\frac{4}{11}} \end{array}$$

(ii) *Arithmetical Method.* (Preferable in this case.)

The water is to constitute 22 per cent. of the total.

Then 22 hundredths of the new total = 16.8 gal.

$$\therefore 100 \text{ hundredths} = 16.8 \text{ gal.} \times \frac{100}{22} = 76\frac{4}{11} \text{ gal.}$$

Hence $16\frac{4}{11}$ gal. must be added.

(k) The deposits in a certain bank fall from £2,371,486 to £2,324,618. Calculate the percentage decrease to two decimal places.

$$\begin{array}{r} \text{£} \\ 2,371,486 \\ 2,324,618 \\ \hline \text{Decrease } 46,868 \end{array}$$

(i) *Equational Method*

Then if this decrease amounts to r per cent. we have

$$\frac{r}{100} \text{ of } 2,371,486 = 46,868.$$

$$\begin{aligned} r &= \frac{46,868 \times 100}{2,371,486} \\ &= 1.97. \end{aligned}$$

Note that the decrease is on £2,371,486, and not on the later figure.

(Advanced pupils should employ contracted division.)

Forecast:

$$\frac{4.686800}{2.371486} = 2 \text{ approx.}$$

Hence the answer will contain three digits—i.e., the units and two decimal places.

Hence retain four figures in the divisor.

$$\begin{array}{r} 2371(4) \overline{) 4686800(1.97} \\ \underline{2371} \\ 23158 \\ \underline{21343} \\ 18150 \end{array}$$

(ii) *Arithmetical Method*

Expressing the decrease as a fraction of the original total deposit, we have $\frac{46,868}{2,371,486}$.

$$\begin{aligned}\text{Hence required percentage} &= \frac{46868}{2,371,486} \text{ of } 100 \\ &= 1.97, \text{ as above.}\end{aligned}$$

IMPORTANT FACTS TO BE MEMORIZED

I. 50 per cent of any quantity equals $\frac{1}{2}$ of that quantity.

25	$\frac{1}{4}$
$12\frac{1}{2}$	$\frac{1}{8}$
20	$\frac{1}{5}$
10	$\frac{1}{10}$
5	$\frac{1}{20}$
$2\frac{1}{2}$	$\frac{1}{40}$
$1\frac{1}{4}$	$\frac{1}{80}$

II. In calculating percentages of sums of money it is often useful to remember that

50 per cent. corresponds to 10s. in the £.

25	5s.
$12\frac{1}{2}$	2s. 6d.
20	4s.
10	2s.
5	1s.
$3\frac{3}{4}$	9d.
$2\frac{1}{2}$	6d.
$1\frac{1}{4}$	3d.

ORAL EXERCISES

1. Give in decimal form the results of the following examples of division :

- | | |
|---------------------|---------------------|
| (a) $754 \div 100.$ | (d) $427 \div 100.$ |
| (b) $329 \div 100.$ | (e) $619 \div 100.$ |
| (c) $813 \div 100.$ | (f) $333 \div 100.$ |

2. Give in shillings, pence, and farthings the value of

- | | |
|------------|------------|
| (a) £0.54. | (d) £0.27. |
| (b) £0.29. | (e) £0.19. |
| (c) £0.13. | (f) £0.33. |

3. Combine the steps you have been taking in the preceding exercises, Nos. 1 and 2, and so give the values of

- | | |
|--------------------------|--------------------------|
| (a) 1 per cent. of £754. | (d) 1 per cent. of £427. |
| (b) 1 per cent. of £329. | (e) 1 per cent. of £619. |
| (c) 1 per cent. of £813. | (f) 1 per cent. of £333. |

4. Give *in decimal form* the value of 2 per cent. of each of the following sums:

- | | |
|-----------|-----------|
| (a) £330. | (d) £418. |
| (b) £610. | (e) £327. |
| (c) £240. | (f) £436. |

5. Give, correct to within a farthing, the value of 2 per cent. of each of the sums given in the preceding question.

6. Give, correct to within a farthing, 3 per cent. of each of the following sums:

- | | |
|-----------|-----------|
| (a) £130. | (d) £133. |
| (b) £312. | (e) £243. |
| (c) £216. | (f) £171. |

7. Revise paragraph II immediately preceding this set of exercises, and then attempt the following:

(a) Give 5 per cent. of (i) £636, (ii) £219, (iii) £83, (iv) £17, (v) £183, (vi) £1728.

(b) Give $2\frac{1}{2}$ per cent. of (i) £69, (ii) £380, (iii) £296, (iv) £187, (v) £365, (vi) £1387.

(c) Give $1\frac{1}{4}$ per cent. of (i) £44, (ii) £116, (iii) £316, (iv) £435, (v) £413, (vi) £737.

(d) Give $3\frac{3}{4}$ per cent. of (i) £80, (ii) £144, (iii) £326, (iv) £612, (v) £37, (vi) £83.

8. Give

- (a) $2\frac{1}{2}$ per cent. of £37.
- (b) 5 per cent. of £71 10s.
- (c) $1\frac{1}{4}$ per cent. of £86.
- (d) $7\frac{1}{2}$ per cent. of £140.
- (e) $2\frac{1}{2}$ per cent. of £73 10s.
- (f) $3\frac{3}{4}$ per cent. of £142.

9. With regard to the following cases consider what fraction each sum is of £1 (approximately), and then evaluate the required percentage:

- (a) 5 per cent. of 14s. 3d.

(Proceed thus: 14s. 3d. is fully $\frac{7}{10}$ of £1. Now, 5 per cent. corresponds to 1s. in the £. Hence the required percentage is $\frac{7}{10}$ of 1s.—*i.e.*, $\frac{1}{10}$ of 7s.—say, 8d.)

- (b) 5 per cent. of 16s. 2d.
- (c) $2\frac{1}{2}$ per cent. of 8s.
- (d) $2\frac{1}{2}$ per cent. of 13s.
- (e) $1\frac{1}{4}$ per cent. of 7s. 1d.
- (f) 5 per cent. of 9s. 4d.
- (g) $1\frac{1}{4}$ per cent. of 12s.

10. Evaluate

- (a) 3 per cent. of £86.
- (b) 7 per cent. of £16.
- (c) 6 per cent. of £137.
- (d) 5 per cent. of £38 16s.
- (e) $2\frac{1}{2}$ per cent. of £151 10s.
- (f) 4 per cent. of £57.
- (g) 2 per cent. of £43.
- (h) $3\frac{3}{4}$ per cent. of £70.
- (i) $12\frac{1}{2}$ per cent. of £63.
- (j) 8 per cent. of £24.

EXERCISES XIV

(*Note.* In money calculations retain four, and not more, decimal places.)

1. (a) Find 7 per cent. of each of the following sums: £39 14s. 6d., £194 11s. 3d., £86 14s. 10d.

- (b) Find 3 per cent. of each of the sums in (a).
- (c) Find $4\frac{1}{2}$ per cent. of each of the sums in (a). (Find 3 per cent., then $1\frac{1}{2}$ per cent.)
- (d) Find 11 per cent. of each of the sums in (a). (Find 10 per cent., then 1 per cent.)
- (e) Find 14 per cent. of each of the sums in (a).
- (f) Find $22\frac{1}{2}$ per cent. of each of the sums in (a).
- (g) Find $3\frac{3}{4}$ per cent. of each of the sums in (a). (Find $2\frac{1}{2}$ per cent.—*i.e.*, $\frac{1}{40}$ —then $1\frac{1}{4}$ per cent.)
- (h) Find $13\frac{1}{2}$ per cent. of each of the sums in (a). (Find $12\frac{1}{2}$ per cent.—*i.e.*, $\frac{1}{8}$ —then 1 per cent.)
- (i) Find $6\frac{1}{4}$ per cent. of each of the sums in (a). (Find 5 per cent., then $1\frac{1}{4}$ per cent. as $\frac{1}{4}$ of the foregoing.)
2. Find 13 per cent. of £43 11s. 10d.
 3. Find 27 per cent. of £61 14s. 9d.
 4. Find $31\frac{1}{2}$ per cent. of £79 18s. 7d.
 5. Find 26 per cent. of £77 15s. 1d.
 6. Find 19 per cent. of £437 16s. 8d.
 7. Find 43 per cent. of £186 9s. 5d.
 8. Find 36 per cent. of 11 tons 18 cwt., to within 1 lb.
 9. Find 14 per cent. of 7 miles 6 fur., to within 1 yd.
 10. Add 11 per cent. of £61 17s. 4d. and $9\frac{1}{2}$ per cent. of £48 13s. 2d.
 11. From £68 11s. 3d. deduct 6 per cent., and then take away 8 per cent. of the remainder.
(Compare the result of taking away 8 per cent., then 6 per cent. of the remainder.)
 12. In 78 gal. of a certain solution 63 per cent. is acid. How many gallons of water are present?
 13. An agent receives 15 per cent. commission on sales. If he sells on one day a car valued at £225 and a cycle at £55 10s., what does his total commission for the day amount to?
 14. A customer is allowed $3\frac{3}{4}$ per cent. discount off an account of £78 13s. 4d. What must he pay in cash to settle the account?
 15. A man's annual salary amounts to £750. On this he pays $6\frac{1}{2}$ per cent. to a pension fund. He banks 20 per cent. of the remainder. What sum has he left yearly?

16. Find the difference between 8 per cent. of £37 11s. 2d. and 9 per cent. of £33 17s. 6d.

17. Add $3\frac{1}{2}$ per cent. of £27 11s. 8d., $8\frac{3}{4}$ per cent. of £19 14s. 7d., and $14\frac{1}{2}$ per cent. of £31 13s. 6d. (Note that $8\frac{3}{4}$ per cent. = 7 per cent. + $1\frac{3}{4}$ per cent., which is a fourth of 7 per cent., and that $14\frac{1}{2}$ per cent. = $\frac{12\frac{1}{2}}{100} + \frac{2}{100} = \frac{1}{8} + \frac{1}{50}$. Keep the sums in decimals until after the addition.)

18. Find the difference between $9\frac{1}{4}$ per cent. of £11 18s. 5d. and $8\frac{1}{4}$ per cent. of £13 17s. 2d. (Note. Take $9\frac{1}{4}$ per cent. as 8 per cent. + $1\frac{1}{4}$ per cent.)

19. Add $1\frac{3}{4}$ per cent. of £68 19s., $3\frac{1}{4}$ per cent. of £51 13s. 4d., and $5\frac{1}{2}$ per cent. of £39 15s. 7d.

20. What percentage of £86 14s. 10d. is £2 15s. 6d.?

21. What percentage of £194 11s. 3d. is £5 16s. 9d.?

22. What percentage of £39 14s. 6d. is £4 7s. 4d.?

23. What percentage of £78 13s. 4d. is £3 14s. 9d.?

24. What percentage of £37 11s. 2d. is £5 1s. 4d.?

25. An agent's sales amount to £389 2s. 6d. He receives as commission £14 11s. 10d. What is the rate per cent. of commission?

26. An agent is paid commission at the rate of $4\frac{1}{2}$ per cent. What must be the amount of his sales to earn a commission of £330?

27. How much water must be added to 36 gal. of alcohol to make a 40 per cent. solution—i.e., a solution of which alcohol constitutes 40 per cent.?

28. There are 357 names on a school roll. If only 313 children are present, what is the percentage of attendance? (Give the answer to the nearest unit.)

29. One mixture contains 45 per cent. of acid; another contains 60 per cent. If 10 gal. of the first be added to 5 gal. of the second, what is the percentage strength of the final mixture?

30. In a school there are on the roll 340 boys and 380 girls. If 320 boys and 350 girls are present, what is the percentage attendance of (a) boys, (b) girls, (c) the whole school? (Give the answers to the nearest unit.)

31. The rental of an estate is reduced from £8625 to £7050. Calculate the percentage reduction.

32. The circulation of a newspaper is increased from 1,250,000 to 1,262,500. Calculate the percentage increase.

33. One solution contains $22\frac{1}{2}$ per cent. water; another contains 27 per cent. If 5 pints of the first is mixed with 7 pints of the second, find the percentage of water in the final mixture.

34. A sum of money, after deduction of $22\frac{1}{2}$ per cent., amounts to £15 10s. What was the original sum?

35. From a sum of money 20 per cent. is deducted, and the remainder is further reduced by 10 per cent. If the sum remaining is now £4 10s., calculate the original sum.

36. The price of an article is reduced from £7 15s. to £6 16s. 3d. By what percentage has the price been reduced?

37. A cask of spirits contains 36 gal., of which 50 per cent. is alcohol. How much water must be added to reduce the percentage of alcohol to 40 per cent.?

38. A man's yearly income is £655. He spends £60 on rent, £28 10s. on taxes, and £45 on clothes. What percentage of his income remains?

39. A cask contains 36 gal. of mixture, of which 65 per cent. is alcohol. If 6 gal. of pure alcohol are added, calculate the percentage of alcohol in the final mixture.

40. A tourist purchases in Switzerland a clock which is invoiced to him thus: movement 225 francs, case 255 francs. He takes it with him into an adjoining country, where he is required to pay import duty of $27\frac{1}{2}$ per cent. on the movement and $35\frac{1}{2}$ per cent. on the case. Calculate the total tax, and express it as a percentage of the total price.

41. Complete the following table:

AIR FORCE ESTIMATES

—	YEAR 1924-25	YEAR 1923-24	INCREASE OR DECREASE	PERCENTAGE INCREASE OR DECREASE (TO TWO DECIMAL PLACES)
Pay	£ 2,491,000	£ 2,459,260		
Quartering stores	1,452,000	1,289,600		
Technical	5,700,000	3,867,300		
Medical services	195,000	232,740		
Totals				

(*Note.* Forecast the quotient approximately in each case, and in dividing retain one more figure in the divisor than the number of figures to be found in the quotient.)

42. Complete the following table:

—	POPULATION IN 1931	POPULATION IN 1911	INCREASE OR DECREASE	PERCENTAGE INCREASE OR DECREASE
England	37,354,917	34,043,117		
Scotland	4,842,554	4,759,521		
Wales	2,593,014	2,027,375		

43. After a reduction of 15 per cent. a man's salary amounts to £867. What was his original salary?

44. From the following data calculate for each year given the British output as a percentage (to one decimal place) of the world's output:

SHIPBUILDING TONNAGE

YEAR	GREAT BRITAIN AND IRELAND	WORLD
1928	1,444,000	2,660,000
1927	1,225,873	2,285,479
1926	639,568	1,674,977
1913	1,932,153	3,332,882

45. Complete the following table:

LIQUOR CONSUMPTION (RETAIL COST) OF GREAT BRITAIN

LIQUOR	1929	1930	DECREASE	PERCENTAGE DECREASE
	£	£		
Spirits	69,946,000	64,318,000		
Beer	185,025,000	182,090,000		
Wine . . .	23,211,000	22,151,000		
Totals				

46. Complete the following table. (The data are taken from the *Glasgow Herald* of November 4, 1927.)

Calculate the percentage to one decimal place in each case.

(*Note.* The pupil will find similar tables for adaptation in the commercial columns of the daily newspapers.)

REPORT ON GOLD-CRUSHINGS

MINE	YIELD			PROFIT			PROFIT AS PERCENTAGE OF YIELD		
	October	September	August	October	September	August	October	September	August
	£	£	£	£	£	£			
Geduld Prop.	117,433	114,759	116,103	48,745	47,523	47,979			
Gort Areas	388,926	384,293	393,465	225,045	225,006	230,607			
Langlaagte Est.	102,014	100,148	100,948	21,038	21,075	21,063			
Modder Deep	99,923	99,641	103,090	65,940	65,745	67,988			
New Primrose	14,348	15,472	16,264	1,029	1,205	1,554			
Randit'n Est.	197,767	193,529	208,497	25,528	22,495	30,066			
Sub-Nigel.	82,942	83,478	83,204	41,852	43,742	42,299			
Van Ryn Deep	101,863	107,576	108,542	47,531	50,395	50,589			
Witwatersrand	46,930	45,049	47,593	3,556	2,513	4,092			

(Data in Questions 47, 48, 49, and 50 are taken from "*The Times*" *Annual Financial and Commercial Review*, February 10, 1931.)

47. The following is a comparison of the bank clearing totals for the years 1929 and 1930 :

—	1930	1929	DECREASE	PERCENTAGE DECREASE
Grand Total	43,558,354,000	44,896,677,000		
Town Clearing	38,782,577,000	39,935,924,000		
Metropolitan Clearing	1,812,146,000	1,881,989,000		
Country Cheque Clearing	2,963,631,000	3,078,764,000		

Complete the last two columns, calculating the percentage to two decimal places.

48. In the following table calculate the percentage decrease to two decimal places; also complete the totals.

BRITISH IMPORTS

—	TOTAL 1930	DECREASE ON 1929	TOTAL IN 1929	PERCEN- TAGE DECREASE
Food, drink, tobacco. Raw materials . Manufactured articles Miscellaneous and parcel post	£ 475,551,761 250,808,124 307,499,025 10,981,284	£ 59,923,074 88,768,868 26,862,539 370,625		
Totals				

(N.B. Remember that the decrease is calculated on 1929. Hence the need for a *third column* before you can calculate the percentage decrease.)

49. Complete the return in the table below, giving the percentage decrease in each case to two decimal places.

BRITISH EXPORTS

—	TOTAL IN 1930	DECREASE ON 1929	TOTAL IN 1929	PERCEN- TAGE DECREASE
Food, drink, and to- bacco . Raw materials . Manufactured articles Miscellaneous and parcel post	£ 48,252,296 63,815,024 439,751,039 18,734,587	£ 7,403,773 15,085,614 134,048,450 2,258,539		
Totals				

50. Complete the return in the following table:

BRITISH RE-EXPORTS

—	TOTAL IN 1930	INCREASE OR DECREASE ON 1929	TOTAL IN 1929	PERCEN- TAGE INCREASE OR DECREASE
Food, drink, and to- bacco .	23,807,740	- 2,204,818		
Raw materials .	38,375,668	- 15,917,064		
Manufactured articles	24,161,770	- 4,735,525		
Miscellaneous	635,092	+ 135,858		
Totals				

CHAPTER XV

INTEREST

SIMPLE INTEREST

MODEL EXAMPLES

(a) Find the interest for 1 year on £739 at 4 per cent. per annum.

$$\begin{aligned}\text{Required interest} &= \frac{4}{100} \text{ of } £739 \\ &= £29.56 \\ &= £29 \text{ 11s. } 2d.\end{aligned}$$

(b) Find the simple interest for 3 years on £739 at 4 per cent. per annum.

$$\begin{aligned}\text{Required interest} &= \frac{4}{100} \text{ of } £739 \times 3 \\ &= £88.68 \\ &= £88 \text{ 13s. } 7d.\end{aligned}$$

(c) Find the simple interest on £463 for 3 years at $3\frac{1}{2}$ per cent. per annum.

$$\begin{aligned}\text{Required interest} &= \frac{7}{200} \text{ of } £463 \times 3 \\ &= £48.615 \\ &= £48 \text{ 12s. } 3d.\end{aligned}$$

$$\begin{array}{r} 4.63 \\ 7 \\ \hline 32.41 \\ 3 \\ \hline 2)97.23 \\ \hline 48.615 \end{array}$$

(d) Find the simple interest on £729 13s. for $2\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. per annum.

$$\begin{aligned}\text{Required interest} &= \frac{9}{200} \text{ of } £729.65 \times \\ &= £82.0856 \\ &= £82 \text{ 1s. } 8d.\end{aligned}$$

$$\begin{array}{r} 72.965 \\ 9 \\ \hline 8)656.685 \\ \hline 82.085 \end{array}$$

Note. The following example should be carefully studied, as it is a type that occurs commonly in practical life.

(e) Find the interest on £281, deposited May 3, withdrawn September 4 of the same year, at 3 per cent. per annum.

Required interest

$$\begin{aligned}
 &= \frac{3}{100} \text{ of } £281 \times \frac{124}{365} \\
 &\left[= £\frac{6}{200} \text{ of } 281 \times \frac{124}{365} \right] \\
 &= £\frac{6 \times 281 \times 124}{73000} \\
 &\left[= £\frac{209.064}{73} \right] \\
 &= £2.864 \text{ almost} \\
 &= £2 \text{ } 17s. \text{ } 3d.
 \end{aligned}$$

May	28 days
June	30
July	31
August	31
September	4 „
	<u>124 days</u>

$$\begin{array}{r}
 281 \\
 \times 124 \\
 \hline
 281 \\
 562 \\
 1124 \\
 \hline
 34844 \\
 6 \\
 \hline
 209064 \\
 \hline
 2.864 \\
 73 \overline{) 209.064} \\
 \underline{146} \\
 630 \\
 \underline{584} \\
 466 \\
 \underline{438} \\
 284
 \end{array}$$

Notes. (i) The steps within brackets should after very little practice be omitted by the pupil. (ii) In the first of these steps note the advantage of the *doubled* rate—viz., 6 instead of 3. This converts the 365 in the denominator into 365×2 —i.e., 730.

Use of the Formula. The formula comes to be

$$I = \frac{r}{100} \text{ of } P \times t,$$

where

I = interest,

r = rate per cent. per annum,

P = principal,

t = time (in years)

or

$$I = \frac{P \times t \times r}{100}.$$

Whence $100 I = P \times t \times r.$

$$r = \frac{100 I}{P \times t}.$$

$$t = \frac{100 I}{P \times r}, \text{ etc.}$$

(See Chapter X, on changing the subject of a formula.)

(f) At what rate per cent. per annum will £739 gain £88 13s. 7d. of interest in 3 years (simple interest)? (See Ex. (b).)

$$\begin{aligned} r &= \frac{100 I}{P \times t} \\ &= \frac{100 \times 88.679}{739 \times 3} \\ &= 4. \end{aligned}$$

$$\begin{array}{r} 4 \\ 2217 \overline{)8867.9} \\ \underline{8868} \end{array}$$

Quotient is obviously 4
for all practical purposes.

(g) What principal will amount in $3\frac{1}{2}$ years to £309 8s. at 3 per cent. per annum?

Note that the formula cannot be applied directly here.

We calculate first the amount of £100 for the given time and rate.

Clearly £100 would amount to £110½.

The required result is now found by proportion.

$$\begin{array}{lcl} \begin{array}{cc} \text{£100} & \text{£110}\frac{1}{2}. \\ ? & \text{£309}\cdot 4. \end{array} & & \begin{array}{r} 280 \\ 221 \overline{)61880} \\ \underline{442} \\ 1768 \\ \underline{1768} \end{array} \\ \text{Required principal} = \text{£100} \times \frac{309\cdot 4 \times 2}{221} & & \\ = \text{£280}. & & \end{array}$$

EXERCISES XV_A

1. Find the interest for a year on £648 at 3 per cent. per annum.

2. Find the interest for a year on £268 18s. 7d. at 6 per cent. per annum.

3. Find the simple interest on £415 for 3 years at 3 per cent. per annum.

4. Find the simple interest on £168 10s. for 4 years at 2 per cent. per annum.

5. Find the simple interest on £317 for 3 years at $2\frac{1}{2}$ per cent. per annum.

6. Find the interest on £281 from May 5 till September 10 (same year) at 3 per cent. per annum.

7. Calculate the interest on £560, deposited May 3, withdrawn October 20 (same year), at $3\frac{1}{2}$ per cent. per annum.

8. Find the interest on £561, lodged January 17, 1918,

withdrawn July 8 of the same year, the rate being 4 per cent. per annum.

9. Find the interest on £345, deposited June 11, withdrawn December 13, at $3\frac{1}{2}$ per cent. per annum.

10. Find the interest on £542 from July 6 to October 16 of the same year at 4 per cent. per annum.

11. £54 15s. is lent on March 10 at the rate of $3\frac{1}{2}$ per cent. per annum. If repayment is made on September 26 of the same year, what is the total amount paid?

12. Find the interest on £34 10s. from August 10 to October 21 of the same year at 4 per cent. per annum.

13. £1250 was borrowed on April 7 and repaid on August 25 of the same year, with interest at the rate of $3\frac{3}{4}$ per cent. per annum. What was the amount of the repayment?

14. Find the interest on £520 5s. from January 2 till July 17, 1927, at 7 per cent. per annum.

15. A man lodges £450 in the bank on January 17, 1926. What interest has accrued by September 6 of the same year, the rate being 3 per cent. per annum?

16. Calculate the interest on £385, deposited April 4, withdrawn October 2, same year, at $2\frac{1}{2}$ per cent. per annum.

17. Find to within a penny the simple interest on £79 15s. for $2\frac{1}{2}$ years at 3 per cent. per annum.

18. Find the interest on £680 from October 10 till December 13 of the same year at $1\frac{1}{2}$ per cent. per annum.

19. Calculate the simple interest on £131 10s. for 1 year and 30 days at 2 per cent. per annum.

20. A man lodges £55 15s. in the bank on June 10. What interest will be due to the depositor on November 11 of the same year if the rate allowed is $2\frac{1}{2}$ per cent. per annum?

21. Find the simple interest in the following cases:

(a) £370 for 30 days at $4\frac{1}{2}$ per cent. per annum.

(b) £365 for 80 days at $3\frac{1}{2}$ per cent. per annum.

(c) £264 deposited on April 10 and withdrawn on October 7 following, at $5\frac{1}{2}$ per cent. per annum.

(d) £55 deposited on March 4, withdrawn on September 17 following, at 3 per cent. per annum.

(e) £430 for 219 days at $2\frac{1}{2}$ per cent. per annum.

- (f) £145 deposited on August 17, withdrawn on February 15 following, at 2 per cent. per annum.
- (g) £87 10s. deposited on July 20, withdrawn on December 14 following, at $1\frac{1}{2}$ per cent. per annum.
- (h) £1135 deposited on May 10, withdrawn on October 16 following, at 4 per cent. per annum.
- (i) £37 15s. for 100 days at 3 per cent. per annum.
- (j) £460 deposited on March 27, withdrawn on November 19 following, at $3\frac{1}{2}$ per cent. per annum.

EXERCISES XV_B

1. What sum must be lodged in the bank to earn £30 of interest in 73 days if the rate of interest is 4 per cent. per annum?
2. What principal will gain £2 19s. 1d. in 168 days if the rate of interest is 3 per cent. per annum?
3. At what rate per cent. per annum will £281 amount to £283 19s. 1d. in 146 days? Give the rate to one decimal place.
4. How long will it take for £565 to amount to £644 2s., reckoning simple interest at $3\frac{1}{2}$ per cent. per annum?
5. Calculate the interest on £135 18s. lodged on May 10 and withdrawn on November 22 of the same year, at a rate of interest of $3\frac{1}{2}$ per cent. per annum.
6. What will £437 10s. amount to in 4 years 2 months at $1\frac{1}{2}$ per cent. per annum simple interest?
7. In what time will £625 amount to £633 19s. 9d. at $3\frac{3}{4}$ per cent. per annum?
8. At what rate per cent. per annum will £225, lodged January 17, 1921, amount by September 6 of the same year to £229 5s. 9d.?
9. In what time will £1040 10s. amount to £1060 4s. 10d. at $3\frac{1}{2}$ per cent. per annum?
10. At what rate per cent. per annum will £172 10s., if lodged in the bank on June 11, amount to £175 8s. 7d. by December 14 of the same year?
11. What sum lodged in the bank now at 5 per cent. per annum will gain £20 of interest in 100 days?

12. £292 is lodged in the bank on the last day of February. If the rate of interest is 4 per cent. per annum, find the date on which the deposit will have amounted to exactly £300.

13. £1000 was borrowed on March 10, the agreed rate of interest being $3\frac{1}{2}$ per cent. per annum. The borrower makes a payment of £600 on August 7. What payment made on December 31 will discharge the debt in full?

14. What sum must be deposited now to produce £75 of interest 100 days hence, the rate being 4 per cent. per annum?

15. Find the interest on £375 for 70 days at 4 per cent. per annum. At what rate per cent. would £300 in the same time produce exactly the same interest? (Can you answer the second question before answering the first?)

16. The National Debt amounted in April 1929 to £7,716,024,047. Calculate the interest (a) for 1 day, (b) for 1 week, assuming the rate to be $4\frac{1}{2}$ per cent. per annum.

17. A man lodges £480 in the bank on July 3. On July 23 he deposits £127 10s. On August 16 he draws £180. Calculate the interest due to him on October 31, assuming the rate to be $1\frac{1}{2}$ per cent. per annum.

18. In what time will a sum of money double itself at 3 per cent. per annum simple interest?

19. What sum of money must be lodged now to earn £39 in 73 days hence, interest being at $4\frac{1}{2}$ per cent. per annum?

20. What principal will amount to £397 10s. in 4 years at 3 per cent. per annum simple interest?

21. Calculate the principal which will amount to £240 in $3\frac{1}{2}$ years at 4 per cent. per annum.

22. A man wishes to present to a school a sum sufficient to yield £54 12s. of interest yearly. Calculate the necessary sum, allowing that the rate of interest is $3\frac{1}{2}$ per cent. per annum.

23. Find the interest on £71 lodged in the bank on July 13 and withdrawn on October 4 of the same year, the rate being $1\frac{1}{2}$ per cent. per annum.

24. A boy wins a prize of £5 5s. He decides to lodge it in the bank until it amounts to £6. How long will it require to lie in the bank if the rate of interest (simple) is 5 per cent. per annum? Give your answer to the *necessary* day.

25. £86 10s. is lodged in the bank on January 1, 1930, and is withdrawn on September 13 of the same year. Calculate the interest earned, the rate being $2\frac{1}{2}$ per cent. per annum.

26. If £200 is to amount to £207 10s. in $7\frac{1}{2}$ months, what must be the rate per cent. per annum?

27. What principal must be lodged in the bank on March 31 in order that the interest earned may amount to £4 10s. on August 13, the rate being 4 per cent. per annum?

28. Find the interest on £268 from May 3 till October 14 of the same year at $3\frac{1}{2}$ per cent. per annum.

29. How long will £365 take to amount to £390 if lodged in the bank at $2\frac{1}{2}$ per cent. per annum?

30. What sum will amount to £869 in 73 days at $2\frac{1}{2}$ per cent. per annum?

COMPOUND INTEREST

In each of the examples in Exercises XVA and XVB the interest has been reckoned once for all at the end of the time. Such interest is called *simple* interest. But it is the practice in banks to calculate at the end of the financial year the interest due to each depositor. Each bank fixes its own financial year, which may run, for example, from October 1 till September 30 following. This interest is now added on to the principal, or, as the banker says, the interest is *compounded* with the principal. Accordingly we speak of interest being *convertible* yearly, and in some cases convertible half-yearly or quarterly. In such cases we are said to be dealing with *compound* interest.

The student should recognize that in the case of simple interest the original principal remains unchanged throughout the whole time, but that in the case of compound interest the principal is augmented at regular periods by the interest that has been earned in the period.

MODEL EXAMPLES

(*Note.* Do not retain more than four decimal places in working.)

(a) Find the compound interest on £643 for 2 years at 3 per cent. per annum, convertible yearly.

	£	
	643	
3 per cent.	19·29	(Multiply by 3, and move figures two places down.)
	662·29	
	19·8687	
	682·159	
Subtract	643	

Required interest = £39 3s. 2d.

(b) Find the compound interest on £719 14s. for 2 years at $4\frac{1}{2}$ per cent. per annum, convertible yearly.

	£	
	719·7	
$2\frac{1}{2}$ per cent.	17·9925	($\frac{2\frac{1}{2}}{100} = \frac{1}{40}$. Hence divide by 40.)
2 per cent.	14·394	
	752·0865	
	18·8021	
	15·0417	
	785·930	
Subtract	719·7	

Required interest = £66·230
= £66 4s. 7d.

(c) Find the compound interest on £728 for $2\frac{1}{2}$ years at 3 per cent. per annum, convertible yearly.

	£	
	728	
3 per cent.	21·84	
	749·84	
	22·4952	
	772·3352	
	[23·1700]	(3rd year's interest, <i>not</i> to be added in.)
	11·5850	(Half the above.)
	783·920	
Subtract	728	

Required interest = £55 18s. 5d.

(d) A man lodges £420 in the bank on January 1, 1920, and the same sum on the first day of 1921 and 1922 successively. Assuming that the rate of interest is 4 per cent. per annum, convertible on the first day of each year, what sum should lie to the depositor's credit on January 1, 1923?

	£	
	420	Principal for 1st year (1920)
4 per cent.	16·8	
add	420	
	<u>856·8</u>	2nd (1921)
4 per cent.	34·272	
add	420	
	<u>1311·072</u>	3rd (1922)
4 per cent.	52·4429	
	<u>1363·515</u>	Amount at end of 1922.

Required sum = £1363 10s. 3d.

(e) What sum must be invested at 4 per cent. per annum (interest convertible yearly) to amount in 3 years to £4393 9s. 10d.?

First find the amount of £100 under the same conditions, and then calculate the required sum by proportion.

Thus:

	100	112 ⁴ 4 ⁸ 6 ⁴ 4)4393492(39058
4 per cent.	<u>4</u>	3374592
	104	1018900
	<u>4·16</u>	1012378
	108·16	<u>6522</u>
	<u>4·3264</u>	5624
	112·4864	<u>898</u>

I.e., £100 amounts to £112·4864.

$$\begin{aligned}
 \text{required sum} &= £ \frac{100 \times 4393 \cdot 492}{112 \cdot 4864} \\
 &= £3905 \cdot 8 \\
 &= £3905 \text{ 16s.}
 \end{aligned}$$

EXERCISES XVc

1. Find the interest in the following cases:

- (a) £216 for 2 years at 3 per cent. per annum, convertible yearly.
- (b) £364 for 3 years at 4 per cent. per annum, convertible yearly.
- (c) £762 for 2 years at $2\frac{1}{2}$ per cent. per annum, convertible yearly.
- (d) £368 for $2\frac{1}{2}$ years at 3 per cent. per annum, convertible yearly.
- (e) £375 14s. for 3 years at $3\frac{1}{2}$ per cent. per annum, convertible yearly.
- (f) £3255 4s. for 3 years at 4 per cent. per annum, convertible yearly.
- (g) £670 16s. 8d. for 3 years at 3 per cent. per annum, convertible yearly.
- (h) £504 16s. for 3 years at $3\frac{1}{2}$ per cent. per annum, convertible yearly.
- (i) £525 for $3\frac{1}{2}$ years at $2\frac{1}{2}$ per cent. per annum, convertible yearly.
- (j) £680 for $3\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. per annum, convertible yearly.
- (k) £1627 for 2 years at $4\frac{1}{2}$ per cent. per annum, convertible half-yearly.

2. A man banks £156 on April 1, 1924, and the same amount on the same date in 1925 and 1926. Calculate the amount lying to his credit on April 1, 1927, assuming interest at a constant rate of $3\frac{1}{2}$ per cent. per annum, convertible yearly on April 1.

3. The population of a town in 1891 was 37,426. Assuming an average increase of 10 per cent. every 10 years, estimate the population in 1931.

4. Find the interest on £2261 18s. 9d. for 2 years at 5 per cent. per annum, convertible half-yearly.

5. What sum must be invested at $2\frac{1}{2}$ per cent. per annum (interest convertible yearly) to amount to £1402 12s. at the end of 2 years?

6. Find the amount of £4206 for 18 months at 7 per cent. per annum, convertible half-yearly.

7. What sum will amount to £101 4s. 9d. 3 years at 6 per cent. per annum, convertible yearly?

8. A man deposits in the bank £80 on the last day of the year 1928. If he repeats this at intervals of exactly a year till December 31, 1931, what sum will lie to his credit at the end of the year 1932 if interest accrues at 3 per cent. per annum, convertible yearly?

9. Calculate to within a penny the difference between the simple and the compound interest on £375 for 3 years at 5 per cent., the interest being converted yearly in the second case.

10. In what time will £128 amount to £148 3s. 6d. if interest at the rate of 5 per cent. per annum is added yearly?

11. Find the interest on £468 for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. per annum, convertible yearly.

12. Calculate the interest on £587 13s. for $2\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. per annum, convertible yearly.

13. A sum of £320 is loaned at 3 per cent. interest per annum, convertible yearly. Repayment is to be made by three successive yearly payments of £100 each and a final payment at the end of the fourth year. Calculate the final payment.

14. £750 is lodged in the bank on September 30, 1929. What sum should the depositor receive if he withdraws his money on December 10, 1931, assuming interest at $4\frac{1}{2}$ per cent. per annum, convertible yearly on September 30?

15. £653 is lodged in the bank on April 6, 1926, the rate of interest being $3\frac{1}{2}$ per cent. per annum, convertible yearly on April 6. What sum should the depositor receive if he withdraws his money on July 10, 1928?

16. A certain bank allows on money deposited interest at $2\frac{1}{2}$ per cent. per annum, convertible yearly on October 31. Calculate the interest on £56 10s. lodged on July 10, 1929, and withdrawn on December 27, 1930.

17. Find the interest on £561 for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. per annum, convertible yearly.

18. Find the interest on £196 15s. for $2\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. per annum, convertible yearly.

19. Calculate the interest on £775 for 2 years at $3\frac{1}{2}$ per cent. per annum, convertible half-yearly.

20. Find the interest on £134 15s. for years at $4\frac{1}{2}$ per cent. per annum, convertible yearly.

21. Calculate the interest on £685 for 2 years at $3\frac{1}{2}$ per cent. per annum, convertible half-yearly.

22. Find the interest on £364 for 2 years at $5\frac{1}{2}$ per cent. per annum, convertible half-yearly.

23. Calculate the interest on £372 for 2 years at $3\frac{1}{4}$ per cent. per annum, convertible yearly.

24. Find the interest on £268 10s. for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. per annum, convertible yearly.

25. A loan of £750 is granted on the following conditions: £275 to be repaid at the end of the first year; £275 at the end of the second year; and the balance, together with all the accrued interest, at the end of the third year. Calculate the amount of the final payment, assuming that the rate of interest is $3\frac{1}{2}$ per cent. per annum, convertible yearly.

26. A man lodges £25 in the bank. At the end of a year he again lodges £25, and again at the end of the next year he lodges £25. If the bank rate of interest is 3 per cent., convertible yearly, what sum will lie to the credit of the depositor at the end of the third year?

27. In order to meet a debt of £100 due three years from now what sum must be immediately lodged in the bank, the rate of interest being $4\frac{1}{2}$ per cent. per annum, convertible yearly? Give your answer correct to the necessary pound.

28. Find to the necessary year the time required for £300 to amount to £365, the rate of interest being 5 per cent. per annum, convertible yearly.

29. Calculate the interest on £137 5s. for $2\frac{1}{2}$ years at 5 per cent. per annum, convertible half-yearly.

30. A deposits £150 in a bank paying interest at 5 per cent. per annum. At the same time B lodges £175 in a bank paying 3 per cent. per annum. After how many years will A's amount exceed B's? Interest is convertible yearly in each case.

(Note. Further exercises in compound interest are given in the chapter on logarithms.)

CHAPTER XVI

PROFIT AND LOSS

BEFORE beginning this chapter the student should revise carefully the model examples given in Chapter XIV, on percentages, especially those on pages 142–145.

Preliminary Notes

- (i) The gain per cent. means
the number of pounds gained on a cost price of 100 pounds,
or shillings 100 shillings,
or pence 100 pence,
or „ „ florins „ „ „ „ 100 florins,
and similarly for half-crowns, francs, dollars, etc.
- (ii) The actual gain (or loss) is reckoned as a percentage of the *cost price*, and NOT of the *selling price*.
- (iii) The student should become quite familiar with the step in reasoning given in the following example.

If $\frac{5}{7}$ of a sum of money amounts to 17s. 2d., find the total sum.

We have
$$\begin{aligned}\frac{5}{7} \text{ of sum} &= 17s. \ 2d. \\ \text{sum} &= 17s. \ 2d. \times \frac{7}{5}.\end{aligned}$$

MODEL EXAMPLES

(a) An article is bought for 15s. and sold for 16s. 10½d. Find the gain per cent.

$$\begin{aligned}\text{The total gain} &= 1s. \ 10\frac{1}{2}d. \\ &= 1\frac{7}{8}s.\end{aligned}$$

(i) By the equational method.

If the rate of profit is r per cent.,

then
$$\frac{r}{100} \text{ of } 15s. = 1\frac{7}{8}s.$$

$$\begin{aligned}
 \text{I.e.,} \quad \frac{r}{100} \text{ of } 15 &= \frac{15}{8}. \\
 r &= \frac{15}{8} \times \frac{100}{15} \\
 &= 12\frac{1}{2}.
 \end{aligned}$$

The gain is $12\frac{1}{2}$ per cent.

(ii) By the purely arithmetical method.

Taking the gain as a fraction of the cost price, we have

$$\frac{1s. \ 10\frac{1}{2}d.}{15s.} = \frac{1}{8}.$$

$$\begin{aligned}
 \text{Then required percentage} &= \frac{1}{8} \text{ of } 100 \\
 &= 12\frac{1}{2}.
 \end{aligned}$$

(b) An article is bought for 15s. Find the selling price that will yield $12\frac{1}{2}$ per cent. gain.

$$\begin{aligned}
 \frac{12\frac{1}{2}}{100} \text{ of } 15s. &= \frac{1}{8} \text{ of } 15s. \\
 &= 1s. \ 10\frac{1}{2}d.
 \end{aligned}$$

Hence the required selling price will be 16s. $10\frac{1}{2}d.$

(c) An article is sold for 16s. $10\frac{1}{2}d.$, thereby yielding $12\frac{1}{2}$ per cent. gain. What was the cost price? (*Note.* It will be quite wrong to find $\frac{12\frac{1}{2}}{100}$ of the *selling price* and subtract it.)

If we call the cost price C shillings,

$$\begin{aligned}
 \text{then} \quad C + \frac{12\frac{1}{2}}{100} \text{ of } C &= 16\frac{7}{8}. \\
 \text{I.e.,} \quad C + \frac{1}{8} \text{ of } C &= 16\frac{7}{8}. \\
 \text{I.e.,} \quad \frac{9}{8} \text{ of } C &= 16\frac{7}{8}. \\
 C &= \frac{16\frac{7}{8}}{\frac{9}{8}} \times \\
 &= 15.
 \end{aligned}$$

Hence the cost price is 15s.

(d) A merchant buys 1 cwt. of goods at 2s. 6d. per lb. He sells 50 lb. at 2s. 9d. per lb. and 40 lb. at 2s. $10\frac{1}{2}d.$ per lb. At what rate per lb. must he sell the remainder to clear 25 per cent. profit on the whole transaction?

$$\text{Total cost of 112 lb. at 2s. 6d. per lb.} = \text{£}14.$$

$$\begin{aligned}
 \text{Required gain} &= 25 \text{ per cent. of } \text{£}14 = \text{£}3 \ 10s. \\
 \text{total selling price} &= \text{£}17 \ 10s.
 \end{aligned}$$

Receipts from 50 lb. at 2s. 9d. per lb. = £6 17s. 6d.

40 2s. 10½d. „ = £5 15s. 0d.

I.e., receipts from 90 lb. = £12 12s. 6d.

selling price of remainder (22 lb.) = £4 17s. 6d.

Hence rate per lb. = $\frac{£4\ 17s.\ 6d.}{22}$

= $\frac{£2\ 8s.\ 9d.}{11}$

= 4s. 5½d.

Say, 4s. 5¼d.

(e) A merchant allows 10 per cent. off the marked prices of his goods by way of discount to customers. How should he mark an article that cost him £2 5s. so that he may make an actual profit of 25 per cent. by its sale?

Since actual selling price is 10 per cent. less than marked price we have

SP = 90 per cent. of MP

= $\frac{9}{10}$ of MP

and so MP = $\frac{10}{9}$ of SP.

Now,

SP = CP + 25 per cent. of CP

= $\frac{5}{4}$ of CP.

MP = $\frac{10}{9}$ of ($\frac{5}{4}$ of CP)

= $\frac{10}{9}$ of $\frac{5}{4}$ of 45s.

= 62½s.

I.e., marked price = 62s. 6d.

(f) By selling an article for 55s. a merchant makes 10 per cent. profit. At what price must he sell it to make 30 per cent. gain?

55s. = cost price + 10 per cent. of CP

= CP + $\frac{1}{10}$ of CP

= $\frac{11}{10}$ of CP.

CP = 55s. $\times \frac{10}{11}$.

(*Note.* It is loss of time to *evaluate* this—the cost price is not wanted.)

$$\begin{aligned}\text{To make 30 per cent. gain } SP &= CP + \frac{30}{100} \text{ of } CP \\ &= \frac{130}{100} \text{ of } (55s. \times \frac{10}{11}) \\ &= 65s.\end{aligned}$$

Or, more briefly (for pupils who have mastered the principles),

$$\begin{array}{rcll} 110 & \text{per cent. of the cost price corresponds to} & 55s. & \\ 130 & & & ? \end{array}$$

$$\begin{aligned}\text{Required price} &= 55s. \times \frac{130}{110} \\ &= 65s.\end{aligned}$$

(g) A dealer bought 50 cattle at a uniform price. He sold 20 of them at a loss of 10 per cent. How must he sell the remainder to clear 5 per cent. on the whole transaction?

Take the cost per head as 1 unit.

If he sells remaining 30 cattle at r per cent. gain, this gain must equal not only the loss on 20, but, in addition, the 5 per cent. gain on the whole.

$$\begin{aligned}\text{Thus, } \frac{r}{100} \text{ of } 30 &= \frac{10}{100} \text{ of } 20 + \frac{5}{100} \text{ of } 50. \\ 30r &= 200 + 250. \\ \therefore r &= \frac{450}{30} = 15.\end{aligned}$$

I.e., he must sell remainder at 15 per cent. gain.

(h) By reducing his rate of profit from 30 per cent. to 20 per cent. a shopman sells 50 per cent. more goods than before. Does he gain or lose by the change?

We have to determine whether 30 per cent. of 100 or 20 per cent. of 150 is greater.

$$\begin{aligned}30 \text{ per cent. of } 100 &= 30. \\ \frac{20}{100} \text{ of } 150 &= 30.\end{aligned}$$

Hence there is neither gain nor loss (financially).

• ORAL EXERCISES

(On finding the selling price)

1. I buy an article for £12. What selling price will give me 50 per cent. profit?

State the selling price that will yield (a) 25 per cent. profit, (b) 20 per cent. profit, and (c) 100 per cent. profit.

2. If the cost price of an article is £30, state the selling prices that will yield (a) 10 per cent. profit, (b) 5 per cent. profit.

(Note that the answer to (b) is not *half* of the answer to (a). Explain why.)

3. A merchant buys tea at 1s. 6d. per lb. What selling price will bring him 25 per cent. profit?

4. A house costs me £750. For what sum must I sell it to earn 20 per cent. profit?

5. An agent buys a motor-car for £140. At what price must he sell it to clear a profit of 15 per cent.?

6. An agent buys a cycle for £8 10s. For what sum must he sell it to make $12\frac{1}{2}$ per cent. profit?

7. Potatoes cost a dealer $7\frac{1}{2}$ d. per stone. What selling price will yield 20 per cent. profit?

8. A dealer pays £12 10s. for a wireless set. What selling price will yield 40 per cent. profit?

9. If I buy magazines at 15s. a dozen, at what rate must I sell to realize $33\frac{1}{3}$ per cent. profit?

10. A salesman buys golf-club shafts at £12 10s. per hundred. At what price per shaft must he sell to make 20 per cent. profit?

11. Books costing 3s. 9d. each having become shop-soiled are to be sold at a loss of 20 per cent. What will be the selling price?

12. Cycles costing originally £10 10s. are to be sold at a reduction of 15 per cent. Give the selling price.

EXERCISES XVIIA

(On finding the selling price)

1. Articles are bought at 16s. 8d. each. At what price must they be sold to yield $22\frac{1}{2}$ per cent. profit?

2. What selling price per lb. will yield 40 per cent. profit on goods purchased at 35s. per cwt.?

3. A shopkeeper sells shop-soiled goods at $7\frac{1}{2}$ per cent. below the 'marked' prices. What will be the actual selling price of a picture marked at £5 15s.?

4. Goods bought at £45 per ton are to be sold so as to yield 15 per cent. profit. Find the selling price per cwt.
5. A dealer allows his customers $4\frac{1}{2}$ per cent. discount off his catalogue prices. What will be the actual selling price of an article catalogued at 25s.?
6. Ribbon is bought at 108s. per piece of 36 yds. Calculate the selling price per yard to yield 25 per cent. profit.
7. Pencils are bought at 54s. per gross. Calculate the retail price per pencil if the seller is to make $33\frac{1}{3}$ per cent. profit.
8. A man buys a house for £620, and spends £140 on alterations and repairs. He wishes to sell it so as to make 30 per cent. profit on his total outlay. Calculate the necessary selling price.
9. I buy goods at £9 16s. per cwt. Calculate the retail price per lb. (to the necessary halfpenny) that will yield me 30 per cent. profit.
10. A dealer buys goods at 161s. per cwt. Carriage and other expenses amount to 14s. per cwt. At what price per lb. must he retail in order to realize 30 per cent. profit on his outlay?

ORAL EXERCISES

(On finding the percentage profit or loss)

Find the rate per cent. of profit or loss in each of the following cases:

1. Cost price £40; selling price £60.
(Proceed thus: profit is £20. £20 is $\frac{1}{2}$ of cost price.
Hence rate of profit is 50 per cent.)
Cost price £18; selling price £24.
3. Cost price £30; selling price £33.
4. Cost price 12s. 6d.; selling price 15s.
5. Cost price 44s.; selling price 49s. 6d.
6. Cost price 1s. 3d.; selling price 1s. $4\frac{1}{2}$ d.
7. Cost price 36s.; selling price 40s. 6d.
8. Cost price 15s.; selling price 12s.
9. Cost price 3s. 9d.; selling price 4s. 6d.
10. Cost price 11s. 8d.; selling price 10s. 6d.

EXERCISES XVI_B*(On finding the percentage profit or loss)*

1. An article is bought for 96s. and is sold for £5. Calculate the rate of profit per cent.

What rate per cent. of loss is suffered by a shopkeeper who buys at £6 and sells for £4 13s.?

3. Goods are bought at 70s. per cwt. Calculate the percentage profit or loss resulting from selling at 8d. per lb.

4. 3 tons of butter is bought at 105s. per cwt. If 36 cwt. is sold at 1s. 2d. per lb., and the remainder is sold at 1s. 1½d., calculate the rate per cent. of profit over the whole transaction.

5. Concentrated acid is bought at 3s. 10½d. per gallon. 2 pints of water is added to every gallon of acid, and the mixture is retailed at 6½d. per pint. Calculate the rate per cent. of profit or loss.

6. 36 lb. of tea costing 1s. 9d. per lb. and 42 lb. costing 1s. 6d. per lb. are mixed, and the mixture is sold at 1s. 10½d. per lb. Calculate the rate per cent. of profit.

7. Goods are bought at £58 10s. per ton, and sold at 8½d. per lb. Calculate the percentage of profit.

8. If I sell at 1s. each magazines which I purchase at 10s. 9d. per dozen, what is my rate of profit per cent.?

9. Work out the preceding question on the assumption that in purchasing I receive 13 as a dozen.

10. Cigarettes are bought at 13s. 4d. per lb., and retailed at 20 for 10½d. Allowing 320 cigarettes to the pound weight, calculate the percentage profit or loss.

ORAL EXERCISES

(On finding the cost price)

1. At what price must I buy fountain-pens in order to be able to retail them at 18s. each, thereby making 50 per cent. profit?

(Proceed thus: 18s. must equal the cost^a price and half as much again.

Thus, $\frac{3}{2}$ of cost price = 18s.

Hence cost price = $\frac{2}{3}$ of 18s. = 12s.)

What cost price will allow a selling price of 12s., realizing 20 per cent. profit?

3. A dealer means to sell a particular motor-car for £250. At what price must he buy if he is to realize a profit of 25 per cent.?

4. A dealer wishes to be able to sell goods at 2s. 4d. per stone, thereby making 25 per cent. profit. At what rate must he buy?

5. What cost price will enable a merchant to sell articles at 2s. 2d. each and make a profit of $12\frac{1}{2}$ per cent.?

6. A sum of money, increased by 20 per cent., amounts to £21. What was the original sum?

7. A boy's wages after being increased 25 per cent. amount to £50 a year. What was the original amount?

8. A man's salary after being reduced $12\frac{1}{2}$ per cent. amounts to £350. What was the original amount?

9. A shopman sells a book for 7s. 6d., thereby making a gain of 25 per cent. What did it cost him?

10. By selling a house for £600 a man loses 20 per cent. What price did he pay for it?

EXERCISES XVIc

(On finding the cost price)

1. If I wish to sell at 24s. 2d., making a profit of 45 per cent., at what price must I buy?

2. Margarine is sold at 1s. $1\frac{1}{2}$ d. per lb., yielding 80 per cent. profit to the seller. What must be the cost price per cwt.?

3. A dealer wishes to allow customers $7\frac{1}{2}$ per cent. discount off the marked price. At what price must he mark an article which he intends to sell for 74s.?

4. If I lose 5 per cent. by selling an article for 15s. 10d., what did it cost me?

5. An article is sold for £5 9s. $4\frac{1}{2}$ d. If this entails a loss of $12\frac{1}{2}$ per cent. for the seller, what must have been the cost price?

6. If an agent makes 40 per cent. gain by selling eggs at 7s. a hundred, at what rate did he buy?

7. A grocer loses 30 per cent. by selling at 5*d.* per lb. What must have been the cost per cwt.?

8. At what rate per gallon must I buy in order to allow me to make a profit of 20 per cent. by retailing at 3*d.* per pint?

9. Calculate to within a penny the cost price per cwt. to allow me to retail at 4½*d.* per lb., thereby making a profit of 30 per cent.

10. At what rate per score must I purchase eggs to be able to retail them at 1*s.* 9*d.* per dozen, making a profit of 40 per cent.? Give your result correct to the necessary penny.

EXERCISES XVII

(General)

1. A book is bought for 17*s.* 6*d.* and sold for £1. Find the gain per cent.

2. Books are bought at 10*s.* 6*d.* per dozen and sold at 1*s.* 2*d.* each. What is the rate of profit per cent.?

3. A merchant's rate of profit is 30 per cent. What will be the selling price of an article which costs him £45 15*s.*?

4. A grocer buys 1 cwt. of goods at 1*s.* per lb. He sells 70 lb. at 1*s.* 9*d.* per lb., 30 lb. at 6*d.* per lb., and the remainder at 2½*d.* per lb. Calculate his gain or loss per cent.

5. An article is sold for 25*s.* 1½*d.*, thereby yielding a gain of 12½ per cent. What was the cost price?

6. A grocer bought 1 cwt. of sugar for £1 3*s.* 4*d.* At what price per lb. must he sell to gain 10 per cent.? (Give answer correct to the necessary farthing.)

7. A house cost £900. At what annual rent should the buyer let it so as to realize 6 per cent. revenue yearly on the cost, after deducting £25 for repairs?

8. By selling eggs at 2*s.* 6*d.* per doz. a grocer makes a profit of 5 per cent. What price per dozen must he charge to make a profit of 26 per cent.?

9. By selling an article at £1 4*s.* a merchant makes a profit of 12½ per cent. Find the cost price.

10. A salesman allows as discount 10 per cent. off the

marked price of his goods. How should he mark an article that cost him 25s. to make an actual profit of 5 per cent.

11. A dealer buys 15 articles at a uniform price. He sells 9 of them at a loss of 7 per cent. What rate of profit must he make on the remainder to gain 10 per cent. over all?

12. A man buys an article at £5 13s. 4d., and sells it at a loss of 10 per cent. What was the selling price? What selling price would have yielded 5 per cent. gain?

13. By selling a horse for £63 a man loses 16 per cent. At what price should he have sold to gain 20 per cent.?

14. A man bought 30 doz. eggs at a uniform price. He sold 22 doz. at a loss of $2\frac{1}{2}$ per cent. What rate of profit must he make on the remainder to clear 5 per cent. over all?

15. If ore sells at 27s. per ton, and the expense of mining it averages 21s., what percentage of the receipts is profit?

16. A clerk invoices goods at 2s. 7d. per lb. instead of 2s. 1d. per lb. What is the percentage error?

17. Oranges are bought at 2 a penny, and sold at 3 for 2d. What is the gain per cent.?

18. A man buys 1000 eggs at 1s. 6d. per doz. Through breakage, etc., he loses 8 doz. At what price per dozen must he sell the remainder to clear 12 per cent. on his outlay? (Give result to the necessary penny.)

19. A man buys 12 houses at a uniform price. He sells 3 at a loss of 5 per cent. and another 5 at a loss of 10 per cent. What percentage of profit must he make on the remainder to gain 15 per cent. over all?

20. A merchant decides to reduce his rate of profit from 15 per cent. to 10 per cent., and finds that in consequence he sells 40 per cent. more goods than formerly. (a) Is the alteration to his advantage? (b) By what percentage is his profit increased or diminished?

21. A dishonest dairyman adds 1 pt. of water to every gallon of milk. If he buys at 6d. per gal., and sells at 1d. per pt., find his profit per cent.

22. A shopkeeper allows his customers 10 per cent. off the marked price of goods as discount, and realizes an actual

profit of 20 per cent. What must be the cost of an article which is marked at 60s.?

23. By selling goods at 24s. per cwt. a salesman makes a profit of 20 per cent. At what rate should he sell in order to double his profit?

24. If 12 lb. of strawberries at $5\frac{1}{2}d.$ per lb. and 10 lb. of sugar at $4d.$ per lb. are used to make 20 lb. of jam, find to the necessary farthing the rate per lb. at which the jam must be sold to clear 40 per cent. profit, after deducting $1d.$ per lb. for expenses.

25. If whisky is bought at 30s. per gal., what proportion of water must be added to allow of making 20 per cent. profit by selling at 32s. per gal.?

26. A dishonest shopkeeper uses a weight of 15.3 oz. for 1 lb. What percentage of profit results from his dishonesty?

27. A merchant buys concentrated acid at 3s. 9d. per gal. He dilutes it with $1\frac{1}{2}$ pts. of water to the gallon, and retails the solution at 6d. per pt. Find his percentage of profit.

28. A merchant buys from abroad goods costing £6, and in addition has to pay import duty of 40 per cent. At what price must he sell to gain 25 per cent. on his whole outlay?

29. A shopkeeper buys an article for £12. At what price must he mark it so as to allow him to realize an actual profit of 25 per cent. after allowing 10 per cent. discount for cash?

30. A number of apples were bought at 2 a $1d.$, and the same number at 3 a $1d.$, and the lot was sold at 5 for $2d.$ Find the gain or loss per cent. (*Note.* In solving assume a convenient number of each—say, 30.)

31. If I buy 7 cwt. 1 qr. 21 lb. for £1 14s. 9d., and sell at $8\frac{1}{2}d.$ per st., what is the rate per cent. of profit?

32. A commodity is sold at 55s. $5\frac{1}{2}d.$ per cwt. after passing through the hands of three dealers, each of whom in succession has made 10 per cent. profit on it. What did it cost the first dealer?

33. A man buys a house for £3500. As owner he is obliged to pay annually a property tax of 5s. 5d. in the pound of rental. At what yearly rent must he let the house in order to clear a net profit of 10 per cent. on the purchase price?

34. A shopkeeper adds a pint of water to every gallon of ammonia, and sells the mixture at 1s. 8d. per gal. If he is to make 25 per cent. profit on his dealing, at what rate must he buy the ammonia?

35. A merchant buys goods at £6 13s. per cwt. At what price per lb. must he retail in order to allow purchasers 5 per cent. discount for cash and yet make a profit of 25 per cent.?

36. A dealer buys 24 cycles at a uniform price. He sells 10 at a profit of 15 per cent. What rate of profit must he make on the remainder in order to gain 25 per cent. on the whole transaction?

37. How should a shopkeeper price an article costing him £9 so that it may yield him 60 per cent. profit after allowing his customer discount at the rate of 20 per cent.?

38. A certain import duty is reduced from $5\frac{1}{2}$ per cent. to $4\frac{1}{2}$ per cent. The result is that the total amount of the commodity imported is increased by 30 per cent. Calculate the percentage loss or gain to the revenue.

39. If a selling price of 12 guineas yields a profit of $12\frac{1}{2}$ per cent., what price must be charged in order that the seller may make a profit of 25 per cent.?

40. What price must I pay for a case of 195 apples in order that I may be able to sell them at 3 for twopence and make a profit of 30 per cent.?

41. A bookseller buys magazines at the rate of 13 copies for 12 shillings, and is allowed also 20 per cent. discount. If he wishes to make 25 per cent. gain, calculate to the necessary halfpenny the rate at which he must retail his magazines.

42. By selling an article at 27s. 6d. a dealer makes $7\frac{1}{2}$ per cent. more profit than if he had sold it at 25s. 9d. Calculate the price at which he bought the article.

43. A shopkeeper marks his goods at 20 per cent. above the cost price, but in selling he allows his customers $7\frac{1}{2}$ per cent. discount. Calculate the amount of profit he makes on an article for which he receives £18 10s.

44. A certain import tax is raised from 30 per cent. to 35 per cent. If, moreover, the amount of goods imported is increased by 7 per cent., calculate the percentage gain in the revenue.

(*N.B.* Your answer should be in the form “the revenue is increased by — per cent.”)

45. A shopkeeper wishes to allow his customers $6d.$ in the $\pounds 1$ as discount for cash, and yet to be able to make a net profit of 30 per cent. On what principle must he make up his catalogue sale prices?

46. A dealer buys goods from a maker abroad at a discount of $33\frac{1}{3}$ per cent. off the catalogue price, and has to pay an import duty of 20 per cent. on the net cost. If he is to realize 25 per cent. profit on his outlay, what must he charge for an article priced in the maker's catalogue at $\pounds 15$?

47. A buys an article and sells it to B at 30 per cent. profit. B sells it to C at 10 per cent. loss to himself. C sells it to D for $19s. 6d.$, thereby incurring a loss of 20 per cent. Calculate the price at which A bought the article.

48. A certain country imposes a tax of 30 per cent. on imported machinery. If the costs of collection amount to 5 per cent. of the sums collected, calculate the net percentage of the value of the total machinery import actually received by the State Treasury.

49. In a certain country the import tax is reduced from 30 per cent. to 25 per cent., and in consequence the total amount of goods imported is increased by 5 per cent. By what percentage is the total revenue from the tax increased or decreased?

50. A merchant buying goods abroad finds that in addition to an import tax of $27\frac{1}{2}$ per cent. on the cost he has to pay freight amounting to $4s. 3d.$ per kg. At what rate per kg. must he be able to buy if he is to clear a net profit of 30 per cent. on his outlay by retailing at $221s.$?

CHAPTER XVII

BILLS OF EXCHANGE AND DISCOUNT

A BILL of exchange is an order to pay a sum of money within a given time. The bill is written out (or 'drawn') by the creditor, and agreed to (or 'accepted') by the debtor.

The form of a bill is illustrated below :

[]	£250 0s. 0d. LONDON 20 Sept., 1927 Three months after date pay to my order the sum of two hundred and fifty pounds, value received. To Mr CLAUDE HATTER Buchanan Street Glasgow JOHN SMITHSON
---	--

Accepted at Bank
Payable to Claude Hatter
Claude Hatter

In the above Smithson is the creditor (the drawer of the bill), and Hatter is the debtor (the drawee, or the acceptor).

If Smithson requires cash before December 23 (the legal date of expiry of the bill) he may present the bill to his banker, who, if he considers Hatter to be financially sound, will pay the £250, less discount for the unexpired time at the rate that happens to rule at the time.

DISCOUNT

When a bill is paid before it is due it is customary to offer a less sum than the face value. The money actually paid in such a case is the *present value* of the bill.

The difference between the bill and the present value is the *discount*.

In commercial practice the discount is just the interest on the bill for the number of days by which payment has been advanced, plus 3 extra days. Thus if a bill for £250 is paid 20 days before it is nominally due, commercial discount is just the interest on £250 for 23 days at the rate that happens to rule at the time of payment. The 3 extra days are known as “days of grace,” and can be legally demanded. Commercial discount is known also as banker’s discount.

On the other hand, the *true* present value of, say, a bill for £250, on a date 20 days before it is due, is that sum which, with interest on it at the ruling rate, would amount in 23 days to £250.

The *true* present value in this case at, say, 4 per cent. per annum turns out to be £249 7s. 5d. approximately, the true discount being 12s. 7d., which equals the interest on £249 7s. 5d. The bank discount is approximately 12s. 7¼d., being the interest on £250.

Note. In ordinary commercial transactions banker’s discount only is used. True discount is employed in theoretical calculations, and also in some cases of long public loans.

MODEL EXAMPLES

(a) A 3 months’ bill for £600, drawn on May 3, is discounted on July 10 at 5 per cent. per annum. Find the (commercial) discount.

Due date of payment is August 6.

Unexpired time is 27 days.

$$\begin{aligned}\text{Discount} &= \frac{5}{100} \text{ of } £600 \times \frac{27}{365} \\ &= £\frac{162}{73} \\ &= £2 \text{ 4s. 4d.}\end{aligned}$$

July	21
August	6
	27
	2·219
73)	162
	160
	140
	670

(b) (To be omitted by beginners.) Find the true present value in the above example.

BILLS OF EXCHANGE AND DISCOUNT 185

Interest on £100 for 27 days at 5 per cent.

$$= \frac{27}{365} \text{ of } £5$$

$$= £\frac{27}{73}.$$

Hence £100 is the true present value of £100 $\frac{27}{73}$.

required true present value

$$= \frac{100}{100\frac{27}{73}} \text{ of } £600$$

$$= £597.789$$

$$= £597 \text{ 15s. 9d.}$$

(This means that the true discount would be £2 4s. 3d.)

$$\begin{array}{r} 100 \times 600 \times 73 \\ \hline 7327 \\ 597.789 \\ 7'3'2'7)4380000 \\ \underline{36635} \\ 71650 \\ \underline{65943} \\ 57070 \\ \underline{51289} \\ 5781 \\ \underline{5129} \\ 652 \\ \underline{586} \\ 66 \end{array}$$

(c) A merchant owes £400, due immediately. He agrees to accept a 3 months' bill. For what sum must the bill be drawn (reckoning discount at 4 per cent.)? (Note that 'accept' means 'to become liable for'.)

The face value must be such that, if the bill is discounted immediately, the present value will equal the debt of £400.

Now 3 months' discount on £100 at 4 per cent. = £1. *I.e.*, present value of £100 in these circumstances would be £99.

I.e., present value = $\frac{99}{100}$ of the bill.

Hence if F denotes the required face value,

then $\frac{99}{100}$ of F = 400.

$$F = £\frac{400 \times 100}{99}$$

$$= £404 \text{ os. 10d.}$$

(Verify by discounting this bill.)

$$\begin{array}{r} 9 \overline{)40,000} \\ 11 \overline{)4444.4444} \\ \underline{404.0404} \end{array}$$

(d) A merchant holds a 6 months' bill for £350, dated March 10. When must he present it to receive from the bank £340, reckoning discount at 6 per cent.?

The discount is £10, and the problem is just an easy example in compound proportion—viz., how long will £350 take to earn £10 of interest at 6 per cent. per annum?

Required time

$$= 365 \text{ days} \times \frac{100}{350} \times \frac{10}{6}$$
$$= 174 \text{ days.}$$

6 7300

7 1216 +

174

March21 days

April30

May31

June30

July31

August31

September13,,

187 days

Now, from date of drawing to due date is a period of 187 days. But bill should be presented 174 days before due—i.e., 13 days after date of drawing—i.e., March 23.

(The student should verify by discounting the bill on this date.)

EXERCISES XVIIIA

(On finding the discount, or the present value)

(Note. Banker's discount is to be understood, except where true discount is expressly stated.)

1. John Smithson sells goods to Claude Hatter, who asks Smithson to draw on him at 3 months. The following is the bill :

Stamp

£436 os. od.

Accepted at Bank

Payable to my order

24 April, 1930

London

Claude Hatter

Three months after date pay to my order the sum of four hundred and thirty-six pounds value received.

JOHN SMITHSON

To Mr CLAUDE HATTER

Buchanan Street

Glasgow

If Smithson discounts the bill on June 10, what sum should he receive, the rate being 4 per cent.?

2. In the following assume the bill to be that shown in Question 1 above, and calculate the discount in each case:

- (a) Bill discounted April 30, rate $3\frac{1}{2}$ per cent.
- (b) Bill discounted June 21, rate $4\frac{1}{2}$ per cent.
- (c) Bill discounted May 18, rate 4 per cent.
- (d) Bill discounted July 3, rate 3 per cent.
- (e) Bill discounted June 5, rate 5 per cent.
- (f) Bill discounted June 30, rate $5\frac{1}{2}$ per cent.
- (g) Bill discounted May 31, rate $4\frac{1}{2}$ per cent.
- (h) Bill discounted April 30, rate 4 per cent.
- (i) Bill discounted May 4, rate $3\frac{1}{2}$ per cent.

EXERCISES XVII_B

(On finding the face value of a bill)

1. A debt of £350 is payable immediately. The debtor offers to accept a 3 months' bill. For what sum should the bill be drawn, assuming a discount rate of 4 per cent. per annum?

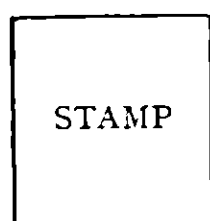
2. Calculate the face value of a '2 months' bill which if discounted on the date of acceptance would yield £140. Assume the discount rate to be 6 per cent. per annum.

3. In the following questions calculate the face value of the bill from the particulars given:

DATE OF DISCOUNTING	NOMINAL DATE OF MATURITY	PRESENT VALUE (i.e., ON DATE OF DISCOUNTING)	RATE OF DISCOUNT
		£ s. d.	
(a) January 20, 1931	November 11, 1931	296 3 3	4 per cent.
(b) November 22, 1928	January 4, 1929	1690 6 3	4
(c) November 4, 1928	February 26, 1929	199 16 1	4
(d) May 8, 1930	June 17, 1930	1046 17 10	$4\frac{1}{2}$
(e) November 24, 1930	December 7, 1930	746 6 8	$3\frac{1}{2}$
(f) October 6, 1929	January 18, 1930	692 18 6	4
(g) March 8, 1929	June 2, 1929	749 1 6	4
(h) September 11, 1929	November 9, 1929	1454 8 0	4

EXERCISES XVIIc

1. What discount at 4 per cent. should be allowed on a bill for £375 settled 40 days before it is legally due?
2. Find the discount on a bill for £280, due nominally on August 23, negotiated July 15 at 3 per cent. (*Note.* A bill due nominally on August 23 is due legally on August 26.)
3. A 3 months' bill for £620 is drawn on May 10. If it is discounted at 4 per cent. on July 4, what sum will the holder receive?
4. Calculate the discount on a bill of £340 of 3 months' currency, dated October 24, discounted December 6 at 4 per cent.
5. Find the discount on a 2 months' bill for £250, dated June 13, discounted June 26 at $3\frac{1}{2}$ per cent.
6. Find the present value on September 25 of a 4 months' bill for £450, drawn June 1, the discount rate being 4 per cent.
7. A 3 months' bill for £340 dated June 11 is discounted on August 21. What sum does the holder receive if the discount rate is $6\frac{1}{2}$ per cent.?
8. If the following bill is discounted on November 3 at $5\frac{1}{2}$ per cent., what will the holder receive?



£430 os. od.

Accepted at Bank
Payable to Bank
Parish & Co. Ltd. London
Claude Hatter

LONDON

Sept. 20, 1930

Three months after date pay to my order the sum of four hundred and thirty pounds, value received.

JOHN SMITHSON

To Mr CLAUDE HATTER
 Buchanan Street
 Glasgow

9. Calculate the discount on a 6 months' bill for £335, drawn and accepted May 20, discounted July 10, the rate being $6\frac{1}{2}$ per cent.

10. Find the present value of a 4 months' bill for £275, drawn on April 10, discounted on May 17, the rate being 6 per cent.

11. For what sum must a 3 months' bill be drawn to meet an immediate debt of £200, allowing discount at 4 per cent.? (Note that it is entirely wrong to find 4 per cent. of £200 for 3 months, since £200 is the present value. Such a calculation would give true discount, which is not employed in commercial life. See Model Example (c).)

12. Find the present value of a 4 months' bill for £700, drawn October 25, discounted on November 14 at 5 per cent.

13. Find the discount on a 6 months' bill for £320, drawn March 10, discounted June 13 at 5 per cent.

14. What sum payable now is sufficient to discharge a debt of £420 due 4 months hence, allowing interest at 6 per cent.?

15. A merchant holds a 3 months' bill for £550 dated September 11. On what date must he discount it to receive £545 for it, reckoning interest at 6 per cent.?

16. Find the present value on July 17 of a 3 months' bill for £440, dated June 20, the discount rate being $4\frac{1}{2}$ per cent.

17. For what sum should a 20 days' bill be drawn to meet an immediate debt of £330, the discount rate being 4 per cent.?

18. Calculate the discount on a 4 months' bill for £140, drawn October 10, discounted November 15, the rate being 5 per cent.

19. A 6 months' bill for £625, dated April 28, is discounted on July 5, the rate being $4\frac{1}{2}$ per cent. What sum does the holder receive?

20. Find the discount on a bill of £655 13s., of 4 months' currency, drawn August 14, discounted October 3 at 4 per cent.

21. Find the present worth of a 6 months' bill for £294 6s. 4d., drawn March 6, discounted June 11 at 6 per cent.

22. To meet an immediate debt of £597 7s. 6d. a 3 months' bill is accepted by the debtor. For what sum should it be drawn if the discount rate is $4\frac{1}{2}$ per cent.?

23. A 6 months' bill for £388 2s. 6d., drawn December 8,

1922, is discounted March 25, 1923, at 6 per cent. What sum does the holder receive?

24. To meet an immediate debt of £75 18s. 10d. a 100 days' bill is agreed to. What must be the face value of the bill if the discount rate is 5 per cent.? (Days of grace not to be reckoned.)

25. For what sum must a 4 months' bill be drawn to meet an immediate debt of £470, assuming the discount rate to be $5\frac{1}{2}$ per cent.?

26. A debt of £535 is due immediately. If a 60 days' bill is offered, what should be the face value of the bill if the discount rate is $4\frac{1}{2}$ per cent.? (Neglect days of grace.)

27. I hold a 6 months' bill for £600 dated March 10. On what date must I discount it in order to receive £590 for it, allowing the rate to be 5 per cent.?

28. A 6 months' bill for £750 dated March 3 is discounted on June 25. If I receive £741 for it, calculate the discount rate per cent.

29. Find the present value on July 10 of a 90 days' bill for £980 dated June 1, discount rate being 6 per cent.

30. What sum payable now is sufficient to discharge a debt of £670 due 2 months hence, allowing that the rate is $4\frac{1}{2}$ per cent.?

31. A merchant owing £386 due immediately accepts a 4 months' bill. For what sum should the bill be drawn, allowing discount at 4 per cent.?

32. What sum paid immediately will be sufficient to discharge a debt of £384 due 60 days hence, if discount rate is $4\frac{1}{2}$ per cent.? (Days of grace not to be reckoned.)

33. Calculate the discount on a 6 months' bill for £600, drawn April 10, discounted August 14, assuming the discount rate to be 4 per cent.

34. The holder of a 6 months' bill for £850, drawn May 1, discounts it on June 11 at $4\frac{1}{2}$ per cent., and deposits the proceeds in a bank paying 5 per cent. How much does he lose or gain by this policy?

35. From the following particulars calculate the discount: Amount of Bill £735; date July 11, 1930; currency 4 months'

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date ; date of discounting September 12, 1930 ; rate of discount $4\frac{1}{2}$ per cent.

36. A bill for £1080 is discounted 100 days before maturity. If the holder receives for it £1071, what discount rate has been observed?

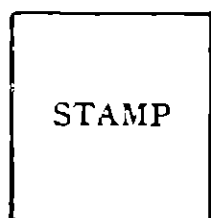
37. A man holds a 6 months' bill, dated June 12, 1930, for £860. On what date must it be discounted in order to realize £850, assuming a discount rate of 5 per cent.?

38. For what sum should a 4 months' bill be drawn to meet an immediate debt of £1250, if the discount rate be $4\frac{1}{2}$ per cent.?

39. Calculate the present value on April 10 of a 3 months' bill for £560 dated March 2, assuming the discount rate to be $3\frac{1}{2}$ per cent.

40. For what sum should a 6 months' bill be drawn to meet an immediate debt of £880 if the discount rate be $5\frac{1}{2}$ per cent.?

41. Assuming the following bill, calculate the discount in each of the questions (a) to (j).



£780 os. od.

LONDON

March 28, 1931

Six months after date pay to my order the sum of seven hundred and eighty pounds, value received.

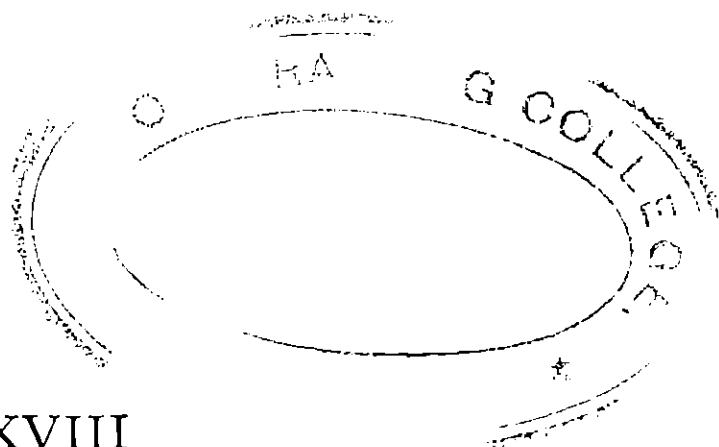
JOHN SMITHSON

To Mr CLAUDE HATTER
Buchanan Street
Glasgow

- (a) Discounted August 11, rate $3\frac{1}{2}$ per cent.
- (b) Discounted July 21, rate 3 per cent.
- (c) Discounted June 9, rate 4 per cent.
- (d) Discounted April 24, rate $4\frac{1}{2}$ per cent.
- (e) Discounted May 17, rate $5\frac{1}{2}$ per cent.
- (f) Discounted June 30, rate 5 per cent.

- (g) Discounted July 10, rate $4\frac{1}{2}$ per cent.
- (h) Discounted August 31, rate 4 per cent.
- (i) Discounted May 4, rate $3\frac{1}{2}$ per cent.
- (j) Discounted April 3, rate $2\frac{1}{2}$ per cent.

42 (a)–(j). Work as examples in true discount Questions (a)–(j) above.



CHAPTER XVIII

STOCKS

TERMS WITH WHICH THE STUDENT SHOULD BECOME FAMILIAR

Shares. The definite portions into which the capital of a company is divided for purposes of buying and selling. Thus, if there are, say, £10 shares, then an investor may purchase a number of such shares, but could not, for example, purchase £73 of shares.

Stock. In Government, Corporation, and other similar funds one may invest any sum of money at all. Thus, in Consols (which are various Government stocks issued at different times, but afterward merged or 'consolidated' into one single stock) one may invest any sum without fractions of a penny.

Inscribed Stock. A particular class of stock, mostly of Government issue—*e.g.*, Consols and certain forms of War Loan. No certificate of possession is issued to a holder of such stock, but a register is kept at the bank of issue wherein names of holders and the respective amounts held are *inscribed*. Sale of such stock necessitates the personal attendance of the holder at the office of the bank of issue (usually the Bank of England).

Dividend. The amount of profits *divided* among the shareholders as interest on their investment.

Cum div. (meaning "with dividend"). The term implies that the purchaser of the stock is entitled to receive the dividend which is almost due.

Ex div. (meaning "without dividend"). The term implies that the dividend when due goes to the seller and not to the new purchaser.

Debenture. A stock having first claim on the assets of a company for payment of both capital and interest.

Preference. A stock that ranks for dividend before ordinary shares.

Brokerage. The fee charged by the agent for buying or selling stock. For example, $\frac{1}{8}$ per cent. amounts to 2s. 6d. for every £100 of *stock* negotiated, regardless of the price.

Note. Where two prices of a stock are quoted—e.g., “ $4\frac{1}{2}$ per cent. Treasury Bonds, $99\frac{1}{8}$ — $99\frac{3}{8}$ ”—the lower is the price at which the dealer (or jobber, as he is sometimes termed) is willing to buy, while the higher is his selling price.

MODEL EXAMPLES

(a) How much 3 per cent. stock quoted at $80-80\frac{7}{8}$ can I buy for £360, the brokerage being $\frac{1}{8}$ per cent.?

Price of £100 of stock = $\pounds(80\frac{7}{8} + \frac{1}{8}) = \pounds 81$.

Hence amount of stock bought for £360

$$\begin{array}{l}
 = \pounds 100 \times \frac{360}{81} \\
 = \pounds 444 \text{ 8s. 10d.}
 \end{array}
 \quad \left| \begin{array}{r}
 \pounds \\
 9 \overline{) 36000} \\
 9 \overline{) 4000} \\
 \hline
 444 \text{ 8s. 10d.}
 \end{array} \right.$$

(b) A 5 per cent. stock is bought at $83\frac{1}{2}$. What rate per cent. does the holder get for his money, allowing brokerage at $\frac{1}{2}$ per cent.?

The £100 of stock costs the buyer $\pounds 83\frac{1}{2} + \frac{1}{2} = \pounds 84$.

Hence he receives £5 on an investment of £84.

on £100 he receives $\pounds 5 \times \frac{100}{84}$

$$\begin{array}{l}
 = \pounds 5.95.
 \end{array}
 \quad \left| \begin{array}{r}
 7 \overline{) 500} \\
 12 \overline{) 71.42} \\
 \hline
 5.95
 \end{array} \right.$$

I.e., the yield is nearly 6 per cent.

(c) A man decides to buy 4 per cent. War Loan when it is yielding actually 5 per cent. on the cash paid. At what price must he buy the stock? (Neglect brokerage.)

If P is the price, then

$$\begin{aligned}
 \frac{5}{100} \text{ of } P &= 4. \\
 P &= 4 \times \frac{100}{5} \\
 &= 80.
 \end{aligned}$$

I.e., he must buy when the 4 per cent. War Loan is quoted at 80. The pupil should verify this.

(*Note.* If brokerage of $\frac{1}{8}$ per cent. is to be considered, then the purchaser should buy when stock is at $79\frac{7}{8}$.)

(*d*) Which is the better investment— $2\frac{1}{2}$ per cent. Consols at 54, or 4 per cent. Victory Bonds at 93 (reckoning brokerage at $\frac{1}{8}$ per cent.)?

Reduce each to a percentage interest on cash.

Consols

Interest on $\pounds 54\frac{1}{8} = \pounds 2\frac{1}{2}$.

$$\begin{aligned}\pounds 100 &= \pounds \frac{5}{2} \times \frac{100}{54\frac{1}{8}} \\ &= \pounds \frac{5}{2} \times \frac{100 \times 8}{433} \\ &= \pounds 4.62 \text{ approx.}\end{aligned}$$

$$\begin{array}{r} 4.62 - - \\ 433 \overline{) 2000} \\ \underline{1732} \\ 2680 \\ \underline{2598} \\ 820 \end{array}$$

Victory Bonds

Interest on $\pounds 93\frac{1}{8} = \pounds 4$.

$$\begin{aligned}\pounds 100 &= \pounds 4 \times \frac{100}{93\frac{1}{8}} \\ &= \pounds 4 \times \frac{100 \times 8}{745} \\ &= \pounds 4.29 \text{ approx.}\end{aligned}$$

$$\begin{array}{r} 4.29 \\ 745 \overline{) 3200} \\ \underline{2980} \\ 2200 \\ \underline{1490} \\ 7100 \end{array}$$

I.e., Consols give .33 per cent. better yield on the money invested.

(*Note.* .33 = $\frac{1}{3}$ approx.—*i.e.*, the interest is higher by about 6s. 8d. per cent.)

ORAL EXERCISES

(*Brokerage is not to be reckoned in these exercises*)

1. How much 5 per cent. War Loan at 95 can I buy for $\pounds 380$?

2. How much $3\frac{1}{2}$ per cent. Conversion Stock at 70 can I buy for $\pounds 210$? How much for $\pounds 245$?

3. What will it cost me to acquire $\pounds 600$ 5 per cent. War Loan at 95?

4. A man has $\pounds 1000$ to invest. How much 6 per cent. stock at 125 can he buy?

5. Calculate the price of $\pounds 750$ 4 per cent. stock at 80.

(*Note.* This is confusing for the beginner. He must observe carefully that “ $\pounds 750$ stock” means $7\frac{1}{2}$ hundred-pound shares; this is why we are able to speak of the *price* of $\pounds 750$ stock.)

6. What amount of interest will a man receive yearly if he buys £750 4 per cent. stock at 80?

7. If I buy £100 4 per cent. stock at 80, am I receiving £4 on £100 of cash or £4 on £80 of cash?

8. What is the price of a 4 per cent. stock if I can buy £600 stock for £450?

9. What income shall I derive from holding £800 5 per cent. stock at 95?

10. What income shall I derive if I invest £380 in a 5 per cent. stock at 95?

EXERCISES XVIII_A

(Buying and selling stock)

1. How much $4\frac{1}{2}$ per cent. stock, quoted at $93-93\frac{5}{8}$ can I purchase for £700, reckoning brokerage at $\frac{1}{8}$ per cent.?

2. A man sells out £500 $4\frac{1}{2}$ per cent. stock, quoted at $93-93\frac{5}{8}$, brokerage being $\frac{1}{8}$ per cent. What sum will he realize?

3. A speculator buys £700 of $3\frac{1}{2}$ per cent. stock, quoted at $87-88$. What will it cost him, brokerage being $\frac{1}{8}$ per cent.?

4. Calculate the loss if the speculator in the preceding question has to sell when the price has fallen to $85\frac{1}{2}-86\frac{1}{4}$ (brokerage $\frac{1}{8}$ per cent.). Can you find a short way of answering this question without working out the answer to Question 3?

5. What will it cost to purchase £8000 of 4 per cent. stock quoted at $95\frac{1}{2}-96\frac{1}{4}$, brokerage being $\frac{1}{8}$ per cent.?

6. How much 4 per cent. stock at $95\frac{1}{2}-96\frac{1}{4}$ can I buy for £1285, brokerage being $\frac{1}{8}$ per cent.?

7. A man sells £7000 of 3 per cent. stock, quoted at $75\frac{1}{8}$, and invests the proceeds in $3\frac{1}{2}$ per cent. stock standing at $87\frac{3}{8}$, brokerage being $\frac{1}{8}$ per cent. in both transactions. How much of the latter stock does he acquire?

8. How many £1 'savings certificates', at 16s. each can I buy with the proceeds of the sale of £175 5 per cent. Conversion Loan at 106? Brokerage, applying to the sale only, of course, is $\frac{1}{8}$ per cent.

9. How much 4 per cent. Consolidated Loan at $92\frac{3}{4}$ can I

buy with the proceeds of the sale of £3715 $4\frac{1}{2}$ per cent. War Loan at $100\frac{1}{8}$, brokerage being $\frac{1}{8}$ per cent. in both cases?

10. Calculate the total cost of purchasing £350 of Consols at $58\frac{1}{16}$ and £1250 4 per cent. Victory Bonds at $96\frac{7}{8}$, allowing for brokerage of $\frac{1}{8}$ per cent. in both cases.

EXERCISES XVIII_B

(Percentage yield on money invested)

Calculate correct to two decimal places the percentage return on money invested in each of the following cases (brokerage $\frac{1}{2}$ per cent.):

1. London C.C., 3 per cent., at $68\frac{7}{16}$.
2. City of London, 3 per cent., at $77\frac{3}{4}$.
3. Belfast, 5 per cent., at $103\frac{1}{2}$.
4. Bournemouth, 6 per cent., at 106.
5. Coventry, 6 per cent., at $107\frac{1}{2}$.
6. Glasgow, 5 per cent., at $104\frac{1}{2}$.
7. Manchester, 3 per cent., at $67\frac{1}{2}$.
8. Liverpool, $3\frac{1}{2}$ per cent., at 79.
9. Ipswich, 5 per cent., at $105\frac{1}{4}$.
10. Newcastle, $3\frac{1}{2}$ per cent., at $97\frac{1}{2}$.

EXERCISES XVIII_C

1. In each of the following cases calculate the actual rate of interest per cent. on the purchase price, allowing $\frac{1}{8}$ per cent. brokerage in each case:

- (a) Conversion Loan, $3\frac{1}{2}$ per cent., at $74\frac{3}{8}$.
- (b) War Loan (Insc.), $3\frac{1}{2}$ per cent., at $99\frac{7}{16}$.
- (c) War Loan (Insc.), $4\frac{1}{2}$ per cent., at $96\frac{1}{16}$.
- (d) Funding Loan (Insc.), 4 per cent., at $86\frac{1}{16}$.
- (e) Treasury Bonds, $4\frac{1}{2}$ per cent., at $99\frac{1}{8}$.

2. Which is the better investment, a 4 per cent. stock at 93, or a 5 per cent. stock at 112? (Neglect brokerage.)

3. A man buys a $3\frac{1}{2}$ per cent. stock at 78, holds it until he draws a year's dividend, then sells at 84. What profit per

cent. does he make on his outlay? (Reckon $\frac{1}{2}$ per cent. brokerage on each transaction—*i.e.*, on purchase and sale.)

4. At what price must $4\frac{1}{2}$ per cent. Treasury Bonds be purchased to yield 5 per cent. interest on the purchase price? (Reckon brokerage at $\frac{1}{8}$ per cent.)

5. Which is the better investment, a 4 per cent. stock (free of income tax) at 96 or a 5 per cent. stock (subject to tax) at 90? Express the difference as a percentage on money invested. (Reckon income tax as 4s. in the £ on income, and neglect brokerage.)

6. The same problem as in the preceding question, reckoning brokerage in each case at $\frac{1}{2}$ per cent.

7. How much money must I invest in New South Wales 3 per cent. stock at 88 to ensure an income of £180? (Reckon brokerage at $\frac{1}{4}$ per cent.)

8. Find the necessary sum if the income in the preceding question is to be £180 after paying income tax at 4s. in the £.

9. The capital of a company consists of (a) 10,000 ordinary shares of £1 each and (b) 25,000 debentures at $4\frac{1}{2}$ per cent. If the available profit for a year amounts to £2275, what is the maximum dividend that can be declared on the ordinary shares?

10. What is the income from *buying* £1000 $5\frac{1}{2}$ per cent. stock at 95? What is the income from *investing* £1000 in $5\frac{1}{2}$ per cent. stock at 95 (brokerage $\frac{1}{2}$ per cent.)?

11. What sum must a man expend to acquire £10,000 4 per cent. stock at 103, reckoning brokerage at $\frac{1}{8}$ per cent.?

12. Which is the more profitable stock to acquire, $5\frac{1}{2}$ per cent. at 108 or 5 per cent. at 99, allowing for brokerage at $\frac{1}{2}$ per cent. in each case?

13. What sum does a man realize by selling out £12,500 stock at $75\frac{1}{4}$, allowing brokerage at $\frac{1}{2}$ per cent.?

14. I hold £10,000 5 per cent. War Loan, which is quoted at 102–102 $\frac{1}{8}$. What change of income will be involved by selling out and investing in 4 per cent. Consols at $88\frac{1}{8}$ – $88\frac{1}{4}$? (Neglect brokerage.)

15. Calculate the change in income resulting from selling out £7000 5 per cent. National War Bonds, quoted at $104\frac{1}{16}$ –

$105\frac{1}{16}$, and investing in 4 per cent. War Bonds, quoted at $99\frac{7}{8}$ – $100\frac{1}{8}$, brokerage in each case being $\frac{1}{8}$ per cent.

16. A man holds £12,000 4 per cent. Victory Bonds. He sells when the rate quoted is $94\frac{3}{4}$ – $94\frac{1}{16}$ and invests in $4\frac{1}{2}$ per cent. Conversion Loan at $98\frac{5}{8}$ – $98\frac{3}{4}$, the brokerage in each case being $\frac{1}{8}$ per cent. Calculate the resulting difference in his income.

17. When the £5 shares of a certain 7 per cent. preference stock are £4 12s.–£4 13s. a man buys 1000 shares. After holding for a year, he sells when the shares stand at £4 18s.–£4 19s. 6d. What percentage of profit has the whole transaction secured for him? (Reckon brokerage at 1 per cent.)

18. The Government of a certain country converts a 5 per cent. War Loan to a 4 per cent. Loan, allowing holders £105 of the new stock for every £100 of the old stock. If the total of the original 5 per cent. stock amounted to £1,000,000, what annual saving to the country is effected if the entire stock is converted?

19. At what price must I purchase a $4\frac{1}{2}$ per cent. stock to secure $5\frac{1}{2}$ per cent. interest on my investment, allowing brokerage at $\frac{1}{4}$ per cent.?

20. What sum must be invested in 4 per cent. Consols standing at $88\frac{1}{8}$ – $88\frac{1}{4}$ to secure an annual income of £400 after paying income tax at 4s. in the £1? (Neglect brokerage.)

21. Show that, reckoning the rate of interest as $4\frac{1}{2}$ per cent., the price of a 10 per cent. bond (for £100) redeemable at par one year hence should be £105 5s. 3d. approximately. (Neglect brokerage.)

22. A man sells his holding in a $3\frac{1}{2}$ per cent. stock standing at $75\frac{1}{8}$ – $75\frac{3}{8}$, realizing £9000. He invests the proceeds in a $4\frac{1}{2}$ per cent. stock standing at $96\frac{1}{4}$ – $96\frac{5}{8}$, brokerage in each case being $\frac{1}{8}$ per cent. Calculate the consequent change in his income.

23. A man wishes to secure an income of £500 by investing £10,000 in a $4\frac{1}{2}$ per cent. stock. At what price should he purchase the stock, reckoning brokerage at $\frac{1}{8}$ per cent.?

24. A 7 per cent. bond (for £100) is redeemable at par in a year from now. What price should I pay for it, assuming

that money is worth 4 per cent. per annum? (Neglect brokerage.)

25. A corporation offers £10 shares at par for public subscription, but calls up only £7 10s. on each share. If a dividend of 15s. per share is paid at the end of a year, calculate (a) the declared rate per cent. of dividend and (b) the actual rate of interest received by a subscriber.

26. What price should be given for 6 per cent. bonds redeemable at par three years hence, allowing simple interest at 5 per cent. per annum? (Neglect brokerage.)

27. Work the problem given in the preceding question allowing compound interest at 5 per cent. per annum, convertible yearly.

28. At what price must I buy a 4 per cent. stock in order that I may receive exactly $3\frac{1}{2}$ per cent. interest on my investment, free of income tax at 5s. in the £1? (Reckon brokerage at $\frac{1}{8}$ per cent.)

29. I invest £6000 in $3\frac{1}{2}$ per cent. stock at $74-74\frac{7}{8}$ and the same sum in a $4\frac{1}{2}$ per cent. stock standing at $84\frac{1}{4}-84\frac{3}{8}$. Calculate the total income therefrom, neglecting brokerage.

30. In the case of the preceding question, if the first stock rises $\frac{1}{8}$ and the second falls $\frac{1}{8}$, calculate the sum realized by selling out, reckoning brokerage at $\frac{1}{8}$ per cent.

31. A man invests £760 in stock at 95. At the end of the year his dividend, free of income tax at 4s. in the £, amounts to £35 4s. What is the rate of dividend per cent.? (Neglect brokerage.)

32. A man invests £760 in a $5\frac{1}{2}$ per cent. stock. At the end of the year his dividend, free of income tax at 4s. in the £, amounts to £35 4s. At what price did he buy the stock? (Neglect brokerage.)

33. A man proposes to invest his money in a 4 per cent. stock at $94\frac{1}{2}$. He is advised by a friend to choose rather a $3\frac{1}{2}$ per cent. stock at $89\frac{1}{4}$. By what percentage would his annual income be increased or diminished if he were to choose the second stock instead of the first? (Neglect brokerage.)

34. If I buy $4\frac{1}{2}$ per cent. stock at 91, hold it for a year and draw the dividend, then sell at $92\frac{3}{8}$, what percentage return

on my outlay have I received? (Reckon brokerage at $\frac{1}{8}$ per cent. on both purchase and sale.)

35. At what price must I buy $4\frac{1}{2}$ per cent. bonds (£100) redeemable at par one year from now in order to make a profit of 8 per cent. on my outlay? (Allow brokerage at $\frac{1}{2}$ per cent.)

36. I wish to purchase sufficient $4\frac{1}{2}$ per cent. stock, quoted at $93\frac{1}{2}$ – $94\frac{1}{2}$, to ensure an income of £250. If I can do this only by disposing of another stock which is quoted at 105–106, how much of this latter stock must I dispose of? (Neglect brokerage.)

37. Work out the preceding question assuming brokerage at $\frac{1}{2}$ per cent. in each transaction.

38. Certain 7 per cent. bonds are redeemable at par in two years from now. Assuming that a man has his money at present in the bank earning 3 per cent. interest per annum, calculate the price at which he may purchase the bonds without loss or gain. Reckon simple interest throughout.

39. Work out the preceding question reckoning compound interest throughout.

40. A bank issues £50 shares at par, but only £20 per share is called up. At the end of the financial year an investor buys some of these shares at $32\frac{1}{2}$ “*cum div.*,” and presently the bank declares a dividend of $5\frac{1}{2}$ per cent. on the face value of the shares. Calculate the rate per cent. of yield to the investor.

THE CHAIN RULE—AVERAGES—THE RULE OF MIXTURES

THIS is simply a convenient mechanical application of the principle of *direct* proportion in problems involving quantities which are *continuously* related—*i.e.*, as the links of a chain.

MODEL EXAMPLES

An easy way is to begin with the *question*, thus:

Notice that shillings, lb., grammes, francs, appear on both sides of our statement.

$$\begin{array}{r} 2.18 \\ 125 \overline{) 272.4} \\ \underline{250} \\ 22.4 \\ \underline{125} \\ 99 \end{array}$$

(b) Find a multiplier that will convert a price given in shillings per yard into francs per metre. (Take £1 = 125 francs.)

Suppose a given price of p shillings per yard. Then

? francs correspond to 1 metre
 1000 metres „ 1095 yds.
 1 yd. corresponds to p shillings
 20 shillings correspond to 125 francs

$$\text{Required number of francs} = \frac{1 \times 1095 \times p \times 125}{1000 \times 20}$$

$$= p \times \frac{219}{32}.$$

$$\begin{array}{r} 4 \overline{) 219} \\ 8 \overline{) 54 \cdot 75} \\ \hline 6 \cdot 84375 \end{array}$$

Hence the required multiplier is $\frac{219}{32}$, or 6.84375.

EXERCISES XIX_A

1. If 1 kilometre = 1095 yds. and 6080 ft. = 1 nautical mile, how many kilometres are in 1 league, which is 3 nautical miles?
2. A locomotive travels at the rate of 45 miles per hour. Express this speed in metres per minute.
3. A man requires 625 sq. yds. of carpet, which he orders from a foreign agent whose quotation is 65 francs per sq. m. Calculate the total cost in British money. (Take the exchange rate as 125 francs to £1 sterling.)
4. Express a rate of 10s. 6d. per gal. in francs per litre. (Assume that 125 francs = £1 and 1 litre = 1.76 pts.)
5. Find (correct to one decimal place) a multiplier to convert a rate of pence per yard to francs per kilometre, assuming the exchange to be 120 francs to £1. (See Model Example (b).)
6. According to quotations on September 24, 1928, rates of exchange were 20.4 Reichsmarks = £1 sterling = 124 francs. Hence find the equivalent of 6 marks per kilogramme in French money.
7. If 1 rupee is worth 1s. 6d., and £1 purchases 4.85 dollars, how many dollars should be exchanged for 1000 rupees?
8. If 50 electric lamps burn electric current costing 1s. 3d.

in the same time as 100 gas-lamps burn gas costing 2s. 1d., how many of the former lamps must be installed to replace 36 gas-lamps without altering the cost of burning?

9. A piece of land in France is sold at 12,350 francs per hectare. Express this price in British money per acre, assuming that 1 hectare = 2.47 acres and that the exchange is 125 francs to £1.

10. If 4.85 dollars = £1 = 20.43 marks, and 1 kilogramme = 2.2 lb. (approximately), express in dollars per lb. a price of 22.7 marks per kilogramme.

11. A motor-car runs 27 miles to a gallon of petrol. If petrol costs 1s. 6½d. per gal., what should be the cost of running 450 miles?

12. If lead is 11.4 times as heavy as water, and 1 cubic foot of water weighs 62½ lb., what should be the weight of 3 cu. yds. of lead?

13. Assuming that 1 gal. measures approximately $\frac{4}{5}$ cu. ft., that 1 cu. ft. of water weighs 62½ lb., that 10 gal. of alcohol weigh as much as 8 gal. of water, calculate the weight of 60 gallons of alcohol.

14. If 1 metre = 39.4 in. and £1 = 124.2 francs, what should it cost me in sterling to procure 45 yds. of a material which is quoted at 9.85 francs per metre?

15. If 1 litre = 1.76 pts. and £1 = 20.45 marks, what should it cost me in sterling to procure 8 gal. of a substance which is quoted at 17.5 marks per litre?

16. Work out Question 14 assuming that in addition to the quoted price of 9.85 francs it is necessary to add 20 per cent. extra to cover transport costs.

17. Express a price of 11.65 lire per kilogramme in British money per lb., assuming that 1 kilogramme = 2.2 lb. and that £1 = 89.2 lire.

18. Express a rate of 225 metres per minute in miles per hour, assuming that 1 kilometre = 4.98 fur.

19. A dealer buys at 88 francs per decalitre—*i.e.*, 10 litres. Calculate in British money the price per gallon at which he must sell to make a profit of 30 per cent. (Take £1 = 123.2 francs and 8 gal. = 36.4 litres.)

20. Work out the preceding question assuming that freight increases the cost by 40 per cent., and that the dealer wishes to make 20 per cent. profit on his outlay.

AVERAGES

The average of a series of values is that value which may be substituted for each without affecting the total. Thus, if we know that the average weight of 8 oarsmen is 10 st. 9 lb., then we know that while some of the men may be heavier and some less, yet their combined weight must be 10 st. 9 lb. $\times$ 8—*i.e.*, 85 st. 2 lb.

MODEL EXAMPLES

(a) What will be the total sum contributed to the school fund by a class of 60 boys if the average contribution is 1s. 6d.?

$$\begin{aligned}\text{Total sum} &= 1s. 6d. \times 60 \\ &= 90s.\end{aligned}$$

(b) 12 men work at a job for 7 hrs., 4 other men for 6 hrs., and finally 1 other man for 8 hrs. Calculate the average time spent at the job by each of the 17 men.

$$\begin{array}{rcll}12 \text{ men in 7 hrs.} & \text{give} & 84 \text{ hrs.} & \text{'work.} \\ 4 \text{ ,, } 6 & & \text{,, } 24 & \\ 1 \text{ man ,, } 8 & & \text{gives } 8 & \end{array}$$

Hence 116 hrs.' work is contributed by the 17 men.

$$\begin{aligned}\text{Hence the average time for each man} &= \frac{1}{17} \text{ of } 116 \text{ hrs.} \\ &= 6\frac{1}{17} \text{ hrs.}\end{aligned}$$

(c) In a club there are 143 full members, 27 half-members, and 10 honorary members. In arranging to raise funds by means of a subscription the secretary suggests that on an average the full members might contribute 7s. 6d. per head, the half-members 2s. 6d. per head, and the honorary members £1 1s. per head. What sum would he raise if his expectations were realized? Also find what average contribution per head over all members would amount to the same total.

From 143 full members at £
7s. 6d. per head, 53·625

From 27 half-members at
2s. 6d. per head, 3·375

From 10 honorary members
at £1 1s. per head 10·5
£67·5

Total sum suggested = £67 10s.

Total number of members of all classes
= 180

average contribution over all

$$= £ \frac{67·5}{180}$$

$$= £·375$$

$$= 7s. 6d.$$

$$\begin{array}{r} £ \\ 143 \\ \hline 5s. \quad 35·75 \\ 2s. 6d. \quad 17·875 \\ \hline 53·625 \end{array}$$

$$\begin{array}{r} 3·675 \\ 6 \overline{) 2·25} \\ \hline ·375 \end{array}$$

(d) 12 men work at a job for 7 hrs., 4 men for 6 hrs., and finally 1 man for 8 hrs. If £9 16s. is paid for the whole work, find the average rate per head per hour.

12 men in 7 hrs.

contribute 84 hrs.' work.

4 men in 6 hrs.

contribute 24 hrs.' work.

1 man in 8 hrs.

contributes 8 hrs.' work.

Total 116

$$\text{rate per head per hour} = £ \frac{9·8}{116}$$

$$= £·084$$

$$= 1s. 8\frac{1}{4}d.$$

$$\begin{array}{r} ·084 \\ 116 \overline{) 9·80} \\ \hline 520 \\ \hline 56 \end{array}$$

ORAL EXERCISES

1. Ten people are to make speeches at a meeting. If the chairman expresses the hope that the speeches will not exceed

ten minutes on an average, what is the total time he proposes to allow for speech-making?

2. If the average annual rainfall at a certain place is 31.5 in., what should be the total rainfall for 5 yrs.?

3. A lady proposes to entertain a party of 23 people to afternoon tea. If she decides to allow on an average $3\frac{1}{2}$ cakes to each person, how many cakes altogether must she provide?

• 4. A boy walks 15 miles on Sunday, 12 on Monday, 18 on Tuesday, 15 on Wednesday, 10 on Thursday, 14 on Friday, and 11 on Saturday. Calculate his average daily distance for the week.

5. In a game of golf a player takes 2 strokes at one of the holes, 3 strokes at each of four holes, 4 strokes at each of nine holes, 5 strokes at each of three holes, and 7 strokes to one of the holes. Calculate the total number of strokes for the eighteen holes, and also the average per hole.

6. What is the average length of a calendar month?

• 7. A book contains 240 pages. If the average number of words on a page is 250 words, calculate the total number of words in the book.

8. Calculate the average height of 6 men from the following particulars: 3 of them measure 5 ft. 6 in. each, 2 measure 5 ft. 9 in. each, and one measures 6 ft.

9. A certain school bank has 70 depositors, 30 girls and 40 boys. If in a year the girls lodge on an average 13s. 4d. per head and the boys 11s. per head, calculate (a) the total sum deposited in the year and (b) the average sum deposited.

10. A clock strikes once at 1 o'clock, twice at 2 o'clock, and so on. What is the average number of strokes per hour?

EXERCISES XIX_B

1. A man reckons that he smokes on an average 12 cigarettes per day. What does this cost him per annum, at the rate of 20 cigarettes for 1s. 3d.?

A man spends on an average six minutes per day in shaving. Calculate the time spent thus in a year.

3. A man catches 18 fish. One weighs 5 lb. 2 oz., three

others weigh $3\frac{1}{2}$ lb. each, two others weigh $2\frac{1}{2}$ lb. each, three others weigh 1 lb. each, and the remaining fish weigh altogether $6\frac{3}{4}$ lb. Find the average weight of his catch.

4. In a dictation exercise given to a class of 50 pupils 22 are correct, 14 have one error, 8 have two errors, one has 4 errors, one 5, one 7, two 8, and one 10 errors. Calculate the average number of errors per head.

5. 100 men are measured with regard to height. 2 are 5 ft. 4 in. high, 6 are 5 ft. 5 in., 11 are 5 ft. 6 in., 30 are 5 ft. 7 in., 28 are 5 ft. 8 in., 9 are 5 ft. 9 in., 6 are 5 ft. 10 in., 4 are 5 ft. 11 in., 2 are 6 ft., 1 is 6 ft. 1 in., and 1 is 6 ft. 3 in. Find the average height to within a tenth of an inch.

6. Of a company of soldiers 12 are 5 ft. 7 in. in height, 23 are 5 ft. $7\frac{1}{2}$ in., 38 are 5 ft. 8 in., 18 are 5 ft. $8\frac{1}{2}$ in., and 9 are 5 ft. 9 in. Calculate the average height to within a quarter of an inch.

7. The daily rainfall at a certain place for the month of November 1926 was as follows: 11 days, nil; 4 days, $\cdot 1$ in.; 3 days, $\cdot 2$ in.; 2 days, $\cdot 3$ in.; 8 days, $\cdot 4$ in.; 1 day, $\cdot 5$ in.; 1 day, $\cdot 7$ in. Calculate the average daily rainfall for the month.

8. A ship makes the outward voyage of 1000 miles in 3 days $3\frac{1}{2}$ hrs., and the return voyage in 3 days. Calculate the average rate per hour for the whole trip.

9. A vessel makes the outward voyage of 1000 miles at the rate of 14 miles per hour, and the return voyage at 16 miles per hour. Find the average rate per hour for the whole trip. (Verify your result.)

10. A sample of ore weighing 1 lb. 2 oz. is found to contain 21 per cent. of silver. Another sample, weighing 2 lb. 1 oz., contains 18 per cent. silver. A third sample, weighing 9 oz., contains 23 per cent. silver. Calculate the average percentage of silver over the three samples.

11. A motorist on a journey of 72 miles does the first 40 miles at an average rate of 25 miles per hour. At what rate must he travel the remaining distance in order to have an average rate of 30 miles per hour for the whole journey?

12. A man buys three horses, the respective prices being £40,

£50, £60. If he sells so as to make 10 per cent. profit on the first, 25 per cent. profit on the second, and 15 per cent. loss on the third, calculate his average gain or loss per cent. over all.

13. In one district the average weekly wage of 1297 engineers is 51s. 6d.; in another the average wage of 4362 engineers is 53s. 9d.; and in a third district that of 3975 engineers is 49s. 6d. Calculate the average weekly wage over the total number in the three districts.

14. 7 planks of wood have an average thickness of $1\frac{1}{4}$ in.; 18 others an average of $\frac{7}{8}$ in.; 13 others of $1\frac{3}{4}$ in.; 2 others of $\frac{5}{8}$ in.; 1 other of $1\frac{1}{2}$ in. Calculate to the nearest eighth of an inch the average thickness of the lot.

15. In connexion with the preceding question, how many two-inch-thick planks must be included with the given lot to bring the average thickness to $1\frac{1}{2}$ in.? (Employ the equation.)

16. A ship sails at an average rate of 18 miles per hour on its outward voyage, and at an average of 17 miles per hour on its return voyage. Calculate its average rate for the double passage. (*Note on method.* Assume as the number of miles in the voyage a common multiple of 18 and 17.) (Verify your result.)

17. A ship does on an average 16 knots on its outward trip of 1000 miles, and 15 knots on its return trip. Calculate its average speed (in knots) for the double voyage. (*N.B.* 16 knots means a speed of 16 *nautical miles* per hour, and a nautical mile equals 6080 ft.)

18. A man, walking uniformly, covers 600 yds. in 8 min., allowing 6 paces to 4 sec. If he keeps the same time to the pace as before, by how much must he increase the length of his pace in order to cut down his time for the next 600 yds. by 1 min.?

19. The hotel expenses of a party of 6 men for 5 days amount to £26 5s. Those of another party of 5 men for 7 days amount to £39 7s. 6d. Calculate the average charge per head per day.

20. A man invests £3500 in a 4 per cent. stock at $87\frac{1}{2}$ and £4000 in a $3\frac{1}{2}$ per cent. at 75. Calculate the average percentage return for his money. Give your result correct to two decimal places.

THE RULE OF MIXTURES

MODEL EXAMPLES

N.B. Where more than two components enter into the problems there may be an infinite number of solutions. The so-called Rule of Mixtures gives only a limited number.

(a) In what proportion should tea costing 2s. 3d. per lb. be mixed with tea costing 1s. 7d. per lb. in order that the mixture may sell at 2s. per lb. without gain or loss?

Necessarily, if one price is greater than the selling price the other must be less.

Take higher price minus selling price, and write this difference after the *lower* price.

Take selling price minus lower price, and write this difference after the *higher* price, thus:

$$24d. \left\{ \begin{array}{l} 27d. \\ 19d. \end{array} \right. \begin{array}{l} 5 \\ 3 \end{array}$$

The required ratio, then, is 5 to 3.

The student should note that on the dearer tea there will be a loss of 5 times threepence, while on the cheaper tea there is a gain of 3 times fivepence.

(b) How should a 56 per cent. solution, a 63 per cent. solution, and a 65 per cent. solution be mixed to produce a 60 per cent. solution?

Method. 60 per cent. is the required standard.

Link each item that is greater than the required standard with at least one item that is less, and *vice versa*.

$$60 \left\{ \begin{array}{l} 56 \\ 63 \\ 65 \end{array} \right) \begin{array}{l} 3 \\ 4 \\ 4 \end{array} \begin{array}{l} 5 \\ \\ 4 \end{array}$$

The required ratio is 8 : 4 : 4, or 2 : 1 : 1. (Verify this.)

(c) In what proportion should teas costing 1s. 8d., 1s. 9d., 2s. 5d., and 2s. 6d. per lb. be mixed in order that the mixture may sell at 2s. 11d. per lb., thereby yielding 25 per cent. gain?

First reduce the *profitable* selling price to a *neutral* selling

price—*i.e.*, a price which will yield neither profit nor loss. Call this price N pence.

Thus,

$$2s. 11d. = N \text{ pence} + 25 \text{ per cent. of } N \text{ pence.}$$

$$\begin{aligned} I.e., \quad 35 &= N + \frac{N}{4} \\ &= \frac{5}{4} \text{ of } N. \\ N &= 35 \times \frac{4}{5} \\ &= 28. \end{aligned}$$

I.e., neutral selling price is 28*d.*

Then we have :

$$28 \left\{ \begin{array}{l} 20 \\ 21 \\ 29 \\ 30 \end{array} \right\} \begin{array}{cc} 1 & 2 \\ 8 & 7 \end{array}$$

Thus the teas, in the order given above, should be mixed in the ratio of 1 2 8 7. (Verify.)

N.B. We can find various proportions by linking differently, or by linking a 'less' with more than one 'greater.' For example :

$$28 \left\{ \begin{array}{l} 20 \\ 21 \\ 29 \\ 30 \end{array} \right\} \begin{array}{cc} 2 & 1 \\ 8 & 7 \end{array}$$

which gives us the ratio 2 : 1 7 : 8. (Verify.)

Or, again,

$$28 \left\{ \begin{array}{l} 20 \\ 21 \\ 29 \\ 30 \end{array} \right\} \begin{array}{ccc} 1 & & 2 \\ 8 & & 7 \\ & 7 & 8 \end{array}$$

which gives the ratio 3 2 8 : 15.

(*d*) In what proportion must tea at 3*s.* 1*d.* and tea at 2*s.* 10*d.* be mixed with 5 lb. of tea at 2*s.* 8*d.* to make a mixture worth 2*s.* 11*d.* per lb.?

For the moment ignore the given quantity, 5 lb.

$$35 \left\{ \begin{array}{l} 37 \\ 34 \\ 32 \end{array} \right\} \begin{array}{cc} 1 & 3 \\ 2 & \\ & 2 \end{array}$$

Required ratio then is 4 lb. 2 lb. 2 lb., or 2 1 1.

Now, to raise to 5 the quantity at 2*s.* 8*d.*—which is represented by the last term in the proportion—we must multiply by 5. Multiplying all along by 5, we get 10 5 5.

The answer, then, is that 10 lb. of the first tea and 5 lb. of the second must be taken.

(*e*) What amount of tea at 3*s.* 1*d.* per lb. must be mixed with 6 lb. at 2*s.* 6*d.* and 5 lb. at 2*s.* 8*d.* to produce a mixture worth 2*s.* 10*d.* per lb.?

Cost of 6 lb. at 2*s.* 6*d.* + 5 lb. at 2*s.* 8*d.* = 28*s.* 4*d.*

I.e., cost of the 11 lb. = 28*s.* 4*d.*
average cost per lb. = 2*s.* 6 $\frac{10}{11}$ *d.*

Now find the proportion in which tea at 2*s.* 6 $\frac{10}{11}$ *d.* may be mixed with tea at 3*s.* 1*d.* to produce a mixture worth 2*s.* 10*d.*

Thus:

$$2s. 10d. \left\{ \begin{array}{l} 2s. 6\frac{10}{11}d. \\ 3s. 1d. \end{array} \right. \begin{array}{c} 3 \\ 3\frac{1}{11} \end{array}$$

The ratio, then, is 3 to 3 $\frac{1}{11}$, or 33 to 34, or 11 to 11 $\frac{1}{3}$.

Hence with the specified 11 lb. we must mix 11 $\frac{1}{3}$ lb. of the tea at 3*s.* 1*d.* (Verify.)

EXERCISES XIXc

1. In what proportion should tea at 1*s.* 9*d.* per lb. and tea at 2*s.* 1*d.* per lb. be mixed to sell at 1*s.* 10*d.* per lb. without gain or loss?

2. It is required to obtain a blend of tea worth 2*s.* 2*d.* per lb. by mixing tea at 1*s.* 11*d.* with tea at 2*s.* 4*d.* Find the proportion.

3. A photographer has a bottle of developing solution 75 per cent. strong and another 82 per cent. strong. How must he mix them to make a 78 per cent. solution?

4. In what proportion should alcohol costing 24s. per gal. be mixed with water to allow of selling the mixture at 18s. per gal. without gain or loss? (Assume the cost of water to be negligible.)

5. How much water must be added to 8 gal. of a 90 per cent. solution to reduce it to 70 per cent. strength?

6. It is required to sell at 2s. 9d. per lb., without gain or loss, a mixture of coffee costing 3s. 3d. per lb. and chicory costing 1s. per lb. In what proportion must the two be mixed?

7. In what proportion should tea costing 1s. 9d. per lb., tea costing 2s. per lb., and tea costing 2s. 1d. per lb. be mixed if the mixture is to sell at 1s. 10½d. per lb. without gain or loss?

8. How should the teas in the preceding question be mixed if the mixture is to sell at 2s. 3½d., thereby yielding 25 per cent. profit to the seller?

9. From four solutions of different strengths, one 60 per cent., one 65 per cent., one 75 per cent., and one 80 per cent., how can I make up 1800 gal. of 72 per cent. strength? (Give as many ways as you can.)

10. At an auction sale a dealer buys a mixed lot of 4-oz. and 5-oz. rubber balls. He finds there are 140 balls altogether, and that the total weight is 36 lb. 12 oz. How many of each must there be?

11. I have 15 gal. of a 70 per cent. solution of acid. How much water must I add to reduce the strength to 60 per cent.?

12. I have 12 lb. of tea worth 1s. 10d. per lb. and 15 lb. worth 3s. 4d. per lb. How many lb. of tea worth 4s. per lb. must I mix with my 27 lb. in order to produce a final mixture worth 3s. per lb.?

13. I buy apples, some at 20 for 1s. and some at 15 for 1s. In what proportion must I mix them in order to be able to sell at 18 for 1s. without gain or loss? (*N.B.* If the rule is used in working with numbers of apples the resulting ratio numbers will be the number of shillingsworths to be mixed.)

14. Work the preceding question assuming that I wish to sell at 15 for 1s., making thereby a gain of 20 per cent.

15. One stock offers a return of 4 per cent. on money invested. Another offers a return of 7 per cent. In what proportion should I invest money in these two stocks in order to secure an average return of 5 per cent.? (Verify your result.)

16. An estate of £12,000 is to be invested partly in a 4 per cent. stock at 96, and partly in a 5 per cent. stock at 90. If an average return of 5 per cent. on the money invested is to be realized, how must the estate be allocated to the two stocks? (Neglect brokerage, and verify your result.)

17. I wish to invest £2500 partly in a 4 per cent. stock at 96, and partly in a 5 per cent. stock at 105, so as to secure an average return of $4\frac{1}{2}$ per cent. on my money. What sum must I invest in each stock? (Neglect brokerage, and verify your result.)

18. In what proportion must I mix magazines bought at 5 for 1s. with others bought at 3s. a dozen to be able to sell at 3 for $7\frac{1}{2}d.$ without gain or loss?

19. Work out the preceding question assuming that I am content to lose 10 per cent. by selling at the rate stated. (Verify your result.)

20. £12,200 is to be invested partly in a 4 per cent. stock at 90, partly in a $4\frac{1}{2}$ per cent. at 96, and partly in a $5\frac{1}{2}$ per cent. at 120. How may the money be apportioned if the average return on the total investment is to be $4\frac{1}{2}$ per cent.?

CHAPTER XX

FOREIGN EXCHANGE

(THE student should revise Chapter XVII, on bills of exchange.)

Suppose a London merchant buys goods from New York. It would be very unusual to pay his debt by sending gold. There are two ordinary methods to choose from.

(1) The debtor may buy in London from a bank or a broker a bill payable in New York and remit this to his creditor. This bill may be payable on demand, or it may be payable after a specified time—*e.g.*, 3 months. The price of such a bill, as in the case of ordinary merchandise, depends on the demand, and will be found quoted in commercial columns of newspapers in the form “London on New York for cheques, 4·87” or “London on New York, 3 months, 4·91.”

If the bill is not payable at sight—*i.e.*, on demand—the creditor may get his money at once from his own banker if he is willing to lose discount for the unexpired time.

(2) The debtor may instruct his creditor to draw a bill on him and send it to him in London to be accepted and returned. The bill will be payable at the debtor's bank in London. The New York creditor, having received the bill after acceptance, will hand it to his own banker for collection. If the bill is a ‘cheque’ he will receive cash for it at the current cheque rate of exchange between the two countries. If it is a ‘long’ bill he may wait for payment till the due date of collection, or he may have it discounted at once at the current ‘long’ rate.

The rates of exchange which regulate this form of procedure will be quoted in the form “New York on London for cheques, 4·87” or “New York on London, 3 months, 4·82,” as the case may be.

MODEL EXAMPLES

(a) London on New York is 4·87 for cheques. Calculate the rate for 3 months' bills, the discount rate in New York being $3\frac{1}{2}$ per cent.

London on New York sight rate is	4·87	$\frac{7}{100}$ of $4\cdot87 \times \frac{1}{4}$
Add 3 months' discount	·0426	$= \frac{34\cdot09}{800}$
	4·9126	

Required 3 months' rate = 4·9126, which would probably be quoted as $4\cdot91\frac{1}{4}$.

Note. "London on New York for cheques, 4·87" means that £1 paid now is entitled to 4·87 dollars paid in New York on demand.

"London on New York 3 months' rate" implies that while £1 sterling is paid now, the dollars are not to be paid in New York till 3 months hence. Accordingly, we should demand *more* dollars.

(b) New York on London for cheques is 4·87. Calculate the 3 months' rate if London discount is at 4 per cent. (per annum).

New York on London for cheques	4·87
Subtract 3 months' discount	·0487
Hence 3 months' rate should be	4·8213

Referring to the case discussed already in paragraph marked (2), it seems clear that if the London merchant can discharge 4·87 dollars of debt by an immediate payment of £1, he should discharge less than 4·87 if he is to be allowed to delay payment of his £1 for 3 months.

(c) London on Paris 3 months' rate is 123·6. Calculate the cheque rate if discount in Paris is 4 per cent.

London on Paris is	123·6	$\frac{4}{100}$ of $123\cdot6 \times \frac{1}{4}$
Subtract 3 months' discount	1·236	
	122·364	
<i>I.e.</i> , required cheque rate =	122·36	

(d) A London merchant wishes to place £600 in Paris for the discharge of debts due there by him.

Paris rate, 3 months, is 127·4; London, 3 months, 129·4. Discount rates: London, 4 per cent.; Paris, 3 per cent. (per annum).

Find whether it is more profitable to remit or to be drawn upon.

We have London on Paris,		$\frac{1}{4}$ of $\frac{3}{100}$ of 129·4
long,	129·4	400)388·2
Subtract 3 months' discount		·9705
at 3 per cent.	<u>·9705</u>	
London on Paris, short,	128·4295	
Paris on London, long,	127·4	$\frac{1}{4}$ of $\frac{4}{100}$ of 127·4
Add 3 months' discount at		= 1·274
4 per cent.	<u>1·274</u>	
Paris on London, short,	128·674	

Comparing the short (or cheque) rates, we choose the latter as more profitable for the merchant in London.

Being drawn upon, he saves ·2445 franc $\times$ 600—*i.e.*, 146·7 francs. That is, by being drawn upon he is able to discharge 146·7 francs more than by remitting.

(e) A London merchant holds a bill on London for £600, due in 3 months. What amount of debt in Paris can he discharge by means of it, if the London rate of exchange is 120 francs for cheques and the rate of discount in London is $4\frac{1}{2}$ per cent.?

This is simply a case of discounting the bill and buying francs with the proceeds.

Present value of bill is	£600	$\frac{1}{4}$ of $\frac{9}{100}$ of £600
less 3 months' discount	<u>6·75</u>	= £6·75
<i>I.e.</i> ,	£593·25	
Hence number of francs he can buy at the		
given rate of exchange is 120 $\times$ 593·25, or		
71,190.		

(*f*) *Definition.* If we know the rate of exchange between, say, London and Paris, and also that between Paris and New York, we can calculate what *should be* the rate between London and New York. A rate calculated by this means is known as an *arbitrated* rate.

Calculate the arbitrated rate of exchange between Berlin and New York from the following: Berlin on London, 20.30, London on New York, 4.86. This is an easy exercise in the chain rule.

(*Note.* For Berlin on New York we require the equivalent in marks for 100 dollars, this being the conventional form in Berlin quotations.)

$$\begin{aligned} ? \text{ marks} &= 100 \text{ dollars} \\ 4.86 \text{ dollars} &= \text{£}1 \\ \text{£}1 &= 20.30 \text{ marks} \end{aligned}$$

Whence required number of marks

$$= \frac{100 \times 20.3}{4.86}$$

$$= 417.7, \text{ very approximately.}$$

$$\begin{array}{r} 417.7 \\ 486 \overline{)203000} \\ \underline{860} \\ 3740 \\ \underline{338} \end{array}$$

(*g*) (“Buying within limits.”) An agent has instructions to buy bills on Paris at 123 or on Berlin at 20.48. He finds that the rates actually ruling on ‘Change are Paris, 123.3, Berlin, 20.49. How should he act in the interest of his client?

$$\text{Paris} \quad \text{the ratio} \frac{\text{actual rate}}{\text{prescribed rate}} = \frac{1233}{1230}$$

$$\text{Berlin:} \quad = \frac{2049}{2048}$$

The former is obviously greater. (The student should note that $\frac{1233}{1230}$ exceeds unity by $\frac{3}{1230}$, while $\frac{2049}{2048}$ exceeds unity by $\frac{1}{2048}$.) Accordingly, the agent should buy on Paris.

(*h*) *Definition.* The *Mint* par of exchange between, say, the sovereign and the dollar is the relation between the coins having regard only to the intrinsic values of the metals in the coins.

Find the Mint par of exchange between Britain and Ger-

many from the following data: 1869 sovereigns are coined from 40 lb. (Troy) of standard gold, $\frac{1}{12}$ fine, and 279 marks are coined from 100 grm. of gold, pure.

(*Note.* By " $\frac{1}{12}$ fine" is meant that such standard gold consists, by weight, of 11 parts of pure or fine gold and 1 part alloy. The *carat* was a small weight used by goldsmiths, and 24 carats' weight of standard gold contained 22 carats pure gold and 2 carats alloy. Hence the phrase "22-carat gold," which comes to be a description of quality. Similarly, 9-carat gold contains $\frac{9}{24}$ pure gold (by weight) and $\frac{15}{24}$ alloy.)

$$\begin{aligned} ? \text{ marks} &= \text{£}1 \\ \text{£}1869 &= \frac{1}{12} \text{ of } 40 \text{ lb. (Troy)} \\ 1 \text{ lb. (Troy)} &= 5760 \text{ grains} \\ 15.432 \text{ grains} &= 1 \text{ grm.} \\ 100 \text{ grm.} &= 279 \text{ marks} \end{aligned}$$

Required number of marks $= \frac{11 \times 40 \times 5760 \times 279}{12 \times 1869 \times 15.432 \times 100}$ $= \frac{16368}{801.178} (= 20 \text{ approx.})$ $= 20.43.$ <i>I.e.</i>, the Mint par is 20.43 marks to £1.	<table style="border: none; margin: auto;"> <tr> <td style="text-align: right;">31</td> <td style="text-align: right;">1.286</td> </tr> <tr> <td style="text-align: right;">12</td> <td style="text-align: right;">623</td> </tr> <tr> <td style="text-align: right; border-top: 1px solid black;">372</td> <td style="text-align: right; border-top: 1px solid black;">771.6</td> </tr> <tr> <td style="text-align: right;">4</td> <td style="text-align: right;">25.72</td> </tr> <tr> <td style="text-align: right; border-top: 1px solid black;">1488</td> <td style="text-align: right; border-top: 1px solid black;">3.858</td> </tr> <tr> <td style="text-align: right;">11</td> <td style="text-align: right;">801.178</td> </tr> <tr> <td style="text-align: right; border-top: 1px solid black;">16368</td> <td></td> </tr> <tr> <td colspan="2" style="height: 20px;"></td> </tr> <tr> <td style="text-align: right;">8'0'1'1'7)</td> <td style="text-align: right;">2043</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">16368</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">16023</td> </tr> <tr> <td></td> <td style="text-align: right;">345</td> </tr> <tr> <td></td> <td style="text-align: right; border-top: 1px solid black;">320</td> </tr> <tr> <td></td> <td style="text-align: right;">25</td> </tr> </table>	31	1.286	12	623	372	771.6	4	25.72	1488	3.858	11	801.178	16368				8'0'1'1'7)	2043		16368		16023		345		320		25
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EXERCISES XXA

1. From the following cheque rates calculate the quotations for bills at 3 months, assuming the discount rate to be 3 per cent. per annum in all places concerned:

(a) London on Paris, 123.71.

(*Note.* The student should reason thus: £1 in London buys 123.71 francs in Paris immediately. If pay-

ment of the francs in Paris is delayed 3 months, should 123·71 be raised or lowered? And similarly in every case, until the process becomes mechanical.)

- (b) Paris on London, 126·83.
- (c) London on New York, 4·86.
- (d) New York on London, 4·85.
- (e) London on Buenos Aires, 47·58 (pence to the dollar).
- (N.B. The student should note the different form of quotation, and should reason thus: 47·58 pence paid now commands 1 dollar now. If the dollar is not to be paid till 3 months later, should the 47·58 be raised or lowered?)
- (f) Buenos Aires on London, 45·62 (pence to the dollar).
- (g) London on Montreal, 4·865.
- (h) Montreal on London, 4·91.

2. From the following 3 months' rates calculate the cheque rate, assuming the discount rate in each case to be 4 per cent.:

- (a) Amsterdam on London, 12·12 (florins to £1).
- (b) Brussels on London, 34·93 (Belgas to £1).
- (c) London on Copenhagen, 18·15 (kronen to £1).
- (d) London on Berlin, 20·41.

3. The following rates are for 3 months. Calculate the cheque rate in each case.

- (a) London on Berlin, 20·47. Discount rate, 4 per cent. (per annum).
- (b) London on Paris, 125. Discount rate, 4 per cent. (per annum).
- (c) London on New York, 4·71. Discount rate, 4 per cent. (per annum).
- (d) London on Lisbon, $2\frac{1}{3}\frac{5}{2}$ (pence to escudos). Discount rate, 4 per cent. (per annum).
- (e) London on Brussels, 34·95 (Belgas to £1). Discount rate, 4 per cent. (per annum).
- (f) London on Copenhagen, 18·18 (kronen to £1). Discount rate, 4 per cent. (per annum).

- (g) London on Italy, 89.20 (lire to £1). Discount rate, 4 per cent. (per annum).
- (h) London on Montevideo, 50.50 (pence to dollar). Discount rate, 4 per cent. (per annum).
- (i) London on Amsterdam, 12.14 (florins to £1). Discount rate, 4 per cent. (per annum).
- (j) London on Bombay, 1s. 5 $\frac{1}{2}$ d. (sterling to rupee). Discount rate, 4 per cent. (per annum).

4. Take the rates given in the preceding question. Assume them to be cheque rates, and calculate the 3 months' rate, assuming discount at 3 $\frac{1}{2}$ per cent. per annum.

EXERCISES XXB

1. Find what it will cost a London merchant to settle a debt of 70,000 kronen in Copenhagen if he instructs his creditor to draw on him at 3 months, the rate Copenhagen on London for cheques being 24.50, and the discount rate in London being 3 $\frac{1}{2}$ per cent. per annum.

2. If London on New York, 3 months, is quoted at 4.70, find the short rate for bills on New York if the discount rate there is 4 per cent.

3. A London merchant holds a 3 months' bill, Paris on London, for £100. What debt in Paris can he discharge immediately with it? London on Paris, short, 127.4; discount in London, 4 per cent. (See Model Example (e).)

4. If the sight rate London on Paris is 123.6, and the Paris discount rate is 4 per cent., find the London rate for 3 months' bills on Paris.

5. Find the arbitrated rate of exchange between London and New York if London on Berlin is 20.43—*i.e.*, marks per £1—and Berlin on New York is 419.5—*i.e.*, marks per 100 dollars.

6. A debt of 30,000 francs is due immediately by a London merchant to a firm in Paris. What amount in bills must the debtor remit to discharge the debt? The London rate—*i.e.*, London on Paris—is 90.45 for 3 months' bills, and the

discount rate in Paris is $5\frac{1}{2}$ per cent. per annum. (Calculate the cost in sterling.)

7. Find what it would have cost the merchant in the preceding question to allow himself to be drawn upon at 3 months. The Paris rate was 89.75 for cheques, and the discount rate in London was $4\frac{1}{2}$ per cent. per annum.

8. A London merchant owes 20,000 marks to a Berlin creditor. London on Berlin at 3 months is 20.53 (marks per £1); Berlin on London, 3 months, is 20.27; the discount rate in both places is 4 per cent. Find whether the debtor should remit or instruct his creditor to draw on him; and calculate in sterling the advantage of the cheaper method.

9. What debt due immediately in Paris can be discharged by a London merchant who holds a 3 months' bill on Paris for 8500 francs, the sight rate between London and Paris being 124, and the Paris discount rate being $4\frac{1}{2}$ per cent.?

10. The sight rate London on Amsterdam is 11.57 (florins to £1), the discount rate in Amsterdam 4 per cent. What debt due in Amsterdam 3 months hence can a London merchant settle by a 3 months' bill on Amsterdam for which he pays £600?

11. A merchant in New York wishes to pay a debt of £1050 in London. Assuming the Mint par to be 4.867, what will he require to pay for a bill on London when such bills are at a premium of $9\frac{1}{2}$ per cent.—*i.e.*, $9\frac{1}{2}$ per cent. above Mint par—in New York?

12. A London merchant required to settle an immediate debt of 10,000 francs in Paris when the exchange was for London on Paris, 3 months, 122.075, and for Paris on London, 3 months, 120.375, the discount rate in both places being $3\frac{1}{2}$ per cent. Calculate the cost of (*a*) remitting and (*b*) meeting, a draft. (Answer to be given in sterling.)

13. If 98 francs = 15s. 10d., and £1 = 4.875 dollars, how many francs will be equal to 100 dollars?

14. The French Stabilization Law of June 1928 fixed the gold equivalent of the new franc at 65.5 milligrammes of gold $\frac{9}{10}$ fine. Assuming that 1869 British sovereigns are coined from 480 oz. (Troy) of standard gold (which is $\frac{11}{12}$ fine), calculate

the Mint par of exchange between Britain and France. (Take 1 oz. Troy = 31.103 grammes.)

15. Find the Mint par of exchange between Britain and America, given that the *eagle*—*i.e.*, the gold 10-dollar piece—is coined from 258 grains of gold $\frac{9}{10}$ fine.

16. Find the Mint par of exchange between America and Germany (in marks to 100 dollars) if 279 twenty-mark pieces are coined from 2000 grammes of fine gold. (Assume 1 gramme = 15.432 grains.)

17. Calculate the Mint par between (a) France and America (in francs to 100 dollars), (b) between Germany and France (in marks to 100 francs). (See data in Questions 14, 15, and 16.)

18. A London merchant owes 25,000 francs in Paris. If he instructs his creditor to draw on him at 3 months, calculate what it will cost him to discharge the debt, assuming that Paris on London cheque rate is 123.85, and the London discount rate is $3\frac{1}{2}$ per cent. (*N.B.* Here, and in all such questions, consider the *creditor's* point of view.)

19. Referring to the preceding question, find what it will cost the debtor to discharge his obligation by a 3 months' bill on Paris, assuming that London on Paris is 123.9 for cheques, and that the Paris discount rate is 4 per cent.

20. The sight rate London on Amsterdam is 12.06 (florins to £1), and the discount rate in Amsterdam is $4\frac{1}{2}$ per cent. What will it cost a London merchant to discharge a debt of 15,000 florins due immediately in Amsterdam, if he decides to settle by a 3 months' bill on that town?

CHAPTER XXI

SQUARE ROOT

MODEL EXAMPLES

$$\begin{array}{ll}
 \sqrt{61} & = 7 \text{ plus (and less than 8).} \\
 \sqrt{61,00} & = 70 \quad (80). \\
 \text{But } \sqrt{6,10} & = 20 \quad (30). \text{ (Verify.)} \\
 \sqrt{7,13,94} & = 200 \quad (300). \\
 \text{Also } \sqrt{.36} & = .6. \\
 \sqrt{.34} & = .5 \text{ plus (and less than .6).} \\
 \sqrt{.9} & \text{is not .3. (Verify.)} \\
 \sqrt{.9} & = \sqrt{.90} = .9 \text{ fully. (Verify.)} \\
 \sqrt{.4} & = \sqrt{.40} = .6 \text{ fully. (Verify.)} \\
 \sqrt{.47} & = \sqrt{.40} \text{ approx.} = .6 \text{ plus (and less than .7).} \\
 \sqrt{.04} & = .2. \text{ (Verify.)} \\
 \sqrt{.06} & = .2 \text{ plus (and less than .3).} \\
 \sqrt{.064} & = .2 \\
 \sqrt{.068197} & = .2 \quad \text{(Verify this.)}
 \end{array}$$

Hence the rule: Beginning at the point, mark off in pairs the digits to left or to right or in both directions, as the case may be. Take approximately the root of the *first period* (not necessarily the root of the first *digit*). This will give the first approximation to the required root.

$$\begin{array}{ll}
 \sqrt{3,61} & = 10 \text{ plus.} \\
 \sqrt{.36,1} & = .6 \\
 \sqrt{3.61} & = 1 \\
 \sqrt{3,61,27} & = 100 \\
 \sqrt{36.12,7} & = 6 \\
 \sqrt{.03,61,7} & = .1
 \end{array}$$

(a) To find $\sqrt{617}$ to within a unit.

$$\begin{array}{r} 2 \overline{) 617} \underline{4} \\ 4 \overline{) 217} \end{array}$$

Taking the root of the first period—*i.e.*, $\sqrt{6}$ —we have 2 plus.

Subtract the square of 2, leaving 2.

Annex the next period, making 217.

Double the 2 in the divisor. Divide 21 by 4, obtaining 5.

$$\begin{array}{r} 2 \overline{) 617} \underline{4} \\ 45 \overline{) 217} \underline{225} \end{array}$$

Place 5 in the ‘answer’ and in the divisor, and multiply out. Clearly 5 is too great. Try 4.

$$\begin{array}{r} 2 \overline{) 617} \underline{4} \\ 44 \overline{) 217} \underline{176} \end{array}$$

I.e., $\sqrt{617} = 24$ (to within a unit).

(b) To find $\sqrt{.07134}$, to three decimal places.

Mark off the digits in pairs from the decimal point.

$$\begin{array}{r} 2 \overline{) .071340} \underline{4} \\ 47 \overline{) 313} \end{array}$$

Taking the root of the first *period*, we have .2 plus.

Take the square of 2 from 7, leaving 3, to which annex the next period, 13, making 313.

Double the 2 in the divisor, making 4.

Divide, not 313, but 31, by 4, obtaining 7.

Place the 7 in the ‘answer’ and in the divisor, and multiply out if you do not see beforehand that 7 is too great.

$$\begin{array}{r}
 2 \cdot 07 \overline{) 13 \cdot 40} \underline{.27} \\
 \underline{4} \\
 47 \overline{) 313} \\
 \underline{329}
 \end{array}$$

Clearly 7 is too great. Try 6.

$$\begin{array}{r}
 2 \cdot 07 \overline{) 13 \cdot 40} \underline{.267} \\
 \underline{4} \\
 46 \overline{) 313} \\
 \underline{276} \\
 527 \overline{) 3740} \\
 \underline{3689}
 \end{array}$$

Multiply out by 6 and subtract, leaving 37. Annex the next period, making 3740.

Add 6 to the divisor, making 52.

Divide, not 3740, but 374 by 52. This suggests 7.

Place this 7 in the 'answer' and in the divisor, and multiply out by 7.

Evidently 7 is not too great.

Thus we have the required root = .267 approximately.

(c) A rectangle is 15 ft. long and 13 ft. broad. Find the length of the diagonal.

This example involves the well-known theorem that in a right-angled triangle the square on the side opposite the right angle equals the sum of the squares on the other two sides.

$$\begin{array}{rcl}
 15^2 & = & 225 \\
 13^2 & = & \underline{169} \\
 & & 394 \\
 \text{Length of diagonal} & = & \sqrt{394} \text{ ft.} \\
 & = & 19.8 \text{ ft.}
 \end{array}
 \quad
 \begin{array}{r}
 1 \overline{) 394} \underline{19.8} \\
 \underline{1} \\
 29 \overline{) 294} \\
 \underline{261} \\
 388 \overline{) 3300}
 \end{array}$$

EXERCISES XXI

1. Without any figuring state approximately the square roots of 44, 93, 99, 61, taking care not to exceed the root.

2. Estimate without calculation (and without exceeding the root) $\sqrt{473}$, $\sqrt{4719}$, $\sqrt{9000}$, $\sqrt{6437}$, $\sqrt{64379}$.

3. Similarly estimate $\sqrt{\cdot 4}$ and $\sqrt{\cdot 1}$, and verify by squaring your estimated root.

4. Estimate at sight $\sqrt{\cdot 417}$, $\sqrt{\cdot 16}$, $\sqrt{\cdot 164}$, $\sqrt{\cdot 121}$.

5. Estimate at sight $\sqrt{3}$, $\sqrt{3\cdot 6}$, $\sqrt{3\cdot 619789}$.

6. Estimate at sight $\sqrt{174}$, $\sqrt{174\cdot 8}$, $\sqrt{10\cdot 317}$.

7. Calculate to within one unit $\sqrt{625}$, $\sqrt{629}$, $\sqrt{638}$, $\sqrt{609}$.

8. Calculate to two decimal places $\sqrt{\cdot 4137}$, $\sqrt{\cdot 612}$, $\sqrt{\cdot 0612}$, $\sqrt{\cdot 0064}$, $\sqrt{\cdot 0069}$, $\sqrt{\cdot 0038}$.

9. Calculate to one decimal place $\sqrt{71\cdot 46}$, $\sqrt{68}$, $\sqrt{39\cdot 7}$.

10. Find to three *significant* figures $\sqrt{186\cdot 4}$, $\sqrt{31297}$, $\sqrt{\cdot 00684}$.

(*Note.* Obviously in the last example the root begins with $\cdot 0$. It will be necessary to find three more figures in order to have three significant figures.)

11. Find $\sqrt{\cdot 11397}$ and $\sqrt{\cdot 011397}$, each to three decimal places.

12. A rectangle is 71·3 ft. long and 48·6 ft. broad. Find to within an inch the length of the diagonal.

13. The diagonal of a square is 304 ft. long. Calculate to a tenth of a foot the length of the side.

14. Calculate the diagonal of a rectangle having a length of 601 ft. and breadth of 403 ft. (Give result to $\frac{1}{10}$ ft.)

15. Calculate $\sqrt{50793}$, to one decimal place.

16. From your result in Question 15 write down $\sqrt{507\cdot 93}$, $\sqrt{5\cdot 0793}$, $\sqrt{\cdot 05793}$, $\sqrt{\cdot 0005793}$, $\sqrt{5079300}$.

17. Calculate $\sqrt{5079\cdot 3}$.

18. From your result in Question 17 write down $\sqrt{50\cdot 793}$, $\sqrt{\cdot 50793}$, $\sqrt{507930}$, $\sqrt{\cdot 0050793}$.

19. From the results obtained in Question 11 write down $\sqrt{\cdot 0011397}$, $\sqrt{\cdot 00011397}$, $\sqrt{1\cdot 1397}$, $\sqrt{11\cdot 397}$, $\sqrt{113\cdot 97}$, $\sqrt{1139\cdot 7}$.

20. Calculate to a hundredth of an inch the diagonal of a square whose side is 7.04 in.

21. From your result in Question 20 write down the length of the diagonal of a square whose side is (a) 704 in., (b) 70400 in., (c) .704 in., and (d) 70.4 in.

22. The length of a rectangle is 17.03 in. and the breadth 11.1 in. Calculate to the hundredth of an inch the length of the diagonal.

23. The diagonal of a rectangle is 40 in. and the length is 20 in. Calculate the breadth to within a tenth of an inch.

24. The diagonal of a square is 131 ft. Calculate the length of the side to within a tenth of a foot.

25. One side of a rectangle is 62.3 ft. long, and the diagonal measures 90.4 ft. Calculate to within a tenth of a foot the length of the other side.

26. The top of a ladder rests against a vertical wall at a point 37 ft. above the ground. The foot of the ladder is on the ground and 10 ft. out from the wall. Find the length of the ladder to within a tenth of a foot.

27. If the foot of a ladder is to rest on the ground 11 ft. out from a wall, find the length of the ladder required to reach a point on the wall 46 ft. above the ground. Give your answer to within a tenth of a foot.

28. A 30-ft. ladder rests against a wall with its foot 9 ft. 6 in. out from the wall. Find to within an inch the height of the top of the ladder above the ground.

29. The diagonal of a rectangle is 7.4 in. and one side is twice as long as the other. Find the lengths of the sides. Give your results to within a tenth of an inch. (Verify your result.)

30. PQ is a straight road, 15 miles long, running alongside the shore of a lake. R is a point on the other side of the lake exactly opposite P, and the distance RP (which is at right angles to PQ) is 5 miles. If a man can row at $3\frac{1}{2}$ miles an hour, and can walk 4 miles an hour, will he take longer to row from R to Q direct or to row from R' to P and then walk from P to Q?

CHAPTER XXII

INDICES

THE student is already familiar with the short way of indicating a power of a number—*e.g.*, 7×7 is written 7^2 .

The small figure which *indicates* the *number* of sevens to be multiplied together is called the *index*—*i.e.*, the *index of the power* of 7.

MODEL EXAMPLES

$$(a) \quad a^3 \times a^2 = (a \times a \times a) \times (a \times a) \\ = a \times a \times a \times a \times a$$

which is written a^5 .

$$\text{Similarly,} \quad a^4 \times a^3 = a^7. \\ a^2 \times a^8 = a^{10}. \\ 3^2 \times 3^5 = 3^7.$$

Thus, to multiply two powers of the same number we add the indices.

E.g., to multiply, say, 9^4 by 9^2 we add the indices, 4 and 2. This gives us 6, which is the index in the product.

We may express this by the general formula

$$a^p \times a^q = a^{p+q}.$$

This is called the Index Law, and all operations in indices are based on this law.

$$(b) \quad \text{If} \quad a^7 \times a^3 = a^{10} \\ \text{then} \quad a^{10} \div a^7 \text{ must be } a^3.$$

This gives us the rule for dividing one power of a number by another power of the same number. The rule is: Subtract the index in the divisor from the index in the dividend. And this is universal, no matter how *strange* the result may appear.

Thus,
and

$$7^6 \div 7^2 = 7^4,$$

$$7^3 \div 7^5 = 7^{-2},$$

which result, we shall presently find, has a perfectly intelligible meaning.

(c) To find the meaning of 7^0 .

Clearly,

$$\begin{aligned} 7^0 &= 7^0 \times \frac{7^1}{7^1} \\ &= \frac{7^{0+1}}{7^1} \text{ (according to our Index Law)} \\ &= \frac{7^1}{7^1} \\ &= 1. \end{aligned}$$

The student must understand and memorize this apparently curious result.

We have, similarly,

$$10^0 = 1, 365^0 = 1, \text{ and } a^0 = 1.$$

(d) To find the meaning of a *negative* index—say, in the expression 7^{-2} .

As in Example (c), we have

$$\begin{aligned} 7^{-2} &= 7^{-2} \times \frac{7^2}{7^2} \\ &= \frac{7^{-2+2}}{7^2} \text{ (Index Law)} \\ &= \frac{7^0}{7^2} \\ &= \frac{1}{7^2} \text{ (since } 7^0 = 1). \end{aligned}$$

Similarly,

$$7^{-1} = \frac{1}{7}$$

$$10^{-3} = \frac{1}{10^3}.$$

(e) To find the meaning of a fractional index—say, in the expression $7^{\frac{1}{2}}$.

Raising $7^{\frac{1}{2}}$ to the second power, we have

$$7^{\frac{1}{2}} \times 7^{\frac{1}{2}} = 7^{\frac{1}{2} + \frac{1}{2}} \text{ (Index Law)} \\ = 7^1.$$

Clearly, then, $7^{\frac{1}{2}}$ is the square root of 7.

I.e.,
$$7^{\frac{1}{2}} = \sqrt{7}.$$

Again, to find the value of $7^{\frac{1}{3}}$ we raise $7^{\frac{1}{3}}$ to the *third* power.

$$7^{\frac{1}{3}} \times 7^{\frac{1}{3}} \times 7^{\frac{1}{3}} = 7^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} \text{ (Index Law)} \\ = 7^1.$$

Obviously, then, $7^{\frac{1}{3}}$ must be the cube root of 7.

I.e.,
$$7^{\frac{1}{3}} = \sqrt[3]{7}.$$

Again, to find the value of $7^{\frac{1}{4}}$ we raise this quantity to the fourth power.

$$7^{\frac{1}{4}} \times 7^{\frac{1}{4}} \times 7^{\frac{1}{4}} \times 7^{\frac{1}{4}} = 7^{\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}} \text{ (Index Law)} \\ = 7^1.$$

So that
$$7^{\frac{1}{4}} = \sqrt[4]{7^1}.$$

Or, if we prefer,

$$7^{\frac{1}{4}} = \sqrt[4]{343}.$$

Note. These results are quite intelligible, even although the student may not yet know how to find a cube root, or a fourth or fifth or sixth root, etc.

(f) Now that we can find the meaning of a fractional index it will be obvious that it will make no difference to the ultimate value of an expression whether the index be expressed as a vulgar fraction or as a decimal.

E.g., $7^{\frac{3}{4}}$ is the same as $7^{0.75}$

and
$$6^{.397} = 6^{\frac{397}{1000}}.$$

I.e.,
$$1000 \sqrt[1000]{6^{397}}.$$

Note. The decimal form of index is often very useful; and, as the student will find later, is the usual form of expressing *logarithms*, which are just a special collection of indices.

(g) Certain useful steps in simplification should be mastered at this stage.

$$(i) \quad \begin{aligned} 6^5 &= (3 \times 2)^5 \\ &= 3^5 \times 2^5. \end{aligned}$$

$$(ii) \quad \begin{aligned} \sqrt{12} &= \sqrt{4 \times 3} \\ &= 2\sqrt{3}. \end{aligned}$$

$$(iii) \quad \begin{aligned} (7^2)^3 &= 7^2 \times 7^2 \times 7^2 \\ &= 7^6 \text{ (Index Law)}. \end{aligned}$$

Similarly,
$$\begin{aligned} (a^4)^5 &= a^{20}. \\ (a^p)^q &= a^{pq}. \end{aligned}$$

N.B. Distinguish this latter from $a^p \times a^q$, which gives a^{p+q} .

(h) Here are some applications of the foregoing principles:

$$(i) \quad 3^2 \times 2^3 \times 2 \times 2^3 \times 3 = 3^3 \times 2^7.$$

$$(ii) \quad \begin{aligned} \frac{6^5 \times 2^3}{3^4} &= \frac{(3^5 \times 2^5) \times 2^3}{3^4} \\ &= \frac{3^5 \times 2^8}{3^4} \\ &= 3(2)^8. \end{aligned}$$

$$(iii) \quad \begin{aligned} \sqrt{6 \times 200} &= \sqrt{2 \times 3 \times 2 \times 100} \\ &= \sqrt{2^2 \times 10^2 \times 3} \\ &= 20\sqrt{3}. \end{aligned}$$

$$(iv) \quad \begin{aligned} 2^{\frac{1}{2}} \times 2^{\frac{2}{3}} &= 2^{\frac{7}{6}} \text{ (Index Law)} \\ &= 2^{\frac{1}{6}} \times 2^{\frac{1}{2}} \\ &= 2(2^{\frac{1}{6}}), \text{ or } 2(\sqrt[6]{2}). \end{aligned}$$

$$(v) \quad \begin{aligned} 16^{-\frac{3}{2}} &= \frac{1}{16^{\frac{3}{2}}} \\ &= \frac{1}{(2^4)^{\frac{3}{2}}} \\ &= \frac{1}{2^6} \text{ or } \frac{1}{64}. \end{aligned}$$

(vi) Given $a = 10^{2.4103}$, $b = 10^{3.9197}$, and $c = 10^{3.1814}$, evaluate $\frac{a^2b}{c}$.

$$\begin{aligned}\frac{a^2b}{c} &= \frac{(10^{2.4103})^2 \times 10^{3.9197}}{10^{3.1814}} \\ &= 10^{4.8206 + 3.9197 - 3.1814} \\ &= 10^{5.5589}.\end{aligned}$$

ORAL EXERCISES

Simplify

- | | |
|--|--|
| 1. $\frac{3^7 \times 3^4}{3^5}$. | 10. $45^{\frac{1}{2}}$. |
| 2. $\frac{5^2 \times 5^6}{5 \times 5^3}$. | 11. $5^{\frac{1}{2}} \times 5^{\frac{1}{3}}$. |
| 3. $\frac{7^4 \times 7^3}{2 \times 7^2}$. | 12. $(2^3)^2$. |
| 4. 6^{-2} . | 13. $(1.7^2)^5$. |
| 5. $2(6^{-2})$. | 14. $(a^3)^7$. |
| Note. This becomes $\frac{2}{6^2}$. | 15. $\sqrt{2 \times 6}$. |
| 6. $\frac{2(6^{-2})}{3}$. | 16. $\sqrt{3 \times 6}$. |
| 7. $9^{\frac{1}{2}}$. | 17. $\frac{7^{\frac{1}{2}}}{7^{\frac{1}{5}}}$. |
| 8. $\sqrt{18}$. | 18. $\frac{7}{7^{\frac{1}{2}}}$. |
| 9. $\sqrt{75}$. | 19. $\frac{7^{\frac{5}{2}}}{.7^3}$. |
| | 20. $\left(\frac{3}{7}\right)^{\frac{1}{2}} \times \left(\frac{3}{7}\right)^{\frac{1}{4}}$. |

EXERCISES XXII

- Simplify $27^{\frac{2}{3}}$, $81^{-\frac{3}{2}}$, $3^{\frac{2}{3}} \times 15^{-\frac{1}{3}}$.
- Simplify $6^{\frac{3}{4}} \times 150^{\frac{1}{4}} \times 35^{\frac{1}{2}}$.
- Simplify $\sqrt[3]{9} \times \sqrt[3]{4} \times \sqrt[3]{6}$.
- If $a = 10^{2.04}$, $b = 10^{.08}$, and $c = 10^{1.34}$, simplify
 - $\frac{ab^3}{\sqrt{c}}$,
 - $\sqrt[3]{\frac{a^2b^3}{c^3}}$,
 - $\sqrt[4]{c^2}$.

5. Simplify $\sqrt[3]{24} + \sqrt[3]{375}$.
6. Simplify $6^{\frac{1}{2}} \times 9^{\frac{1}{2}} \times 4^{\frac{1}{2}}$.
7. Simplify $\sqrt{12} + \sqrt{27} + \sqrt{75}$.
8. Multiply $2(3)^{\frac{1}{2}} + 6^{\frac{1}{2}} + 3(2)^{\frac{1}{2}}$ by $6^{\frac{1}{2}}$.
9. Simplify $10^{1.4162} \times 10^{2.3174} \times 10^{.2664}$.
10. Simplify $\frac{10^{3.7142} \times 10^{1.2163}}{10^{3.3426}}$.
11. If $K = \frac{3}{4}ab^2c^3$, find K when $a = 2^{\frac{1}{2}}$, $b = 6^{\frac{1}{2}}$, $c = 12^{\frac{1}{2}}$.
12. Simplify $(10^{3.2164} \div 10^{3.0496})^{\frac{1}{2}}$.
13. Simplify $\sqrt{45} + \sqrt{245}$.
14. Multiply $2^{\frac{1}{2}} + 3^{\frac{1}{2}} + 6^{\frac{1}{2}}$ by $6^{\frac{1}{2}}$.
15. Multiply $3^{\frac{1}{2}} + 2^{\frac{1}{2}}$ by $3^{\frac{1}{2}} - 2^{\frac{1}{2}}$.
16. Find to two decimal places the value of $3^{-2} + 5^{-2} + 7^{-2}$.
17. Find to two decimal places the value of $\frac{7^{5.5443}}{7^{3.2174} \times 7^{1.8269}}$.
18. Simplify $75^{\frac{1}{2}} + 363^{\frac{1}{2}} - 147^{\frac{1}{2}}$.
19. Simplify $135^{\frac{1}{3}} + 40^{\frac{1}{3}}$.
20. Taking the values given in Question 4, simplify $\left(\frac{ac}{b^2}\right)^{\frac{1}{2}}$.

CHAPTER XXIII

LOGARITHMS—COMPOUND INTEREST

EVERYBODY knows that 10, 100, 1000, 10,000, 100,000, and 1,000,000 are powers of 10. These particular numbers are *integral* powers of 10.

$$\begin{array}{ll} 10 = 10^1 & 10,000 = 10^4 \\ 100 = 10^2 & 100,000 = 10^5 \\ 1000 = 10^3 & 1,000,000 = 10^6 \end{array}$$

When any number is expressed as a power of ten the index is called the *logarithm* of the number in question. Thus the logarithm of 1000 is 3, and $\log 100 = 2$. Now it is possible (by algebra) to express *any number whatsoever* as a power of ten. This has actually been done to various degrees of accuracy by mathematicians; and the results—*i.e.*, the indices—have been tabulated and printed. At the end of this book there is a table giving the indices—*i.e.*, the logarithms—for all numbers of not more than four digits.

It is obvious that most numbers will have *fractional* logarithms.

For example, since $100 = 10^2$ and $1000 = 10^3$, all numbers between 100 and 1000 will have logarithms between 2 and 3—*i.e.*, their logarithms will be 2.—. Printed tables of logarithms give only the decimal parts of the logarithms, since the whole-number part can be written down by inspection. For instance, it is obvious that 47 is greater than 10^1 and less than 10^2 , and hence its logarithm will be 1.—. The tables give the decimal part—namely, 6721. Hence $\log 47 = 1.6721$.

Similarly, 729 is between 10^2 and 10^3 , and hence $\log 729 = 2.—$. The tables give the decimal part—namely, 8627. Accordingly we have $\log 729 = 2.8627$.

The whole-number part of the logarithm is called the

characteristic. For example, all the numbers between 1000 and 10,000 will have logarithms of the form 3.—, and so 3 is the characteristic of all these logarithms.

The decimal part of a logarithm is called the *mantissa*. Before attempting to use the printed tables of mantissæ the student must master the following all-important principle.

We have noted above that $\log 729 = 2.8627$.

$$\text{I.e.,} \quad 729 = 10^{2.8627}.$$

$$729 \times 10 = 10^{2.8627} \times 10^1.$$

$$\text{I.e.,} \quad 7290 = 10^{3.8627} \text{ (Index Law).}$$

$$\text{Hence} \quad \log 7290 = 3.8627.$$

$$\text{Again,} \quad 729 \div 10 = 10^{2.8627} \div 10^1.$$

$$\text{I.e.,} \quad 72.9 = 10^{1.8627}.$$

$$\text{Hence} \quad \log 72.9 = 1.8627.$$

Thus the logarithms of 729, 7290, and 72.9 have the same mantissa.

From the foregoing consideration it is easy to recognize the fact that all numbers having the *same series of digits*, differing only in the position of the decimal point, have the *same mantissa* in their logarithms.

Observe that since $10^0 = 1$ (see chapter on indices, page 230) we have $\log 1 = 0$.

It follows that all numbers less than 1 have logarithms less than 0; or, to put it otherwise, the logarithms of all proper fractions are negative. Thus the real logarithm of .7290 is -0.1373 . Note carefully that, for convenience in manipulation, negative logarithms are not written directly. For example, $\log .7290$ is not written -0.1373 , but as $\bar{1}.8627$, which means $-1 + .8627$.

The student should now revise the principle already stated.

$$\begin{array}{ll} \text{Thus:} & \log 729 = 2.8627 \\ & \log 7290 = 3.8627 \\ & \log 72.9 = 1.8627, \\ & \log 7.29 = 0.8627 \\ & \log 0.729 = \bar{1}.8627 \\ & \log 0.0729 = \bar{2}.8627, \text{ and so on.} \end{array}$$

Revise also the following summary of the operations discussed in the chapter on indices.

$$10^a \times 10^b = 10^{a+b}.$$

I.e., to *multiply* powers of a number *add* the indices.

$$10^a \div 10^b = 10^{a-b}.$$

I.e., to *divide* one power of a number by another power of the same number *subtract* the index of the divisor from the index of the dividend.

$$(10^a)^b = 10^{ab}.$$

I.e., to find the b^{th} power of 10^a *multiply* the indices.

$$\sqrt[3]{10} = 10^{\frac{1}{3}}$$

and

$$\sqrt[3]{10^a} = 10^{\frac{a}{3}}.$$

I.e., to find a root of any power of a number *divide* the index of the power by the figure that indicates the root.

From the foregoing it is evident that we shall frequently have to add, subtract, multiply, and divide logarithms (which, as the student should remind himself, are just indices).

Here are illustrations of these processes in cases where the characteristic of the logarithm is negative and the mantissa (as always) is positive.

$$\begin{array}{r} \text{(1) Add:} \\ 2.3641 \\ \bar{1}.7036 \\ \bar{1}.3047 \\ \hline 1.3724 \end{array}$$

Note. The carriage from the mantissæ is always positive.

Hence the characteristic of our required result is got from $+1 - 1 - 1 + 2$, and is accordingly 1.

$$\begin{array}{r} \text{(2) Add:} \\ 1.3641 \\ \bar{1}.7036 \\ \bar{2}.3047 \\ \hline \bar{1}.3724 \end{array}$$

Note. After carrying $+1$ from the sum of the mantissæ the index in the required result is found from $+1 + 1 - 1 - 2$, and is therefore -1 .

(3) From $3\cdot4278$ take $4\cdot6237$.

$$\begin{array}{r} 3\cdot4278 \\ 4\cdot6237 \\ \hline \bar{2}\cdot8041 \end{array}$$

Note. Carriage from the mantissæ is $+ 1$. To find the index in the required result consider $3 - (4 + 1)$, which gives $- 2$.

(4) From $\bar{3}\cdot4278$ take $4\cdot6237$.

$$\begin{array}{r} \bar{3}\cdot4278 \\ 4\cdot6237 \\ \hline \bar{8}\cdot8041 \end{array}$$

Note. The index in the required result is found from $\bar{3} - (4 + 1)$, and is therefore $- 8$.

(5) From $\bar{3}\cdot4278$ take $\bar{4}\cdot6237$.

$$\begin{array}{r} \bar{3}\cdot4278 \\ \bar{4}\cdot6237 \\ \hline 0\cdot8041 \end{array}$$

Note. The index in the required result is found from $- 3 - (- 4 + 1)$, and is therefore $- 3 - (- 3)$, that is, 0.

(6) Multiply $\bar{3}\cdot8276$ by 2.

$$\begin{array}{r} \bar{3}\cdot8276 \\ 2 \\ \hline \bar{5}\cdot6552 \end{array}$$

Note. The index in the required result is found from $(- 3) \times 2 + 1$, *i.e.*, from $- 6 + 1$, and is therefore $- 5$.

(7) Divide $\bar{3}\cdot8276$ by 4.

Make the negative index great enough to contain 4 exactly, and compensate thus:

$$\bar{4} + 1\cdot8276.$$

It is now easy to divide.

$$\begin{array}{r} 4)\bar{4} + 1\cdot8276 \\ \hline \bar{1} + \cdot4569 \end{array}$$

I.e., the result is $\bar{1}\cdot4569$.

The following examples show how the drudgery in many calculations is avoided by the use of logarithms.

N.B. The practical form is shown later.

(i) *Multiplication*

$$\begin{aligned} 63.97 \times 0.218 \times 7.946 \\ &= 10^{1.8060} \times 10^{\bar{1}.3385} \times 10^{0.9001} \text{ (from tables)} \\ &= 10^{2.0446} \text{ (Index Law)} \\ &= 110.9 \text{ (from tables).} \end{aligned}$$

(ii) *Division*

$$\begin{aligned} \frac{81.73}{31.72} &= \frac{10^{1.9124}}{10^{1.5014}} \text{ (from tables)} \\ &= 10^{0.4110} \text{ (Index Law)} \\ &= 2.576 \text{ (from tables).} \end{aligned}$$

(iii) *Involution*

$$\begin{aligned} 73.94^3 &= (10^{1.8688})^3 \text{ (from tables)} \\ &= 10^{5.6064} \\ &= 404000 \text{ (from tables).} \end{aligned}$$

(iv) *Evolution*

$$\begin{aligned} \sqrt[3]{62.74} &= \sqrt[3]{10^{1.7976}} \text{ (from tables)} \\ &= 10^{\frac{1.7976}{3}} \\ &= 10^{0.5992} \\ &= 3.974 \text{ (from tables).} \end{aligned}$$

USEFUL NOTE ON THE USE OF THE TABLES

I. Finding the Characteristic of a Logarithm

An easy rule for finding the characteristic is: Note how many places the first significant digit of the given number is removed from the units. This gives the characteristic.

For example, consider 743. The first digit is two places to the *left* of the units.'

Hence the characteristic of $\log 743$ is 2.

Consider 1468.29. The first digit is three places to the *left* of the units. Hence $\log 1468.29$ has characteristic 3.

Consider 0.0638. The first significant digit—*i.e.*, the 6—is two places to the *right* of the units. Hence $\log 0.0638$ has the characteristic $\bar{2}$.

II. Finding the Antilogarithm

(*N.B.* At first the beginner should use only the table of logarithms for all purposes. Later he should use the table of antilogarithms.)

In the reverse process—*i.e.*, finding the number when its logarithm is given—we are said to be finding the antilogarithm.

Example (a): What number has 1.6778 for its log?

Looking up .6778 in the antilogarithms table, we find the digits 4762.

But is the required number 4762, or 476.2, or 47.62, or 4.762, or .4762, or .04762, and so on? All these will have the same mantissa—*viz.*, .6778.

The answer to this question is given by the characteristic. The original problem was, Find the number whose log is 1.6778. The characteristic, 1, tells us that the first significant digit of the number must be one place to the left of the units.

Hence the number is not 4762 or 476.2, but 47.62 (where the first digit, 4, is one place to the left of the units).

Example (b) Find the number whose logarithm is $\bar{2}.3064$.

Looking up the mantissa, .3064, in the table, we find the digits 2025. Now the characteristic, $\bar{2}$, tells us that the first significant digit must be two places to the right of the units.

Hence, our required number is 0.02025, where the 2 is two places to the right of the units. (Note that 2 is the first *significant* digit.)

Practical Form of Logarithmic Work

N.B. Logarithms are to be used to lighten labour, and only practical forms of manipulating them should be tolerated. It is an inexcusable waste of time to attempt to set down the working so as to *explain the theory*, and it almost invariably engenders confusion of statement and of thought. The tabular form is the only practical form.

MODEL EXAMPLES

(a)	$63.97 \times 0.218 \times 7.946$ $= 110.9$	No. 63.97 0.218 7.946	Log 1.8060 1.3385 0.9001	Antilog 2.0446	110.9	
(b)	$\frac{81.73 \times 24.69}{317.2}$ $= 6.360$	No. 81.73 24.69 317.2	Log 1.9124 1.3925 3.3049 2.5014	Antilog 0.8035	6.360	
(c)	73.94^3 $= 404,000.$	No. 73.94 73.94 ³	Log 1.8688 5.6064	Antilog 404000		
(d)	$\frac{60.45^3 \times 3.48^2}{79.43^3 \times .996}$ (Note. Employ extra columns wherever advantageous.) Given expression = 5.359.	No. 60.45 3.48 79.43 0.996	Log 1.7814 0.5416 1.9000 5.7000 1.9983 5.6983	Log 5.3442 1.0832 6.4274 5.7000 1.9983 5.6983	Antilog 0.7291	5.359
(e)	$\sqrt[3]{62.74}$ $= 3.974.$	No. 62.74	Log 1.7976	Log 0.5992	Antilog 3.974	
(f)	$\sqrt[4]{\frac{42.79 \times 1.86}{94.32^2}}$ $= .3075.$	No. 42.79 1.86 94.32	Log 1.9746	Log 1.6313 0.2695 1.9008 3.9492 3.9516 1.4879	Antilog .3075	
(g)	$\sqrt[2]{\frac{43.9}{2.748} + \frac{18.73}{.4629}}$ (Note. Addition and subtraction cannot be performed by logarithms.)	No. 43.9 2.748 18.73 .4629 56.44	Log 1.6425 0.4391 1.2034 1.2725 1.6654 1.6071 1.7516	Log 0.8758	Antilog 7.513	15.97 40.47

Expression

$$\begin{aligned} &= \sqrt[2]{15.97 + 40.47} \\ &= \sqrt{56.44} \\ &= 7.513. \end{aligned}$$

COMPOUND INTEREST

An Alternative Method (adaptable to logarithmic calculation). Suppose an annual rate of 5 per cent. interest, convertible yearly.

In 1 year £1 amounts to £1.05. To find *interest* on £1.05 for 1 year we take

$$\begin{aligned} &\frac{5}{100} \text{ of } \pounds 1.05 \\ &= \pounds 1.05 \times .05. \end{aligned}$$

Hence the *amount* at the end of 2 years is the principal, £1.05, + the interest, £1.05 × .05.

I.e., we have amount = 1.05 taken once, + 1.05 taken .05 ‘times’—that is, 1.05 taken 1.05 times altogether.

I.e., amount at end of 2 years = 1.05².

Collecting, we have

Amount of £1 for 1 year	= 1.05.
2 years	= (1.05) ² .
Similarly, 3	= (1.05) ³ .
And so n	= (1.05) ⁿ .

(h) Find the interest on £300 for 3 years at 4 per cent. per annum, convertible yearly.

Amount of £1 at given rate for given time = £(1.04)³.

amount of £300	No.	Log	Log	Antilog
= £300(1.04) ³	300		2.4771	
= £337.4.	1.04	0.0170	0.0510	
required interest			2.5281	337.4
= £37 8s.				

N.B. For greater accuracy one must use more extended logarithms.

The correct result is

£37 9s. 2d.

Note. It is obvious that the *general* formula is

$$A = P \left(1 + \frac{r}{100} \right)^t$$

In connexion with this the student should revise Chapter X, on changing the subject in a formula.

Thus, from the general formula we can obtain

$$P = A \div \left(1 + \frac{r}{100} \right)^t.$$

Also $\left(1 + \frac{r}{100} \right) = \sqrt[t]{\frac{A}{P}}$, whence we easily find r .

Further, since $\log A = \log P + t \log \left(1 + \frac{r}{100} \right)$,

$$t \log \left(1 + \frac{r}{100} \right) = \log A - \log P,$$

and $t = (\log A - \log P) \div \log \left(1 + \frac{r}{100} \right).$

N.B. This last formula shows a case of actual division of a logarithm by a logarithm.

EXERCISES XXIII

Note. In many of these exercises four-figure logarithm tables are not sufficient for reliable results. The student should have access to more extended tables.

1. Given $\log 7916 = 3.8985$, write down the logs of 7.916, 791.6, 79160, 0.7916, 0.007916.

2. Given $\log 82.74 = 1.9177$, write the logs of 8.274, 82740, 827.4, 0.8274.

3. Given $\log 410.8 = 2.6136$, write the logs of 0.4108, 4108, 41.08, 41080, 0.004108.

4. Given that $\log 4689 = 3.6710$, write down the anti-logarithms of 1.6710, 2.6710, 0.6710, 1.6710, 2.6710, 3.6710.

5. Find the logarithms of 36.7, 0.8109, 4.377, 0.52, 0.06187, 2.104, 137, 88.8, 163.7.

6. Take the logarithms found in the preceding exercise, and find the antilogarithm in each case.

7. Evaluate $79.63 \times 3.182 \times .0496$.

8. Evaluate $\frac{63.41 \times 12.86}{9.743}$.

9. Find the value of $\frac{64.97}{93.08} \times 23.64 \times .0891$.

10. Evaluate $\frac{472.9}{13.09 \times 86.27}$.

11. Find 7.938^3 , 12.72^3 , 23.45^2 , 0.8174^3 .

12. Calculate $\sqrt{60.83}$, $\sqrt{2.83}$, $\sqrt{0.8167}$, $\sqrt[3]{0.0268}$.

13. Find 0.8197^2 , 11.08^3 , 0.093^2 , 0.8069^3 .

14. Evaluate $\frac{0.879 \times 3.925}{0.7649^2}$.

15. Evaluate $\frac{0.4873^2 \times 13.92}{1.837}$.

16. Evaluate $\frac{77.83}{69.47 \times 3.816}$.

17. Evaluate $\frac{.4903 \times 17.06 \times 3.444}{84.23 \times 0.7747}$.

18. Evaluate $4.897^2 + 13.29^2 + 5.435^2$.

19. Evaluate $\frac{0.8963^2}{0.7386^3} + 0.3788^2$.

20. Evaluate $\frac{\sqrt[3]{4.296} + \sqrt[3]{77.39}}{.1.86}$.

21. Find $.4097^3 \times 3.971^2 \times .9074$.

22. Evaluate $\frac{68.97 \times 18.94^2 \times 3.097^3}{12.86 \times 491.7}$.

23. Find the value of $\sqrt{2.698} \times 8.923^2$.

24. Evaluate $\sqrt[2]{21.97 \times 3.862 \times 74.09}$.

25. Evaluate $\frac{.0974 \times .3969 \times .7342}{.9154 \times 3.26 \times 1.7}$.

26. Evaluate $\sqrt[3]{\cdot 09746} \div \sqrt{\cdot 8109}$.

27. Find the value of $\sqrt{\frac{\cdot 7293 \times \cdot 0864}{\cdot 2376^3}}$.

28. Evaluate $\sqrt{47 \cdot 93^2 + 8 \cdot 649^2}$.

29. Evaluate $\frac{43 \cdot 87 \times \cdot 1129 \times 6 \cdot 97}{379 \cdot 4 \times \cdot 0178}$.

30. Find $\sqrt[3]{793 \cdot 8 \times \cdot 04963}$.

31. Find $\sqrt{\frac{432 \cdot 6 \times 18 \cdot 79}{19 \cdot 38^3 \times 43 \cdot 06}}$.

32. Find the amount of £973 14s. for 5 years at $3\frac{1}{2}$ per cent. per annum, convertible yearly.

33. In what time will a sum of money double itself at 4 per cent. per annum, interest convertible yearly? (Give the answer correct to the necessary year.)

34. Find the amount of £682 17s. for 4 years at 3 per cent. per annum, convertible half-yearly.

35. Calculate the interest on £463 for 7 years at $3\frac{1}{2}$ per cent. per annum, convertible yearly.

36. Calculate the interest on £580 for 10 years at 3 per cent. per annum, convertible yearly.

37. In what time will £490 amount to £700 at 4 per cent. per annum, convertible yearly? Give the answer correct to the necessary year.

38. What sum must be deposited now to amount in 10 years' time to £1000, reckoning interest at 4 per cent. per annum, convertible yearly?

39. Evaluate $\sqrt{6 \cdot 239^2 - 3 \cdot 817^2}$.

40. Find the amount of £73 15s. for 6 years at $4\frac{1}{2}$ per cent. per annum, convertible yearly.

41. What sum will amount in 5 years to £105 at $4\frac{1}{2}$ per cent. per annum, convertible half-yearly?

42. Calculate the time, correct to the necessary year, in which £86 will amount to £150 at 3 per cent. per annum, convertible yearly.

43. What sum will earn £76 in 7 years, the rate being 3 per cent. per annum, convertible yearly?

(*Note.* Calculate the *interest* on £1 under the given conditions, and then find the required result by simple proportion.)

44. At what rate of interest per cent. per annum, convertible yearly, will a sum of money double itself in 20 years? Calculate your answer correct to within .01.

45. Which is the more profitable from the investor's point of view, $3\frac{1}{2}$ per cent. per annum, convertible yearly, or 3 per cent. per annum, convertible half-yearly? Show the difference in the *amount* of £1 at the end of 10 years.

46. Two sums, one of £80, the other of £81, are loaned. The rate of interest in both cases is 5 per cent. per annum; but in the first case interest is convertible yearly, while in the other case interest is convertible half-yearly. After how many years will the amounts be equal? Give your answer correct to the nearest year.

47. What principal will amount in 20 years to £500, allowing interest at 6 per cent. per annum, convertible yearly?

48. How many years will be necessary for a sum of money to treble itself if it is earning interest at $4\frac{1}{2}$ per cent. per annum, convertible yearly?

49. At what rate per cent. per annum will £60 amount to £100 in 9 years, if interest is convertible half-yearly? Give your answer to within .01.

50. What principal will amount to £365 in 10 years at 3 per cent. per annum, convertible half-yearly?

CHAPTER XXIV

MENSURATION

(Continued from Chapters VIII and XII)

VOLUME—SUPPLEMENTARY NOTES ON AREA

MENSURATION OF VOLUME

The Prism. If we place a rectangular sheet of paper on, say, the table, and imagine the sheet to rise vertically upward, remaining parallel to its original plane—*i.e.*, to the surface of the table—without rotating, the space through which the sheet passes is a right rectangular prism. Similarly, a triangular sheet would generate a right triangular prism, a square sheet a right square prism, a hexagonal sheet a right hexagonal prism, a circle a right circular prism (or cylinder), and so on. Thus, a prism is named from its ends.

The Rectangular Prism (or brick-shaped solid). Reverting to the experiment in the last paragraph, suppose the area of the rectangular sheet to be 6 sq. in. If the sheet rises through a height of 1 in. the volume of the prism will be exactly 6 cu. in. If the height be 2 in. the volume will be 6×2 cu. in., and, in general, if the base be b sq. in. and the height be h in. we have

$$\text{volume} = bh \text{ cu. units.}$$

It will readily be understood that this formula holds for every right prism.

Thus, in the case of the cylinder the base is a circle, having area πr^2 , and the volume is $\pi r^2 h$.

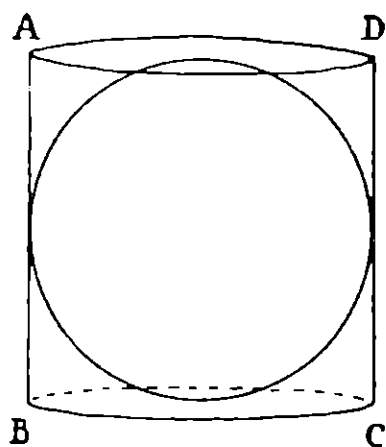
Tapering Figures. At this stage the pupil must simply accept the fact that the volume of a tapering figure is *one-third* that of the corresponding prism. This may easily be verified if we use a hollow cylinder and a hollow cone having the same base and the same height. It will be found that if we fill the cone

with sand, and pour this into the cylinder, we must repeat the process three times in order to fill exactly the cylinder.

Hence, the formula for a right cone or a right pyramid is $V = \frac{1}{3}bh$.

The Sphere. The pupil at this stage must accept without proof the formula for the volume of the sphere:

$$V = \frac{4}{3}\pi r^3.$$

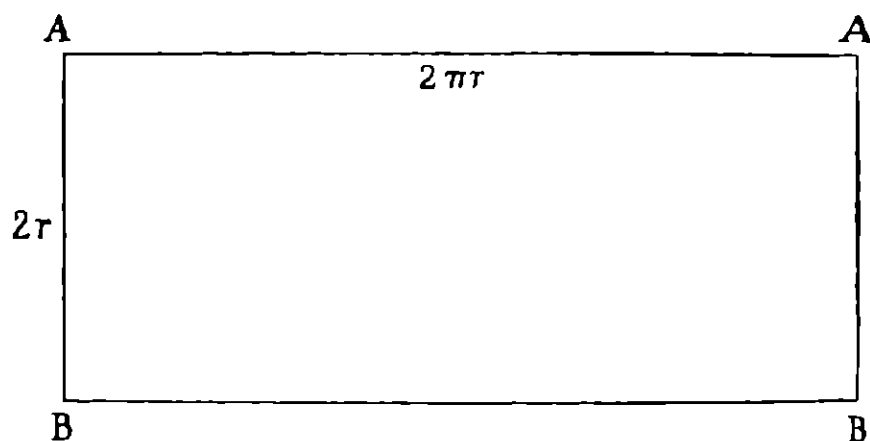


Useful Note. The *surface* of a sphere is exactly equal to the curved surface of the circumscribing cylinder.

Consider a hollow cylinder ABCD, of which the height equals the diameter of the end (made, say, in paper or tin). Imagine the ends removed. Now cut down along the line AB and unroll the curved surface. The resulting plane figure is obviously a rectangle whose base equals the circumference of the cylinder—*i.e.*, $2\pi r$.

Now, the altitude of the rectangle is just AB (the same as before the unrolling), and in this case $AB = 2r$.

Hence curved surface of cylinder $= 2\pi r \times 2r = 4\pi r^2$.



This is exactly the area also of the surface of the enclosed sphere. (The proof of this is beyond the scope of this work.)

Note. These formulæ should be tabulated and frequently revised.

$$\begin{array}{l} \text{Circle} \left\{ \begin{array}{l} \text{circumference} = 2\pi r. \\ \text{area} = \pi r^2. \end{array} \right. \\ \text{Sphere} \left\{ \begin{array}{l} \text{surface} = 4\pi r^2. \\ \text{volume} = \frac{4}{3}\pi r^3. \end{array} \right. \end{array}$$

Triangle—Area in Terms of the Sides. Let a , b , and c units be the lengths of the sides.

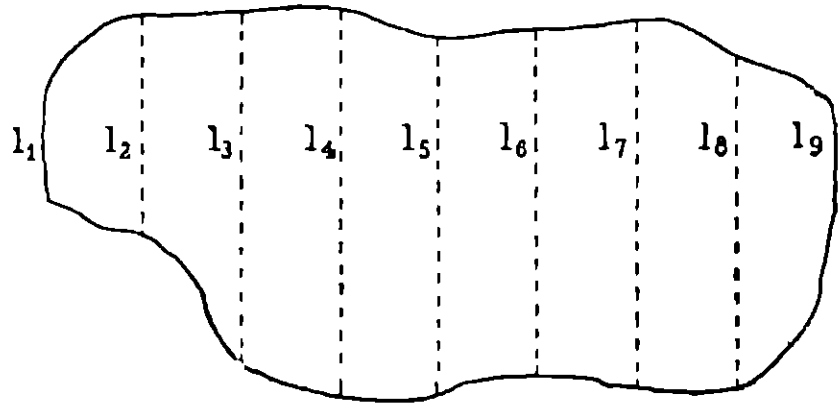
The perimeter, then, is $a + b + c$.

Let s denote the semi-perimeter—*i.e.*, $\frac{1}{2} (a + b + c)$.

Then area = $\sqrt{s (s - a) (s - b) (s - c)}$.

(The student should simplify this formula for the equilateral triangle—*i.e.*, when $a = b = c$.)

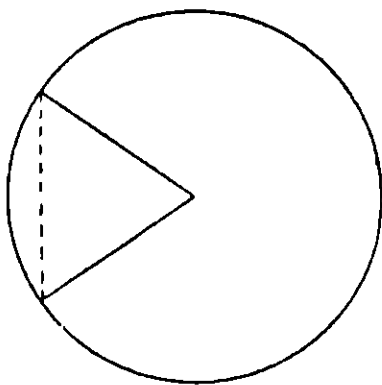
An Irregular Figure—Simpson's Rule for Area. Divide the figure into an *even* number of strips by drawing a series of equidistant parallel ordinates.



Suppose nine ordinates, whose lengths are respectively $l_1, l_2, l_3, \dots, l_9$. Let the constant distance between the ordinates be d . (Note that l_1 and l_9 are boundary ordinates.)

Then

$$\text{area} = \frac{d}{3} \{ (l_1 + l_9) + 2(l_3 + l_5 + l_7) + 4(l_2 + l_4 + l_6 + l_8) \}.$$



Sector of a Circle. Let the angle be, say, 70° .

Then area of sector

$$= \frac{70}{360} \text{ of circle}$$

$$= \frac{70}{360} \text{ of } \pi r^2.$$

Another Method. If the length of the arc is l , then the sector is the same fraction of the circle as the arc is of the circumference.

$$\text{I.e., area of sector} = \frac{l}{2\pi r} \text{ of circle}$$

$$= \frac{l}{2\pi r} \text{ of } \pi r^2$$

$$= \frac{1}{2}lr.$$

Segment = area of sector – area of triangle.

Ellipse

$$\text{Area} = \pi ab,$$

where a is the semi-major axis and b is the semi-minor axis.

MODEL EXAMPLES

(a) The internal dimensions of a rectangular tank are 9 ft. 6 in. $\times$ 7 ft. $\times$ 12 ft. 6 in. Calculate its capacity in gallons, allowing $6\frac{1}{4}$ gal. per cubic foot.

Capacity of tank = base $\times$ height

$$\begin{aligned} &= (9\frac{1}{2} \times 7) \times 12\frac{1}{2} \text{ cu. ft.} \\ &= \frac{19}{2} \times 7 \times \frac{25}{2} \times \frac{25}{4} \text{ gal.} \\ &= 5196 \text{ gal. approx.} \end{aligned} \quad \left| \begin{array}{r} 133 \\ 25 \\ \hline 3325 \\ 25 \\ \hline 4 \overline{)83125} \\ 4 \overline{)20781\frac{1}{4}} \\ \hline 5195\frac{5}{8} \end{array} \right.$$

(b) Find the volume of a sphere of radius 4.26 in.

Volume

$$\begin{aligned} &= \frac{4}{3} \pi r^3 \\ &= 1.333 \times 3.142 \times 4.26^3 \\ &\quad \text{cu. in.} \\ &= 323.8 \text{ cu. in.} \end{aligned} \quad \left| \begin{array}{c|c|c|c} \text{No.} & \text{Log} & \text{Log} & \text{Antilog} \\ \hline 1.333 & & 0.1249 & \\ 3.142 & & 0.4972 & \\ 4.26 & 0.6294 & 1.8882 & \\ \hline & & 2.5103 & 323.8 \end{array} \right.$$

(c) A cylindrical tank is 8 ft. 9 in. high, and stands on a base 3 ft. 4 in. in diameter. How long will it take to fill the tank with water delivered by a pipe, of $1\frac{1}{2}$ in. bore—*i.e.*, internal diameter—through which water flows at the rate of 4 ft. per second?

Volume of tank

$$= \pi r^2 h$$

$$= \pi \times 1.667^2 \times 8.75 \text{ cu. ft.}$$

Cross-section of pipe

$$= \pi r^2$$

$$= \pi (\cdot 75)^2 \text{ sq. in.}$$

$$= \frac{\pi (\cdot 75)^2}{144} \text{ sq. ft.}$$

$$\text{Diameter} = 3 \text{ ft. } 4 \text{ in.}$$

$$r = 1 \text{ ft. } 8 \text{ in.}$$

$$= 1\frac{2}{3} \text{ ft.}$$

$$= 1.667 \text{ ft.}$$

Capacity of 4 ft. of pipe—*i.e.*, delivery of water per sec.

$$= \frac{\pi (\cdot 75)^2 \times 4}{144} \text{ cu. ft.}$$

time to fill tank

$$= \frac{\pi \times 1 \cdot 667^2 \times 8 \cdot 75 \times 144}{\pi (\cdot 75)^2 \times 4} \text{ sec.}$$

$= \frac{1 \cdot 667^2 \times 8 \cdot 75 \times 36}{\cdot 75^2} \text{ sec.}$	<i>No.</i> 1·667 8·75 36	<i>Log</i> 0·2219	<i>Log</i> 0·4438 0·9420 1·5563	<i>Antilog</i>
$= 1555 \text{ sec.}$			2·9421 1·7502	
$= 25 \text{ min. } 55 \text{ sec.}$	·75	1·8751	3·1919	1555

(*d*) Calculate the weight of a stone pyramid of square base, side 7 ft. 10 in., and height 11 ft. 6 in., if the material weighs 243 lb. per cu. ft.

$$\begin{aligned} \text{Volume} &= \frac{1}{3}A \times h \text{ (where } A \text{ is area of base)} \\ &= \frac{1}{3} \times (7\frac{5}{6})^2 \times 11\frac{1}{2} \text{ cu. ft.} \end{aligned}$$

$\text{weight} = \frac{47^2 \times 23 \times 243}{3 \times 6^2 \times 2 \times 112 \times 20} \text{ tons}$	$\frac{2240}{6}$ 13440
---	---------------------------------

$= \frac{47^2 \times 23 \times 243}{36 \times 13440} \text{ tons}$	<i>No.</i> 47 23 243	<i>Log</i> 1·6721	<i>Log</i> 3·3442 1·3617 2·3856	<i>Antilog</i>
(Shorten in this way where advantageous.)			7·0915	
$= 25 \cdot 51 \text{ tons.}$	36 13440		1·5564 4·1284 5·6848	
			1·4067	25·51

(*e*) A metal sphere (solid) weighs 6 lb. 13 oz. If the metal weighs 372 lb. per cu. ft., calculate the radius of the sphere.

1 cu. ft. metal weighs 372 lb.

$$1 \text{ cu. in.} \quad \frac{372}{1728} \text{ lb.}$$

Sphere weighs $6\frac{13}{16}$ lb.

volume of sphere = $6\frac{13}{16} \div \frac{372}{1728}$ cu. in.

I.e., $\frac{4}{3}\pi r^3 = 6\frac{13}{16} \div \frac{372}{1728}$

radius = $\sqrt[3]{\frac{109}{16} \times \frac{1728}{372} \times \frac{1}{1.333 \times 3.142}}$ in.

	No.	Log	Log	Antilog
	109	2.0374		
	1728	3.2375		
		5.2749		
	16	1.2041		
	372	2.5705		
	1.333	0.1249		
	3.142	0.4972		
		4.3967		
= 1.962 in.		0.8782	0.2927	1.962

(f) Find the area of a triangular enclosure having sides 219 yds., 335 yds., 396 yds.

Area

$= \sqrt{s(s-a)(s-b)(s-c)}$
 $= \sqrt{475 \times 256 \times 140 \times 79}$ sq. yds.
 $= \frac{\sqrt{475 \times 256 \times 140 \times 79}}{1210}$ rds.
 $= 30.31$ rds.
 $= 7$ acres 2.31 rds.

$a = 219 \quad s - a = 256$
 $b = 335 \quad s - b = 140$
 $c = 396 \quad s - c = 79$
 $2s = 950$
 $s = 475$

No.	Log	Log
475	2.6767	
256	2.4082	
140	2.1461	
79	1.8976	
	9.1286	4.5643
1210		3.0828
		1.4815

EXERCISES XXIV

1. A rectangular cistern measures 10 ft. 6 in. × 5 ft. 6 in. × 4 ft. (internal measurement). Calculate its capacity in gallons, reckoning $277\frac{1}{4}$ cu. in. to the gallon.
2. A rectangular cistern stands on a base 8 ft. 4 in. long and 6 ft. 6 in. broad (inside measurement). How many gallons of water does it contain if the depth of water is 3 ft.?

3. Find the weight of a square copper plate, $\frac{3}{4}$ in. thick, of 14 in. side, assuming that the metal is 8.9 times as heavy as water and that water weighs 62.4 lb. per cu. ft.

4. A cubic foot of lead is made into a rectangular plate 4 ft. 3 in. long, 2 ft. 10 in. broad, and of uniform thickness. Calculate the thickness.

5. A classroom 25 ft. long, 22 ft. broad, 15 ft. high, is seated for 49 pupils. What cubic space is allowed per pupil?

6. Find the volume of a cylinder 20 in. long and of diameter 9 in.

7. A cylindrical measure is 9 in. high and $5\frac{1}{2}$ in. in diameter. How often can it be filled from a tank standing on a rectangular base 3 ft. 6 in. long and 2 ft. 6 in. broad which is filled with water to a depth of 18 in.?

8. Calculate the area of the entire external surface of a cylinder 3 ft. 4 in. long and 21 in. in diameter.

9. Find the volume of a sphere of radius 7 in.

10. A hollow cylinder standing on its base, which is 15 in. in diameter (inside measurement), is filled with water to a depth of more than 15 in. Into the cylinder is introduced a solid lead sphere exactly 15 in. in diameter. Calculate the rise in the level of water.

11. A cylindrical tank is 6 ft. high, and stands on a base 3 ft. 6 in. in diameter. Find its capacity in gallons, assuming 6.23 gal. per cu. ft.

12. Calculate the cost of gilding the surface (external) of a sphere 18 in. in diameter at 7s. 6d. per sq. ft.

13. The base of a hollow cylinder is 4 ft. 9 in. in diameter, and the height is 7 ft. 6 in. How long will it take to fill the cylinder with water delivered at the rate of 3 gal. per min.? (Take 6.23 gal. to the cu. ft.)

14. Calculate the cost of digging a trench (of rectangular section) 123 yds. long, 2 ft. 6 in. broad, and 3 ft. 6 in. deep at 1s. 3d. per cu. yd.

15. Find the weight of a rectangular stone block, 6 ft. 6 in. long, 2 ft. broad, and 2 ft. 3 in. thick, hollowed out on the upper side into a trough to hold 10 gal. of water. Assume that

water weighs 10 lb. per gallon, and that the stone is 6.5 times as heavy as water.

16. A cylindrical well 8 ft. in diameter and 30 ft. deep is dug in ordinary soil. Calculate the weight of soil taken out, assuming the usual rate of 21 cu. ft. to the ton.

17. Calculate the weight of a hollow brass sphere 18 in. in external diameter, the metal being $\frac{1}{2}$ in. thick. Assume that 1 cu. ft. of brass weighs 525 lb.

18. Calculate the weight of a rectangular box made with wood $\frac{3}{4}$ in. thick, the external measurements being 3 ft. 6 in., 2 ft. 9 in., 2 ft. 4 in. Assume that the material weighs 55.5 lb. per cu. ft.

19. Find the weight of a solid brass cone 10 in. high and having a base 8 in. in diameter. (Take 1 cu. ft. of brass as weighing 525 lb.)

20. Calculate the weight of 30 ft. of lead pipe of $\frac{3}{4}$ in. bore, the thickness of the lead being $\frac{1}{8}$ in. Assume the weight of lead to be 690 lb. per cu. ft.

21. The cylinder of a Chandler Six motor-car measures 3 in. bore, $4\frac{1}{4}$ in. stroke; that of a Daimler Six 81.5 mm. bore, 114 mm. stroke. Express the capacity of the less as a percentage of the greater. (The stroke may be taken as the length of the cylinder for purposes of this problem. Assume that 1 metre = 39.4 inches.)

22. From a cone having height 8 ft. and diameter of base 6 ft. a slice is cut parallel to the base, and at a distance of 2 ft. 6 in. from the base. Calculate the volume of this slice. (See Exercises XIb, Question 11.)

23. Calculate in gallons the capacity of a cylindrical tank 4 ft. 6 in. high, having a base of 3 ft. diameter. (Allow $6\frac{1}{4}$ gal. to the cu. ft.)

24. A reservoir is in the form of an inverted cone, having diameter of base 5 ft. 9 in. and height 6 ft. Calculate the number of gallons of water required to fill it up to within 1 ft. of the upper edge. (Allow $6\frac{1}{4}$ gal. to the cu. ft.)

25. Round the outside of a circular plot 90 ft. in diameter it is required to dig a trench 3 ft. broad and 3 ft. 6 in. deep. Calculate the cost of digging at the rate of 1s. 3d. per cu. yd.

26. In a cylindrical tank having a base 4 ft. in diameter the water is standing at a certain level. If 11 rectangular bricks 9 in. $\times$ $4\frac{1}{2}$ in. $\times$ $3\frac{1}{2}$ in. are submerged in the water, calculate to within $\frac{1}{10}$ in. the rise in the water-level. (Take π as $3\frac{1}{7}$.)

27. A rectangular cistern standing on a base 4 ft. 6 in. $\times$ 3 ft. 6 in., is partly filled with water. If now twenty-one metal spheres, each 6 in. in diameter, are submerged in the water, calculate to within $\frac{1}{10}$ in. the rise in the water-level.

28. A circular swimming-pond having the bottom perfectly horizontal and the walls vertical is 60 ft. in diameter. How long will it take to fill the pond to a depth of 6 ft. 6 in. if the feed pipes deliver 390 gal. per minute? (Take 1 cu. ft. = $6\frac{1}{4}$ gal.)

29. In the preceding question it is suggested to supply water to the pond by a single circular pipe 4 in. in bore, through which the water travels at 5.2 ft. per second. In this case calculate the time required to fill the tank to a depth of 5 ft. 9 in.

30. Calculate the weight of a hollow spherical shell, internal diameter 6 in., external diameter 7 in., if the material weighs 380 lb. per cu. ft.

31. Find the area of a triangular enclosure the sides of which measure 63.6 ft., 55.9 ft., 48.7 ft.

32. Calculate the area of a triangle of which each side measures 17.28 metres.

33. Two diameters at right angles to each other are drawn in a circle. The ends of the diameters are joined to form a square. If the radius of the circle is 17.5 cm., calculate the area of the part of the circle outside the square.

34. The sides of a triangle measure 43.82 ft., 39.76 ft., and 33.04 ft. Calculate the area of the triangle.

35. Draw a circle having radius 3 in. Draw a diameter, and divide it into six equal parts. Use Simpson's Rule to find the area of the circle. Compare your result with that obtained by the formula πr^2 .

36. The base of an isosceles triangle measures 3.418 in. The equal sides measure 5.297 in. each. Calculate the area of the triangle.

37. The base of a right prism is an equilateral triangle having sides measuring 18.38 in. The height of the prism is 31.02 in. Calculate (*a*) the area of the whole surface, (*b*) the volume of the prism.

38. The base of a cone is 23.46 in. in diameter. The slant height is 41.34 in. Calculate the area of the whole surface of the cone. (See note on sector of a circle.)

39. The dial of a clock is a circle 14.36 in. in diameter. If one hand points to XII, and the other to V, calculate the area of the part of the dial bounded by the two hands and the arc of the circle between XII and V

40. The side of a regular hexagon measures 11.32 in. Calculate the area of the hexagon. (Imagine it divided up into six equilateral triangles by lines drawn from the mid-point of the figure to the vertices.)

CHAPTER XXV

DUODECIMALS

THIS is really a system of notation employed by certain trades—*e.g.*, by builders, painters, glaziers.

Length. The fundamental unit of length is the foot.

Just as the foot contains 12 in., so the inch (called the *prime*, because it is the *first* division of the foot) is subdivided into 12 equal parts called *seconds*; and the second is divided into 12 *thirds*.

A length of 3 ft. 5 in. 7 sec. 8 thirds is written $3^{\circ} 5' 7'' 8'''$. These symbols are called *indices* (each being an *index*).

Area. Similarly, the square foot (or superficial foot) is divided into 12 *superficial primes*, each of these again into 12 *superficial seconds*, and so on.

An area of 3 superficial feet, 5 superficial primes, 8 superficial seconds, is written $3^{\circ} 5' 8''$.

An examination of these units of area will give the key to the manipulation of duodecimals.

(a) Consider a rectangle 1 ft. long and 1 prime broad, or, as we may write it, $1^{\circ} \times 1'$.

Now this area is clearly a twelfth of a superficial foot—*i.e.*, 1 superficial prime—and is written $1'$.

Hence we have

$$1^{\circ} \times 1' = 1',$$

where the last-mentioned measurement is understood to be superficial.

(b) Consider a narrow rectangular strip 1° long and $1''$ broad. This area is clearly $\frac{1}{144}$ of a square foot. (*N.B.* The student must continually bear in mind that in duodecimals the linear unit $1''$ is not 1 in. (or prime), but 1 sec.—*i.e.*, $\frac{1}{12}$ of an inch.)

Now $\frac{1}{144}$ of a square foot is a superficial second. Hence we have

$$1^{\circ} \times 1'' = 1''—i.e., 1 \text{ superficial second.}$$

(c) Consider a rectangle 1 in. (or prime) long and 1 in. (or prime) broad. This area is clearly $\frac{1}{144}$ of a square foot—*i.e.*, 1 superficial second.

Hence we have

$$1' \times 1' = 1''—i.e., 1 \text{ superficial second.}$$

From the results of (a), (b), and (c) we may conclude that in multiplying two lengths to find an area we *add* the indices. For example,

$$\begin{aligned} 3^{\circ} \times 2^{\circ} &= 6^{\circ}. \\ 4' \times 4^{\circ} &= 16' = 1^{\circ} 4'. \\ 7' \times 5'' &= 35''' = 2'' 11''' \end{aligned}$$

Example (a): Find the area of a rectangle $3^{\circ} 6' 2''$ long and $3'$ broad.

$$\begin{array}{r} 3^{\circ} \quad 6' \quad 2'' \\ \quad 3' \\ \hline 10' \quad 6'' \quad 6''' \end{array}$$

Explanation:

$$\begin{aligned} 2'' \times 3' &= 6''' \\ 6' \times 3' &= 18'' \\ &= 1' 6'' \end{aligned}$$

We carry the $1'$ to the next partial product.

$$3^{\circ} \times 3' = 9',$$

to which we add the $1'$ (carried), obtaining $10'$.

Example (b): Find the area of a rectangle $4^{\circ} 4' 3''$ by $2^{\circ} 3'$.

$$\begin{array}{r} 4^{\circ} \quad 4' \quad 3'' \\ 2^{\circ} \quad 3' \\ \hline 8^{\circ} \quad 8' \quad 6'' \\ 1^{\circ} \quad 1' \quad 0'' \quad 9''' \\ \hline 9^{\circ} \quad 9' \quad 6'' \quad 9''' \end{array}$$

Explanation: We multiply first by 2° .

$$\begin{aligned} 3'' \times 2^\circ &= 6'' \\ 4' \times 2^\circ &= 8' \\ 4^\circ \times 2^\circ &= 8^\circ. \end{aligned}$$

Next we multiply by $3'$.

$$\begin{aligned} 3'' \times 3' &= 9''' \\ 4' \times 3' &= 12'' \\ &= 1' 0''. \end{aligned}$$

We carry the $1'$.

$$4^\circ \times 3' = 12',$$

to which we add the carried $1'$, obtaining $13'$ —*i.e.*, $1^\circ 1'$.

The addition is straightforward.

Example (c): Find the area $4^\circ 5' 7'' \times 2^\circ 6' 4''$.

$$\begin{array}{r} \begin{array}{r} 4^\circ \quad 5' \quad 7'' \\ 2^\circ \quad 6' \quad 4'' \\ \hline 8^\circ \quad 11' \quad 2'' \end{array} \\ \begin{array}{r} 2^\circ \quad 2' \quad 9'' \quad 6''' \\ \quad 1' \quad 5'' \quad 10''' \quad 4'''' \\ \hline 11^\circ \quad 3' \quad 5'' \quad 4''' \quad 4'''' \end{array} \end{array}$$

It is sufficient to explain how the third line is obtained—*i.e.*, $4^\circ 5' 7'' \times 4''$.

We have $7'' \times 4'' = 28'''' = 2''' 4''''$.

We carry the $2'''$

$$5' \times 4'' = 20''',$$

to which we add the carried $2'''$, obtaining $22'''$, which equals $1'' 10''''$. We carry the $1''$

$$4^\circ \times 4'' = 16'',$$

to which add the carried $1''$, obtaining $17''$ —*i.e.*, $1' 5''$

The addition is easy if we bear in mind that we must divide the sum wherever possible by 12, set down the remainder, and carry the quotient.

Cubic Units. In exactly the same way as above, the cubic foot may be divided into twelve equal slices called primes, and each of these again into 12 seconds, and so on.

Example. Find the volume (called frequently by tradesmen the “cubical content”) of a slab $1^{\circ} 3' \times 11' \times 9'$.

1°

3'

11'

1°

1'

9''

9'

10'

3''

9'''

The student should verify this by finding the volume (in the ordinary way) of a slab 15 in. $\times$ 11 in. $\times$ 9 in.

EXERCISES XXV

1. Find the areas of the following rectangles, employing the duodecimal method:

- (a) $6^{\circ} 3' 4'' \times 2^{\circ}$.

(b) $6^{\circ} 3' 4'' \times 2'$

(c) $4^{\circ} 2' 8'' \times 4^{\circ}$.

(d) $3^{\circ} 4' 7'' \times 4^{\circ} 1'$.

(e) $2^{\circ} 8' 5'' \times 5^{\circ} 2'$.

(f) $4' 3'' \times 2^{\circ} 7'$

2. Calculate the areas of the triangles in the following cases:

- (a) Base $4^{\circ} 5' 10''$, altitude $2^{\circ} 7'$

(b) Base $3^{\circ} 10' 7''$, altitude $5^{\circ} 9'$

(c) Base $2^{\circ} 6' 8''$, altitude $4^{\circ} 11' 3''$.

(d) Base $1^{\circ} 3' 9''$, altitude $3^{\circ} 7' 11''$

3. The following measurements give in each case the length, breadth, and thickness of a rectangular solid. Calculate the volumes.

- (a) $4^{\circ} 3' \times 1^{\circ} 7' \times 2^{\circ} 5'$.

(b) $5^{\circ} 7' \times 5' \times 3' 2''$.

(c) $2^{\circ} 3' 2'' \times 4' \times 2' 7''$

4. The base of a rectangular prism measures $1^{\circ} 3' 5'' \times 1^{\circ} 2'$. The height is $3^{\circ} 7' 11''$. Calculate the total surface area.

5. Calculate the total surface area of a rectangular slab $2^{\circ} 9' 8'' \times 1^{\circ} 10' \times 2' 7''$.

6. Find the area of a circle having radius $4^{\circ} 7' 9''$.
7. Calculate the area of a trapezium, the parallel sides measuring $7^{\circ} 8' 11''$ and $5^{\circ} 10' 9''$, and the perpendicular distance between these being $8^{\circ} 5' 7''$.
8. Calculate the area of a rectangle $7^{\circ} 8' 9'' \times 3^{\circ} 11' 5''$, and then find $\frac{1}{7}$ of this area.
9. In the middle of a rectangular metal sheet $8^{\circ} 4' 9'' \times 6^{\circ} 7' 8''$ a semicircular hole is cut, the diameter being $3^{\circ} 7' 6''$. Calculate the area of the part left.
10. Find the area of a trapezium of which the parallel sides measure $5^{\circ} 7' 4''$ and $8^{\circ} 5' 10''$, the perpendicular distance between these being $7^{\circ} 7' 5''$.

CHAPTER XXVI

MISCELLANEOUS PROBLEMS

A. Time and Work

MODEL EXAMPLES

(a) A can mow 3 acres in 4 days, B 4 acres in 5 days, C 7 acres in 10 days. How long should all three, working together, take to mow 15 acres?

A can mow	$\frac{3}{4}$ acre in	1 day.
B	$\frac{4}{5}$	1
C	$\frac{7}{10}$	1

Hence their combined work for 1 day

$$\begin{aligned}
 &= \frac{3}{4} + \frac{4}{5} + \frac{7}{10} \text{ acres.} \\
 &= \frac{45}{20} \\
 &= \frac{9}{4}
 \end{aligned}$$

Hence time required for 15 acres

$$\begin{aligned}
 &= 1 \text{ day} \times 15 \times \frac{4}{9} \\
 &= 6\frac{2}{3} \text{ days.}
 \end{aligned}$$

(b) A can mow a field in 12 days. A and B together require 8 days. How long should B take working alone?

A alone does $\frac{1}{12}$ of the field in 1 day.

A and B do $\frac{1}{8}$ of the field in 1 day.

B alone does $\frac{1}{8}$ less $\frac{1}{12}$ of the field per day.

I.e., B alone does $\frac{1}{24}$ of the field in 1 day.

Hence he should require 24 days to do the whole field.

EXERCISES XXVIA

1. A can mow a field in 16 hrs. A and B together take 12 days. How long should B take working alone?

2. A tank can be filled in 90 min. by one tap. Another tap would fill the tank in 100 min. What time will it take to fill the tank if both taps are turned on?

3. How long will it take to fill a swimming-pond by turning on three taps simultaneously if one of the taps alone would require 30 hrs. to fill the pond, another 25 hrs., and the third 35 hrs.?

4. One man requires $2\frac{1}{2}$ hrs. to mow a lawn. Another man requires $3\frac{1}{2}$ hrs. What time will the two men require when working together?

5. A can mow a field in 18 hrs. B takes 20 hrs. If A works alone for 3 hrs., and is then joined by B, how long will it take to finish the work?

6. A and B together can mow a field in 9 days, B and C in 6 days, C and A in 8 days. Find what time B would take if working alone.

(Find what part would be overtaken in 1 day (*a*) by A and B jointly, (*b*) by B and C jointly, and (*c*) by C and A jointly. The sum of (*a*) and (*b*) will give a day's work by A and C jointly, together with *two* days' work by B. Subtracting from this total the result (*c*), we find the part overtaken by B in *two* days. Hence find B's time for the whole work.)

7. A, B, and C, working together, can dig a certain trench in 6 days. A and B jointly require 9 days. What time would C require if working alone?

8. A can do a piece of work in 12 hrs., B in 15 hrs. If A works at it for 3 hrs., how long should B, working alone, take to finish it?

9. After 3 hrs.' work by A alone, in the preceding question, how long would A and B, working together, take to finish the work?

10. A can do a piece of work in 18 hrs., B in 20 hrs., C in 24 hrs. A works alone for 3 hrs., and is then joined by B. An hour later C joins A and B. Calculate the additional time required to complete the job.

11. A, with 6 days' help from B, can mow 12 acres in 10 days. B, with 3 days' help from A, can mow 10 acres in 15 days. In what time will A and B together mow 22 acres?

B. Race Problems

MODEL EXAMPLES

(a) A can give B 20 yds. in a race of 1 mile, and B can give C 88 yds. What can A give C?

Simplify the problem thus:

- (i) A can run 1760 yds. in the time that B takes to run 1740 yds.
- (ii) B can run 1760 yds. in the time that C takes to run 1672 yds.
- (iii) How many yds. can C run while A runs 1760 yds. —i.e., while B runs 1740 yds.?

From (ii) and (iii) we obtain the following exercise in simple proportion:

C runs 1672 yds. while B runs 1760 yds.
C ? yds. 1740

$$\begin{aligned} \text{Required distance} &= \frac{\overset{19}{\cancel{152}} \overset{87}{\cancel{1740}} \times 1672}{\underset{16}{\cancel{1760}} \underset{2}{\cancel{1760}}} \text{ yds.} \\ &= 1653 \text{ yds.} \end{aligned}$$

I.e., while A runs 1760 yds. C runs 1653 yds.

Hence A can give C 107 yds. in a mile race.

Alternative method by Chain Rule:

A's time for 1760 yds. = B's time for 1740 yds.
B's 1760 = C's 1672
C's x = A's ,, 1760

Whence $x = \frac{1740 \times 1672 \times 1760}{1760 \times 1760} = \dots, \text{ etc.}$

(b) A and B start simultaneously from the same point to race in the same direction round a circular track, A at 7 miles per hour, B at $7\frac{1}{2}$ miles per hour. In what time will they meet again if the circumference is $\frac{1}{4}$ mile?

Since B goes ahead of A at once he must make up $\frac{1}{4}$ mile on A before he overtakes him again.

Now B makes up $\frac{1}{2}$ mile in 1 hr.

$\therefore \frac{1}{4}$ „ „ $\frac{1}{2}$ hr.

I.e., required time is 30 min. (Verify this.)

(c) The same data and problem as in Example (b), except that the men run in opposite directions.

As before, distance separating the men is 1 round— $\frac{1}{4}$ mile.

Now, A and B are approaching each other at $14\frac{1}{2}$ miles per hour. Hence the problem:

1 hr. is required for $14\frac{1}{2}$ miles.

? $\frac{1}{4}$ mile.

$$\begin{aligned} \text{I.e.,} \quad \text{required time} &= 1 \text{ hr.} \times \frac{\frac{1}{4}}{14\frac{1}{2}} \\ &= 1 \text{ hr.} \times \frac{2}{4 \times 29} \\ &= \frac{60}{58} \text{ min.} \\ &= 1 \text{ min. } 2\frac{2}{29} \text{ sec.} \end{aligned}$$

N.B. By finding now how far either runner travels in 1 min. $2\frac{2}{29}$ sec. we get at once the distance of the meeting-point from the starting-point. (Verify the above result.)

(d) Two trains, one 360 ft. long, and travelling at 30 miles per hour, the other 300 ft. long, and travelling at 25 miles per hour, are approaching each other on parallel tracks. If there is exactly $\frac{1}{4}$ mile between the fronts of the two engines at the instant of observation, how long will it be (i) before the engine fronts are abreast of each other, (ii) before the trains have completely passed each other?

(i) The distance $\frac{1}{4}$ mile is being reduced at the rate of $30 + 25$ miles per hour. Hence this distance is quite abolished in

$$1 \text{ hr.} \times \frac{\frac{1}{4}}{55}.$$

$$\text{I.e.,} \quad \frac{1}{220} \text{ hr.}$$

$$\text{or} \quad \frac{3600}{220} \text{ sec.}$$

$$\text{I.e.,} \quad 16\frac{4}{11} \text{ sec.}$$

- (ii) The distance separating the rears of the trains is
 $\frac{1}{4}$ mile + (360 + 300) ft.

I.e., 660 yds.

Time to abolish this distance is

$$1 \text{ hr.} \times \frac{660}{55 \times 1760}.$$

$$\begin{array}{r} 90 \quad 3 \\ 450 \quad 6 \\ \hline 3600 \times 660 \\ \hline 55 \times 1760 \end{array} \text{ sec.}$$

$$\begin{array}{r} 270 \\ 11 \end{array} \text{ sec.}$$

$$\begin{array}{r} 270 \\ 11 \end{array} \text{ sec.}$$

EXERCISES XXVI_B

1. In a race between two runners, A and B, find when the faster will overtake the slower in each of the following cases:

- (a) A's speed $7\frac{1}{2}$ miles per hour.

B's „ $7\frac{1}{4}$ „

B receives 40 yds.

- (b) A's speed $6\frac{3}{4}$ miles per hour.

B's „ 7 „

A receives 30 yds.

- (c) A's speed $7\frac{3}{4}$ miles per hour.

B's „ $7\frac{1}{2}$ „

B receives 50 yds.

2. One cyclist travels at $12\frac{1}{2}$ miles per hour, another at 11 miles per hour. What start can the faster man concede in a mile?

3. Two clocks show correct time at midnight Sunday. One clock loses 1 min. every 24 hrs., the other gains $1\frac{1}{2}$ min. What time will elapse before the clocks show an hour of difference? What time will each clock show at that instant?

4. A and B start from opposite ends of a road 15 miles long. Each walks along the road to the other end and back again. If A's rate is 4 miles per hour, and B's rate $3\frac{1}{2}$ miles per hour, find the points and the times of meeting.

5. A train 160 yds. long passes through a station at 15 miles per hour. If the platform is 240 yds. long, what time will the train take to clear it?

6. A and B working together can mow a field in 45 hrs. If they work together for $16\frac{1}{2}$ hrs. A can then finish alone in 57 hrs. more. In what time could each mow the entire field if working alone?

7. Assuming that in a game of billiards A can give B 20 in 100, and that B can give C 30 in 100, what should A concede to C in a match of 1000 up?

8. One pipe can fill a tank in 36 hrs. An outlet can empty the tank in 24 hrs. In order to keep the tank full when both these pipes are open it will obviously be necessary to fit another inlet pipe. In what time must this new pipe be able to fill the tank when working alone?

9. A and B can mow a field in 18 hrs., B and C in 20 hrs., C and A in 21 hrs. If all three work together, and if the wages paid be in proportion to each man's output, find (a) in what time the work will be done, (b) what should be the wage of each when 60s. is paid for the whole job?

10. A clock keeping accurate time shows exactly 5 o'clock. In what time will the two hands exactly coincide?

11. A man can arrive at his destination at the same time either by setting out at noon and walking uniformly at 3 miles an hour, or by catching the 6 P.M. train, which travels at 15 miles per hour. Calculate the length of his journey.

12. Three pipes, A, B, C, can fill a cistern separately in 48 min., 20 min., and 54 min. respectively. If they are all opened simultaneously, and kept open for 9 min., in what time will pipe A alone complete the filling of the cistern?

13. One clock gains 2 min. in 24 hrs., another gains 10 min. in 24 hrs. If they are both right at noon on January 1, find what actual time will elapse before the clocks agree again, and what time each clock will be showing at that instant.

14. A train 330 ft. long meets a man walking at 5 miles an hour, and passes him in 10 sec. A quarter of an hour later it meets a cyclist and passes him in 8 sec. Find (*a*) the speed of the train, (*b*) the rate of travelling of the cyclist, and (*c*) the time the cyclist will require to overtake the pedestrian.

15. A, who walks at $3\frac{1}{2}$ miles per hour, sets out at noon. B, walking at 4 miles per hour, follows at 1.30 P.M., stops at 3 o'clock for 1 hr., and then continues. At what time will B overtake A?

CHAPTER XXVII

GRAPHS

IN its simplest form a graph is a pictorial representation of the rate at which a variable quantity changes its value. Fig. 1 enables us to realize at a glance the successive values of the height of a growing plant measured every fifth day.

O is any conveniently chosen point called the *origin*. OX and OY are two straight lines, called axes, drawn through O

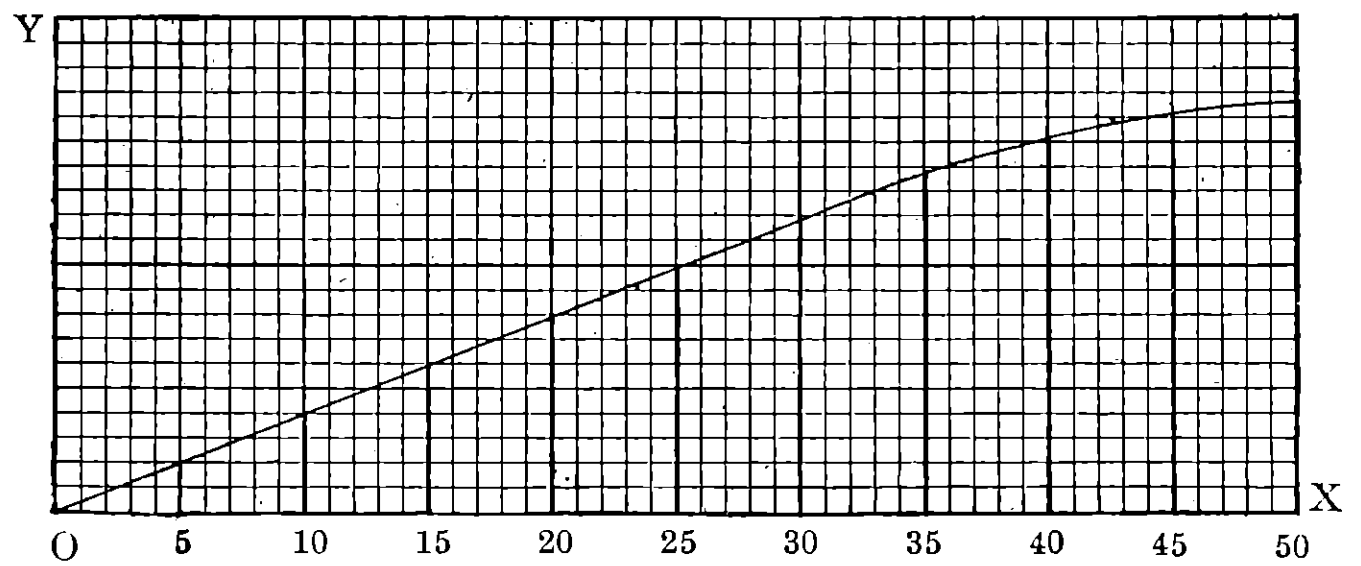


FIG. 1

at right angles to each other. It is customary, though not necessary, to draw OX from left to right, parallel to the top edge of the page. The usual position for OY is found by rotating OX counter-clockwise through a right angle.

Every half-inch along OX represents a period of five days; and the perpendiculars, called *ordinates*, represent the successive values of the height of the plant. These ordinates may be drawn exactly equal to the heights, or they may be drawn to scale. In the diagram given above they are made exactly double the actual heights.

The free line drawn through the tops of the ordinates is

called the graph. In practice it is customary to mark the top points only, and to leave the ordinates undrawn.

Fig. 2 shows the variation in the height of the mercury column of the barometer as observed daily at noon for three weeks.

Successive days are indicated along OX by intervals of one small division. It saves space to make the ordinates represent

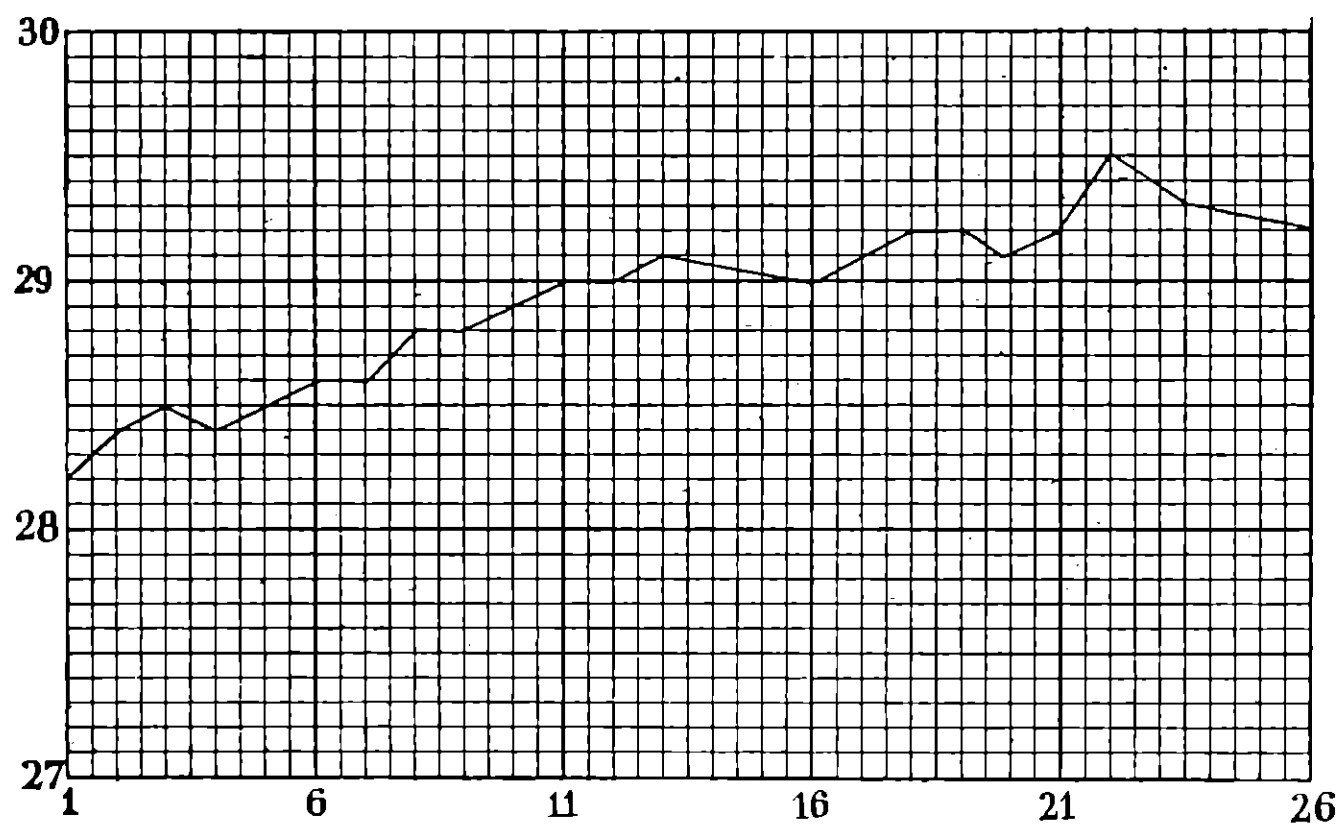


FIG. 2

merely the excess of the heights above 27 in.—*e.g.*, a barometric reading of 29.4 in. is represented by an ordinate of 2.4 in. Note that instead of finding the top points of the ordinates by measuring upward from the axis OX in every case, it saves labour to indicate the scale once for all along the axis OY, and to make reference to this scale as required.

EXERCISES XXVIIA

In each of the following questions decide, first, which of the variables is the one in which we are interested; then indicate the scale for that variable along OY. The scale for the other variable is then marked along OX.

1. Show graphically the variation of the rainfall at a certain

place throughout the year from the following monthly totals. The measurements are in inches.

JAN.	FEB.	MARCH	APRIL	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
4.1	3.1	3.7	2.6	1.1	0.5	0.2	0.6	2.0	3.8	4.0	3.8

Mark the months along OX, taking $\frac{1}{2}$ in. to represent a month.

2. Draw a graph to show how the price of tin kettles varies as the capacity changes:

Capacity in pints	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
Price	1s. 9d.	2s. 1d.	2s. 5d.	2s. 8d.	2s. 11d.	3s. 1d.	3s. 3d.

Mark the capacity in pints along OX, taking 1 in. to represent 1 pt.; and along OY take $\frac{1}{10}$ in. to represent 1d.

3. Draw a graph to show how the charge for a telephone trunk-call varies with the distance:

Miles	10	$12\frac{1}{2}$	15	20	25	35	50	75	100	125	150	200	250	300	350	400
Pence	3	3	4	5	7	9	12	18	24	27	33	39	39	48	48	57

We are to have a range of 400 miles along OX. Hence a suitable scale will be 1 in. to 100 miles. Also we have a range of 57d. along OY; hence a suitable scale will be $\frac{1}{10}$ in. to 1d.

4. Show graphically the rate of increase in the number of fatal street accidents for the years 1921 to 1928.

Year	1921	1922	1923	1924	1925	1926	1927	1928
Accidents	571	655	668	844	840	1003	1056	1244

Take $\frac{1}{2}$ in. intervals along OX to denote the years, making the origin 1920. Note that the range of accidents from 571 to 1244 is 750 approximately. We wish to use, say, 6 in. approximately of the OY axis; hence we may use the scale 1 in. to 150 accidents, making the origin correspond to 500.

5. Show graphically how the amount of income tax for single persons varies with the income.

Income	£175	£180	£190	£200	£210	£225	£250	£270
Tax	£1 2s.	£1 10s.	£2 6s.	£3 4s.	£4 0s.	£5 5s.	£7 6s.	£9 0s.

Income	£285	£300	£350	£400	£450	£500	£550
Tax	£10 5s.	£11 10s.	£15 14s.	£19 16s.	£25 10s.	£33 18s.	£42 6s.

Income	£600	£650	£700	£800	£900	£1000
Tax	£50 10s.	£58 18s.	£67 2s.	£83 18s.	£100 10s.	£117 2s.

For income we have to show a range of £1000 along OX; a suitable scale is 1 in. to £200. For tax the range is about £120. Assuming that we have 6 in. of space above the OX axis, we may take the scale 1 in. to £20.

6. Show the variation in the rate of income tax over the period 1907 to 1931.

Year	1907	1908	1909	1910	1911	1912	1913	1914	1915	1916
Rate	1s.	1s.	1s.	1s. 2d.	1s. 2d.	1s. 2d.	1s. 2d.	1s. 2d.	1s. 8d.	3s.

Year	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926
Rate	5s.	5s.	6s.	6s.	6s.	6s.	5s.	4s. 6d.	4s. 6d.	4s.

Year	1927	1928	1929	1930	1931
Rate	4s.	4s.	4s.	4s.	4s. 6d.

7. Show by a graph the variation in the average price per quarter of British corn over the period 1908 to 1928.

Year	1908	1909	1910	1911	1912	1913	1914
Price	32s.	36s. 11d.	31s. 8d.	31s. 8d.	34s. 9d.	31s. 8d.	34s. 11d.

Year	1915	1916	1917	1918	1919	1920	1921
Price	52s. 10d.	58s. 5d.	75s. 9d.	71s. 11d.	72s. 11d.	80s. 10d.	71s. 6d.

Year	1922	1923	1924	1925	1926	1927	1928
Price	47s. 10d.	42s. 2d.	49s. 3d.	52s. 2d.	53s. 3d.	50s. 1d.	42s. 10d.

8. Show graphically the increase in speed in flying as indicated by the attainments in the Schneider Trophy contests from 1914 to 1931.

Year	1914	1920	1921	1922	1923	1925	1926	1927	1929	1931
Speed in m.p.h.	55.3	102.3	117.4	146.5	177.4	232.6	246.5	281.5	328.6	379.0

9. Illustrate graphically the variation in the average weight of boys in relation to variation in height. (The figures given relate to boys aged 15 to 19 yrs.)

Height in inches	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
Weight in lb.	108	110	112	114	117	120	123	126	130	134	138	141	145	150	155

10. Draw a graph to show how the logarithm of a number increases as the number increases.

Numbers	10	15	20	25	30	35	40	45	50	55	60
Logarithms	1.00	1.18	1.30	1.40	1.48	1.54	1.60	1.65	1.70	1.74	1.78

Numbers	65	70	75	80	85	90	95	100
Logarithms	1.81	1.84	1.87	1.90	1.93	1.96	1.98	2.00

Along OY take the scale for the logarithms as 5 in. to represent unity.

From your graph write down the logarithm of each of the following numbers: 17, 22, 33, 47, 62, 87, 92.

MATHEMATICAL GRAPHS

In mathematical graphs the relation between the two variables can be expressed by an equation.

(a) Thus, the relation between the area of a square and the length of the side may naturally be expressed by the equation

$$A = s^2.$$

In this case we may choose any values we please for s , and for each value we can calculate the corresponding value of A . s^2 is said to be a *function* of s , since it depends entirely on s for its value. Hence, of course, we are entitled to speak of the area of a square as a *function* of the length of the side. s is called the *argument* of the function.

(b) Similarly, if one side of a rectangle is 3 units longer than the other, we may represent the lengths of the sides by s and $s + 3$; and the area in square units will be $s(s + 3)$, or $s^2 + 3s$. Our equation will be $A = s(s + 3)$, i.e., A is a *function* of s . Again, s is the *argument* of the function $s(s + 3)$.

(c) Similarly, if the length, breadth, and depth of a rectangular box be $s + 3$, $s + 1$, and s , respectively, we have the equation

$$V = s(s + 1)(s + 3),$$

where V is the volume in cubic units.

$s(s + 1)(s + 3)$, or $s^3 + 4s^2 + 3s$, is the function, and s is the *argument* of the function.

(d) Again, we know that the circumference of a circle is found by multiplying the diameter by a constant factor (3.1416 approximately), denoted by the Greek letter π , so that we have the equation $C = \pi D$ —i.e., the circumference is a *function* of the diameter. Here D is the *argument* of the function πD . Alternatively, C equals the *function* $2\pi r$, and r is the *argument* of this function.

(e) Here is an example of a kind totally different from any of the foregoing:

A train travels over one part of its course at a uniform rate of 40 miles per hour. If the length of this part of the course is 180 miles we can find the required time (in hours) thus:

$$t = \frac{180}{40}.$$

We may put this relation more generally by supposing any uniform speed, say, v miles per hour.

Then we have
$$t = \frac{180}{v}.$$

I.e., the time is a *function* of the speed, and v is the *argument* of the function.

N.B. In cases (a) to (e) mentioned above, and in all similar cases, if we draw a graph to show the relation between the *function* and the *argument*, we are said to have drawn a *graph of the function*, or, more briefly, we are said to have *graphed the function*. It is customary to mark the values of the argument along OX and the values of the function along OY.

Important Precautions to be observed in drawing any Mathematical Graph

(a) Before drawing up your table of values make a rough forecast to ensure that the unit you are choosing is such that your page will accommodate the graph you propose to draw.

(b) When the points have been marked—preferably by pricking with a sharp point—endeavour to sweep a smooth curve through the points. If any point seems to lie outside of such a smooth curve investigate the accuracy of the values for that point.

(c) If at any critical part of your graph you feel that a closer series of plotted points is desirable, go back to your table of values and insert the necessary values.

(d) The graph should be a *fine, smooth* line.

EXERCISES XXVII_B

1. Draw a graph to illustrate how the area of a square increases as the side increases.

From your graph read off the area of a square the side of which measures (a) 1.5 units, (b) 1.9 units, (c) 2.3 units.

Choose for yourself a series of suitable values for the side, s , and then calculate the corresponding values for the area, s^2 .

Tabulate your values as below, and then plot points on the graph, taking 1 in. as the unit of length along OX, and 1 in. to represent the unit of area along OY.

<i>s</i>	0		·4	·6	·8	1	1·2	1·4	1·6	1·8	2	2·2	2·4
<i>s</i> ²	0	·04	·16	·36	·64	1	1·44	1·96	2·56	3·24	4	4·84	5·76

2. One side of a rectangle is 2 units longer than the other. Draw a graph to show how the area increases as the breadth increases.

(If *s* represents the length of the shorter side, then the length of the other side will be represented by *s* + 2, and the area will contain *s* (*s* + 2) square units, or *s*² + 2*s* square units. Thus you are asked to show how the function *s*² + 2*s* varies as *s* varies. The method is similar to that indicated in Example (a) above.)

3. The area of a circle is given by the formula πr^2 , where *r* is the length of the radius. Illustrate by a graph how the area varies as the radius varies. (Take π as $\frac{22}{7}$.)

4. Draw a graph to illustrate how the volume of a cube increases as the edge increases.

From your graph read off the volume of the cube of which the edge measures (a) 3·5 units, (b) 3·8 units, (c) 4·1 units.

(If *s* be the length of the edge, then the volume will be given by *s*³. Let $\frac{1}{10}$ in. along OX represent a unit of length, and $\frac{1}{10}$ in. along OY represent a unit of volume.)

5. Draw a graph to illustrate how the volume of a brick-shaped solid, of which the depth, breadth, and length are respectively *s*, *s* + 1, and *s* + 2 units, increases as *s* increases.

(The volume is given by *s* (*s* + 1) (*s* + 2), or *s*³ + 3*s*² + 2*s*. This is the function to be graphed.)

6. Calculate the respective times required to travel a *distance* of, say, 72 miles at the following speeds in miles per hour: 1, 1 $\frac{1}{4}$, 1 $\frac{1}{2}$, 1 $\frac{3}{4}$, 2, 2 $\frac{1}{2}$, 3, 3 $\frac{1}{2}$, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 24, 30, 40, 60. Tabulate speeds and corresponding times, and draw a graph to show how the time decreases as the speed increases.

(Let $\frac{1}{10}$ in. along OX represent a speed of 1 mile per hour, and let $\frac{1}{10}$ in. along OY represent 1 hr. of time required for the journey.)

Notes on Typical Curves. In Questions 1 and 2 of the foregoing the curves are identical. This curve is called the *parabola*, and is inevitably the curve of a function which involves the *square* of the argument. In Question 3 the curve is also a parabola, but is somewhat drawn 'out'—the result of the square—viz., r^2 —being multiplied by the factor π .

In Questions 4 and 5 the curve is the *cubic* curve.

In Question 6 the curve is the *hyperbola*.

Note that this curve is inevitable when one of our variables is inversely proportional to the other. In this case the *time* is inversely proportional to the *speed*—i.e., if we double the *speed* we halve the *time*; if we take three times the *speed* we get a third of the time; and so on.

EXERCISES XXVII_B—continued

7. Assuming that pencils cost $1\frac{1}{2}d.$ each, draw a graph to illustrate how the cost of a number of pencils increases as the number of pencils increases.

From your graph read off the cost of (a) 13 pencils, (b) 29 pencils, (c) 43 pencils, (d) 58 pencils.

(Take $\frac{1}{10}$ in. along OX to represent one pencil, and $\frac{1}{10}$ in. along OY to represent a penny.)

8. A car is running at a constant rate of 2 min. to the mile. Draw a graph to illustrate how the distance increases as the time increases.

From your graph read off the distance travelled in (a) 1 hr., (b) 83 min., (c) an hour and a half.

9. Assuming that 1 litre = 1.76 pts., draw a graph to illustrate the relation between litres and pints.

From your graph write down correct to two decimal places the number of pints in (a) 1.5 litres, (b) 1.86 litres, (c) 4.37 litres; also the number of litres in (d) 3 pints, (e) 4.5 pints, (f) 5.75 pints, (g) 6.39 pints.

(Take 1 in. along OX to represent 1 litre, and 1 in. along

OY to represent 1 pt. Make a table of corresponding values, thus :

Litres	1	2	3	4
Pints	1·76	3·52	5·28	7·04

Draw the graph accordingly.

10. An examiner in marking a set of worked examination papers fixes his possible mark at 48, and then wishes to express the mark awarded to each paper as a percentage mark.

Draw a graph to connect the actual marks and the corresponding percentages.

Write down from your graph the percentage mark corresponding to the following actual marks: (a) 31, (b) 16, (c) 43, (d) 29.

(Make your table of values from the following considerations: 48 marks corresponds to 100 per cent., 24 will correspond to 50 per cent., 12 to 25 per cent., 6 to 12·5 per cent., 36 to 75 per cent., etc. Take ·1 in. along OX to represent each actual mark, and ·1 in. along OY to represent each percentage mark.)

Important Note. Notice that in each of the foregoing questions, Nos. 7 to 10, the two variables involved are directly proportional to each other; and notice the constant nature of the graph. Remember that wherever direct proportionality is involved the graph is *invariably a straight line*.

EXERCISES XXVIIc

Miscellaneous Graphs

1. Complete the following table of numbers and square roots :

Number	0	·2	·4	·7	·9	1	1·5	2	3	4	5	6	7	8
Square root	0	·45	·63						·6					

Draw a graph connecting numbers and square roots. From your graph write down $\sqrt{·8}$, $\sqrt{1·9}$, $\sqrt{3·6}$, $\sqrt{5·75}$.

2. Make out a table showing a few thermometer readings, Fahrenheit scale, and the corresponding Centigrade readings. Draw a graph connecting the two scales.

F.	32°	77°	122°	167°	212°
C.	0°	25°	50°	75°	100°

From your graph write down the Fahrenheit readings corresponding to (a) 60° C., (b) 12° C., (c) 41° C.; also the Centigrade readings corresponding to (d) 40° F., (e) 60° F., (f) 133° F.

3. The following particulars are taken from a Scottish railway time-table:

—	ABER- DEEN	PERTH	CAR- LISLE	STIR- LING	CREWE	BRISTOL	NEW- CASTLE	BIR- MING- HAM
Distance in miles from Glasgow	153	63	102	30	243	385	172	296
Single fare	19s. 1d.	7s. 11d.	12s. 9d.	3s. 8d.	30s. 6d.	47s. 8d.	20s. 9d.	37s.

Draw a graph to show the relation between distance and single fare.

From your graph estimate the single fare to (a) Oban, 117 miles from Glasgow; (b) Preston, 192 miles; (c) York, 252 miles; (d) London, 425 miles.

(Take 1 in. to represent 100 miles along OX, and 1 in. to represent 10s. along OY. If your plotted points do not seem to lie along a continuous line draw the curve which seems best to conform to the points.)

4. The following table gives the amount of £1 at 2½ per cent. interest convertible yearly for periods up to 50 yrs. Draw a graph to show the relation between the amount and the time.

Amount	£1·13	£1·28	£1·45	£1·64	£1·85	£2·10	£2·37	£2·69	£3·04	£3·44
Time in years	5	10	15		25	30	35	40	45	50

From your graph write down (a) the amount at the end of 25 yrs., (b) the time required for a sum of money to double

itself, and (c) the time required for a sum of money to treble itself.

5. From the following table draw a graph to illustrate the variation in temperature from midnight on a certain date to midnight on the following date:

Time	Mid-night	2 A.M.	4 A.M.	6 A.M.	8 A.M.	10 A.M.	Noon
Temperature in degrees Fahrenheit	60°	59·5°	59·3°	59·5°	60·6°	62°	63·4°

Time	2 P.M.	4 P.M.	6 P.M.	8 P.M.	10 P.M.	Mid-night
Temperature in degrees Fahrenheit	64·2°	64·1°	63°	61·7°	60·7°	59·6°

From your graph estimate the temperature at (a) 11 A.M., (b) 5 P.M., (c) 7 P.M.

(Take $\frac{1}{2}$ in. along OX to represent 2 hrs., and take 1 in. along OY to represent 1 degree of temperature above 59°—i.e., the origin marks 59° on OY. Since the temperature is likely to have gone through these variations gradually it will *probably* be right to draw a continuous curve through your plotted points.)

6. At a certain constant temperature the volume of a quantity of air is related to the pressure (measured in inches of mercury) in accordance with the equation $P \times V = 180$. Make a table giving P values from 30 to 90 by intervals of 5 in., and calculating the corresponding values of V Draw a graph to show the relative variation of P and V

7. The following table gives the distances through which a stone, falling freely, will pass in the appended times:

Time in seconds	0	1	2	3	4	5	6	7
Distance in feet	0	16	64	144	256	400	576	784

Draw a graph to show the relation between distance and time. From your graph write down the relative position of the stone after it has fallen for (a) $\frac{1}{2}$ sec., (b) $2\frac{1}{2}$ sec., (c) $5\frac{1}{2}$ sec. Write down also the time required for the stone to fall (d) 50 ft., (e) 100 ft., (f) 200 ft.

8. In marking examination papers the total mark is fixed at 80. Draw a graph to enable you to read off the percentage figure corresponding to any actual mark gained. What percentage mark does a candidate receive if his actual mark is (a) 57, (b) 63, (c) 70? What actual mark corresponds to (d) 45 per cent., (e) 60 per cent., (f) 70 per cent.?

9. The following table shows different lengths of pendulums and the appropriate times required to make one swing:

Time of swing in seconds	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$1\frac{1}{2}$	
Length in inches	0.6	2.4	9.8	22	39	88
						156

Write down the length of pendulum which takes (a) .4 sec. to swing, (b) .8 sec.; and also the time of swing of a pendulum (c) 30 in. long, (d) 60 in.

10. The breadth of a rectangle is half its length. Draw a graph to show how the area varies as the length varies.

(The area is a function of the length; for if l denotes the length, then $\frac{l}{2}$ denotes the breadth, and hence we have $A = \frac{1}{2}l^2$.)

11. In the preceding question we may take b to denote the breadth. Then length will be $2b$ and $A = 2b^2$, i.e., the area is a function of the breadth.

Draw a graph connecting area and *breadth*, and try to explain the reason why the curve differs in form from the curve connecting area and *length*.

12. Employing the same axes, draw three graphs (a) one connecting the *perimeter* of a square and the *side*, (b) one connecting the *area* of the square and the *side*, (c) one connecting the *volume* of a cube and the *edge*.

What do we learn from observing the differences of the three curves?

(If s denotes the side of the square or the edge of the cube, we have three functions of s :

$$(a) \text{ Perimeter} = 4s.$$

$$(b) \text{ Area} = s^2.$$

$$(c) \text{ Volume} = s^3.$$

Take the following values of s : 0, .2, .4, .6, .8, 1, 1.2, 1.5, 1.7, 2.)

13. The following table gives (a) the dial settings on a certain wireless instrument corresponding to the given stations, and also (b) the wave-lengths on which the stations operate.

Find from an accurate graph whether the dial figures are directly proportional to the wave-lengths.

Station	Cork	Lon- don	Heils- berg	Bor- deaux	Göte- borg	Stras- bourg	Glas- gow
Dial	19	33	41	52	60	70	80
Wave-length in metres	224	261	276	304	322	345	376

Station	Sot- tens	Rome	Lan- gen- berg	Milan	Vien- na	Buda- pest	Ljubl- jana
Dial	91	109	124	136	145	163	180
Wave-length in metres	403	441	473	501	517	550	574

From your graph write down the dial settings you would expect to employ to get the following stations:

Prague (487 m.), Madrid (424 m.), London Regional (356 m.), Turin (296 m.), Leipzig (253 m.), Warsaw No. 2 (214 m.).

GRAPHICAL SOLUTIONS OF PROBLEMS

14. A starts walking at 12 noon at the average rate of 4 miles an hour. At 2.30 P.M. B sets out from the same starting-place and cycles along the same road at 9 miles an hour.

Draw, with the same axes, two graphs connecting *distance covered* and *time* (*a*) for A's journey, (*b*) for B's journey.

Find at what o'clock, and how far from the starting-place, B will overtake A. Check your result by calculation.

(Take $\frac{1}{10}$ in. along OX to represent 1 mile, and 1 in. along OY to represent 1 hr.—the origin corresponding to 12 noon.)

15. If in the preceding question B punctures a tyre half an hour after setting out, and is delayed 20 min., what will be the time and place of overtaking?

16. A pedestrian setting out at 9 A.M. walks from P to Q at a uniform rate of $3\frac{1}{2}$ miles per hour, arriving at Q at 6.30 P.M., having taken half an hour's rest from 1 to 1.30 P.M. A cyclist setting out from P at 12.30 P.M., and, riding continuously at a uniform speed, arrives at Q at the same time as the pedestrian.

Draw with the same axes two graphs connecting *distance covered* and *speed* for the two journeys. From your graph find (*a*) the speed of the cyclist, (*b*) the distance from P to Q.

(Along OX take 1 in. to 5 miles. Along OY let the origin correspond to 9 o'clock, and take $\frac{1}{2}$ in. to represent 1 hr.)

N.B. These last three examples are interesting rather as illustrations than as practical methods of solution.

APPENDIX

MENSURATION FORMULÆ

Plane Figures

Rectangle. Area = $b \times h$.

Parallelogram. Area = $b \times h$

Triangle. Area = $\frac{1}{2}(b \times h)$.

Also area = $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c , are the sides and s is the semi-perimeter.

Trapezium. Area = $\frac{1}{2}(a + b)h$, where a and b are the measures of the parallel sides.

Circle. Circumference = $2\pi r$. Area = πr^2 .

Arc. Length = $\frac{a}{360}$ of $2\pi r$, where a° is the angle of the arc.

Sector. Area = $\frac{a}{360}$ of πr^2 ,

Solid Figures

Any Prism (including the cylinder). Volume = $A \times h$.

Tapering Figure. Volume = $\frac{1}{3}A \times h$.

Sphere. Surface = $4\pi r^2$. Volume = $\frac{4}{3}\pi r^3$.

WEIGHTS AND MEASURES

Weight

Avoirdupois Weight

16 drams	=	1 ounce
16 ounces	=	1 pound
14 pounds	=	1 stone
2 stones or 28 pounds	=	1 quarter
4 quarters	=	1 hundredweight
20 hundredweights	=	1 ton

Note. 1 pound (avoir.) = 7000 grains.

Troy Weight

(used by goldsmiths)

24 grains	=	1 pennyweight
20 pennyweight	=	1 ounce (Troy)
12 ounces	=	1 pound (Troy)

Note on the Carat. Early writers disagree regarding the original value of the carat as a definite weight, one valuing it as $\frac{1}{24}$ lb. Troy and another at $\frac{1}{24}$ oz. Troy. It is now employed as a term of *quality* only, in assaying gold. Thus *any weight at all* of 18-carat gold contains $\frac{18}{24}$ of that weight of pure gold. 9-carat gold contains $\frac{9}{24}$ of its own weight of pure gold, and so on.

On the other hand, jewellers use a weight called a carat in weighing stones, but it is not uniform.

Important. The grain has a constant value, whether it be used in avoirdupois, Troy, or apothecaries' weight.

Apothecaries' Weight

20 grains	=	1 scruple ($\mathfrak{S}$)
3 scruples	=	1 drachm ($\mathfrak{Z}$)
8 drachms	=	1 ounce ($\mathfrak{Z}$)
12 ounces	=	1 pound

Capacity*Liquid Measure*

4 gills	=	1 pint
2 pints	=	1 quart
4 quarts	=	1 gallon

Apothecaries' Fluid Measure

60 minims (M)	=	1 drachm (f $\mathfrak{Z}$ —i.e., 1 fluid drachm)
8 drachms	=	1 ounce (f $\mathfrak{Z}$)
20 ounces	=	1 pint
8 pints	=	1 gallon

Notes. A minim is a drop. A drachm is usually reckoned as one average teaspoonful.

Dry Measure

2 pints	=	1 quart
4 quarts	=	1 gallon
2 gallons	=	1 peck ^c
4 pecks	=	1 bushel
8 bushels	=	1 quarter
5 quarters	=	1 load
140 lb. (avoir.)	=	1 boll

Length

(12 lines	1 inch)
12 inches	= 1 foot
3 feet	= 1 yard
$5\frac{1}{2}$ yards	1 pole
22 yards	= 1 chain
40 poles or 10 chains	= 1 furlong
8 furlongs	= 1 mile

Notes. Nautical or Admiralty mile = 6080 feet.

A knot is not a unit of length, but is a measure of speed. Thus, 1 knot means a *speed* of a nautical mile per hour. Hence in the thoughtless phrase "4 knots per hour" the words "per hour" are superfluous.

Area

144 square inches	= 1 square foot
9 square feet	= 1 square yard
$30\frac{1}{4}$ square yards	= 1 square pole
40 square poles or	
1210 square yards	= 1 rood
4 roods	= 1 acre
640 acres	= 1 square mile

Volume

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard

MISCELLANEOUS MEASURES IN
COMMON USE

Water

A cubic foot of water equals approximately $6\frac{1}{4}$ gallons and weighs about $62\frac{1}{2}$ lb.

1 gallon of water weighs 10 lb.

Electrical Units

Ohm—a unit of resistance. 1 *megohm* = 1,000,000 ohms.

Ampere—a unit of current.

Volt—a unit of electromotive force. The fundamental equation is

$C = \frac{V}{R}$, where C is the current in amperes, V is the E.M.F. in volts, and R is the resistance in ohms.

Farad—a unit of capacity (of a condenser). 1 *micro-farad* = $\frac{1}{1000000}$ farad.

Watt—a unit of power of a current—*e.g.*, for lighting or for motor purposes. 746 watts = 1 horse-power. 1 *kilowatt* = 1000 watts.

UNITS EMPLOYED IN THE METRIC SYSTEM

In measuring length the fundamental unit is the *metre*, which is a little more than a yard, being equal to 39.37 in. approximately.

In measuring weight we employ the *gramme*, which equals about half a dram (avoirdupois), there being nearly 454 grammes to 1 lb.

In measuring capacity we use the *litre*, equal approximately to $1\frac{3}{4}$ pints, or, more accurately, 1.76 pts. The litre holds exactly a cubic decimetre.

The table of length is formed from the metre thus:

10^1 metre	= 1 decimetre	10 metres	= 1 decametre
10^2 „	= 1 centimetre	100 „	= 1 hectometre
10^3 „	= 1 millimetre	1000 „	= 1 kilometre

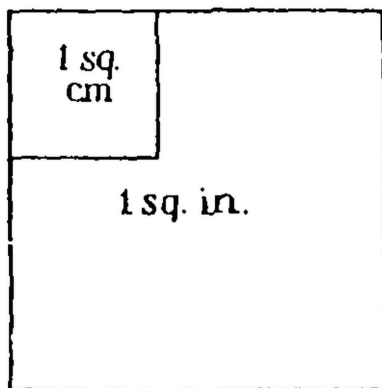
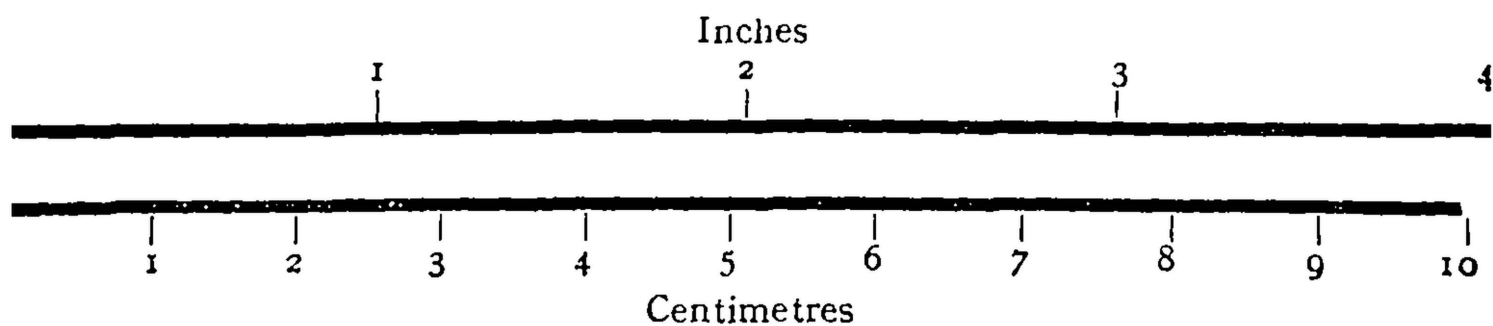
Similarly, from the gramme we have decigramme, centigramme, milligramme, and decagramme, hectogramme, kilogramme.

From the litre we have decilitre, centilitre, millilitre, and decalitre, hectolitre, kilolitre.

In area we employ a special unit, the *are*, which is the square on a line of 1 decametre and contains therefore 100 sq. metres. The *hectare* (nearly $2\frac{1}{2}$ acres) is used in land-measurement. Also, of course, there are used the square metre, square decimetre, etc.

Useful relations to memorize:

1 kilogramme	= 2.205 lb. (nearly $2\frac{1}{4}$ lb.)
1 litre	= 1.76 pints (fully $1\frac{3}{4}$ pints)
1 metre	= 39.37 in. (approx.)
$2\frac{1}{2}$ centimetres	= 1 in. „
1 kilometre	= 1095 yds. „



TABLES OF LOGARITHMS
AND ANTILOGARITHMS

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 8 12	17 21 25	29 33 37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 11	15 19 23	26 30 34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 10	14 17 21	24 28 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 6 10	13 16 19	23 26 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9	12 15 18	21 24 27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 8	11 14 17	20 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8	11 13 16	18 21 24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2 5 7	10 12 15	17 20 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7	9 12 14	16 19 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7	9 11 13	16 18 20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4660	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7

LOGARITHMS—continued

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

ANTILOGARITHMS—continued

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3398	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20