The Elementary Theory of the Internal-Combustion Engine

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BLACKIE & SON LIMITED
LONDON AND GLASGOW
1931

The Internal-Combustion Engine

BLACKIE & SON LIMITED
50 Old Bailey, LONDON
17 Stanhope Street, GLASGOW
BLACKIE & SON (INDIA) LIMITED
Warwick House, Fort Street, BOMBAY
BLACKIE & SON (CANADA) LIMITED
TORONTO

Printed in Great Britain by Blackie & Son, Ltd., Glasgow

PREFACE

Engineering apprentices and others commencing a serious study of the internal-combustion engine frequently encounter much difficulty in the theoretical chapters of advanced text-books. It is for such students, often none too well equipped mathematically, that this little book is intended. The author hopes that it will help to clear the way.

A fairly large number of exercises, with answers, has been included. Most of these are original; a few, marked (*Lond. Univ.*), have been taken from examination papers set at London University, by kind permission of the Senate; and some have been compiled from data taken from the columns of *The Motor Ship*, a journal with which all students of the Diesel engine should be closely acquainted, and to whose proprietors the author owes grateful acknowledgments.

F. W. LUDLAM.

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THE INTERNAL-COMBUSTION ENGINE

CHAPTER I

The Laws of Gases

Imagine a long glass tube of perfectly even bore, and closed at one end, to contain a quantity of any permanent gas partitioned off from the outer air by a tiny piston—say a spot of mercury. (Fig. 1.)

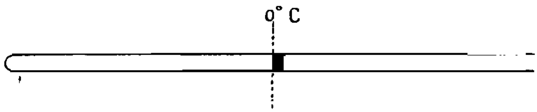


Fig. 1

Warming or cooling the tube will cause the spot of mercury to move outwards or inwards.

Suppose we mark on the tube the position occupied by the inner face of the piston, or spot of mercury, when the temperature of the enclosed gas is that of the freezing-point of water, i.e. 0° C.

Now let us mark off the length of tube between this position and the closed end into 273 equal divisions, and continue these divisions right along the tube to the other end. (Fig. 2.)

If now we warm the tube (by immersing it in a bath of water, keeping the open end above the surface), until the temperature becomes 1° C., we shall find that the spot of mercury has moved outwards exactly one division; and so on, until at 100° C. it will have moved outwards exactly 100 divisions.

Again, if, starting once more from the freezing-point, we cool the contained gas by one Centigrade degree, the piston moves *inwards* one division, and so on for each succeeding degree of cooling. But, of course, this process of contraction cannot

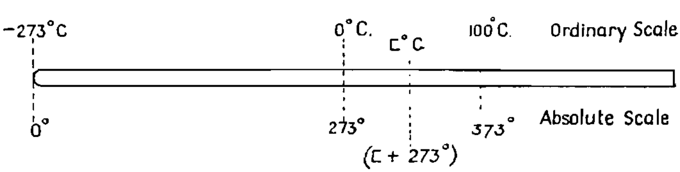


Fig. 2

go on indefinitely, or eventually, at a temperature of −273° C., i.e. 273 Centigrade degrees of frost—supposing we have means of achieving such intense coldness—we should have no gas left at all, which is clearly impossible.

But before we had cooled the gas to such an extent it would have ceased to be a gas: it would have become liquid, or perhaps solid.

But so long, however, as we are dealing with it as gas the following law is true:

When a quantity of gas is warmed or cooled by one Centigrade degree, *the pressure being kept constant*, the volume of the gas increases or decreases by $\frac{1}{273}$rd of its volume at 0° C.

This is termed **the Law of Charles.**

It doesn't matter what the permanent gas is—oxygen, hydrogen, air, or any other—the fraction is the same for all, $\frac{1}{273}$.

The condition italicized must be observed. In our experiment above, one end of the tube was open to the atmosphere, and so the gas behind the piston was always at atmospheric pressure, i.e. 14·7 lb./sq. in. If the tube had been closed at both ends, then as the gas warmed throughout its pressure would have increased equally on both sides of the piston and the latter would not have moved at all.

But although, for Charles' law to hold, the pressure must not change during the experiment, it does not matter what that pressure is: if we performed the experiment at the bottom of a deep mine, where the pressure is greater than 14·7 lb./sq. in., or at the top of a very high mountain, where it is less, we should still get the same result, $\frac{1}{273}$.

Our tube, the contained gas, and the spot or plug of mercury would make a very good thermometer. We could graduate it just as ordinary centigrade thermometers are graduated, marking the freezing-point 0° C., and the boiling-point 100° C. But there is a much more convenient way: we could just number the divisions from the closed end of the tube. Then the closed end would be 0, the freezing-point 273, and the boiling-point 373; in fact, every point would be 273 greater than it would if we numbered it in the ordinary way.

Our new scale is called the *Absolute scale*, and temperatures measured by it are called *Absolute temperatures*; the zero on it, at the closed end, is called the *Absolute zero*.

If t = ordinary temperature centigrade, then *Absolute Temp. Centigrade* $= t + 273$

We shall denote ordinary temperatures by t, and absolute temperatures by T.

Fahrenheit temperatures we shall not use at all.

Whenever we mean "absolute" temperature, we had better write abs. after it, thus: 240° C. abs.

Going back to the tube again, we see that at say 400° C. abs. the gas occupies twice the volume it occupies at 200° C. abs., and four times the volume at 100° C. abs.

That is, *at constant pressure the volume occupied is simply proportional to the absolute temperature.*

Or, to put it another way, *the ratio between any two volumes* *is the same as the ratio between the two corresponding absolute temperatures.*

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This is evidently *not* true if we work with ordinary temperatures.

At 10° C., the gas occupies 283 divisions of the tube;

At 20° C., the gas occupies 293 divisions of the tube;

and $\frac{20}{10}$ is not equal to $\frac{293}{283}$.

So it simplifies the statement of the facts—Charles' law—to work with *absolute* temperatures: i.e. add 273 to the ordinary temperatures, and the ratio between the numbers obtained is the same as the ratio between the volumes occupied by the same quantity of gas at these temperatures; *always provided, however, that the pressure of the gas is unaltered.*

When a gas is warmed and prevented from expanding, its pressure increases; and it increases in exactly the same way that the volume increases when the pressure is kept constant, viz. is proportional to the abs. temperature. Thus, when the volume is kept constant,

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

But we shall have to deal with problems in which all three quantities, pressure, volume, and temperature, change their values.

It has been found experimentally that in such changes the number obtained by multiplying the pressure and volume together, and dividing by the absolute temperature, *is the same at every moment during the change.*

That is, $\frac{PV}{T}$ is constant.

Denoting this constant by the letter R, we have

$$\frac{PV}{T} = R,$$

or

$$PV = RT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This very important equation is called the "Characteristic Equation" of a perfect gas.

The value of the constant R will depend upon the quantity or mass of gas being dealt with, and the units in which the pressure and volume are measured. But before going on to this let us have a look at the equation in a general way.

Don't just think of symbols, P's and V's and T's, but think of an actual cylinder and its piston, and the gas enclosed within. When you think of a change of volume, think of the piston moving in or out, for example.

Since $\frac{PV}{T}$ is constant, if a gas changes from the state P_1, V_1, T_1 to the state P_2, V_2, T_2, then

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3')$$

First, this law contains Charles' law, for if the pressure does not change, P_1 and P_2, being equal numbers, cancel out, and so

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

i.e. $$\frac{V_1}{V_2} = \frac{T_1}{T_2} \text{ which is equation (1).}$$

Again, if the pressure changes, but the volume does not, then V_1 and V_2 *cancel out, and so*

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

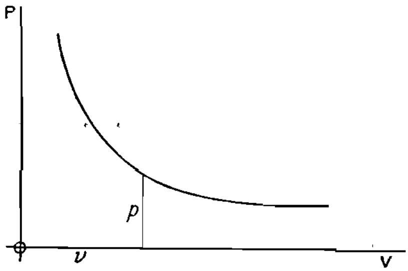
i.e. $$\frac{P_1}{P_2} = \frac{T_1}{T_2} \text{ which is equation (2).}$$

Thirdly, take the case when the pressure and volume change, but not the temperature. We shall see later that when a gas is compressed the temperature rises as well as the pressure. But at the end of the process of compressing we can wait until the gas cools again to its former temperature—that of the surroundings, say; this will cause the pressure to ease off somewhat, to a value depending upon the amount of compression and the original pressure.

If, then, in the general relation $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$, we put T_1 and T_2 equal, the equation becomes

$$P_1V_1 = P_2V_2, \quad \ldots \ldots \quad (4)$$

that is to say, the product of the pressure and volume is constant, the same at the end as at the beginning.



pv = constant

Fig. 3

This is termed **the Law of Boyle.**

We may state it thus:

If the temperature of a quantity of gas does not change, then during expansion or compression the product of the pressure and volume remains constant,

i.e. $$PV = C \text{ (constant)}. \quad \ldots \ldots \quad (5)$$

The relation between P and V is shown graphically in fig. 3.

It simply comes to this, that halving the volume doubles the pressure, and so on.

This kind of expansion or compression, because it takes place at one constant temperature, is said to be **Isothermal.**

It occurs, for example, when the process takes place slowly, for then there is time for any slight change of temperature to be neutralized by heat flowing inwards, or outwards, between the gas and the surroundings.

As a simple example of the use of the general relation between P, V, and T, take the following:

In a cylinder 8 c. ft. of air at 20 lb./sq. in. and temperature 27° C. are enclosed behind a piston. The piston is then forced in so as to halve the volume. What is the final pressure if the temperature of the gas is brought to

(*a*) 137° C.

(*b*) its original value, 27° C.?

Here (*a*) $p_1 = 20$ lb./sq. in.

$v_1 = 8$ c. ft.

$T_1 = 273 + 27 = 300°$ C. abs.

and $p_2 = ?$

$v_2 = 4$ c. ft.

$T_2 = 273 + 137 = 410°$ C. abs.

$$\therefore \frac{p_2 \times 4}{410} = \frac{20 \times 8}{300}$$

$$\therefore p_2 = \frac{410 \times 20 \times 8}{4 \times 300} = 54\tfrac{2}{3} \text{ lb./sq. in.}$$

(*b*) In this case we have Isothermal expansion, so Boyle's law is sufficient; and as the volume is halved, the pressure will be doubled,

$$\therefore p_2 = 40 \text{ lb./sq. in.}$$

Let us now go back to the characteristic equation (3), viz.

$$PV = RT.$$

We will find the value of R for air, taking one pound as the quantity, and measuring pressure in pounds per square foot, volumes in cubic feet.

At the freezing-point 0° C., or 273° C. abs., and at atmospheric pressure 14·7 lb. per square inch or 2117 lb. per square foot, one pound of air has a volume of 12·39 c. ft.

So $2117 \times 12{\cdot}39 = R \times 273$

$\therefore R = 96$ (practically).

Thus the characteristic equation for air is

$$PV = 96T. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3'')$$

We can use this to find the volume at any other pressure and temperature, but, of course, it only gives us the volume of one pound. If we want our equation to give us the volume of m lb. of air we must evidently multiply the right-hand side by m. Then

$$PV = 96mT.$$

Remember.—Pressures in lb. per square *foot*, volumes in cubic *feet*, and temperatures in *absolute* °C.

EXAMPLE.—What volume will 6 lb. of air occupy at a pressure of 100 lb./sq. in. and temperature 300° C.?

Here $P = 100 \times 144 = 14400$ lb./sq. ft.

$T = 300 + 273 = 573°$ C. abs.

$$\therefore\ 14400 \times V = 96 \times m \times 573$$

$$\therefore\ V = 6 \times \frac{96 \times 573}{14400}$$

$$= 6 \times 3{\cdot}82$$

$$= 22{\cdot}92 \text{ c. ft.}$$

One pound would have occupied a volume of 3·82 c. ft.

The simple law PV = constant will not usually hold good for the expansions and compressions which occur in engine cylinders, because the temperature will change too.

But, even then, experience shows that the relation between P and V can usually be expressed in the simple form

$$PV^n = \text{Constant}, \quad \ldots \ldots \quad (6)$$

n being a constant index which usually has a value between 1 and 1·4, the actual value depending upon how far from Isothermal the process really is.

Knowing the pressure, volume, and temperature at one particular moment, perhaps at the beginning of a stroke, we shall want to calculate the temperature at another moment, perhaps at the end of the stroke, when the pressure and volume have different values.

Knowing the change in volume, the law

$$PV^n = \text{Constant, i.e. } P_1V_1{}^n = P_2V_2{}^n,$$

will enable us to calculate the changed pressure; and then by means of the general relation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

we shall be able to calculate the changed temperature.

We had better illustrate this by working an example.

EXAMPLE.—Suppose we have a cubic foot of air in a cylinder, its pressure being 200 lb./sq. in., and its temperature 500° C., and allow it to expand, pushing the piston along until the volume becomes 4 c. ft. Let us calculate the final temperature, assuming the law of expansion to be

$$PV^{1 \cdot 1} = \text{Constant}.$$

Here $P_1 = 200$, and $V_1 = 1$; $V_2 = 4$.

$$\therefore\ 200 \times 1^{1 \cdot 1} = P_2 \times 4^{1 \cdot 1}$$

$$\therefore\ \frac{200}{4^{1 \cdot 1}} = P_2 = 43 \cdot 52 \text{ lb./sq. in. \dots (by logs.).}$$

Now

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore\ T_2 = \frac{P_2V_2}{P_1V_1} . T_1,$$

and

$$T_1 = 500 + 273 = 773° \text{ C. abs.}$$

$$\therefore\ T_2 = \frac{43 \cdot 52 \times 4}{200 \times 1} \times 773$$

$$= 673° \text{ C. abs.}$$

$$\underline{273}$$

$$\underline{400° \text{ C.}}$$

Thus the temperature has fallen by 100° C. Had the index *n* been greater than 1·1, there would have been a bigger fall—try it; had the index been equal to 1, there would have been *no* fall of temperature at all, for then we should have

$$PV = \text{Constant (Boyle's Law)},$$

and had the index been less than 1 there would have been a *rise* of temperature.

EXAMPLES I

On the law PV = RT

1. A certain mass of gas occupies a volume of 200 c. in. at 15° C. and atmospheric pressure. What volume will it occupy at 100° C. and the same pressure?

2. 6 c. ft. of gas at a temperature of 120° C. are cooled at constant pressure until the temperature is 60° C. What is the final volume?

3. A cylinder contains 4 c. ft. of air at a pressure of 15 lb./sq. in. and temperature 60° C. If the air be compressed into a clearance space of 1 c. ft. and the final temperature is found to be 80° C., what is the final pressure?

4. In a certain test a gas engine was supplied with 500 c. ft. of gas; the pressure at the meter was 15 lb./sq. in., temperature 10° C. What volume would the same mass of gas have occupied at " normal temperature and pressure ", i.e. at 0° C. and 14·7 lb./sq. in.?

5. The cylinder of a gas engine is 9·5 in. diameter; stroke 19 in.; clearance volume 270 c. in.. At the commencement of compression of the charge the temperature and pressure are 100° C. and 14·7 lb./sq. in. If at the end of compression the pressure is 130 lb./sq. in., what is the temperature?

(*Hint*: V_1 = stroke volume + clearance volume.
V_2 = clearance volume.)

6. In Question 5, if, on ignition, the pressure rises to 350 lb./sq. in., what does the temperature become, supposing explosion occurs at constant volume?

(*Hint*: $\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2}$ or $= \frac{P_1V_1}{T_1}$; and $V_3 = V_2$.)

7. The compression ratio in a certain Diesel engine is 14. If the temperature and pressure of the air at the end of the suction stroke are 80° C. and 15 lb./sq. in. respectively, and the pressure at the end of the compression stroke is 480 lb./sq. in., what is the compression temperature?

8. Find the value of R in the characteristic equation PV = RT for Carbon Dioxide, given that one pound of this gas, at normal temperature and pressure, occupies a volume of 8·104 c. ft.

9. Given that the characteristic equation for Hydrogen is PV = 1384T, calculate the volume occupied by 1 lb. of hydrogen at N.T.P.* (i.e. 0° C., 14·7 lb./sq. in.).

10. A gas engine was supplied, during an hour's test, with 382 c. ft. of gas at 15° C. and 771 mm. of mercury. Reduce this volume to N.T.P.

* N.T.P. = normal temperature and pressure, of 0° C. and 1 atmosphere.

(*Note*: Pressures are frequently stated in this way, as so many inches or millimetres of mercury. Atmospheric pressure, or 14·7 lb./sq. in., corresponds with a barometric height of 760 mm., or 30 in.

In this example, the student will write $\frac{P_1}{P_2} = \frac{771}{760}$.)

11. A cylinder contains 1·5 c. ft. of air at 12° C. and atmospheric pressure. Find the increase in volume if (*a*) the temperature be doubled, (*b*) the absolute temperature be doubled, the pressure in each case being kept constant.

12. What volume will be occupied at N.T.P. by a mass of air which occupies 300 c. in. at 40° C. and 20 lb./sq. in.?

13. In a certain engine the clearance volume is 0·183 c. ft. and the compression ratio 14·2. At the end of the suction stroke the pressure is 13·7 lb./sq. in. and the temperature 30° C. What volume would the contents of the cylinder occupy at N.T.P.?

14. Gas occupying a volume of 6 c. ft. at atmospheric pressure and temperature 14° C. is compressed into a space of 1 c. ft. and allowed to cool to its original temperature. What will the pressure be? It is then expanded to 6 c. ft. again, and its pressure is observed to fall to 12 lb./sq. in. What is the corresponding temperature?

15. Find the value of R in the equation PV = RT for hydrogen, given that 1 c. ft. of hydrogen at N.T.P. has a mass of 0·00559 lb.

CHAPTER II

The Laws of Gases (*continued*)

The Unit of Heat

A definite quantity of heat is required to warm 1 lb. of water from the freezing-point to the boiling-point, i.e. from 1° C. to 100° C.

The $\frac{1}{100}$th part of this quantity is the

Centigrade Heat Unit, or C.H.U.

So 1 C.H.U. is the average amount of heat required to warm 1 lb. of water through one centigrade degree. It is very very nearly, but not quite, the same for each degree between the freezing- and boiling-points.

The Unit of Work

The amount of work done, or energy expended, when you exert a push equal to the weight of one pound through a distance of one foot is *one foot-pound.*

The Mechanical Equivalent of Heat

Dr. Joule of Manchester found that if he expended energy by agitating water with revolving paddles, the water became warmer. Taking care to avoid all losses, or at any rate to make allowance for them, he found that the same expenditure of energy always produced the same amount of warming, and that twice the expenditure produced twice as much warming, and so on. He showed, in fact, that there is a definite equivalence between work and heat, that work and heat are both forms of energy, and that one can be converted into the other. Students usually have difficulty in grasping this idea —pushing a truck and warming water seem such totally different things! But so did some of the most highly placed scientific men of Joule's time, and only after much patient persistence on Joule's part, backed up by a rising young scientist who afterwards became the famous Lord Kelvin, were his ideas understood and accepted.

Joule's experiments have been repeated over and over again since his time, with many different forms of apparatus and the fact established that

1400 foot-pounds are equivalent to 1 C.H.U.

It is an easy matter to convert mechanical work into heat—you do that when you rub your hand vigorously to and fro on the table before you; but to convert heat into work is not such an easy matter—we have to construct complicated machinery such as Otto cycle and Diesel cycle engines to do that.

Specific Heat

It is remarkable that water requires much more heat to warm it up (and gives out much more heat in cooling down) than other substances do. Iron, for example, requires only about $\frac{1}{9}$th as much; mercury only about $\frac{1}{30}$th; and air about $\frac{1}{5}$th as much.

Thus, to warm 18 lb. of water from 10° C. to 20° C., would require the supply of

$$18 \times 10 = 180 \text{ C.H.U.},$$

whereas to similarly heat up 18 lb. of iron would require only

$$\frac{1}{9} \times 180 = 20 \text{ C.H.U.}$$

The fractional part of a heat unit required to raise the temperature of 1 lb. of a substance by 1 degree is termed the Specific Heat of that substance.

Energy

Energy is stored-up capacity to perform work.

Imagine a man pushing hard at a truck which in some way is prevented from moving. The man does no work. He possesses within him a store of energy. Now let the truck move: he begins to do work at the expense of what we might call his " internal " energy, and by the time he has exerted a push of 20 lb. weight for a distance of 10 ft. he will have done 200 foot-pounds of work, and his store of internal energy will be so much the less. By the time he has done several thousand foot-pounds of work he will probably be exhausted —unless some sort of nourishment can be administered to him continuously as he pushes.

It is much the same with gas at a pressure within a cylinder. The molecules of gas bombard furiously the surfaces of the cylinder and the piston; the hotter the gas the more intense is the bombardment. Allow the piston to move, and work is done, and by so much is the " *internal energy* " of the gas diminished—unless by additional warming the strength of the bombardment is maintained.

We cannot see the vibratory movement of the molecules; but we feel it—as heat.

Heat is Energy.—Imagine heat, i.e. energy, to be " supplied " to a gas which at the same time is allowed to expand and push a piston along. What happens to its internal energy? At the end of the expansion its internal energy may be less, greater than, or equal to its original store.

It will be less if the energy supplied is less than the work done on the piston.

It will be greater if the energy supplied is greater than the work done on the piston.

It will be unaltered if the energy supplied is equal to the work done on the piston.

In fact, the amount of energy E supplied to the gas is accounted for in two ways: part of it goes to make up the loss due to the external work W done on the piston; the remainder I goes to increase the internal energy of the gas.

Energy supplied = Gain in Internal Energy + External work done.

$$E = I + W. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The energy is supplied in the form of heat, measured in C.H.U.'s; the external work done is mechanical energy measured in foot-pounds.

The three quantities E, I, and W must, of course, all be expressed in the same way when we form an equation, either all in C.H.U.'s, or all in foot-pounds.

The student should try to form a clear notion of these balancing flows of energy in and out. An analogy might assist him: *imagine flood water pouring into a reservoir at* one end, discharge or overflow water flowing out at the other end, and the level of the water in the reservoir rising at the same time. There will always be balance between the quantity that has flowed in, the quantity that has flowed out, and the increase in quantity contained in the reservoir. The first amount will always be equal to the sum of the other two.

Specific Heats of a Gas

Imagine 1 lb. of gas enclosed in a cylinder behind a piston.

Warm the gas until its temperature has increased by 1° C., at the same time *preventing the piston from moving*. The fractional part of a C.H.U. supplied is termed the *specific heat at constant volume*.

Its value for *air* is about 0·169, different authorities giving slightly differing values.

Again warm the pound of gas through 1° C., but this time allow the gas to push the piston outwards so as to *keep the pressure constant*. *More heat* will have to be supplied this time, to make up for the external work done as the gas expands. The fractional part of a C.H.U. now supplied is termed the *specific heat at constant pressure*. Its value for *air* is about 0·238.

These specific heats are not quite constant, but are slightly higher at higher temperatures. More advanced theory takes account of this slight variation, but we shall not bother about it.

We may express the above specific heats in work units:

$$0{\cdot}169 \text{ C.H.U.} = 1400 \times {\cdot}169 \text{ ft. lb.}$$
$$= 236{\cdot}6 \text{ ft. lb.}$$
$$0{\cdot}238 \text{ C.H.U.} = 333{\cdot}2 \text{ ft. lb.}$$

Let us denote the two specific heats by C_p and C_v when they are in heat units, and by K_p and K_v when they are in work units.

EXAMPLE.—A cylinder contains 6 c. ft. of air at 40 lb./sq. in. and 10° C. How much heat must be supplied to warm it up to 60° C.,

(*a*) keeping the piston fixed;

(*b*) allowing it to move so that the pressure remains the same?

First find the weight of air.

For 1 pound, $PV = 96T.$

$$P = 144 \times 40 \text{ lb./sq. ft.}$$
$$T = 10 + 273 = 283° \text{ C. abs.}$$
$$\therefore\ 144 \times 40 \times V = 96 \times 283$$
$$\therefore\ V = 4{\cdot}717 \text{ c. ft.}$$

Actually we have 6 c. ft., so the weight must be

$$\frac{6}{4{\cdot}717} = 1{\cdot}273 \text{ lb.}$$

(*a*) Heat Supplied = No. of lb. × No. of degrees rise × Spec. Ht.

$$= 1{\cdot}273 \times 50 \times 0{\cdot}169$$
$$= 10{\cdot}75 \text{ C.H.U.}$$

or, in work units, $= 10{\cdot}75 \times 1400$

$$= 15050 \text{ foot lb.}$$

(*b*) Heat Supplied $= 1{\cdot}273 \times 50 \times 0{\cdot}238$

$$= 15{\cdot}15 \text{ C.H.U.}$$
$$= 21190 \text{ foot lb.}$$

It is worth going into this problem a little further.

Let us find, in case (*b*), how much the gas expanded.

$$\left.\begin{array}{l} P_1 = 40 \text{ lb./sq. in.} \\ V_1 = 6 \text{ c. ft.} \\ T_1 = 283° \text{ abs.} \end{array}\right\} \text{ and } \begin{array}{l} P_2 = 40 \\ V_2 = ? \\ T_2 = 333° \text{ abs.} \end{array}$$

By (3′): $\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$, or as P_1 and P_2 are equal,

$$\frac{V_2}{T_2} = \frac{V_1}{T_1} \therefore V_2 = V_1 \cdot \frac{T_2}{T_1}$$
$$= 6 \times \frac{333}{283}$$
$$= 7{\cdot}06 \text{ c. ft.}$$

So the gas expanded by 1·06 c. ft.

Now if the cross-section of the cylinder, or area of piston face, was 1 sq. ft., then the piston was pushed forwards 1·06 ft.; and all the time the gas was pushing on the piston face with a total force of

$$144 \times 40 = 5760 \text{ lb.}$$

Hence the external work done was

$$5760 \times 1{\cdot}06$$
$$\mp 6105 \text{ ft. lb.}$$

The two amounts of energy supplied in (*a*) and (*b*) were 15050 and 21190 ft. lb., a difference of *6140 ft. lb.*, not very different from *6105*. Had we used quite accurate values of the two specific heats, and of any other constants, such as the value of R, 96, our two results would have agreed exactly. If you have thought about things you should expect them to be in agreement.

In the course of our work above, we supposed the area of the piston face to be 1 sq. ft., in which case the movement of the piston was 1·06 ft. But the actual size of the piston doesn't matter: if we had taken a piston of only half the area, then the movement of the piston would have had to be twice as much, i.e. 2·12 ft. And half the force acting for twice the distance does the same amount of work as before.

In general, suppose the area of the piston face to be A sq. ft., the pressure P lb. per square foot, and the movement of the piston L ft. Then the work done

$$= \text{Total force} \times \text{Distance}$$
$$= \quad PA \quad \times L \text{ foot-pounds,}$$

which is the same as

$$P \times AL.$$

Now AL is the *increase in volume*. So we have this result: If a gas expands from V_1 to V_2 cubic feet, the pressure being continually P lb./sq. ft., then the work done is

$$P(V_2 - V_1) \text{ foot-pounds. } \quad . \quad . \quad . \quad . \quad (8)$$

Remember that your units in this must be *pounds* and *feet*:
Pressures in pounds per square *foot*;
Volumes in cubic *feet*.

EXAMPLES II

On Specific Heat

1. How much heat must be supplied to 5 lb. of gas of specific heat 0·172 (C_v) to increase its temperature by 300° C., the volume remaining constant? What is the equivalent of your answer in ft./lb.?

2. What will be the increase in temperature of 0·6 lb. of gas of specific heat C_v = 0·16 when it receives heat equivalent to 12,000 ft./lb., at constant volume?

3. At a constant pressure of 30 lb./sq. in., 2 c. ft. of gas expand to a volume of 6 c. ft. How much external work is done, and how much heat, in C.H.U., must have been supplied to the gas to do this external work?

4. The specific heat of air at constant pressure being 0·24, how much energy, in C.H.U., must be supplied to 10 lb. of air to heat it from 0° C. to 100° C., the change of volume taking place at constant atmospheric pressure?

5. In Question 4, using the characteristic equation PV = $96m$T for air, calculate the initial and final volumes, and so find how much of the total amount of energy supplied was used externally in overcoming the resistance of the atmosphere during expansion.

6. A cylinder contains 5 c. ft. of air at 50 lb./sq. in. and 15° C. How much heat must be supplied to increase its temperature to 100° C., (*a*) keeping the volume constant, (*b*) allowing the piston to move so that the pressure remains constant?

Take C_v = 0·17, C_p = 0·24.

(*Hint*: first find the mass of air present, using PV = $96m$T.)

7. What is the volume of 1 lb. of air at a pressure of 40 lb./sq. in and temperature 50° C.? It is supplied with heat energy equivalent to 120,000 ft. lb., the volume remaining constant. Taking C_v = 0·172, calculate the new temperature and pressure.

8. The specific heats of a gas are higher at higher temperatures.

The following table gives the values of C_v and C_p for air at various temperatures:

Temp. $t°$ C	0	200	500	800	1000
C_v	0·162	0·17	0·181	0·192	0·20
C_p	0·23	0·238	0·249	0·26	0·268

Show these results (the average of results obtained by a number of investigators) graphically.

9. 0·8 lb. of air is to be heated from 100° C. to 800° C., at constant volume. Read from your graph of Question 8 the mean specific heat for this temperature range, and use it to calculate the amount of heat that must be supplied.

10. The specific heat of air at 0° C. is 0·162 (at constant volume). What does this mean exactly? 1 lb. of air at N.T.P. occupies a volume of 12·39 c. ft., or 1 c. ft. weighs 0·0807 lb. What would the "specific heat" of *1 c. ft.* of this air be, i.e. what fraction of a C.H.U. is required to warm a mass of air which at N.T.P. has a volume of 1 c. ft., by 1° C.? Also express this amount of heat in ft. lb.

Note: Your answer is the specific heat of air in *ft. lb. per cubic foot of air* (at N.T.P.). It is often convenient to state the specific heat in this way when dealing with gases. It is termed the *volumetric heat*, and, as you have seen, is obtained by multiplying the ordinary specific heat by the weight of a cubic foot of the gas, and then by Joule's equivalent, 1400.

11. 0·5 c. ft. of gaseous mixture, at N.T.P., is exploded, and 18 C.H.U. thereby liberated. Calculate the final temperature, given that the volumetric heat = 19·5 ft. lb. per cubic foot.

(*Hint*: 0·5 × 19·5 × rise in temperature = the energy liberated in ft. lb.)

12. A cubic foot of gas is exploded and the temperature rises from 100° C. to 2000° C. Given that for this range of temperature the mean volumetric heat is 26·5, calculate how much energy was liberated, in C.H.U.

13. The specific heat, in C.H.U. per lb., of a certain gas is 0·15. Its volumetric heat is 26 ft. lb. per cubic foot. Calculate the density of this gas in pounds per cubic foot.

14. Calculate the weight of a cubic foot of normal gas engine mixture, given specific heat = 0·243, volumetric heat = 26·6.

A very important law that has been established is that *the store of energy possessed by a gas depends almost entirely on the mass of gas and its temperature.* The first of these statements is obviously true: twice as much gas, twice as much energy; but the second is not at all obvious.

It states that so long as you keep the temperature the same, you can vary the volume and pressure as much as you like, but the internal energy remains just the same.

That is to say, *in a Boyle's law, or Isothermal change, for which* PV = *Constant, there is no change in the internal energy of the gas*—it is the same at the end as at the beginning.

[The student will find detailed accounts of experiments which show this law to be true, in advanced books on Heat Engines, such as Prof. Ewing's.]

Now, if a gas expands and pushes a piston, it loses some of its energy in so doing. Therefore, if at the end it possesses as much energy as at first, some energy must have been supplied to it during the process, just as much, in fact, as the work done in pushing the piston along. This is what happens in an Isothermal expansion.

If, on the other hand, the piston is pushed inwards and the gas compressed, and yet at the end the gas possesses no more energy than at the beginning, some energy must have flowed out—squeezed out, as it were.

These compensating *flows of energy* inwards or outwards are, of course, *flows of heat.*

In heat engine theory, we have only to reckon *changes of energy*, not total values, and these changes can be found at once when we know the mass of gas and the *change of temperature.*

Whether a gas is warmed up or cooled down at constant pressure, or at constant volume, it matters not—*the temperature change is all we need know to calculate the change of internal energy.*

If a gas expanded and pushed a piston along, and during the action no heat was allowed to flow in, it would lose energy to the extent of the work done on the piston, which is the same as saying that its temperature would fall.

Conversely, if, during compression, no heat was allowed to flow out, the gas would gain energy to the extent of the work done on it, i.e. its temperature would rise.

Thus:

the tendency during expansion is Cooling,
the tendency during compression is Heating.

In practice, the processes outlined above, during which no heat flows in or out, can be imitated by expanding or compressing very rapidly, because then there is little time for any appreciable flow to take place.

Such expansions and compressions, in which absolutely no heat flow inwards or outwards occurs, and in which, therefore, temperatures rise and fall fully, the change in internal energy being exactly equal to the external work done *by* the gas, if expanding, or *on* it, if compressing, are said to be "*Adiabatic*".*

What we might call the two extreme cases, therefore, in expansions or compressions, are:

1. *Isothermal Changes.*

Boyle's law PV = Constant. No change in temperature, no change in internal energy. Heat is "sucked in," or "squeezed out" to compensate exactly for the mechanical work done.

2. *Adiabatic Changes.*

The temperature changes. Maximum change in internal energy equal to mechanical work done.

These matters are very important, and you should think about them—picturing the processes taking place in engine cylinders, pistons moving in and out—until they are perfectly clear to you.

Expansions and compressions in practice are neither isothermal nor adiabatic as a rule, but fall between these limits of change.

Having clear notions, we ought now to find it easy to put some of these things into the shape of formulæ.

When a gas expands from V_1 to V_2 cubic feet, at constant pressure P lb. per square foot, the external work done is, by equation (8):

$$P(V_2 - V_1) \text{ ft. lb.}$$

* This word is of Greek origin—*a*, not, *diabaino*, pass through.

If at the same time its temperature has gone up from T_1 to T_2, then the energy supplied to the gas must have been

$$m.K_p(T_2 - T_1),$$

m being the mass of gas in pounds.

Hence the increase in internal energy is (equation (7)), Energy Supplied—External Work done,

$$= mK_p(T_2 - T_1) - P(V_2 - V_1) \text{ ft. lb.}$$

Now, if the gas had been heated from T_1 to T_2 at constant volume, no external work would have been done, and the internal energy would have increased by exactly the amount of energy supplied, i.e.

$$mK_v(T_2 - T_1) \text{ ft. lb.}$$

The gain in *internal energy* is the same in each case, because the temperature change has been the same. More energy was supplied in the former process, but some of it was spent in doing external work. So

$$mK_p(T_2 - T_1) - P(V_2 - V_1) = mK_v(T_2 - T_1)$$

or $$m(K_p - K_v)(T_2 - T_1) = P(V_2 - V_1)$$

$$\therefore\ m(K_p - K_v) = \frac{P(V_2 - V_1)}{T_2 - T_1} \quad . \quad . \quad . \quad . \quad (9)$$

Now, for m lb. of gas, $PV = mRT$

$$\therefore\ PV_1 = mRT_1$$

$$\text{and } PV_2 = mRT_2$$

$$\therefore\ P(V_2 - V_1) = mR(T_2 - T_1).$$

So the above formula becomes

$$m(K_p - K_v) = \frac{mR(T_2 - T_1)}{T_2 - T_1}$$

$$\therefore \qquad K_p - K_v = R, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

i.e. we have the very interesting and important result that the difference of the two specific heats, expressed in work units, is the value of R in the characteristic equation (3).

The ratio of the two specific heats is often required. Let us denote it by γ.

$$\frac{C_p}{C_v} \text{ or } \frac{K_p}{K_v} = \gamma.$$

For air $$\frac{C_p}{C_v} = \frac{0{\cdot}238}{0{\cdot}169} \doteq 1{\cdot}408 = \gamma.$$

The following bit of work will come in useful:

$$K_p - K_v = R$$

$$\therefore \frac{K_p}{K_v} - 1 = \frac{R}{K_v}$$

$$\therefore \gamma - 1 = \frac{R}{K_v}$$

$$\therefore K_v = \frac{R}{\gamma - 1}$$

The general law of expansion or compression being taken as (v. Eq. (6)),

$$PV^n = \text{Constant},$$

when a gas expands from pressure and volume P_1 and V_1 to pressure and volume P_2 and V_2, expressed in lb./sq. ft. and cubic feet, the external work done is accurately expressed by:

$$W = \frac{P_1V_1 - P_2V_2}{n - 1} \text{ ft. lb.} \quad . \quad . \quad . \quad . \quad (11)$$

This is a simple formula, but its proof involves rather harder mathematics which you can go into later on. Take it for granted just now.

Before going further it will be best to work out a problem on this and other preceding formulæ.

EXAMPLE.—A cylinder contains 2 c. ft. of air at a pressure of 100 lb./ sq. in., and temperature 200° C. The air expands and pushes the piston along until its volume is 4 c. ft. Let us find out all we can about the matter, assuming the law of expansion to be

$$PV^{1{\cdot}2} = \text{Constant}.$$

We can find what mass of air is being dealt with. For one lb. of air

$$PV = 96T.$$

$$P_1 = 100 \times 144 = 14400 \text{ lb./sq. ft.}$$

$$T_1 = 200 + 273 = 473^\circ \text{ C. abs.}$$

$$\therefore\ V = \frac{96 \times 473}{14400} = 3{\cdot}154 \text{ c. ft.}$$

We have only 2 c. ft., so the mass of air is

$$\frac{2}{3{\cdot}154} = 0{\cdot}634 \text{ lb.}$$

Next we can find the value of the constant in the expansion formula. Its value is

$$P_1V_1^{1\cdot2} = 14400 \times 2^{1\cdot2} = 33090.$$

Using this, we can go on and find the final pressure P_2, because

$$P_2V_2^{1\cdot2} \text{ also equals } 33090.$$

$$\therefore\ P_2 \times 4^{1\cdot2} = 33090$$

$$\therefore\ P_2 = \frac{33090}{4^{1\cdot2}} = 6267 \text{ lb./sq. ft.} = 43{\cdot}52 \text{ lb./sq. in.}$$

Now we can find the external work done by the expanding air.

$$W = \frac{P_1V_1 - P_2V_2}{n - 1}$$

$$= \frac{14400 \times 2 - 6267 \times 4}{1{\cdot}2 - 1}$$

$$= 18650 \text{ ft. lb.}$$

If the expansion had been adiabatic (which it wasn't—or at least we don't know that it was), the air would have lost internal energy to the extent of 18650 ft. lb., because no heat would have flowed in to compensate this amount of external work done. In that case, as the mass of air is 0·634 lb., we should have

$$0{\cdot}634\ K_v(473 - T_2) = 18650,$$

because, remember, $K_v(T_1 - T_2)$ represents the change in internal energy even if the volume has not been constant.

$$K_v = 236{\cdot}6 \text{ (see page 16),}$$

$$\therefore\ 473 - T_2 = \frac{18650}{0{\cdot}634 \times 236{\cdot}6}$$

$$= 124{\cdot}4,$$

that is, the temperature would have fallen by 124·4 Centigrade degrees.

$$473 - T_2 = 124{\cdot}4$$
$$\therefore\ T_2 = 348{\cdot}6^\circ \text{ C. abs.}$$
$$\underline{273{\cdot}0}$$
$$\text{Final temperature} = \underline{\underline{75{\cdot}6}} \text{ C.}$$

To find what the final temperature really was we must use the two formulæ

$$PV^{1\cdot 2} = 33090 \text{ and } \frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}.$$

We have already used the first of these, to find P_2, which was 43·52 lb./sq. in., and the second formula then gives us

$$T_2 = \frac{P_2V_2}{P_1V_1} \cdot T_1 = \frac{43{\cdot}52 \times 4}{100 \times 2} \times 473$$
$$= 411{\cdot}7^\circ \text{ C. abs.}$$
$$\underline{273{\cdot}0}$$
$$\therefore \text{ True final temperature} = \underline{\underline{138{\cdot}7^\circ}} \text{ C.}$$

This is very different from 75·6° C., and so the expansion was not adiabatic. It lies between isothermal, in which there would have been no fall in temperature, the final being 200° C., and adiabatic, in which there would have been a fall down to 75·6° C.

Now, as the temperature did not fall as far as 75·6° C., but only to 138·7° C., some heat, or energy, must have flowed in during the expansion process to keep the temperature up. We can easily find how much.

The temperature was kept up by

$$138{\cdot}7 - 75{\cdot}6 = 63{\cdot}1 \text{ degrees,}$$

and to produce this change of temperature in 0·634 lb. of air of specific heat 0·169 (C_v) requires a supply of

$$0{\cdot}634 \times {\cdot}169 \times 63{\cdot}1 \text{ C.H.U.}$$
$$= 6{\cdot}761 \text{ C.H.U.}$$
$$= 9464 \text{ ft. lb.}$$

Thus we have found:

(*a*) Quantity of Air = 0·634 lb.
(*b*) Final Pressure = 43·52 lb./sq. in.
(*c*) Final Temperature = 138·7° C.
(*d*) Work done by the air = 18650 ft. lb.
(*e*) Energy as heat flowing into the air from the cylinder walls during expansion = 9464 ft. lb.
(*f*) ∴ Loss of internal energy = 9186 ft. lb.

Item (*f*) by subtraction.

It comes natural now to ask what the value of n must be, in the law of expansion

$$PV^n = \text{Constant},$$

for the expansion to be truly adiabatic. The last exercise has given us a clue: the external work as given by the W formula is to be equal to the loss of internal energy as given by the formula $m.K_v(T_1 - T_2)$.

Now $$W = \frac{P_1V_1 - P_2V_2}{n - 1}$$
$$= \frac{mR(T_1 - T_2)}{n - 1} \quad \text{(as on page 22).}$$

The loss of internal energy is $mK_v(T_1 - T_2)$, and if this is to be the same as W, evidently

$$K_v = \frac{R}{n - 1}.$$

But $$K_v = \frac{R}{\gamma - 1} \quad \text{(page 23).}$$

$\therefore$ *n must be equal to γ.*

Thus we have the very interesting result that *if the law*

$$PV^n = C$$

is to represent adiabatic expansion, or compression, the value of the index n must be equal to the ratio of the two specific heats of the gas, $\frac{C_p}{C_v}$ or $\frac{K_p}{K_v}$. For air this will be 1·4 very nearly, as on page 23.

This is a very important result.

It will be necessary to have a formula for *the ratio between the initial and final temperatures* of a gas when it expands, or when it is compressed.

If it starts to expand from volume V_1 and ends with volume V_2, the ratio

$\frac{V_2}{V_1}$ is called the *Ratio of Expansion*,

and is denoted by r.

The process is represented graphically in the diagram, in which the curve shows the falling pressure as the expansion proceeds. Below is represented the cylinder and its piston.

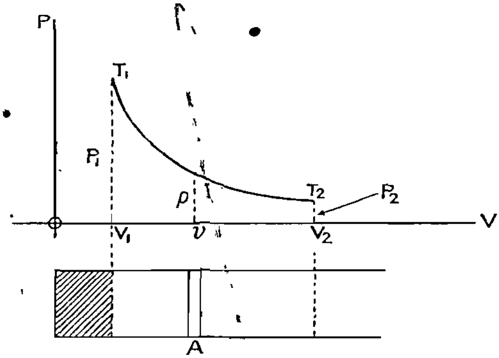


Fig. 4

The shaded area represents the original volume V_1. When the piston is at A, the pressure of the gas is represented by the ordinate p, its value being calculated by means of the formula

$$PV^n = C. \quad \ldots \quad (6)$$

For temperatures, we have the formula

$$PV = RT, \quad \ldots \quad (3)$$

i.e.

$$\frac{P_2V_2}{P_1V_1} = \frac{T_2}{T_1}. \quad \ldots \quad (v.\ \text{Eq. } (3'))$$

We have

$$P_1V_1^n = P_2V_2^n$$

$$\therefore \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n = r^n$$

or

$$\frac{P_2}{P_1} = \frac{1}{r^n}.$$

But

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} \cdot \frac{V_2}{V_1}$$

$$= \frac{1}{r^n} \cdot r = \frac{1}{r^{n-1}}, \left[\text{just as } \frac{1}{r^3} \cdot r = \frac{1}{r^2}\right]$$

which is the same as $\left(\frac{1}{r}\right)^{n-1}$.

Thus we have the formula

$$\frac{T_2}{T_1} = \left(\frac{1}{r}\right)^{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Suppose, for example (taking simple numbers for the sake of clearness), a gas expanded from 1 to 3 c. ft., the law of expansion being $PV^3 = C$.

Then $r = 3, \ \frac{1}{r} = \frac{1}{3}, \text{ and } n - 1 = 2.$

So we should have

$$\frac{T_2}{T_1} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

or $T_2 = \frac{1}{9}T_1$, the final temp. $\frac{1}{9}$th of the initial.

Perhaps we had better remember it like this:

In an expansion or compression

$$\left.\begin{array}{l} \dfrac{\text{Lowest Temp.}}{\text{Highest Temp.}} = \left(\dfrac{1}{r}\right)^{n-1} \\ \text{or, if you like,} \\ \dfrac{\text{Highest Temp.}}{\text{Lowest Temp.}} = r^{n-1} \end{array}\right\} \quad . \quad . \quad . \quad (12')$$

EXAMPLE.—Four cubic feet of air compressed into a space of 1 c. ft., adiabatically.

The value of n will be 1·4 (p. 23).

The ratio of compression $r = \frac{4}{1} = 4$.

$$\therefore \frac{\text{Highest Temp.}}{\text{Lowest Temp.}} = 4^{1\cdot4-1} = 4^{0\cdot4} = 1\cdot741.$$

So if the lowest temperature, at the beginning of the compression, was 10° C., or 283° abs., then the final temperature was

$$\begin{array}{rl} 1\cdot741 \times 283 = & 492\cdot7^\circ \text{ C. abs.} \\ & \underline{273\cdot0} \\ & \underline{\underline{219\cdot7^\circ}} \text{ C.} \end{array}$$

It is worth while remembering also that the ratio

$$\frac{\text{Highest Pressure}}{\text{Lowest Pressure}} = r^n \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

EXAMPLES III

On Expansions and Compressions

1. What volume is occupied by 1 lb. of air at 15° C. and 14·7 lb./sq. in.?
If this air is compressed adiabatically ($PV^{1\cdot 4} = C$) to one-fifth of this volume, what will the pressure and temperature become?

2. Evaluate P_1V_1 and P_2V_2 (units lb./sq. ft. and cubic feet) in Question 1, and so calculate the work done on the air during compression.

3. Taking $C_v = 0\cdot 173$, calculate the increase in internal energy of the air in Question 1 in C.H.U. and also in ft. lb. Compare your answer with that to Question 2.

4. If, after compression, the air of Question 1 is allowed to cool to its original temperature, what will the pressure fall to?

5. If a cubic foot of gas at 400 lb./sq. in. expands to 6 c. ft., the law of expansion being $PV^{1\cdot 3}$ = Constant, find the final pressure and the work done during expansion.

6. If the initial temperature of the gas in Question 5 was 500° C., what was the final temperature?

7. Find the resulting pressure and temperature when 4 c. ft. of air at atmospheric pressure and 60° C. are compressed into a space of ½ c. ft., (*a*) suddenly, so that heat has no time to escape, (*b*) very slowly.

8. In a gas engine cylinder, the pressure and temperature of the charge immediately after ignition were 350 lb./sq. in. and 2000° C. If expansion then took place according to the law $PV^{1\cdot 25}$ = Constant, what were the pressure and temperature when the volume was 5·2 times as great? Also calculate the work done, given that the initial volume was 0·157 c. ft.

9. 10 c. ft. of air at 14 lb./sq. in. and 30° C. are compressed to 1 c. ft., the law of compression being $PV^{1\cdot 3}$ = Constant. Find
(*a*) the mass of air present;
(*b*) the final pressure and temperature;
(*c*) the work done during compression;
(*d*) the amount of heat which flowed in, or out, through the cylinder walls.

(*Hint*: In working part (*d*) take $C_v = 0\cdot 1714$ and find what the final temperature would have been had all the energy (*c*) gone to increase the internal energy of the air.)

10. What relation exists between C_v, R, and the value of n in the law PV^n = Constant for adiabatic changes? If R = 96 and $n = 1\cdot 4$, what is the value of C_v? (Compare with Question 9.)

11. If for air $K_p = 330$ and $K_v = 234$, what is the value of R? Calculate also the value of n for adiabatic changes.

Find the volume of 3 lb. of air at a pressure of 40 lb./sq. in. and temperature 25° C.

12. At the end of the suction stroke in a gas engine, the pressure is 15 lb./sq. in., temperature 32° C. The clearance volume is 4·5 c. ft. and the stroke volume 7·5 c. ft. Find the temperature at the end of the compression stroke, assuming $PV^{1·4} = C$.

(*Hint*: Use the relation between T_2, T_1, and r.)

13. If in Question 12 the pressure on ignition rises to 250 lb./sq. in., find the temperature in the cylinder.

14. An internal-combustion engine has the following dimensions:

Stroke volume = 6·6 c. ft.

Compression ratio = 14.

The temperature and pressure at the end of the suction stroke are 45° C. and 14 lb./sq. in. Compression follows the law $PV^{1·35} = C$.

Find (*a*) the pressure and temperature at the end of the compression stroke;

(*b*) the weight of the charge;

(*c*) the work done;

(*d*) the heat loss through the cylinder walls during compression.

Take $C_v = 0·17$, $C_p = 0·24$.

Note: Determine the correct value of R corresponding to the given values of C_v and C_p.

15. 1 c. ft. of gas expands according to the law $PV^{1·2}$ = Constant until the volume is 5 c. ft. The initial pressure being 200 lb./sq. in., calculate the pressures corresponding to volumes 2, 3, 4, 5 c. ft. Plot a diagram, showing pressures vertically and volumes horizontally. Estimate from your graph the *mean* pressure in pounds per square foot, and so find the work done.

Note: Work done = Mean Pressure (lb./sq. ft.) × Increase in Volume (cubic feet).

16. Calculate the work done in Question 15, using the formula.

17. Repeat Question 15, but supposing the expansion to be isothermal, i.e. PV = Constant.

Note: The work done during isothermal expansion may be calculated by means of the formula

$W = P_1V_1 \log_e r$, where r = ratio of expansion,

P_1 = initial pressure in pounds per square foot,

V_1 = initial volume in cubic feet.

$\log_e r$ is 2·303 times the ordinary log of r.

Find the work done in this problem by calculation.

CHAPTER III

The Otto and Diesel Cycles

Efficiency

It is the duty of all heat engines to utilize as much as possible of the total amount of energy stored up in the fuel supplied to them. All of them waste quite a lot of it unavoidably, and some waste more than others.

The Diesel engine uses heavy oil as fuel, each pound of which may have stored up in it, chemically, energy of amount

$$\begin{aligned} &10{,}000 \text{ C.H.U.} \\ &= 14{,}000{,}000 \text{ ft. lb.} \end{aligned}$$

All this energy is liberated if the fuel is completely burnt; but a really good engine may utilize only about

$$\begin{aligned} &4000 \text{ C.H.U.} \\ &= 5{,}600{,}000 \text{ ft. lb.} \end{aligned}$$

All the rest of it is wasted, much of the waste being accounted for by the heat contained in the exhaust gases: you must have an exhaust, and the loss is unavoidable. The amount lost in this way may be as much as

$$\begin{aligned} &3000 \text{ C.H.U.} \\ &= 4{,}200{,}000 \text{ ft. lb.} \end{aligned}$$

So, you see, the energy passing away in the exhaust may be almost as much as the engine utilizes and returns to us as useful mechanical work. Yet even so, the performance would be better than the very best steam engines are capable of. In the above example, the ratio

$$\frac{\text{Energy utilized}}{\text{Energy supplied}} \text{ is } \frac{4000}{10{,}000} = 0{\cdot}4.$$

This ratio is called the *Absolute Thermal Efficiency* of the engine; expressed as a percentage, it is 40 per cent.

EXAMPLES IV

On Efficiency

Note: *One Horse-power* is that *rate* of working which corresponds to *33,000 ft. lb. per minute*. In the exercises which follow, the horse-power referred to is the "brake" horse-power, i.e. the horse-power exerted at the crankshaft. The efficiency is the over-all heat-efficiency —or "brake-thermal efficiency"—of the engine.

In *one hour*, a 1 H.P. engine does 33,000 × 60 = 1,980,000 ft. lb. of work, and this amount of work is spoken of as a "H.P.-hour".

1. What is the equivalent of 1 H.P. hour in C.H.U.?

2. In an hour's test a 25 H.P. engine used 400 c. ft. of gas of "calorific value" 360 C.H.U. per cubic foot, i.e. each cubic foot of gas liberates 360 C.H.U. when completely burnt. How many cubic feet of gas were used per horse-power per hour? What is the brake-thermal efficiency?

3. At full load, a certain oil engine used 0·405 lb. of oil per horse-power hour. The calorific value of the oil was 10,020 C.H.U. per lb. Calculate the efficiency.

4. A Ruston Vertical Oil Engine developing 419 H.P. used 172 lb. of oil per hour, of calorific value 10,070 C.H.U. per lb. Calculate its efficiency.

5. Calculate the efficiency of a Diesel engine using 0·41 lb. of oil of calorific value 10,850 C.H.U. per horse-power hour.

6. The brake-thermal efficiency of a petrol engine was 21%. If the calorific value of the petrol used was 10,280 C.H.U., what was the fuel consumption per B.H.P. hour?

The "Cycle" of Operations

Let us now consider carefully the process which goes on in the cylinder of a gas or oil engine working on the famous four-stroke, or de Rochas or "Otto" cycle:

(*a*) A charge of explosive gaseous mixture is drawn into the cylinder as the piston makes its first outstroke, at something like atmospheric pressure 14·7 lb./sq. in.

(*b*) On the return stroke this charge is compressed into the clearance space, the pressure rising to a value depending upon the compression ratio. A ratio of 5 would bring the pressure up to about 140 lb./sq. in.

The temperature rises also, perhaps to as much as 600° C., this depending upon how hot the charge was to begin with: for, remember, the cylinder is hot, and although the temperature of the charge as it is drawn in may be only about 10° C., by the time the cylinder is full and compression is about to begin, the temperature may be nearly 200° C.

Well, as the gas gets hotter during compression its internal energy is increasing. Energy is being supplied to it—but not from the fuel: the momentum of the engine itself is supplying it by compressing the gas. On the next stroke, however, this energy is returned to the engine, at any rate most of it. To help understand this, imagine the gas to be replaced by a strong spring. Then during the second stroke the piston compresses the spring, and so the motion of the engine is retarded as it loses some of its energy. This energy is not lost to the engine, however, but is stored up in the spring; and when the piston passes the "dead point" and moves the opposite way, the spring helps to push it along, and by the end of the stroke will have returned to the engine the energy which it borrowed from it, so to speak, in the previous stroke. It is a process of give and take. In this process, however, the spring would probably behave more efficiently than the gas does. Because—and this is very important—*as the gas gets hotter some of its heat will flow out through the cylinder walls* and be radiated away, or conveyed away by the cooling water. In such an event this energy would be lost altogether, and the gas would not fully repay what it had borrowed.

So we see that *the ideal type of compression to aim at is adiabatic*. Then there is no heat flow, and no loss.

In practice, however, we deliberately cool the cylinder walls, and so actually assist heat to flow away; not because we want to, but because if we didn't we should very soon have trouble with the lubrication. In endeavouring to improve the engine thermally we should spoil it mechanically, and cause it to seize up.

(*c*) Towards the end of the compression stroke an electric spark ignites the charge, explosion occurs, and there is a very rapid rise of pressure and temperature. The pressure might rise to say 350 lb./sq. in. and the temperature to 2000° C.

Thus the energy of the charge is liberated, and it remains for the engine to make the best possible use of it.

The piston is driven forcibly forwards on the third stroke, and during this *a continuous transformation of heat energy into mechanical energy—work—is taking place*, and machinery is being driven. Part of the energy is stored up in the flywheel and other moving parts, and this reserve is drawn on to help the engine over the other three strokes.

As the gas expands, its pressure and temperature fall in accordance with the loss of internal energy; and here again *the ideal type of expansion would be adiabatic*, in which none of the energy lost would be lost as heat flowing outwards through the cylinder walls, but every bit of it would reappear as mechanical work on the other side of the piston. This ideal cannot be achieved in practice.

By the time the piston arrives at the end of its stroke the pressure will have fallen to something like 40 lb./sq. in., and the temperature to 900° C.; thus a good deal of energy remains, but as the piston cannot go on farther, no further energy transformation can occur.

(*d*) The piston returns on the fourth and last stroke of the "cycle", and during this the heated burnt charge is ejected into the atmosphere and is lost.

The cycle of operations is then repeated indefinitely during the running of the engine. Thus only one stroke in every four is a working stroke, if the engine be of the single-cylindered, single-acting, four-stroke cycle type.

The ideal state of affairs, then, would be one in which the expansions and compressions were adiabatic.

Fig. 5 is a diagrammatic representation of the "ideal" Otto cycle. AB and CD are the compression and expansion

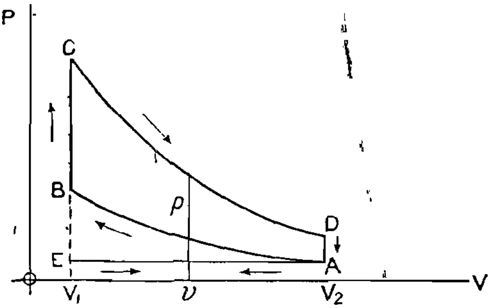


Fig. 5.—Theoretical Otto Four-stroke Cycle Diagram

curves respectively, the relation between p and v in each case being

$$PV^n = C, \quad \ldots \ldots \quad (6)$$

in which n is the proper index for an adiabatic change.

BC represents an "ideal" instantaneous rise of pressure following ignition at B, and DA represents the sudden fall of pressure at exhaust. The horizontal line EA is the atmospheric line for the gaseous fuel drawn in during the first stroke of the piston, and exhausted during the exhaust stroke.

What we have now to do is to determine what the absolute thermal efficiency of such an ideal engine would be.

We suppose the gaseous mixture to behave, as it is heated and cooled, just as if it were air, for which $n = 1{\cdot}4$, and the thermal efficiency so deduced is called the "Air-standard" efficiency. The nearness of approach to this efficiency of an actual engine will be a measure of the quality of its performance.

It is quite easy to deduce the efficiency formula.

Let the temperatures at the four points A, B, C, and D be, respectively, T_1, T_2, T_3, and T_4 degrees absolute.

We need only consider each pound of fuel.

The energy liberated by the explosion BC is

$$C_v(T_3 - T_2),$$

of which none is supposed to escape. There is no loss in the expansion CD. The energy lost in the exhaust DA is

$$C_v(T_4 - T_1).$$

Therefore the amount transformed into mechanical work is

$$C_v(T_3 - T_2) - C_v(T_4 - T_1),$$

and the efficiency is the ratio of this to the amount liberated in the explosion or combustion of the fuel.

$$\therefore \text{ the efficiency} = \frac{C_v(T_3 - T_2) - C_v(T_4 - T_1)}{C_v(T_3 - T_2)}$$
$$= 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Now by Eq. (12) $\frac{T_4}{T_3} = \left(\frac{1}{r}\right)^{n-1}$ and $\frac{T_1}{T_2} = \left(\frac{1}{r}\right)^{n-1}$.

So $\frac{T_4}{T_3} = \frac{T_1}{T_2}$, and a bit of elementary algebra shows that both of these ratios are equal to $\frac{T_4 - T_1}{T_3 - T_2}$, the second term in the efficiency formula. We had better do it.

$$\frac{T_4}{T_3} = \frac{T_1}{T_2},$$
$$\therefore \frac{T_4}{T_1} = \frac{T_3}{T_2},$$

$$\therefore \frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1,$$

$$\therefore \frac{T_4 - T_1}{T_1} = \frac{T_3 - T_2}{T_2},$$

$$\therefore \frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2}.$$

So we can replace $\frac{T_4 - T_1}{T_3 - T_2}$ by $\frac{T_1}{T_2}$ or $\frac{T_4}{T_3}$, or by $\left(\frac{1}{r}\right)^{n-1}$ which these are equal to.

Thus the air-standard efficiency of the four-stroke Otto cycle is

$$\left.\begin{aligned} E &= 1 - \frac{T_4 - T_1}{T_3 - T_2}, \\ \text{or } E &= 1 - \frac{T_1}{T_2}, \\ \text{or } E &= 1 - \frac{T_4}{T_3}, \end{aligned}\right\} \quad \ldots \quad (14)$$

$$\text{or } E = 1 - \left(\frac{1}{r}\right)^{n-1} \quad \ldots \quad (15),$$ in which $n = 1{\cdot}4$, and $r =$ the compression ratio.

More exact formulæ than these have been devised which make allowance for the fact that the specific heats of a gas are not quite constant, but vary slightly with the temperature. These simple formulæ, however, will be good enough to help us to understand broad principles, and in order to be quite clear on them it will be as well to work in full detail a numerical example.

EXAMPLE.—Take the case of an engine with a compression ratio of 5, and suppose the temperature of the charge at the commencement of compression to be 190° C., and the pressure 14·7 lb./sq. in.

First find P_2 and T_2 at the end of compression.

By Equation (13): $P_2 = P_1 r^n$
$= 14{\cdot}7 \times 5^{1{\cdot}4}$
$= 140$ lb./sq. in.

and by Equation (12): $\dfrac{T_1}{T_2} = \left(\dfrac{1}{r}\right)^{n-1}$

$\left[\text{or we could use } \dfrac{P_2V_2}{T_2} = \dfrac{P_1V_1}{T_1} \text{ as we have found } P_2\right]$

i.e. $\dfrac{T_2}{T_1} = r^{n-1}$; $\therefore$ $T_2 = (273 + 190) \times 5^{0{\cdot}4}$
$= 463 \times 5^{0{\cdot}4}$
$= 881{\cdot}4°$ C. abs.
$273{\cdot}0$
$608{\cdot}4°$ C.

Let us now suppose that the richness of the fuel and compression pressure result in a rise of temperature, on ignition, to 2000° C.

$$T_3 = 2273° \text{ C. abs.}$$

There has been no change of volume (line BC in the diagram), so we find the explosion pressure P_3 by Equation (2); thus:

$$\frac{P_3}{P_2} = \frac{T_3}{T_2} \therefore P_3 = 140 \times \frac{2273}{881{\cdot}4}$$
$$= 361 \text{ lb./sq. in.}$$

Now find P_4 and T_4 at the end of expansion.

$$P_4 = \frac{P_3}{r^n} = \frac{361}{5^{1{\cdot}4}}$$
$$= 37{\cdot}92 \text{ lb./sq. in.}$$

$$\frac{T_4}{T_3} = \left(\frac{1}{r}\right)^{n-1} = \left(\frac{1}{5}\right)^{0{\cdot}4}$$

$$\therefore T_4 = 2273 \times \left(\frac{1}{5}\right)^{0{\cdot}4}$$
$$= 1194° \text{ C. abs.}$$
273
$921°$ C.

So $$\frac{T_1}{T_2} = \frac{463}{881{\cdot}4} = 0{\cdot}5253$$

$$\frac{T_4}{T_3} = \frac{1194}{2273} = 0{\cdot}5253$$

$$\frac{T_4 - T_1}{T_3 - T_2} = \frac{731}{1391{\cdot}6} = 0{\cdot}5253$$

and $$\left(\frac{1}{r}\right)^{n-1} = \left(\frac{1}{5}\right)^{0 \cdot 4} = 0 \cdot 5253$$

Thus each of the four forms of the efficiency formula given on p. 37 gives

$$E = 1 - 0 \cdot 5253$$
$$= 0 \cdot 4747 \text{ or } 47 \cdot 47\%.$$

It is important to note that the actual rise of temperature on ignition makes no difference to the final result; the ratios

$$\frac{T_1}{T_2} \text{ and } \frac{T_4}{T_3}$$

will be just the same whether we suppose the explosion temperature to be 2000° or 4000° C.

Now we can go on to the Diesel engine cycle.

Looking again at the Otto Cycle efficiency formula

$$E = 1 - \left(\frac{1}{r}\right)^{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

evidently *the higher the compression ratio r, the greater will the efficiency be*, because if you substitute in the formula a bigger number for r, then $\frac{1}{r}$ is less, and so there is less to stub-tract from 1, the first term.

But, if you use a very high degree of compression, the gas becomes correspondingly hotter *and may ignite itself before the completion of the compression stroke*. Then you would have what is called " pre-ignition ", to the detriment of the working of the engine and perhaps to the engine itself. So in the Otto engine the compression ratio has to be limited—the temperature of compression must be well below the self-ignition point of the fuel used.

The Diesel engine gets over this difficulty in the following way.

In the first stoke, *air only* is drawn into the cylinder, and air can be compressed to any degree you like. In the second stroke this air is compressed very highly, to 500 or 600 lb./sq. in. Then, at the end of this stroke, or towards the end, the oil fuel is gradually admitted, either forced in by means of a pump, or blown in by a blast of very highly compressed air. This blast air must of course be at a greater pressure than the 500 or 600 lb./sq. in., in order to force itself and the fuel in; its pressure would be more like 1000 lb./sq. in. So blast-air Diesel plants have to be equipped with special air-compressors.

In this manner, not only is the danger of pre-ignition got over, but the necessity of an electric spark is done away with; for *the great heat of the high compression is quite sufficient to ignite the fire spray of oil fuel as it enters the cylinder.*

Now, the needful quantity of fuel is not injected all in one sudden splash, as it were, but gradually; if this were not so, there would be a further rise of pressure—a sudden rise, just as there is in the Otto engine when explosion occurs. Not that this would matter thermally. In fact, as you will see presently, combustion at constant volume is a more efficient process than combustion at constant pressure and increasing volume. But the sudden rise of pressure would be bad mechanically, for in the Diesel cylinder the pressure is already high enough for the easy and smooth working of the engine.

The fuel is sprayed in at such a rate that, theoretically, there is no rise in pressure; and by the time the last particle of oil has entered and been burnt, the piston will have moved some distance forward on its third and working stroke. The point at which the supply of fuel thus ceases is called the point of " cut-off ", and the student will notice that the working stroke in the Diesel engine bears a close resemblance to the working stroke in the steam engine.

From the point of cut-off, the hot gases expand, adiabatically in preference, until the end of the stroke. Then the exhaust valves open, and the pressure drops rapidly to atmospheric—the more rapidly the better, because then the returning piston will not have to move against an excess of back pressure.

The ideal and theoretical pressure or indicator diagram will now be clear; see fig. 6. Note carefully its points of difference from the Otto diagram.

The ratio $\frac{V_2}{V_1}$ is the compression ratio.

The *ratio* $\frac{V_2}{V_3}$ *is the expansion ratio.*

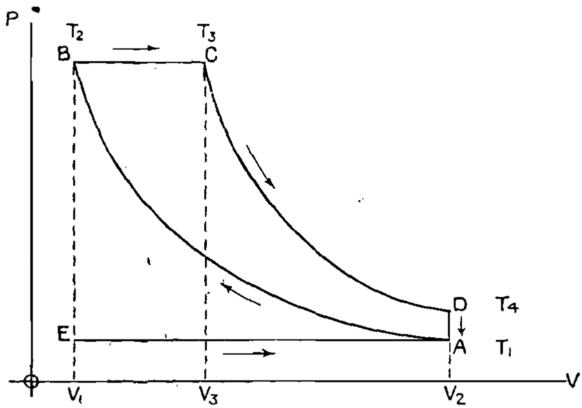


Fig. 6.—Theoretical Diesel Four-stroke Cycle Diagram

EA, Suction line. AB, Compression line.
BC, Constant pressure line. CD, Expansion line.
DA, Exhaust drop. AE, Exhaust line.

BC is the constant pressure combustion line. AB, the compression curve, and CD, the expansion curve, are, theoretically adiabatics.

Let us now derive the efficiency formula.

Heat liberated by the combustion BC

$$= C_p(T_3 - T_2) \qquad \text{[Not } C_v\text{, notice].}$$

Heat lost in the exhaust DA

$$= C_v(T_4 - T_1)$$

∴ the amount converted into mechanical work

$$= C_p(T_3 - T_2) \quad C_v(T_4 - T_1).$$

Dividing this by $C_p(T_3 - T_2)$, the heat liberated, we get the efficiency formula

$$E = 1 - \frac{C_v}{C_p} \cdot \frac{T_4 - T_1}{T_3 - T_2} \quad . \quad . \quad . \quad . \quad (16)$$

Thus the specific heats do not cancel out as they did in the Otto formula.

Denoting the ratio $\frac{C_p}{C_v}$ by γ, as on page 23,

$$E = 1 - \frac{1}{\gamma} \cdot \frac{T_4 - T_1}{T_3 - T_2} \cdot \quad . \quad . \quad . \quad . \quad (16')$$

Now we have to bring in the compression and expansion ratios, which we will denote by r_c and r_e respectively.

$$r_c = \frac{V_2}{V_1} \text{ and } r_e = \frac{V_2}{V_3} \quad \therefore \quad \frac{r_c}{r_e} = \frac{V_3}{V_1}.$$

But, since the pressures at B and C are the same, the ratio of the volumes is equal to the ratio of the absolute temperatures—Charles' law.

Hence $$\frac{r_c}{r_e} = \frac{T_3}{T_2} \text{ or } T_3 = \frac{r_c}{r_e} \cdot T_2.$$

We go on to express T_4 and T_1 also in terms of T_2. Remembering our results on page 28, we have

$$\frac{T_1}{T_2} = \left(\frac{1}{r_c}\right)^{n-1}, \quad \therefore \quad T_1 = \frac{1}{r_c^{\,n-1}} \cdot T_2.$$

Also,

$$\frac{T_4}{T_3} = \frac{1}{r_e^{\,n-1}}, \qquad \therefore \quad T_4 = \frac{1}{r_e^{\,n-1}} \cdot T_3$$

$$= \frac{1}{r_e^{\,n-1}} \cdot \frac{r_c}{r_e} \cdot T_2$$

$$\therefore \quad T_4 = \frac{r_c}{r_e^{\,n}} \cdot T_2.$$

Thus, $$\frac{T_4 - T_1}{T_3 - T_2} = \frac{\frac{r_c}{r_e^{\,n}} \cdot T_2 - \frac{1}{r_c^{\,n-1}} \cdot T_2}{\frac{r_c}{r_e} \cdot T_2 - T_2}$$

which, dividing top and bottom by T_2, becomes

$$\frac{\dfrac{r_c}{r_e^{\,n}} - \dfrac{1}{r_c^{\,n-1}}}{\dfrac{r_c}{r_e} - 1} = \frac{\dfrac{1}{r_c^{\,n-1}}\left\{\dfrac{r_c^{\,n}}{r_e^{\,n}} - 1\right\}}{\dfrac{r_c}{r_e} - 1}$$

The last step perhaps puzzles you; but if you multiply the quantity inside the wavy brackets again by $\frac{1}{r_c^{\,n-1}}$, notice that

$$\frac{r_c^{\,n}}{r_c^{\,n-1}} = r_c^{\,n-(n-1)} = r_c^{\,1} = r_c.$$

We now have our efficiency formula in about as simple a shape as it can be put into.

For the Diesel cycle, then,

$$E = 1 - \frac{1}{\gamma} \cdot \frac{\dfrac{1}{r_c^{\,n-1}}\left\{\left(\dfrac{r_c}{r_e}\right)^n - 1\right\}}{\dfrac{r_c}{r_e} - 1.} \quad . \quad . \quad . \quad (17)$$

The ratio $\frac{r_c}{r_e}$ will of course be greater than 1; and it is useful to note that it is the *constant pressure expansion ratio*, $\frac{V_3}{V_1}$; let us denote it by K. At the same time let us give γ the proper value for air, that is, 1·4, and we have, since n is also 1·4,

The air-standard efficiency of the Diesel cycle is

$$E = 1 - \frac{\dfrac{1}{r_c^{\,0\cdot4}}(K^{1\cdot4} - 1)}{1\cdot4(K - 1)} \quad . \quad . \quad . \quad . \quad (18)$$

where r_c is the compression ratio,

and K = the ratio $\frac{r_c}{r_e}$, i.e. $\frac{\text{Compression ratio}}{\text{Expansion ratio}} = \frac{V_3}{V_1}$ = the constant pressure expansion ratio.

Let us work an example.

EXAMPLE.—Take the case of a Diesel engine having a compression ratio of 12 and an expansion ratio of 6. Let us suppose the pressure of the air at commencement of compression to be 15 lb./sq. in., and its temperature 127° C.

Here $$K = \frac{12}{6} = 2 \text{ and } r_c = 12$$

$$\therefore\ E = 1 - \frac{\frac{1}{12^{\cdot 4}}(2^{1\cdot 4} - 1)}{1\cdot 4(2 - 1).}$$

$$2^{1\cdot 4} = 2\cdot 638 \text{ and } 12^{\cdot 4} = 2\cdot 7$$

$$\therefore\ E = 1 - \frac{1\cdot 638}{2\cdot 7 \times 1\cdot 4}$$

$$= 1 - 0\cdot 433$$

$$= 0\cdot 567 \text{ or } 56\cdot 7\%.$$

Let us study the pressures and temperatures.

$$P_1 = 15 \text{ and } T_1 = 273 + 127 = 400.$$

$$P_2 = P_1 \times r_c^n = 15 \times 12^{1\cdot 4} = 486\cdot 4° \text{ lb./sq. in.}$$

$$T_2 = T_1 \times r_c^{n-1} = 400 \times 12^{0\cdot 4} = 1080° \text{ C. abs.}$$

$$\underline{273}$$

$$\underline{\underline{807°}} \text{ C.}$$

The pressure P_3 at the point C = 486·4 lb./sq. in. The temperature must be greater, however, to maintain this pressure at the increased volume.

$$\frac{T_3}{T_2} = \frac{V_3}{V_1}, \text{ and } \frac{V_3}{V_1} = \frac{V_2}{V_1} \div \frac{V_2}{V_3}$$

$$= r_c \div r_e = K = 2.$$

$$\therefore\ \frac{T_3}{T_2} = 2 \quad \therefore\ T_3 = 2T_2 = 2160° \text{ C. abs.}$$

$$\underline{273}$$

$$\underline{\underline{1887°}} \text{ C.}$$

$$P_4 = \frac{P_3}{r_e^n} = \frac{486\cdot 4}{6^{1\cdot 4}} = 39\cdot 6 \text{ lb./sq. in.}$$

and $$T_4 = \frac{T_3}{r_e^{n-1}} = \frac{2160}{6^{0\cdot 4}} = 1054° \text{ C. abs.}$$

$$\underline{273}$$

$$\underline{\underline{781°}} \text{ C.}$$

Using the efficiency formula as on p. 42,

$$E = 1 - \frac{1}{1{\cdot}4} \cdot \frac{1054 - 400}{2160 - 1080}$$

$$= 1 - \frac{654}{1{\cdot}4 \times 1080}$$

$$= 1 - 0{\cdot}433$$

$$= 0{\cdot}567 \text{ or } 56{\cdot}7\% \text{ as before.}$$

It will be interesting now to calculate the ideal efficiency of the Otto cycle for a compression ratio of 12.

$$E = 1 - \left(\frac{1}{12}\right)^{1{\cdot}4-1}$$

$$= 1 - 0{\cdot}37$$

$$= 0{\cdot}63, \text{ or } 63 \text{ per cent.}$$

So the Otto cycle is more efficient than the Diesel. But the student will want to know what effect the expansion ratio has upon the efficiency of the Diesel cycle. He should try this by working the above example again for greater and lesser expansion ratios than 6. He will find that a greater expansion ratio, i.e. an earlier cut-off, results in a higher thermal efficiency, but that this is always less than the efficiency of the corresponding Otto cycle. The important point, however, is that *the Otto engine is unable to use a high compression ratio*, for the reason already explained, and so the Diesel engine is able to gain an advantage.

The student will observe that the Otto cycle efficiency formula $E = 1 - \left(\frac{1}{r_c}\right)^{n-1}$, where $n = 1{\cdot}4$, is converted into the Diesel formula by multiplying the second term by the factor

$$\frac{K^{1{\cdot}4} - 1}{1{\cdot}4(K - 1)}, \text{ where } K = \frac{r_c}{r_e}, \text{ the constant pressure expansion ratio.}$$

In the table below the value of this factor, denoted by y, is shown for values of K from 1 to 2 (the student should himself perform these calculations). Note that as K becomes more nearly equal to unity, so also does y.

K	2	1·8	1·6	1·4	1·2	1·1
y	1·17	1·14	1·109	1·075	1·04	1·021
E	0·567	0·578	0·59	0·602	0·615	0·622

For a compression ratio $r_c = 12$, we found the efficiency of the Otto cycle to be

$$E = 1 - 0{\cdot}37 = 0{\cdot}63, \text{ or } 63 \text{ per cent.}$$

If we multiply the term 0·37 by each of the values of y in the table, then by subtraction from unity we obtain the corresponding Diesel efficiencies, as shown in the third line of the table. Note how these efficiencies approach the Otto efficiency, 63 per cent, as K approaches the value 1; but the Diesel efficiency is always less than the Otto, *for the same compression ratio.*

The accompanying graph (fig. 7) will perhaps show these things more clearly. AB is the curve giving y in terms of K; while CD is the graph of the resulting Diesel efficiency.

Now $K = \frac{r_c}{r_e}$, and the less this ratio differs from unity, i.e. the shorter the period of combustion at constant pressure, the more efficient is the Diesel cycle. K would be equal to 1 if $r_e = r_c$; but the expansion ratio must of course be less than the compression ratio. However, for a given value of r_c, the higher the expansion ratio, i.e. *the shorter the process of constant-pressure combustion* (line BC in fig. 6), the higher will be the efficiency. Thus, in theory, the Diesel engine becomes more efficient as its load is reduced.

The reader should bear in mind that all our figures have been only theoretical, and that the real state of affairs in the cylinder of an actual engine differs in many respects from the ideal and simple conditions we have described. All this the student will find no difficulty in reading up in the

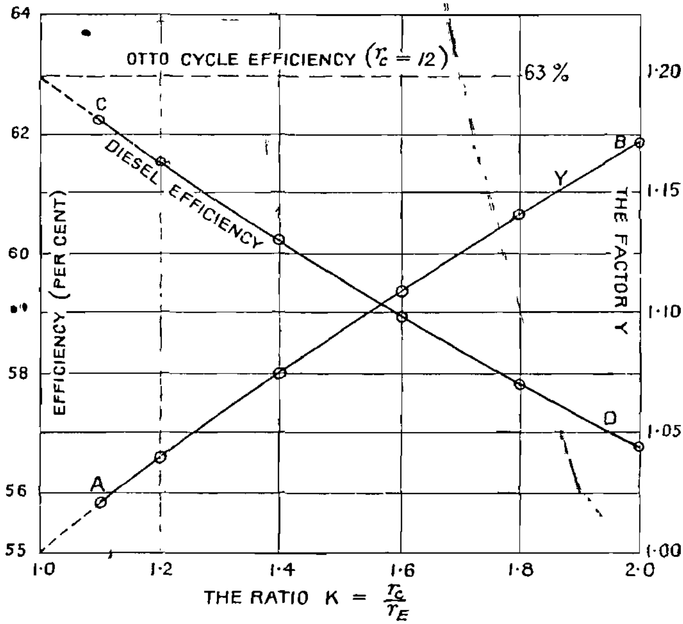


Fig. 7

standard works; it has been our object merely to explain as simply as possible, and starting right from the beginning, the elementary thermodynamic principles.

The "Two-stroke Cycle"

In a single-cylinder, single-acting engine working on the Otto cycle, there is only one working stroke to every four of the piston, that is, to every two revolutions of the crank.

When we studied this cycle of events in detail, it probably occurred to you that this small proportion of working strokes was a bad feature, and that—for one thing—it would tend to produce unsteadiness in the rate of revolution from moment to moment during the progress of each cycle. So it does, in fact, and to minimize this defect it is necessary to provide a comparatively heavy flywheel. The effect is not so noticeable in larger engines with two or more cylinders, because then matters can be so arranged that the working strokes in the various cylinders occur one after another, not simultaneously, and the resultant driving effect on the crank-shaft is steadier. Even so, however, if, in each cylinder, we could contrive to obtain one working stroke in every *two*, instead of in four, a further gain in steadiness would result; besides which, supposing each working stroke of our " two-stroke " engine to be equally as powerful as each one of the " four-stroke " engine, we should have a doubly powerful engine—supposing the rate of revolution to be the same. Or, if we did not desire an engine of greater power, we could reduce the size or number of cylinders, and so secure a great gain in *compactness*. This is a point of much importance, especially in marine engines, because space in ships is very limited and valuable.

Large, highly successful engines working on this two-stroke or " Clerk " cycle—so named after Sir Dugald Clerk, to whom internal-combustion engineering owes so much—have been constructed. Some are even double-acting, i.e. work is done on both sides of the piston, as in the steam engine, and thus every stroke is a working stroke. For detailed information of such engines the student will turn to the standard works.

In the case of the two-stroke Diesel, the action in the cylinder, briefly, is as follows. Towards the end of the working stroke, the outward-moving piston first uncovers a belt of exhaust ports in the cylinder walls, through which the burnt gases, still at a pressure considerably above atmospheric, rapidly discharge themselves. Shortly after these exhaust ports are uncovered, and when the pressure within the cylinder has fallen to a low value, the further motion of the piston uncovers a belt of inlet ports in the cylinder wall through which air at a pressure slightly above atmospheric flows in. This air " scavenges " the cylinder of the waste products of combustion. The piston, moving inwards again, covers up the inlet and exhaust ports, and the trapped air is then subjected to the usual high compression. Oil fuel is then injected and another working stroke at once follows. The action in the two-stroke gas engine is much the same. A point to notice is that in these two-stroke engines the effective compression ratio is not equal to

$$\frac{\text{Total cylinder volume}}{\text{clearance volume}}.$$

For the numerator of this fraction we must substitute the cylinder volume up to the position occupied by the piston face when the exhaust ports are just being uncovered.

Two advantages of the two-stroke engine over the four-stroke have been noted. It has other advantages; but it has its disadvantages also: its more intensive action results in a tendency to more noise and more rapid wear, it is difficult to scavenge effectively, and again there is a higher mean temperature over the cycle which increases the danger of overheating.

EXAMPLES V

On the Otto and Diesel Cycles

1. Taking $n = 1{\cdot}4$, calculate the value of the air-standard efficiency of the Otto cycle, $E = 1 - \left(\frac{1}{r}\right)^{n-1}$, for each of the following values of r: 3, 4, 5, 6, 7, 8, 9, 10, 15.

Show E and r in a graph, and keep the curve for reference.

2. A four-stroke gas engine working on the Otto cycle has a compression ratio of 4. Assuming that the pressure and temperature of the charge at the end of the suction stroke are 14 lb./sq. in. and 100° C., that the expansion and compression curves are adiabatics, and that the rise of temperature, at constant volume, on explosion, is 1000° C., tabulate the pressures and temperatures at the four points of the cycle.

3. Calculate the efficiency of the cycle of Question 2 by means of the temperature formula $E = 1 - \frac{T_4 - T_1}{T_3 - T_2}$, and compare with your result for $r = 4$ in Question 1.

4. If the stroke volume of the engine of Question 2 is 3 c. ft., calculate the work done during (*a*) compression, (*b*) expansion. Thus find the net work done per cycle.

5. A gas engine uses 15 c. ft. of gas of calorific value 300 C.H.U. per cubic foot per indicated horse-power per hour. ("Indicated" horse-power is the power developed in the cylinder.) What is the indicated thermal efficiency? The compression ratio of the engine is 4·85. What is the air-standard efficiency? (You may use your graph of Question 1 for this.) What ratio does the indicated efficiency bear to the air-standard efficiency? (This shows how near the engine's actual performance approaches to the ideal.)

6. Calculate the air-standard efficiency of an internal-combustion engine, working on the Otto cycle, having a piston diameter 15 in., stroke 18 in., and a clearance volume of 520 c. in.

If the indicated thermal efficiency is 60% of the above ideal, how much gas of calorific value 300 C.H.U. per cubic foot would the engine use per indicated horse-power hour?

7. In an engine working on the Otto cycle, the temperature of suction was found to be 100° C., and the compression temperature 310° C. What was the compression ratio, assuming $n = 1·4$?

Note: The student should be able to solve equations like $1·563 = r^{0·4}$.

8. In Question 7, having found the value of r, calculate the air-standard efficiency. Check your result, using the formula $E = 1 - \frac{T_1}{T_2}$.

9. If in Question 7 the maximum temperature of explosion was 1145° C., and expansion to the end of the stroke followed the law $PV^{1·4} = C$, what was the temperature at the end of expansion?

10. A gas engine draws in during the suction stroke 1 c. ft. of gaseous mixture containing 10% by volume of coal gas of calorific value 300 C.H.U. per cubic foot. How much energy in ft. lb. will be liberated on ignition, supposing complete combustion?

If at the end of compression the pressure and temperature are 130 lb./sq. in. and 400° C., and explosion takes place at constant volume, calculate the maximum explosion temperature and the corresponding pressure, assuming a mean volumetric heat of 27 ft. lb. per cubic foot. (See Question 10, Examples II.)

11. A Diesel engine has a compression ratio of 13·5. Calculate the air-standard efficiency when the expansion ratio is (*a*) 7·5, (*b*) 8·5. What is the efficiency of the Otto cycle for the same compression ratio? (Use your graph.)

12. The ignition temperature of the vapour of the oil used by a Diesel engine is 520° C. If the temperature and pressure at the commencement of compression are 100° C. and 14 lb./sq. in. and the compression follows the law $PV^{1·35} = C.$, what must the compression ratio be if the temperature produced is to be (*a*) just sufficient to cause ignition; (*b*) 100° C. above the ignition temperature of the oil vapour?

13. Calculate the pressures corresponding to (*a*) and (*b*) in Question 12. Further, if the piston diameter = 12 in., and stroke = 15 in., calculate in each case the work done in compressing the air.

14. A Diesel engine has a compression ratio of 13, and an expansion ratio of 6·5. Starting with a pressure of 14 lb./sq. in. and temperature 80° C. at the end of the suction stroke, calculate and tabulate pressures and temperatures at the four points of the ideal cycle. Take $n = 1·4$.

15. In Question 14, assuming $C_p = 0·238$ and $C_v = 0·17$, and that the mass dealt with is 1 lb., calculate (*a*) heat supplied during the constant pressure combustion period; (*b*) heat rejected at exhaust. Thus calculate the amount of heat converted into mechanical work, and the efficiency of the cycle.

16. Calculate the efficiency of the above cycle, using the formula involving r_c, r_e, and n.

17. If the engine of Question 14 has a clearance volume of 1 c. ft., calculate (*a*) work done in compressing the air; (*b*) work done during constant pressure combustion; (*c*) work done during expansion. Thus find the net work done per cycle, and determine the mean effective pressure in lb./sq. in.

18. Repeat Question 14 for an expansion ratio of 8, find the efficiency as in Question 15, and the work done per cycle as in Question 17.

CHAPTER IV

The Actual Performance of Internal-Combustion Engines

The Measurement of Work and Power

The work done by an unvarying force F lb. when it moves against the resistance through a distance s ft., in its own direction, is Fs foot-pounds. When the force is a variable one, such, for example, as the driving force on a piston as it moves along its stroke, we must substitute for F the *mean value* of the force during the whole of the movement.

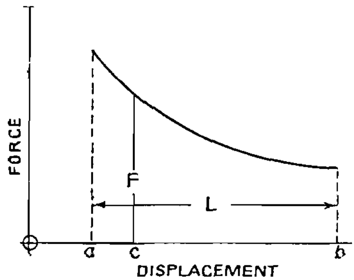


Fig. 8

If, for each position occupied by a piston as it moves along, we know the magnitude of the driving force at that moment, we are able to draw a diagram, such as that in fig. 8, termed a *diagram of work*.

In this diagram the line *ab* represents, to some scale, the total movement, or displacement as it is often called, of the piston; and any ordinate, such as F, represents to scale the driving force corresponding to the piston position C. Now the area between the curve and the line *ab* could be found by multiplying the mean height of the curve by the length L of the diagram; but the mean height of the curve represents the mean driving force, and the length lb represents the displacement of the piston, hence *the area of the diagram represents the work done on the piston.* That is why it is termed a diagram of work.

Suppose in such a diagram we measured the area spoken of—say by a planimeter—and found it to be 5 sq. in. If the length ab were 2 in., then clearly the mean height of the curve above the base, or zero line, is $2\frac{1}{2}$ in., obtained by division. And if the force scale employed were 1 in. = 20 lb., then the mean driving force $= 2\frac{1}{2} \times 20$

$= 50$ lb.

Again, suppose each inch of the base line represents a piston movement of one foot. Then 1 sq. in. of diagram represents

$$20 \times 1 = 20 \text{ ft. lb.},$$

and the total area being 5 sq. in., the total work done is

$$5 \times 20 = 100 \text{ ft. lb.}$$

It is most important to obtain diagrams which show the changes of pressure which occur in an engine cylinder from moment to moment during the progress of a complete cycle, or of a succession of cycles. Instruments called Indicators have been devised for this purpose, and the diagrams so obtained are called *Indicator Diagrams*.

In the indicator a pencil point moves up and down in accordance with the rise and fall of pressure in the cylinder, and a rectangular sheet of paper, or " card ", moves to and fro horizontally. Thus a closed curve is drawn, which, if the engine be of the four-stroke Otto-cycle type, will take somewhat of the form shown in the figure. (Fig. 9.)

As the piston moves outwards on the charging or suction stroke, the indicator pencil draws the line *ab*, which, note, lies below the atmospheric pressure line. This fall of pressure happens because, as a result of the restricted area of the inlet passages, the entering gases are unable to keep up, as it were, with the rapidly advancing piston. (The effect has, however, been greatly exaggerated in the figure for the sake of clearness.) If, at some particular point of this stroke, the pressure should be only 14 lb./sq. in., then at that moment the progress of the piston is being retarded to the extent of 0·7 lb. for each square inch of its area, assuming that the pressure on the other side of the piston is atmospheric. Thus the diagram area between the atmospheric line and the suction line *ab* is *negative work*, i.e. work done *against* the engine.

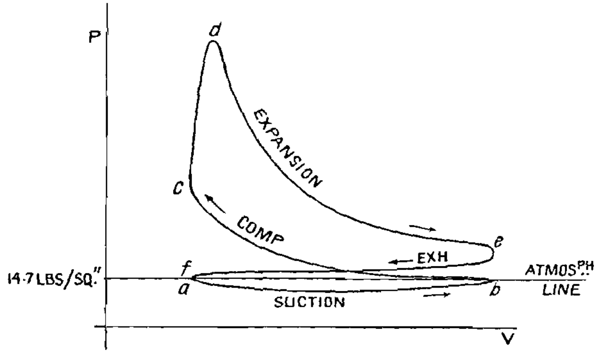


Fig. 9

The line *bc* shows the rise of pressure during compression of the charge, and of course *the area between bc and the atmospheric line also represents work done against the engine.*

At *c* explosion occurs and the pressure line rises rapidly, but not instantaneously, to the point *d*, the piston having meanwhile commenced its working stroke. (The point *c* may not correspond with the end of the stroke exactly, but we will assume here that it does, for simplicity.) *de* is the expansion line, and *the area between the line cde and the atmospheric line represents the work done in urging the piston forward on the working stroke.*

Then exhaust occurs; and again, because of the restricted area of the exhaust passages, the gases exert a back pressure on the piston, and the exhaust line, *ef*, rises slightly above the atmospheric line, the included area being negative work. Thus *the whole of the loop at the bottom of the diagram is negative work.*

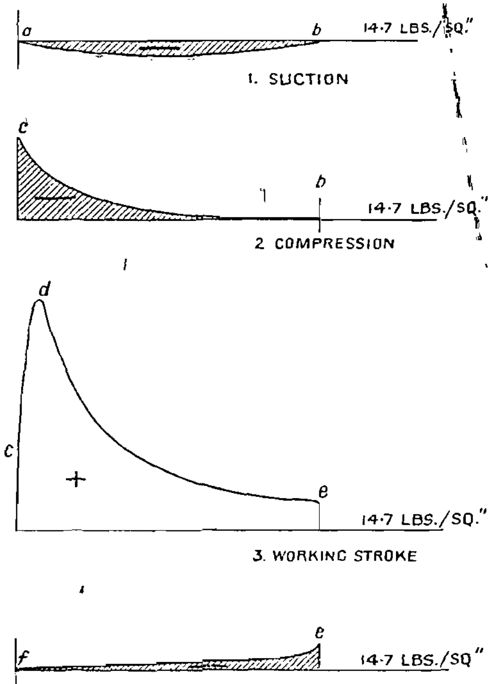


Fig. 10.—The negative work of strokes 1, 2, and 4 must be deducted from the positive work of stroke 3 to obtain the effective work done per cycle.

Fig. 10 shows the indicator diagram " dissected ", as it were, into its component parts. If the student considers this carefully, in conjunction with fig. 9, he will see that the net effective work done on the piston during the complete cycle is represented by the area of the upper loop of the indicator diagram, *minus* the area of the lower, or exhaust-suction loop.

The pressures in internal-combustion engine cylinders rise to such high values that very stiff springs (which regulate the up and down movement of the pencil) have to be used. The consequence is that the exhaust-suction loop becomes almost undistinguishable from a straight line, and is disregarded.

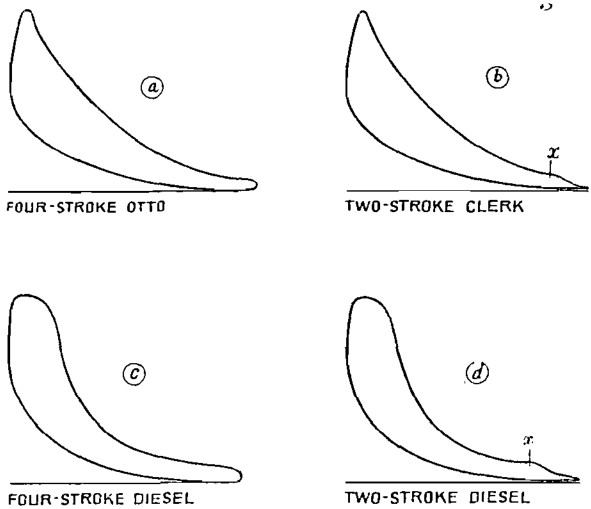


Fig. 11

a, *b*, *c*, and *d* in fig. 11 indicate the kind of diagram obtained from four-stroke Otto, two-stroke Clerk, four-stroke Diesel, and two-stroke Diesel engines respectively.

In diagrams *b* and *d* the point *x* marks the uncovering of the exhaust ports. From here to the end of the stroke and for an approximately equal portion of the return stroke, exhaust, scavenging, and recharging of the cylinder with fresh air take place.

The area of the loop is divided by the length of the diagram, and then, knowing the stiffness or "scale" of the indicator spring employed, we are able to compute the *Mean Effective*

Pressure for the complete cycle. For example, if the area of the diagram were 1·92 sq. in., and its length 3 in., then the mean height of the diagram would be $\frac{1 \cdot 92}{3} = 0 \cdot 64$; and if the spring used were of such strength that each inch of height represented a pressure of 100 lb./sq. in., then the mean effective pressure

$$= 100 \times \cdot 64 = 64 \text{ lb./sq. in.}$$

The determination of this value is the first step—and the most difficult step—in the calculation of the *Indicated Horse-Power* of the engine.

A strong horse working its very hardest can do about 33,000 ft. lb. of work in a minute, i.e. in that time it could lift a 330 lb. weight through a vertical height of 100 ft. A smaller horse might perhaps take twice as long, two minutes, to perform the same task, in which case we should say that it was only half as powerful as the other.

Power is the *rate* at which work is done, and a machine which is able to do exactly 33,000 ft. lb. of work in one minute is said to develop one horse-power.

1 H.P. = 33,000 ft. lb. per minute.

If p_m = the mean effective pressure in pounds per square inch, and

A = the area of the piston face in square inches, then

p_mA = the total mean driving force per cycle in pounds.

If L = the length of stroke in feet,

p_mAL = the work done per cycle in ft. lb.,

then, if there are *N cycles*, or explosions, per minute, the effective work done per cylinder per minute is p_mALN ft. lb., and the cylinder, or Indicated Horse Power as it is called, is the number of times that this amount of work contains 33,000, i.e.

$$\text{I.H.P.} = \frac{p_m\text{ALN}}{33{,}000}.$$

This is a very simple formula, but the student must be careful not to make a slip in units—he must not make the mistake, for instance, of substituting for L the length of the stroke in inches: that would give a result twelve times too large. The calculation is so easy to do from first principles that it is perhaps hardly worth while bothering with a formula at all.

EXAMPLE.—In a test on a gas engine the mean effective pressure was found to be 60 lb./sq. in., the number of explosions per minute 80. If the diameter of the cylinder was 10 in., length of stroke 18 in., what was the I.H.P.?

Area of piston face $= \frac{\pi}{4} \times 10^2$ sq. in. (or use area tables)

$= 78{\cdot}54$ sq. in.

Total mean pressure $= 78{\cdot}54 \times 60$

$= 4712$ lb. $(= p_m A)$.

Work per cycle $= 4712 \times 1\frac{1}{2}$

$= 7068$ ft. lb. $(= p_m AL)$.

Work per minute $= 7068 \times 80$ ft. lb. $(= p_m ALN)$.

$$\therefore \text{ I.H.P.} = \frac{7068 \times 80}{33{,}000} = 17{\cdot}1$$

Of course it would be quicker to pop the various numbers in the formula and log it out directly; but an elementary student who wishes to get a real grasp of his subject had better not do this, but go to the little extra trouble of working his problems out from first principles.

An engine drives machinery through the medium of its crankshaft, and the horse-power exerted here is somewhat less than that developed in the cylinder, because between the latter and the shaft there are frictional resistances to be overcome. This absorbs power.

The useful horse-power exerted at the crankshaft may be measured by applying to the rim of the flywheel a frictional load which can be adjusted to the desired amount. The simplest means of doing this is the rope brake, shown diagrammatically in fig. 12.

The rope is coiled once round the wheel and held in position on the rim by several wooden blocks (not shown in the figure). One end of the rope is attached to the hook of a spring balance S hung from a beam in the ceiling, and the other end carries a load W.

If there were no friction between the rope and the flywheel rim, the register of the spring balance would be equal to W; the difference between W and the actual reading, i.e. (W — S), is the amount of friction in pounds, and in one revolution of

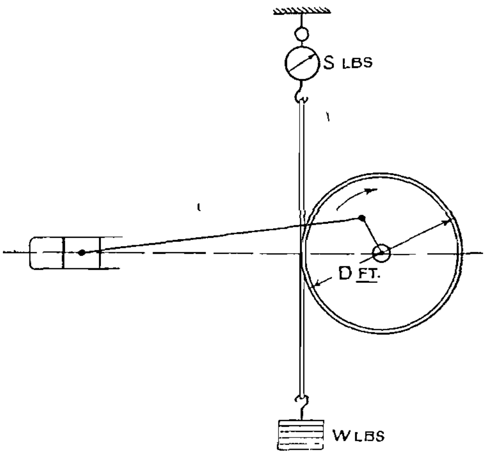


Fig. 12

the wheel this resistance is overcome through a distance equal to the wheel's circumference, that is, πD *feet*. Thus the work done per revolution is (W — S)πD ft. lb., and if the number of revolutions per minute is N (obtained by means of a revolution counter), the work done per minute is (W — S)πDN ft. lb. So the horse-power exerted, termed the *Brake Horse-Power*, is

$$\text{B.H.P.} = \frac{\pi D(W - S)N}{33{,}000}.$$

Here again the calculation is so easy from first principles that the student need not try to remember the formula.

EXAMPLE.—Dead load, 128 lb.; spring balance reading, 28 lb.; diameter of wheel, 4 ft.; revolutions per minute, 200.

$$\begin{aligned}
\text{Frictional load} &= 128 - 28 = 100 \text{ lb. wt.} \\
\text{Circumference of brake wheel} &= 4\pi \text{ ft.} \\
\text{Work done per revolution} &= 100 \times 4\pi \text{ ft. lb.} \\
\text{Work done per minute} &= 100 \times 4\pi \times 200 \text{ ft. lb.} \\
\therefore \text{B.H.P.} &= \frac{80{,}000\pi}{33{,}000} \\
&= 7 \cdot 62.
\end{aligned}$$

In this test the whole of the power developed is absorbed in friction. Part of the I.H.P. is absorbed in overcoming the frictional resistances between the various sliding surfaces in the mechanism (it would be worth the student's while to draw up a list of such surfaces). What remains is the B.H.P., and this is absorbed by the friction between rope and rim. The energy so lost reappears as heat. Wherever frictional losses occur heat is generated. The flywheel rim gets very hot, so hot, indeed, in tests on larger engines, that cooling water has to be supplied within the rim to carry the heat away. In very large engines the rope brake method of determining the horse-power becomes impracticable, and other less simple means, as e.g. hydraulic and electrical brakes, have to be employed.

Mechanical Efficiency

Suppose that in a test of an engine working at full load the I.H.P. and B.H.P. were found to be 80 and 70 respectively. The difference, 10 H.P., is the mechanical loss; so that even when running " light ", that is, driving no external machinery whatever, something like 10 H.P. would be required for the engine to keep itself in motion at the required speed against its own internal resistances. Not quite so much, perhaps, because at lighter loads the pressures between bearing surfaces are not so heavy, and so the friction is less. The ratio which the B.H.P. bears to the I.H.P., in our example $\frac{70}{80}$ or

0·875, is termed the *Mechanical Efficiency* of the engine. It is usual to express it as a percentage—87·5%.

$$\text{Mechanical Efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}} \times 100\%.$$

This is a quite different thing from *Thermal Efficiency*, which depends so much on the kind of heat cycle employed. The mechanical efficiency depends rather on what we might call the constructive merit of the engine—the mechanical design.

When we were explaining the manner of obtaining the mean effective pressure from the indicator card, we mentioned that the area of the exhaust-suction loop was usually disregarded in this calculation. This being so, it follows that the difference between the indicated and brake horse-powers is not really all friction loss: it includes the power loss in the exhaust and suction strokes. It also includes what is called " flywheel windage " loss—air resistance to its motion—and the power loss involved in compression and expansion in " idle cycles ", i.e. cycles in which ignition does not occur.

In certain very careful tests of a 25 B.H.P. gas engine, the various losses in the engine when working at *full load* were as follows:

Actual friction losses	.. .. ..	4·04
Flywheel windage loss	.. .. ..	0·53
Pumping { Suction loss	.. .. ..	1·30
Pumping { Exhaust loss	.. .. ..	0·80
Power loss in idle cycles	.. .. ..	0·11
	Total ..	6·78 H.P.

The corresponding total when the engine was running light was 6·16 H.P., not very much less. Although this mechanical loss includes losses other than that due to friction, it is, nevertheless, usual to speak of it as the " friction horse-power ". In the above test the B.H.P. was 25; adding on the mechanical loss at full load, the

$$\text{I.H.P.} = 25 + 6{\cdot}78 = 31{\cdot}78.$$

Hence the mechanical efficiency was

$$\frac{25}{31 \cdot 78} \times 100 = 78 \cdot 6\%.$$

At full load and normal working speed, the average mechanical efficiencies usually attained are:

Gas Engines	80%
Diesel Engines	78%
Petrol Engines	85%

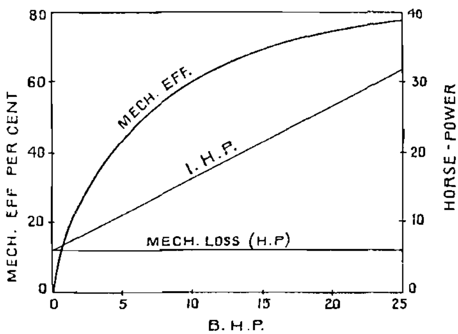


Fig. 13

At lighter loads the mechanical loss, which, as we have seen, remains fairly constant, bears a larger proportion to the total power developed; hence the mechanical efficiency is less. At no-load the mechanical efficiency is zero, for then the I.H.P. is all accounted for by mechanical loss, the B.H.P. being zero.

Typical curves of I.H.P., mechanical loss (H.P.), and mechanical efficiency are shown in fig. 13.

EXAMPLES VI

On Indicated and Brake Horse-power

1. The indicator diagram taken from a small oil engine has an area of 1·47 sq. in. and a length of 3 in., and the indicator spring is of such strength that 1 in. height in the diagram represents a pressure of 150 lb./sq. in. Calculate the mean effective pressure.

2. If the engine of Question 1 gives 600 explosions per minute, what is the indicated horse-power? Diameter of cylinder = 3·5 in., stroke = 4 in. Supposing the mechanical efficiency to be 85%, what is the brake horse-power?

3. Estimate the average mean effective pressure in an engine, given that the average of the cards taken in a trial had an area of 4·26 sq. in., and a length of 3 in., the scale of the indicator spring being $\frac{1}{60}$.

(*Note*: The "scale" of the spring of Question 1 would be said to be $\frac{1}{150}$.)

4. Calculate the I.H.P. of the engine of Question 3, given that the diameter of the piston = 15 in., stroke = 20 in., number of working cycles per minute = 120.

If the B.H.P. was measured and found to be 73·5, what was the mechanical efficiency of the engine?

5. The mechanical efficiency of a gas engine was 82%, and the corresponding B.H.P. was 240. What was the I.H.P.?

6. In a rope brake test of an engine, the diameter of the brake wheel was 26 in., the friction load was 220 lb., and the speed 1200 revs. per minute. Calculate the B.H.P.

7. How much heat in C.H.U. was generated by friction at the brake per minute in Question 6?

8. What B.H.P. is an engine developing at 250 r.p.m., if the rope brake carries a load of 200 lb. at one end, the spring-balance reading at the other end is 20 lb., diameter of brake-wheel 5 ft.?

9. If, in Question 8, 90% of the heat generated by friction at the brake is absorbed by a stream of water which rises in temperature by 12° C., how many gallons of water are used per minute?

10. A test of an oil engine gave the following data:

Mean effective pressure	80 lb./sq. in.
Diameter of cylinder	15 in.
Length of stroke	2 ft.
No. of explosions per minute	90.
Diameter of brake wheel	8 ft.
Friction load (W − S)	390 lb.
Revolutions per minute	210.

Calculate the I.H.P., B.H.P., and mechanical efficiency.

11. A four-stroke gas engine develops 20 B.H.P. at 240 r.p.m. Assuming a mechanical efficiency of 80%, what is the I.H.P.?

If the mean effective pressure is 82 lb./sq. in. and the length of stroke is 14 in., what is the diameter of the piston? Assume that there are no idle cycles.

12. The I.H.P. and B.H.P. of a gas engine were measured at various loads as follows:

I.H.P.	6·2	11·4	17	28	31·28.
B.H.P.	0	5·1	10·5	21·3	24·5.

Tabulate the mechanical efficiency and friction horse-power, and show your results in a graph with a B.H.P. base. Show also the I.H.P. line (see fig. 13).

13. The average diagram taken from a gas engine on test had an area of 1·125 sq. in., length 3 in., scale of spring $\frac{1}{100}$. The revs. per min. were 180, and on the average the engine (four-stroke) missed 5 explosions per minute.

Diameter of cylinder = 9 in., stroke 15 in.

The rope brake was coiled round the flywheel, 58 in. diameter, and the friction load (W — S) was 150 lb.

Calculate the I.H.P., B.H.P., and mechanical efficiency.

14. A four-stroke cycle oil engine had a cylinder 12 in. diameter, stroke 18·25 in. In a rope-brake test the diameter of the brake wheel was 9 ft.; r.p.m., 200; and the friction load 195 lb.

The mean effective pressure was found to be 89·7 lb./sq. in.

Calculate the I.H.P., B.H.P., and mechanical efficiency.

The Calorific Values of Fuels

When *one pound* of petrol is completely burnt, about 11,000 centigrade units of heat are liberated, i.e. sufficient heat to raise the temperature of nearly a cwt. of water from the freezing- to the boiling-point.

The complete combustion of a mass of ordinary town gas which at 14·7 lb./sq. in. and 0° C. occupies a volume of *1 c. ft.*, liberates about 330 C.H.U.

These figures, 11,000 C.H.U. and 330 C.H.U., are the calorific values—per pound and per cubic foot respectively—of petrol and town gas.

Gas-meters register gas consumption in cubic feet, not in pounds, so it is more convenient to have the calorific value of a gaseous fuel in C.H.U. per cubic foot. Knowing the density of the gas we could of course easily convert this figure to C.H.U. per pound—in the above case it would work out to about 10,650 C.H.U. per pound.

The calorific values of some common fuels used in internal-combustion engines are, in round figures:

Crude petroleum	10,800 C.H.U./lb.	Diesel.
Heavy fuel oil	10,000 C.H.U./lb.	Diesel and semi-Diesel.
Paraffin oil	10,200 C.H.U./lb.	Oil engines with vaporizers.
Petrol	10,600 C.H.U./lb.	Oil engines with carburettors.
Town gas	330 C.H.U./c. ft.*	Small gas engines.
Suction producer gas	70 C.H.U./c. ft.*	Gas engines.

* At standard temp. (0° C.) and press. (14·7 lb./sq. in.)

Heat Losses in Internal-combustion Engines

The exhaust gases and the cooling water carry off the major portion of the fuel energy supplied which is not converted into mechanical work in the cylinder. It is a comparatively simple matter in a test to determine how much heat is conveyed away by the cooling water. Then, knowing how much has been transformed into work, the amount lost in the exhaust together with the small amount lost by radiation from hot surfaces, is found by subtraction. Representative "heat-balances", as they are termed, for gas and Diesel engines are given in tabular form below:

	Gas.	Diesel.
Heat converted into work	33%	39%.
Heat carried away by cooling water	25%	25%.
Heat carried away by exhaust gases; radiation ..	42%	36%.
Heat supplied	100%	100%.

These figures correspond with full-load conditions.

Thermal Efficiency

A Diesel engine which showed such a heat balance as that in the table would be said to have an Indicated Thermal Efficiency of 39 per cent—"indicated" because this figure refers to the mechanical work done on the piston. The "Brake" Thermal Efficiency, i.e. the ratio of energy delivered at the crankshaft to energy supplied, would of course be less than 39 per cent—probably 35 per cent—

exactly how much less depending upon the *mechanical* efficiency of the engine.

Again, the indicated thermal efficiency is itself less than the absolute theoretical thermal efficiency as calculated from considerations of compression and expansion ratios.

The ratio $\dfrac{\text{Indicated Thermal Efficiency}}{\text{Ideal ``Air-standard'' Thermal Efficiency}}$

is termed the *Relative Efficiency*: it shows how closely an engine's actual performance approaches the ideal.

Let us now work out fully the following record of a test made on a four-stroke Otto-cycle gas engine:

Duration of test, 1 hour.
Diameter of cylinder, 9·5 in.; stroke, 19 in.
Clearance volume, 270 c. in.
Circumference of brake-wheel, 12 ft. 9 in.
Load on brake (i.e. W — S), 275 lb.
Mean speed, 227 r.p.m.
Number of explosions per minute, 77.
Mean effective pressure, 105 lb./sq. in.
Gas used, 438 c. ft.
Calorific value of gas, 330 C.H.U. per c. ft.
Cooling water, 1380 lb., raised 34·2° C.

(*a*) *B.H.P.*

Energy absorbed by brake

$$= 275 \times 12{\cdot}75 \text{ ft. lb. per } rev.$$
$$= 275 \times 12{\cdot}75 \times 227 \text{ ft. lb. per } min.$$

$$\therefore \text{ B.H.P.} = \frac{275 \times 12{\cdot}75 \times 227}{33{,}000} = 24{\cdot}12.$$

(*b*) *I.H.P.*

$$\text{Work done per cycle} = \left(105 \times \frac{\pi}{4} \times 9{\cdot}5^2\right) \times \frac{19}{12} \text{ ft. lb.}$$
$$= 11{,}790 \text{ ft. lb.}$$

The number of working cycles per minute

$$= 77.$$

$$\therefore \text{ I.H.P.} = \frac{11{,}790 \times 77}{33{,}000}$$
$$= 27{\cdot}5.$$

(*c*) *Mechanical Efficiency.*

$$\text{M.E.} = \frac{24\cdot12}{27\cdot5} \times 100 = 87\cdot7\%.$$

(*d*) *Indicated Thermal Efficiency.*

Gas supplied per hour = 438 c. ft.

Gas supplied per I.H.P. hour $= \frac{438}{27\cdot5} = 15\cdot93$ c. ft.

∴ Heat supplied per I.H.P. hour $= 15\cdot93 \times 330$
$= 5257$ C.H.U.

∴ Indicated Thermal Efficiency

$$= \frac{\text{Heat equivalent of 1 H.P. hour}}{\text{Heat supplied per I.H.P. hour}}$$

$$= \frac{1415}{5257} = \cdot2691, \text{ or } 26\cdot91\%.$$

(*e*) *Brake Thermal Efficiency.*

Since the mechanical efficiency is 87·7 %, the brake thermal efficiency is

$$0\cdot877 \times 26\cdot91\% = 23\cdot6\%.$$

Thus only 23·6% of the energy supplied is usefully delivered at the crankshaft.

(*f*) *The Air-standard Efficiency.*

We must first calculate the compression ratio.

Stroke volume $= \frac{\pi}{4} \times 9\cdot5^2 \times 19$
$= 1347$ c. in.

Clearance volume = 270 c. in.

∴ Volume of charge
before compression = 1617 c. in.

$$\therefore\ r = \frac{1617}{270} = 5\cdot987.$$

$$\therefore \text{ Air-standard efficiency } = 1 - \left(\frac{1}{r}\right)^{n-1}$$
$$= 1 - \left(\frac{1}{5\cdot987}\right)^{0\cdot4}$$
$$= 1 - 0\cdot4888$$
$$= 0\cdot5112, \text{ or } 51\cdot12\%.$$

Thus the Relative Efficiency, or " Efficiency Ratio " as some term it, is

$$\frac{0\cdot2691}{0\cdot5112} = 0\cdot5265, \text{ or } 52\cdot65\%,$$

i.e. the engine's performance is just over one-half of the theoretical ideal.

We can now calculate the heat loss due to cooling, and then draw up a " heat balance ".

(*g*) Heat carried away by the cooling water per hour

$$= 1380 \times \text{rise of temperature}$$
$$= 1380 \times 34\cdot2 \text{ C.H.U.},$$

or, per I.H.P. hour

$$= \frac{1380 \times 34\cdot2}{27\cdot5} = 1716 \text{ C.H.U.}$$

Thus the percentage loss of heat by cooling

$$= \frac{1716}{5257} \times 100 = 32\cdot64\%.$$

Heat Balance:

Converted into indicated work	..	26·91%
Carried away in cooling water	..	32·64%
Exhaust, and radiation, losses (by difference)	..	40·45%
Total ..	..	100·00%

Another example, taken from the record of a full test of a four-stroke Diesel oil engine:

Duration of test	1 hour.
Diameter of cylinder, 12 in.; stroke, 18¼ in.	
Clearance volume	150 c. in.
Revolutions per minute	204.
Mean effective pressure	95 lb./sq. in.
Diameter of brake-wheel	9 ft.
Friction load (W — S)	174 lb.
Amount of oil used	14·3 lb.
Calorific value of oil	10,000 C.H.U.
Cooling water:	
Quantity flowing through jackets	1110 lb.
Inlet temperature	12° C.
Outlet temperature	53° C.

The air-blast for injecting the fuel into the cylinder was supplied by a compressor driven from the engine shaft.

We will assume the expansion ratio (not given) to be 6·25.

(*a*) $$\text{I.H.P.} = \frac{95 \times \left(\frac{\pi}{4} \times 12^2\right) \times \frac{18{\cdot}25}{12} \times 102}{33{,}000}$$
$$= 50{\cdot}47$$

(*b*) $$\text{B.H.P.} = \frac{174 \times 28{\cdot}27 \times 204}{33{,}000}$$
$$= 30{\cdot}4$$

(*c*) Mechanical Efficiency $= \dfrac{30{\cdot}4}{50{\cdot}35} \times 100$
$$= 60{\cdot}3\%.$$

(*d*) Heat supplied per I.H.P. hour
$$= \frac{14{\cdot}3 \times 10{,}000}{50{\cdot}35} = 2833 \text{ C.H.U.}$$

∴ Indicated Thermal Efficiency
$$= \frac{1415}{2833} \times 100 = 49{\cdot}94\%.$$

(*e*) Brake Thermal Efficiency

$$= 0{\cdot}603 \times 49{\cdot}94\%$$
$$= 30{\cdot}11\%.$$

(*f*) Stroke volume $= \frac{\pi}{4} \times 12^2 \times 18{\cdot}25$

$= 2063$ c. in.

Clearance volume $= 150$ c. in.

Total $= 2213$ c. in.

$$\therefore \text{ Ratio of compression, } r_c = \frac{2213}{150} = 14{\cdot}75$$
$$r_e = 6{\cdot}25.$$

Substituting these values of r_c and r_e in the air-standard efficiency formula, we obtain:

Air-standard efficiency $= 58{\cdot}31\%$.

$$\therefore \text{ Relative efficiency } = \frac{{\cdot}4994}{{\cdot}5831} \times 100$$
$$= 85{\cdot}65\%.$$

(*g*) Heat carried away by cooling water per I.H.P. hour

$$= \frac{1110 \times (53 - 12)}{50{\cdot}47}$$
$$= \frac{1110 \times 41}{50{\cdot}47} = 901{\cdot}8 \text{ C.H.U.}$$

$\therefore$ Percentage loss by cooling

$$= \frac{901{\cdot}8}{2833} \times 100 = 31{\cdot}83\%.$$

Heat Balance:

Converted into indicated work	..	49·94%
Carried away in cooling water	..	31·83%
Exhaust and radiation losses (by difference)	..	18·23%
Total ..	..	100·00%.

There is a point in the above calculation which calls for notice. The mechanical loss is 50·47 − 30·4 = 20·07 H.P. In the above engine, however, this loss includes the power used in driving the compressor, which might be, say, 6 per cent of the gross I.H.P. Now the air blast helps to do work in the engine cylinder, and the true or net I.H.P. derived from the energy of the fuel supplied is really less than that shown by the indicator diagram, in our example 50·47. If we suppose that the true I.H.P. is 6 per cent less than this figure, then the true or net thermal efficiency is only 0·94 of 49·94% = 47% nearly. And the real mechanical efficiency is higher than 60·3%, for strictly speaking—if we wish our result to indicate only frictional and similar losses in the mechanism—we ought to reckon it on the real or net I.H.P. Doing this we get

$$\frac{30{\cdot}4}{0{\cdot}94 \times 50{\cdot}47} \times 100 = 64{\cdot}2\%.$$

It is usual, however, to reckon the indicated thermal and the mechanical efficiencies as has been done above.

EXAMPLES VII

On Engine Performance

1. The following data refer to a series of tests made on a four-stroke single-acting oil engine built by the North Eastern Marine Engineering Co., Ltd. (See *The Motor Ship*, March, 1930.)

Dimensions: 6 cylinders, 24·4 in. diameter, 51·2 in. stroke.	
Duration of test	1 hour.
Mean r.p.m.	115·6.
Mean indicated effective pressure per cycle ..	154 lb./sq. in.
B.H.P. (by Heenan and Froude dynamometer)	2620
Quantity of fuel oil used	1066 lb.
Calorific value of oil	10,060 C.H.U.

Calculate the indicated horse-power mechanical efficiency, and the indicated and brake thermal efficiencies.

2. The above engine was tested at different loads, and the following results obtained:

I.H.P.	2145	2465	2935	3230
B.H.P.	1485	1845	2314	2620
Fuel per hr. (lb.)	612	740·5	926·5	1066.

Tabulate the fuel used per I.H.P. hour and per B.H.P. hour, the mechanical efficiency, and the indicated and brake thermal efficiencies, at each load. Plot curves, on a B.H.P. base, of I.H.P., mechanical efficiency, fuel per I.H.P. hour, and fuel per B.H.P. hour. In this example take the calorific value of the oil = 10,740 C.H.U.

3. In Question 1, what mean effective pressure in the cylinders would develop an indicated power equal to the actual brake power, i.e. 2620? Your result is termed the "Brake Mean Effective Pressure", abbreviated to B.M.E.P.

Calculate the B.M.E.P. in each of the four tests of Question 2, given that the indicated mean pressures (M.I.P.) in lb./sq. in. were, in order, 101, 118, 140, 154.

4. The following data were obtained in a test of a four-cylinder double-acting two-stroke oil engine.

Dimensions: 4 cylinders, 28 in. diameter, 40 in. stroke; piston rod 8·5 in. diameter.

Mean r.p.m.	96·4
Mean B.H.P.	2907.
Top of cylinder, M.I.P.	93·2 lb./sq. in.
Bottom of cylinder, M.I.P.	83·7 lb./sq. in.
Quantity of fuel used in 24 hr.	15·14 tons.
Calorific value of oil per lb.	9820 C.H.U.

Calculate (*a*) the total I.H.P.; (*b*) the mechanical efficiency; (*c*) the brake thermal efficiency.

Note: In calculating the I.H.P. developed in the lower part of the cylinder, allowance must be made for the presence of the piston rod, which reduces the effective area of the piston. (In connexion with this example, see *The Motor Ship*, May, 1926, p. 40.)

5. In a test of a six-cylinder double-acting two-stroke Diesel engine the I.H.P. and B.H.P. at full load were found to be 5320 and 4460 respectively. The quantity of oil, of calorific value 10,125 C.H.U. per lb., consumed per hour was 1798 lb., and during this time 14,720 gallons of cooling water were raised 40° C. in temperature. Draw up a simple percentage heat balance, showing (1) heat converted into effective work, (2) heat carried away in cooling water, (3) heat absorbed by friction and compressors, taking this to be the heat equivalent of (I.H.P. − B.H.P.), (4) heat loss in exhaust and by radiation, (5) heat in fuel. State also the mechanical and indicated thermal efficiencies.

6. The following results were obtained during a test of a gas engine loaded by a friction brake:

Cylinder diameter 8 in., stroke 17 in., dead weight 165 lb., spring balance reading 23·8 lb., brake-wheel diameter 5 ft., revolutions per minute 215, explosions per minute 98, m.e.p. of indicator card 82 lb. per square inch, gas per minute 7·16 c. ft. at 29·9 in. of mercury and 14·8° C., cooling water per minute 37·7 lb. raised 25·8° C., calorific value of gas 275 C.H.U. per cubic foot, measured at N.T.P.

Calculate the indicated and brake horse-powers of the engine, and find the mechanical and thermal efficiencies.

Draw up a heat balance sheet for the engine per minute. (Lond. Univ.)

7. Calculate the diameter and stroke of a gas engine which can develop 25 brake horse-power at 300 revolutions per minute, assuming a mechanical efficiency of 80 per cent, a mean effective pressure of 85 lb. per square inch, and a ratio of stroke to diameter of 1·5.

If the clearance volume is one quarter of the swept cylinder volume and the actual thermal efficiency (I.H.P. basis) is 55 per cent of the ideal efficiency, what will be the consumption per brake-horse-power-hour of gas having a calorific value of 275 C.H.U. per cubic foot? (Lond. Univ.)

8. A four-stroke gas engine has a piston diameter of 8·5 in., a stroke of 14 in., and runs at 250 revolutions per minute. Assuming a mean effective pressure of 82·5 lb. per square inch, what is the maximum horse-power the engine can develop, and what will be its gas consumption per minute if the engine has a mechanical efficiency of 85 per cent and consumes 22 cubic feet of gas per brake-horse-power-hour?

What is the thermal efficiency of this engine if the calorific value of the gas is 265 C.H.U. per cubic foot?

9. Explain the difference in method of working in oil engines working on the Otto and Diesel cycles, and illustrate this difference by reference to typical indicator diagrams.

10. The following figures relate to a test of an oil engine:

Duration of trial 30 min.; oil used, 9·72 lb. of calorific value 10,000 C.H.U. per lb.; cooling water, 718 lb. with a temperature rise of 39° C.; revolutions per minute, 204; area of mean indicator card, 1·15 sq. in.; length 2·8 in.; scale of spring $\frac{1}{200}$; rope brake dead weight, 305 lb.; spring balance reading, 43 lb.; diameter of brake-wheel, 6 ft.; diameter of cylinder, 12 in.; stroke, 18 in.

Calculate (*a*) M.E.P., (*b*) I.H.P., (*c*) B.H.P., (*d*) mechanical efficiency, (*e*) indicated thermal efficiency.

Draw up a percentage heat balance.

11. A four-stroke Diesel engine had six cylinders of diameter 15 in., stroke 30 in. The mean indicated pressure was 104·8 lb./sq. in., speed 165 r.p.m. The B.H.P. was 502. Calculate the indicated horse-power and the mechanical efficiency.

If the oil consumption was 0·4 lb. per B.H.P. hour, and cooling water was supplied at the rate of 3·5 gallons per B.H.P. hour, the temperature rise being 35° C., what percentage of the heat supplied was carried away by the cooling water?

Calorific value of oil = 10,000 C.H.U. per lb.

12. An oil engine has a compression ratio of 10 : 1. It burns 0·4 lb. of oil per B.H.P.-hour. The oil has a calorific value of 10,330 C.H.U. per lb. What is the thermal efficiency of the engine, and what is the efficiency of the corresponding air cycle? (Lond. Univ.)

13. The mean indicated pressures in the six cylinders of one of a pair of Diesel engines for a 2050-ton vessel were 113·2, 103·3, 105·5, 119·6, 102·0, and 108·9 lb./sq. in. Speed 168·1 r.p.m. Cylinders 15 in. diameter by 30 in. stroke. The engine was of the four-stroke type. Find the average M.I.P. and compute the I.H.P.

14. In a 3-hour test of a Camellaird-Fullagar oil engine, the I.H.P. and B.H.P. were found to be 1526 and 1126 respectively. The indicated horse-power of the air-compressor was 97·2, and of the scavenge pumps 87·6. Calculate the mechanical efficiency of the plant (*a*) in the usual way, i.e. B.H.P./I.H.P.; (*b*) deducting the air-compressor I.H.P. from the power cylinders I.H.P.; (*c*) deducting also the scavenge power.

Calculate the indicated thermal efficiency, given that the weight of fuel oil, of calorific value 10,145 C.H.U. per lb., used per hour was 479·2 lb.

15. If, in the engine of Question 14, the cylinder jacket water and piston cooling oil carried away together 19,920 C.H.U. per minute, what percentage of the heat supplied by the fuel was lost in the exhaust gases and by radiation?

16. Calculate the B.H.P. of:

(*a*) A Werkspoor four-stroke engine, given: Number of cylinders, 6; bore 23 in., stroke 41 in.; r.p.m. 125; "brake" mean effective pressure, 77 lb./sq. in.

(*b*) A Sulzer two-stroke engine, given: Number of cylinders, 4; bore 23·6 in., stroke 37 in.; r.p.m. 102; brake M.I.P., 76·3 lb./sq. in.

(*c*) A Vickers four-stroke engine, given: Number of cylinders, 6; bore 24·5 in., stroke 39 in.; r.p.m. 118; brake M.I.P., 76 lb./sq. in.

17. Calculate the indicated thermal efficiencies of engines A and B, given:

A.—B.H.P., 1840; mechanical efficiency, 73·8 per cent; oil per B.H.P.-hour, 0·386 lb. of calorific value 10,000 C.H.U. per lb.

B.—B.H.P., 1760; mechanical efficiency, 78 per cent; oil per B.H.P.-hour, 0·42 lb. of calorific value 10,000 C.H.U. per lb.

18. An eight-cylinder four-cycle single-acting oil engine was tested at different loads, with the following results:

Load	1/2	3/4	1/1	Overload
B.H.P.	1410	2120	2825	3040
I.H.P.	2261	2982	3731	4099
Oil per B.H.P.-hr. (lb.)	0·48	0·439	0·436	0·443
Exhaust temp. °C. ..	326	384	450	470

Take the calorific value of the oil per lb. as 10,000 C.H.U., and tabulate the indicated thermal and mechanical efficiencies. Show all your results, including the above, graphically, on a B.H.P. base. Write down a brief general observation on the results of the trial.

19. If, in the engine of Question 18, the bore of the cylinders is 29·14 in., stroke 46·17 in., calculate the mean indicated pressure in the cylinders at full load, the speed then being 125 r.p.m. What is the " brake " M.E.P.?

20. Derive all the information you can from the following record of an hour's test of a four-stroke gas engine:

One cylinder, bore 7·5 in., stroke 12 in., clearance volume 140 c. in.

Mean effective pressure 78 lb./sq. in.; mean number of explosions per minute, 116; mean r.p.m. 240.

Diameter of brake-wheel, to rope centre, 60·5 in.; dead weight, 100 lb.; spring balance reading, 14 lb.

Gas used 190 c. ft. measured at N.T.P., of calorific value 285 C.H.U. per c. ft.

Cooling water 290 lb.; inlet temp. 12° C.; outlet temp. 60° C.

Miscellaneous Examples

1. Find the volume of 2 lb. of air when at a pressure of 120 lb./sq. in. and temperature 80° C.

(Take $C_v = 0{\cdot}169$, $C_p = 0{\cdot}238$.)

2. The temperature and pressure of 8 c. ft. of air are 72° C. and 45 lb./sq. in. respectively. What will the volume become when the temperature and pressure are changed to 144° C. and 90 lb./sq. in.?

3. In a certain oil engine the piston displacement is 0·395 c. ft. and the volume of the clearance space 0·210 c. ft., and the pressure of the charge at the instant compression begins is 13 lb./sq. in. Find the compression pressure and the temperature reached at the end of the compression stroke if the temperature of the charge at the instant compression began was 129° C. Assume the law of compression to be $pv^{1\cdot39}$ = constant. (Lond. Univ.)

4. If 1 lb. of air is expanded from an initial pressure of 200 lb./sq. in. and temperature 1100° C. to a final pressure of 40 lb./sq. in. (*a*) adiabatically, (*b*) by a process defined by the equation $PV^{1\cdot2}$ = constant, calculate the final temperature and the work done in each case. (Lond. Univ.)

5. Explain why compressing a gas adiabatically warms it, and expanding it adiabatically cools it.

6. Find the difference between the work done in compressing 4 c. ft. of air at 15 lb./sq. in. to a volume of 1 c. ft. adiabatically and isothermally. You may use a graphical method if you wish.

7. *Air at a temperature of 15° C. is compressed in a cylinder from* 15 lb./sq. in. pressure to 120 lb./sq. in. The equation of the compression curve is $PV^{1\cdot25} = C$. Find the work done in compressing a pound of air, and the heat that escapes through the cylinder walls.

(Lond. Univ.)

8. An internal-combustion engine has the following dimensions: diameter of cylinder 22 in., stroke 30 in., compression ratio 13·5. At the end of the suction stroke the pressure is 14 lb./sq. in and the temperature 43° C. Compression follows the law $PV^{1\cdot37} = C$. Determine (*a*) the pressure and temperature at the end of compression; (*b*) the weight of the charge; (*c*) the work done; and (*d*) the heat rejected during compression.

Assume $C_p = 0{\cdot}238$, $C_v = 0{\cdot}169$.

(Lond. Univ.)

9. Taking R = 96, determine the temperature of 1 lb. of air if its volume is 5 c. ft. and its pressure 50 lb./sq. in.

If to this mass of air 126 C.H.U. of heat are added at constant volume, what will the temperature and pressure become? Take $C_v = 0{\cdot}18$.

10. Explain why the specific heat of a gas at constant pressure is greater than at constant volume.

If R for air is equal to 96, find the density of air at 0° C. and 14·7 lb./sq. in., measured in pounds per cubic foot.

11. Calculate the ideal efficiency of a Diesel engine with a compression ratio 14 and fuel cut off at $\frac{1}{10}$ stroke. If the efficiency ratio for the engine is 0·72 at full load, find the oil consumption per horse-power-hour when using oil of calorific value 10,000 C.H.U. per pound.

What change in efficiency ratio would you expect as the load on the engine is reduced? (Lond. Univ.)

12. Prove that the ideal efficiency of an internal-combustion engine working on the Otto cycle is $1 - \left(\frac{1}{r}\right)^{\gamma-1}$, where r is the ratio of *compression. Calculate the efficiency in the case of an engine* having a stroke 16 in., a piston diameter 12 in., and a clearance volume of 485 c. in.; and find the gas consumption per indicated horse-power-hour if the gas has a calorific value of 260 C.H.U. per c. ft. and the efficiency ratio of the engine is 56 per cent. (Lond. Univ.)

13. Without using the usual efficiency formula in terms of the compression and expansion ratios, calculate the ideal efficiency of a Diesel engine in which (*a*) $r_c = 12, r_e = 6$; (*b*) $r_c = 12, r_e = 10$.

Assume that the pressure and temperature at the commencement of compression are 15 lb./sq. in. and 100° C. respectively.

14. Ten cubic feet of air at a pressure of 100 lb./sq. in. and a temperature of 50° C. are allowed to expand to a volume of 30 c. ft. If the law of expansion is $PV^{1{\cdot}2}$ = constant, find the final temperature and pressure. Find also from first principles the amount of heat absorbed from an external source during the expansion. Take $C_v = 0{\cdot}17$, and R = 96. (Lond. Univ.)

15. A cubic foot of a mixture of air and carbon-dioxide at a pressure of 300 lb./sq. in. expands adiabatically to a volume of 3 c. ft. Assuming that the ratio of the specific heats for this mixture remains stationary at 1·35 over the whole range of expansion, calculate the work done during expansion.

Deduce any expression you may use. (Lond. Univ.)

16. The density of CO_2 at 0° C. and 14·7 lb./sq. in. pressure is 0·124 lb./c. ft., and its specific heat at constant pressure is 0·216. A quantity of this gas is compressed adiabatically from a condition of 15° C. and 13 lb./sq. in. to a pressure of 50 lb./sq. in. What is the final temperature? (Lond. Univ.)

17. A quantity of air is compressed adiabatically in a cylinder. Calculate the ratio of expansion if (*a*) the pressure rises from 14 lb./sq. in. to 84 lb./sq. in.; (*b*) the temperature rises from 50° C. to 200° C.

18. A gas engine working on the Otto cycle has a cylinder 8·5 in. diameter × 18 in. stroke. The clearance volume is 0·2467 c. ft. The weight of mixed gas and air per stroke is 0·0484 lb. and its temperature at the end of the suction stroke is 62·8° C. at a pressure of 13·8 lb./sq. in. abs. The absolute pressures are: at the end of compression, 67·8 lb./sq. in.; at the end of constant volume explosion, 240 lb./sq. in. The volume at the end of constant pressure expansion is 0·2617 c. ft.

Find: (1) The temperatures at the end of (*a*) compression, (*b*) explosion, (*c*) constant pressure expansion. (2) Heat added as shown by the diagram, (*a*) during explosion, (*b*) during constant pressure expansion. $C_p = 0{\cdot}26$, $C_v = 0{\cdot}19$. (Lond. Univ.)

19. A Diesel engine has a relative efficiency of 0·55 on the brake. If the compression ratio is 13·8, and the expansion ratio 7·4, and the lower calorific value of the oil is 10,500 C.H.U., find the consumption of oil in pounds per B.H.P.-hour. (Lond. Univ.)

20. An ideal air engine works on the following cycle: air is taken in at 15 lb./sq. in. and temperature 17·2° C., and is compressed adiabatically, the pressure at the end of the stroke being 500 lb./sq. in. Heat is taken in at constant pressure, and expansion afterwards takes place adiabatically, the ratio of expansion being 5. The air is exhausted at the end of the stroke, the heat assumed to be rejected at constant volume. Find the efficiency. Take the specific heats of air $C_p = 0{\cdot}238$, and $C_v = 0{\cdot}17$. (Lond. Univ.)

Answers to Examples

Examples I

1. 259 c. in. **2.** 5·084 c. ft. **3.** 63·62 lb./sq. in. **4.** 492·2 c. ft. **5.** 277·9° C. **6.** 1211° C. **7.** 533·9° C. **8.** 62·84. **9.** 178·5 c. ft. **10.** 367·5 c. ft. **11.** 0·063 c. ft.; 1·5 c. ft. **12.** 356 c. in. **13.** 2·183 c. ft. **14.** 88·2 lb./sq. in.; −38·7° C. **15.** 1387.

Examples II

1. 258 C.H.U.; 361,200 ft. lb. **2.** 89·3° C. **3.** 17,280 ft. lb.; 12·34 C.H.U. **4.** 240 C.H.U. **5.** 123·8 c. ft.; 169·1 c. ft.; 68·5 C.H.U. **6.** (*a*) 18·78 C.H.U.; (*b*) 26·52 C.H.U. **7.** 5·383 c. ft.; 548·4° C.; 101·7 lb./sq. in. **9.** C_v = ·1793; 100·3 C.H.U. **10.** 0·01307 C.H.U.; 18·3 ft. lb. **11.** 2584° C. **12.** 36 C.H.U. **13.** 0·1238 lb. **14.** 0·0782 lb.

Examples III

1. 13·06 c. ft.; 140 lb./sq. in.; 275·7° C. **2.** 62,525 ft. lb. **3.** 45·09 C.H.U.; 63,130 ft. lb. **4.** 73·5 lb./sq. in. **5.** 38·94 lb./sq. in.; 79,870 ft. lb. **6.** 178·5° C. **7.** (*a*) 270 lb./sq. in.; 492° C.; (*b*) 117·6 lb./sq. in.; 60° C. **8.** 44·58 lb./sq. in.; 1232° C.; 10,690 ft. lb. **9.** (*a*) 0·6933 lb.; (*b*) 279·4 lb./sq. in.; 331·5° C.; (*c*) 66,920 ft. lb.; (*d*) 12 C.H.U. rejected. **10.** C_v = 0·1714. **11.** 96; 1·41; 14·9 c. ft. **12.** 178·5° C. **13.** 1633° C. **14.** (*a*) 493·5 lb./sq. in.; 527·7° C.; (*b*) 0·46 lb.; (*c*) 62,220 ft. lb.; (*d*) 6·7 C.H.U. **15.** 87·06, 53·52, 37·88, 28·98 lb./sq. in.; P_m = 144 × 68·6 lb./sq. ft.; W = 39,520 ft. lb. (Author's results, using average care.) **16.** W = 39,680 ft. lb. **17.** 100, 66·7, 50, 40 lb./sq. in.; P_m = 144 × 79·8 lb./sq. in., W = 45,970 ft. lb. (Author.) By calculation, W = 46,340 ft. lb.

Examples IV

1. 1415 C.H.U. **2.** 16 c. ft.; 24·6%. **3.** 34·9%. **4.** 34·2%. **5.** 31·8%. **6.** 0·655 lb.

Examples V

1. ·356, ·426, ·475, ·512, ·541, ·565, ·602, ·661.

2.	Pressures (lb./sq. in.)	14	97·5	247·6	35·55
	Temperatures °C.	100	376·4	1376	674·1

3. Eff. = 0·4259. **4.** (*a*) −14,940 ft. lb.; (*b*) + 37,950 ft. lb.; Net work = 23,010 ft. lb. **5.** Indic. eff. = 0·3145; Air-std. eff. = 0·4682; relative eff. = 0·6716. **6.** 0·5439; 14·45 c. ft. **7.** *r* = 3·055. **8.** 0·3603. **9.** 634° C. **10.** 42,000 ft. lb.; 1955° C.; 430·3 lb./sq. in. **11.** (*a*) 0·5986; (*b*) 0·6093; 0·6469. **12.** (*a*) 8·584; (*b*) 12·11. **13.** (*a*) 255·5 lb./sq. in.; 5716 ft. lb.; (*b*) 405·8 lb./sq. in.; 7450 ft. lb.

14.	Pressures (lb./sq. in.)	14	507·7	507·7	37
	Temperatures °C.	80	711·9	1697	659·6

15. (*a*) 234·4 C.H.U.; (*b*) 98·54 C.H.U.; 135·9 C.H.U.; eff. = 0·58. **16.** 0·58. **17.** (*a*) −117,200 ft. lb.; (*b*) +73,100 ft. lb.; (*c*) +192,300 ft. lb. Net work = 148,190 ft. lb.; M.E.P. = 85·8 lb./sq. in.

18.	Pressures (lb./sq. in.)	14	507·7	507·7	27·63
	Temperatures °C.	80	711·9	1327	423·6

Efficiency = 0·6010; net work = 96,193 ft. lb.

Examples VI

1. 73·5 lb./sq. in. **2.** 4·285; 3·642. **3.** 85·2 lb./sq. **4.** 91·22; 80·6%. **5.** 292·7. **6.** 54·5. **7.** 1283 C.H.U. **8.** 21·4. **9.** 3·784 galls. **10.** 77·2; 62·1; 80·4%. **11.** 25; 9·566 in.

12.	Mech. eff.	0	44·7	61·8	76·07	78·3.
	F.H.P.	6·2	6·3	6·5	6·7	6·78.

13. 15·37; 12·43; 80·9%. **14.** 46·7; 33·4; 71·5%.

Examples VII

1. 3230; 81·1%; 42·6%; 34·6%.

2.

Oil per I.H.P. hr. (lb.)	0·29	0·3	0·32	0·33
Oil per B.H.P. hr. (lb.)	0·412	0·401	0·4	0·407
Mech. eff. per cent	69·2	74·8	78·8	81·1
Ind. th. eff. per cent	46·3	43·8	41·7	39·8
Brake th. eff. per cent	32	32·8	32·9	32·3

3. 125 lb./sq. in.; 69·9; 88·2; 110·3; 125 lb./sq. in. **4.** (*a*) 4058; (*b*) 71·6%; (*c*) 29·64%. **5.** (i) 34·64%; (ii) 32·32%; (iii) 6·68%; (iv) 26·36%; (v) 100%.

Mech. eff. = 83·8%; Ind. th. eff. = 41·32%.

6. I.H.P. = 17·4; B.H.P. = 14·5

Mech. eff. = 83·5%; Th. eff. = 22%.

Heat balance:—

	C.H.U.	%
Heat converted into work/min.	410	22
Heat carried away in cooling water per minute	973	52
Heat lost in exhaust gases, by radiation, &c. per minute	483	26
Heat supplied per minute	1866	100

7. Dia. 9·38 in., stroke 14·07 in.; 24·6 c. ft. **8.** 20·68 H.P.; 6·446 c. ft./min.; 28·55% (ind.). **10.** (*a*) 82·13 lb./sq. in.; (*b*) 42·97; (*c*) 30·52; (*d*) 70·84%; (*e*) 31·27%.

Heat converted into work	31·27%
Heat carried away by cooling water ..	28·80%
Exhaust and other losses	39·93%
Heat supplied ..	100·00%

11. 694·2; 72·31%; 30·63%. **12.** 34·25%; 60·2%. **13.** 735 I.H.P. **14.** (*a*) 73·8%; (*b*) 79%; (*c*) 84%; 44·1%. **15.** 31·3%. **16.** (*a*) 1242; (*b*) 1260; (*c*) 1249. **17.** A: 49·7%; B: 43·2%.

18. Mech. eff.: 62·3 70·8 75·7 74·2 per cent.

Th. eff.: 47·3 45·4 45·4 43·1 per cent.

19. 95·9 lb./sq. in.; 72·6 lb./sq. in. **20.** I.H.P. = 12·12; B.H.P. = 9·91; mech. eff. = 81·8%; indic. th. eff. = 31·64%; brake th. eff. = 25·89%; air-standard eff. = 46·54%; efficiency *ratio* = 67·98%.

Heat converted into ind. work.. ..	31·64%
Heat carried away by cooling water ..	25·70%
Exhaust and other losses	42·66%
Heat supplied	100·00%

ANSWERS TO MISCELLANEOUS EXAMPLES

1. 3·947 c. ft. **2.** 4·835 c. ft. **3.** 56·58 lb./sq. in.; 334·3° C. **4.** (*a*) 594° C.; 121,680 ft. lb.; (*b*) 777° C.; 155,200 ft. lb. **5.** See text. **6.** 4050 ft. lb. **7.** 57,150 ft. lb.; 15·3 C.H.U. **8.** (*a*) 495 lb./sq. in.; 555° C.; (*b*) 0·471 lb.; (*c*) 62,900 ft. lb.; (*d*) 4·185 C.H.U. **9.** 102° C.; 802° C.; 143·3 lb./sq. in. **10.** 0·08076 lb. **11.** 0·6114; 0·3215 lb. **12.** 0·463; 21 c. ft. **13.** (*a*) 0·567; (*b*) 0·615. **14.** −13·7° C.; 51·1 C.H.U. **15.** 39,410 ft. lb. **16.** 108·4° C. **17.** (*a*) $r = 3·596$; (*b*) $r = 2·595$. **18.** (1) (*a*) 213° C.; (*b*) 688° C.; (*c*) 714·4° C.; (2) (*a*) 4·37 C.H.U.; (*b*) 0·37 C.H.U. **19.** 0·408 lb. **20.** 0·547.

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