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PRACTICAL PHYSICS

BY

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IN COLLABORATION WITH

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PREFACE.

In this handbook of Practical Physics various exercises in the measurement of physical quantities are described. These are adapted to the capabilities of matriculated students, and are therefore more advanced than those of the First Courses of Physics followed in the junior classes of schools. It has, however, been assumed that the student has done no previous work in Practical Physics, and hence the handbook is a complete elementary manual of the subject.

The general course deals with work for which the student should attend a well-equipped laboratory. The experiments printed in small type are more advanced or less important than those in ordinary type. The descriptions of apparatus and methods have been generalised rather than referred to special instruments and conditions. It is hoped that this will not unduly increase the initial difficulties, and that when these are overcome the student will have really gained the power of tackling unfamiliar varieties of work, and of adapting himself to novel conditions. If students do not acquire this flexibility their practical work loses considerably in value as an educational instrument.

No attempt has been made to divide the subject-matter into lessons of equal length, and the chapters are moderately independent of one another. The order of experiments is not essential. It is an advantage to many students to change the subject frequently, and to make a broad acquaintance with the important parts of each before going into detail in any one. The order of the practical work should, however, be largely influenced by the sequence of the theoretical lessons.

Most of the exercises are arranged to be done by students working singly; partnerships are generally inadvisable.

Certain experiments in the book may be readily performed at home at a trifling cost; these have been indicated by an asterisk, and form a course which should prove especially useful to those whose opportunities for work in a well-equipped laboratory are limited. A student who conscientiously and intelligently works through this Home Course will gain experience and knowledge that will greatly raise the character and standard of his laboratory work. In the laboratory he will be able to omit many of the usual experiments with simple apparatus, and devote himself to those that more especially require the guidance and criticism of a teacher and costly instruments. These home experiments are, however, a definite part of the whole course, and if not done elsewhere should be performed in the laboratory. (See Introduction.) In any case it is not anticipated that a student will work through the whole course of 266 experiments. Some of these are alternative to others: the teacher should select those suited to the requirements and accomplishments of the student.

The Appendix contains a table of four-figure logarithms and some lists of physical constants. No instructions in the use of logarithms or of squared paper have been given; these matters should be dealt with in a Mathematical or tutorial class.

Great care has been taken to provide suitable and instructive illustrations. Many of these are printed from blocks which have been kindly lent by the following firms:—Messrs. J. J. Griffin and Co. (Figs. 26, 71, 78, 106, 140, 175, 179, 230), Harvey and Peak (Fig. 27), Nalder Bros. (Figs. 209, 218, 235, 236, 237), R. W. Paul (Figs. 32, 200, 208), W. G. Pye and Co. (Figs. 28, 40, 202, 207, 217, 232, 234), W. Wilson (Figs. 29, 131).

I venture to draw attention here to certain special features of the book. In the Home Course the cost of apparatus has been kept very low, but trivial experiments have been avoided. The construction, calibration, and standardisation of the set of masses (Exps. 1, 11, 12), and the experiments on the Principle of Archimedes (Exps. 28, 29, 30), demonstrate very important principles. In the measurement of mass by coarser balances it is recommended that fractions of a gramme should be obtained by placing a one-gramme rider on the graduated balance beam: it is then only necessary to supply a set of masses ranging from one gramme upwards, and no pliers need be used. Each mass should be arranged to fit a hole in a wooden block: the teacher is then able to see at a glance whether any have gone astray. In the experiments with converging lenses emphasis is laid on the fact that there is a minimum distance between object and image. The simple methods and apparatus used in the earlier experiments in Electrostatics will not be found to make them less effective than the more usual arrangements. In current Electricity the earlier measurements are electrolytic rather than electromagnetic: this has permitted an early demonstration of Ohm's law (Exps. 238, 239). The value of the reflecting galvanometer as a current measurer has been emphasised and some important practical difficulties incidental to the tangent galvanometer have been indicated. The use of the latter by inexperienced students is not recommended; the reflecting galvanometer gives better results in a simpler manner. Several original methods and modifications of apparatus have been introduced into the text: it is hoped that these will be found valuable.

I gratefully record my obligations to my collaborator, Mr. J. Satterly, B.Sc., and to the following, all of whom have given valuable advice, criticism and assistance in preparing the work: Messrs. C. W. C. Barlow, M.A., B.Sc., University Tutorial College, London; (Mrs.) M. W. Bower, B.A.; G. H. Broom, B.Sc., A.R.C.Sc, Technical College, Derby; A. Griffiths, D.Sc., A.R.C.Sc. Birkbeck College, London; and J. Stephenson, B.Sc., A.R.C.Sc.

WILLIAM R. BOWER.

TECHNICAL COLLEGE,
HUDDERSFIELD,
February 1906.

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INTRODUCTION.

1. Qualitative and Quantitative Experiments.—An experiment may be *qualitative*, that is, it may be restricted to the observation of the sequence and relations of phenomena, or it may be *quantitative*, that is, require the measurement of some quantity involved. The facts, for instance, that water when sufficiently heated becomes steam, that a stone returns after being thrown up, that matter has properties like hardness, fluidity, colour, etc., are qualitative; exact measurement is not necessary to gain such information. But the volume, weight, etc., of a body are determined by comparison with units of volume, weight, etc. The information then is quantitative; to obtain it careful measurement is essential.

2. Measurement of Quantities.—In ordinary affairs a distance is measured by applying a foot rule or yard tape, a period of time is compared with a clock or watch indicating hours, minutes, etc., a volume of milk is delivered in pints and quarts, a quantity of coal in tons and hundredweights. In these cases the *physical quantities*, length, duration, volume, weight, are being dealt with, and the nature of the body or substance is not under consideration. Also each quantity is measured by stating the number of times it contains a particular magnitude of its own kind; *e.g.* a length of 100 yards, a delay of 35 minutes, 1½ pints of milk, 2 tons of coal. (See Appendix for a list of Physical Quantities, etc.)

Every quantity is measured by comparing it with a special or selected magnitude, called the **unit**, *of its own kind. The* **ratio** *of the quantity to its unit is called the* **measure** *or* **numerical value** *of the quantity.*

To determine the conditions under which the comparison of a quantity with its unit may be accurately made is one of the important aims of Practical Physics.

Specification of a Quantity.—Consider the following :—

1. Quantity—a *length*, say 10 yards :

When the *unit* of measurement is the	Yard	Foot	Inch	**Mile**
then the *numerical value* is	10	30	360	$\frac{10}{1760}=0{\cdot}0057$

2. Quantity—a *period of time*, say $2\frac{1}{2}$ days :

When the *unit* is the	Day	Hour	Minute	Week
then the *numerical value* is	2·5	60	3600	2·5/7 = 0·36.

3. Quantity—a *speed*, say 30 miles per hour :

When the *unit* is	1 mile per hr.	1 yd. per min.	1 ft. per sec.
then the *numerical value* is	30	880	44.

Thus the *numerical value of a quantity depends upon the unit measuring it, the larger the unit the smaller being the numerical value.* Hence **to specify a quantity** two data must be mentioned: (i) The *numerical value* or number of units in the quantity; (ii) The *name of the unit measuring it.*

The numerical value of a certain quantity is generally deduced by calculation from the data obtained by measuring quantities directly or indirectly associated with it. For instance, the volume of a body may be measured directly by the use of a graduated vessel, indirectly by finding its dimensions in certain directions, or its mass and density, or by Archimedes' Principle.

3. **Two systems of units.—I. British Imperial Weights and Measures.**—These are based upon three standards kept at the Standards' Office of the Board of Trade, Westminster.

The Imperial Standard **Yard** is the distance at 62° F. between the centres of two gold plugs in a certain bronze bar.

The Imperial Standard **Pound** is the mass of a certain piece of platinum.

The Imperial Standard **Gallon** is a brass measure that contains ten imperial standard pounds of distilled water, weighed in air with brass weights, when the water and the air are at 62° F., and the mercurial barometer, 30 inches high.

The important multiples and submultiples and their relations are shown in the Appendix.

II. **Metric Weights and Measures.**—These are based upon two standards kept in the Archives at Paris.

The Standard **Metre** is the distance between the ends of a certain rod of platinum when its temperature is that of melting ice.

The Standard **Kilogramme** is the mass of a certain piece of platinum.

It was originally intended that the Metre should be the ten-millionth part of a quadrantal arc on the Earth's surface, and that the mass of 1 Kilogramme should be that of 1000 cubic centimetres of water at 4° C. These values were not exactly realised in the standards constructed.

The important multiples and submultiples and their relations are shown in the Appendix.

The British and Metric systems of weights and measures are the most important with which we have to deal. Our scales for measuring length are copies, more or less exact, of the standard yard or metre or their subdivisions; and the "sets of weights" are copies of the standard pound or kilogramme or their subdivisions.

Time.—The standard interval of time, called the **Day**, is the period during which the Earth turns once upon its axis.

The *Solar Day* is the interval between the moment when the sun crosses the southern meridian of the place of observation and the moment when it next crosses (also called the *Apparent Solar Day*).

The *Sidereal Day* is the interval between two successive transits of the same fixed star across the southern meridian of the place of observation.

Definition.—The **Mean Solar Day** is the average value of the consecutive Solar Days throughout a year.

Definition.—The **Second**, or **Mean Solar Second**, is the $\frac{1}{86400}$th part of the mean solar day.

The sidereal day is very approximately equal to the standard day. All sidereal days have the same length. Solar days are longer than sidereal, and are not equal in length.

The length of the mean solar day is practically invariable. Ordinary clocks and watches are adjusted to indicate mean solar seconds.

4. **Fundamental and Derived units.**—Although each physical quantity is measured by a unit of its own kind it is found that most of these can be expressed in terms of units of three kinds of quantity only. The three quantities **length, mass**, and **time**, form a very convenient basis for a system of units. When so adopted they are regarded as *fundamental quantities* and their units are called *fundamental units.* Those of other quantities that can be defined in terms of the fundamental are called *derived units.*

From the fundamental unit of length, the foot, the derived unit of volume, the cubic foot, is obtained. Also the unit of speed, one foot per second, is derived from the fundamental foot and second, etc. (See Appendix.)

The position of length, mass, and time as fundamental is not essential but convenient. On the Continent, length, weight, and time are adopted. Some quantities, like candle-power and temperature, cannot be measured in terms of the fundamental units, but require independent ones. (See Appendix.)

Absolute Units and Measurements.—The various multiples and sub-multiples are adopted, the larger units to measure greater, and the smaller units lesser magnitudes, in order that the numerical value of the quantity may be a number neither too large nor too small for convenience. But in scientific work the results of measurement, whether the quantity is large or small, are, for the sake of uniformity, very frequently specified in terms of one unit only of each fundamental quantity. The particular units chosen are called **Absolute Units**, and a quantity measured in terms of them is said to be expressed in **Absolute Measurement.** There are two systems of absolute units, (1) the **C.G.S.** or **Centimetre-Gramme-Second**, (2) the **F.P.S.** or **Foot-Pound-Second.** (See Appendix.)

Practical and Engineers' Units.—In Engineering a strict adherence to Absolute Units has been found inexpedient. Certain *Practical Units* are used in electrical work: these are multiples or sub-multiples of the C.G.S. units. In Mechanical Engineering the **Gravitation Unit of Force, the Pound Weight,** is frequently adopted as a fundamental measure. For particulars see Appendix.

5. Calculation of Results.—Use 4-figure logarithm and anti-logarithm tables (see Appendix). A slide rule is also useful for the less exact calculations. Express all fractions as decimals. Do not carry arithmetical calculations further than is justified by the closeness of the measurement.

For instance, suppose a length measured by a scale, graduated in sixteenths of an inch, is found to be 44 whole divisions and a part estimated as three-tenths, that is, the measurement is 44·3 scale divisions. Now $44{\cdot}3 \div 16 = 2{\cdot}76875$. Practically the result is expressed as 2·77 inches. Had the estimate of the fraction of the scale division been two-tenths, the result ($44{\cdot}2 \div 16 = 2{\cdot}7625$) would have been expressed as 2·76 inches. Thus there is a difference in the result of 0·01 inch consequent upon a difference in the estimate of one-tenth of a scale division. Hence as one-tenth of so small a division cannot be safely judged, it is absurd to express the result in this case to more than two places of decimals.

Generally speaking, work out to one place beyond that which denotes the percentage of accuracy that is anticipated; if, for instance, a result should be correct to one per cent. of the whole, then the calculations should be taken to one-tenth per cent.

When the calculations have been performed we obtain a series of numbers representing the numerical value of the same quantity. Each having been obtained by an independent observation should be equally trustworthy. All will be nearly equal, none, owing to unavoidable errors of experiment, is necessarily the true value. The *arithmetic mean* of all the values obtained is likely to be the best measure of the quantity.

Hence, after taking a series of observations, first apply the proper formulae of *correction* and *reduction* and calculate the respective numerical values of the magnitude concerned; next *find the mean* by adding together all the values and dividing by the number of them.

6. To conduct an Experiment.—Read the instructions and examine the apparatus provided, *note what special precautions are necessary in using it.* Conduct the operations deliberately, methodically, and conscientiously. Be prepared to take all necessary trouble. Especially do not be satisfied with merely "knowing the principle" of an experiment. A single result to an experiment is generally insufficient; repeat it several times until there is satisfactory agreement between the values obtained. If possible the repetitions should not be precisely similar—vary the quantity of material, size of body, measure with different parts of a scale, or use another instrument, etc.

Record observations, calculations, and results in a laboratory note-book, in such a manner that everything may, if necessary, be read and understood at a later date. Avoid using loose sheets or odd pieces of paper. Make a note of observations *when* and *exactly as* they are made. Apply corrections, etc., afterwards, and show clearly in the record the manner in which they have been introduced. Put down *all* observations even though some may be numerically the same. The calculated mean is more satisfactory when it is obtained by taking account of a number of observations agreeing very closely in value, provided that each has been made conscientiously and *without bias,* and that the conditions of the experiment have been varied when possible.

Make a liberal use of simple diagrams and sketches, tabulate data and plot results when possible. Plan these to as large a scale as the paper permits.

Indicate by their reference number or description the particular instruments used. Give descriptions of special pieces of apparatus, novel methods, etc. Record the place, date, and time at which the experiment is performed. In many experiments mention also the temperature of the room, the height of the barometer, and, especially in magnetic experiments, the bench or position at which the observations are made.

The object of the laboratory note-book is thus to give a full account of an experiment and the results obtained. Highly important parts of the laboratory work, viz. those of the adjustment of apparatus and preparation of material cannot be recorded. The student should, however, give very careful attention to these; he should regard "results" not as the real object of the experiment but the test of the care and skill exercised in performing the whole of the operations. In this respect an experiment requiring skilful manipulation and yielding comparatively poor results may be of a higher order than one that can be done by the mere reading of the indications of elaborate instruments. The time spent in "getting things ready" is by no means wasted.

Home experiments.—A careful record of these should be kept and submitted to the criticism of the Superintendent of the laboratory. (See Preface and p. 8.)

Laboratory Note-book.—The following type is recommended: size of page, 9″ × 7″, both sides of page ruled by *faint* lines (blue) into squares, distance between lines, 1/5″, or 1/10″, or 1/5 cm., every *fifth* line a little more prominent than the others.

Use the left page to record, in pencil, the observations immediately they are made, and the necessary calculations, also to *plan* diagrams, sketches, etc., and the tables and curves for exhibiting the results. On the right page place a brief, accurate account of the experiment, with diagrams and descriptions of the apparatus, the data (those directly observed and those obtained by calculation) tabulated, and the curves obtained by plotting the numbers. Write up this report, in ink or pencil, during the progress of, and immediately after completing the experiment.

If certain experiments require more finely ruled or larger sheets of squared paper than those provided in the book, these can be obtained separately. Fix such sheets into the note-book by gummed slips of paper (stamp). If several curves are drawn with reference to the same axes, use inks of different colours.

The note-book as used above is a combined rough and fair book; the student should, however, avoid carelessness on the rough side. The right side should show a lucid and readable digest of the work on the left, yet the condition of the rough page should be such that the details could be followed and understood by another person if required.

7. Laboratory rules.—Rules similar to the following have generally to be obeyed.

1. No student is allowed to take apparatus from the cupboards, workshop, or storerooms; the necessary instruments, tools, etc., will be supplied when requested.

2. Apparatus must be used with every precaution, and left, when finished with, in good condition, ready to be put away: glass and tin vessels swilled out, inverted and put to drain: connecting wires rolled up: gas, water and electricity turned off, lamps out.

3. All breakage and damage must be immediately reported and will be charged for. Each student is required to deposit 2s. 6d. on joining a class. The balance after deducting the cost of breakages, etc., will be returned at the end of the session.

4. Students are expected to avoid the following bad habits:—Unnecessarily touching surfaces of glass, ebonite, etc., with fingers, etc.: fidgeting with screws and handles: marking or scratching scales, or apparatus: playing with magnets, switches, gas taps, etc.: using mercury except with a tray: interfering with fittings and apparatus.

5. Each student is expected, when necessary, to adjust or prepare the apparatus he is about to use.

6. Each student must supply himself with a laboratory note-book. No student can pass from one experiment to another until the former has been completed, written up in the note-book and passed as satisfactory by the Superintendent.

Home Experiments.

In the following chapters all experiments marked by an asterisk may be conveniently performed by the student at home. If not done at home they should be worked in the laboratory: these experiments deal with very important parts of Experimental Physics and therefore should not be neglected.

PART I.

MECHANICAL QUANTITIES.

CHAPTER I.

WEIGHING AND MEASURING.

8. In this chapter a short course of experiments requiring only simple and inexpensive apparatus is described. The work may, with advantage, be done at home. While working these experiments the student should read the account of the methods of weighing and measuring as given in Chapters II.–V.

The following pieces of apparatus are required :—

(1) A boxwood scale, twenty inches or half a metre long, divided into tenths of inches on one edge, centimetres and millimetres on the other. Cost, 9d.

(2) A boxwood scale, one foot long, divided into tenths of an inch (cost, 4d.) : or a second scale like (1).

(3) Half-a-pound of wire nails (1″ long, $\frac{1}{8}$″ diameter). Cost, 2d.

(4) Tin tacks. Cost, 1d.

(5) Several cylindrical canisters (cigarette tins).

(6) Cylindrical lamp chimney and cork or plug. Cost, 2d.

(7) A flat-bottomed glass tube (specimen tube), (a lamp chimney plugged as in Exp. 22 can be used). Cost, 2d.

(8) A rough pipette (piece of wide glass quill tubing, 9″ long, one end nearly closed). Cost, 1d.

(9) A glass tumbler or bottle, cotton, pins, a few screws, etc.

(10) Compasses, set-squares, protractor and drawing-board.

(11) A rough box.

(12) Glass stopper, pieces of coal, stone, salt, etc.

(13) A medicine bottle divided into tea or table spoons.

(14) A large "marble" (stone). Cost, 1d.

***Exp. 1.**—*Prepare the boxwood scales.* (1) In a 20″ scale bore two holes as in Fig. 2, one at the middle division, the other 1 inch from an end. To make a hole long enough to take the blade of a penknife, bore two holes, $\frac{1}{4}''$ diameter, close together, then cut away the division between them. (2) Bore the 12″ scale as in Fig. 3, viz. a large elongated hole at the middle division, a small hole through division 1 and another through 11. Number each inch division as shown. Or prepare a 20″ scale as in Fig. 2.

Make a set of weights from the large nails. Tie together in bundles, two of 7 nails each, two of 14, one of 35. (To be adjusted as in Exp. 11.) Call these weights P, $2P$, $5P$. (If the nails are 1″ long, $\frac{1}{8}''$ diam., then P is about 14 grammes or half-an-ounce.)

Make a scale pan (three required). Pierce the edge of a canister lid with three equidistant holes: pass a string, knotted at one end, through each hole, tie together the other ends, and form a loop to hang from the boxwood scale. Make a small hole through the centre of each pan.

LENGTH.

***Exp. 2.**—*Find the ratios of the sides of a right-angled triangle.*

I. Measure in inches and centimetres each side of a 60° set-square.† Show that when measured by either unit the sides are as 1 : 1·73 : 2.

Similarly measure a 45° set-square.‡ Show that the sides are as 1 : 1 : 1·41.

* All experiments indicated by an asterisk may be performed by the student at home and in this chapter require only the simple apparatus mentioned in § 8. Also see the Appendix, the Preface, and p. 8.

† Or draw an equilateral triangle (side 6″, say). From one angular point drop a perpendicular on to the opposite side.

‡ Draw two lines at right angles: then make an isosceles right-angled triangle.

II. Draw any two lines intersecting at A. From a point, B, on one line draw BC perpendicular to the other line. Measure AB, BC, CA. From any point, b, on AC draw bc perpendicular to AB. Measure Ab, bc, cA. Calculate, in decimals, the proportion $AB:BC:CA$, also $Ab:bc:cA$. (These should be equal.) Calculate $\sin A$, $\cos A$, $\tan A$.

Confirm by measuring the *angle* A (with protractor) and obtaining the trigonometrical ratios from mathematical tables.

Repeat with other values of *angle* A.

***Exp. 3.**—*Comparison of the Yard and Metre.* Open the dividers so that the distance between the pointed ends of the legs is a definite number of inches, then apply to the metric scale, and measure the distance to a fraction of a millimetre: similarly measure a definite number of centimetres in fractions of an inch. Plot the results, inches horizontally, centimetres vertically (the graph will be a straight line), and deduce the relations between the units.

RECORD

	length in cms.: 5 7 9 11 13 . . . . .
	length in ins.: 2 3 4 5 6
Hence	1 cm. = . . inch and 1 metre = . . yard
	1 inch = . . cms. and 1 yard = . . metre.

***Exp. 4.**—*Find from a map the distances between places— (1) direct, (2) by road, (3) by rail.* Use the methods of §§ 12, 13 for straight lines, and of § 16 for curved. Refer the lengths to the scale of the map.

***Exp. 5.**—*Measure the circumference and diameter of a circle. Find the value of* π.

From the same centre describe three quadrants of circles, of radii, say, 8, 10, 12 cms.

I. Measure arcs and radii by bow-compasses only (no scale), counting the steps and judging fractional parts. (Repeat once.)

II. Measure with thread and scales. (Repeat once.)

RECORD as below—

Quadrant A.	Arc.	Radius.	Arc / Radius.	π.
By spring bows, Exp. (i)	.. steps	.. steps	....	....
,, ,, Exp. (ii)	.. steps	.. steps	....	....
By thread,....Exp. (i)	.. cms.	.. cms.	....	....
,, Exp. (ii)	.. cms.	.. cms.	....	....

Similarly for quadrants B and C.

***Exp. 6**.—*Measure the width of a neck, or diameter of a cylinder* (Fig. 1). Rest a scale and set-squares on books. Place the cylinder or neck (of a bottle) between the set-squares. Press together so that one side of each set-square is in contact with the edge of the scale, and the second side of each square touches the cylinder. Then the distance between the set-squares equals the width of neck or diameter of cylinder.

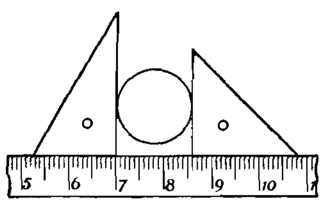


Fig. 1.

The diameter of a tube, or cylinder, should be measured at different positions along its surface in order to test the uniformity of its figure.

This illustrates the principle of the *sliding calipers* (§ 20).

***Exp. 7**.—*Find the circumference of a disc by rolling it along a scale. Deduce the value of π.* A large coin or canister lid may be used. Make a mark (by gummed paper) on the edge of the disc, rest the edge on a scale with the mark against a unit division, then roll the disc (without slipping) one revolution along the scale: note the division reached by the mark on the disc. Deduce the circumference of the disc. Repeat several times. Calculate the mean value. Measure the diameter. Calculate *circumference* $\div$ *diameter*. The result is an experimental value of π.

***Exp. 8**.—*Measure the girth and diameter of a canister. Deduce the value of π.* Measure girth (§ 17) and diameter (Exp. 6) in several places. Calculate from mean values.

THE LEVER.

9. Apparatus.—Fig. 2 shows an arrangement for experiments with the lever. A penknife with open blade is held, edge upwards, against the side of a box by three screws so

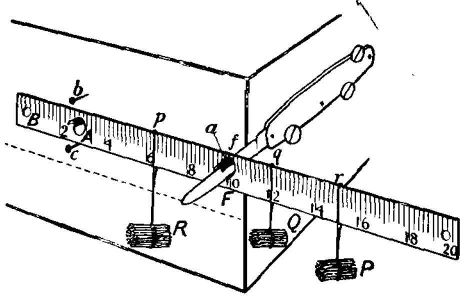


Fig. 2.

that it does not shake. The shank of a pen (a) is squeezed into the large hole of the scale. Weights (bundles of iron nails, Exp. 1) are hung on the scale by loops of cotton. These may be pushed along the beam. Two nails, *b*, *c*, act as stops: when the scale lies evenly between these it may be considered to be in equilibrium. (The face of the box below the dotted line should be removed so that the scale pan, etc., does not rub against the box.)

A simpler arrangement is to drive a nail into the box, and pass it through a large hole in the scale. The preceding is more satisfactory, the movement of the beam being less affected by friction.

*__Exp. 9.__—*Demonstrate the Principle of Moments.*† Arrange that the fulcrum (f) is at the centre of the scale. Place a weight, P, successively at the 2 and 5 inch marks. Find in each case where (i) an equal weight must be placed to produce equilibrium, (ii) a weight $2P$, (iii) a weight $3P$, (iv) three equal weights in different positions, (v) three unequal weights in different positions, and so on. In each case note the values of the weights used and their respective positions.

CALCULATE the force (F) through the fulcrum (= the sum of all the weights hung on the scale); the moment of each weight and of the force, F, about the fulcrum (moment of a weight about the fulcrum = weight × distance from fulcrum). Of these moments add together those that tend to produce rotation of the lever round the fulcrum in a counter-clockwise sense, and then those that tend to produce clockwise rotation. (The former moments are conveniently indicated by +, the latter by —.) The sum of the positive or counter-clockwise moments should be equal to the sum of the negative or clockwise moments.

Similarly calculate the moment of each weight and the force, F, about one end of the scale (the moment of a force about any point = force × distance from the point), and show that the total clockwise or negative moment equals the total counter-clockwise or positive moment.

Repeat the calculation, using another reference point, say, a point 3″ from the right-hand end.

† **The Principle of Moments.**—When a body acted on by several forces in one plane is in equilibrium the algebraic sum of the moments of the forces about *any* point in that plane is zero.

Record of an experiment. Weights (bundles of iron nails) wer balanced on a boxwood scale, 20″ long, fulcrum at the middle. (Th numbers in thick type were obtained by experiment after arrangin under the conditions recorded in ordinary type.)

		Exp. number.	1.	2.	3.	4.	5.	6.
Experimental results.	Value of	*P*	2	2	2	2	2	2
		Q	2	2	2	2	2	2
		R	—	—	4	4	4	4
		S	—	—	—	1	1	1
		F	4	4	8	9	9	9
	Position of	*P*	2	5	2	2	2	2
		Q	**17·9**	**14·95**	5	5	7	**4·7**
		R	—	—	**16·55**	**14·55**	**13·8**	15
		S	—	—	—	18	17	17
		f	10	10	10	10	10	10
Calculated results.	Distance between	*P* and *f*	8	5	8	8	8	8
		Q and *f*	7·9	4·95	5	5	3	5·3
		R and *f*	—	—	6·55	4·55	3·8	5
		S and *f*	—	—	—	8	7	7
		F and *f*	0	0	0	0	0	0
	Moment about *f*	of *P*	+16	+10	+16	+16	+16	+16
		of *Q*	−15·8	− 9·9	+10	+10	+ 6	+10·6
		of *R*	—	—	+26·2	−18·2	−15·2	−20
		of *S*	—	—	—	− 8	− 7	− 7
		of *F*	0	0	0	0	0	0
	Total.†	+ moment	16	10	26	26	22	26·6
		− moment	15·8	9·9	26·2	26·2	22·2	27
	Moment about end	of *P*	− 4	−10	− 4	− 4	− 4	− 4
		of *Q*	−35·8	−29·9	−10	−10	−14	− 9·4
		of *R*	—	—	−66·2	−58·2	−45·2	−60
		of *S*	—	—	—	−18	−17	−17
		of *F*	+40	+40	+80	+90	+90	+90
	Total.†	+ moment	40	40	80	90	90	90
		− moment	39·8	39·9	80·2	90·2	90·2	90·4

† The practical equality of the + and − moments demonstrates th truth of the Principle of Moments.

THE BALANCE.

***Exp. 10.**—*Prepare a balance.* Take a boxwood lath, say a ruler, 12″ long, graduated in one-tenths of an inch. Ink in plainly each inch and ½-inch division. Prepare the lath as in Fig. 3, and mount on a knife-edge as in Fig. 2. If when the pans are unloaded the lath does not balance horizontally, bind to the lighter side a piece of wire sufficient to make equilibrium. One of the weights, P, is hung by a

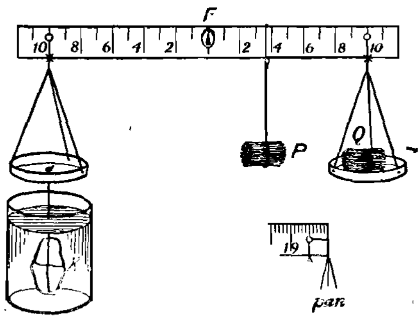


Fig. 3. Fig. 3*a*.

loop from the balance beam and can be adjusted to different positions upon it. It acts as a *rider* (§ 45): *e.g.* since the half-beam is divided into tenths, on division 3 the rider is equivalent to $0{\cdot}3\ P$ in the scale pan, on division 4 to $0{\cdot}4\ P$, etc.

Fig. 3 shows a body, X, being weighed in a liquid. The body is conveniently suspended by a thread, the upper end of which is knotted after being passed through the hole at the centre of the pan.

The amount is $Q + 0{\cdot}36P$. If $Q = P$, then $X = 1{\cdot}36P$; if $Q = 2P$, then $X = 2{\cdot}36P$, etc.

A 20-inch ruler, subdivided into $\frac{1}{10}$ inch, does very well; it will turn with less than 0·1 gramme. The scale pans are fixed to the ends as in Fig. 3*a* (the string ends go through a hole and then under the beam, where they are tied). Each scale division is in this case one-hundredth of the half-length of beam. If then P is 10 grammes, a change in its position on the beam of one scale division will be equivalent to the addition or withdrawal of $\frac{1}{100}$ of $10 = \frac{1}{10}$ gramme from the scale pan.

***Exp. 11.**—*Adjust the set of masses to be relatively correct:* **Method of substitution** (§ 50). (1) Place the heavier, P_2, of the two smallest bundles of nails, P_1, P_2, in the right pan; add nails, tin tacks, etc., to the left pan until there is equilibrium. (2) Substitute the second bundle, P_1, in the right pan, cut down a piece of wire until the (wire + P_1) is in equilibrium with the left side. Bind the wire round P_1. The two weights P_1, P_2 are now equal in value, $= P$ *grammes*, say. (3) Place both bundles, P, in the right pan, equilibrate by adding nails, etc., to the left. (4) Substitute in the right pan one of the bundles nearly equal to $2P$, adjust it by wire, etc., into equilibrium with the load in the left. (5) Substitute in the right pan the second bundle nearly equal to $2P$, and adjust similarly. The doubles are now each twice the singles. (6) Put $2P$, $2P$, P in the right pan, equilibrate on the left, substitute the bundle nearly equal to $5P$ in the right pan and adjust it to equilibrium with the left. The absolute values (in lbs. or gms.) of the masses are found in Experiment 12.

***Exp. 12.**—*Find the values of the masses in grammes.*

METHOD I. Measure in cms. the internal diameter and depth of a small cylindrical canister: calculate its volume, V, in cb. cms. Weigh it, using the bundles of nails, empty $(= m_1 . P)$ and full of water $(= m_2 . P)$. Then numerically

$$V = m_2 . P - m_1 . P, \quad \therefore\ P = V/(m_2 - m_1) \text{ grammes.}$$

Care must be taken in filling the vessel. Add water until the surface appears above and bending down to the edges of the can. Then remove water until the surface is flush with the edge. This can be done by repeatedly touching the surface with a finger, then withdrawing, and drying the finger. Keep the outside of the can dry.

METHOD II. It is better to measure the buoyancy on an immersed body (Exp. 60). Measure in cms. the proper dimensions of a regular solid (glass cube, prism, large marble, etc.), calculate its volume, V, in cb. cms., weigh it, using the bundles of nails, in air $(= m_1 . P)$ and in water $(= m_2 . P)$. Then

$$P = V/(m_2 - m_1) \text{ grammes.}$$

To find a mass in grammes, first weigh it by **the bundles** of nails, P, $2P$, etc. Multiply the value found **by the** value of P in grammes.

The value of P in grammes may be called the *constant* of the set of weights. It is the factor that converts an arbitrary measure, P, into a conventional one; viz. the gramme. Such constants are frequently used in connection with all kinds of measuring instruments.

***Exp. 13.**—*Find the masses of coins, etc.*

***Exp. 14.**—*Find the specific gravity of glass (stopper), stone, coal, paraffin-wax (candle), beeswax, methylated spirit, petroleum, salt solution.* The necessary operations,† etc., are described in § 65.

***Exp. 15.**—*Find the specific gravity of methylated spirit, petroleum, salt solution, etc., by means of a bottle.* For the necessary operations† see § 64. For bottle use a medicine phial. Mark its neck by twisting a thread or fine wire twice round and tying tightly, or stick on a strip of stamp-paper.

***Exp. 16.**—*Find the specific gravity of glass (stopper), a piece of coal, stone.*‡ Arrange the knife-edge to be in the hole near the end. Suspend a body (whose weight need not be known, *e.g.* a bottle filled with nails) somewhere on the short arm. Suspend the body whose specific gravity is to be determined, on the long arm, adjust into equilibrium, and note its distance from the fulcrum when weighed (i) in air, (ii) in water (bring up in a glass). For the necessary formula see § 66.

***Exp. 17.**—*Find the specific gravity of candle wax, methylated spirit, petroleum, salt solution*‡. For the necessary operations see § 65; formulae, § 66. Compare the results with those of Exps. 14, 15.

† Note that in many of these experiments the values of P, $2P$, etc., in grammes or lbs. need not be known.

‡ This illustrates Walker's Specific Gravity Balance or Steelyard.

***Exp. 18.**—*Graduate a steelyard* (§ 51). (1) Support the scale on a knife-edge through the large hole, A (Fig. 2). (2) Hang a pan from the small hole, B. Add nails to the pan until the right side is moderately overbalanced. Bind the nails into a bundle and attach it to the scale pan (this is then equivalent to the heavy end of a steelyard). (3) Make another bundle (W) of nails and suspend from the long arm (this is equivalent to the movable weight of a steelyard); adjust into equilibrium with the scale pan, etc., on the short arm. Note the reading of W on the scale: this is the *zero* of the steelyard. (4) Adjust W into equilibrium, and note its position when $2P$, $4P$, $6P$, $8P$, $10P$ are successively put in the pan. (5) Plot the added masses, $2P$, $4P$, etc., vertically and the scale readings horizontally. The graph obtained is practically a straight line, but does not pass through the origin.

To find the weight, X, of a body in the scale pan. Adjust W into equilibrium, note its reading, x, and refer to the graph. The ordinate through the reading, x, gives the weight, X, in terms of P.

Since the graph is a straight line, X can also be calculated: let y be the distance of W from the zero of the steelyard when in equilibrium with X in the pan, q the distance when a known mass, P, is in the pan. Then

$$X = P.y/q.$$

EXPLANATION.—Let the weight of the scale pan and its accessories be S, and its moment (arm, s) about the fulcrum be $S.s$.

Let U, V be the respective weights of the short and long arms of the steelyard; $U.u$, $V.v$, their moments about the fulcrum.

Let the distance of the movable weight, W, from the fulcrum be

x_0, when there is no body in the scale pan;

x, when the pan contains an unknown weight (mass), X;

p, when the pan contains a known weight (mass), P.

Take moments about the fulcrum, then the following relations, corresponding to each of the conditions above, are obtained:

$$S.s + U.u = V.v + W.x_0$$

$$X.s + S.s + U.u = V.v + W.x$$

$$P.s + S.s + U.u = V.v + W.p$$

$$\therefore X.s = W(x - x_0) \qquad \therefore P.s = W(p - x_0)$$

$$\therefore X/P = (x - x_0)/(p - x_0) \qquad \therefore X/P = y/q.$$

***Exp. 19.**—*Obtain the masses of coins, etc., by the steelyard.* Compare the results with those of Experiment 13.

AREA.

***Exp. 20.**—*Measure the area of a triangle.* On a sheet of squared paper draw an acute-angled triangle, ABC (sides, say, 7″, 6″, 5″).

METHOD I. Estimate the area by counting squares (§ 56).

METHOD II. Draw AD, BE, CF respectively perpendicular to BC, CA, AB. Measure the length of each perpendicular; record and calculate as below:

$$\begin{array}{ll} AB = \ldots\ldots\ldots\ldots & CF = \ldots\ldots\ldots\ldots \\ BC = \ldots\ldots\ldots\ldots & AD = \ldots\ldots\ldots\ldots \\ CA = \ldots\ldots\ldots\ldots & BE = \ldots\ldots\ldots\ldots \end{array}$$

$$\left.\begin{array}{l} \tfrac{1}{2} \times AB \times CF = \ldots\ldots\ldots\ldots \\ \tfrac{1}{2} \times BC \times AD = \ldots\ldots\ldots\ldots \\ \tfrac{1}{2} \times CA \times BE = \ldots\ldots\ldots\ldots \end{array}\right\} \text{Each of these will be equal to the area of the triangle.}$$

METHOD III. Calculate $s = \frac{1}{2}(AB + BC + CA)$,
$(s - AB)$, $(s - BC)$, $(s - CA)$, and show that
$\sqrt{s(s - AB)(s - BC)(s - CA)} = \textit{Area of triangle.}$

***Exp. 21.**—*Deduce the rule that the area of a circle is proportional to the square of its radius or diameter.*

METHOD I. On a sheet of squared paper draw 5 quadrants from the same centre, say 6, $5\frac{1}{2}$, 5, $4\frac{1}{2}$, 4 inches radius respectively. Obtain the area of each quadrant by counting the squares within its figure. Tabulate (i) radius, (ii) $(\text{radius})^2$, (iii) area. Plot $(\text{radius})^2$ horizontally and area vertically. The graph will be a straight line: hence

(radius)^2 *is proportional to area.*

The slope of the line will be such that an *ordinate* = 3·14 (its *abscissa*): hence, practically,

area of a circle = 3·14 *(radius)*^{2}.

METHOD II. On a sheet of thin cardboard draw 5 quadrants from the same centre. Cut out the largest one and weigh, similarly the next largest, and so on. Finally weigh a rectangle cut from the same cardboard. Calculate as in § 56, METHOD II.; plot as in Exp. 21, METHOD I.

VOLUME AND DENSITY.

***Exp. 22.**—*Make and graduate a measuring vessel.* (1) Close the small end of a cylindrical lamp chimney with a cork. (To avoid leakage put some pieces of candle wax in the tube, and expose to a fire so that the wax melts and soaks into the cork.) Gum a strip of stamp paper along the outside. Mark every fifth notch of the stamp paper, beginning to count near the cork. Number every tenth notch, 1, 2, 3, etc. Read intermediate notches as decimals. (2) Mark the stem of a rough pipette (§ 8 (8)) (bind a piece of cotton or fine wire round). Fill up to the mark with water (§ 57). Discharge the water into the measuring vessel, allow the pipette to drain, blow out the last drop. Note the reading of the water surface in the vessel. (3) Again fill the pipette, add the water as before to the measuring vessel, and note the height to which it rises. Repeat until the vessel is full. The measuring vessel has thus been *calibrated* or divided into parts of *equal volumes.*

RECORD as below:—

Volumes	1	2	3	4	5	etc.
Readings of water surface	.	.	.	.	.	

Plot volumes vertically, readings horizontally.

The graph will be a straight line through the origin if the bore of the tube is uniform. This is unlikely: then the curve will bend. In either case the graph shows the volume for any scale reading in terms of that of the pipette as unity.

***Exp. 23.**—*Find the volume, k, of the pipette in cubic centimetres.* Weigh a small vessel (express in grammes). Fill the pipette up to the mark with water, allow it to discharge into the vessel, blow out the last drop; weigh the vessel again. The increase in weight is that of the water required to fill the pipette. It is numerically equal to the volume (k) of the water and of the pipette in cubic centimetres.

The quantity, k, may be called the *constant* of the pipette. It is the factor that converts volumes measured in an arbitrary unit (that of the pipette) into a conventional measure (the cubic centimetre). For a similar reason k is also the constant of the measuring vessel. Such constants are frequently used in connection with all kinds of measuring instruments.

To express in cubic centimetres the volume of a quantity of liquid, etc., contained in the measuring vessel.—The scale reading of the liquid surface having been observed, refer to the graph obtained in Exp. 22 and find the volume corresponding to the reading. Multiply the volume by k.

To find the volume of a body by the measuring vessel.—Put water in the vessel, read its surface. Slip in the body, and again read the water surface. Obtain the corresponding volumes from the graph obtained in Exp. 22. The difference between them multiplied by k is the volume of the body in cubic centimetres.

***Exp. 24.**—*Obtain the volume and mass of a piece of coal, a stone, etc. Calculate the density of each.* The masses can be determined in grammes by the balance and weights of Exp. 10, the volumes in cubic centimetres by the measuring vessel. Divide the mass of each by its volume, the quotient is the density of the material in grammes per cubic centimetre.

Also find the volumes of the bodies by weighing in air and water (§ 61).

***Exp. 25.**—*Obtain the volume of a cylindrical canister.*

Method I. Pour water into it from the measuring vessel of Exp. 22 (see § 59 (1), also § 59 (1*a*)).

Method II. Measure length and diameter in centimetres. Calculate the volume in cubic centimetres (§ 57).

Method III. Weigh the can (i) empty, (ii) full of water (§ 59 (2)).

***Exp. 26.**—*Obtain the volume of a spherical ball.* Use glass or stone "marbles."

(1) By displacement in the measuring vessel (§ 60).
(2) Measure a diameter (§ 14). Calculate the volume.

***Exp. 27.**—*Obtain the volumes of a medicine bottle marked in tea or table spoons.* Fill with water up to one of the marks, then empty into the measuring vessel. Deduce the value of the tea or table spoon. Repeat two or three times, using different marks.

PRINCIPLE OF ARCHIMEDES.†

10. Apparatus.—A tube (Fig. 4) closed flat at one end (a specimen tube, 8 inches long, ½ inch diameter), open at the other, is provided.‡ Take a strip of ruled paper, ink in every fifth line, number 0, 5, 10, etc.; roll it (lines perpendicular to length), press it close to the inner surface of the tube, and fix by stamp paper.

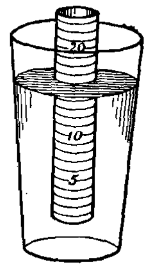


Fig. 4.

***Exp. 28.**—*Demonstrate the Principle of Archimedes.* Add sufficient nails to the flat-bottomed tube (§ 10) to sink it in water up to mark 20, say. Suspend the tube from the pan of the balance. Weigh in grammes the tube and nails, first in air, and then when immersed in water as far as the lines 4, 8, 12, 16, 20 in succession. (Hang the tube from the scale pan by cotton.) Calculate the difference between each weight when partially immersed in water and the weight in air: in each case the difference is the magnitude of the *buoyancy* acting on the body.

Measure in centimetres the outside diameter of the tube and the lengths between the closed end and the lines 4, 8, 12, etc. Calculate the volumes immersed in the several cases.

Tabulate (i) length of tube immersed, (ii) apparent weight of tube when immersed, (iii) buoyancy acting on the tube, (iv) volume of immersed part. The volumes immersed should be *numerically* equal to the respective buoyancies in water. Plot volumes horizontally, buoyancies vertically. The graph will be a straight line, showing that the buoyancy acting on the tube is proportional to the volume of the immersed part.

Repeat substituting, say, methylated spirit for water.

† For the Principle of Archimedes and Law of Flotation see Exp. 60.

‡ A lamp chimney, plugged as in Exp. 22, can be used. Immerse in water in a large bottle or jug.

***Exp. 29.**—*Law of Flotation.* I. Measure the length (L cms.) of the tube from the end (outside) to a line, m, on the paper (say, 16). Measure the outside diameter (D cms.). Place the tube in a vessel (tumbler) of liquid and put in nails until the liquid surface outside stands at the mark, m. Remove and dry the tube, then weigh (tube + nails) in grammes. Also find the specific gravity (S) of the liquid by the bottle (§ 64) or common hydrometer (§ 68). Calculate the volume of liquid displaced ($= \frac{\pi}{4} LD^2$ cu. cms.), and the weight of liquid displaced ($= \frac{\pi}{4} SLD^2$ gms.). The latter should be equal to the weight of the tube and nails.

Float in turn in water, methylated spirit, salt solution.
Repeat by working to, say, mark 20.
Compare Nicholson's hydrometer (§ 67).

***Exp. 30.**—*Law of Flotation.* II. Add nails so that the tube floats upright in salt solution, say, at mark 10. Weigh (tube + nails) in grammes. Then place in turn in water, methylated spirit, etc. In each case note the reading of the liquid surface (estimate tenths by eye) on the paper scale. Measure in centimetres the length from the end of the tube to the scale reading, also the diameter of the tube; then calculate in *cubic centimetres* the volume of the tube immersed. Obtain the $Sp.G.$ of each liquid. Multiply together the volume immersed in one of the liquids and the specific gravity of the liquid. The product should be numerically equal to the weight of the (tube + nails). Proceed similarly with each liquid.

Plot the scale readings horizontally and the specific gravities vertically. The instrument can then be used as a **common hydrometer** (§ 68). *To find the specific gravity of a liquid* by means of it immerse the tube in the liquid and observe the reading of the liquid surface. The ordinate of the graph having the reading as abscissa denotes the value of the specific gravity.

CHAPTER II.

MEASUREMENT OF LENGTH AND ANGLE.

LENGTH.

11. Measurement of Length.—This is usually done by applying to the length some kind of measure, *e.g.* a 2-foot-rule, a yard stick, or a tape, on which lengths of a foot are marked and subdivided into inches and smaller parts. In the laboratory a steel rule, about a foot long, graduated in British and Metric units is of value as a standard. Boxwood rules (less accurate) half a metre long are also convenient. A flexible tape measure, 7 or 8 feet long, is serviceable. The tape may be of linen or steel. In the former case the accuracy of its graduation should be tested by comparing it with a standard scale.

A **steel tape** is generally attached to a spindle contained in a metal case. The spindle is controlled by a spring and ratchet. The tape may be unwound, more or less, by pulling at the free end. It winds up, owing to the action of the spring, when a button at the side of the box is pushed in. In winding up let the tape run and be gently pressed between the finger and thumb, so that it does not move very quickly.

The units of length and the relations between them are shown in the Appendix.

Observe that on a length measure (i) the graduation marks are at equal distances, that is the *scale is one of equal parts:* (ii) certain marks are numbered; the lengths along the edge of the scale between the numbered divisions are either inches or centimetres: (iii) each inch is subdivided; if into 20 parts, then the scale measures to $\frac{1}{20}$ inch: (iv) each centimetre is subdivided; if into 10 parts, then the scale measures to $\frac{1}{10}$ centimetre, that is to a millimetre.

In deciding whether a measure, if not specified, is British or Metric, remember that a halfpenny is one inch in diameter, that 2·5 centimetres = 1 inch nearly, and beware that the difference between $\frac{1}{50}$ inch and $\frac{1}{2}$ millimetre cannot be detected by the naked eye.

Exp. 31.—*Examine the measures provided and describe them as below.*

A. *Examination of a boxwood rule.* The measure is just 1 metre long. One edge is divided into centimetres and millimetres. Every second cm. division is numbered. The other edge is divided into inches and $\frac{1}{10}$ inch. Each inch division is numbered. There are nearly 39·4 inches on the scale.

B. *Examination of a steel rule.* The rule is about a foot long. On each edge there is a scale of equal parts.

One edge marked "metre" is divided into centimetres and millimetres. Each centimetre division is numbered. A length of 10 cm. is subdivided into $\frac{1}{2}$ mm. Total length of scale 30·5 cm.

On the other edge there is a scale of inches divided into tenths. Each inch line is numbered. Total length 12 inches. The first two inches are further subdivided into fortieths of an inch, the next three into twentieths, the eleventh inch into fiftieths, and the last into hundredths.

On the reverse side the scale on one edge is of inches divided into twelfths. The first inch is further subdivided into forty-eighths, the next three into twenty-fourths, and the last into ninety-sixths. Each inch line is numbered. Total length 12 inches.

The remaining edge is of inches divided into eighths. The first three inches are subdivided into thirty-seconds, the last inch into sixty-fourths. Each inch line is numbered. Total length 12 inches.

***Exp. 32.**—*Find and note which of your fingers is practically* 10 *cm. long.*

12. To measure the distance between two marks.—Place the rule against the marks as in Fig. 6, not as in Fig. 5. If the distance to be measured is not an exact number of scale divisions the length of the bit over should be estimated by the eye. (Exp. 34.)

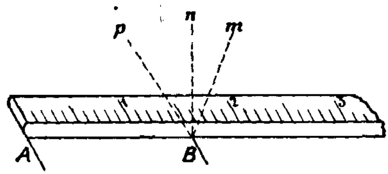


Fig. 5.

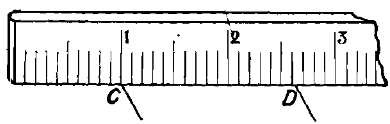


Fig. 6.

In Fig. 5 observe (1) that there would be inaccuracy at A because the end of the scale is not "square." The end of a scale becomes, in time, worn in this manner. Also observe (2) that owing to the thickness of the rule the reading of B would be judged differently when the position of the eye is changed. In the figure it is 1·63 when viewed along mB, 1·58 along nB, 1·48 along pB. To avoid this uncertainty (called a *parallax error*; § 120) the graduated edge is placed (Fig. 6) on CD. The position of C is 1·00, of D, 2·64; $\therefore AB = 2{\cdot}64 - 1{\cdot}00 = 1{\cdot}64.$

Use of and limit to subdivision. Although the steel scale is partly divided into 1/100 inch, it is difficult to use so fine a subdivision unless the eye is aided by a lens. The naked eye cannot easily read fractions less than 1/50 inch, or 1/2 mm. In order that measurements to smaller values may be readily made, devices such as the diagonal scale (§ 14), vernier (§ 18), and graduated screw head (§ 23) are adopted.

13. Use of compasses, calipers, etc.—A short length is conveniently measured by compasses. The legs are opened out and adjusted until the distance between their ends equals that under measurement. This is tested by placing, *always after their adjustment*, the point of one leg at an end of the length to be measured, and seeing if the point of the second leg just reaches the other end of the length.

In compasses the adjustment is made by trial, the distance between the legs being slightly but indefinitely

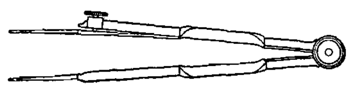


Fig. 7.

altered by hand. It is better to use **hair-dividers** (Fig. 7) or **bow compasses** (Fig. 8) in which the adjustment is quickly and accurately performed by slightly turning the nut at the side.

Fig. 8.

The compasses, after being adjusted to the unknown length, are next made to span the divisions of the steel or diagonal scale and the distance noted.

Calipers.—Fig. 9 shows the compass calipers used for gauging the diameters, widths, etc., of rods, pipes, flanges, etc., of machines. The "inside" end measures internal, and the "outside" external dimensions. The opening between the tips is adjusted until they touch the surfaces, the distance between which is to be measured. For instance, the diameter of a cylinder or sphere is obtained by adjusting the "outside" gap until the body is very slightly pinched. Finally the distance between the ends is measured by laying them on a centimetre or inch scale.

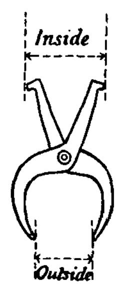


Fig. 9.

14. To measure the diameter of a sphere or altitude of a cone or pyramid.—Pinch the figure between rectangular blocks, press the blocks against the edge of a scale (Figs. 10, 11).

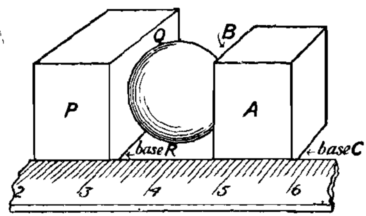


Fig. 10.

The blocks need not be similar nor equal, but each must have three adjacent faces, A, B, C, and P, Q, R, mutually perpendicular. This should be tested by a set-square, or carpenter's square.

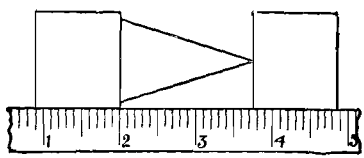


Fig. 11

The following method is sometimes used **to test whether the faces of rectangular blocks are actually perpendicular.** (1) Take two blocks: put a face of each in contact; press the blocks on a flat surface. If the faces do not keep in contact, but allow light to pass between them, then the block is not rectangular. If they do keep in contact, then (2) turn one block through two right angles and press the faces, as before, on the flat surface. If now the faces do not keep in contact, the block is not rectangular. If they do, repeat the tests with other faces in contact.

The second test is necessary because two blocks cut with equal angles fit in one position (Fig. 12), but not in the other (Fig. 13).

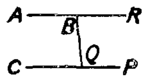


Fig. 12.

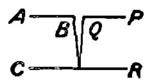


Fig. 13.

If all the faces and angles of each block exactly fit *both* blocks are rectangular. The method by the carpenter's square is less confusing.

15. Measurement of the length of a curved line.—METHOD I. *By spring bow compasses* or *dividers*. Open the points a short distance (about $\frac{1}{4}$ inch, or *less* if the curvatures are sharp) and step along the curve from end to end.† Count the number of steps. Then take, say, 20 steps along a measuring scale, and note the distance traversed. Calculate by assuming

$$\frac{(\textit{length of curve})}{(\textit{length on scale})} = \frac{(\textit{number of steps on curve})}{(\textit{number of steps on scale})}.$$

METHOD II. *By thread*. Place an end of the thread at the left extremity of the line,† and hold it there by pressing it with the nail of the left hand forefinger. Adjust a short length ($\frac{1}{2}$ inch, say) of the thread *along the curve* and fix by the nail of the right hand forefinger. Place the nail of the left hand forefinger close to the right and press the thread by it. Then adjust another short length of thread along the curve and repeat the operations until the end is reached. Finally, cut the thread at the end of the curve and stretch it over a scale. Its length will be equal to that of the curve. A *flexible tape measure* (§ 11) can sometimes be substituted for the thread with advantage.

† If the curve is *closed*, *e.g.* a circle or ellipse, start from a mark on it, and continue round until the mark is again reached.

16.—Measurement of circumference or girth.—This is conveniently done by a tape measure or string.

METHOD I. *By string.*

Closely wind a piece of fine string round the body, say, 10 times. Cut the string at the beginning and end of the number of turns; then unwind and measure the distance between the ends of the string in inches or centimetres. Calculate

$$girth = \frac{\textit{length of string}}{\textit{number of turns}}.$$

METHOD II. *By a strip of paper.*

Wrap a strip of paper tightly round the figure so that the ends overlap. Prick through both thicknesses of the paper somewhere on the overlap. Unroll, and measure accurately the distance between the pin-holes: *e.g.*, place a finely divided scale over the points as Fig. 6.

METHOD III. *By a flexible tape.*

Wrap a tape measure round the body so that several inches of the tape overlap and the graduated edges nearly coincide. Read the position on the upper lap of division (1) on the lower, then of divisions (2), (3), etc. Deduce the mean value of the girth.

***Exp. 33.**—*Make a centimetre scale.*† Draw a straight line on cartridge paper and lay a metric scale edgewise along it. Prick the line at the points under the centimetre divisions of the scale.

Or better, open the dividers to span exactly 4 cm. measured on a standard scale, prick along the line divisions 0, 4, 8, etc. Reduce the width of dividers to 3 cm. exactly and prick off the remaining points.

Mark the points by short lines, and number every second, counting from left to right: 0, 2, 4, 6, etc.

In a similar manner *make a scale of half-centimetres.*†

† These scales are required in Exp. 34.

***Exp. 34.**—*Eye estimation of tenths of a division.* **1.** Draw a line, *AB*, about half the width of the page. Measure its length in millimetres by the paper centimetre scale (Exp. 33). Since this is not subdivided into *mm.*, the fractions of the cm. must be judged, in tenths, by the eye. *After* estimating the value, confirm by applying a millimetre scale. Similarly measure other lengths.

2. Similarly measure various lengths by the half-centimetre scale, estimating to tenths, that is half-millimetres.

This exercise is of *great* importance: *eye-estimation should be conscientiously and frequently practised.*

THE DIAGONAL SCALE.

17. The Diagonal Scale (Fig. 14).—The points *A*, *B*, *C*, are, say, 1″ apart. The lines *AP*, *BQ*, *CR*, etc., are parallels, practically (but not necessarily) perpendicular to *AC*. *AP* is any convenient length. *AB* is divided into, say, tenths; *AP* into, say, fifths. The first division, *M*, of *AB* is joined to *P*; through the others are drawn parallels to *MP*. Through each division of *AP* are drawn parallels to *AC*. The *least count* or *smallest difference indicated* by the scale is $\frac{1}{5}$ of $\frac{1}{10} = \frac{1}{50}$ inch; length (see below) of $xy = 1{\cdot}84$.

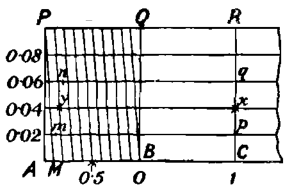


Fig. 14.

In any diagonal scale if the unit, *AB*, is divided into *p* parts and *AP* into *q* parts, then the *least count* $= 1/p.q.$ Thus when the least count is 1/40, there are six ways of drawing the scale; when $p = 20$, $q = 2$; $p = 10$, $q = 4$; or $p = 8$, $q = 5$; and when *p* and *q* are interchanged. Hence when the least count is given, select two convenient factors of the denominator, take *p* equal to one of these, and *q* to the other.

In *numbering* the scale it is necessary to divide only the first unit. Call the right-hand end of the first unit length, 0, number the units from left to right, the subdivisions from right to left.

To use the diagonal scale. Suppose the unknown length is finally found to be $xy = 1{\cdot}84$. Adjust the dividers to be equal to the unknown length (§ 13). Put the point of one leg of the adjusted compass at A, and observe that the other point will lie on AC, between the unit divisions 0, 1. If now the compass point is placed on division 1, the other point will lie on BA between divisions 0·8 and 0·9. Thus the base line, AB, shows that the length is between 1·8 and 1·9. Now keeping the compass point on the line, CR, through division 1, place it in turn at the points of intersection of this line with the several parallels to the base line, AB.

Observe the position of the other compass point on the same parallel: thus, when one point is at p the other will be at m; when at q, then at n; at x, then at y. In the first and second positions it is between diagonals, in the last it is at the intersection of the diagonal through 0·8 and the horizontal through 0·04. Hence the reading is 1·84.

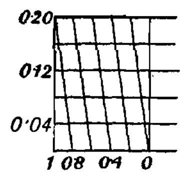


Fig. 15.

***Exp. 35.**—*Make a diagonal scale; least count* 1/25 *inch.* (See Fig. 15.) Make a unit scale of inches (Exp. 33). Take $p = 5$, $q = 5$. Number the divisions of AB, 1, 0·8, 0·6, 0·4, 0·2, 0, putting 1 at A, 0 at B; and those of AP, 0·20, 0·16, 0·12, 0·08, 0·04, putting 0·20 at P.

THE VERNIER.

18. The Vernier is a device, invented by Pierre Vernier, for readily estimating the fractions of the smaller parts of a measuring scale; its use avoids the necessity for minute subdivision. It is an *auxiliary* scale that slides along the *principal* one, and is graduated so that a number (n) of its divisions is equal to one less ($n - 1$) or one more ($n + 1$) than those of the principal scale.

See § 33, Note, for hints concerning the illumination and reading of finely divided scales.

*Exp. 36.—*Construct a scale and verniers as follows* (see Fig. 16):

(1) *Principal scale.* On a piece of paper draw a straight line and on it mark off (by dividers) equal parts, say about $\frac{1}{2}$ inch long. Number from left to right 0, 1, 2, 3, etc.

(2) *Vernier A.* On another strip draw a length equal to, say, 5 divisions of the principal scale. Divide this into six equal parts. Number from left to right 0, 1, 2, 3, 4, 5, 6. Then each part of the vernier = $\frac{5}{6}$ths of a division of the principal scale, and the difference between the two is $\frac{1}{6}$th of the same.

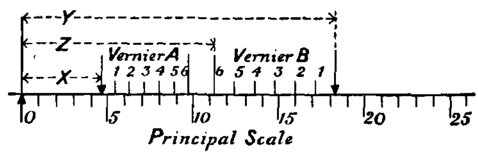


Fig. 16.

(3) Cut out the vernier along the line of graduation and arrange it to slide on the principal scale. Put the vernier zero, or mark 0, in line with 10 of scale, vernier division 6 will then coincide with 15 of scale. Now move the vernier until its line 1 coincides with 11 of the scale: the zero of the vernier will then have moved through a distance equal to the difference between a vernier and scale division ($\frac{1}{6}$th in this case). Next bring vernier mark 2 up to 12 of scale; the movement is equal to the previous step and the total displacement of the zero from its first position is $\frac{1}{6}+\frac{1}{6}=\frac{2}{6}$. Bring in turn the succeeding marks of the vernier into coincidence with 13, 14, etc., respectively of the principal scale and verify the numbers below.

Vernier mark coincident with a scale mark,	0,	1,	2,	3,	4,	5,	6
Scale mark coincident with a vernier mark,	10,	11,	12,	13,	14,	15,	16
Displacement in scale divisions of zero of vernier from its first position	0,	$\frac{1}{6}$,	$\frac{2}{6}$,	$\frac{3}{6}$,	$\frac{4}{6}$,	$\frac{5}{6}$,	1

(4) *Vernier B.* On another strip draw a length equal to, say, seven divisions of the principal scale. Divide this

into six equal parts. Number the marks 0, 1, 2, 3, 4, 5, 6 from right to left. Each part of the vernier is $\frac{7}{6}$ths of a division of the principal scale, and the difference between the two is $\frac{1}{6}$th of the same.

(5) Cut out the vernier along the line of graduation and arrange it to slide on the principal scale. Put vernier zero, or mark 0, in line with 10 of scale, the vernier line 6 will then coincide with 3 of scale. Then, as in the case of Vernier A, bring in turn the vernier line 1 and succeeding marks into coincidence with 9, 8, 7, etc., respectively of the principal scale and verify the numbers below.

Vernier mark coincident with a scale mark ..	0, 1, 2, 3, 4, 5, 6
Scale mark coincident with a vernier mark ..	10, 9, 8, 7, 6, 5, 4
Displacement in scale divisions of zero of vernier from its first position	0, $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, 1

In both verniers the movement of the zero is for each step equal to $\frac{1}{6}$th of a scale division. Each of these verniers is thus equivalent to a subdivision into sixths.

In every case the fractional value of the zero displacement has the number (6) of divisions on the vernier for its denominator, and for its numerator the number of the vernier mark that coincides with a line on the principal scale. To effect this, however, the vernier (A) in which the parts are less than the scale divisions has to be numbered in the same direction as the principal scale, and the vernier (B) in which the parts are greater than the scale divisions has to be numbered in the reverse way.

***Exp. 37.**—*Measure with the scale and verniers* constructed as above the length of a pencil, length and breadth of an envelope or card, diameter of a penny, etc. The results will be in terms of the divisions of the principal scale. Find the values† of these in inches and centimetres by comparison with a standard measure. Then express the above lengths in inches and centimetres.

† The ratio of the length of the scale division to a centimetre may be called the metric *constant* of the scale (see Exps. 12, 23, etc.).

***Exp. 38.**—*Construct a vernier to* work with the above principal scale and to read to $\frac{1}{10}$th of its divisions. Measure the above lengths by means of it and express them in inches and centimetres.

METHOD OF EXAMINING A VERNIER.

19. (1) Find the *value* in fractions of an inch or centimetre (degrees if the measurement is angular) *of the smallest parts of the principal scale.* To do this either compare it with standard measures directly or use dividers (§ 13).

(2) Count the *number of parts* into which the vernier is divided.

(3) Find the *relation between the divisions of the vernier and of the principal scale.* Bring the zero of vernier into coincidence with a scale division: the relationship can then be observed. Generally

$$n \text{ (vernier divisions)} = (n-1) \text{ (scale divisions)}$$

or

$$n \text{ (vernier divisions)} = (n+1) \text{ (scale divisions)}.$$

In the former case the vernier divisions are smaller than those of the scale, larger in the latter. Also the vernier reads forwards with the scale in the former case and backwards in the latter.

In some cases, *e.g.* the barometer, 10 vernier divisions = 19 scale. Imagine each vernier division bisected. Note that the vernier reads forwards with scale (§ 22, 4).

(4) Calculate the *least count* of vernier, that is the smallest difference that can be read by it. When

$$n \text{ (vernier divisions)} = (n \pm 1) \text{ (scale divisions)}$$

$$\text{the least count} = \frac{1}{n} \text{ (scale divisions)}.$$

(5) Show in diagrams the numbering of the scale and vernier.

In actual instruments the numbering of the graduation marks given by the maker frequently requires some practice in reading on the part of the observer. Until he can readily decide the reading by a simple observation of scale and vernier he should check his guesses by calculations similar to those of § 22.

(6) **To measure a length by the vernier.**—(i) Adjust one end of the length to be measured to the zero of the principal scale, and the zero of the vernier to the other end.

(ii) Observe the numbers, p, and $p+1$, of the lines of the principal scale between which the zero of the vernier lies.

(iii) Note the number of the mark, q, on the vernier that most nearly coincides with a scale division.

Then, n being the number of the divisions on the vernier, the length $= (p + q/n)$ scale divisions. Reduce from scale divisions to inches or centimetres as required.

In the more fully divided verniers two or even three graduations may appear equally coincident with lines on the principal scale. In the former case take the mean of the two, and in the latter the reading of the middle line.

In Fig. 16, which illustrates a scale and vernier constructed as in Exp. 36, $X = 4\frac{4}{6} = 4\frac{2}{3}$, $Y = 18\frac{2}{6} = 18\frac{1}{3}$.

The "model verniers for students" of the reverse-reading type are often arranged to take the length under measurement between the scale zero and the nearer end of the vernier (its zero being at the far end). In Fig. 16, Z is such a length: $Z = 11\frac{2}{6} = 11\frac{1}{3}$. The verniers on instruments however do not measure in this way.

20. Vernier Calipers (Fig. 17).—The principal scale is graduated on a steel strip, and the vernier on a frame that slides along the strip. A steel jaw projects at right angles to the strip and is fixed at one end of it. Another steel jaw, also at right angles to the strip, forms one end of the slider. The construction is such that the two jaws are in contact when

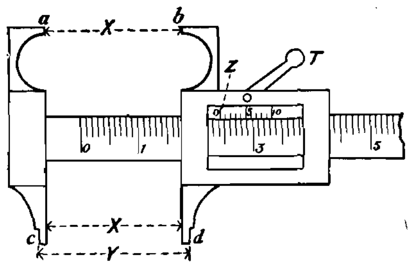


Fig. 17.

the zero of the vernier, Z, coincides with the zero of the principal scale. Hence the scale reading at any time measures the gap between the jaws. In Fig. 17 the distance between the tips, a, b, is equal to that between the jaws, X. These, *a*, *b*, can be introduced into tubes, and measure thicknesses that cannot be reached by the jaws, *c*, *d* (see Fig. 19). The distance, Y, between the outside edges of the projections, *c*, *d*, is 2 mm. (usually) greater than that between the jaws. These serve to measure the internal diameter of tubes, etc. (see Fig. 18). The catch, T, clamps the slider to the strip when required. *Be sure that the slider is free before trying to move it.*

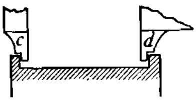


Fig. 18.

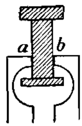


Fig. 19.

The reading in Fig. 17 is 2·43 cm.

NOTE.—If a cylindrical body is slowly turned while between the jaws the feeling of alternate tightness and looseness experienced will enable the observer to judge whether the figure is fairly circular or not.

Exp. 39.—*Find the external and internal diameters, and the length of a piece of tubing, by the vernier calipers.* Take five or six measurements in several positions and across different diameters. Tabulate external diam., D; internal diam., d; length, L. Calculate the means. Calculate the

$$volume = \frac{\pi}{4}(D^2 - d^2)L.$$

21. EXAMPLES. (1) Vernier on a *travelling microscope* (§ 31). The parts of the principal scale are millimetres. There are 10 divisions on the vernier. 10 vernier divisions are equal to 9 scale divisions. Therefore the vernier reads to $\frac{1}{10}$ mm.

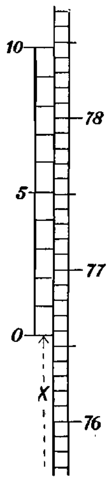
(2) *Barometer verniers.*

	Metric (Fig. 20).	British (Fig. 21).
Principal scale ..	millimetres	$\frac{1}{20}$ inch
Number of vernier divisions	10	25
Relation between principal scale and vernier divisions	10 vernier = 19 scale. Imagine each vernier division bisected, then 20 vernier = 19 scale	25 vernier = 24 scale
Least count ..	$\frac{1}{20}$ mm.	$\frac{1}{25}$ of $\frac{1}{20}$ = $\frac{1}{500}$ inch

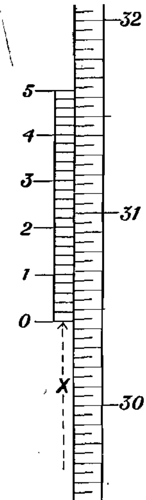
Metric vernier. (i) *By inspection* (Fig. 20):

Reading by principal scale, 76·5 cm. = 765 mm.

A mark midway between 6 and 7 of vernier would coincide with a scale division.



Metric
Fig. 20.



British
Fig. 21.

∴ reading by vernier = 0·65 mm.,

∴ total reading is 765·65 mm.
= 76·565 cm.

(ii) *By calculation:* It is seen that

$X + 6{\cdot}5$ vernier divs. = 77·8 cm.
= 778 mm.

By construction

1 vernier div. = 1·9 scale divs.,

$\therefore\ X + 6{\cdot}5 \times 1{\cdot}9$ scale divs.
= 778 scale divs.

$\therefore\ X = 778 - 12{\cdot}35$
= 765·65 scale divs. = 76·565 cm.

British vernier. (i) *By inspection:*

Reading by principal scale
= 30·4 inches (Fig. 21).

22nd mark on vernier coincides with a scale division,

∴ reading by vernier
= 0·044 inch,

∴ total reading = 30·4 + 0·044 = 30·444 inches.

(ii) *By calculation:* It is seen that

$$X + 22 \text{ vernier divs.} = 31{\cdot}50 \text{ inches.}$$

By construction 25 vernier divs. = 24 scale divs.,

$\therefore$ 1 vernier div. $= \frac{24}{25}$ scale div. $= \frac{24}{25} \cdot \frac{1}{20} = 0{\cdot}048$ inch,

$\therefore\ X + 22 \times 0{\cdot}048 = 31{\cdot}50.$

$\therefore\ X = 31{\cdot}50 - 1{\cdot}056 = 30{\cdot}444$ inches.

Exp. 40.—*Read the Barometer Verniers.* Set the vernier slide in any position. Read the Metric and British scales. Deduce the relation between inches and centimetres. Repeat several times with the slide in different positions.

22. Examples. (1) Verniers on a *table cathetometer* (§ 32).

	Vertical slide.	Circle.
Divisions of principal scale	millimetres	degrees
Divisions on vernier	20	6
Relation between vernier and principal scale ..	20 vernier = 19 scale.	6 vernier = 11 scale. Bisect each vernier division: then 12 vernier = 11 scale.
Least count of vernier	$\frac{1}{20}$ mm.	$\frac{1}{12}$ of $1^\circ = 5'$

(2) *Circular verniers on spectrometer* (§ 151).

Principal scale.	Number of vernier divisions.	Relation between vernier and scale divisions.	Least count of vernier.
A. $\frac{1}{2}^\circ$	30	30 vernier = 29 scale	$\frac{1}{30}$ of $\frac{1}{2}^\circ = \frac{1}{60}^\circ = 1'$
B. $\frac{1}{3}^\circ$ (Fig. 22)	40	40 vernier = 39 scale	$\frac{1}{40}$ of $\frac{1}{3}^\circ = \frac{1}{120}^\circ = \frac{1}{2}'$

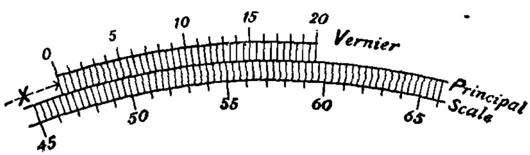


Fig. 22.

Vernier B (*Fig.* 22). (i) *By inspection:* Reading by principal scale $= 46\frac{1}{3}$. Division 23 of vernier is coincident; therefore, since the least count $= \frac{1}{2}'$, the reading by vernier $= 23 \times \frac{1}{2}$ minutes $= 11'\ 30''$,

$\therefore$ total reading $= 46^\circ\ 20' + 11'\ 30'' = 46^\circ\ 31'\ 30''$.

(ii) *By calculation:* It is seen that $X + 23\,V = 54^\circ$.

By construction 40 vernier divs. = 39 scale divs. or $V = \frac{39}{40}$ scale,

$\therefore\ X + 23 \times \frac{39}{40}$ scale divs. $= 54 \times 3 = 162$ scale divs.,

$\therefore\ X + 22{\cdot}425$ scale divs. $= 162$ scale divs.,

$\therefore\ X = 139{\cdot}575$ scale divisions $= 139{\cdot}575 \times 20$ minutes $= 46^\circ\,31'\,30''$.

GRADUATED SCREW HEAD.

***Exp. 41**.—*Principle of Screw.* Take a large screw and piece of wood. Lay the edge of a measuring scale against the screw thread and count the number of threads per inch or centimetre. Then calculate the *pitch* or distance between adjacent threads. (1) Turn the screw a little way into the wood. Measure accurately the distance between the wood and the edge of the screw head. (2) Rotate the screw, say, five times. Measure the distance between the wood and the edge of head. Observe that the head of the screw moves through a distance

$$= (pitch) \times (number\ of\ revolutions).$$

(3) Turn the screw through, say, $\frac{1}{3}$ or $\frac{1}{2}$ revolution. Measure the distance between the wood and the edge of head. Observe that the head moves through a distance

$$= (pitch) \times (fraction\ of\ turn\ of\ screw).$$

Thus in any case the end of the screw moves through a distance equal to the pitch multiplied by the number of revolutions and parts of a revolution through which the screw has been turned.

23. When short lengths have to be measured with considerable accuracy, a screw gauge, spherometer, etc., may be employed. In these instruments an accurately cut screw moving in a fixed nut is turned by hand until the end is felt to be in contact with the body to be measured. The adjustment is thus effected by the sense of touch, not by sight. It affords an example of *end* measurement.

The instruments are provided with two scales:

(1) A *pitch-scale*,† running parallel with the axis of the screw, and showing the number of complete revolutions.

(2) A *head-scale*,† round the edge of the screw-head, by which the fractions of a turn are indicated.

† *It is not usual to give names to the scales. The above are* convenient.

24. Micrometer Calipers or Screw Gauge.—A pattern of screw gauge or caliper is illustrated in Fig. 23. It consists of a rigid U-shaped frame having a tubular

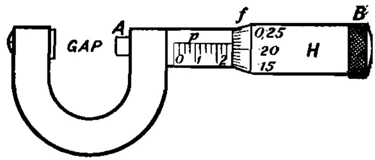


Fig. 23.

hub fixed to the right hand limb. The measuring screw works through a nut fixed inside the hub. The screw head or thimble, H, moves over the outside of the hub, and is attached to the screw spindle at the right hand end. By turning the screw head, H, the end, A, moves and makes a wider or narrower gap. The end, B, of the thimble is often milled. The body to be measured is put into this gap and the screw-head turned until the body is *just* held. Care must be taken not to screw up tightly, nor to force or hurry the movement. The instrument will soon deteriorate if used harshly.

In some instruments the milled end, B, is not fixed to the head, H, but turns it by a spring and ratchet arrangement. Then when the body in the gap is pinched sufficiently, the spring gives, and the milled end turns without moving the measuring screw.

On the outside of the hub there is a scale, p, of fractions of an inch or centimetre. This is the *pitch scale.* Its base line running along the hub is called the *line-of-graduation.* The equidistant marks, f, on the left hand bevelled edge of the head, H, form the *head scale* for measuring the incomplete turns of the screw.

In some instruments there are two pitch scales along the hub, one of half-millimetres ($= 0{\cdot}0197''$), the other of fiftieths of an inch ($= 0{\cdot}02$). Their lines of graduation are not coincident, but slightly diverge after starting from the same zero-point. If the pitch of the screw is a $\frac{1}{2}$ mm. then the line of graduation of its pitch scale is parallel to the axis of the hub, and the line of graduation of the scale of one-fiftieth inch slowly spirals round the cylindrical hub.

TO EXAMINE A SCREW GAUGE.

25. (1) Turn the head so as to *expose the pitch scale, p* (Fig. 23). Find what fraction of an inch or centimetre is equal to its smallest parts. To do this compare by dividers (§ 13) the length of, say, 12 divisions of the pitch scale with the subdivisions of the standard measure. Beware that $\frac{1}{50}$ inch is practically equal to $\frac{1}{2}$ mm.†

(2) Find the *pitch of* the measuring *screw.*—Bring the zero mark, 0, of head scale, *f*, into coincidence with the line of graduation. Observe now the position on the pitch scale, *p*, of the graduated edge of the head, *H*, and also after turning the head round once. The difference in the readings is the pitch of the screw. Express it in fractions of the inch or centimetre.

(3) Count the *number* (*n*) *of divisions of the head scale.* When the head is turned each division of its scale, *f*, as it passes the line of graduation on the hub indicates that the screw has made *one-nth* of a turn, and that the width of the gap has altered by *one-nth* of the pitch of the screw. This quantity is the smallest reading or *least count* of the instrument.

(4) *Zero error.* Screw up until the gap is closed. If the division, 0, of the head scale is on the line of graduation of the pitch scale, then the reading of the instrument when measuring a body equals the width of the gap. If the lines are not coincident the reading is subject to a *zero error.* The value of the zero error is obtained by observing what mark of the head scale coincides with the line of graduation when the gap is closed. The number of divisions between this mark and the zero of the head scale gives the fraction of a revolution required to bring about the coincidence of the scale zeros. The zero error is added to the readings of the instrument if the gap is slightly open when the zero of the head scale coincides with the line of graduation, and subtracted if it is closed before this is the case.

† $\frac{1}{2}$ mm. = 0·01969 inch.

26. EXAMPLES. The following are data of two screw gauges—

	Metric.	British.
Smallest parts of pitch scale ..	$\frac{1}{2}$ millimetre	$\frac{1}{40}$ inch.
Pitch of measuring screw	$\frac{1}{2}$ millimetre	$\frac{1}{40}$ inch.
Divisions of head scale	50	25
Least count of instrument	$\frac{1}{50}$ of $\frac{1}{2} = \frac{1}{100}$ mm.	$\frac{1}{25}$ of $\frac{1}{40} = \frac{1}{1000}$ inch.
Zero error	$-\cdot 03$ (Note 1)	$+\cdot 004$ (Note 2)
Numbering of pitch scale ..	Every tenth division, 0, 5, 10, 15, etc.	Every fourth division, 0, 1, 2, 3, etc.†
Numbering of head scale	Every tenth division, $\underbrace{0,\ 5}$, 1, 2, 3, 4	Every fifth division, $\underbrace{0,\ 25}$, 5, 10, 15, 20†

Note.—(1) The gap is closed 3 divisions of the screw head before the zeros would coincide. Hence, subtract $\frac{3}{50}$ of $\frac{1}{2} = \cdot 03$ mm. from the readings of the instrument. (2) The gap is closed 4 divisions of the screw head after the zeros come into coincidence. Hence, add $\frac{4}{25}$ of $\frac{1}{40} = 0\cdot 004$ inch to the readings of the instrument.

Exp. 42.—*Find the diameter of a wire.* Measure the diameter at five or six positions along the wire (1) by the *Metric* screw calipers, (2) by the *British.* Record each value: calculate the means.

Use wires of different substances and diameters.

Exp. 43.—*Find the density of the material of a wire.* I. Measure (1) the mean diameter in cm. by screw calipers, (2) length in cm., (3) mass in grms. Calculate volume in cb. cm.: then *density = mass/volume.* See § 69.

† The numbering of the scales is unsatisfactory. The instrument could be read more easily if every fourth division of pitch scale was numbered ·1, ·2, ·3, etc., and every fifth division of head scale, $\underbrace{0,\ \cdot 025}$, ·005, ·010, ·015, ·020.

THE SPHEROMETER.

27. A metal frame, *A* (Fig. 24), resting on three equidistant feet, has a nut at its centre through which the measuring screw passes. The screw spindle can be turned by a knob, *B*, at its upper end. It carries a disc, *C*, graduated in equal parts; this is the *head-scale* for measuring the incomplete turns of the screw. The movements of the disc are referred to the *pitch scale*, on the side arm, *D*, that rises parallel to the screw spindle. The parts of the pitch

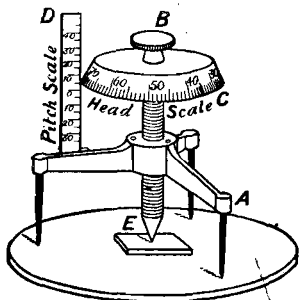


Fig. 24.

scale are usually a fraction of an inch or centimetre. When the knob, *B*, is turned the screw end, *E*, rises or falls. In one position it is in the same plane as the outer feet. The zeros of the scales are then in or near coincidence.

The plane of the three outer feet is the *reference* or *ground plane* of the spherometer. It is practically realised by resting the instrument on the flat surface of a piece of plate glass, and adjusting (§ 28, 4) the screw until its end touches the surface.

28. To examine a Spherometer:—(1) Find what fraction of an inch or centimetre is equal to the smallest parts of the *pitch scale*. To do this compare by dividers (§ 13) the length of, say, 12 divisions of the pitch scale with the subdivisions of the standard measure. Beware that $\frac{1}{50}$th inch is practically equal to $\frac{1}{2}$ mm.

(2) Find the *pitch of the measuring screw*. Bring the zero mark of the head scale up to the edge of *D*. Observe now the position on the pitch scale of the edge of disc *C*, and also after turning the screw round once. The difference in the readings of the pitch scale is the pitch of the screw. Express it as a fraction of an inch or centimetre.

(3) Count the number, *n*, of divisions of the head scale. When the head is turned each division of its scale, as it passes the edge *D*, indicates that the screw has made *one-nth* of a turn, and that the end

of the screw has moved through *one-nth* of its pitch. This quantity is the smallest reading, or *least count* of the instrument.

(4) *To find when the end of the screw is in the same plane as the three outer feet.* Rest the instrument on a piece of plate glass or other flat surface. Turn the screw until its end touches the surface. When this is the case a *gentle* knock against one of the legs of the instrument with a loosely held pencil will make it spin round a little way, or on touching the frame *lightly* with the finger it can be felt to rock. Find the division of head scale at which the instrument *just* begins to rock or spin, and the division at which it *just* sticks. The mean of these may be taken as the reading for which the end of the screw comes into contact with the surface.

Place the instrument on other parts of the surface, and determine the readings as above. If all agree to within one or two divisions it may be concluded that the surface is sufficiently flat.

Zero error. If in the above adjustment the 0 mark of the head scale is against the 0 of the pitch scale, the instrument has no zero error. If this is not the case the number of divisions between the zero of the head scale and the edge of the arm, *D*, is the zero error. Note whether it should be added to or subtracted from the scale reading.

In a similar manner the end of *the screw is adjusted to touch any other surface.* The exactness of a measurement depends considerably on the precision with which the point of contact is determined.

(5) When at the zero position ((4) above) measure, by placing the feet on a standard scale, the distance between (i) each pair of the outer feet, (ii) the centre and each outer foot. The former should be equal to one another, also the latter. The latter $= \frac{1}{\sqrt{3}}$ (former) $= 0{\cdot}58$ (former).

29. To find the thickness of a lamina by the spherometer.—(1) Place the spherometer with its three outside feet resting on the flat reference surface; also the lamina whose thickness is to be measured (Fig. 24). Adjust the measuring screw until its end touches the upper surface of the lamina. Note the reading of the scales.† Repeat three times.

(2) Remove the lamina, adjust the screw until its end touches the flat surface. Note the reading of the scales.† Repeat three times.

(3) Calculate the mean scale readings. The mean difference is the required thickness.

Exp. 44.—*By the spherometer find the thickness of a piece of* (1) *plate glass,* (2) *patent plate,* (3) *microscope cover-glass.*

Exp. 45.—*Test the flatness of a piece of common window glass by the spherometer.* Place the instrument in turn on each surface. Adjust, etc., as in § 28 (4). Repeat at various positions on the surfaces.

---

† NOTE.—It is better to note in (1) what division of the head scale is against the edge of the pitch scale, then count from this the whole and fractional turns of the screw needed to realise the adjustment in (2).

30. To find the radius of a spherical surface by means of a spherometer.—The surface may be either convex or concave.

(1) Place the instrument on the spherical surface and adjust the screw until its end touches the surface. Read the scales. Repeat three times, placing the instrument in different positions. Record each reading and calculate the mean.

(2) Place the instrument on the flat surface and adjust the screw until its end just touches the surface. Read the scales. Repeat three times, placing the instrument in different positions. Record each reading and calculate the mean.

Find the difference (a) between the mean readings.

(3) Measure the distances between the centre and outer legs and calculate the mean (b). Then

$$R = \frac{b^2}{2a} + \frac{a}{2}.$$

Or measure the distances between the outer legs and calculate the mean (c). Then

$$R = \frac{c^2}{6a} + \frac{a}{2}.$$

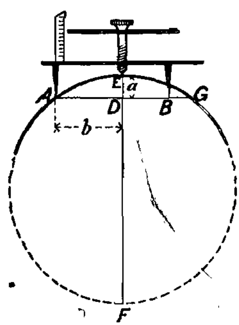


Fig. 25.

PROOF OF FORMULAE.—The diagram (Fig. 25) represents a side view of a spherometer resting on a spherical surface. The third leg is hidden by B. E is the position of the screw end on the spherical surface. D would be its position if the surface was flat.

DE is therefore the quantity (a) measured by the spherometer. EDF is a diameter of the spherical surface ($= 2 . R$). DA is the distance (b) between the centre and outer feet when on a flat surface.

By Euc. iii. 35

$$ED \,.\, DF = AD \,.\, DG.$$

Also EF bisects AG at right angles,

$$\therefore\ ED\ (EF - ED) = AD^2, \qquad \therefore\ a\ (2R - a) = b^2,$$

$$\therefore\ R = \frac{(a^2 + b^2)}{2a}.$$

Since D is the centre of the equilateral triangle formed by the three feet A, B, C, if the distance between the feet is c, then

$$\frac{b}{\frac{1}{2}c} = \frac{2}{\sqrt{3}}, \qquad \therefore\ b = \frac{c}{\sqrt{3}}, \qquad \therefore\ R = \frac{3a^2 + c^2}{6a} = \frac{c^2}{6a} + \frac{a}{2}.$$

The distance, DE, depends on the curvature of the surface, being smaller or greater as the surface is flatter or sharper. It is sometimes called the *sagitta*.

Exp. 46.—*Find the radius of a large lens, concave or convex, by the spherometer.* (See § 30.)

31. The Travelling Microscope (Fig. 26).—A microscope is mounted on a rigid metal frame so that it

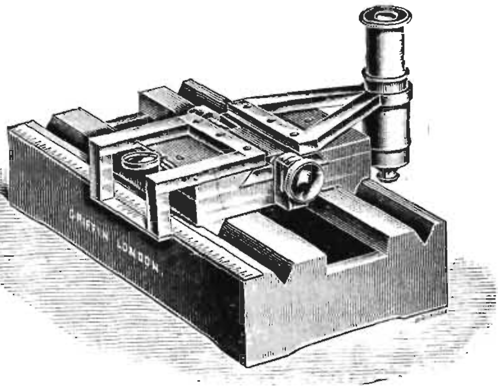


Fig. 26.

can be moved (by a rack and pinion†) horizontally along a substantial base. The microscope is usually set with its axis vertical. Sometimes it can also be placed horizontally.

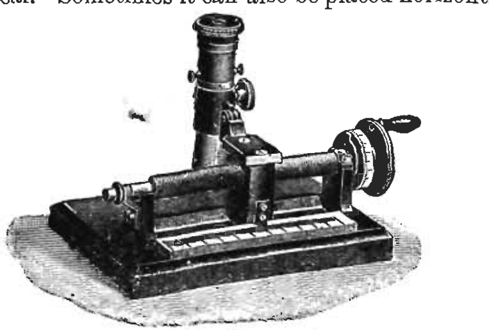


Fig. 27.

To measure the displacement of the microscope a vernier is fixed to the slider, and the principal scale to the base.

† The frame carrying the microscope (Fig. 26) is slid by hand along the base. A fine adjustment may be effected by turning the screw head seen to the right of the centre.

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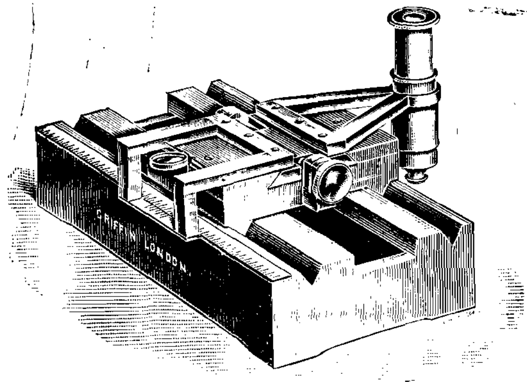

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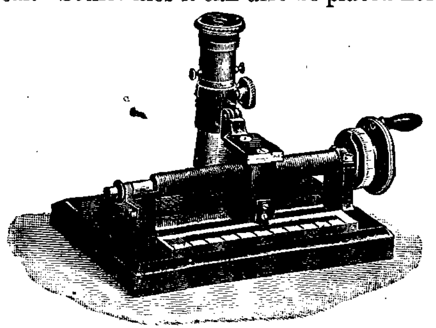

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† The frame carrying the microscope (Fig. 26) is slid by hand along the base. A fine adjustment may be effected by turning the screw head seen to the right of the centre.

In some forms the slider is moved by a screw with graduated head (Fig. 27). The eye-piece of the microscope is provided with a spider-line (§ 147). The length to be measured is clamped parallel to the principal measuring scale, and within view of the microscope; it is focussed, the spider-line in the eye-piece adjusted to one end of the length, and the *reading* (x_1) of the scales taken. Next the microscope frame is moved until the spider-line is adjusted to the other end of the length, the scales are then read (x_2). The length $= x_2 \backsim x_1$.

32. The Kathetometer.—This is an instrument for measuring vertical heights. It consists of a small astronomical telescope arranged to slide along and be clamped to a long vertical graduated rod. The tube of the telescope is adjusted horizontally by a spirit level. Its eye-piece is provided with a spider-line. Its position with reference to the vertical principal scale is read by a vernier on the telescope holder. The telescope holder is attached to the slider by a screw with a large head, so arranged that a slight movement of the telescope can be made by turning the screw after the slider has been clamped. This forms a *fine adjustment.*

Table Kathetometer (Fig. 28).—This is a short pattern of the former instrument suitable for the bench. The rod carrying the scale, etc., rests on three levelling screws, and can be adjusted vertically by a plumb-line.

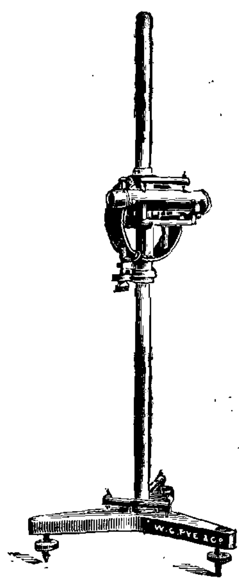

Fig. 28.

Fig. 29 shows a useful combination instrument. Either a microscope or telescope can be set with its axis horizontal, vertical, or oblique. The frame supporting it may be slid up and down a vertical

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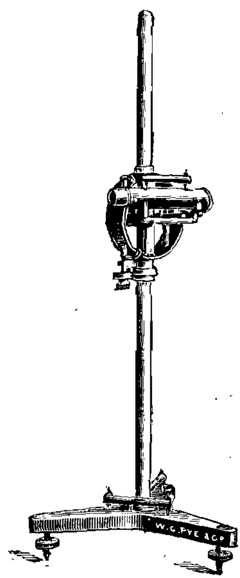

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Fig. 29 shows a useful combination instrument. Either a microscope or telescope can be set with its axis horizontal, vertical, or oblique. The frame supporting it may be slid up and down a vertical graduated pillar. The pillar also may be slid horizontally along the base. The positions of the sliders on the vertical or horizontal scales

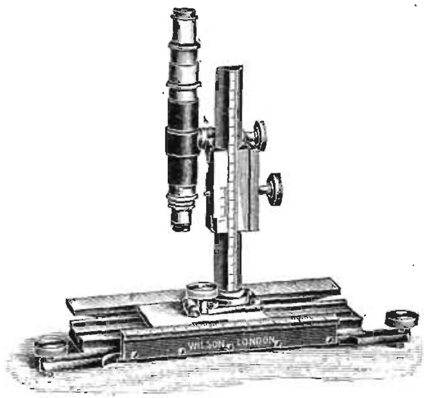


Fig. 29.

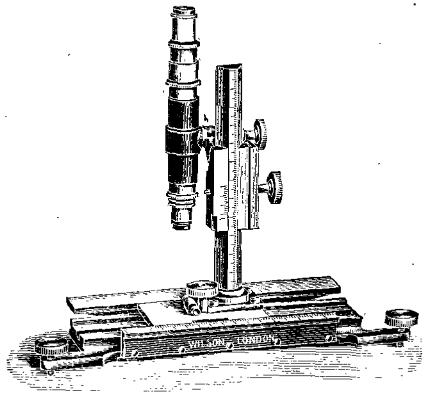
are read by verniers. The rotation of the axle carrying the telescope tube can, in some patterns, be measured on a graduated circle.

33. Note.—Scales and verniers should be well illuminated. A *naked* flame, *e.g.* a lighted match, must *on no account* be brought near them. It is usually sufficient to reflect light on them by a piece of mirror or white card. A lens is often useful in reading minute subdivisions.

Exp. 47.—*Measure by a travelling microscope* the distances between the following divisions of a thermometer: 0 to 100, 0 to 50, 50 to 100, 0 to 30, 30 to 60, 60 to 100. Plot thermometer readings horizontally, distances vertically.

Exp. 48.—*Measure by a travelling microscope* the distances between the graduations of a common hydrometer; 0·800 to 0·900 to 1·000 to 1·100 to 1·200. Plot specific gravities horizontally, distances from one end vertically.

graduated pillar. The pillar also may be slid horizontally along the base. The positions of the sliders on the vertical or horizontal scales



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MEASUREMENT OF ANGLE.

34. Angles.—The practical standard is the *right angle.* The working unit is the *degree*: this is 1/90 of a right angle (see Appendix for subdivisions, etc.). Angles are measured by a *protractor,* an instrument consisting of a sheet of metal, paper, or celluloid upon which is engraved a circle or semicircle divided into degrees, etc. The marks all radiate from a conspicuous *centre.* Sometimes the protractor is rectangular in shape, there is then no circle, but the radiating degree divisions show round the edges.

To measure an angle, place the centre of the protractor at the angular point, bring the zero line into coincidence with one arm of the angle, and note the mark at or near which the other arm crosses the divided circle.

An angle may also be obtained by measuring one of the trigonometrical ratios (Exp. 2).

In instruments of the highest accuracy there is a complete graduated circle. An arm carrying a vernier and telescope turns about the centre (see the spectrometer, § 151). (For circular vernier see § 22 (2).)

35. In some instruments the angular movement of a suspended body has to be measured, *e.g.* that of the magnet in a galvanometer. The body then has attached to it a long pointer that can move close to a divided circle, which to avoid parallax (§ 120) is mounted in front of a plane mirror. The axis about which the body turns should pass through the centre of the graduated circle. As, however, this adjustment is not likely to be exact, it is important to note the position of each end of the pointer, and to take the mean of the two readings.

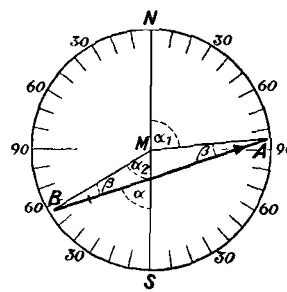


Fig. 30.

PROOF.—(See Fig. 30.) Let NS be the diameter of the graduated circle that passes through the zero, and M the centre. Let AB be the pointer attached to the suspended body.

Join MA, MB. Then the reading of A will be the angle NMA or α_1; and the reading of B will be the angle SMB or α_2. The actual angle between AB and NS is α.

Now $\alpha_1 = \alpha + \beta$ and $\alpha = \alpha_2 + \beta$, $\therefore \alpha = \frac{1}{2}(\alpha_1 + \alpha_2)$.

To measure an angular deflection read the position of each end of the pointer over the graduated circle (1) before (let the values be d_1, d_2), (2) after the deflection has been produced (let the values be D_1, D_2). The difference between the mean of the latter $[= \frac{1}{2}(D_1 + D_2)]$ and the mean of the former $[= \frac{1}{2}(d_1 + d_2)]$ is the value of the angular displacement. It is generally an advantage to obtain a deflection to the other side of the initial or zero position. When this can be done, again read both ends (D_3, D_4) of the pointer, calculate the mean deflected position of the pointer,

$$D = \frac{1}{4}(D_1 + D_2 + D_3 + D_4),$$

and subtract from it the mean initial position $= \frac{1}{2}(d_1 + d_2)$.

The pointer should be stiff, and as light and long as possible. It is frequently a thin doubled strip of aluminium placed edgewise, or a fine glass tube. A reflected pencil of light (§ 36) is an ideal index: it can be made as long as convenient, it is massless, and always straight.

36. Measurement of small angles.—If light is caught on a mirror it can be reflected to form a bright patch on a surface in another direction. If the mirror is turned, the patch of light is displaced; and it can be shown (Exp. 137) that *the reflected ray is deflected through an angle equal to twice that through which the mirror is turned.* Also, if the spot of light moves along a straight scale, *its displacements are proportional to the angular deflections of the mirror when these are small* in value.

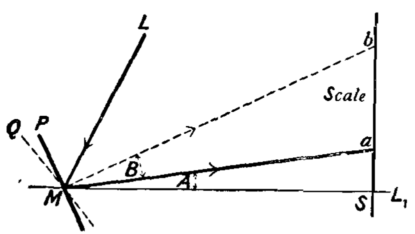


Fig. 31.

PROOF.—Suppose (Fig. 31) a light at L, mirror at M, and a scale along Sb. Let SM be perpendicular to Sb, and ML an incident ray. When the mirror is along MP, let the reflected ray, Ma, produce a spot of light at a; when the mirror is along MQ, let the spot be displaced to b. Then the *angle* $(aMb) = 2$. *angle* (PMQ). Now

$$\tan(aMb) = \tan(bMS - aMS) = \frac{\frac{bS}{SM} - \frac{aS}{SM}}{1 + \frac{bS}{SM} \cdot \frac{aS}{SM}}$$

$$= \frac{ab}{SM}\left(\frac{1}{1 + \frac{bS}{SM} \cdot \frac{aS}{SM}}\right).$$

When the angles are small (less than about 12°) their tangents are equal to their circular measures; also the product of two tangents is negligible in comparison with unity. Hence the angle, B between the reflected rays $= \tan(aMb) = ab/SM$, and the angle between the two positions of the mirror $= ab/2 \, . \, SM$. Hence, when SM is kept constant, the displacement of the spot of light, ab, is proportional to the angle (aMb), and therefore to the angle (PMQ) through which the mirror is turned.

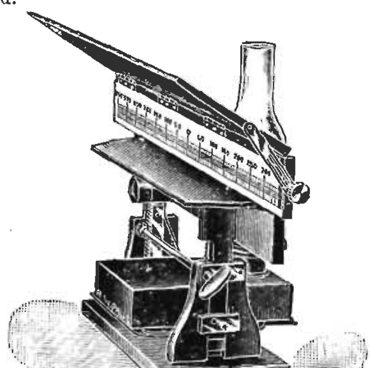


Fig. 32.

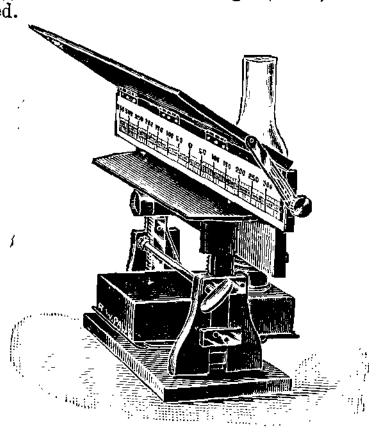
The light may be in any convenient position: usually it is along MS, a little below and behind the scale (say at L_1) and shines through a slit or hole (see Fig. 32). In order that definite measurements may be made, a clear image of a wire or slit must be produced on the screen; to effect this certain optical arrangements are necessary (§ 37). The above method is frequently employed for galvanometers, etc.

PROOF.—Suppose (Fig. 31) a light at L, mirror at M, and a scale along Sb. Let SM be perpendicular to Sb, and ML an incident ray. When the mirror is along MP, let the reflected ray, Ma, produce a spot of light at a; when the mirror is along MQ, let the spot be displaced to b. Then the *angle* $(aMb) = 2$. *angle* (PMQ). Now

$$tan\ (aMb) = tan\ (bMS - aMS) = \frac{\frac{bS}{SM} - \frac{aS}{SM}}{1 + \frac{bS}{SM} \cdot \frac{aS}{SM}}$$

$$= \frac{ab}{SM}\left(\frac{1}{1 + \frac{bS}{SM} \cdot \frac{aS}{SM}}\right).$$

When the angles are small (less than about 12°) their tangents are equal to their circular measures; also the product of two tangents is negligible in comparison with unity. Hence the angle, B between the reflected rays $= \tan (aMb) = ab/SM$, and the angle between the two positions of the mirror $= ab/2 . SM$. Hence, when SM is kept constant, the displacement of the spot of light, ab, is proportional to the angle (aMb), and therefore to the angle (PMQ) through which the mirror is turned.



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37. Common arrangements for obtaining a clear image on the scale of an illuminated slit, wire, scratch, or other object.

I. *A plane mirror with a short-focus converging lens some way off* (Fig. 33). The lens is placed about two feet from mirror, between it and the scale, and would produce a magnified inverted image of the slit behind the mirror, but the light is reflected to and the image

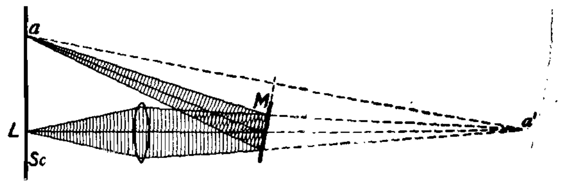

Fig. 33.

formed on the screen. Hence the slit and the screen are related like conjugate foci (§ 135). The image is considerably magnified—a disadvantage; the object and scale need not be equally distant from the mirror—an advantage.

II. *A long-focus concave mirror* (Fig. 34). The object and scale are both near the centre of curvature of the mirror.

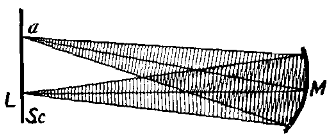

Fig. 34.

III. *A plane mirror and long-focus converging lens close to it* (Fig. 35). The object and scale are both near the principal focus of lens.

In both II. and III. the image is the same size as the object—an advantage; the object and scale are equally distant from the mirror—may be a disadvantage.

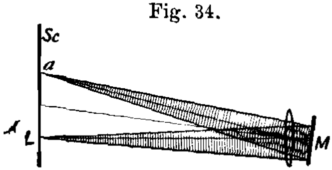

Fig. 35.

The scale is generally of millimetres, or 1/40 inch. Often the zero of the scale is at the middle, and the divisions numbered consecutively to the right and left of it. It is, however, more convenient to have the zero at one end—say, the left. The scale may be engraved on nearly transparent paper. The spot of light can then be seen at the back, *i.e.* when the scale lies between the mirror and the observer.

An effective way of getting a well defined line imaged across a round patch of light on the scale is to scratch a line across the centre of a short focus lens; mount this in a hole in the screen (Fig. 32), and place it near the light with the scratch vertical, and its side of the lens away from the light. The lens acts as a lantern condenser, and the scratch as the object on the slide.

The light is usually that of an oil or gas flame, or an electric glow lamp may be used. The last may be placed so that a straight portion of the filament is focussed by the mirror as a bright line of light on the screen: the lens and scratch, etc., mentioned above will then not be necessary.

A poorly lighted room or corner is required for the measurements.

IV. **Poggendorff's method with plane mirror and telescope.**—A telescope is arranged at L_1 (Fig. 31), and focussed so that the image of the scale is seen in the plane mirror. The eye-piece of the telescope must be provided with spider-lines. The scale should be well illuminated. The scale and telescope may be at different distances from the mirror. A darkened room is not necessary, but the telescope and mirror may need to be screened from bright lights. A bright well-defined image is seen in the telescope, it is not distorted but is "laterally inverted."

38. To set up scale, wire, etc., for measuring the deflections of a suspended mirror.—(i) See that the suspended body and mirror can turn freely: if not, make the necessary adjustments of levelling screws, etc. If the body is a magnet it can be easily deflected by bringing a pocket-knife near to it.

(ii) Place about two yards in front of the mirror (which must be at rest) a lighted candle and look for its image. *When found keep an eye on it.* If the arrangement is that of II. or III. the image will be in front of the mirror, inverted, and should be diminished (if not, place the candle further off). If there is a plane mirror only, the image will be behind the mirror, erect and of the same size.

(iii) Get, by small, carefully followed movements, the candle, its image, and the mirror nearly in line.†

A. *If the arrangement is that of II. or III.*,

(iv) Hold a sheet of card or paper near the candle and slowly move it towards the mirror, until the image is defined. (If not found, place the candle further off.) Take care that the card does not prevent the light from the candle reaching the mirror.

(v) *Slowly* push the candle towards the mirror and follow the image (it should move from the mirror) with the card until the candle and its image are side by side.† Then mark the position.

† Move, in turn, each thing a little bit at a time. It is well to keep one hand in the pocket.

(vi) Substitute lamp and scale for candle and paper, and adjust until a well focussed and illuminated image is got on the scale, over the object.† The lamp flame should be put nearly edgewise to the scale.

(vii) Set the scale perpendicular to the line joining the mirror and image by making its ends equidistant from the mirror (measure with string).

B. *If the arrangement is that of I.*,

(iv) Place the lighted lamp and scale perpendicular to the line obtained in (iii) and as far from the mirror as it finally should be. Put a piece of card beside the mirror. Place the lens between the object and mirror, and close to the former. Slowly move it *away* from the light until an image is formed on the card.

(v) Adjust the lens without altering its distance so that the light that passes through it falls on the mirror; then move it, say, an inch *towards* the light, and catch the image reflected from the mirror on a card. (Be careful that the card does not cut off the light from the mirror.) Then again move the lens slightly towards the light, and follow up the image with the card† until the image is near the scale.

(vi) and (vii). Adjust as in A(vi) and A(vii) above.

† Move, in turn, each thing a little bit at a time. It is well to keep one hand in the pocket.

CHAPTER III.

MEASUREMENT OF MASS.

THE BALANCE.

39. Weighing.—The balance is used for the comparison of masses. For convenience and accuracy it is made with arms of practically equal lengths. The operation of comparing masses is called *weighing,* and a body whose mass is required is usually compared with certain bodies of known mass, *e.g.* the members of a *set of* "*weights.*"

The balance compares forces. But the forces being weights are proportional to the masses used. The set of weights is really one of masses; the mass of each is the same everywhere, the weight varies slightly from place to place. However, the ratios of the members of the set are constant everywhere whether we consider the weights of each, or their masses, and the result of a comparison by the balance is the same wherever it is performed.

Other weighing machines are the *steelyard* (§ 51)—a balance with unequal arms—and the *spring balance* (§ 52).

The units of mass and the relations between them are given in the Appendix. Simple experiments on the measurement of mass, etc., are given in Chapter I.

40. The Balance (Fig. 36).—The following is a useful form for elementary work. The *beam, AB,* has a central *knife-edge* or fulcrum, *C,* of agate or steel. This rests on a flat surface of hard material fixed to the top of a support, *Q.* It is midway between, and about 5 or 6 inches from *two* other *knife-edges,* at *A* and *B,* close to the ends of the beam. The edges of the latter point upwards and support the stirrups, *D,* from which the *scale pans, S,* are hung.

The *masses to be compared* are placed in these pans. A *pointer* or *index* is fixed to the beam. This, when the beam is swinging, moves in front of a short scale of equal parts (unit unimportant) fixed at the foot of the pillar. Thus the index marks the position of the beam, which is practically horizontal when the end of the pointer is in front of the middle line of the scale. The *adjusting nuts, n* (sometimes there is one only), consist of a screw stem upon which the nut can travel. Turning either nut so that it moves towards the right or left displaces the pointer in the opposite

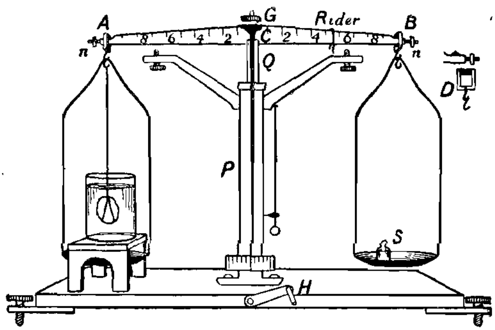

Fig. 36.

way owing to the change in the equilibrium position of the beam. For instance, if, when the beam is free, the end of the pointer is opposite division 6 on the left, then it may be brought nearer the middle mark, that is *displaced to the right* by moving *either nut to the left*. It is convenient to have the beam graduated in equal parts (§ 45).

The Arrestment. The central support, Q, can be moved slightly up and down within the pillar, P, by the handle, H. When the handle is over to the left the support is in its lowest position, and the beam is lifted from it by the rods and frame springing from and fixed to the top end of the pillar,† also the scale pans rest on the base-board. The beam is now *arrested*, or the balance is *out of action*. The arrangement saves wear and tear of the knife-edges. Turning the handle *completely over to the right* (as in Fig. 36) raises the support, and lifts the beam and pans. The beam is now *free* and the balance is *in action*.

The pillar is fixed upright to a base-board, and is sometimes provided with a plummet and the base-board with levelling screws. By these the pillar may be adjusted into a vertical position.

41. In practice.—The balance is in working *adjustment* if, when both pans are unloaded, and the beam free, the end of the pointer moves to and fro in front of the short scale.

The to and fro or vibratory motion of the pointer gradually diminishes in range or *amplitude*, and presently the movement stops. The scale division in front of which the end of the pointer comes to rest is called the *resting point* when the pans are loaded, or the *zero point* or *equilibrium position* of the balance when the pans are unloaded. **To determine a resting point at any load** it is convenient not to wait until the swinging stops, but to proceed as follows: Observe the end of the moving pointer from a position in front of, and about two feet away from the short scale. Note the *turning points* or the two scale divisions on the right and left that mark the ends of the movement of the pointer. The resting point may be assumed to be midway between these divisions.‡

The zero point of the unloaded balance is determined before bodies are weighed. The mass of a body is obtained by manipulating the standard masses, etc., as described later, until the pointer makes equal excursions to the right and left of the zero position. The balance is then said to be *in equilibrium*, and the mass of the body is practically equal to the sum of the known masses in the other scale pan.

† In some balances the beam is not lifted off the central knife-edge, the scale pans are merely let down on to the base-board.

‡ An accurate method of finding the resting point of a covered balance is described in § 43.

42. **More sensitive balances**, capable of measuring a difference of 0·0001 gramme between two masses of 100 grammes each (that is one-millionth of the mass measured), are built in a similar manner to the rougher balance shown in Fig. 36. The maker, however, pays greater attention to the keenness of the knife-edges, and the smoothness and accuracy with which the arrestment works. The arrestment handle is often at the left hand side of the base board. The balance is enclosed in a glass case with sliding doors in order that its vibration may be undisturbed by air currents. The set of weights, each of which must have, as exactly as possible, the value specified upon it, consists of brass units of mass and aluminium or platinum fractions. A centigramme rider is used to determine the third and fourth decimal places. The weighing is performed with the case open until it is necessary to use the rider. The doors are then closed. The rider is manipulated by means of a sliding rod. The left end of this carries a fork by which the rider may be picked up from or laid upon the beam; the right end projects through the side of the balance case, it can be pushed and twisted by the hand so that the rider may be placed on any part of the beam.

43. **The resting point of a very sensitive balance** is not determined quite so simply as in § 41. Owing to the falling off in the range of vibration an excursion on one side is slightly further than that of the succeeding excursion on the other side. Hence the resting point is not exactly midway between two turning points. **To determine a resting point at any load** set the balance vibrating, observe three or five successive turning points, calculate the mean of the turning points (i) on the left side, (ii) on the right, (iii) the mean of these two means; this is the required resting point (Exp. 49).

To read the position of the pointer on the scale.—Call the middle division 100, and number from left to right .... 70, 80, 90, 100, 110, 120, 130, ... Watch the moving pointer and estimate the position of a turning point by mentally dividing the space in which it occurs into tenths (Example, § 50). To avoid parallax error (§ 120) it is convenient to place a strip of mirror behind the pointer, close to, and parallel with the scale.

44. Set of Metric Weights.—These are usually as follows:

(i) Brass weights (gilded)

100,	50,	20,	10,	10,	5,	2,	2,	1	gms.

(ii) Platinum

	0·5,	0·2,	0·2,	0·1	gm.
marked	500,	200,	200,	100	mgms.

(iii) Platinum or aluminium

	0·05,	0·02,	0·02,	0·01	gm.
marked	50,	20,	20,	10	mgms.

In a "box of weights" each member of (i) fits into a hole, and there are compartments for the fractional values. Tweezers or pliers are provided for handling the weights.

By set (i) above, any mass from 1 to 200 grammes may be measured in multiples of the gramme.

By sets (ii) and (iii), decimals of a gramme may be measured in multiples of 10 milligrammes.

Examples:

176·35 = (100+50+20+5+1) gms. + (200+100+50) mgms.

99·08 = (50+20+10+10+5+2+2) gms. + (50+20+10) mgms.

In *weighing a body* it is best not to add the weights haphazard but in descending order of magnitude, the equilibrium being tested by releasing the balance after each addition. Consider the following:

Weights in pan.	Equilibrium test.
100	too little
100 + 50	too much
100 + 20	too much
100 + 10	too little
100 + 10 + 5	too little
100 + 10 + 5 + 2	too much
100 + 10 + 5 + 1	too much

Thus the mass is greater than 115, less than 116. The fraction can be found by adding the smaller weights in a similar manner, and finally by using a rider.

45. Weighing with a rider.—The *rider* is a piece of wire of definite mass (say 0·01 gramme, or 1 gramme) bent to straddle over the beam, which between the central and right hand knife-edges is divided into equal parts, conveniently 10. The dividing lines are numbered 1, 2 . . . 9, the mark nearest the centre being 1.

The use of the rider may be understood if the moment of its *weight* about the centre is considered.

Thus, if its mass is 1 gramme and the half-beam is divided into tenths, then when the rider is

on division	5	*it is equivalent to*	0·5	*grm. in the*	*scale pan*
on division	4	,,	0·4	,,	,,
midway between 6 and	7	,,	0·65	,,	,,
$\frac{3}{10}$ of length from 7 to	8	,,	0·73	,,	,,

A rider of mass 0·01 gramme is used in a sensitive balance to estimate fractions less than 0·01 gramme.

For balances like Fig. 36 it is very convenient to have a rider of mass 1 gramme and the half-beam graduated into tenths. The rider may be pushed along the beam by a pencil. With such a rider weights less than 1 gramme—so easily lost—need not be used.

The rider may be a U-shaped piece of stout brass wire, about 2″ long, cut down to a mass of exactly 1 gramme. The division marks may be determined geometrically and scratched across the beam.

RULES TO BE OBSERVED IN USING A BALANCE.

46. (1) Before commencing to weigh see that the pans are clean and dry, the rider off the beam, and the stirrups not dislodged; also that the plummet or spirit level shows the balance to be correctly levelled. Use a camel-hair brush for dusting the scale pans, etc., if necessary.

(2) *While weighing* observe the short scale of the balance from a position directly in front, and about two feet off. Press the left hand on the balance base, work the arrestment handle with the right.† *Hold the handle until it is turned completely over to the right or left.* Release or arrest the balance slowly and without jerking.

(3) *Determine the zero or resting point* of the unloaded balance (§ 41).

(4) Place the box of weights near the right hand end of the balance base; the body to be weighed in the left hand

† This is important to beginners: the right hand should work the handle and manipulate the weights in turn. Two pupils should not be permitted to cooperate in weighing. When there is a partnership one should do the weighing, the other look on.

pan; add the weights to the right hand pan. Weights, except small ones, or those of the greatest accuracy, may be carried by the head between the finger and thumb. The fingers, etc., must be clean. *On no account should weights be kept in the hand:* they must be either on the balance or in the box. *The balance must be arrested when a weight is to be added to or taken from the pan.*

(5) A body must not be weighed when hot. All substances liable to injure the pans must be put in appropriate vessels.

(6) Add the known masses in descending order of magnitude (§ 44). The larger masses should be put in the centre of the pan. On no account must the balance be loaded with a weight greater than the maximum it is constructed to carry. (This is sometimes stamped on the instrument.)

(7) If the balance does not swing when released, either arrest and release again, or, by moving the hand in the neighbourhood, beat some *air* down on a pan. *The pointer must not be touched.*

(8) When equilibrium (referred to the zero point of the balance) has been obtained sum up the masses in the scale pan, and confirm by observing what spaces in the box are empty. Add to this the amount indicated by the rider. Note the total value. Finally *replace the weights in the box.*

47. Requirements of a balance.—It is important that a balance should be

(1) *true,* that is, the mass in one pan should be equal to that in the other;

(2) *stable,* that is, whatever the load, within limits, the balance should vibrate about its original resting point or nearly so;

(3) *sensitive* or *sensible,* that is, a very small overweight should produce a perceptible displacement of the resting point.

A balance should swing somewhat quickly (period, 10 to 15 secs.) in order that weighing may be done rapidly.

A balance is constructed, as nearly as possible,

(i) With arms of equal length and scale pans of equal mass, to secure truth.

(ii) With a long and light beam to attain sensibility, but shaped and stiffened so as to be rigid. When rigid it will be in stable equilibrium for all loads, within limits, if it is stable for any load.

(iii) With the central and end knife-edges in one plane. The sensibility is then practically constant for all loads.

(iv) With the centre of gravity of the beam below, but very close to the central knife-edge. The smaller the distance between these, then the greater the sensibility, the slower the swing, and the less the stability. In practice some sensibility is sacrificed in order that the period of vibration may be moderately rapid.

(v) With three keen knife-edges working on hard surfaces. This ensures that when bodies are put in the scale pans the forces exerted on the beam always act vertically through the knife-edges, and therefore the forces through the end knife-edges are at constant distances from the central knife-edge. The positions of the masses in the scale pans are then unimportant.

48. Testing a balance.—(1) *Stability.* A balance when unstable will not vibrate about its central knife-edge. The *gravity bob* on the top of the beam (*G*, Fig. 36) influences the stability. The nut can be screwed up and down the vertical stem: the lower the nut the lower the C. G. of the beam. Hence, when a balance is unstable, the gravity bob should be lowered.

A change in the position of the gravity bob also affects the sensibility. The gravity bob should be altered as seldom as possible.

(2) *Time of vibration.* This should be from 10 to 15 secs. To find it set the balance oscillating and note the time of 12 complete swings.

The vibration period increases, that is, the beam swings more slowly, as the sensibility increases.

(3) *Constancy of zero point.* Arrest and release the balance two or three times: then determine the zero point. Repeat several times. Also load the balance with, say, 25 grammes, release and arrest two or three times, then remove the masses in the pans, release, and determine the zero point. Replace the load and operate as before. Also use loads of 50, 75, etc., grammes, and repeat the operations. The differences in the values will indicate the degree of inconstancy of the zero point. This should be observed frequently.

(4) *Equality of masses of scale pans.* Determine the resting point of the unloaded balance before and after interchanging the scale pans. If their masses are equal the resting points should not differ more than would be expected from the observed inconstancy of the zero point.

(5) *Ratio of the balance arms* (see § 50).

(6) *Sensibility of a balance at load, P.* (1) Place the mass, *P*, in left pan: adjust masses and rider on the right-hand

side so that the resting point will be near one end of scale. (2) Determine the resting point (x). (3) Shift the rider through d (two or three) divisions of the beam so that the resting point will be near the other end of the scale. (This may be done, if *care is taken*, without arresting the balance.) (4) Determine the resting point (y).

Thus, when the load is P, the resting point is displaced $(x-y)$ divisions when the rider is shifted d divisions. If the mass of the rider is m, and the half-beam is divided into tenths, then the shift of the rider is equivalent to a change of mass in the pan of $\frac{1}{10}d.m.$ Then, since a mass $\frac{1}{10}\,d.m$ displaces the resting point $(x-y)$ divisions, unit mass will displace the resting point $(x-y)/(\frac{1}{10}\,d.m)$ divisions. The displacement per unit mass measures the sensibility. In a delicate balance the sensibility is reckoned as the displacement of the resting point due to a milligramme. In practice it is also convenient to calculate the mass required to produce a displacement of 1 division. It is equal to $\frac{1}{10}\,d.m/(x-y)$, that is, it is the reciprocal of the sensibility.

EXAMPLE.—Load, 50 gms	Resting point, (i) 6·2.
Shift of rider (0·01 gm.), 4 divs.	Resting point, (ii) 15·7.
	Displacement of resting point = 9·5.

Mass in pan equivalent to shift of rider

$$= \tfrac{1}{10} \times 4 \times 0{\cdot}01 = 0{\cdot}004 \text{ gm.} = 4 \text{ mgm.}$$

Then displacement of resting point due to 1 mgm. in pan or *sensibility* $= 9{\cdot}5/4 = 2{\cdot}4$ divisions.

Mass in pan that would produce a displacement of 1 scale division $= 4/9{\cdot}5 = 0{\cdot}42$ mgm.

PRACTICE.—The sensibility of a balance should be determined for several loads (0, 25, 50, 75, 100, etc.), and two curves plotted, viz.:

(1) The *sensibility curve*: divisions per unit mass as ordinate, load as abscissa.

(2) The *weighing curve*: mass per unit displacement (one division) as ordinate, load as abscissa. See weighing by vibration (§ 49).

49. Weighing by Vibration.—The determination of the sensibility leads to a simple and accurate method of *weighing* by *vibration*, as follows:

(1) Determine the zero point, a.

(2) Obtain approximate equilibrium between the unknown mass, X, and standard masses + rider (total $= P$). Determine the resting point, x.

(3) Shift the rider (mass, m) through 1 division ($\frac{1}{10}$ of half-beam). Determine the resting point, y.

Suppose the resting points are in the order a, x, y.

Now $(y - x)$ is the displacement of the resting point due to a

$$\text{mass} = \tfrac{1}{10}\, m.$$

Therefore unit displacement is due to a

$$\text{mass} = \tfrac{1}{10}\, m/(y - x).$$

Hence to displace the resting point from x to a would require a mass

$$p = \tfrac{1}{10}\, m\,(x - a)/(y - x).$$

Then
$$X = P + p.$$

NOTE.—In an actual experiment care must be taken to decide whether p is to be added or subtracted. Work from first principles.

EXAMPLE.—Balance unloaded; zero point, 10.

Mass of rider, 1 gramme.

X grms. on left side; 47·4 grms. on right side; resting point, 15·7.

X grms. on left side; 47·5 grms. on right side; resting point, 13·3.

Then difference between the zero point and one of the resting points $= 13{\cdot}3 - 10 = 3{\cdot}3$.

Displacement of the resting point due to 0·1 grm. = 2·4.

∴ To displace the resting point one division requires $0{\cdot}1/2{\cdot}4 = 0{\cdot}04$ grm.

∴ To displace the resting point 3·3 divisions requires $0{\cdot}04 \times 3{\cdot}3 = 0{\cdot}132$ grm.

$$\therefore\ X = 47{\cdot}5 + 0{\cdot}13 = 47{\cdot}63 \text{ grammes.}$$

Exp. 49.—*Find the mass of a coin by the method of vibration.* Use a less sensitive balance and gramme rider.

Exp. 50.—*Find the sensibility of a balance at loads 0, 25, 50, 75, 100, etc.* (§ 48 (6)). For a more sensitive balance find the several resting points as in § 43; other balances as in § 41.

RECORD of an experiment on the sensibility of a balance. The index scale was counted as in § 43, the sensibility determined as in § 48 (6).

Load.	Turning Points.		Resting Point.	Added to one side.	Turning Points.		Resting Point.	Sensibility for 1 mgm.
0		160		0·003 gm.	15			
	95					65		
		156			17			
	98					62		
		154			19			
	96·5	156·7	126·6		17	63·5	40·3	28·8
25 gms.	17			0·002 gm.	120			
		129				137		
	21				120			
		125				136		
	26				121			
	21·3	127	74·2		120·3	136·5	128	27·1
50 gms.	80			0·002 gm.		72		
		161			50			
	83					71		
		159			50			
	86					70		
	83	160	122		50	71	61	31·0

SUMMARY: Load	0	25 gms.	50 gms.	75 gms.	100 gms.
Scale divs. per mgm.	2·9	2·7	3·1		
Gms. per scale div.	0·00034	0·00037	0.00032		

50. Exact Weighing.—The determination of mass is affected by three important errors.

(1) Inequality of the arms of the balance.
(2) Difference of density between the weights and the body weighed.
(3) Errors in the relative values of the weights.

(1) The effect of *inequality of the arms* is eliminated by adopting the method (A) of Borda, or (B) of Gauss.

A. Method of Borda (Substitution, or Taring).—(1) The body to be weighed is placed in the right pan, and counterpoised by lead shot and clean dry sand in the left. (2) The body is removed from the right pan, known masses substituted, and the rider adjusted until equilibrium is obtained. The substituted mass equals that of the body. This method is applied in the steelyard and spring balance.

IN PRACTICE adjust so that any resting point, x, may be found for (1), and any resting point, y, for an approximate value of (2). Calculate from the sensibility what mass is required to displace the resting point from y to x.

B. Method of Gauss.—(1) The unknown mass, X, in, say, the left pan is balanced against known masses, P, in the right. (2) The masses, P, are then removed, X is put into the right pan, and balanced by known masses, Q, in the left.

If L and R are the lengths of the right and left arms respectively, then $X.L = P.R$, $Q.L = X.R$;

$$\therefore X = \sqrt{(P.Q)}.$$

Thus the true mass is the geometric mean of the observed masses. Usually Q and R are nearly equal. Hence the arithmetic and geometric means are practically equal. Hence

$$X = \tfrac{1}{2}(P + Q).$$

Ratio of the balance arms.—From the above equations it follows that $L/R = \sqrt{(P/Q)}$. Hence the ratio of the balance arms may be calculated. One of the weights from the box may conveniently be used for X.

IN PRACTICE adjust so that any resting point (x) may be found for (1) and any resting point (y) for an approximate value of (2). Calculate from the sensibility what mass is required to displace the resting point from y to x.

(2) **Difference of density between the weights and body weighed.**—Owing to the buoyancy due to the displacement of air the so-called *apparent weight* (= *true weight* − *buoyancy*) of a body is not quite proportional to its mass. Let M be the mass of a body, Δ, its density, then, in *C.G.S.* units,

$$volume = M/\Delta.$$

If the air has density, δ, then

$$mass\ of\ air\ displaced\ by\ the\ body = \delta M/\Delta\ ;$$

$$\therefore\ the\ apparent\ weight\ of\ body = \left(Mg - \frac{\delta Mg}{\Delta}\right) = Mg\left(1 - \frac{\delta}{\Delta}\right).$$

If a standard mass, P, of density, D, is used, then

$$the\ apparent\ weight\ of\ P = Pg\left(1 - \frac{\delta}{D}\right).$$

Hence when M and P equilibrate on the balance, assuming that the apparent weight of the body = apparent weight of standard masses, then

$$Mg\left(1 - \frac{\delta}{\Delta}\right) = Pg\left(1 - \frac{\delta}{D}\right).$$

Since M and P are nearly equal and δ/Δ is small, assume $M\delta/\Delta = P\delta/\Delta$,

$$\therefore\ M = P + P\delta\left(\frac{1}{\Delta} - \frac{1}{D}\right).$$

Assuming $\delta = 0{\cdot}0012$, and for brass (weights) $D = 8{\cdot}4$, then $\delta\left(\frac{1}{\Delta} - \frac{1}{D}\right)$ may be calculated for different values of Δ. The results give the *amount per gramme* to be added to the observed mass, P.

TABLE OF CORRECTIONS FOR BUOYANCY.

Density of substance weighed.	Add per gramme to mass observed.
0·8	+ 0·00136
1·0	+ 0·00106
1·5	+ 0·00066
2·5	+ 0·00034
5·0	+ 0·00010
8·4 (brass)	0
12·0	− 0·00004
13·6 (mercury)	− 0·00005
20·0	− 0·00008

The above values should be plotted by taking densities as abscissae, and the increments per gramme as ordinates.

(3) **Errors of weights.**—Their determination is difficult (see a more advanced work on Practical Physics). In a first class set of weights the errors are practically negligible for ordinary work.

Exp. 51 —*Find the true mass of a sovereign and the ratio of the balance arms.* Use a more sensitive balance. Weigh (i) by Borda's, (ii) by Gauss' method. Correct for buoyancy.

51. **The Steelyard** is a lever balance with unequal arms. In Fig. 37, the beam, *AB*, is movable about a knife-edge,

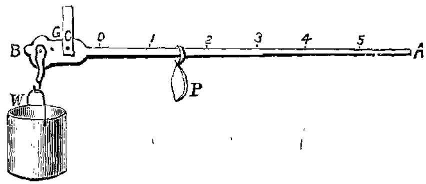

Fig. 37.

C, fixed near one end. From *B* a scale pan is suspended, in this the body to be weighed is placed; the movable weight, *P*, is pushed along the arm, *CA*, which is graduated and numbered, so that the division at which *P* rests, when there is equilibrium, indicates the mass of the body in the scale pan in lbs., etc.

Sometimes the scale pan containing the body to be weighed is adjusted on the graduated arm, and a weight is kept at one position on the short arm (Exps. 16 and 17). In the Danish steelyard the whole beam is shifted with regard to the fulcrum.

Exps. 16, 17, 18, 19, in Chap. I., illustrate the steelyard.

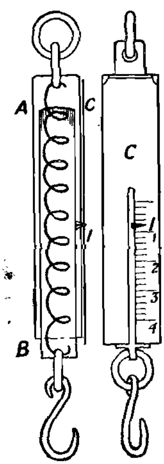

Fig. 38.

52. **The Spring Balance.**—In this the weight of a body is measured by the extent to which it lengthens a spring. (See Exp. 76.) A simple form is shown in Fig. 38. The tube, *B*, moves easily inside another, *A*. A spring connects the ends of the tubes. A plate, *C*, is fixed to the outer tube. The plate is slotted: an index, *I*, fixed to the inner tube passes through the

slot. When the spring is stretched the index moves downwards, the extension being indicated on the scale graduated on the plate. The scale (one of equal parts) is *direct reading;* it is graduated so that the value of the force applied to the spring in lbs. wt., etc., is indicated by the number of the division to which the index is pulled. When the force is due to a body in the scale pan the spring balance indicates the mass of the body in lbs., etc., because the weight of unit mass is the unit (gravitation) of force in terms of which the instrument is graduated.

By using springs of different lengths and thicknesses a wide range of forces may be measured. The spring balance, however, is far less sensible than a lever balance. It has, however, the advantages of being direct reading, quick, compact, and portable.

CHAPTER IV.

TIME. THE PENDULUM.

53. Measurement of Time.—This is generally done by clocks and watches. In these a wound-up spring or lifted weight is prevented from running down freely, and compelled to move very slowly and regularly by a *controlling* agent, *e.g.* a vibrating *pendulum* or *balance-wheel*. Experiments show that for either of these agents the time of vibration is practically constant. In timekeepers the swinging pendulum or balance-wheel moves, at regular intervals, an *escapement;* this is an arrangement by which a toothed wheel of the mechanism is alternately released and arrested at each tooth in succession. Such proportions are taken, and things are so arranged that the hands and face of the instrument measure the hours, etc., of the *mean solar day* (§ 3). We shall assume that the student can tell the time, and will *check* his watch by comparison with a standard clock or chronometer showing *mean solar seconds*. As a rule it is not important in a physical experiment to know the exact time of day. It is, however, necessary *to rate* the watch, that is, *to find accurately what fraction of the mean solar second* is indicated by it. To do this, observe simultaneously, at a convenient moment, the time by the watch, and a standard clock. Repeat the observation about 24 hours afterwards. If the interval of time by the standard clock is t mean solar seconds, and by the watch x seconds, then 1 second by the watch $= t/x$ mean solar seconds.

The **stop-watch** is frequently convenient for measuring intervals of time. A common kind is wound, like a keyless watch, by twisting the head. Pressing down the head first sets the watch working, pressing down the head a second time stops the watch, a third time, brings the hands back to zero. The dial over which the large hand moves is divided into fifths of a second. A *small* hand records the complete turns of the large, that is to say minutes. Hours are not shown. In an accurate experiment the stop-watch must be *rated* (see below) by comparison with a standard clock.

The **metronome** (Fig. 39).—In this instrument a spring is wound up by a key at the side. A vertical bar is kept

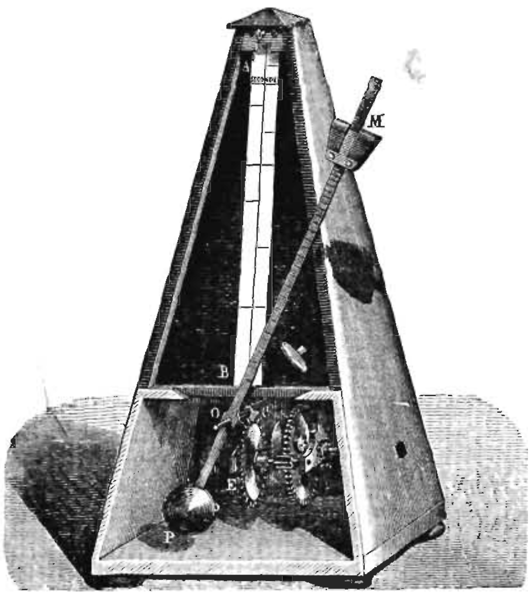

Fig. 39.

in vibration, and at regular intervals a loud *tick tack* is heard. The interval is altered by sliding a small weight, M, along the vibrating bar. By this means the instrument can be adjusted to beat seconds, half-seconds, etc. In some metronomes a bell is struck, say, every second, or fourth beat, etc., as arranged.

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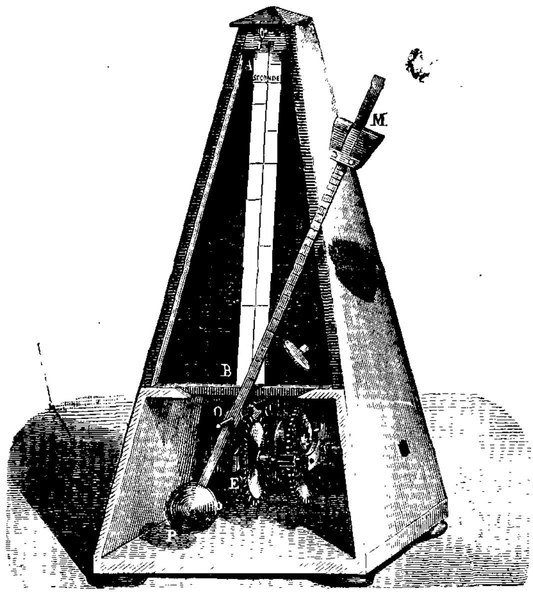

Fig. 39.

in vibration, and at regular intervals a loud *tick tack* is heard. The interval is altered by sliding a small weight, *M*, along the vibrating bar. By this means the instrument can be adjusted to beat seconds, half-seconds, etc. In some metronomes a bell is struck, say, every second, or fourth beat, etc., as arranged.

The metronome is a fairly accurate time-keeper. It must be frequently wound up. Its beating should be *rated.* To do this put the instrument some distance off where it can be heard but not seen, look at the seconds hand of a standard clock, count the number of ticks of the metronome in a given time by the clock (5 minutes, say). If there are n ticks in s seconds, then the time between two ticks is $s/(n-1)$ seconds.

Tuning-fork.—To measure minute intervals of time, say $\frac{1}{100}$*th second,* a *tuning-fork* is used. This is kept in vibration by an electro-magnet (Fig. 40). A stiff bristle or fine needle-point attached to one prong is adjusted to touch a smoked surface. A waved line is traced when the

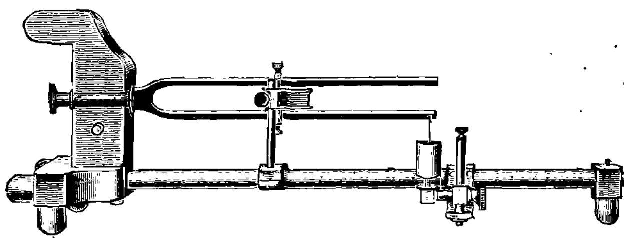

Fig. 40.

fork vibrates and the surface moves simultaneously. *Electro-chronographs* are based on this principle.

Standard clock and chronometer.—These instruments, which serve to measure time with very great accuracy, are notable for the care and skill that have been devoted to their design and construction.

The pendulum of the *standard clock* is about a metre long, and has a period of two seconds. It is compensated for temperature, usually by Graham's method (mercurial). The minute hand moves over a large dial, the second and hour hands each over a smaller dial. The final regulation of the pendulum is best done, not by adjusting screws, etc., but by loading with small pieces of metal.

The *chronometer* is practically a large watch. The mechanism is worked by a spring, and controlled by a compensated balance wheel.

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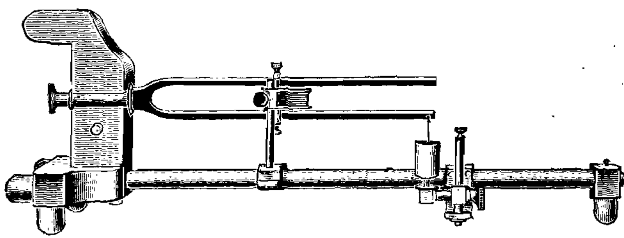

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54. The Simple Pendulum.—Mathematically this consists of a material particle at the end of an inextensible massless string. It is practically realised by hanging a heavy ball (called the *bob*) at the end of a thread or fine wire.

To set up a simple pendulum.—Get a heavy lump of metal (preferably a small lead ball), hang it from a nail by a *fine* string or wire, five or six feet long. The attachment needs care. Lay about an inch of the wire along the nail, and bind it *tightly* there by several turns of fine wire or strong thread. If not fixed closely enough the point about which the bob swings will be indefinite.

This can be shown by making a loop an inch or two long, hanging it over the nail, and setting the pendulum swinging: its plane of vibration soon gets askew. The experiment is especially worth trying with a long loop hung over two nails a foot or two apart. Vary the length of the loop. The arrangement illustrates a *Blackburn pendulum.*

If the bob of the pendulum is pulled aside and then let go, it begins and maintains a to-and-fro movement or *vibration* in the arc of a circle whose radius is the distance between the point of suspension and the centre of the ball. This distance is practically the so-called *length of the pendulum.* Half the distance between the ends of the arc is the *amplitude of vibration.* The *equilibrium position,* or point on the arc vertically beneath the point of suspension, is the lowest position through which the bob passes and the one at which it will eventually rest. It very nearly bisects the arc, but not quite so, for the amplitude gradually diminishes, owing mainly to the resistance of the air to the movement of the bob. If the bob is light for its size the extent of its vibration will *decay,* or *be stilled* or *damped,* sooner than if heavy (Exp. 52).

In vacuo the position of equilibrium would be more exactly the mid point of the arc, and the vibration would keep up much longer. (If an electric glow lamp with a snapped filament is shaken, the filament vibrates much longer when air has not leaked in than when it has. This well illustrates the *damping* or *stilling* effect of the air.)

The time taken by the bob in swinging to and fro is called the *time of a complete vibration,* or *the period* of the pendulum. The time of going to *or* fro is sometimes taken, and called the *time of semi-vibration*, or ***time*** *of vibration*, or *time of swing*.

It can be shown mathematically † that when the arc of vibration is anything less than about 10°, the

$$\text{Period } (t) = 2\pi \sqrt{\frac{\text{length of pendulum } (l)}{\text{acceleration of gravitation } (g)}}.$$

Hence (i) for *any small arc the swings are isochronous* (equal-timed).

(ii) *The period is independent of the mass of bob.*

(iii) *The square of the period is proportional to the length*, or l/t^2 is a constant quantity $= g/4\pi^2$.

(iv) $g = 4\pi^2 \, l/t^2$ and $\log g = \log 4\pi^2 + \log l - 2 \log t$.

g will be in *feet sec. sec.* when t is in *secs.* and l in *feet.*
g ,, *cms. sec. sec.* ,, t ,, *secs.* ,, l in *cms.*

55. To measure the period of a pendulum.—When the pendulum is at rest make a mark on the wall or erect a rod (retort stand) behind the bob. Set the pendulum swinging. Stand 4 or 5 yards off and note by a watch, or start a stop-watch at the moment when the bob makes a *transit*, that is, passes in front of the mark. Again observe the time of transit when the pendulum has performed, say, 100 complete swings. This can be done by counting either each transit from right to left *or* the reverse way.

The experimenter will find it convenient to go to the point of observation, write down a time of day in hours and minutes slightly ahead of the actual time, then observe how many seconds after this *noted* time a transit occurs. Add the seconds to the noted time, and call the *next* transit in the same direction, *one*, the following transit, *two*, etc.

† Expressions of the same form as that for the simple pendulum give the periods of bodies whose vibrations are due to forces other than gravitation. (See §§ 76, 170.)

***Exp. 52.**—*Show that the period is independent of the mass of the bob, but dependent on the length of the pendulum.* Hang up 3 pendulums (say 1 yard long) with bobs of different materials (say iron, glass, wood). Push them aside (by a book): release them at the same moment (withdraw the book). Adjust the lengths so that each bob passes its equilibrium position simultaneously in successive swings, that is the periods will have been made equal. Each will then be equal in length. (There will be a slight difference in length if the bobs have different shapes and sizes; it is best for each to be spherical.) If after a time the pendulums are no longer *in step,* it shows that the adjustment in length has not been exactly done. Readjust by *slightly* lengthening the quicker pendulums. Even when each is adjusted to have the same period a difference in amplitude will soon be apparent: the lighter wood bob will be *damped* or *stilled* sooner than the heavier iron.

Repeat the experiment with shorter lengths (say 30 inches). Observe also that now all the pendulums vibrate more quickly.

***Exp. 53.**—*Show that the square of the period is proportional to the length of pendulum.* Hang up a bob by a fine wire or string. Make determinations of (1) the time (read in half-seconds) of 100 swing-swangs, or complete swings, and (2) the length of pendulum when the wire is about (i) 9 feet, (ii) 7 feet, (iii) 5 feet, (iv) 4 feet long. Calculate the mean period in each case. Tabulate (i) Exp. number, (ii) length of string, (iii) t, (iv) t^2, (v) l/t^2.

From the mean value of l/t^2, calculate g.

Plot l and t: the graph should be a parabola.

Plot l and t^2, also $log\ l$ and $log\ t$: each graph should be a straight line.

***Exp. 54.**—From the graphs find the lengths of the pendulums whose periods are respectively (i) 2 secs., (ii) 1 sec. Set up one of each of these lengths. Observe that the shorter goes to and fro while the longer goes to or fro, and confirm the values of the periods.

Exp. 55.—*Find the value of* g *by a simple pendulum.* Two observers, A and B, are required. Suspend a heavy ball, about 5 cm. diameter, by a fine wire, say, 4 metres long. Arrange two rods (say retort stands), one about 6″ in front, the other as far behind the bob, so that from a convenient place of observation, or *station,* several yards away, the two rods and suspending wire appear in line. Set the pendulum swinging. This is conveniently done by catching the bob in a loop on a silk thread, pulling it aside through a small arc, fixing the end of the thread and finally *burning* the silk.

I. *Find the period roughly.* Let A stand at the station, and B watch the clock.† A gives a signal by *sharply* knocking a piece of wood just when the bob makes a *transit,* that is passes between the rods: B notes the exact time when he hears the sound. A however must prepare B by saying "ready" a second or two before the transit occurs. In a similar manner the moment when, say, 50 complete swings have been done is noted. See § 55.

Let $t_1 =$ (Interval of time) $\div$ (Number of complete swings); then t_1 is, roughly, the period.

II. *Find the mean period.* The observers resume their positions. A gives signals when 6 successive transits occur, and B notes the times. (This requires practice.) The pendulum is now allowed to swing for 10 or 15 minutes. Then again A signals any 6 successive transits, and B notes the times. Tabulate as in the record below.

Exp. I. is conveniently done while waiting between the two series of observations of transits in II.

Calculate the interval of time between the first transits of each series, between the second transits, etc. Divide the mean of these intervals by t_1. The *nearest whole number* (n) to the quotient‡ is the probable number of complete swings in the interval.

† This experiment can be done by one student if he uses a stop-watch, but the above is better practice.

‡ If the value of the quotient comes practically midway between two whole numbers a fresh determination of t_1 is necessary. The new value of t_1 must be determined from the time of a larger (60 or 70 per cent.) number of vibrations than that used in the earlier observation: *e.g.* 80 or 85 vibrations instead of 50.

Divide the mean interval by n. The quotient is the *period* (t) very approximately.

Carefully measure the length (L) of the wire and diameter (d) of bob; then approximately

$$\textit{length of pendulum, } l = L + \frac{d}{2}.$$

Calculate (use logs) $g = 4\pi^2 l/t^2$.

Record of an Experiment—

Determination of g *by a simple pendulum.*

I.—*Experiment giving rough value of period.*

Time at start, 4h. 6m. 16·8s.; finish, 4h. 9m. 5·3s.; interval, 168·5s.

Number of swings, 50.

Therefore period = 3·37s. nearly.

II.—*Successive transits.*

Signals by [Observer]. Clock by [Observer].

Transits, 1st series.	Time by clock, *h. m. s.*	Transits, 2nd series.	Time by clock, *h. m. s.*	Interval, n vibrations, *m. s.*
0	3 22 42·5	n	3 34 17·8	11 35·3
1	3 22 49·3	$n+1$	3 34 24·6	11 35·3
2	3 22 56	$n+2$	3 34 31·4	11 35·4
3	3 23 2·8	$n+3$	3 34 38·1	11 35·3
4	3 23 9·6	$n+4$	3 34 44·8	11 35·2
5	3 23 16·4	$n+5$	3 34 51·6	11 35·2

Mean interval for n vibrations, 695·3 secs.

695·3/3·37 = 206·29. Nearest whole number = 206.

$$\therefore \text{ period} = \frac{695{\cdot}3}{206} = 3{\cdot}375 \text{ secs. approx.}$$

length of wire, 281 cm.; diameter of bob, 3 cm.

$\therefore$ length of simple pendulum = 282·5;

$$\therefore g = \frac{4 \times (3{\cdot}142)^2 \times 282{\cdot}5}{(3{\cdot}375)^2} = 979 \textit{ cm. sec. sec.}$$

CHAPTER V.

VOLUME, DENSITY, ELASTICITY, ETC.

56. Measurement of Area.—Areas enclosed by *regular* figures are most readily obtained by calculation. In every case the area is the product of two lengths and numerical constants. (For units and their relations, and formulae for calculating areas of regular figures, see the Appendix.)

To find the area enclosed by a figure.—METHOD I. *By squared paper.* (1) Find the number, N, of squares on the paper that make a square inch, or the number, n, to a square centimetre. (2) Outline the figure to be measured on tracing paper. Fix the latter on the squared paper, and count the number, A, of complete squares within the figure. Count each part of a square equal to, or greater than a half as a whole square. Let B be the number of them. Neglect each part less than a half. Then

$$\text{Area of figure} = \frac{A + B}{N} \text{ sq. in.} = \frac{A + B}{n} \text{ sq. cm.}$$

NOTE. If possible transfer the figure direct to the squared paper without using tracing material.

METHOD II. *By weighing.* Cut a piece of tin plate, or foil, or cardboard to fit the figure or its tracing: also a rectangle from the same material. Find the weight of each piece. Calculate the area of the rectangle from its dimensions. Then

Area Irregular ÷ Area Regular
= Weight Irregular ÷ Weight Regular.

Experiments on the measurement of area are given in Chapter I.

Exp. 56.—*An oblique circular cylinder is provided. Measure* (i) *girth* (Methods II. and III., § 16), (ii) the major and minor *axes of the elliptical end* (§ 14). Also *measure the area of the elliptical end* by § 56, Method I.

Measure the angle of slope of the cylinder. This is conveniently done by first cutting a sheet of paper half-way across, then folding back the cut edges until the angle between them fits that between the end and slanting surface of the cylinder (one edge of the paper must rest on the flat end of the cylinder and pass through its centre, the other touch the surface along a generating line): finally measure by a protractor the angle (a) between the edges of the paper. Find $\sin a$ from trigonometrical tables.

Calculate (see Appendix) (i) the area of a right section (perpendicular to the axis of the cylinder), (ii) area of the elliptical end. Calculate (*Area right section*) $\div$ (*Area elliptical end*); this should be equal to $\sin a$. Compare the measured area or the elliptical end with the calculated.

The relation is usually expressed in terms of the obliquity (θ) or angle between the axis of the cylinder, and the perpendicular to the base: θ is the complement of a above. Hence the measurements show that (*Area right section*) $\div$ (*Area elliptical end*) $= \cos\theta$.

MEASUREMENT OF VOLUME.

57. Volume.—The volume of a *regular* solid may be calculated when the appropriate dimensions have been measured. In every case the volume is the product of *three* lengths and numerical constants. (For units and their relations, and formulae for calculating volumes of regular figures, see the Appendix.)

The volume of an *irregular solid* is obtained experimentally by immersing it in a fluid, and measuring either the volume of the fluid displaced by it, or the force of buoyancy acting upon it. For the former measuring vessels are used, and for the latter a balance.

The volume of a quantity of *liquid* is determined by some form of measuring vessel (Figs. 41, 42), or less directly by the balance. Similarly, the cubical content or capacity of a vessel may be obtained.

Volume of a gas, see § 70.

Measuring Vessels (Fig. 41).—For scientific purposes these are usually made of glass; trade measures of wood (bushel, etc.) or metal (quart, etc.).

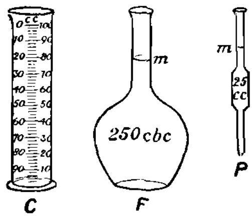

Fig. 41.

Measuring flask, F; when filled up to the mark, *m*, it contains a definite volume (500, or 250 cb. cm., etc.) of a liquid.

Measuring or graduated cylinder, C: the marks on the side show the volume between a division and the bottom. These are graduated in cb. cm., one-eighth of an ounce, one tenth of a cubic inch, etc.

Burette, B (Fig. 42): a narrow tube with a tap or pinch-cock (indiarubber tube and clip), by means of which the liquid may be let out; these are frequently graduated in tenths of a cu. cm. and read downwards, so that the volume of liquid delivered can be measured. Burettes are clamped in a vertical position to a wood or iron stand.

Fig. 42.

Pipette, P (Fig. 41): when filled to the mark, *m*, it holds a definite volume (100, 75, 50, or 25 cb. cm.) of fluid. A pipette is useful for adding or removing a small quantity of liquid to or from a vessel.

To fill a pipette by suction.—Hold the upper stem between the middle finger and thumb. Immerse the lower end in the liquid, apply the mouth to the upper end and withdraw air so that the bulb fills with liquid. Continue until the liquid rises a little above the mark. Then press the forefinger on the end in the mouth, and remove the pipette from the mouth. Relax the pressure of the forefinger so that the liquid *drops* from the pipette. When it has reached the mark on the stem, increase the pressure of the forefinger to stop the dropping; transfer the pipette to the place required, remove the forefinger, and allow the liquid to run out. Finally blow out the last drop.

Readings of measuring vessels should not be hurriedly done; the liquid requires a little time to drain down from the wet walls of the vessels. The reading, especially of the burette, is very liable to parallax error. The eye should look along the liquid surface (*ab*, Fig. 43) and the reading of the lowest part taken. If a piece of white paper is held behind and close to the tube a well defined dark band is seen (Fig. 44). The reading of the lowest edge of this may be taken as the position of the surface. The curvature of the whole of the liquid surface is more pronounced in narrow tubes.

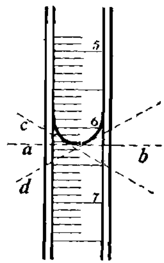

Fig. 43.

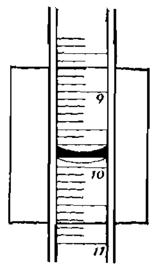

Fig. 44.

58. To find the denomination of a measuring vessel, that is, to determine whether its graduation marks represent cubic centimetres or cubic inches, etc.

A. (1) Weigh a clean, dry beaker or flask ($= W_1$ grms.).

(2) Fill the measuring vessel with water up to a definite mark ($=$ volume X).

(3) Pour the water from the measure into the weighed vessel. Weigh water + vessel ($= W_2$ grammes).

The volume is then

$$(W_2 - W_1) \quad \text{cubic centimetres,}$$
or
$$(W_2 - W_1)/16{\cdot}4 \text{ cubic inches,}$$
or
$$(W_2 - W_1)/28{\cdot}3 \text{ fluid ounces.}$$

Work out each of these expressions in turn. Probably a simple numerical relation will then be indicated between X and one of the above common units of capacity. If not, the relations between the cubic centimetre and other units of capacity must be tried.

B. If a cu.cm. measure is provided discharge the water into it from the graduated vessel of unknown denomination, and thus measure X directly in cu. cm. Then divide as above by the several conversion factors.

59. To find the volume of a vessel.—(I) Fill the measuring cylinder up to a mark with water. Note the position. Pour water from it, without spilling, into the vessel whose volume is to be found. Observe how much water is left in the measure. Deduce the volume of water poured out: this equals the volume of the vessel.

(I*a*) Fill the vessel with water. Pour the water from it into a measuring cylinder, and note the volume.

(II) Weigh the vessel (i) empty, (ii) full of water. The increase in weight (i) in grammes is numerically equal to the volume in cu. cm., (ii) in ounces is numerically equal to the volume in thousandths of a cubic foot. The volume in cubic inches is obtained by multiplying the increase of weight in ounces by 1·73.

NOTE.—Since 1 cb. ft. = 1728 cb. ins. and 1 cubic foot of water has a mass of nearly 1000 ozs. $\therefore$ 1 ounce of water has a volume of 1728/1000 = 1·73 cb. ins. nearly.

60. To find the volume of a body (which may be fragmentary) **by displacement.**—(I) Add some water to a measuring cylinder. Note the reading of the surface, (i) before, (ii) after the body is introduced. The difference is the volume of the body. The water introduced at the beginning must be sufficient to cover the body.

(II) Place the body in a dry measuring cylinder. Run in water from a burette until the body is completely immersed, and the surface stands at a definite division of the measure. Read the burette before and after discharging the water, and deduce the volume delivered. The difference between the volume delivered *from the burette* and the reading of the measuring vessel is the volume of the body.

(II*a*) Make a mark (strip of stamp paper) on a convenient vessel (if the body is fragmentary, use a flask and place the mark on the neck). Find the volume up to the mark of the vessel (§ 59), introduce the body, add water from a burette until it is filled to the mark. Read the burette before, and after, discharging the water. The volume of the body is the difference between the volume of the vessel and that of the water from the burette.

61. To find the volume of a body by the balance.—Weigh first in air, then in water. The difference in the values measures the buoyancy, and (i) when in *grammes* is numerically equal to the volume in cubic centimetres, (ii) when in ounces, is numerically equal to the volume in *thousandths* of a cubic foot. The volume in cubic inches is obtained by multiplying the numerical value of the buoyancy in ounces by 1·73.

Exp. 57.—*Find the denominations of the measuring vessels provided* (§ 58).

DENSITY, SPECIFIC GRAVITY.

62. Density.—Definition: The density of a substance is the mass of it contained in unit volume, *e.g.* the number of *grammes per cu. cm.* or of *lbs. per cu. ft.*

Note that in the specification of the density of a substance there is no reference to any other material.

To measure the density of a substance, take a body composed of the material, measure its mass (by weighing in air) and its volume. Express these quantities in absolute units. Then the

(*density of substance*) = (*mass of body*) ÷ (*volume of body*).

63. Specific gravity.—Definition: The specific gravity (Sp. G.) of a substance is the ratio of the weight of any volume of the substance to the weight of an equal volume of water.

Thus specific gravity is a numerical quantity; its specification for a particular material makes reference to a standard substance, *e.g.* water.

Since 1 cu. cm. of water has a mass of 1 grm., the *Sp. G. of a substance* is numerically equal to its *density in grms. per cu. cm.*

Since 1 cu. ft. of water has a mass of 62·4 lbs., the (*Sp. G. of a substance*) × 62·4 is numerically equal to the *density of the substance in lbs. per cu. ft.*

Also since 62·4 lbs. = 1000 ozs. nearly, the *density* of a substance in *ozs. per cubic foot* = 1000 × (*Sp. G. of substance*

Hence the $\left(\begin{array}{c}\textit{Volume of a body}\\ \textit{in cubic feet}\end{array}\right) = \dfrac{\textit{Mass of body in ounces}}{1000 \times \textit{Sp. G. of substance}}.$

The *specific gravity of a liquid* is readily determined by a specific gravity bottle (§ 64) or a direct reading hydrometer (§ 68).

The *specific gravity of a solid* is obtained by a balance (§ 65), advantage being taken of Archimedes' Principle. At least two weighings are necessary.

64. The specific gravity, or density bottle (Pyknometer), (Fig. 45).—This is a glass bottle with a carefully fitted stopper. A small hole, *ab*, is bored through the stopper. Sometimes it carries a thermometer. (In very accurate experiments the temperature should be noted and allowed for.) The bottle is filled with liquid, which overflows through the hole when the stopper is put in. The outside is then wiped dry.

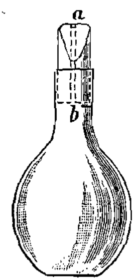

Fig. 45.

For many experiments it is sufficient to use a flask, about 3″ diameter, or a common bottle. Mark the neck by twisting a piece of *fine* wire twice round it, or stick a piece of stamp paper on it.

NOTE.—The bottle must not be held in the palm of the hand, but by the neck between finger and thumb. This is to avoid warming and expanding the liquid. To clean and dry the bottle, see Appendix.

A. To find the specific gravity of a liquid by the bottle.

(1) Weigh the empty bottle, clean and dry ($= W_1$).

(2) Fill the bottle with some of the liquid whose Sp. G. is required and weigh ($= W$).

(3) Remove liquid from bottle, clean, fill with water, and weigh ($= W_2$).

Then *Sp. G. of liquid* $= \dfrac{W - W_1}{W_2 - W_1}$.

B. To find the specific gravity of a solid by the bottle.—The solid may be in small fragments or a powder.

I. *When the fragments are numerous and small:*

(1) Weigh the bottle empty, clean, and dry ($= W_1$).

(2) Put in the solid: weigh ($= W$).

(3) Add water to fill the bottle: weigh ($= w$).

NOTE.—Use the bottle about half full of fragments. Cover them with water. Hold the bottle slantwise, and slowly rotate it: air bubbles will then be dislodged. Finally fill up with water.

(4) Remove all fragments, and fill the bottle with water only: weigh ($= W_2$).

Then $Sp.\ G.\ of\ solid = \dfrac{W - W_1}{W - W_1 + W_2 - w}$.

Any other liquid may be used instead of water. If a substance that dissolves in water is to be experimented with, use some liquid that has no chemical action upon it (Exp. 61*a*).

PROOF.—Weight of solid $= W - W_1$.

Weight of liquid displaced by solid

$$= W_2 - \{w - (weight\ of\ solid)\} = W_2 - w + W - W_1.$$

II. *When the fragments can be weighed separately:*

(1) The weight of the bottle need *not* be found.

(2) Weigh fragments apart from bottle ($= M$).

(3) and (4) as I. (3) and I. (4) above.

$$Sp.\ G.\ of\ solid = \frac{M}{M + W_2 - w}.$$

Exp. 58.—*Find the Sp. G. of methylated spirit, salt solution (saturated and half-saturated) by the bottle.*

Exp. 59.—*Find the Sp. G. of brass (nails), iron (nails), lead (shot), sand by the bottle.*

Exp. 59a.—*Find the Sp. G. of mercury.* Proceed as in B. Method I. (§ 64). Put about 50 grammes of mercury in the bottle instead of a solid. (A bottle full of mercury is likely to break or to be too heavy for the balance.)

Exp. 60.—*Demonstrate the* **Principle of Archimedes**: † *that when a body is wholly or partly immersed in a fluid, a force, called the buoyancy, acts upon it, whose direction is vertically upwards, and whose magnitude is equal to the weight of the fluid displaced by the body.*

Law of Flotation.—*A body floats when the force of buoyancy acting upon it is equal to the weight of the body.* This can be easily deduced from the Principle of Archimedes.†

(1) Determine the volume of a solid body (glass stopper "marble"). If regular, measure the proper dimensions and calculate the volume. Express in cb. cm.

Weigh in (2) air, (3) methylated spirit, (4) water, (5) concentrated salt solution, (6) concentrated solution of zinc sulphate. (Hang, in turn, in glasses, each containing one of the solutions: as in § 65. Well rinse and dry after each weighing.) Express weights in grammes.

(7) Find the Sp. G. of each of these solutions (by Sp. G. Bottle or by a Hydrometer (§ 68)).

(8) Calculate the weight of each liquid displaced by the body (= *volume of body* × *Sp. G. of liquid*).

(9) Calculate the buoyancy on the body due to each liquid (= *Weight of body in air* — *Weight of body in liquid*).

The values of (8) and (9) should be equal.

RECORD of Experiments:

A right cylinder (iron) was used.

Measured lengths .. .. .. mean length = .. cm.
Measured diameters .. .. .. mean diam. = .. cm.
Volume of cylinder .. .. = .. cu. cm.
Mass of cylinder (weight in air) .. .. .. = .. grms.

	Methylated Spirit.	Water.	Salt Solution.	Zinc Sulphate Solution.
Apparent weight of body in liquid............				
Sp. G. of liquid........				
Buoyancy due to liquid..				
Vol. of body × Sp. G. of liquid				

† See Experiments 28, **29**, 30 on **Flotation**, **etc.**, in **Chap. I.**

65. To find the specific gravity of a substance by means of the balance.—The body provided is hung by a thread (loop at upper end) from the hook at the end of the balance beam (Fig. 36) so that it can be immersed in a vessel of water. The water should have been well boiled to remove air, but must be cold at the time of the experiment. The body should lie, *when the pan is raised,* half an inch below the surface of the liquid and free from the walls of the vessel. If air bubbles appear on its surface these should be brushed off just before the completion of the weighing. Use a small brush or narrow folded strip of paper.

A solid body that sinks in water.—(1) Weigh clean and dry in air ($= W$).

(2) Weigh in water ($= W_1$).

$$\begin{pmatrix}\textit{Sp. G. of}\\ \textit{substance}\end{pmatrix} = \frac{\textit{Weight in air}}{\begin{pmatrix}\textit{Weight}\\ \textit{in air}\end{pmatrix} - \begin{pmatrix}\textit{Weight}\\ \textit{in water}\end{pmatrix}} = \frac{W}{W - W_1}.$$

A solid body that floats in water.—(1) Weigh in air ($= W$). (2) Attach the body to another so that the combination sinks in water. Weigh body + sinker ($= W_2$). (3) Weigh the sinker only in water ($= W_3$).

$$\textit{Sp. G. of substance} = \frac{W}{W - (W_2 - W_3)} = \frac{W}{W + W_3 - W_2}.$$

A liquid.—Take a body that is not chemically affected by the liquid, weigh it (1) in air ($= W$), (2) in liquid ($= W_4$), (3) in water ($= W_1$). Then

$$\textit{Sp. G. of liquid} = \frac{W - W_4}{W - W_1}.$$

A substance chemically affected, or dissolved, by water should be weighed in a liquid that has no action on it. The specific gravity of the liquid must be determined.

CALCULATION.—Let S be the Sp. G. of the solid; D, the Sp. G. of the liquid. Let W be the weight of the solid in air, W' its apparent weight in the liquid. Let V cb. cm. be the volume of the solid, and hence of liquid displaced. Then (§ 62) $W = S \, . \, V$, and (Exp. 60) $W - W' = D \, . \, V$.

$$\therefore S/D = W/(W - W').$$

Exp. 61.—*Determine the Sp. G. of glass, sulphur, iron, copper, lead, aluminium, wax, alcohol, ether, etc.*

Exp. 61a.—*Find the Sp. G. of copper sulphate crystals.* If two or three large crystals are provided, tie them into a bunch and suspend from the arm of a balance as in § 65. Weigh (i) in air, (ii) in a liquid, say, petroleum oil, in which the crystals do not dissolve. If the crystals are small put them into the Sp. G. bottle and proceed as in § 64, B., filling the bottle however with petroleum oil instead of water.

66. Modification of formulae.—Let numerator and denominator in each of the above be divided by the weight in air (W), then (see table below) the values of Sp. G. will be expressed in terms of the ratios of the several apparent weights to the weight in air.

In Jolly's spring balance (Exp. 77) the weights are proportional to the respective elongations (e, e_1, e_2, e_3, e_4) of a spring.

In Walker's steelyard (Exps. 16, 17) the weights are *inversely* proportional to the respective distances (l, l_1, l_2, l_3, l_4) on the long graduated arm.

The formulae in the table below are the expressions required in the several cases. The proofs are left to the student. He should not, however, attempt to remember these, but should in all cases work from the formulae printed in bold type.

REMEMBER the formulae in bold type.

	Heavy Solid.	Light Solid.	Liquid.
Ordinary Balance. Weight measured directly.	$\dfrac{\mathbf{W}}{\mathbf{W} - \mathbf{W}_1}$	$\dfrac{W}{W + W_3 - W_2}$	$\dfrac{\mathbf{W} - \mathbf{W}_4}{\mathbf{W} - \mathbf{W}_1}$
ditto. (alternative formulae)	$\dfrac{1}{1 - \dfrac{W_1}{W}}$	$\dfrac{1}{1 + \dfrac{W_3}{W} - \dfrac{W_2}{W}}$	$\dfrac{1 - \dfrac{W_4}{W}}{1 - \dfrac{W_1}{W}}$
Spring (Jolly). Extension proportional to Weight.	$\dfrac{e}{e - e_1}$	$\dfrac{e}{e + e_3 - e_2}$	$\dfrac{e - e_4}{e - e_1}$
Steelyard (Walker). Arm inversely proportional to Weight.	$\dfrac{l_1}{l_1 - l}$	$\dfrac{l_2 l_3}{l_2 l_3 + l l_2 - l l_3}$	$\dfrac{l_1 l_4 - l l_1}{l_1 l_4 - l l_4}$

67. Nicholson's Hydrometer (Fig. 46).—A closed hollow vessel, C, carrying two scale pans is arranged to float in a liquid with one scale pan, B, immersed, the other, A, not. The *hydrometer* is one *of constant immersion*. It is *adjusted* by adding known weights to the *upper* pan until a mark, m, on the stem is at the surface of the liquid. This is best observed by looking at the stem below the surface. Adjust so that the mark and its image (by reflection from the surface) come *into coincidence*.

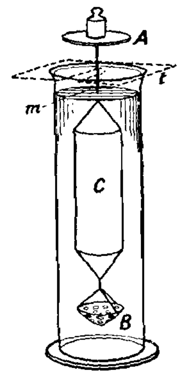

Fig. 46.

The stem should have been well cleaned by rubbing it with a rag moistened with alcohol, to remove grease. A slotted tinplate cover, t, is laid across the top of the tall glass cylinder containing the liquid, to prevent weights from falling in and the hydrometer from touching the sides. If air bubbles adhere to the instrument or substance, remove them by rubbing with a folded strip of paper. When water is used it should have been boiled, or distilled.

To find the specific gravity of a solid by Nicholson's Hydrometer.—Float the hydrometer in water.

(1) Adjust the hydrometer by adding weights, W_1, to the upper pan. The *substance* should not be in either pan.

(2) Place the *substance* in the *upper* pan, adjust to the mark by adding weights, W_2, to upper pan.

(3) Place the *substance* in the *lower* pan, adjust to the mark by adding weights, W_3, to upper pan.

$W_1 - W_2$ = the apparent weight of substance in air.

$W_3 - W_2$ = difference due to moving the body from the upper pan to lower = weight of liquid displaced by the body.

$$\therefore \textit{Sp. G. of substance} = \frac{W_1 - W_2}{W_3 - W_2}.$$

NOTE.—If the substance is lighter than water it must be attached to the lower pan by wire. The wire must be kept attached to the lower pan during all operations.

To find the specific gravity of a liquid by Nicholson's Hydrometer.

(1) Weigh the hydrometer, clean and dry ($= M$).

(2) Float the hydrometer in the liquid and adjust by adding weights, W_4, to the upper pan.

(3) Float in water and adjust by adding weights, W_1, to the upper pan. Since the hydrometer displaces equal volumes

$$Sp.\ G.\ of\ liquid = \frac{M + W_4}{M + W_1}.$$

NOTE.—If the hydrometer will not float upright in the liquid, put a piece of glass or lead in the lower pan. Consider this to be a part of the hydrometer. Then go through the operations above.

Exp. 62.—*Find Sp. G. of glass, iron, sulphur, etc., by Nicholson's Hydrometer.*

Exp. 63.—*Find Sp. G. of alcohol, petroleum, salt solution (saturated and half-saturated) by Nicholson's Hydrometer.*

68. The Common Hydrometer (Fig. 47).—These are usually made of glass, and consist of a long tube, AE, closed at one end and having a bulb, G, containing mercury or shot at the other. A little above the lower bulb the tube is considerably enlarged, B. The whole is arranged to float vertically with more or less of the stem showing above the surface of the liquid. The less the Sp. G. of the liquid the greater the amount submerged. The instrument is hence a *hydrometer of variable immersion.* A scale is fixed within the tube by which the Sp. G. of the liquid may be read off *directly*, the value being the reading of the scale division against the surface of the liquid. When, for instance, the hydrometer is floated in water the reading is 1000, *i.e.* Sp. G. = 1·000. If in another liquid the reading is, say, 1125, then Sp. G. of liquid = 1·125. If the instrument is graduated to show equal increments in the value of specific gravity

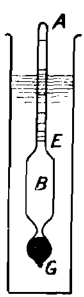

Fig. 47.

the scale is then not one of equal parts, the lengths of the divisions nearer the bulb are shorter than those more remote.

A hydrometer to measure a range of values of Sp. G., say from 0·7 to 2·0, would be either very long or very insensitive. Generally a set of hydrometers is used, each member measuring a part of this range, *i.e.* (i) 0·7 to 1·0, (ii) 1·0 to 1·35, (iii) 1·35 to 1·7, (iv) 1·7 to 2.

Exp. 64.—*Find by the common hydrometer the Sp. G.* of (1) concentrated sulphuric acid, (2) saturated copper sulphate, (3) saturated salt solution, (4) petroleum, (5) methylated spirit, (6) glycerine.

After each observation the hydrometer *must be well rinsed and wiped dry.*

Exp. 65.—*Find the mass of liquid in a vessel.* (1) Obtain the Sp. G. of the liquid. (2) Measure the volume of the part of the vessel filled by the liquid by determining the appropriate dimensions if regular, or as in § 59. (3) Calculate the *mass of liquid* = *Vol. of liquid* × *Sp. G. of liquid* × *Mass of unit vol. of water.*

Find the mass of acid in a "Winchester" (bottle), of petroleum or spirit in a drum, etc.

69. I. To find the mean diameter of a wire indirectly from its mass, length, and density.—(1) Stretch the wire across the bench by hanging weights from its ends. Make two fine file marks on it about a metre apart. Accurately measure the length (l) in *cm.* between the marks, and cut the wire at these points with pliers. (2) With a very sensitive balance weigh the length of wire, in air ($= W$ grm.), and (3) in distilled or boiled water ($= w$ grm.). (4) Calculate

$$diam.\ in\ cm. = \sqrt{\frac{W - w}{0{\cdot}7854 \times l}},$$

and density (D) of substance $= W \div (W - w)$.

Also, for comparison, measure the mean diameter by screw calipers (Exps. 42, 43), and calculate the density.

PROOF OF FORMULA.—Since 1 *grm.* of water has vol. 1 *cu. cm.*, the vol. of wire = $(W - w)$ *cu. cm.* But for a round wire of diam., d *cm.*, and length, l *cm.*,

$$volume = 0{\cdot}7854\ d^2 l \qquad \therefore\ 0{\cdot}7854\ d^2 l = W - w.$$

Also if D is the density in *grms. per cu. cm.*, $W - w = W/D$

$$\therefore\ d = \sqrt{\frac{W}{0{\cdot}7854\ l.\ D}} = \sqrt{\frac{W - w}{0{\cdot}7854\ l}}.$$

NOTE.—If very carefully done, the result of this experiment is likely to be more accurate than the direct measurement with calipers.

II. To find indirectly the mean thickness of a lamina.—Measure the area (§ 56) in *sq. cm.*, weight in air (W *grm.*), weight in water (w *grm.*). Then *thickness* $= (W - w)/area$.

Exp. 66.—*Find the diameters* of pieces of wire—copper, steel (piano), etc. Use Method I., § 69.

Exp. 67.—*Find the mean thickness* of a coin, a piece of assay lead foil, tin foil, copper foil, etc. Use Method II., § 69.

Exp. 68.—*Find the mean diameter of a capillary tube.* Introduce into it a thread of mercury. (Use a mercury tray. Attach india-rubber pipe to one end of the tube; place its other end under mercury, slope the tube, suck sufficient mercury in, close the rubber piping by a pinch cock.) Measure the length of the thread (by a finely divided scale or travelling microscope). Alter the position of thread several times and repeat the measurements of length. Run the mercury into a weighed vessel. Weigh (vessel + mercury). Deduce the mass of mercury. Calculate as in § 69, I. (4).

70. Volume of a gas.—To express this completely it is necessary to observe (1) the volume, V, of the quantity of gas, (2) the pressure, P, exerted by it, (3) its temperature, t. It is usual finally to specify the volume (V_0) in *cu. cm.* when the pressure (P_0) is that of the standard atmosphere (see Appendix), and the temperature, $0°C$. To effect this reduction assume

$$PV/(273 + t) = P_0\ V_0\ /\ 273.$$

To measure a volume of gas.—Fill a measuring tube graduated in cu. cm. with water. Cover the end of the tube with the thumb (take care that no air enters) and immerse it in water. Pass the gas into the measuring vessel by introducing the end of the delivery tube. Completely immerse the measuring tube in water contained in a wide cylindrical glass vessel, allow it to remain there, say, ten minutes. Finally observe (i) the volume (V cu. cm.) of the gas, (ii) the temperature (t) of the water (the temperature of the gas is assumed to be equal to this), (iii) the vertical distance (a cm.) between the surface of the water exposed to the atmosphere and the surface in the measuring tube, (iv) the height (H cm.) of the barometer, (v) if the liquid used is not water, observe its specific gravity, D.

The pressure, P, exerted by the gas equals that of the atmosphere, H, plus that due to a depth, a, of liquid of density, D. The latter equals $aD/13{\cdot}6$ cm. of mercury.† Hence

$$P = H + aD/13{\cdot}6 \text{ cm. of mercury.}$$

If the level of the liquid inside the collecting vessel is above that outside, then

$$P = H - aD/13{\cdot}6.$$

Then at *normal temperature and pressure* (N.T.P.), that is, at 0°C. and 76 cm. of mercury, the volume of gas ‡

$$V_0 = \frac{H \pm aD/13{\cdot}6}{76} \times \frac{273}{273 + t} \times V.$$

Exp. 69.—*Find the volume at N.T.P., and mass of the quantity of air provided.*

71. Mass of a volume, V cu. cm., of dry air at temperature, t° C., and pressure, P cm. (mercury at ice-point).

Density of dry air at 0°C. and 76 cm. = 0·001293 grms. per cu. cm. Hence mass of V *cu. cm.* of dry air at t°*C.* and P *cm.*

$$= 0{\cdot}001293 \times \frac{273}{273 + t} \times \frac{P}{76} \times V \text{ grammes.}$$

† The effect of temperature on the densities of the liquids is neglected. 13·6 is the density of mercury.

‡ The proofs of these formulae involve the laws of Boyle (§ 83) and Charles (§ 97). See also the account of the Barometer (§ 78 *et seq.*)

Mass of a volume, V cu. cm., of aqueous vapour at temperature, t° C., and pressure, f cm. (mercury at ice-point).

The mass of a volume of aqueous vapour is practically five-eighths (actual value = ·622) of that of the same volume of air at the same temperature and pressure. Hence mass of *V cu. cm.* of aqueous vapour at t° *C.* and *f cm.*

$$= \frac{5}{8} \times 0{\cdot}001293 \times \frac{273}{273 + t} \times \frac{f}{76} \times V \text{ grammes.}$$

Mass of a volume, V cu. cm., of moist air at temperature, t° C., pressure, P′ cm. Pressure of aqueous vapour present, f cm.

There is a volume, V, of dry air at $t^\circ C.$ and $(P' - f)$ *cm.*

There is a volume, V, of water vapour at $t^\circ C.$ and f *cm.*

Hence total mass

$$= \frac{0{\cdot}001293 \times 273}{(273 + t) \times 76} \left\{ P' - \frac{3}{8} f \right\} V \text{ grammes.}$$

CENTRES OF GRAVITY.

72. Centre of gravity or mass centre. To find the C.G. of a uniform sheet (lamina) of material of any shape.—Method I. (1) Suspend from the same point (i) the lamina so that its plane is vertical, (ii) a plummet in front of, and close to the lamina. Mark on the lamina the positions of two points (as far apart as possible) on the plummet thread. Draw a straight line through the two points. (2) Alter the suspension of the lamina so that it hangs with the line of (1) inclined. Proceed as in (1). (3) Two intersecting lines are obtained. The C.G. is behind the point of intersection, half-way through the lamina. (4) Similarly determine other lines. All should intersect at the C.G.

Practice. *To suspend the lamina:* (I) pass a stout needle through a slightly larger hole in the lamina, push the needle into a support or clamp it. (II) Tie a piece of cotton round the lamina, leave an end by which it may be suspended. *Plummet:* loop the end of its thread and put over the needle, etc., from which the lamina is suspended.

Method II. Balance the lamina on the bevelled edge of a ruler, mark two points at which the edge touches the lamina, draw a line through them. Repeat for another position. The point of intersection of the two lines indicates the C.G. Repeat for other positions.

***Exp. 70.**—*Find the C.G. of a triangle.* Use a cardboard figure. Use Method I. or II. (§ 72).

Join the mid point of each side with the opposite angle. Observe that the three lines (medians) intersect at the C.G. Show, by actual measurement, that the C.G. is one of the points of trisection of each median.

***Exp. 71.**—*Find the C.G. of a four-sided figure* (in cardboard). Use Method I. or II. (§ 72). Mark the C.G. when found. Then divide the figure into two triangles, find (i) the C.G., (ii) the weight of each. Reconstruct the figure, and show by measurement that the C.G. of the whole is on the line joining the C.Gs of each triangle, and divides this line into two parts whose ratio is inversely proportional to the weights of the triangles.

ATWOOD'S MACHINE.

73. In Atwood's machine (Fig. 48) a light pulley is placed at a considerable height (at least 7 feet) above the ground. A fine cord passes over it; at the ends of this are hung equal masses *P*, *Q*. The axle of the pulley must turn with as little friction as possible. A scale of $\frac{1}{4}$ ft., or decimetres, is arranged vertically beneath the pulley so that the distances through which *P* and *Q* move may be measured. *A* is a platform by which *Q* may be supported. It is arranged so that the projecting part may be pulled down when required, leaving *Q* free to fall.† *B* is a ring through which *Q* can pass; *C*, a platform that arrests its motion. *A*, *B*, and *C* can be clamped in different positions. An additional mass or *rider*, *R*, is provided: this if placed on *Q* falls with it, but, being made too wide, will not pass through the ring.

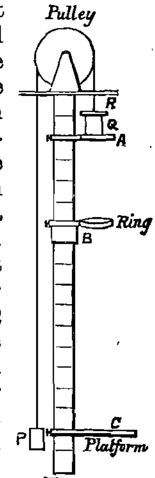

Fig. 48.

P and *Q* being equal they remain at rest wherever they are placed, or when set in motion move with

† An electromagnetic arrangement is sometimes provided.

uniform velocity.† When R is on Q and the system moves, the motion is uniformly accelerated, for the constant *mass*, $P + Q + R$, is being moved by the constant *weight* of R. Let f be the acceleration of $(P + Q + R)$ and g the acceleration of gravitation, then $(P + Q + R)f = Rg$. After R is lifted off Q the system moves with uniform velocity ($= v$, say). Between A and B the system moves with uniform acceleration, f. Let s be the distance and t the time taken. Then $s = \frac{1}{2}ft^2$.

Between B and C the system moves with uniform velocity (v). Let S be the distance, and T, the time taken. Then $S = vT$. By measuring s, S, t, T, P, Q, R determinations of f, v, and g may be obtained experimentally.

PRACTICE.—Support Q + *rider* on the platform, A, at the top of the machine; place C near the bottom, and the ring, B, between. Be careful that P does not acquire a swinging movement.

I. *Time* should be measured by observing the seconds hand of a watch, the experimenter noting the moments (estimate fractions of a second) when he permits Q to start, and hears R strike the ring or Q the lower platform. The time for each distance must be measured by four or five observations, and the mean value taken.

II. Another method is to set a metronome going, and rate it (§ 53). Adjust the positions A, B, or C so that Q starts and the blows with B, or C, are heard simultaneously with beats of the metronome. The moments may be accurately judged by counting successive beats, one, two, three, etc.; let Q start at eight, the ring be struck at nine, the platform at ten.‡ Four or five adjustments of the distance must be made for each value of the time and the mean taken.

I. **To find the acceleration of the moving bodies.**—Adjust the distance (s *cm.*) between A and the ring, B, so that Q is one second in falling to B from rest at A. Then the acceleration, $f = 2.s$ *cm. sec. sec.*

The acceleration of gravitation, $g = (P + Q + R)f/R$.

II. **To find the velocity of the moving body.**—Adjust the distance (S *cm.*) between B and C so that Q is one second in falling (rider left on ring) from B to C. Then the velocity, $v = S$ *cm. per sec.*

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† This is not quite true in practice owing to the friction at the axle of the pulley, etc. To compensate for this, cut a piece of wire so that when placed on Q it is *slightly* too small to set P and Q in motion. Fix the wire to Q, regard it as a part of Q.

‡ Do not look at the metronome or the falling bodies; use the ear, not the eye.

Exp. 72.—*Show that when the total mass moved is constant the acceleration is proportional to the weight of rider.* Two riders are supplied, one (R) having thrice the mass of the other (r). Adjust the metronome to beat seconds.

1. Place $(R + r)$ on Q. Find the acceleration (f_1).
2. Leave R on Q. Place r on P. Find the acceleration (f_2).

In both cases the mass moved $= P + Q + 4r$. The acting force or overweight in (1) is $Q + 4r - P = 4r$, in (2) is $Q + 3r - (P + r) = 2r$. The acting forces being as $2 : 1$, then the accelerations should be as $2 : 1$.

Exp. 73.—*Show that when the applied force is constant the acceleration is inversely proportional to the mass moved.* Change P and Q for values P' and Q'. Place the riders $R + r$ on Q'. Find the acceleration (f'). Also find f_1 as in Exp. 72 (1).

Calculate f'/f_1, and $(P + Q + R + r) \div (P' + Q' + R + r)$.

Since the acting force ($=$ weight of $R + r$) is the same in both cases, these ratios should be equal.

Exp. 74.—*Show that after the rider is removed the velocity is uniform.* Set the metronome to beat seconds. Adjust (1) so that Q starts, and the ring and lower platform are struck at three successive ticks of the metronome; (2) so that the interval between A and B is one stroke of the metronome, and that between B and C, two strokes; (3) A to B, one stroke; B to C, three strokes. The times taken in going from B to C being as $1 : 2 : 3$, the respective distances should be as $1 : 2 : 3$.

Exp. 75.—*Show that $s \propto t^2$ and $v \propto t$.* (1) Set the metronome to beat seconds. Adjust so that Q starts, and the ring and lower platform are struck, at three successive ticks. Measure the distances, AB $(= s_1)$, and $BC (= S_1)$.

(2) Alter the metronome to beat faster than seconds: rate it (let the interval $= t_2$). Adjust the positions, A, B, C, as in (1). Measure AB, BC ($= s_2$, S_2, respectively).

(3) Again alter the metronome to beat more slowly than seconds: rate it (interval, t_3). Adjust A, B, C as in (1). Measure AB, BC ($= s_3$, S_3, respectively).

Show that $$s_1 : s_2 : s_3 = t_1^2 : t_2^2 : t_3^2.$$

Also that in each case the distance $BC = 2 . AB$, or $S = 2 . s$.

Then $$v_1t_1 : v_2t_2 : v_3t_3 = S_1 : S_2 : S_3 = s_1 : s_2 : s_3 = t_1^2 : t_2^2 : t_3^2,$$
$$\therefore v_1 : v_2 : v_3 = t_1 : t_2 : t_3.$$

ELASTICITY OF SPRINGS, ETC.

***Exp. 76.**—*Extension of a spring.* A long spiral spring is provided. Fig. 49 shows a convenient arrangement. A stout wooden rod, held vertically by a clamp and stand, carries a nail *A*. A boxwood measure is hung from the nail, then the spring in front of it. The lower end of the spring after forming a loop is twisted back, and finally turned (*p*) to point to the scale divisions; or the end is passed axially through a cork, that also carries a needle (*n*) horizontally (Fig. 50). A scale pan is hung from the lower loop by a length of string so that it lies clear of the support, etc.

Observe the reading of the pointer, *p*, when the pan is unloaded, and when loaded successively with 20, 40, 60, etc., grammes.

Plot the readings with reference to loads. The graph is practically a straight line, and shows that the extension of the spring is proportional to the load or force applied. From the graph deduce the extension per gramme, and the mass required to produce unit (1 cm.) extension.

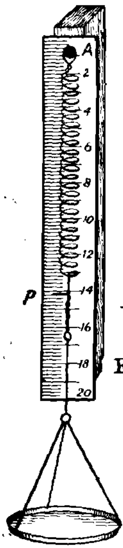

Fig. 49.

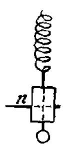

Fig. 50.

***Exp. 77.**—*Find the specific gravity of glass, iron, coal, stone, sulphur, wax, methylated spirit, petroleum, salt solution, etc., by means of a spring.* (The experiment illustrates the use of Jolly's specific gravity balance.) Hang the spring as in Fig. 49. (Remove the scale pan.) Observe the readings of the pointer, *p*, (i) when the spring is unloaded, (ii) when a body is hung from it in air, (iii) in water. The difference of the readings is the extension due to the load on the spring.

The necessary operations and formulae are given in §§ 65, 66.

***Exp. 78.**—*Graduate a spring balance.* Remove the boxwood scale, and fix, with drawing pins or small nails, a strip of paper behind the spring. Mark on the paper the position of the pointer for no load, and loads 10, 20, 30, etc., grammes. Number the lines 0, 10, 20, etc., and subdivide the spaces between into halves or fifths.

Find by the spring the weights of coins, etc. Place the body in the scale pan, note the number of the division to which the pointer is drawn.

***Exp. 79.**—*Extension of an india-rubber cord.* Make loops at both ends (Fig. 51), and push two pins, A, B, through the cord about an inch away from the loops.

Measure the diameter, in several places, with screw calipers at the beginning and the end of the experiment. (Care must be taken not to squeeze the cord unduly.)

Suspend the cord in front of a measure as in Exp. 76. Hang a scale pan from the lower loop. (i) Add weight sufficient to stretch the cord straight; note the scale readings of both A and B (call the difference of these the *initial length*). (ii) Increase the load in scale pan by 20, 40, 60, etc., grammes above the initial value, note the scale readings of A and B for each load. Put the weights into the pan carefully. Take the readings a minute or two after adding weights to the pan.

Fig. 51.

Tabulate (i) load, (ii) scale reading of A, (iii) scale reading of B, (iv) length AB, (v) extension of cord (= actual length of AB—initial length of AB).

Plot the lengths, AB, with regard to the loads. The graph will be a slightly curved line, indicating that the extension of the cord is roughly proportional to the load.

DEDUCE the mean cross-section of cord in sq. cm.;

the mean extension in cm. per gramme-wt. of load;

the mean stress = (one gramme-wt.) ÷ (cross-section)
= (981 dynes) ÷ (cross-section);

the mean strain = (mean extension per gramme-wt.) ÷ (initial length in cm.);

the mean modulus of elasticity for tensile stress = (stress) ÷ (strain)

TORSION. MOMENTS OF INERTIA.

74. Torsion.—If a body is hung by a filament (silk twist, cotton, thread, string, wire, etc.) its movements are more or less affected by the elasticity of the suspension, for whenever a filament is twisted forces come into play that tend to make it untwist. These are called *torsional forces*, and the body is said to be affected by a *torsional stress*. Their magnitudes are greater when the angle through which the filament is twisted is larger.

A suspended body when affected only by forces, *e.g.* weight, parallel to the axis of suspension† rests in an equilibrium position in which the filament is not twisted. Let the body be turned from this position, the suspension is more or less twisted and torsional forces come into play: these oppose the turning movement of the body. If the body is turning freely, then as the angular space traversed increases, the opposing torsional forces increase and presently stop the movement of the body. The next moment the body, in obedience to the torsional forces, begins to return towards the equilibrium position. The continued action of the torsional forces makes the body gain velocity from moment to moment, and therefore it does not stop at, but passes through the equilibrium position; the suspension now becomes twisted in the reverse way, and torsional forces again oppose the angular movement, presently stop it, and then bring the body back towards the equilibrium position. Again velocity and momentum are acquired, the body passes through the equilibrium position and swings over to the first side. The movements are then repeated and so on. Thus the body oscillates about its equilibrium position; it forms a *torsion pendulum*.

NOTE.—The small magnets and mirrors used in reflecting galvanometers (§ 205) and magnetometers (§ 172) are hung by a long single fibre of cocoon or unspun silk. The torsional forces then called into play when the magnet is deflected are insignificant.

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† If the suspended body is a magnet it is affected by the vertical and horizontal components of the Earth's magnetism. The latter tends to twist the suspension, *except when the equilibrium position lies in the magnetic meridian*; the former does not affect it.

75. Moment of Inertia.—A quantity whose value depends (i) on the mass of the vibrating or rotating body, (ii) on the distribution of the mass about and its distance from the axis of rotation.

DEFINITION.—*The moment of inertia of a particle about an axis of rotation is the value of the product*

(*Mass of particle*) (*Square of its distance from the axis*).

The moment of inertia of a *body* is the sum of the moments of inertia of its particles.

Parallel axes. If I is the moment of inertia of a body, of mass, M, about an axis through its centre of gravity, and I', about a parallel axis whose distance from the first is a, then

$$I' = I + Ma^2.$$

The moments of inertia of *regular bodies* about definite axes can be calculated. The formulae may be found in physical and engineering table books; several cases are given below. The moments of inertia of *irregular bodies* are determined by experiment (§ 77).

VALUES OF MOMENTS OF INERTIA.

Right cylinder, length, l, radius, r, mass, M;

(i) About an axis continuous with the axis of cylinder	$\frac{1}{2} Mr^2$.
(ii) About an axis through the mid point, and perpendicular to axis of cylinder	$M\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$.

Circular disc, thickness (l) negligible, radius, r, mass, M;

(i) About an axis through the centre, and perpendicular to the surface	$\frac{1}{2} Mr^2$.
(ii) About a diameter as axis	$\frac{1}{4} Mr^2$.

Rectangular prism, edges a, b, c, mass, M;

About an axis through the centre, and perpendicular to the face contained by edges a, b	$M\frac{a^2+b^2}{12}$.

Rectangular lamina, sides, a, b, mass, M;

(i) About an axis through its centre, and perpendicular to the surface	$M\frac{a^2+b^2}{12}$.
(ii) About an axis in the plane of the lamina through the centre of, and perpendicular to the edge, a	$M\frac{a^2}{12}$.

Thin rod, length, a, mass, M;

About an axis through its centre, and perpendicular to its length $\quad M\frac{a^2}{12}$.

Sphere, radius, r, mass, M;

About an axis through its centre $\quad \frac{2}{5}Mr^2$.

NOTE.—The values for a *disc* are derived from those for the right cylinder by assuming the thickness of the disc, l, to be small in comparison with the radius, r. Hence neglect the term $l^2/12$. Similarly a *rectangular lamina* and *thin rod* are derived from the prism.

Exp. 80.—*The torsion pendulum.* Fig. 52 shows a convenient apparatus for illustrating moment of inertia. A bar, AB (boxwood $\frac{1}{2}$ metre-scale) is suspended horizontally by a wire, C. (The figure at the right side shows details of the clamp, viz., a brass or iron strip bent to hold the

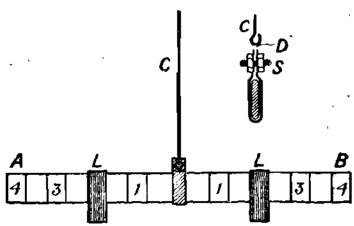

Fig. 52.

bar; a screw stem, S, carrying two nuts; the wire is introduced at D, looped round the stem, and screwed up tightly. A good clamp is also required at the upper end.) Two lead strips, L, ∩-shape, of equal mass, rest on the bar. These are placed at a definite distance from the axis of suspension, the bar set vibrating, and the interval of ten or a dozen swings obtained (see § 55).

The masses are then shifted to another position, or lead strips of different mass are substituted. See the record below.

RECORD OF AN EXPERIMENT:—

Bar of wood, 20″ × 1″ × 1·5″. Lead strips each of 1 lb. mass, 2″ wide. Suspension of fine copper wire, 12″ long.

Experiment.	1	2	3	4	5	6
OBSERVED DATA.						
Mass each side ..	0	1 lb.	2 lbs.	1 lb.	2 lbs.	1 lb.
Distance between wire and centre of lead strips ..	0	3″	3″	6″	6″	9″
Relative distances	0	1	1	2	2	3
Period (in seconds)	12	18	22·7	29·4	40	42·2
CALCULATED.†						
Square of period..	144	324	515	864	1600	1781
$(\text{Period})^2 \div 12^2$ ..	1	2·26	3·58	6	11·14	12·37
Moment of inertia of the system ..	I	$I + I_2$	$I + I_3$	$I + I_4$	$I + I_5$	$I + I_6$
(Total moment of inertia) $\div I$. ..	1	2·26	3·58	6	11·14	12·37
Therefore		$\frac{I_2}{I} = 1·26$	$\frac{I_3}{I} = 2·58$	$\frac{I_4}{I} = 5$	$\frac{I_5}{I} = 10·14$	$\frac{I_6}{I} = 11·37$
Relative moments of inertia for lead masses only		$\frac{I_2}{I_2} = 1$	$\frac{I_3}{I_2} = \frac{2·58}{1·26}$	$\frac{I_4}{I_2} = \frac{6}{1·26}$	$\frac{I_5}{I_2} = \frac{10·14}{1·26}$	$\frac{I_6}{I_2} = \frac{11·37}{1·26}$
		$= 1$	$= 2·05$	$= 3·97$	$= 8·05$	$= 9·02$
or approximately		1	2	4	8	9

Thus doubling the mass doubles the moment of inertia, doubling the distance increases the moment of inertia fourfold. Also the moment of inertia is trebled when the mass is trebled, and increased ninefold when the distance is trebled, etc.

† METHOD OF CALCULATION.—The moment of inertia of the bar, I, is constant throughout, and is not necessarily related in a simple manner to the moments of inertia when the lead strips are added. Hence, if I is subtracted from each of the latter, the respective remainders are the moments of inertia of the lead strips in their several positions, and should roughly vary as

(*mass of lead strips*) × (*mean distance from axis of rotation*)^2.

The subtraction of I is effected by assuming (§ 76) the moments of inertia to vary as the squares of the respective periods. Hence express the ratio to I of the total moment of inertia in each case; then deduce the ratio to one another of the moments of inertia of the added strips.

76. Torsion Pendulum.—It can be shown that $t = 2\pi \sqrt{I/c}$, where t is the period of oscillation, I the moment of inertia of the body about the axis of vibration, and c the moment of the torsional couple† called into operation by a unit twist (one radian). The form of this expression is the same as that for the common pendulum (§ 54). The torsional oscillations are isochronous whatever the angle (within wide limits) of vibration.‡ It therefore follows that the force of torsion is proportional to the angle of twist. When there is equilibrium the moment of the torsion couple equals that of the applied couple that produces the twist. Therefore the angle of twist is proportional to the moment of the *applied couple.*

If the dimensions and material of the suspension are varied, then the magnitude of the torsion-couple

(i) is inversely proportional to the length of the wire;
(ii) is directly proportional to the fourth power of the diameter of the wire;
(iii) depends on the material of the wire.

77. To find the moment of inertia of an irregular body about an axis through its centre of gravity. (1) The body, moment of inertia, I', is suspended at the end of a firmly clamped wire, and set in oscillation. The period (t) is determined. (2) A *regular body*, whose moment of inertia, I, can be calculated, is (i) added to or (ii) substituted for the irregular body, set in oscillation, and the period (T) determined. In case (i) I' is calculated from the relation

$$I'/(I + I') = t^2/T^2.$$

In case (ii) the relation is

$$I'/I = t^2/T^2.$$

THEORY. The arrangement is a torsion pendulum in which the moment of inertia of the suspended body is altered. The suspension is unchanged; hence the value of c (see § 76) is constant.

In case (i), $\quad t = 2\pi \sqrt{I'/c} \quad$ and $\quad T = 2\pi \sqrt{(I + I')/c}$

$$\therefore \; t/T = \sqrt{\{I' \div (I + I')\}}.$$

Exp. 81.—*Compare the moments of inertia of a thin circular disc* (1) *about an axis through its centre and perpendicular to its face,* (2) *about a diameter as axis.* (The former moment of inertia is twice the latter.)

Exp. 81a.—*Compare the moments of inertia of a cylinder when vibrating about an axis* (1) *continuous with the axis of the cylinder,* (2) *through the centre and perpendicular to the axis of the cylinder.*

Exp. 82.—*Find the moment of inertia of an irregular body.*

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† The forces called into play when a filament is twisted, compound into a couple, called the *torsion couple.*

‡ In this respect there is a difference from the common pendulum, whose oscillations are isochronous for small angles of swing only.

THE BAROMETER.

78. The *atmospheric pressure* at any time and place is usually measured by the vertical height of the column of mercury it can support. The height is expressed in either inches, or centimetres when the temperature of the mercury is that of melting ice. It varies between 28 to 32 inches, and 70 to 80 centimetres. The values, 30 *inches of mercury at* 32° *F.* and 760 *mm. of mercury at* 0° *C.*, are called the *normal pressures.*

Exp. 83.—*Set up a simple barometer.* Select a very clean glass tube, not less than 32″ long, nor $\frac{1}{4}$″ diameter, and closed at one end; fill it with mercury up to nearly half an inch from the open end.† Cover this end with the thumb and hold the tube slanting so that a bubble of air *slowly* moves along it to the closed end, then so that it returns. This sweeps out small bubbles of air that would otherwise be left. Now fill the tube completely with mercury, cover the open end firmly with the thumb, invert the tube and immerse the end, still covered by the thumb, in a basin of mercury. Hold the tube in a vertical position, and remove the thumb: the mercury then falls a little way in the tube leaving the top part empty.‡ Carefully incline the tube so that the mercury runs against the end: if all has been well done a sharp sound should be heard, there being no cushion of air to deaden it. Adjust the tube, by a plumb line, in a vertical position and fix it. Determine the distance in inches and cms. between the mercury surfaces in the tube and basin. This distance is the *height of the barometer.* The *vertical height* is the same when the tube is sloping, and is unaffected by changes in the bore of the tube. It increases, or diminishes as the atmospheric pressure increases, or diminishes. As the mercury rises, or falls in the tube it (slightly) falls, or rises in the basin or cistern. In less accurate barometers the measuring scale is fixed and the change of position of the mercury surface in the cistern is neglected.

† In making a barometer for permanent use the tube and mercury should be well heated to remove moisture.

‡ The empty space at the top end of the tube of a well made barometer is called the *Torricellian vacuum.*

79. The Syphon Barometer (Fig. 53).—A glass tube, *PQ*, that has been carefully filled with mercury, has the upper end of the longer limb closed, that of the shorter open. Both limbs are graduated† in inches or centimetres, reading as in I. or II. below:—

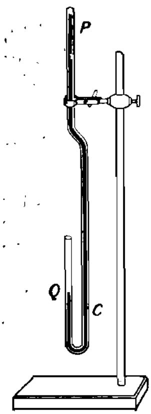

Fig. 53.

I. The scale on each limb has the same zero, situated near the middle of the tube. The scale on the long limb reads upwards, the other downwards. When the tube is set with the limbs vertical, the height of the barometer is found by adding the readings of the scale positions of the mercury surfaces.

II. The scale on each limb has the same zero situated near an end of the tube. The barometric height is then equal to the difference of the readings of the scale positions of the mercury surfaces.

In this barometer the error at the cistern is avoided: the mercury rises as high at one end as it falls at the other. Hence a change, x, in the barometric height produces a movement, $\frac{1}{2}x$, at both ends.

The tube is held by a retort-stand and clamp. The barometer is then portable. At *C* is an air-trap, constructed as Fig. 54. Air very slowly creeps along the tube between the mercury and glass. This is caught at *D*.

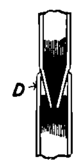

Fig. 54.

80. Fortin's Barometer (Fig. 55).—This is a cistern barometer with a fixed scale. The base of the cistern, being made partly of leather, is flexible, and may be altered in shape by the thumb-screw at the bottom in such a way that the surface of the mercury can be brought into contact with the tip of the fixed, ivory peg seen through the

† Sometimes the tube is mounted on a board, and the scales are also fixed to this.

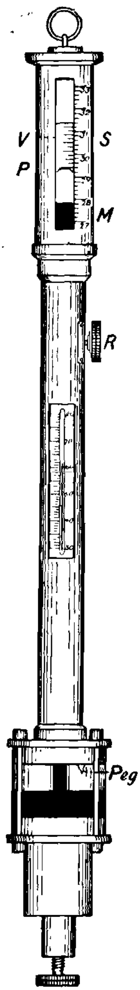

Fig. 55.

glass side of the cistern. The tip marks the zero of the barometer scale.

The *adjustment of the cistern* is done by looking along the surface so as to see the peg and its reflected image in the mercury. Turn the thumb-screw until the tips of the image and object just touch. A second method is to place a piece of white paper or glass behind the cistern. Gradually raise the surface of the mercury, by turning the screw, until the space between the tip and the surface just disappears.

The *reading of the upper surface* of the mercury is obtained by a vernier engraved on a metal slider, *PQ* (Fig. 56). Adjust as follows:—Look above the upper surface of the mercury, *M*, so as to see the front edge of the slider, *P*, and the back edge, *Q*. Move the slider by turning the side screw, *R*, and get the eye into such a position that *P* just hides *Q*, and the line of sight touches the mercury surface, *M*. Two white triangles (at *aa*) will then show, the mercury surface being convex. The position of *P* may be read by the scale and vernier (§ 21 (2)). By using the two edges, *P*, *Q*, in this way, parallax error (§120) is avoided.

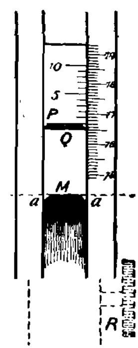

Fig. 56.

The best barometers are very frequently of the Fortin type. The internal diameter of the upper end of tube is not less than about $\frac{1}{2}''$. The scales are usually of silvered brass, sometimes of glass. Some barometers are provided with but one vernier and scale of centimetres or inches; others with two, so that the height may be measured in both units.

81. Corrections.—1. *Capillarity, etc.* A slight depression of the mercury is caused by *capillarity.* The error is nearly eliminated by using a tube of wide diameter (more than $\frac{1}{2}''$). This error, and others due to the setting of the scales, ivory point, etc., are determined by comparison with a standard barometer, say at the National Laboratory. The results of the comparison are shown in a certificate; from this the necessary correction may be found.

2. *Temperature.* The pressure of the atmosphere being conventionally expressed as the length in inches or centimetres of a mercurial column when the temperature of the mercury is that of melting ice, the value read at any time should, when accuracy is required, be corrected for temperature. A rise of temperature (i) lengthens the scale, (ii) diminishes the density of the mercury. The former tends to make the reading too low, the latter too high. It is shown (§ 82) that if h is the scale reading (metric) of the barometer, and t° *C.*, its temperature, then the height (H) expressed in terms of a column of mercury at 0° C. can be calculated from the formula

$$H = h(1 - 0{\cdot}000164\,t)\ mm.$$

3. *Latitude.*† If the height of the mercurial barometer is H *mm.* at a place of observation in latitude, ϕ, then its height, A, at the sea level, and at latitude 45°, is nearly

$$H - 2 \,.\, \cos 2\phi \textit{ millimetres.}‡$$

4. *Altitude.*† The correction is practically negligible.

5. *Atmospheric pressure in C.G.S. units.* A barometric height, A *mm*, is practically a pressure of

$$1\tfrac{1}{3} \times 10^3 \times A \textit{ dynes per sq. cm.}‡$$

Rules to be observed in using the Barometer.

1. Note the temperature, F.° or C.°, before opening the case of the barometer.

2. Gently tap the upper end of the tube.

† The need for these corrections arises from the fact that the weight of, and therefore the pressure due to a column of mercury depends to some extent on its position on the earth's surface.

‡ These rules are simpler than usual, but sufficiently accurate.

3. Gently tap the cistern and adjust the surface of the mercury to the tip of the peg.
4. Set the vernier.
5. Read the vernier.
6. Correct the reading for capillarity or zero error by reference to the certificate.
7. Correct for temperature by calculation.
8. Correct for latitude by calculation.

Exp. 84.—*Obtain the height of the Fortin barometer* on several occasions. Mention in each case the date and time of observation. Correct and reduce the readings. Calculate the atmospheric pressures in *dynes per sq. cm.*

82. To correct the actual reading of a barometer for temperature.

Let h be the scale reading, and t° C. the temperature,

λ, coefficient of linear expansion of scale per 1° C.,

μ ,, cubical ,, mercury ,,

The scale has been so graduated that the divisions would be millimetres at 0° C. Hence a scale length, h, at t° C. will be h mm. at 0° C.

$$\therefore \text{ length of } h \text{ divs. at } t^\circ = h(1 + \lambda \, . \, t) \text{ mm.}$$

Let Δ be the density of mercury at t°,

Δ_0 ,, ,, ,, 0°,

g ,, acceleration of gravity.

Then the pressure due to a column of mercury, $h(1 + \lambda \, . \, t)$ mm. high and of density, Δ,

$$= \tfrac{1}{10} h(1 + \lambda \, . \, t) \times \Delta \times g \quad \text{dynes per sq. cm.}$$

Let H mm. be the height of a column of mercury of density, Δ_0, which will give the same pressure as above. The pressure of this column is $\tfrac{1}{10} H \, . \, \Delta_0 \, . \, g$ dynes per sq. cm. Therefore

$$h \, . \, \Delta(1 + \lambda \, . \, t) = H \, . \, \Delta_0, \quad \therefore \; H = (1 + \lambda \, . \, t) h \Delta/\Delta_0.$$

$$\text{But} \qquad \frac{\Delta}{\Delta_0} = \frac{1}{1 + \mu \, . \, t}, \quad \therefore \; H = \frac{1 + \lambda \, . \, t}{1 + \mu \, . \, t} h.$$

Since λ and μ are comparatively small

$$H = \{1 - (\mu - \lambda) t\} h \quad \text{practically.}$$

For a brass scale $\lambda = 0{\cdot}000018$,

and for mercury $\mu = 0{\cdot}000182$;

$$\therefore \; \mu - \lambda = {\cdot}000164,$$

$$\therefore \; H = h(1 - 0{\cdot}000164 \times t) \text{ mm.}$$

BOYLE'S LAW.

83. Boyle's law.—This expresses the relation that exists between the pressure and volume of a quantity of gas when the temperature does not alter.

If a definite mass of gas is kept at a constant temperature then the pressure, P, varies inversely as the volume, V.

DEDUCTIONS.—Since P varies as $1/V$, then (i) when P is doubled, V is halved, when P is trebled, V is one-third, etc.;

(ii) $P \times V$ is a *constant* quantity. The graph of P and V is a rectangular hyperbola;

(iii) $\log P + \log V$ is a constant quantity. The graph of $\log P$ and $\log V$ is a straight line;

(iv) *the density of a gas varies directly as its pressure;*

(v) the higher the constant temperature the greater the value of $P \times V$ (see § 97).

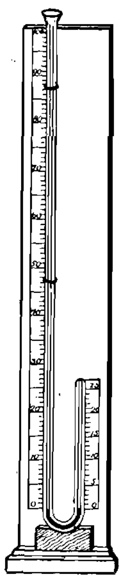

Fig. 57.

Exp. 85.—*Demonstration of Boyle's law.* (I.) Use a *Boyle's tube* (Fig. 57). The short branch (about 15″ in length) is closed, the long one (about 40″) is open. Both rest over a scale graduated, say, in centimetres (that under the short limb should be sub-divided into, say, millimetres). The apparatus is only suitable for pressures greater than atmospheric. Fix so that the limbs are vertical. Work over a mercury tray (see Appendix) on a low stool. Hang a thermometer near the short limb.

The gas used is *air*. Pour mercury in at the funnel in small amounts: (i) until its surfaces stand at the zeros of the scales (this is not essential, but convenient); (ii) add more mercury, and read the positions of the surfaces in the limbs; (iii) repeat five or six times. Observe first the height, H, of the barometer (syphon); second, the temperature after adding the mercury, but before reading the positions of the surfaces. Record as below.

Observation number ..	1	2	3	4	5	6
Barometric height ..						
Temperature	If variations are serious, work in a more sheltered spot.					
Position of the mercury surface—						
In the closed limb ..						
In the open limb ..						
Total mercury column ..						
Length or volume of air enclosed						
$\begin{pmatrix}\text{Total}\\\text{column}\end{pmatrix} \times \begin{pmatrix}\text{Volume}\\\text{of air}\end{pmatrix}$	(Numbers should be practically constant.)					
Log (total column) ..						
Log (volume of air) ..						

Plot (total column of mercury) and (volume of air).

Plot log (total column of mercury) and log (volume of air).

Show from the graphs that when the pressure is doubled or halved, the volume is halved or doubled, etc.

EXPLANATION: See Fig. 58. When, as in (i.), both surfaces are at 0, the pressure of the air in the closed limb equals that of the atmosphere. It is therefore expressed by the barometric height (H) in *mm.* of mercury. The volume of air enclosed is measured by the length, AO.

When the surfaces are at P,Q, the pressure of the enclosed air in *mm.* of mercury is $H +$ (length, QP, in *mm.*). The value of this is called the total mercury column. The volume of air enclosed is expressed by the length, AP.

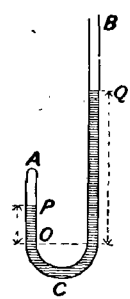

Fig. 58.

NOTE.—In order that volumes may be expressed by lengths the bore of the short limb must be uniform. In an accurate experiment it would be graduated into parts of equal volume (say *cu. cm.*). The bore of the long limb need not be uniform.

(II.) In a *better form of apparatus* (Fig. 59) two glass tubes of about the same length and diameter are joined by a long flexible pipe (thick-walled india-rubber). Sufficient mercury for the whole experiment is kept in the apparatus. One glass tube is closed at an end and fixed, the other is open and can be raised or lowered. The positions of the mercury surfaces are by this means altered: these can be read on the scales. Values for pressures *less* than atmospheric are obtained conveniently.

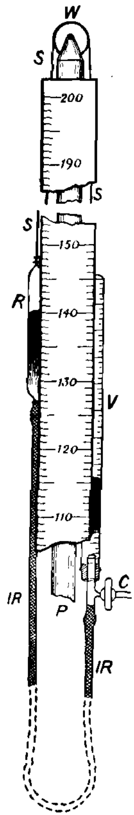

Fig. 59.

The closed tube, V, is graduated in cb. cm. Hence the volumes are measured directly. A stop-cock (C) is provided; by connecting this with a drying apparatus (see Appendix) dry air, or another gas may be drawn in for use in the experiment.

The reservoir, R, must be *moved with caution* by means of the cord, S. When R is adjusted the cord, S, must be firmly attached to the stud at the back of the apparatus.

In working with the apparatus use dry air. Note the readings of the mercury surfaces in R and V on the long scale of centimetres. The difference in the readings must be added to or subtracted from the barometric height, H, when the surface in R is above or below that in V. Read also the volume of the air in *cu. cm.* by means of the scale graduated on V. Observe and record as above (p. 113).

PART II.—HEAT.

CHAPTER VI.

THERMOMETRY, EXPANSION.

84. Thermometers.—The common thermometer† consists of a glass capillary tube having a *cylindrical* or *spherical* bulb at one end, and sealed at the other. The bulb is full of mercury (sometimes coloured alcohol). When its temperature rises or falls, the mercury advances or recedes along the capillary. The stem is usually graduated into either Centigrade or Fahrenheit degrees. The scale is practically, but not essentially, one of equal parts. (The divisions, however, are not simple fractions of the inch or centimetre.) The *reading* of a thermometer is the position on the scale at which the liquid surface in the capillary stands.

The temperature of melting ice is conveniently called the *ice-* or *freezing-point* (0° C., 32° F.); that of water boiling at the standard temperature and pressure is called the *steam-* or *boiling-point* (100° C., 212° F.) (see Exp. 95).

To identify the scale of a thermometer.—This is likely to be either Centigrade or Fahrenheit. Note the reading (1) when the thermometer is exposed in the room, (2) when the bulb is placed in the mouth. If the indication is about 60 in the room, and nearly 100 in the mouth, the scale is Fahrenheit; if about 16 in the room, and nearly 40 in the mouth, the scale is Centigrade.‡

† For a general account of Temperature and Thermometers see the Text-Book of Heat.

‡ The normal temperature of the body is 98·4° F. or 36·7° C.; that of a room is 62° F. or 16·7° C.

Frequently a thermometer has either the freezing-point or boiling-point, or both, named: the scale can then be readily identified.

Fig. 60.

Beckmann's Thermometer (Fig. 60). In this the bulb is comparatively large. The other end of the capillary tube is blown into a small bulb, bent over, and sealed. The scale is usually one of hundredths of a Centigrade degree. The range is only 4 or 5 degrees. This thermometer is especially useful for measuring *small changes* of temperature. If these are to be observed at *high* values, the instrument is heated until sufficient mercury has overflowed into the side bulb. The experiment is then done with what is left. At lower values more or less mercury is returned to the stem from the side bulb.

85. Maximum and Minimum Thermometers.—*A maximum thermometer* (Fig. 61, thermometer, X) indicates the *highest* temperature to which it has been exposed. Usually the liquid in the bulb is mercury. The stem is laid horizontally. An iron index (c) lies in the capillary, but outside the mercury. When the liquid expands sufficiently the index is pushed by the mercury surface further from the bulb, but, because it does not break through the surface, it is not drawn back on contraction. Hence the *end* of the index *nearer* the bulb marks the greatest advance of the mercury. This instrument also shows the temperature at the time of observation.

To read the maximum temperature in degrees, note the position on the scale of the end of the index *nearer* to the bulb. *To set the index* for an observation, slope the instrument and tap it gently

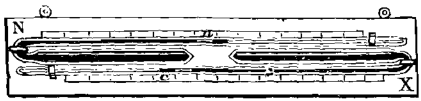

Fig. 61.

until the index has slipped as far as the surface of the liquid.

Negretti and Zambra's form. In this there is a constriction in the capillary close to the bulb that the mercury in expanding readily passes, but at which the thread breaks on contraction. This instrument does not show the temperature at the time of observation.

To read the maximum thermometer, slightly tilt the instrument so that the mercury that has passed the constriction slowly moves towards, and is stopped by it. When adjusted in this way the scale reading of the end of the thread further from the bulb is the maximum temperature. *To set the instrument* for a fresh observation shake the mercury past the constriction until the bulb is full.

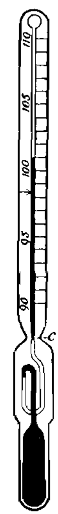

Fig. 62.

Clinical or medical thermometers (Fig. 62). These are sensitive short-range (95° F. to 110° F.) instruments (normal temperature of body = 98·4° F.). Some are made with a constriction (c) near the bulb, and will then indicate the maximum temperature.

A minimum thermometer (Fig. 61, thermometer, N) indicates the *lowest* temperature to which it has been exposed. The liquid in the bulb is usually alcohol. The stem is laid horizontally. A glass index (n) lies in the capillary immersed in the alcohol. When the liquid contracts sufficiently the index is drawn by the alcohol surface towards the bulb, but, as it does not break through the surface, it is not carried forward on expansion. Hence the *end* of the index *farther* from the bulb indicates the greatest contraction of the alcohol. This instrument also shows the temperature at the time of observation.

To read the minimum temperature in degrees, note the scale position of the end of the index *farther* from the bulb. The index is set similarly to that of the maximum thermometer, X (Fig. 61).

Six's maximum and minimum thermometer (Fig. 63). In this instrument, which indicates both the maximum and minimum temperatures which occur during a period of observation, there is a U-tube terminated at one end, *B*, by a large bulb, at the other end, *C*, by a small one. The bulb, *B*, and stem, as far as *m*, are filled with alcohol, a thread of mercury, *mm'*, occupies the bend, and the remainder of the stem adjacent to *C* contains alcohol. The branches of the tube are fixed over scales of Fahrenheit degrees. In each limb there is an iron index, *i*, *i'*, immersed in the alcohol, but not in the mercury.† If the alcohol in *B* expands, the mercury surface in the left hand branch rises and pushes the index, *i'*, upwards; if it contracts, the mercury surface in the right hand branch rises and pushes the index, *i*, upwards. The position of the mercury surface, *m*, shows the temperature at the time of observation.

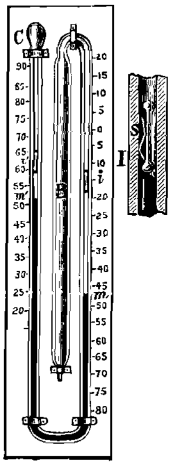

Fig. 63.

To read the minimum temperature, note the position on the scale of the end of the index, *i*, nearer the mercury thread. *To read the maximum temperature*, note the position on the scale of the end of the index, *i'*, nearer the mercury thread. *To set the instrument*, bring each index by means of a small horse-shoe magnet down to the mercury surfaces.

Exp. 86.—*Use of thermometers, etc.* At a definite time of day, say 10 A.M., note the temperatures indicated by various thermometers. Set the indexes of the maximum and minimum instruments. Some hours later, say 10 P.M., again note the temperatures, also the maximum and minimum values that have been reached during the period. Re-set the instruments and repeat the observations from day to day.

† Each index has a fine spring attached to it (see Fig. 63, right hand side). This by pressing against the walls of the tube just holds the index in place.

TRANSFERENCE OF HEAT.

86. To obtain a curve showing the rate of cooling of a liquid in a vessel.—Fill a vessel about three-quarters full of the hot liquid, immerse a thermometer in it, keep stirring gently, and note the temperatures after equal intervals, say of one minute, have elapsed. Also note the temperature of the room. *Tabulate* the observations in two columns, (i) time, (ii) temperature, corresponding values being on the same line. *Plot* temperature with regard to time.

The calorimeter may be hung from a ring by threads, or supported on three small corks, or a cross made as follows:—Two pieces of cardboard are cut as in Fig. 64; one is slotted along *a*, the other along the dotted line *a′*. The pieces are put together to form a cross. The calorimeter may be surrounded by a cylinder or water-jacket (§ 106.)

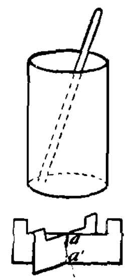

Fig. 64.

Exp. 87.—*Obtain the cooling curves for water in a tin can,* whose outside is (i) polished, (ii) jacketed with cotton wool, (iii) coated with lamp-black (hold it in the smoky flame of a lamp, or of burning camphor), (iv) when the blackened can is suspended by cotton loops within a larger one whose inside is bright, (v) as (iv) but when the inside of the larger can is blackened. Use the same quantity of water in each experiment, begin observations when its temperature is about 80° C., and continue to about 40° C.

Tabulate each case and *plot* all to a large scale on *one* sheet of paper. The curves will show roughly the relative *emissive* or *radiating powers* of the surfaces when polished, (i); blackened, (iii); lagged, (ii); when screened by an absorptive surface, (v), and a reflecting surface, (iv).

***Exp. 88.**—*Obtain the cooling curves for water in a tin can (polished),* (1) when nearly full, (2) three-quarters full, (3) half full.

Tabulate each case and plot all on one sheet. The observations will show that the smaller quantities cool more quickly than the larger.

* Home experiments are marked by an asterisk. See p. 8.

Exp. 89.—*Obtain and compare the cooling curves for different liquids.* Fill a polished tin can three-quarters full of (1) water, (2) methylated spirit, (3) glycerine, (4) petroleum. Begin observations at about 65° or 70° C. Get the methylated spirit and petroleum hot by putting the flasks holding the liquids in hot water. Being *very inflammable* they must not be heated directly over a flame. *Tabulate* each case, *plot* all on one sheet.

The curves will show that the rates of cooling of different liquids are slower for those whose specific heats or thermal capacities are higher.

***Exp. 90.**—*Warming curve of water.* Light a Bunsen burner, keep it burning steadily during the experiment. Place a tripod over it. Put about a pound of ice, pieces the size of a walnut, in a can; also a thermometer. At a definite time, place the can over the Bunsen flame, observe the temperature at starting, and after intervals of one minute have elapsed. *Stir gently with the thermometer* throughout the experiment and continue the observations until the whole of the water has just boiled away. Also note the times and temperatures at which (i) all the ice is melted, (ii) the water begins to "sing," (iii) the water begins to boil.

The thermometer should be removed and the observations of time and temperature suspended several minutes before the end. But the time when the water disappears must be noted. Then remove the Bunsen burner.

Tabulate (1) times, (2) temperatures, and plot the latter with regard to the former as abscissae.

REMARKS.—Since the Bunsen flame is kept burning steadily, it can be assumed that equal quantities of heat per unit time are produced by the combustion.

The graph consists of (i) a short horizontal branch that rounds easily into (ii) a middle, nearly straight, sloping part, (iii) a horizontal line.

The part (i) shows that the temperature of ice and water is at first constant, but when the proportion of ice present is inconsiderable, the heat absorbed from the flame warms the water more quickly than the ice can melt, and hence the temperature rises (compare Exp. 91).

The straightness of part (ii) indicates that the thermal capacity † of the water is, roughly speaking, constant, the rise of temperature of the water being proportional to the quantity of heat obtained from the flame. The line slopes a little less when vaporisation becomes appreciable—the "singing" stage; heat is then absorbed in producing vapour.

The straightness and horizontality of branch (iii) demonstrate the constancy of the temperature of boiling. The heat absorbed from the flame now no longer affects the thermometer; it is all used in making steam. The fact is generally expressed by saying that the heat becomes "latent." The constancy of the temperatures of melting ice and boiling water is taken advantage of in graduating a thermometer.

Observe that the interval of time taken to heat the water from ice- to steam-point is not quite one-fifth of that required to boil away the whole of the water. Thus it takes more than five times as much heat to convert a quantity of boiling water into steam, as to raise the water from ice-point to boiling. This is a rough measurement of the *latent heat of vaporisation* (Exp. 119).

***Exp. 91.**—*Warming curve of ice.* Put about a pound of ice into a vessel exposed to the warm air of the room: note the temperature at intervals of, say, five minutes; note the time when all the ice has melted.

Tabulate and plot time and temperature.

The graph is first horizontal, but at the time when all the ice has melted turns upwards and continues as a slightly sloping straight line. Hence the *temperature of melting ice is constant:* the heat absorbed becomes "latent," and is used in producing the change of state from ice to water.

† The thermal capacity of a mass of water has a minimum value at about 40° C., and is about 0·7 per cent. larger at the ice- and steam-points.

GRADUATION OF THERMOMETERS.

Exp. 92.—*Mark the fixed points of a mercurial thermometer.*† Use an ungraduated thermometer.

Ice-point: place the thermometer in a large glass, pack small pieces of ice round it. After a little time twist a piece of fine wire or tie cotton (a double turn) tightly round the stem and push it to the point where the mercury surface stands when the bulb is in ice.

Steam-point: Boil water in a flask. Hang the thermometer in the steam (not in the water). After a little time mark, as before, the position of the mercury surface.

Immerse the thermometer in (1) a hot and (2) a cold bath, mark the positions of the mercury surface. Note also the temperatures of the baths with a Centigrade thermometer.

Measure the distances between the ice-point (A) and each of the three marks, B, C, and steam-point, D. Calculate AB/AD, AC/AD. Deduce the temperatures of the baths on the Centigrade and Fahrenheit scales. Compare the results with the observations by the Centigrade thermometer.

Exp. 93.—*Comparison of two graduated thermometers.* Place the thermometers, A,B, together in a bath. One of the thermometers, A, is to be considered as the reference or standard thermometer. Warm the bath to definite temperatures as indicated by A, say 0°, 10°, 20°, etc. Note the respective readings of the other thermometer, B. Between the temperatures of observation the bath may be heated quickly, but when near to one of them adjust the Bunsen flame so that the temperature changes very slowly: stir well. Compare a Fahrenheit thermometer with a Centigrade. *Plot* (i) the readings of the Fahrenheit thermometer with regard to those of the Centigrade, also (ii) Fahrenheit temperatures equivalent to Centigrade. (The latter is a straight line: 50° F. = 10° C., 140° F. = 60° C.)

If the curve obtained in (i) is approximately a straight

† This experiment is a rough, illustrative one.

line, then the bore of *B* is practically as uniform as the standard. If the curve is definitely above or below the straight line of equivalent temperatures (ii), it is likely that the scale has been badly fixed to the stem.

Exp. 94.—*Graduate an alcohol thermometer.* An ungraduated alcohol thermometer is provided. As alcohol boils at 78° C., temperatures higher than this (amongst these is the steam-point) cannot be determined. The instrument is graduated by direct comparison with a standard mercurial thermometer. Immerse (see Exp. 93) the thermometer successively in baths at 70°, 45°, 20°, 0° (ice and water), − 10° (ice and salt), as indicated by the standard thermometer. In each case mark the position of the alcohol surface with cotton. Measure off by compasses the distances between the marks, transfer them to a card, and divide the spaces so that the scale may read to 5°. Bind the card to the stem by fine wire.

Exp. 95.—*Correct the fixed points of a thermometer.* (1) *The ice- or freezing-point.* Hold the thermometer by a clamp in a can (Fig. 65) or funnel. Surround the thermometer with small pieces of ice,† heaping it round the stem until the mercury surface is only just exposed at the front. Allow the thermometer to remain in the ice bath for 10 or 15 minutes. Then estimate its reading in tenths of a degree. The difference between the reading and the mark 0 (32 on a Fahrenheit thermometer) is the *error at the ice-point.* The error is reckoned + if the observed position is below the ice-point mark on the scale, − if above.

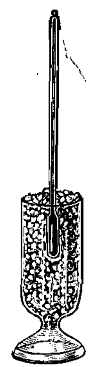
Fig. 65.

(2) *The steam- or boiling-point.* The *hypsometer* (Fig. 66) is usually employed. (An easily constructed form is shown in Fig. 68.) Water is boiled in the lower part, the steam rises through the central tube, down between the centre and outside, and issues at the side. (Use a tin

† Well wash the ice to remove salt, the presence of which sensibly lowers the melting-point.

to catch the water that drips from this.) Thus the central tube is steam-jacketed. The gauge (left side) shows the difference, if any, between the pressure of the steam and that of the atmosphere. The thermometer passes through a cork in the lid, and is suspended in the steam in the central tube. Adjust it so that the end of the mercury column just shows above the cork. The thermometer should pass easily through a hole in the cork. (If loose put a small piece of rubber tubing round it to prevent it from falling through.) (i) Note its indications every two minutes, the reading being estimated to, at least, tenths of a degree. Between these times observe (ii) the temperature, (iii) the height of the barometer every two minutes. When several practically constant readings of the three quantities have been obtained the experiment is completed. The stem may conveniently be viewed with a telescope placed with its axis horizontal some distance off. Remember in making the reading that the image in the telescope is inverted. Tabulate (i) thermometer reading, (ii) barometer temperature, (iii) barometer reading.

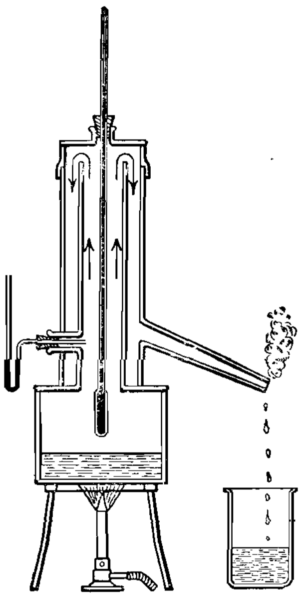

Fig. 66.

From the final barometer reading calculate the atmospheric pressure, A, in *mm.* of mercury at sea level, latitude 45°, and 0° C. (§ 81).

Calculate the boiling-point (t) of water at pressure, A, by assuming that the temperature of boiling alters at the rate of 1 C.° per 27·3 mm. (of mercury) change of pressure, or 0·037 C.° per 1 mm. (of mercury) change of pressure.

Hence $(A - 760) : 27{\cdot}3 = (t - 100) : 1$

$\therefore\ t - 100 = (A - 760) \div 27{\cdot}3 = 0{\cdot}037\ (A - 760).$

The difference between the reading of the thermometer and the actual temperature, t, is the *error at the steam-point*. The correction is reckoned + if the reading is less than t, and − if greater.

TABLE.

The following are corresponding values (Regnault, Brock) of boiling-points of water and atmospheric pressures :—

Boil.-pt. water	99·0	99·5	100·0	100·5	101
Cm. of mercury Lat. 45°, sea level	73·316	74·648	76·000	77·373	78·767
Dynes per sq. cm.	977 500	995 300	1 013 300	1 031 600	1 050 200

MELTING-POINTS.

87. To find the melting-point of a substance.—METHOD I. Close and blow one end of a length of quill tubing into a bulb ($\frac{1}{4}$ inch diameter) with thin walls, or draw out the quill tubing to form a coarse capillary tube with thin walls. Fill the tube with small pieces of the substance whose melting-point is to be determined. Fasten it to the thermometer by fine wire, so that its closed end is against the bulb of the thermometer. Immerse in a water bath. (Use an oil or hot air bath if higher temperatures are required.) Heat and stir the bath; note the temperature when the substance appears to melt. Remove the flame and allow to cool, stir; note the temperature when the substance appears to solidify. These temperatures indicate the position of the melting-point. Replace the Bunsen flame and turn it low, so that the bath heats very slowly; stir gently. Note as before the temperatures when the substance appears (i) to melt on warming, (ii) to solidify on cooling; repeat until the temperatures differ very slightly in several successive determinations. Assume the melting-point to be the mean of these temperatures.

METHOD II. In the case of a wax immerse the thermometer in a quantity of the melted substance, on withdrawing some adheres to it. When this solidifies the bulb assumes a dull appearance: note the temperature when this shows itself. Next hold the bulb in the hot air, say, a foot above a Bunsen flame. Note the temperature when the dulling disappears. Repeat until the temperatures of the appearance and disappearance of dulling differ only slightly. Assume the melting-point to be the mean of these.

METHOD III. Place a quantity of the substance in a vessel and melt it.† Then place the vessel within an enclosure (§ 106) and make observations of time and temperature. Tabulate the values and plot the cooling curve.

The first part of the graph due to higher temperatures slopes steeply downwards, the next is less steep or horizontal, and rounds into a third part sloping downwards.

When the second portion is straight and horizontal, it is inferred that the substance melts at a definite temperature, viz. that temperature on the vertical axis in line with the horizontal part. When the second portion slopes more or less, then the substance has no definite melting-point, but is one that is in a plastic, or partially fluid condition over a more or less considerable range of temperature.

When alloys are melted and allowed to cool, the curve obtained often shows more than one nearly horizontal portion. Each of these marks the melting-point of one of the metallic constituents, or a stable alloy formed between two or more of them.

To investigate the *melting-point of an alloy* it is convenient to use a cylindrical iron vessel. An iron rod is welded into its base and rises from it internally and centrally. The rod is bored with an axial hole large enough to take a thermometer bulb easily. The space between the bulb and tube is filled with mercury, or iron or copper filings. By using special thermometers high melting-points may be determined.

Exp. 96.—*Find by using small tubes the melting-points of paraffin wax, beeswax, sulphur, etc.*

***Exp. 97.**—*Find by a cooling curve the melting-points of paraffin wax (candle), and beeswax.*

Exp. 98.—*Find by a cooling curve the melting-point of naphthalene.*

Exp. 99.—*Investigate by cooling curves the melting-points of lead, tin, solder, and an alloy of tin and lead (say 6 lead to 1 tin).*

---

† Melt the substance by placing the vessel holding it in (i) hot water, (ii) hot glycerine, or a hotter liquid, if necessary.

BOILING-POINTS.

88. To find the boiling-point of a liquid.—Boil the liquid in a flask or large test-tube and arrange a thermometer so that its bulb is in the *vapour* from the liquid. To make the boiling regular put fragments of glass or earthenware in the vessel. Note the highest temperature (which should remain constant for five minutes) indicated by the thermometer: this is the boiling-point of the liquid at atmospheric pressures. Observe also the height of the barometer.

To determine the *boiling-point of a saline solution*, place the bulb of the thermometer in the *liquid* and read the temperature when boiling begins.

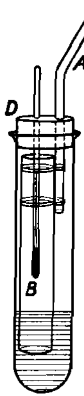

Fig. 67.

APPARATUS.—I. Fit a cork to a flask or large test-tube, bore two holes through it, pass a thermometer through one, and a discharge tube through the other.

II. It is an improvement to arrange so that the thermometer bulb lies in a tube jacketed by the vapour. A satisfactory method of doing this is to fix by fine wire a thin-walled glass tube, *B* (Fig. 67), to the inside part of the delivery tube. Its upper end should be a little below the cork, its lower should dip into the liquid.

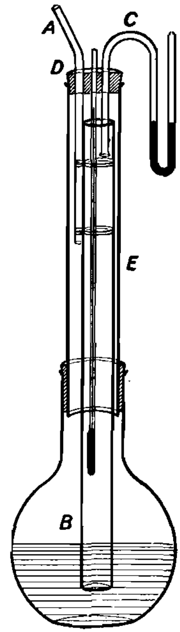

Fig. 68.

Fig. 68 shows a similar apparatus consisting of a wide-necked flask fitted with a lengthening tube, *E*. It can be used with long thermometers or as a hypsometer (Exp. 95). A pressure gauge, *C*, may also be fitted. The tube, *E*, may be fixed in the neck of the flask by rag, the joint need not be very tight. Support the apparatus on a retort stand; hold the tube, *E*, by a clamp.

***Exp. 100.**—*Find the boiling-point of methylated spirit*, etc. Beware of *inflammable* vapours. A condenser should in such cases be attached to the delivery tube, *A*.

***Exp. 101.**—*Find the boiling-point of a solution of salt.* Weigh 50 grms. of salt, dissolve in 500 cu. cm. of water. Find the boiling-point of (i) the above solution, (ii) equal volumes of the solution and water, (iii) one volume of solution, two volumes of water, etc.

VAPOUR PRESSURE.

89. Vapour pressure (sometimes called *vapour tension*). —Dalton showed that the pressure exerted by a vapour cannot exceed a certain maximum, the value of which depends on the nature of the vapour, and the temperature of the space in which it exists; it is unaffected by the pressure of other gases, vapours, etc., present in the same space. A table showing the values for water at different temperatures is given in the Appendix. The graph obtained by plotting these values is a curved line rising more rapidly at higher temperatures. In a given space at a temperature, t, the pressure of the vapour present may have any value less than the maximum for $t°$. The actual value is practically proportional to the density (mass per unit volume) of the vapour within the space; it may be increased *up to the maximum* by augmenting the quantity of vapour, diminishing the space containing it, or increasing the pressure. When the vapour pressure has the maximum value the vapour is said to be *saturated*, when below this value it is *unsaturated* or *dry*. A saturated vapour is in such a condition that the slightest increase of pressure, diminution of volume, or lowering of temperature condenses a portion into liquid. The maximum vapour pressure at $t°$ is called the *saturation pressure at $t°$*; the maximum vapour density at $t°$ is called the *saturation density at $t°$*. A vapour is always saturated if some of the liquid from which it is formed is in the same space. If the space containing vapour and liquid is increased more liquid evaporates, so that the saturation or maximum pressure may be maintained. The rate of evaporation depends on the pressure on the liquid surface, whether due to the vapour or other gases. If the space continues to increase after all the liquid has evaporated the pressure will fall, and the vapour becomes unsaturated.

Exp. 102.—*Find the vapour pressure of ether, etc., at the temperature of the air.*

Work over a mercury tray on a low table.

Carefully fill *two* barometer tubes with mercury as in Exp. 83. Clamp each to a *heavy* retort stand, side by side in a vertical position, with the open ends immersed in a vessel of mercury. One tube (B) is to act as a barometer; no liquid is to be put into it. Into the other (A) introduce a little ether. This is conveniently done by using a pipette whose jet is bent up so that it can be poked into the open end of the tube while under the mercury. Observe that as soon as the ether reaches the top it evaporates and the mercury column is depressed. Mark on B, by stamp paper, the level of the depressed surface in A. Unclamp the tube A, hold it *firmly* by hand and slowly slope it. The depression will increase a little, but the volume of tube above the mercury will diminish. As soon as a layer of liquid ether shows on the surface in A, clamp the tube, and mark on B, by stamp paper, the level of the surface in A.

Again clamp the tube vertically, add a little more ether, and repeat the observations.

Continue to add small quantities of ether, and repeat the observations until a *thin* layer of the liquid remains when the tube is in the vertical position.

It will be found that the initial depression is greater as more ether is added, but the second mark is always the lower and keeps at a constant level. Hence the pressure of the unsaturated vapour (when *no* liquid is present) rises as the quantity of ether is increased to a maximum, viz., the pressure of the saturated vapour, but does not increase beyond this. Note that, in the last stages, sloping the tube largely increases the layer of condensed ether.

Finally clamp the tube vertically, and measure the difference of level of the mercury in the tubes, by applying a cm. scale, or using compasses.

Note the temperature, t, of the air.

Find similarly the vapour pressure at $t°$ C. of alcohol; also of water. Use separate tubes and pipettes for each substance.

HYGROMETRY.

90. Hygrometry deals with the measurement of the quantity of aqueous vapour present in the air. A determination is made of the *relative humidity* (also called the *humidity* or *fraction of saturation*): this is defined as the ratio of the actual pressure of the water vapour present in the air at any time to the maximum or saturation pressure for the same temperature. Practically the fraction-of-saturation is also equal to the ratio of the mass of vapour present per unit volume to the saturation density.

If moist air is cooled a temperature will be reached when vapour begins to condense. This temperature is called the *dew-point*. It is the temperature at which the vapour actually present would be saturated. Now the saturation pressure at the dew-point equals the pressure of the aqueous vapour at higher temperatures. Hence the

$$\textit{fraction of saturation} \text{ or } \textit{relative humidity}$$
$$= \frac{(\textit{saturation-pressure at the dew-point})}{(\textit{saturation pressure at the actual temperature of the air})}.$$

The simplest way to measure the humidity is to find (i) the dew-point (T), and (ii) the temperature of the air (t), then (iii) calculate the fraction-of-saturation by looking out from a table (§ 92) the vapour or saturation-pressures at t° and T°.

***Exp. 103.**—*The Dew-point* (§ 91). Half fill a thin glass (plain ruby or green) with water from the tap. Add pieces of ice, stir with thermometer, and note the temperature (T) when a deposit of moisture begins to show on the glass. This is the dew-point. Dry the thermometer and note the temperature (t) of the air. Look up in a table (§ 92) the saturation pressures at these temperatures and calculate the fraction of saturation.

***Exp. 104.**—*Wet and dry bulb.* Suspend the thermometer (clean and dry) and expose to the air, note temperature (t). Loosely wrap a piece of string round the bulb and well moisten it. After a little time note the temperature (t_1) of the wet bulb. Obtain the dew-point and fraction of saturation as in § 92.

91. Hygrometers.—These are instruments for determining the humidity of the air.

Chemical hygrometer.—In this an aspirator draws a measured volume of air through two drying tubes. Of these the one further from the aspirator is weighed at the beginning and at the end of the experiment: the increase in weight is that of the water vapour absorbed from the air that has passed through. The second drying tube absorbs the vapour that may diffuse from the aspirator. (See *Text-Book of Heat.*)

Dew-point hygrometers.—(1) *Daniell's hygrometer* (Fig. 69). A closed glass tube bent at right angles has a large bulb at each end. One of these (B) is wrapped in muslin. The other (A) holds ether, into which a thermometer (t) dips: this bulb is frequently gilded or made of black glass, in order that the first deposit of dew may be easily seen. The tube is held by a stand, to which is also fixed a thermometer, t', for showing the temperature of the air.

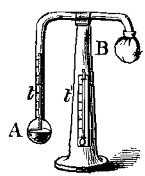

Fig. 69.

To find the dew-point, tilt the instrument so that all the ether passes into A, moisten the muslin well with ether and carefully observe the surface of the bulb, A, in a good light (incident obliquely). Note the temperature indicated by t, as soon as (i) dew appears, and (ii) disappears. The mean of these is the *dew-point*. Note also the temperature of the air. Note whether the thermometer readings are $C.^\circ$ or $F.^\circ$

NOTE.—The observer should place a large sheet of glass between himself and the instrument. If a fine wire is drawn over the surface of the bulb it is easy to detect the first traces of moisture.

The Daniell's hygrometer is unsatisfactory in practice, because (i) it is difficult to detect the first appearance of dew, (ii) the surface layer of ether from which the evaporation takes place is colder than the bulk of the liquid of which the temperature is indicated by the thermometer, and (iii) as the observer stands near the instrument his presence is likely to affect the hygrometric conditions there.

(2) *Regnault's hygrometer* (Fig. 70). A smooth, well-polished, thin-walled silver thimble, *S*, is cemented to the end of a glass tube, *T*. A cork carries a thermometer, *t*, and a tube, *d*, whose inside end is near the bottom of the thimble. The side tube, *c*, is joined to an aspirator (*A*). There is usually a second thimble, *S′*, and tube, not connected however with the aspirator. The thermometer, *t′*, shows the temperature of the air.

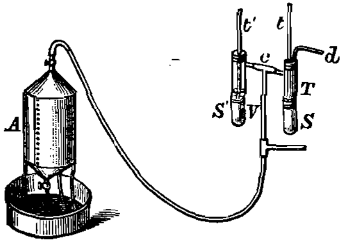

Fig. 70.

To find the dew-point, arrange a telescope to view the hygrometer from a distance. Place ether, or alcohol in *S*, and adjust the aspirator so that air freely bubbles *through the spirit. Note the temperature of t* when (i) dew appears, (ii) disappears (after stopping the bubbling); also the temperature of air (by *t′*). Observe whether the readings are *C.*° or *F.*°

By watching and contrasting the two silver surfaces it is easy to see when the dulling due to the deposition of moisture begins and disappears. Also, by adjusting the aspirator, the rate of cooling may be controlled so that the temperatures of the appearance and disappearance of dew are very nearly the same.

92. Wet and dry bulb, or Mason's hygrometer (Fig. 71).—Two thermometers are suspended side by side. One of these *indicates the temperature of the air.* The bulb of the other is covered with muslin kept moistened by a tail of lamp-wick† that dips into an adjacent cup of water. This wet bulb thermometer shows, except when the air is saturated, a lower temperature than that of the dry bulb.

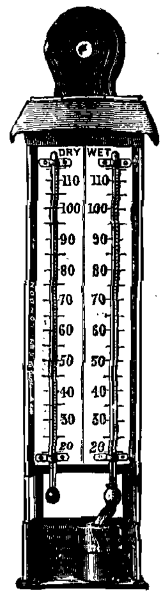

Fig. 71.

To find the dew-point, observe the temperature of (i) the dry, (ii) the wet thermometer (note whether the readings are *C.*° or *F.*°). *Refer to the table below:* from the top line look out the *difference* between the wet

† The wick should be boiled in washing soda to remove grease and then in water, before attaching as above.

and dry readings, follow down the column under this, and pick out the number (f) that is on the same line as the air or dry bulb temperature in the first column. This number (f) is the pressure in *mm.* of the vapour actually present in the air at the time of observation. The second column, under difference 0, gives the saturation pressures of aqueous vapour at the temperatures indicated in the first column. Hence look down the column under 0 for the number equal to f: the temperature beside this is the dew-point (calculation may be necessary).

To calculate the fraction of saturation.—Write as the numerator of a fraction the number (f) in the second column beside the dew-point ($T°$ C.) in the first, and as denominator the number in the second column beside the air temperature ($t°$ C.) in the first. Express the fraction as a decimal: then the result as a percentage (example, Exp. 105).

WET AND DRY BULB HYGROMETER (from Smithsonian Table 170).

Reduce observations to C.° and mm. pressure.

Air Temp.	Difference between Wet and Dry Bulbs in C°.										
	0	1	2	3	4	5	6	7	8	9	10
C.°	Saturation Pressures in mm. of Mercury.										
0	4·6	3·7	2·9	2·1	1·3						
2	5·3	4·4	3·6	2·7	1·9	1·1	0·3				
4	6·1	5·2	4·3	3·4	2·6	1·8	0·9				
6	7·0	6·0	5·1	4·2	3·3	2·4	1·6				
8	8·0	7·0	6·0	5·0	4·1	3·2	2·3	1·4	0·6		
10	9·2	8·1	7·0	6·0	5·0	4·0	3·1	2·2	1·3		
12	10·5	9·3	8·2	7·1	6·0	5·0	4·0	3·0	2·1	1·2	0·3
14	11·9	10·7	9·4	8·3	7·1	6·1	5·0	4·0	3·0	2·0	1·1
16	13·5	12·2	10·9	9·7	8·4	7·3	6·0	5·0	4·0	3·0	1·9
18	15·4	13·9	12·5	11·2	9·9	8·6	7·4	6·3	5·1	4·0	3·0
20	17·4	15·9	14·3	12·9	11·5	10·2	8·8	7·6	6·4	5·2	4·1
22	19·7	18·0	16·4	14·8	13·3	11·9	10·5	9·1	7·8	6·6	5·4
24	22·2	20·4	18·6	17·0	15·3	13·8	12·3	10·9	9·4	8·1	6·8

For intermediate temperatures take proportional values.

Exp. 105.—*Find the dew-point by the hygrometers supplied, and the fraction of saturation of the atmosphere. Calculate the mass of a cubic metre of the moist air, and of the aqueous vapour present in it.* Observe the dew-point (T),† temperature of air (t), reading of barometer (h), temperature of barometer (t'). Record and calculate (using logs.) as below:

Observations—

Hygrometer: temperature, wet bulb, 12° C.; dry bulb, 15° C.
Barometer: temperature, 15° C.; height, 77·41 cm.

Calculation of dew-point: difference between wet and dry bulb = 3°.
Then vapour pressure = 9 mm. Then dew-point = 9·7° C.
Saturation pressure at 15° C. = 12·7 mm.,
∴ fraction of saturation = 9/12·7 = 0·71 or 71 per cent.

Correction of barometer: height in centimetres
$= (1 - 0{\cdot}000162 \times 15) \times 77{\cdot}41 = 77{\cdot}22.$

Calculation of mass of a cubic metre of moist air:
the total pressure = 77·22 cm., vapour pressure = 0·9,
∴ pressure of dry air = 76·32 cm.
Therefore mass of 1 cubic metre of (§ 71)

$$\begin{cases} \text{dry air} = 0{\cdot}001293 \times \dfrac{273}{288} \times \dfrac{76{\cdot}32}{76} \times 10^6 = 1231 \text{ gm.,} \\ \left.\begin{matrix}\text{aqueous} \\ \text{vapour}\end{matrix}\right\} = \dfrac{5}{8} \times 0{\cdot}001293 \times \dfrac{273}{288} \times \dfrac{0{\cdot}9}{76} \times 10^6 = 9 \text{ gm.,} \\ \text{moist air} = 1231 + 9 = 1240 \text{ gm.} \end{cases}$$

EXPANSION.

93. Expansion.—Most bodies when heated increase their dimensions proportionally in every direction: that is, if for a certain temperature range the length increases by 1/100th per cent., the breadth and depth will also increase by the same fraction.

Def.: **Coefficient of linear expansion.**—*The ratio of the increase in length, in any direction, produced by a rise of temperature of 1°, to the original length in the same direction is called the coefficient of linear expansion* (symbol α).

† With the wet and dry bulb hygrometer the dew-point need not be determined.

Similarly the coefficients of superficial (β) and of cubical (γ) expansion are the proportional enlargements of area and volume respectively.

If L_0, S_0, V_0 are respectively the length, area, and volume to begin with, and L, S, V, the same quantities after heating through $t°$, then the three coefficients of expansion are

(i) *Linear*, $a = \dfrac{L - L_0}{L_0 . t}$; $\therefore$ $L = L_0(1 + at)$.

(ii) *Superficial*, $\beta = \dfrac{S - S_0}{S_0 . t}$; $\therefore$ $S = S_0(1 + \beta t)$.

(iii) *Cubical or Volume*, $\gamma = \dfrac{V - V_0}{V_0 . t}$; $\therefore$ $V = V_0(1 + \gamma t)$.

The

cubical coefficient = 3 (*linear coefficient*); or $\gamma = 3a$.
superficial coefficient = 2 (*linear coefficient*); or $\beta = 2a$.

For liquids and gases the volume or cubical coefficient is the only one considered. With these it is frequently most important to deal with change of density rather than change of volume (see below).

Relation between density (D) and the volume coefficient (γ).—Let D_0 and D be the densities at the initial and final temperatures (difference $= t°$). Since the volume of a given mass is inversely proportional to the density, then

$$\gamma = \frac{V}{V_0 . t} - \frac{1}{t} = \frac{D_0}{D . t} - \frac{1}{t} = \frac{D_0 - D}{D . t},$$

whence $D_0 = D(1 + \gamma t)$.

94. To fill with a liquid a bulb having a capillary stem.—(1) Attach a small thistle funnel by a short length of caoutchouc tubing to the stem. To stiffen the arrangement bind a piece of glass tubing partly to the stem, partly to the funnel. Pour some of the liquid into the funnel. For a short time ($\frac{1}{4}$ min.) hold the bulb over, not in, a Bunsen flame; move it about while doing so. Withdraw the bulb from above the flame, and allow to cool. Liquid passes down the capillary and partly fills the bulb. Now boil the liquid in the bulb, then allow to cool: the vapour in the bulb condenses, liquid flows down the capillary, fills the bulb and part of stem. Remove the funnel and tubing, then allow to stand.

(2) When it is inconvenient to attach a funnel, warm the bulb, then immerse the end of the stem in the liquid: some of it rises into the bulb. Boil the liquid in the bulb, and again immerse the end of the stem in the liquid: the bulb cools, the vapour condenses, and the liquid passes into and fills the bulb.

(3) When the liquid is mercury or glycerine, it is well not to boil it. Repeat the warming and cooling of the bulb until it is full.

If bubbles appear shake them out. To do this safely, hold the stem close to the bulb, lay it roughly parallel to the arm with bulb outwards, then jerk smartly from the elbow. By holding it with the bulb inwards liquid may be jerked out and bubbles in.

(4) Bulbs may also be filled by placing them and the liquid in a bottle from which the air is exhausted by an air or vacuum pump. If the end of the stem is immersed in the liquid, this will be forced into the bulb when air is admitted into the vessel after exhaustion.

Exp. 106.—*The Volume Dilatometer. Find the apparent expansion of water.* Use a glass bulb with a capillary stem. Make several fine file marks on the stem. (Bulb, $1\frac{1}{2}''$ diameter; capillary bore, $\frac{1}{10}''$ diameter; stem, $20''$ long; marks about 2 inches apart.) Slightly warm the bulb, immerse the end of stem in mercury, pick up a thread of mercury, 6 or 7 inches long. Then obtain the mean cross section (a) of the bore of tube (Exp. 68). Get the mercury out of tube by warming the bulb. Weigh the empty tube (W_1). Fill it with distilled water that has been boiled some time (to remove air). Arrange so that at ordinary temperatures the liquid stands just below the first file mark nearest the bulb: clean and dry the outside, and weigh (W_2). Measure the distances between the first and succeeding marks with a finely divided scale or travelling microscope. Let l_2, l_3, etc., be the values of these lengths.

Immerse the bulb in a water bath, and adjust the source of heat (a small flame some distance below) so that the liquid surface in the bulb remains at the first mark on the stem for several minutes. Keep the bath well stirred. Finally note the temperature, t_1.

Similarly for the second (temperature, t_2) and succeeding marks (temperatures, t_3, t_4, etc.).

Tabulate and plot lengths and temperatures. The graph will not be quite straight. Find the mean length (l) expanded per degree from 25° to 50°, from 50° to 75°.

Calculate the

coefficient of apparent expansion between any two temperatures

$$= \frac{\text{(increase of volume)}}{\text{(initial volume) (change of temperature)}}$$

$$= \frac{\text{(length between two marks)} \times a}{(W_2 - W_1)\text{ (difference of temperature)}}.$$

Similarly *find the apparent expansion of alcohol, glycerine*, etc. The specific gravities of liquids other than water must be determined. Then initial volume $= (W_2 - W_1)/(Sp.g.)$.

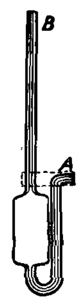

Fig. 72.

A convenient form of volume dilatometer is illustrated in Fig. 72. Two capillary tubes pass from the bulb. The open end, *A*, is immersed in the liquid to be experimented upon. The bulb is filled by suction at *B*. The end, *A*, is ground flat, and is closed by holding a small glass plate against it by a rubber band (dotted).

95. Absolute expansion of a liquid.—To find the coefficient it is necessary to obtain (i) the cubical expansion of the vessel holding the liquid, (ii) the *apparent* expansion of the liquid relative to the vessel. *The absolute or actual coefficient of expansion* of the liquid is equal to the coefficient of cubical expansion of the vessel added to the coefficient of apparent expansion of the liquid.

(i) **To find the expansion of the vessel** it is filled at a definite temperature, t_1, with mercury, a liquid whose absolute coefficient of expansion is accurately known

(= 0·0001815 per C.°). It is then heated to a temperature, t_2, the mercury that flows out is collected and weighed. The weight of mercury in the bulb is also determined. Then the

coefficient of apparent expansion of mercury in the vessel

$$=\frac{\text{mass of overflow}}{\begin{pmatrix}\text{range of}\\ \text{temp., } t_2 - t_1\end{pmatrix} \times \begin{pmatrix}\text{mass of mercury in}\\ \text{vessel at higher temp.}\end{pmatrix}}.$$

Then the

(cubical coefficient of the vessel)

= (absol. coef. of mercury) — (apparent coef. of mercury).

(ii) **To find the absolute expansion of the liquid.**—The vessel is filled at a definite temperature with the liquid. It is then warmed to a second temperature, and the liquid that flows out is collected and weighed, or the overflow is determined as the difference between the weights of *bulb* + *liquid* at the two temperatures. Then the

coefficient of apparent expansion of liquid in vessel

$$=\frac{\text{mass of overflow}}{\begin{pmatrix}\text{range of}\\ \text{temperature}\end{pmatrix} \times \begin{pmatrix}\text{mass of liquid in vessel at}\\ \text{higher temperature.}\end{pmatrix}}.$$

Then the

(absolute coefficient of the liquid)

= (apparent coef. of liquid) + (cubical coef. of vessel).

96. Apparatus.—A stout cylindrical bulb with capillary stem, like Fig. 73, must be used when mercury is worked with. Such bulbs are frequently called **weight thermometers**, because the temperature may be deduced from the amount of mercury that overflows from the bulb. A specific gravity bottle is sometimes used for finding the apparent expansion of lighter liquids.

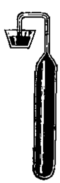

Fig. 73.

The vessel is put in a basket of wire gauze which is suspended in a bath of water or oil, arranged so that it can be heated to different temperatures

To heat the bulb.—Use the heater described in § 99. The bulb is put into the inner tube with its stem projecting, and so arranged that the end can be immersed in mercury in a small vessel (crucible). Pack, by placing metal discs at the bottom to make the height right: fill in the sides *loosely* with small pieces of copper foil or brass nails. Heat is then readily conducted to the bulb. Cover the neck with cotton wool. Fill the heater with water (glycerine or oil for higher temperatures), and warm over a Bunsen flame.

To fill the weight thermometer with mercury.—Use a mercury tray throughout the operations. Put the bulb in a wire gauze basket (flat bottom) attached like a butterfly net to a ring, rod, and handle. Place a lighted Bunsen and a vessel of distilled mercury in the tray: also some wooden blocks. Arrange the latter to support the rod and basket, and also the vessel of mercury so that the end of the stem of the bulb is immersed in the liquid. Warm the bulb (do not make it hot, it is liable to break) and mercury by playing the Bunsen flame round. Put the end of the stem under mercury and allow to cool. Repeat until the bulb is full up to the stem (each time warm only the upper portion where the air is). Gently tap the bulb several times. Finally well warm it, and when all the air has been expelled put the end of the stem under mercury in a small crucible, again warm, then leave to cool. When cold lift the bulb with one hand, the crucible by the other, and arrange in the heater, taking care during the adjustment *to keep the end of the stem always under the mercury.*

To get the mercury out of the bulb, first well warm it, then allow to cool with the end of the stem in air. Get the air bubble into the bulb, warm it, allow to cool. Repeat until empty.

Exp. 107.—*Determine the coefficient of expansion of the glass of a weight thermometer.*

(1) Weigh the bulb, empty, clean, and dry.
(2) Fill the bulb with mercury.
(3) Make a bath at 0° C. (water and ice). Put a thermometer and wire stirrer in the bath. When the bulb, etc., has remained in this for some time, remove the crucible containing the mercury and substitute a small empty porcelain crucible that has been weighed beforehand.

(4) Warm the bath carefully to about 50°. (It may be heated pretty rapidly to, say, 45°, and then very slowly.) Adjust the Bunsen flame so that the bath keeps at one temperature for, say, 5 minutes. This can be assumed to be the case if no drop of mercury issues from the weight thermometer during the period. Note the actual temperature of the bath. Remove the crucible containing the overflow and weigh on a sensitive balance.

(5) Replace the crucible (still containing the overflow), and heat the bath up to boiling. Keep as before the bath and bulb at one temperature for, say, 5 minutes; note the temperature of the bath. Remove the crucible containing the overflow, and weigh on a sensitive balance.

(6) Finally weigh the mercury and bulb (the balance must be strong: do not use a sensitive balance except by permission of the laboratory Superintendent).

(7) Calculate the coefficient between 0 and 50°, and 0 and 100°.

(8) Work to higher temperatures by using a glycerine bath.

Exp. 108.—*Determine the apparent expansion of water with reference to the weight thermometer.* Weigh the bulb, clean, dry, and empty, on a sensitive balance. Boil distilled water for some time to remove air from it. Fill the bulb with the water (§ 94). Put into the heater (§ 99). Arrange so that the bath temperatures are successively about 20° C., 50° C., 80° C. In each case weigh the *bulb* + *water* (the bulb must be dry externally).

Similarly measure the coefficient for *glycerine.* Bath temperatures, say, 25° C., 100° C., 150° C., 200° C. (Select a bulb with a coarse capillary bore.)

97. Dilatation of Gases.—When a quantity of gas is heated, both the volume and pressure generally increase. Experimentally however it is usual to arrange for one of these quantities to remain constant while the change in the other is measured: that is, there is a determination of the coefficient (i) of increase of volume due to rise of temperature when the pressure is kept constant, (ii) of increase of pressure due to rise of temperature when the volume is kept constant.

Charles' law.—*The volume of a definite mass of any gas, maintained at constant pressure, increases per unit rise of temperature by a constant fraction of its volume at a standard temperature.* Practically for many gases the constant fraction is, per degree centigrade, 1/273 of the volume at ice-point (0° C.). This is then the value of the conventional coefficient of expansion for gases.

The coefficient of increase of pressure follows a similar law and has the same value.

Definition : *Absolute temperature.* A temperature of $t°$ C. is $(273 + t)$ degrees on the absolute scale.

Law connecting the pressure, volume, and temperature of a mass of gas. By combining the laws of Boyle and Charles (see *Text-Book of Heat*) it can be proved that for the same mass of gas under any conditions the value of

$$\frac{(pressure) \times (volume)}{(absolute\ temperature)}$$

is a constant number.

Exp. 109.—*Find the coefficient of increase of volume at constant pressure* (Fig. 74). A glass tube, A, about a metre long, having a narrow bore, is closed at one end and open at the other. Several pieces of fine copper wire are bound

Fig. 74.

round the tube at various distances from the closed end (say about 60, 65, 70, 75, 80 cm.). The tube is placed horizontally on two blocks in a water or oil bath. It contains dry air, and a plug of mercury, M, two or three inches long. A thermometer is bound to the tube. The bath is placed on two or three tripods and warmed by several Bunsens. The liquid must be well stirred, and the flames adjusted so that the bath is at such a temperature that the end of the thread of mercury nearer the closed end of the tube keeps against the nearest wire mark for several minutes. Note the temperature (t_1).

Similarly adjust the bath, etc., for the second mark, and note the temperature, t_2; for the third, etc. Measure the distances between the closed end of the tube and the successive marks. (These distances are proportional to the volumes of the air at the respective temperatures if the bore of the tube is uniform.) Tabulate and plot lengths and temperatures.

The graph is practically straight. Note that it cuts the temperature axis at about $-273°$ C. (The temperature of $-273°$ C. is called the absolute zero.) Deduce from the graph that the coefficient of increase of volume

$$= \frac{1}{273} \text{ (\textit{volume of gas at ice point})}.$$

Exp. 110.—*Find the coefficient of increase of pressure at constant volume* (Fig. 75). A cylindrical bulb, A, has a long capillary stem, B, attached to it. This is bent twice at right angles. The end, D, is made large enough to be joined to the tube (Fig. 59) of the Boyle's law apparatus. There is a mark, m, on the tube, D, to which the mercury is adjusted. The bulb, A, filled with dry air, is put into the heater, copper filings, or nails are filled in, and finally cotton wool. Water, oil, or glycerine is used in the bath.

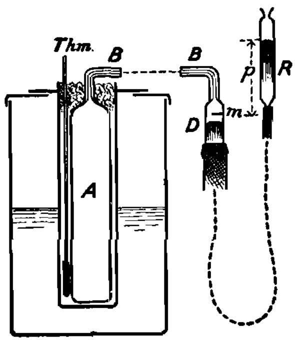

Fig. 75.

1. The apparatus being arranged, fill the bath with melting ice. By manipulating R, adjust the mercury in D to stand at m. When the mercury, etc., needs no further adjustment during an interval of, say, 10 minutes, note the position of its surface in R. Also the distance, p, between the mercury surfaces, and the height, H, of the barometer.

2. Keep the temperature of the bath constant for, say, 10 minutes, at about 25°, adjust the mercury in D to the mark, m, and observe as before the position of the mercury in R, and the actual temperature of the bath.

3, etc. Also when the bath is kept at about 50°, 75°, 100°.

The differences between the readings of the mercury surface in R, at the successive temperatures and its reading at the ice-point, are the increments of pressure (in cm. of mercury) for the respective rises of temperature. The pressure exerted by the gas $= H + p$. Tabulate and plot (1) temperature of gas, (2) pressure of gas.

The graph is practically a straight line. Note that it cuts the temperature axis at $-273°$ C. Deduce from the graph that the

coefficient of increase of pressure $= \frac{1}{273}$ (*pressure of gas at 0° C.*).

CHAPTER VII.

CALORIMETRY.

98. Specific Heat, Thermal Capacity, etc.—The *specific heat of a substance* is the number of heat units required to change the temperature of unit mass of the substance by one degree. (For heat units see Appendix.)

The *thermal capacity of a body* is the number of heat units required to change the temperature of the body by one degree. It is generally equal to the product (*mass of body*) (*specific heat of its material*). The *thermal capacity* is frequently called the *water-equivalent* of the body, because its value is practically that of the mass of water that would absorb or emit the same quantity of heat as the body when its temperature changes through 1°.

If a body of mass, m, composed of material of specific heat, s, has its temperature raised or lowered by $\theta°$, it absorbs or emits $ms\theta$ units of heat.

Quantity of heat in calories
= (*mass in grammes*) (*sp. ht.*) (*range in C.°*).

Quantity of heat in British thermal units
= (*mass in pounds*) (*sp. ht.*) (*range in F.°*).

Latent Heat.—While a substance is passing from the solid to the liquid condition, or from the liquid to the gaseous, the heat absorbed does not alter the temperature.

The *latent heat of vaporisation* is the number of heat units required to change unit mass of a substance from the liquid to the vaporous condition without change of temperature. The *latent heat of fusion* is the number of heat units required to change unit mass of a substance from the solid to the liquid condition without change of temperature.

The quantities of heat emitted by a substance on condensation or solidification are respectively equal to the quantities absorbed on vaporisation or fusion.

***Exp. 111.**—*Find roughly the activity or power of a Bunsen flame.* Arrange a Bunsen burner under a tripod and keep the flame burning steadily during the experiment. Place in a can a measured quantity of water, say 1 pint or 500 cu. cm. Stir the water and note its temperature (t). At a definite moment place the can of water over the Bunsen flame. Finally, after an interval of time, d, say five minutes, stir, and note the temperature (T) of the water.

CALCULATION. Heat units absorbed by the water $= M(T-t)$; where M is the mass of water.

Let J be the mechanical equivalent of the heat unit, then energy absorbed by the water $= JM(T-t)$.

$\therefore$ energy absorbed per unit time $= JM(T-t)/d$.

Assume this to be equal to the power or activity of the flame. To express the power in *ergs per second,* measure M in grammes, d in seconds, temperature in degrees centigrade. Assume $J = 42{,}000{,}000$ ergs. To express the power in *horse-power,* measure M in lbs., d in minutes, temperature in degrees centigrade. Assume $J = 1400$ foot pounds. Finally divide by 33,000. (See Appendix.)

EXAMPLE.—A pint ($= 1·25$ lb.) of water was heated by a Bunsen flame from 20° to 60° in 4 minutes.

Heat units absorbed by the water $= 1·25 \times 40 = 50$.

Then energy absorbed $= 1400 \times 50 = 70000$ ft. lb.

Then ft. lb. absorbed per minute $= 70000/4 = 17500$.

Then, since one H.P. is an activity of 33,000 ft. pds. per min., the horse-power of the Bunsen flame

$$= 17500/33000 = 0·53 = \tfrac{1}{2} \text{ H.P. nearly.}$$

***Exp. 112.**—*Find the specific heat of iron.* (1) Weigh ($= W_1$) a piece of iron (a 7 lb. weight, say). Tie a piece of string to it. Keep it in boiling water for, say, 5 minutes; its temperature is then raised to 100°. (2) Into a tin put a measured quantity (say 1 quart $= 2·5$ lb.) of water. Note its temperature (t). (3) Quickly lift the iron by the string out of the boiling water into the cold, stir the water by moving the iron about in it. After a minute or two note the temperature (t_1) to which the cold water has been raised. Calculate the specific heat (s) of iron as follows:

Heat emitted by iron in cooling

$$= \textit{wt. of iron} \times \textit{sp. ht.} \times \textit{fall of temp.}$$

Heat absorbed by cold water in warming

$$= \textit{wt. of water} \times \textit{rise of temp.}$$

Assume (heat absorbed) = (heat emitted), and deduce s from

$$W_1 \,.\, s\,(100 - t_1) = (\textit{wt. of water})(t_1 - t).$$

99. Specific Heat: Apparatus.—*Heater.* The following (Fig. 76) is convenient:—A tin can, A (6″ long, 4″ diameter), is half filled with water. The water is boiled by placing the tin on a tripod over a lighted Bunsen burner. The lid, B, is pierced by a large central hole and two or three small ones ($\frac{1}{4}$ inch diameter) nearer the edge. A tin tube, C (5″ long, $1\frac{1}{2}$″ diameter), contains the substance to be experimented upon. It passes freely through the lid and is supported by the flange, D.

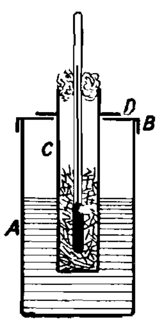

Fig. 76.

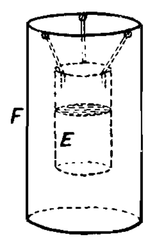

Fig. 77.

Calorimeter and shield (Fig. 77). These are of tinplate, but the calorimeter may with advantage be made of thin brass or spun copper. The calorimeter, E, is hung inside the shield, F, by 3 loops of cotton that catch in hooks on E. The inside of the shield, and the outside of the calorimeter especially, should be brightly polished.

METHOD OF MIXTURE.

100. Method of Mixture.—It is better when the *substance* to be experimented upon is in fragments. Place a *known mass* of it, and a thermometer, in the tube *C*. Plug the upper end of *C* loosely with a pad of cotton wool—all in one piece. Boil the water in the heater. When the reading of the thermometer remains constant for, say, five minutes, it may be assumed that all the substance has attained the temperature indicated.

A thermometer should be used when available, but is not essential. When a thermometer is not used keep the substance in the tube and the water of the heater boiling for at least ten minutes. Then assume that the temperature of the substance is 100° C.

Weigh the calorimeter (1) empty, (2) about two-thirds full of water. The difference is the mass of water used.

Tip the hot substance into the cold water.

Note the temperature of the water in the calorimeter by a *sensitive* thermometer a little *before*, and *after* the substance has been introduced. Slowly and *cautiously* stir the water by the thermometer immediately before a reading is taken.

To tip the substance into the calorimeter: grip the projecting portion of the tube with the clamp from a retort stand, place the calorimeter conveniently near, hold the clamp with the right hand, remove the thermometer and plug of cotton wool with the left, at once lift out the tube, and tip the substance into the calorimeter, taking care not to splash out any water.

After the experiment, *dry the substance* by placing it on a piece of tin plate over a small Bunsen flame; empty water from the calorimeters, boilers, etc., invert them, and leave to drain.

To calculate the specific heat of the substance. Let W be the mass of the empty calorimeter, and s its specific heat. Then $W \times s$ is its water equivalent.

Let M be the mass of the substance, x its specific heat.

Let w be the mass of cold water in the calorimeter. Let T be the temperature of the substance, t_1 of the cold water before adding the substance, and t_2 **after mixing**. The temperature of the calorimeter is assumed **to be that** of the water in it.

Calculate as follows:—

Heat *emitted* by substance in cooling from T to t_2

$$= M \, . \, x \, (T - t_2) = \ldots .$$

Heat *absorbed* by water in warming from t_1 to t_2

$$= w \, (t_2 - t_1) = \ldots .$$

Heat *absorbed* by calorimeter in warming from t_1 to t_2

$$= W \, s \, (t_2 - t_1)\dagger = \ldots .$$

Assume *heat emitted* = *heat absorbed*. An equation is obtained in which x is the only unknown.

Exp. 113.—*Find the specific heat of a solid by the method of mixtures.* Use iron nails (150 gms.), brass nails (150 gms.), lead shot (300 gms.), pieces of glass rod and tube (100 gms.), etc.‡

Exp. 114.—*Find the specific heat of a liquid by the method of mixtures.*‡ Put the *liquid* in the calorimeter *instead of* water ; then perform the experiment as above. Use methylated spirit, petroleum, glycerine, etc., and a solid like iron (nails).

CALCULATION. Assume the specific heat of the solid (S) ; let x be that of the liquid.

Heat *emitted* by substance in cooling from T to t_2

$$= M \, . \, S \, (T - t_2) = \ldots$$

Heat *absorbed* by the liquid in warming from t_1 to t_2

$$= w \, . \, x \, (t_2 - t_1) = \ldots$$

Heat *absorbed* by the calorimeter in warming from t_1 to t_2

$$= W \, . \, s \, (t_2 - t_1) = \ldots$$

Assume *heat emitted* = *heat absorbed*. Calculate x from the equation obtained.

† For iron, copper, and brass calorimeters, assume $s = 0{\cdot}1$. The heat absorbed by the calorimeter may sometimes be neglected.

‡ Several determinations of the specific heat of each substance should be made until there is approximate agreement in the results obtained.

101. Method of Mixtures : A modification of Regnault's apparatus (Fig. 78).—*The heater :* This is a copper cylinder closed at both ends. Steam from a boiler can enter by a narrow pipe near the top, and leave by one at the bottom. A wide copper tube passes axially through the cylinder; its top end is closed with a cork, its bottom rests on the stool. Thus the tube is a closed steam-jacketed chamber. The body to be experimented upon is suspended in this chamber side by side with a thermometer. The latter is held by the cork; its bulb

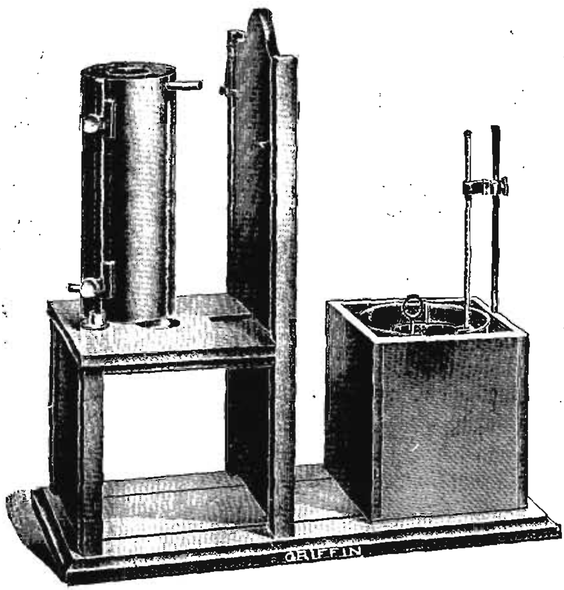

Fig. 78.

should be close to the body. The thread suspending the body passes through a short tube fixed in the cork, and is tied outside. The heater can be rotated until the central tube is over the large hole in the stool.

The *calorimeter* is a cylinder of thin copper, suspended by threads within a larger copper vessel. The latter rests inside an open box that can slide along the base so that it may be easily pushed below the stool, and the calorimeter brought under the hole. A small clamp for holding a thermometer in the calorimeter is fixed to the side of the box. A shutter is let down between the heater and calorimeter during the warming of the former. This can be raised when the time arrives for sliding the box with calorimeter, etc., under the stool. There is also a copper stirring rod. Other details will be understood on examining the apparatus. The several operations should be tried, and adjustments tested, before the actual experiment is attempted.

102. To find the water equivalent (e) of the calorimeter and accessories.†

(1) Weigh the calorimeter (clean and polished), thermometer, P, and stirrer ($= W$). Place in position in box. Half-fill with warm water.

(2) Nearly fill a vessel with tap water. Immerse a sensitive thermometer, Q, in it and weigh all ($= W_1$).

(3) Stir the warm water in the calorimeter, and when its temperature (which should be falling *slowly*) is about 30°, note the exact value of the temperature, 1st, of the tap water ($= t_1$); 2nd, of the warm water ($= t_2$). Immediately, and without spilling, pour tap water into the calorimeter, stir and note the temperature (t_3) of the mixture.

(4) Weigh the vessel containing the remaining tap water and thermometer, Q, ($= W_2$).

(5) Weigh the calorimeter, thermometer, P, stirrer, and water ($= W_3$).

(6) CALCULATION, etc.—

Mass of cold water $= (W_1 - W_2)$ $= \ldots$ Temp. (t_1)...
Mass of hot water $= \{W_3 - (W_1 - W_2) - W\} = \ldots$ Temp. (t_2)...
Temperature of mixture (t_3) $= \ldots$
Fall of temp. of hot water, calorimeter, etc. $= \ldots$
Rise of temperature of cold water $= \ldots$
Heat absorbed by cold water
$= $ (Mass cold water) (Its temp. rise) $= \ldots$
Heat emitted by hot water, calorimeter, etc.
$= $ (Mass of hot water $+ e$) (Its temp. fall) $= \ldots$

Assume *heat emitted = heat absorbed*: an equation is obtained in which the water equivalent (e) is the only unknown.

103. To find the specific heat (x) of a solid substance.—It is preferable to use the material in the form of wire, or sheet rolled into a cylinder. A lump, especially of a bad conducting substance, takes considerable time both to warm and cool. If the material is fragmentary put it in a wire gauze basket of known water-equivalent.

OPERATIONS.—(1) Find the mass of substance to be used ($= M$).

(2) Suspend it in the heater, close to the bulb of the thermometer. Use a long thread fixed outside. Join the heater to the steam supply, and condense the waste steam. The thermometer will rise very slowly, perhaps not higher than about 98°. The substance should not be

† It is better to obtain e by calculation;
$e = $ (*Mass of calorimeter*) (*Sp. ht. of its material*).

put into the calorimeter until the thermometer has been *observed* to give a steady maximum reading for about ten minutes. During heating the lower end of the hot chamber must rest on the stool, and thus be closed.

(3) Weigh the calorimeter and stirrer, clean and empty ($= W$).

(4) Fill the calorimeter about two-thirds with water. Weigh ($= W_1$). Clamp thermometer in position.

(5) When the temperature of the heater is quite steady note its actual value (T) and also that of the water in the calorimeter (t_1), lift the shutter, slide the calorimeter underneath the stool, and rotate the heater over the hole, let the hot body down into the calorimeter, and cut the cotton. Withdraw the calorimeter, stir, and note the highest temperature (t_2) reached by the water in it.

(6) CALCULATION, etc.

Mass of cold water ($W_1 - W$) = . . . temperature (t_1) = . . .
Water value of calorimeter,† etc. (e) = . . .
Mass of hot substance (M) = . . . temperature (T) =
Temperature of mixture (t_2) = . . .
Fall of temperature of substance ($T - t_2$) = . . .
Rise of temperature of water and calorimeter ($t_2 - t_1$) = . . .

Heat absorbed by cold water and calorimeter = (*mass of cold water* + *water equivalent of calorimeter*) (*rise of temperature*) = . . .

Heat emitted by hot substance = (*mass substance*) (*specific heat of substance*) (*fall of temperature*) = . . .

Assume *heat emitted* = *heat absorbed*: an equation is obtained in which the *specific heat of the substance* is the only unknown.

Specific heat of a liquid. Use the liquid instead of water and proceed as above (see Exp. 114).

NOTE.—A little heat will be lost during the fall of the body into the calorimeter. Also some will pass from the calorimeter by radiation and convection *before* the highest reading of the thermometer is attained. Hence the highest reading is a trifle lower (say $p°$) than it would be if no heat had been lost. The value of p may be nearly obtained (§ 104) by observing the rate of cooling of the calorimeter (Regnault), or the loss of heat may be compensated for (Rumford).

104. Rumford's compensation method.—Find roughly by a preliminary experiment the rise of temperature ($t_2 - t_1$) of the water. Then perform the experiment, taking an equal mass of water at a temperature $\frac{1}{2}(t_2 - t_1)$ degrees below the temperature of the room. Thus the calorimeter will gain heat at first and afterwards lose it. The compensation, however, is not exact, because the period of gain is shorter than that of loss.

† It is better to obtain e by calculation;
e = (*Mass of calorimeter*) (*Sp. ht. of its material*).

When this method is adopted care must be taken that the water in the calorimeter is not cooled below the dew-point. If dew is deposited heat will be absorbed in evaporating it.

Radiation correction, R (Regnault's method). If the rise of temperature is *small* the loss of heat is nearly equal to that which would occur if the calorimeter had been at its mean temperature $\frac{1}{2}(t_2 + t_1)$ during the period of time (d) between the introduction of the body and the moment when the highest reading was attained. Then, if z is the rate of cooling, e the total water equivalent of (calorimeter + thermometer + substance, etc.), then in one second the heat radiated $= ez$, and in d secs. the *total heat radiated* $(R) = d.e.z.$

The total heat radiated is the value of the *radiation correction* (R). In the calculation of specific heat, R is added to the other quantities of heat absorbed.

To determine R, observe (1) the time, d, between the moment at which the body is introduced and that at which the highest temperature is attained, (2) the fall of temperature, $q°$, of the calorimeter and its contents in d' seconds (5 minutes, say). Then

$$z = q/d' \quad \text{and} \quad R = d.e.z.$$

Exp. 115.—*Find the water equivalent of the calorimeter, etc.*

Exp. 116.—*Find the specific heats of copper, brass, iron, glycerine, etc., by Regnault's apparatus.*

METHOD OF COOLING.

105. Method of Cooling. Specific heat of a liquid.—The amount of heat radiated from a surface at a temperature (t_1) to another at a lower temperature (t) depends upon (i) the difference of temperature ($t_1 - t$), (ii) the chemical nature and physical condition—whether dull, polished, etc.—of the surface (see Exp. 87), and (iii) the area of the surface. Hence if equal volumes of different liquids are put in the same or similar calorimeters and hung within an enclosure whose walls are maintained at a constant temperature (t), then when the liquids have the same temperature (t_1) the *rates of emission* of heat (quantities per unit time) are equal and independent of the nature of the liquid. Hence the temperatures of those having smaller thermal capacities drop more quickly, the time taken by each to fall through the same range of temperature being proportional to its thermal capacity. Then for any substance

(Time of cooling from t_1 to t_2) $\propto$ {(Mass) $\times$ (Specific heat)}.

Hence, if a mass, m_1, of a substance of sp. ht., s, takes T_1 seconds to cool between any two temperatures (t_1, t_2), and a mass, m, of water takes T seconds to cool between the same temperatures, then

$$\frac{m_1 s\,(t_1-t_2)}{T_1}=\frac{m\,(t_1-t_2)}{T} \quad \therefore \frac{T_1}{T}=\frac{m_1 \cdot s}{m} \quad \therefore \; s=\frac{m}{m_1} \cdot \frac{T_1}{T}.$$

The influence of the vessel may be taken account of by adding its water equivalent (e) to each thermal capacity above. Then

$$\frac{T_1}{T}=\frac{m_1 \cdot s+e}{m+e} \qquad \therefore \; s=\frac{m}{m_1} \cdot \frac{T_1}{T}+\frac{e}{m_1}\left\{\frac{T_1}{T}-1\right\}.$$

***Exp. 117.**—*Find the specific heat of petroleum by the method of cooling.*† (1) Weigh ($= W$) a calorimeter (blackened outside), support it on a cardboard cross (§ 86) or large cork.

(2) Fill the calorimeter about three-quarters full of hot water, place a thermometer in it, and stir gently. Note the time (T secs) of cooling from 55° to 45°. Weigh *calorimeter + water* ($= W_1$). Empty and dry.

(3) Repeat operation (2), but use, instead of water, an equal volume of petroleum. Heat the petroleum by placing a quantity of it in a large test tube immersed in hot water. Avoid a flame, **petroleum is inflammable.** Note the time (T_1) of cooling from 55° to 45°. Weigh *calorimeter + petroleum* ($= W_2$). Then calculate (§ 105) the specific heat (s) of petroleum.

106. To find the specific heat of a liquid by the method of cooling.—*A vessel containing a known mass of the hot liquid is hung within an enclosure, the walls of which are kept at a constant temperature.*

Calorimeter: Use a copper or iron canister with its outside lamp-blacked. Cut two holes in the lid to pass the thermometer and stirring rod. Support on a cardboard cross (§ 86).

† This experiment is a simple illustrative one.

Enclosure: (1) For rougher experiments the vessel may be surrounded by a large cardboard cylinder—say 1 ft. long, 1 ft. diam., edges overlapping and pinned together by paper-fasteners. Two small openings are cut in the side, one to let light in, the other to view the thermometer. The inside is painted with lamp-black. The base and top are movable. The stirring-rod handle passes through a small hole in the top.

(2) *Water jacket, etc.* This consists of a large metal canister fixed coaxially inside another so that the walls and bases are an inch or two apart: the space between can be filled with ice, water, or oil at any temperature.

OPERATIONS.—(1) Weigh the calorimeter empty ($= W$).

(2) Fill the calorimeter about three-quarters full of water. Tip the water into a large test-tube, mark the height to which it rises by stamp-paper. Use the test-tube to deliver equal volumes of hot liquids to the calorimeter.

(3) Arrange the calorimeter in the enclosure and add a measured volume of hot water from the test tube. Stir carefully and note the times when the temperatures are 60°, 55°, 50°, etc.†

(4) Weigh the *calorimeter* + *water* ($= W_1$). Then empty and dry the calorimeter.

(5) Repeat operation (3), but use the liquid under investigation instead of water.

(6) Finally weigh the *calorimeter* + *liquid* ($= W_2$).

† NOTE.—*Demonstrate Newton's law of cooling.* Hang a thermometer against the walls of the enclosure (or in the water of the wter-jacket). Note its indications (which should be practically constant) just after observing the several temperatures of the cooling liquid. *Tabulate* (i) temperature of liquid, (ii) time, (iii) temperature of enclosure, (iv) excess of temperature of liquid over that of the enclossure. *Plot* the logarithm of the excess of temperature (column iv.) against *time* (*column* ii.). The graph is nearly a straight line.

So far as the graph is straight it shows that log. (temp. excess) is proportional to the time of cooling to the temperature of the surroundings. Newton's law that the rate of cooling is proportional to the temperature excess then follows. The graph indicates that the law is not true if the temperature excess is greater than about 20°.

Tabulate (i) temperature, (ii) time for water, (iii) time for liquid.

Plot temperature of water with regard to time, and to the same axes of reference the temperature of the liquid with regard to time. Calculate as in § 105. (See example below.)

EXAMPLE.—*Determination of the specific heat of petroleum oil, by the method of cooling.*

Observed temperature	60°	55°	50°	45°	40°
Time for water . .	26m 45s	32m 0s	38m 50s	47m 30s	57m 55s
Time for petroleum .	14m 30s	17m 0s	20m 15s	24m 20s	29m 20s

Mass of calorimeter .. .. 44·6
,, calorimeter + water .. 168·7 ∴ Mass of water .. 124·1
,, calorimeter + petroleum 140·2 ,, petroleum 95·6

Temperature Range.	Time of Cooling.		Relative thermal capacity ($= T_1/T$).
	Water (T).	Petroleum (T_1).	
60—55	315 secs.	150 secs.	0·477
55—50	410	195	0·476
50—45	520	245	0·471
45—40	625	300	0·480
Then mean relative thermal capacity = 0·476.			

Assume water-equivalent of calorimeter (iron, sp. ht. = 0·1) is $0{\cdot}1 \times 44{\cdot}6 = 4{\cdot}5$. Then, s being the sp. ht. of petroleum,

$$\frac{95{\cdot}6\,s + 4{\cdot}5}{124{\cdot}1 + 4{\cdot}5} = 0{\cdot}476, \qquad \therefore\ s = 0{\cdot}59.$$

Exp. 118.—*Find the specific heat of petroleum,† alcohol,† glycerine, castor oil, etc., by the method of cooling.*

† **Inflammable.** Heat by immersing large test-tubes containing the liquids in water that has just boiled. Avoid the neighbourhood of flames.

LATENT HEAT.

***Exp. 119.**—*Find roughly the latent heat of vaporisation of steam.* A value may be calculated from the data obtained in Exp. 90. Or proceed directly as follows:

METHOD I. (No thermometer is required.) Place in a can a mixture of ice (one part by weight) and cold water (about four parts by weight). Arrange a Bunsen burner under a tripod, keep it burning steadily throughout the experiment. When all the ice in the mixture has melted note the time and at once place the can over the Bunsen flame. Note also the time when (ii) boiling begins, (iii) the water has just boiled away.

METHOD II. (A thermometer is required.) Put a can, half filled with water, over a Bunsen burning steadily. Note the time when (i) the temperature is, say, 25°, (ii) boiling begins, (iii) the water has just boiled away.

CALCULATION. Assume that the heat absorbed from the flame by the water is proportional to the time during which it has been heated. Find (i) the time, T, taken to boil away the whole of the water, (ii) the time, t, taken to warm the water from ice-point to boiling. In Method I. the initial temperature is the ice-point; in Method II. it is, say, 25°. In the latter case calculate the whole time it would have taken to heat the water from 0° to 100° if the initial temperature had been 0°. Finally

$$\frac{\text{“\textit{Latent heat}” \textit{absorbed in vaporisation}}}{\text{“\textit{Sensible heat}” \textit{absorbed from ice- to steam-point}}} = \frac{T}{t}.$$

EXAMPLE. Time taken to heat a quantity of water from 25° to 100° = 3 mins. 30 secs = 210 secs.

Time taken to boil away the water = 23 mins. 20 secs. = 1420 secs.

Then time taken to heat the water from 0° to $100° = \frac{4}{3} \times 210$ = 280 secs.

∴ latent heat)/(sensible heat) = 1420/280 = 5·1.

Exp. 120.—*Find the latent heat of vaporisation of steam at* 100° *C.* Fig. 79 shows a useful form of apparatus. Water is boiled in the vessel, *K* (a copper bottom to this is an advantage), the steam is led through a bent glass tube (which should be covered with cotton wool) into the calorimeter, *E*, hung in the screening can, *F*. The outside of *E* and of *F* should be bright.

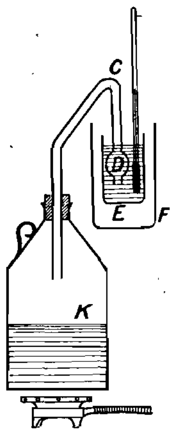

Fig. 79.

Fig. 80.

Sometimes a *trap*, like Fig. 80, is placed at the delivering end of the steam tube. It is designed to catch the steam that condenses in the tube. In doing this, however, it forms a cold belt at *b*. The delivery tube, *H*, coming from the trap itself, needs to be longer than *CD* in Fig. 79. A piece of rubber tubing and pinchcock, *R*, may be fitted. The condensed steam can then be drawn off when desired. This, when a trap is used, should always be done a little before the time arrives for passing steam into the calorimeter. There is no real advantage in using a trap.

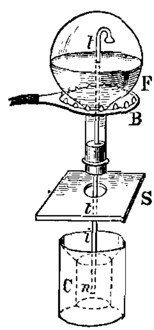

Fig. 81.

Fig. 81 shows a second form of apparatus—inverted flask, *F* (or tin as above), ring burner, *B*, delivery tube, *t*, screen, *S*. The flask is clamped to a retort stand at such a height that the calorimeter can pass freely under the nozzle, *n*. In performing the experiment lift and hold the calorimeter so that the steam passes into, and is absorbed by the cold water. The operations are similar to those described below. The arrangement is improved by using a flask with a very short neck, or by pushing the cork into the neck as far as the globular part. If this is not done the water in the neck is likely to be at a temperature less than boiling. The delivery tube also should be shorter. No india-rubber tube and nozzle should be used.

Operations. (1) Boil the water briskly in the kettle, K.

(2) Weigh the calorimeter empty ($= W$).

(3) Fill the calorimeter two-thirds full of water. Weigh ($= W_1$).

(4) Use a sensitive thermometer, carefully stir the water in the calorimeter with it, and note the temperature of the water ($= t_1$).

(5) Remove the thermometer from the water, holding it with the right hand. With the left hand quickly raise the screening can and calorimeter, until the end of the glass tube from which the steam issues is under the surface of the water in the calorimeter. Replace the thermometer in the water and stir gently. When it indicates about 40° quickly remove the calorimeter from the steam tube, and note, after stirring, the temperature ($= t_2$) of the water.

(6) Note the height of barometer. Deduce the temperature (t) of steam to $\frac{1}{2}°$ (Exp. 95).

(7) Remove the thermometer from the water, letting its bulb trail along the inside (above the water) of the calorimeter so that drops of water may be removed from it. Weigh ($= W_2$).

Calculate the mass of steam condensed ($= W_2 - W_1$), and of cold water used ($= W_1 - W$).

Also x the required latent heat, as below:

Heat *emitted* by $(W_2 - W_1)$ grammes of steam at $t°$ in condensing to water at the same temperature $= (W_2 - W_1)\,x$.

Heat *emitted* by $(W_2 - W_1)$ grammes of condensed steam, or water at $t°$ in cooling to $t_2° = (W_2 - W_1)(t - t_2) = \ldots.$

Heat *absorbed* by $(W_1 - W)$ grammes of water in warming from $t_1°$ to $t_2° = (W_1 - W)(t_2 - t_1) = \ldots.$

Heat *absorbed* by the calorimeter† (specific heat, s) in warming from $t_1°$ to $t_2° = W.s.\ (t_2 - t_1) = \ldots.$

† Its temperature is assumed to be that of the water it contains. (This quantity of heat may sometimes be neglected.)

Assume *heat emitted* = *heat absorbed.* Then solve the equation, in which x is the only unknown quantity.

Several determinations of the latent heat of vaporisation should be made.

EXAMPLE. Determination of the latent heat of vaporisation of steam at atmospheric pressure.

Mass calorimeter, 96 *gm.*

Mass calorimeter + water, 290·7 *gm.* ∴ mass of water = 194·7 *gm.*

Mass calorimeter + water + condensed steam, 300·1 *gm.*

∴ Mass of steam condensed = 9·4 *gm.*

Barometric height, 762 *mm.* ∴ Temperature of steam = 100° C.

Temperature of cold water, 14° C.

Temperature after mixing with steam, etc. = 41·5° C.

Heat emitted by steam in condensing to water at 100° C = $9·4x$.

Heat emitted by 9 *gms.* of hot water (condensed steam) cooling from 100° to 41·5° = 9·4 × 58·5 = 550.

Total heat emitted = $9·4x$ + 550.

Heat absorbed by 194·7 *gm.* water in warming from 14° to 41·5° = 194·7 × 27·5 = 5354.

Heat absorbed by calorimeter (specific heat 0·1) in warming from 14° to 41·5° = 96 × 0·1 × 27·5 = 260.

Total heat absorbed = 5354 + 260 = 5614.

Assume *heat emitted* = *heat absorbed.* Then $9·4x$ + 550 = 5614.

∴ Latent heat of steam at atmospheric pressure = 539 *calories.*

Exp. 121.—*Find the latent heat of fusion of ice.* A shielded calorimeter, well polished outside, is required.

OPERATIONS. (1) Weigh the calorimeter empty ($= W$).

(2) Half fill the calorimeter with water. Weigh ($= W_1$).

(3) Note the temperature of the water in the calorimeter ($= t_1$). It is an advantage to make this about 25° C.

(4) Place, say, half-a-dozen thimble-sized pieces of ice on blotting paper, carefully and quickly dry each piece and drop it into the calorimeter. Stir slowly with the thermometer. Note the temperature ($= t_2$) as soon as all the ice has melted.

(5) Remove the thermometer from the water (allow its bulb to trail along the inside—above the water—of the calorimeter). Weigh ($= W_2$).

CALCULATE the mass of ice melted ($= W_2 - W_1$),

and of cold water used ($= W_1 - W$).

Also x the required latent heat as below:

Heat emitted by $(W_1 - W)$ grammes of water in the calorimeter in cooling from t_1 to t_2

$$= (W_1 - W)\ (t_1 - t_2) = \ldots.$$

Heat emitted by the calorimeter (specific heat s) in cooling from t_1 to $t_2 = Ws\ (t_1 - t_2) = \ldots.$

Heat absorbed by the ice at 0° in melting to water at $0° = (W_2 - W_1)x$.

Heat absorbed by $(W_2 - W_1)$ grammes of water in warming from 0° to $t_2 = (W_2 - W_1)t_2$.

Assume *heat emitted* = *heat absorbed*, then solve the equation, in which x is the only unknown quantity.

PART III.—SOUND.

CHAPTER VIII.

VELOCITY. FREQUENCY.

107. Pitch and frequency.—A sound or note is described as high or low (deep), shrill or grave, according to the kind of sensation it produces in the mind of the hearer. Loudness or softness expresses the intensity of the sensation. The sensation produced is, by a trained ear, easily relegated to a position amongst a certain sequence of musical notes called the *Diatonic Scale*, and the position is defined by giving the name of the note of this scale that *sounds like the one under consideration: this name is* called the *pitch* of the note. Thus a note from a violin string or an organ pipe is said to have a pitch c'' if it agrees with the note of this name of a standard tuning fork.

A sounding body is always a vibrating body and the pitch of the note emitted depends upon the rate of vibration, the note or pitch getting higher as the rate of vibration quickens. The number of vibrations executed per second by a sounding body is called its *frequency* or *vibration number*. The ratio of the frequency of a note, X, to that of another, Y, is called the *frequency ratio* of the notes. On it depends that musical relation which is called the *interval*.

A person with a trained ear is able not only to identify equality of pitch, but can also tell when the frequencies of notes are in certain ratios by the concord or agreement between them. The following are common concords:—

Name of interval:	Octave.	Fifth.	Fourth.
Frequency ratio:	2 : 1.	3 : 2.	4 : 3.

The frequencies of the successive notes, C, D, E, F, G, A, B, C', of the diatonic scale are as 24, 27, 30, 32, 36, 40, 45, 48, giving, between the consecutive notes, the respective intervals 9/8, 10/9, 16/15, 9/8, 10/9, 9/8, 16/15.

In *Standard pitch* (adopted 1899) the frequency of $c'' = 522$, and of $a' = 439$. c'' and a' tuning-forks have these frequencies at 68° F. (The value of g' is then 391·1.)

In scientific work $c'' = 512$, $a' = 426\frac{2}{3}$ $g' = 384$.

The tuning-fork (sometimes called a *pitch fork*) consists of a steel bar of rectangular section doubled into a U shape. From the bend a shank projects. A vibrating tuning-fork has nearly constant frequency, and hence emits a note of practically unchanging pitch.

Smaller forks, held at the shank by the hand, are made to vibrate by striking one of the prongs against a pad of india-rubber, or soft wood. Larger forks are screwed by their shanks to a box, called the *resonance box*. This is made of thin wood. It considerably increases the intensity of the sound produced. A fork is made to vibrate by drawing a resined fiddle bow across the prongs, in a direction perpendicular to their length.

Longitudinal waves.—In these the particles of the vibrating body move to and fro in a line *parallel* to that in which the wave is travelling. (§ 115.)

Transverse waves.—In these the particles move *at right angles* to the line of propagation of the disturbance. (Exp. 124.)

* **Exp. 122.**—*Pitch of a note.* Stretch strips of corded silk of different "grain" on a thin piece of wood. Place a finger-nail lightly in contact with the silk and run it quickly along the strip. Observe that the more rapidly the nail moves the higher the note heard; also if it is passed with about the same speed over different strips, then the note is of higher pitch when the cording is fine than when it is coarse. Practise to produce the notes of the musical scale.

By moving the nail over the cording a quick succession of taps is produced, and the observations show that the more rapid the taps the higher the pitch.

* Home experiments are marked by an asterisk. See p. 8.

***Exp. 123.**—*Transverse waves on a rope.* Stretch a long rope (clothes line) and fix its ends. Strike the rope near one end with the hand held edgewise. Observe that a depression is formed, travels to the other end, is there reflected, returns, is again reflected, and thus travels to and fro along the rope; the depth of the depression diminishes, its length does not alter. Pull the rope tighter, observe that the wave travels more quickly. (The ropes that hold up a tall flagstaff are suitable.)

***Exp. 124.**—*Stationary waves.* Fix one end of a rope or light brass chain. Move the other end to and fro by the hand at such a rate (found by trial) that the rope vibrates in two or more segments. Observe that each *vibrating segment* or *loop* (also called a *ventral segment*) is separated by a portion, called a *node,* that is nearly stationary. Each part of the rope in any vibrating segment is moving in the same direction, but the parts of adjacent segments are moving in opposite senses. The complete wave is formed by two adjacent segments, and the wave-length is twice the distance between consecutive nodes. The part midway between two nodes is called an *antinode.*

Increase the rapidity of movement of the hand, the rope will then divide into a larger number of vibrating segments, each of shorter length.

***Exp. 125.** *Stationary waves.* Stretch a fine string, six or eight feet long, and fix its ends.

(1) Pluck it between the fingers near the centre, then release it. The string forms one vibrating segment, the fixed ends being nodes. This is the fundamental mode of vibration. The wave-length of the transverse wave along the string is twice the length of the string. The note heard is of unvarying pitch, but of diminishing intensity. Hence the intensity is dependent on the amplitude, but the frequency is not, *i.e.,* vibrations of different amplitudes are isochronous.

(2) Press the centre of the string with a folded strip of paper, pluck one side, let go; the string vibrates in two segments, the fixed ends and centre being nodes. The wave-length is equal to the length of the string.

(3) Press the paper strip at $\frac{1}{3}$ (length of string), pluck the shorter segment, let go; the string vibrates in three segments. Wave-length = $\frac{2}{3}$ (length of string).

(4) Press at $\frac{1}{4}$ (length of string), etc., get four vibrating segments and so on.

If paper riders, Λ-shaped, are placed on the string when at rest, they, when the string is vibrating, are tossed off at the antinodes, but remain on at, or near, the nodes.

Exp. 126.—*Compare the frequencies of two forks.* Attach with as little soft wax as possible a short bristle to one prong of each fork. Slightly smoke a plate of glass by holding it over a candle flame (burning camphor is better); lay it on the bench with the smoked surface uppermost. Take a fork in each hand, and after striking them against a piece of wood to make them vibrate hold them over the plate with the bristles close together and just touching the smoked surface. Quickly draw the bristles over the plate (or slide the plate under the bristles). Two wave lines will be traced on the plate. Draw two parallel lines (several inches apart) at right angles to the wave lines and count the number of undulations in each wave line between the parallels. The respective frequencies are in the ratio of the numbers of undulations. The relative frequency of a third fork may be found by comparing it with either of the above.

VELOCITY OF SOUND.

108. Relation between Velocity, Wave-length, and Frequency.—If λ is the wave-length, and n the frequency of undulation, then the wave travels with a velocity $v = n\lambda$. Thus the velocity in a medium may be calculated by determining the frequency of undulation and the wave-length. The frequency of undulation is assumed to be that of the sounding body whose vibrations cause the wave-motion: *e.g.* a vibrating tuning-fork of frequency, n, sets up undulations of the same frequency, n, in the air by which it is surrounded, or in a string attached to its prong, or in its sounding box, etc. (The wave-lengths, however, in the several cases are unequal.)

Relation between Velocity, Density, and Elasticity of Medium.—When a wave travels along a wire of density, D, its velocity $v = \sqrt{E/D}$, where E is a coefficient of elasticity (Exp. 79). Express the quantities in absolute units.

If *longitudinal waves* are propagated the coefficient of elasticity concerned is Young's Modulus.

For a gaseous medium, the coefficient of elasticity is numerically equal to the product of the pressure, P, and a constant, γ. For air, $\gamma = 1{\cdot}41$. Hence for air

$$v = \sqrt{1{\cdot}41 \times P/D} = 1{\cdot}187 \sqrt{P/D}.$$

The expression $v = \sqrt{E/D}$ is called *Newton's formula*, and $v = \sqrt{E\gamma/D}$ is *Laplace's*. The constant, γ, is the ratio of the specific heat of the gas at constant pressure to that at constant volume. Dynamical proofs of these formulae are given in Treatises on Sound.

The same gaseous medium at different temperatures. Let P, D, and v; P', D', and v' be the respective pressures, densities, and sound velocities at the centigrade temperatures t, t'; then

$$\frac{v}{v'} = \sqrt{\frac{P}{D} \cdot \frac{D'}{P'}}.$$

By the laws of Boyle and Charles

$$\frac{P}{D\left(1 + \frac{t}{273}\right)} = \frac{P'}{D'\left(1 + \frac{t'}{273}\right)} \qquad \therefore \frac{v}{v'} = \sqrt{\frac{273 + t}{273 + t'}}.$$

The ratio of velocities is thus independent of the change of pressure.

When one temperature is $t°$ C. and the other $0°$ C.,

$$\begin{pmatrix}\textit{Velocity}\\ \textit{at } t°\text{ C.}\end{pmatrix} = \begin{pmatrix}\textit{Velocity}\\ \textit{at ice-pt.}\end{pmatrix} \sqrt{\left\{1 + \tfrac{1}{273}t\right\}}$$

When the medium is air the velocity at the ice-point $= 33240$ cm. per sec. Hence writing

$$\sqrt{\{1 + \tfrac{1}{273}t\}} = 1 + \tfrac{1}{546}t$$

$$\therefore \textit{velocity at } t°\text{ C.} = 33240 + 60\,t \quad \textit{cm. per sec.}$$

Different gaseous media at the same temperature and pressure. Let v_1, D_1; v_2, D_2 be the respective velocities and densities, then, if γ is the same for both gases, $v_1/v_2 = \sqrt{D_2/D_1}$, or the velocities are inversely proportional to the square-roots of the densities.

109. Transverse waves.—The velocity, v, with which a transverse wave travels along a stretched wire $= \sqrt{P/m}$, where P is the force stretching the wire and m the mass of unit length of the wire. The quantities must be expressed in absolute units.

Transverse vibrations of strings.—If a stretched string vibrates transversely, since

$$v = 2\,l\,n \quad \text{and} \quad v = \sqrt{P/m},$$

$$\therefore n = \frac{1}{2\,l}\sqrt{\frac{P}{m}}$$

l, length vibrating segment
P, stretching force
m, mass unit length of string

} Each quantity must be expressed in absolute units.

This important expression may also be written in terms of r, the radius of the wire, and D, the density of its material. For

$$m = D\,\pi\,r^2, \qquad \therefore\ n = \frac{1}{2\,l\,r}\sqrt{\frac{P}{\pi\,D}}.$$

The expressions above show that if only one of the quantities l, P, m, r, D, varies at a time, the frequency is

(1) inversely proportional to the length of the vibrating segment,

(2) directly proportional to the square root of the stretching force,

(3) inversely proportional to the square root of the mass per unit length of string,

(3a) inversely proportional to the radius, or diameter of the wire, or to the square root of the density of the substance of the string. These relations are derived from (3).

The elasticity of the material does not directly affect the rate of transverse vibration. A given value of m may be made up by twisting together wires of different metals: for example, the bass wires of a piano are loaded with copper.

110. To find by resonance the wave-length of a note (in **air**) (Fig. 82).—A tall glass cylinder, C, is nearly filled with water. A metal or glass tube, B (about 1″ wide), is held vertically by a clamp, A (on a retort stand), so that its lower end dips in the water.

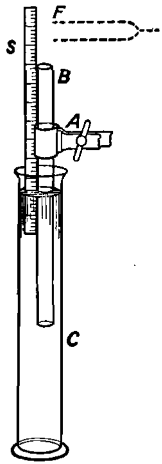

Fig. 82.

The sounding body, a tuning-fork, say, giving the note to be experimented on, is held close to the upper end of the tube, and the length of tube outside the water so adjusted that the column of air within the tube responds to, and strongly reinforces the note.

This is conveniently done by holding the tube with the left hand, loosening the clamp, and letting the tube down until the upper end is near the water. Sound the fork, bring it with the right hand over and close to the upper end of the tube, raise the tube with the left until resonance is heard, then clamp the tube somewhat loosely. Resound the fork, and twist the tube to move it a little up or down, adjust in this manner until the maximum resonance is obtained. The fork should be sounded several times, and the position carefully confirmed. Finally clamp firmly, and measure the distance (a) between the water surface and the top of the tube. If the scale is dipped in the water it must be *removed and dried* as soon as the measurement is completed.

Repeat the whole experiment half-a-dozen times. Calculate the mean value of a.

Finally measure the diameter (d) of the tube, and note the temperature (t) of the air.

Then at temperature, t, the wave-length

$$\lambda = 4\,(a + 0{\cdot}3d).$$

λ increases as the temperature rises. Observe that λ is not equal to $4a$. Theory indicates that $0{\cdot}3d$, more or less, should be added to a. The value of this correction, however, is not very definite; it is quoted sometimes as $0{\cdot}4d$, or as $0{\cdot}5d$.

A *second position* of resonance (of weaker intensity, however) may be found by lifting the tube further out of the water (length, $3a$). If necessary the tube can be lengthened by a paper extension. (Make this by rolling stout paper round the experimental tube, slide it off, and fix the edges with stamp paper, or gum.) Obtain, as before, half-a-dozen values for the position of maximum resonance. Calculate the mean value A.

Since in the first case, $\frac{1}{4}\lambda = a + 0{\cdot}3d$, and in the second case, $\frac{3}{4}\lambda = A + 0{\cdot}3d$, therefore the difference, $A - a = \frac{1}{2}\lambda$.

The wave-length is used in connection with the expression $v = n\lambda$: either the frequency, n, of the fork is assumed and the velocity of sound in air, v, calculated, or the velocity is assumed and the frequency of the fork determined (Exps. 127, 128).

Exp. 127.—*Find the frequency of a tuning-fork by means of a resonance column.* Obtain the wave-length, λ, at temperature t (§ 110). Calculate the velocity of sound (v) at temperature t, then frequency, $n = v/\lambda$.

Find the frequencies of the forks of pitches, g', a', c'' respectively.

EXAMPLE. Fork g'. Temperature, 15° C. Tube diameter, 4·5 cm. Observed lengths of resonance column, 19·7, 19·4, 19·3, 19·6, 19·6, 19·5. Mean length of resonance column, 19·5 cm.

Corrected length $= 19{\cdot}5 + 0{\cdot}3 \times 4{\cdot}5 = 20{\cdot}9$ cm.

$\therefore$ wave-length $= 83{\cdot}6$ cm.

velocity of sound at $15° = 33240 + 60 \times 15 = 34140$ cm. secs.

$\therefore$ frequency of fork $g' = 34140/83{\cdot}6 = 408$.

Exp. 128.—*Find the velocity of sound in air at 0° C. by means of a tuning-fork and resonance column.* Proceed as in § 110, using a fork of known frequency.

EXAMPLE. Frequency of fork, 520.

Temperature, 15° C. Tube diameter, 4·5 cm.

Mean length of resonance column, 14·9 cm.

Corrected length $= 14{\cdot}9 + 0{\cdot}3 \times 4{\cdot}5 = 16{\cdot}3$ cm. $\therefore \lambda = 65{\cdot}2$ cm.

$\therefore v = 65{\cdot}2 \times 520 = 33904$ cm. secs.

$\therefore v_0 = 33904 - 60 \times 15 = 33000$ cm. secs.

111. The organ pipe (Fig. 83) consists of a wide tube of wood or metal of rectangular or circular section. One end is provided with a flute mouthpiece, the other may be open or closed. The mouthpiece is so constructed that on joining the narrow tube projecting at the bottom to a bellows, and blowing air through, the blast is directed against the sharp edge of a rectangular slit in the wall of the pipe. When the blast is properly adjusted (found by trial) the pipe "speaks" or sounds; that is, the air column in it is thrown, by resonance, into stationary vibration. The fundamental note is sounded when the blast is moderate; by blowing harder the harmonics are successively obtained. In every case the open end and mouthpiece are antinodes.

The bellows used for a blow-pipe will also do for the organ pipe. To ensure a steady blast, weight the boards of the bellows, and partly close the india-rubber tube with a clip. Work the treadle slowly and as little as necessary.

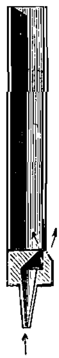

Fig. 83.

112. The Sonometer.—A common form is illustrated in Fig. 84. *AA* is a long sounding box. It often rests on short rubber feet or tubes instead of the substantial supports shown in the figure. *BB'* are two fixed and *C* a

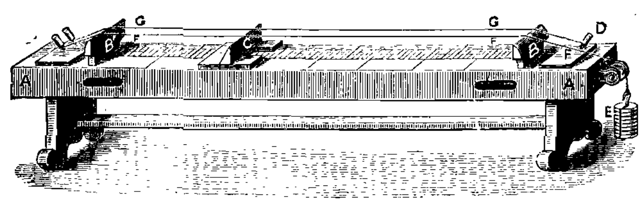

Fig. 84.

movable bridge whose edge is a little higher than *BB'*. A wire, *F*, looped round a screw at the left end, passes over the bridges and a pulley; it is stretched by weights *E*. Another wire, *G*, looped round a screw at the left end, is attached to a wrest pin, *D*, that can be turned by a key so that the wire may be tightened or slackened. By altering the position of the central bridge longer or shorter segments of the string can be made to vibrate. The lengths are measured by the scale fixed to the sounding box.

A *monochord* is a simpler form provided with one string.

Fig. 85 shows, by plan and elevations, a sonometer, better adapted for the laboratory. *A* is a thin sounding board, *B* a fixed bridge, *C* a movable bridge. The distance between the bridges is measured by directly applying a scale, *S*. The stretching force is supplied by a spring

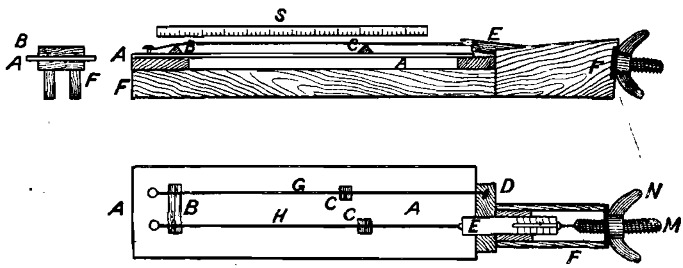

Fig. 85.

balance, *E*, instead of weights. A screw stem, *M* (of square section to prevent rotation) carries a large winged nut, *N*, that bears against an iron plate at the end of the frame, *F*. When the nut is turned the stem is moved, and the pull of the spring altered. *G* is a second wire looped to a screw at the left end, and attached to a wrest pin, *D*, at the right.

The spring balance should read to 50 lb. by $\frac{1}{2}$ lb. steps. The length scale should measure in $\frac{1}{10}$ inch; length, *AD*, about 40″.

113. To tune a wire to a note.—Stretch a piano-wire (steel) tightly over the bridges of the sonometer. The note, *e.g.* a tuning-fork, being sounded, pluck the wire to set it in vibration, and adjust the movable bridge until the notes emitted by the wire and fork seem to be the same in pitch. To tell this, sound first one and then the other.

To a musical ear tuning is not a difficult matter, but the unmusical is easily deceived, especially with notes of high pitch. The concord of a note (n) with its octave ($2n$) or fifth $\left(\frac{3n}{2}\right)$ is liable to be mistaken for that of two notes of the same pitch. Hence when the experimenter has found a point at which the agreement seems right he should move the bridge so as to make the vibrating segment first somewhat longer, then somewhat shorter. If in the former case the note of the wire is unmistakably lower in pitch, and in the latter higher, then the original position is nearly the correct one. If these observations are not confirmed, one note is a harmonic of the other, hence the bridge must be moved to another part of the wire and tuning recommenced.

It is an assistance to use a rough "stethoscope" consisting of a wooden rod, $\frac{3}{8}$ inch square, 8 inches long, fixed at one end into a slab of wood, 3 inches square, $\frac{1}{2}$ inch thick. The end of the rod is put on the sonometer board, and the ear is placed against the surface of the slab. Sound from the sonometer is conducted along the wood, and is easily heard in a noisy room.

Beats.—When two notes (frequencies n, n') are very nearly in tune, they will, if sounded together, produce a throbbing effect due to a rapid rise and fall in the intensity of the sound. The notes are then said to *beat*. The *beats per second* $=(n \sim n')$; hence they are slower and slower as the unison is closer, and cease altogether when it is exactly reached.

With a wire and fork of moderate pitch the beats are readily heard, especially when the sound is dying away, by pressing the shank of the fork on the board of the monochord. The two may be adjusted to agree so closely that a beat takes several seconds to complete itself. In obtaining unison by beats find the positions of the bridge, one on each side of the correct point, at which say one beat per second can be counted: the mean of these positions gives the value of l.

Sympathetic vibration.—When the fork and wire are in tune, if the fork is sounded and its shank pressed on the board of the sonometer, the string will be set in vibration; its note will be heard if the fork is removed and silenced. The movement of the string can also be detected by placing a finger just in contact with it; also small paper ∧-shaped riders placed on the wire will be thrown off.

The wire begins to respond to the fork only when unison is very nearly attained. This can be shown by first tuning the notes as nearly as possible, then gradually moving the bridge and noting the positions, one on each side of the correct point, at which the response cannot be detected.

In tuning a wire to a fork obtain first the beats, and when these have been reduced in rapidity look for the sympathetic vibration. The latter is not obtained easily if the sounding body cannot be placed on the sonometer. Beats may be heard with any two notes nearly in unison, but are not readily detected with loud or high notes.

Fainter beats and weaker responses are sometimes obtained between harmonics of the notes instead of the fundamentals.

Exp. 129.—*Compare the frequencies of two tuning-forks of nearly the same pitch, and adjust them to unison.* Sound the forks simultaneously, listen for the beats, and watch the seconds-hand of a clock. Count the number of beats in a given time and deduce the number per sec. This will be the difference in the frequencies of the forks.

Usually the fork of higher pitch may be distinguished from the other by ear. Confirm by loading one of the prongs (bind a turn of fine wire round it): this will lower the pitch slightly. Hence (i) if the higher fork has been loaded the beats will be fewer, (ii) if the lower they will be increased. The loading (iii) may have been sufficient to reduce the pitch of the higher fork so much below the lower that rapid beats are produced. If this is suspected place the load about half-way along the prong and test again. The effect of the load in reducing the frequency is greatest when at the end of the prong.

To adjust the forks to unison.—METHOD I. Bind several turns of wire round each prong of the higher fork, adjust the wires along the prongs until unison is obtained

METHOD II. Take a third fork, P, of nearly the same pitch as the two (Q, R) under adjustment. Load P so that there are, say, four beats per second when it is sounded with the lower (Q) of the two forks (Q, R). Next load R until there are four beats per second when it is sounded with P.

To test the unison, rest the shanks of the two forks on a board, one of the two being excited. After a few seconds remove and silence the excited fork. The other should be emitting its note, it having been excited by sympathetic vibration.

Exp. 130.—*Demonstrate the laws of the transverse vibrations of strings* (§ 109). Use three or more tuning-forks (pitches g', a', c'' for instance) and the sonometer. Find the relative frequencies of the forks (Exp. 126 or 127).

(1) *Frequency is inversely proportional to the length of the vibrating segment.* Keep the stretching force constant, find and measure the lengths successively in tune with each fork. Calculate for each case (*Frequency of fork*) × (*length of vibrating segment*). The values of the product should be the same.† Plot log n and log l. The graph will be practically straight.

Repeat with other values of the stretching force.

(2) *Frequency is directly proportional to the square root of the stretching force.* Keep the length of the string constant, observe the values of the stretching force that will bring the wire successively into tune with each fork. Calculate for each case (*Frequency of fork*)^2 ÷ (*Stretching force*). The values of the quotient should be the same.‡ Plot (i) n^2, and P, or (ii) log n and log P. In either case the graph will be practically straight.

Apply first (in Fig. 85, turn N) a high value of the stretching force (20 kg., or 40 lb., if the wire will sustain it), and adjust the movable bridge until the length of wire is in tune with the fork of lowest pitch. Proceed with higher forks.

Repeat with other initial values of the length.

† For if $n \propto 1/l$, then $nl = const.$

‡ For if $n \propto \sqrt{P}$, then $n/\sqrt{P} = const.$ $\therefore n^2/P = const.$

114. To find the frequency of a note by means of a monochord and fork of known frequency.—Tune the wire of the monochord (1) to the note and (2) to the fork. Record the respective lengths. Keep the stretching force constant throughout. Then

$$\frac{\text{length of wire for note } (l\,)}{\text{length of wire for fork } (l')} = \frac{\text{frequency of fork } (n')}{\text{frequency of note } (n)}.$$

$$\therefore\ n = n'\,l'/l.$$

The frequency of the fork may be found by Exp. 127.

Exp. 131.—*Find the frequency of the notes, fundamental and harmonic, of an organ pipe.* Proceed as in § 114.

Exp. 132.—*Find the frequency of a tuning-fork by the "absolute" monochord.* Fix the board of a monochord vertically. Hang a steel wire (piano) over the top bridge so that it *just* touches the lower movable one. (If the wire rattles during vibration press it by the finger *lightly* on the lower bridge.) Suspend a known mass (P, say 20 *kg.*) from the wire. Carefully adjust the movable bridge until the vibrating segment is in unison with the fork: note the mean length (l) between the bridges. Make a mark by a blunt knife on the wire near each bridge. Measure the length, l', between the marks. Remove the wire from the monochord, cut it at the marks and weigh the portion ($= w$).

Calculate (i) $m = w/l'$, (ii) the frequency, n.

In the formula (§ 109) $n = \sqrt{P}/2l\sqrt{m}$ express

either P in dynes, l in *cm.*, m in *gm.* per *cm.*

or P in poundals, l in *feet*, m in *lbs.* per *foot*.

If the stretching force is due to the weight of Q *lbs.* then

$$P = Q \times 453{\cdot}6 \times 981 = 445000\ Q \quad \textit{dynes}.$$

If the stretching force is the weight of Q *lbs.*, l the length in *cms.*, and m the mass in *gms.* of 1 *cm.* of the string, then

$$\log n = \tfrac{1}{2} \log Q - \log l - \tfrac{1}{2} \log n + 2{\cdot}52314.$$

The monochord should be vertical in order to minimise the friction between the wire and the lower bridge. If this is considerable, the stretching force on the vibrating segment is not approximately equal to the weights hung on the wire.

EXAMPLE. Frequency of a fork, c''. Mean length of vibrating segment, 39·6 *cm.* Stretching force = 56 lb. wt. = 56 × 453·6 × 981 dynes. Mass of 36·2 *cm.* of wire = 0·5025 *gm.*:

$\therefore$ mass of 1 *cm.* of wire = 0·01387 gm.

$$\therefore \text{frequency} = \frac{1}{2 \times 39{\cdot}6}\sqrt{\frac{56 \times 453{\cdot}6 \times 981}{0{\cdot}01387}} = 535.$$

115. To find the velocity of sound in the substance of a rod or wire. I) ROD. The rod should be 5 or 6 feet long. Grasp the rod at its centre by the left hand. Rub it by the right hand with a well resined leather. (A glass rod requires a cloth moistened with water, or, better, alcohol.) Obtain the frequency (n) of the note emitted (§ 114). Measure the length, l, of the rod in *cms.*

The note (fundamental) obtained is that of the rod when vibrating longitudinally and having a node at the middle and an antinode at each end. Hence the wave-length, $\lambda = 2l$. Hence velocity, $v = 2\,ln$.

(II) WIRE. The wire is stretched over two bridges, 9 or 10 feet apart. The stretching force need not be known: the note emitted is independent of this. Proceed as in (I).

When giving the *fundamental* note there is a node at each end and an antinode in the middle.

The value of the velocity obtained above is that in a rod or wire. The velocity in an unlimited medium is about 1·1 times the velocity in a rod or wire.

To find Young's modulus (Y) of elasticity for the substance of a rod or wire.—The velocity of sound (v) in the substance and its density (D) being obtained, since $v = \sqrt{Y/D}$, and $v = 2ln$, then $Y = 4Dl^2n^2$.

Exp. 133.—*Determine the velocities of sound in the substances below, and the values of Young's modulus:* glass (rod), oak (rod), deal (rod), brass (rod), iron (piano wire), copper (wire).

116. To find the velocity of sound in a gas by means of Kundt's tube.—Thoroughly clean and dry a long glass tube of about 2 inches diameter. Fix to one end of a stick, a cardboard disc a little smaller in diameter than the tube. *Arrange so that* the disc lies in the glass tube. Clamp a long rod (say 6 ft.) of wood, or glass, or brass firmly in the middle. Fix a cardboard disc to one end of the rod and arrange that this lies within the glass tube, 9 or 10 inches from one end. The axes of the rod and tube must be in line. The *clamp* is conveniently made of two stout pieces of wood slightly hollowed out to hold the rod between them (clamp between lead strips). The pieces are screwed tightly together, and the bottom one fixed to the table.

Thoroughly dry a small quantity of lycopodium powder, place it in, and distribute it evenly along the glass tube.

Rub the free end of the clamped rod with a resined cloth, and move the stick at the other end of the glass tube until the lycopodium settles into distinct figures. At the nodes the lycopodium will not be disturbed. Measure the distance between p nodes and divide by $(p - 1)$. The quotient is a value of the half wave-length in air of the note emitted by the rod.

Repeat the experiment several times by pushing the end of the rod an inch or two further into the tube. Calculate the mean wave-length from the measurements of $\lambda/2$. Note the temperature.

Find the frequency of the note emitted by the rod.

Finally calculate the velocity of sound in air at 0° (§ 108).

Exp. 134.—*Find the velocity of sound in air by means of a Kundt tube.*

PART IV.—LIGHT.

CHAPTER IX.

REFLECTION. REFRACTION. PHOTOMETRY.

117. Refraction and Reflection.—When light falls upon the surface of a body a part is *reflected* or returned into the medium from which it came, the remainder enters the body and is *transmitted* and *refracted.* Of the reflected part some is *regularly reflected,* the remainder is *scattered* (often said to be "irregularly" reflected); of the transmitted part a portion is *regularly refracted,* the remainder is *scattered.*

Relative intensities.—The proportion of the light reflected to that refracted depends upon the nature of the surface and its polish: *e.g.* much from clean mercury, polished silver, little from a dead black surface, etc. The amount reflected increases and that refracted diminishes as the angle of incidence increases: *e.g.* the proportion reflected from polished glass is small when the angle of incidence is small, but considerable when the angle of incidence is large. The transmitted portion is rapidly absorbed if the body is opaque, slowly when the body is transparent. The light scattered during refraction is that which renders surfaces visible and gives them colour. Its effects are more or less modified by the light scattered on reflection.

In experiments with prisms, lenses, etc., most of the light undergoes refraction: a trifle however is reflected at each surface, this is often made use of in optical measurement.

See the *Text-Book of Light* for definitions of ray and pencil; converging, diverging and parallel pencils or beams; incident, transmitted, refracted and emergent rays; a point of incidence, a normal to the surface; planes of incidence, of reflection and of refraction; angles of incidence, reflection, refraction, emergence and deviation.

REMEMBER that the angles of incidence, reflection, refraction and emergence are those between the *normal* to the surface and the respective rays.

Snell's law of refraction (law of sines).—*When light passes from a medium A to a medium B, the ratio of the sine of any angle of incidence (ϕ) to the sine of the resulting angle of refraction (ϕ') is a constant number.*

The constant is called the *index of refraction* (symbol μ) between the two media. When light passes from medium B to medium A, the index of refraction is the *reciprocal* of its value when the light passes the reverse way, viz. from medium A to medium B. In symbols

$$\sin\phi/\sin\phi' = \mu_{AB}$$
$$\sin\phi'/\sin\phi = \mu_{BA}$$
$$\mu_{BA} = 1/\mu_{AB}.$$

NOTE that the value of the index of refraction depends upon the nature of *two* media: these should both be specified.

Critical angle.—When ϕ is such that ϕ' is a right angle, then the value of ϕ is called the *critical angle* (symbol θ) between the two media. For angles of incidence *less than* the critical angle (θ) a proportion of the light is refracted; for angles of incidence greater than the critical angle there is no refraction, the light is *totally reflected.*

Since $$\sin\phi = \mu \;.\; \sin\phi'$$
when $\phi' = 90°$, then $\phi = \theta$, and $\sin\theta = \mu$. Thus if the critical angle is found by experiment, the value of μ is that of its sine, this may be found from trigonometrical tables.

REMEMBER

μ (air to glass) = 3/2 practically	μ (air to water) = 4/3
μ (glass to air) = 2/3 ,,	μ (water to air) = 3/4
critical angle = 38° to 41°	critical angle = 48°·5.

118. Definitions (optical).—A *slab* or *plate* is a transparent body having two parallel plane faces.

A *prism* is a transparent body having two plane faces inclined to one another.

The angle between the plane faces of the prism is called the *refracting angle,* and the straight line in which they meet is called the *refracting edge.* (If the section of the prism perpendicular to the edge is an isosceles, an equilateral, or a right-angled triangle, the prism is said to be *isosceles,* or *equilateral,* or *right-angled.*)

Lens, see § 127.

119. Refraction at plane surfaces: to find μ.—I. In some of the following experiments values of ϕ and ϕ' can be measured. From these μ may be calculated by reference to trigonometrical tables (4 figure), assuming

$$\mu = \sin\phi / \sin\phi'.$$

If the complements of the angles of incidence $(90° - \phi)$ and refraction $(90° - \phi')$ have been measured, then

$$\mu = \frac{\cos(90° - \phi)}{\cos(90° - \phi')}.$$

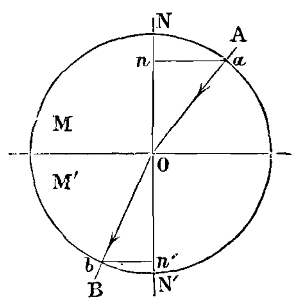

Fig. 86

II. The value of μ may also be obtained by a graphic construction (Fig. 86). Draw a line NN' to represent the normal: at O, a point on it, make the angles NOA, $N'OB$ equal to those of incidence and refraction respectively: with O as centre, any convenient radius (several inches long), describe a circle cutting OA, OB at a and b respectively: draw an, bn' perpendicular to NON': measure an, bn' in, say, millimetres. Then μ is the ratio of an to bn'.

III. If an object is viewed *directly,* that is along the normal to the surface of the refracting medium that passes through the object, then the image of the object lies on the normal, and the index of refraction (for light passing to the eye from the medium through which the object is viewed) will be equal to the ratio (*distance of image from surface*) $\div$ (*distance of object from surface*).

PROOF. Let mm be the surface separating the media a, b (b optically denser than a): let A be the object, AN the normal to mm, and ABC a ray. Produce CB to cut the normal at A'. The image of A will be along CBA'. Let ϕ, ϕ' be the angles of incidence and refraction respectively. Then

$$\mu = \frac{\sin\phi}{\sin\phi'} = \frac{\sin BAN}{\sin BA'N} = \frac{BN/BA}{BN/BA'} = \frac{BA'}{BA}.$$

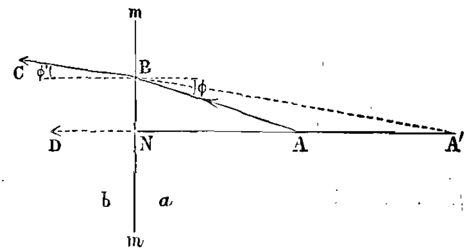

Fig. 87.

When ϕ and ϕ' are small NA, NA' are practically equal to BA, BA' respectively, and A' is the position of the image of A. Hence when the image is seen on the normal, $\mu = NA'/NA$ practically.

IV. Note that $\mu = BA'/BA$ for *any* angle of incidence. Hence a value of μ may be obtained as follows: track (§ 121) any ray from the object. Draw *any* parallel to the normal at the point of incidence (B). Measure the distances between the point of incidence (B) and the points of intersection of this parallel with the refracted and incident rays respectively (produced if necessary). Then μ equals the ratio of these distances.

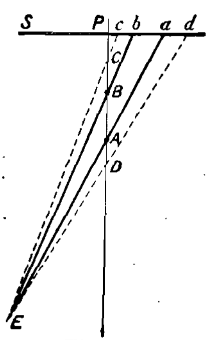

Fig. 88.

120. Parallax (See Exp. 135).—Let AB (Fig. 88) represent two objects. When the eye of the observer is in the line BA, B is hidden by A; when the eye is, say, at E, on the left of BA, then B appears left of A. If B is further off, say at C, it appears further on the left of A; if B is in front, say at D, it appears on the right of A. The relative positions and amounts of separation are indicated in the figure by c, b, a, d. Similarly if the eye is on the right of BA the relative positions, reckoning from left to right, are d, a, b, c.

Thus A, B appear closer together as the eye is nearer the line AB. The greater the actual distance between A, B, the greater their apparent separation when viewed from a definite position. When the line of sight is along AB the objects appear in line, whatever the actual distance between them may be. When the distance between them is negligible they appear in coincidence in every position of the eye.

DEFINITION. The apparent change in the position of an object due to a change in the point of view of the observer is called *parallax*.

Parallax error.—Whenever the position of an object with regard to a scale, *e.g.* the end of the pointer of a galvanometer, has to be observed it is important to beware of parallax. The liability to *parallax error* is minimised by the devices described in §§ 12, 35, 80, etc.

Parallax method.—In some optical experiments it is possible to fix accurately the position of an image by adjusting so that the parallax between the image and a pointer is eliminated. A needle (or cross wires), for instance, may be brought up to the image, and adjusted so that, when the observer's eye is moved (one or two feet) from side to side, no apparent separation between the image and pointer can be detected, the two seem fixed together. The needle is then in the same position as, or in coincidence with the image. Remember that the *further of two objects will appear to move, relatively to the nearer, in the same direction as the eye of the observer.*

***Exp. 135.**—Fix a sheet of paper on a drawing board (Fig. 88). Stick two pins, A, B, into it. Draw a line, PS, perpendicular to AB; place a scale (edgewise) along it.

(I.) Note the scale division hidden by each pin (called the scale reading of the pin) when the eye is (1) over, (2) left of, (3) right of BA. Mark the positions of the scale readings along PS, join with A, B respectively.

(II.) Alter the position of B. Obtain the scale readings as before, placing the eye, however, so that those for A are the same as in (I.). Mark scale readings, and join with *A*, *B*, as before.

(III.) Put *B* in front of *A*. Make observations, etc., similar to (II.).

The lines across the paper should intersect at the respective positions of the eye.

REFLECTION AND REFRACTION.

121. Apparatus.—1. Plane mirror (Fig. 89): a strip (about 3″) is fixed with round-headed screws and washers of brass or card to a rectangular piece of wood so that it stands with its reflecting surface vertical. The mirror should be thin and of very good quality.

2. A cube (2″ edge, say) and a prism (1″ long, say) of solid glass.

The prism is frequently equilateral (1″ edge) with three polished faces. Paint one face black.

3. Some large pins (3″ long, say) and common pins.

Method (see Fig. 89): A sheet of paper is fixed to a drawing board, and a straight line (which may be called

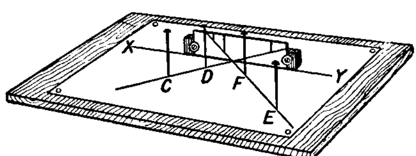

Fig. 89.

the *base line*) drawn on it: the mirror, cube, or prism is placed on the paper with an edge lying along the base line. The *paths of rays* whether incident, reflected, or refracted *are tracked* by sticking pins vertically into the board so that they, their images, or the images of other pins are in one line. This is *judged* by arranging them so that in one position of the eye the several pins and images are hidden by the front one. The pins should be placed as far apart as possible. The *image* of a pin may be *identified* by slightly shaking each pin in turn: the corresponding image will move similarly. An image by *reflection* is obtained by putting the *object in front of the mirror*, etc., that is on the same side as the eye of the observer; an image by *refraction* by putting the object behind or on the opposite side of the surface, etc. Refracted images, especially those with the prism, are coloured, reflected ones are not coloured.

Arrange so that objects are well illuminated by a side light. When the images are faint, screen the cube, etc., forming them by placing dark surfaces (brown or grey book covers, cards, etc.) so as to form a dull background.

In the diagrams the position of a pin is indicated by • with a capital italic: that of its image by × with the corresponding small italic. The base line is indicated by *XY*.

In an experiment *outline the figure* of cube, prism, etc., by running a finely pointed pencil along the edges of the face resting on the paper. Mark the pin-points with reference letters, and sketch in the rays as soon as the pins are adjusted. When the observations are completed remove the paper from the board, place it on a page of the note-book, prick through the outline of the figure and the several pin-points as determined, then join neatly with ruled lines, etc. Measure the necessary angles with a protractor, mark the values on the diagram.

***Exp. 136.**—*Prove the law of reflection*, viz. *that the angles of reflection and incidence are equal.* Adjust the *silvered* or back surface of the mirror to the *base line* on the paper. (1) *Find the normal to the mirror* (Fig. 90). Put a pin *A* in front of mirror. Put *B* so that *B*, *A*, *a*, *b* are in line. The straight line through *BA* is then a normal to the mirror. Find the angles between it and the base line (should be each 90°). (2) *Track the incident and reflected rays.* Put *C* and *D* anywhere in front of mirror. Put first *F*, then *E*, in line with *c*, *d*. Then *e*, *f* are also in line with *C*, *D*. Join *CD*, and produce: similarly *EF*. Measure the shaded

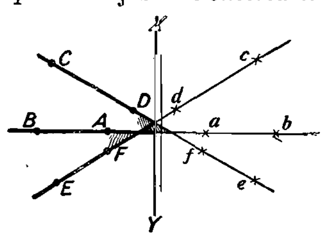

Fig. 90

angles. Observe that these are equal: one is the angle of incidence, the other that of reflection. (Also observe that the intersection of the slanting lines lies on the reflecting surface.) If the normal has not been found measure the angles between (i) CD and XY, (ii) EF and XY: these are the complements of the angles of incidence and reflection, and hence should be equal.

***Exp. 137.**—*Prove that if the direction of a ray incident upon a plane mirror is not altered, then when the mirror is displaced through any angle, the reflected ray is turned through twice that angle* (Fig. 91). Draw three lines, M_1, M_2, M_3. (1) Place the reflecting surface of the mirror coincident with M_1. Put two pins A, B in front of the

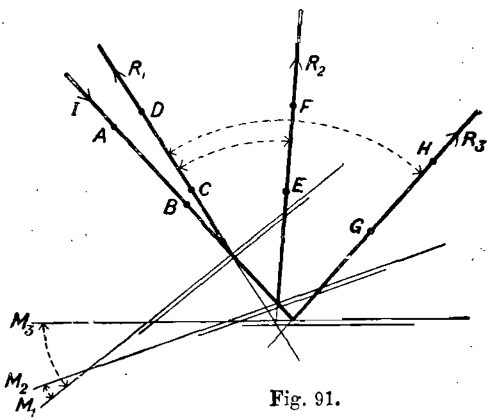

Fig. 91.

mirror. Adjust C, then D, in line with the images a, b.† (2) Place the mirror along M_2. Adjust E, a, b in line, then F, a, b. (3) Place mirror along M_3. Adjust G, a, b in line, then H, a, b. Measure the angles between (i) EF and CD, (ii) GH and CD, (iii) GH and EF, (iv) M_2 and M_1, (v) M_3 and M_1, (vi) M_3 and M_2. The first three should be respectively the doubles of the last three.

† The images are not shown in Fig. 91.

*Exp. 138.—*Measure the angle of a prism or the angle between two reflecting surfaces* (Fig. 92). Draw two parallel lines (about $\frac{1}{2}''$ apart). Place the prism with its refracting edge perpendicular to the paper and between the parallels. Put two pins *A*, *B* on one, and two others *C*, *D* on the second parallel. Adjust the first pin *E*, then *F*, in line with the images (*a*, *b*) of *A* and *B* (these will be very faint); also *G*, *H* in line with *c*, *d*. Observe also that *A*, *B*, *e*, *f* are in line, and *C*, *D*, *g*, *h*; and that the intersections of *AB*, *EF* and *CD*, *GH* are on the reflecting surfaces of the prism.

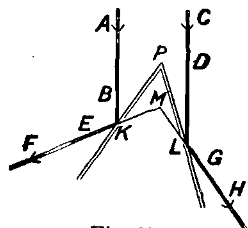

Fig. 92.

Draw lines through *FE* and *HG* and measure the angle between them: this angle is twice that of the prism.

Confirm as follows: cut a piece of paper to fit the angle above, fold it in halves, show by superposition that it then equals the angle of the prism.

NOTE: The geometrical proof that the angle *KML* between the reflected rays is twice the angle *KPL* between the prism faces is as follows. Let *BK*, *DL* be two parallel rays, and *KE*, *LG* the corresponding rays reflected from the prism faces *PK*, *PL* respectively. Produce *EK* and *GL* to meet at *M*.

Now $\wedge BKP = \wedge PKM$, each being equal to the complement of the angle of incidence or reflection.

Hence $\qquad \wedge BKM = 2 . \wedge BKP.$

Similarly $\qquad \wedge DLM = 2 . \wedge DLP.$

$\because$ *BK* and *DL* are parallels, $\wedge KPL = \wedge BKP + \wedge DLP$: also

$$\begin{aligned} \wedge KML = \wedge BKM + \wedge DLM & \\ &= 2(\wedge BKP + \wedge DLP) \\ &= 2 . \wedge KPL. \end{aligned}$$

*Exp. 139.—*Find the position of the image of an object due to a plane mirror* (Fig. 93). Place as object a large pin (*A*) in front of a plane mirror: then *B*. Put first *C*, then *D*, in line with *a*, *b*. Similarly place *E* somewhere in front, and first *F*, then *G*, in line with *a*, *e*. Set up *H*, then *K*, *L* in line with *a*, *h*. (Observe also that *A*, *B*, *c*, *d* are in line, also *A*, *E*, *f*, *g*, also *A*, *H*, *k*, *l*.) Produce *DC*, and *GF*, *LK*;

these lines should intersect at one point (a), which is the position of the image of A. Join Aa: show that XY is *perpendicular to* and *bisects* Aa. (If carefully done the

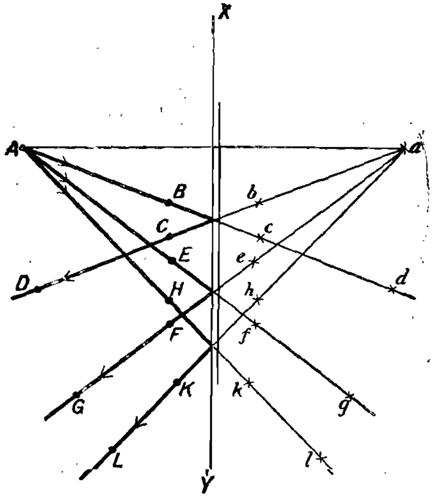

Fig. 93.

points of intersection of the pairs of slanting lines will be practically on the reflecting surface.)

***Exp. 140.**—*Find by the parallax method the position of the image of an object due to a plane mirror* (Fig. 94). Place

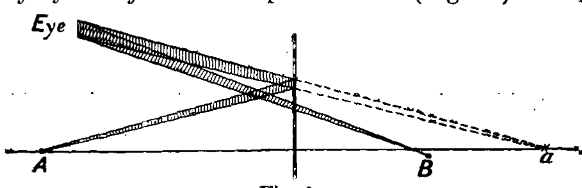

Fig. 94.

a pin, A, some distance (two feet, say) in front of a plane mirror. Put another pin, *B*, behind the mirror so that the upper part of it and the lower part of the image of *A* can be seen simultaneously. Adjust *B* until it appears to coincide with the image of A, and continues to do so when the observer moves his head to the right or left, up or down. There is *one* position in which this can occur: in all others the pin and the image will separate. When the adjustment is correct the pin, *B*, occupies the same position as the *virtual* image of *A*. Measure the distance (i) of *A*, (ii) of *B*, from the mirror. (The mirror lies practically midway between *A* and *B*.) Repeat with *A* at other distances.

Exp. 140 gives an indirect proof of both laws of reflection, because it shows that in *one position* only the pin, *B*, and image of *A* remain in coincidence from *every* point of view, and that then the reflecting surface is midway between *A* and *B*. (See *Text-Book of Light.*)

***Exp. 141.**—*Show that when a ray of light goes through a slab the emergent and incident portions are parallel* (Fig. 95).

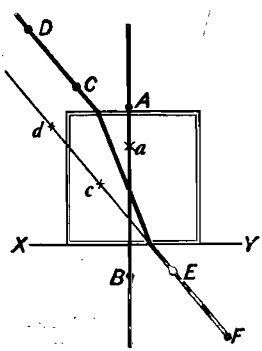

Fig. 95.

Adjust an edge of a glass cube or slab to a base line, *XY*, and outline its figure.

First, find the normal to the surface. Put a large pin, *A*, a little behind the cube. Adjust another, *B*, in front so that *A*, *a*, *B* are in line (the top of *A* should be seen above the cube). Then *B*, *b*, *A* will be in line. The line *A B* is normal to the surfaces.

Secondly, put pins *C* and *D* behind the cube on a line inclined (say 30°) to its face. Adjust in front of the cube first *E*, then *F*, in line with images *c*, *d*. Then *C*, *D*, *e*, *f* are in line. Show that *CD* is parallel *EF*. Measure the angles of incidence and refraction at the two surfaces; calculate μ (§ 119).

Repeat with *CD* inclined at several angles, say 45°, 60°.

Tabulate (1) angle of incidence (ϕ_1) at first face, (2) angle of refraction (ϕ'_1) at first face, (3) angle of incidence (ϕ_2) at second face, (4) angle of emergence (ϕ_2') at second face, (5) sin ϕ_1, (6) sin ϕ_1', (7) sin ϕ_2, (8) sin ϕ_2', (9) μ at first face, (10) μ at second face.

***Exp. 142.**—*Find the index of refraction of glass using a cube or thick slab* (Fig. 96). I. (1) Arrange as in Exp. 141. Place a large pin, A, in contact with the back surface. (2) Find the normal (AB) to the surface (Exp. 141). (3) Put a pin Q about $\frac{1}{4}''$ from the front surface, then another, R, 4″ or 5″ off, in such a position that Q,R and the image of A are in line. Produce RQ to cut the normal at J and XY at p. Join pA. Measure pJ, pA. Then

$$\mu \text{ (glass to air)} = pJ/pA.$$

II. Repeat (1) and (2) above. (3a) (i) Put a pin C about 5″ from A and $\frac{1}{2}''$ from the normal, then another, D, about 8″ from A and in line with C and image of A. (ii) Similarly place pins E, F on the other side of AB. Produce DC and FE; these should intersect on the normal at a. The position of a is that of the image of A when viewed directly. Measure An, an. Calculate μ (glass to air) $= an/An$.

III. (1) Adjust as in I. (1). (2) Lay a drawing pin (point upwards) on the top surface of the glass. Place the eye so that it can see the point, k, of the drawing pin, and the image of A in line (Exp. 140). Adjust these into coincidence by the method of eliminating parallax. Measure the distance (d) of k from the front surface and the thickness (D) of the block. Calculate μ (glass to air) $= d/D$.

IV. (1) Place a strip of stamp paper on the back surface of the slab. Put a pin, B', in front, look for the reflected image (faint), b', of B'. (Mount a paper flag on B', its image may then be more readily detected.) Adjust the position of the glass with regard to B' so that b' and the image of the edge of the stamp paper are in coincidence (parallax eliminated). Measure the distance (d) of B' from the front surface (this equals the distance of b') and the thickness (D) of the block. Calculate μ (glass to air) $= d/D$.

Exp. 143.—*Find the refractive index of the substance of a thin slab or lamina.* Use a microscope (§ 32) having a scale and vernier or equivalent arrangement fitted to the tube so that the movement of the tube can be measured. Use a flat disc of glass or metal as a base or reference surface. Lay the lamina upon it.

Note the scale readings r_1, r_2, r_3 respectively, of the index mark on the tube when the microscope is focussed, (1) on the reference surface (a fine mark or scratch), (2) on the same when the lamina is laid on it, (3) on the top surface (a bit of dust on it) of the lamina. Then

$$\mu \text{ (substance to air)} = (r_3 - r_2) / (r_3 - r_1).$$

Use a piece of plate glass. (The thickness, $r_3 - r_1$, may be measured by calipers.)

Exp. 144.—*Find the refractive index of a liquid, water, petroleum, ether, etc.* Use the method of Exp. 143. Place the liquid in a shallow vessel. Focus on a mark on the bottom (inside) of the vessel (1) before, (2) after the liquid is introduced, (3) on dust (lycopodium) on the surface.

***Exp. 145.**—*Find the caustic by refraction at a plane surface* (Fig. 96). Use a glass slab. Find a normal, Exp. 141, then, in turn, 5 or 6 emergent rays, Q_1 R_1, Q_2 R_2, etc. Produce each ray to cut the normal. Draw a curve to touch the produced parts of the rays: this is the required *caustic*. The *cusp*, *a*, is the point at which the image of *A* is seen when it is viewed directly. (Since $\mu = an/An$; $\therefore$ for glass and air, $an = \frac{2}{3} . An$.) If *A* is viewed obliquely, its image is at that point on the caustic at which the line through the centre of the eye touches the caustic (Exp. 150).

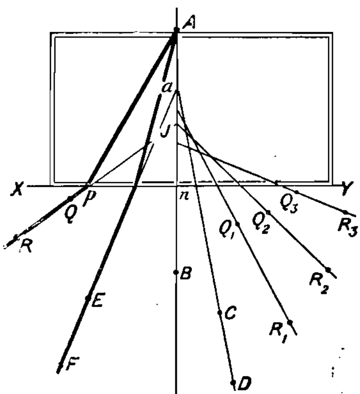

Fig. 96.

***Exp. 146.**—*Track a ray of light through a prism* (Fig. 97). Place the glass prism provided with its refracting edge at right angles to the paper. Outline its figure. Put pins, A, B, in front of one refracting face (the line joining them inclined to it, say, 60°), look into the other face, find their images, a, b, and adjust first C, then D,

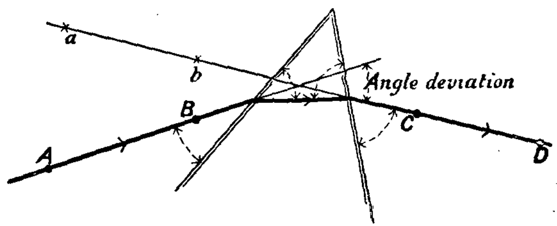

Fig. 97.

in line with a, b. Observe that then A, B, c, d are also in line. Measure the complements of the angles of incidence and refraction at the first face, of incidence and emergence at the second face; also the angle of deviation (the angle between the *incident* and emergent rays). Calculate μ at the two faces (§ 119).

Repeat with AB inclined at other angles, say 45°, 30° Tabulate as in Exp. 141, adding (11) angle of deviation.

Exp. 147 shows that in going through a prism a ray of light is deviated from its original direction, and the images produced are coloured. Exp. 142 shows that a slab produces displacement, but *no* deviation. Also that light incident normally is neither deviated nor displaced.

***Exp. 147.**—*Total reflection in an equilateral or right-angled prism.* I. On the paper draw two lines at right angles. Arrange the glass prism with one face coincident with one of the lines. On the other line set up two pins A, B (Fig. 98, 99) ; *look for the images* a, b (these will *not* be coloured), and arrange first a pin C, then D, in line with them. Outline the figure of the prism. Show that DC and AB meet on a face of the prism, and that CD is at right angles to the face from which the ray emerges. (See § 151 (5) : comparison prisms of spectroscopes.)

II. The above experiment may be varied by arranging the pins A, B (Fig. 99a) so that the line AB is inclined at, say, 80^o with the face of the prism (if the angle is less

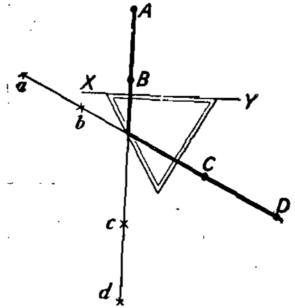

Fig. 98.

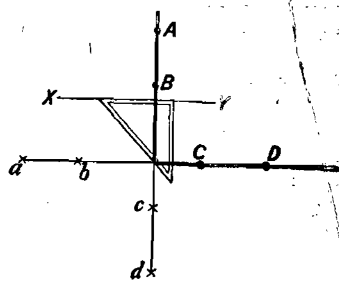

Fig. 99.

than this the ray may not be totally reflected, but undergo refraction as in Fig. 97). Then set up C, D, as before.

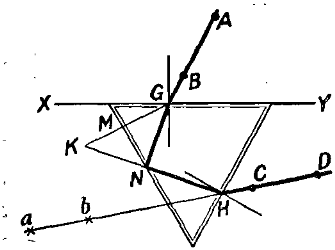

Fig. 99a.

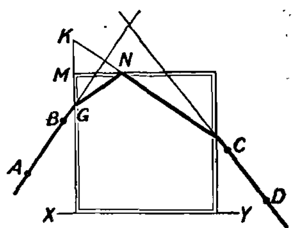

Fig. 100.

To find the path of the ray in the prism, produce AB and DC to intersect the faces of the prism at G and H respectively. Draw GK perpendicular to MN, make $MK = MG$, join KH, GN. Then the angles GNM and KNM are equal; hence the lines GN, NH are equally inclined to the prism face, and GNH is therefore the path of a ray that undergoes total reflection.

III. Similarly the *total reflection in a cube* may be investigated (Fig. 100: lettered like Fig. 99a).

PHOTOMETRY.

122. Luminosity.—The *illuminating power, luminosity,* or *luminous intensity of a light* is its particular value (both brilliancy and size are involved) considered with reference to certain standard lights.

The *brightness* or *intrinsic brilliancy* of a light is estimated as the luminosity of unit area.

The common British standard of luminosity is the candle; the unit is called the *candle power.* SPECIFICATION: A *candle power* is the luminosity of a spermaceti candle burning 120 grains per hour (6 to the lb.).

The *carcel* (French), *Hefner-Alteneck* (German), and *pentane* standards are the lights produced by burning certain substances under specified conditions. The *Congress international standard* is the luminosity of a *sq. cm.* of molten platinum at its temperature of solidification. It is about 18·5 candle power. The *international candle* or *bougie décimale* is 1/20 of the above Congress standard.

21 Hefner units = 2 carcels = 20 international candles = 18·5 British candles. The pentane standard = 10 British candles.

Surface illumination.—The degree to which a surface can be illuminated by a source of light is (1) directly proportional to the illuminating power (L) of the source, (2) inversely proportional to the square of the distance (d) between the surface and the source, (3) directly proportional to the cosine of the angle of incidence (θ) of the rays on the surface.

The *brightness, illumination,* or *intensity of illumination* (I) of a surface is defined as the flux of light incident normally per unit area.† *Unit intensity of illumination* is the flux of light per unit area† received from a small source of unit luminosity at unit distance when the axis of the incident pencil is normal to the surface upon which the light falls.

Hence $$I = L. \cos \theta / d^2.$$

The British unit of illumination, called the *candle-foot,* is that produced by one standard candle on a white surface at a distance of one foot.

† $\left(\begin{array}{c}\text{Quantity of light}\\ \text{received by a surface}\end{array}\right) = \left(\begin{array}{c}\text{Flux of}\\ \text{light}\end{array}\right) \times \left(\begin{array}{c}\text{Time during which the}\\ \text{surface is exposed.}\end{array}\right).$

123. *If two lights equally illuminate a surface, their illuminating powers are directly proportional to the squares of their respective distances from the surface, provided that the rays from each light incident upon the surface are practically normal to it.*

PROOF. If a surface is illuminated by a light of candle-power, L, at a distance, d, the intensity of illumination, $I = L/d^2$. Hence if the surfaces are equally illuminated by two lights, L_1, L_2, at distances d_1, d_2, respectively then

$$L_1/d_1^2 = I = L_2/d_2^2 \qquad \therefore L_1/L_2 = d_1^2/d_2^2.$$

124. Photometry has two aims, viz., the comparison (i) of illuminating powers, (ii) of intensities of illumination. **Photometers** are instruments by which these comparisons may be made. The distances from the photometer of the lights to be compared are adjusted so as to *equally illuminate* adjacent portions of the same surface. The equality of illumination is judged by the eye of the observer; this may be done with considerable accuracy, because the eye is able to detect a slight inequality in the intensities of illumination, but it cannot estimate the ratio of two unequal intensities.

PRACTICE. Photometric experiments should be conducted in a room with blackened walls and ceiling, also the bench, stands, etc., should be blackened. The bench should be from 10 to 20 feet long. The eye should be prevented by opaque screens from viewing the lights, and the photometer surfaces should be similarly shielded from all stray illumination.

In photometric determinations adjustments at three or four different distances should be made, another series when the photometer is reversed, another series when the lights are changed over from one side to the other, and finally when the photometer is again reversed.†

125. Rumford or Shadow photometer (Fig. 101).—Place a stick, R, vertically in front of a white screen, S. Arrange the lights to be compared, L_1, L_2, so that each casts a shadow of the stick on the screen. Adjust them until the shadows, S_1, S_2, are close together and of equal

† The reversal cannot be effected with the shadow photometer.

darkness. Finally measure the distance of each light from the screen. Calculate as in § 123.

Evidently the shadow S_1, due to L_1, is illuminated by L_2, and S_2 by L_1. Thus the adjacent shadowed strips of the screen are exposed to one light only, and the intensities of illumination can be adjusted equal.

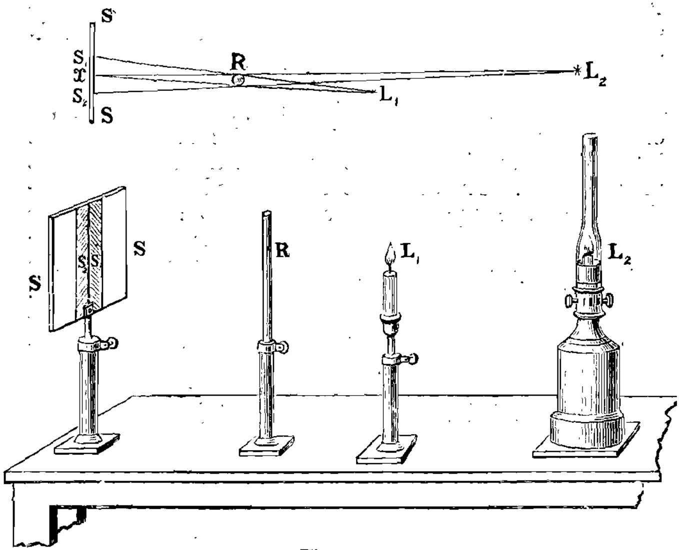

Fig. 101.

A *small* white card on a black board is an effective screen: for the stick use a pencil or the rod of a retort stand. The comparison by this method may be done in a poorly lighted room.

Unless the lights are of the same kind, *e.g.* candles, the shadows will be different in colour and the adjustment to equality is not so readily done.

Bunsen or Grease-spot Photometer.—A piece of paper, having a grease-spot, often star-shaped, at its centre—frequently called the *Bunsen disc*—is placed between the two lights and its position adjusted, until, when viewed from a little distance, the grease-spot cannot readily be distinguished from the rest of the surface. The distance of each light from the disc is then measured. As the disappearance of the grease-spot does not occur at the same position if viewed from both sides, make the observation and measurements first from one side, next from the other, and calculate the mean. Finally calculate the illuminating powers as in § 123.

In other positions of the disc the grease-spot appears lighter or darker than the surrounding paper. In the former case the amount of light per unit area that gets through the grease-spot from the light behind is greater than the intensity of illumination of the paper by the light in front: in the latter it is less. The quantities are equal when the spot is indistinguishable. Thus the Bunsen disc indicates when the lights illuminate the surfaces with equal intensities. A little light is, however, absorbed in passing through the grease-spot; also the scattering of the light is uneven; *e.g.*, when the spot disappears from one point of view it often reappears when looked at from another position.

The wax of the spot should be very evenly spread. A piece of white tissue paper sandwiched between two sheets of thin cardboard, each with a central hole, is a good substitute.

Frequently two mirrors are placed one on each side of the disc, and inclined to it at an angle of about 45°. The observer can then see the images of both surfaces of the disc at the same time.

The paraffin-slab photometer.—This consists of two slabs of paraffin-wax, about $\frac{1}{4}$-inch thick, pressed together with a piece of tin-foil between them. The block is held between the two lights to be compared, so that the plane of the tin-foil is perpendicular to the line joining the lights. The position of the block is adjusted until its edges appear equally bright, then the distance of each light from the wax is measured. Calculate as in § 123.

It is a good plan to put a black screen in front of, and close to the edges of the wax block. Make in it a rectangular slot nearly $\frac{1}{2}''$ wide, so as to expose a portion of the edge. The lights should be screened from the eye.

Exp. 148.†—*Practise the balancing or adjustment of photometers (often called the "Proof of the law of inverse squares").* Compare a group of two or three candles with a single one, using different photometers. Make half-a-dozen observations. Tabulate (1) distance of single candle from photometer, (2) ditto of group, (3) square of distance of single candle, (4) square of distance of group, (5) ratio of squares of distances.

The ratio of the squares of the distances should be practically equal to that of the illuminating powers, that is, the ratio of the numbers of the candles in the groups.

Exp. 149.—*Find by the photometer the candle-power of a gas-flame,*† *oil-flame,*† *electric glow-lamp, argand burner, Welsbach burner, Nernst lamp, etc.* Compare each light with a group of two or three candles. The last three are of considerable luminosity and may be compared more conveniently with an electric glow lamp, whose candle-power can be determined by a separate experiment. Make ten or twelve determinations for each light, and use different photometers. Tabulate the values for each photometer and light as in Exp. 148. Finally tabulate (1) kind of light, (2), (3), etc., the determinations by the several photometers.

† May be done partly as home-work by using the Rumford photometer.

CHAPTER X.

MIRRORS AND LENSES.

126. Reflecting surfaces are, as regards their shape, either plane or curved. When the latter they are called *concave* if hollow, *convex* if bulging. Curved surfaces are usually *spherical* or *cylindrical*.

Spherical surfaces.—DEFINITIONS. The radius of the sphere is called the *radius of curvature*, its centre the *centre of curvature*. The middle point of the spherical surface is the *pole, vertex,* or *centre* of the mirror. The straight line through the pole and centre of curvature is the *principal axis*. The value of the solid angle subtended by the mirror surface at the centre of curvature is called the *aperture* of the mirror.

CAUTION.—*The surfaces of mirrors, lenses, prisms, etc., must on no account be touched by the fingers.*

To maul a mirror or lens with the fingers is a dirty and destructive habit, for if the finger marks are not cleaned off the efficiency is impaired, and if the piece is rubbed with a cotton handkerchief, it becomes scratched and ultimately spoilt; lenses, etc., should be cleansed as seldom as possible, and then soft silk should be used.

To determine the shape of a reflecting surface.—The rules are given below. Practise each. Remember those in thick type.

I. Look into the surface and observe the image produced, whether larger or smaller than the object, whether erect or inverted.

Image.	*Reflecting surface.*
Diminished, erect-and-virtual.	**Convex.**
Same size as object, erect-and-virtual.	Plane.
Magnified, erect-and-virtual.	Concave.
Diminished, inverted-and-real.	Concave.
Magnified, inverted-and-real.	Concave.

II. Hold the surface near the eye and look along it towards a window or bright light. Place a straight edge (folded piece of paper) across and close to the surface. Arrange so that the eye sees the image of the straight edge. Observe the shape of the image carefully.

Image.	*Reflecting surface.*
Straight, parallel to the object.	Plane.
A waved line.	Poor surface.
Curved, bent towards the straight edge.	Convex.
Curved, bent away from the straight edge.	Concave.

RULE.—**The form of the image is in each case practically similar to that of the section of the surface.**

PRACTICE. The straight edge should be placed across the surface in several directions. If the indications are the same in each direction the surface is, most likely, spherical, if in one direction (parallel to the axis of cylinder) the image is straight, and in other directions more or less bent, the surface is a **cylindrical surface.**

The rules, II., above, are of special importance in determining the shape of a lens surface. In these, however, the two surfaces are usually close together, and hence to avoid mistakes caution and practice are necessary.

127. Lenses.—These are transparent (usually glass) bodies having one surface curved, the other either plane or curved. The curved surfaces may be cylindrical or spherical, and concave or convex. For the method of identifying a *surface* see § 126.

Lenses are primarily distinguished as **convex**—these are thicker in the middle than at the edges—and **concave**—these are thinner in the middle than at the edges. They are further subdivided (Fig. 102), the former into *double convex* (a), *plano-convex* (b), *concavo-convex* or *convex meniscus* (c), and the latter into *double concave* (d), *plano-concave* (e), and *convexo-concave* or *concave meniscus* (f).

The above is a geometrical classification. There is also an optical distinction. When the conditions are such that lenses can produce erect-and-diminished images only (like a concave glass lens *in air*) they are called *diverging lenses.* When the conditions are such that the lenses can produce inverted, or magnified-erect images (like a convex glass lens in air) they are called *converging lenses.* These terms should ***not*** be used in describing a particular lens because the convergency or divergency depends not only on (i) the shape of the lens, but also upon (ii) its substance, and (iii) the medium in which it is immersed.

The lenses used in optical instruments are generally compound, that is, a combination of several constituents: *e.g.* the object glass of a telescope (Fig. 128) consists of a convex lens of crown glass and a concave lens of flint glass. The combination is (in air) converging; it is also achromatic, that is, the images produced are not fringed with colour.

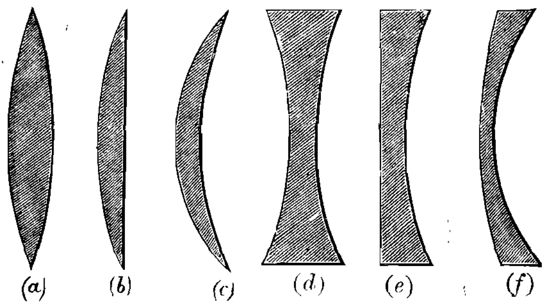

Fig. 102.

To identify glass lenses in air.—METHOD I. Do not feel the face of the lens: hold it by the edge close to the eye and look through it at an object, say a printed page, near enough to give an erect image. If the image is erect and diminished the lens is diverging, *e.g.*, a glass concave; if erect and magnified the lens is converging, *e.g.*, a glass convex. If the image is apparently the same size as the object, the lens is of low power, or the piece may be a glass plate. To decide the matter wave it before the eye. If the piece is a lens the image will also move, if a plate the image will remain at rest.

METHOD II. Move the lens from left to right or *vice versâ* in front of, and close to a printed page: if the image moves in the *opposite way* to the lens the lens is *convex*, if in the *same way* the lens is *concave*; no movement, not a lens.

***Exp. 150.**—*Draw a caustic to a circle* (Fig. 103). Describe a circle of radius not less than, say, 9 or 10 inches. Draw any diameter, aCP, and parallel to it 9 or 10 equidistant (about $\frac{3}{4}''$) lines, intersecting the arc at Q, R, S, etc.

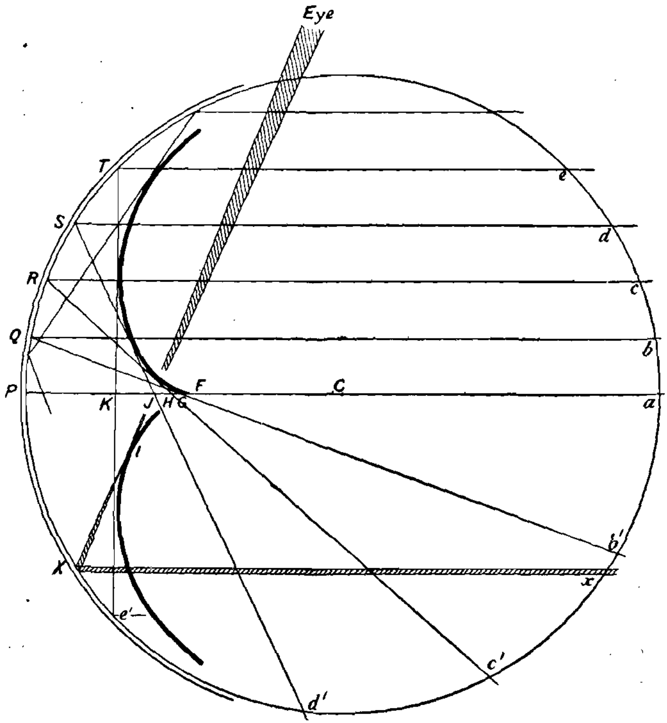

Fig. 103.

Find the mid point, F, of PC. Draw Qb', Rc', etc., so that the radii QC, RC, etc. (these are normal to the circle), are the bisectors of the angles bQb', cRc', etc., respectively.†

† A very convenient construction is to draw with Q as centre and radius Qb, a circular arc cutting the circumference, aQP, in b'. Simi-

Draw a curve to touch aP, $b'Q$, $c'R$, etc., in succession: this curve is a *caustic* to the circle.

G will be very close to F; FG, GH, HJ, etc., are in increasing order of magnitude.

EXPLANATION. If aP, bQ, cR, dS, etc., are incident rays of a parallel beam of light, Pa, Qb', Rc', Sd', etc., will be the respective rays reflected from the arc PS. The construction shows that all the incident rays between aP and bQ will on reflection cut the axis, CP, between FG; all those incident between bQ and cR will on reflection cut the axis between GH, and so on. Now if the $\angle QCP$ is not more than 5°, FG is small compared to GP: thus when the aperture ($2\angle QCP$) of a spherical mirror is less than 10° all the rays of an incident parallel beam are reflected very nearly through a point (called the *principal focus*) midway between the centre of curvature, C, and the pole of the mirror, P. When the aperture of the mirror is considerable, this is no longer the case, the so-called *spherical aberration* largely increases.

Whatever the position of the observer he sees an image in the direction of that point (I) on the caustic at which the line (XI) through the centre of the eye touches the caustic. Hence when the eye is on the principal axis PC, it sees an image at the cusp F; hence when the mirror is of small aperture images are only seen directly when the eye is close to the principal axis.

To avoid appreciable distortion of images due to spherical aberration, mirrors are made of small aperture. Lens surfaces too are generally of small aperture and the lens thin. (See § 129.)

Exp. 151.—*Identify various lenses, concave and convex, spherical and cylindrical, short, moderate, and long focus. Also a disc of thin plate glass.*

Exp. 152.—*Identify the shapes of the surfaces of various lenses, etc.*

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larly make $Rc' = Rc$, $Sd' = Sd$, etc. The whole must be done with great care and accuracy, especially for the points, b', d'.

Exp. 153.—A glass disc, having a scale graduated on one face and fixed inside a short tube, is provided. *Find which face is graduated.* Hold the tube so that light (from a window) is incident on a face of the disc and is reflected to the eye. Look obliquely at the glass disc: if two scales are seen, the graduated face is nearer the eye than the ungraduated one; if only one scale is seen, the graduated face is further from the eye. (In the former case the second scale is an image due to partial reflection from the further surface of light travelling in the glass.)

128. To track a ray through a lens.—It is useful to adopt the method of § 121. The principal axis of the lens should lie on the surface of the paper and board into which the pins are stuck. (1) This may be effected with a complete lens by making a slot in the board into which the lens may be fixed so that a half of it appears above the surface. (2) A half lens—one divided along a diameter—is most convenient. It is placed so that this diametral section rests on the paper.

On one side of the lens draw a line on the paper to mark an incident ray. Stick two pins, *A, B,* into this line. Looking at the other side of the lens set up first a pin, *C,* then a pin, *D,* so that *C, D* and the images (*a, b*) of *A, B* are in line. Join the points *C, D*; the line marks the emergent ray. Outline the lens section on the paper. Join the points of incidence on, and emergence from the respective lens surfaces; the line indicates the path of the ray in the lens. Note that when *C, D, a, b* are in line when viewed from one side, then *A, B, c, d* are in line when viewed from the other side.

Exp. 154.—*Track rays through a lens as follows:*—

1. *Parallel beam.* Draw, say, six equidistant parallel lines on the paper about ¼″ apart. Arrange the plane of the lens to be at right angles to them. Let each line represent an incident ray, track it through the lens as in § 128. The emergent rays should intersect at one point, viz. the principal focus.

2. *Diverging beam.* Draw, say, 6 lines radiating from a point, P, at about 5° inclination to one another. Place the lens at a distance from P equal, say, about half the focal length. Let each line between P and the lens represent a ray diverging from P and incident on the lens, track it as in § 128. The emergent rays should intersect at one point, viz. the conjugate focus of P.

3. *Converging beam.* Draw lines and set the lens as in (2). Let each line on the side of the lens opposite to P represent a ray converging to P and incident on the lens. Track it as in § 128. The emergent rays should intersect at one point, viz. the conjugate focus of P.

Repeat (2), (3) for distances of P two or three inches greater than the focal length. Also for distances greater than twice the focal length.

Use glass "half-lenses" (focal lengths about one foot), 1st concave, 2nd convex.

129. Mirrors and lenses.—Mirrors are generally of *small aperture* (§ 126): lenses are *thin*, and bounded by curved or plane surfaces. Such mirrors and lenses produce images of objects placed at positions *on or close to their principal axes.* In relation to the object the image is (a) enlarged or diminished, (b) inverted-and-real or erect-and-virtual. A *real* image is produced at the position to which rays of light are converged by the mirror or lens (Figs. 34, 129); a *virtual* image is at the position from which rays appear to diverge, the position from which they actually diverge being elsewhere (Figs. 93, 94, 120). The rays themselves pass through a *real* image, but not through a virtual one. *Hence only real images can be focussed on a screen.*

When rays of light are incident on a mirror or lens in a direction parallel to its principal axis they after reflection or refraction, (i) in the case of a concave mirror or converging lens converge to, or (ii) in the case of a convex mirror or diverging lens diverge from, a point on the principal axis. This point is the *principal focus* of the mirror or lens, and its distance from the optical centre measured along the principal axis is the *focal length* of the mirror or lens (see Figs. 103, 122, and Exp. 154, 1).

Magnification.—By the magnification is meant the ratio of the length of any line across the image to the length of the corresponding line in the object: or, shortly,

magnification = (*length of image*)/(*length of object*).

The term magnification is used in a general sense and does not necessarily mean enlargement. An image that is larger than the object is, however, very frequently called a *magnified image*, while one that is smaller is called a *diminished* or *minified image*. Thus when the magnification is > 1 the image is magnified, when the magnification is < 1 the image is minified. Note that magnification is the ratio of lengths, not of areas.

130. Formulae for mirrors and lenses.—The optical quantities (f, m, r, see below) of mirrors of small aperture and thin lenses are associated in several formulae of great importance. To express these in a simple way it is convenient to consider that each quantity has not only a numerical value but may be $+$ or $-$ according to the following

CONVENTION. Distances measured from the mirror or lens in a sense opposed to that of the incident light shall be positive ($+$), in the same sense as the incident light shall be negative ($-$).

Let u and v be the distances of the object and image respectively from the lens or mirror, f its focal length, m the magnification, r the radius of curvature of the mirror, r_1 that of the lens surface upon which the light is incident, r_2 that of the lens surface from which it emerges, and μ the refractive index when light passes into the lens from the medium in which it is immersed.

MIRRORS.	LENSES.
$r = 2f$	$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$
$\frac{2}{r} = \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$	$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$
$m = -\frac{v}{u}$	$m = \frac{v}{u}$

COMBINATION OF LENSES. If thin lenses of focal lengths $f_1, f_2, f_3, \ldots$ are put close together the focal length, f, of the combination is such that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots$$

IT FOLLOWS from the above that

(i) The focal length is a + quantity for a concave mirror, or a diverging lens, or diverging combination.

(ii) The focal length is a − quantity for a convex mirror, or a converging lens, or converging combination.

(iii) The magnification is a + quantity when images are erect-and-virtual.

(iv) The magnification is a − quantity when images are inverted-and-real.

(v) The distance of the image is a + quantity when an inverted-and-real image is produced by a mirror.

(vi) The distance of the image is a − quantity when an inverted-and-real image is produced by a lens.

In the above formulae (which should be remembered) all the quantities are subject to the convention and hence involve both numerical value and sign. In some cases, however, it is useful to consider relations between numerical values only without regard to sign. *These relations should be deduced from the fundamental equations when required.* To do this it is satisfactory to use as symbols for the numerical values the capital letters corresponding to the small italics and to prefix the − sign when necessary.

EXAMPLES. (i) The formula $1/f = 1/v - 1/u$ becomes, when real images are produced by a lens,

$$\frac{1}{-F} = \frac{1}{-V} - \frac{1}{U} \quad \therefore \frac{1}{F} = \frac{1}{U} + \frac{1}{V},$$

(ii) that for the focal length $(-F)$ of a converging combination of a convex lens $(-F_1)$ and a concave (F_2) becomes

$$\frac{1}{-F} = \frac{1}{-F_1} + \frac{1}{F_2} \quad \therefore F = \frac{F_1 \cdot F_2}{F_2 - F_1}.$$

In calculating use should be made of a table of reciprocals.

131. Apparatus for optical measurements with mirrors and lenses.—(1) *Lens holders*. These should hold the lens or mirror without shaking. It is an advantage when the piece can be fixed at different heights.

In Fig. 104 the lens is placed in a hole of slightly greater diameter in a wooden ring or board fixed at right angles to a base. A springy piece of brass wire bent nearly into a complete circle is pushed into the hole against the lens, and keeps it from moving. The wooden ring is fixed at the end of a rod that fits into a stand. This arrangement is satisfactory when lenses are of the same diameter. If

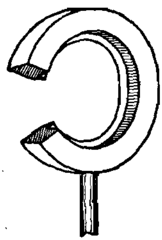

Fig. 104.

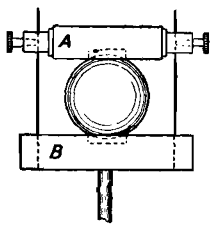

Fig. 105.

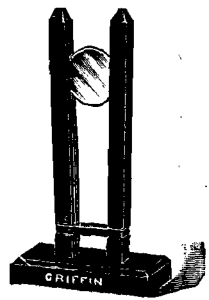

Fig. 106.

the lenses need not be moved from their holders, a very simple plan is to make a hole (it need not be turned) through a piece of wood, and keep the lens against it by three screws (Fig. 114). (Put a small washer or a half-inch disc of cardboard between the screw head and the lens.) This may be mounted on a rod or fixed to a base. Fig. 105 shows a holder arranged to take lenses of different sizes. The horizontal pieces *A*, *B* are of wood. The vertical rods are steel (knitting needles), passing through binding screws. Half-inch holes are bored into the wood; the edges of these grip the lens. Fig. 106 illustrates a form in which the lens is held in the angle between two pieces of wood hinged to a base, and pulled together by an elastic band placed an inch or two above the base.

A small lens may be fixed by wax to a loop of stout brass wire, or into a cork (Fig. 107; see § 140).

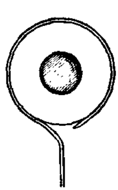

Fig. 107.

(2) *Objects*. For experiments on a large scale a candle (liable to spill wax), gas flame or electric lamp is good. For measuring purposes, however, the object should be flat

with well-defined lines, *e.g.* wire gauze (Fig. 114) or cross wires (Fig. 108). Illuminate them by putting a light behind.

For the cross-wires (Fig. 108) make a 1″ hole in a piece of tin-plate or stout cardboard, say 6″ square. Screw the plate to a strip of wood (W) at the back. Stretch 4 thin iron wires across the hole, as shown (back view). A small cross-wires is also useful (*Fig. 115*): make the hole $\frac{3}{8}$″ diameter, paint the front white.

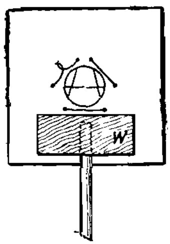

Fig. 108.

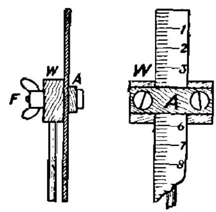

Fig. 109.

A short mm. scale is a useful object. The graduations may be on translucent paper or glass. Fig. 109 shows how it may be clamped. The wood strip, W, is fixed to a rod; the scale is between W and a wood strip A; two screw-bolts, with wing-nuts, F, pass through the strips. By screwing up the wing-nut, F, the scale may be clamped.

For parallax experiments, a rod (knitting needle) with a white paper flag a little below the point (Fig. 110) is convenient. If a sharper point is needed, push a fine sewing needle through a cork and mount on the knitting needle (Fig. 111).

In *parallax experiments* it is a good plan to use a scale, as above, for object, and a needle to mark the position of the image of the scale.

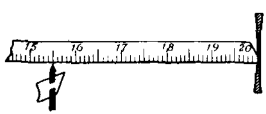

Fig. 110.

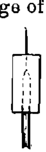

Fig. 111.

(3) *Images*. These are focussed on a screen of white cardboard attached to a rod, or a wooden stand (Fig. 114).

For several experiments it is convenient to have a piece of ground glass, mounted vertically in a large ring, so that both sides are exposed.

(4) *Accessories.* Black screens that can be placed in the neighbourhood of the images in order to cut off extraneous illumination.

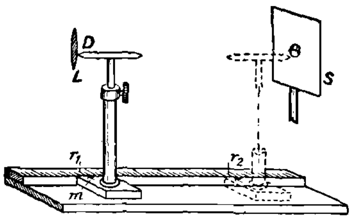

Fig. 112.

A *plane mirror* (of very good quality) mounted vertically (in a large ring).

The various pieces are fitted with rods that can be fixed in stands (Fig. 112). The stands have a rectangular (iron) base, and carry a vertical brass tube. The rods of the pieces fit easily into the tube and are clamped by the thumb screw at the top. The base has marks on it to aid in measuring.

Optical bench, or optical bank. The stands are frequently arranged to slide along a horizontal board or frame graduated into centimetres or inches. Thus the distances between them are directly indicated. Fig. 112 shows an arrangement (short length). The use of these cumbersome benches can very frequently be avoided.

Tape measures, and *boxwood scales* (100 cm. and 50 cm. long) are required for measuring the distances between the lens, object, image, etc. Fig. 114 shows a tape fixed to the bench by drawing pins. Fig. 110 shows an application of a boxwood scale, useful especially in a parallax adjustment. It can be held by hand in such a position (rest the elbows on the bench).

A *distance piece* (Fig. 112) is useful for more accurate measurements. It consists of a rod, D, with pointed ends, and of definite length (say 20 cm.). It is carried by a stand having a mark, m, on its base.

Fig. 113.

I. *To measure the distance between two pieces, L and S* (Fig. 113). (i) Bring one end of D into contact with L, note r_2, the scale reading of m; (ii) bring the other end of D into contact with S, note r_1, the scale reading of m. If a is the distance $(=r_2 \backsim r_1)$ through which the stand has moved, and b the length of the distance rod, then the length between L and $S=a+b$. It is sometimes more convenient to work as in II.

II. *To measure the distance through which a piece L has been displaced* (Fig. 113). Let L_1 be the first and L_2 the second position of the piece. (i) Bring one end of the distance rod into contact with the piece in its initial position, note the reading (r_1) of m. (ii) Bring the *same* end of the distance rod into contact with the piece in its second position, note the reading (r_2) of m. Then $r_2 \sim r_1$ is the distance, p, through which L has been displaced. The length of the distance rod (b) need not be known.

If L itself is carried by one of the stands the distance rod is unnecessary. In this case observe the two readings of a mark on the base.

132. The simpler optical experiments do not require a very dark room. If the laboratory is provided with green blinds it can be darkened sufficiently. Experiments by the parallax method can generally be done in ordinary light. Arrange the principal axis of the lens or mirror parallel to the window and place a dark background in view of the eye.

RULES.

(1) In experiments with lenses, mirrors, etc., the planes of the lens, mirror, screens, etc., must throughout the observations be kept *at right angles to the base line* or optical bench as nearly as can be judged; also the principal axis of the lens must pass through the object, and be parallel to the base line, optical bench, etc.

(2) In making measurements be careful not to disturb the pieces. Observe whether the plane of the lens, etc., passes through the index-mark on the base of the stand. If not, decide whether the measurement to be made, or the way of making it, needs that the distance, if any, between these should be known. If so, use the distance piece.

CONVERGING LENSES.

133. To find roughly the focal length of a converging lens.—Get as far as possible from a well-illuminated window or bright object (lamp). Hold a piece of paper (envelope) in one hand—the palm of the hand itself is often good enough—the lens in the other. Place the lens close to the paper and slowly draw it away until a well-defined image of the window is seen on the paper. Guess the distance of the lens from the paper: (it is a little greater than the focal length).† *This preliminary experiment should be done invariably when an unknown lens is provided for measurement.* Remember that a real image cannot be obtained on a screen unless the distance between the object and screen is at least four times the focal length of the lens (Exp. 156).

134. To obtain a real image of an object by a converging lens (Fig. 114).—First obtain the focal length roughly as in § 133. Draw a *base line* (in chalk), or pin (by drawing pins) a tape measure on the table, or use an optical bench. Arrange object, lens, and image-screen at equal

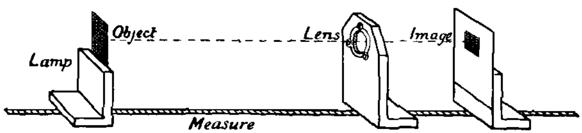

Fig. 114.

heights; also the principal axis of the lens parallel to the base line or bench, and passing through the object. Place the object and the image-screen at a distance apart greater than four times the focal length of the lens: put the lens between, close to the object. Slowly move the lens from the object until an image is seen‡: then leave the lens and adjust the position of the image-screen until a well-defined image is obtained. Note that the image is inverted.

To determine the magnification produced by a converging lens.—Measure by compasses (§ 13), as accurately as possible (to 1/50 inch), the distance between two points of the object and the same two points of the image. The magnification is the ratio of the latter to the former. The width of the hole or distance between the wires of the object, or the distance between, say, ten wires of the gauze may be measured.

† It is of course more accurate to measure by a scale. A precise determination is, however, not the aim of the experiment.

‡ Remember rule 1 (§ 132).

THEORY.—*To show that the magnification* $= V/U$. In Fig. 115, F is the principal focus, C the optical centre of the lens, L. AL is parallel the principal axis, UCV. Let the object move from a distance up to the lens. When at AU it produces the image BV (obtained by drawing ACB to cut LF at B). FB is the locus of the image of that part of the object that moves along AL. Then as A gets nearer L, B gets further from F, *i.e.* the image gets larger. From the figure, since AUC and BVC are similar triangles, $BV/AU = VC/UC$. Hence *magnification* = (*length of image*) ÷ (*length of object*) = (*Distance of image*) ÷ (*Distance of object*).

For the relation between focal length and magnification see § 139.

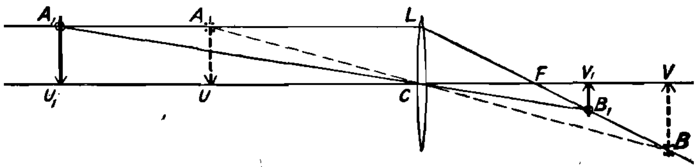

Fig. 115.

135. To find the focal length of a converging lens by the method of conjugate foci (also see Exp. 165).—Arrange lens, etc., as in § 134, to give a real image. Determine the distance between the lens and (i) the object, (ii) the image. Calculate the focal length from $1/f = 1/v - 1/u$ (§130).

Exp. 155.—*Find the focal lengths of glass convex lenses by conjugate foci* (§ 135). Make three determinations for each lens. Record *kind of lens, U, V, F.*

Exp. 156.—*Investigate the relation between the distance of an image from a converging lens and that of the object. Find the minimum distance between image and object.* Use a glass convex lens of, say, 15 ins. focal length. Place the image screen four yards, say, from the object, and obtain a clear enlarged image. Measure the distance between (i) image and object ($U + V$), and (ii) lens and object (U).

Increase the distance (U) of the lens from the object by about 1 *inch*. Focus the image and measure the distances as before. Again increase U, focus and measure: repeat ten or a dozen times. In the later steps the lens may be advanced a foot, then 2 *ft.* instead of 1 *inch*. In each case note whether the image is enlarged or diminished.

(a) Tabulate the values of U and corresponding $(U+V)$, V, $1/U$, $1/V$, $(1/U+1/V)$, also mention whether the image is magnified or diminished. The values of $(1/U+1/V)$ should be practically constant. Calculate the mean of them, then its reciprocal, the latter is the value of F. (b) Plot U horizontally, $(U+V)$ vertically.

Note that as U increases, V diminishes, and practically equals the focal length when U is very large. Also $(U+V)$ has a minimum value, it is *never less than four times the focal length.* When $(U+V)$ is a minimum then $U=V$, and *size of image = size of object.*

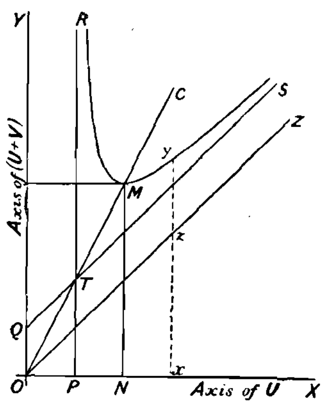

Fig. 116.

THE CURVE obtained by plotting U and $U+V$ is a hyperbola situated as in Fig. 116. There is a minimum value of

$$(U+V) = MN = 4F.$$

To find the minimum distance geometrically. Take any point, C, whose ordinate equals twice its abscissa. Draw a line, CO, through the point and the origin, O. From the point M where CO cuts the curve draw MN parallel to OY. MN is the minimum distance.

To find the focal length. Bisect ON in P. Then OP is the focal length.

To find a value of V. Draw OZ bisecting the right angle XOY. Then V is the length (yz) of the portion of the ordinate (xy) of an abscissa, Ox, intercepted between the curve and OZ.

Make $OQ=OP$. Through P draw PR parallel to OY, through Q draw QS parallel to OZ. These lines are asymptotes to the hyperbola. The curve gets closer and closer to these, that is, the smallest values of U and V are ultimately (but not simultaneously) $=F$.

Exp. 157.—*Measure the magnification produced at different distances by a glass convex lens.* Arrange lens, object, image-screen, etc., and proceed as in Exp. 156. For measurements, etc., see § 134. Tabulate (1) U, (2) V, (3) size of object, (4) size of image, (5) magnification, (6) V/U. The corresponding values in (5) and (6) should be equal.

136. To find the focal length of a converging lens by the minimum distance method (a simple and accurate method).—(i) Arrange object, lens, and screen to obtain a *diminished* and real image. (ii) Move lens a little further from the screen, then adjust the screen to focus the image (be careful to move only one thing at a time). (iii) Repeat (ii) until in focussing the image the movement of the screen is *away from the object* instead of towards it. (iv) Finally, find where the screen is nearest the object. Measure the distance between the screen and object, and divide by four.

Note.—The position of the lens may not be midway between the object and image. Theoretically it should be exactly so. The curve obtained in Exp. 156 indicates however that considerable variation in U produces little alteration in the minimum distance. Hence in practice calculate the focal length, not by dividing either U or V by 2, but by *dividing the minimum distance* $(U + V)$ *by* 4.

Exp. 158.—*Find the focal lengths of convex glass lenses by the minimum distance method.*

137. To find the focal length of a converging lens by the "displacement method."—(1) Place the object and image screen at a distance apart, say, about five times the focal length of the lens. (2) Put the lens between and adjust it to give a real image. Note the position of the lens stand on the tape, or by sticking a piece of stamp paper on the bench and pencilling on it the position of the index mark on the stand. (3) Do not alter the positions of either the object or image screen, but move the lens until a second image is clearly focussed. Mark as before the position of the lens stand. (If the former image was enlarged, the latter is diminished, or conversely.) (4) Measure the distance (a) between the two positions of the lens, and the distance (l) between the object and image screen. (5) Calculate

$$\textit{focal length of lens} = (l^2 - a^2)/4l.$$

Magnification. In § 137, (3), (4) measure the lengths of a suitable part of the image, also of the object.

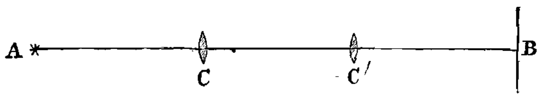

Fig. 117.

PROOF.—Let A and B (Fig. 117) represent respectively the positions of the object and screen. Let a magnified image be got at B when the lens is at C, and a minified one when the lens is at C', then A and B are conjugate foci for either position of the lens. $\therefore BC' = AC$.

$$\therefore AC + CB = l, \text{ and } CC' = a\text{; also } AC' - AC = a,$$

$$\therefore AC = \tfrac{1}{2}(l - a), \text{ and } BC = \tfrac{1}{2}(l + a).$$

Since $$\frac{1}{F} = \frac{1}{AC} + \frac{1}{BC} \qquad \therefore F = \frac{l^2 - a^2}{4l}.$$

Minimum distance.—The minimum value of a^2 is 0. Then $l = 4F$. Thus the minimum distance method (§ 136) is a special case (when $a = 0$) of the above.

Magnification relation.—Let O be the length of object, I of the magnified image, i of the minified image. Then $I/O = (CB)/(AC)$ and $i/O = (C'B)/(AC')$. Multiply together the left sides, also the right, then $I.i = O^2$.

Exp. 159.—*Find the focal lengths of glass convex lenses by the displacement method* (§ 137). *Prove the magnification relation.* Make two determinations for each lens. Record (i) kind of lens, (ii) l, (iii) a, (iv) F, (v) size of object, O, (vi) magnified image, I, (vii) minified image, i, (viii) Ii, (ix) O^2. All values of the two last should be equal.

138. To find the focal length of a converging lens by a direct method.—I. *Using a telescope.* Adjust the telescope to focus parallel rays (§ 147): this must not be altered during the experiment. Direct the telescope towards a printed page or other object with small details: clamp it in this position. Place the lens close to and concentric with the object glass of the telescope.† Put the object close to the lens. Move the object away from the lens by short steps of an inch, say; look through the telescope after each step, continue until a position is obtained at which a good image of the object is seen in the telescope. Then measure the distance between the lens and object; this is the focal length.

II. *a. Using a plane mirror.* Place in order a plane mirror, lens,† and object (small cross wires). Arrange that the principal axis of the lens is practically horizontal, passes through the centres of the object and mirror and is at right angles to them. Put the object near the lens, slowly move it away until a well-defined inverted image is obtained side by side with and equal in size to the object (see Fig. 118). Measure the distance between the lens and object; this is the focal length. Repeat the determination.

II. *b. Parallax method.* Place in order a needle (Fig. 110), lens, and plane mirror.‡ Arrange that the needle is, as nearly as can be judged, on, and the mirror at right angles to the principal axis of the lens. Place the needle at a distance from the lens roughly equal its focal length. Look along the principal axis towards the needle, from a position three or four feet off, for an inverted image of the needle: when found adjust the needle, by eliminating parallax, until its point is in exact coincidence with the point of the image. The point is then at the principal focus of the lens. Measure its distance from the lens (Fig. 110): this is the focal length. Repeat the determination.

Frequently in II. *a.*, II. *b.*, images are got which turn out to be wrong, being produced by reflection from the lens surfaces (§ 143 *a*). Remember that the right image moves if the mirror is shaken.

The *definition* depends very considerably on the flatness of the reflecting surfaces: a good piece of mirror or plate-glass is necessary.

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† It is a good plan to fix the lens to the telescope, in front of and concentric with the object glass.

‡ It is convenient to clamp the lens to the face of the mirror, or put them in the same holder.

Exp. 160.—*Find the focal lengths of glass convex lenses* by methods of § 138.

Exp. 161.—*Test the flatness of a reflecting surface.* Use an object with well-defined lines. Work in a dark room. Using a *good* mirror and glass convex lens adjust as in § 138, II. *a.* Substitute in turn the several surfaces to be examined for the mirror. The bad surfaces give blurred images.

NOTE.—For another test of flatness, see Exp. 45.

139. To obtain the focal length of a converging lens by measuring the magnification produced.—Let a lens, focal length f, at a distance u_1 from the object, produce an image, magnification m_1, at a distance v_1. Also let the same lens and object at a distance u_2 produce an image of magnification m_2 at a distance v_2. The lens may be thick or thin.

$$\frac{1}{f}=\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{v_2}-\frac{1}{u_2} \qquad m_1=\frac{v_1}{u_1} \qquad m_2=\frac{v_2}{u_2},$$

$$\therefore \frac{v_1}{f}=1-\frac{v_1}{u_1}=1-m_1 \quad \therefore v_1=(1-m_1)f.$$

Similarly $\quad v_2=(1-m_2)f,$

$$\therefore v_1-v_2=f(m_2-m_1) \qquad \therefore f=\frac{v_1-v_2}{m_2-m_1}.$$

Exp. 162.—*Determine the focal length of a glass convex lens by measuring magnification.* (1) Open the legs of some compasses so that the tips are exactly, say, half-an-inch apart. Arrange the compasses, lens, and a finely divided ($\frac{1}{100}''$) steel scale on stands on an optical bench, and adjust so that a magnified image of the scale rests on the compass-points (by the parallax method).† Note the number (n_1) of image-divisions bridged by them. (2) Move the compasses towards the lens through a measured distance (d, say 1 inch for a short-focus to 6 or 9 inches for a long-focus lens). Adjust the scale (the lens must not be moved) until an image of it rests on the compass points.† Note the number (n_2) of image-divisions bridged by them. Let n be the number of divisions of the steel scale bridged by the compass-points. In the first position the magnification $m_1 = n/n_1$, in the second $m_2 = n/n_2$. Focal length $= d/(m_2 - m_1)$.

140. The surfaces of lenses reflect sufficient light to form images visible in a dimly lighted room. Lens-surfaces can therefore be used as concave or convex reflectors.

† Examine with a magnifying glass so that the adjustment may be very accurate.

SPHERICAL MIRRORS OR REFLECTING SURFACES.

For the experiments with concave surfaces it is convenient to use small mirrors (like those of galvanometers). *To hold them* fit each into a hole in a thin, broad cork (Fig. 107); make a loop at the end of a piece of stout, springy brass wire and push the cork into it so that the loop and mirror are in the same plane. Bend the brass so that when the stem is held in a holder the axis of the mirror is horizontal.

141. To find roughly the focal length of a concave mirror.—An experiment similar to § 133 may be attempted; the paper is held between the mirror and object, a little to one side, in order that it may not cut off all the light.

To obtain a real image of an object by a concave mirror.—(1) Obtain the focal length roughly. (2) Draw a chalk line—the *base* line—on the table, or use an optical bench, etc. (3) Arrange the illuminated cross-wires and mirror (centre) at equal heights, also the principal axis of the mirror parallel to the base line or bench and to pass through the object. (4) Place the object and mirror at a distance apart somewhat greater than twice the focal length. Push the object screen slowly towards the mirror until an image is obtained upon it side by side with the object. Note that the image is inverted and equal to the object in size. (5) Place a screen between the object and mirror, close to the former, arrange it to catch the image near to its edge. (6) Move the object a little (two or three inches) away from the mirror. Push the image screen slowly towards the mirror and adjust until a well-defined image is obtained.† (7) Repeat (6) until object and image are obtained in convenient positions. Note that the image is minified and inverted.

142. To find the centre of curvature and focal length of a concave surface.—I. Place the small cross-wires (illuminated) (Fig. 118) in front of and close to the centre of the surface. Move the surface away from the object until a clear image of the cross-wires is obtained side by side with the cross-wires themselves; the cross-wires will then be practically at the centre of curvature. Measure the distance of the cross-wires from the surface: its value

† Remember rules, § 132.

is that of the radius of curvature. The focal length is one-half the radius of curvature.

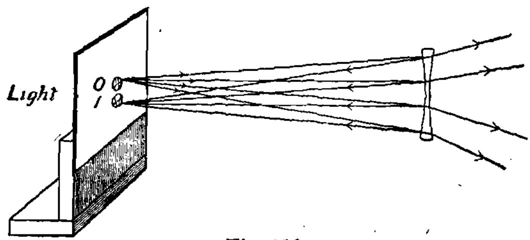

Fig. 118.

II. *By eliminating parallax.* Place a needle (Fig. 110) in front of the reflecting surface so that its point is roughly on the principal axis. Arrange that the distance separating them is about twice the focal length. From a position several feet off look along the principal axis at the needle: an inverted image of it will also be seen. Adjust the position of the object until the point of the image is exactly in coincidence with the point of the needle. The points are now at the centre of curvature of the surface. Measure (Fig. 110) the radius, calculate the focal length.

III. The *focal length and radius of curvature* may be found by the *method of conjugate foci.* Arrange as in § 141 to get a real image, measure U, V, calculate F and R (§ 130). See Exp. 164.

Exp. 163.—*Find the radii of curvature of the concave surfaces of lenses, and of long focus galvanometer mirrors by the above methods.*

Exp. 164.—*Demonstrate conjugate foci with a concave reflecting surface: Parallax method.* (1) Clamp in stands two needles, P, Q, so that the points are at the same height and on the principal axis of the lens. (2) Using one needle, P, find the centre of curvature by § 142, Method II. (3) Place the needle, P, say 2″ further from the mirror. From a position three or four feet off look towards P for its inverted image: bring up the needle Q

and adjust by the method of eliminating parallax, until the point of the image of P is exactly in coincidence with the point of Q. Observe that the image of Q is in coincidence with the object P when the image of P coincides with the object Q.

Measure U, V and calculate $F = UV/(U + V)$.

Repeat at other distances.

Exp. 165.—*Demonstrate conjugate foci with a convex lens: Parallax method.* Find the focal length (F) of lens roughly. Clamp a knitting needle in each of the two stands so that the points are the same height and on the principal axis of lens. Place them apart at a distance $= 5F$† nearly. Put the lens between so that the needle points are roughly on its principal axis and at a distance from one needle $= 3F$† nearly. From a position three or four feet off look along the principal axis towards one of the needles, Q, for the inverted image of the other needle, P. Adjust, by the method of eliminating parallax, the point of Q into coincidence with the point of the image of P. Observe that the image of P is exactly in coincidence with the object Q, when the image of Q is exactly in coincidence with the object P; that is, P and Q are at conjugate foci. Also if the image of Q is magnified, that of P is minified, or conversely.

Measure the distances (U, V) of the lens from each needle point and calculate $F = UV/(U + V)$.

Repeat, *making U two or three inches larger.*

143. To find the radius of curvature of a convex surface. (See Fig. 119.) Let the object at O and a glass convex lens at L produce a

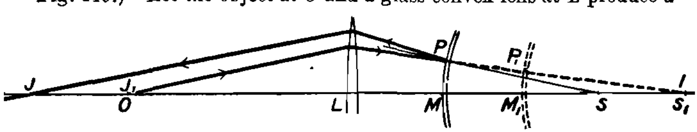

Fig. 119.

real image at I. Let M be a position of the convex mirror, and S of its centre. A ray from O after passing through the lens is incident

† These values are convenient: others of course can be adopted.

on the mirror at P (normal PS), is reflected, returns through the lens and forms an image at J. If the mirror is in such a position, M_1, that its centre, S_1, coincides with I, then the rays emerging from L, since they converge to I, fall normally on the mirror; they are therefore reflected normally and return along the path by which they came. Hence they form an image J_1 at O. Thus when the mirror is adjusted to give an image J_1 side by side with the object, O, the position is discovered for which its centre of curvature becomes coincident with I.

Exp. 166.—*Find the radius of a convex reflecting surface.* (1) Arrange the small cross-wires, convex lens, and image screen, so that a magnified real image is produced (roughly). (2) Place the convex surface (carefully observe the rule, § 132) between the image screen and the lens and close to the former. Slowly move it towards the lens, and adjust to produce an inverted real image side by side with the cross-wires themselves. (See (4) below.) (3) Remove the surface. Adjust the image screen (do not move either object or lens) so that a well-defined image is focussed on it by the lens. (See (4) below.) (4) Measure (by the distance piece) the length between the surface in (2) and the image in (3): this is equal to the radius of the convex surface.

143a. To find the radius of the convex surface of a lens.—Let O (Fig. 120) be an object (distance u) in front of a lens. Consider a pencil of rays incident, say, at M_1, generally it gives rise to four pencils in different directions, viz. a pencil (i) reflected at M_1, (ii) refracted at M_1, going through the lens and incident at M_2, (iii) reflected at M_2, (iv) refracted at M_2 and emerging from the lens. The last gives rise to the virtual image at a distance, v. Also $1/f = 1/v - 1/u$.

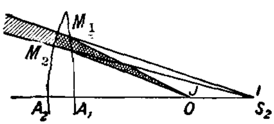

Fig. 120.

If O is at such a distance, D, that the portion (ii) strikes at M_2 normally, then the emerging part (iv) is also normal and the virtual image is at S_2 the centre of curvature of A_2M_2: hence $v = r_2$; the portion (iii) returns normally, is refracted at M_1 along M_1O, and forms an image at O. Hence when an image, J, is obtained side by side with the object, then the virtual image, I, of the object formed by refraction through the lens is at the centre of curvature of the second surface. Hence $1/f = 1/r_2 - 1/D$. Note that if $D = -f$, $r_2 = \infty$, that is the surface is plane. Compare § 138, II. *a* and *b*.

PRACTICE.—Place a needle in front of that surface of the lens whose radius is not being determined. Look for a real inverted image of the needle. Adjust, by eliminating parallax, the point of image and needle into coincidence. Measure the distance (D) of the needle point from the lens. Find the focal length of the lens. (This is conveniently done for a convex lens by § 138, II. *b*.)

144. To find the index of refraction of the material of a lens.—Measure the radii of curvature (r_1, r_2) of both surfaces, and the focal length (f). Calculate (§ 130) from the expression

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

Exp. 167.—*Find the radii of curvature of the surfaces of a glass lens and calculate the index of refraction.*

DIVERGING LENSES.

145. *To find the focal length of a diverging lens.*—None of the preceding methods is immediately applicable because a diverging lens always gives *virtual images.* If, however, the diverging lens is put with a converging lens sufficiently powerful to form a converging combination; and the focal lengths, (i) of the combination (f), (ii) of the converging component (f_1), are found, then that of the diverging lens (f_2) can be calculated from the relation $1/f = 1/f_1 + 1/f_2$. Attention must be paid to the signs. For instance if F and F_1 are the *numerical* values of the focal lengths of the combination and converging lenses respectively,

$$f_1 = -F_1, \quad f = -F, \quad \therefore f_2 = F.F_1/(F - F_1).$$

Exp. 168.—*Find the focal length of a glass concave lens.* Combine with a glass convex lens. Test whether the combination is convergent (§ 127). If so find the focal length by § *136 or* § 138, II. *a.*, (i) of the combination, (ii) of the convex constituent. Then calculate the focal length of the concave lens as in § 145.

145a. Other methods for a diverging lens by using an auxiliary converging lens.†—Let (Fig. 121) a real image of an object at P be formed at P' by the convex lens, L. If a concave lens L' is placed between L and P', then the image is no longer formed at P but (1) at a further point P'', provided that $L'P'$ is *less* than the focal length of the concave lens, or (2) the light emerges from the concave lens parallel to the axis when $L'P'$ is equal to the focal length (Fig. 122).

In case (1) a real image is formed on a screen placed at S'. If $L'P'' = U$, $L'P' = V$, and the focal length is F, then

$$1/F = 1/V - 1/U \qquad \therefore F = UV/(U - V).$$

† The method, § 145, is a special case of this; when $LL' = 0$.

In the second case, $F = L'P'$. To test when the emergent light is parallel (i) use a telescope focussed for viewing a distant object, or (ii) put a plane mirror at right angles to the principal axis.

The axes of the lenses and telescope should be in line, and the planes of the lenses and mirror perpendicular to it.

Exp. 169.—*Find the focal length of a glass concave lens using an auxiliary convex lens* (Fig. 121). (1) Arrange an object and convex lens to give a real image. Mark the position (P') of the screen when the image is focussed on it. (Do not move the object or convex lens again during the experiment.)

(2) Put the concave lens between the convex lens and screen, several inches from the latter. Move the screen until an image is focussed on it: then mark its position (P''). Measure $L'P'$ ($= V$), and $L'P''$ ($= U$). *Calculate the focal length.*

Repeat the experiment several times with different distances.

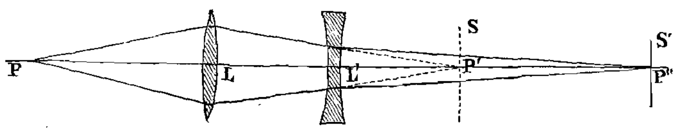

Fig. 121.

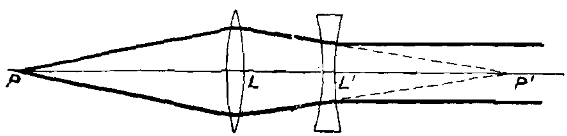

Fig. 122.

Exp. 170.—*Find the focal length of a glass concave lens using an auxiliary convex lens and a telescope.*

(1) Adjust the telescope for parallel rays (§ 147).

(2) Arrange the object and convex lens to give a real image. Mark the position (P') of the screen when the image is focussed. (Do not again move the object and convex lens during the experiment.)

(3) Take away the screen. Arrange the telescope to look towards the object. Introduce the concave lens between the convex and telescope. Move the concave lens until a well defined image of the object is seen in the telescope. Mark the position of the concave lens (L'). Measure distance ($L'P'$) between the marks: this is the focal length of the concave lens. Repeat several times.

Exp. 171.—*Find the focal length of a glass concave lens using an auxiliary convex lens and a plane mirror.*

(1) Arrange an object (small cross wires), convex lens, and screen to give a real image on the screen.

(2) Place the concave lens between the convex lens and screen. Put a plane mirror between the concave lens and screen. Move the concave lens until an image of the cross wires appears side by side with the object. Adjust until the image is well defined. Then mark the position of the concave lens (L').

(3) Do not move the object or convex lens, take away the concave lens and mirror, bring in the image-screen and mark its position (P') when a well defined image due to the convex lens is obtained on it. Measure the distance ($L'P'$) between the marks: this is the focal length of the concave lens. Repeat several times.

CHAPTER XI.

OPTICAL INSTRUMENTS. THE SPECTROSCOPE.

146. Optical Instruments.—The following section describes how to arrange lenses, etc., to illustrate the construction of the commoner optical instruments. A concave lens (2″ or 3″ focus) and two convex lenses (long focus, say 10″ to 20″; short focus, about 3″ or 4″) are required, and stands to hold them; also a ground-glass or cardboard screen, a candle flame or electric light, an object with prominent details (a poster), another with minute details (a microscope- or lantern-slide, a millimetre scale on glass, an inked diagram on ground glass). The room should be dimly lighted.

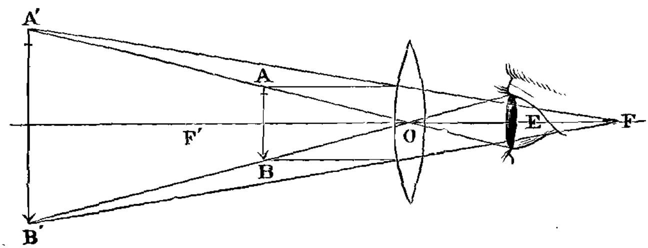

Fig. 123.

The Simple Microscope (Fig. 123).—In this a *converging lens* is arranged to produce a magnified erect image. An ordinary magnifying or reading glass is arranged similarly to the above.

Exp. 172.—Place an object close to a short-focus convex glass lens. A magnified erect image is seen. Slowly move the object away from the lens, observe that the magnification increases; presently the image blurs—this occurs when the distance of the object is greater than the focal length. Any position short of this will satisfy the conditions for a simple microscope. In practice the lens is adjusted to produce a well-defined, magnified, erect image when the eye is placed close to the lens.

The Compound Microscope (Fig. 124).—In this a *converging object-glass* forms a magnified real-inverted image; this image is viewed through a *converging eye-piece* in a similar way to that in which the object is viewed in a simple microscope. A highly magnified, inverted image of the object is obtained.

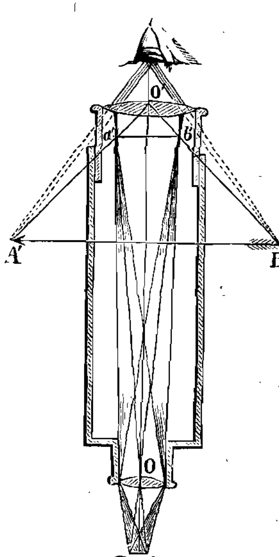

Fig. 124.

Exp. 173.—Place a short-focus convex lens—this represents the object-glass—so that its principal focus comes between the lens and object (candle flame). Obtain the inverted image on a cardboard screen.† Place, as in Exp. 172, a long-focus convex lens—this represents the eye-piece—so that a magnified erect image of a mark on the back of the cardboard screen is obtained.† Then remove the screen, and substitute a microscope slide (well illuminated) for the candle. Adjust.

The Astronomical Telescope (Fig. 125).—In this a *converging object-glass* forms near its principal focus a real-inverted diminished image of a distant object; this image

† If a ground glass screen is used the image of the object can be seen from the back.

is viewed through a *converging eye-piece* in a similar way to that in which the object is viewed in a simple microscope. An inverted image of the distant object is obtained.

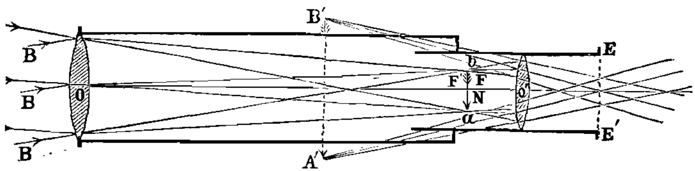

Fig. 125.

Exp. 174.—Place a long-focus convex lens—object-glass—to focus the image of a distant object (light) on a cardboard screen. (Note, p. 223.) Place a short-focus convex lens—eye-piece—so that a magnified erect image of a mark on the back of the screen is obtained. Remove the screen, and substitute a poster for the light.

Opera Glasses (Galileo's Telescope) (Fig. 126).—In this a *converging object-glass* forms near its principal focus a

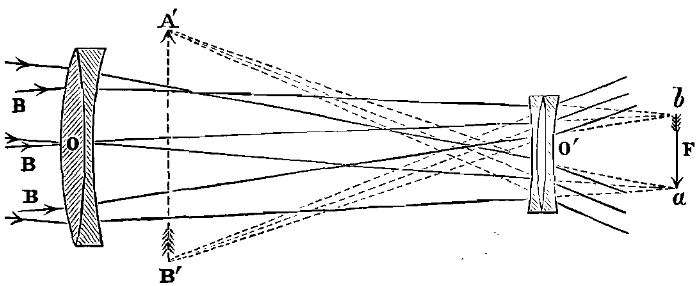

Fig. 126.

real-inverted diminished image of a distant object. The *eye-piece is diverging*, and lies between the object-glass and its focus. An erect image of the distant object is obtained.

Exp. 175.—Place a long-focus convex lens—object-glass—to focus an image of a distant light on a cardboard screen. Place a short-focus concave lens—eye-piece—close to the screen, between it and the object-glass. Remove the screen, substitute a poster for the light, and adjust the concave lens until a well-defined image of a distant poster is obtained.

The Optical (Magic) Lantern (Fig. 127).—In this a converging lens—the *objective*—forms a highly magnified real-inverted image of an object on a screen some distance

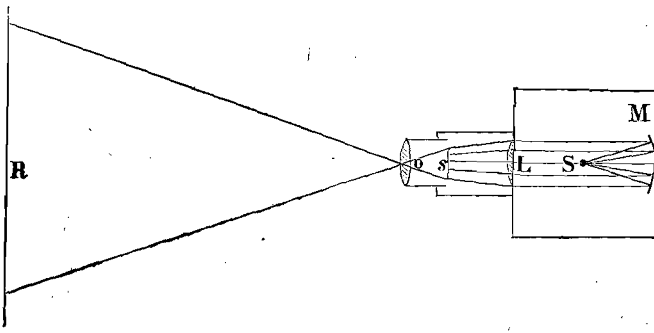

Fig. 127.

off. The object needs to be well illuminated. To effect this a bright light (lime-light or electric) and a *condenser* are used. The condenser is a large, short-focus converging lens: the source of light is situated beyond its principal focus. Hence the light after passing through it forms a convergent beam in which the lantern slide is placed close to the condenser (the condenser is between the light and slide).

Exp. 176.—Place a moderate-focus (say 10″) convex lens—objective—to produce a large image of a candle flame on a screen. Replace the candle flame by a lantern slide. Place a short-focus convex lens—condenser—close to the object. Place a candle flame on the other side of it and adjust until a well illuminated image of the lantern slide is obtained on the screen.

147. Reading or observing telescope.—A small astronomical telescope is very useful for reading distant thermometers, scales, etc. A long tube, A (Fig. 128), carries an achromatic object-glass, O, at one end. A tube, B, slides into A, C into B, and D into C. At F is fixed a ring, S, across which two fine hairs or wires—called the *cross wires* or *spider lines*—are stretched. Two convex lenses are fixed in D to form a Ramsden (+) eye-piece. (This consists of two lenses of equal focal length, at a distance of 2/3 focal length—see *Text-Book of Light.*) The tubes

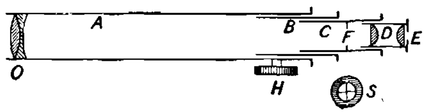

Fig. 128.

B, C, D can be moved by turning the milled head, H—the so-called *focussing screw*. Thus the distance between the objective, O, and the spider lines can be adjusted without affecting the distance between the eye-piece and spider lines.

To focus a telescope.—(1) Adjust the tube, D, by sliding it in or out until the cross wires appear clear and distinct to the eye placed at E. (2) Direct the tube to the object, and turn the milled head, H, until the image of the object and cross wires appear distinct *simultaneously*. Also move the eye about in front of the eye-piece: there must be no movement of the image relatively to the spider lines (the best test).

The adjustment is not easy. Frequently either the cross wires or the image can be seen clearly, but not both at the same time. It is well not to look into the telescope while the final focussing is being done. In focussing the cross wires, remove the tubes C, D from A, B, roughly adjust the eye-piece, then put it to the eye and look through it: if the cross wires are not visible *at once*, remove the tube from the eye, slightly alter the adjustment, and again look through it. Repeat several times. The focussing is right when *the eye sees the cross wires as soon as the tube is put to it*. There is no need to shut the other eye. Replace the tubes in the telescope, and similarly focus the object on the cross wires by turning the focussing screw. *The adjustment is right* *when both object and cross wires are seen as soon as the eye looks into the tube.* The observations are then taken with the eye at its normal accommodation, and work becomes less tiring.

Frequently the focussing that suits one person will not do for another. When this is the case, one observer should focus first the cross wires, then the image of the object as above. A second observer must not touch the focussing screw, H, but should *adjust the eye-piece* until he sees the cross wires and image simultaneously. If this is not possible, the original focussing by the other observer may be at fault, and should be done again.

A microscope is adjusted similarly.

A telescope is said to be *focussed for infinity* or *for parallel rays* when it has been adjusted for viewing a very distant object.

148. Magnifying power of an optical instrument.—The apparent size of an object depends upon the angle it subtends at the eye. An ordinary object at a distance and a minute one close at hand both appear small because the angles subtended by them are small. The function of a telescope or microscope is to change the directions of the rays coming from the object so that their angular separation when they enter the eye is considerably increased. The magnifying power of the instrument is the ratio of the respective angles subtended at the eye by the image seen through the instrument and by the object viewed directly.

For a telescope the magnifying power† is equal to the ratio (i) of the focal lengths of the object-glass and eyepiece respectively, or (ii) of the size of the image to that of the object when the telescope is focussed so that these are at the same distance (considerable) from the eye.

For a microscope the magnifying power† is equal to the ratio of the size of the image to that of the object when the instrument is focussed so that these are at the distance of most distinct vision (average value about 10 inches, but varies for different eyes).

The magnifying power† of a simple microscope (thin convex glass lens) may be calculated roughly from its focal length. Let D be the distance of most distinct vision; then the *magnification* $= 1 + D/F$.

† For proofs see the *Text-Book of Light.*

Exp. 177.—*Find the magnifying power of a telescope.* Set up a scale vertically at a distant end of the room.† Focus the telescope so that the image of the scale can be seen in its field by one eye, and the distant scale by the other. Adjust so that the object appears to lie on its image. Note how many divisions (n) of the image are equal to the length of the scale. The ratio of the total length (N divs.) of the scale to n is the magnification.

Exp. 178.—*Find the magnifying power of a microscope.* Focus the instrument on a finely divided scale, A; place a second scale, B, at the distance of most distinct vision (usually about 10″). Arrange that one eye looks at the image in the microscope, the other at the scale B. Adjust so that the two appear side by side. Find the number of divisions (n) of the image that equals N divisions of the scale (B) seen directly. The magnifying power is N/n. (N and n must be expressed in terms of the same unit.)

149. Micrometer scale.—Microscopes are often provided with a minutely divided scale, called a *micrometer scale*, in the eye-piece, for measuring small dimensions.

To find what size of object corresponds to one of the micrometer scale divisions, place a finely divided scale, A, before the object-glass, focus, and note the number, N, of the divisions of the micrometer scale equal n divisions of the image of A. Then one division of the micrometer scale $= n/N$ divisions of A.

To find the size of an object, focus the microscope upon it, note the number (x) of divisions of the micrometer scale covered by the image. Then the size of object is x times the length corresponding to one scale division as previously determined.

Exp. 179.—*Find the diameter of a capillary tube.* Clamp it so that it may be viewed end on through the microscope. Measure both ends. Calculate the mean.

† A brick wall is convenient. Number the layers (by chalk marks) to distinguish them. Or view a slate roof.

THE SPECTROSCOPE.

150. The Spectrum.—If light from an illuminated slit passes through a prism, it is *deviated* from its original path and *dispersed* into its constituent colours. A number of coloured images of the slit lying side by side, sometimes overlapping, sometimes separate, may be obtained on a screen. The group of images is called the *spectrum* of the light that enters the slit. If the coloured images lie side by side without overlapping the spectrum is said to be *pure*, if they are more or less superposed the spectrum is *impure*. Various lights have characteristic spectra (see § 155). The light from a gas flame (ordinary or with incandescent mantle), candle, or electric lamp gives a ribbon-like spectrum in which seven colours may be distinguished: viz. violet, indigo,† blue, green, yellow, orange, red (least deviated). Such a spectrum is called *continuous* because it is not broken up into lines or bands. Spectra when continuous or made up of bright lines or bands are called *radiation spectra*. If light is passed through media like coloured glasses, liquids, etc., the spectra produced are crossed by dark lines and bands: these are called *absorption spectra* (§ 153).

150*a*. The Solar Spectrum.—Sunlight gives an absorption spectrum, it is crossed by many dark lines. These are called Fraunhofer lines (Fraunhofer first investigated them, 1814). The more prominent ones are designated *A*, *a* (a group), *B* and *C* in the red; *D*, orange; *E*, yellow; ***b*** (a group), green; *F*, on the border of green and blue; *G*, indigo; *H*, violet.

Exp. 180.—*Obtain a spectrum on a screen. Apparatus:* Slit, about 1 cm. long and $\frac{1}{2}$ mm. wide, cut in a plate of metal, supported vertically; convex lens, say 6″ focus; glass prism; cardboard screen. Arrange that the centres of the slit, lens, and prism shall be in one line. A darkened room is important.

† Indigo is frequently not specified, it then being regarded as part of the blue.

*1. Place an electric lamp or a luminous flame (edgewise behind the slit. Arrange the convex lens to produce a sharp image of the slit on the screen when the latter is 2 or 3 feet off. Place the prism (with its refracting edge parallel to the slit) near the lens (Fig. 129), to catch the light that emerges from it: the image then disappears from the screen, the light being deviated from its original path, in a direction away from the refracting edge of the prism. Arrange the cardboard screen so that a well-defined spectrum is focussed upon it. Observe the colours of the spectrum, that red is less deviated than violet, and that the screen is now

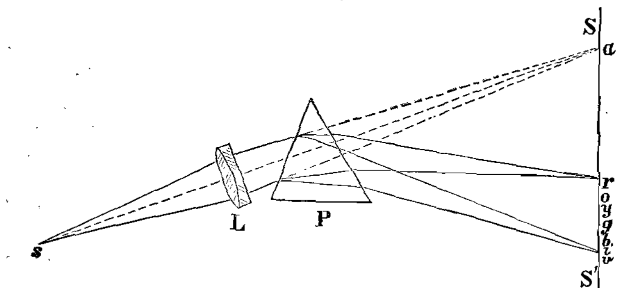

Fig. 129.

as far from the prism as at first. Turn the prism so that the deviation of the spectrum increases, observe also that the spectrum lengthens, and is not so well defined. Then *slowly* turn the prism in the reverse way, the spectrum moves, its deviation diminishes, and its definition improves. Presently the spectrum becomes stationary, looks sharp and bright, then begins to retrace its path, and gets shorter in length. Note that the stationary position is the one in which the deviation of the light from its direction of incidence is *least*. In this position the prism is said to be set roughly for *minimum deviation.*

NOTE.—*The deviation is a minimum when the angle of incidence on one face of the prism equals that of emergence from the other.* For the same angle of incidence each ray, reckoning from red to violet, has a slightly greater angle of emergence. Hence the prism can only be exactly at minimum deviation for one ray at a time. When, however, it is at minimum deviation for one ray it is nearly so for all.

2. *Arrangement illustrating the optical system of a spectroscope.* Adjust the convex lens (method I. or II. *a*, § 138) so that the illuminated slit is at its principal focus. Place a second convex lens (say 6″ focal length) about 3″ from the first, so as to catch the emerging parallel beam, and, further off, a cardboard screen so that the slit is well focussed on it. Place the prism between the two lenses, move the second lens and screen so that the light emerging from the prism is focussed to form a spectrum on the screen. Adjust for minimum deviation.

3. Place coloured glasses between the slit and the prism. Observe the resulting spectra (see § 153). Also illuminate the slit with coloured flames produced by placing metallic salts in a Bunsen flame (see § 153).

4. Repeat experiment (1) without the convex lens; also using a round hole, and broad slit instead of the narrow one.

The experiments show that to obtain a pure spectrum, it is important to use a narrow slit placed parallel to the refracting edge of the prism and to arrange that a parallel beam of light from the slit shall be incident on the prism. It is an advantage to set the prism at minimum deviation.

151. The Spectroscope (Figs. 130, 131).—The parts of this instrument are arranged for producing and examining a pure spectrum. (1) *A slit,* the width of which can be varied by a thumb screw, is placed at one end of a tube, an achromatic converging lens at the other. The distance between the two can be altered, and adjusted so that the slit is at the principal focus of the lens. The tube so fitted is called a *collimator.* The tube is supported on an arm that can turn about a vertical axis at one end, or it is clamped to the large table of the instrument.

(2) The *prism* is usually of glass, often a "heavy" variety, that is having a high index of refraction (these are somewhat yellowish in appearance). The section is triangular. If two faces only are polished it has one refracting angle, if three faces there are three refracting angles. The latter is an advantage when the angles are unequal, say 45°, 60°, 75°. The prism is either clamped to the table of

the instrument (Fig. 130) or to a small platform or table resting on levelling screws (Fig. 131).

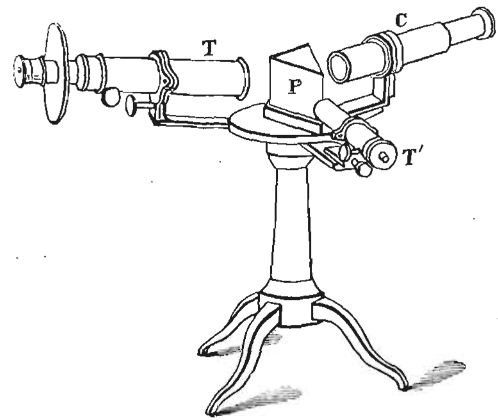

Fig. 130.

(3) The *telescope* is an astronomical one and is provided with cross wires. Its objective forms a real image of the

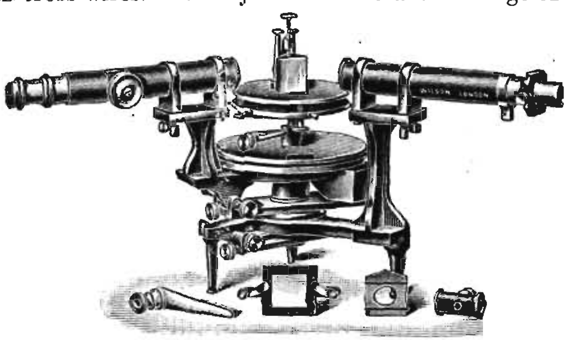

Fig. 131.

spectrum, which the eye-piece magnifies. The telescope is supported on a movable arm that can turn about a vertical axis at one end.

the instrument (Fig. 130) or to a small platform or table resting on levelling screws (Fig. 131).

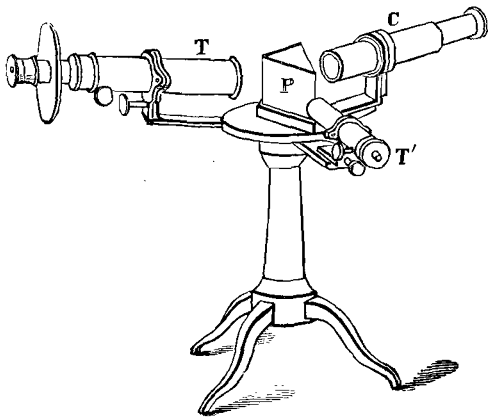

Fig. 130.

(3) The *telescope* is an astronomical one and is provided with cross wires. Its objective forms a real image of the

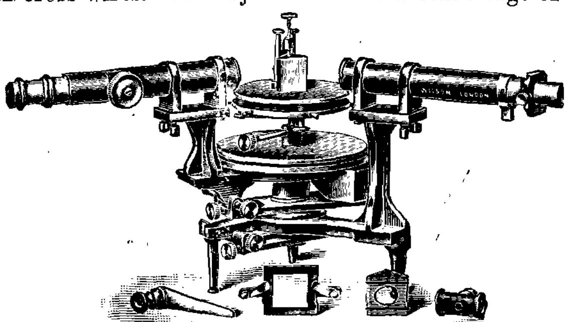

Fig. 131.

spectrum, which the eye-piece magnifies. The telescope is supported on a movable arm that can turn about a vertical axis at one end.

(4) As it is often necessary to compare two spectra *the instrument* is provided either with a comparison prism (§ 151 (5)) or each spectrum is mapped (§ 157).

(5) The slit (Fig. 132) should have sharp straight edges. These may be cleaned by rubbing them with a slip of soft wood; *e.g.* a match end, bevelled. A metallic piece, *Q*, with a *V*-slot can be slid over the slit, *S*, and thus make its working part longer or shorter. A comparison prism, *P*, either equilateral or right-angled, can be placed over half the slit. By its means two spectra may be compared (Fig. 133): light from one source, L_1, enters the collimator through the uncovered portion of the slit, that from the second source, L_2, is caught by the comparison prism and reflected into the collimator. The spectrum of the one appears below that of the other.

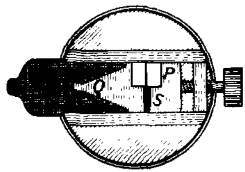

Fig. 132.

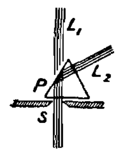

Fig. 133.

The spectrum, for instance, of an unknown substance may be obtained by putting one of its salts (§ 153) in a flame at L_1. Salts of known substances are similarly placed in turn at L_2, and their spectra *compared with the unknown. When the lines of the spectra coincide* it is inferred that the substances at L_1 and L_2 have a common constituent.

A **spectrometer** (Fig. 131) has the same parts as a spectroscope: viz. (Fig. 134) collimator, *C*, prism, *P*, and telescope, *T*. In addition the supporting table is graduated

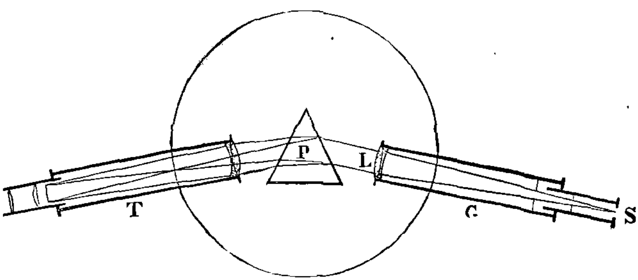

Fig. 134.

into degrees, etc. (§ 22 (2)), and the movable arms of telescope and collimator are provided with verniers. Also

(4) As it is often necessary *to compare two spectra the instrument* is provided either with a comparison prism (§ 151 (5)) or each spectrum is mapped (§ 157).

(5) The slit (Fig. 132) should have sharp straight edges. These may be cleaned by rubbing them with a slip of soft wood; *e.g.* a match end, bevelled. A metallic piece, Q, with a V-slot can be slid over the slit, S, and thus make its working part longer or shorter. A comparison prism, P, either equilateral or right-angled, can be placed over half the slit. By its means two spectra may *be compared (Fig. 133):* light from one source, L_1, enters the collimator through the uncovered portion of the slit, that from the second source, L_2, is caught by the comparison prism and reflected into the collimator. The spectrum of the one appears below that of the other.

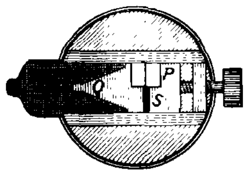

Fig. 132.

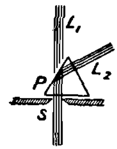

Fig. 133.

The spectrum, for instance, of an unknown substance may be obtained by putting one of its salts (§ 153) in a flame at L_1. Salts of known substances are similarly placed in turn at L_2, and their spectra *compared with the unknown. When the lines of the spectra coincide* it is inferred that the substances at L_1 and L_2 have a common constituent.

A **spectrometer** (Fig. 131) has the same parts as a spectroscope: viz. (Fig. 134) collimator, C, prism, P, and telescope, T. In addition the supporting table is graduated

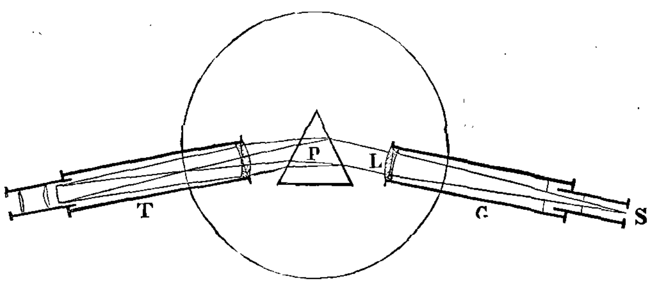

Fig. 134.

into degrees, etc. (§ 22 (2)), and the movable arms of telescope and collimator are provided with verniers. Also the prism platform can rotate about an axis through the centre of the graduated circle. It is also provided with a vernier. Thus the position of each can be accurately expressed, or its angular displacement measured. The telescope is provided with *cross wires* and a *fine adjustment.* To effect the latter a screw with a large milled head works through a nut fixed to the telescope arm. The end of the screw is held in a piece that can be clamped to the table of the instrument. The telescope is roughly got into position, and the piece clamped: the milled head is then slowly turned until the intersection of the cross wires is brought into coincidence with the line or part of the spectrum under observation. The reading of the zero of the telescope vernier is then taken: the reading is considered to specify the position of the vertical spider line.

152. Adjustment of a Spectroscope or Spectrometer.— *Do as much as possible with the unaided eye. Use the telescope, etc., for the final adjustments.* (1) Focus the telescope for parallel rays. (2) Arrange the telescope so that the axes of the collimator and telescope are in line. Remove the prism† from its platform. Illuminate the slit‡ (opened moderately wide) by a gas-flame, lamp flame, electric light, or reflect daylight on it by a mirror or white card. Adjust the collimator (do not touch the telescope) until a well-defined image of the slit is seen in the telescope. Replace the prism† on its platform.

In some instruments the prism is clamped directly to the platform. Its faces should have been ground perpendicular to its base: therefore when put on the platform the faces should be perpendicular to the axes of the telescope and collimator without further adjustment. In other instruments the platform has to be "levelled" (as below), that is the faces of the prism are adjusted to be at right angles to the axes of the tubes.

† Hold the prism at the edges, *its faces must not be touched.*

‡ The light (edgewise, if possible) should be placed six or more inches from the slit. Glare due to reflection from the inside of the collimator tube is then avoided. If there is a third tube focus it (see 5, § 152) before adjusting the collimator.

(3) *To level the prism.* (i) Place the prism with one face (M) perpendicular to the line joining two (P, Q) of the three screws (P, Q, R) for levelling the platform. (ii) Turn the *platform* so that the prism is in the position suitable for measuring its refracting angle (Fig. 93). (iii) Find the image of the slit formed by the light reflected from the face, M, of the prism. Bring up the telescope, and adjust the levelling screws, P, Q, until the image is seen at the centre of the field of the telescope. (iv) Similarly, find and adjust, by the *third* levelling screw, R, the image of the slit due to reflection from the second face of the prism.

(4) *To set the prism roughly at minimum deviation.* Turn the telescope out of the way. Arrange the prism† so that the light from the collimator falls on one face at an angle of incidence of about 30°. Look for the spectrum with the unaided eye. When found slowly turn the prism platform, and observe the movement of the spectrum. The deviation diminishes (if its deviation increases rotate the prism the reverse way), presently the movement of the spectrum ceases for a moment, then recommences in the opposite direction. Adjust the prism so that the spectrum is at its turning point. The prism is now roughly set at minimum deviation. Bring round the telescope so that the light emerging from the prism may enter its object-glass. Cover prism, end of telescope, etc., with a black velvet cloth (be careful not to disturb the prism). Look into the telescope for the spectrum. If it is not seen move the telescope *slowly* to the right or left until the spectrum is found. Narrow the slit until the spectrum is well defined. A slight refocussing of the telescope is sometimes necessary. In accurate work the prism must be placed so that a specified spectrum line has minimum deviation (§ 159). Use a monochromatic light (§ 156) or a hydrogen tube (§ 154, II.).

(5) If the instrument is provided with a third tube (Fig. 130) its scale should be focussed in a similar manner to the slit of the collimator. It is convenient to put the scale-tube in the place of the collimator, do the focussing, then return it to its own holder.

When the adjustments have been completed, look into the telescope and slowly turn the arm of the scale-tube until the image of the scale is seen in the field of the telescope. A slight refocussing of the scale is then sometimes necessary.

153. To obtain flame spectra.—The spectra of some metals are conveniently obtained by placing their metallic salts in a non-luminous Bunsen flame. To effect this, a narrow elongated loop is formed at the end of a platinum wire, the loop is moistened with dilute hydrochloric acid, dipped into the salt (powdered in a watch glass, or bottle kept for the purpose), and then heated in the flame. The salt fuses and fills the loop, then volatilises, and gives a characteristic colour to the flame. It is sometimes sufficient to use clean iron wire instead of platinum: or the salt may be put at the end of a narrow boat-like strip of fine wire gauze or platinum foil and held in the flame.

To obtain a continuous spectrum illuminate the slit with a luminous gas-flame, a lighted candle, oil, or electric lamp.

To obtain absorption spectra of coloured glasses, gelatine strips, etc., hold the coloured media in turn between a luminous flame and the slit. If the slit is only partly covered the continuous spectrum of the flame will show above or below the absorption spectrum due to the medium; thus the two may be compared.

Liquids should be placed in a test-tube held against the slit. A cell with parallel glass sides is preferable. A liquid should be observed when in several known degrees of dilution.

To obtain the spectrum of sunlight, arrange a lens to focus an image of the sun on the slit: a cylindrical lens—a test-tube full of water—is suitable. Or reflect sunlight from a mirror on to the slit.

154. To obtain spark or vacuum tube spectra.—I. *Spark spectrum of* (say) *iron.* The ends of two pieces of iron wire are adjusted $\frac{1}{4}$ inch apart, and set opposite and close to the slit. The other ends are joined by thin wires (copper), each to one of the secondary terminals of an induction coil (§ 202) or to the prime conductors of a Wimshurst machine (§ 184). The intensity of the spark is increased by having another gap (*E*, Fig. 136) in the secondary circuit.

The spectrum obtained is largely that of the material of the wire between the ends of which the spark passes. Some lines (faint), however, are due to the air, this becomes incandescent where the spark occurs.

II. *Vacuum-tube spectrum.* A Plücker's tube, *CD* (Fig. 135), should be used. This consists of two bulbs joined by a fine-bore tube, *S*. The tube contains a small trace of gas, hydrogen for instance. The electrodes, metal rings at the ends of the tube, are joined each to one of the secondary terminals of an induction coil (§ 202). When an electric discharge passes through the tube the rarefied gas becomes incandescent, the capillary, *S*, shows as a brilliant line of light whose spectrum may be examined by placing the capillary part against the slit.

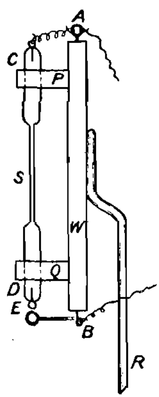

Fig. 135.

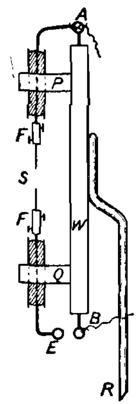

Fig. 136.

Figs. 135, 136 show convenient arrangements. *W* is a strip of wood supported vertically in a stand by the rod *R*. *P* and *Q* are two side pieces, each bored with a hole. The Plücker tube (Fig. 135) is loosely wedged in the holes with paper or a short piece of string. *A* and *B* are stout rings to which the wires from the induction coil are joined. The electrodes are thus not damaged by pulling and twisting due to connecting wires. For the spark spectrum (Fig. 136) two rods are passed each through a glass tube, and fixed in it by sealing or paraffin wax or sulphur so that a portion projects at each end. The glass tubes are held by the pieces *P*, *Q*. There is a spark gap at *E*. A connector, *F*, carrying a short length of a wire, made out of the material whose spectrum is to be examined, is clamped to the end of each rod.

155. Flame spectra.†—*Sodium.* Use common salt. A brilliant orange line—D, sometimes split into two, D_1(5896), D_2(5890)—is obtained.

Potassium. Use nitre. A line (α, 7699) in the extreme red, another (γ, 4047) in the *extreme* violet, will be visible.

The nitre melts into a bead, then suddenly flares. A number of observations must be made before the positions of the lines can be fixed with certainty. A companion should feed the flame while the observer is looking for the lines.

Thallium. Use thallium chloride. A strong green line (α, 5351).

Lithium. Use lithium chloride. Red (α, 6708) and orange (6104) lines are usually seen.

Strontium, use strontium nitrate ; *Barium,* use the chloride ; *Calcium,* use the chloride. The spectra obtained are complex. The strontium blue line (4607), and the calcium blue line (4226) should be noted.

Spark Spectra.—*Iron* and *copper* give brilliant, complex spectra.

Plucker-tube spectra.—*Hydrogen.* Red (C or α, 6563), green (F or β, 4861), blue (f or γ, 4340), and violet (h or δ, 4102) lines are obtained.

Air, Oxygen, Nitrogen, Iodine give complex spectra.

Mercury. Use an electric discharge in vacuo between two electrodes of mercury. A brilliant green (5461) line is obtained, and very faint ones in the yellow and violet.

156. Monochromatic light.—By this is meant light of one wave-length. As the spectrum of the sodium flame consists of two lines very close together, the light from it is very nearly of one wave-length. It affords the most convenient source of monochromatic light.

A bright and persistent sodium flame is conveniently obtained by holding in a Bunsen flame a thick piece of asbestos that has been soaked

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† The numbers in brackets are the wave lengths of the lines in air expressed in tenth-metres, *e.g.* sodium, $D_1 = 5890 \times 10^{-10}$ metres. The Roman letters are the designation by Fraunhofer, etc., of the corresponding lines of the solar spectrum. (See § 150*a*.)

in salt solution. Or, better, wind a piece of stout iron wire, two or three turns close together, round the metal tube of a Bunsen burner. Moisten the ring with water, dip in salt, place in the flame, the salt fuses and runs between the turns of wire. Repeat until the ring is well charged. Spiral the other end of the wire round the metal tube of the burner so that the salted ring is held in the flame.

The most effective monochromatic illumination is obtained by passing, *in vacuo*, a powerful electric discharge between electrodes of mercury. The light produced is almost entirely green and of wave length 5461 tenth-metres.

157. To map a spectrum.—METHOD I. A third tube (Fig. 130) is provided: in this there is a finely divided scale of equal parts at the outer end, (T'), and a convex lens at the inner. Like the telescope, it is supported on a movable arm. The scale is well illuminated (by a mirror or white card), and the tube arranged so that the light from it strikes the face of the prism and is reflected down the tube of the telescope. When focussed (see § 152 (5)), an image of the scale is projected on the spectrum. Hence the position of a line or band can be specified by noting its scale reading.

METHOD II. If the instrument is a spectrometer (§ 151), the cross wires of the telescope are successively brought into coincidence with the spectrum lines, and the respective readings of the zero of the telescope vernier noted.

By either method care must be taken that for the spectra observed the *D*-line (the orange line due to sodium, this can usually be seen) shall always have the same scale reading or be set at minimum deviation.

Finally, the readings of the lines of a spectrum are plotted along a horizontal axis, and short vertical lines drawn at the points. The next spectrum is drawn underneath the first, with the *D*-line vertically below, and so on.

As the scale readings are arbitrary it is better to refer to the various lines by their wave lengths. To do this obtain the readings for the important lines of known spectra, plot them horizontally and their respective wave lengths (see § 155) vertically. From the curve obtained the wave lengths of other lines may be deduced after their scale readings have been determined.

158. To measure the angle of the prism.—Adjust the spectrometer as usual (§ 152). METHOD I. Arrange the prism with its refracting edge towards the end of the collimator (Fig. 92). Illuminate the slit: look for its image (uncoloured) formed by reflection from (i) the right face of prism, (ii) the left. In each case adjust the spider lines of the telescope into coincidence with the image of the slit, and read the position of the telescope vernier on the graduated circle. The difference between the readings is *twice* the angle of the prism (see Exp. 138).

METHOD II. Place the telescope at an angle of about 90° with the collimator (Fig. 137). Turn the prism table or platform until the image of the slit formed by the light reflected from one face is on the spider lines: then read the vernier attached to the prism table. Do not alter the telescope and collimator, but turn the prism table until the image of the slit formed by the light reflected from the second face is on the spider lines: then read the table vernier. The difference between the angle, B, through which the prism has been rotated and 180° is the angle of the prism. B is equal to the difference of the readings of the positions of the zero of the vernier.

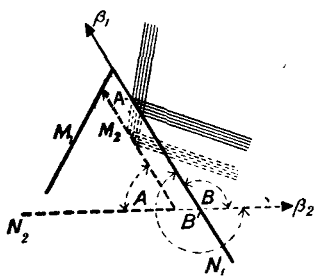


Fig. 137.

PROOF. Let (Fig. 137) the prism in its first position be represented by the thick continuous lines, and in its second position by the dotted lines. The rays reflected in the second case are parallel to those reflected in the first, hence M_2 is parallel to N_1. Then $A + B$ = two right angles. But A equals the angle of the prism, and B the angle through which the prism table has been turned.

$$\therefore A = 180° - B.$$

If the prism is turned the other way, the difference of the readings of its positions

$$= B' = 360° - B = 180° + A.$$

159. To measure the minimum deviation of a line of the spectrum.—For the same prism the angle of minimum deviation is slightly different for each line of the spectrum. The sodium line, and the red and violet hydrogen lines are convenient to adopt (see §§ 153, 154). First obtain the reading (α), called the *direct reading*, of the position of the telescope when the telescope and collimator are in line and the image of the slit on the cross wires. Second, set the prism roughly at minimum deviation (§ 152 (4)), then slightly turn the prism so that the selected line moves across the field of the telescope and begins to turn back just when it reaches the intersection of the cross wires. To do this stop turning the prism when the spectrum is stationary: then bring up the cross wires to the selected line. Rotate the prism slightly in both directions, and readjust the spectrum line to the cross wires. Repeat until the cross wires are exactly at the turning-point of the line. Finally note the reading (α_1) of the position of the telescope. Calculate the minimum deviation, $D = \alpha_1 - \alpha$. Turn the prism over and repeat the determination (the deviation is now to the other side). The mean of the two is the minimum deviation of the selected ray (which should be specified).

160. To find the angle of incidence of the light on one face of the prism.—The direct reading (α) of the telescope must be known. (1) The prism being in position on its platform, turn the telescope so that the image of the slit formed by the light reflected from the face of the prism is obtained on the spider lines. Note the reading (γ) of the telescope position.

The reading of the collimator would be $180 + \alpha$. Then the angle between the incident and reflected rays is $180 + \alpha - \gamma$. Therefore the angle of incidence $= \frac{1}{2}(180° + \alpha - \gamma)$.

To set one face of the prism perpendicular to the axis of the collimator. Obtain an angle of incidence as above. Note the reading of the prism vernier. Turn it through an angle equal to that of incidence.

A more usual method is as follows: (1) Obtain the direct reading (α) of the telescope. (2) Set the telescope at right angles to the direct position of (1). (3) Turn the prism until an image of the slit due to light reflected from its face appears on the cross wires of the telescope. The angle of incidence is now 45°. Note the reading of the position of the prism. (4) Turn the prism through 45° from the position of (3).

161. To find the refractive index of a ray through the prism.—Measure the angle of the prism (A) and the minimum deviation (D) of a selected line of the spectrum, then $\sin \frac{1}{2}(A + D) / \sin \frac{1}{2}A$ is the index of refraction of the selected line for light passing from air into the material of the prism.

A second method of finding a refractive index is as follows: (1) Find the angle (A) of prism. (2) Find the direct reading, a, of the telescope. (3) Set a face of the prism accurately perpendicular to the incident light (§ 160). (4) Adjust the cross wires of the telescope into coincidence with a line of the spectrum. Note the reading, a_2, of the position of the telescope. Calculate the deviation, $d = a_2 - a$. Since the light is incident normally on the first face of the prism, it passes into the substance of the prism without deviation, and is therefore incident on the second face at an angle equal that of the prism (A). The corresponding angle of refraction is equal ($d + A$). Hence the index of refraction from air to glass $= \sin\ (A + d) / \sin\ A$. Evidently the value of A must not be greater than the critical angle of the prism material. The method is suitable for prisms of small angle.

To find the refractive index of a liquid.—The liquid is put into a hollow prism. Determine the angle of the prism and that of minimum deviation of a selected ray. Frequently the hollow prism is a bottle or tube having two faces of thin plate glass inclined to each other.

162. To measure the dispersive power of a medium.—If μ_r and μ_v be the respective indices of refraction of a medium for the extreme red and violet rays of the spectrum, and μ the mean value, then $(\mu_v - \mu_r) / (\mu - 1)$ is the measure of the dispersive power (usually denoted by ω) of the medium.

To find the dispersive power of a substance a portion of it should be formed into a prism, and the refractive indices determined (§ 161). A liquid is placed in a hollow prism.

If the prism has a small refracting angle it is sufficient, after setting the prism at minimum deviation, to measure D_r, D_v, the angular deviations of the extreme red and violet rays.† Then, D being the mean deviation,

$$\textit{dispersive power} = (D_v - D_r) / D.$$

† Often the extreme blue is adopted instead of the violet. The position of the blue is better defined.

Exp. 181.—*Observe by the spectroscope, and map the spectra of sodium, potassium, hydrogen, lithium, and a few characteristic lines of barium, strontium, calcium, iron, copper, air, etc.*

Exp. 182.—*Observe the absorption spectra of uncoloured glass, cobalt-, ruby-, green-, and black-glass.*

Note that blue glasses transmit a little red, and ruby a little blue.

Exp. 183.—*Observe the absorption spectra of solutions of potassium chromate, potassium permanganate, blood.* Use solutions of different strengths (known).

Note that the potassium chromate gives a narrow band, that is the transmitted light is nearly monochromatic.

The other substances give spectra that vary with the degree of dilution of the liquid. Map the spectra, and arrange them so that the effects of dilution may be compared, *e.g.* strongest solution first, below it the next strongest, etc.

Exp. 184.—*Measure the angle of the prism of the spectrometer by two methods, and the minimum deviation of the sodium, hydrogen, and potassium lines. Calculate the index of refraction of the prism for these rays.* If the prism has three polished faces measure its three angles. Show that their sum is 180°. Also find in each case the minimum deviation of, say, the sodium line, and calculate the index of refraction from each determination.

Exp. 185.—*Obtain a curve of wave lengths for use with the spectroscope or spectrometer.*

Exp. 186.—*Adjust a prism face so that the light is incident normally. Find the deviation of the sodium line. Calculate the index of refraction for the sodium line.*

Exp. 187.—*Measure the dispersive power of glass, water, carbon bisulphide.*

PART V.

MAGNETISM AND ELECTRICITY.

CHAPTER XII.

MAGNETISM.

163. Bar-magnets and keepers.—These must be replaced in the box with keepers across the poles as in Fig. 138. Note the order of the poles. A horse-shoe magnet should also be provided with a keeper when not in use.

Caution.—*Magnets should on no account be played with or laid carelessly on one another. If incautiously used they soon become irregularly magnetised.*

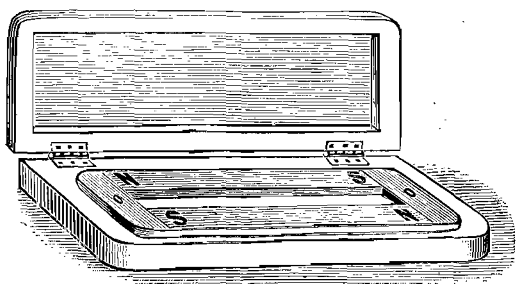


Fig. 138.

The *magnetic needle* or *compass* (Fig. 139) is a small bar magnet that can turn freely in a horizontal plane. It is either suspended by a fibre or has an agate or glass central cap resting on a steel point.

A small compass ($\frac{3}{4}$ inch long) in a case with a glass top and bottom is very useful. It is sometimes provided with a pointer set at right angles to its length.

In the *mariner's compass* the card upon which the cardinal points are marked is attached to the needle, so

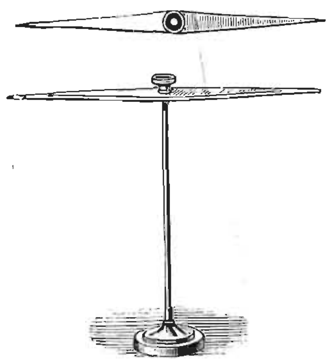


Fig. 139.

that the north-point of the card indicates the magnetic north. Fig. 140 shows a laboratory form. A wire points

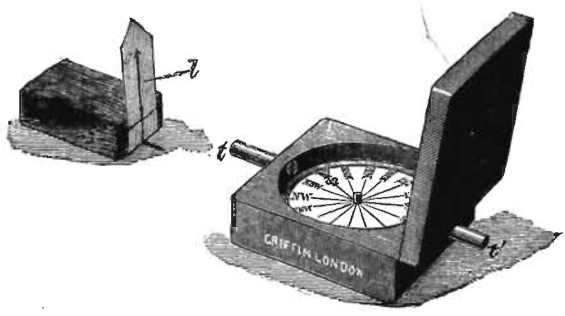


Fig. 140.

up from *N*. Arrange the case so that this can be seen through the sighting tubes, *t*, *t'*, when the lid is closed. Each tube has a wire across it. Arrange all in line with two pins outside (Exp. 200); or the "sighter," *l*.

In the *mariner's compass* the card upon which the cardinal points are marked is attached to the needle, so

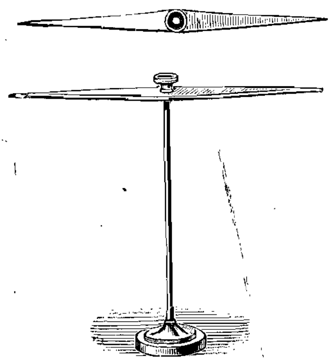


Fig. 139.

that the north-point of the card indicates the magnetic north. Fig. 140 shows a laboratory form. A wire points

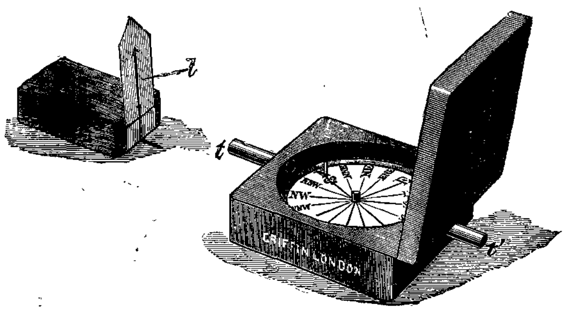


Fig. 140.

up from *N*. Arrange the case so that this can be seen through the sighting tubes, *t*, *t'*, when the lid is closed. Each tube has a wire across it. Arrange all in line with two pins outside (Exp. 200); or the "sighter," *l*.

***Exp. 188.**—*Magnetic and non-magnetic substances.* Bring the ends of a horse-shoe magnet (without keeper) against pieces of various substances—glass, wood, iron, bone, copper, steel, paper, a stair-rod (these are often of iron covered with brass), etc. Classify these into (1) magnetic substances (those attracted), and (2) non-magnetic (those unaffected).

***Exp. 189.**—Lay the magnet on a table, place on it, in turn, a sheet of glass, card, thin wood, tin-plate (lid of a tin), sprinkle iron filings on the sheet, and *gently* tap it with a long pencil loosely held between finger and thumb. The filings arrange themselves in definite chains stretching from end to end of the magnet, except in the case of the tin-plate (iron coated with tin). Hence non-magnetic substances are, in a sense, "transparent" to magnetism, that is permit the magnetic forces to act through them without modification. The curves with the tin-plate will not be well defined, because the iron partially "screens" off the induction due to the magnet. A thick plate of iron will do this more effectively.

***Exp. 190.**—*Poles and equator.* When the magnet is placed in tin-tacks or iron filings these readily adhere at its ends, but not at its middle. Also more filings adhere to the edges, especially the inside ones, than to the surfaces. The ends, where the attraction is strongest, are called *poles*; the middle, where there is no attraction, is called the *equator* of the magnet.

***Exp. 191.**—*Make a magnet* (Fig. 141). Stroke a knitting-needle 30 or 40 times from one end (A) to the other (B) with one pole (say that marked N) of the horse-shoe magnet. Always stroke one way, never reverse: carry the magnet back at a distance from the needle. Slowly rotate the needle about its longitudinal axis during the operations so that the stroking pole may move along different parts of the surface. Test as in Exp. 190.

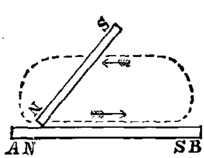


Fig. 141.

Observe, by testing as in § 164, that the polarity of the end at which stroking began is like that of the stroking pole.

The above is the method of *single touch*. For *double* and *divided* touch see a text-book of Electricity.

***Exp. 192.**—*Naming the poles of a magnet.* (i) Hang two magnetised needles horizontally each by a fibre of silk† (unspun) at a distance from one another and away from masses of iron. Note that each comes to rest in a nearly north and south direction. Mark on each magnet (by stamp paper) the end that points northwards. Observe that the needle always comes to rest along the same line with the marked end pointing northwards after being (ii) disturbed: (iii) set so that the marked end points southwards.

***Exp. 193.** *Mutual action of poles.* Show (i) that the marked or north-seeking ends of each needle repel one another, (ii) the unmarked ends repel one another, (iii) the marked end and unmarked end attract one another.

164. Definitions.—The end of a magnet that points north is conveniently called the *north*, *north-seeking*, or *N end* or *pole* of the needle, the other end then being the *south*, *south-seeking*, or *S end* or *pole*.

The vertical plane in which a suspended magnetic needle comes to rest is called the *magnetic meridian*.

In a laboratory the line along which a compass sets is likely to vary in direction owing to the presence of masses of iron (gas pipes, etc.). The line is then a magnetic reference line, not necessarily the meridian. It is however usually called the meridian (Exps. 200, 201).

To test whether a piece of iron is a magnet or not.—Hold the piece horizontally east and west (note Exp. 202) and bring each end of it, in turn, *slowly* towards, say, the N pole of a compass needle. If the piece is not a magnet

† If the experiment works unsatisfactorily, the fibre is likely to be at fault, probably too stiff. Try another. See § 173.

both of its ends will attract the pole of the compass: if it is a magnet one end will attract, the other repel the pole of the compass: if it is a *weak* magnet one end may at first slightly repel the pole of the compass, and then attract it when brought closer. Confirm by using the other pole of the compass.

To distinguish the poles of a magnet.—Observe which end of the magnet repels the N pole of a compass. This end will be the N pole of the magnet. Confirm by testing the other end with the S pole of the compass.

To demagnetise a weak magnet hold it horizontally east and west and strike it irregularly (note Exp. 202).

***Exp. 194.**—*Magnets and magnetic substances.* Show that the same end of a rod of iron (i) attracts *both* ends of the magnetic needle if unmagnetised, (ii) attracts one end and repels the other if magnetised.

Show that a soft-iron keeper cannot be permanently magnetised, that is, after being treated as in Exp. 191, it does not act as in Exp. 194 (ii), but as in (i).

***Exp. 195.**—*Breaking a magnet.* (1) Break one magnetised needle in halves. Show that each half is a magnet with two unlike poles. Also that the original poles remain unaltered in nature (also in strength). (2) Break each piece into halves, and smaller. Show that each piece is a magnet.

Arrange at each stage the pieces to reform the bar; obtain and draw the filing chains (Exp. 207).

***Exp. 196.**—*Making red-hot destroys a magnet.* Use one of the magnetised needles. Test by Exp. 195 after heating red-hot.

***Exp. 197.**—*Make a magnet with consequent poles:* that is, having a pole or poles intermediate to those at the ends. Stroke a needle as in Exp. 191, beginning at each end and finishing at the middle. Obtain and draw the filing chains (Exp. 207).

***Exp. 198.**—*Make a compound magnet.* Separately magnetise seven or eight knitting needles. Test each by filing chains for consequent poles. Neglect any needle that has consequent poles. Bind the magnetised needles together, so that the N poles are all at one end.

***Exp. 199.**—*Magnetic induction.* Suspend the keeper horizontally (use a paper or wire stirrup) about half an inch from the ends of the horseshoe magnet. Bring a pole of the compound magnet near an end of the keeper, and show by attraction or repulsion that the end of keeper near the N end of magnet is a south pole, the other end a north.

Exp. 200.—*Find the magnetic meridian or reference line by a compass needle.* Remove all movable iron or steel. Set the needle oscillating. When at rest adjust two brass pins vertically and in line with, but a little distance from, the ends of the needle. (Tap the pivot base *gently* with a loosely held pencil.) The line through the two pins is the magnetic reference line. (Exp. 203 gives a more accurate determination.)

Exp. 201.—*Trace the magnetic meridian or laboratory reference line across the room.* Proceed as in Exp. 200, but instead of setting up pins, make chalk marks, A, B, on the table just under the ends, P, Q respectively, of the needle. Move the needle so that P is over B, make a mark C under Q. Move the needle so that P is over C, make a mark D under Q. Proceed by similar steps. Draw a line through the points. The laboratory reference line is not necessarily coincident with the "real" magnetic meridian (§ 164). To find the latter do the experiment in a place free from iron; *e.g.* in a field.

***Exp. 202.** *Induction by Earth.*—(I.) Hold a poker or soft iron rod vertically (say handle downwards), bring (1) its upper end near one pole of a suspended magnetised needle, (2) its lower end near: (3) and (4) repeat when the poker is reversed in position (handle upwards). Show *that the upper end always repels the* S-pole and the lower repels the N-pole of the magnet.

(II.) Hold the poker or soft-iron rod horizontally, east and west, and bring its ends in turn up to the poles of a suspended magnet. Observe that each end attracts both poles, that is, the poker is not polarised at its ends.

(III.) Hold the poker vertically, hit it once or twice with a mallet. Test as in (II.). Observe that the end that was lower is now an N-pole, the other end an S-pole.

(IV.) Throw the poker down, or hit it violently. Test as in (II.). Observe that it no longer has polarity (§ 164).

EXPLANATION.—The east and west position (II.) is one in which the Earth's induction produces no polarity. The induction is greatest along a line parallel to the dipping needle (§ 165).

Note that the important position described as east and west is strictly one perpendicular to the magnetic reference line: hence in practice begin by holding the bar horizontally, some distance from and at right angles to the compass needle.

Note that the lower and more northern end is magnetised N. In the southern hemisphere the lower end is magnetised S. In (III.) the temporary magnet is made permanent, but is destroyed by violence in (IV.).

Exp. 203.—*Find the magnetic meridian or reference line by a bar magnet.* Fix (by soft wax) two pins, *A*, *B*, per-

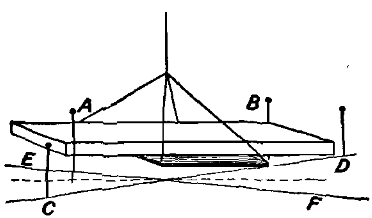


Fig. 142.

pendicular to the edges of a bar magnet (Fig. 142). Suspend the magnet by a long fibre above paper fixed to the table: the points of A, B should just clear the surface.† When the magnet is at rest set a pin, C, in line with A, B: then another, D. Turn the magnet right over so that A, B point upwards instead of downwards. Then, as before, fix pins E, F in line with A, B. Draw the bisector (dotted) of the angle between CD and EF: it lies in the magnetic meridian.‡ Regarding the suspension of the magnet see § 173.

An adjustment similar to the above is done when using the dip-circle or the declination compass.

*** Exp. 204.**—*Find the magnetic axis of a magnetised disc, or combination of magnets.* The latter may be several magnetic needles fixed to a card.

(1) Find roughly the direction of the magnetic meridian at the place where the experiment is to be done.

(2) Mark any diameter AB across the disc.

(3) Suspend so that the plane of the disc or card is horizontal (say, in a large stirrup of copper or brass wire hung by a fibre) and near the surface of the table.

(4) When the disc is at rest stick two pins in the table close to the ends of the marked diameter, say, a near A and b near B.

(5) Reverse the disc, and when at rest make, close to the pins, two marks on what is now the underside of the disc, say a' near A and b' near B.

(6) Remove the disc and join ab, $a'b'$. Bisect the angle between ab and $a'b'$. This bisector marks the position in which the magnetic axis of the combination sets and from this the magnetic axis can easily be marked on the disc.

165. Dip-needle and dip-circle (Fig. 143).—A *dip-needle* is a small bar magnet that can turn freely in a vertical plane about a horizontal axis that passes through its centre of gravity.

A *dip-circle* is an instrument for observing the magnetic dip. It consists essentially of a dip-needle and two

† The apparatus, Fig. 154, is suitable. Use the marks on the shade instead of pins C, D. Put the magnet flatwise.

‡ Note that the angle to be bisected is that over which an end of the magnet lies.

graduated circles, one vertical, the other horizontal. The axle of the needle is supported on knife-edges, at the centre of the vertical circle: hence its inclination may be measured. The plane of the vertical circle turns about a line that passes through the centre of the horizontal circle·

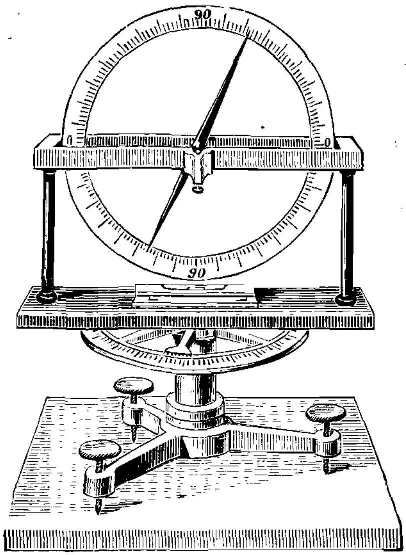


Fig. 143.

hence the latter measures the angular movement of the vertical circle, that is, of the axis of the needle.

The instrument is adjusted by altering the levelling screws until the spirit level or plummet fixed to it indicates that the proper position is attained. If there is no spirit level or plummet arrange a fine plummet close to the face of the vertical circle, and adjust so that the plumb line lies parallel to the plane of the circle and over the two divisions marked 90°.

Exp. 205.—*Investigation of the change of inclination of a dip-needle as its plane of vibration alters from a position perpendicular to the magnetic meridian to another coincident with it.* (1) Adjust the instrument as in § 165. (2) Rotate the vertical circle until the needle stands vertically. Read the position of the vertical circle on the horizontal scale. (3) Turn the vertical circle through 15°, then observe the readings of both ends of the needle. (4, etc.) Repeat, taking steps of 15°, until 180° have been turned through. Record as below, and deduce the angle of dip. Plot the mean inclinations of the needle as ordinates to the angular displacements of the vertical circle as abscissae.

Angle between plane of vibration of needle and magnetic meridian.	Reading of		Mean inclination.
	Upper end of needle.	Lower end of needle.	
90°	......	......	......
75°	......	......	......
......			
......			
15°	[The inclination diminishes to a minimum (about 70°) at 0 and then increases to 90°. The *minimum* value is the *magnetic dip.*]		
0			
15°			
......			
......			
75°	......	......	......
90°	......	......	......

Exp. 206.—*Determine the magnetic dip.* The following is the direct method of determining dip: (i) Set the instrument and get the needle vertical as in Exp. 205. (ii) Turn the vertical circle through a right angle. (iii) Read the positions of both ends of the needle. The mean is the *dip*.

Accurate determination of dip is not an easy matter. (See a more advanced work.)

MAGNETIC FIELDS.

166. To map a magnetic field by iron filings.—Place the magnet under a large card. Scatter iron filings (through muslin or by a pepper-pot) *scantily* over the card. *Gently* tap the card *for some time* by a long pencil loosely held between finger and thumb. Draw a diagram showing the shapes of the filing chains: these are also the shapes of the magnetic lines of force.

To fix the filing chains. Spray gum mastic over the paper, allow to dry; or use paper that has been soaked in paraffin wax, brush over with a Bunsen flame, or warm in hot air. Allow to cool or dry before removing the magnets.

***Exp. 207.**—*Obtain and draw the filing chains for a horse-shoe magnet and keeper.* For method see § 166. Observe the following cases—(1) horse-shoe without keeper; (2) horse-shoe with keeper half an inch away; (3) horse-shoe with keeper on.

NOTE.—(i) The chains lie between points situated along the limbs as well as on the ends. This shows that the "polarity" is not confined to the ends, although most concentrated there. Thus some of the internal lines of force that cross the equatorial section of the magnet leak out before reaching the ends. (In a *uniformly* magnetised bar all the lines pass through from end to end.)

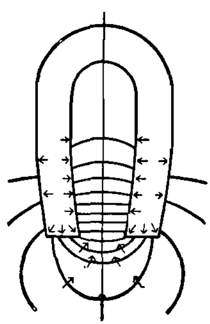


Fig. 144.

(ii) The chains are symmetrical about a middle line between the limbs: but the magnet is not uniformly magnetised. Similarly bar magnets are found to be symmetrically but not uniformly magnetised.

(iii) *Observe the movements of the filings* (indicated by arrows in Fig. 144). Those scattered in the neighbourhood of the magnet do not travel along the curves, but at *right angles* to them, and tend to form flatter and *shorter* chains. Those on the surfaces of the limbs stand aslant and during the tapping march towards the edges (they are really the ends of chains that tend to be formed above the magnet). If the tapping is prolonged the filings concentrate along the shortest lines between the poles.

The formation of the chains and the movements of the filings show that at every point of the field magnetic induction is effected and forces called into play which tend to turn the magnetised filings into definite lines, and to translate them in a direction perpendicular to these lines.

The soft-iron keeper considerably modifies the field. Notice in (3) the short chains bridging the minute gaps between the keeper and the ends of the limbs. They bulge out more as the keeper is separated further: compare (2).

Exp. 208.—*Obtain filing chains* and draw illustrative diagrams for the following cases—(1) a bar magnet; (2) two bar magnets, parallel, an inch apart, unlike poles adjacent; (3) as (2), but like poles adjacent; (4) ⊢ arrangement of two bar magnets; (5) a bar magnet and soft-iron rod of equal length, placed parallel, an inch apart; (6) ditto, half an inch apart; (7) ⊢ arrangement of bar magnet and soft iron; (8) as (7), but positions interchanged; (9) two bar magnets in line, unlike poles facing, two inches apart, soft-iron ring between; (10) magnet with consequent poles.

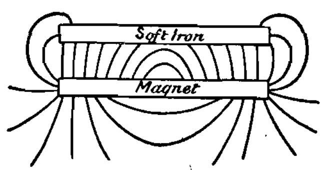


Fig. 145.

Compare the filing chains obtained in (2) and (5). In both cases there are parallel magnets with unlike poles adjacent, but in (2) two independent magnetic fields are superposed, while in (5) the field of the soft-iron induced magnet is dependent upon that of the inducing magnet. Observe that no chains are formed behind the soft-iron rod, the magnet's influence fails to work through the soft-iron. This illustrates the value of a thick iron plate as a *magnetic screen*.

167. To map or trace the lines of force in a magnetic field.—Place a small compass needle (§ 163) in the field, and mark the positions, A, B, of its ends. Move the compass until the end that was against A is over B, mark the position, C, of the other end; again move the compass until the end that was over A is against C, mark the position, D, of the other end; continue the process until the line has been sufficiently traced. Join AB, BC, etc., as *each step is completed*. The curve through ABC... marks a magnetic line of force. Obtain others by starting the compass from different positions. Indicate by an arrow-head the *+ sense* (that in which a north-pole *tends* to travel) *of the line of force.*

The magnetic field may be due to a magnet or magnetic combination placed on a *sheet of paper fixed to a board*, and the survey is restricted to the horizontal plane of the paper.

Filing chains afford a somewhat rougher and more restricted picture of the lines of force, but are more quickly obtained. Observations of them, too, have a *kinetic* value (Exp. 207 (iii)) as well as static. By the compass method, however, the lines can be traced in those portions of the field where the forces are very small, areas where the filings would not be visibly affected. Practically when filing chains are produced the effect of the earth is negligible, and the curves show the lines of force due to the magnets alone. The lines obtained by the compass needle in the weaker parts of the field are those that result from the combination of the earth's forces with those due to the magnet.

Exp. 209.—*Trace magnetic lines of force due to the earth* (I) *alone,* (II, etc.) *in combination with a bar magnet.*

(I) Fix a sheet of paper to the table with an edge, PQ, lying roughly east and west. Start the compass from positions along PQ about an inch apart.

(II) On a large sheet of drawing paper fixed to a drawing board set a *bar magnet in the magnetic meridian with its N-end pointing south.* Place its centre about a foot from a corner of the paper. Map the field.

Fig. 146 shows the sort of map obtained. *X* is a *null* or *neutral* position. The complete map would be symmetrical about the axis of the magnet, and a line through its centre perpendicular to its axis. Hence there is a second null point on the lower side.

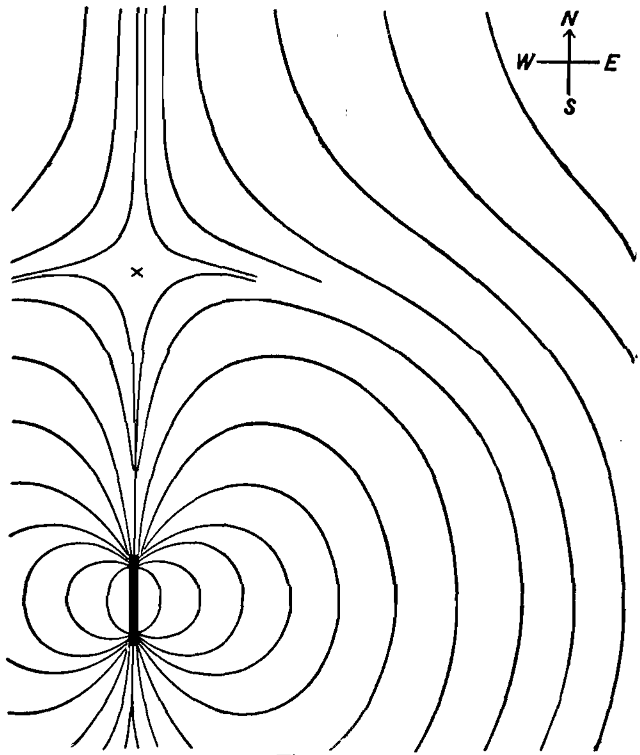


Fig. 146.

(III) Similarly a *bar magnet in the magnetic meridian with its N-end pointing north.* The two null points are found equidistant from the magnet on the line through its centre perpendicular to its axis.

(IV) Similarly a *bar magnet at right angles to the magnetic meridian* (Fig. 147. There are two null points diagonally opposite).

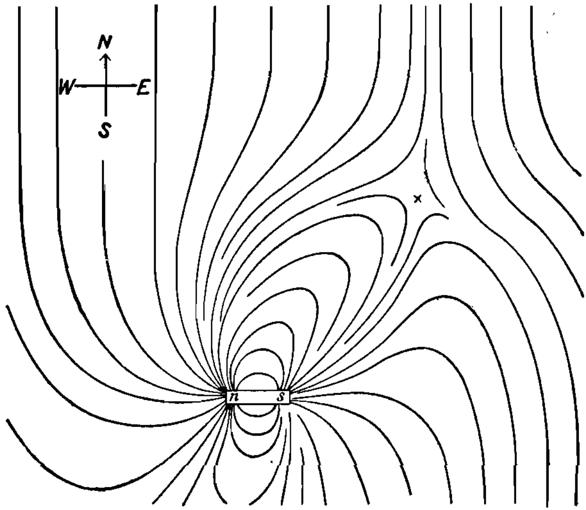


Fig. 147.

Figs. 146, 147 show *null* or *neutral* positions, obtained under certain conditions, at which the horizontal field due to the magnet is neutralised by that due to the earth. At these points the only magnetic force is practically the earth's vertical component. The maps show a fairly well defined region in which the forces due to the magnet are predominant; at greater distances the effects of the magnet are subordinate, and the lines appear to be those of the earth's field more or less distorted.

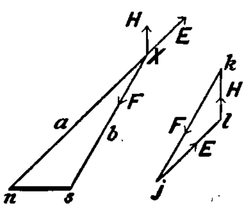


Fig. 148.

168. Relation between moment of magnet and the horizontal intensity of the earth's field.—Let (Fig. 148) *ns* be a magnet of moment M, pole strength p, length L. Let X be a null point at distances a, b, respectively from the N and S poles of the magnet. Imagine a unit N-pole at X: it will be in equilibrium, and acted on by three forces, H due to the earth, a repulsion, E, an attraction, F. Draw the force-triangle, jkl. Then $E/H = jl/lk$, $F/H = jk/lk$.

$$\text{Also } E = p/a^2,\ F = p/b^2, \text{ and } M = p.L,$$

$$\therefore\ M = La^2.E, \qquad \text{and} \qquad M = Lb^2.F.$$

$$\therefore\ \frac{M}{H} = L.a^2\frac{jl}{lk} \text{ and } \frac{M}{H} = L.b^2\frac{jk}{kl}.$$

Hence by measuring L, a, b, jl, lk, kj, two values of M/H may be determined. If H is known, M and p may be calculated.

MAGNETIC MOMENTS AND FIELD INTENSITIES.

169. The tangent relation.—If two uniform magnetic fields are superposed on one another, the resultant field is also uniform. Its direction and intensity are determined by the *parallelogram of forces.*

Let OP (Fig. 149) represent in length and direction the intensity (F) and direction of one field, and similarly OQ, the intensity (H) and direction of the second field. Then on completing the parallelogram, OR represents the resultant field in intensity and direction.

Now $\quad F/H = \sin\delta/\sin ORQ.$

Hence when the fields are at right angles $\quad F/H = \tan\delta.$

Also *intensity of resultant field*

$$(OR) = \sqrt{(F^2+H^2)}.$$

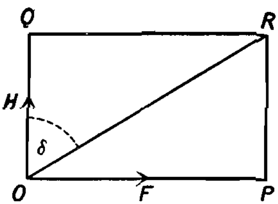


Fig. 149.

If a freely suspended magnet is placed in the field it will set itself with its magnetic axis in the direction of the resultant, or of either component if the other is not acting. If then H is a known field, *e.g.* the earth's, the intensity of an unknown field, F, can be compared with it by observing the angle, δ, between the position of a compass needle when due to H only, and its position when acted upon by both F and H. If, by arrangement, F is set at right angles to H, then $F = H.\tan\delta$. This is effected in the Gauss positions (§ 171) and in the tangent galvanometer (§ 208).

Note that the position assumed by the compass does not depend upon its moment, pole-strength, or length. It is, however, frequently necessary to use short magnets, because the magnetic fields obtained in practice are only approximately uniform within narrow limits (§ 208). The magnets, too, should be strong in order that the effects of friction at the pivot, or the torsion and stiffness of the suspending fibre, may be insignificant.

170. Vibrating Magnet.—Let a bar magnet of magnetic moment M, suspended in a *uniform* magnetic field of intensity H, be set in vibration. When the angle of vibration is *small*, if t is the period and I the moment of inertia (§ 75), then

$$t = 2\pi\sqrt{I/MH} \text{ (the magnetic pendulum).}$$

Now t can be measured and I calculated or found by experiment, hence obtain (express quantities in *C.G.S. units*)

$$MH = 4\pi^2 \, I/t^2 = 39{\cdot}48 \, I/t^2.$$

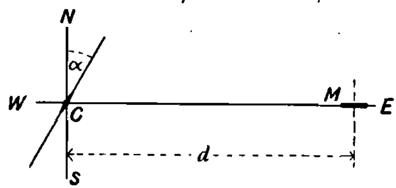


Fig. 150.

171. Compass needle deflected by a magnet.—Let a small compass needle lie in a *uniform* magnetic field of intensity, H. Place a bar magnet (length, $2l$, magnetic moment, M) with its magnetic axis perpendicular to the magnetic meridian and magnetic north or south, east or west of the compass (Figs. 150, 151). Let an angular deflection α of the compass be produced, and let d be the distance between the centres of the deflecting magnet and the compass.

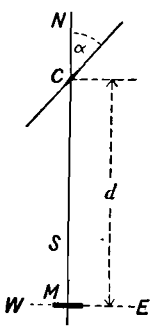


Fig. 151.

When $2l$ is *small* in comparison with d the relations are as below:

Figure..	150	151
Name of position	End on.	Broadside on
Value of M/H (1st approximation, l neglected)	$\frac{1}{2} d^3 \tan a$	$d^3 \tan a.$
Value of M/H (closer approximation)	$\frac{(d^2 - l^2)^2}{2d} \tan a$	$(d^2 + l^2)^{\frac{3}{2}} \tan a.$

The above are called the *Gauss positions.* After d and a have been observed the value of M/H can be calculated.

NOTE.—The same magnet at an equal distance will produce a bigger deflection when *end on* than when *broadside.*

The formulae for the stationary and vibrating magnets form the basis of the methods of measuring the moments of magnets, and the intensities of fields.

172. Magnetometers.—These are instruments arranged for magnetic measurements.

By *deflection magnetometers* the angular displacement of a suspended compass needle is observed.

By *vibration magnetometers* the period of vibration of a suspended magnet is determined.

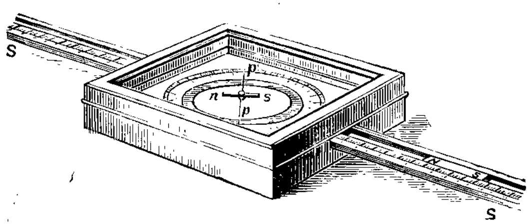
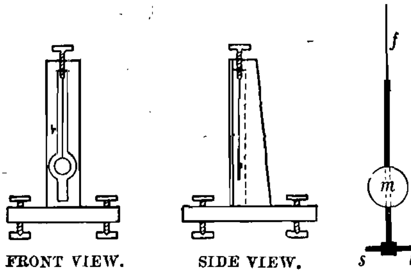


Fig. 152.

Deflection magnetometer (Fig. 152). The short compass needle, *ns*, is supported on a pivot, and has attached to it at right angles a long pointer, pp', that can move over a graduated circle. Some plane mirror is exposed at the bottom of the box. The parallax error (§ 120) in reading the position of the pointer may then be avoided. The graduated metric scales, S, S, enable the bar magnet, NS, to be put at known distances from the compass needle. *The instrument is adjusted* so that, when no magnets or pieces of iron are in the neighbourhood, the pointer lies either over the 0 and 180° divisions of the graduated circle, or over the 90° and 270°. The scales may then be assumed to lie in the magnetic meridian or at right angles to it.

In the *reflecting or mirror magnetometer* (Fig. 153) several very short magnets are fixed to the back of a small mirror, which is suspended by a long fibre of unspun silk in a brass chamber or cell only slightly too big for it. (By this means the oscillations of the mirror are quickly damped.) The fibre passes up a tube and is fixed to the cap at the top. A glass window covers the mirror cell. The suspended needle is adjusted by the levelling screws so that it moves freely. The small deflections of the mirror are measured by one of the methods of § 37, very frequently by Method IV.



FRONT VIEW. SIDE VIEW.

Fig. 153.

173. Vibration magnetometer.—Fig. 154 illustrates a convenient arrangement. A wooden stool, A, supports a tube, B, and cork, C. A rod, D, passes through the cork. A fibre carrying a metal or cardboard (Fig. 142) stirrup for holding the magnet is fixed to the rod. A glass shade, E, covers the whole (this shields from draughts). Two strips of stamp paper are placed on the shade, with their edges vertical and diametrically opposite.

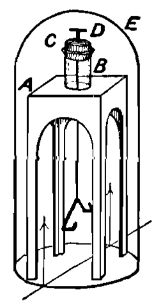


Fig. 154.

A rough arrangement is to hang the magnet in a wide-mouthed bottle. The upper end of the suspending fibre is tied to a hook on the under side of the cover (wooden).

Another arrangement consists of a wooden box, say, 8″ long, 3″ wide, 3″ deep. In each of the two long sides there is a glass strip that can be slid in and out. A glass tube rises from the top of the box. The suspending filament is attached to a cap at the upper end of this, passes down the tube, and supports the stirrup and magnet in the box. The cap can be twisted.

ADJUSTMENTS, ETC., OF THE VIBRATION MAGNETOMETER.

1. Mark roughly the magnetic reference line (Exp. 200). Set the magnetometer so that the reference line passes, roughly, midway between its legs.

2. There must be *no twist to begin with* of the suspending filament. To avoid this place a small brass bar in the stirrup (the brass stirrup itself may be sufficient) and leave until vibration ceases—the fibre may then be assumed to be untwisted.

3. Observe the angle between the bar and reference line. Turn the cork through an angle equal to this so that the brass bar may rest over the reference line. Hold the stirrup so that no further twisting takes place, put the magnet in it and remove the brass bar. Now allow the magnet to hang freely and let it come to rest—this may be assisted by touching it lightly and carefully with the fingers. Place the glass shade over it, and adjust so that the marks on it (edges of paper strips) are on the magnetic reference line. Care must be taken not to disturb the magnet when the cover is put on (certain magnetometers having sliding covers seem designed to give a bad start).

4. *Start the suspended magnet swinging* by bringing slowly towards it a bar magnet presented end on (when done remove this bar magnet from the neighbourhood). If there is an appreciable side to side movement of the suspension and magnet, stop and start again. *A twist only is required.*

5. Make the observations when the *arc of vibration* is *small.*

174. To compare magnetic moments: (I.) By deflecting a compass needle.—If magnets of moments M_1, M_2 are placed in turn either *broadside* or *end on*† at distances d_1 and d_2 from a compass needle, then (§ 171) if the position of the compass needle is not changed, H is constant, and

$$M_1 : M_2 = d_1^3 \,.\, tan\, \alpha_1 : d_2^3 \,.\, tan\, \alpha_2.$$

† *End on* is the better position (§ 171: Note).

If the deflections are equal $M_1 : M_2 = d_1^3 : d_2^3$, or *the moments of the magnets are directly proportional to the cubes of their distances when the deflections are equal.*

In the last case the magnets can be arranged one on each side of the compass needle and at such distances that they tend to produce equal deflections in opposite senses and hence the needle remains undeflected. The distance should not be less than five times the length of the magnet. This is the most convenient way of comparing magnetic moments.

(II.) **By determining the period of vibration.**—Each magnet is in turn suspended in the same place (hence H is constant) and its period measured. Its moment of inertia is also calculated. Then (§ 170)

$$\frac{M_1}{M_2} = \frac{I_1 \cdot t_2^2}{I_2 \cdot t_1^2}.$$

If the magnets are practically of the same dimensions and mass, and vibrate about the same axis, $I_1 = I_2$, then $M_1 : M_2 = t_2^2 : t_1^2$, or *when the moments of inertia are equal the magnetic moments are inversely proportional to the squares of the periods of vibration.*

If the magnets are of the same cross-section then the mass is proportional to the length and the moment of inertia varies as the *length cubed.* When thin magnets are hung edgewise their breadth and depth may generally be neglected, but their lengths (l) and masses (m) should be measured. Their moments of inertia are then proportional to

$$(mass)\ (square\ of\ length),$$

or
$$I_1 : I_2 = m_1 \cdot l_1^2 : m_2 \cdot l_2^2.$$

Then
$$\frac{M_1}{M_2} = \frac{m_1}{m_2} \cdot \left(\frac{l_1}{l_2}\right)^2 \cdot \left(\frac{t_2}{t_1}\right)^2.$$

(III.) **Sum and Difference Method.** The measurement of the moments of inertia, etc., may be avoided by proceeding as follows:—Suspend both magnets together with like poles pointing (i) the *same* way; (ii) in *opposite* ways. In each case determine the vibration-period of the combination.

Let M and m be the magnetic moments of the magnets, t_1 the period of the combination in the first case, t_2 in the second, then, since the total moment of inertia, I, is the same in both,

$$\frac{M+m}{M-m} = \left(\frac{t_2}{t_1}\right)^2 \quad \therefore \quad \frac{M}{m} = \frac{t_1^2 + t_2^2}{t_2^2 - t_1^2}.$$

If round and thin it is convenient to push the magnets through a cork: if flat and large they may be bound by thread, one on each side of a strip of wood (Fig. 155). First bind one magnet to the strip, then the other. Mark the positions of the magnets on the wood. To reverse the positions of the poles cut the binding of the second magnet, reverse, place in coincidence with the marks on the wood, and rebind. The combination may be supported in a wire stirrup or cradle hung by a fine wire or fibre (Fig. 154).

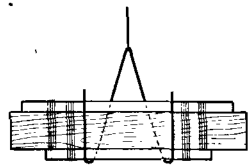


Fig. 155.

To compare pole strengths, let p_1, p_2 be the pole strengths and l_1, l_2, the respective lengths of the magnets. By definition $M = l \cdot p$, therefore $p_1/p_2 = M_1 \cdot l_2/M_2 \cdot l_1$.

Exp. 210.—*Compare the moments of magnets.*—I. Marl the magnetic meridian (Exp. 203). Draw a line at right angles to it. Place a compass at the point of intersection. Note the positions of its ends on the divided circle, or fix a pointer (brass pin) close to one end. Lay the magnets to be compared along the east and west line, and adjust the poles and distance of one of them until there is no deflection of the needle. Note the distances of the compass from the *equator* of each magnet. Repeat at other distances. Calculate as in § 174 (I.).

The magnetometer shown in Fig. 152 is adapted for this experiment. Arrange the instrument so that to begin with the centimetre scales are in line with the pointer.

II. Suspend each magnet in turn edgewise as in § 173. Measure the time of fifty swings by a stop-watch, also the length, breadth (often negligible), and mass. Calculate as in § 174 (II.).

III. Compare by the sum and difference method (§ 174, (III.)).

175.—Comparison of magnetic moments by a reflecting magnetometer. Adopt a **difference method** as follows: let two magnets of moments M and m be adjusted to *balance*, that is produce no deflection, when at distances D_1 and d_1 respectively: also when at distances D_2 and d_2. Then

$$\sqrt[3]{M/m} = D_1/d_1 = D_2/d_2 = (D_1 - D_2)/(d_1 - d_2).$$

It is possible to measure the difference in the distances more accurately than the actual distances, in which there are uncertainties regarding the positions of the compass and the equator of the magnet. The differences above are determined by reading the positions on the scale of *any* mark on the magnets.

Exp. 211.—*Compare the moments of two magnets by means of the reflecting magnetometer.*

176. To compare field intensities: (I.) By deflecting a compass needle.—If the *same* bar magnet is placed either *broadside* or *end on* to a compass needle (§ 171) first at one place (field intensity H_1), next at a second (field intensity H_2), and if $\alpha_1\alpha_2$, d_1d_2 are the respective deflections and distances, then

$$H_1 \, . \, d_1^3 \, . \, tan \ \alpha_1 = H_2 \, . \, d_2^3 \, . \, tan \ \alpha_2.$$

The calculation is simplified

(i) if $\alpha_2 = \alpha_1$ when $H_1 : H_2 = d_2^3 : d_1^3$

(ii) if $d_2 = d_1$ when $H_1 : H_2 = tan \ \alpha_2 : tan \ \alpha_1$.

(II.) **By determining the vibration period.**—If the *same* magnet is vibrated at the two places, and t_1, t_2 are the respective periods, then since M and I are constant $H_1 : H_2 = t_2^2 : t_1^2$ or the *field intensities are inversely proportional to the squares of the periods.* This is an important and convenient method of comparing field intensities.

Exp. 212.—*Compare field intensities by a compass needle.*

At each of the places (say in different rooms) at which the field intensities are to be compared, (i) adjust the deflection magnetometer (Fig. 152) so that the scales lie perpendicular to the magnetic meridian.

(ii) Place the deflecting magnet along the scale on one side of the compass needle, and with its equator (marked) at a definite distance from it (obtain a deflection of about 30°). Note the distance (d) between the equator and the compass, and the deflection (α) (read both ends of the pointer, the deflection, α, is the *mean* of these).

(iii) Keeping the distance (d) the same value throughout, read the deflection when the magnet is reversed, when

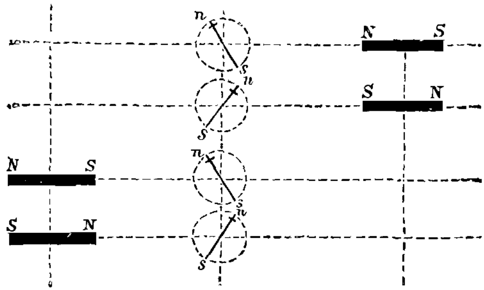


Fig. 156.

on the other side of the compass, when again reversed. Fig. 156 illustrates the successive positions.

Repeat for another value of d. Record each as below:

Station.	[Specify room, bench, etc.]			[Specify room, bench, etc.]		
Position.	Reading of pointer. W. end.	E. end.	a_1	Reading of pointer. W. end.	E. end.	a_2
1. 2. 3. 4. (Draw small sketches as Fig. 156.)						
$d =$	mean $a_1 =$ tan $a_1 =$			mean $a_2 =$ tan $a_2 =$		
Hence Field at . . .: Field at . . . = 1· . . . : 1						

Exp. 213.—*Compare field intensities by vibration.*

Suspend the same magnet at each place in turn and find the respective periods of fifty swings.

GAUSS ABSOLUTE METHOD.

177. To find the field intensity in dynes.—The formulae (§§ 170, 171), $M/H=\frac{1}{2}d^3.\ tan\ a$, and $M.H=4\pi^2I/t^2$, show that the *quotient* of the two magnetic quantities, M and H, can be measured by length and angle, while the *product* is in terms of time, mass, and length (I involves mass and length), also $M^2=(M.H)(M/H)$ and $H^2=(M.H)\div(M/H)$. Hence by using absolute units of the fundamental quantities, M and H can be expressed in absolute units (usually C.G.S.).

Exp. 214.—*Find the field intensity, H, at a place (in the laboratory).* Two experiments are necessary: (1) the determination of $M\times H$ by vibration, (2) the determination of M/H by deflection.

1. Adjust a deflection magnetometer at the place of observation and proceed as in Exp. 213. Finally remove the magnetometer.
2. Suspend the deflecting bar magnet used in (1) at the place of observation, find its vibration period.
3. Measure mass, length, and width of bar magnet and calculate the moment of inertia (§ 75).
4. Calculate M/H, $M.H$, and H. Use C.G.S. units. Record as below.

Exp. 215.—*Find by the Gauss absolute method the value of H at a place in the laboratory.*

Determination of H, at [bench....], [....lab.], [....Institution].

I. *Deflection observations.* End on method.

	Position.	Reading of pointer. East end.	West end.	Deflection.
1.	(Show by	30	29	29·5
2.	small	30·5	29·5	30
3.	sketches	28	29	28·5
4.	as Fig. 156)	29	30	29·5

Distance 30 cm. Mean 29·4°

tan 29·4° = 0·563

$$M/H=\tfrac{1}{2}\times 30^3\times 0{\cdot}563=7600.$$

II. *Vibration observations.*

	h. m. s.
Time at start	8.56.5
Time after 50 complete vibrations..	9. 4.1

Interval .. .. 7.56 = 476 seconds,

∴ vibration period = 476/50 = 9·52 seconds.

III. *Deflecting magnet.*

Mass 62·5 gm., length 10·2 cm., width 1·3 cm., swung with its two greatest dimensions horizontal round a vertical axis through the centre of the magnet.

$$\therefore I=\tfrac{1}{12}(10{\cdot}2^2+1{\cdot}3^2)\times 62{\cdot}5=551,$$

$$\therefore MH=39{\cdot}48\times 551/(9{\cdot}52)^2=243,$$

$$\therefore H=\sqrt{(243)\div(7600)}=\sqrt{0{\cdot}0320}=0{\cdot}18 \text{ dyne per unit pole.}$$

CHAPTER XIII.

ELECTROSTATICS.

178. Experiments in Electrostatics.—Of the following those marked * are perhaps more conveniently done at home than in the laboratory. The glasses, rubbers, etc., *must be dry and warm.* To make them so place them on the hearth in front of a good fire. In the laboratory use an oven.

Oven (Fig. 157). A piece of thin sheet iron is bent to the shape *ABCD*. Another piece bent like *EF* is placed inside. The base *AC* rests on four feet or laboratory tripods, and is heated by a Bunsen burner. Glass rods and paper are placed on *EF*, silk, flannel, and fur rubbers on the dome *ABC*. Ebonite, sulphur, sealing-wax, glass insulating stands, tumblers, etc., are placed on the table near but not directly underneath the oven. Care must be taken not to overheat them.

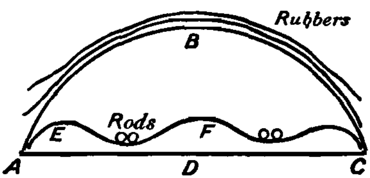


Fig. 157.

Electroscope.—Paper balls (Fig. 159), made as below, are very satisfactory for simple experiments.

Fold a piece of tissue paper about three inches square into halves, then into quarters, finally into eighths. Cut with sharp scissors a circular arc *BC* (Fig. 158), then as shown by the intermediate arcs. Open out very carefully. Make a second disc in a similar manner. Touch the edge of one disc in four or five places with a match-end that has been dipped in gum, superpose the other, press at the gummed places, and set aside to dry: or (a better plan) fix two discs together at the edges by four or five very small pieces of stamp-paper. Pass a length (two or three feet) of silk twist, knotted at one end, through a pinhole in a small piece of paper, then through a hole at the centre A. Four of these balls are required. (It is useful to make the pairs of papers of different colours.) Hang them in pairs from a gas-pendant, a stretched string or a stick: they should be at least two or three feet from a wall.

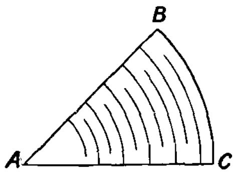


Fig. 158.

***Exp. 216.**—*Electrification by friction.* 1. Rub a glass tumbler† with silk.† Bring the glass near the paper balls, observe that they are attracted, then, after touching the glass, are (i) repelled from the glass, and (ii) repel each other. Also (iii) they repel each other when the glass is removed.

2. Bring the finger near the balls, they are attracted, but after touching the finger are not repelled from it, nor do they repel each other.

Explanation. In (1) the glass becomes electrified when rubbed with silk, and the balls become similarly electrified after touching the excited glass: the similarly charged bodies repel. In (2) the electricity on the balls induces a charge of the opposite kind on the finger; these neutralise when contact occurs.

***Exp. 217.**—*Repulsion of similarly electrified bodies.* Lay two strips (8″ by 2″) of thin paper on the table, rub them with the palm of the hand. Observe that the paper clings strongly to the table. This is owing to the attraction between the electrification of the paper and the induced charge on the table. Lift the strips from the table, one by the right, the other by the left hand, hold the top edges in contact, observe that the strips do not hang vertically, but at an angle with one another.

† Must be warm and dry.

179. Definition. A *positive* (+) *electrification* is that developed on polished glass when rubbed with dry silk.

A *negative* (−) *electrification* is developed on ebonite, sulphur, or sealing wax when rubbed with flannel or fur.

The experiments also show that a positive electrification is developed on brass when struck with india-rubber; and a negative electrification is produced when the brass is beaten with fur or flannel. (See § 181.)

Remember that the kind of electrification produced on a body depends upon the nature of the rubber as well as that of the body rubbed.

***Exp. 218.**—*Show that similarly electrified bodies repel one another, dissimilar attract.* (1) Touch a pair of suspended paper balls with electrified paper† (rubbed by hand). Each ball becomes negatively electrified.

(2) Touch the second pair of suspended paper balls with electrified glass† (rubbed with silk). Each ball becomes positively electrified.

(3) Bring the −ly electrified paper near the +ly electrified balls ((2) above); the balls are attracted.

(4) Bring the +ly electrified glass near the −ly electrified balls ((1) above); the balls are attracted.

(5) Bring in turn a piece of sulphur, resin, charcoal, metal, stoneware, or china, etc., near the paper balls. Observe that each ball is attracted. The bodies are in this case being tested in their normal unelectrified, or neutral, condition.

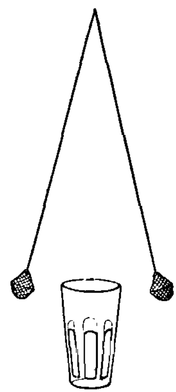


Fig. 159.

(6) Rub each body with flannel or fur, and bring it near the balls. Note whether the +ly or −ly charged balls are repelled. If the former then the electrification of the body is +, if the latter −. The charcoal and metal will attract each pair of balls.

† Must be warm and dry.

180. To test whether a body is electrified or not present it first to a +ly charged ball or electroscope, next to a −ly charged one. If the body is electrified there will be attraction in one case and repulsion in the other, if unelectrified attraction occurs in both cases. *The repulsion of a body similarly charged is the sure proof of electrification on the piece tested.* **To determine the kind of electrification** on the body remember that when the +ly charged ball is repelled the body is +ly charged, when the −ly charged ball is repelled the body is −ly charged.

***Exp. 219.**—*Conductors and non-conductors.* Electrify one pair of paper balls +ly. Touch the balls with a body held in the hand (be careful that the fingers do not come into contact with the balls). If the balls fall together the body is a conductor, if not it is a non-conductor or insulator. Try various substances, *e.g.* metals, paper,† silk, glass, carbon, wood,† cotton,† sulphur, string,† etc.

Repeat experiment using negatively charged balls.

Record the results making a list of conductors and of non-conductors.

***Exp. 220.**—*Induction or Influence.* Two insulated conductors are required. Each may be realised in a simple manner by covering the mouth of a dry and warm glass tumbler with a metal lid or canister (inverted). Lift or move the conductor by holding the glass at the base. Charge one pair of paper balls −ly, the other +ly.

Place the two conductors in *contact*. Hold an electrified rod close to one of them (not however over the centre where the bodies touch), then separate the conductors and bring each in turn near the charged balls. The +ly charged balls will be repelled by a +ly charged conductor, and the −ly charged balls by a −ly charged conductor.

Perform the experiment (1) with +ly electrified glass; (2) with −ly electrified paper. Show that in either case the charge on the conductor nearer the inducing electrification is of the opposite kind to the inducing, while on the further conductor it is of the same kind.

† Test before and after drying.

***Exp. 221.**—*Make a paper electrophorus.* 1. Group four tumblers (mouth upwards) on the table in order to support and insulate a metal tray (bottom upwards), put a sheet of brown or note paper on the tray, pinch it to the tray between the forefinger and thumb of the left hand, rub the paper with the right hand (palm) or a dry clothes-brush.

2. Lift the paper from, and then present a knuckle to the tray—an electric spark is obtained.

3. Replace the paper on the tray, taking care that the fingers do not touch the metal, then present the knuckle to the tray—a spark is obtained.

4, etc. Repeat operation (2), then (3), several times—a spark is obtained on each occasion.

The tumblers and paper (use three or four sheets of foolscap, some to be warming while one is being used) must be well dried before a fire or in an oven. *To lift the paper* bring together two opposite corners, and pinch between finger and thumb. The paper may be *re-electrified* by rubbing with the hand as in (1). (When this fails substitute a freshly warmed piece and rub it by the hand.)

EXPLANATION. The friction with the hand or brush electrifies the paper −ly. The tray is charged by induction; the bound-charge being +, the free charge negative. The spark of (3) is due to the latter. When the paper is removed the bound + electricity is released. This gives the spark in (2) and (4). The arrangement is thus a source of both kinds of electrification.

181. The gold-leaf electroscope (Fig. 160).—This is a sensitive form of electroscope in which there are two† gold-leaves (g,g) that hang vertically—"collapsed" or closed—when unelectrified, and, when electrified, at an angle with one another—"divergent" or open—the angle being greater as the degree of electrification is greater. The gold leaves are hung at the lower end of a metal rod, *b*, whose upper end, *D*, is terminated by a ring, knob, or disc—sometimes called the *cap*. The rod passes through a

† A single gold-leaf resting against a vertical strip of metal is frequently used.

plug of insulating material—shellac, paraffin-wax, sulphur—which is fitted in the mouth of a glass flask or shade, or (Fig. 160) a hole in the top of a box, B, having a glass front (and back). The sides and base of the box should be coated inside with tinfoil, connected with the terminal, t. When using the instrument join t to the gas-pipes by a wire. The tinfoil coating is then earth-connected.

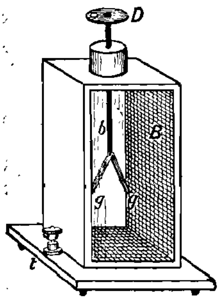


Fig. 160.

Sometimes calcium chloride or sulphuric acid is put inside the case so that the air inside the flask may be kept dry. This is unimportant. The upper parts of glass shades are often varnished with a solution of shellac or sealing-wax in alcohol. This is unnecessary. A plug of sulphur or paraffin wax gives excellent insulation. Provided the plug insulates well it is an advantage for the glass surface to be damp and therefore conducting. When the plug does not insulate well substitute a better one. A dirty wax plug may be cleaned by scraping.

To charge an electroscope.—When the cap of the neutral electroscope is beaten (1) with flannel or fur, it acquires a *negative* charge, (2) with a strip of india-rubber (split gas tubing) it acquires a *positive* charge.

It is better to (1) beat a brass ball, held by an insulating (ebonite) handle, with the fur or india-rubber, then (2) touch the cap of the electroscope with the ball. The electroscope takes some of the charge from the ball.

CAUTION.—A strongly excited rod ought not to be brought into contact with or even close to the cap of an electroscope—the leaves may be torn off.

REMEMBER that the electroscope when charged by *contact* with an excited body acquires an electrification of the *same* kind; when charged by *induction*, its electrification is of the *opposite* kind to the inducing charge. Also that the leaves of a charged electroscope diverge more widely if an electrification of the same kind is brought near, but less widely for one of the opposite kind. Also that a slight closing occurs when an earth-connected body (the hand, say) is put near the disc. *Increased divergence is the sure test* of electrification on a body brought near the electroscope. These facts are demonstrated in the experiments below.

Exp. 222.—*Charge an electroscope by induction.* (i) Hold an electrified body near the electroscope and while there (ii) touch the cap with the finger for a moment, (iii) remove the electrified body—the leaves diverge. The electrification of the electroscope is now of the opposite kind to the inducing charge on the electrified body.

I. Use +ly electrified glass (rubbed with silk). Show that after operating as above the leaves diverge more widely if a −ly charged body is brought near, but less widely for one +ly excited. Also that the leaves collapse slightly if the hand is placed parallel with and close to the disc, but reopen on removing the hand. Finally that the leaves close completely and remain so if the disc is touched for a moment by the finger.

II. Use −ly electrified ebonite (rubbed with fur, on the coat, etc.). Show that after operating as above the leaves diverge more widely if a + charge is brought near, but less widely when the electrification is −. Also the effects with the hand and the finger are the same as in the previous case.

Exp. 223.—*Observe the phenomena of induction.* **I.** Slowly bring a +ly electrified glass rod towards an electroscope. Observe that the leaves increase in divergence as the distance diminishes, but collapse completely when the glass rod is removed.

Explanation.—As the inducing charge gets nearer each of the induced charges increases in amount: the divergence widens because the quantity of free + gets greater. Collapse occurs because the induced + and − charges are exactly equivalent and neutralise one another when the inducing charge is removed.

II. (1) Place the +ly electrified rod, say, three inches from the disc: the leaves diverge (+ electrification). (2) Touch the disc for a moment with the finger: the leaves collapse. (3) Put the rod further (four inches, say) from the electroscope: the leaves diverge slightly (negative electrification). (4) *Slowly* move the rod towards the disc of the electroscope: the divergence diminishes, is nothing when the rod is in the position of (1) and (2), after this the leaves reopen (with +) and the divergence increases as the rod approaches the cap. (5) Quickly withdraw the rod to a distance, the leaves momentarily collapse, reopen, and remain divergent (negative electrification).

Explanation.—The tabular statement below shows the variations in the several quantities involved in the preceding experiment, II. Let the inducing charge $= Q$.

Case.	Bound charge.		Free charge.	Surplus charge.	Potential (see § 182) of electroscope.	
1	$-q_1$	$<Q$	$+q_1$	*none*	$+$	Because + tends to flow to earth.
2	$-q$	$\begin{cases} >q_1 \\ <Q \end{cases}$	none	$-q$	0	That of earth.
3	$-$	$<q$	$-$	$-q$	$-$	− tends to flow to earth.
4	$-$	$>q$	$+$	$-q$	$+$	+ tends to flow to earth.
5	none		$-q$	$-q$	$-$	− tends to flow to earth.

The experiment shows that the bound charge is always opposite in kind to the inducing charge but varies in quantity, the free charge may be of either kind and determines the potential, whether − or +. The potential is therefore not necessarily that of the surplus electrification of the conductor.

182. Potential.—The *potential* of a conductor is a relation between it and a second conductor which determines whether + electricity shall flow from it to the other conductor or in the reverse direction.

Definitions.—(i) If two conductors, A, B, are so related that + electricity tends to flow from A to B, then A is said to be at a *higher* potential than B, and B at a *lower* potential than A. (ii) Two conductors are said to be at the *same* or *equal* potentials when electricity does not tend to flow from one to the other. (iii) The locus of all the points for which the value of potential is the same is called an *equipotential surface* or *electric level.*

Also negative electricity tends to flow from a conductor at lower potential to one at higher.

CONVENTIONS.—The earth is considered to be at potential zero (0). Bodies at a higher potential than the earth are said to be at a + potential, those at lower potential than the earth are said to be at a − potential.

Hence potential has the same value at every point of a conductor however electrified; or the surfaces of all conductors are equipotential surfaces.

All conductors connected with the earth are at zero potential.

183. The Electrophorus (Fig. 161).—A simple form of electric generator.† Its action depends on induction. It generally consists of an ebonite or resinous *cake* or *slab, E,* usually resting on a metal base or *sole*, and a metal *cover* or *plate, B,* provided with an insulating handle (glass rod). *To work the electrophorus,* (1) beat the cake with flannel or catskin, (2) place the cover on the cake, (3) touch the cover with the finger, (4) lift the cover from the cake by the handle. (The handle should be warm and dry, and held near the top end.) The cover is now +ly charged. On placing it in contact with insulated conductors (knob of Leyden jar (§ 185), etc.) it gives up a portion of its charge, or on putting the knuckle of a finger against it a spark is produced. (5) After discharging the cover replace it on the cake, touch it, lift it, the cover will again be +ly charged. (6) Repeat several times, then excite the cake again.

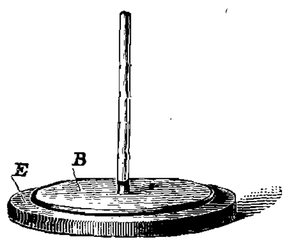


Fig. 161.

If there was no loss of electrification by contact, leakage, etc., from the cake, the cover could be charged an indefinite

† Compare with the "paper electrophorus," Exp. 221.

number of times. Its electrification is +, while that of the cake is −; hence it does not share the charge of the cake, but is electrified by induction.

The sole is not an essential part of the instrument: it frequently consists of a disc of tinfoil stuck to the under side of the ebonite. To avoid touching the cover with the finger it is convenient to have a metal pin fixed to the sole and passing through the cake until its point is practically flush with the upper surface. The free induced negative charge on the plate sparks across to the pin and passes to earth.

The theory of the action of the electrophorus is described in text-books of Electricity. The paper electrophorus (Exp. 221) works similarly. In it, however, the cake (electrified paper) is lifted from the cover (tray).

Exp. 224.—*Use an electrophorus to charge a Leyden jar, or an insulated conductor.*

Exp. 225.—*Show* that (i) The cake is −ly excited, (ii) the cover before being touched gives − electricity if kept on the cake, and nothing if lifted from it, (iii) the cover, after being touched, gives nothing if left on the cake, but + if lifted from it.

Use paper balls (§ 178). Charge a pair +ly, another −ly. Bring in turn near to the electrophorus.

184. The **Wimshurst Machine** (Fig. 162) is a convenient and reliable means of producing electricity. It is a form of influence machine. It consists of two circular glass plates, each of which, by turning the handle at the base of the machine, can be made to rotate, but in opposite directions. The glass plates are placed parallel and close together, and each turns about the same axis. On the external or exposed surface of each plate fifteen or twenty metal strips are arranged at equal distances, and with their lengths radial. Two prime conductors supported on glass or ebonite pillars are provided. The rakes or combs of these are placed usually parallel to the horizontal diameters of the plates. The prime conductors are not connected with one another. When the machine is at work one becomes +ly, the other −ly electrified. Generally a rod terminated by a knob is attached to each conductor. These are bent and long enough to permit the knobs to be brought into contact. The rods can be moved by the straight handles shown on the right and left of the figure, so that the distance—called the *spark gap*—between the knobs may be adjusted. Small brushes of soft metallic filaments attached to each prime conductor are arranged to touch the sectors. There are also two earth-connected conductors, one for each plate. Each carries a

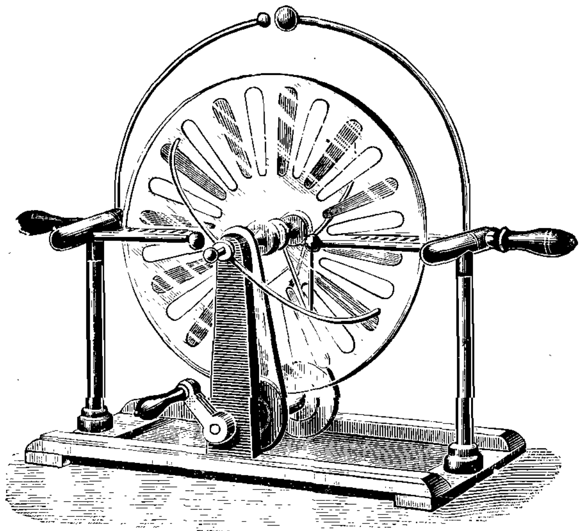


Fig. 162.

metal brush at its ends. These conductors are arranged at right angles to each other, and at about 45° with the combs of the prime conductors.

To get the machine to work, nearly close the spark gap, and repeatedly turn the handle at the base. Almost at once sparks will appear in the gap. The machine generally excites itself. If, however, it fails to do so, rotate the plates, and hold an electrified conductor near one of the prime conductors. (For the theory of influence machines see the *Text-Book of Electricity*.)

To increase the intensity of the spark the prime conductors of the Wimshurst machine may be connected with Leyden jars, as below :—(I.) A Leyden jar has its outer coat insulated, *e.g.* it stands, say, on a slab of wax. The inner coat is connected with one prime conductor, the outer coat with the other prime conductor. (II.) Two jars are taken, the outer coat of each is earth-connected or joined with the uninsulated conductors of the machine; the inner coat of one jar is joined with a prime conductor, the inner coat of the second jar is joined with the other prime conductor.†

Exp. 226.—*The "ice-pail" experiment* (Fig. 163). Place a narrow deep canister (say 12″ by 3″) on a piece of paraffin wax (I). Connect the canister by a fine wire with the disc of an electroscope. Fix a small ball (1″ diam.) or tin (B) to the end of a long (2 ft.) glass (G) or ebonite rod.

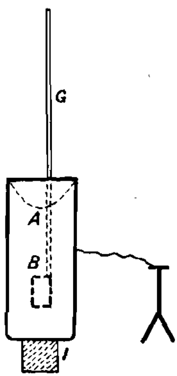


Fig. 163.

(1) Charge the body (B), say +ly. Bring it slowly towards the mouth, and finally place it within the canister, but not to touch it. Observe that the divergence of the leaves increases more and more as B approaches, but attains a maximum when it reaches a position A a little below the mouth (as indicated by the dotted line). This maximum divergence is maintained whatever the position of B in the canister, provided it is below the dotted line. Withdraw B, the leaves completely collapse. Also bring B into several positions short of A and withdraw it without touching the canister: in each case the leaves will diverge while B is near, collapse when it is removed.

The increasing divergence of the leaves shows that the induced charges grow to a maximum which is attained when the inducing charge is well within the canister. The complete collapse on withdrawal shows that the induced charges (+ and −) are always produced in equal quantities.

† Small jars connected in this way are often fitted to the frame of the machine. This is an example of condensers joined in *series* or "*cascade*." The Leyden battery is an example of condensers joined in *parallel*.

(2) Introduce the charged body and let it touch the bottom of the vessel, observe that the divergence of the leaves is maintained without a blink at the moment of contact. Remove *B*: the divergence is unaltered. Discharge the canister by touching it: disconnect it from the electroscope. Place *B* in contact with the electroscope: observe that the leaves remain closed; hence the body has been completely discharged.

The experiment shows that the maximum induced charge equals the inducing charge, for the "bound-charge" just neutralises and is neutralised by the inducing. If either had been in excess the divergence of the leaves would have altered.

(3) (i) Hold the charged body, *B*, in the canister, but do not let it touch: the leaves diverge. (ii) Touch the canister for a moment with the finger: the leaves completely collapse. (iii) Lift the body, *B*, slowly out of the canister: the leaves diverge more and more as *B* is withdrawn further—the divergence is due to the freeing of some of the bound-charge. (iv) Replace *B* in the vessel: the leaves collapse, and remain so whatever the position of *B* (below the dotted line). (v) Touch *B* against the inside of the vessel, the leaves remain collapsed. (vi) Withdraw *B*: the leaves remain collapsed—hence the canister is unelectrified. (vii) Test the body, *B*, by placing it on the disc of the electroscope—it is unelectrified.

The experiment shows that the inducing charge and maximum bound-charge just neutralise one another.

Exp. 227.—*Obtain on a canister a quantity of electricity equal to that on a non-conducting electrified rod.* 1. Place the electrified end of the rod inside, but not touching, a deep canister. Touch momentarily the outside of the canister by the finger, then remove the rod. The canister will be charged with a quantity of electricity equal to that on the rod, but of the *opposite* kind.

2. *To get an equal charge of the same kind* as that on the rod, support a canister, *A*, inside, but not touching, a larger one, *B*. Hold the electrified end of the rod inside but not touching, the smaller canister, *A*. Touch both

canisters simultaneously and for a moment, then remove the excited rod. Next touch momentarily the outer canister. Finally withdraw the inner one. The outer canister is now charged as required.

185. Condensers.—A *plate condenser* (sometimes called a *fulminating* or *Franklin's pane*) consists of two sheets of tinfoil (called the *coats*) separated by a sheet of insulator, *e.g.* a glass plate (see § 233).

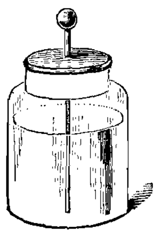


Fig. 164.

A Leyden jar (Fig. 164) consists of a jar, usually of glass, coated inside and outside with tinfoil. A metal rod terminated by a knob is connected (by a chain) with the inner coat. *To charge the jar* (i) earth-connect the outer coat: *e.g.* stand the jar on a wire joined to gas or water pipes, or hold it in the hand by the outer coating. (ii) Connect the inner coat by a wire or chain to, or place the knob in contact with, the prime conductor of an electrical machine.

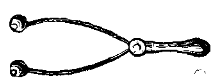


Fig. 165.

To discharge the jar. Use the *discharge tongs* (Fig. 165). These consist of two rods hinged together at one end, and terminated by knobs at the other ends. They are held by a glass handle. Separate the knobs sufficiently. First place one in contact with the outer coat, then bring the other into contact with the inner coat.

CHAPTER XIV.

ELECTROMAGNETICS, ELECTROLYSIS, ELECTROMAGNETIC INDUCTION.

VOLTAIC CELLS.

186. Voltaic cells.—Descriptions of common types are given below. For details of the chemistry of the cell a text-book of Electricity should be consulted.

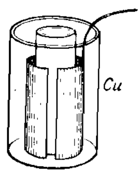


Fig. 166.

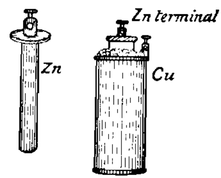


Fig. 167.

Daniell cell.—*Portable form* (Fig. 167). This consists of a copper outside vessel† in which a saturated solution of copper sulphate is placed; a porous pot in which dilute sulphuric acid or a semi-saturated solution of zinc sulphate is placed; a rod of zinc (amalgamated, see § 188) held in the acid by a wooden plug that loosely fits the mouth of the porous pot. A terminal is fixed to the zinc, another to the copper. Round the inside of the copper vessel a shelf is often fitted. Crystals of copper sulphate are placed on this: they serve to keep the solution saturated.

† Sometimes the external vessel is of glass or earthenware. There is then a cylindrical plate of copper (Fig. 166).

NOTE.—The constancy of a Daniell cell during an experiment is better ensured if it is kept short-circuited (§ 190) for a little time before it is used in the test.

Leclanché cell (Fig. 168).—A plate of carbon is placed in a porous pot and packed round with black oxide of manganese (MnO_2) and pieces of carbon. This is put in

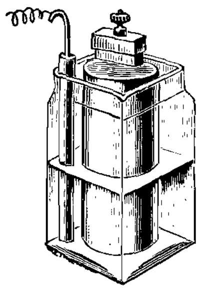


Fig. 168.

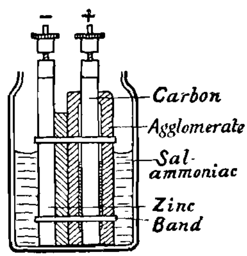


Fig. 169.

an outer glass vessel, which also holds an amalgamated zinc rod and solution of sal-ammoniac (NH_4Cl). Fig. 177 shows four Leclanché cells joined in parallel.

Leclanché agglomerate form (Fig. 169). The pot and packing are replaced by porous blocks composed of carbon, black oxide of manganese, with a little shellac, and potassium sulphate. The pieces are kept together by rubber bands. A zinc cylinder is sometimes used instead of a rod (Fig. 170).

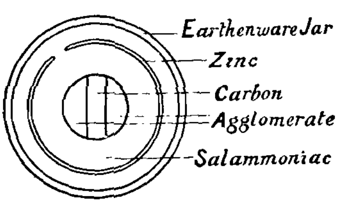


Fig. 170.

Dry cells. These are modifications of the Leclanché, moist pastes being used instead of liquid. The E.C.C. or Burnley form, shown in section in Fig. 171, is typical. A carbon rod, *C*, is surrounded by a black paste, *B*, of manganese dioxide, carbon, zinc chloride, and sal-ammoniac. Outside this is an enveloping layer, *W*, of a white paste of plaster of Paris, flour, zinc chloride, and ammonium chloride; next a zinc cylinder, Zn, and finally the whole is encased in a cardboard tube, which insulates and protects the cell. The top is closed by pitch through which a tube passes (this permits gas to escape from the cell).

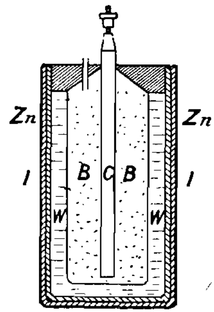


Fig. 171.

187. Bunsen cell (Fig. 172).—The carbon stick, *C*, with clamp terminal, stands in a porous pot containing strong nitric acid. An amalgamated zinc cylinder, Zn, rests in the outside earthenware jar which contains dilute sulphuric acid. Frequently the cell is flat, like Fig. 173, a thin plate of carbon being placed in the inner porous

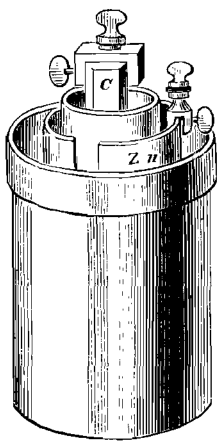


Fig. 172.

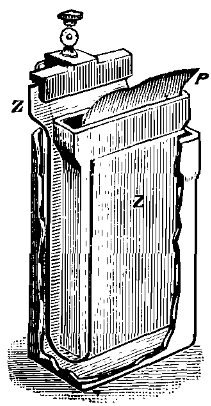


Fig. 173.

pot, and the plate of zinc, Z, is U-shaped. (The preparation of a Bunsen battery is described in § 188.)

Grove cell (Fig. 173). This is the same as the Bunsen, except that a sheet of platinum (*P*) takes the place of the carbon.

Bichromate cell (Fig. 174). Two plates of carbon dip into the "bichromate solution" contained in a vessel, frequently a globular bottle. Both carbon plates are joined to a terminal *a*. A central amalgamated zinc plate is fixed to a rod so arranged that the zinc can be raised out of or lowered into the liquid as required. *The zinc must be out of the liquid when the cell is not being used.* The zinc is connected through the rod and holder to the terminal, *b*. The plates, terminals, etc., are fixed to an ebonite plug that fits the mouth of the bottle. The solution consists of bichromate of potash (1 part by weight), strong sulphuric acid (2 parts), water (12 parts). Bichromate of sodium or chromic acid is sometimes used instead of the potassium salt.

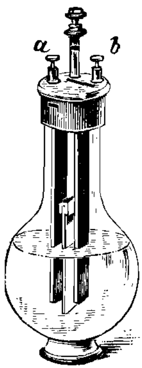


Fig. 174.

The Clark Cell.—This type of cell when carefully made according to a definite specification has an E.M.F. of 1·434 volts at 15° C. It is used as a standard of E.M.F. (For particulars see *Higher Text-Book of Magnetism and Electricity*, **141**.)

188. To prepare a cell (Bunsen).†—(1) *Clean* by a file and emery paper the parts of the *clamps and binding screws* against which plates or wires are to be fixed.

(2) *Amalgamate the zinc plate.* Nearly fill a jar with battery sulphuric acid, and immerse the zinc plate in it until effervescence begins, then remove it and lay it in the dish of amalgamation mixture. Pour the mercury over the zinc with a wooden spoon and rub until the zinc acquires a bright appearance. Then hold the zinc a little above the

† Bottles of "Battery Nitric" (Sp. Gr. 1·38), of "Battery Sulphuric" (one of acid to ten of water, by volume), and a glass dish containing "amalgamation mixture" (waste mercury and dilute sulphuric acid) should be provided. When the "Battery Nitric" has acquired a greenish-blue colour it should be replaced by fresh acid.

dish, tap it gently with the spoon, and let the mercury thoroughly drain off. Amalgamation should not be done often, it tends to make the zinc rotten.

(3) Put together the jar, porous pot, zinc and carbon plates. Fill, through a funnel, the porous pot, containing carbon, about three-quarters full, with strong battery nitric acid, and similarly the jar containing zinc with dilute battery sulphuric acid. Fix the binding screws.

Fig. 176 shows a battery of four Bunsen cells joined in series.

As the working battery produces pungent gaseous oxides of nitrogen, it should be put in a fume closet away from the experimenter and apparatus.

(4) When the cell is no longer required, it must be taken to pieces, the parts well rinsed with water (soak the porous pot for twelve hours) and set aside to dry. Put the nitric acid back into the bottle labelled "Battery Nitric," and throw away the dilute sulphuric acid (it is contaminated with nitric). Put waste mercury from the battery vessels into the amalgamation dish.

189. Secondary or Storage Cell.—This consists of two plates of lead, more or less oxidised, dipping in dilute sulphuric acid. The positive (of which the outside lug is painted red) or more highly oxidised plate is usually dark brown, and the negative (lug painted black) a slate colour.

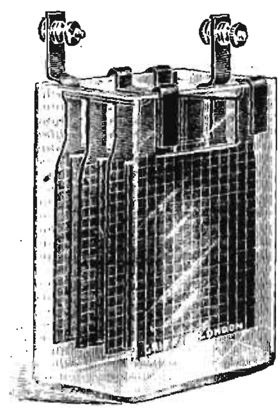


Fig. 175.

Portable cells (Fig. 175) can be obtained from manufacturers. A simple form suitable for many laboratory requirements can be made by placing a pair of plates (6″ × 2½″, "chloride" + and − : these may be bought) in a jar. Their lower ends are kept apart by a piece of wood (prism-shaped) well soaked in paraffin-wax. The projecting lead lugs at the upper ends pass through round holes in the wooden (soaked in paraffin) lid of the jar. The terminals consist of a screw-stem with fly-nuts.

Dilute sulphuric acid (Sp. G., 1·215; one part acid, five of water, by volume) is put into the jar.

The terminals of a secondary cell should be removed from the lead lugs when not in use, rinsed, and put aside in a box. If, however, the connections of a battery are not to be interfered with, the terminals should be well smeared with vaseline.

To charge a battery of secondary cells join it in circuit with a dynamo† so that current enters the secondary cell by the positive plate, and leaves by the negative. The voltage of the battery of secondary cells must be less than that of the generator charging it: *e.g.* assuming the maximum E.M.F. of a secondary cell to be 2·4 volts, a battery of 50 cells would require a generator developing more than 120 volts to charge it.

The charging current should be about one-half that which is specified as the maximum discharge current of the cell. Charging should be continued until the solution in the cell appears milky owing to the evolution of minute bubbles of hydrogen and oxygen. During charging these substances are deoxidising and reoxidising the negative and positive plates respectively, and during discharge the positives are deoxidised, the negatives oxidised. When charging is complete the E.M.F. of each cell is nearly 2·4 volts. On working, however, it soon drops to nearly two volts, and remains at that value for a considerable time. With prolonged work the E.M.F. gradually lessens, it should never be allowed to get lower than 1·7 volts. The positive plate of a secondary cell, when fully charged, is of a dark brown colour, the negative a dark slate: during discharge these colours get lighter. The specific gravity of the acid when the cell is charged is 1·21, when sufficiently discharged about 1·15.

190. Nomenclature, etc.—As soon as a cell has been built up from its constituent parts a difference of potential between the terminals is produced. The value of this, measured in volts, is called the *electromotive force* (abbreviated to E.M.F. or Emf.) of the cell. The plate at which the potential is the higher is called the *positive* (+) *pole*, the other plate is called the *negative* (−) *pole*. In diagrams a cell is generally represented by two parallel lines, a long thin one to indicate the + pole, a short thick one to indicate the − pole (Fig. 178).

As soon as the two plates are connected by a conductor, such as a copper wire, a magnetic field is produced around, and heat within the conductor: also chemical decomposi-

† It is instructive to charge one or two secondary cells by a Bunsen battery; use two Bunsens to charge one secondary, three or four Bunsens to charge two secondaries (Exp. 240).

tions occur in the liquids of the cell. These effects are considered to be associated with an *electric current* or flow of electricity. The current is considered to pass round a circuit of which two parts are distinguished, (1) the *external circuit*, consisting of the apparatus, wires, etc., joined to the terminals of the cell; (2) the *internal circuit*, consisting of the plates and liquids of the cell or battery of cells. The direction or + sense of the current is along the external circuit from the positive pole to the negative, and through the cell from the negative pole to the positive. The direction of the current is determined in the external circuit by deflecting a compass needle (Exp. 229), in the internal circuit by observing what substances are produced at the respective plates by the electrolysis of the liquids. An evolution of hydrogen, or deposition of metals is characteristic of the positive pole, while oxygen, oxides, and acids are produced at the negative.

The cell or battery is said to be on *open* circuit when its terminals are not joined by a conductor, on *closed* circuit when they are so connected: it is *short-circuited* when the connection is a conductor of very low resistance, *e.g.* a short piece of thick copper wire.

Parts of a circuit are said to be *in series* when so connected that the current which passes through one of them passes in turn through each of the others: they are said to be *in parallel* when so connected that the current in one part does not pass through any of the others (see §§ 192, 218).

191. Behaviour, etc., of cells.†—From the electrical standpoint cells are considered with regard to electromotive force (E.M.F.), resistance, and the constancy of these quantities. The values of these quantities depend upon the nature of the plates and liquids used, and upon the degree of dilution of the latter: also slightly upon the temperature. For cells of the same kind and of similar proportions the resistance of smaller sizes is greater than that of larger; and, for the same cell, the resistance is diminished if the plates are increased in size, or brought

† This section will be better understood when experiments on resistance and E.M.F. have been done.

closer together, and conversely. The resistance varies considerably with the degree of dilution of the liquids in the cell. Broadly speaking, it is increased when the proportion of water is increased.

For cells of the same kind the E.M.F. of all sizes is the same. The E.M.F. varies a little with the degree of dilution of the liquids.

Both E.M.F. and resistance alter with temperature.

A particular cell should have specified (i) its E.M.F. in volts, (ii) its resistance in ohms at ordinary temperatures. A type of cell may be specified by its approximate E.M.F. in volts, and its resistance by a descriptive term. It is only when cells are made very carefully by a detailed method that the resistances of a number of them are likely to be nearly equal in value.

Frequently the current yielded by a cell is the only quantity mentioned. The value then given is that of the *maximum current* obtained by short-circuiting the cell (§ 190). This datum is not of much advantage, for the current produced depends not only on the cell but also upon the resistance of the external circuit (§ 221).

The *value* of a cell is judged with reference to—

(1) Its initial electrical E.M.F. and resistance (§ 221).

(2) Its freedom from "polarisation," that is, the constancy of its E.M.F. and resistance.

(3) Its ability to recover from polarisation.

(4) Its ability to withstand deterioration if set aside.

(5) The emission of corrosive gases; smell; trouble in preparing it for, and keeping it at work.

(6) The cost and durability of its parts, both to begin with and to replace.

(7) Its portability.

Cell.	E.M.F.	+ pole and liquid.	− pole and liquid.	Current.†	Remarks.
Daniell	1·1 volts. Constant (no polarisation).	Copper in a saturated solution of copper sulphate.	Amalgamated zinc in dilute sulphuric acid or semi-saturated zinc sulphate.	Very constant for a long time. Strength small.‡	Useful in telegraphy, etc. Deteriorates slightly.§
Leclanché Dry	1·4 volts. Inconstant. Recovers when not used.	Carbon and manganese dioxide.	Zinc in a solution of ammonium chloride.	Inconstant. Moderate strength to begin with.‡	Very useful for intermittent work (testing, bells, etc.). Deteriorates very slightly.§
Bunsen Grove	1·9 volts. Constant (little polarisation).	Carbon in strong nitric acid. Platinum in strong nitric acid.	Amalgamated zinc in dilute sulphuric acid.	Constant. Considerable strength for some time.‡	Deteriorates.§
Bichromate	2 volts. Inconstant. Recovers when not used.	Carbon. Solution of bichromate of potash and sulphuric acid.	Amalgamated zinc.	Inconstant. Considerable strength for a short time.‡	
Secondary	About 2 volts. Constant.	Highly oxidised lead (PbO_2). Dilute sulphuric acid.	Lead.	Constant. Considerable strength for some time.‡	Very useful for regular work. Deteriorates slightly.§
Clark	1·43 volts. Constant.	Mercury and mercurous sulphate.	Zinc and zinc sulphate.	Very small.‡	

† When the cell is short circuited. ‡ These particulars are necessarily indefinite (p. 290). § If put aside when charged.

192. Combinations of cells.†—Cells, usually of the *same* kind, are joined together to form batteries, the object being to attain higher E.M.Fs. or greater currents.

I. *Cells joined in series* (Fig. 176). The negative of the 1st cell is joined to the positive of the 2nd, negative of 2nd to positive of 3rd, etc. The positive of 1st and negative of last are the free terminals to which the external

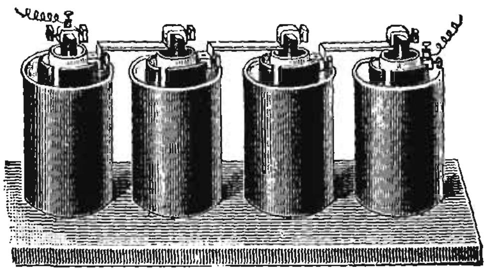


Fig. 176.

cuit is joined. *The E.M.F. of the battery is the sum of E.M.Fs. of the constituent cells. The resistance of the battery is the sum of the resistances of the constituent cells.*

II. *Cells joined in parallel* (sometimes called *multiple arc*) (Fig. 177). Each cell must have the same E.M.F. The positives of each cell are joined together, also the negatives. The external circuit has one end connected with the positive group, the other end with the negative. *The E.M.F. of the battery is the same as that of one cell. The resistance of*

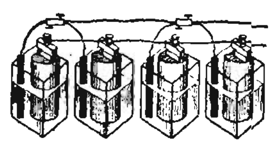


Fig. 177.

† To understand this section the student should be familiar with Ohm's law (§ 216).

the battery is equal to the resistance of one cell divided by the number of cells.

n Cells each of the *same* E.M.F., E, and resistance, B.	In *series.*	In *parallel.*
Battery E.M.F. Battery resistance Current from battery	$n.E$ $n.B$ Current through one cell	E $B \div n$ $n \times$ (Current through one cell)

III. *Cells joined partly in series, partly in parallel.* Let the cells, each of E.M.F., E, and resistance, B, be divided into a number, y, of groups, each group consisting of x cells. Let the cells of each group be joined in series, the groups themselves in parallel. Of each group the E.M.F. $= x.E$, the resistance $= x.B$. Therefore of the whole battery the E.M.F. $= x.E$ and the resistance $= x.B/y$. The current from the whole battery $= y \times$ (current through one cell).

Fig. 178 shows 12 cells, in 3 groups each of 4 in series. E.M.F. of battery $= 4E$, resistance of battery $= 4B/3$.

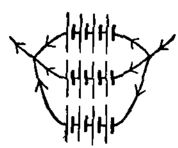


Fig. 178.

To obtain the maximum current through a given resistance (R) from a certain number of cells, they should be joined so that the internal resistance, $x.B/y$, equals the external resistance, R, as nearly as possible.

When R is high compared to B, join the cells in series, when low join them in parallel. For intermediate values adopt a mixed arrangement. Calculate from $x.B/y = R$ and $x.y =$ *number of cells.*

193. Resistance.—As regards their capability of discharging electrified bodies substances are classified into insulators, partial conductors, and conductors. In dealing, however, with electric currents, the various conducting substances are found to differ considerably in conductivity. It is usual to express the degree of obstruction offered by conductors to the flow of electricity by speaking of their greater or less resistance. For wires the resistance increases as the length increases, diminishes as the cross-section of the wire increases, and depends on the material.

The following are average values of the resistances, compared with silver, of conductors of the same length and cross-section:—

Silver	1.	Mercury	64.
Copper	1·04.	Bismuth	73.
Iron	6·6.	Carbon	about 3,000.
German silver	13 to 20.	Water	more than 1,000,000.
Platinoid	about 23.	Wood (dry)	more than 10,000,000.
Manganin	about 29.	Glass (cold)	more than a billion.

194. Rheostats, etc.—Wires differing in length, cross-section, and material are introduced into electric circuits to control the strength of current. Such wires are, when fine, insulated and wound on reels, their ends are joined to terminals (Fig. 181); when coarse, the bare wire is laid over a board (Fig. 221) or wound in a spiral and stretched across a frame. A *rheostat* or *regulating resistance* is a conductor so arranged that the amount of it through which the current flows can be readily altered. *Resistance boxes* (§ 219) are rheostats in which definite amounts of resistance may be introduced or withdrawn as desired.

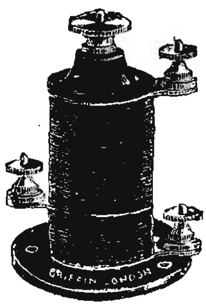


Fig. 179.

Varley's Carbon Rheostat (Fig. 179). This consists of a number of discs of carbonized cloth, strung on a vertical rod. Three brass plates with terminals attached are introduced, one at the top, one at the bottom, and one intermediate. The connecting wires of the circuit are joined to two of these, current may then be passed through the cloth. A thumb-screw working at the upper end of the vertical rod presses the cloth discs together. By tightening, the resistance is diminished; by relaxing, it is increased.

When the current is likely to be considerable carbon plates in a box or frame are used. These are pressed together, more or less, by a screw that can be turned by hand.

Liquid rheostats are satisfactory. In these plates of copper are immersed in a solution of copper sulphate, or plates of zinc in a solution of zinc sulphate.

Fig. 180 shows a convenient arrangement: A wood strip, *A* (soaked in paraffin), rests on a glass vessel. The central part of the wood is cut away. Two terminals at *B* are fixed to a copper strip: similarly a pair at *C*. Each copper plate is supported by a stout wire which passes through and is clamped by one of the terminals. The connecting wires of the circuit are joined to the second terminal of each pair. Put copper sulphate solution in the glass. The resistance between the copper plates can be increased by increasing their distance from each other, or diminishing the depth to which they are immersed, or diluting the solution with water. For large currents a long wooden trough (which should be lined with pitch) is used. A copper plate is fixed at one end; a second plate can be adjusted at any position between the ends.

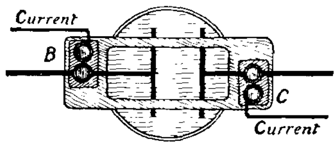


Fig. 180.

195. Accessories, etc. — 1. *Connecting wires.* Stout copper wire covered with cotton or insulating material: *e.g.* bell-wire; gauge 18 (0·048 inch diam., 0·0018 sq. in. section) is a useful size; this will carry five ampères without undue heating; resistance per 1000 feet = 4·36 ohms. When larger currents have to be used join two or more wires of equal length in parallel. The insulating material round a wire is removed (by scraping with a blunt knife) from the ends of wires over a length of about $\frac{3}{4}$ inch. The bare metal ends are clamped by binding screws, etc.

2. *Binding and connecting screws* (Fig. 181). These are of various shapes and sizes: *a* is more suitable for joining thin wires to instruments, *b* for thick (the hole prevents slipping): Fig. 182 shows a connector for two wires.

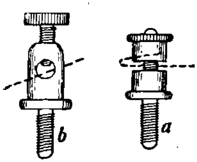


Fig. 181.

3. *Tap or Press key* (Fig. 183). The terminals, *T*, *T′*, are screwed to metal strips, *B*, *B′*, of which *B* is "springy." The base, *C*, is of wood or ebonite. (Ebonite is used when high insulation is required.) The ends of the connecting wires are joined

to the terminals. On pressing *H* with the finger, the break at *D* is closed and current flows through the key. When the finger is removed the piece *B* springs back and breaks the circuit. Thus the tap key is useful when current is required for a very short time.

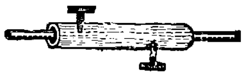


Fig. 182.

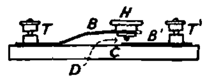


Fig. 183.

4. *Plug key.* The terminals are screwed to metal plates mounted separate from one another on a wood or ebonite base. A plug (Fig. 184) can be pushed into the gap between the plates. When the plug is out the circuit is broken, when pushed in, it is made and current flows until the plug is removed. Thus the plug key is useful when a current has to be maintained for some time.

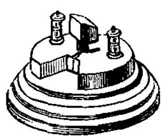


Fig. 184.

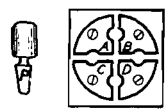


Fig. 135.

5. *Commutator:* any arrangement that permits the direction of the current in a section of the external circuit to be reversed when required. Fig. 185 illustrates a convenient form. Four metal plates, *A*, *B*, *C*, *D*, separated from one another, are fixed to an ebonite or wooden base. Each plate carries a terminal. There are two plugs, *P*, that can be pushed into the gaps between the plates. Fig. 186 illustrates a second form. A thick piece of wood has four holes, *A*, *B*, *C*, *D*, bored half-way through it. Thin sides are

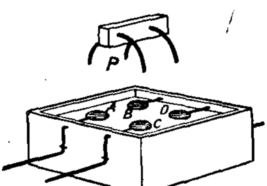


Fig. 186.

fixed to it, making a shallow box. From each hole a stout copper wire passes through the sides. These are joined into the circuit by connecting screws (Fig. 182). Mercury is put in the holes. The piece *P*, is a strip of wood carrying two copper arcs, the ends of which are wide enough to reach between adjacent holes. Then the piece may be placed so that (i) hole *A* is connected with *B*, and *C* with *D*, or (ii) *A* with *C*, and *B* with *D*. The circuit is broken by lifting the piece out of the holes.

The method of joining either commutator with the circuit is shown in Fig. 187, where *S* represents a battery, *G* a galvanometer. Join the battery wires to one pair of opposite (not adjacent) terminals of the commutator, the galvanometer wires to the remaining pair. The arrows in the diagrams indicate the direction of the current. In I. the adjacent plates or holes *A*, *C* and *B*, *D* are joined, in II. *A*, *B* and *C*, *D*. The directions of the current in the battery section are the same in both cases, in the galvanometer section they are opposite.

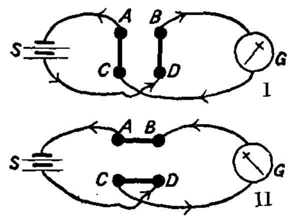


Fig. 187.

ELECTROMAGNETIC EXPERIMENTS.

Exp. 228.—*Arrange a Leclanché cell, press-key, and electric trembling bell so that the bell may be rung when the key is pressed.* Join in series the bell, press-key, and battery. Make a diagram of the bell, and describe the working of it.

The essential parts of an electric bell are the gong, the electromagnet, the keeper of the magnet to which the striker is attached, and the arrangement called a *make and break* for starting and stopping the current.

Exp. 229.†—*Relation between the sense or direction of a current and the deflection of a magnet produced by it.* (*Ampère's rule.*) Join the terminals of a cell (Leclanché or dry) each with one terminal of a tap-key. For one of the connections use a long stiff wire.

First, arrange (Fig. 188) a straight portion of the wire parallel to a compass needle in the positions indicated

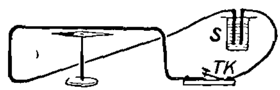


Fig. 188.

below. Then complete the circuit by pressing the tap-key.‡ Observe whether the N-end of the compass turns to the east or west, up or down. Also whether the current flows from north to south or south to north (the wire being set parallel to the compass needle will lie nearly north and south).

Wire placed	Current from	N-end of compass deflected.
Above the compass ..	North to south	
Below the compass ..	North to south	
Level with the compass on the east side §	North to south	
Level on the west side §	North to south	
Above the compass ..	South to north	
Below the compass ..	South to north	
Level on the east side §	South to north	
Level on the west side §	South to north	

† Note that Exp. 229 gives methods of determining direction of current.

‡ Or join a long wire to one terminal of the cell, arrange with regard to the compass needle, then press the free end of the wire against the free terminal.

§ A strong current is required, and the wire must be carefully adjusted near the compass.

2. Arrange a portion of the wire vertically. Adjust one end of the compass close to the wire and note whether on pressing down the key the N-end is deflected east or west.

Wire against	Current	N-end of compass deflected.
N-end of compass ..	up wire	
S-end of compass ..	,,	
N-end of compass ..	down wire	
S-end of compass ..	,,	

The relation is given by either of the rules below.

Ampère's rule.—Imagine a man to lie in the wire so that the current flows from his heels towards his head, and to face the N pole of the compass needle; then the N pole tends to be deflected towards his left hand.

Maxwell's rule.—Imagine a screw (ordinary right-handed) to lie in the wire, and to be twisted so that its end moves with the current, then a north pole tends to go round the wire in the same sense as that in which the head of the screw turns.

Confirm the results of the experiment by applying Ampère's and Maxwell's rules to each case.

Exp. 230.—(1) *Identify the parts of cells provided. Build from them the complete cells.* (2) *Determine the + pole of each cell.* (1) See §§ 186, 187, 189. (2) Arrange cell, compass needle, wire, etc., in one position of Exp. 229: observe the direction in which the N pole of the compass is deflected. Deduce the direction of the current by applying Ampère's or Maxwell's rule. Remember that in the external circuit the current flows from the + pole.

For example, if when the compass is placed above the wire its N pole is deflected east, then the man lies on his back with his head northwards: the current flows from heel to head.

Exp. 231.—*Prepare a Bunsen Battery. (Two cells in series.)* See § 188.

Exp. 232.—*Test the polarity of a solenoid.*† (1) Wind a piece of insulated copper wire (No. 18, say) round a pencil or a glass, brass, or cardboard tube (half-inch diameter, six inches long), so that the turns lie closely together (*tie with string at the ends* if necessary). Hang from a wooden clamp and connect with a Bunsen or secondary cell, *S*. It is convenient to introduce a commutator, *K*, into the circuit (Fig. 189).

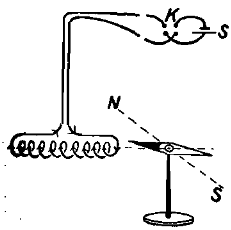


Fig. 189.

(2) Place one end of the solenoid against an end of the compass needle, complete the circuit, note whether there is attraction or repulsion of the pole of the compass, and thus determine the polarity of the end of the solenoid.

(3) Move a finger along the wire of the solenoid in the direction in which the current is flowing: look at the end of the solenoid tested in (2): note if the finger appears to move round the solenoid in the same way as the hands of a clock (clockwise) or the reverse (counter or anti-clockwise).

(4) Similarly test the other end of the solenoid.

(5) Reverse the current and again test both ends.

(6) Confirm the results by testing the ends of the suspended solenoid with the poles of a bar magnet. Observe which pole of the magnet repels one end of the solenoid.

The observations will show that when the current is circulating round the solenoid it acts like a weak bar magnet, and exhibits a north pole at one end, a south at the other. The polarity is reversed when the current is reversed. The *relation between direction of current and polarity* is given by the following Rule: If when looking

† A *solenoid* is an electrical term for the spiral obtained by winding a wire round a cylinder: it may be right-handed (Fig. 189) or left-handed (Fig. 196). The threads of common screws are right-handed. The direction of a current may be inferred from the polarity imparted to a solenoid.

at one end of a solenoid and following the current with the finger, the finger passes round (i) clockwise, then the end is a south end, (ii) counter-clockwise, then the end is north (Fig. 189).

Confirm the results by rewinding the solenoid in the reverse direction: *e.g.* if wound right-handedly at first, rewind left-handedly. The coil shown in Fig. 189 is right-handed, that of Fig. 196 is left-handed.

Exp. 233.—*Test the polarity of a solenoid with an iron core.* Rewind the solenoid of Exp. 232 round an iron rod. Repeat the tests of Exp. 232. Observe that the iron considerably increases the strength of magnetisation but does not alter the nature of the poles.

Exp. 234.—*Excite an electro-magnet.* Join a Bunsen or secondary battery of two or more cells with the terminals of a horse-shoe electro-magnet. Include a plug key in the circuit. Note that the armature or keeper is easily pulled off before the current is started, but not when the current is flowing. Also that the keeper when separated from one pole readily adheres to the other along one of its edges, and turns like a hinge about this edge.

Excite the magnet by the current, place the keeper across the poles, stop the current. Observe that the keeper now sticks to the poles. Pull it off, then replace it across the poles, it is no longer held on by them. Repeat the experiment. Also repeat it with a sheet of paper between the keeper and the poles: after the circuit is broken the keeper is found to be held with much less force than before. Repeat with double and treble thicknesses of paper.

The *experiment shows* that (i) when the keeper is in contact with the poles the residual magnetisation left after stopping the current is considerable. A large proportion of this, however, disappears when the keeper is pulled off. (ii) When there is a thin gap (the thickness of a sheet of paper) between the keeper and polar surfaces, the residual magnetisation is much less than when the keeper is in metallic contact with the poles.

Exp. 235.—*Map the magnetic field through a circular bobbin when carrying a current.* The bobbin of a tangent galvanometer (high resistance preferable) may be used. Arrange it with its plane vertical and in the magnetic meridian. Fix a sheet of paper to a board by pieces of stamp paper at each corner. Arrange the board so that the paper lies horizontally and passes through the centre of the hoop.

1. Trace the magnetic lines of force by the method described in § 167, when no current passes round the hoop.
2. Substitute a fresh sheet of paper for that used in 1. Join the terminals of the hoop to a voltaic cell (Daniell or secondary) and rheostat. Adjust the resistance so that a compass needle placed at the centre of the hoop is deflected 40° or 50° from the meridian. Then map the field thoroughly.

Explanation. The map obtained in the first case will be that of the Earth's field alone and will therefore consist of a number of parallel lines. If this is not found to be the case it will indicate disturbance of the Earth's field due to iron pipes, etc., in the laboratory.

The map obtained in the second case will be not unlike Fig. 210 as regards the general disposition of the lines, but they will lie obliquely to the plane of the coil, not perpendicular to it as in the figure, which shows the lines due to the current in the coil only. The map obtained in the experiment is that of the resultant field due to (i) the current in the coil and (ii) the Earth. The obliquity of the lines at the centre of the coil indicates the angle through which a compass needle would have been deflected by the current if it had been placed in that position.

EXPERIMENTS ON ELECTROLYSIS.

Exp. 236.—*Electrolyse copper sulphate solution.*† A glass containing copper sulphate solution is required. Join a stout copper wire to each terminal of a Bunsen cell. Dip their ends (two or three inches) parallel to one another,

† The experiment affords a means of determining the direction of a current.

near, but not touching, into the copper sulphate solution. Examine the ends after the current has passed for two or three minutes. A deposit of copper will be found on the wire (cathode) joined with the negative pole of the battery. Reverse the direction of the current through the liquid. Copper is deposited on the other wire (now joined to the zinc of the cell).

196. NOTE ON NOMENCLATURE.—*Electrolysis* is the chemical decomposition of a liquid by the electric current; the *electrolyte* is the liquid that undergoes decomposition; the *voltameter* is the vessel containing it. The *electrodes* are the plates by which the current enters or leaves the voltameter, the *positive* (+) or *anode* being that by which it enters, and the *negative* (−) or *kathode* being that by which it leaves; thus the anode and + pole of battery are connected together, also the kathode and − pole of battery. The *ions* are the products of the decomposition. Hydrogen and metallic ions are often called *electro-positive radicals*, they appear at the kathode; Oxygen, acid radicals, and the halogens are electronegative and appear at the anode. (Note that a *voltameter* is a measurer of quantity of electricity, sometimes of current (Exp. 239); a *voltmeter* (§ 204) is a measurer of voltage, that is of difference of potential or electric pressure.)

197. Laws of Electrolysis.—Faraday showed that the *amount of decomposition of an electrolyte is directly proportional to the quantity of electricity* that has passed through it. Hence if a mass, m, of a metal (say silver, or copper) is deposited in a voltameter, the quantity of electricity

$$q \propto m,$$

or

$$m = w.q$$

where w is a constant. If a constant current, C, is maintained for t seconds, the quantity of electricity $q = C.t$.

Hence

$$m = C.w.t.$$

The constant, w, is called the *electro-chemical equivalent of the metal.*

The value of the electro-chemical equivalent is different for each radical. A relation, however, exists between them, for the electro-chemical equivalents of radicals are directly proportional to their respective chemical equivalents. Hence, if one is known, others may be deduced if the molecular weights and valencies are known (see a text-book of Electro-Chemistry).

The electro-chemical equivalent of silver has been very carefully determined: it is 0·001118 gramme per coulomb.

TABLE.

Substance.	Silver.	Hydrogen.	Copper.	Zinc.
Chemical equivalent	107·94	1	$\frac{1}{2}$(63·44)	$\frac{1}{2}$(65·38)
Electro-chemical equivalent in *grms. per coulomb*	0·001118	0·00001035	0·0003286	0·0003386

When a current of 1 ampère is maintained a coulomb of electricity flows past any section of the circuit per second. Therefore 1 *ampère* deposits 0·0003286 *gramme of copper per second* from a solution of copper sulphate.

Hence

$$\begin{pmatrix}\text{Mass in grammes}\\ \text{of copper}\\ \text{deposited}\end{pmatrix} = 0{\cdot}0003286 \times \begin{pmatrix}\text{Current}\\ \text{in}\\ \text{ampères}\end{pmatrix} \times \begin{pmatrix}\text{Time in seconds}\\ \text{during which}\\ \text{the current flows}\end{pmatrix}.$$

Then if a constant current is passed through a solution of copper sulphate for a definite time (*t secs*), and a mass (*m grms.*) of copper is deposited on the kathode, then

$$C = m/0{\cdot}0003286 \times t \textit{ ampères}.$$

If the electrolyte is silver nitrate, then

$$C = m/0{\cdot}001118 \times t \textit{ ampères}.$$

A current of *one ampère* when passed through dilute sulphuric acid decomposes 0·00009326 *grms. of liquid per second, and liberates* 0·1733 *cu. cm. at N.T.P. of mixed hydrogen and oxygen.* When the temperature is $t°$ C. and the barometric height H. mm., then the volume in cu. cm. of mixed hydrogen and oxygen is

$$0{\cdot}1733 \times 760 \times (273+t)/H \times 273 \textit{ per ampère per second.}$$

198. To arrange a voltameter for the electrolysis of water.—1. *The electrolyte.* Ordinary water is a very poor conductor of electricity, hence it electrolyses very slowly. If it is acidulated with sulphuric acid the result of the electrolysis is the same as for water alone, but the rate of decomposition is quickened considerably. An electrolyte consisting of concentrated sulphuric acid, 1 part; water, 5 parts (by volume), is the best to use—its conductivity is then greatest. For many purposes, however, the proportions, acid, 1, water, 10, are very satisfactory.

2. Fig. 190 shows a *Hofmann voltameter.* The three vertical tubes join at the bottom. The electrolyte is poured into the bulb, *B*; on slightly opening the taps at *S*, *S*, it rises in the tubes, *T*, *T*. When these are full the taps are closed. *E*, *F* are two platinum electrodes, each joined to a terminal, *t*. On connecting these to the plates of the battery, current flows between the electrodes and decomposition of the liquid occurs. When the electrolyte is acidulated water, oxygen collects in the tube over the *anode* or + *electrode*, hydrogen over the *cathode* or – *electrode*. Note that the volume of hydrogen is practically twice that of the oxygen. The identity of the gases may be verified by showing that the hydrogen burns with a pale blue flame if a lighted taper is held near when the tap is slightly opened, while the oxygen rekindles a glowing splinter of wood held similarly.

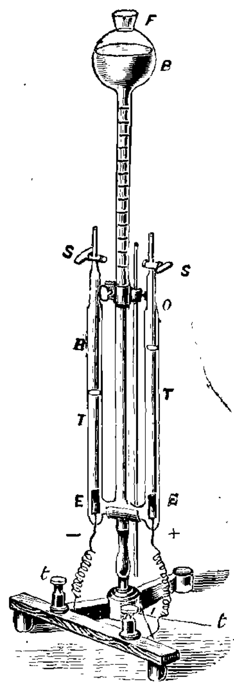


Fig. 190.

3. Fig. 191 shows a common form of voltameter. The vessel, *V*, is filled with the electrolyte. Each of the tubes, *T*, *T* (it is convenient to have these graduated into parts of equal volume), is separately filled with the liquid. The open end, closed by the thumb, is immersed in the liquid† in *V*, adjusted, the thumb being removed, over the electrode, *E*, and held in place by the crosspiece, *C*. The terminals, *t*, *t*, are joined to the battery.

† The acid, if sufficiently dilute, will do no real harm to the skin.

4. The voltameter (Fig. 192) described below can be readily and cheaply constructed.† A wide-mouthed bottle, *B*, is fitted with a bung (india-rubber). This is bored with four holes. The electrodes are passed through two of the holes, a pipette through a third, and a short delivery tube (shown dotted) through the fourth. There are two marks,

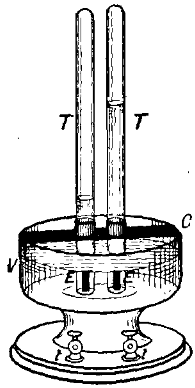


Fig. 191.

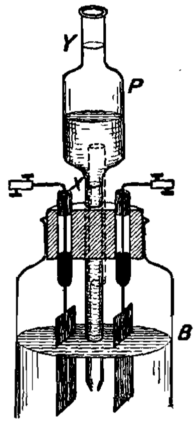


Fig. 192.

X, *Y*, one above, the other below, the bulb of the pipette. The upper end of the delivery tube is provided with a piece of rubber tubing and a pinch cock so that the interior of the bottle may be opened to the air or closed as required. The bottle is filled about three-quarters full with the electrolyte.

To make an electrode (two required) take a strip of platinum (1″ by $\frac{3}{8}$″) and weld (see Appendix) it to a piece of platinum wire (1″ long). Fuse the wire through a piece of glass quill tubing so that $\frac{1}{4}$ inch of the wire projects into the tube. Pass the tube through a hole in the bung, put into it a little mercury. Bend a length (3″) of bare copper wire at

† A convenient and simple modification of a common form. The mercury connections are very satisfactory. Voltameters like Figs. 190, 191 are usually not well fitted for meeting the wear and tear of laboratory life.

right angles. Arrange one end to dip in the mercury in the tube. Fix the wire (not tightly) by wedges of soft wood (match ends) and wax.

Method of measurement. Close the delivery tube. Join the electrodes by connectors to an electric circuit. The gases produced by the electrolysis accumulate in the bottle and the liquid is forced up the tube of the pipette. Note the time in seconds taken by the liquid surface to travel from the mark below the bulb to the one above.

Exp. 237.—*Faraday's law* (§ 197). *Show that if a constant current is maintained through a water voltameter the volume of hydrogen or oxygen produced is proportional to the time during which the current flows.* Join in series a Bunsen or secondary battery (2 cells), S (Fig. 193), a water-voltameter, V, having graduated (say in cu. cm.) collecting tubes, a carbon rheostat, PR, and plug-key, K (plug out).†

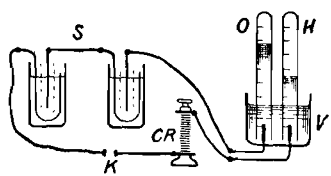


Fig. 193.

(1) Complete the circuit by pushing in the plug of the key, adjust the carbon rheostat until there is a rapid production of gas (say 5 cu. cm. of hydrogen per minute). Break the circuit. Refill the tubes with acidulated water.

(2) Place a watch where the seconds hand can be readily seen. Start the current through the voltameter just when the seconds hand is beginning a minute. Note the number of seconds that have elapsed when, say, 10, 15, 20, 25, 30, 35, 40 cu. cm. of hydrogen have been collected, and 6, 8, 11, 13, 16, 18, 21 cu. cm. of oxygen.‡

Tabulate (i) volumes of hydrogen, (ii) time in seconds; (iii) volumes of oxygen, (iv) time in seconds. Plot on the same sheet (1) volumes of hydrogen with regard to

† Note that the direction of current through the circuit may be inferred by identifying the gas liberated at either electrode.

‡ The readings for oxygen can thus be taken between those for hydrogen.

time, (2) volumes of oxygen with regard to the same time axis.

In each case the graph will be practically straight, and for any abscissa the ordinate of the hydrogen graph is twice that of the oxygen. Each graph passes through the origin. The results are thus in agreement with Faraday's law of electrolysis.

Exp. 238.—*Resistance of conductors.* **1.** Join in series two Bunsen or secondary cells, a water-voltameter, and plug-key (plug out). Make a mark on a collecting tube five or six inches from the top. (Bind a piece of fine copper wire round it, or a graduated tube may be used.) Fill the voltameter and collecting tube with the electrolyte (water, 5 by vol., sulphuric acid, 1). Place the tube over the cathode. Start the current through the voltameter at a definite time. Note the number of seconds taken for the tube to fill to the mark with hydrogen. Then stop the current. (The pattern, Fig. 192, is satisfactory.)

2. Refill the hydrogen tube with the electrolyte. Place a length of German silver wire (say a yard, No. 20) in series with the battery, voltameter, etc. Again observe the time taken to collect the same amount of hydrogen as before.

3, etc. Repeat with wires of equal lengths and diameters of platinoid, manganin, etc.

The times taken to collect the same quantity of hydrogen will be different, the shorter the time the greater the current through the circuit.

The experiment shows that the strength of the current depends on the conductors in the circuit, some of these offer more resistance than others and less current flows.

Exp. 239.—*Demonstration of Ohm's law* (§ 216). Arrange in series several Bunsen or secondary cells, a water voltameter (preferably like Fig. 192), a stretched wire (Fig. 221) or resistance frame (up to 2 or 3 ohms), and a plug-key (plug out). Note the times, in seconds, taken to collect the same volume of hydrogen or of mixed gases, when various lengths of the wire are respectively in circuit: viz.

(i) the whole wire, (ii) 0·8 of the length, (iii) 0·6, (iv) 0·4, (v) 0·2, (vi) 0·1, (vii) none.

Tabulate (i) lengths of wire, (ii) times. Plot the latter as ordinates to the former as abscissae. The graph is a straight line (it does not pass through the origin).

EXPLANATION. The intercept on the axis of abscissae is the length of wire equivalent to the resistance of voltameter, battery, connections, etc. The graph indicates that times are proportional to lengths of wire, or total resistance of circuit. Since equal masses of gas are produced in each case, the quantities of electricity that have passed through the electrolyte are also equal (§ 197), and the respective currents are inversely proportional to the times. Hence the currents are inversely proportional to the lengths of wire in or the resistances of the circuit. (See Ohm's law, § 216.)

Exp. 240.—*Make a secondary cell.* The lower ends of two strips of lead are immersed in dilute sulphuric acid; each strip is joined to a terminal of a battery (two Bunsen cells in series). As the current passes the electrolyte decomposes, the lead plate joined with the carbon of the battery oxidises (it acquires a brownish appearance), and hydrogen bubbles off from the other plate. The cell may hence be regarded as a voltameter for electrolysing acidulated water with lead electrodes.

Pass the current for some time, then disconnect the battery. Arrange wires, etc., in one position of Exp. 229, join up with the lead plates. Observe the deflection of the compass needle, and deduce the direction of the current. It will be found that the current from the secondary cell passes round the external circuit from the oxidised plate to the unoxidised one. Thus the current from the secondary on discharging passes through the cell in the opposite direction to that of the charging current from the Bunsen battery. Hence the E.M.F. developed in the secondary cell during charging is in the opposite sense to that applied by the battery, *i.e.* it is a *counter-* or *back-E.M.F.*

199. Copper voltameter (see Exp. 243).—(1) Fig. 194 shows a form of copper voltameter. The kathode, *K*, is a strip of moderately thin copper. The anode consists of two thick copper strips, *A*, *A'*, fixed to a wooden fork. A copper wire is soldered to each strip, the ends are bent round the fork and twisted together. A connecter, *C*, joins the anode with the + pole of the battery. A stiff wire is also fixed to the kathode; the end of this is clamped by the terminal, *T*, so that *K* is supported between *A* and *A'*. (In the lower figure *A'* is only partly drawn in order to show the position of *K*.) The wooden fork rests on the top of a wide-mouthed earthenware jar or glass, into which the electrolyte is put. The copper plates should be rounded at the corners and edges.

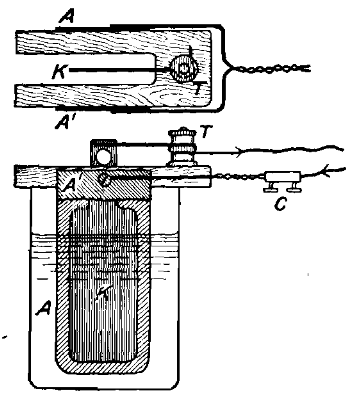


Fig. 194.

(2) The *copper plates.*—(i) Before use thoroughly clean† the copper plates with glass paper. Then dip them for, say, half a minute in moderately strong nitric acid (conduct the operation in a fume closet), lift out and immerse in a jug of tap water, finally rinse in distilled water. Do not touch the plates with the fingers; always interpose a doubled strip of paper when it is necessary to handle them. Lift them by the attached wire, or by a copper hook introduced into the hole in the projecting lug. (ii) When the kathode is removed from the copper sulphate solution, plunge it at once into water very slightly acidulated with sulphuric acid (this prevents oxidation of the copper), then immerse it in a jug of tap water, finally rinse with distilled water, drain, and lay it on filter paper. Hold the plate in the warm air two or three feet above a Bunsen burner; then allow it to cool. The area of the kathode surface must not be less than 25 (it is well to allow 50) square centimetres per ampère passing: if it is less, the copper adheres to the plate poorly.

(3) *The electrolyte* is prepared by adding one per cent. by volume of strong sulphuric acid to a solution of density 1·18 (about) of recrystallised copper sulphate in tap water.

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† Unless well cleaned the copper deposit will not adhere. There should be no trace of brown oxide on either side.

ELECTROMAGNETIC INDUCTION.

Exp. 241. *On Electromagnetic Induction.* PRELIMINARY. (1) Wind a length (say ten yards) of cotton-covered copper wire to form a ring-like coil (about an inch diameter). (2) Arrange a sensitive galvanometer (preferably a moderately dead-beat reflecting instrument), adjust the index near to the centre of the scale. (3) Join, say, the left terminal of the galvanometer with the negative pole of a cell. Touch the other terminal lightly with a wire connected to the positive of the cell. Observe whether the index moves to the right or left.† Record the direction in which the index moves, and the terminal at which the current leaves the galvanometer. (4) Disconnect the cell. Move a bar magnet to and fro at a distance of three or four yards from the galvanometer: observe whether the galvanometer index is affected. If so, find a position further off where the magnet has no influence. (5) Clamp the coiled wire at the position thus determined, and by long wires join its ends to the galvanometer terminals.

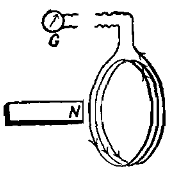
I. (1) Quickly push nearly half the length of the magnet into the coil: note whether the index moves to the right or left.† Observe that the index presently comes to rest in its original position, thus showing the induced current of electricity that produces the deflection to be *momentary*, to exist during the movement of the magnet. (2) Now withdraw the magnet rapidly, and observe that the deflection is in the opposite direction to the previous one. (3) Repeat the operations, but move the magnet more slowly. Note that each deflection is in the same direction as before, but is smaller. (4) Next repeat the operations, using the other end of the magnet.

Finally, determine for each case, by applying the rule of Exp. 232, the polarity, due to the induced current, of the face of the coil towards or from which the magnet is moved. If, for instance, the preliminary test (3) shows that the index moves to the right when current leaves the

† With an indicator like that of the galvanometer shown in Fig. 205 describe the deflection as clockwise or counter-clockwise.

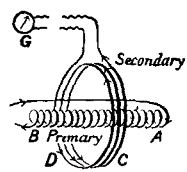
galvanometer at the left-hand terminal, and the observations of I. show *that the current induced when the* N-end of the magnet approaches the coil (Fig. 195) produces a deflection to the right, then, on moving the finger along the wire of the coil in the direction of the current, it will be observed that it passes counter-clockwise round the near end of the coil. Therefore the induced polarity is north. Hence there is repulsion between the coil and the approaching N-pole. Record as in the table below.

	Operation.	Deflection.	Polarity of face of coil.
1	N-end approaching	(right or left)	(north or south)
2	N-end receding		
3	S-end approaching		
4	S-end receding		



Magnet approaching Coil

Fig. 195.



Primary current increasing

Fig. 196.

II. Join one or two cells (Bunsen or secondary) in series with a plug key (plug out) and a solenoid† (spirals, ½-inch diameter. Wind insulated copper wire on a brass or glass tube into which an iron rod can be slipped). Complete the circuit through the plug key, and determine the polarity of the primary solenoid. Next push about half the length of the primary through the secondary coil, and note the direction of deflection: also on withdrawing. Repeat, using the other end of the primary. Also when the solenoid has an iron core. In each case determine the

† The solenoid joined with the battery is called the *primary*, that joined with the galvanometer is called the *secondary*.

polarity of the face of the secondary coil towards or from which the primary is moved. Record as in table below.

	Operation with primary.	Deflection.	Polarity of face of secondary coil.
1	N-end pushed in	(right or left)	(north or south)
2	N-end pulled out		
3	S-end pushed in		
4	S-end pulled out		
5 etc.	[When the primary has an iron core the deflections are the same in direction as above, but are of much greater magnitude.]		

III. Arrange the primary (withdraw plug from key) so that the secondary lies over one end of the primary. Note the direction of deflection when the current is (1) made, (2) broken. Determine the polarity of an end of the primary, and the face of the secondary nearer to it. Repeat with the other end of the solenoid. Record as below.

In Fig. 196 the end, *B*, of the primary is a south pole, that of the nearer face, *D*, of the secondary is a north pole.

	Operation with primary.	Deflection.	Polarity of face of secondary.
1	Current made so that the end, *A*, of primary is an N-pole	(right or left)	(north or south)
2	End, *A*, of primary an N-pole (current broken)		
3	Current made so that the end, *A*, of primary is an S-pole		
4	End, *A*, of primary an S-pole (current broken)		

Hence in an induction coil (§ 202) the induced currents are alternately in opposite senses as the primary circuit is made and broken. When, however, the coil is provided with a condenser, only the induced current at break discharges across the spark gap.

IV. Arrange as in III. Close the circuit. Note the direction of deflection when the soft-iron core is (1) slid in, (2) withdrawn. Determine as in III. the polarity of the end of the primary and the face of the secondary nearer to it. Repeat with the other end of the solenoid. Record as below.

	Operation with primary.	Deflection.	Polarity of face of secondary.
α	Current flowing so that the end of primary is an N-pole		
1	Iron core introduced.. ..		
2	Iron core withdrawn ..	(right or left)	(north or south)
β	End of primary an S-pole ..		
3	Iron core introduced.. ..		
4	Iron core withdrawn ..		

200. Explanation. The experiments show that the induced current flows *one way* (sometimes called the *inverse induced current*) for (1) an approaching N-pole,† (2) a receding S-pole,† (3) an N-pole increasing in strength, (4) an S-pole decreasing in strength; *the reverse way* (sometimes called the *direct induced current*) for (1) a receding N-pole,† (2) an approaching S-pole,† (3) an N-pole decreasing in strength, (4) an S-pole increasing in strength.

The induced current is in each case momentary, it is called into play while the magnet is moving or the pole changing in strength (§ 201).

The following summarises the observations of Exps. I. and II. The induced current in the secondary always magnetises it in such a way that the force called into play between it and the inducing pole of the primary is one that opposes the relative motion of the primary and secondary; that is, if the two are being separated the force is attractive: if the two are being brought together the force is repulsive (see § 201). The induced pole, for instance, is North if a North pole is approaching, it is South if a North pole is receding.

† Either of a permanent or electro magnet.

The following summarises the observations of Exps. III. and IV. The induced current in the secondary always magnetises it in such a way that it opposes the change of magnetisation of the primary;† that is, if the magnetisation of the primary is increasing in strength (by making or increasing the current or adding the iron core), then the induced polarity of the secondary is in the *opposite* direction to that of the primary, and in the *same* direction when the primary is decreasing in strength (§ 201). In Fig. 196, if the connections are such that A becomes an N-pole and B an S-pole, then the magnetisation of the secondary is related to it as below.

Coil	*Primary magnetisation increasing.*	*Primary magnetisation decreasing.*
Primary	End B, an S-pole.... End A, an N-pole.	End B, an S-pole.... End A, an N-pole.
Secondary	End D, an N-pole.... End C, an S-pole.	End D, an S-pole.... End C, an N-pole.
Polarity	Unlike poles adjacent.	Like poles adjacent.
Total magnetisation	Less than the magnetisation of primary.	Greater than the magnetisation of primary.

201. Laws of Electromagnetic Induction.—A current is induced in a circuit while the flux through the circuit is changing.

The magnitude of the electromotive force to which the induced current is due is dependent on the change per unit time in the flux through the circuit.

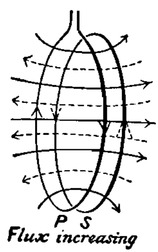


Fig. 197.

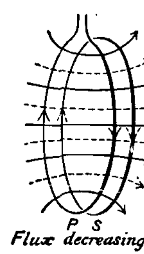


Fig. 198.

The *flux* is *the number of magnetic lines of force.* The magnitude of the induced E.M.F. also depends on the number of convolutions of the solenoid.

The direction of the induced current is such that the magnetic field associated with it acts in conjunction

† When a current is made or broken in a circuit it does not reach its maximum strength nor become zero instantaneously: a finite time is required to effect the change, and it is during this short interval that the induced E.M.Fs are developed.

(Fig. 198) with the inducing field when the latter is diminishing in intensity, in opposition (Fig. 197) to the inducing field when the latter is increasing in intensity. The following generalised statement is important:—

Lenz's Law. *If a constant current flows in the primary circuit, P, and if, by the motion of P, or of the secondary circuit, S, a current is induced in S, then the direction of this induced current is such that it tends, by its electromagnetic action on P, to oppose the relative motion of the circuits.*

Note.—It is considered that when any conductor or part of a conductor moves so as to cut magnetic lines of force a difference of potential is produced between points on it. The direction of the P.D. depends on the relative direction of motion of the conductor, and that of the magnetic lines of force. The magnitude of the P.D. depends on the number of magnetic lines cut per unit time. The total P.D. acting along any conductor is the algebraic sum of the P.Ds. induced in its various parts. To produce a current the algebraic sum must therefore not be zero. In order that the resultant E.M.F. in a closed circuit may not be zero the flux through the circuit must alter.

202. The Induction or Ruhmkorff Coil.—Fig. 199 shows a common form. The cylinder, AB, consists of a core of soft-iron wires, round this a primary solenoid of thick copper wire is coiled. The secondary solenoid is outside the primary: it is made of fine copper wire, a considerable length (ten to twenty miles in large coils) being used so that there may be hundreds of turns. The ends of the primary solenoid are connected through a commutator, D, and an automatic "make and break," S, with the terminals, a, b. The ends of the secondary are connected to brass pieces, S, S, fixed at the top of ebonite pillars. These pieces also carry discharging rods, pointed at one end and with ebonite handles at the other. The points are directed towards one another; and the distance between them—called the *spark gap*—can be adjusted. When the coil is at work sparks pass between the points if their distance apart is not too great.

The function of the "make and break" is to magnetise and demagnetise the primary rapidly and regularly. The stiff metal pillar, *c*, is joined with one of the terminals. A flexible metal strip, *d*, carries a soft-iron piece at its upper end, close to the iron core of the primary. The end of the screw, *C*, can be adjusted into contact with the flexible strip, *d*, by turning the head.

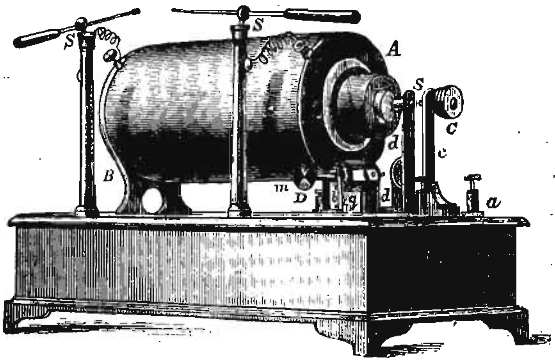


Fig. 199.

The function of the commutator is to reverse the direction of the current through the primary when desired. It is also usually arranged to set or stop the apparatus working. In the commutator shown at *D* (Fig. 199) two brass plates, insulated from one another, are fixed to an ivory or ebonite cylinder. By turning the screwhead, *m*, so that the plates come into contact with the springs, *p*, *q*, the circuit is completed. Rotating the cylinder through two right angles reverses the direction in which the current flows through the primary. Rotating it through one right angle breaks the circuit.

To set the coil working.—(1) See that the commutator is *off*, so that the circuit is broken. (2) Connect two or three Bunsen or secondary cells with the terminals, *a*, *b*. (3) Make the length of the spark gap small. (4) Adjust the head, *C*, so that the screw end touches the spring, *d*. Longer sparks may be obtained by screwing more tightly. (5) Turn the commutator so that the circuit is completed. *Remember* that the immediate neighbourhood of the spark gap is **dangerous**; do not touch the metal mountings or any part of the apparatus, except the handle of the commutator, while the coil is working.

CHAPTER XV.

GALVANOMETERS.

203. Galvanometers.—These are instruments for comparing primarily the strengths of electric currents. There are two important types of galvanometer, (1) the *moving magnet* type, in which the current is passed through a fixed coil, and produces a movement or deflection of a magnet, (2) the *moving coil* type (also called *D'Arsonval, Despretz,* or *suspended coil*), in which the current is passed through a movable coil placed between the poles of a powerful fixed magnet.

In the so-called *electro-dynamometers* the wire through which the current is passed is wound in two coils, one fixed, the other movable: there is no permanent magnet. In *hot-wire* instruments the expansion due to the heating of a wire by a current is indicated.

Galvanometers are also distinguished as (1) *pointer galvanometers*, in which a pointer is attached to the needle† and arranged so that its end passes over a scale fixed to the instrument, and (2) *reflecting or mirror galvanometers*, in which the needle† is provided with a mirror from which a pencil of light is reflected.

The method of supporting the needle gives rise to another difference. (1) A *suspended needle galvanometer* is one in which the needle is hung by a fibre of silk, or quartz, or a thin strip of metal. (2) A *pivoted needle galvanometer* is one in which the needle is provided with (*a*) an agate centre and lies on a vertical pivot, or (*b*) an axle whose pointed ends are held in jewelled centres fixed to the instrument.

† The part that undergoes deflection, whether compass or coil, may be conveniently spoken of as a *needle*.

A further classification refers to the behaviour of the needle† when deflection occurs. If it continues to vibrate about its equilibrium position for a long time the galvanometer is *periodic*; if it rapidly comes to rest the movement is *damped*; if it glides up to and assumes its position of rest without vibration the galvanometer is *dead-beat* or *aperiodic*.

Ballistic Galvanometer.—A periodic galvanometer whose needle has a comparatively large time of vibration (more than 10 seconds) is called *ballistic*, and is of value in measuring the total *quantity* of electricity discharged from a condenser. Such a needle will practically not move from its initial position during the short period of discharge, and hence receives the total impulse due to the momentary current.

A light needle may be made ballistic by loading it with a bit of lead, or weakening the magnetic field in which it swings (alter the control magnet so that the period of vibration is increased).

204. Galvanometers and galvanoscopes: Calibration.—The magnitude of the *deflection* or angle through which the needle of a galvanometer is turned depends partly on the strength of the current through the instrument. The greater the current strength the larger the deflection. There is not, however, always a simple relation between the angle of deflection and current strength. The current strength in the case of the reflecting galvanometer is proportional to the displacement or deflection of the index when small. In the tangent galvanometer the current strength is proportional to the tangent of the angle of deflection. Those instruments in which the relation rule or *law* connecting deflection and current is simple are spoken of as *galvanometers*, in distinction to others, called *galvanoscopes*, that are detectors or indicators of the existence of current.

A galvanoscope may, however, be converted into a current measurer or galvanometer by *calibration*. To effect this, currents of known value or ratio are passed through it, and the resulting positions of the pointer on the scale of the instrument noted. The observations are plotted, currents as ordinates, scale readings as abscissae. The graph obtained is called the *calibration curve*. By means of the curve the instrument may be made *direct-reading*, that is have its scale marked to indicate a unit current, its multiples and sub-multiples. This may be done by finding from the calibration curve the deflections corresponding to currents 1, 2, 3, etc., then making marks on and numbering the scale of the instrument at the values of deflection so determined. The scale is not necessarily one of equal parts.

Direct reading instruments often have a vertical face and scale (Fig. 209) and the needle turns about a horizontal axis.

An *ampère meter* (or *ammeter*) is usually a direct-reading instrument whose scale divisions are numbered to indicate *ampères* or practical units of current. When the pointer is deflected its scale reading is the number of ampères passing through the instrument. Similarly a *voltmeter* (Fig. 209) is direct-reading, it is graduated so that when a current passes through it the scale reading of the deflected pointer is the value in *volts* or practical units of the difference of potential between the terminals of the instrument.

Generally a voltmeter is a high resistance instrument and an ampère-meter one of low resistance.

Frequently a voltmeter is converted into an ampère-meter by providing it with a shunt across its terminals (§ 212). High resistance galvanometers are sometimes called *potential galvanometers* because they can be used to measure differences of potential.

205. Varieties of the moving magnet type.—In these the wire through which the current is passed is coiled into a number of loops. A stiff frame of brass or ebonite is made, round this the wire is wound evenly and closely in one or more layers. (In Fig. 205 the bobbin lies under the circular card.) Sometimes the wire is wound on two or more bobbins. A terminal is connected to each end of the wire.

Many galvanometers are wound with two independent lengths of wire on the bobbin, a pair of terminals being supplied for each wire. Usually one winding, called the *low resistance* coil, consists of comparatively few turns of thicker wire, the other, called the *high resistance* coil, of many turns of fine wire.

Differential Galvanometer. In this there are two independent windings, of equal length, of the same size of wire, wound side by side, and so arranged that when equal currents are passed simultaneously through the coils, each tends to produce equal deflections, but in opposite ways; hence the needle remains undisturbed. If one current is stronger than the other the deflection produced depends on the difference between them.

The magnet is small, and is either supported on a pivot or suspended by a fibre of silk. Regarding the first method, the friction between the magnet and pivot is a defect, and instruments in which it is adopted require gentle tapping with the fingers. Pivoted magnets are used in the less sensitive instruments, especially in direct-reading ones. In the more sensitive galvanometers the magnet is suspended by a silk fibre.

The advantage of silk fibre is that, when twisted, it has little tendency to untwist. To minimise what there is the fibre should be long and fine. Fibres, however, are easily broken.

Single needle galvanometer (Fig. 200). The magnetic needle lies entirely inside the bobbin upon which the wire is wound. If the magnet rests on a pivot it is provided with an index set at right angles to the magnet and with its end or ends projecting outside the bobbin, over a circular scale. By winding the bobbin with a large number of turns of fine wire a sensitive and useful galvanometer can be constructed.

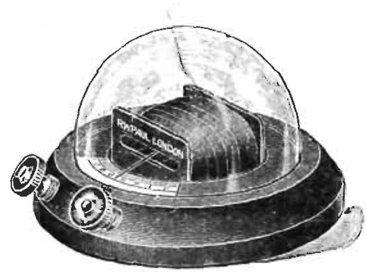


Fig. 200.

Reflecting single needle galvanometers are shown in Figs. 201, 202. The bobbin, *R*, surrounds a mirror, *m* ($\frac{1}{4}$ inch diameter), at the back of which is placed a short magnet. The mirror is suspended by a silk fibre. The optical arrangements and adjustments are described in §§ 37, 38. The large magnet *NS* is the *control magnet*; its

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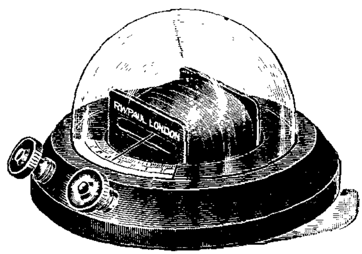

Fig. 200.

Reflecting single needle galvanometers are shown in Figs. 201, 202. The bobbin, *R*, surrounds a mirror, *m* ($\frac{1}{2}$ inch diameter), at the back of which is placed a short magnet. The mirror is suspended by a silk fibre. The optical arrangements and adjustments are described in §§ 37, 38. The large magnet *NS* is the *control magnet*; its use is explained in § 210. It can be pushed higher or lower along the vertical rod, or rotated by hand. The tangent screw, *T*, is a fine adjustment: by turning its head the control magnet can be twisted very slightly.

The *Tangent galvanometer* (described in § 208) is an important example of a single needle galvanometer.

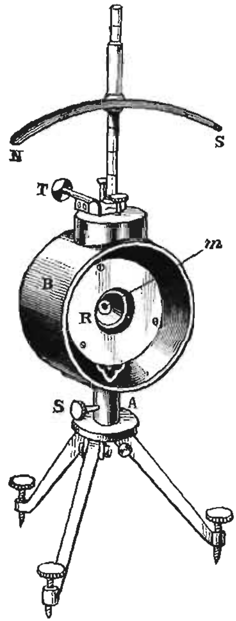

Fig. 201.

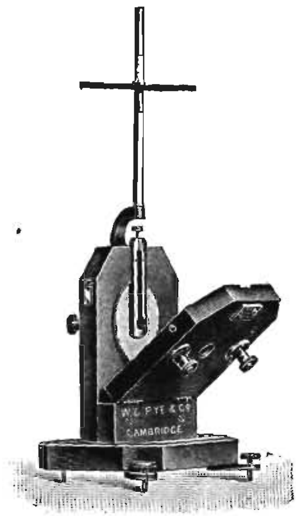

Fig. 202.

Astatic galvanometers. These are so called because the needle is a combination of two magnets forming a roughly astatic pair. The ideal astatic combination is one that would be in entirely neutral equilibrium, and would therefore not set itself along any particular line. This ideal can only be roughly realised. One method is to make two needles of practically equal pole strength and length, fix use is explained in § 210. It can be pushed higher or lower along the vertical rod, or rotated by hand. The tangent screw, *T*, is a fine adjustment: by turning its head the control magnet can be twisted very slightly.

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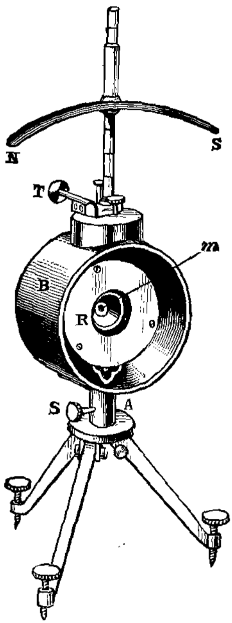

Fig. 201.

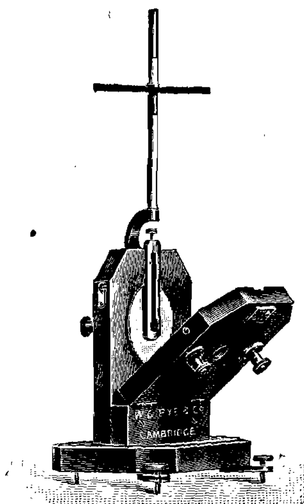

Fig. 202.

Astatic galvanometers. These are so called because the needle is a combination of two magnets forming a roughly astatic pair. The ideal astatic combination is one that would be in entirely neutral equilibrium, and would therefore not set itself along any particular line. This ideal can only be roughly realised. One method is to make two needles of practically equal pole strength and length, fix them parallel to one another with the N-end of one and

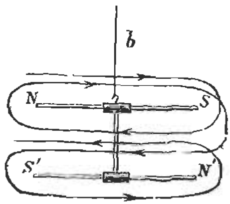

Fig. 203.

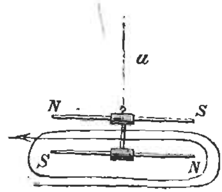

Fig. 204.

the S-end of the other pointing in the same direction (Fig. 203). The combination, after construction, will set itself along a definite line, the direction of which varies with different needles. It may, by judicious twisting of the wire that holds the needles apart, be made to set practically at right angles to or in the meridian.

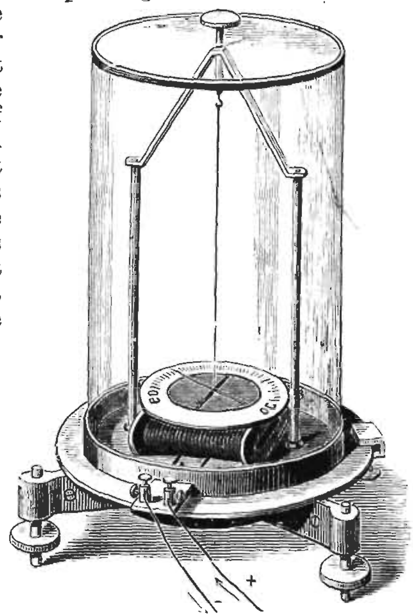
Fig. 205.

An astatic galvanometer may have one bobbin surrounding one needle (say the lower (Fig. 204)), or a bobbin surrounding each needle (Fig. 203) (often adopted in reflecting astatic galvanometers). Note that in these cases the forces due to the current in the portions of the bobbins above and below the needles all tend to deflect the combination the same way.

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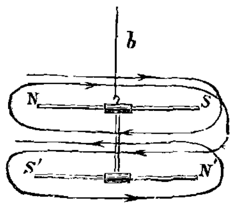
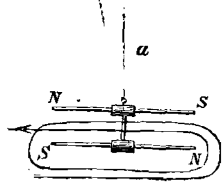


Fig. 203.



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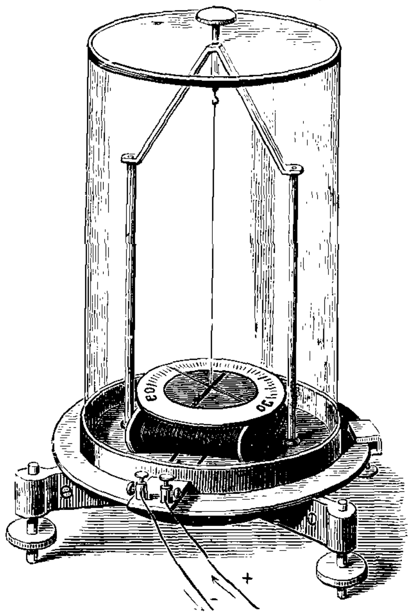

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Fig. 205 illustrates *Nobili's astatic galvanometer.* There is a single bobbin round the lower needle. The upper needle (sometimes lengthened by a light piece of wire) moves over a graduated circle. Frequently the bobbins are mounted so that they can be turned independently of the base. It is then possible to adjust the plane of the bobbins parallel to the needles without disturbing the whole instrument.

206. Moving-coil galvanometers.—These are very useful and important. Fig. 206 shows the essential features of the type. A coil of wire is suspended by a thin metal strip, *w*, between the poles, *M*, *M*, of a permanent magnet. The upper end of the rectangular coil is connected with the suspension, *w*, and thence with a terminal of the instrument. The lower end of the coil is joined to the wire,

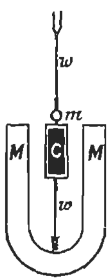

Fig. 206.

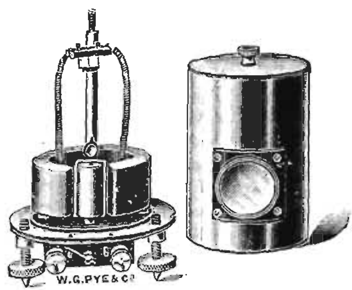

Fig. 207.

w', and spring; from this a wire passes to the other terminal of the instrument. The mirror, *m*, fixed to the coil serves in connection with lamp and scale to measure the deflection produced. *C* is a cylinder of soft-iron held within, but detached from, the movable coil. Fig. 207 is an illustration of a moving-coil instrument.

In moving-coil galvanometers the suspension is of fine metal wire or strip; this is twisted when deflection occurs. Sometimes a bifilar suspension is adopted: that is, the coil is hung by two wires placed parallel and close together (Fig. 207).

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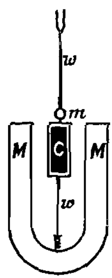

Fig. 206.

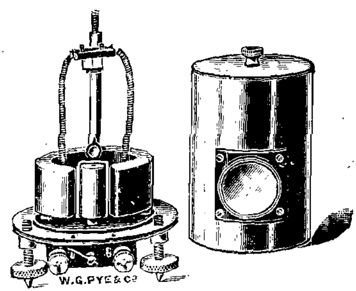

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Fig. 208 shows another pattern of reflecting D'Arsonval galvanometer. The magnet is ring-shaped, the distance between its poles is short. The suspended coil consists of a number of very narrow long rectangular loops: there is no iron core. The coil is placed inside a tube, which is of silver when the galvanometer is to be dead-beat, and of ivory when it is to be ballistic. In the figure the part containing the suspended coil is shown separately on the right. The large brass cylinder on the left is arranged to cover the magnet and fittings. (Generally this form of galvanometer should be insulated by resting each levelling screw on an ebonite disc.)

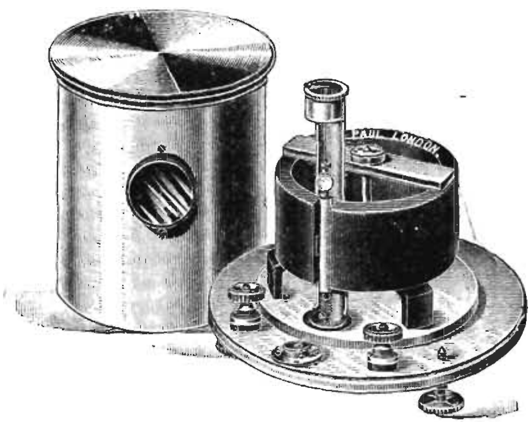

Fig. 208.

Fig. 209 shows a voltmeter of the moving-coil type. The ends of the steel horse-shoe magnet are provided with soft-iron pieces projecting inwards. The moving coil is wound on a copper frame and has a pointer attached to it. The frame is mounted on pivots working in jewels and arranged to turn about a horizontal axis: its motion is controlled by small "hair" springs. Inside the coil and concentric with it there is a fixed soft-iron cylinder. This in conjunction with the soft-iron projections on the poles of the horse-shoe magnet

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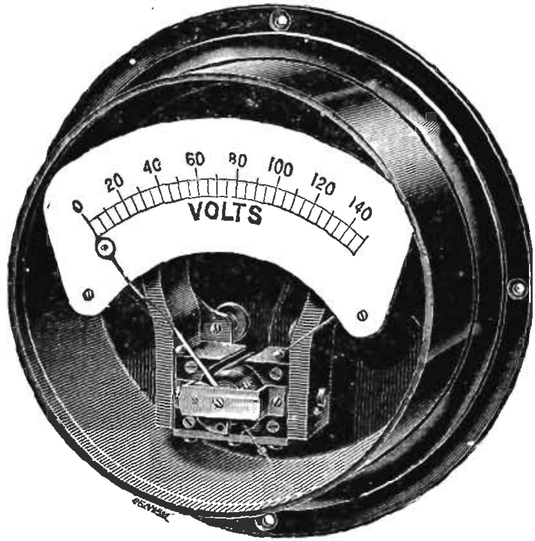

Fig. 209.

Such an instrument can also be used indirectly as an ampère-meter (§ 204).

207. Magnetic field due to a current flowing round a circular hoop.—The magnetic field produced is of nearly uniform intensity at and near the centre of the hoop, and along the perpendicular through the centre to the plane of the hoop. The width of the uniform field at the centre of the hoop is not greater than one-tenth the diameter. (Fig. 210 shows the lines of force produced by a current in a circular hoop.† The space, *a b c d*, is that of approximate uniformity.)

The *intensity of the magnetic field at the centre* of a hoop, 1 cm. in radius and carrying unit current (C.G.S.) is by definition 2π *dynes*.

When the radius is r cm. unit current gives at the centre of the hoop

$$\textit{field-intensity} = 2\pi r/r^2 = 2\pi/r.$$

† The figure is the field of the coil uninfluenced by the earth. A resultant of coil and earth is obtained in Exp. 235.

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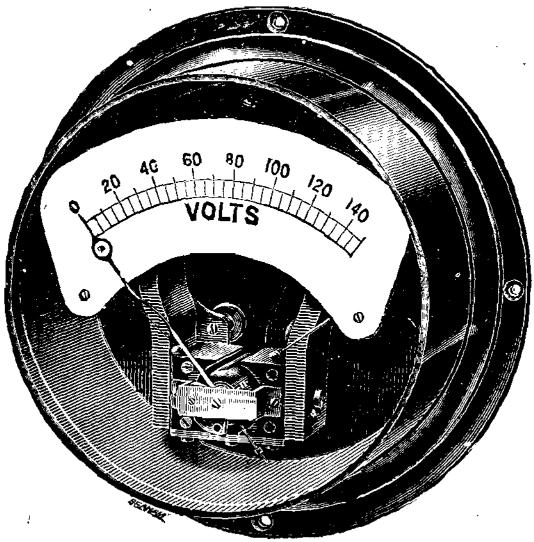

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Therefore a current of C absolute (C.G.S.) units round the hoop gives a field-intensity at the centre,

$$F = 2\pi C/r.$$

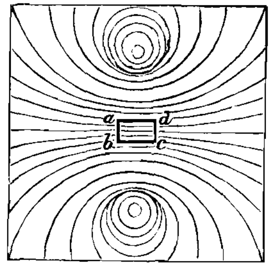

Fig. 210.

If there are n circular coils of equal radius, r, in the winding, then at the centre, *the field intensity*, $F = 2\pi n C/r$.

The direction of the magnetic field at the centre is perpendicular to the plane of the hoop. Hence when the hoop is placed parallel to the direction of a field of strength, H (that due to the earth for instance), if current passes round the hoop there will be at its centre two uniform fields superposed at right angles.

Let D be the angle between the direction of the resultant field and the one of strength, H; F, the field strength due to the current, C, round the coil,

then (§ 169) $$F = H \,.\, \tan D.$$

But $$F = 2\pi n.C/r\,; \qquad \therefore\ C = r.H \,.\, \tan D/2\pi n.$$

If the current be A ampères, since 1 C.G.S. unit of current = 10 ampères,

$$\therefore A = 10\, r.H \,.\, \tan D/2\pi\, n.$$

Now r and n depend entirely on the construction of the coil, the quantity $r/2\pi\, n$ is therefore invariable. The value of its reciprocal $2\pi n/r$ is called the *coil constant*.

208. The tangent galvanometer is constructed in such a manner that the above expressions may be applied. A simple form of it is shown in Fig. 211. The wire is wound in a circular bobbin. A magnet is placed at the centre or at some point along the perpendicular through the centre to the plane of the coil. In a tangent galvanometer the length of the magnet must not be greater than one-tenth the diameter of the hoop. Fixed to the magnet at right angles is a long light pointer of wire or aluminium ("lozenge"-shape, like Fig. 139). The ends of this move over a circle graduated in degrees, etc. A strip of mirror is usually fitted to the base of the compass-box to avoid parallax error

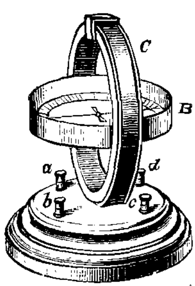

Fig. 211.

(§ 120). The figure shows several pairs of terminals fixed to the base-board. To these are joined the ends of independent coils. These may be used separately or in combination. Sometimes the compass-box is arranged to slide so that its magnet moves in a line perpendicular to the hoop and passing through its centre.

The method of adjusting a tangent galvanometer is given in § 214.

Galvanometer constant.—It was shown (§ 207) that

$$A = 10\, r H \,.\, \tan D / 2\,\pi\, n.$$

When a properly adjusted tangent galvanometer is used, D is the measured deflection of the needle.

If the galvanometer is used at one place and H is the intensity there of the earth's magnetic field, then the whole factor

$$10\, r\, H / 2\,\pi n = 1{\cdot}592\, r\, H/n$$

will be constant. The value of this is called the *working constant* or *reduction factor* of the galvanometer. Denoting it by K, then the

$$\textit{current in ampères} = K \,.\, \tan D.$$

Thus when K is constant, A is proportional to *tan* D.

Rule of the tangent galvanometer.—When the instrument is in adjustment *the strength of the current in the coil is proportional to the tangent of the angle of deflection it produces.* Thus the tangent galvanometer affords a means of comparing currents, for two currents will be in the same ratio as that of the tangents of the respective angles of deflection, or

$$C_1 : C_2 = \tan D_1 : \tan D_2.$$

The value of the constant (K) need not be known in making a comparison of currents, *provided that the position of the galvanometer is not altered.*

If the value of K is known, then the strength of the current producing a deflection, D, may be calculated, either in ampères or C.G.S. units, by assuming A ampères $= K \,.\, \tan D$. The value of K may be obtained in two ways, (1) by electrodeposition, (2) from the data of the coil and value of H.

1. **To find the constant by electrodeposition.**—Pass a constant current through the galvanometer and a copper (or silver) voltameter in series with it. Find the mass (m) of copper deposited in t seconds. Note the angle of deflection (D) of the galvanometer. Then

$$A = m/0{\cdot}000328\, t, \qquad \therefore\ K = m/0{\cdot}000328\, t \,.\, \tan D.$$

The method is not convenient if the galvanometer has a high resistance.

2.† **To find the constant from the data of the coil.**—Apply the expression

$$K = 1 \cdot 592\, r\, H/n.$$

Count the number (n) of turns, and measure the mean radius (r), then calculate $1 \cdot 592\, r/n$. Assume the value of H (it may be determined as in § 177), calculate K.

3.† The following is called an **absolute method of measuring current.** Use a well-made tangent galvanometer. Count n, measure r (mean). Determine the value of H at a convenient place in the laboratory. Set up and adjust the galvanometer at that place. Pass current through the galvanometer, note, D, the deflection produced. Then the

current in absolute (*C.G.S.*) *units* $= r\, H \,.\, \tan D/2\, \pi n.$

4. The value of H may be obtained by determining K by electro-deposition, measuring r,† and counting n. Then

$$H = K\, n/1 \cdot 592\, r.$$

209. Reflecting galvanometers.—These, whether of the moving-magnet or moving-coil type, can be used for comparing the strengths of small currents. (For the method of measuring large currents see § 212.) The moving-coil type is more satisfactory, however, because its indications are practically unaffected by small changes in the magnetic field. In moving-magnet reflecting galvanometers the tangent relation between deflection and current practically holds. In the moving-coil type the angle of deflection itself is proportional to the current. As however the angular deflections are in either case small, the displacement of the index on the scale, the angular deflection, and its tangent are proportional (§ 36), hence the rule below.

RULE OF THE REFLECTING GALVANOMETER.—*The currents through a reflecting galvanometer are proportional to the displacements of the index produced by them.*

210. Methods of control.—In the various measuring instruments it is important to consider what forces tend to produce deflection of the needle, and what tend to prevent it. The position assumed by the needle depends on the resultant of these. Generally the forces form a deflecting couple and a controlling couple, and equilibrium

† The bobbins must be accurately circular, and very carefully wound.

occurs when things have so adjusted themselves that the moments of these couples are equal.

In the moving-magnet type of galvanometer the deflecting couple is due to the current in the coils, the controlling couple to the magnetic field of the Earth or of the Earth and another magnet. In the latter case the added magnet is called *the controlling or governing magnet.* It may be arranged so that its field acts in conjunction with or in opposition to the Earth's field. Suppose it placed magnetic north and south a little distance from the galvanometer (whether above or to the side does not matter). The centre of the galvanometer magnet should lie on the line passing through the centre of and perpendicular to the control magnet. Hence when the S-end of the control magnet points northwards it acts in conjunction with the Earth's field, and therefore the controlling forces are increased. When the N-end of the control magnet points northwards it acts in opposition to the Earth's field. If now the control magnet is some way off from the galvanometer magnet its field may merely weaken that of the Earth, if it is brought near the direction of the controlling field may be reversed. REMEMBER that *a given current produces a greater deflection when the controlling field is weaker.*

In the moving-coil type of galvanometer the loops of the suspended coil should when undeflected lie roughly parallel to the lines of force between the poles of the fixed magnet. When current flows round the coil the magnetic lines of force associated with it tend to coincide with those of the permanent field, and thus deflecting forces act upon the coil. When deflection occurs controlling forces are called into play by the twisting of the suspending wire. This is an example of a *torsion control.*

In many ampèremeters and voltmeters the control is due to a hair spring like that of the balance wheel of a watch.

A controlling magnet has no effect on the suspended coil when no current is passing through it. Also when the coil is carrying current there is practically no effect, because the field of the permanent magnet is so powerful that very little change is produced in it by bringing a magnet close to its poles. This is an important advantage of the suspended-coil type, as the indications are therefore unaffected by magnetic changes in the neighbourhood, as, for instance, the running, etc., of dynamos. Instruments of the moving-magnet type are very susceptible even to small magnetic changes, and are therefore troublesome to work with when moving machinery is near.

211. Methods of damping.—A freely swinging needle may be damped by putting it in a narrow cell (§ 172) or attaching to it a small vane of mica immersed in oil, or to a larger vane exposed to the air.

Electro-magnetic damping is most satisfactory, and is effected by allowing the magnet to swing close to a piece of copper or other good conductor. In a moving-coil galvanometer the coil is often put inside a silver tube (§ 206). Electro-magnetic damping is due to the induced currents (eddy currents) set up when there is relative motion between a conductor and a magnetic field (§ 201).

212. Shunts, etc.†—Suppose the resistance of the coil of a galvanometer to be G and that when it carries a current, C_1, the index is displaced to the limit of the scale: C_1 is therefore the maximum current that can be measured by the galvanometer. Let V_1 be the P.D. at the terminals of the instrument, then $V_1 = C_1G$. Since G is constant, V_1 is therefore the maximum P.D. that can be applied at the terminals. Thus if $C_1 = 1/1000$ and $G = 200$, then $V_1 = 1/5$. The range of the instrument may be extended by using (1) a controlling magnet; (2) a shunt or resistance in parallel with the galvanometer coil; (3) a resistance in series with the galvanometer coil.

1. The *control* or *governing magnet* (§ 210).

2. *Shunts*. A galvanometer is said to be *shunted* when its terminals are joined by a wire (called the *shunt*). Fig. 212 indicates the arrangement. The galvanometer wire of resistance G is in parallel with the shunt of resistance, S. If C is the current round the circuit, and c_1 that through the galvanometer, then (§ 218) $c_1/C = S/(G + S)$. If G and S are known, then the ratio of C to c_1 can be calculated. Thus if $G = 200$, $S = 40$, then $C = 6c_1$.

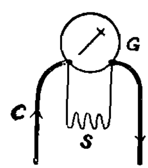

Fig. 212.

Usually a galvanometer is provided with several shunts of resistances $G/9$, $G/99$, $G/999$, etc. With these the main currents are respectively 10, 100, 1000, etc., times the galvanometer currents. If then a galvanometer indicates a maximum current of 1/1000 ampère,

† To understand this section the student should be familiar with Ohm's law, etc.

it will indicate currents in the main circuit up to 1/100 ampère, with a shunt whose resistance is $G/9$, up to 1/10 ampère with shunt $G/99$, and up to 1 ampère with shunt $G/999$. Fig. 213 shows an arrangement of resistances in a *shunt-box*. A plug is placed in one of the gaps a, b, or c, between the brass pieces. The numbers 1, 10, 100 indicate the ratio of the main current, C, to the galvanometer current. (The respective shunt resistances are ∞, $G/9$, $G/99$.)

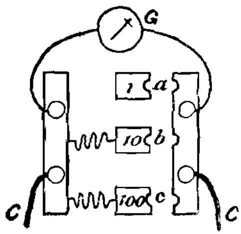

Fig. 213.

Note that the combined resistance of galvanometer and shunt is less than that of the galvanometer itself. Hence the main current will increase in value owing to the diminished resistance of the circuit. When it is desired to keep the main current constant, a *compensating resistance* equal to $G-\{GS/(G+S)\}$ must be added to the main circuit.

Galvanometer Resistance (G)			
Shunt Resistance (S) =	$G/9$	$G/99$	$G/999$
Combined Resistance = $GS/(G+S)$	$G/10$	$G/100$	$G/1000$
Compensating Resist. = $G-\{GS/(G+S)\}$	$9G/10$	$99G/100$	$999G/1000$

3. *Resistance* added *in series* with the galvanometer wire. Fig. 214 indicates the arrangement; the galvanometer coil of resistance, G, is in series with conductors of resistance, R. The ends, A, B, of the galvanometer circuit are joined with points between which there is voltage $= V$, produced, say, by a flow of current along a conductor. If c_1 is the maximum current that the galvanometer can measure, $v_1 = c_1 G$ is the greatest voltage between its terminals. Since

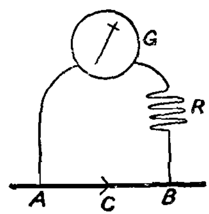

Fig. 214.

$$V/v_1 = (G + R)/G$$

if G and R are known then the ratio V/v_1 can be calculated. Thus $V = 6\,v_1$ when $R = 5G$.

If several resistances $9G$, $99G$, $999\,G$ are provided, then $V = 10\,v_1$ or $100\,v_1$ or $1000\,v_1$ respectively. Hence if the maximum reading of the galvanometer is 1/10 volt it will indicate up to 1, 10, or 100 volts when these resistances are respectively joined in circuit.

213. Adjustments and tests of a galvanometer.—1. Join the appropriate parts of the circuit to the terminals of the instrument, which should be placed away from the rest of the apparatus. Twist the wires leading to it loosely together.

2. Generally a pointer instrument is so constructed that the needle is in its proper position with regard to the coil when the indicator is over the zero. Hence adjust so that the needle swings freely and the index rests over the zero of the scale. (This is not always necessary.) Levelling and other screws are provided so that the adjustment may be effected.

In moving-magnet instruments since the needle tends to set itself along a definite line, *e.g.* the magnetic meridian, the coils and scale must be adjusted to the needle (§§ 205, 214).

In moving-coil instruments the head to which the suspending wire is attached can be twisted, and thus the coil adjusted until the index is in position. (In Fig. 20 turn the milled edge at the top of the long tube.)

When the needle is pivoted the instrument should b[e] *[g]ently* tapped with the finger whenever the position o[f] [t]he pointer is to be observed.

For the optical adjustment of a *reflecting galvanometer* [s]ee § 37.

3. When an instrument is provided with several independent coils, [t]he relation between the windings and terminals should be examined. [N]umber the terminals 1, 2, 3 .... from left to right. Join 1 to the + pole of a cell, join 2, 3 .... in turn to the − pole: observe the [v]alues and directions of the deflections of the needle. Next join termi[n]al 2 to the + pole, and 3, 4 .... in turn to the − pole: observe the [d]eflections; and so on until all have been tested. Record the results [a]nd deduce the arrangement of the windings.

4. *Test whether the circuit* external to the galvanometer *[a]ffects the needle* of the instrument. Join both wires lead[i]ng from the circuit to *one* of the terminals of the galvano[m]eter. Observe the position of the pointer: then complete [t]he circuit. If the pointer moves, the connecting wires and [p]ositions of apparatus must be altered and the test [r]epeated until there is no effect on the magnet.

214. Adjustments and tests of a tangent galvanometer.—1. Join in series (Fig. 187) a constant battery, resistances, commutator, and galvanometer.

2. *Set the hoop* roughly *in the magnetic meridian* by arranging the instrument so that the pointer of the needle lies with its ends over the zeros of the scale.

When the magnet is pivoted the compass box should be gently tapped with the finger whenever the position of the pointer is to be observed.

3 and 4. See 3 and 4, § 213.

5. *Set the hoop accurately in the magnetic meridian.* Adjust the resistance of the circuit so that the current produces a deflection of about 45°. Read both ends ($=D_1, D_2$) of the pointer. Reverse the commutator and again read both ends ($=D_3, D_4$) of the pointer. If the readings differ by more than 1°, slightly turn the hoop until they agree as closely as possible.

If the readings persistently differ by more than 1°, the instrument is probably out of adjustment; a responsible person should then be consulted.

6. *To find the deflection produced* when a current is passed through the coil, read the positions (D_1, D_2) on the graduated circle of both ends of the pointer, then reverse the commutator and again read both ends of the pointer (D_3, D_4). The mean $D = \frac{1}{4}(D_1 + D_2 + D_3 + D_4)$ is the value of the deflection produced.

7. In working with a tangent galvanometer the value of resistance in the circuit should be arranged so that if possible the deflection is about 45°. If the test involves the observation of more than one deflection these should lie between 30° and 60°. In any case avoid small or large angles. The unavoidable errors of observation have at deflection 45° less influence on the results of the determination than at any other value. When the deflections are less than 20° or more than 70° these errors are relatively so considerable that the experimental results are unreliable.

8. Determine the resistances R_2, R_1 for which the deflections are about (i) 35°, (ii) 55°. ($R_2 - R_1$) is then equal (roughly) to the total resistance of the circuit ($= R_1$ + battery + galvanometer)

when the larger deflection is obtained (§ 225). Note also the E.M.F. of the battery.

Repeat for the other coils of the instrument.

The value of $(R_2 - R_1)$ is of importance when a galvanometer has to be selected for certain tests. For the comparison of E.M.Fs (§ 228) the galvanometer selected must permit large values (500 to 1000 ohms) for the resistances: in measuring battery resistance (§ 226) only small values (less than 10 ohms) are permissible.

215. Direct-reading ampère-meters and voltmeters (Fig. 209).—The faces of some instruments must be placed vertically, of others horizontally. Before using observe if the pointer is over the zero of the scale (tap the side gently), if not it will require adjustment, possibly levelling, or packing at the back, or underneath. Adjusting screws are generally fitted to the instrument.

To measure a current by an ampère-meter, join it in series with the circuit, and observe the scale-reading to which the index is deflected. The resistance of the ampère-meter should be so low that its introduction into the main circuit does not appreciably affect the current.

To measure the voltage between two points by a voltmeter, join its terminals each with one of the points and observe the scale-reading to which the index is deflected. The resistance of the voltmeter should be so high that when connected with two points on the main circuit it does not appreciably diminish the current between the two points.

Exp. 242.—*Investigate the effect of the control magnet of a galvanometer.* Join in series a Daniell cell, resistance box, tap key, and reflecting galvanometer with control magnet. Adjust the value of the resistance so that there is a small displacement of the galvanometer index when the circuit is completed through the key.

Place the control magnet close to the galvanometer. Set it so that the index is at a definite mark (near the centre) of the scale, close the key, note the displacement of the index. Observe the direction in which the N-end of the control magnet points.

Repeat with the control magnet at the far end of the rod on which it slides; also when midway between these positions.

Clearly record the observations.

The test indicates the extent to which the sensitiveness of the galvanometer may be changed by altering the strength of the control field.

Exp. 243.—*Find the constant of a tangent galvanometer by electro-deposition.* Join in series a constant cell (Daniell), carbon rheostat, copper voltameter, commutator, and tangent galvanometer (low resistance). The galvanometer must be adjusted as in § 214, and the voltameter, etc., prepared as in § 199. The method is described in § 208 (1).

First make a preliminary experiment. Join a low-reading ampère-meter in the circuit in addition to the apparatus mentioned above. Adjust resistances to give a deflection of about 50°, then note the current shown by the ampère-meter. Assuming that roughly speaking 1 ampère deposits 1 grm. of copper in 50 minutes, calculate how long the experiment must continue in order that $\frac{1}{4}$ grm. of copper may be deposited. If, for instance, the current is $\frac{1}{3}$ ampère the time required to deposit $\frac{1}{4}$ grm. will be $\frac{1}{4}(50 \times 3) = 38$ minutes nearly. Hence run the experiment for 40 minutes. Remove the ampère-meter and proceed with the experiment. It is well to pass the current round the galvanometer for half the time one way, and for the remaining time in the reverse way. This can be effected by reversing the commutator.

Exp. 244.—*Find the constant of a tangent galvanometer from the data of the coils, and assuming the value of H.* A high resistance galvanometer may be used.

CHAPTER XVI.

RESISTANCE: OHM'S LAW.

216. Ohm's Law.—This expresses the relation between the strength of the current that flows along a conductor and the difference of potential between the extremities of the conductor. The relation is that the *current along the conductor and the potential difference between its ends are proportional.* The ratio of the potential difference to the current measures the *resistance* (§ 193) of the conductor.

NOTE ON SYMBOLS. The strength of current along a conductor is conveniently represented by C, the resistance of the conductor by R, the potential-difference between its ends by V. The expression potential-difference is abbreviated to P.D. or Pd. The term electromotive force (abbreviated to E.M.F. or Emf.) is frequently used instead of potential-difference. The term electric pressure or voltage is sometimes substituted for potential-difference.

Ohm's law is an inference from experimental results and is true after the circuit has been closed for a moment or two. (If the current is alternating, its magnitude does not generally depend only on potential difference and resistance.) The resistance of a conductor is a quantity that depends on its length, and area of cross section, its temperature, and material. As long as the values of these remain unaltered the resistance is constant. The resistance is thus independent of the strength of the current through the conductor, and of the intensity of the magnetic field in which it lies.† The resistance of a conductor may hence be considered as one of its own constants, like its

† The resistance of selenium, however, decreases on exposure to light, and that of bismuth is considerably affected by the magnetic field.

volume, mass, etc. It is therefore possible to set apart a conductor as a permanent standard of resistance, just as a selected rod is used as a standard of length or a particular solid body as a standard of mass. The practical unit of resistance is called the *ohm* (see Appendix).

Ohm's Law expressed in terms of Practical units.—Let the current along a conductor be C ampères, the P.D. between its ends V volts, and its resistance R ohms, then

$$V = C \times R$$

or $$\textit{Volts} = \textit{Ampères} \times \textit{Ohms}.$$

It may also be written

$$\textit{Ampères} = \textit{Volts} \div \textit{Ohms},$$

and $$\textit{Ohms} = \textit{Volts} \div \textit{Ampères}.$$

The reciprocal ($= 1/R$) of the resistance is called the *conductance* (see note, § 217) of the conductor.

Unit conductance is sometimes called the *mho*. A conductor having resistance R ohms has a conductance $1/R$ mhos.

217. Resistivity or Specific Resistance.—For conductors of the same material and at the same temperature the resistance (R) is directly proportional to the length (l) and inversely proportional to the cross-section (a) of the conductor: that is,

$$R \propto l/a \text{ or } R = Sl/a$$

where S is a constant. When l and a are both unity, $R = S$, that is, S represents the resistance of a piece of the conductor of unit length and unit cross-section.

DEF.—The *resistivity* (see note, § 217) *or specific resistance* of a substance is the resistance at 0° C. between opposite faces of a cube of the material when the edge of the cube is one centimetre long.

NOTE that the figure specified is a cube whose edge is 1 cm., this is frequently called the *centimetre cube*. To call it a cubic centimetre would be misleading, because the resistance of a cubic centimetre of the material would be much greater when drawn out into a fine wire than when formed into a thick conductor.

The *resistivity per inch cube* is similarly defined, the edge of the cube being, however, 1 inch.

Resistivity is usually expressed in microhms † per unit cube.

The resistivity of copper is nearly two-thirds of a microhm per inch-cube, or eight-fifths of a microhm per centimetre cube.

The following relations hold

$$\begin{pmatrix}\textit{Resistance}\\ \textit{of conductor}\\ \textit{in ohms}\end{pmatrix} = \begin{pmatrix}\textit{Resistivity of}\\ \textit{material in}\\ \textit{ohms per}\\ \textit{centimetre cube}\end{pmatrix} \frac{\textit{Length in centimetres}}{\begin{pmatrix}\textit{Cross section in}\\ \textit{square centimetres}\end{pmatrix}}.$$

$$\text{Since} \quad \begin{pmatrix}\textit{area of cross-section}\\ \textit{in square centimetres}\end{pmatrix} = 0{\cdot}7854 \begin{pmatrix}\textit{diameter in}\\ \textit{centimetres}\end{pmatrix}^2$$

$\therefore$ *Resistivity of material in microhms per cm. cube*

$$= \begin{pmatrix}\textit{Resistance of wire}\\ \textit{in microhms}\end{pmatrix} \times 0{\cdot}7854 \times \frac{(\textit{Diam. of wire in cm.})^2}{(\textit{Length of wire in cm.})}.$$

Note on Nomenclature. The following older terms are frequently used instead of the ones in the text:

Newer.	*Older.*
Resistance.	Resistance.
Resistivity.	Specific Resistance.
Conductance.	Conductivity.
Conductivity.	Specific Conductivity.

The first part of the newer words indicate the quantity dealt with, the ending *-ity* (*-ivity*, *-ility*) being used to express values for a specified volume (e.g. one cubic centimetre) of the substance, and the ending *-ance* being used to express the value for particular bodies without reference to their size, mass, etc. Other examples of this nomenclature are permeability (for specific magnetic conductivity), reluctance (for magnetic resistance), inductance (for coefficient of self-induction), impedance, reactance, etc.

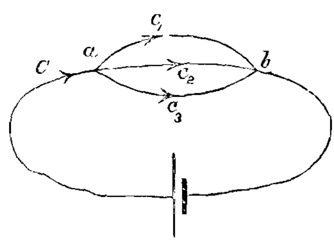

Fig. 215.

218. Combinations of conductors. — Conductors are said to be joined in *series* when an end of one is joined to an end of a second, the other end of the second to an end of a third, the other end of the third to an end of a fourth, and so on. Hence the same current passes through each conductor.

† The microhm is one millionth of an ohm (see Appendix).

Conductors are said to be joined in *parallel* (or *multiple arc*) (Fig. 215) when an end of each conductor is joined to one point (clamped, say, to a terminal or copper strip), and each of the other ends is joined to a second point (terminal or strip). Thus a current entering at one end divides, a part flowing along each conductor.

RULES FOR COMBINATIONS OF CONDUCTORS.

1. When conductors are joined in **series**

(i) *the same current passes through each;*

(ii) *the total P.D. is divided amongst the conductors, the P.D. between the ends of each being proportional to the resistance of the conductor;*

(iii) *the total resistance is the arithmetical sum of the resistances of each conductor.*

The constancy of the current through each part of a circuit of conductors in series, and the drop of voltage along the circuit are two facts of the highest importance.

Exps. 254, 255 show that if the ends of a conductor are joined to two points on an electric circuit, the P.D. between the two points maintains a current along the conductor. Also that if the resistance (R) of the conductor is high (more than a thousand times, say) compared with that between the points on the main circuit, then the P.D. between these points ($= V$) is not appreciably affected, and the current through the conductor $= V/R$. If however the resistance of the conductor is not comparatively large, the P.D. between the points diminishes in value (to V_1, say). The current round the conductor is then $= V_1/R$.

2. When conductors are joined in **parallel**

(i) *there is the same potential difference between the ends of each conductor;*

(ii) *the total current is divided amongst the conductors, each sharing in proportion to its conductance;*

(iii) *the total conductance is the arithmetical sum of the conductances of each conductor.* (Hence note that whenever conductors are joined in parallel, the total conductance of the whole is greater than that of any member, therefore the total resistance is less than that of any member, therefore the total resistance is less than that of the conductor of smallest resistance.)

The relations between current, potential-difference, and resistance are shown below. Capital letters are used in connection with the series arrangement, small letters with the parallel. The total values have no subscript, the individual values for the respective conductors are indicated by a subscript number.

	Series.	Parallel.
Total current ..	C	$c=c_1+c_2+c_3$
Total potential difference	$V=V_1+V_2+V_3$	v
Division of current	C same throughout	$\begin{cases} \dfrac{c_1}{1/r_1}=\dfrac{c_2}{1/r_2}=\dfrac{c_3}{1/r_3}=\dfrac{c}{1/r}= \\ c_1r_1=c_2r_2=c_3r_3=v=cr \end{cases}$
Division of potential	$\dfrac{V_1}{R_1}=\dfrac{V_2}{R_2}=\dfrac{V_3}{R_3}=C=\dfrac{V}{R}$	v same for each
Total resistance ..	$R_1+R_2+R_3=R$	$r=1/$(total conductance)
Total conductance	$\dfrac{1}{R}$	$\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}=\dfrac{1}{r}$

219. Resistance boxes.—The resistances used in practical work are generally lengths of insulated wire wound on bobbins and having their ends joined to terminals. The lengths, etc., are such that the resistance of the wire is some multiple of the ohm. In order that there may be no electro-magnetic or self-inductive effect the length of wire is first doubled back, and then wound on the bobbin (Fig. 216). Resistances of

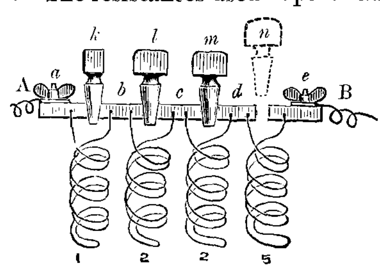

Fig. 216.

various values are joined up to form a *resistance box* or *set*.

Fig. 216 illustrates one method: *a, b, c, d, e* are brass pieces, usually fixed to an ebonite slab. Brass plugs, *k, l, m, n,* slightly conical in shape, and with ebonite or wooden heads, can be pushed between the pieces, or withdrawn as desired. The wire resistance has an end fixed to each of two adjacent pieces. Hence if a plug, *n*, is out the resistance wire is the only conducting connection between the pieces; if a plug is in it forms a path of negligible resistance, it is said to "short-circuit" the resistance, or "cut it out" from the circuit. The number expressing the resistance of the wire in ohms† is engraved close to the space between the brass pieces. Thus in Fig. 216 the resistance between the terminals, *A, B,* is 5. If *n* is put in and *k, m* are pulled out it will be 3.

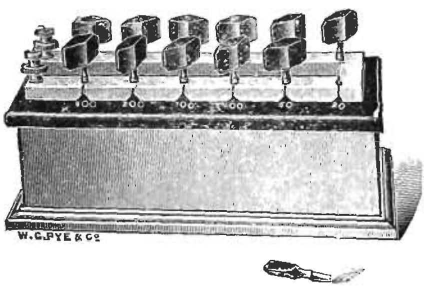

Fig. 217.

Linear arrangement. Fig. 217 illustrates a resistance box by which any integral number of ohms up to 1000 can be obtained. Fig. 233 is a somewhat similar arrangement. Notice that the values are in multiples 1, 1, 2, 5, or 1, 2, 2, 5, like a set of weights. The multiples at present in favour are 1, 2, 3, 4. In each case totals 1, 2, ..., 7, 8, 9, 10 are readily obtained; *e.g.*

$$8 = 5 + 2 + 1 = 4 + 3 + 1.$$

† Manufacturers now supply *international ohms*. The values in older boxes are likely to be *legal ohms*. For conversion factors see the Appendix.

various values are joined up to form a *resistance box* or *set*.

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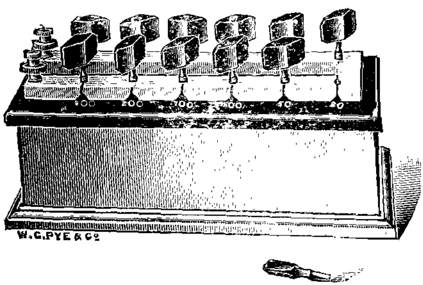


Fig. 217.

Linear arrangement. Fig. 217 illustrates a resistance box by which any integral number of ohms up to 1000 can be obtained. Fig. 233 is a somewhat similar arrangement. Notice that the values are in multiples 1, 1, 2, 5, or 1, 2, 2, 5, like a set of weights. The multiples at present in favour are 1, 2, 3, 4. In each case totals 1, 2, ..., 7, 8, 9, 10 are readily obtained; *e.g.*

$$8 = 5 + 2 + 1 = 4 + 3 + 1.$$

† Manufacturers now supply *international ohms*. The values in older boxes are likely to be *legal ohms*. For conversion factors see the Appendix.

Dial arrangement. In the linear arrangement above many plugs are used. To reduce the number or to avoid them altogether a dial form (Figs. 218, 219, 220) is adopted. In each dial ten brass pieces are arranged round the circumference of a circle. Resistance wires connect

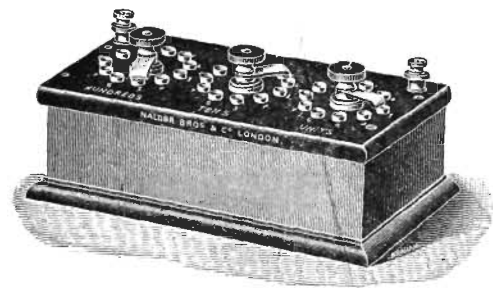


Fig. 218.

adjacent pieces, except one pair. There is also a central brass piece. In a dial marked UNITS each of the nine resistances is of 1 ohm, in the dial marked TENS each is of 10 ohms, in the dial marked HUNDREDS

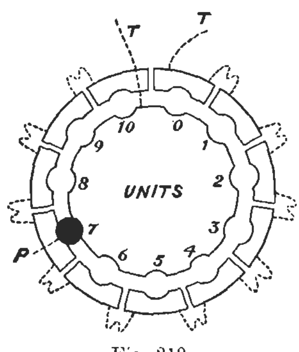


Fig. 219.

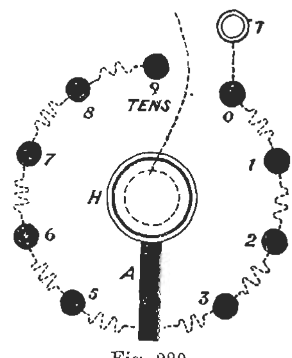
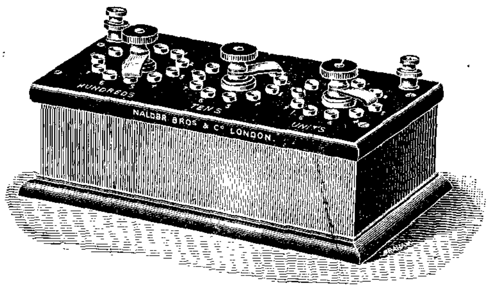


Fig. 220.

each is of 100 ohms. Connection can be made between the central and each circumferential brass piece in turn; either a plug (Fig. 219) or movable arm, *A* (Fig. 220), turned by a head, *H*, being employed. Note that in the former only one plug per dial is required.

Dial arrangement. In the linear arrangement above many plugs are used. To reduce the number or to avoid them altogether a dial form (Figs. 218, 219, 220) is adopted. In each dial ten brass pieces are arranged round the circumference of a circle. Resistance wires connect



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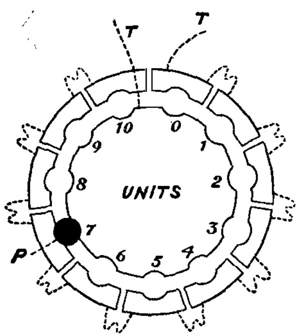
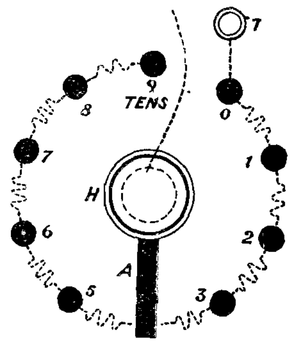


Fig. 219.



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Post-office Resistance Box and Bridge.—This is a set of resistances constructed as described in § 219, combined with extra resistances and keys in order that Wheatstone bridge tests may be carried out conveniently. Fig. 233 shows a diagram of the arrangement. In this there are generally two tap keys fixed to the top (ebonite) of the box.

RULES TO FOLLOW IN USING RESISTANCE BOXES.

(1) When plugs are withdrawn from their positions between brass pieces they *must not be kept in the hand*, but should be put in the lid of the box, or some other safe place.

(2) When plugs are placed in position between brass pieces press them "home" gently, then twist them slightly.

(3) Before commencing a test try each plug and terminal to make sure that the connection is sufficiently tight and good.

(4) The cleaning of ebonite surfaces, plugs, etc., should not be attempted.

Fractions of the Ohm.—Fig. 221 shows a convenient form of resistance capable of taking a current of 2 or 3 ampères. A length of wire (manganin or German silver), of resistance 1 ohm, is laid zigzag on a stout varnished board to form 10 segments of equal length and resistance. A binding screw is placed at each angular point, one also is placed midway between the ends of the first segment. Any resistance from 0·05 to 1 ohm (by steps of 0·05) can be introduced into the circuit; for example, by joining the wires to 0·05 and 0·6, a resistance of 0·65 is introduced.

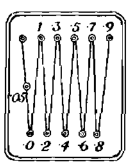

Fig. 221.

220. Uniform wire.—Fig. 229 shows a piece of apparatus that is of considerable use as a potentiometer, Wheatstone's bridge, etc. A wire (manganin or German silver), uniform in diameter, etc., 200 inches long, is stretched over a base board, the ends being soldered to copper strips *H*, *K*. The wire is bent round small brass screws. (A slip of paper is put between the wire and the screw to insulate the wire from the screw.) The top surface of the base board is covered with a sheet of varnished paper upon which the wire rests. A scale of half inches is marked under the wire. The inch divisions are numbered and reckoned from one of the copper strips (H). There is a central strip of copper, P; this enables a Wheatstone's bridge test to be easily arranged (§ 234). Each of the copper strips H, K, has two terminals, and P has three. The potential at any point of the wire can be "tapped" by the jockey J. This is a piece of wood, $6'' \times \frac{1}{2}'' \times \frac{1}{2}''$; one end is sharpened to an edge over which a piece of copper foil is fixed, the other end carries a terminal. A wire joins the terminal to the copper: it is convenient to run this through the wood.

221. General expression for a series circuit.—Let a battery (E.M.F., E, resistance, B) be in series with a galvanometer (resistance, G) and conductors of known resistance, R. Let C be the current in the circuit, then

$$C = E/(B + G + R).$$

If the battery consists of n cells, each of E.M.F., E, and resistance, B, in series, then

$$C = nE/(nB + G + R).$$

(i) When an *ampère-meter* is used, the scale reading, A, of the deflected pointer is the value of the current in ampères. Then

$$A = E/(B + R + G).$$

(ii) When a *tangent galvanometer* is used, the deflection, D, is in degrees, and $C = k \,.\, \tan D^\circ$ where k is a constant. Hence

$$k \,.\, \tan D^\circ = C = E/(B + R + G).$$

(iii) When a *reflecting galvanometer* is used, the displacement, d, is in scale divisions and is proportional to C, or $C = K.d$ where K is a constant. Hence

$$K.d = C = E/(B + R + G).$$

222. To measure resistance by substitution.—Join in series a constant cell, galvanometer, the unknown resistance, X, resistance box, and plug key (plug out). (1) Make the box resistance of such a value ($= R_1$) that when the circuit is completed through the plug key an adequate deflection (about half the scale) of the galvanometer needle is obtained. (2) Remove X: alter the box resistance to a value ($= R_2$) such that the deflection is the *same* as before. Then

$$X = R_2 - R_1.$$

(3) Repeat (1). If the values are unaltered it may be assumed that the cell is constant. If the values are different repeat the experiment with another cell.

Any galvanometer that gives an adequate deflection can be used, provided that when X is small, the galvanometer resistance and the value of $(R_1 + B)$ are also small.

EXPLANATION.—The deflections being the same in both cases, the currents through the circuit are equal. Also the E.M.F. *of the battery does not change. Hence the* total resistances of the circuit in the two cases are equal; that is,

$$X + R_1 + G + B = R_2 + G + B.$$

Then

$$X = R_2 - R_1.$$

223. *To measure resistances* by means of a curve obtained from a galvanometer.†—Arrange the galvanometer so that the undeflected index is at the zero of the scale, or in the case of a reflecting instrument near one end. Record its position.

Join in series 5 or 6 known resistances $(=R_1)$, the galvanometer, plug key (plug out), and a constant cell. Complete *the circuit through* the plug key, note the position to which the index is displaced, and the value of the resistance, R_1.

Remove one of the known resistances, note the value, R_2, of the remainder, and the position to which the index is deflected.

Remove another of the known resistances, note the value left, and the deflection.

Proceed similarly until the deflection is too large to be measured.

Tabulate (i) Resistances (R_1, R_2, etc.) in circuit, (ii) Deflection or displacement of index (= Difference between the deflected and undeflected positions).

Plot deflections or displacements with regard to resistances as abscissae.

When a reflecting galvanometer is used, the graph obtained is nearly a hyperbola.

To obtain the value of an unknown resistance X.—Join it in series in the circuit. Note first the undeflected position of the index, then its deflected position. *Calculate its displacement. Find from the graph* the value of resistance corresponding to this displacement: this is the value of X.

---

† An extension of the substitution method. A resistance box is more convenient than separate resistances, but is not essential.

224. To investigate the relation between the current through a galvanometer and the deflection produced.—Make experimental observations as in § 223.

I. When a reflecting galvanometer is used: tabulate (i) resistances (R_1, R_2, etc.), (ii) displacements of index, (iii) reciprocals of displacements: plot the reciprocals of the displacements with regard to resistances as abscissae. The graph will be practically straight.†

The deviation of the graph from the straight line indicates to what extent the assumption that the displacements are proportional to the currents (§ 209) through the galvanometer is not justified.

II. When a tangent galvanometer is used: tabulate (i) resistances (R_1, R_2, etc.), (ii) angular deflections, (iii) tangents of angular deflections, (iv) co-tangents of angular deflections: plot *cot* (*angle of deflection*) with regard to resistances (R_1, R_2, etc.) as abscissae. The graph will be a straight line.†

The graph shows that since *cot* D *is proportional to total resistance, tan* D *is proportional to the total conductance.* Now by Ohm's law *current = conductance* × *E.M.F.* Since the E.M.F. is constant, *current is proportional to conductance* and therefore to *tan* D. Thus the rule or law of the tangent galvanometer (§ 208) is demonstrated.

III. In either case plot (i) R_1, R_2, etc., (ii) deflections. The graph can be used similarly to the one obtained in § 223.

225. To measure resistance by a tangent galvanometer.—Join in series a constant cell (E.M.F., E, resistance, B) galvanometer (G), commutator (plugs out), known resistance, R_1, and unknown resistance, X. Adjust the galvanometer (§ 214). Close the circuit and obtain the mean deflection, D_1, of the needle.

Alter the value of resistance, R_1, to another, R_2. Obtain the mean deflection, D_2, of the needle.

† Note.—The graph cuts the resistance axis on the negative side of the origin. The length of the intercept on this axis is the value of ($B + G$ + resistances not included in R_1, R_2, etc.).

Exp. 248.—*Find the resistance of an incandescent lamp when glowing.*

NOTE AND CAUTION.—A wired lamp-holder, and a wired wall plug or lamp plug are required for this experiment. Before testing, the connections, etc., must be examined by a responsible demonstrator.

I. Join an ampère-meter in series with the lamp, and arrange to connect the two with the electric lighting mains. Join a voltmeter in parallel with the ampère-meter and lamp. Close the circuit; observe the volts and the ampères. Then

$$\textit{resistance in ohms} = (\textit{volts}) \div (\textit{ampères}).$$

The value of resistance obtained is that of (lamp + ampère-meter). The resistance of an ampère-meter is generally negligible. Hence the value is practically the resistance of the lamp.

II. Proceed as in I., but join the voltmeter in parallel with the lamp only. Calculate (*ampères*) ÷ (*volts*). The value obtained is that of the total conductance of (lamp + voltmeter). Subtract the conductance of the voltmeter (previously determined): the remainder is the conductance of the lamp. Then deduce its resistance.

Exp. 249.—*Find by the method of substitution the resistance of the coil of an electric bell, an electric glow lamp, a coil of wire, a high resistance.*† See §§ 222, 223. A galvanoscope is provided.

Exp. 250.—*Find by a tangent galvanometer the total resistance of a cell and galvanometer.*† See § 225.

Exp. 251.—*Find by a tangent galvanometer the resistance of a Daniell cell,† etc.* See § 226.

Exp. 252.—*Investigate the relation between current and deflection*† for (1) a reflecting, (2) a tangent galvanometer, (3) a galvanoscope. See § 224.

Exp. 253.—*Find by an ampère-meter the resistance of wires, etc., of a Daniell cell,† etc.*

227. Case of a uniform wire.—Suppose a long wire, uniform in diameter, homogeneous in material, and at the same temperature throughout. Then the resistances of different lengths are proportional to the lengths.

Let a current flow along the wire. Consider any three points on it, mark them, in order, l, m, n. Then since

† For a constant cell use a Daniell that has been short-circuited (§ 186), or a secondary.

electricity flows from points at higher potential to those at lower, the potential at m will be higher than at n, but lower than at l. Let V_{mn}, R_{mn} be, respectively, the potential difference and the resistance between mn, also V_{lm} and R_{lm} the same quantities for lm.

Since the current, C, is of constant strength throughout the wire,

$$C = V_{lm}/R_{lm} \quad \text{and} \quad C = V_{mn}/R_{mn}.$$

Hence $V_{lm}/V_{mn} = R_{lm}/R_{mn} =$ length lm/length mn,

that is, *when a current flows along a uniform wire the P.D. between any two points of the wire is proportional to the length of wire between the points.* (See Exp. 255.)

Exp. 254.—*Find on a wire carrying a current points such that the difference of potential (P.D.) between adjacent ones is constant: or to graduate a wire into parts of equal resistance.*

Join in series (Fig. 222) a constant cell, S,† plug key, Pk, and uniform wire, AB, stretched over a scale (§ 220). To one of the terminals of a sensitive galvanometer join a wire and jockey, J, and to the other a resistance box (RB), wire and jockey K. Complete the stretched-wire circuit through the plug key.

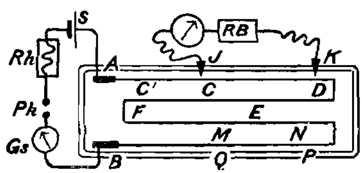

Fig. 222.

Make resistance of (RB) as high as possible, put J in contact with A, and K at a point distant about one-fifth the length of the wire. Adjust the resistance of (RB) until an adequate deflection of the galvanometer needle (about half or two-thirds the scale) is produced. Note the position of the index of the needle, also of the points A, C where J and K are pressed.

Next press J at C and K at different points until one, D, is found such that the deflection is the *same* as before. Note the reading of D.

† Use a Daniell (after short circuiting) or a secondary. It is advisable to introduce a galvanoscope (Gs) and rheostat (Rh). Adjust (Rh) to keep the deflection of (Gs) constant during the experiment.

Press J at D, with K find the point E such that the deflection is the same as before. Note the reading of E.

Similarly find other points F, etc. Finally break the circuit.

Note the distances AC, CD, DE, EF, etc.

Repeat beginning at some other point (C').

THEORY.—Since the same deflection of the galvanometer is obtained in each case, the currents through the galvanometer are equal. Also the resistance of the galvanometer circuit between J and K is unaltered. Hence the P.D. producing the current (= current × resistance) is constant. But this P.D. is also that between the points of the stretched wire at which the jockeys are pressed. Then if the current through the stretched wire is constant the points found divide the wire into portions of equal resistance (by Ohm's law, § 216). If the lengths are found to be practically equal it may be inferred that the wire is of uniform cross section, etc.

Exp. 255.—*Determine the fall of potential along a wire carrying a current.* Arrange the apparatus as in the previous experiment. Use a reflecting galvanometer. When no current is passing adjust its index (spot of light) to a division near one end of its scale. Make resistance of (RB) as high as possible. Join J to A, press K on B. Complete the circuit through the plug key. Alter the resistance of (RB) until the deflection is nearly the whole length of the scale. Break the circuit. Finally note the scale-reading of the galvanometer index when no current is passing.

Complete the circuit at the plug key. Press jockey, K, at B (the connection between A and the galvanometer is retained throughout the experiment). Note the scale reading of the deflected index and the length, AB.

Press K at a point, Q, on the wire several inches from B. Note the positions of the galvanometer index and of Q.

Similarly press K successively at points P, N, M, etc. In each case note the respective positions of the galvanometer index and of the points on the wire. Finally break the circuit.

Tabulate (i) the positions on the wire of the points Q, P, N, M, etc., (ii) the corresponding readings of the galvanometer index, (iii) the lengths of wire between A and the successive points Q, P, N, M, etc., (iv) the corresponding displacements of the index (equal to the difference between the readings when deflected and when undeflected).

Plot displacements (col. iv.) with regard to lengths (col. iii.) as abscissae.

Repeat, using two Daniell cells instead of one.

EXPLANATION.—The currents through the galvanometer are proportional to the P.Ds between A and the points on the stretched wire at which the jockey K is pressed; the P.Ds are proportional to the resistances of the respective lengths of the stretched wire reckoned from A. If the displacements of the index are proportional to the currents through the galvanometer and the resistances of the lengths of wire are proportional to their lengths, then the displacements of the index are proportional to the lengths of the wire, and the graph obtained by plotting them is a straight line.

If the graph is not straight it indicates that (i) the wire may not be uniform, or (ii) the displacements of the galvanometer index are not proportional to the currents through the instrument. The previous experiment shows whether the wire is practically uniform; if it is so the graph now obtained shows the relation between displacement of the galvanometer index and the current through the instrument. Confirm as in § 224.

ELECTROMOTIVE FORCES.

228. Comparison of electromotive forces.—(i) If a ce. or battery of E.M.F. E_1, resistance, B_1, is joined in serie with resistances equal to R_1 and a galvanometer of resist ance, G, then the current (C_1) round the circuit is such that

$$E_1 = C_1 \ (B_1 + G + R_1).$$

(ii) If a second cell of E.M.F. E_2, resistance, B_2, is now substituted for the first and the resistance altered to another value, R_2, then the current (C_2) round the circuit is such that

$$E_2 = C_2 \ (B_2 + G + R_2).$$

Therefore $$\frac{E_1}{E_2} = \frac{C_1 \ (B_1 + G + R_1)}{C_2 \ (B_2 + G + R_2)}.$$

I. *General method.* If the resistance of the external circuit is, in each case, so high that the resistances of the batteries may be neglected, then

$$\frac{E_1}{E_2} = \frac{C_1}{C_2} \cdot \frac{G + R_1}{G + R_2}. \ldots\ldots\ldots\ldots\ldots \text{II.}$$

II. *Constant resistance method.* If in the second case the resistance is not altered from the value R_1, then

$$E_1/E_2 = C_1/C_2.$$

III. *Equal deflections method.* When the value of R_2 is adjusted so that the same deflection of the galvanometer needle is obtained in the two cases, then $C_2 = C_1$ and

$$\frac{E_1}{E_2} = \frac{R_1 + G}{R_2 + G}.$$

Practice of the Methods. No galvanometer is suitable for the determinations unless an adequate deflection (half or two-thirds the scale) is obtained when the resistance of the external circuit is at least 200 times that of any cell tested.

The requirements of the three methods are compared in the table below :—

<table>
<tr><th>Method.</th><th>General.</th><th>Constant resistance.</th><th>Equal deflections.</th></tr>
<tr><td>Relation between galvanometer current and deflection .. ..</td><td colspan="2">Must be known.
Use a sensitive tangent or reflecting galvanometer.</td><td>Need not be known.</td></tr>
<tr><td>Galvanometer resistance .. ..</td><td>Must be known.</td><td>Need not be known.</td><td>Must be known.</td></tr>
<tr><td>Circuit resistance must be of relatively high value</td><td>Resistance box required.</td><td>High resistance: value need not be known.</td><td>Resistance box required.</td></tr>
<tr><td>Cell resistance ..</td><td colspan="3">Must be negligible.</td></tr>
</table>

Preliminary test. Join in series, galvanometer, resistances, plug key (plug out), and one of the cells. Make the resistance high, complete the circuit through the plug key and observe roughly the deflection of the galvanometer needle. Then substitute each of the other cells in turn, and find which gives the largest deflection, this will be the cell of highest E.M.F.

In the accurate determination join in series the several items and the cell of highest E.M.F., then proceed with one of the methods (the general method need not be done).

II. *Constant resistance method.* The cell of highest E.M.F. being in action adjust the circuit resistance until an adequate deflection is obtained. Note the position of the deflected galvanometer index. Join in turn the other cells in the circuit. In each case obtain and note the position to which the galvanometer index is deflected. Between each of these observations read and record the position of the undeflected index (this "zero position" may vary slightly).

A. If a reflecting galvanometer has been used, tabulate (1) type of cell used, (2) zero position of index, (3) deflected position of index, (4) displacement of index (equal to the distance between the deflected and zero positions).

Calculate the E.M.F. of each cell relatively to a standard cell (Daniell). Assume

$$\frac{\text{E.M.F. of a cell}}{\text{E.M.F. of standard cell}} = \frac{\text{Displacement for cell}}{\text{Displacement for standard}}.$$

B. If a tangent galvanometer has been used in the experiment, tabulate the observations under (1) type of cell used, (2) readings (four) of ends of pointer, (3) mean angular deflection; (4) tan (deflection).

Calculate the E.M.F. of each cell relative to one (say, the Daniell) considered as a standard cell by assuming

$$\frac{\text{E.M.F. of a cell}}{\text{E.M.F. of standard cell}} = \frac{\tan\ (\text{deflection for cell})}{\tan\ (\text{deflection for standard})}.$$

In either case (A or B) calculate the voltage of each cell, assuming that of the standard (Daniell, 1·1 volts).

III. *Equal deflections method.* The cell of highest E.M.F. being in action, adjust the resistance of the circuit until an adequate deflection is obtained. Note the position of the deflected index, and the value of the circuit resistance. Join in turn the other cells in the circuit. In each case adjust the circuit resistance, noting the values for the respective cells, until the galvanometer index is deflected to the same position as for the first cell. Between each of these observations obtain and record the position of the undeflected index (zero position). Tabulate (1) type of cell used, (2) zero position of index, (3) deflected position of index, (4) circuit resistance, (5) total resistance (= circuit + galvanometer).

Calculate the E.M.F. of each cell relatively to a standard cell (Daniell), assuming

$$\frac{\text{E.M.F. of a cell}}{\text{E.M.F. of standard cell}} = \frac{\text{Total resistance for cell}}{\text{Total resistance for standard}}.$$

Calculate the voltage of each cell, assuming the value of the standard (Daniell = 1·1 volt).

Exp. 256.—*Compare the E.M.Fs of Daniell, Leclanché, dry, Bunsen, and secondary cells.*—Work the constant resistance method with (i) a reflecting galvanometer, (ii) a tangent galvanometer. Then do the equal deflections method with a sensitive astatic or single needle galvanometer.

Tabulate (1) type of cell, (2) relative E.M.F. by constant resistance method, (3) relative E.M.F. by equal deflections method, (4) voltage by constant resistance method, (5) voltage by equal deflections method.

229. Wiedemann's sum and difference method of comparing E.M.Fs.—Two of the cells whose E.M.Fs (E_1, E_2) are to be compared are joined, as in Fig. 223, to a tangent galvanometer or ampère-meter, a commutator (*abcd*), and if necessary a resistance (*RB*). The commutator is arranged so that the cells act, 1st, in *conjunction*, that is, send a current in the same direction round the circuit; 2nd, in *opposition*, that is, tend to send currents in opposite directions round the same circuit. The value of the resistance (*RB*) should be arranged so that suitable deflections (§ 214 (7)) of the galvanometer needle are obtained. It must not be altered in value during the two parts of the experiment.

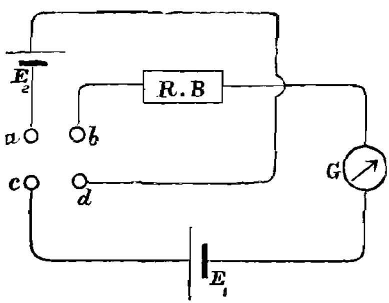

Fig. 223.

Since the resistance $(R + G + B_1 + B_2)$ is the same in both parts of the experiment, the currents produced will be proportional to the actual E.M.F. acting. The value of this when the cells are in conjunction is $(E_1 + E_2)$, and $(E_1 - E_2)$ when in opposition. Hence

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{C_1}{C_2} = \frac{\tan D_1}{\tan D_2},$$

$$\therefore \frac{E_1}{E_2} = \frac{\tan D_2 + \tan D_1}{\tan D_1 - \tan D_2}.$$

230. The requirements of three methods of measuring E.M.Fs are compared below :—

<table>
<tr><th>Method.</th><th>Wiedemann (§ 229).</th><th>Poggendorff (§ 237).</th><th>Condenser (§ 238).</th></tr>
<tr><td>Relation between galvanometer current and deflection.</td><td>Must be known.</td><td>Need not be known. Null method. Very accurate.</td><td>Must be known.</td></tr>
<tr><td>Galvanometer resistance.</td><td colspan="3">Need not be known.</td></tr>
<tr><td>Cell resistances.</td><td colspan="3">Need not be known nor negligible.</td></tr>
<tr><td>Circuit resistance.</td><td>Resistance of unknown value may be added.</td><td>Potentiometer required. Unknown high resistance.</td><td>No resistance required.</td></tr>
</table>

For *Lumsden* or *Bosscha's method* see p. 428 (*Higher Text-Book of Electricity*).

Exp. 257.—*Compare the E.M.Fs of Daniell, Bunsen, and secondary cells' by Wiedemann's method.* Use a tangent galvanometer or an ampère-meter.

231. Relation between the E.M.F. of a cell and the P.D. between its terminals when a current is flowing through it.—When a cell is on open circuit (no current flowing) the value of the voltage between its poles is called the E.M.F. of the cell. The experiment described below shows that as soon as the circuit is closed, and a current flows, the value of the voltage between the terminals falls below that of the E.M.F. of the cell, the difference between the two becoming greater as the current strength is increased. When the cell is short-circuited the voltage between its poles is negligible.

THEORY. Let the cell have E.M.F., E, and resistance, B. Join its terminals by a conductor of resistance, R. Let C be the current that then flows through the conductor and cell, and V the P.D. between the poles. Then by Ohm's law

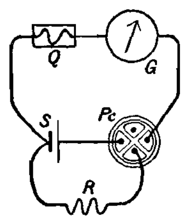

Fig. 224.

$$E = C(B + R) \quad \text{and} \quad V = CR,$$

$$\therefore \frac{V}{E} = \frac{R}{R+B}$$

and
$$\frac{E-V}{E} = \frac{B}{B+R}.$$

Thus the *fall in voltage* $(E - V)$ *is to the total E.M.F. as the battery resistance to the total resistance of the circuit.*

The following values can be calculated from the above :—

$E - V$	$0{\cdot}1E$,	$0{\cdot}2E$,	$0{\cdot}5E$,	$0{\cdot}6E$,	$0{\cdot}75E$,
R	$9B$,	$4B$,	B ,	$\frac{2}{3}B$,	$\frac{1}{3}B$.

Hence, since E and B are constant, $(E - V)$ is increased, when R is diminished (therefore the current increased).

To obtain the battery resistance. From the expression above it follows that

$$B = \left(\frac{E}{V} - 1\right)R.$$

Hence, if the ratio is obtained of E on open circuit to V when the circuit is closed by a known resistance, R, then B may be calculated. Note that $B = R$ when $E/V = 2$.

Experimental demonstration.—Use a constant cell (Daniell or secondary). Join (Fig. 224) a reflecting galvanometer in series with a high resistance, Q (the value need not be known), and connect with the terminals of cell. Also join a conductor of comparatively low resistance, R, between the cell terminals. (The conductor should carry a current of one or two ampères without appreciable heating: its resistance should be capable of variation by known amounts, say from 1/2 to 20 ohms. A stretched wire similar to Fig. 221 is convenient.) A plug commutator (Pc) is useful.

Observe when the R circuit is broken (value of R is then infinite), first, the deflected position of the galvanometer index (galvanometer circuit completed), second, the undeflected position (galvanometer circuit broken).

Similarly observe the deflected and undeflected positions of the galvanometer index when R is successively made equal to 20, 10, 5, 4, 2, 1 ohms. (The R circuit as well as the galvanometer circuit should be broken after each reading of the deflected index.)

Tabulate (1) value of R, (2) deflected position of galvanometer index, (3) undeflected position, (4) displacement.

Plot the displacements with regard to the values of R as abscissae. The graph will not be straight. Find the value of R for which the displacement is one-half the maximum displacement. The battery resistance equals this value of R.

Experimental proof of formula. If y is written for E/V and x for $1/R$ the expression $E/V = (R + B)/R$ becomes $y = 1 + Bx$, the graph of which is a straight line. Hence if the values of E/V are taken as ordinates with respect to $1/R$ as abscissae the graph should be a straight line. Confirm this from the observations above.

Exp. 258.—*Find the resistances of Daniell, Leclanché, and secondary cells. Investigate the drop of voltage on open and closed circuit.*

232. To compare resistances of low value.—This is readily done by joining the conductors in series with a constant cell, and comparing the P.Ds between their ends by one of the methods for the comparison of E.M.Fs. Since the conductors are joined in series the same current (C) passes through each. If R_1, R_2, R_3 . . . are the resistances and V_1, V_2, V_3 . . . the respective P.Ds between their ends, then

$$V_1 = CR_1, \quad V_2 = CR_2, \text{ etc.} \qquad \therefore R_1 : R_2 = V_1 : V_2, \text{ etc.}$$

PRACTICE.—When the constant resistance method of comparing E.M.Fs is adopted the circuit is arranged as in Fig. 225. (It is an advantage to introduce a rheostat (Rh), and galvanometer, G_1, in series with the resistances and cell. Keep the deflection of G_1 constant, then the current in the circuit will be constant.) A reflecting galvanometer in series with a high resistance, Q, is used to measure the P.Ds. If J, K are the terminals of the galvanometer circuit, join them in turn with the ends of the resistances, AB, CD, etc. In each case complete the main circuit through the plug key (Pk), and note the displacement (= deflected position − initial position) of the galvanometer index. The displacements are proportional to the P.Ds and therefore to the resistances. If the value in ohms of one resistance is known, the others may be calculated.

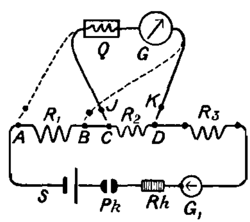

Fig. 225.

Exp. 259.—*Compare the resistances of German silver, iron, and manganin wires of equal lengths and diameters with a 1 ohm resistance.*

CHAPTER XVII.

WHEATSTONE'S BRIDGE, POTENTIOMETER, AND CONDENSER.

233. The Wheatstone's network or bridge.—This is an arrangement of wires, galvanometer, battery, etc., by means of which resistances may be compared with great accuracy.

Let two conductors, HPK, HQK, be joined in parallel. Let H be connected with the $+$, and K with the $-$ plate of a battery. Then current (strength, A) flows in at H, and divides along the two branches (strengths, A_1, A_2). The divided current recombines at K and thence returns to the battery. Hence there is a difference of potential between the extremities, H, K, and a continuous fall of potential along each branch.

Take any point, P, in HPK: there will be another point, Q say, on the other branch at which the potential will be the *same* as at P. Join one terminal of a galvanometer to P, to the other terminal attach a wire whose free end (J) can be pressed to any position on the branch, HQK. When J is pressed at the point Q, at which the potential is the same as at P, then no current passes through the galvanometer, and its needle is not disturbed. When J is pressed at a point, T, between Q and H, the galvanometer needle is deflected, because current flows through the instrument from T to P (T is at a higher potential than Q, therefore higher than P); if J is pressed at a point, U, between Q and K, the galvanometer needle is deflected, but in the reverse way to the preceding, because current flows through it from

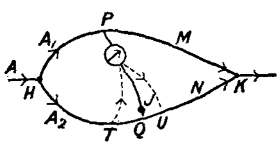

Fig. 226.

P to *U*. Hence **to find Q,** press *J* into contact with the wire at different positions until one is found at which there is no deflection of the galvanometer needle. Note that on either side of it the deflections are reverse, also that the amount of deflection becomes less as the point touched is nearer the required point. Similarly a point, *N*, may be found at which the potential is the same as at *M*, also other pairs of points.

Now (§ 216)

$$V_{HP}=A_1 \,.\, R_{HP}, \qquad V_{PK}=A_1 \,.\, R_{PK},$$
$$\therefore \quad V_{HP}/V_{PK}=R_{HP}/R_{PK}.$$

Also

$$V_{HQ}=A_2 \,.\, R_{HQ}, \qquad V_{QK}=A_2 \,.\, R_{QK},$$
$$\therefore V_{HQ}/V_{QK}=R_{HQ}/R_{QK}.$$

Since *potential at P = potential at Q,*

$$\therefore \quad V_{HP}=V_{HQ} \quad \text{and} \quad V_{PK}=V_{QK},$$
$$\therefore \quad R_{HP}/R_{PK}=R_{HQ}/R_{QK}.$$

Thus *Q* divides the branch, *HQK*, into two parts whose resistances are in the same ratio as the resistances of the two parts into which the branch, *HPK*, is divided by the point *P*. This is the **rule of the Wheatstone's bridge or network.**

The Wheatstone's network may be also regarded as a closed ring of four resistances, *W, X, Y, Z* (Fig. 227), in which there are four joints, *H, P, K, Q*. Of these the pair, *H, K* may be called *opposite joints*, or corners, so also *P* and *Q*: any other pair may be called *adjacent*. Then note that the battery terminals are connected with either pair (*H, K*) of opposite joints, the galvanometer with the other.

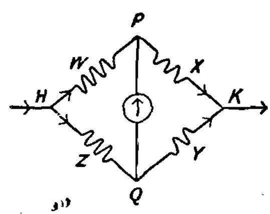

Fig. 227.

Finally, when on closing the circuit there is no deflection of the galvanometer, the *resistances are* said to be *balanced*, and their values form the proportion

$$W : X = Z : Y \quad \text{or} \quad W . Y = X . Z,$$

that is the *product of one pair of opposite resistances equals the product of the other pair.*

In the practical application of the Wheatstone's network two plans are adopted. In the *slide-wire* or *metre bridge* (§ 234) the branch, HQK, is a wire, uniform in cross-section, etc., W a known resistance, X an unknown resistance. The ratio between X and W is then directly measured as the ratio of the lengths of the two segments into which the wire is divided by the point Q (determined as in § 234). In the *P.O. Box* (§ 236) various known resistances are used for W, Z, Y, the ratio of Z to Y is made a definite value, W is then adjusted by known amounts with regard to X, and when there is "balance" the ratio of W to X is equal the known ratio of Z to Y.

In practice the comparison of resistance may be made by the Wheatstone's network with very great facility and accuracy. As it is a null† method a very sensitive galvanometer may be used. There need be no heating of the conductors, due to prolonged flow of current. The test is independent of variations of E.M.F. of the cells: if change occurs the several resistances share proportionally.

234. To measure resistance by a slide-wire bridge (Fig. 228).—Apparatus. A uniform wire is stretched over a scale of equal parts on a base board, the ends of the

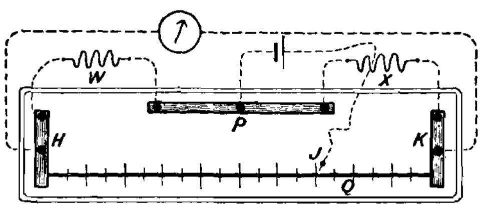

Fig. 228.

wire are attached to copper strips, H, K, to each of which two terminals are fixed. There is also a copper strip, P,

† A *null* method is one in which the adjustments are correct when there is no deflection of the galvanometer.

with three terminals. Fig. 229 shows a convenient but less usual form of slide-wire bridge.

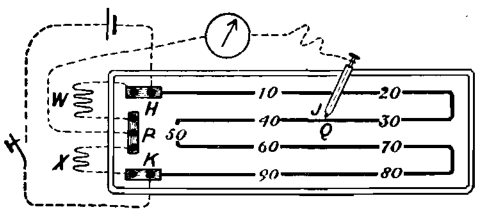

Fig. 229.

CONNECTIONS.—Join (Figs. 228, 229) a Leclanché cell, with a tap key in its circuit, to H, K. Connect one terminal of the galvanometer to P, the other to a jockey. (The end of the wire itself may be used, but is not satisfactory in practice.) The unknown resistance, X, is joined across one gap, a known resistance, W, across the other. The galvanometer should be sensitive, a single-needle or astatic pointer galvanometer is usually sufficient.

In Fig. 228 the cell is joined between P and J (jockey), the galvanometer between H, K, and there is no tap key in its circuit. This is simpler but less satisfactory than the preceding.

The apparatus being joined up, the jockey feels† at points on the stretched wire, until one (Q) is found at which when contact is made there is no deflection of the galvanometer. Note the scale reading of Q.

Then $$\frac{W}{X} = \frac{\text{resistance between } HQ}{\text{resistance between } QK} = \frac{\text{length } HQ}{\text{length } QK}.‡$$

Interchange the resistances, X and W, and repeat the determination.

Note the value of W. Calculate X from each of the two observations: then its mean value.

† RULE.—*Press down the tap key in the battery circuit before placing the jockey in contact with the wire.*

‡ Because the wire is uniform the resistance of a length of it is proportional to the length.

235. The Metre Bridge.—Fig. 230 shows the usual form and Fig. 231 a diagrammatic view of this apparatus. There are five stout strips of copper fixed in line on a base-board. Four movable copper straps are provided for bridging the gaps between the strips when required. Between the ends of the extreme strips a stout wire (uniform cross section, etc.) of German silver is stretched: this is 1 metre long. Parallel with it a boxwood millimetre scale is fixed. The

Fig. 230.

jockey serves to make contact with the stretched wire and to indicate the point of contact on the measuring scale. A wire from the circuit is joined to its head. When a test is to be made place the jockey over the wire and press the head down, an edge on the under side of it then comes into contact with the wire. When the head is not pressed, a spring inside the jockey lifts the edge from the wire.

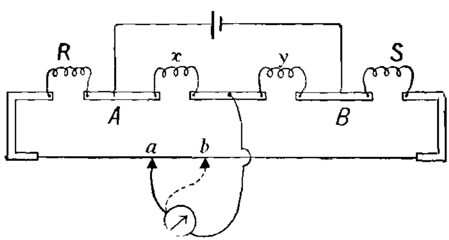

Fig. 231.

Fig. 231 shows the metre bridge joined for Carey Foster's method (see *Higher Text-Book of Electricity*, § 150). For the ordinary test, copper straps are placed across the two outside gaps instead of the resistances, *R*, *S*.

The metre bridge is really designed for the Carey Foster test and serves that purpose admirably. The low resistance (less than an ohm) of the stretched wire and the importance then of knowing the errors due to imperfect adjustment at the ends, etc. (these can be only roughly measured), diminish the value of the metre bridge for determinations by the ordinary method. A finer and longer slide wire is better for this.

Exp. 260.—*Demonstrate the rule of the Wheatstone's Bridge.* Arrange a slide-wire bridge as in § 234, placing a known resistance, say 5 ohms, for X, and a resistance box or known resistances for W. Make W, in turn, equal to, say, 2, 4, 6, 8, 10 ohms. In each case find and note the position of J when balance is obtained. Tabulate values of (1) W, (2) X, (3) HQ, (4) QK, (5) W/X, (6) HQ/QK. The numbers in (6) should be equal to those in (5).

Repeat the experiment with another value of X.

Exp. 261.—*Measurements of resistance by means of the Wheatstone's bridge.* Wires of manganin, German silver, platinoid, brass, copper, iron, may be used. It is convenient to cut definite lengths of these (50 inches, 100 cm.) and clamp to a board. Leave some straight portions exposed so that the diameter may be measured.

1. Measure in ohms the resistances of wires of different materials, but of equal lengths and diameters.

2. Measure in ohms the resistances of wires of the same material, of equal lengths, but of unequal diameters. Measure the diameters by the screw gauge. Prove that the conductances are in the same ratio as the cross sections of the wires, or as the squares of their diameters.

3. (*a*) Measure in ohms the resistance of a wire, its diameter in centimetres by the screw gauge, its length in centimetres. Calculate (§ 217) the resistivity (or specific resistance) per cm. cube of the material.

(*b*) Measure resistance in ohms, diameter in inches, length in inches. Calculate (§ 217) the resistivity per inch cube of the material.

4. Three wires are provided. Find the resistance in ohms of (i) each separately, (ii) the three in series, (iii) each pair in series (three arrangements), (iv) the three in parallel, (v) each pair in parallel (three arrangements).

Assuming the individual resistances found in (i) above, calculate (see § 218) what the result should be in each of the other cases. Compare with the experimental results.

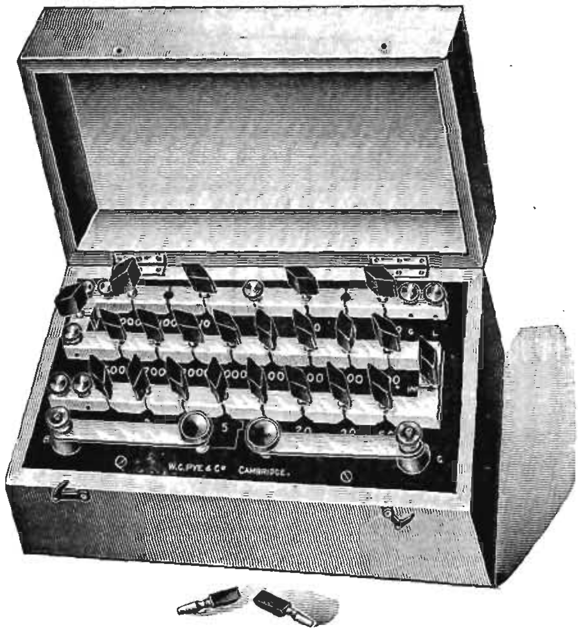

Fig. 232.

236. To measure a resistance by means of the Post Office Box or Bridge.—The connections and details of the P.O. box are shown in Fig. 233, X being the resistance to be measured. The lettering is similar to Fig. 229. The four resistances forming the closed ring (§ 233) are, when plugs are pulled out, between HP, PK (the unknown), KQ, QH. PH and HQ are called the *ratio or proportional arms*, QK the *rheostat* arm, PK the *gap* for the resistance, X. Fig. 232 shows the common pattern of P.O. box (of which Fig. 233 is a plan) and Fig. 234 a special form.

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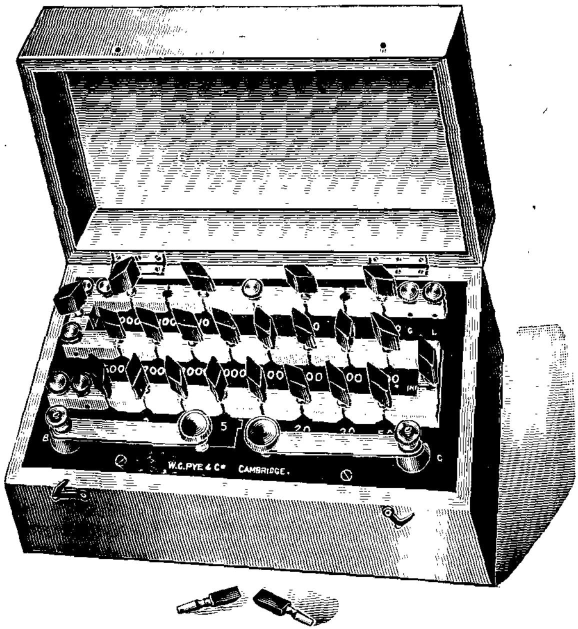

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USE OF THE P.O. BOX.

(1) See general rules for resistance boxes, § 219.

(2) Connect the galvanometer, and the unknown resistance, X, to the proper parts of the box. If the box is not provided with tap keys, one should be introduced in the cell connections and another into the galvanometer connections.

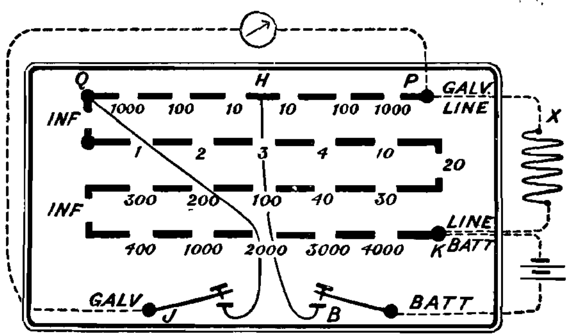

Fig. 233.

The connections are generally indicated on the box. The words LINE at P and LINE or EARTH at K have been adopted because the apparatus was originally designed for testing telegraphic lines, etc.

(3) The galvanometer should be sensitive and as nearly dead-beat as possible. Join a resistance of one or two ohms across its terminals, to act as a shunt.

(4) Join in a Leclanché cell.

(5) The battery tap key must be put down a moment or two earlier than the galvanometer one, and kept down while the latter is pressed.

(6) Make the resistance of each ratio arm, HP, HQ, equal to 1000 ohms, and withdraw the infinity plug from its hole. Press the keys as in (5). Note whether the galvanometer index moves to the right or left. Whenever during the test it moves the same way as it does now it signifies that the resistance of the rheostat arm is too much, and when it moves in the reverse way that it is too

little. Replace the infinity plug in its hole, see that all holes in the rheostat arm are plugged. Leave the 1000 ohm resistances of the ratio arms unplugged. Press the keys as in (5). The index should move in the direction signifying too little.

These tests show whether the apparatus is working correctly. If deflections are not obtained the connections, battery, etc., must be carefully examined and the fault rectified.

(7) Keep the resistance of each ratio arm $= 1000$. Alter the resistance of the rheostat arm systematically (see example below) and note what value $(= n)$ gives a small deflection one way, and a reverse deflection when increased by one ohm. Then

$$X \text{ lies between } n \text{ and } (n+1) \text{ ohms.}$$

(8) Remove the shunt from the galvanometer. Make ratio arm, $PH = 100$, $HQ = 1000$ (then $PH : HQ = 1 : 10$), rheostat arm, $QK = 10n$. The previous test shows that balance should occur between the values $10\,n$ and $10(n+1)$. Increase the resistance of the rheostat arm by one ohm at a time and note what value (n_1) gives a small deflection one way, and a reverse deflection when increased by an ohm. Then

$$X \text{ lies between } n_1/10 \text{ and } (n_1+1)/10.$$

(9) Make the ratio arm, $PH = 10$, $HQ = 1000$ (then $PH : HQ = 1 : 100$), rheostat arm, $QK = 10\,n_1$, and proceed similarly† to (8). If n_2 is the balancing value of the rheostat arm, then

$$X = n_2 / 100.$$

(10) When the null value cannot be obtained exactly by adjusting the resistances, it may be determined by observing the final small deflections obtained in (9).† Having adjusted the rheostat arm as nearly as possible, read the position at which the index rests when the keys are (i) up, (ii) pressed down. The deflection in scale divisions is the difference in the readings of the index. Alter the resistance of the rheostat arm by one ohm so as to get a deflection the opposite way. Deduce as before the value of the deflection in scale divisions. If, for the smaller resistance of the rheostat arm the deflection is x, for the larger, y, then

† Two or three Leclanché cells in series may now be added to the battery circuit.

$x+y$ is the deflection due to a change of 1 ohm in the rheostat arm. Then $1/(x+y)$ *ohm* would produce a deflection of 1 scale division. Therefore for x scale divisions the increment in resistance $=x/(x+y)$. Hence if $x/(x+y)$ *ohm* is added to the resistance of the rheostat arm that gives the deflection x, then the deflection would be reduced to zero (see example below).

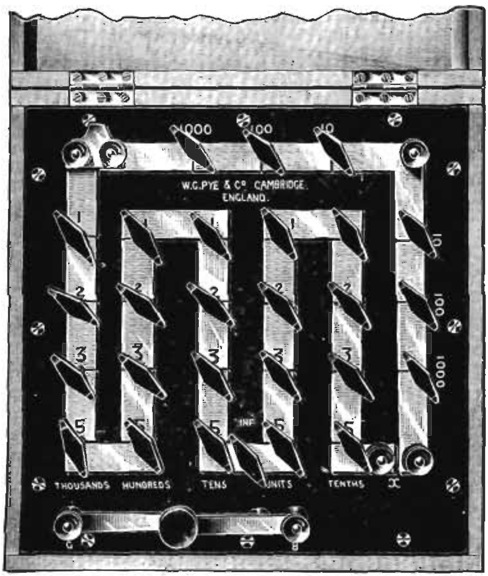

Fig. 234.†

(11) The range of resistance of the rheostat arm is generally from 0 to 11,110. Thus any resistance up to 111·10 can be measured to two decimal places, and estimated to three. If the value is less than one

† Fig. 234 shows another arrangement of the P.O. box. The top and right-hand rows are the ratio arms. The resistances of the rheostat arm are in five rows, of which the respective units are 0·1, 1, 10, 100, 1000 ohms, and the multiples, 1, 2, 3, 5. The unknown is introduced at x. One finger works both keys: the upper closes the battery circuit, the lower closes the galvanometer circuit.

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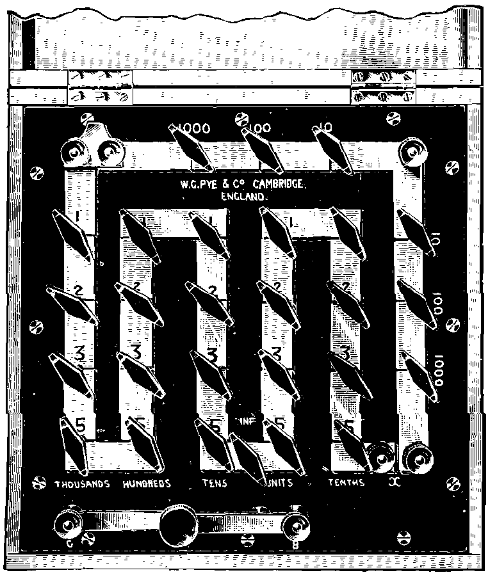

Fig. 234.†

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or two ohms the second decimal place is hardly reliable. Resistances from 11,000 to 110,000 can be measured by making the ratio arms $HP=1000$, $HQ=100$. Then the unknown is 10 times the resistance of the rheostat arm. The measurement, however, would not be closer than 10 ohms. Resistances from 110,000 to 1,110,000 can be measured by making the ratio arms, $HP=1000$, $HQ=10$. The measurement, however, would not be closer than 100 ohms.

EXAMPLE.—A resistance measured by the P.O. box proved to be 67·123 ohms. This value was obtained by the following observations:—

Ratio.	Resistance of Rheostat Arm	Deflection.	Inference.
1000/1000 $=1$ $X=Y$	∞	to right	too large
	0	to left	too small
	1000	to right	too large
	100	to right	too large
	10	to left	too small
	various	——	——
	$10+20+30+4+3=67$	to left	too small
	$10+20+30+4+3+1=68$	to right	too large
1000/100 $=10$ $X=\frac{1}{10}Y$	$670+4=674$	to right	too large
	$670+3=673$	to right	too large
	$670+2=672$	to right	too large
	$670+1=671$	to left	too small
1000/10 $=100$ $X=\frac{1}{100}Y$	$6710+1=6711$	to left	too small
	$6710+2=6712$	to left	too small
	$6710+3=6713$	to right	too large
	6713	9 divs. left	6712·3 is the value for zero deflection†
	6712	4 divs. right	
$\therefore X=67{\cdot}123$.			

† Difference of 1 ohm in rheostat arm gives deflection of 13 divisions.
$\therefore$ deflection of 1 division is equivalent 1/13 ohm in rheostat arm.
$\therefore$ deflection of 4 divisions is equivalent 4/13 ohms $= 0{\cdot}3$.
$\therefore$ 0·3 added to 6712 should make the deflection zero.

Exp. 262.—*Find by the P.O. box the resistance of an electric lamp (cold), the coil of an electric bell, of a galvanometer, of a telephone, etc.*

Exp. 263.—*Find the resistance* (1) *of a moving-magnet galvanometer,* (2) *of a moving-coil galvanometer* (*control the latter by method* (ii) *below*)‡ *by its own deflection* (*Lord Kelvin's method*). Join up the circuit in the usual way for a Wheatstone's bridge test, placing, however, the galvanometer whose resistance is required in the position for the unknown resistance, and connecting each of the opposite corners, between which the galvanometer is usually joined, by a wire to a terminal of a tap key (no galvanometer is now required in this part).† Also put a plug key (plug out) in the battery connection instead of a tap key. Use a constant cell (Daniell).

Operate as in the ordinary method of comparing resistances by the Wheatstone's bridge.

Test.—When things are arranged, put in the plug of the plug key, B, the galvanometer needle is then deflected. Bring its index to a convenient part of the scale ;‡ when it is at rest, press the jockey or tap key, J, if there is *a change in the deflection* of the galvanometer needle the resistances are not balanced. Readjust their values, and repeat the test until there is *no change* in the deflection. The resistances are then related as in § 233.

EXPLANATION.—See § 233, Fig. 226. The galvanometer being placed between P and K, its needle is deflected by the current that flows in PK. When J is pressed at T or U or Q, the current along PK is respectively greater or less than or equal to A_1. Hence the galvanometer carries the same current after J is pressed at Q as before, and there is therefore no change in its deflection. Thus the point Q on the branch HQK having the same potential as P on HPK, can be discovered by adjusting until there is *no change* in the current in a branch, this being indicated by no change in the deflection of a galvanometer needle placed in the branch.

237. Comparison of potential-differences: Poggendorff's method.—Suppose a current, C, to flow along a uniform wire, HK (Fig. 235). Between a point, H, and any other point, Q, on the wire there is a P.D., the value of which is proportional to the length, HQ, between the points. If the P.D. between HK is, say, 2 volts, then that

---

† In Fig. 229, join P to a jockey, J, or tap key by a wire; connect the galvanometer between PK; substitute a plug key for the tap key, B.

‡ If the control magnet is not strong enough, (i) adjust a bar magnet close to the galvanometer: or (ii) introduce a rough resistance box in the battery connection, between the plug key and a terminal of the cell (generally a better plan). When the box resistance is, say 200, the working P.D. between HK is small. Reducing the box resistance increases the working P.D.

between HQ is less than 2, also that between HT is less, and HU greater than HQ.

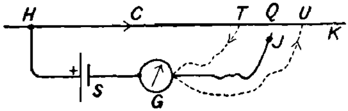


Fig. 235.

Suppose a cell, S, of E.M.F., E, has its + pole joined to H. Then the + pole and H will be at the same potential. The jockey, J, and galvanometer coil will have the same potential as the − pole of the cell, and will therefore be E volts below H (the circuit of the cell being open).

Suppose the point, Q, on the conductor, HK, to be at such a distance from H that the P.D. between $HQ = E$ volts. Then the potential at Q equals that of J. Hence on placing J in contact with Q *no electricity flows*, the galvanometer needle is not disturbed. If, however, J is pressed at a point T between H and Q, then since the P.D. between HT ($=v_1$, say) is less than that between HQ, $\therefore v_1$ is less than E. Hence round the circuit, $HTGH$, the acting E.M.F. $=E-v_1$, and current flows in the direction, TGH. Similarly, if J is pressed at U, beyond Q, so that the P.D. between HU ($=v_2$, say) is greater than E, then round the circuit, $HUGH$, the acting E.M.F. $=v_2-E$, and current flows in the direction, HGU, reverse to the preceding. Hence to find Q, press J into contact with the wire at different points until the position is found at which there is no deflection of the galvanometer needle. This is the required point. Note that on either side of it the deflections are reverse, also that the amount of deflection gets less as the point touched is nearer the required point.

The preceding highly important principle was first applied by Poggendorff to compare the E.M.Fs of cells. The cells are, in turn, substituted for S, and the respective lengths of wire that "balance" the cell determined. If for E.M.Fs, E_1, E_2, E_3 . . . the lengths are HQ_1, HQ_2, HQ_3 . . . then

$$E_1 : E_2 : E_3 = HQ_1 : HQ_2 : HQ_3, \text{ etc.}$$

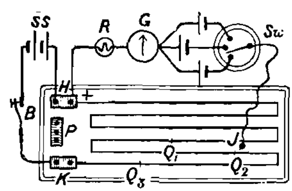


Fig. 236.

The advantages of the method are (i) it is a null method (a very sensitive galvanometer may therefore be used); (ii) the comparison is realised when no current is passing, hence the cells are practically on open circuit, and the relations obtained are independent of their resistances; (iii) the comparison is also independent of the actual strength of current in the potentiometer wire provided it remains constant during the test.

Potentiometer.—This consists of a uniform wire stretched over a scale of equal parts, and is suitable for determinations like the above. (See Figs. 228, 229.)

Practice.—The battery (SS) joined to the ends of the potentiometer wire must be of practically constant E.M.F. The voltage between the ends of the potentiometer wire must be greater than the E.M.F. of any cell to be tested. It is well to join a high resistance, R (10,000 ohms or more) in series with the galvanometer; keep it in circuit until the several balancing points are nearly found, then remove it and find the balancing points for each cell as exactly as possible.

The P.Ds between the ends of resistances, etc., may be compared in a similar manner, and hence the ratio of the resistances deduced (compare Exp. 232).

Exp. 264.—*Compare the E.M.Fs of Daniell, Leclanché, and Bunsen cells by means of the potentiometer.* Fig. 236 shows the arrangement of the circuit: potentiometer wire (HK), jockey (J), tap key (B), secondary cell (SS), galvanometer (G), high resistance (R), 3-way switch (Sw). Record the lengths (HQ_1, etc.) of the stretched wire as determined. Express the E.M.Fs relatively to the Daniell or, if provided, a Clark or other standard cell.

THE CONDENSER.

238. Condensers.—These consist of a number of sheets of tinfoil separated by insulating substance. The insulator is mica in the standard or highest class of condenser, and paraffined paper in others. The first, third, fifth, etc., sheets of tinfoil are connected together, also the second, fourth, sixth, etc. Each set forms a *coat* or *armature* of the condenser. Each set is joined to a terminal on the outside of the box (wood or metal), into which the condenser is put. Fig. 237 shows a standard condenser. The box is circular, made of brass, and has an ebonite top. The terminals are fixed each to a brass piece. A plug fits between these pieces. When the plug is withdrawn the coats are insulated from one another, when it is pushed in the coats are connected.

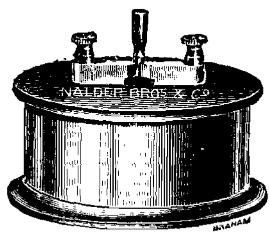


Fig. 237.

Fig. 238 shows a cheaper, but efficient, form. The box is of wood, rectangular in shape. The terminals are fixed to an ebonite plate. There is no plug.

Explanation.—Suppose a body, M, to be kept at a higher potential than another, N: for instance, M and N may be the poles of a battery or two positions on a circuit round which an electric current flows. Let one coat of a condenser be joined with M, the other with N. Electricity will flow into the condenser until the potentials of its coats are equal to those of the points with which they are respectively connected.

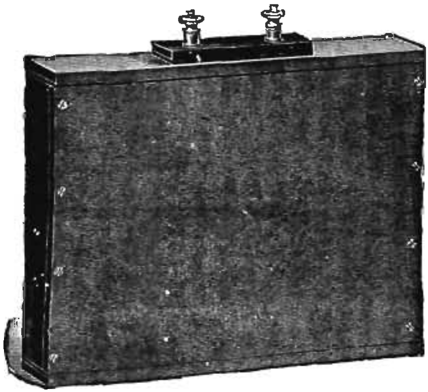


Fig. 238.

Capacity. The ratio of the quantity of electricity (Q) in the condenser to the P.D. between its coats (V) is constant in value: or

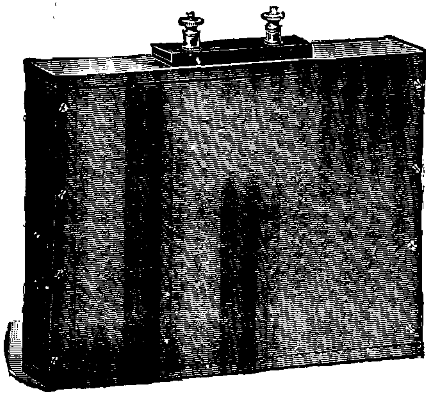
$$Q=F.V,$$

where F is constant. The value of F is called the *capacity* of the condenser. If $V=1$, then $F=Q$, or the capacity is numerically equal to the quantity of electricity that will charge the condenser to unit P.D. The practical unit of capacity is called the *farad*, and is such that a condenser of 1 farad capacity is charged by 1 coulomb to a P.D. of 1 volt. Hence the capacity of any condenser in farads is the number of coulombs of electricity in the condenser when the P.D. between the coats is 1 volt.

Then $$Q \textit{ coulombs}=(F \textit{ farads})\times(V \textit{ volts}).$$

In practice the standard condenser is 1 *micro-farad* (*one millionth farad*). The condensers used for experimental work are generally fractions of the micro-farad : *e.g.* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{10}$, etc.

PRACTICE.—The following experiments with a condenser are done by first charging it, and then discharging it through a ballistic galvanometer. The needle is deflected by the momentary current, oscillates to and fro, and finally comes to rest in its original position. The extent of the *first displacement* or *throw* of the needle is measured by the scale of the instrument. If the galvanometer is a reflecting one the first displacement of the index is proportional to the quantity of electricity discharged. *To measure the throw* note (1) the zero or initial position of the image, (2) the position roughly to which it is coats are equal to those of the points with which they are respectively connected.



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Then $$Q \textit{ coulombs} = (F \textit{ farads}) \times (V \textit{ volts}).$$

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The charging and discharging is done in a moment or two. It is therefore readily effected by including a tap key in the charge and another in the discharge circuit. Two separate tap keys are, however, dangerous, because they are liable to be pressed down simultaneously and thus to connect the galvanometer and battery directly. A *Morse key* (Fig. 239) makes a good charge and discharge key, but does not permit the condenser to be insulated from the circuit. *Ka* is a stiff

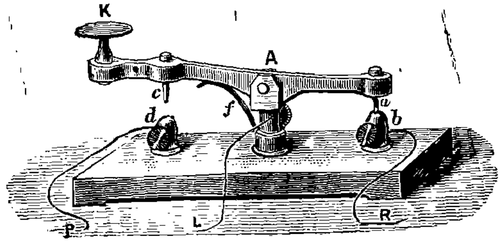

Fig. 239.

metal rod, carrying metallic contact points *a*, *c*. Wires from the circuit are joined to *d*, *f*, *b*, the battery and condenser between *L* and *R*, the condenser and galvanometer between *L* and *P* (Fig. 240). The spring, *f*, keeps *a* in contact with *b*; therefore the condenser is kept charged. Pressing *K* separates *a*, *b*, and then brings *c*, *d* into contact, the condenser then discharges. When *K* is not pressed the spring, *f*, breaks contact at *c*, *d* and restores it at *a*, *b*.

The circuit may be arranged as in Fig. 240: condenser, *F*, charge and discharge key, *KA*, reflecting galvanometer (ballistic), *BG*, and constant cell *E*.

Exp. 265.—*Compare the E.M.Fs of cells by means of a condenser.* Compare secondary, Leclanché, and Daniell cells. Use a condenser of large capacity. (1) Arrange as in Fig. 240, with one of the cells to be tested at *E*. Charge the condenser by connecting it through the key with the cell, then discharge it through the galvanometer. Note the magnitude of the throw produced. (2) Substitute each cell in turn at *E* and repeat (1).

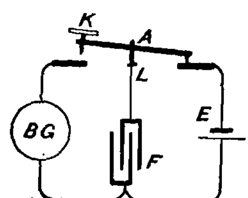

Fig. 240.

The E.M.Fs of the cells are proportional to the respective throws.

thrown on discharge; (3) recharge the condenser, and (4) while looking at the part of the scale to which the image was thrown in (2), discharge and note exactly the scale reading reached by the image. Confirm by repeating the charge and discharge. Calculate the displacement.

The charging and discharging is done in a moment or two. It is therefore readily effected by including a tap key in the charge and another in the discharge circuit. Two separate tap keys are, however, dangerous, because they are liable to be pressed down simultaneously and thus to connect the galvanometer and battery directly. A *Morse key* (Fig. 239) makes a good charge and discharge key, but does not permit the condenser to be insulated from the circuit. *Ka* is a stiff

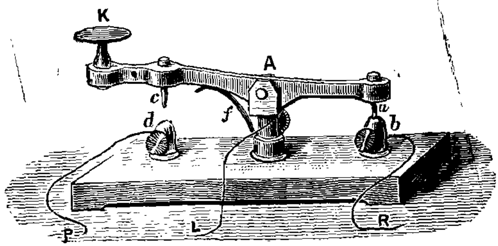

Fig. 239.

metal rod, carrying metallic contact points *a*, *c*. Wires from the circuit are joined to *d*, *f*, *b*, the battery and condenser between *L* and *R*, the condenser and galvanometer between *L* and *P* (Fig. 240). The spring, *f*, keeps *a* in contact with *b*; therefore the condenser is kept charged. Pressing *K* separates *a*, *b*, and then brings *c*, *d* into contact, the condenser then discharges. When *K* is not pressed the spring, *f*, breaks contact at *c*, *d* and restores it at *a*, *b*.

The circuit may be arranged as in Fig. 240: condenser, *F*, charge and discharge key, *KA*, reflecting galvanometer (ballistic), *BG*, and constant cell *E*.

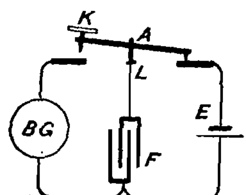

Fig. 240.

Exp. 265.—*Compare the E.M.Fs of cells by means of a condenser.* Compare secondary, Leclanché, and Daniell cells. Use a condenser of large capacity. (1) Arrange as in Fig. 240, with one of the cells to be tested at *E*. Charge the condenser by connecting it through the key with the cell, then discharge it through the galvanometer. Note the magnitude of the throw produced. (2) Substitute each cell in turn at *E* and repeat (1).

The E.M.Fs of the cells are proportional to the respective throws.

THEORY.—If E_1, E_2, E_3 are the E.M.Fs of the cells and F the *capacity of the condenser, then the respective quantities of electricity* are

$$Q_1 = FE_1,\ Q_2 = FE_2,\ Q_3 = FE_3,$$
$$\therefore\ E_1 : E_2 : E_3 = Q_1 : Q_2 : Q_3.$$

When these quantities are discharged through the galvanometer the respective throws are proportional to them, and therefore to the respective E.M.Fs.

Exp. 266.—*Compare the capacities of condensers.*—Arrange as in Fig. 240. Use at E, if necessary, several Leclanché cells in series. (1) Place one of the condensers to be tested at F. Charge, and discharge, note the throw. (2) Substitute each condenser in turn at F and repeat (1).

The capacities are proportional to the respective throws.

THEORY.—Let E be the voltage of the battery, F_1, F_2, the capacities of the condensers. When charged by the battery the quantities of electricity in them are

$$Q_1 = F_1 E,\ Q_2 = F_2 E,$$
$$\therefore\ F_1 : F_2 = Q_1 : Q_2.$$

The quantities are proportional to the throws, therefore the capacities are proportional to the throws.

APPENDIX.

PRACTICAL DETAILS.—TABLES OF FOUR-FIGURE LOGARITHMS, ANTILOGARITHMS AND TRIGONOMETRICAL FUNCTIONS.—TABLE OF ABSOLUTE UNITS.—MENSURATION.—BRITISH AND METRIC WEIGHTS AND MEASURES.—CONVERSION FACTORS.—MECHANICAL DATA AND UNITS.—DATA AND UNITS IN HEAT, SOUND AND LIGHT.—DATA AND UNITS IN ELECTRICITY AND MAGNETISM.

Apparatus for home experiments.—The following pieces of apparatus are required in addition to those mentioned on p. 9:

(15) Glass prism. Cost, 6d.
(16) Glass slab. Cost, 1s. 0d.
(17) Lead ball. Cost, 2d.
(18) Spring. Cost, 6d.
(19) Rubber cord. Cost, 1d.
(20) Thermometer. Cost, 1s. 6d.
(21) Bunsen burner, tubing and tripod. Cost, 2s. 6d.
(22) Horseshoe magnet and keeper. Cost, 4d.
(23) Iron filings. Cost, 2d.
(24) Piece of mirror, mounted. Cost, 6d.
(25) Note-book (pages squared). Cost, 6d.

The cost of the whole, if items 5, 9, 10, 11, 12, 13 and 21 are excluded, is practically 7s. 6d.

Mercury tray.—When mercury is employed in an experiment, the apparatus should be placed in a large shallow tray; the spilt mercury is then prevented from spreading and damaging other apparatus; it can also be recovered. The tray may be made of cardboard, or, better, wood, say $\frac{3}{8}''$ thick. A tray 2 ft. square by 3 ins. deep is useful.

To clean and dry bottles, etc.—Well rinse the bottles with (i) hot, (ii) distilled water: jerk out as much of the water as possible. Hold the bottle by the neck and keep it moving in the warm air above the flame of a Bunsen burner. Remove it when it feels hot. Introduce into the bottle the end of a piece of glass quill tubing and slowly suck air through it; continue doing this until the bottle is cold.

To dilute Sulphuric Acid.—When sulphuric acid and water are mixed together considerable heat is produced. Measure the required volumes of strong acid and water in separate vessels. Add the acid in small portions to the water and stir thoroughly. Allow the mixture to stand until cool.

To weld Platinum.—Pieces of platinum at a white heat can be readily welded. To weld platinum wire to a piece of foil, lay the foil on a smooth thick iron plate resting on a brick. Lay the end of the wire on the foil, direct the flame of a foot blowpipe on the foil and end of wire, and when these are white hot hammer them together with a small hammer.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	25	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969					3	7	11	14	18	21	25	28	32
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	28
					1584	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903					3	6	9	11	14	17	20	23	26
							1931	1959	1987	2014	3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2405	2430					3	5	8	10	13	15	18	20	23
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

Angle. Degrees.	Angle. Radians.	Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
0°	0	0	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine.	Co-tangent.	Tangent.	Sine.	Chord.	Angle. Radians.	Angle. Degrees.

TABLE OF ABSOLUTE UNITS.

Quantity.	Physical Relation, etc.	Absolute Unit. C. G. S.	Absolute Unit. F. P. S.	Relations between the Quantities when measured in Absolute Units.	Other Units. † See pp. 388-394.
Length, l, s Mass, M Time, t	Regarded as fundamental quantities (see § 4)	centimetre gramme second	foot pound second		yard, inch, metre ounce, grain minute, hour
Area, A	square of length	1 square centimetre	1 square foot	$A=l_1\times l_2$	square inch
Volume, V	cube of length	1 cubic centimetre	1 cubic foot	$V=l_1\times l_2\times l_3$	cubic inch
Density, D	mass per unit volume	1 gramme per cu. cm.	1 pound per cu. ft.	$V=MD$	[specific gravity]†
Velocity, v	rate of moving	1 cm. per sec.	1 foot per sec.	$s=vt$	ft. per min.
Angular velocity, w	rate of rotation	1 radian per sec.	1 radian per sec.	w × radius = arc traversed	revs. per min.
Acceleration, a	rate of change of velocity	1 cm. sec. per sec.	1 ft. sec. per sec.	$v=at$	
Momentum	mass × velocity	1 grm. moving with 1 cm. sec.	1 lb. moving with 1 ft. sec.	Mv	
Force, F (Weight)†	rate of change of momentum, or mass × acceleration	1 grm. accelerated 1 cm. sec. per sec. The unit is called a **dyne**.	1 lb. accelerated 1 ft. sec. per sec. The unit is called a **poundal**.	$F=M.a$ $M_2v_2-M_1v_1$ $=F.t$	pound weight†
Impulse (Ft)	force × time	1 dyne for 1 sec.	1 poundal for 1 sec.		
Work or energy	force × distance	1 dyne through 1 cm. The unit is called an **erg**.	1 poundal through 1 ft. The unit is called a ***foot-poundal***.	work $=F.s$ kinetic energy $=\frac{1}{2}Mv^2$	foot-pound† Joule† Kilowatt-hour†
Power or activity	rate of working, or work per unit time	1 erg per sec.	1 ft.-poundal per sec.	power $=\frac{\text{work}}{\text{time}}$	Horse power† Watt†
Stress (pressure tension)	force per unit area	1 dyne per sq. cm.	1 poundal per sq. ft.	F/A	lbs. per sq. in.†, atmosphere†, etc.

QUANTITY.	PHYSICAL RELATION, ETC.	ABSOLUTE UNITS: C. G. S.††	RELATIONS BETWEEN ABSOLUTE UNITS.	OTHER UNITS.
Magnetism Electricity	Quantities of magnetism or electricity are measured by the forces called into play when the quantities are at known distances from one another	The quantity that repels an equal amount at the distance of 1 cm. in air with a force of 1 dyne. Unit quantity of magnetism is called **unit pole**.	$F = \frac{Q_1 \times Q_2}{d^2}$	Coulomb† Ampère hour†
Field intensity, H	Force exerted by the field on unit quantity	1 dyne per unit quantity. Unit intensity of magnetic field is called the **gauss**.	$F = HQ$	
Magnetic moment, M	Strength of pole × distance between the poles	Solenoidal magnet, poles of strength 1 C.G.S. unit, distance 1 cm.	$M = Ql$	
Potential difference, V	Work done in moving unit quantity	1 erg per unit quantity	work $= V.Q$	Volt†
ELECTROMAGNETIC UNITS.††				
Current, c	Measured by the intensity of the magnetic field produced under specified conditions (§ 207)	The current flowing along 1 cm. of wire bent into a circular arc of 1 cm. radius that produces a field intensity of 1 gauss at the centre of the circle.	$F = \frac{Qlc}{r^2}$	Ampère†
Quantity, Q	Amount of electricity that flows through a section of a circuit in unit time	The quantity carried per sec. by unit current through a section of the circuit.	$Q = ct$	Coulomb† Ampère hour†

† See p. 393.

†† F.P.S. units are not specified. For Electrostatic and other units of Electric and Magnetic quantities see Stewart's *Higher Text-Book of Magnetism and Electricity*.

All statements in pages 385-394 which are marked by an asterisk should be committed to memory.

MENSURATION, ETC.

Base of hyperbolic logarithms, $e = 2{\cdot}7183$.

$$\text{Hyp. log } X = 2{\cdot}3026\,(\log_{10} X).$$

$\pi = 3{\cdot}1416$ (= $3{\cdot}14$* or $22/7$* roughly):

$$\log \pi = 0{\cdot}497\ 15:$$
$$\text{hyp. log } \pi = 1{\cdot}1447.$$
$$\pi^2 = 9{\cdot}870,$$
$$1/\pi = 0{\cdot}3183,$$
$$\sqrt{\pi} = 1{\cdot}772.$$

TRIANGLE. Area = $\frac{1}{2} \times \text{base} \times \text{height}$.*

CIRCLE. Circumference = $\pi \times$ diameter.*

$$\text{Area} = \pi(\text{radius})^{2*} = \frac{1}{4\pi}(\text{circumf.})^2.$$

ELLIPSE, semiaxes, a, b. Area = $\pi.\ a.\ b$.

SPHERE. Surface = $4\pi(\text{radius})^{2*} = 12{\cdot}57\,(\text{radius})^2$.
Volume = $\frac{4}{3}\pi\,(\text{radius})^{3*} = 4{\cdot}19\,(\text{radius})^3$.

RIGHT CYLINDER OF PRISM. Volume = (Area base) $\times$ (height).*

RIGHT CONE OR PYRAMID. Volume = $\frac{1}{3}$ (Area base) $\times$ (height).*

MEASURES OF TIME.

1 mean solar day*
= 24 hours (*hrs.*) = 1440 minutes (*m.*) = 864 00 seconds (*s.*).
1 sidereal day = 86 164·1 mean solar seconds (*s.*).

MEASURES OF ANGLE.

A revolution* = 4 right angles = 360 degrees (°) = 2π radians.
1 right angle = 90 degrees* = 5400 minutes = 324 000 seconds.
1 degree (°)* = 60 minutes (′) = 3600 seconds (″).
1 minute (′)* = 60 seconds (″).

Conversion factors for circular measure (radians).

1 right angle = 1·5708 radians.
1° = 0·017 45 radian.
1 radian = 57·3°* = 3437·7′ = 206 265″.

BRITISH MEASURES OF LENGTH.

1 yard* (*yd.*) = 3 feet (*ft.*) = 36 inches (*ins.*).
1 inch (*in.*) = 0·0833 feet.
A mil is one-thousandth of an inch.
The halfpenny is an inch in diameter.*

METRIC MEASURES OF LENGTH.

1 metre* (*m.*)
= 10 decimetres (*dm.*) = 100 centimetres (*cm.*)
= 1000 millimetres (*mm.*).

A micron (μ) is one-thousandth of a millimetre.

A millimicron or micromillimetre ($\mu\mu$) is one-millionth of a millimetre.

An angström or tenth-metre = 10^{-10} metre = 0·000 000 01 cm.

Conversion factors.

1 inch = 2·54 centimetres.* 1 centimetre = 0·3937 inch = 0·0328 ft.
1 foot = 30·48 centimetres. 1 metre = 39·37 inches.*

One-fiftieth of an inch is very nearly half a millimetre.*

One foot is nearly thirty centimetres.*

BRITISH MEASURES OF MASS.

1 pound (*lb.*) avoirdupois* = 16 ounces (*oz.*) = 7000 grains (*grs.*).
0·0625 lb. = 1 ounce = 437·5 grains.

10 pounds is practically the mass of a gallon of water.*

An ounce is practically the mass of one-thousandth of a cubic foot of water.*

The mass of a halfpenny is one-fifth of an ounce.

The grain is the same mass in Troy, Apothecaries and Avoirdupois measures.

METRIC MEASURES OF MASS.

1 kilogramme* (*kgm.*, *kilog.*, *kg.*) = 1000 grammes (*gm.*).
1 gramme* (*gm.*)
= 10 decigrammes (*dgm.*) = 100 centigrammes (*cgm.*)
= 1000 milligrammes (*mgm.*).

One gramme is the mass of a cubic centimetre of water.*

Conversion factors.

1 grain = 0·0648 gramme. 1 gramme = 15·432 grains.
1 oz. avoirdupois = 28·35 grammes. 1 gramme = 0·0353 ounce.
1 lb. avoirdupois = 453·6 grammes.* 1 kilogramme = 2·205 lbs.*

BRITISH MEASURES OF AREA OR SURFACE.

1 square foot* (*sq. ft.*) = 144 square inches (*sq. in.*).
1 square inch = 0·006 944 square feet.

METRIC MEASURES OF AREA OR SURFACE.

1 square centimetre* (sq. cm., $cm.^2$) = 100 square millimetres (sq. mm., $mm.^2$).

Conversion factors.

1 square inch = 6·451 *sq. cm.*	1 *sq. cm.* = 0·155 *sq. in.*
1 square foot = 929 *sq. cm.*	1 *sq. cm.* = 0·001 076 *sq. ft.*

BRITISH MEASURES OF VOLUME OR CAPACITY.

1 cubic foot (*cb. ft.*, $ft.^3$) = 1728 cubic inches* = 6·228 gallons.
1 cubic inch (*cb. in.*, $in.^3$) = 0·000 578 7 cb. ft. = 0·003 604 gallon.
1 gallon = 0·1606 cubic foot = 277·5 cubic inches.
1 fluid ounce (*fl. oz.*) = 0·006 25 gallon = 0·001 003 5 *cb. ft.*

METRIC MEASURES OF VOLUME OR CAPACITY.

1 litre* (*l.*) or 1 cubic decimetre (*cb. dm.*, $dm.^3$) = 1000 cubic centimetres (*cb.c.*, *c.cm.*, *c.c.*, $cm.^3$).

Conversion factors.

1 cubic inch = 16·39 *cb. cm.*	1 cubic centimetre = 0·061 *cb. in.*
1 cubic foot = 28 317 *cb. cm.*	1 litre = 0·035 32 *cb. ft.*
1 gallon = 4541 *cb. cm.*	1 litre = 1·76 pints = 0·2202 gallon.
1 fluid ounce = 28·4 *cb. cm.*	1 litre holds a kilog. (2·2 lbs.) of water.

DENSITY, ETC.

1 gramme per cubic centimetre = 62·43 pounds per cubic foot = 0·0362 pound per cubic inch.

1 pound per cubic foot = 0·016 02 gramme per cubic centimetre.

1 pound per cubic inch = 27·65 grammes per cubic centimetre.

1 cb. ft. water at 62° F. weighs 62·32 lbs. = 997·1 ozs. = 282 69 grammes.

1 cb. in. water at 62° F. weighs 0·036 lb. = 0·5766 oz. = 16·35 grammes.

1 cubic centimetre of water at 4° C. weighs 1·000 013 grammes.

1 pound of water at 62° F. occupies 0·016 cb. ft.* = 27·67 cb. in. = 453·6 cubic centimetres.

1 cb. cm. of mercury weighs at 0° C. 13·596 gms. [13·6*]; at .5° C., 13·56 grammes.

1 cubic foot of air at N.T.P.‡ weighs 0·0807 pound.

1 litre of air at N.T.P.‡ weighs 1·293 grammes.*

1 gramme of air at N.T.P.‡ occupies 773·3 cubic centimetres.

1 litre of hydrogen at N.T.P.‡ weighs 0·090 01 gramme [0·09*].

SPECIFIC GRAVITY (Sp. G.). For the distinction between specific gravity and density see §§ 62, 63.

$$\left(\begin{matrix}\textit{Mass of}\\ \textit{a body}\end{matrix}\right) = \left(\begin{matrix}\textit{Volume of}\\ \textit{the body}\end{matrix}\right) \times \left(\begin{matrix}\textit{Sp. G. of its}\\ \textit{material}\end{matrix}\right) \times \left(\begin{matrix}\textit{Mass of unit}\\ \textit{volume of water}\end{matrix}\right)^*$$

TABLE OF SPECIFIC GRAVITIES.

alcohol	0·8	iron, cast	7·2
aluminium	2·6	,, steel	7·7
brass	8·3	lead	11·3
coal	1·3	marble	2·7
copper	8·7	mercury	13·6
German silver	8·5	petroleum	0·8
glass	2·5	sulphur	2·0
,, flint	3·1	sulphuric acid	1·8
glycerine	1·3	turpentine	0·9
gold coinage	17·49	wax (bees)	0·9
gold, pure	19·3	wood	0·5 to 1
granite	2·7	,, ebony	1·1

Mean density of the Earth, 5·53.

FORCE, WORK, ETC.

VELOCITY. 1 foot second = 30·48 cm. sec. 1 cm. sec. = 0·0328 ft. sec.

ACCELERATION. 1 ft. sec. sec. = 30·48 cm. sec. sec.
32·185 ft. sec. sec. = 981 cm. sec. sec.

ACCELERATION OF GRAVITATION, g, in cm. sec. sec., at an altitude of h cm., and in latitude, ϕ. $g = 980{\cdot}62 - 2{\cdot}6 \cos 2\phi - h/330\ 000$.

Greenwich (lat. $51\frac{1}{2}°$). $g = 32{\cdot}191$ ft. sec. sec. $= 981{\cdot}17$ cm. sec. sec. Length of seconds pendulum = 99·413 cm.

Latitude 45°. $g = 980{\cdot}6$ cm. sec. sec. Length of seconds pendulum = 99·356 cm.

MOMENTUM. 1 ft. lb. sec. = 138 25 cm. gm. sec.
1 cm. gm. sec. = 0·000 072 33 ft. lb. sec.

FORCE. 1 poundal = 138 25 dynes. 1 dyne = 0·000 072 33 poundals. A megadyne is a million dynes.

WEIGHT. The weight of a body at any place is the force with which the earth attracts it at that place.

(*Weight of body*) = (*Mass of body*) × (*Acceleration of gravitation*).*

‡ N.T.P. = normal temperature and pressure.* The normal temperature is 0° C. For the normal pressure see p. 389.

GRAVITATION UNITS OF FORCE.—The weights of unit masses. (The variation in the value of g is neglected.)

The British unit, viz. the weight of a pound of matter, is called a force of **one pound**, or **one pound-weight**.*

1 pound weight = 32·2 poundals‡* = 445 000 dynes.*

1 gramme weight = 981·2 dynes‡* = 0·71 poundals‡.

STRESS (Pressure, tension, shear).

dynes per sq. cm.		lbs. wt. per sq. in.
1	0·0672 poundals per sq. ft.	
14·88	1 poundal per sq. ft.	
68 971	4636·8 poundals per sq. ft.	1
479	1 lb. wt. per sq. ft.‡	0·006 944
$1{\cdot}545\times10^8$	1 ton wt. per sq. inch‡	2240
33 880	1 inch mercury column at 0° C.‡	
1 016 300	30″ (= 76·2 cm.) ditto‡**	
13 338	1 centimetre ditto‡	
1 013 800	76 cm. (= 29·922″) ditto‡**	14·7
1 000 000	74·96 cm. (= 29·513″) ditto	
(megadyne)	A proposed standard atmosphere	

WORK AND ENERGY. POWER.—Gravitation units of work and power are derived from the gravitation units of force. These are used by engineers. They are less exact than absolute units because the variations at different places of the weight of the same mass are neglected.

The British unit quantity of work, called the **foot-pound**, is the work done when a body acted on by a force of one pound-weight moves through a distance of one foot.*

$$\left(\begin{matrix}\textit{Kinetic energy or accumulated}\\ \textit{work in foot-pounds}\end{matrix}\right)^{*} = \frac{1}{2}\,\frac{\textit{mass of body in lbs.}}{32{\cdot}2}\left(\begin{matrix}\textit{speed in}\\ \textit{ft. sec.}\end{matrix}\right)^2.$$

The British engineers' unit power, called the **horse-power** (H.P.), is the power of an agent that can do 33000 foot-pounds of work per minute.*

$$\left(\begin{matrix}\textit{Work done}\\ \textit{in ft. lbs.}\end{matrix}\right) = 33000\times\left(\begin{matrix}\textit{H.P. of}\\ \textit{agent}\end{matrix}\right)\times\left(\begin{matrix}\textit{Time in}\\ \textit{minutes}\end{matrix}\right).$$

In Electrical Engineering other units are also used (definitions, p. 394), viz.:—

FOR ENERGY OR WORK: the joule and kilowatt-hour (B.T.U.).

FOR POWER OR ACTIVITY: the watt and kilowatt.

In thermal work energy is expressed in calories or British thermal units (definitions, p. 390).

** "Normal" atmospheric pressure; also called an *atmosphere*.

‡ Approximate values for English Midlands; g = 981·3 cm. sec. sec. or 32·2 ft. sec. sec.

Work or Energy: Conversion factors.

ergs.		*foot-pounds.*‡
$4 \cdot 214 \times 10^5$	1 foot poundal	0·03106
1	....	$7 \cdot 375 \times 10^{-8}$
$1 \cdot 356 \times 10^7$	....	1
10^7*	1 Joule = 0·24 calorie	0·7375
$3 \cdot 6 \times 10^{13}$.	1 Kilowatt-hour (B.T.U.) = 3600 Joules	$2 \cdot 655 \times 10^6$
$4 \cdot 2 \times 10^7$..	1 Calorie or therm (J.) = 4·2 Joules* ..	$3 \cdot 1 \times 10^{-7}$
$1 \cdot 06 \times 10^{10}$	1 British thermal unit (B.Th.U.) is about 250 calories.......................	775

ergs per sec.		*ft.-pds. per min.*‡
1	....	$4 \cdot 43 \times 10^{-6}$
10^7	1 Watt or Joule per sec. = 0·00134 H.P.	44·3
10^{10}......	1 Kilowatt = 1000 Watts = 1·34 H.P...	44300
$7 \cdot 46 \times 10^9$.	1 Horse power (H.P.) = 746 Watts* ..	33000

HEAT, SOUND, AND LIGHT.

TEMPERATURE.—Measured in (i) degrees Centigrade, or (ii) degrees Fahrenheit.

A degree Centigrade is $\frac{1}{100}$ of the range of temperature between ice-point and steam-point; a **degree Fahrenheit,** $\frac{1}{180}$ of this range.*

The **ice-point** is the temperature of a mixture of pure ice and water.*

The **steam-point** is the temperature of the steam from boiling water when the atmospheric pressure is *normal* (p. 389, also Exp. 95).*

UNITS OF HEAT.—A quantity of heat is measured in terms of the amount absorbed or emitted when unit mass of a standard substance rises or falls one degree in temperature. Two heat-units are common.

(i) The **calorie** (or *therm*).‡‡ The quantity of heat required to raise the temperature of 1 gramme of water by 1° Centigrade.*

(ii) The **British thermal unit** (B.Th.U.). The quantity of heat required to raise the temperature of one pound of water by 1° Fahrenheit.*

The initial temperature of the water is usually not specified. As, however, the thermal capacity of water alters with its temperature, the heat units as defined above are inexact.

Mechanical equivalent of a unit of heat.—Fundamentally, quantity of heat is measured by the units of work. Hence the erg is the C.G.S. unit quantity of heat. The calorie is then regarded as a derived unit. The calorie is practically equal to $4 \cdot 2 \times 10^7$ ergs or 4·2 Joules. (For conversion factors see above.)

‡ $g = 32 \cdot 2$. Hence these are correct for the Midlands of England.

‡‡ The **therm** is sometimes defined more exactly, viz. as the quantity of heat equivalent to 42 000 000 ergs.

THERMAL DATA.—

	Melting-point: °C.	Boiling-point: °C.	Specific Heat.	Coefficient Cubical Expansion.
Alcohol (ethyl) ..	—	78·3	0·61	0·001 08
Benzene	—	80·8	0·45	0·001 38
Brass	1040	—	0·094	0 000 057
Carbon bisulphide ..	—	48	0·22	0·001 47
Chloroform	—	61·2	0·23	0·001 4
Copper	1080	—	0·095	0·000 05
Ether (ethyl) ..	—	35·5	0·517	0·002 1
Glass	—	—	0·2	0·000 024
Glycerine	—	290	0·58	0·000 53
Ice	0	—	0·5	0·000 16
Iron	1500	—	0·112	0·000 035
Lead	325	—	0·031	0·000 084
Marble	—	—	0·2	—
Mercury	−39·5	350	0·033	0·000 180 2
Paraffin	—	370	0·68	—
Sulphur	114·5	444	0·23	—
Sulphuric Acid ..	—	326	0·33	0·000 49
Turpentine	—	156	0·47	0·001
Water	—	100	1	0·000 21 (20°)

AIR.—Specific heat at constant volume, 0·1684.
,, ,, ,, pressure, 0·2375.
Coefficient of expansion at constant volume or pressure
$= 0{\cdot}003\ 665$ approx. $= \frac{1}{273}$ approx.*

ALCOHOL: VAPOUR PRESSURE.

Temp. °C.	−20	0	10	20	30
Dynes per sq. cm.	4455	16 940	32 320	59 310	104 800

ETHER: VAPOUR PRESSURE.

Temp. °C.	−20	0	10	20	30
Dynes per sq. cm.	91 900	246 000	382 600	577 200	846 800

MERCURY: VAPOUR PRESSURE.

Temp. °C.	0	50	100	200	300
V.P. mm. Merc.	negligible	0·012	0·28	17·81	246·8

WATER.—Latent heat of fusion .. .. .. 80 approx.*
,, ,, vaporisation at 100° C. 540 approx.*
Temperature of maximum density .. 3·98° C.

STEAM.—Specific heat at constant volume .. 0·37
,, ,, ,, pressure . 0·48.

PRESSURE OF AQUEOUS VAPOUR (also see p. 125).

Temp. °C.	V.P. mm. Merc.	Temp. °C.	V.P. in Atmospheres.
−10	2·1	100	1
−5	3·1	150	4·7
See cols. 1, 2, page 133.		200	15·4
50	92		
75	289		
100	760		

DIATONIC SCALE and Standard Pitch: see pp. 160, 161.

VELOCITY OF SOUND.

Air at 0° C.* = 1091 ft. sec. = 33 200 cm. sec. (see p. 164).
Various woods.. 300 000 to 600 000 cm. sec.
Glass .. .. 400 000 to 500 000 cm. sec.
Brass .. .. about 400 000 cm. sec.
Steel .. .. about 500 000 cm. sec.

PHOTOMETRIC units, etc. (see p. 190).

WAVE LENGTHS of spectrum lines (see p. 238).

INDICES OF REFRACTION and critical angles (see p. 176).

VELOCITY of light in vacuo = 3×10^{10} cm. sec. = 186000 miles sec.

VIBRATIONS PER SEC., *of waves corresponding to Faunhofer* lines: A (red), $3·945 \times 10^{14}$; D (orange), $5·092 \times 10^{14}$; F, $6·172 \times 10^{14}$; G, $6·965 \times 10^{14}$; H (violet), $7·6 \times 10^{14}$.

ELECTRICAL UNITS, ETC.

These are related to C.G.S. units. The so-called **practical units** are indicated below by §.

I. Primary Units.

Resistance. (1) Unit called the international **ohm** ($= 10^9$ C.G.S. units).§* The resistance of a cylinder of mercury, temperature 0° C.; length, 106·3 cm.; mass, 14·4521 grm. (Cross section, 1 sq. mm.)

The megohm is one million ohms. The microhm is one-millionth of an ohm.

(2) Unit called the **legal ohm** § ($= 10^9$ C.G.S. approx.). Cylinder of mercury, 0° C.; 106 cm.; 1 sq. mm.

(3) Unit called the **B.A. unit of resistance**. The resistance of a certain piece of platinum-silver wire (equals that of a cylinder of mercury, 0° C.; 104·87 cm.; 1 sq. mm.).

Conversion factors.

1 International ohm	= 1·0028 legal ohms	= 1·0136 B.A. units	
0·9972 ,,	= 1 ,,	= 1·0108 ,,	
0·9866 ,,	= 0·9893 ,,	= 1 ,,	

Current.—Unit called the **ampère**§* ($= \frac{1}{10}$ C.G.S. unit). The electric current of constant strength which when passed through a solution of silver nitrate deposits silver at the rate of 0·001118 grm. per sec. (p. 304).

II. *Derived Units of Electrical Quantities.*

Electromotive Force (E.M.F.), Difference of potential or electric pressure (P.D.). Unit called the **volt** §* ($= 10^8$ C.G.S. units). The potential difference that produces a constant current of 1 ampère in a conductor of 1 ohm resistance.

$$\therefore \textit{Volts} = \textit{Ampères} \times \textit{ohms}.*$$

Quantity of Electricity or Charge. (1) Unit called the **coulomb***§ or **ampère-second** ($= \frac{1}{10}$ C.G.S. unit). The quantity of electricity that flows per second across any section of the circuit when a constant current of one ampère is maintained.

$$\therefore \textit{coulombs} = \textit{ampères} \times \textit{seconds}.*$$

The **ampère-hour** (= 3600 coulombs). In the definition of the coulomb substitute per hour for per second.

$$\therefore \textit{ampère-hours} = \textit{ampères} \times \textit{hours}.$$

Capacity.—Unit called the **farad**§* ($= 10^{-9}$ C.G.S. unit). The capacity of a condenser which when charged with 1 coulomb has a P.D. between its coats of 1 volt.

The **microfarad*** is a capacity of one-millionth of a farad,

$$\therefore \textit{farads} = \textit{coulombs} \div \textit{volts}.*$$

III. Derived Units of Mechanical Quantities.

Power or Activity.—Unit called the **watt**§* (= 10^7 ergs per sec., or 1 joule per sec.). The power supplied when a current of 1 ampère and a P.D. of 1 volt are maintained in a circuit,

$$\therefore \textit{watts} = \textit{ampères} \times \textit{volts} = (\textit{ampères})^2 \times \textit{ohms} = (\textit{volts})^2 \div \textit{ohms}.$$

The **kilowatt** is a power of one thousand watts.

Energy or Work.—(1) Unit called the **joule** or **watt-second**§* (= 10^7 ergs). The energy absorbed per second by an electric circuit when the power supplied to it is 1 watt.

(2) Unit called the **kilowatt-hour** or **Board of Trade unit**, B.T.U. The energy absorbed per hour by an electric circuit when the power supplied to it is one kilowatt.

$$\textit{Joules} = \textit{ampères} \times \textit{volts} \times \textit{time in seconds.*}$$

$$\textit{Kilowatt-hours} = \tfrac{1}{1000}\ \textit{ampères} \times \textit{volts} \times \textit{time in hours}.$$

Electrical and Magnetic Data, etc.

Electrochemical equivalents. (See p. 304.)

Electromotive forces of cells. (See p. 291.)

Magnetic elements for 1910.

Place.	*Declination.*	*Dip.*	*Horizontal intensity of C.G.S. units.*
London	16° 0′ W.	67° 0′	0·18
Exeter	17 15	66 50	0·17
Manchester	17 20	68 30	0·17
Edinburgh	18 40	69 55	0·18

Relative resistances. (See p. 294.)

Resistance of a metal generally increases as the temperature rises. For small changes of temperature,

$$R_{t'} = R_t\{1 + \alpha(t' - t)\},$$

α is called the *temperature coefficient*. For pure metals α is about 0·004. For alloys it may be less, in some cases negligible, *e.g.* manganin.

Resistivity per centimetre cube at 0° C. in **microhms**.

copper	1·63	manganin	about	44
iron	9·0	platinoid	,,	40
mercury	94·07	carbon (arc lamps)	,,	3500
platinum	10·90	saturated $CuSO_4$	,,	29×10^6
eureka	about 40	,, $ZnSO_4$	,,	34×10^6
German silver	,, 30	H_2SO_4 (specific gravity 1·2)	,,	$1{\cdot}24 \times 10^6$

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