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Paretianism, Justice and Liberty in the Theory of Social Choice

by

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Paretianism, Justice and Liberalism in the Theory of Social Choice

Abstract

Two celebrated theorems in the theory of social choice are those of Sen (the "impossibility of a Paretian liberal") and Kelly ("the impossibility of a just liberal"). The former could be regarded as a damaging critique of the Pareto-principle and the latter as a damaging critique of the "Arrow-Sen" programme of enriching utility-based information through the use of extended preference orderings. Such a view would, however, be convincing only if the principles of liberalism used by the two authors were unexceptionable. Without in any way holding a brief for welfare judgements founded solely on individual utility orderings, it is argued in this note that the source of Kelly's and Sen's liberal paradoxes is to be located more proximally in their respective principles of liberalism than in, respectively, the "Arrow-Sen" programme of extended sympathy and the Pareto principle.

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Paretianism, Justice and Liberalism in the Theory of Social Choice

Introduction

An important aspect of the framework of Professor Arrow's (1963) General Possibility Theorem relates to the informational base exploited by the framework in guiding social welfare judgements. In a paper of wide-reaching significance, Professor Sen (1979) has criticized the remarkably impoverished informational base which traditional welfare economics employs in the aid of evaluating the welfare-content of alternative states-of-the-world. Arrow's own impossibility result is shown to be amenable to the interpretation that it is a somewhat inevitable outcome of the use of restricted information - specifically information solely on individual utility orderings - which, furthermore, is not rich enough to accommodate interpersonally comparable individual utility orderings. Arrow's (1967) notion of "extended sympathy" has been used by Sen (1970a) to give a precise meaning to the way in which information on individual utility orderings *can* be used t effect interpersonal comparisons of utility. This would entail employing individual *extended* preference orderings which permit the use of statements such as: "individual i prefers being person j in state x to being person k in state y". The Arrow-Sen programme of enriching utility-based information with the aid of the instrument of "extended sympathy" might have been expected to serve as a way out of various "Arrow-type" impossibility results.

What happens when the informational base is permitted to be infected by some measure of *non*-utility information? This constitutes the content, from one possible interpretation, of Sen's (1970b) celebrated theorem on the "impossibility of a Paretian Liberal", in which he demonstrates that the Pareto principle is incompatible, in a specific way, with a principle of liberalism that appears to demand no more than a very

minimal recognition of what constitutes liberalism. Sen's liberal principle requires, broadly, that if two states of the world differ from one another only in a feature which is of exclusive personal relevance to some person, then these states should enter into the person's "protected sphere", in the sense that the person should be allowed to be decisive in the social choice between the two states. Another principle, apart from Paretianism, with which the liberal principle can be shown to be irreconcilable is what may be called the "principle of non-discrimination", which will be examined in some-what greater detail at a later stage. For the present, it may be noted that liberalism in the Sen sense is compatible, under certain conditions, with conferring upon a single individual the power to determine social choices over pairs of alternatives which do not fall into anybody's "protected sphere". It could be argued that while it is best that no single individual should be entitled to such an overriding privilege, if any one individual is allowed this privilege, then non-discrimination should require that every individual be allowed a similar matching privilege. Subject to some conditions, it can be demonstrated that a social choice rule whose range is confined to acyclic social preference relations may be able to satisfy the liberal and non-discrimination principles only at the cost of declaring Pareto non-comparable alternatives to be collectively indifferent, while a social choice rule whose range is confined to quasitransitive social preference relations may not exist at all, if it is required to satisfy the liberal and non-discrimination principles. Now these results would seem to highlight the tension attendant upon evaluating the welfare-content of states-of-the-world when the underlying informational base is a mixed bag of utility and non-utility information; the severe inadequacies of the former, it can be argued, clash with the differing motivation underlying the latter, leading thereby to contradiction. This could be seen as an argument for restricting the scope of welfare judgements founded solely on individual utility orderings.

Is there a way out of the difficulty if the Arrow-Sen programme of enriching utility-information through the use of "extended" preference orderings is implemented? It would appear not: Professor Kelly (1976) has demonstrated that one other principle with which (a modified version of Sen's) liberalism conflicts is Suppes' (1966) "grading principle of justice", a principle which is based on the use of extended preference orderings. Kelly's liberalism, which he calls "weak just liberalism", requires that each of at least two persons should be assigned a pair of social states such that the relevant person's preference over his assigned pair of alternatives should be socially respected, provided nobody believes the opposite preference to be more just in the sense of Suppes; this principle of liberalism is shown to be inconsistent with the requirement of justice which demands that if every individual considers some state x to be more just than some state y (in the sense of Suppes), then y should not be socially chosen in the presence of x[1]. It would appear then that the Arrow-Sen programme is also a failure in terms of ensuring that utility information - however enriched through the accommodation of interpersonal comparisons of utility - is adequate for effecting meaningful social-welfare judgements.

In short, Sen's and Kelly's theorems on, respectively, the impossibility of a Paretian liberal and the impossibility of a just liberal seem to furnish good grounds for questioning the Pareto principle and, more generally, the soundness of any welfare-judgement based exclusively on individual utility orderings, extended or otherwise. Indeed, in reviewing an immense literature spawned by the "Paretian-Liberal Paradox", Sen (1976, p.219) noted: "..... a mechanical use of the Pareto rule irrespective of context seems questionable. One

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1. It should be noted that Kelly's impossibility theorem is presented in choice-theoretic terms while that of Sen's is presented in relation - theoretic terms; in this note, I shall preserve the authors' respective approaches.

of the issues that will be examined in this note in the light of subsequent contributions is whether this questioning of the Pareto principle stands, since many of the contributions ... have revealed a preference for resolving the conflict by weakening condition [Sen's liberal principle] rather than by weakening the Pareto principle". Again in the same paper, we have (p.226): "The fundamental issue really is whether individual preference orderings alone provide enough of a basis for a social judgement without going into the causation of and motivation behind these preferences ...". The burden of Kelly's own impossibility result is also made to be borne by the inadequacy of individual preference orderings for contributing to a sound basis for social judgements: Kelly (1976 op.cit., p.71) remarks: "... [The impossibility of a just liberal] does seem to be a rather damaging critique of the Arrow-Sen programme of introducing extended sympathy as a resolution of the Arrow problem".

Now the question I shall be addressing myself to is whether the principles of liberalism used by Sen and Kelly are unexceptionable enough to warrant the view that the "villain" of the paradoxes that have been discussed must be sought elsewhere than in these principles themselves. It will be contended in this note that while acknowledging the normative inadequacies of the Pareto principle or more generally of any strictly "welfarist" approach to the evaluation of states-of-the-world, it is still possible to question the appropriateness of demonstrating the embarrassment to one's moral intuition caused by such a "welfarist approach" on grounds of its alleged inconsistency with liberalism when the latter itself, as will be argued, has not been interpreted in a "reasonable" enough way. Specifically, it will be argued in the context of Kelly's "weak just liberalism" that the latter is not even self-consistent, leave alone consistent with the justice principle. Insofar as Sen's principle of liberalism is concerned, it will be contended that there is room for modification of the principle along lines

that will preclude the possibility of conflict with the Pareto and the non-discrimination principles - and further that such modification is motivated by the internal logic of liberalism itself, and not just by an *ad hoc* impulse to resolve a paradox.[2] In particular, it will be held that it is of the essence of liberalism to require that the fact of one individual's exercising his right does not lead to a state of the world which precludes the possibility of some *other* individual's exercising his right: Sen's principle of liberalism fails to ensure the necessity of seeing individual rights as being "co-possible" (on this see Nozick, 1974). For example, Sen liberalism would support the oddly contradictory claim that my liberty and my neighbour's liberty in the matter of choice of clothes, say, are protected if my desire to wear a white shirt is secured in a world in which my neighbour is condemned to wearing a green shirt which he despises, and my neighbour's desire to wear a pink shirt is secured in a world in which I am condemned to wearing a ghastly purple shirt: in securing *my* liberty, no cognizance seems to be taken of my neighbour's condition, and in securing *my neighbour's* liberty, no cognizance seems to be taken of my condition. In short, a proper acknowledgement of our respective liberties should ensure that each person is decisive over a pair of alternatives in which the *other* person's subjective preference is not violated. Where this is not ensured, liberal paradoxes of the type we have described are an unsurprising consequence.

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2. For example, Karni's (1978) suggestion for weakening liberalism by requiring semi-decisiveness rather than decisiveness over pairs of alternatives removes the problem of a Paretian-Liberal conflict by ensuring that cycles in the strict social preference relation are not generated; it is not clear though how requiring semi-decisiveness rather than decisiveness takes us closer to the spirit of liberalism.

The present note is divided into six sections. Section I introduces all the important concepts and definitions that shall be employed in the note. Section II states Kelly's result on the impossibility of a just liberal, and demonstrates how his concept of liberalism is self-inconsistent. Sections III(a) and III(b) present, respectively, Sen's result on the impossibility of a Paretian liberal and a result on the impossibility of a non-discriminating liberal. Sections IV(a) and IV(b) present illustrative sketches of how to circumvent the corresponding paradoxes of Sections III(a) and III(b) respectively, through appropriate specification of the agenda over which choices are to be made. The treatment throughout is informal, and relies heavily on the use of examples and illustrations. In Section V, the source of the paradoxes is identified and discussed, and the paradoxes are sought to be eliminated through appropriate reformulation of Sen's principle of liberalism. Concluding observations are offered in Section IV.

I. Concepts and Definitions

In this Section, I get most of the formalities involving the definitions of important terms and concepts out of the way, leaving the remaining sections free for more-or-less informal discussion.

Let X be the set of all available social states and H the set of all individuals in society, with the individuals being designated by the running index $i = 1, 2, \ldots, n$. Let $\mathcal{A}$ be the set of all non-empty subsets of X, viz., the null set-excluded-power set of X. Every $A \in \mathcal{A}$ is called an agenda (see Plott, 1973). For every $A \in \mathcal{A}$, let R_A be the set of all preference orderings defined on A, i.e. let R_A be the Cartesian product of A with itself. Let R_A^n be the n-fold Cartesian product of R_A. Then the n-tuple $\{R_i\}_{i \in H}$ is an element of the set R_A^n, and R_i is individual i's preference ordering on A where, for all $x, y \in A$ and all $i \in H$, $x R_i y$ is interpreted

to mean that x is regarded by i to be at least as good as y. P_i and I_i are, respectively, the asymmetric and symmetric parts of R_i. For every n-tuple of individual preference oderings - one ordering for each individual - there is assumed to exist a unique social preference relation R. For any two alternatives $x, y \in A$, xRy is interpreted to mean that x is regarded as socially at least as good as y. P and I are the asymmetric and symmetric parts respectively of R. The notion of the dependence of social preference on individual preference orderings is captured formally by the statement $R = f(\{R_i\}_{i \in H})$. A Collective Choice Rule (or CCR) is a functional relation f which, for every n-tuple of individual preference orderings $\{R_i\}_{i \in H}$ in its domain, specifies a unique social preference relation R. Two notions which will be of relevance (to the discussion of the impossibility of a just liberal) are those of an extended preference ordering and a General Collective Choice Function, to which we now turn. For every $A \in \mathcal{A}$, let R_A be the set of all preference orderings defined on the set A x H, and let $\tilde{R}^n_A$ be the n-fold Cartesian product of $\tilde{R}_A$. Then the n-tuple $\{\tilde{R}_i\}_{i \in H}$ is an element of the set $\tilde{R}^n_A$ and $\tilde{R}_i$ is individual i's extended preference ordering on the set A x H, where for all $x, y \in A$ and all i, j, $k \in H$, $(x, j)\ \tilde{R}_i\ (y, k)$ is interpreted to mean that person i considers being person j in state x to be at least as good as being person k in state y. $\tilde{P}_i$ and $\tilde{I}_i$ are, respectively, the asymmetric and symmetric parts of $\tilde{R}_i$. A General Collective Choice Function (or GCCF) is a function C(.) which for every agenda A and the corresponding n-tuple of extended preference orderings defined on A x H in its domain, specifies the (socially) chosen element(s) as some non-empty subset of A:

$$C(A, \{\tilde{R}_i\}_{i \in H}) = S(\neq \{\emptyset\}) \subset A\ ,\ [A \in \mathcal{A}, \{\tilde{R}_i\}_{i \in H} \in \tilde{R}^n_A]$$

Other relevant definitions are provided below:

Definition 1 (Transitivity): A binary relation R is said to be transitive if for every triple of alternatives x, y, z A, x R y and y R z implies x R z.

Definition 2 (Acyclicity): Acyclicity of a binary relation R requires that for any subset of m (≥ 3) social states $(x_1, x_2, \ldots, x_m)$ in A,

$$x_1 P x_2 \ldots P x_m \rightarrow \sim(x_m P x_1) \; (\rightarrow x_1 R x_m).$$

Definition 3 (Quasitransitivity): A binary relation R is said to be quasitransitive if for every triple of alternatives $x, y, z \in A$, $x \, P \, y$ and $y \, P \, z$ implies $x \, P \, z$.

Notice that acyclicity is a weaker requirement than quasitransitivity and quasitransitivity a weaker requirement than transitivity of R.

Definition 4 (Acyclic Social Decision Function): An acyclic social decision function (or ASDF) is a CCR the range of which is confined to acyclic social preference relations.

Definition 5 (Quasitransitive Social Decision Function): A quasitransitive social decision function (or QTSDF) is a CCR the range of which is confined to transitive strict social preference relations.

Definition 6 (Almost Decisive Individual): An individual $i \in H$ is said to be almost decisive over some ordered pair (x,y) of alternatives in A if whenever by virtue of his preferring x to y with the rest of the individuals preferring y to x, x is declared to be socially preferred to y.

Definition 7 (Almost Semidecisive Individual): An individual $i \in H$ is said to be almost semidecisive over some ordered pair (x,y) of alternatives in A if whenever by virtue of his preferring x to y with the rest of the individuals preferring y to x, x is declared to be socially at least as good as y.

Definition 8 (Quasidecisive Individual): An individual $i \in H$ is said to be quasidecisive over some ordered pair (x,y) of alternatives in A if whenever by virtue of his preferring x to y with at least one other individual preferring y to x, x is declared to be socially at least as good as y.

(Note: Clearly, almost semidecisiveness implies quasidecisiveness, but not vice versa; further, in a two-person community, almost semidecisiveness and quasidecisiveness would be equivalent).

Certain conditions which can be imposed on a CCR, as well as certain concepts of relevance to the problem of a just liberal are now described:

a) Condition U (Unrestricted Domain): The domain of the CCR is unrestricted, viz., the domain admits every logically possible n-tuple of individual preference orderings.

b) Condition P* (the Weak Pareto principle): If all individuals of a community prefer some alternative x in A to some alternative y in A, then so should society:
$$[\forall x,y \in A, \forall i \in H : x P_i y] \rightarrow x P y$$

c) Condition L (Liberalism): I quote from Sen (1970 a, op.cit): "For each individual i, there is at least one pair of alternatives, say (x,y), such that if this individual prefers x to y, then society should prefer x to y, and if this individual prefers y to x, then society should prefer y to x."

d) Condition L* (Minimal Liberalism): Again I quote from Sen (1970a, op.cit.): "There are at least two individuals such that for each of them there is at least one pair of alternatives over which he is decisive, that is, there is a pair x,y, such that if he prefers x (respectively y) to y (respectively x), then society should prefer x (respectively y) to y (respectively x)".

e) Condition I (Independence of Irrelevant Alternatives): If $\forall x,y \in A$, $\forall i \in H$: $x R_i y \leftrightarrow x R'_i y$ and $y R_i x \leftrightarrow y R'_i x$, then the ranking of x and y vis-a-vis each other by $f(\{R_i\}_{i \in H})$ and $f(\{R'_i\}_{i \in H})$ should be the same.

In other words, condition I demands that the social ranking of any two alternatives x,y should depend on individual orderings on only those two alternatives x,y.

f) Condition ND (Non-Discrimination): If any one individual is allowed almost semidecisiveness over some m ($\geq$ 1) distinct ordered pairs of alternatives not falling in anybody's protected sphere, then every individual should be allowed quasidecisiveness over m distinct ordered pairs of alternatives not falling in anybody's protected sphere.

g) Condition WND (Weak Non-Discrimination): If any one individual is allowed almost decisiveness over some m ($\geq$ 1) distinct ordered pairs of alternatives not falling in anybody's protected sphere, then every individual should be allowed quasidecisiveness over at least one ordered pair of alternatives not falling in anybody's protected sphere.

h) The Justice Criterion: Let T be a set of all one-to-one correspondences from H onto itself. Then $x J_i y$ (interpreted to mean that x is regarded by i to be more just then y) will hold if and only if $\exists f \in T$ such that $\forall j \in H : (x, j) \widetilde{R}_i (y, f(j))$ and $\exists i : (x, j) \widetilde{P}_i (y, f(j))$.

i) Just General Collective Choice Function (JGCCF) (following Kelly (1976, op.cit.)): A GCCF $C(A, \{\widetilde{R}_i\}_{i \in H})$ is said to be just if and only if $[\forall x, y \in A, \forall i \in H : x J_i y] \rightarrow y \notin C(A, \{\widetilde{R}_i\}_{i \in H})$. That is, if everybody considers x to be more just than y (in the sense of definition (h) above), then society should not chose y in the presence of x.

j) The Axiom of Complete Identity: The axiom of complete identity requires that $\forall i, j \in H : \widetilde{R}_i = \widetilde{R}_j$. The axiom ensures (see Sen, 1970b, op.cit.) that the Pareto Principle does not conflict with the justice criterion.

k) Condition WJL (Weak Just Liberalism) (following Kelly (1976, op.cit.)): Weak just liberalism requires that $\exists A = \{x,y,z,w\} \in \mathcal{A}$ & $\exists\, i,j \in H$ such that

(i) $[(x,i)\,\tilde{P}_i\,(y,i) \;\&\; \sim(\exists k : yJ_k x)] \rightarrow y \notin C(A, \{\tilde{R}_i\}_{i\in H})$.

(ii) $[(y,i)\,\tilde{P}_i\,(x,i) \;\&\; \sim(\exists k : xJ_k y)] \rightarrow x \notin C(A, \{\tilde{R}_i\}_{i\in H})$.

(iii) $[(w,j)\,\tilde{P}_j\,(z,j) \;\&\; \sim(\exists k : zJ_k w)] \rightarrow z \notin C(A, \{\tilde{R}_i\}_{i\in H})$.

(iv) $[(z,j)\,\tilde{P}_j\,(w,j) \;\&\; \sim(\exists k : wJ_k z)] \rightarrow w \notin C(A, \{\tilde{R}_i\}_{i\in H})$.

In other words, condition WJL requires that there be at least two individuals i and j and two pairs of alternatives (x,y) and(z,w) such that i's preference over the pair (x,y) be reflected by society provided nobody considers the opposite preference to be more just, and similarly j's preference over the pair (z,w) be reflected by society, again provided nobody considers the opposite preference to be more just.

Given the preceding inventory of terms, definitions and concepts, one is now free to proceed with the substantive issues of this note.

II. The Self-inconsistency of Kelly's Liberalism

It will be demonstrated in this section that Kelly's concept of liberalism suffers from the problem of internal inconsistency, which renders the result of an incompatibility between his liberal principle and the justice criterion somewhat empty of significance.

Kelly (1976, op. cit.) has proved a theorem to the effect that there exists no just social choice function satisfying weak just liberalism and the domain restriction that every n-tuple of extended preference orderings $\{\tilde{R}_i\}_{i\in H}$ must satisfy the axiom of complete identity.

The proof of this theorem is available in Kelly's paper and we need not go through it again here, save to note that the manner of proof consists in positing a configuration of extended individual preference orderings (which respects the axiom of complete identity) such that this enables one to systematically empty out the choice set on grounds either of liberalism or of both liberalism and justice until the choice set is rendered null, and the domain restriction of a non-empty choice set is violated. Kelly's proof covers three cases: a) the case where the pairs of alternatives (x,y) and (z,w) are the same; b) the case where the pairs of alternatives (x,y) and (z,w) have exactly one alternative in common; and c) the case where all of the four alternatives x,y,z,w are distinct. In seeking a self-contradiction in the concept of weak just liberalism we do not have to look beyond Kelly's proof for case (a), which demonstrates that there exists no general collective choice function (note that one does not require a <u>just</u> collective choice function) satisfying condition WJL. This result carries over also to the case where more than one individual is not allowed decisiveness over the <u>same</u> pair of alternatives, if we strengthen Kelly's weak just liberalism to what may be called just liberalism whereby at least <u>three</u> individuals are allowed liberal privileges in the Kelly sense.

<u>Definition (Just Liberalism)</u>: <u>Just liberalism</u> (or JL) requires that

$\exists A = \{x,y,z,w,u,v\} \in \mathcal{A} \;\&\; \exists\, i,j,R_i \in H$ such that

(i) $[(x,i)\,\tilde{P}_i\,(y,i)\;\&\sim(\exists \ell : yJ_\ell x)] \rightarrow y \notin C(A, \{\tilde{R}_i\}_{i\in H})$;

(ii) $[(y,i)\,\tilde{P}_i\,(z,i)\;\&\sim(\exists \ell : xJ_\ell y)] \rightarrow x \notin C(A, \{\tilde{R}_i\}_{i\in H})$;

(iii) $[(w,j)\,\tilde{P}_j\,(z,j)\;\&\sim(\exists \ell : zJ_\ell w)] \rightarrow z \notin C(A, \{\tilde{R}_i\}_{i\in H})$;

(iv) $[(z,j)\,\tilde{P}_j\,(w,j)\;\&\sim(\exists \ell : wJ_\ell z)] \rightarrow w \notin C(A, \{\tilde{R}_i\}_{i\in H})$;

(v) $[(u,k)\widetilde{P}_k(v,k) \;\&\; \sim(\exists \ell : v J_\ell u)] \rightarrow v \notin C(A,\{\widetilde{R}_i\}_{i\in H})$;

(vi) $[(v,k)\widetilde{P}_k(u,k) \;\&\; \sim(\exists \ell : u J_\ell v)] \rightarrow u \notin C(A,\{\widetilde{R}_i\}_{i\in H})$.

Consider now the case where (x,y) and (z,w) have exactly one element in common, say y = w; (z,w) and (u,v) have exactly one element in common, say z = u; and (u,v) and (x,y) have exactly one element in common, say x = v.

Under these conditions it can be shown that there exists no GCCF satisfying JL and the domain restriction that every n-tuple of extended preference orderings must satisfy the axiom of complete identity.

To see this, consider the following configuration of extended individual preference orderings. Suppose, $\forall i \in H$:

(II.1) $(z,3)\widetilde{P}_i(x,1)\,\widetilde{P}_i(x,3)\,\widetilde{P}_i(z,1)\,\widetilde{P}_i(y,3)\,\widetilde{P}_i(y,1)\,\widetilde{P}_i(y,2)\,\widetilde{P}_i(x,2)\widetilde{P}_i(z,2)$

It is useful to note here that for a society consisting of three individuals i.e. for $H = \{1,2,3\}$, the set T of all one-to-one correspondences from H onto itself is given by

$T = \{(1\to1, 2\to3, 3\to2), (1\to1, 2\to2, 3\to3), (1\to2, 2\to3, 3\to1), (1\to2, 2\to1, 3\to3),$
$(1\to3, 2\to2, 3\to1), (1\to3, 2\to1, 3\to2)\}$.

Recalling the definition of the Justice Criterion from Section I, it is clear that for $y J_i x$ to hold, (at least) one of the following ((a1)- (a6)) must hold:

(a1): $(y,1)\widetilde{P}_i(x,1), (y,2)\widetilde{P}_i(x,2), (y,3)\widetilde{P}_i(x,3)$; (a2): $(y,1)\widetilde{P}_i(x,2), (y,2)\widetilde{P}_i(x,1), (y,3)\widetilde{P}_i(x,3)$;

(a3): $(y,1)\widetilde{P}_i(x,3), (y,2)\widetilde{P}_i(x,2), (y,3)\widetilde{P}_i(x,1)$; (a4): $(y,1)\widetilde{P}_i(x,1), (y,2)\widetilde{P}_i(x,3), (y,3)\widetilde{P}_i(x,2)$;

(a5): $(y,1)\widetilde{P}_i(x,2), (y,2)\widetilde{P}_i(x,3), (y,3)\widetilde{P}_i(x,1)$; (a6): $(y,1)\widetilde{P}_i(x,3), (y,3)\widetilde{P}_i(x,2), (y,2)\widetilde{P}_i(x,1)$.

Given (II.1), it is easy to check that none of ((a1)- (a6)) holds.

For $z J_i y$ to hold (at least) one of the following ((b1) - (b6)) must hold.

(b1) : $(z,1)\tilde{P}_i(y,1), (z,2)\tilde{P}_i(y,2), (z,3)\tilde{P}_i(y,3)$; (b2): $(z,1)\tilde{P}_i(y,2), (z,2)\tilde{P}_i(y,1), (z,3)\tilde{P}_i(y,3)$;

(b3) : $(z,1)\tilde{P}_i(y,3), (z,2)\tilde{P}_i(y,2), (z,3)\tilde{P}_i(y,1)$; (b4): $(z,1)\tilde{P}_i(y,1), (z,2)\tilde{P}_i(y,3), (z,3)\tilde{P}_i(y,2)$;

(b5) : $(z,1)\tilde{P}_i(y,2), (z,2)\tilde{P}_i(y,3), (z,3)\tilde{P}_i(y,1)$; (b6): $(z,1)\tilde{P}_i(y,3), (z,2)\tilde{P}_i(y,1), (z,3)\tilde{P}_i(y,2)$.

Given (II.1), it is easy to check that none of ((b1) - (b6)) holds.

For $x J_i z$ to hold (at least) one of the following ((c1) - (c6)) must hold.

(c1) : $(x,1)\tilde{P}_i(z,1), (x,2)\tilde{P}_i(z,2), (x,3)\tilde{P}_i(z,3)$; (c2): $(x,1)\tilde{P}_i(z,2), (x,2)\tilde{P}_i(z,1), (x,3)\tilde{P}_i(z,3)$;

(c3) : $(x,1)\tilde{P}_i(z,3), (x,2)\tilde{P}_i(z,2), (x,3)\tilde{P}_i(z,1)$; (c4): $(x,1)\tilde{P}_i(z,1), (x,2)\tilde{P}_i(z,3), (x,3)\tilde{P}_i(z,2)$;

(c5) : $(x,1)\tilde{P}_i(y,2), (x,2)\tilde{P}_i(y,3), (x,3)\tilde{P}_i(y,1)$; (c6): $(x,1)\tilde{P}_i(y,3), (x,2)\tilde{P}_i(y,1), (x,3)\tilde{P}_i(y,2)$.

Given (II.1) it is easy to check that none of((c1) - (c6)) holds.

Now, since $y J_i x$ fails for all i, condition JL gives us, by individual 1,

(II.2) $\quad y \notin C(\{x,y,z\}, \{\tilde{R}_1, \tilde{R}_2, \tilde{R}_3\})$.

Since $z J_i y$ fails for all i, condition JL gives us, by individual 2,

(II.3) $\quad z \notin C(\{x,y,z\}, \{\tilde{R}_1, \tilde{R}_2, \tilde{R}_3\})$

Since $x J_i z$ fails for all i, condition JL gives us, by individual 3,

(II.4) $x \notin C(\{x,y,z\}, \{\tilde{R}_1, \tilde{R}_2, \tilde{R}_3\})$.

From (II.2), (II.3), (II.4), we have: $C(\{x,y,z\}, \{\tilde{R}_1, \tilde{R}_2, \tilde{R}_3\}) = \varnothing$, which violates the domain restrictions.

It should be noted here that if x,y,z,w,u,v are all distinct alternatives, one does not obtain an impossibility result of the above type; but then it seems to be motivationally arbitrary to restrict oneself to such cases. One must conclude then that Kelly's liberalism is self-inconsistent - and this does tend to considerably soften the blow of a result which calls the fruitfulness of the "Arrow-Sen programme" into question by establishing incompatibility between an internally inconsistent principle of liberalism and a criterion of justice which is a derivative of the "Arrow-Sen" programme.

A final remark may be in order here. If one were to characterize the impossibility result of this section, as "the impossibility of just liberalism", semantics may mislead one into believing that what has been demonstrated is an incompatibility between liberalism and justice, whereas what in fact has been done is to establish an internal inconsistency in a concept of liberalism which one has chosen to call "just liberalism".

III. Paretianism, Non-Discrimination and Liberalism: Paradox Gained

(a) Paretianism and Liberalism

A formal statement of the "impossibility of a Paretian Liberal" result would be: there exists no ASDF satisfying conditions U, L* and P*. Proof of this proposition will be furnished in terms of a couple of illustrations of which one is due to Sen (1970a, op.cit.). Before proceeding with a sketch of the manner of proof, it should be noted that in general we can have two types of situation which may be called Type 1

and Type 2 situations: a Type 1 situation is one in which for the two distinct pairs of alternatives over which each individual is granted decisiveness under condition L*, each pair has exactly one element in common; and a Type 2 situation is one in which all four of the alternatives are distinct. Consider first a Type 1 situation, for which I invoke an example due to Sen.

The dramatis personae in Sen's example are constituted by two individuals - Mr.Prude and Mr.Lewd -- of whom, as their names suggest, the first is devoted to the pursuit of purity and the second to the pursuit of prurience. The example is constructed in such a way as to precipitate a tension between the liberal right due to each individual to read or not read a "dirty" book (Lady Chatterly's Lover, as it happens) and the Pareto principle which seeks a reflection (in its weak form) of unanimous preferences in social preference over any pair of social states. To see this, consider the agenda $A_1 = a^1, a^2, a^3$, where the social states a^1, a^2, a^3 are defined as follows:

a^1 is a state in which only Mr.Prude reads the book

a^2 is a state in which neither Mr.Prude nor Mr.Lewd reads the book; and

a^3 Is a state in which only Mr.Lewd reads the book.

Designate Prude and Lewd by the subscripts 1 and 2 respectively. Note now that by condition L*, Prude ought to be granted decisiveness over the pair of alternatives (a^1, a^2), and Lewd over the pair of alternatives (a^2, a^3). Given their respective tastes, and condition L*, we have:

(III.1) $a^2 P_1 a^1 \rightarrow a^2 P a^1$

(III.2) $a^3 P_2 a^2 \rightarrow a^3 P a^2$

Consider now each individual's preference over the pair of alternatives (a^1, a^3). We are told that although Prude hates reading the book, he would much rather read it himself than allow Lewd to be corrupted by it; and although Lewd loves reading the book, he derives a greater "kick" from seeing Prude squirm under the impact of the book than from reading it himself. In short, both individuals prefer state a^1 to state a^3; the Pareto principle will then translate this unanimous preference into a social preference for a^1 over a^3:

(III.3). $a^1 P_1 a^3 \;\&\; a^1 P_2 a^3 \rightarrow a^1 P a^3$

From (III.1), (III.2), (III.3), we generate the cycle $a^3 P a^2 P a^1 P a^3$.

Consider now a Type 2 situation for which one needs to invoke four distinct alternatives - say y^1, y^2, y^3, y^4. The following example is supplied to highlight the nature of the problem. Define the following social states in the context of a two-individual community, where the individuals are designated 1 and 2 respectively:

y^1 is a state in which 1's compound-wall is painted pink, and 2's compound-wall is painted yellow;

y^2 is a state in which 1's compound wall is painted green, and 2's compound wall is painted yellow;

y^3 is a state in which 1's compound-wall is painted pink, and 2's compound-wall is painted orange; and

y^4 is a state in which 1's compound-wall is painted pink, and 2's compound wall is painted blue.

By condition L*, 1 ought to be granted decisiveness over the pair of states (y^1, y^2), and 2 over the pair (y^3, y^4). Assume that 1's preferred colour for his compound wall is pink, and that 2's preferred colour for <u>his</u> compound wall is orange.

One can then say:

(III.4) $y^1 P_1 y^2 \rightarrow y^1 P y^2$

(III.5) $y^3 P_2 y^4 \rightarrow y^3 P y^4$

Consider now in turn each individual's preferences over the pairs (y^1, y^4) and (y^2, y^3). If they (i.e. the two individuals) are neighbours and have identical views on the appropriate colour combinations of their compound-walls, it is possible that they will both prefer a combination of pink-blue to a combination of pink-yellow and a combination of green-yellow to a combination of pink-orange. Such a configuration of preferences enables one to write:

(III.6) $y^4 P_1 y^1 \;\&\; y^4 P_2 y^1 \rightarrow y^4 P y^1$

(III.7) $y^2 P_1 y^3 \;\&\; y^2 P_2 y^3 \rightarrow y^2 P y^3$

From (III.4), (III.5), (III.6), (III.7) we generate the cycle: $y^1 P y^2 P y^3 P y^4 P y^1$.

Thus, in both the Type 1 and Type 2 situations, acyclicity of the social preference relation is violated - whence the impossibility of a Paretian Liberal.

(b) Non-Discrimination and Liberalism

As noted in the introduction to this note, Sen's liberal principle is sometimes compatible with conferring upon a single individual the right to dictate on social preferences over pairs of alternatives which do not fall in anybody's protected sphere. To give an example, consider again the Prude-Lewd example outlined earlier: suppose that individual 1 preferred a^1 to a^3, and individual 2 preferred a^3 to a^1, and suppose further that the CCR is required to satisfy condition I (independence of

irrelevant alternatives). Now given (by condition L*) $a^3 P a^2$ and $a^2 P a^1$, acyclicity of the social preference relation requires $a^3 R a^1$. By condition I, social preference over the pair (a^1, a^3) must depend only on individual preferences over the pair (a^1, a^3), and since only individual 2 prefers a^3 to a^1, clearly he is being allowed almost semi-decisiveness over the ordered pair (a^3, a^1) which falls in nobody's protected sphere. (A protected sphere will be understood to mean a pair of alternatives which has been assigned to some individual under the liberal principle). It can be argued that individual 1 is being unfairly discriminated against and that, in general, if anybody at all is permitted to rule on social preferences over "impersonal" features of the world, such a privilege should be available to all members of society. This is the motivation underlying the definitions of the ND (Non-Discrimination) and WND (Weak Non-Discrimination) principles in Section I.

Now it can be shown that for a society consisting of more than 2 individuals, the following two propositions hold:

(i) any ASDF satisfying conditions U, I, L and ND renders all pairs of Pareto non-comparable states not falling within anybody's protected sphere socially indifferent - which as Sen (1976, op.cit. p.227) himself has remarked, makes for " ..... a very peculiar system indeed";

(ii) there exists no QTSDF satisfying conditions U, I, L and WND.

To see this, consider a set of n individuals of whom the first n-1 individuals are all prudes and the nth individual is given to lewd propensities. Without loss of generality, let n=3. Let d_i stand for individual i not reading a copy of Lady Chatterly's Lover and r_i stand for his reading it. Then, for example, a social state b^1, described by $d_1 d_2 d_3$ will be

IV. Paretianism, Non-discrimination and Liberalism : Paradox Lost

(a) Paretianism and Liberalism

Harking back to the Prude-Lewd example of the previous section, suppose now that the agenda $A_1 = \{a^1, a^2, a^3\}$ were to be replaced by the agenda $A_1' = \{a^{1'}, a^2, a^3\}$, where the social states a^2 and a^3 are as described earlier, and $a^{1'}$ is defined to be a state in which both Prude and Lewd read Lady Chatterly's Lover. Then, it is clear, given condition L*, that Prude ought to be granted decisiveness over the pair of alternatives $(a^3, a^{1'})$, and Lewd over the pair (a^3, a^2). Given the two individuals' respective tastes, we have:

(IV.1) $a^3 P_1 a^{1'} \rightarrow a^3 P a^{1'}$; and

(IV.2) $a^3 P_2 a^2 \rightarrow a^3 P a^2$

Since a^3 dominates each of the other two alternatives $a^{1'}$ and a^2, it is immediately clear that it is a matter of complete irrelevance how $a^{1'}$ and a^2 are socially ranked vis-à-vis each other to the fact that a^3 - the "Liberal solution" - must emerge as the most preferred alternative in the triple of alternatives $\{a^{1'}, a^2, a^3\}$. A specific (and straightforward) implication of this is that liberalism will not conflict with the Pareto principle. For note that if, for example, Lewd were to be extra-generous - to the point of caring more for Prude's comfort than his (Lewd's) own - then both individuals would favour state a^2 over state $a^{1'}$, leading, through condition P*, to:

(IV.3) $a^2 P_1 a^{1'} \;\&\; a^2 P_2 a^{1'} \rightarrow a^2 P a^{1'}$

From (IV.2), (IV.3), we have the social preference sequence $a^3 P a^2 P a^{1'}$ - which is perfectly consistent with $a^3 P a^{1'}$ (from IV.1). Similarly, a magnanimous gesture on Prude's part would lead to, given (IV.1) and (IV.3), the social preference sequence $a^3 P a^{1''} P a^2$, which again is perfectly consistent with $a^3 P a^2$ (from (IV.2)).

In short, with the agenda A_1', no demonstrable tension between Paretianism and Liberalism emerges.

(b) Non-Discrimination and Liberalism

Going back to the example furnished in Section III(b), suppose that the agenda $A_2 = \{b^1, b^2, b^3, b^4\}$ were to be replaced by the agenda $A_2' = \{b^1, b^2, b^{3'}, b^{4'}\}$, where the states b^1 and b^2 are as described earlier and the states $b^{3'}$ and $b^{4'}$ are defined as follows:

$b^{3'}$: $d_1 r_2 r_3$

$b^{4'}$: $r_1 d_2 r_3$

Then it is clear that by condition L the assigned pair of alternatives for individual 1 is $(b^1, b^{4'})$ for individual 2 is $(b^1, b^{3'})$, and for individual 3 is (b^1, b^2). Given the respective tastes of the three individuals, we have, by condition L:

(IV.4) $b^1 P_1 b^{4'} \rightarrow b^1 P b^{4'}$

(IV.5) $b^1 P_2 b^{3'} \rightarrow b^1 P b^{3'}$

(IV.6) $b^1 P_3 b^2 \rightarrow b^1 P b^2$

Since b^1 uniquely dominates each of the other three alternatives, it is the most preferred alternative in the quadruple $\{b^1, b^2, b^{3'}, b^{4'}\}$: further, b^1 is also the "liberal solution". Notice however that the emergence of the most preferred alternative as the "liberal solution" does not

also confer upon any individual the privilege of decisiveness over pairs of alternatives not falling in anybody's protected sphere - unlike in the example furnished in section III.(b) : there is no scope, here, for conflict with the principle of non-discrimination. In short, the agenda A'_2 ensures the non-emergence of any demonstrable conflict between non-discrimination and liberalism.

In this section, I have shown how the specification of agenda different from "Sen-type" agenda, and yet fulfilling Sen's principle of liberalism, precludes the possibility of emergence of the paradoxes discussed in section III. The results of this section are discussed in the next section.

V. The Source of the Liberal Paradoxes

What is the crucial difference between, on the one hand, the agenda A'_1 and A'_2 of section IV and, on the other, the "Sen-type" agenda A_1 and A_2 of section III? The difference between the agenda A_1 and A'_1 is to be located in the fact that in the former case Prude is permitted to be decisive over a pair of states in which Lewd's subjective preference (for reading Lady Chatterly's Lover) is violated: In the latter case, however, each individual is allowed decisiveness over a pair of states in which the other individual's subjective preference is preserved. Similarly, the difference between the agenda A_2 and A'_2 is to be located in the fact that in the former case, each of the two Prudes (individuals 1 and 2) is allowed decisiveness over a pair of states in which at least one other individual's subjective preference is violated, while in the latter case (that of agenda A'_2), each individual is offered decisiveness over a pair of states in which the subjective preferences of the remaining two individuals are preserved. It seems therefore that the condition for the emergence of a conflict between the Pareto principle and liberalism, or between the principle of

non-discrimination and liberalism, is that one individual's liberty be sought to be secured in a state of the world in which the other individual's subjective preference is violated. This is suggestive, although of what, is not quite clear yet. This can perhaps be deduced from a consideration of a variant of Sen's Prude-Lewd example.

In this variant, the actors are constituted by two vulgarians with identical and mutually unsurpassed tastes in the matter of 'muck'. We may call the two individuals Lewd the First and Lewd the Second, and index them by 1 and 2 respectively. Three states of the world are defined: C^1 is a state in which only 1 reads the book (Lady Chatterly's Lover); C^2 is a state in which neither individual reads the book; and C^3 is a state in which only 2 reads the book. Condition L* can be invoked to support the following statements:

(V.1) $C^1 P_1 C^2 \rightarrow C^1 P C^2$

(V.2) $C^3 P_2 C^2 \rightarrow C^3 P C^2$

To uphold Lewd the First's liberty, society should permit him to read the book (i.e. uphold his preference for C^1 over C^2); to uphold Lewd the Second's liberty, society should permit him to read the book (i.e. uphold his preference for C^3 over C^2). All this is routine. But note now that the states C^1 and C^3 are mutually exclusive: from any larger set (i.e. larger than the pairs (C^1, C^2) and (C^2, C^3)) containing both C^1 and C^3, society can chose one, and only one, of the states C^1 and C^3. (A possible way out, it may be argued, is to declare C^1 and C^3 socially indifferent; but then professing indifference as to which of the two individuals - 1 or 2 - secures his liberty is surely a very different proposition from the liberal sentiment that both individuals' liberties are equally important). In other words, the two individuals' liberties are mutually incompatible : by the very terms of the problem, at least one individual's liberty must necessarily be

flouted. This gives a rather hollow ring to Sen's principle of liberalism : as required by condition L*, we have granted each individual decisiveness over a pair of states in which what he does is entirely his own business; and yet, in practice, at best only one individual can enjoy the privilege of his decisiveness. The kinds of agenda from which choices are to be made are clearly crucial to an appropriate interpretation of liberalism. A society which acknowledges each individual's right to two square meals a day is still an illiberal society if the agenda it offers for eliciting individual preferences over social states includes states in which any members of the society have to go hungry and such states cannot be expelled from the choice set though the exercise of individual rights in combination with some other principle of social choice. The trouble with Sen liberalism seems to reside in regarding a liberal right as the right to hold certain preferences rather than the right to realize these preferences in concrete acts of choice. To invert, with some variation, a well-known legal maxim: "Liberty must not only be seen to be granted, but must in fact be secured". Where this is not the case, there is something fundamentally paradoxical about a notion of liberalism which grants each (of at least two) individual(s) his liberal right, but ends up actually securing only one individual's liberal right. A characteristic feature of the variant of Sen's Prude-Lewd example I have just discussed and in which the paradox arises is that each individual is being granted decisiveness over a pair of states in which the other individual's subjective preference is being violated. It seems therefore that, in general, Sen liberalism is perfectly compatible with the existence of agenda which do not contain a "liberal solution" at all.

The source of the above difficulty can perhaps be located from a consideration of the following two propositions:

(i) when an individual is granted the right to choose between two social states, there is already an implication

to the effect that the individual's choice can be realized - else it is pointless to offer him the right of "choice";

(ii) if an individual's choice can be realized, then presumably - since it is his choice, after all - his choice will be realized.

The implications of these two propositions for Sen's Prude-Lewd example in the context of a Type I situation (the three - distinct - alternatives - case) are immediately clear. Insofar as Lewd is granted the liberty of choosing between reading and not reading Lady Chatterly's Lover, from proposition (i) it follows that his preference can be attained. If it can be attained, then from proposition (ii), it follows that it will be attained. If that be the case, then, it is difficult to see how Prude can be granted decisiveness over a pair of states (a^1, a^2) in both of which Lewd is not reading the book. Sen's example, in the light of the preceding comments, appears to be a contrived and logically suspect illustration of a Type I situation; in a three-distinct-alternatives case, the appropriate illustration seems to be the one I have provided using the agenda A_1'.

It should be noted, however, that Prude's decisiveness over a pair of states in which Lewd's preference is violated can be interpreted to mean that Lewd cannot realize his preference (owing, perhaps, to an interfering literary mentor or to the presence of too many commitments in a packed daily schedule, or whatever). But if Lewd cannot realise his preference, then an agenda such as A_1 already reflects a failure of liberty (for Lewd), and it is quite meaningless to grant him decisiveness in the matter. The story must stop with securing Prude his liberty and acknowledging that there has been a failure in securing Lewd's liberty: there is no scope for any subsequent development of the plot.

In similar fashion, one can examine the implications of propositions (i) and (ii) for the example I have furnished in the context of a Type 2 situation (the four-distinct-alternatives-case). Propositions (i) and (ii) ensure that in so far as individual 2 has been granted the liberty to choose between orange and blue walls, and insofar as his preference runs to orange, his walls will be orange; yet individual 1's liberty is being sought to be secured in a state of the world in which 2's wall is yellow! To rectify this, 1's decisiveness ought to be confined to a pair of alternatives in which 2's wall is orange, but this will mean working with an example in which there are only three distinct alternatives - e.g. one in which 1's wall is green and 2's orange, a second in which 1's wall is pink and 2's orange, and a third in which 1's wall is pink and 2's blue. This, of course, is subject to the assumption that orange is 2's most preferred colour. If orange is not his most preferred colour, then yellow must be, since, by virtue of his preferring orange to blue, blue manifestly is not his most preferred colour. But then if yellow is 2's favourite colour in the triple of colours {blue, orange, yellow}, it is surely perverse to confine his decisiveness to a pair of alternatives (y^3, y^4) in which his compound wall is not painted with his favourite colour (yellow). Depending, then, on whether orange or yellow is 2's favourite colour, the appropriate "liberal" agenda would be respectively (say) $\{y^{1'}, y^3, y^4\}$ and $\{y^1, y^2, y^{4'}\}$, where $y^{1'}$ is a state in which 1's wall is painted pink and 2's is painted orange, and $y^{4'}$ is a state in which 1's wall is painted green and 2's is painted blue. Briefly, the four-distinct-alternatives-case must collapse to a three-distinct-alternatives-case, where the three alternatives constitute a 'liberal' agenda of the type A_1'. Sen's 'paradox', in short, is illusory, and owes its origin to a definition of liberalism which, while it appears to be entirely reasonable, is not quite so. I state below a modified version of his L* condition (which we may call L'*), which removes the

paradox of an impossible Paretian liberal, and also, in a sense, sums up the entire argument of this section:

Definition: Condition $L^{\star}$ (Modified Minimal Liberalism)

There are at least two persons k and j and two distinct pairs of alternatives (x,y) and (z,w) with exactly one alternative in common such that k and j are decisive over (x,y) and (z,w) respectively, each pair taken in either order. This however is subject to the caveat that if it is possible to secure k's (respectively j's) liberty in a state of the world in which j's (respectively k's) subjective preference is not violated, then k (respectively j) should not be permitted to be decisive over a pair of states in which j's (respectively k's) subjective preference is violated; where it is not possible to secure k's (respectively j's) liberty in a state of the world in which j's (respectively k's) subjective preference is preserved, k's (respectively j's) liberty must certainly be secured, but it is pointless to pretend to proceed to grant j (respectively k) the liberal right of decisiveness.

(It may be noted that a similar modification of condition L to condition L' will remove the paradox of an impossible non-discriminating liberal).

Sen's liberal principle offers to each individual his liberty in principle and denies it to at least one in effect: what it bestows with one hand, it withdraws with the other. But the liberal principle modified as above rectifies this, and betrays no demostrable incompatibility with the Pareto principle or the principle of non-discrimination.

It may be noted that the argument in this section is similar to (and yet distinct from) Nozick's (1974, p. 165-166) comment on Sen's Paretian-Liberal dilemma:

> "Individual rights are co-possible; each person may exercise his rights as he chooses. The exercise of these rights fixes some features of the world. Within the constraints of these fixed features, a choice can be made by a social choice mechanism based upon a social ordering, if there are any choices left to make!"

This view is exemplified by means of an illustration (Nozick, 1973): "If I have a right to choose to live in New York or in Massachussetts, and I choose Massachussetts, then alternatives involving my living in New York are not appropriate objects to be entered in a social ordering".

My own contention, in terms of Sen's Prude-Lewd example, would be "if Lewd has a right to choose between reading and not reading Lady Chatterly's Lover, and he chooses to read, then alternatives involving his not reading the book are not appropriate objects to constitute any other individual's assigned pair." It is clear that both Nozick's argument and the argument presented in this section demand certain kinds of restrictions on the types of agenda that might be construed as being "liberal". Nozick's restrictions are, however, drastic in the extreme: he will simply exclude from any agenda those social states that will not come to pass by virtue of individuals exercising their "rights" against the possibility of occurence of these social states. With every individual exercising what he chooses to call his "rights", society might well be confronted by a situation wherein the area of choice is whittled down to nothingness! The difficulty with Nozick's argument is that it treats a social ordering purely as what Sen (1976, op.cit. p.229) calls a "mechanism of choice", and not at all as something that "reflect(s) a view of social welfare". As Sen remarks (p.230): But one can also argue that, if I believe that it is a better society which - given other things - lets Nozick decide where he wishes to live, then I must assert that it is socially better that Nozick should be permitted to live in Massachussetts

as desired by him". If my interpretation is correct, then Sen seems to be suggesting that before we can allow the exercise of liberal rights to serve as a constraint on the set of alternatives which may be regarded as "appropriate objects to be entered in a social ordering"., there must exist some clearly enunciated principle of liberalism which acknowledges the legitimacy of these liberal rights. Nozick seems to take no account of this requirement. On the other hand, if Nozick neglects to interpret a "social ordering" as something that "reflects a view of social welfare", Sen seems to neglect to interpret a social ordering as a "mechanism of social choice". Condition L* however, takes both interpretations of a social ordering into account; it does enable one to assert that it is "socially better that Nozick should be permitted to live in Massachussetts"; at the same time, it takes Nozick's liberal right to live in Massachussetts seriously enough to arrange matters such that alternatives involving his not living in Massachussetts do not enter any other person's assigned pair, and this is ensured through suitable restrictions on the agenda which amount to an appropriate interpretation of a "social ordering" as a "mechanism of choice".

VI. Concluding Observations

I have tried to demonstrate in this note that while there are doubtless very many difficulties with a strictly "welfarist" approach to the evaluation of social states, these difficulties are not necessarily exposed by impossibility results such as those of Kelly and Sen. Kelly's sceptimism about the fruitfulness of enriching utility-based information through the implementation of the "Arrow-Sen" programme of making inter-personal welfare-comparisons with the use of extended preference orderings is based on an alleged incompatibility between Suppes' justice principle and his (Kelly's) liberal principle. I have attempted to demonstrate that Kelly's concept of liberalism is not even compatible with

itself, leave alone with the justice principle. Sen questions the "mechanical use" of the Pareto principle by establishing an incompatibility of the former with his principle of liberalism; one can similarly question the appeal of Arrow's condition of "independence of irrelevant alternatives" by pointing to the central role it occupies in ensuring an incompatibility between a principle of "non-discrimination" and Sen's principle of liberalism. The case for such a questioning in this context, however, would be weakened if it could be demonstrated that Sen's principle of liberalism is itself not above reproach. This I have attempted to do by arguing that there is a specific sense in which the kinds of agenda which Sen employs in his impossibility result are incompatible with the spirit of liberalism; suitable modification of Sen's liberal principle, entailing particular agenda-restrictions, removes the paradoxes of an impossible Paretian liberal and an impossible non-discriminating liberal.

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