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INVESTIGATION OF UNSTEADY GROUND WATER FLOW WITH A FREE SURFACE

By

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SYNOPSIS

Results of analytical and experimental investigations on the evolution of the transient free surfaces inside a homogeneous and isotropic permeable medium overlying an inclined impermeable bed, resulting from infiltration and a uniform rise of water level in an adjoining surface source have been presented in this paper. The case of an initial horizontal and inclined water tables have been considered.

Analytical work involves the solution of the basic non-linear partial differential equation using a method of linearisation. The linearised equation is solved for the given initial and boundary conditions by Laplace transforms. The resulting solution has been used to compute the theoretical free surfaces on a digital computer. Experimental studies have been carried out on a viscous flow model. The effect of the initial water table height and the bed slope on the accuracy of the linearised solution and on the bank storage has been discussed.

1. INTRODUCTION

A major field of investigation in ground water relates to the problem of transient free surface flows which is an important consequence of fluctuations in any surface source such as stream or reservoir, natural or artificial recharge from the surface or withdrawal by drainage from the medium. The analysis of problems of unsteady ground water flow in a stream-aquifer system assumes significant importance in view of the need to understand the response of the aquifer to such fluctuations. The prediction of the transient free surfaces as a function of time would particularly be useful in evaluating precisely the ground water component in the total flow. In view of the importance of the problem, innumerable investigations have been carried out (1,2,3,4,5) in the past on various aspects of the problem, employing different techniques to obtain solutions to problems with different boundary and initial conditions. However, it appears that the effect of the slope of the impervious bed both on the accuracy of the linearised solution and on the bank storage has not been considered in the earlier investigations.

The present study is concerned with analytical and experimental investigations of unsteady ground water flow resulting from a uniform rise of water level in one of the surface sources (Fig.1). The region between the two surface sources is, in addition, subjected to uniform infiltration from the surface and the impervious bed rock below the medium is inclined.

The cases of initial horizontal and inclined water tables have been considered. The aim of the investigation is to verify the validity of the proposed linearised solution by comparing the theoretical results with those from studies on a viscous flow model, with particular reference to the effect of the slope of the impervious bed and that of the initial water table height on the accuracy of the mathematical model and on the non-dimensional bank storage.

2. ANALYTICAL INVESTIGATIONS

2.1 Basic Equation : Based on Dupuit's assumptions of small slope for the free surface and of near horizontal flow (neglecting the vertical component of the velocity) the basic equation for unsteady free surface flows may be written as (1)

$$\frac{\partial H}{\partial t} = \frac{k}{m}\left(H \pm ix\right)\frac{\partial^2 H}{\partial x^2} \pm \frac{ki}{m}\frac{\partial H}{\partial x} + \frac{\omega_0}{m} \qquad (1)$$

Where H (x,t) is the ordinate of the free surface,

k and m are the coefficient of permeability and porosity respectively

i is the slope of the impervious bed

w_0 is the rate of infiltration-discharge per unit area in plan

Eqn.1 is a non-linear partial differential equation which is difficult to solve for the given boundary and initial conditions. The Eqn.1 is therefore linearised by replacing the variable $(H \pm ix)$ by a constant $\bar{h} = (H_0+Hm)/2$ where H_0 and Hm represent the initial and maximum water depths. Thus one has

$$\frac{\partial H}{\partial t} = \frac{k\bar{h}}{m}\frac{\partial^2 H}{\partial x^2} \pm \frac{ki}{m}\frac{\partial H}{\partial x} + \frac{\omega_0}{m} \qquad (2)$$

This is the modified Boussinesq Equation for unsteady flow with a free surface when the underlying bed rock has a slight slope.

2.2 Initial and Boundary Conditions : Refering to Fig.1, the initial and boundary conditions can be formulated as follows. In the general case when the initial free surface is not horizontal, the same is assumed to be a straight line, with a slope $j = (H_0-H_1)/L$ (for $L \gg H_0-H_1$) where H_0 and H_1 are the initial water depths in the upstream and downstream reservoirs respectively above a horizontal datum.

$$\begin{aligned} H(x,0) &= H_0 - jx \\ H(0,t) &= H_0 + \alpha t \\ H(L,t) &= H_0 - jL = H_1 \text{ (Constant).} \end{aligned} \qquad (3)$$

The first case of initial horizontal water table is defined by the condition $H(x,0) = H_0$ $(j = 0)$.

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2.3 Solution : The solution of the basic linearised Eqn.2 subject to the boundary conditions given by Eqn.3 is obtained through the use of Laplace transforms.

Introducing the new variable $\tau = \frac{k\bar{h}}{m} t$ and with $\frac{i}{\bar{h}} = 2\lambda$

Eqn.2 reduces to

$$\frac{\partial H}{\partial \tau} = \frac{\partial^2 H^2}{\partial x^2} \pm 2\lambda \frac{\partial H}{\partial x} + \frac{\omega_0}{k\bar{h}} \qquad (4)$$

Multiplying each of the terms of Eqn.4 by $(e^{-p\tau} d\tau)$ and integrating between limits 0 and ∞, one obtains the Laplace transform of Eqn.4 as

$$\frac{d^2 \bar{H}^2}{d x^2} \pm 2\lambda \frac{d\bar{H}}{dx} + \frac{\omega_0}{p k \bar{h}} - p\bar{H} = - H(x,0) \qquad (5)$$

$$\text{Where} \quad \bar{H} = \bar{H}(x,p) = \int_0^\infty H(x,\tau)\, e^{-p\tau}\, d\tau$$

The solution of Eqn.5 using the transforms of the boundary conditions given by Eqn.3 is

$$\bar{H} = C_1 \left[\frac{\sinh (L-x)\sqrt{\lambda^2+p}}{\sinh L \sqrt{\lambda^2+p}} \right] e^{-\lambda x} + e^{\lambda(L-x)} \left[\left(\frac{2\lambda j}{p^2} - \frac{\omega_0}{k\bar{h} p^2} \right) \frac{\sinh x\sqrt{\lambda^2+p}}{\sinh L\sqrt{\lambda^2+p}} \right] + \left[\left(\frac{H_0 - jx}{p} \right) + \left(\frac{\omega_0}{k p^2 \bar{h}} - \frac{2\lambda j}{p^2} \right) \right] \qquad (6)$$

$$\text{Where} \quad C_1 = \left[2\lambda j - \frac{\omega_0}{k\bar{h}} + \alpha' \right] / p^2$$

By evaluating the inverse Laplace transforms of the terms of Eqn.6, one obtains the final solution for the free surface as

$$H(x,\tau) = \left(2\lambda j - \frac{\omega_0}{k\bar{h}} + \alpha'\right) e^{-\lambda x} \left[\tau \frac{\sinh \lambda(L-x)}{\sinh \lambda L} - 2\pi L^2 \sum_{n=1}^{\infty} \frac{n \sin(\frac{n\pi x}{L}) \left\{ 1 - e^{-(\lambda^2 + \frac{n^2\pi^2}{L^2})\tau} \right\}}{(L^2\lambda^2 + n^2\pi^2)^2} \right]$$

$$+ e^{\lambda(L-x)} \left(2\lambda j - \frac{\omega_0}{k\bar{h}}\right) \left[\tau \cdot \frac{\sinh \lambda x}{\sinh \lambda L} + 2\pi L^2 \sum_{n=1}^{\infty} \frac{(-1)^n n \sin(\frac{n\pi x}{L}) \left\{ 1 - e^{-(\lambda^2 + \frac{n^2\pi^2}{L^2})\tau} \right\}}{(L^2\lambda^2 + n^2\pi^2)^2} \right]$$

$$+ (H_0 - jx) + \left(\frac{\omega_0}{k\bar{h}}\right)\tau - 2\lambda j \tau \qquad (7)$$

Where $\alpha_1 = \alpha m / k\bar{h}$ and $\alpha = (H_m - H_0)/\tau$

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The theoretical free surfaces were computed by programming Eqn.7 on a digital computer.

3. EXPERIMENTAL INVESTIGATIONS

3.1 Description of the set up: The experimental studies have been carried out on a viscous flow analog model which utilises the analogy between ground water movement and laminar flow of a viscous fluid between two closely spaced parallel plates. The set up consists of two glass plates 180x60x1.25 cm placed 3.17 mm apart. Two reservoirs 20.3x19x60cm fixed at the two ends simulate the two surface sources. Threaded overflow pipes are used to maintain the desired constant fluid level in the tanks. The plate and reservoir assembly is fixed on a cast iron base plate supported on bearings at one end and nuts moving over threaded rods at the other end. With this arrangement any desired slope can be given to the model. The closed circuit for oil comprises of a sump, gear pump, by-pass and a 3/4" supply line that feeds the oil to the reservoirs and infiltration tank. Infiltration is introduced into the model from the top of plates through 1.6 mm diameter copper tubes 10 cms long connected at 1.25 cms apart to a 1.875 cm dia. G.I. header pipe 180 cm long, in turn connected to the constant head infiltration tank. This tank could be fixed at any level so as to vary the infiltration rate.

3.2 Schemes of experiments : The parameters of the problem may be expressed in a non-dimensional functional relationship by

$$\left(\frac{H}{Hm-H_0}\right) = f\left[\frac{X}{L}, \frac{t}{T}, \frac{W_0}{K/m}, \frac{(Hm-H_0)/T}{k/m}, \frac{H_0}{(Hm-H_0)}, \frac{L}{(Hm-H_0)}, j, i\right]$$

Where $\alpha = (Hm-H_0)/T$ is the rate of rise kept constant at 0.333 cms/Sec and w_0 = 0.2025 cm/sec. (constant for all the runs)

$(L/Hm-H_0) = 15$ for a constant 12 cm rise.

Three values of the bed slope i were considered viz. 1 in 60, 1 in 40 and 1 in 20. In each case four values of the initial depth H_0 viz. 3,4,6 and 12 cms were considered for the first series with j = 0. For the second series ($j \neq 0$), the same bed slopes and zero slope were studied. The downstream level (H_1) was constant at 3 cms while the upstream depth H_0 considered were 6, 9,12 and 15 cms. The equivalent coefficient of permeability of the model for the spacing and oil used worked out to be 49.5 cm/sec. The temperature of oil was kept nearly constant by using a heat exchanger. The free surface were recorded at intervals of 9 secs.(corresponding to t/T = 0.25, 0.5, 0.75 and 1.0) by a 35mm Kodak Retinette Camera.

4. RESULTS AND DISCUSSIONS

Fig.2 shows a plot of the experimental and theoretical free surfaces at t/T = 0.75 for series 1 (j = 0) H_0 = constant at 3 cms, for the three bed slopes studied. It may be observed that the trend of agreement between the experimental and theoretical results is nearly the same for all

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the three slopes. It may therefore be concluded that the effect of variation of bed slope on the accuracy of the linearised solution is only marginal.

The experimental and theoretical free surfaces at $t/T = 0.75$ for a given bed slope ($i=1/60$) for different values of H_c (viz 4, 6 and 12 cms) are presented in Fig.3 in addition to $H_o = 3$ cms presented in Fig.2(a). It may be noted that for a given slope the magnitude of discrepancy between the theoretical and the experimental results is very nearly the same irrespective of the value of H_c. But in the case of horizontal impervious bed (4), it was found that there was definite improvement in the agreement with increase in H_o and as such it was possible to suggest a range of validity of the linearised solution as $H_o/(Hm-H_o) > 1$. It appears therefore that the range of validity suggested for horizontal bed is adversely affected by the bed slope. In view of the discrepancy existing for values of $H_o/Hm-H_o$ as high as 1, more experimental studies covering a higher range of $H_o/Hm-H_o$ would be needed for establishing a range of validity of the linearised solution in case of sloping beds.

The experimental and theoretical results for the case of non-horizontal initial water table have been presented in Fig.4, for the case of horizontal bed ($i=0$) corresponding to $t/T = 0.25$. Each of the Figs. 4(a), 4(b) and 4(c) corresponds to a particular value of $H_c = 6$, 9 and 12 cms respectively (or $j = 1/60, 1/30, 1/20$). It may be seen that the deviation between the experimental and theoretical results increases with increase in H_c (or j) and the agreement is far from satisfactory. Further, the same trend has been observed for all the other cases with sloping beds also (6). It appears therefore that unsymmetrical initial conditions have an adverse effect on the accuracy of the proposed theoretical solution. The discrepancy may be attributed to (i) the Dupuit's assumptions which are not satisfied in this case in view of high slopes of the free surface (ii) the assumption of a straight line for the initial water table (iii) effect of linearisation.

The nature of variation of non-dimensional bank storage ($\Delta V/V_o$) [ΔV=increment in bank storage and V_c = initial volume] as a function of the bed slope is presented in Figs. 5 (a) ($j= 0$ and $H_c = 4$ cm) and 5 (b) ($j=1/30$). It may be seen that $\Delta V/V_o$ decreases with increasing bed slopes and this reduction increases with time.

5. CONCLUSION

Results of the theoretical and experimental studies reported highlight the inadequacy of the range of validity suggested for horizontal bed to cover the case of sloping impervious bed suggesting that indiscriminate use of the linearised solution might lead to considerable errors. Studies on the unsymmetrical case with a non-horizontal initial water table indicate that significant departures from Dupuit's assumptions would result in considerable errors in the theoretical solution. The studies demonstrate the limitation of the proposed mathematical model and reiterate the versatality and usefulness of the viscous flow model.

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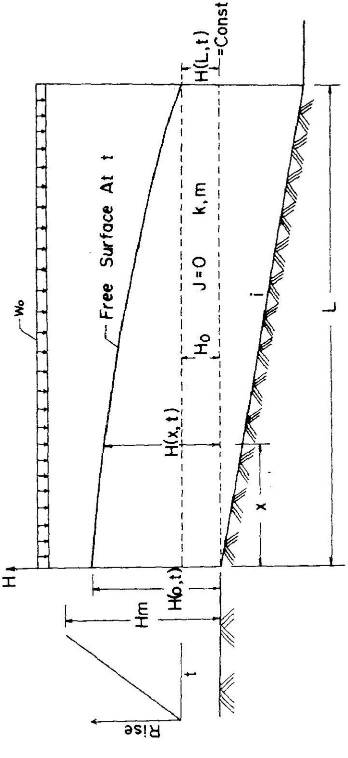

FIG. 1. DEFINITION SKETCH OF THE PROBLEM

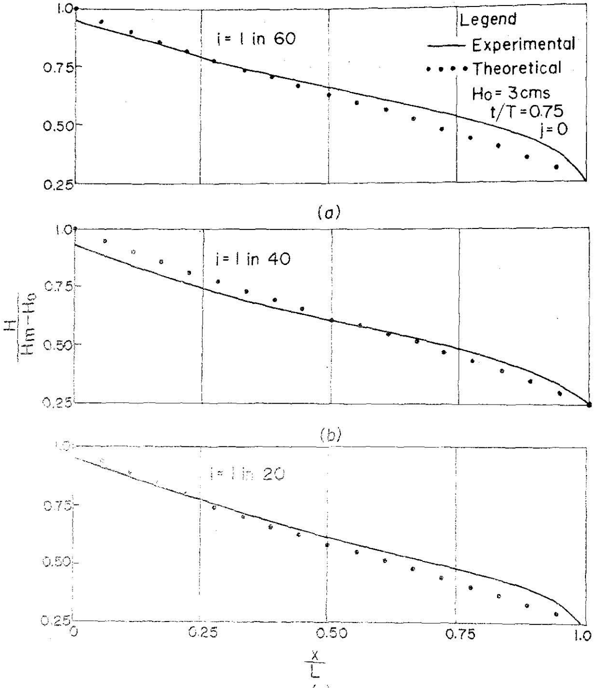

FIG. 2 PLOT OF EXPERIMENTAL AND THEORETICAL FREE SURFACES

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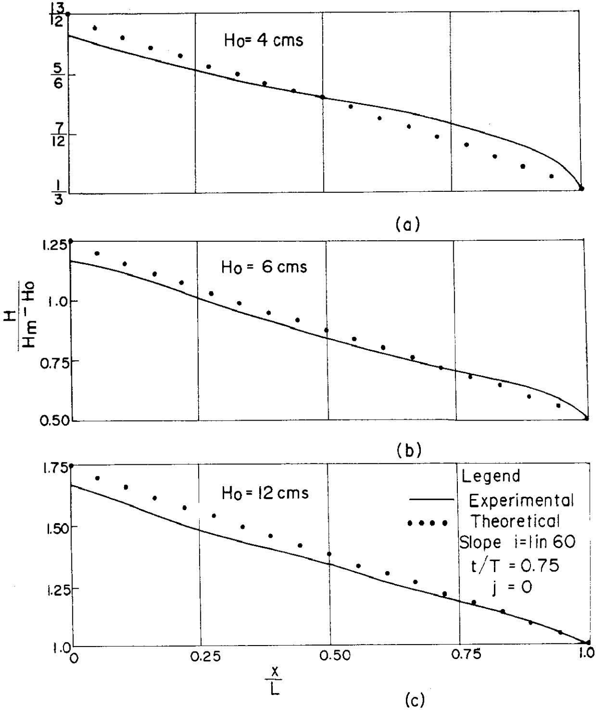

FIG. 3. PLOT OF EXPERIMENTAL AND THEORETICAL FREE SURFACES

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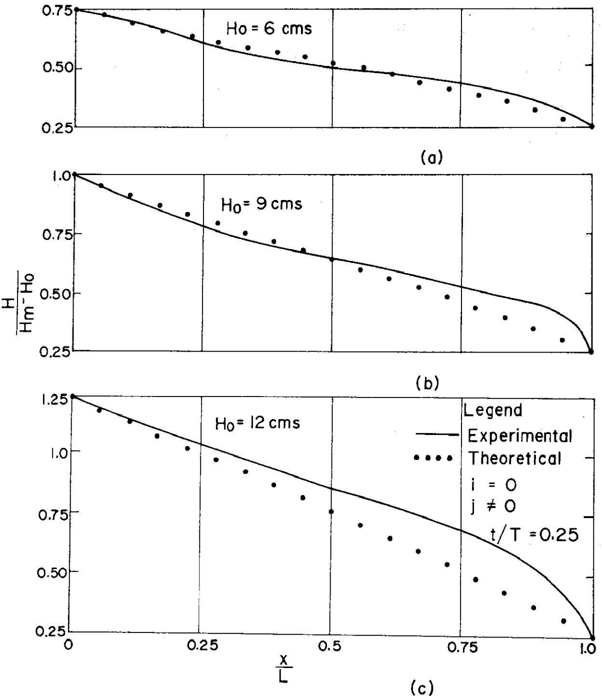

FIG. 4. PLOT OF EXPERIMENTAL AND THEORETICAL FREE SURFACES

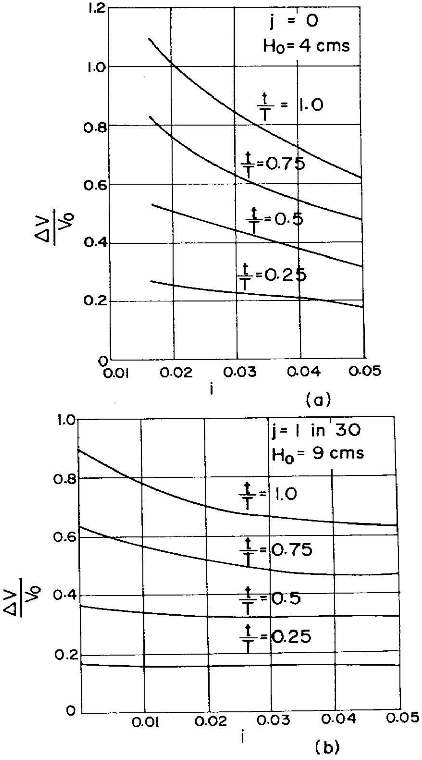

FIG. 5. PLOT OF BANK STORAGE Vs TIME

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GROUND WATER FLOW OBEYING NON-DARCY LAWS

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SYNOPSIS

There are many real situations in the field of ground water flow where a linear relationship between velocity and hydraulic gradient is not valid. In this paper numerical procedures are outlined which enable the analysis of steady-state problems for any arbitrary law governing the flow. Comparisons made for a problem for which a closed-form solution is available show that fairly accurate results are obtained. Further, ground anisotropy can also be taken into consideration simultaneously with non-linear flow behaviour.

1. INTRODUCTION

Isotropy of ground and the validity of Darcy's law are important assumptions made in the solution of many complex problems involving the flow of ground water. Anisotropy can, in general, be taken into consideration provided the flow of water obeys the linear law. However, there is considerable evidence in the literature indicating departures from Darcy's law (1,2,3,4,5). In coarse-grained materials this behaviour has often been attributed to turbulence although Forchheimer (6) and Risenkampf (7) have indicated that such deviations may occur even for laminar flow. For very fine-grained soils, it has been suggested (5,8) that non-linear behaviour could be due to the mechanism of adsorbed layers and other molecular forces. Polubarinova-Kochina (9) has indicated that a threshold gradient may also exist and this must be exceeded before flow can occur.

Non-linear relationships have practical significance in problems involving flow through rockfill dams and fissured rock and also for flow in areas subject to disturbance caused by drilling, boring of holes etc. (9). Influence on problems of consolidation and settlement is particularly important when flow through very fine-grained materials is considered (4,5).

A review of different mathematical relationships for non-Darcy flow through isotropic ground has been made by Chowdhury (10). No general solutions for any of these are available. An elaborate method proposed by Khristianovich (11) for a particular class of problem was successfully used by Sokolovsky (12) for the solution of two specific cases governed by a non-linear flow law. These closed-form solutions were found by the writer to be valuable for comparison of solutions obtained by methods given in this paper.

2. BASIS OF NUMERICAL SOLUTIONS

The proposed techniques are based on the calculus of variations which aims at finding those functions in a prescribed class called admissible functions for which a functional has extreme values. Zienkiewicz and Cheung (13) derived the functional that has to be minimised in order that the governing differential equation (the Laplace Equation) for linear seepage through isotropic ground is obeyed in the same region. An extension was developed to take ground anisotropy into consideration but departures from Darcy's law were not considered.

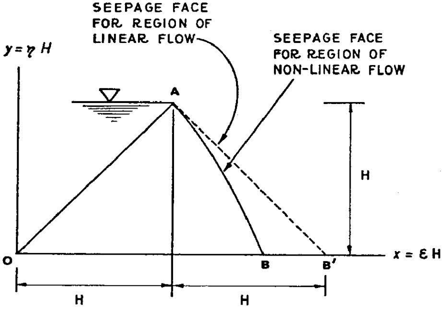


FIG. 1. SHAPE OF DAM ANALYSED BY SOKOLOVSKY (1949)

3. TECHNIQUE FOR ANALYSIS OF NON-DARCY FLOW

The differential equation for steady flow of an incompressible fluid through an incompressible porous medium may be derived by combining the equation of continuity with the particular law of flow. In an isotropic medium the direction of the hydraulic gradient J coincides with the direction of flow (or velocity v) at any point. Considering two dimensional flow, the following relationships can, therefore, be written for the corresponding components:

$$v_x = v\, J_x / J, \quad v_y = v\, J_y / J \qquad \ldots . (1)$$

The equation of continuity may be written in the form:

$$\frac{\partial}{\partial x}\left(v_x \right) + \frac{\partial}{\partial y}\left(v_y \right) = 0 \qquad \ldots . (2)$$

The flow law considered by Sokolovsky is:

$$J = v / k \left(1 - \frac{v^2}{m^2}\right)^{\frac{1}{2}} \qquad \ldots . (3)$$

in which k and m are arbitrary constants with the dimensions of velocity. Therefore, the governing differential equation for flow is:

$$\frac{\partial}{\partial x}\left(\frac{J_x\, k}{(1+J^2k^2/m^2)^{\frac{1}{2}}} \right) + \frac{\partial}{\partial y}\left(\frac{J_y\, k}{(1+J^2k^2/m^2)^{\frac{1}{2}}} \right) = 0 \qquad \ldots . (4)$$

It can now be shown (10) that the corresponding functional I (h) that must be minimised in accordance with the principles of variational calculus is:

$$I(h) = \iint_R \frac{m^2}{k} \left(1+k^2J^2/m^2\right)^{\frac{1}{2}} dx\, dy \qquad \ldots . (5)$$

where integration is over region R with given boundary conditions. The process of minimisation yields the solution in terms of total head h over the region and is accomplished by using the finite-element technique. The region (e.g. that in Fig. 1) is subdivided into a number of triangular elements (Fig. 2); within each element a linear variation of h is assumed. The process of minimisation of the functional is carried out for each element with respect to the total head at its three nodal points. Assemblage over all the elements yields a set of n simultaneous non-linear equations where n is the total number of nodal points. These are solved using a simple iterative technique.

4. ALTERNATIVE PROCEDURE

The functionals to be minimised can be formulated for some well-known flow laws and have been given elsewhere (10). It has been demonstrated, however, that functionals can not be formulated for all types of flow law that have been suggested in the literature. Obviously, there is need for an alternative procedure.

An homogeneous region in which flow obeys a non-linear law is regarded as a non-homogeneous assemblage of elements in which the linear flow law is obeyed. Each element must have a co-efficient of permeability depending upon the hydraulic gradient in accordance with the given non-linear law. Consider arbitrary relationships between velocity and

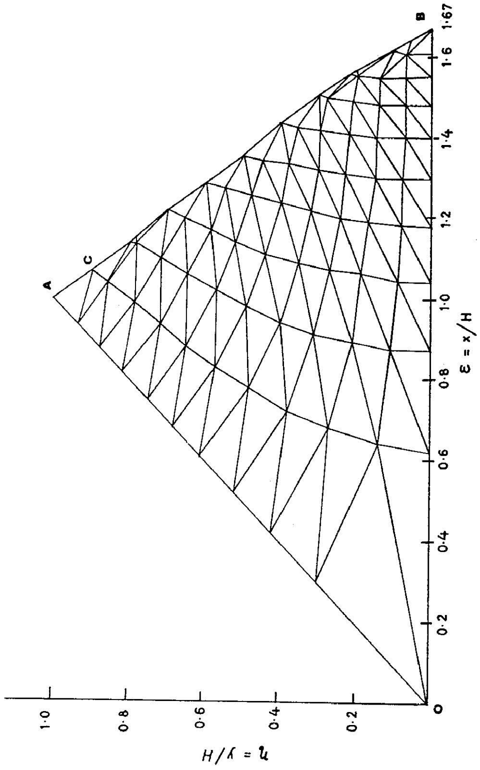
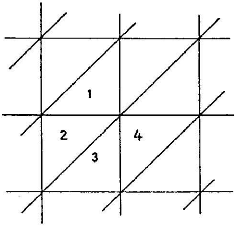
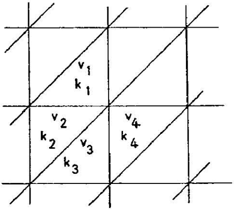


FIG. 2. SUBDIVISION OF SECTION OAB INTO ELEMENTS FOR NUMERICAL SOLUTIONS.



(a) SUBDIVISION OF A HOMOGENEOUS REGION INTO FINITE ELEMENTS – NON LINEAR FLOW LAW – $J = \phi(v)$



(b) ASSEMBLAGE OF ELEMENTS WITH DIFFERENT PERMEABILITIES – LINEAR FLOW LAW –

$$k_n = V_n / (\phi(v))_n = V_n / J_n$$

FIG. 3. SUGGESTED MODEL FOR ISOTROPIC NON-LINEAR FLOW

hydraulic gradient in the form:

$$J = \phi_1(v) \quad \text{or} \quad v = \phi_2(J) \qquad \ldots . (6)$$

The corresponding velocity / hydraulic gradient ratios may be expressed as the variable co-efficients of permeability (see Fig. 3)

$$k_1(v) = v \;/\; \phi_1(v)\;, \quad k_2(J) = \phi_2(J) \;/\; J \qquad \ldots . (7)$$

A given problem is solved assuming the region to be homogeneous and the flow to obey the linear flow law. The values of k_1 (v) or k_2 (J) are then calculated for each element and the resulting non-homogeneous assemblage of finite elements is solved again.

The procedure is repeated and the iterations stopped when the maximum difference of head values in successive solutions is below a tolerable value at each of the nodes.

5. COMPARISON OF RESULTS FOR FLOW THROUGH A DAM

The problem solved by Sokolovsky concerns seepage through section OAB of height H shown in Fig. 1. The dam section rests on an impermeable base and water is level with the crest on the upstream side. In his solution, summarised elsewhere (10), the symmetrical section OAB' has to be considered first and the configuration OAB is determined after involved mathematical manipulation. The closed form solution is then obtained within this new region OAB.

The subdivision of OAB into elements is shown in Fig. 2. The solution was obtained for unit hydraulic head (H = 1) across the dam for comparison with Sokolovsky's solution. The maximum difference of head at any node after successive iterations was:

Iteration	1	2	3	4	5	6
Method 1	0.0388	0.0104	0.0035	0.0014	0.0007	0.0005
Method 2	0.0387	0.0104	0.0037	0.0014	0.0007	0.0005.

The quantity of seepage was computed and found to be about 6 per cent higher than the value obtained by Sokolovsky. This order of error is expected for the size of mesh used (14) particularly when one of the extreme flow lines is a point and the line joining nodes A and C (Fig. 2) has to be used instead.

Head values along the impermeable base are compared in Fig. 4. The four curves are for three non-linear solutions and one Darcy flow solution. For the same problem and mesh, solutions were obtained for different values of m. The variation of discharge with m is shown in Fig. 5 (a). Head values at a point on the base are plotted in Fig. 5 (b). A comparison of head values along the base for two values of m is made in Fig. 6.

6. CONCLUSIONS AND EXTENSION FOR ANISOTROPY

Reliable solutions to problems of ground water flow governed by non-linear flow laws may be obtained using the methods outlined in this paper. As regards anisotropy of ground, there is no mention in the literature of relevant flow laws. Generalised Darcy's law (15) is also for linear seepage. Elsewhere (10) the writer has proposed a valid form of non-linear relationship which takes ground anisotropy into consideration provided the relevant constants are known. The alternative solution method given herein is found to be applicable.

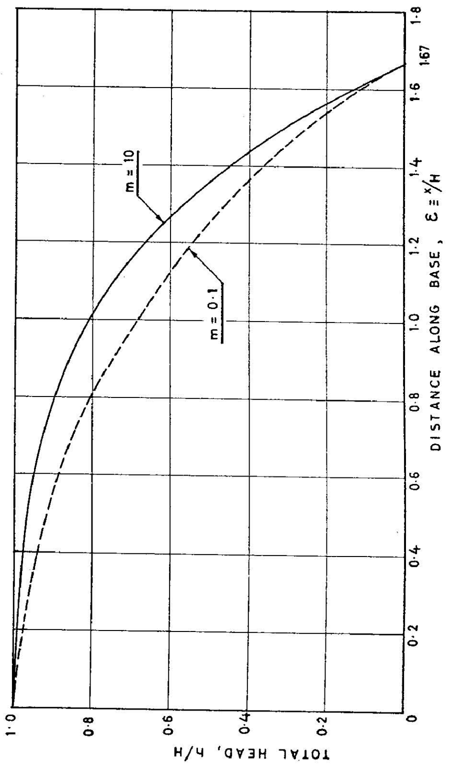
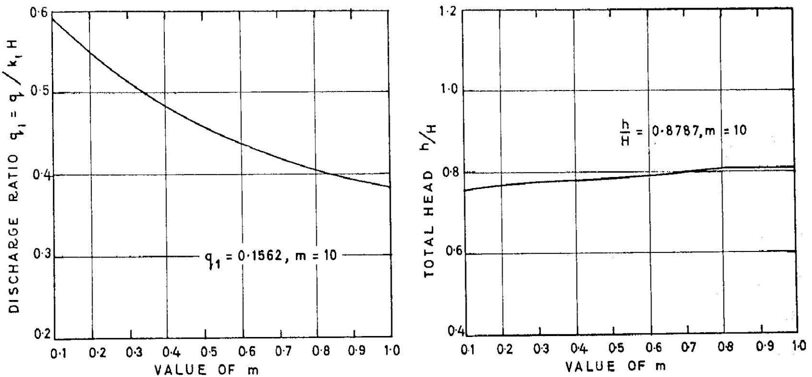


FIG. 6. COMPARISON OF HEAD VALUES ALONG IMPERMEABLE BASE FOR TWO VALUES OF m.



FLOW LAW: $J = \dfrac{v}{k_1 \sqrt{1 - \left[\frac{v}{m}\right]^2}}$

FIG. 5. (a) VARIATION OF DISCHARGE WITH m. (b) VARIATION OF TOTAL HEAD AT $\varepsilon = 0·87, \eta = 0$ WITH m.

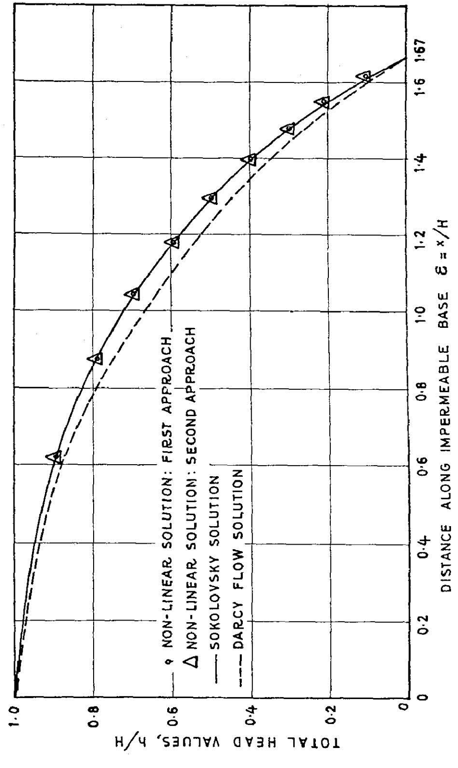


FIG. 4. COMPARISON OF LINEAR AND NON-LINEAR SOLUTIONS FOR TOTAL HEAD ALONG BASE OF DAM.

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THEORY AND APPLICATION OF THE SKIN EFFECT CONCEPT TO GROUND WATER WELLS

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SYNOPSIS

The concept of skin effect and its significance in ground water hydrology is discussed. Two mathematical models to evaluate the skin effect are formulated and subsequently solved. The solutions for the drawdown at the pumped well of the two models show excellent agreement. The second model is preferred because its solution is simpler to use. A trial and error technique is recommended for the determination of transmissibility and the radius of the zone where skin effect is present.

INTRODUCTION

Field analyses have shown that a zone often exists in the vicinity of a well bore whose permeability is substantially lower than that of the rest of the formation. HURST (1) termed the increased head loss or pressure drop that results from the lower permeability the skin effect. The skin effect is often evident from a comparison of observed and theoretical drawdown curves. In many cases actual drawdown curves at the pumped well lie below the theoretical ones and thus suggest that the additional drawdown may be the result of a zone whose permeability or storage coefficient is less than that of the rest of the formation.

Numerous papers in the petroleum literature have been concerned with the skin effect concept. VAN EVERDINGEN (2) proposed a method for determining the skin effect from recovery or pressure build-up data of the pumped well. RUSSELL (3) presented techniques that extended the range of applicability of pressure build-up analysis for determining the skin effect. Because water wells normally exhibit very rapid build-up or recovery curve once the well is shut off, the skin effect can not be readily determined from water well recovery data. JONES (4) demonstrated that the skin effect is a result of a decrease in permeability in the vicinity of the well bore, and that the magnitude of this loss can be evaluated from known flow equations provided that the permeability of the "damaged" zone is known.

The purpose of this paper is to present two unsteady state flow equations to a well where the skin effect is present and to demonstrate how the value of the skin effect can be evaluated from a pumping test data. The letter symbols adapted for use in this paper are defined where they first appear and are arranged alphabetically in the Appendix.

2. THE MATHEMATICAL MODEL I

The model proposed herein assumes that a well is located at a center of a finite radial confined aquifer and pumping a constant discharge. This finite radial aquifer is surrounded by another confined aquifer of infinite areal extent. Initially, when the well started to produce, the aquifer characteristics of the inner and outer regions are considered the same. But, after some time, the permeability of the inner zone decreases due to the migration of fines, encrustation and other phenomena. Consequently, after the well has been in operation for some time, the aquifer characteristics of the two regions are different but uniform, i.e., each region is considered as a homogeneous, isotropic aquifer. The boundary value problem to be solved is:

$$\frac{\partial^2 s_i}{\partial r^2} + \frac{1}{r}\frac{\partial s_i}{\partial r} = \frac{1}{\omega_i}\frac{\partial s_i}{\partial t}; \quad i=1,\ 2 \tag{1}$$

$$s_i(r,0) = 0 \tag{2}$$

$$s_2(\infty,t) = 0 \tag{3}$$

$$T_2\frac{\partial s_1}{\partial r}\bigg|_{r=R} = T_2\frac{\partial s_2}{\partial r}\bigg|_{r=R} \tag{4}$$

$$s_1(R,t) = s_2(R,t) \tag{5}$$

$$-\frac{Q}{2\pi r_w T_1} = \frac{\partial s_1}{\partial r}\bigg|_{r=r_w} \tag{6}$$

Where s_i (r,t) is the drawdown at any time from the start of pumping at a distance r from the well, and $i=1,2$ indicates the inner and outer regions, respectively; $\omega=T/S$ where T is the transmissibility and S is the storage coefficient; Q is the constant discharge, r_w is the radius of the well and R is the radius of the inner region where skin effect is present.

The above boundary value problem can be solved using Laplace transformation and approximate inverse transformation.

The Laplace transformation of equations (1) - (6) is

$$\frac{d^2\bar{s}_i}{dr^2} + \frac{1}{r}\frac{d\bar{s}_i}{dr} - \frac{p\bar{s}_i}{\omega_i} = 0 \qquad (7)$$

$$\bar{s}_2 (\infty, p) = 0 \qquad (8)$$

$$T_1 \left.\frac{d\bar{s}_1}{dr}\right|_{r=R} = T_2 \left.\frac{d\bar{s}_2}{dr}\right|_{r=R} \qquad (9)$$

$$\bar{s}_1 (R,p) = \bar{s}_2 (R,p) \qquad (10)$$

$$\frac{-Q}{2\pi r_w T_1 p} = \left.\frac{d\bar{s}_1}{dr}\right|_{r=r_w} \qquad (11)$$

Applying the boundary conditions, i.e., equations (8) - (11) to equation (7) results in the transformed drawdown equation for each of the two circular regions;

$$\bar{s}_1 (r,p) = - C_1^* \left[\frac{C_2^* I_0(\delta^*) + K_0(\delta^*)}{C_2^* I_1(\gamma^*) - K_1(\gamma^*)}\right] ; \quad r \leqslant R \qquad (12)$$

$$\bar{s}_2 (r,p) = - C_1^* \left[\frac{C_2^* I_0(\alpha^*) + K_0(\alpha^*)}{C_2^* I_1(\gamma^*) - K_1(\gamma^*)}\right] \frac{K_0(\rho^*)}{K_0(\beta^*)} ; \quad r \geqslant R \qquad (13)$$

where:

$$C_1^* = - \frac{Q}{2\pi T_1 p \gamma^*}$$

$$C_2^* = \frac{\theta . K_1(\alpha^*) K_0(\beta^*) - K_0(\alpha^*) K_1(\beta^*)}{\theta . I_1(\alpha^*) K_0(\beta^*) + I_0(\alpha^*) K_1(\beta^*)}$$

$I_0(x)$, $I_1(x)$ = zero and first order modified Bessel function of the first kind respectively

$K_0(x)$, $K_1(x)$ = zero and first order modified Bessel function of the second kind respectively

$$\alpha^* = R\sqrt{p/\omega_1}$$

$$\beta^* = R\sqrt{p/\omega_2}$$

$$\gamma^* = r_w\sqrt{p/\omega_1}$$

$$\delta^* = r\sqrt{p/\omega_1}$$

$$\rho^* = r\sqrt{p/\omega_2}$$

$$\theta = \sqrt{\frac{T_1 S_1}{T_2 S_2}}$$

Equations (12) and (13) do not have a simple inverse transformation. An approximate inversion for Laplace transforms was developed by SHAPERY (5) and applied by STERNBERG (6) yielded excellent agreement between the exact and the approximate solutions for a number of radial flow problems.

The approximation is based on the observation that in cases where

$$d\,(pf(p))\;/\;d(\log p)$$

where

$$f(p) = \int_0^\infty f(t)\,\exp(-pt)dt$$

is a slowly varying function of log p, f(t) can be approximated by

$$f(t) \approx pf(p)\Big|_{p=\frac{1}{2}t} \tag{14}$$

Applying the approximation given by equation (14) to equations (12) and (13) yields

$$s_1(r,t) = -\frac{Q}{2\pi T_1\gamma}\left[\frac{C_2\,I_0(\delta) + K_0(\delta)}{C_2\,I_1(\gamma) - K_1(\gamma)}\right]\;;\quad r \leqslant R \tag{15}$$

$$s_2(r,t) = -\frac{Q}{2\pi T_1\gamma}\left[\frac{C_2\,I_0(\alpha) + K_0(\alpha)}{C_2\,I_1(\gamma) - K_1(\gamma)}\right]\frac{K_0(\rho)}{K_0(\beta)}\;;\quad r \geqslant R \tag{16}$$

where

$$C_2 = \frac{\theta_* K_1(\alpha)\ K_0(\beta) - K_0(\alpha)\ K_1(\beta)}{\theta_* I_1(\alpha)\ K_0(\beta) + I_0(\alpha)\ K_1(\beta)}$$

$$\alpha = \sqrt{R^2 / 2t\omega_1}$$

$$\beta = \sqrt{R^2 / 2t\omega_2}$$

$$\gamma = \sqrt{r_w^2 / 2t\omega_1}$$

$$\delta = \sqrt{r^2 / 2t\omega_1}$$

$$\rho = \sqrt{r^2 / 2t\omega_2}$$

It may be noted that when $r=R$, $\alpha=\delta$, and $\beta=\rho$, equations (15) and (16) are equal as required by equation (5). Equation (15) relates the drawdown at any $r \leqslant R$ to the aquifer characteristics of the two regions and time. The drawdown at the well, i.e. at $r=r_w$ is given by

$$s\ (r_w,t) = -\frac{Q}{2\Pi T_1 \gamma}\left[\frac{C_2 \cdot I_0(\gamma) + K_0(\gamma)}{C_2 \cdot I_1(\gamma) - K_1(\gamma)}\right] \tag{17}$$

and is a function of Q, T_1, S_1, T_2, S_2, R and time. The aquifer chracteristics of the outer region can be determined from data obtained at an observation well located sufficiently far from the pumped well. HURST (7) demonstrated that in cases where the aquifer characteristics in the vicinity of the well are markedly different from those in the rest of the aquifer, the former do not affect the drawdown characteristics in an observation well not located near the inner zone. Thus, values of T_2 and S_2 can be obtained by various techniques such as those developed by THEIS (8) and others.

During the first few minutes of a pumping test the drawdown in the well is very rapid and collection of data at that time is difficult. Even if drawdown data can be secured from the pumped well, graphical techniques such as type curves cannot be used to evaluate the three unknowns of equation (17), i.e., S_1, T_1 and R. In order to overcome the difficulty involved in determining the above three unknowns another model was formulated to approximate the model of equation (17).

3. THE MATHEMATICAL MODEL II

The equation for the drawdown distribution due to a fully penetrating well operating in a confined aquifer is:

$$s\,(r_w,t) = \frac{Q}{4\Pi T_2}\left[\ln \frac{2{\cdot}25\ T_2\ t}{r_w^2\ S_2}\right]; \qquad u \leqslant 0.01 \tag{18}$$

In practice one often observes that the actual drawdown at the well is larger than that predicted by equation (18) because of the skin effect. A better agreement between equation (18) and the observed drawdown may be obtained by modifying equation (18) to include the additional head loss resulting from the skin effect, i.e.,

$$s(r_w,t) = \frac{Q}{4\Pi T_2}\left[\ln \frac{2{\cdot}25\ T_2\ t}{r_w^2\ S_2}\right] + 2\ S_e \tag{19}$$

where S_e is the added drawdown or the skin effect.

In a study on pressure distribution in oil reserviors BROWNSCOMBE and COLLINS (9) demonstrated that in the vicinity of the well there was no apparent difference in the pressure distribution between compressible and incompressible fluids under steady state flow conditions, and because of the small volume of fluid in the vicinity of the well bore, the non-steady state solution can be approximated by the steady state solution. HAWKINS (10) suggested that the value of Se may be evaluated from known steady state flow equations. Under steady state conditions, the additional head loss, Δs, due to the skin effect is

$$\Delta s = \frac{Q}{2\Pi T_1}\ln\frac{R}{r_w} - \frac{Q}{2\Pi T_2}\ln\frac{R}{r_w} \tag{20}$$

and the total drawdown in the pumped well is

$$s(r_w,t) = \frac{Q}{4\Pi T_2}\left[\ln \frac{2{\cdot}25\ T_2 t}{r_w^2\ S_2}\right] + 2\left[\frac{T_2 - T_1}{T_1}\ln\frac{R}{r_w}\right] \tag{21}$$

where

$$S_e = \frac{T_2 - T_1}{T_1}\ln\frac{R}{r_w}$$

4. DISCUSSION

Equations (17) and (21) both describe the drawdown in a well where skin effect is present. Numerical evaluation of the two equations for a wide range of data is given in Table I. The results show that equation (21), although based on a simplified mathematical model, is an excellent approximation to equation (17). The storage coefficient of the inner zone, S_1, does not appear in equation (21) but the results indicate that this term has little influence on the drawdown in the pumped well.

Because equation (21) is simpler to use than equation (17), its use is preferred when computing the additional drawdown due to skin effect. Equation (21) contains two terms, namely, $2.25\ T_2t/r_w^2 S_2$ and S_e. The first term can be evaluated from data collected at an observation well located in the outer zone. The second term contains the two unknowns, R and T_1 and their values can be evaluated most efficiently by a trial and error procedure. Values of drawdown as a function of time obtained at the pumped well can be compared to the theoretical values given by equation (21) with $S_e=0$. If the observed drawdown values are greater than the ones obtained from equation (21) with $S_e=0$, one may conclude that $S_e > 0$ and that the skin effect is present. Once the value of S_e has been obtained, the values of R and T_1 can be estimated by trial and error noting that the term R/r_w appears in logarithmic form.

Thus, the condition of a well can be determined by comparing actual drawdown at the pumped well to the calculated values using the aquifer characteristics determined from an observation well data. Positive values of S_e indicate a damaged zone near the well bore while negative values indicate higher hydraulic conductivity zone which may be a result of acidizing, fracturing or gravel pack. Results of acidizing or other treatment to reduce the skin effect can be quickly checked by calculating the value of S_e before and after treatment. The method is applicable only to wells affected by losses due to laminar flow (both the normal losses and those in the "damaged" zone) and does not account for turbulent losses that may be present in large capacity wells.

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TABLE I

Comparison between Equation (17) and (21) for various values of T_1, T_2, S_1, S_2 and R.

t = 0.1 days, r_w = 0.25 ft

T_1, $\frac{ft^2}{min}$	T_2, $\frac{ft^2}{min}$	$S_1 x 10^4$	$S_2 x 10^4$	R, ft	$\frac{2\pi s}{Q}$ from eq. (17)	$\frac{4\pi s}{Q}$ from eq. (21)
0.1	1.0	1.0	1.0	0.5	5.769	5.742
o.1	5.0	1.0	1.0	0.5	4.006	4.001
0.1	500.0	1.0	1.0	0.5	3.473	3.473
0.5	100	1.0	1.0	0.5	0.727	0.727
0.5	1.0	1.0	1.0	1.0	3.344	3.316
0.1	5.0	1.0	1.0	1.0	7.399	7.397
0.1	10.0	1.0	5.0	1.0	7.141	7.141
0.5	5.0	1.0	10.0	1.0	1.743	1.737
0.5	1.0	5.0	1.0	1.0	3.333	3.316
0.5	5.0	10.0	1.0	1.0	1.854	1.852
0.5	1.0	1.0	5.0	1.0	2.948	2.913
0.5	10	1.0	10.0	1.0	1.582	1.579
1.0	10.0	5.0	1.0	1.0	0.946	0.948
0.1	5.0	10.0	1.0	1.0	7.361	7.397
0.1	1.0	1.0	1.0	2.0	11.975	11.980
0.5	10.0	1.0	1.0	2.0	2.296	2.295
0.5	5.0	1.0	10	2.0	10.576	10.679
1.0	10.0	10	1.0	2.0	1.253	1.255
0.1	10.0	5.0	1.0	3.0	12.364	12.620
1.0	5.0	5.0	10.0	3.0	1.487	1.483
0.1	5.0	10.0	10.0	3.0	12.641	12.746
1.0	10	10.0	10.0	3.0	1.375	1.380
1.0	1.0	10.0	1.0	3.0	2.544	2.622

APPENDIX NOTATION

$$C_2 = \frac{\theta \cdot K_1(\alpha)\, K_0(\beta) - K_0(\alpha)\, K_1(\beta)}{\theta \cdot I_1(\alpha)\, K_0(\beta) + I_0(\alpha)\, K_1(\beta)}$$

$I_0(x)$, $I_1(x)$ = Zero and first order modified Bessel functions of the first kind respectively

$K_0(x)$, $K_1(x)$ = Zero and first order modified Bessel function of the second kind respectively

Q = Discharge

R = Radius of the inner zone where skin effect is present

r = Radial distance from the well

r_w = Radius of the well

$s(r,t)$ = Drawdown at any time from the start at a distance r from the well

S = Storage coefficient

S_e = Drawdown due to skin effect

T = Transmissibility

$\alpha = \sqrt{R^2/2t\omega_1}$

$\beta = \sqrt{R^2/2t\omega_2}$

$\gamma = \sqrt{r_w^2/2t\omega_1}$

$\delta = \sqrt{r^2/2t\omega_1}$

$\rho = \sqrt{r^2/2t\omega_2}$

$\theta = \sqrt{\dfrac{T_1 S_1}{T_2 S_2}}$

PARTIALLY PENETRATING WELL IN A CONFINED AQUIFER

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SYNOPSIS

A critical review of the existing methods to analyse the flow toward a partially penetrating well in a confined aquifer is made and their limitations, assumptions implied and applicability are discussed. The method of electrical analogy is employed to analyse the flow towards a partially penetrating well in a confined aquifer. The effect of partial penetration on drawdown and discharge is demonstrated, making use of the test results. The influence of the various parameters on the phenomenon is analysed.

1. INTRODUCTION

A partially penetrating well is one whose length of water entry portion is less than the thickness of the aquifer. Flow to such a well is three-dimensional, some times with radial symmetry. Partially penetrating wells are often constructed for the following reasons:

1. Water requirement being less than the yield of fully penetrating wells.
2. Economic considerations: increase in the cost of drilling with depth, and increase in yield being not proportional to the increase in penetration.
3. Technical Considerations: capacity of the drilling machine on hand, special problems encountered in well construction which prevent further progress.

A greater resistance to flow is encountered in the case of partially penetrating wells owing to the increase in the average length of a flow line and hence an additional amount of drawdown is created in comparison with fully penetrating wells for the same discharge. However, it was observed that the effect of partial penetration is negligible on the flow pattern beyond a distance of about 1.5 to 2 times the thickness of the aquifer. The additional drawdown created in a partially penetrating well is of great interest from the point of view of aquifer test analysis on partially penetrating wells. Also, one would normally like to know the increase in yield per unit additional depth of penetration in order to carry out an economic analysis.

The scope of the present work is limited to the analysis of partially penetrating wells in confined aquifers under steady state conditions. The problem is analysed using a conductive liquid analog model and the experimental results are compared with those from the existing theoretical and semi-empirical expressions. The various theories so far available on the subject are reviewed. The assumptions implied, the parameters influencing the phenomenon, the applicability and limitations of the various theories are discussed. The following assumptions are generally made in analysing the problem:

1. The aquifer is composed of homogeneous and isotropic material.
2. The aquifer is horizontal and of uniform thickness.
3. There is no contribution to flow from the confining strata.
4. Steady-state conditions are established over a reasonably large area around the wells.
5. The well penetrates only a part of the aquifer from the top.

2. CRITICAL REVIEW OF THE EXISTING METHODS

Forchheimer (2) was probably the first investigator who considered the effect of partial penetration in an aquifer. Cases of flow through plane and hemispherical bottomed wells having water tight walls were also analysed by him. His analysis of the problem was semi-empirical and the discharge ratio for a partially penetrating well is given by

$$Q_p / Q_f = \sqrt{p} \; \sqrt[4]{2-p} \qquad (1)$$

where Q_p and Q_f are the discharges from partially penetrating and fully penetrating wells respectively and p is the penetration factor defined as the ratio of the well penetration depth, l, to the aquifer thickness, M(Fig.1)

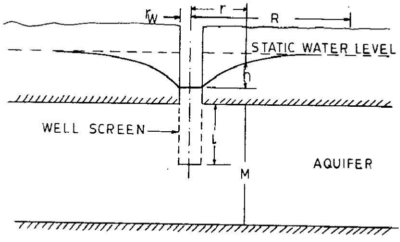

FIG.1. DEFINITION SKETCH

The discharge ratio is plotted as a function of penetration fraction in Fig.2. de Glee's eduation (7) for the discharge ratio, of a partially penetrating

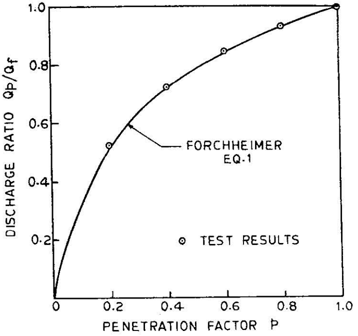

FIG. 2. DISCHARGE RATIO FOR A PARTIALLY PENETRATING WELL

well is of the form

$$\frac{Q_p}{Q_f} = \frac{\ln(R/r_w)}{\frac{1}{p}\ln\left(\frac{\pi l}{2 r_w}\right) + 0.1 + \ln\left(\frac{R}{2M}\right)} \tag{2}$$

where R is the radius of influence, r_w is well radius and ln is natural logarithm. The above equation is stated to be valid for $1.3\,l \leq M$ and $l/2r_w \geq 5$. Muskat (5) treated the problem of fluid flow towards partially penetrating wells in confined aquifers as analogous to the flow of electricity in large cylindrical disks and evaluated the potential distributions and flow rates for various conditions. This stands to be the first exact theoretical analysis of the problem. An approximate formula developed by Muskat (6) for discharge ratio is of the form

$$\frac{Q_p}{Q_f} = \frac{\ln\left(\frac{R}{r_w}\right)}{\frac{1}{2p}\left\{2\ln\frac{4M}{r_w} - A\right\} - \ln\frac{4M}{R}} \tag{3}$$

where A is a function of p. Eq.3 is evaluated and presented graphically in Figs. 3 and 4 for different values of p, R/r_w, and M/R. Eq.3 is based on the assumption that the flux is uniform all along the axis of the well which is not true. Even in its approximated form, the equation is a bit cumbersome to handle and it was observed from computations that the results of the equation for $p < 0.3$ are not consistent with those for the rest of the range and hence the equation can be recommended only for penetrations of more than 30 per cent.

Based on Muskat's analysis of the problem and experiments, Kozeny (3) presented a still simplified approximate formula for the discharge ratio

$$\frac{Q_p}{Q_f} = p\left(1 + 7\sqrt{\frac{r_w}{2M}}\sqrt{\frac{1}{p}}\cos\frac{\pi p}{2}\right) \qquad (4)$$

Eq.4 does not satisfy the Theim's equation for large values of r and hence is not perfect in its form.

Wen Hsiung Li et al.(8) evolved a simple empirical formula for the drawdown in a partially penetrating well and for the discharge ratio, based on the data obtained from a series of tests on an electric analog model. The ratio of the drawdown h_p in a partially penetrating well to the drawdown h_f in a fully penetrating well in the same aquifer, for the same discharge is given by

$$\frac{h_p}{h_f} = 1 + \frac{\log(M/r_w)}{\log(r/r_w)}\left[\left(\frac{1}{p}\right)^n - 1\right] \qquad (5)$$

and

$$\frac{Q_p}{Q_f} = \frac{1}{1 + \frac{\log(M/r_w)}{\log(R/r_w)}\left[\left(\frac{1}{p}\right)^n - 1\right]} \qquad (6)$$

The value of n can be taken as 0.75. These equations are not applicable for penetrations less than 0.1.

Kirkham (1) developed his theory for flow into partially penetrating wells considering the case of top penetration and advanced solutions for flow rate and potential distribution. Though his analysis is an elegant exercise in advanced mathematics, it is too sophisticated for ordinary use unless it is extremely approximated. Jacob (6) recommended a method of computing the additional drawdown in a partially penetrating well which is based on the expressions developed by Muskat. He also presented nomograms which give the relationship between the drawdown correction, penetration fraction and another parameter $\pi r/M$.

3. EXPERIMENTAL INVESTIGATION AND ANALYSIS

In the present study a conductive liquid analog is used to simulate a partially penetrating well field. A sector model is chosen since the flow to a partially penetrating well is axisymmetric. A tank 50 cm long and 20 cm wide is constructed with perspex sheets and the base of the tank is graduated. The potential boundary and the well are simulated by using copper electrodes

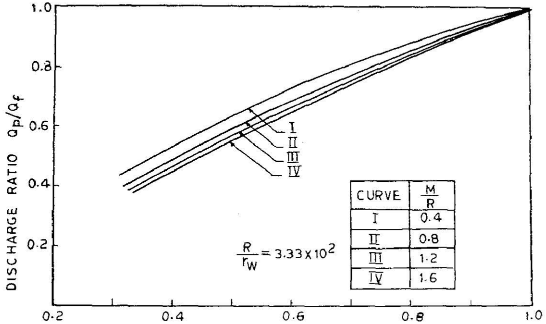

FIG. 3. DISCHARGE RATIO FOR A PARTIALLY PENETRATING WELL IN A CONFINED AQUIFER. MUSKAT'S EQUATION

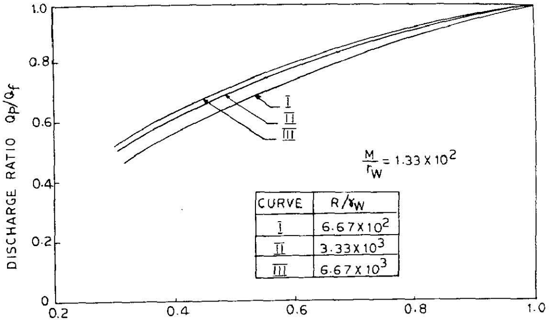

FIG. 4. DISCHARGE RATIO FOR A PARTIALLY PENETRATING WELL IN A CONFINED AQUIFER. MUSKAT'S EQUATION.

(1.6 mm thick) CD and BE respectively. The insulating faces BC and AD represent the impermeable confining beds (Fig.5). A stabilized voltage is derived from an audio frequency generator and a frequency of 1000 cps. is chosen. Ordinary tap water is used to serve as the conducting medium. A potential of 3 V is applied at the potential boundary electrode and the well electrode is earthed. The potential at various points in the field is measured by means of an insulated probe connected to a vacuum tube voltmeter. Finally the resistance of the model is determined. The problem is analysed experimentally for 0, 20, 40, 60, 80 and 100 per cent penetrations of the model. The equipotential lines are plotted for various penetrations. It is observed that the effects of partial penetration are not felt beyond a distance of about 1.5 times the aquifer thickness. From the resistance of the model for each penetration, the current in the circuit is computed, which is analogous to the flow rate in the prototype. Hence the ratios of current for various penetrations to that for full penetration should represent the discharge ratios for various penetrations of the well. Variation of the discharge ratio for various penetration fractions is shown in Fig.2. The variation closely agrees with Forchheimer's equation. To determine the size of the well electrode and the conductivity of the liquid used, the following result for fully penetrating wells is used.

$$\frac{KMh}{Q} = \frac{\sigma MV}{I} = \frac{1}{2\pi} \ln\left(\frac{r}{r_w}\right) \qquad (7)$$

where K is the hydraulic conductivity, σ is the specific conductivity of the medium, V is the potential difference, I is the current, Q is the discharge and h is the hydraulic head between any point and that at the well. The values of σ and r_w were found to be 1.28×10^{-5} mhos/cm. and 1.17 cm respectively.

Making use of the test results, a plot is made of the parameter KMh/Q for different values of r/M and for different penetrations (Fig.6). The curves obtained are parallel to the line for full penetration except in the region of $r/M < (1 - p)$, thus supporting Wen Hsiung Li's (8) conclusion that the additional drawdown created as an effect of partial penetration is essentially constant except in the region where r/M is less than (1 - p). Another interesting feature of this plot is that the lines for penetration fractions of 0.8 and 1.0 almost coincide, supporting the general idea that penetrations of more than 80 per cent can for all practical purposes be considered equivalent to full penetration.

Another plot of KMh/Q for various penetrations is made on logarithmic scales at a convenient value of r = M.(Fig.7). The result is a straight line with a slope of -0.59, which is in close agreement with the values arrived by Wen Hsiung Li.

The equations reviewed in the earlier section can be expressed in terms of functions of the parameters in their non-dimensional form as under:

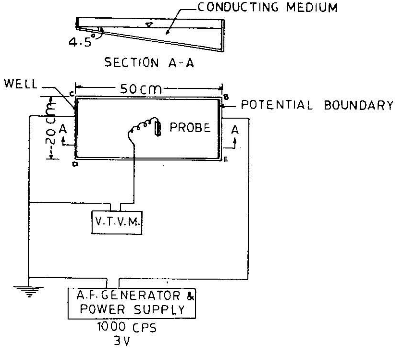

FIG. 5. LIQUID CONDUCTOR ANALOG SET-UP

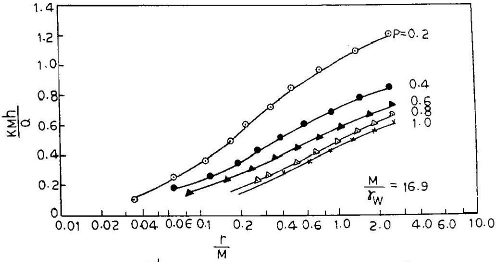

FIG. 6. $\frac{KMh}{Q}$ PLOTTED AS A FUNCTION OF $\frac{r}{M}$ FOR VARIOUS PENETRATIONS

1. Forchheimer: $Q_p/Q_f = f(p)$

2. de Glee: $Q_p/Q_f = f\left(p, \frac{R}{r_w}, \frac{l}{r_w}, \frac{R}{M}\right)$

3. Muskat: $Q_p/Q_f = f\left(p, \frac{R}{r_w}, \frac{M}{r_w}, \frac{M}{R}\right)$

4. Kozeny: $Q_p/Q_f = f\left(p, \frac{M}{r_w}\right)$

5. Wen Hsiung Li: $Q_p/Q_f = f\left(p, \frac{R}{r_w}, \frac{M}{r_w}\right)$

In its most general form the discharge ratio can be expressed as:

$$Q_p/Q_f = f\left(p, \frac{R}{r_w}, \frac{M}{r_w}, \frac{l}{r_w}, \frac{M}{R}\right)$$

It is worthwhile to discuss the extent to which each of the above parameters influence the discharge and the drawdown.

1. Penetration Fraction , p :

Other conditions being the same, penetfation fraction has a significant effect on the discharge and additional drawdown in a partially penetrating well. In general, an increase in penetration does not give rise to a proportional increase in yield, this being especially true beyond a penetration of half the aquifer thickness, as evident from the plots (Fig.2,3 and 4). So one has to decide carefully the optimum depth of the well or optimum length of well screen from economic point of view.

2. Ratio of Radius of Influence to well radius, R/r_w:

A close study of the plots in Fig.4 reveals that the discharge ratio increases with increase in the ratio R/r_w. In other words, the relative discharge is more in case of extensive aquifers, well radius being the same.

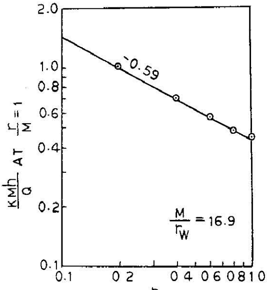

FIG. 7. $\frac{KMh}{Q}$ AT $r = M$

3. Ratio of aquifer thickness to radius of influence, M/R:

It can be observed from Fig.3 that the parameter M/R bears an inverse relationship with the discharge ratio, though the effect is relatively less significant. This is from the simple fact that convergence of flow is more intense in case of thick aquifers for the same penetration percentage and hence a lesser value of discharge ratio.

4. Well Slim-ness, $1/r_w$:

The well slimness appears to have a significant effect on the discharge ratio, as can be seen from de Glee's equation. For the same penetration fraction, the discharge ratio bears an inverse relationship with well slim-ness. In otherwords, the effect of partial penetration is less pronounced in wells of larger size.

5. Ratio of aquifer thickness to radius of the well, M/r_w:

The influence of this parameter can well be appreciated from Muskat's and Wen Hsiung Li's analysis of the problem. A substantial drop in the discharge ratio can be observed for the same penetration when the aquifer thickness is large compared to the well radius.

4. CONCLUSIONS

The results of the tests conducted are in support of Wen Hsiung Li's and Muskat's investigations. The parameters p, R/r_w and M/r_w have significant influence on the discharge and drawdown in a partially penetrating well. The influence of each of these parameters is graphically represented and discussed. The additional drawdown is essentially of a constant magnitude except in a region very close to the well, which is dependent on the penetration. For penetrations of more than eighty percent the effect of partial penetration is so insignificant that it can for all practical purposes be treated equivalent to full penetration. Beyond a penetration of half the aquifer thickness, the increase in yield is not significant. One has to choose the well depth or screen length carefully from economic considerations. Effects of partial penetration are less pronounced in wells of larger size.

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STUDIES ON A WELL PENETRATING A TWO-AQUIFER SYSTEM

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SYNOPSIS

A theoretical formulation for the analysis of a well, penetrating a two-aquifer system with an unconfined aquifer at the top and a confined aquifer at the bottom is indicated. The steady state performance characteristics of such a system have been studied on a sand model. An interference coefficient of discharge is defined and the values of the same are determined from experimental data for different driving head conditions in the two aquifers. These values are found to vary from 10 to 25 percent, indicating that the interference effects in a two aquifer system is significant.

1. INTRODUCTION

In ground water extraction it is quite common that the supply is derived from a multiaquifer system. It may often consist of one unconfined aquifer on top and one or more confined aquifers separated by impermeable layers at the bottom. The connection between the aquifers in many cases is only through the well. The well penetrates the overlying unconfined aquifer completely and in the confined aquifer the penetration may be either full or partial. The yield in such cases is usually considered as the sum of the individual contributions from each of the aquifers under the respective driving head conditions, the interaction between the aquifers being assumed negligible.

2. THEORETICAL FORMULATION

A theoretical analysis for a two aquifer system both of them confined has been given by Papadopolus (1). But a multi-layered system involving an unconfined aquifer has not been analysed. The presence of a free surface in the unconfined aquifer and the likely formation of a surface of seepage at the well face render this problem difficult to analyse, despite the fact that the solution for flow into a well from a single aquifer either confined or unconfined is well known (2,3). An analytical model for a two aquifer system with an unconfined aquifer at the top as shown in Figure 1 is presented herein.

Consider that the aquifers are of infinite areal extent with the initial heads being different in them. At time $t = 0$, the two aquifers are considered to be connected through the well, without pumping the well. Let the well be kept unpumped for a period t_o. During this period internal flow takes place due to the difference in head. After time t_o let the well be pumped at a constant rate Q_T. The diameter of the well is assumed to be small so that storage in the well can be neglected and the problem made tractable. If the aquifers are assumed to be homogeneous and isotropic, the head distributions around the well may be described by the following boundary value problem, using the notation given below: (the subscripts u and c refer to the unconfined and confined aquifers respectively.)

$s_u(r,z,t)$, $s_c(r,t)$	= drawdowns at any radius r and time t
H_u, H_c	= heads at infinity
K_u, K_c	= permeabilities
S_u, S_c	= Specific storages
$\bar{Q}_u$, $\bar{Q}_c$	= contributions to total discharge
r_w	= well radius
ϵ	= specific yield for unconfined aquifer

For the unconfined aquifer the differential equation for drawdown $s_u(r,z,t)$ is

$$\frac{\partial^2 s_u}{\partial r^2} + \frac{1}{r}\frac{\partial s_u}{\partial r} + \frac{\partial^2 s_u}{\partial z^2} = \frac{S_u}{K_u}\frac{\partial s_u}{\partial t} \quad \ldots\ldots (1)$$

The solution should satisfy the conditions

(i) $$\frac{\epsilon}{K_u}\frac{\partial s_u}{\partial t} = \left(\frac{\partial s_u}{\partial r}\right)^2 + \left(\frac{\partial s_u}{\partial z}\right)^2 - \left(\frac{\partial s_u}{\partial z}\right) \quad \text{at the free surface} \ldots (2a)$$

(ii) $$\frac{\partial s_u}{\partial z} = 0 \quad \text{at } z = 0 \quad \ldots\ldots (2b)$$

(iii) $$s_u = 0 \quad \text{at } t = 0 \quad \ldots\ldots (2c)$$

(iv) $$s_u = 0 \quad \text{at } r = \infty \quad \ldots\ldots (2d)$$

and $$2\pi K_u (H_u - b')\, r \frac{\partial s_u}{\partial r} = -\bar{Q}_u \quad \text{as } r \to 0 \ldots (2e)$$

For the case of the confined aquifer, the differential equation for the drawdown $s_c(r,t)$ is

$$\frac{\partial^2 s_c}{\partial r^2} + \frac{1}{r}\frac{\partial s_c}{\partial r} = \frac{S_c}{K_c}\frac{\partial s_c}{\partial t} \qquad \ldots\ldots (3)$$

The solution should satisfy the conditions,

(i) $s_c = 0$ at $t = 0$ ..... (4a)

(ii) $s_c = 0$ at $r = \infty$ ..... (4b)

and (iii) $2\pi K_c b r \frac{\partial s_c}{\partial r} = -\overline{Q_c}$ as $r \to 0$ ..... (4c)

The common conditions to be satisfied by both the solutions for the case of the aquifers performing simultaneously are

(i) $\overline{Q_u} + \overline{Q_c} = 0$ for $0 < t \leq t_0$ ..... (5a)

$= Q_T$ for $t > t_0$

and (ii) $(H_u - s_u) = (H_c - s_c)$ at $r = r_w$ ..... (5b)

neglecting the seepage surface.

The specific storage S_u in the case of unconfined aquifer is very small relative to the specific yield ϵ and therefore the contribution to the flow from aquifer compression may be neglected except during the very early periods of flow (3). With this assumption, the right hand side of Eq.1 may be treated as zero. If the gradients are small the squares viz., $\left(\frac{\partial s_u}{\partial r}\right)^2$ and $\left(\frac{\partial s_u}{\partial z}\right)^2$ in Eq.2a can be neglected, and Eq.1 and Eq.2a may be written as

$$\frac{\partial^2 s_u}{\partial r^2} + \frac{1}{r}\frac{\partial s_u}{\partial r} + \frac{\partial^2 s_u}{\partial z^2} = 0 \qquad \ldots\ldots (6)$$

and $\frac{\partial s_u}{\partial t} + \frac{K_u}{\epsilon}\frac{\partial s_u}{\partial z} = 0$ at $z = H_u$ ..... (7)

The solutions of the differential equations Eq.6 and Eq.3 can be attempted using the Laplace transform and the Hankel transform. The exact nature of the solution is under investigation. Due to the omission of the seepage surface in the unconfined aquifer, a slight deviation in the vicinity of the well is likely; but the error may not be significant as shown by Hantush (4).

3. EXPERIMENTAL STUDIES

The present study on a sand model is an attempt to arrive at a factor for the effect of interaction for various conditions of individual driving heads and differential heads for the two aquifer formation as shown in Figure 1. This factor is designated as the interference coefficient of discharge as is defined as

$$\eta = 1 - \left\{ \frac{Q_T}{Q_c \pm Q_u} \right\} \quad \ldots . (8)$$

in which Q_T = Total discharge from the twin aquifers when both are contributing simultaneously through the well, Q_c and Q_u = independent individual discharges from confined and unconfined aquifers for various driving heads, with each aquifer discharge being collected when the other aquifer is sealed.

The aim of the present study is to experimentally determine the values of the interference coefficient of discharge for specific driving heads conditions for each of the aquifers for three different penetration ratios for the confined aquifer, with the ratio of the diameter of the well to the thickness of the confined aquifer being large.

3.1 Experimental facility and procedure

The experiments were conducted on a sand model the details of which are shown in figures 2a and 2b. The model was prepared in a flume 440 cm x 60 cms connected to two separate reservoir systems on either side. Perforated overflow pipes with sliding collars were provided to maintain desired levels in the tanks. Point gauges were used for measurement. Wire gauge screens were provided at the junctions to prevent entry of sand into the tanks. One of the longer sides of the flume is of glass and the other one is of masonry. Sieved sand was filled in layers for both the aquifers. An aluminium sheet 1.5 mm thick was used to simulate the impermeable layer separating the two aquifers.

Well was simulated using a half cylinder of steel 70 cms high and 5 cms in diameter with circular perforations of 4 mm diameter in rows of 8 mm spacing for the full height. This was placed butting the side of the glass plate. A syphon arrangement was used for simulating pumping from the well. A pet cock fixed on the delivery side was controlling the rate of flow from the well.

A confined aquifer of 17.5 cm thickness and unconfined aquifer of 50 cm thickness were prepared with the aluminium sheet in between. Piezometers with specially prepared ends were placed at selected points on either side of the well at the bottom level of the aquifers. These piezometer tubes were connected to manometers on the sides of the model.

In order to study the individual performance of confined aquifer and unconfined aquifer separately under specific driving heads, water in the concerned tank was maintained at the desired level. The portion of well over the unconfined aquifer region was completely sealed during the study of the confined aquifer performance. In order to avoid the effect of pressure uplift on the aluminium plate the upper aquifer was also kept saturated. Model was allowed to run for some time to make sure that the sand was completely saturated. After completing the readings for the confined aquifer for different penetration ratios, the portion of the well for this aquifer was sealed and the well was screened only for unconfined aquifer and the observation made.

All the experiments on the combined performance were concerned with the steady state conditions and the total discharges were measured for various water level conditions in the two aquifers for which individual discharges were already measured in the previous phase. Here, the well was penetrating the unconfined aquifer completely and the confined aquifer either fully or partially, with the values of penetration ratio as 1.0, 0.7 and 0.5.

3.2 Results

The range of experimental parameters is shown in Figure 3. Discharges as obtained from experiments are used to compute the interference coefficients (Table 1). The discharge-drawdown graphs for various conditions of water levels in the aquifers are also plotted (Fig. 4 and 5).

The plots appear nearly linear indicating that for small drawdowns, the relationship is approximately linear for the unconfined aquifer also. Figure 4 shows the discharge drawdown graphs under differential heads of 5 cms and 17 cms for different penetration ratios in a confined aquifer. For the same driving head the discharge is found to be increasing with increasing penetrating ratios. With equal water levels for both the aquifers, the effect of interference is found to increase with increase in the driving head and decrease in the penetration ratios. The average value of the interference coefficient of discharge is seen to be 0.19. In the case of different water levels for the two aquifers the interference is increasing with the differential head. The average value is around 0.23. Thus the effect of interference in this case is predominantly higher for smaller discharges.

TABLE 1

SUMMARY OF COMPUTED VALUES OF INTERFERENCE COEFFICIENT OF DISCHARGE

H_c	H_u	$\eta = 1 - \left\{\frac{Q_T}{Q_c \pm Q_u}\right\}$		
		1/b = 1.0	1/b = 0.7	1/b = 0.5
46.5	46.5	0.07	0.17	0.18
51.5	51.5	0.04	0.21	0.25
56.0	56.0	0.14	0.22	0.28
63.5	63.5	0.18	0.22	0.22
51.5	46.5	0.11	0.25	0.26
56.0	46.5	0.12	0.26	0.28
60.0	46.5	0.21	-	-
63.5	46.5	0.24	0.29	0.29
56.0	51.5	0.19	-	-
63.5	51.5	0.21	-	-

4. CONCLUSIONS

a) The discharge from a combined aquifer system is less than the sum of the individual discharges from the aquifers for corresponding heads.

b) When water levels for both the aquifers are equal the effect of interference increases with increase in driving head and decrease in penetration ratio.

c) When water levels in both the aquifers are different, the interference increases with increase in differential head.

d) The value of the intereference coefficient of discharge is significant and is found to range from 10 to 25 percent.

In the theoretical analysis an expression for discharge may be obtained in terms of drawdown and initial heads in the two aquifers.

5. ACKNOWLEDGEMENT

The authors are grateful to Dr. V. Sethuraman, Professor of Hydraulic Engineering, I.I.T. Madras, for his encouragement in this study.

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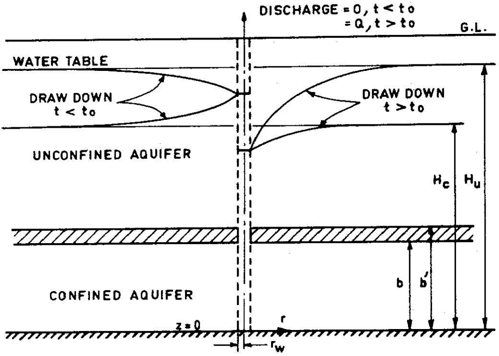

FIG. 1 - WELL PENETRATING A TWO-AQUIFER SYSTEM.

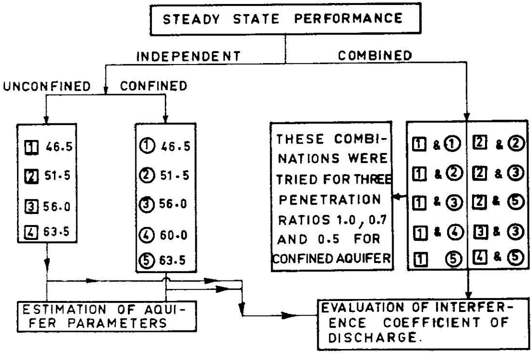

FIG. 3 - DETAILS OF DRIVING HEADS (cm) IN THE SAND MODEL.

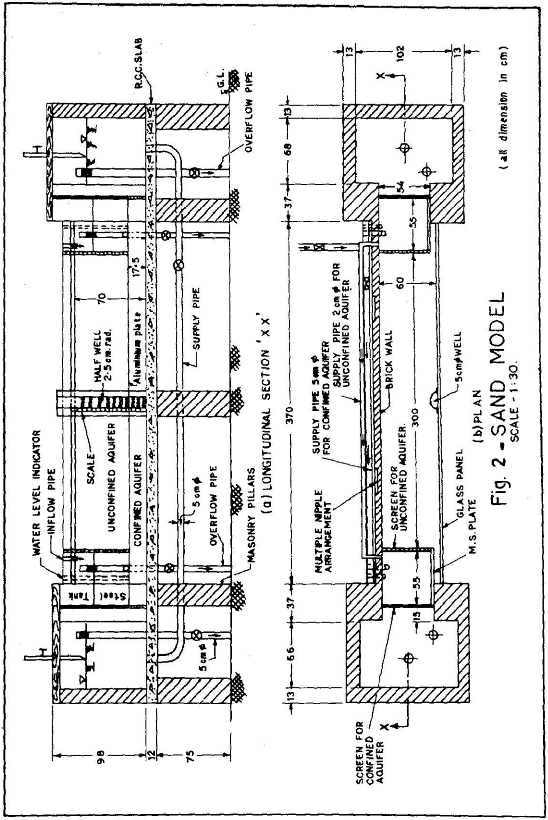

Fig. 2 - SAND MODEL
SCALE - 1:30.

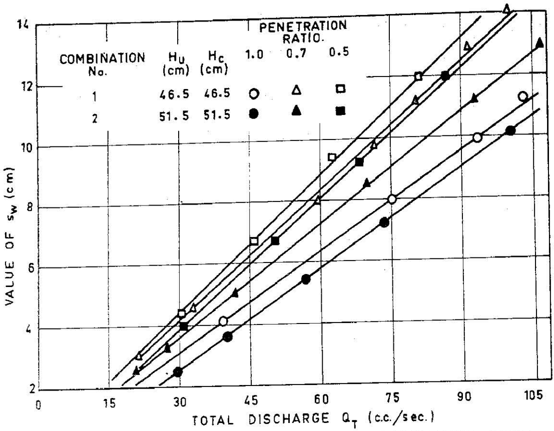

FIG. 4 - A TYPICAL VARIATION OF TOTAL DISCHARGE WITH DRAWDOWN - EQUAL DRIVING HEADS.

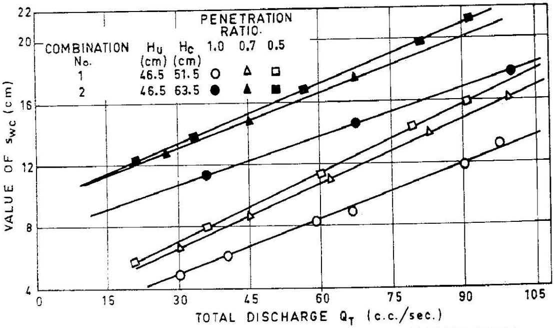

FIG. 5 - A TYPICAL VARIATION OF TOTAL DISCHARGE WITH DRAWDOWN - UNEQUAL DRIVING HEADS.

SEEPAGE IN WELL SYSTEMS NEAR A BARRIER BOUNDARY

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SYNOPSIS

Based on the principle of superposition and the method of images, a mathematical technique is developed to obtain the transient drawdown in the vicinity of well groups located near a barrier boundary. A straight line array of wells penetrating a phreatic acquifer is considered. Each well in the array is assumed to discharge at the same rate. Solutions are obtained by using a digital computer to evaluate the exponential integral. In order to generalise the results, concepts of drawdown ratio and coefficient of interference are introduced. A parametric study is undertaken to show the effect of different variables which include number of wells, well spacing, barrier distance and pumping time. It is shown that these analytical procedures provide useful tools for the engineer who is charged with the responsibility of designing and analysing well fields for ground water extraction.

1. INTRODUCTION

Existing methods for computing the drawdown pattern in the vicinity of well groups do not generally consider the transient nature of flow of ground water. If the pumping time is relatively short or the well spacing is very large, the drawdown in an individual well is unaffected by surrounding wells; however, this seldom occurs in practice, and well interference is commonly encountered. Further, majority of available methods assume seepage from a circular source with a boundary of constant potential. Frequently wells are located near faults, buried rock valleys or dikes that cut the aquifer, thus preventing ground water flow across such barriers.

The present study develops a mathematical technique keeping the above aspects in view. A procedure is obtained to compute the time - dependent drawdown in systems of multiple wells which are located along a straight line, near a barrier boundary.

2. BASIS OF SOLUTION

The principle of superposition is employed to calculate the effect of neighbouring wells in a given array. Although the governing differential equation for flow towards a fully penetrating well in a homogeneous, unconfined, infinite aquifer is nonlinear, it can be replaced by the linearised form,

$$k\bar{h}\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r}\right) = S\frac{\partial U}{\partial t} \quad \cdots (1)$$

in which, U is the drawdown function after time t at a radial distance r in an aquifer of average thickness $\bar{h}$. The hydraulic conductivity and specific yield are k and S, respectively. The drawdown function may be written as

$$U = \frac{1}{2}(H_o^2 - h^2) \quad \cdots \quad (2)$$

where, H_o is the height of initial water level above impermeable base, and h is the height of water level after time t. It must be noted that the linearised form of Eq. 1 assumes a constant $\bar{h}$ in the solution domain. An approximate value for $\bar{h}$ is normally not known a priori, but a value of $0.7\,H_o$ can be used as a first approximation (1).

Since Eq. 1 is linear in the dependent variable, U, a linear combination of its solutions is also a solution and the total drawdown at any point in the zone of influence is equal to the summation of drawdowns caused at that point by each individual well.

The method of images is used to transform a finite aquifer to one of infinite extent. This is illustrated in Fig. 1(a), wherein imaginary discharging wells are placed across the impermeable boundary. A ground water divide would exist at the boundary and the condition of no flow across the boundary is thus fulfilled. The resultant cones of depression are shown in Figs. 1(b) and (c). The drawdown at any given point in the finite aquifer is affected by real as well as imaginary well arrays.

3. THEORETICAL DEVELOPMENT

As shown in Fig. 1(a), a linear array of (2n + 1) wells is considered. Each well of radius r_w and spacing l discharges at rate Q. The distance of the array from the barrier is d. The imaginary array consists of wells which are symmetrically placed with respect to wells in the real array. The origin of the compound system is taken at the central well of the imaginary array and the coordinate axes X and Y are directed perpendicular and parallel to the line of wells. Based on the principle of superposition, the drawdown function at any arbitrary point after time t may be expressed as (2)

$$U = \frac{Q}{4\pi k}\left[\sum_{j=-n}^{n} -E_i\left(-\frac{r_{ij}^2}{4at}\right) + \sum_{j=-n}^{n} -E_i\left(-\frac{r_{rj}^2}{4at}\right)\right] \quad \ldots(3)$$

In the above equation, $a = \bar{k}h/S$ and E_i is the well-known exponential integral function (3), given by :

$$-E_i(-u) = \int_u^{\infty} \frac{\exp(-u)}{u}\, du \quad \ldots\ldots\ (4)$$

The radial distance r_{ij} in Eq. 3 is the distance between an arbitrary point (x,y) and the centre of jth imaginary well; r_{rj} is the distance between the point and jth real well. Based on the geometry of arrays, as shown in Fig. 1(a), the following equations may be written:

$$r_{ij}^2 = x^2 + (y - jl)^2 \quad \ldots\ldots\ (5)$$

$$r_{rj}^2 = (x - 2d)^2 + (y - jl)^2 \quad \ldots\ldots\ (6)$$

In this well system the largest drawdown occurs in the central well and the smallest in the outermost well; hence all subsequent discussion will pertain to either of these two wells, which constitute the limiting cases.

At the face of the central real well,

$x = 2d + r_w$ and $y = 0$. Therefore, for the effect of image wells, Eq.3 gives

$$U_i = \frac{Q}{4\pi k}\sum_{j=-n}^{n}\left[-E_i\left\{-\frac{(2d+r_w)^2}{4at}\right\} - E_i\left\{-\frac{(jl)^2}{4at}\right\}\right] \quad \ldots(7)$$

Substituting the relationships $\alpha = r_w^2/(4at)$, $\alpha_1 = d^2/(at)$, $\alpha_2 = d\,r_w/(at)$, $\beta = l^2/(4at)$,

Eq. 7 takes the form,

$$U_i = \frac{Q}{4\pi k}\sum_{j=-n}^{n}\left[-E_i(-\alpha) - E_i(-\alpha_1) - E_i(-\alpha_2) - E_i(-\beta j^2)\right] \quad \ldots(8)$$

The exponential integral may be expanded in a convergent infinite series as

$$-E_i(-u) = -0.5772 - \ln u - \sum_{n=1}^{\infty}\frac{(-1)^n u^n}{n \cdot n!} \quad \ldots(9)$$

and may be approximated (1) by

$$-E_1(-u) = \ln(\frac{0.562}{u}) \qquad \cdots\cdots\cdots (10)$$

for values of u less than 0.01; ln is the natural logarithm. With this approximation Eq.8 may be written as

$$U_i = \frac{Q}{4\pi k} \left[\ln \frac{0.562}{\alpha} + \ln \frac{0.562}{\alpha_1} + \ln \frac{0.562}{\alpha_2} + 2F(\beta, n)\right] \quad \cdots (11)$$

where,

$$F(\beta, n) = \sum_{j=1}^{n} -E_i(-\beta j^2) \quad \cdots (12)$$

Similarly, the effect of real array is given by

$$U_r = \frac{Q}{4\pi k} \sum_{j=-n}^{n} \left[-E_i\left(-\frac{r_w^2}{4at}\right) - E_i\left(-\frac{j^2 l^2}{4at}\right)\right] \quad \cdots (13)$$

which may be written as

$$U_r = \frac{Q}{4\pi k}\left[\ln \frac{0.562}{\alpha} + 2F(\beta, n)\right] \quad \cdots (14)$$

Summing up Eqs. 11 and 14, the drawdown function for the central well is given by

$$U_c = \frac{Q}{4\pi k}\left[2\ln \frac{0.562}{\alpha} + \ln \frac{0.562}{\alpha_1} + \ln \frac{0.562}{\alpha_2} + 4F(\beta, n)\right] \quad \cdots (15)$$

Proceeding in the same manner, the drawdown function for the exterior well E in Fig. 1(a) is written as

$$U_E = \frac{Q}{4\pi k}\left[2\ln \frac{0.562}{\alpha} + \ln \frac{0.562}{\alpha_1} + \ln \frac{0.562}{\alpha_2} + 2F(\beta, 2n)\right] \quad \cdots (16)$$

where

$$F(\beta, 2n) = \sum_{j=1}^{2n} -E_i(-\beta j^2) \quad \cdots (17)$$

To study the effects of well interaction, a coefficient of interference (2) defined as the ratio of Q_s (the discharge of a single well without interference at time t with a given drawdown function) and Q_I (the discharge of a well in the array at the same time and with the same drawdown function) will be employed. For a single well the drawdown function is given by

$$U_s = \frac{Q_s}{4\pi k}\left[-E_i\left(-\frac{r_w^2}{4at}\right) - E_i\left\{-\frac{(2d+r_w)^2}{4at}\right\}\right] \quad \ldots (18)$$

Approximating the exponential integral function as before,

$$Q_s = \frac{4\pi k U}{2\ln\frac{0.562}{\alpha} + \ln\frac{0.562}{\alpha_1} + \ln\frac{0.562}{\alpha_2}} \quad \ldots (19)$$

From Eq. 15,

$$Q_I = \frac{4\pi k U}{2\ln\frac{0.562}{\alpha} + \ln\frac{0.562}{\alpha_1} + \ln\frac{0.562}{\alpha_2} + 4F(\beta, n)} \quad \ldots (20)$$

Thus, η_c, the coefficient of interference for the central well may be written as

$$\eta_c = \frac{Q_s}{Q_I} = 1 + \frac{4F(\beta, n)}{2\ln\frac{0.562}{\alpha} + \ln\frac{0.562}{\alpha_1} + \ln\frac{0.562}{\alpha_2}} \quad \ldots (21)$$

An expression for the coefficient of interference of the outermost well may be derived in a similar manner.

4. PARAMETRIC STUDY

In order to evaluate the response of various well systems, a parametric study is undertaken in terms of variables such as well spacing, pumping time, barrier distance and number of wells in the line array. All the computations are carried out for wells of 75 mm radius, fully penetrating a 30 m - thick phreatic aquifer with hydraulic conductivity of 5 m per day, specific yield being 10 per cent. Each well is assumed to pump at a rate of 100 cubic metres per day . The aquifer characteristics reasonably represent the average values in poorly permeable formations. Different well systems studied herein consist of 5, 7, 9, 11 and 13 wells, uniformly spaced at 20, 30, 40, 50 and 60 m, at a distance of 30, 60, 90, 120 and 150 m from the barrier. A dimensionless parameter, designated as drawdown ratio, and defined as the ratio of drawdown to the saturated thickness of aquifer, is chosen to exhibit the variation in drawdown. The drawdown ratio and coefficient of interference are computed for pumping times of 0.1, 1, 10 and 100 days.

The exponential integral function involved in the computations was adapted for a digital computer solution by a combination of calculation and control tests. The infinite series form, as expressed in Eq.9, was truncated after reaching an error factor of 10^{-7}. After writing and debugging the computer programme, the drawdown calculations for 500 cases needed approximately two hours of compilation and execution time on an IBM 1620 computer.

5. RESULTS

The effect of well spacing and number of wells on drawdown ratio is shown in Fig.2. For a given spacing, the drawdown increases with an increase in the number of wells and this effect is more pronounced at smaller spacings. Beyond a certain well spacing, the drawdown is essentially independent of number of wells. Fig. 3 relates the drawdown ratio to pumping time for various well spacings. At small values of time, drawdown may be considered to be independent of spacing, since the respective cones of depression of individual wells in the array do not interfere with one another. Continued pumping, however, leads to a rapid increase in drawdown.

The influence of barrier distance on drawdown ratio is shown in Fig. 4 in terms of well spacing. From a knowledge of aquifer characteristics such as hydraulic conductivity, specific yield, aquifer thickness, and by using Fig. 4, the minimum depth of water level to be maintained in a given area can be estimated for an assumed spacing of wells. Fig. 3 helps in determining the transient response of the system.

The co-efficient of interference for the central well is plotted in Figs. 5 and 6 as a function of various parameters considered in this study. Since the co-efficient of interference is a measure of discharge in a single well without interference, compared to that of a well in a group, it approaches unity, as the well spacing increases; that is, the wells do not interfere from a practical point of view. Based on other calculations not presented here, it was found that the coefficient of interference for the central well is higher than that for the outermost well.

6. SUMMARY

A methodology has been developed to compute the time-dependent drawdown in well groups located near a barrier boundary and penetrating a phreatic aquifer. The mathematical technique is very broad and can be readily extended to other types of aquifers and boundary conditions. Choosing representative geohydrological constants, functional curves are obtained to interrelate different combinations of variables. The graphical relationships presented here provide convenient tools to avoid subjective estimates in determining the spacing, number of wells, period of pumping and drawdown in multiple well arrays. It is concluded that these procedures are useful in analysing or designing well fields for extraction of ground water.

7. ACKNOWLEDGEMENTS

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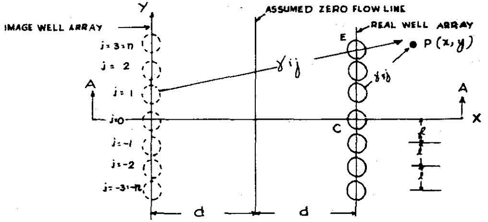
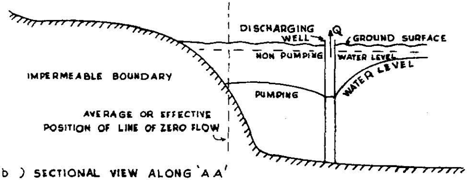
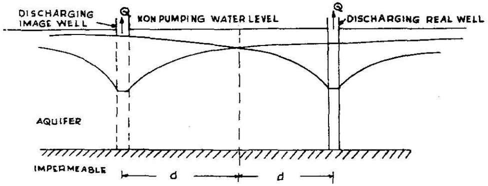


(a) LINEAR SYSTEM ALONG A BARRIER BOUNDARY - PLAN VIEW.



(b) SECTIONAL VIEW ALONG 'AA'



(c) EQUIVALENT HYDRAULIC SYSTEM IN AN AQUIFER OF INFINITE EXTENT

FIG. 1. DEFINITION SKETCH, STRAIGHT LINE ARRAY OF WELLS NEAR AN IMPERMEABLE BOUNDARY.

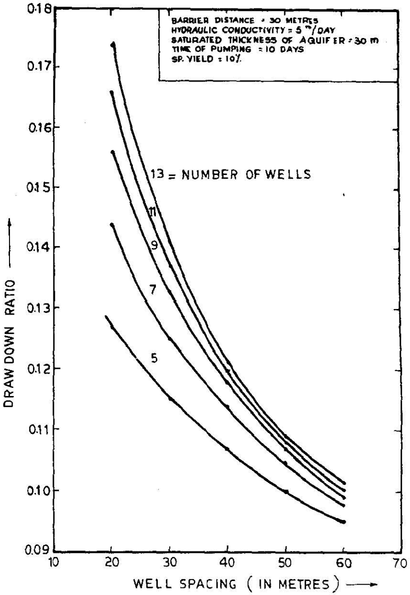


FIG. 2. DRAWDOWN RATIO AS A FUNCTION OF WELL SPACING AND NUMBER OF WELLS

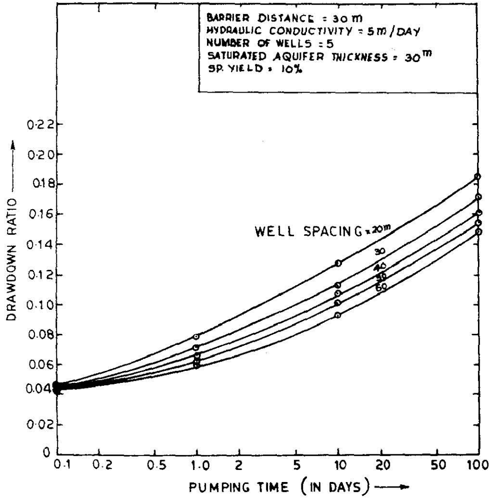


FIG. 3. DRAWDOWN RATIO AS FUNCTION OF PUMPING TIME AND WELL SPACING

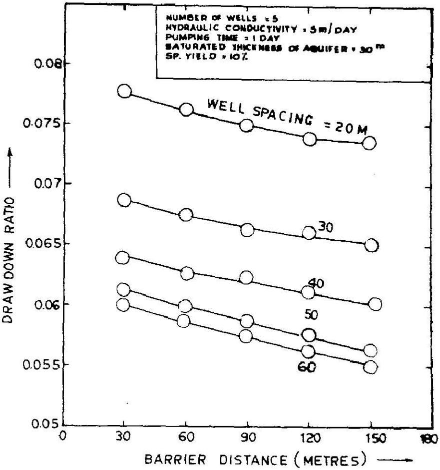


FIG. 4. DRAWDOWN RATIO AS A FUNCTION OF BARRIER DISTANCE AND WELL SPACING

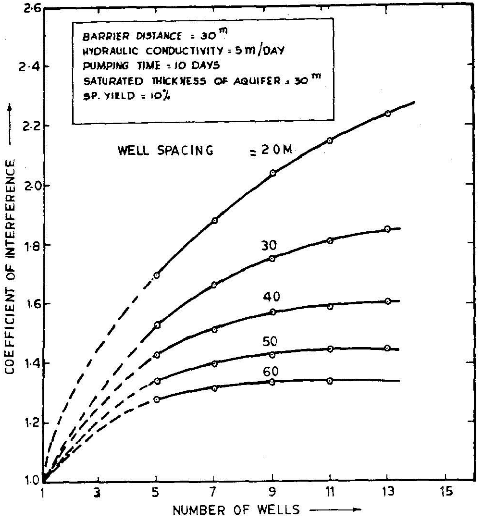


FIG. 5. COEFFICIENT OF INTERFERENCE AS A FUNCTION OF NUMBER OF WELLS AND WELL SPACING

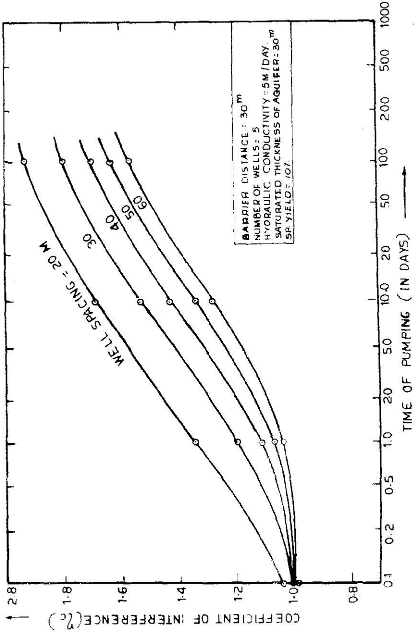


FIG. 6. COEFFICIENT OF INTERFERENCE AS A FUNCTION OF TIME OF PUMPING AND WELL SPACING

AVERAGE HEAD APPROACH TO PARTIALLY PENETRATING WELL GROUPS WITH APPLICATION TO GROUND WATER PROBLEMS

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SYNOPSIS

The paper presents a brief review of the methodologies in use for the analysis of partially penetrating well groups. The concept of average head introduced by the author for the solution of partially penetrating multiple-well systems is explained. Simplified average head expressions are furnished for steady and unsteady flow conditions in anisotropic and isotropic aquifers overlain by leaky and non-leaky confining layers. Approximate average head expression for partially penetrating gravity well under steady state condition is also suggested.

Hantush's examples of fully penetrating well groups in non-leaky isotropic aquifer under unsteady state conditions are extended for the partially penetrating well groups in leaky anisotropic aquifer under unsteady state conditions.

Babbit and Caldwell's examples of fully penetrating gravity well groups within circular recharge boundary are extended for partially penetrating cases. The author's analytical results for partially penetrating well groups in artesian aquifer are extended for use in the case of partially penetrating gravity well groups. Charny's 'equivalent fully penetrating well approach' for artesian aquifer is extended to the water table aquifer.

The utility of the derived results for the hydraulic design of well-point systems is illustrated.

1. INTRODUCTION

It is well known that partially penetrating wells are employed in several ground water problems. The problems of partially penetrating well groups in artesian aquifer have been studied by Segal(1), Charny(2), Middle Brooks and Jervis(3), Mansur and Kaufman(4) and Sharma(5,6). The available literature on partially penetrating gravity well groups is practically nil within the knowledge of the writer but for the lone exception of the paper by Chapman(7) in which an attempt has been made to initiate investigation in this line by equivalent ditch approach. A critical brief review of the methodologies used by the above workers for the analysis of partially penetrating artesian well groups is given below.

Segal applied potential theory approach to obtain rigorous solutions for two specific problems with infinite sets of equally spaced partially penetrating wells in a semi-infinite strip, in which the boundary value problem reduces to a single well problem in a parallelepiped. His general result is too much involved and converges conditionally as noted by Segal. For finite numbers of wells, for example even for a two well system, Segal has not given any solution.

Charny's method of replacing each partially penetrating well by an equivalent fully penetrating fictitious well, may be called ' equivalent fully penetrating well approach'. The method is useful only for large well spacings. Further, this method enables determination of discharge only.

Middle Brooks and Jervis employed experimental method of solving a problem by electrolytic tank model. But his results should be considered of restricted applicability due to inherent limitations of experimental techniques and electrical analog methods.

Mansur and Kaufman applied 'equivalent ditch approach' for the solution of the problem analysed by Middle Brooks and Jervis. Although they improved the application of the 'equivalent ditch approach' by accounting for the effect of point abstractions due to wells, their final results were presented in terms of empirical constants whose values were to be obtained from results of electrical analog experiments. The 'equivalent ditch approach' serves as an useful tool but Mansur and Kaufman's work could not take full advantage of the method due to the presence of empirical constants in their final results.

The author has employed 'average head approach' to analyse problems related to partially penetrating well groups in artesian aquifer. The concept of average head briefly stated transforms the problems which by nature are three-dimensional into virtually two-dimensional problems, by eliminating the vertical co-ordinate z, from the analysis. It should be noted that this method while rendering the analytical solution simplified, does not sacrifice the desired degree of accuracy. Once the problem is reduced to a two-dimensional one this permits the application of the principle of superposition for analysing the interference effects and the method of images to satisfy boundary conditions.

For the water table aquifer, none of the analytical methods employed for the solution of the problems of partially penetrating artesian well groups have been extended as yet. Chapman studied the probem of flow to a partially penetrating ditch under gravity-flow conditions by electrical analog model. He gave expressions in terms of empirical constants to be evaluated by his electrical analog results. Chapman's experimental method was that of 'equivalent ditch approach' in its original form in which correction for point abstraction was not introduced. Hence his expression for the head should be considered applicable to points located quite far off from the ditch centre line.

The present study aims at the objectives stated in the synopsis. The validity of this average head approach for partially penetrating artesian well-groups has already been verified (5,6). For partially penetrating gravity well-groups, the method yields results in conformity with equivalent ditch approach(13).

2. THE AVERAGE HEAD CONCEPT AND EXPRESSIONS

2.1 The Average Head Concept

It is now an established fact that though the head distribution around the fully penetrating gravity well is three-dimensional, Dupuit Forcheimer's assumption of uniform head distribution along the vertical direction yields exact expressions for the discharge(8,10). Muskat(11,12) has shown by rigorous analysis that in the case of fully penetrating artesian well, the total flow through the aquifer remains the same if the nonuniform head distribution over the well surface and external boundary are replaced by uniform head distribution of average head value obtained by averaging the non-uniform head in azimuth. He further used average head along the vertical direction for discharge computation to a fully penetrating gravity well, a fully penetrating artesian-gravity well, and a partially penetrating gravity well.

Solution of ground water problems by considering the average head (or average piezometric pressure) over the internal and external boundaries and by considering variation of such average head in radial direction only, may be called as 'average head approach'. In this method, the actual head distribution in vertical direction is not needed, rather it is sufficient to know the average head in the vertical direction over the boundaries.

In the case of a partially penetrating well too, the head distribution around a well is three-dimensional (axi-symmetric), but the water levels in a pumping well and in an observation well reflect the average heads in the aquifer profile that is occupied by the screened portion. Hence the 'average head approach' will be applied for the solutions of problems related to partially penetrating wells.

2.2 Average Head Expressions

Under steady state flow in a homogeneous isotropic and non-leaky aquifer, the expressions for average head in a partially penetrating artesian well and an observation well having the same penetration as the pumping well are given by following expressions(5,6). Unless otherwise specified well and observation well are assumed to be screened fully between upper confining bed and bottom of the well.

For small values of radial distance r, of the order of the radius of the well r_w,

$$h(r) = -\frac{q}{b}\left[\ln\frac{4B}{r} - f(\bar{b})\right] + C' \qquad ..(1)$$

and for large values of r, not too close to the well centre,

$$h(r) = -q\left[\ln\frac{4B}{r} + S(\bar{b}, \bar{r})\right] + C' \qquad ..(2)$$

where

h(r) = average head over the depth of the penetration, b, of the well (pumping well or observation well),

$q = \frac{Q}{2\pi kB}$, C' - being an arbitrary constant to take care of datum plane, Q represents the flux associated with the well, B = thickness of the artesian aquifer, b = depth of penetration, $\bar{b} = b/B$, the penetration ratio, and $\bar{r} = r/B$, where r represents the radial distance from the well in polar co-ordinate notation, k represents conductivity of the aquifer,

$$f(\bar{b}) = \frac{1}{2} \ln \frac{\Gamma(0.875\,\bar{b})\,\Gamma(0.125\,\bar{b})}{\Gamma(1-0.875\,\bar{b})\,\Gamma(1-0.125\,\bar{b})}, \quad ..(3)$$

a corrective function near the well surface,

$$\text{and } S(\bar{b},\bar{r}) = \frac{2}{\pi^2 \bar{b}^2} \sum_{n=1}^{\infty} \frac{1}{n^2} K_o\,(n\pi\bar{r})\,\sin^2(n\pi\bar{b}), \quad ...(4)$$

a corrective function at any radial distance, r. When the observation well just taps the artesian aquifer, the corrective function $S(\bar{b}, \bar{r})$ simplifies to $S'(\bar{b}, \bar{r})$ where

$$S'(b, r) = \frac{2}{\pi b} \sum_{n=1}^{\infty} \frac{1}{n} K_o(n\pi\bar{r})\,\sin(n\pi\bar{b}) \quad ...(5)$$

Γ denotes Gamma function, K_o represents the zero order modified Bessel function of the second kind, n is a positive integer varying from 1 to ∞.

For unsteady flow to a partially penetrating well in a leaky, anisotropic artesian aquifer, the following expression(8,13) is used both for small and large values of radial distances.

$$h(r) = C' - \frac{q}{2}\left[W(U_r, r/B_r) + f\,(U_r, \bar{b}, \bar{r}\,\bar{k})\right] \quad ...(6)$$

$$\text{where } f(U_r, \bar{b}, \bar{r}\,\bar{k}) = \frac{2}{\pi^2 \bar{b}^2} \sum_{n=1}^{\infty} \frac{1}{n^2} W(U_r, n\pi\bar{r}\,\bar{k})\,\sin^2(n\pi\bar{b}) \quad ...(7)$$

$W(U, x)$ = well function for leaky aquifer.

$U_r = r^2/4\nu_r t$, $\nu_r = k_r B/S$, $B_r^2 = k_r B/(k'/B')$, S= storage coefficient of main aquifer.

$\bar{k} = \sqrt{(k_z/k_r)}$; k', B' being conductivity and thickness of semipervious confining layers, k_r and k_z represent conductivities in radial and z direction respectively of main aquifer, and t is the time variable. For isotropic non-leaky aquifer, average head expression for partially penetrating pumping well and observation well screened from the base of the artesian aquifer upto a height 'b' above the base, may be written as (13,14)

$$h(r) = C' - \frac{q}{2} W(U_r, \bar{b}, \bar{r}) \quad ...(8)$$

where $W(U_r, \bar{b}, \bar{r})$ represents well function for non-leaky isotropic artesian aquifer partially penetrated by wells. For partially penetrating gravity well in an isotropic aquifer under steady state condition following general expression(13) for average head may be used.

$$h(r) = C' + q\left[\ln \bar{r} - S\,(\bar{b}\,,\,\bar{r}\,)\right] \quad ...(9)$$

where $q = Q/2\pi kH$, $\bar{b} = t_w/H$, $\bar{r} = r/H$, t_w = depth of water level in the well above well bottom. H = saturated thickness of water table aquifer.

The value of function $S(\bar{b}, \bar{r})$ for $r = r_w$ in Eq.(9) should be evaluated by Eq.(4a) given below instead of using Eq.(4), which will be used for values of r, not too close to the well centre.

$$S(\bar{b}, \bar{r}_w) = S(\bar{b}, \bar{R}) + \frac{(1-\bar{b})}{\bar{b}} \; \frac{\ln E\, b}{r_w} \qquad \ldots(4a)$$

where E is a function of penetrating ratio $\bar{b}$ and eccentricity of well screen. The values of E are given by Huisman(15).

$\bar{R} = R/H$, where R represents radius of influence(where the drawdown is assumed to be zero).

3. ANALYTICAL SOLUTIONS

3.1 Starting with Eq.(6) and using the principle of superposition, the following general expression for the drawdown, S_{wj}, inside jth well in a group of 'n' partially penetrating artesian wells may be readily written.

$$S_{wj} = H - h_{wj} = \frac{q_j}{2}\left[W_{rw} + f_{rw}\right] + \sum_{i=1}^{n}{}' \frac{q_i}{2}\left[W_{r_{ij}} + f_{r_{ij}}\right] \qquad \ldots(10)$$

where ' prime indicates omission of the term i = j in the summation, h_{wj} represents average head inside the jth well, 'H' representing static piezometric head may be considered as average head at infinitely large radial distance, r_w represents well radius, r_{ij} represents distance between ith and jth well. W_x and f_x denote values of function $W(U_r, r/B_r)$ and $f(U_r, \bar{b}, \bar{r}\,\bar{k})$ for r = x respectively. For a 'n' well-system, n equations may be written by setting j = 1 to n in Eq.(10) and simultaneous solution of 'n' such equations gives discharge in each well. Hantush's(8) examples of fully penetrating well groups in non-leaky isotropic artesian aquifer under unsteady state condition are extended for the partially penetrating well groups in leaky anisotropic aquifer under unsteady state condition.

For two wells 'd_2' apart, from symmetry, $Q_1 = Q_2$ and only one equation need be written. Setting $h_{w1} = h_{w2} = h_w$, one readily obtains

$$Q_1 = Q_2 = \frac{4\pi kB(H - h_w)}{\left[W_{rw} + W_{d2} + f_{rw} + f_{d2}\right]} \qquad \ldots(11)$$

where $k = k_r$.

For three wells forming an equilateral triangle of side d_3

$$Q_1 = Q_2 = Q_3 = \frac{4\pi k\, B\,(H - h_w)}{\left[W_{r_w} + 2W_{d_3} + f_{r_w} + 2f_{d_3}\right]} \qquad \ldots(12)$$

Discharge of each of four wells forming a square of side d, will be given by

$$Q_1 = Q_2 = Q_3 = Q_4 = \frac{4\pi kB\,(H - h_w)}{\left[W_{r_w} + 2W_d + W_{d\sqrt{2}} + f_{r_w} + 2f_d + f_{d\sqrt{2}}\right]} \qquad \ldots(13)$$

For a line of three equally spaced wells distance 'd' apart

$$Q_1 = Q_3 = \frac{4\pi kB\,(H - h_w)\left[W_{r_w} - W_d + f_{r_w} - f_d\right]}{\left[(W_{r_w} + W_{2d} - 2W_d + f_{r_w} + f_{2d} - 2f_d)(W_{r_w} + f_{r_w}) + 2(W_d + f_d) \times (W_{r_w} - W_d + f_{r_w} - f_d)\right]} \qquad \ldots(14)$$

and

$$Q_2 = \frac{4\pi kB\,(H - h_w)\,(W_{r_w} - W_{2d} - 2W_d + f_{r_w} + f_{2d} - 2f_d)}{\left[(W_{r_w}+W_{2d} - 2W_d + f_{r_w} + f_{2d} - 2f_d)(W_{r_w}+f_{r_w}) + 2(W_d+f_d)(W_{r_w} - W_d + f_{r_w} - f_d)\right]} \quad ..(15)$$

when the aquifer is non-leaky, isotropic and homogeneous, corresponding results may be obtained from Eqs.(11) to (15) by replacing well function term $W(U_r, r/B_r)$ by $W(U_r, \bar{r}, \bar{b})$ and setting 'f' function term to zero and writing W_x as value of the function $W(U_r, \bar{r}, \bar{b})$ for $r = x$.

3.2 Staring with Eq.(9) in conjunction with Eq.(4a) and using the principle of superposition, the following general expression for the drawdown in case of partially penetrating gravity well groups within circular recharge boundary may be written.

$$S_{wj} = H - h_{wj} = q_j\, G^* + \sum_{i=1}^{n}{}' q_i \left[\ln\frac{R}{r_{ij}} + S(\bar{b}, \bar{r}_{ij}) - S(\bar{b}, \bar{R})\right] \quad ...(16)$$

where G^*, denoting penetration factor for a gravity well is related to discharge of a single gravity well by the relation

$$Q = \pi k\left[H^2 - h_w^2\right]/\,G^* \quad ...(17)$$

and may be expressed by any of the following expressions

$$G^*_D = \ln\,(R/r_w) + \left(\frac{1-\bar{b}}{\bar{b}}\right)\ln\,(Eb/r_w) \quad ...(18a)$$

$$G^*_S = \ln\,(R/r_w)\,/\left[\frac{H - h_w}{H + h_w} + \frac{2\,h_w}{1 + h_w}\left(\frac{\ln R/r_w}{G}\right)\right] \quad ...(18b)$$

where G, represents penetration factor(5) for the artesian well having aquifer thickness h_w and penetration ratio $\bar{b} = t_w/h_w$. For Eq.(18a), penetration ratio $\bar{b} = t_w/H$, as defined earlier is to be used. G^*_D and G^*_S refer to values of penetration factor G^* after deGlee(16) and the author(13). The impervious base forms the datum for H and h_w.

For a two-well system, d_2 apart, setting $h_{w1} = h_{w2} = h_w$ one readily obtains from (16)

$$Q_1 = Q_2 = \pi k\left[H^2 - h_w^2\right]/\left[G^* + \ln\frac{R}{d_2} + S(\bar{b}, \frac{d_2}{B}) - S(\bar{b}, \bar{R})\right] \quad ...(19)$$

The author's expression(13) for discharge in case of partially penetrating two well system in artesian aquifer is given below.

$$Q_1 = Q_2 = \frac{2\pi k\,B\,\left[H - h_w\right]}{\left[G + \ln\frac{R}{d_2} + S(\bar{b}, \frac{d_2}{B}) - S(\bar{b}, \frac{R}{B})\right]} \quad ...(20)$$

A comparison of Eq.(17) with Eq.(20) suggests that discharge in the case of partially penetrating gravity well groups may be obtained by corresponding formulae of partially penetrating artesian well-groups with the replacement of parameters $[2B\,(H - h_w)]$, B, and G of the case of artesian problems with $[H^2 - h_w^2]$, H, and G^* respectively in terms of water table parameters.

This procedure permits the use of all analytical results obtained by the author (5, 6) for partially penetrating well groups in artesian aquifer for various well arrangements (not covered by Babbit and Caldwell) to corresponding problems in water table aquifer.

3.3 With the radius of fictitious well equal to twice saturated thickness, 2H, Charny's (2) 'equivalent fully penetrating well approach' is extended for water table aquifer(13). The final results are given below.

The head, h_o, on the surface of fictitious well is given by

$$h_o^2 = \frac{[H^2 + \bar{G}^* h_w^2]}{[1 + \bar{G}^*]} \qquad ..(21)$$

and discharge to a partially penetrating well in a group is given by

$$Q = \frac{\bar{G}^*}{1 + \bar{G}^*} (Q_H) = \left(\frac{\bar{G}^*}{1 + \bar{G}^*}\right) \frac{\pi k(H^2 - h_w^2)}{G_f} \qquad ..(22)$$

where $\bar{G}^* = \frac{G_f}{G^*}$; for a given well arrangement G_f is related to the discharge of a fully penetrating well of radius 2H, having water head h_o, inside the well by the relation

$$Q = \frac{\pi k\,(H^2 - h_o^2)}{G_f} \qquad ..(23)$$

The values of G_f for different well arrangements presented by Charny(2) may be used for the gravity cases by a simple replacement of aquifer thickness B in case of artesian well by saturated thickness H in the case of water table well.

'Q_H' denotes discharge of a fully penetrating well of radius 2H, but with the water surface head, h_w.

4. APPLICATIONS

The equations developed in the present study could be applicable for many field problems of drainage or water supply. Their applications for several cases like analytical determination of empirical constants in equivalent ditch approach, design of relief-well systems, effect of partial penetration on shielding, have already been reported(6). Their usefulness in fixing the spacing of partially penetrating wells or well-point systems are illustrated below.

Due to short and limited length of perforated screen provided in a well-point or screened length of less than saturated thickness in case of ordinary tube well, the flow to a well-point or a well correspond to flow to a partially penetrating well. Normally, the layout of wells referred to in well-point systems consists of number of wells in one or two lines array placed on either side of the area to be dewatered. In case of dewatering or drainage problems, the hydraulic design refers to fixing the well spacing and penetration depth of the well-point or well to attain the desired lowering of water table at the site.

When wells are placed in a single line array in the presence of a line source, the artesian flow equations(6) modified for gravity aquifer with the replacement of parameters suggested in this study may be written as follows:

Discharge to a well in the array is given by

$$Q = \frac{\pi k \left[H^2 - h_w^2\right]}{G^{*\prime} + \ln \frac{\sinh(\frac{2\pi d}{a})}{(\frac{2\pi d}{a})} + 2 \sum_{i=1}^{\infty}\left[S(\bar{b},\frac{ia}{H}) - S(\bar{b}, \frac{\sqrt{i^2a^2 + 4d^2}}{H})\right]} \quad ..(24)$$

where $G^{*\prime} = G^*$ with $R = 2d$.

For $a > H$, Eq.(24) simplifies to

$$Q = \frac{\pi k \left[H^2 - h_w^2\right]}{\left[G^{*\prime} + \ln \frac{a}{4\pi d} + \frac{2\pi d}{a}\right]} \quad ..(24a)$$

The maximum residual head h_D downstream of the well array is given by

$$h_D^2 - h_w'^2 = \frac{(H^2 - h_w^2)\left[G^{*\prime} + \ln\frac{a}{4\pi d} + 2\sum_{i=1}^{\infty}\left\{S(\bar{b}, ia/H) - S(\bar{b}, \frac{\sqrt{i^2a^2 + 4d^2}}{H})\right\}\right]}{G^{*\prime} + \ln\frac{a}{4\pi d} + \frac{2\pi d}{a} + 2\sum_{i=1}^{\infty}\left\{S(\bar{b}, ia/H) - S(\bar{b}, \frac{\sqrt{i^2a^2 + 4d^2}}{H})\right\}} \quad ..(25)$$

For well spacing $a < H$, the maximum value of i = minimum of $1/\bar{b}$ or H/a in summation term of Eqs.(24) and (25) are found to be sufficient.

For $a > H$, Eq.(25) simplifies to

$$h_D^2 - h_w'^2 = \frac{(H^2 - h_w^2)\left[G^{*\prime} + \ln\frac{a}{4\pi d}\right]}{\left[G^{*\prime} + \ln\frac{a}{4\pi d} + \frac{2\pi d}{a}\right]} \quad ..(25a)$$

where h_w' represents depth of water just outside the well and d represents distance of the well array from the line source.

For a well point, radius of the tube forming well-point and the length of perforation are usually of standard dimensions. Hence, r_w and t_w are known. For an assumed layout, the distance of the well array from the line source, d is fixed. The top of the well screen may be assumed at the level of desired ground water table. For an assumed well spacing, Eq.(24) or Eq.(24a) may be used to compute the discharge of the well which is required in fixing the capacity of the pumping unit. Eq.(25) or Eq.(25a) may be used to determine maximum residual head which should agree with the desired ground water table. To obtain least value of maximum residual head (i.e. maximum drawdown at the centre of gravity of the area to be dewatered), the drawdown, h_w in the well should be kept at the bottom of the well. For such condition, seepage height t_w may be taken as half of the filter length of the well-point. Thus $h_w'(=h_w+\frac{t_w}{2})$ is fixed up. The impervious base forms datum for each of H, h_w, h_w' and h_D measurements.

5. CONCLUSION

The concept of average head renders very useful mathematical simplification in obtaining analytical solution for the partially penetrating multiple-well system in artesian and water table aquifers without sacrificing the desired degree of accuracy. The existing practice of adopting same penetration factor for artesian and water table aquifer is limited in its applicability for small drawdowns only. With the use of appropriate penetration factor for gravity aquifer and replacement of parameters suggested in this study, all existing artesian flow formulae for partially penetrating artesian well-groups become applicable to partially penetrating gravity well groups without any restriction in magnitude of drawdown and depth of penetration (i.e. $0 \leq h_w < H$ and $0 < t_w \leq H$).

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STEADY SPHERICAL FLOW TO A NONPENETRATING WELL IN A LEAKY ARTESIAN AQUIFER

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SYNOPSIS

An extensively thick artesian aquifer with lower confining layer completely impermeable and an upper confining layer partly permeable capable of permitting some leakage due to head difference has been considered. In such an aquifer, hydraulics of a well with sides impermeable and spherical bottom resting on the boundary of the upper semi-confining layer and the confined aquifer for two possible methods of leakages have been analysed. Laplace equation in spherical coordinates incorporating a term for leakage has been utilized in the analysis to arrive at relationships describing the flow systems for each of the two types of leakages.

1. INTRODUCTION

Construction of wells in an aquifer under heavy artesian pressure requires specialized equipment, know-how and skill. Successful wells have been constructed by direct rotary drilling in such aquifers where strainer assembly has been lowered in the full depth of the water bearing aquifer. Whenever percussion drilling has been employed, difficulty has been experienced in successfully lowering the assembly. In construction of a well by this process, when after lowering the well assembly, the outer casing pipe is lifted, it is generally observed that the annular space between the outer casing and the well gets clogged by the onrushing aquifer material and both the outer casing and the assembly are lifted together resulting in an unsuccessful well. A practical and economic alternative commonly employed for such a case is the use of the liner itself for drilling and leaving it at the interface of the confining layer and the confined aquifer. This results in a well with impervious sides and an open bottom. The type of flow in such a well closely resembles a spherical flow system. If the upper confining layer is not completely impermeable some leakage would occur in the confined aquifer. The hydraulics of a well with impermeable casing and spherical bottom resting on the top of an unconfined layer was studied by For-chheimer(1) and is available in different texts dealing with flow through porous media such as by Muskat (2), Aravin and Numerov (3), etc. Studies on cavity wells which are other forms having similar flow systems have also been carried out by Sanghi (4) and Misra, Anjaneyulu and Lal (5) and Chauhan (6.).

The above studies refer to spherical flow systems in which the layers confining the artesian aquifer are completely impermeable. According to DeWiest (7) extensive studies on the hydrological characteristics of artesian aquifers have indicated that leakage contributed significantly to the yield of artesian aquifers overlain with semipervious layers. Pioneering studies were done on hydraulics of wells constructed in leaky aquifers by Dutch Scientists De-Glee (8) and Steegewentz (9). In U.S.A. an extensive analytical theory of leaky aquifers was developed individually and jointly by Jacob and Hantush through a series of papers too numerous to mention here. Salient aspects of the theory have been consolidated by Hantush (10). Other notable work on leaky aquifers was done by the Russian scientist Mgatiev (11). No analytical or experimental work seems to have been done for a steady spherical flow system in a leaky aquifer. The purpose of this study is to analyse theoretically the hydraulics of a well with impermeable sides and a hemispherical bottom resting at the top of an artesian aquifer with the lower confining layer being impermeable and the upper confining layer being partly permeable permitting some leakage.

2. DEFINITION OF THE PROBLEM

The flow systems to be investigated are described in Figures 1 and 2. In the first system (Figure 1) the permeable bottom of the well is assumed to be spherical in shape, of radius r_o. The aquifer in which this sink is located is assumed to be of extensive thickness. Under such conditions the equipotential surfaces will be concentric hemispheres.

Before pumping is started the head 'h' in the main aquifer is uniform and equal to H, the height of water table. After pumping for sometime a steady state is reached. The piezometric head in the main aquifer develops a cone of depression with its radius of inverted base as the radius of influence R indicated by dotted line. The ponded water in the unconfined aquifer is maintained at a height H. A difference in the head develops and water seeps from unconfined aquifer to confined aquifer. After seeping vertically through this semiconfining layer of thickness b' and conductivity K' water enters the confined aquifer and is subjected to spherical flow conditions. The flow system is assumed to be governed by Laplace equation. For the present study, it is convenient to use it in spherical coordinate system as given below:

$$K\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial h}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial h}{\partial\theta}\right) + \frac{1}{r^2 \sin\phi}\frac{\partial^2 h}{\partial\phi^2}\right] = 0 \quad (1)$$

Where, h is the piezometric head, K is the hydraulic conductivity and r, θ, Ø represent the coordinate system of the flow region with origin at O.

For the flow conditions assumed to have the isopiestic surface as concentric hemispheres, h becomes independent of θ and Ø reducing Equation 1 to:

$$K\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dh}{dr}\right) = 0 \quad (2)$$

Equation 2 has to be modified to include a term representing leakage. In a rigid theoretical analysis, this factor should represent the mass of fluid generated per unit volume per unit time within the flow region. In case of heat generation this has to be heat generated per unit volume per unit time in the spherical element under consideration. In the present case, however, all the replenishment is received at the interface of the two aquifers. The leakage velocity is given by the expression :

$$\frac{K'}{b'}(h - H)$$

The isopiestic surface without much distortion can have the maximum radius equal to thickness b. Considering the flow region to be a hemisphere of radius b and the receiving area to be a circle of radius b the necessary expression for leakage can be obtained as :

$$\frac{3K'}{2Kbb'}(h - H)$$

If however, for obtaining the leakage factor, the flow region is taken as cylindrical, this factor is given by $(K'/b\ b')\ (h - H)$ which has only a minor difference from the previous expression. Including the term for leakage in the aquifer, the flow system is assumed to be governed by the differential equation:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dh}{dr}\right) - \frac{3\,K'}{2\,k\,b\,b'}\,(h - H) = 0 \qquad (3)$$

Instead of considering leakage in this way, it is also possible to consider another flow system (Fig. 2) with replenishment P having the units of velocity occurring in an area of circle of influence because of constant head difference created in unconfined aquifer from a possible rainfall or other inflow. The effect of piezometric drawdown may not be considered separately and net effect of head difference because of sudden rise in water table may be reflected in a constant replenishment in the flow region. This inflow could be converted as net accretion per unit volume in the flow region by considering the volume of a hemisphere of radius of influence R. The accretion may be added to Equation 2 giving another differential euation:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dh}{dr}\right) + \frac{3}{2}\frac{P}{K\,R} = 0 \qquad (4)$$

Thus two differential Equations 3 and 4 are proposed to approximate the effect of leakages in the confined aquifer one assuming the difference of head because of drawdown and the other because of sudden rise of water table in the unconfined aquifer because of rainfall or other inflow. The problems thus consist of solutions of Equations 3 and 4 for the boundary conditions:

$$h(0) = h_o \qquad (5)$$

$$h(R) = H \qquad (6)$$

and the constraint

$$-K\left(\frac{dh}{dr}\right)_{r = r_o} = -\frac{Q}{2\pi r_o^2} \qquad (7)$$

3. SOLUTIONS OF THE PROBLEMS

Putting $\frac{3K'}{2Kbb'} = c^2$, and $\frac{3P}{2KR} = c^2$ in the Equations 3 and 4 respectively and applying the transformation

$$Z = r\,h$$

they can be rewritten as

$$\frac{d^2 Z}{dr^2} - c^2 Z = c^2 H r \qquad (8)$$

and

$$\frac{1}{r}\frac{d^2 Z}{dr^2} + c = 0 \qquad (9)$$

Equations 8 and 9 are ordinary nonhomogeneous equations with constant coefficients the solutions of which can be found utilizing conditions 5 and 6 as :

$$h = \left[H - \frac{r_o\,(H-h_o)}{\sinh C(R-r_o)} \quad \frac{\sinh C(R-r)}{r}\right] \quad \begin{matrix} r_o < r < R \\ h_o < h < H \end{matrix} \qquad (10)$$

$$\text{and } h = H + \frac{C\,(R^2 - r^2)}{6} + \frac{H - h_o + \frac{C\,(R^2 - r_o^2)}{6}}{\frac{1}{R} - \frac{1}{r_o}}\left(\frac{1}{r} - \frac{1}{R}\right). \qquad (11)$$

These relationships describe possible phreatic surfaces at different radial distances within the flow region considered.

Application of condition 7 to Equations 10 and 11 yield

$$Q = 2\pi K\, r_o (H-h_o)\left[1 + C\, r_o \operatorname{Cot} h\, C(R-r_o)\right] \qquad (12)$$

$$\text{and } Q = \frac{2\pi K\, r_o\,(H - h_o)}{r\,\frac{1}{r_o} - \frac{1}{R}} + \frac{\pi K C}{3}\;\frac{(R^2 - r_o^2)}{\frac{1}{r_o} - \frac{1}{R}} - \frac{2\pi K}{3} C\, r_o^3 \qquad (13)$$

Considering R to be much larger than r_o, Equations 12 and 13 can be approximately written as :

$$Q = 2\pi K\, r_o (H - h_o)\,(1 + C\, r_o \operatorname{Cot} h\, C R) \qquad (14)$$

and

$$Q = 2\pi K\, r_o (H - h_o) + \frac{\pi K C\, r_o}{3}\,(R^2 - 3\, r_o^2) \qquad (15)$$

4. DISCUSSION

Equations 12 and 13 can be separated into two parts; one representing normal discharge without leakage as Q_N and the other representing discharge contribution due to lekage as Q_L. So total discharges can be written as :

$$Q = Q_N + Q_L = Q_N + Q_N \; C \; r_o \; \mathrm{Cot\,h} \; C \; R \tag{16}$$

and $$Q = Q_N + \frac{\pi K \; C \; r_o}{3} \left(R^2 - 3 r_o^2 \right) \tag{17}$$

Percentage contribution due to leakage over normal discharge for Equation 16 may be written as :

$$\frac{Q_L}{Q_N} = C \; r_o \; \mathrm{Cot\,h} \; C \; R \tag{18}$$

Let one of the solutions, Equation 14, be applied to a set of assumed conditions within the range of values generally available in the field. Let K' = 0.2 gallons / day/square foot, K = 150 gallons / day/ square foot, b = 40 feet, b = 10 feet, H-b = 15 feet, R = 400 feet and r_o = 1 foot. For such a flow system total discharge is obtained as 14152 gallons per hour, out of which 52 gallons per hour or 0.37 percent is contributed from leakage. Similarly the other solution can also be used for finding leakage contribution.

The validity of mathematicl solution of a physical problem is dependent on the assumptions applied for its solution. Conditions of spherical flow are valid only if the aquifer has an extensive thickness. In nature such aquifers are seldom available. With small thickness of aquifer the equipotential surfaces beyond a radius equal to the thickness of aquifer tend to become coaxial cylinders. Radius of influence which is difficult to estimate and is dependent on the extensive thickness of artesian aquifer plays an important role in this relationship. Similarly, construction of a spherical bottomed artesian well is an impractical proposition. The cavity formed by removal of sand does in some form represent a hemispherical sink. The leakages have been considered according to particular expressions in the differential equations characterising the flow systems. Results obtained are dependent on the validity of these expressions. None the less the study outlines an approach of separating the contribution of leakages in the flow systems under consideration.

5. CONCLUSIONS

Steady flow systems consisting of a nonpenetrating well tapping an extensively thick leaky artesian aquifer has been considered. The flow has been assumed to be characterised by Laplace equation in spherical coordinates incorporating possible terms for leakage. The solutions describing the phreatic surfaces as a function of radial distance have been found within the flow region bound by well diameter and an assumed radius of influence. Considering the well bottom as a hemispherical sink relationships of discharge-drawdown have been developed. From these relationships it is possible to separate the effect of leakages from those of the normal flows.

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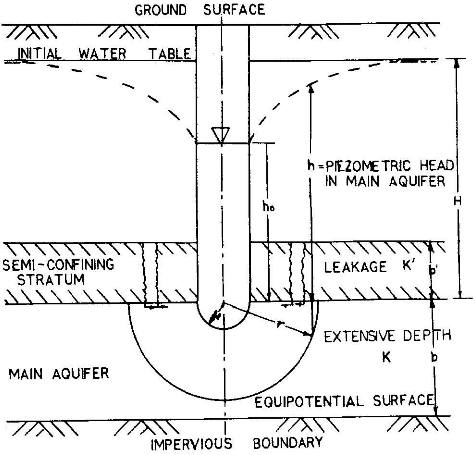

FIG. 1 LEAKAGE DUE TO CONE OF DEPRESSION.

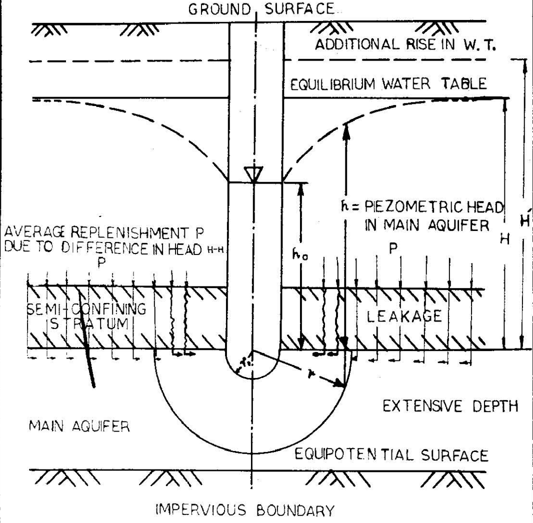

FIG. 2 LEAKAGE DUE TO SUDDEN RISE IN WATER TABLE.

DEVELOPMENT OF CAVITY TYPE TUBEWELL

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SYNOPSIS

The procedure adopted for the development of cavity in a cavity well is described. The forces acting on sand particles in the aquifer are analysed and equilibrium conditions are formulated. The relationship between the critical drawdown needed for the initiation of movement of sand particles and the aquifer parameters is established.

1. INTRODUCTION

Cavity wells are a special type of tubewells without the conventional strainers and are constructed in confined aquifers having a hard clay or rock stratum as the upper confining layer. The well boring is taken through the upper clay stratum just upto the surface of the aquifer. Some quantity of sand is taken out with a sand bailer and thus a small initial cavity is created below the well pipe. Pumping is then started and the drawdown is gradually increased till a stage is reached when sand comes out along with discharge and the initial cavity is enlarged. Mishra, et.al. (1,2,3) studied the problem of flow into a cavity well analytically and experimentally and proposed equations for predicting the size of cavity and discharge. Sanghi (4) made a theoretical analysis of the flow into a cavity well based on the assumption that the surface of cavity was a part of sphere. He also studied the performance of a number of cavity wells in the field. The drawdown required for the initiation of movement of aquifer particles has not been studied earlier. The mechanics of development of cavity are presented in this paper.

2. MOVEMENT OF SAND PARTICLES IN AQUIFER

Let the surface of the initial cavity be hemispherical with the centre at O and radius r_c (Fig. 1). When the well is pumped at a particular drawdown, a pressure gradient exists between a point E_1 at the radius of influence, r_e and another point B_1 at the upper periphery of the initial cavity. An elementary volume a b c d a′ b′ c′ d′ with average sides $rd\theta$, dr and $r.\sin\theta.d\psi$ is considered in spherical coordinates θ, ψ and r (Fig. 2). The flow through the elementary volume is considered to be purely spherical. The pressures on faces a b c d and a′ b′ c′ d′ are $P + \Delta P$ and P respectively. The fluid force on the surface a b c d is F_1 given by

$$F_1 = \left(r + \frac{dr}{2} \right) d\theta \left(r + \frac{dr}{2} \right) \sin\theta.\ d\psi \ \ (P+\Delta P)$$

and the force on the surface $a'b'c'd'$ is F_2 given by

$$F_2 = (r - \frac{dr}{2}) \sin\theta . d\psi . (r - \frac{dr}{2}) d\theta . P$$

The net force on the elementary volume a b c d $a'b'c'd'$ is F given by

$$F = F_1 - F_2$$

$$= d\theta . \sin\theta . d\psi (2r.dr.P + r^2 . \Delta P + \frac{dr^2}{4} . \Delta P + r.dr. \Delta P)$$

Neglecting the product of infinitesimal terms the net force on the elementary volume reduces to

$$F = d\theta . \sin\theta . d\psi . r^2 . \Delta P$$

$$= A . \Delta P$$

where,

$$A = r.d\theta . r.\sin\theta . d\psi$$

= average area perpendicular to the direction of flow.

If the elementary volume considered is at A_1 (Fig. 1) and the particles are at the verge of movement, the upward force, F is equal to the submerged weight of the particles. Therefore

$$\Delta P . A = (1-n) (\rho_s - \rho) g . A . dr \qquad ...(1)$$

where,

ΔP = pressure difference over length dr,
A = average area perpendicular to the direction of flow,
n = porosity of aquifer,
ρ_s = density of solids in the aquifer,
ρ = density of water, and
g = acceleration due to gravity.

If the elementary volume considered is at B_1 (Fig. 1) and the particles are on the verge of movement, the net force, F is equal to the frictional resistance developed due to the submerged weight of the particles. Therefore

$$\Delta P . A = \tan\beta . (1 - n) (\rho_s - \rho) g . A . dr \qquad ...(2)$$

where β is the angle of internal friction of the aquifer material. Since the value of β is usually less than 45°, the particles at point B_1 are made unstable with a lesser force compared to those at point A_1.

3. RELATIONSHIP BETWEEN PRESSURE DROP AND VELOCITY

In the case of pipe flow, the pressure drop ΔP over a length ΔL is given by Hagen-Poiseuille law (5) in the form

$$\Delta P = \frac{K_o .\ \mu .\ u_a .\ \Delta L}{m^2} \qquad ...(3)$$

where,

μ = absolute viscosity,

u_a = mean velocity of flow in a pipe,

m = hydraulic mean radius, and

K_o = a coefficient

The value of coefficient K_o is 2 for circular passages and if the openings are not circular in shape, as in a porous medium, the frictional resistance per unit length will be larger for a given area of cross section due to the increased perimeter and therefore the value of K_o will be larger than 2. The actual velocity, u_a through the passages in a porous medium is given by u_m/n, where u_m is the average or Darcy's velocity. The hydraulic mean radius, m for the passages is given by n/S_d, where S_d is the wetted surface of all the particles in an unit volume of material. Substituting the values of u_a and m equation (3) reduces to the form

$$\Delta P = K_o .\ S_d^2 .\ \mu .\ u_m . \frac{\Delta L}{n^3} \qquad ...(4)$$

The surface area, S_d of all the particles in an unit volume is given by

$$S_d = \frac{6 \lambda (1 - n)}{d_s}$$

where λ is a shape factor. Substituting the value of Sd, equation (4) reduces to the form

$$\Delta P = \frac{36\ K_o .\ \lambda^2\ (1-n)^2\ \mu . u_m . \Delta L}{d_s^2 .\ n^3} \qquad ...(5)$$

Replacing the length dr in equation (1) by L, the velocity necessary to cause instability of particles at A_1, designated as critical velocity, u_{m_1} is obtained by substituting the value of ΔP from equation (5) in equation (1) and is of the form

$$u_{m_1} = \frac{g\ (\rho_s - \rho)\ n^3\ d_s^2}{36\ K_o\ \lambda^2\ (1-n)\ \mu} \qquad ...(6)$$

Similarly, the critical velocity, u_{m_2} for particles at B_1 is obtained from equations (2) and (5) in the form

$$u_{m_2} = \frac{\tan\beta .\ g(\rho_s - \rho)\ n^3\ d_s^2}{36\ K_o\ \lambda^2\ (1-n) \cdot \mu} \qquad ...(7)$$

4. FLOW THROUGH A CAVITY WELL

The solution of Laplace's equation for purely spherical flow (6) is of the form

$$\varphi = \left[\frac{\varphi_e - \varphi_w}{\frac{1}{r_e} - \frac{1}{r_w}}\right]\left[\frac{1}{r} - \frac{1}{r_c}\right] + \varphi_w$$

where,

φ = Potential at a distance r from the centre of the sphere.
φ_e = Potential at the radius of influence, r_e, and
φ_w = Potential at the surface of cavity with radius, r_c.

The velocity v_r, at a distance, r from the centre of the cavity is given by

$$v_r = -\frac{\partial\varphi}{\partial r}$$

The velocity v_c at the surface of a cavity with radius r_c is given by

$$v_c = -\left[\frac{\varphi_e - \varphi_w}{\frac{1}{r_c} - \frac{1}{r_e}}\right]\frac{1}{r_c^2} \quad ...(8)$$

Ignoring the negative sign in equation (8) and assuming that the velocity is the same at all the points on the surface of cavity, the particles at point B_1 (Fig. 1) become unstable when v_c is equal to u_{m_2} in equation (7). Therefore

$$\left[\frac{\varphi_e - \varphi_w}{\frac{1}{r_c} - \frac{1}{r_e}}\right]\frac{1}{r_c^2} = \frac{\tan\beta \cdot g(\rho_s - \rho)\, n^3\, d_s^2}{36\, K_o\, \lambda^2\, (1-n)} \quad ...(9)$$

The potentials φ_e and φ_w can be respectively replaced by $k.h_e$ and $k.h_w$, where k is the hydraulic conductivity, and equation (9) reduces to the form

$$(h_e - h_w) = \frac{\tan\beta.\ g.\ \rho\left(\frac{\rho_s}{\rho} - 1\right) n^3\ .d_s^2}{36\ K_o.\ \lambda^2.\ (1-n)\ \mu.\ k}\left[\frac{1}{r_c} - \frac{1}{r_e}\right] r_c^2 \quad ..(10)$$

Substituting the term critical drawdown, S_h in place of $(h_e - h_w)$ and the term intrinsic permeability, K_i in place of $k\mu/\rho g$, equation (10) reduces to the form

$$S_h = \frac{\tan\beta\left(\frac{\rho_s}{\rho} - 1\right) n^3\ d_s^2\left(\frac{1}{r_c} - \frac{1}{r_e}\right) r_c^2}{36\ K_o\ \lambda^2\ (1-n)\ k_i} \quad ...(11)$$

The above equation clear ly shows that there is a critical drawdown corresponding to the size of the initial cavity and that the aquifer particles cannot be disturbed unless the drawdown in the well is equal to the critical drawdown.

Experiments were conducted on a sand model of well in confined aquifer. The drawdowns at which the movement of particles started for different thicknesses and sizes of aquifer materials were determined. It was observed that the critical drawdown remained constant for all the thicknesses in a particular material. This fact is in conformity with equation (11) which does not contain any term for the thickness of aquifer. The data of the computed and the observed values of critical drawdowns for different aquifer materials are presented in table 1.

The observed value of critical drawdown is some-what larger than the computed value in all the cases which may be attributed to the bridging effect between the particles in the aquifer. However, the minimum drawdown needed for the development of a cavity can be estimated from equation (11) by knowing the properties of the aquifer material.

5. SUMMARY

Cavity wells are a special type of tubewells constructed in confined aquifers. The well boring is carried just upto the surface of the water bearing stratum and some amount of sand is taken out with a sand bailer and thus an initial cavity is created. This cavity is further enlarged by pumping water at a high rate. The movement of sand particles begins only when the drawdown is equal to the critical drawdown which is dependent upon the properties of the aquifer material and the radius of the initial cavity but independent of the thickness of aquifer.

6. ACKNOWLEDGEMENTS

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Table 1: Computed and observed values of critical drawdowns for different aquifer materials.

Sl. No.	Mean diameter of particles d_{50} cm	Intrinsic permeability $K_i \times 10^6$ cm^2	Porosity n	Tangent of angle β Tan β	Computed value of critical drawdown $\bar{s}_h$ cm	Observed value of critical drawdown s_h cm
1.	0.640	2.262	0.385	0.800	5.78	7.0
2.	0.305	0.422	0.361	0.760	5.29	6.0
3.	0.790	4.185	0.397	0.790	5.41	8.0
4.	0.420	0.521	0.333	0.762	6.28	7.0
5.	0.414	0.630	0.362	0.765	6.54	6.5

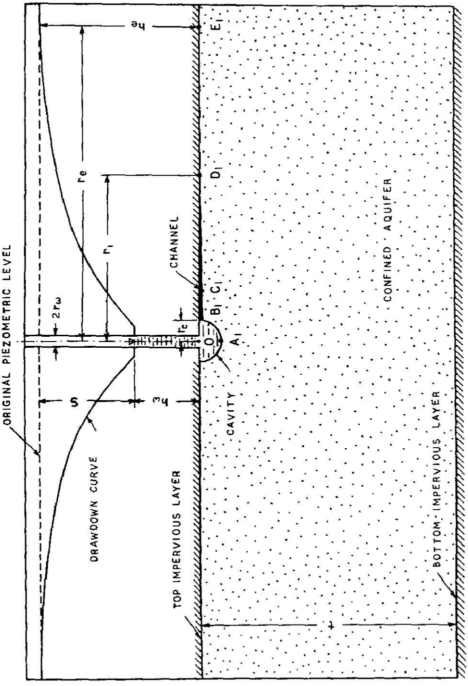

FIG.1 SCHEMATIC DIAGRAM OF CAVITY FORMATION

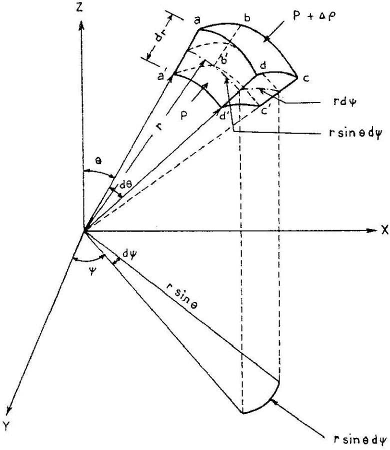

FIG.2 FLOW GEOMETRY IN SPHERICAL COORDINATES

A MATHEMATICAL APPROACH TO SOME ASPECTS OF GROUND WATER FLOW IN HARD ROCK AREAS

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SYNOPSIS

Field conditions in hard rock areas often deviate too far from those assumed for developing standard mathematical equations dealing with ground water flow towards well in non-hard rock areas. Large drawdowns, limited dimensions of aquifers, small permeabilities, large seasonal variations in yield of wells, non-radial flow etc., are some of the specialities of hard rock areas of Central and Southern India, which call for a separate study of ground water flow towards dug wells or open wells.

The authors of this paper (1) have previously attempted explanation of some phenomena observed in field. This paper gives a basic approach towards formulation of a separate mathematical treatment for dug wells in hard rock areas on the basis of parallel flow lines.

1. INTRODUCTION

1.1 Mathematical treatment of ground water flow, which started from Darcy's Law, has been so far applied for field conditions where water flows radially towards pumping wells with small drawdown through a thick, isotropic, semi-infinite aquifer. Such field conditions are met with some reservations in alluvial or sandstone regions while in hard rock areas they often vary considerably from the ideal ones mentioned above. Hard rock areas, being relatively very much less productive, systematic regional development of their ground water resources remained unattended so far. Recent trends of utilizing whatever small resources of ground water in hard rocks by digging open wells for development of irrigated agriculture however necessitate a reconsideration of the mathematical formulae in relations to the observed field conditions.

1.2 This paper is a basic attempt towards mathematical treatment of ground water flow in hard rock areas, based on the field conditions commonly met with in hard rock areas of Central and Southern India. These areas are mainly covered by Basalts, Granites, Gneisses, shales, Slates, Schists and Quartzites etc.

2. AQUIFERS OF HARD ROCK AREAS.

2.1 Ground water in hard rock areas can be classified as water above hard rock and water within hard rock.

2.2 Water above the hard rock occurs in the soft porous mantle overlying the hard rock. This mantle may be of weathered rock (murrom), alluvium, laterite and kankar lime, individually or in combination. The thickness of soft mantle varies froma few inches to about hundred feet or so. In fact, a controversy may arise if areas with a thicker mantle are classified as hard rock areas. The usual thickness of this mantle in Central and Southern India is from 10 feet to 40 feet and dug wells or open wells serve to tap water resources in this mantle. In few cases where the mantle is thicker, open wells are dug upto 80 to 90 feet depth to penetrate the whole thickness of the mantle and a few feet in the hard rock below. The diameter of the open well is between 10 feet to 30 feet. It is also a common practice to dig square or rectangular wells. The average yield of a dug well is from 1,000 to 1,500 cu.ft per day (about 6,000 - 9,000 g.p.d) which is used to irrigate small plots of individual farms.

2.3 Water within hard rock occurs in the fissures, fractures, cooling cracks, joints, cleavages, junctions between lava flows, weathered dykes, fault planes and solution openings in the otherwise sound and impermeable rock. The fissures, fractures etc. mentioned above, when saturated with water, may be termed as 'Water Bearing Planes' within the hard rock (2). Such 'Water Bearing Planes' are in many cases found to occur with close spacing and extensive length along a certain direction in relation to geological and topographical conditions. The permeability is therefore more in this direction than that in others.

2.4 The porous mantle mentioned above many times extends with greater thickness in one direction while in the perpendicular direction it gets pinched out and hard rock gets exposed. Most of the water divides have hard rock exposed on their tops and weathering is pronounced along the valleys of streams. Thickness of weathered rock is therefore more towards the valley and floor of the hard rock is inclined in the same direction. The alluvial deposits and Kankar lime also have more thickness in stream basins (Fig 1). Network of fractures or fissures in hard rock is dense towards the valley centre. All these factors favour occurrence of ground water along the valley. Water table is just localized in the central portion of valley and the water divide being totally dry, the water tables in two adjacent valleys are not connected.

Fig. 1

Cross Section of a Valley

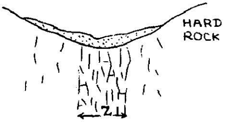
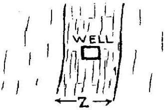


Fig. 2

Banded rocks with high dip (in plan)



Z = Zone of concentration of water bearing planes.

2.5 In addition to their concentration in valley centres, water bearing planes may also show clustering along fault zones, axis of folding and along strike direction in banded rocks dipping at high angle (Fig.2). Not only that the permeability is more along such zones of concentration of water bearing planes, but the occurrence of useful quantity of water is also many times restricted to these zones only.

3. FLOW OF GROUND WATER IN HARD ROCKS.

3.1 In the case of valleys with porous mantle pinching out sidewards as shown in Fig 1, if an open well is dug in the valley, most of the flow towards the well will be along the axis of the valley i.e. along the direction in which the porous mantle extends. The sides will contribute very small supply. It is not therefore correct to assume a radial flow pattern in such cases.

3.2 In the case of a zone of concentration of water bearing planes as shown in figure No. 2, a well dug within the zone will derive most of its supply from water which flows along the strike of the planes. The flow in the direction perpendicular to the strike will be small due to less permeability. Here also the assumption of radial flow may not be justified.

3.3 It will thus be clear that in the case of porous mantle in a valley, the geometrically elongated shape of the mantle and in the case of fissured rock, the directional behaviour of permeability along the zone of concentration of water bearing planes, necessitate a consideration for parallel flow of water towards a dug well.

3.4 Another feature which is commonly found in hard rock areas, is that the saturated thickness of the mantle or of the zone of concentration of water bearing planes is small while the drawdown is comparatively large. The values of permeability of Kankar lime, weathered rock etc., are also small compared to the sand and gravel of the non-hard rock areas. Even if the permeability along a single water bearing plane may appear to be more, the effective permeability of a cluster of planes depends on the number of planes intersected per square foot across the zone of concentration.

3.5 Seasonal variation of yield of water from dug wells in hard rock areas, is of utmost importance while planning a suitable cropping pattern. Dug wells which sustain 10 hours pumping per day with a 5 H.P., pump set, upto December (i.e. upto 2 to 3 months after monsoon rains are over) may sustain only 3-4 hours pumping per day in March, and 1-2 hours pumping per day in May. The limited thickness, extent and permeability of aquifers and limited recharge, cause considerable seasonal fluctuations in the yield of wells.

3.6 While considering pumping from dug wells, it should be noted that the rate of pumping is usually higher than the rate at whichthe water flows to the dug well. The quantity pumped out therefore contains a large proportion of the water pumped from the storage in the dug well. Water level in the dug well does not indicate the water level in the aquifer in its close vicinity, unless pumping is done at a small rate.

4. PARALLEL FLOW OF WATER TOWARDS AN INFILTRATION WELL OR GALLERY.

4.1 Figure 3 shows a longitudinal section referring to flow of water along a stretch of porous mantle or a zone of concentration of water bearing planes towards an infiltration gallery dug across it. The gallery penetrates all the saturated thickness and has its bottom on the unfractured hard rock below. The width of the gallery is not important in calculating the yield of water.

4.2 If Q is the pumping rate in Cft. per day from the gallery under steady state condition and L is the length of the gallery,

$$Q = 2\,q\,L \quad \text{or} \quad q = \frac{Q}{2L} \qquad \ldots (1)$$

where q is the discharge in Cft. per day per unit length along one face of the gallery.

Fig. 3

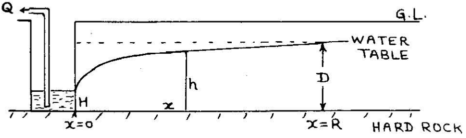


Fig.3 : Flow towards a gallery dug across a valley or a zone of concentration of water bearing planes.

In figure 3, (D - H) is the drawdown, K is the permeability and R is the distance of influence. If at a distance x from the face of the gallery the saturated thickness of the aquifer is h, for unit width of the gallery,

$$q = Kh \left(\frac{dh}{dx}\right) \qquad \dots (2)$$

While integrating for x, the vertical component of the flow is neglected and horizontal flow lines have been assumed for calculation of q with the result that Dupuit's assumption is valid.

$$\therefore\ qx + C = \frac{Kh^2}{2}$$ where C is the constant of the integration.

when $x = 0,\ h = H$

$$\therefore\ C = \frac{KH^2}{2} \quad \text{and} \quad qx = \frac{K}{2}\ (h^2 - H^2) \qquad \dots (3)$$

when $x = R,\ h = D$

$$\therefore\ q = K\ \frac{(D^2 - H^2)}{2\ R} \quad \text{or} \quad Q = \frac{KL\ (D^2 - H^2)}{R} \dots (4)$$

For large draw down H is much smaller than D. Therefore, H^2 may be neglected compared to D^2. Therefore,

$$q = \frac{KD^2}{2R} \qquad \dots (5)$$

4.2 It may be noted that q the discharge per unit length depends upon the permeability K which is constant, saturated thickness D and the distance of influence R.

4.3 Immediately after monsoons the stream is flowing, D has its maximum value and the value of R indicating dewatered zone does not increase because the recharge from flowing stream is readily available. Large quantity can be pumped from the gallery in this season by increasing drawdown. In course of time the stream dries and as dry period approaches, the dewatered zone of the aquifer extends further from the gallery that is the value of R increases and the yield decreases. By end of summer, R spreads maximum from the gallery and water table also depletes to its summer level so that D is reduced. Because of this, Q is at its minimum.

4.4 In hard rock areas the value of D being moderate, large yields are only possible when R is small and/or the value of K is high. In the common cases R plays an important role in yield of water. For the same D and K, small value of R means large yields and large value of R means small yields.

4.5 For example, if the value of L is 100 ft., K is 60 ft. per day, D is 12 ft., H is 3 ft. and R is 50 ft., the discharge Q will be 16,200 Cft. per day or about one lakh gallons per day. This is a typical summer yield of such galleries, which are commonly used for Town water supply or Lift Irrigation schemes. For getting this yield average value of R is about 50 feet which means that the water table in the stream bed has a full saturated thickness D at and beyond 50 feet from the gallery. It is possible to get more yield by increasing the length of the gallery.

5. FLOW TOWARDS DUG WELLS.

5.1 Under the conditions mentioned in section 3.1 to 3.3 above, an open well or dug well in hard rock area will receive its supply from water flowing along one or two mutually perpendicular difections. If the natural ground water flow takes place along x-axis which may coincide with topographical depression, zone of concentration of water bearing planes or such hydrogeological factors, the flow towards a well along x and y directions will be,

$$Q = Q_x + Q_y$$

$$= \frac{2L\,(K_x D^2)}{2R_x} + \frac{2L\,(K_y D^2)}{2R_y} \quad \ldots\ldots \quad (6)$$

where K_x and K_y are permeabilities and R_x and R_y are distances of influences along x and y axes. L is the side of a square well or is 1.41 times the radius of a circular well. D is the saturated thickness of the aquifer in the water bearing zone.

5.2 The spread of porous mantle along y axis is small and in the case of hard rocks the fissures along y axis may have less permeability, with the result that Q_y is less than Q_x. Q_x being the major part of the supply, a well of good yield has more Q_x which means that R_x the distance of influence is small and closer spacing of wells along x-axis is possible. Equipotential line around such a well has been indicated in figure 4.

Fig. 4

Schematic diagram of equipotential line around a dug well.

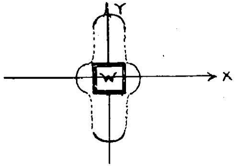
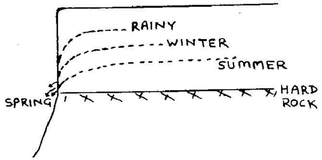


Fig. 5

Position of water table at a natural spring in different seasons.



5.3 It is a fact commonly observed in field that dug wells are closely spaced along x-axis i.e. water bearing zone and still prove themselves to be economically viable for the farmers. Wells spaced away from the zone along y-axis are found to yield smaller supply even if the spacing is large.

5.4 While financing large scale well-digging projects in hard rock areas, it is therefore essential to determine the water availability of a sub-basin and the number of new wells with average yield that can be dug and to locate such wells in the water bearing zones of the sub-basin, with a closer spacing along the zones rather than insist on a grid type scattering of new wells with an arbitrarily fixed spacing.

5.5 If Q_y is neglected and only the flow along x-axis is considered $Q = Q_x = 2Lq$. For steady state condition according to equation No. 3

$$q\,x = K\,\frac{(h^2 - H^2)}{2}$$

$$\therefore\; q\,(x_2 - x_1) = \frac{K}{2}\,(h_2^2 - h_1^2) \qquad (7)$$

where q is the discharge per unit length, h_1 and h_2 are water columns in observation wells located along x-axis at distances X_1 and X_2 respectively. Permeability K along x-axis can thus be determined.

6. FLOW TOWARDS NATURAL SPRINGS & A QUASI STATIC STAGE

6.1 In hard rock areas many natural springs are found on the sides of hills or high level flat lands and steeply cut stream valleys. (Fig. 5)

These springs issue from the junction of soft mantle or fissured rock with hard rock below. Steeply cut lateritic plateaux have springs along the junction of hard rock and laterite.

6.2 Figure 5 shows a section of a steeply cut plateau. Immediately after monsoons the discharge from spring is large as D is at its maximum and R is small because the drainage from the water reserves of the plateau has just started. Gradually by summer season D attains a lower value and R becomes larger because the effect of drainage spreads inwards. During summer many of the springs are reduced to mere tricklings or wet patches.

6.3 For a typical case, if it is assumed that the discharge q per unit width in summer is just one C.ft. per day, the value of D is 10 feet and K is 10 ft. per day for laterite or weathered rock, neglecting H^2 compared to D^2, we get according to equation No. 5, R the distance of influence as 500 feet, meaning that at a distance of 500 feet or more, the effect of drawdown due to spring is not observed.

6.4 If the discharge of one cubic foot per day per unit width in summer as assumed above is just sufficient to support evaporation from wet surface at the mouth of the spring and the transpiration by the vegetation nearby, the actual flow of spring would stop and people may say that the spring and so the porous mantle has dried up. It is however important to note that at a distance of about 500 feet away from the spring the effect of drawdown is not noticeable. This can be called a 'quasi-static' stage of ground water flow in which a small gradient of water table in an aquifer of less permeability is not able to drive a live spring in summer at the exposed end of the aquifer. Although a very small flow may take place according to Darcy's Law, for practical purposes it appears as if the water table is resting with a small gradient corresponding to an angle of response due to friction. The aquifer fully saturated after monsoon is thus only partly emptied till next monsoon even if it is exposed on one side. New wells are many times located at about a furlong's distance or more from the exposed end, for getting water supply.

6.5 Similar consideration will apply to the effluent seepage in the form of springs issuing from the bank cuttings of a river having alluvial banks of large spread and hard rock exposed in the river bed.

7. CONCLUSION

7.1 While considering the flow of water towards dug wells or open wells in hard rock areas, the limited extent of aquifer, directional behaviour of permeability along the zone of concentration of water bearing planes, presence of net work of joints in two directions etc,, necessitate a separate mathematical development on the basis of parallel flow lines. A basic equation of such flow has been developed in this paper and its application to infiltration wells or galleries and dug wells has been indicated. It is also shown that closer spacing of dug wells is possible in certain zones.

7.2 A limit of dessication of an aquifer which gives rise to a natural spring at its exposed end is found by applying the equation to the yield of the spring when it almost dries in summer. A quasi-static stage of ground water flow has been indicated.

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HYDRAULICS OF SHALLOW WELLS IN HARD ROCKS

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SYNOPSIS

Steady, unsteady, confined and unconfined radial ground water flow equations are compared and unified by one general equation in this paper. Validity of the equation is enlarged by applying it for hard rock aquifer having variable hydraulic conductivity with the depth. The equation is employed further for the development of method of hard rock bore and dug well pumping and recovery test data analysis.

The method for bore well data analysis is based on Jacob's principle substituting drawdown by drawdown function and taking into account water table drop at the entrance into the well.

Dug well pumping and recovery test data can be used for the determination of aquifer conductivity as function of ground water inflow which varies during the test because of significant storage capacity of the well. Specific yield for the calculations is assumed and radius of influence is expressed as function of time.

Proposed equations are applicable also for hydraulic computation of shallow water table wells in hard rocks.

1. INTRODUCTION

Character of ground water flow into a shallow well in hard rock is much different from that of flow into a well penetrating unconsolidated formations or deep hard rock aquifer. Variation of hydraulic conductivity with depth is the main reason for this difference. Flow to a dug well in hard rock is different from bore well because of storage capacity of dug well.

Because of the special features of ground water flow usual methods are not applicable for hydraulic computation of wells in shallow hard rock aquifer. Absence of reliable or even approximate methods for the computations leads to major errors determining hydrogeological parameters from well pumping and recovery test data and designing of wells. It results in underdevelopment or overdevelopment of ground water resources.

An attempt is made in this paper to develop methods for analysis and designing of bore and dug wells in shallow hard rock aquifer based on pumping and recovery test data.

2. GENERAL

Using simplified expression of integral exponential function, Theis unsteady confined radial flow equation can be written in the form

$$s = \frac{Q}{4\pi T} \ln \frac{2.25\,Tt}{Sr^2} \qquad (1)$$

where s is draw-down at the distance r from the well, Q is discharge pumped from the well, T is transmissitivity, t is time, S is storativity.

Transmissitivity of confined aquifer characterises capacity of aquifer to transfer ground water flow through the aquifer and is expressed as

$$T = Kb \qquad (2)$$

where K is hydraulic conductivity and b is thickness of aquifer.

Capacity of aquifer to propagate changes of piezometric head is characterised by coefficient of diffusitivity (UNESCO, 5), denoted by a and expressed as

$$a = \frac{T}{S} \qquad (3)$$

Introducing diffusitivity concept equation 1 can be written as

$$s = \frac{Q}{4\pi T} \ln \frac{2.25\,a\,t}{r^2} \qquad (4)$$

If radius of the depression cone is denoted by R, drawdown s at the distance r = R from the well will equal zero. It follows from this condition and equation 4 that

$$\ln \frac{2.25\,a\,t}{R^2} = 0 \qquad (5)$$

and

$$R = 1.5\sqrt{a\,t} \qquad (6)$$

Term R, commonly called as conditional radius of influence, differs from radius of influence at steady flow conditions by it s dependance on time t.

Introducing expression of conditional radius of influence R into equation it obtains from

$$s = \frac{Q}{2\pi T} \ln \frac{R}{r} \qquad (7)$$

Equation 7 can be easily rearranged into the formula

$$Q = \frac{2.73\,s_{\bullet}\,T}{\log R - \log r_w} \qquad (8)$$

used for designing of artesian and water table wells at steady flow conditions where s_e is drawdown in aquifer at well wall, R is radius of influence and r_w is radius of well as shown in Figure 1.

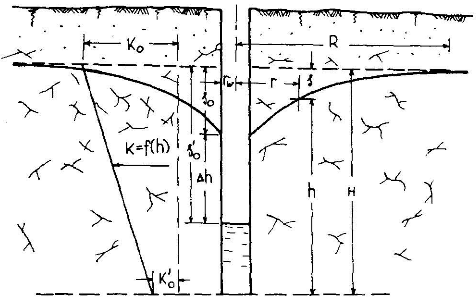

Fig.1 Scheme of ground water flow into shallow well in hard rock

For unconfined aquifer transmissitivity T is expressed as

$$T = KH_a \tag{9}$$

where H_a is average thickness of saturation expressed as

$$H_a = H - \frac{s_e}{2} \tag{10}$$

where H is thickness of saturation at static conditions.

Equation 8 suits for both confined and unconfined flow conditions. Therefore equation 7 may be applied for unconfined flow conditions, if storativity S in expression of diffusitivity a for confined aquifer is substituted by specific yield of unconfined aquifer k_y (Bindeman, Klimentov, 1), that is

$$a = \frac{T}{k_y} = \frac{KH_a}{k_y} \tag{11}$$

Right side of equation 7, when applied to the case of unconfined flow, will contain drawdown, which is included into expression of transmissitivity T and conditional radius of influence R, (equations, 10,9,11 and 6). Equation

7 can be rearranged into more convenient and universal form

$$U = \frac{Q}{2 \pi K} \ln \frac{R}{r} \tag{12}$$

where U is so-called drawdown function, expressed for artesian well as

$$U = sb \tag{13}$$

and for water table well as

$$U = sH_a \tag{14}$$

Relationship between drawdown function U and logarithm of time in equation 12 is nearly linear, therefore the equation is convenient for the analysis of pumping and recovery test data and for hydraulic computation of interacting wells.

Equation 12 is applicable for steady and unsteady, confined and unconfined flow conditions. For unsteady flow conditions radius of influence R is determined from equation 6.

Drawdown in aquifer s_o is different from that in water table well s_o' as it is shown in Figure 1. Relationship between the two parameters can be expressed by Ehrenberger's (2) formula

$$\Delta h = s_o' - s_o = \frac{(s_o')^2}{2H} \tag{15}$$

where Δh is water level drop at the entrance into water table well or the height of so-called seepage face.

3. HETEROGENEOUS UNCONFINED AQUIFER

Hydraulic conductivity of shallow hard rock aquifer varies with depth reaching maximum (K_o) at the top and minimum (K_o')(practically zero) at the bottom of aquifer.

In the case when conductivity is related with depth by linear relationship and $K_o' = 0$, local conductivity K at any level can be expressed as

$$K = K_o \frac{h}{H} \tag{16}$$

Assuming that cross section of ground water flow to a well has the form of vertical cylinder of radius r and height h, average conductivity of entire cross section can be written as

$$K_a = K_o \frac{h}{2H} \tag{17}$$

and discharge of ground water flowing through the cross section

$$Q = AK_a I = 2 \pi r h K_o \frac{h}{2H} \frac{dh}{dr} \tag{18}$$

Solution of equation 18 is equivalent to equation 12. Drawdown function U for this case is expressed as

$$U = sH \left[1 - \frac{s}{H} + \frac{1}{3}\left(\frac{s}{H} \right)^2 \right] \quad (19)$$

Conductivity K in equation 12 must be substituted by 0.5 K_0.

When conductivity is related with depth by linear law and varies within limits from K_0 at static water table level to K_0' at the bottom of aquifer, expression of drawdown function is modified as

$$U = sH \left[1 + \frac{K_0'}{K_0} + \frac{s}{H} + \frac{1}{3} \left(\frac{s}{H}\right)^2 \left(1 - \frac{K_0'}{K_0}\right) \right] \quad (20)$$

and conductivity K in equation 12 must be substituted by $\frac{1}{2}(K_0 + K_0')$.

Equation 15 was derived for water table well in aquifer having homogeneous conductivity. The depth of ground water flow approaching water table well radüces and water percolates through lower part of aquifer. Hydraulic conductivity of lower part of shallow hard rock aquifer is smaller and hence depression curve in hard rock is steeper at the well compared with that in aquifer with homogeneous conductivity. Thus it can be inferred that the entrance drop Δh should be larger than that obtained from equation 15.

Direct measurement of drawdown will always give completely reliable magnitudes of drawdown for hydraulic computations using equations 12 and 19 or 20. The measurements can be performed in observational bore holes or in well of large diameter itself as drawdown of seepage face top line from initial level.

4. ANALYSIS OF BORE WELL PUMPING TEST DATA

Bore wells penetrate fissured zone of rock nearly completely and distance from static water level in the well to the bottom of the well approximately equals to the thickness of saturation H (Figure 1).

Analysis of pumping test data of the well can be performed using Jacob's principle. Analysis of water table well pumping test data differs from that of artesian well by using drawdown function U instead of drawdown s for construction of graph U = f (lnt). Besides this, entrance drop phenomenon must be taken into account if water level in the well during pumping test is measured.

Function U = f (lnt) has graphical expression very close to straight line the gradient of which is

$$\frac{\Delta U}{\Delta \ln t} = \frac{Q}{4\pi K} \quad (21)$$

from where

$$K = \frac{Q}{4\pi} \frac{\Delta \ln t}{\Delta U} \qquad (22)$$

It follows from equations 12 and 6 that at the point of line $t = t_0$ and $U = 0$

$$\frac{2.25 a t}{r^2} = 1 \qquad (23)$$

From equations 23 and 11 specific yield

$$k_y = \frac{2.25 \; KH_a \; t_0}{r^2} \qquad (24)$$

Time t_0 is determined from graphical analysis of pumping test data.

Analysis of bore well pumping test data is performed in the following steps.

(a) Using equation 15 – s_0 and equations 19 or 20 – U are computed.
(b) Drawdown function U is plotted against lnt and straight line is drawn through the plotted points.
(c) Increment of drawdown function U, corresponding to any definite increment of logarithm of time, is read from the plot.
(d) Conductivity K is determined from equation 22.
(e) Time t_0 corresponding U = 0 is determined graphically or analytically extrapolating the line U = f (lnt).
(f) Specific yield k_y is determined from equation 24.

5. ANALYSIS OF BORE WELL RECOVERY DATA

Jacob's method can also be used for analysis of recovery data. As in the case of analysis of pumping test data, entrance drop Δa must be substracted from drawdown s'_0 measured in the well and drawdown function U must be used for graphical analysis of recovery test data.

Ground water flow in recovery stage is characterised by equation

$$U = \frac{Q}{4\pi K} \ln \frac{t + t_p}{t} \qquad (25)$$

where t_p is duration of pumping test and t is recovery time measured from the end of pumping test. Drawdown function U is plotted against $\ln \frac{t + t_p}{t}$ and straight line is drawn.

It is obvious that the function $U = f\left(\ln \frac{t + t_p}{t}\right)$ follows a straight line which has the slope of

$$\frac{\Delta U}{\Delta \ln \frac{t + t_p}{t}} = \frac{Q}{4\pi K} \qquad (26)$$

rom where

$$K = \frac{Q}{4\pi} \frac{\Delta \ln \frac{t + t_p}{t}}{\Delta U} \qquad (27)$$

Specific yield k_y can be determined using equation 24 derived for pumping test data analysis.

Analysis of bore well recovery test data is performed in the following steps:

(a) Using equation 15 - s_0 and equation 19 or 20 - U are computed.

(b) Drawdown function U is plotted against $\ln \frac{t + t_p}{t}$ and a straight line is drawn.

(c) Increment of drawdown function U corresponding to any definite increment of $\ln \frac{t + t_p}{t}$ is read from the chart.

(d) Conductivity K is computed from equation 27.

(e) Time t_0 corresponding $U = U_{max}$ is determined graphically or analytically extrapolating line $U = f (\ln \frac{t + t_p}{t})$.

(f) Specific yield is computed from equation 24.

Hydraulic conductivity and specific yield determined on the basis of pumping and recovery test data represent some average value of the parameters corresponding to approximately local conductivity at the level of middle point of saturated aquifer's layer.

6. GROUND WATER FLOW INTO DUG WELL

Ground water flow into dug well in hard rock is much different compared with that of small diameter bore well. Because of significant volume of water stored in the well and usually low well productivity, discharge of ground flow entering the well Q_1 is always much smaller than discharge pumped out from the well. Ground water flow discharge Q_1 increases with drawdown and reaches maximum value at the end of pumping period. However even at the end of pumping test Q_1 is far less than Q.

Recovery stage is also entirely different from that of bore well. If inflow discharge Q_1 for recovery period of bore well can be considered equal to zero, the discharge for recovery period of dug well of large diameter is of significant value defined by parameters of well and aquifer.

Formation of depression cone in aquifer around dug well is much slower due to necessity to remove water accumulated in the well before pumping. It is shown by Bindemann (1) that parameters of unsteady ground water flow into a well of small diameter after a short period of pumping become equivalent to those of steady flow having the same geometrical parameters (R, s_0, H).

Similarity of steady and unsteady flow parameters of dug well should be even more pronounced because of reasons indicated above.

Equations 12, 6 and 15 can be applied for hydraulic calculation of dug well. However it is necessary to take into account that formation of depression cone commences from initial diameter equal to the diameter of well. Dug well diameter is significant and cannot be neglected like in the case of bore well.

Presence of initial diameter of depression cone can be evaluated introducing time correction t_i necessary for formation of cone of depression having radius $R = r_w$. Correction t_i must be added to the time t in equation 6 used for calculation of conditional radius of influence R. Correction t_i is determined from the same equation 6 applied for the case

$$r_w = 1.5\sqrt{a t_i} \tag{28}$$

from where

$$t_i = \frac{r_w^2}{2.25a} \tag{29}$$

Conditional radius of influence for dug well is determined as

$$R = 1.5\sqrt{a(t + t_i)} \tag{30}$$

Because of the special features of ground water flow to a dug well the method developed above do not suit for analysis of data of pumping and recovery test performed in dug well of large diameter. Conductivity in this case is expressed from equation 12 as

$$K = \frac{Q_i}{2\pi U} \ln \frac{R}{r_w} \tag{31}$$

where discharge of ground water infiltrating into the well Q_i is determined from condition

$$Q - Q_i = \frac{\pi r_w^2 \Delta s_o'}{\Delta t} \tag{32}$$

as

$$Q_i = Q - \frac{\pi r_w^2 \Delta s_o'}{\Delta t} \tag{33}$$

where Q is discharge pumped out from the well during pumping test, r_w is radius of the well, $\Delta s_o'$ is decline of water level in the well during interval of time Δt.

For recovery period discharge of ground water infiltrating into the well is expressed as

$$Q_i = \frac{\pi r_w^2 \Delta s_o'}{\Delta t} \tag{34}$$

where $\Delta s_o'$ is increment of water level in the well during interval of time Δt.

Conditional radius of influence R in equation 31 is determined from equation 30. *Specific yield k_y is assumed according to the type of rock and* degree of weathering. There is no doubt that the assumption introduces some error into calculations of K, however there is no possibility to determine the parameter directly from pumping or recovery test data.

Although the thickness of aquifer saturation H is different from depth of penetration by dug well, it is possible to assume that dug well penetrates shallow aquifer completely. Such assumption is possible because transmissitivity of lower part of aquifer situated below the bottom of the well is much less compared with transmissitivity of the part of aquifer penetrated by the well.

Taking into account that dug well usually penetrates only upper part of rock, variation of hydraulic conductivity with depth in upper part of aquifer *can be neglected assuming K = const. Drawdown function for such case is* determined from equation 14 and 10.

Periodical pumping of water from the well results in formation of deep cone of depression around the well. Conditional radius of influence R in equation 31 applied for such well becomes indefinite and as such it has to be assumed only. This and other assumptions reduce accuracy of proposed method and makes it approximate.

7. ANALYSIS OF DUG WELL PUMPING AND RECOVERY TEST DATA

Analysis of dug well pumping and recovery test data is performed in the following *steps:*

(a) Drawdown(for pumping stage) or recovery (for recovery stage) of water level in the well $\Delta s_o'$ during intervals between measurements Δt are computed.

(b) Infiltration discharges Q_i are determined for each interval from equations 33 and 34.

(c) *Average values of $\bar{s}_o$ are calculated for each interval.* Drawdown s_o for the purpose must be possessed. It is measured during pumping and recovery test as decline of seepage face top line from initial level.

(d) *Average magnitude of drawdown function U corresponding* drawdowns $\bar{s}_o$ are calculated as

$$\bar{U} = \bar{s}_o \left(H_1 - \frac{\bar{s}_o}{2}\right) \tag{35}$$

where H_1 is depth of penetration.

(e) Time from the beginning of pumping test to the middle of each interval between measurement $\bar{t}$ is computed.

(f) Conductivity K and radius of influence R are determined for each interval by trial method from equation 31, 30, 11 and 10. Equation 31 for the convenience of selection of right value of K is used in another form

$$\frac{K}{\ln \frac{R}{r_w}} = \frac{Q_1}{2\pi U} \qquad (36)$$

Specific yield K_y is assumed as it was mentioned according to the type of formation.

8. CONCLUSIONS

The method developed in this paper for analysis of pumping and recovery data is based on approximate equations and some assumptions. Thus accuracy of the method is not high. However, absence of reliable or even approximate methods for hydraulic calculation of shallow wells in hard rocks under complex hydrogeological conditions makes the method applicable for analysis of pumping and recovery test data and designing of wells in hard rock areas.

Equation 12 together with equations [illegible], 19 and 20 may also be used for well interference studies in hard rock areas. Equation 12 is practically linear, therefore drawdown from influence of some closely situated wells in a well system can be determined summing up drawdown function values, computed separately for influence of each well, and then determining drawdown from resultant [illegible] value of the drawdown function. Similar method for interference of [illegible] wells is [illegible] (4).

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AN APPROACH FOR THE OPTIMUM UTILIZATION OF YIELD FROM OPEN WELLS IN HARD ROCKS AREAS.

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SYNOPSIS

Hard rocks include igenous, metamorphic and highly compacted sedimentary rocks in which the intergranular porosity is very low. Groundwater from these rocks is extracted mainly by openwells tapping the weathered mantle or the joints, fissures, vesicular horizons and solution cavities, The yield of openwells is generally very low and the wells get dry after only a few hours of pumping. In this paper an attempt has been made to present a new method for optimum utilization of yields from openwells in hardrock areas. The proposed mathematical equations have been verified by field application during yield test of two openwells and the results were found to be encouraging.

1. INTRODUCTION

A major portion of India is covered by igneous, metamorphic and higly compacted sedimentary rocks. The intergranular porosity is little or absent in these rocks. Groundwater in areas covered by hard rocks occur either in the intergranular pore spaces of the weathered mantle or in the joints, fissures, fault zones, vesicular horizons and solution cavities.The water is extracted in these areas mainly through open wells. The yield of wells is very low. Except the openwells located on fault zones or tapping vesicular horizons, subsurface channels and cavities, the majority of wells get dry after 2 to 3 hours of pumping. To augment the water yielding capacity of openwells two methods can be followed i.e. (1) Improvement in the construction of wells such as boring, enlargement of diameter or construction of infiltration galleries, (2) optimum utilization of yield from existing wells.

In the present paper an attempt has been made to give a new method for optimum utilization of yield from openwells in hard rock areas, without resorting to any improvement in the construction of well.

2. PROPOSED METHOD

From field observations it is observed that the recuperation rate after emptying an openwell is faster in the beginning and gradually decreases with time. The first 50 percent recuperation takes much less time than the next 50 percent. Keeping this fact in view, an openwell can be utilized to the maximum, if after once emptying, it is operated repeatedly as soon as 50 percent recuperation takes place. In this way a well can give the maximum yield. To find out the number of times an openwell can be operated after once emptying, the following procedure of conducting yield test may be followed:

After recording the undisturbed static water level the well should be pumped at a constant discharge of water (Q). The exact time (t_1) for emptying the open well should be found out. Recuperation readings after stopping of pump should be noted at suitable intervals for sufficient time. A simple graph can now be plotted between time since pumping stopped (t), and residual drawdown (s'). The plotted curve will be an asymptote showing faster rate of recuperation in the begining and gradual decrease with time. The 50 percent recuperation point can be obtained from the plotted curve. Let it be denoted by t_2. If the duration of a working day be taken as 12 hours or 720 minutes, giving sufficient time for complete recuperation of well during the night, the following equation can be put forward:

$$t_1 + n(t_2 + t_1/2) = 720 \qquad (1)$$

where t_1 is time in minutes to empty an undisturbed well, t_2 is time in minutes taken for 50 percent recuperation, and 'n' is the number of times the well can be operated after once emptying.

Equation (1) can be simplified into the following working equation.

$$n = \frac{2(720-t_1)}{t_1 + 2t_2} \qquad (2).$$

Thus knowing the values of t_1 and t_2 from yield test of an open-well, the value of 'n' can be found out.

The maximum water yielding capacity of well can be expressed by the following equation.

$$Y = Qt_1 + \frac{nQt_1}{2} \qquad (3)$$

Or $$Y = \frac{Qt_1}{2}(n + 2) \qquad (4).$$

Where Y is the maximum water yielding capacity of the well in cubic metres/day.

Q is the average discharge of water during pump-ing in cubic metres/minute
t1 = time taken in minutes to empty an undisturbed well.

3. APPLICABILITY OF THE METHOD

The proposed method is applicable in openwells which get dry after a few hours of pump-ing and the recuperation is through the entire desaturated zone. If the well taps only a few localised joints or solution cavities the rate of recuperation will show a linear relation with time and hence the method cannot be applied. The method is well suited for openwells tapping weathered zones, valley fills with thin saturated zones, and highly jointed and fissured rocks with a fairly uniform permeability.

To verify the applicability of the method yield tests were conducted on openwells at Naswadi (22°03';73°44', 46F/12) and Bhagwanpura (22°06';73°41', 46F/12) in Baroda district, Gujarat, tapping valley fill and weathered basalt. The data of the yield tests were plotted (Figure-1) and the values of the time for 50 percent recuperation (t_2) were found out. The results obtained, after application of the proposed method, are given in Table-1.

From a perusal of Table-1 it is evident that the openwell at Naswadi yielding at present 81.6 cubic meters of water per day can be operated 5 times a day after once emptying to yield 178.5 cubic metres of water per day. Similiarly, the openwell at Bhagwanpura can be operated 3 times after once emptying and the total water yielding capacity of well can be increased from 92.25 cubic meters per day to 161.44 cubic metres per day. From the values of time taken for emptying an undisturbed well (t_1) and the time for 50 percent recuperation (t_2), a time table for working of pump can also be prepared. A time table for pump operation for the two tested openwells is given in Figure-1.

4. CONCLUSIONS

The proposed method can be profitably applied in hard rock areas where openwells, tapping weathered zones, valley fills and uniformly distributed joints and fissures, get dry after a few hours of pumping. The applicability of the method has been established by yield tests on two open wells.

ACKNOWLEDGEMENT

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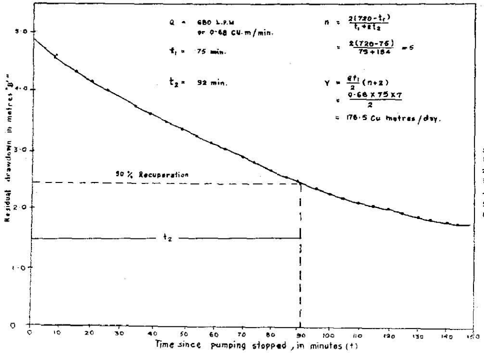
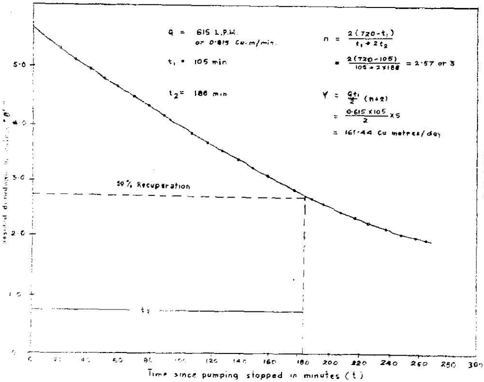
(A) Yield test of well at Naswadi



TIME TABLE FOR PUMPING THE WELL

S.No	TIME OF OPERATION
1	5·00 hrs to 6·15 hrs
2	7·47 hrs to 8·24 hrs
3	9·58 hrs to 10·33 hrs
4	11·05 hrs to 12·42 hrs
5	14·14 hrs to 16·51 hrs
6	16·23 hrs to 17·00 hrs

(B) Yield test of well at Bhagwanpura



TIME TABLE FOR PUMPING THE WELL

S.No	TIME OF OPERATION
1	5·00 hrs to 6·45 hrs
2	9·51 hrs to 10·43 hrs
3	13·49 hrs to 14·41 hrs
4	17·47 hrs to 18·39 hrs

Figure-1 Plots showing the variation of residual drawdown (δ') with the time since pumping stopped (t) for yield test of openwells in hard rock areas.

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TABLE-1. RESULTS OF YIELD TESTS ON OPENWELLS

Sl. No.	Location	Aquifer material	Total depth b.g.l. (m).	Diameter (m)	Static water level b.g.l. (m).	Discharge (Q) in Lit/min.	Time taken for emptying undisturbed well (t_1) (in min).	Time taken for 50% recuperation (t_2) (in min).	Value of 'n' from formula.	Max. yield capacity in cu.m/day (from formula)	Present yield capacity of well in cu.m/day.
1.	[illegible] (22°0[illegible]';73°44')	Alluvium (valley fill)	13.26	3.0	8.40	860	75	92	5	178.5	81.6
2	Bhaguanpura (22°06';73°41')	Weathered basalt.	10.23	4.0	4.03	615	105	186	3	161.44	92.25

TUBE WELLS, OPEN WELLS, AND OPTIMUM GROUND WATER RESOURCE DEVELOPMENT

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SYNOPSIS

Many tube wells drilled for irrigation use fail prematurely because of poor well design, improper sized and placed gravel-pack material, and ineffective slotted-pipe screens. Well and gravel-pack design criteria and an improved slotted-pipe screen described in this paper offer promise of drastically reducing such failures. Properly designed tube wells equipped with this type of screen should have more than double the safe-yield and service-life capability of most irrigation tube wells now in use.

In some countries large-diameter, low-capacity open wells always have been used to obtain irrigation water. Now, small-diameter, high-capacity tube wells are being constructed in the same aquifers. As extensive tube-well development occurs, the water table will drop, drying up many shallow open wells. In such instances, those who can afford the deeper, more expensive tube wells could gain almost exclusive use of the aquifer.

Optimum development of surficial aquifers using both tube and open wells may have to be rigidly controlled to assure every farmer a fair share of available water. Such controls should be based on the potential safe yield capability of aquifers. A graphical procedure used in Illinois to obtain an estimate of this value for planning and initial development purposes is presented in this paper.

1. INTRODUCTION

In many parts of the world, expedited development of available ground water resources for food production now is underway. As this development takes place, many problems associated with well production, aquifer yield evaluation, and ground water resource management will arise. From personal observations made during recent ground water consulting assignments in Central and Northern India, it appears that serious well-failure and aquifer overdevelopment problems may already be occurring in large portions of major aquifers in that country. Also, from technical reports and personal communication with ground water hydrologists from other parts of the world, similar problems seem to be prevalent almost everywhere.

Improper well design, the use of ineffective slotted pipe screens, and a lack of accurate aquifer recharge information create especially serious problems from an optimum aquifer development standpoint. For this reason, the discussion presented in this paper deals primarily with these subjects. It is hoped that the included material and methodology will be applicable and useful in minimizing such problems in other countries as they have proved to be in Illinois.

2. DESIGN CRITERIA FOR SCREENED WELLS

Major ground water developments generally are associated with unconsolidated glacial outwash or alluvial sand and gravel aquifers. In such deposits maximum production and peak efficiency considerations dictate that a screen be used to hold back the water-bearing material yet permit water to freely enter the well with minimum head loss.

Screened wells typically are classified as either natural packed or artificially packed wells. In the natural pack type, a screen slot size is selected which will allow a definite proportion of the finer part of the aquifer adjacent to the screen to pass into the well for removal during development. The remaining envelope of coarser aquifer material around the screen serves as a retainer for surrounding fine-grained deposits. In the artificial pack type, an envelope of materials having a coarser uniform grain size than the aquifer is mechanically placed around the screen to serve as a filter for the finer formation particles.

Illinois State Water Survey design criteria for either type as described by SMITH (1) and WALTON (2) are based on the effective size, uniformity coefficient, and other grain-size distribution considerations determined from a mechanical analysis of the aquifer material. The sieve size that retains 90 percent of the aquifer material is termed the effective size. The uniformity coefficient is the ratio of the sieve size that will retain 40 percent of the aquifer material to the effective size. A natural pack well normally can be justified if the effective grain size of the aquifer is greater than 0.01 inch and the uniformity coefficient is above 3.0. An artificial pack usually proves to be desirable if either the effective size or uniformity coefficient is much below these values. In some aquifers an artificial pack must be employed to stabilize well-graded aquifers containing a large percentage of fine material or to permit the use of a larger screen-slot size.

For natural packed wells, homogeneous deposits overlain by materials which will not easily cave generally are screened with a slot size that will retain from 40 to 50 percent of the aquifer material. Retention of 60 to 70 percent of the aquifer materials is specified if cavey materials overlie the aquifer. Heterogeneous water-bearing material overlain by firm, noncavey beds is developed using a screen which retains from 30 to 40 percent of the aquifer material. Wells finished in such materials capped by cavey deposits are designed to retain 50 to 60 percent of the aquifer material. In cases where it is suspected that adequate well development procedures will not be followed, a screen that will retain about 70 percent of the water-bearing formation may be suggested regardless of the type of aquifer or overlying materials present.

If an artificial pack is used, it is important to select a pack material grain size that will effectively retain fine materials from the aquifer. A uniform gravel-pack size that is from 3 to 5 times the 50-percent size of the aquifer material, a screen size that will retain at least 90 percent of the pack material, and a pack thickness of from 6 to 9 inches is generally used in Illinois. Experience has shown that if these criteria are followed, problems associated with segregation and bridging during artificial pack placement, and plugging or sand pumping during and following development are minimized.

In cases where only large screen slot opening sizes are available and the aquifer is composed primarily of fine-grained deposits, a mixture of two or three uniform pack-material sizes may be required. Providing the pack-aquifer ratio and screen slot-opening size restrictions previously discussed permit a two-size mixture, the coarser-material component selected constitutes about 65 percent of the total pack material volume. In cases where 3 uniform sizes must be mixed, a 50-30-20 percentage combination of coarse to fine sizes usually is employed. Graded pack material is placed using a tremie pipe always kept filled with the pack material to minimize grain-size segregation during placement.

The screen-selection criteria used by the Water Survey are based upon optimum screen entrance velocities considering aquifer permeability and screen area effectively open to the water-bearing material. For natural packed wells the proper screen length and/or optimum discharge rate are determined from the equation:

$$L_s = \frac{Q}{7.48\,A_e V_c} \qquad (1)$$

where:

L_s = length of screen, in ft
Q = optimum discharge, in gpm
A_e = effective open area per foot of screen, in sq ft
V_c = optimum entrance velocity, in fpm

In this equation, the effective screen open area (A_e) used is one-half the actual area provided during fabrication. The remaining open area is assumed to be blocked by the aquifer material. Using an appropriate aquifer coefficient of permeability, an optimum entrance velocity (V_c) value is selected from the following tabulation.

Coefficient of permeability *(gpd/sq ft)*	Optimum screen entrance velocities *(fpm)*
>6000	12
6000	11
5000	10
4000	9
3000	8
2500	7
2000	6
1500	5
1000	4
500	3
< 500	2

V_c values included were determined from studies of actual case histories of well failures due to the partial clogging of well walls and screens by over pumping.

For an artificial pack well, the same procedure is used to select an appropriate screen length and safe pumping rate except that the average of the permeabilities of the aquifer and the pack is used to determine the optimum screen entrance velocity.

Well diameter selection is usually dictated by pump dimension requirements. Where possible, a casing at least 2 inches larger than the nominal pump-bowl diameter is specified to minimize pump-placement and water-level measurement problems. The following pumping rate - well diameter combinations are typically used in Illinois:

Pumping rate *(gpm)*	Diameter of well *(in.)*
125	6
300	8
600	10
1200	12
2000	14
3000	16

3. SLOTTED-PIPE SCREENS

In parts of the world, many irrigation wells are constructed using a maximum hole diameter of 12 inches, an 8-inch diameter casing, and a 6-inch diameter slotted-pipe screen. A large percentage of these wells are artificially packed with a 3-inch thickness of gravel. Commercially made slotted pipe screens normally are available in only one-eighth, one-fourth, and three-eighths inch slot widths. Because of this an artificial pack material large enough to be retained by these wide slot sizes generally is much too large to retain the fine-grained segments of many unconsolidated aquifers. In such cases excessive sand production and short well-service life usually results. Another disadvantage created by the relatively small hole diameter and large screen-slot opening sizes is that a graded rather than a uniform grain-size pack has to be used in the wells. Because tremie-pipe placement of this graded pack material is impractical, the pack material usually is merely shoveled into the two-inch annulus between the hole and casing at land surface and permitted to settle around the 6-inch slotted pipe screen within the aquifer. Considerable segregation and subsequent stratification of sizes may occur as the nonuniform grain size material falls through the water during placement. This in turn readily permits pockets of coarser pack material to form between the aquifer and screen, thereby providing avenues for fine sand from the aquifer to move through the pack material into the well.

In addition to these limitations, the safe pumping rate of slotted-pipe screened wells generally is significantly less than that of wells equipped with comparable lengths of continuous slot screens because of a much smaller effective open area characteristic of typical slotted-pipe screen designs. Practically all 6-inch diameter slotted pipe screens now used provide open areas ranging from less than 3 to about 14 percent with the majority falling between about 6 and 11 percent.

In 1970, the author, working with two Indian engineers at Jabalpur in Central India, devised an improved slotted-pipe well screen design which affords from 20 to 40 percent of actual open area depending upon the slot width and spacing provided. Use of this screen design should practically eliminate problems of well design, construction, and yield limitation normally associated with the old type of slotted-pipe screens. The original work was done with Mr. Dayaram D. Rathod and Mr. Anil G. Khadakkar. Initial fabrication of a screen section incorporating the new design features was done in 1970 in the Pioneer Earthmasters shops at Jabalpur under the supervision of Mr. Rathod. A picture of this screen is shown in Figure 1.

Figure 1. Original Model of Improved Slotted-pipe Screen Design

At the request of Mr. Rathod, refinements of the original design were made by Dr. James E. Stallmeyer, Professor of Civil Engineering, University of Illinois, at Urbana, Illinois. Except for slight differences in the width of columnar and horizontal structural segments in the pipe skeleton framework, and a horizontal rather than a vertical orientation of the slot pattern, the original design illustrated in Figure 1 is identical to the optimized one which resulted from Dr. Stallmeyer's refinements.

In the finalized design, columnar and tensile strength is provided by a series of 6 vertical columns, each three-eighths of an inch wide, equally spaced around the circumference of the pipe used as screen material. Horizontal stresses resulting from the weight of the aquifer against the screened section, plus possible differences of head inside and outside the well screen, are met by a series of hoop sections of the pipe formed during fabrication. The horizontal hoop segments of the pipe skeleton can be arranged in any spacing pattern dictated by construction or other controlling considerations. However, from a structural-strength standpoint, one inch of horizontal band width per foot of screen length must be provided. Thus, in any 1-foot length of screen, hoop strength can be obtained by leaving a 1-inch band of pipe per foot; two one-half inch bands spaced 6 inches center to center; three one-third inch bands spaced 4 inches center to center; or four one-fourth inch bands on a 3-inch center to center spacing. Providing that horizontal slots are cut, a one-inch wide horizontal hoop per foot of screen length should suffice; if vertical slots are cut, a one-half inch wide hoop every 6 inches of pipe length was recommended by Dr. Stallmeyer. Also, he suggested that a horizontal rather than a vertical slot arrangement be used. This was deemed advisable for two reasons. One was that vertical slot cutting might create considerable problems with saw blade wobble, thereby resulting in poor quality control of slot width. The other was that sawing long vertical slots in the pipe might cause adverse heat-strength effects in the thin metal strips left between slots.

In Dr. Stallmeyer's design refinements, laboratory test data for a leading wire-wrapped, continuous-slot screen were used to establish minimum structural limits for the slotted-pipe screen design. This was done to insure that the new design would provide minimum structural limits higher than those of a screen noted for its high percent of open area and structural strength stability under actual field installation and operation conditions. According to Dr. Stallmeyer, a safety factor of 1.67 is realized by the six three-eighths inch vertical columns and an effective one inch per one foot horizontal band configuration. Also, an additional factor of safety would be realized from the added structural strength provided by the slotless portion within each screen window.

The open area actually attainable using this slotted-pipe screen design is primarily dependent upon the degree of quality control employed during construction and the number and width of slots fabricated within each screen window. With proper quality control it is possible to realize 40 percent open area if the slot opening and intervening metal strip widths are the same; 27 percent if the metal-strip spacers are twice the slot-opening size; and 20 percent if the metal-strip space between slots are 3 times the slot-opening size. Considering these open areas in light of those currently available (from 3 to 14 percent), it appears from equation 1 (see page 3) that truly remarkable increases in safe yield and service life expectancy of tube wells could be realized by using this improved slotted-pipe screen design.

4. AQUIFER LONG-TERM SAFE YIELD EVALUATION

The long-term safe yield of an aquifer is defined as the rate at which water can be continuously pumped without eventually lowering water levels below the bottom of shallow open wells or below the top of screens in tube wells. This rate is primarily dependent upon the amount of precipitation recharged to the ground water reservoir each year.

Recent ground water recharge and runoff studies made by the Water Survey (3,4,5,6,7,8,9) indicate that the long-term precipitation recharge rate to unconsolidated aquifers in Illinois is controlled primarily by the thickness of dense, low-permeability clay or glacial till deposits (aquitards) overlying the aquifer in any given area. A graphical expression of the ground water recharge-aquitard relationship developed from data collected during these studies is shown in Figure 2. From this graph and the accompanying precipitation distribution map it is possible to obtain meaningful approximations of the yearly recharge rate to most of the sand and gravel aquifers within the State.

In other countries, where different geohydrologic or climatic conditions prevail, the graph in Figure 2 may not be as dependable as it has proved to be in Illinois. However, it is presented here in hopes that it will at least provide approximate recharge values suitable for initial planning and development purposes. Then, at a later stage, when dependable geohydrologic data needed to accurately define all controlling parameters are available, refinements of the original recharge assumptions still can be made in time to prevent serious overdevelopment of available groundwater resources.

5. THE ROLE OF OPEN WELLS IN OPTIMUM DEVELOPMENT

One of the major dangers in the world today is overdevelopment of ground water resources. It could happen anywhere, but I have seen cases in Asia where extensive construction of tube wells may create hardships for many farmers instead of benefiting them (10). Ground water lies close to the surface in this area and for years millions of farmers have irrigated small-acreage land holdings using large-diameter, low-capacity open wells which penetrate only the upper few feet of the aquifer. Now, small-diameter, high-yielding tube wells are being sunk deeper into these aquifers. They are much more expensive than dug wells and generally out of reach of the small-acreage farmers. As extensive tube-well development takes place, the water table will drop, drying up many of the dug wells just when these farmers need the water most for irrigation. Those who can afford deeper wells might possibly gain almost exclusive use of the ground water. The question arises whether

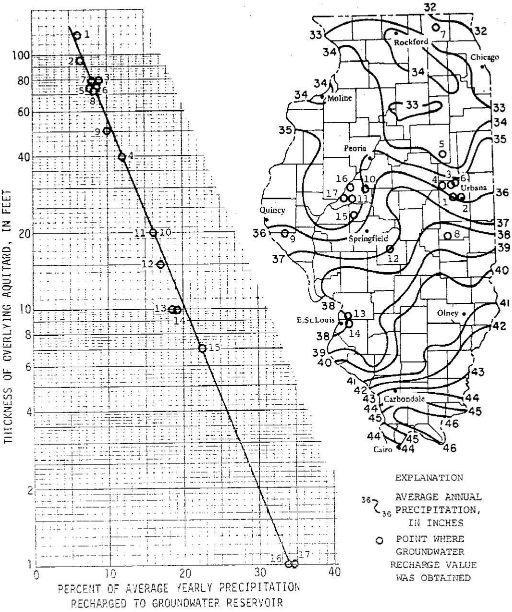

Figure 2. Ground Water Recharge – Aquitard Thickness Relationship for Unconsolidated Aquifers in Illinois

it is wise to permit vast numbers of small-acreage farmers to be forced from the land when the economies of cities cannot support continued influx of relatively unskilled labor.

Methods should be devised of rehabilitating and improving the large-diameter dug wells to improve their yields. Ground water hydrologists in such areas also need to determine the long-term safe yield of each ground water reservoir, taking into account yearly recharge from precipitation. Then the optimum number, spacing and type of wells can be determined for complete and equitable utilization of each aquifer.

Overdevelopment of such ground water reservoirs should not be permitted at this time. Such a drastic measure should be resorted to only if more food needs to be grown to avert major famine.

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A DISCONTINUUM APPROACH TO FLUID FLOW IN FRACTURED ROCKS

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The equations that govern the flow of water through fractured rock depend very closely on the scale of the fracturing with respect to the phenomenon being considered. For example, one geological situation could be viewed from a point of a continuum (say, the predicti..n of piezometric surface over a region) and the same system would behave very differently if one were to examine a local phenomenon (i.e., the performance of a single well). On a small scale, it is very important that the flow-pressure laws are re-examined with a view to a better understanding of the flow regime. Laboratory and field tests have been carried out and the results are incorporated in an analysis. The analytical approach changes as a result of looking at a discontinuum. Flow is confined to the fractures and the matrix is considered to be impermeable. Owing to the anisotropic, nonhomogenious and discontinuous nature of the problem with irregular fracture patterns, classical analytical methods have to give way to Numerical methods. The role of the Finite Element Method is discussed and an application of the method is made to a field problem.

Introduction Early work on the role of ground water, assuming an isotropic homogenious media led Karl Terzaghi (1958) to comment that, "the most unpredictable factor determining slope stability is the hydrostatic pressure in the water flowing out of a reservoir or a leaking pressure tunnel through the joints towards a slope, etc." However, in the past ten years, tremendous strides have been made in this field to justify confidence in analysis of problems where the fluid flow is confined to discrete fractures in hard rock. Since then, a vast amount of work has been carried out and a detailed account can be found in the Proceedings of a Symposium on Rock Mechanics held in Stuttgart (1972).

The percolation of water in fractured rock is primarily governed by the hydraulic characteristics of fractures and the imposed boundary conditions. Further, these characteristics are continuously changing as a result of changing hydraulic and stress fields. The detailed hydraulic characteristics of fractures are also significant in determining the quantity of fluid that can be removed from a rock mass.

A large amount of study on the flow of water through fractures has been based on the application of classical hydrodynamics applied to incompressible fluids. In these studies, it has been further assumed that the flow is laminar within the fractures such that the flow rate per unit width of fracture can be expressed as:

$$q = K_j\ i \qquad \dots. (1)$$

where i is the effective hydraulic gradient within the fracture

K_j is proportionality constant for the fracture under a given geometrical condition and for a given fluid.

If it can be assumed that the fracture is a parallel plate of constant aperture e, the exact solution for steady incompressible viscous flow between two plates is given by Lamb (1879):

$$q = \frac{e^3}{12n}\ .\ i \qquad \dots. (2)$$

where n is the kinematic viscosity of the fluid.

<u>Scale Effect</u> It is beyond the scope of this paper to consider the different types of fracture systems in terms of their genesis, continuity and importance to fluid flow. However, it is necessary to examine the role of scale in fracturing in attempting to analyse fluid flow through fractured systems. Rats and Chernyashov (1967) suggest a simple system based on the size of flow conduits:

(a) 3rd Order Heterogeneity: Form difference, size difference, pore distribution, presence of microjoints, etc.

(b) 2nd Order Heterogeneity: Heterogeneity of structure and composition of the rocks within the limits of one bench rhythm, interstratification of the rocks of different composition, macrojointing, small tectonic dislocation.

(c) 1st Order Heterogeneity: Heterogeneity of the rock massif, magma intrusion, degree of lithification, tectonic faults, zones of hydrothermal action, etc.

Figure 1 shows how the above concept affects the approach one has to take in analysing a problem.

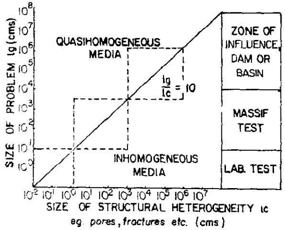

FIGURE 1

Another simple way of looking at the scale effect is to consider the flow of water from a borehole into a set of fractures Maini (1971). As the depth of the hole increases, Figure 2, the flow rate per unit length reaches an asymptotic value. In a homogeniously porous media, this value would always remain constant.

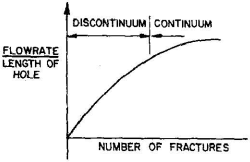

FIGURE 2

It has been demonstrated by Maini (1971) and Louis (1972) that the above factor is a serious problem in the field if due care is not taken in choosing the correct scale for a test with the final analysis in mind. Ideally in ground water basins, the best test would be to observe the piezometric surface under known boundary conditions over a period of time to predict the aquifer characteristics using a mathematical model (Bredehoeft and Pinder, 1970). Such methods do exist but still have to be proved in hard rocks.

<u>Hydraulic Characteristics of Fractures</u> Studies on simulated fractures have generally been modelled on the classical pipe flow experiments of Nikuradse (1930) and others under approximate parallel flow conditions. A notable extension is that of Louis (1967) who carried out experiments on parallel plates made from concrete with a wide range of surfaces and roughnesses. These results are generally correlated using a Reynolds Number-Friction Factor approach. Natural fractures vary widely as far as planarity and surface geometry are concerned. Bedding plane fractures in fine-grained sedimentary rocks may be relatively smooth and parallel; whereas, in granites, rough and meandring tension fractures are more common. The flow through a fracture is generally three-dimensional, as is clearly shown by laboratory tests by Maini (1971) on transparent replicas of rock fracture surfaces. Because of the highly irregular surface geometry of natural fractures, when two surfaces are in contact, dead water areas will tend to form where the opening is small. Often, these dead water areas tend to divide the fracture into discrete channels such that the flow is effectively concentrated within a limited area of the fracture. This effect is commonly observed in the field when boreholes penetrating essentially the same fracture produce vastly different quantities of water.

Little data is available on tests carried out on real fractures in the laboratory. Figure 3 is the result of a detailed series of tests carried out in the laboratory by Jouanna (1972) which shows the relationship between the flow rate and the applied hydraulic field under different normal loads. It is very important to notice (a) the Q-P relationship is nonlinear, and (b) it is highly dependent on the normal load. This implies that any change of stress on a rock mass as a result of, say, pumping or excavation or loading, plays an important role in controlling the flow regime. This factor will be further demonstrated in another section.

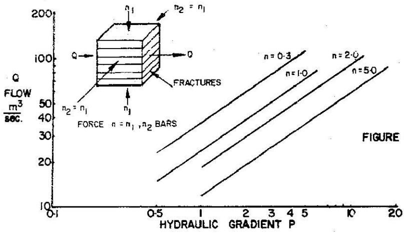

FIGURE 3 (JOUANNA)

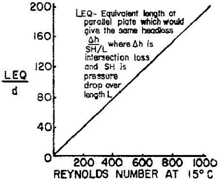

FIGURE 4 INTERSECTION LOSS (MAINI 1971)

Very often fracture sets occur in different orientations and, therefore, intersect each other. In order to find out if these intersections have any serious effect on the pressure distribution in a rock mass, laboratory tests have been carried out by Maini (1971) and Wilson and Witherspoon (1970). Figure 4 shows the results of such a test and it can be seen that under high gradients these losses could play an important role in controlling the flow regime.

Detailed tests on two- and three-dimensional models have been carried out by Maini (1971) and results compared with numerical solutions. The results of these studies show that only at high Reynolds Number do the intersection effects become important.

Numerical Solutions to Problems of Ground Water Flow

General Considerations In order to determine the steady-state distribution of pressures when fluids move through a deformable mass of fractured rock, it is necessary to consider the coupled action of flow forces, body forces and boundary loads. The pressure distribution must, of course, be compatible with the state of stress within the total system. The discussion that follows is restricted to the two-dimensional steady-state case, but the approach can be extended to three dimensions. One obstacle in developing three-dimensional programmes is that of computer storage, which at present seriously restricts the number of elements in a model. With the present rapid growth in the size of computers, it would appear that this restriction is steadily lessening.

The method used in this study employs two Finite Element techniques and a converging iteration process. The procedure is summarized in the form of a flow chart in Figure 5. One must first describe the geometry of the fractured system. This requires carrying out

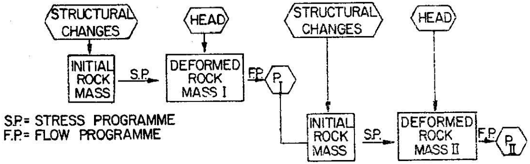

FIGURE 5 FLOW CHART FOR ITERATION PROCESS

sufficient field work so that the fracture sets, i.e., orientation, spacing and distribution of apertures, can be defined in space. One must also have a measure of the mechanical properties of both the intact rock and the rock fractures.

The stress-flow method of analysis to be described is not limited to problems of subsurface injection. The method can also be applied to dams, rock slopes, underground openings, etc. where problems on the effects of deformable fracture flow must be investigated The first step in the procedure is to determine the effect of the structure that is planned for the rock mass under construction. By making a stress analysis of this fractured rock mass, it is possible to determine how the introduction of a perturbation causes the size distribution of apertures to be modified. With a known geometry for the fractures, it is then possible to make a flow analysis from which one has the first approximation for pressure (P_I) at every point within the fracture system (Figure 5).

The second step is to repeat the stress analysis using both the effects of the initial structure and the first approximation for fluid pressure, P_I. This may result in a second modification of the apertures. By making another flow analysis, one then obtains a second approximation for pressure (P_{II}). If the difference between pressures P_I and P_{II} is unacceptable, the process is repeated using the loadings due to the planned structure and the second approximation for fluid pressure. This iterative technique is continued until the differences between two successive values for pressure are everywhere less than some arbitrary chosen limit. When this limit is reached, the fluid pressure distribution is compatible with the state of stress within the deformable rock body, and the effects of seepage throughout the fractured rock mass can be quantitatively evaluated.

The rate of convergence to an acceptable solution for this iterative process has been found to be quite rapid, at least for the initial problems studied. Figure 6 shows the maximum difference between successive calculations of pressure in one particular situation of a single injection well in an orthogonally jointed rock mass. The maximum difference is only 0.6% of the net available pressure by the end of the twelfth iteration.

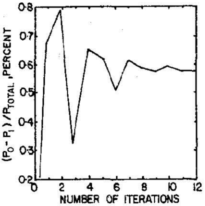

FIGURE 6. Convergence characteristics for test problem.

Procedure for Stress-Flow Analysis: As has been emphasized above, a realistic approach to the stress-flow analysis of fractured rock must take into account the two important factors of (a) fracture deformability, and (b) the coupling between flow pressure and rock stress. In using the perturbational Finite Element technique, the first factor is taken into account directly and the second factor must be satisfied by an iterative procedure that assures compatibility between rock stress and fluid pressure.

The behaviour of the fractured rock mass is governed by the two equations:

$$(K_f) \quad \{h\} \quad - \quad \{Q\} = 0$$

where (K_f) is the flow conductivity matrix

$\{h\}$ is the head vector for the network

$\{Q\}$ is the flux vector

$$(K) \ \{\delta\} - \{F\} = 0$$

where (K) is the total stiffness matrix

$\{\delta\}$ is the nodal point displacement vector

$\{F\}$ is the nodal force vector

These equations may be written more explicitly as:

$$\frac{1}{\gamma}\left[K_f(\delta)\right] \{P\} - \{Q\} = 0 \qquad K_f(\delta)\Big|_{\delta=0} = 0 \qquad \ldots. (3)$$

$$(K)\{\delta\} - \{F(P)\} = 0 \qquad F(P)\Big|_{P=0} \neq 0 \qquad \ldots. (4)$$

The first relationship (3) is the numerical formulation for the Finite Element flow analysis, in which the net effective head at any point in the system has been replaced by its equivalent pressure P. The new term $\left[K_f(\delta)\right]$ is used to indicate that the flow conductivity matrix is dependent on the fracture deformations. The second relationship (4) is the numerical formulation for the Finite Element structural analysis, in which $\{F(P)\}$ is used to indicate that the nodal force vector is dependent, in part, on flow pressure.

It is evident that equations (3) and (4) are coupled implicitly by $\{P\}$ and $\{\delta\}$. Furthermore, it should be remembered that (K_f) varies non-linearly with fracture deformation, i.e., (K_f) is a function of δ^3. As a result, the two equations cannot be solved simultaneously, and therefore an iterative scheme was developed.

The iteration procedure starts with an initial guess for pressure distribution. It is seen from (4) that $F(P) \neq 0$ when $P = 0$, and this suggested that a convenient starting point is to set $P = 0$, which is equivalent to having no flow forces initially in the system. By solving (4) with this assumption, we obtain the first approximation for $\{\delta\}$ which gives the deformations in the rock mass due to the effects only of the engineering structure. These deformations can then be used to modify the conductivity matrix (K_f) and (3) can be solved to obtain the first approximation for $\{P\}$.

In the second iteration, F(P) is first modified to include these new values of $\{P\}$, and the above two-step iteration is repeated to obtain the second approximation for $\{P\}$. This sequential process is continued until successive values of P at any node are less than some arbitrary limit. This assures a compatibility between fluid pressure and rock stress.

In applying the above method of stress-flow analysis, the following information is needed: (1) geometry of the fracture system, (2) initial distribution of apertures, (3) physical boundaries of rock mass from the standpoint of both flow and stress, (4) mechanical properties of intact rock as well as fractures, and (5) boundary conditions from the standpoint of both flow and stress analysis. Knowing the geometry of the fractured rock mass and the locations of boundary loads, two networks of elements must be prepared that are compatible. One network is used for the flow analysis and the other is used for stress analysis. In the former, fracture segments are represented by line elements with two nodal points. In the latter, fractures as well as rock blocks are represented by two-dimensional elements with four nodal points. The method of setting up these networks and preparing the data input to the computer programme that has been developed is described by Noorishad et al. (1971). The computer output provides details of stress, displacement, and fluid pressure at all nodal points of the network as well as flow rates at the boundaries.

The following will describe a few results that have been obtained by this coupled method of stress-flow analysis.

Results with Mutually Perpendicular Systems of Vertical Fractures Several numerical models have been analysed to determine the effect of fluid injection in a fractured rock mass. These cases examine the influence of joint orientation, permeability of the system, residual stresses and material properties of the rock mass. Before attempting to analyse the effects of injecting fluid into complex arrangements of fractures, we have first carried out preliminary studies on relatively simple models. Such studies will be useful in developing a better understanding of how to apply this computer approach to the complex systems that are being planned for further investigation.

The first problem considered was that of injection of a fluid into a vertical and orthogonal set of fractures. In this model, the initial joint aperture of each fracture was given a uniform value of 0.001 feet and the joint normal stiffness was set equal to that of the intact rocks, viz 10^8 p.s.f. The rock matrix was assumed to have a Youngs Modulus of 10^8 p.s.f. and a Poisson's ratio of 0.25. The angle of friction in the joints was set at 20^o and the tangential stiffness was 10^6 p.s.f. Figure 7 shows the water pressure distribution away from the injection point for both the rigid and deformable analysis.

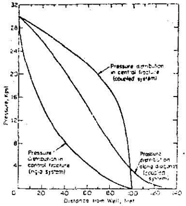

FIGURE 7. Comparison of pressure distributions for rigid and deformable fracture system.

A simulation of injection into a set of fractures inclined to the confining boundaries is illustrated (Figure 8). Figure 9 shows the result of injection into the

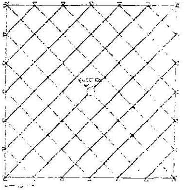

FIGURE 8. Model of square fracture oriented at 45° to confining boundaries.

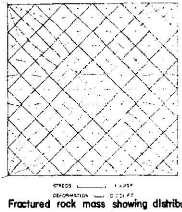

FIGURE 9. Fractured rock mass showing distribution of compressive stresses in rock elements with fracture system oriented at 45° to confining boundaries.

fractured system in terms of deformations and stress vectors within the rock mass. It is interesting to note how the fractures nearest the injection point open up and stress concentrations develop at the corners of the confining boundary.

Conclusions It is evident that analytical tools have been developed which can make use of the field and laboratory data that is far in excess of what is ordinarily available. It is essential that emphasis be placed on collection of detailed structural data, i.e., frequency and orientation of fractures and the presence of large-scale fractures, such as faults. Further detailed examination of flow-through fractures must be carried out in the laboratory for different types of rock. Theoretical studies are now under way at the University of California, Berkeley, to produce numerical techniques capable of solving transient and unsaturated fluid flow problems in fractured rocks.

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PROCEDURES AND EQUIPMENT FOR PUMPING AND FREE-FLOWING TESTS OF WATER WELLS

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SYNOPSIS

Testing of water wells still provides the best method of determining the hydraulic coefficients of aquifers and their confining beds, and determining the efficiency and capacity of production wells.

To ensure that data obtained from tests is accurate and interpretable by known analytical methods it is important that acceptable procedures be followed and that water levels and discharge are accurately measured. This paper briefly reviews the procedures for pumping and flow tests, the installation and operation of various types of pumps, and equipment for measuring discharge and water levels.

1. INTRODUCTION

The testing of yields of water wells should play a crucial role in the development of individual wells and in forecasting potential development of regional groundwater resources.

Provided testing is carried out according to acceptable procedures measurements are obtained from which it is possible to:

(a) Calculate the hydraulic coefficients of the aquifer and the confining beds - inputs for models and analogs for simulation of regional response of aquifers to various patterns of groundwater withdrawal.

(b) Determine the characteristics of production wells; to decide the acceptable yield to be pumped and the size and type of pump to be chosen.

This paper is concerned with obtaining accurate interpretable data, rather than its analysis.

Too often effort and time is wasted on pumping tests giving little or dubious data. To overcome this problem there is a strong need for manuals or handbooks which can be understood by drillers, field assistants, water-system operators, and pump manufacturers.

Some such manuals have already been published (1, 2, 3, 4, 5, 6, 7).

2. PROCEDURES FOR TESTING OF WATER WELLS

The principle of an aquifer test is rather simple. From a well water is pumped for a certain time and at a certain rate. The effect of this pumping on the water level is measured in the pumped well and in observation wells in the vicinity. The hydraulic characteristics of the aquifer are then found by substituting the drawdown measured in these observation wells, their distance from the pumped well, and the well discharge in an appropriate formula.

When observation wells are lacking, it is still possible to perform tests from which to give a value of safe sustained yield of the production well, its efficiency and also an estimate of transmissivity. A single observation well often permits calculation of transmissivity and the storage coefficient. The advantage of two or more observation wells placed at different distances from the discharging well is that the drawdowns measured in these observation wells can be analyzed by studying both the time-drawdown and the distance-drawdown relationships.

In selecting the position of observation wells it is important that they be sufficiently close to the control to give a significant drawdown. Generally observation wells will lie in the range 2 to 400 metres from the control well. The following tabulation indicates qualitatively the influence of variables on the positioning of observation wells, though it is preferrable to position observation wells from drawdowns calculated from estimates of transmissivity, storage coefficient and leakage.

	Aquifer type	Hydraulic conductivity	Thickness	Discharge
Observation wells close to control well	unconfined	low	thin	low
Observation wells distant from control well	confined	high	thick	high

For all tests it is desirable that:

(a) All relevant details be recorded on a form headed similarly to that shown below, and that this data and the analysis be kept on file for future reference.

Pumping test data

Well no. __________ Original water level __________

Distance from control well ________ metres

Date __ / / __ to __ / / __. Measured by: ______

Time of recording	Time since pump started (minutes)	Time since pump stopped (minutes)	Depth to water level (metres)	Discharge (litres/sec)	Remarks

(b) Standards of accuracy should be met, based on Stallman (6) those suggested are:

Control well discharge (± 10 percent), depth of water level (± 0. [illegible] distance from control well to each observation well ($\pm$ 5 percent), elevation of measuring point ($\pm$ 0.003 m), depth of well and screened intervals ($\pm$ 1 percent).

(c) The control well penetrate the entire thickness of the aquifer to be tested and screen at least 80 percent of the thickness of the aquifer.

(d) The control well be sited: away from pumping wells, which are likely to interfere with the test; where discharge can be readily disposed of without returning to the aquifer; away from the boundary of the aquifer.

(e) Observation wells should be tested for efficiency by adding a known volume of water and observing whether the initial rise of water readily dissipates.

(f) The well be developed so that change in its efficiency is unlikely to occur during the test.

(g) The water level be observed for at least several days prior to the test to note the antecedent trend needed to correct drawdown values.

(h) One or several brief preliminary tests be made to derive sufficient information about the time-drawdown relationship to choose a discharge rate which is sufficiently high to induce an appreciable drawdown, but sufficiently low for the water level not to drop down to the depth of the pump.

These preliminary pumping runs should be performed at least one day prior to the full pumping test to allow complete recovery of the water level.

For the actual test it is important that water-level (or discharge) measurements be taken for at least those intervals indicated as follows: 0, ½, 1, 1½, 2, 3, 4, 5, 6, 8, 10, 13, 15, 20, 25, 30, 40, 50, 60, 75, 90, 105, 120, 150, 180, 210, 240, 270, 300 minutes and then hourly. Immediately pumping stops, the recovering water level (or discharge) should be measured on the same schedule as that for the pumping period.

2.1 TEST PROCEDURES

2.11 CONSTANT DISCHARGE PUMPING TEST AND RECOVERY.

Designed with observation wells, the constant discharge pumping test is the most common test. The pumping rate is maintained at a constant level despite its tendency to drop slightly during the test as the pumping lift increases. It is therefore necessary during the test to repeatedly check and if necessary adjust the pumping rate by opening the discharge valve or by speeding up the motor.

2.12 DISCHARGE-RECHARGE TEST

The discharge-recharge test is a variation of the constant discharge test; but no pump is used. This test can only be applied where a natural hydraulic potential gradient exists between two superposed aquifers.

An intra-well device was designed in 1971 by Riha (9), which when installed in a specially constructed well can be used to maintain a constant discharge from the aquifer of higher hydraulic potential to that of lower hydraulic potential.

This method has the disadvantage of being difficult to instrument, and there is difficulty in maintaining constant discharge.

2.13 STEP-DRAWDOWN (MULTIPLE STAGE) TEST

This test (10, 11) is the most important type of pumping test where there are no observation wells. The well is pumped at 3 or 4 different rates for equal periods (e.g. 1 hour). Each pumping rate is greater than the one preceding it. Measurement of the water levels is taken for each step at time intervals given above.

It is possible to calculate well-loss and transmissivity, and predictions of drawdown for various discharge rates or duration of pumping.

2.14 FREE-FLOWING AND RECOVERY TEST

This test, sometimes referred to as the constant drawdown test, can be performed on naturally flowing wells tapping a confined aquifer. Observation wells are not essential. The free-flowing well is "shut in" until the head becomes virtually static, then the valve is opened and the declining discharge, rather than the drawdown, is measured.

The transmissivity and the storage coefficient are determined graphically from this data by equations developed by Jacob and Lohman (12). After the test the well is "shut in" and the recovery in head is measured to enable rechecking of transmissivity.

2.15 BAILING TESTS

Bailing tests are carried out with percussion rigs as aquifers are encountered; both to obtain water samples for chemical analysis and give an indication as to whether a detailed pumping test is warranted. Usually only a qualitative assessment can be made of the data obtained from a bailing test, because at this stage the well has not been screened and only a small proportion of the total thickness of the aquifer may be open to the well.

For that situation where the entire thickness of the aquifer is open to the well and the transmissivity is too low to warrant a pumping test it is possible to perform a bailing test and to calculate an approximate value for the transmissivity from measurements of the rate of recovery (13).

3. PUMPING EQUIPMENT

A variety of pumps are used in water wells. The main types are the single-stage centrifugal pump, multi-stage centrifugal pump, submersible pump, shallow and deep-well jet pumps, air pump and reciprocating pump. Comparison of pumps can be seen in Table 3.1.

4. DISCHARGE MEASUREMENT

Discharge can be measured by a variety of methods.

4.1 CALIBRATED DRUM AND STOP-WATCH

The flow can be calculated by timing the filling of a drum of known volume. This method is simple, but has the disadvantage of being impractical and inaccurate for large flow rates. It may be convenient to install a swinging discharge pipe rather than moving the drum for each measurement.

4.2 WEIRS

There are a variety of types, sizes, and shapes of weirs for which there are empirically derived formulae from which to determine the discharge from the measured crest over the weir. Two types of weir are considered here.

4.21 90° V-NOTCH WEIR

For discharges generally encountered in aquifer tests the 90° V-notch weir is the most versatile, being suitable for flows of 10 to 120 litres/sec

The weir should be composed of non-corrodible metal, be less than 1/16th inch thick with a sharp upstream edge and set vertically. The height of the bottom of the V-notch above the bottom of the channel on the upstream side should be not less than 30 cm for heads up to 23 cm or not less than 46 cm for higher heads. On the downstream side this distance should not be less than 10 cm for heads up to 23 cm.

The head, h cms, should be measured up-stream from the weir plate, at a point equal to 4 times the maximum head. The discharge Q litres/sec can then be calculated from the equation:

$$Q = 2.304\ h^{2.48}.$$

4.22 RECTANGULAR WEIR WITH DOUBLE-END CONTRACTIONS

The rectangular weir can measure the discharge in an open channel. For the rectangular weir with double-end contractions the length of the weir shorter than the width of the channel.

The weir should be vertical, 1/16th inch thick, the upstream edge horizontal and sharply cambered at 45° to the downstream side. The channel should be free of obstructions, at least three times wider than the length of the weir, l cms, and the distance from crest to bottom of the channel should be at least three times the head over the weir. The head, h cms, over the weir should be measured upstream from the weir plate at a distance approximately six times the maximum

TABLE 3.1 PUMPS

Type	Usual Maximum Yield (litres/sec)	Usual Maximum Lift (metres)	Advantages	Disadvantages	Common Use
Single-stage centrifugal	100s	6	Simple, fast installation	Small lift	Irrigation and town
Multi-stage centrifugal (shaft driven)	100	100s	Capable of large yields and lifts, main type of pump used for testing	Need straight and vertical well to minimise vibration	Irrigation and town
Electric submersible	50	100s	For deep wells costs considerably less than for multi-stage centrifugal	Need source of electricity, overheading at low speeds, yields less than centrifugal	Domestic, irrigation, town
Jet - shallow well	3	7		Low discharge and small lift	Domestic and stock
Jet - deep well	3	40		Low discharge and small lift	Domestic and stock
Reciprocating	8	100s	Simple, easy installation	Low discharge, mostly driven by wind and hence cannot regulate flow	Domestic and stock
Air lift	50	50	Simple easy installation	Water level cannot be measured unless have eductor pipe, need air compressor, difficult to control discharge	Well development

head over the weir.

The discharge, Q litres/sec, can be calculated from the equation: $Q = 3.06\ (1-0.1h)^{1.5}$.

4.23 ORIFICE METER

The orifice meter is a simple, compact, easily readable and installed method of measuring flows from pumping wells. It consists of an orifice plate clamped to the end of the discharge pipe to build up head in a piezometer tube tapping the discharge pipe at a known distance (2 feet i.e. 60.96 cm) back from the orifice plate. The orifice plate is of sufficiently small diameter to neatly seal the discharge pipe and be held in position by a standard pipe cap. The orifice itself must be centrally placed, circular, sharp-edged, and of sufficiently small diameter to cause a head in the piezometer tube which is at least 3 times greater than the diameter of the orifice. The piezometer tube should be transparent, of small diameter and attached to a nipple located in a straight and horizontal discharge pipe at the horizontal centre axis.

The head, h cms, in the piezometric tube is measured above a point level with the centre of the orifice. Then by knowing the diameter of the discharge pipe and the diameter of the orifice the flow rate, Q litres/sec, can be calculated from the equation: $Q = KA\sqrt{2gh}$

Where g cm/sec^2 is the acceleration due to gravity, A cm^2 is the area of the orifice, and K is the coefficient of discharge (1).

4.24 PROPELLER FLOW METER

A horizontal propeller-type flow meter can be incorporated in the discharge pipe. As water flows through the working chamber of the meter it rotates the propeller; this movement is transmitted mechanically to a counter which indicates the discharge.

It is important in the use of propeller-type flow meters that:

(a) The gate valve should be located in the discharge pipe down from the meter.

(b) If sand is present in the pumped water the meter should be flushed out with clear water after each test.

(c) The meter be frequently recalibrated.

5. WATER-LEVEL MEASUREMENT

In a pumping test the depth of the water level in affected wells must be measured accurately before, during and after the test.

A variety of water level indicators and recorders are suitable for use in water wells (1, 14, 15). They are remote (surface) or downhole, and may be mechanical, electronic or electro-mechanical.

A number of the more common methods of water level measurement are mentioned below, and comparison is made in Table 5.1.

5.1 CHALKED TAPE

The chalked tape method is one of the simplest to measure the depth

TABLE 5.1 WATER-LEVEL MEASUREMENT

Type	Recorder	Pumping well	Advantage	Disadvantage
Chalked tape		x	Precise, simple	Slow since must remove from well to read.
Fox-whistle			Simple, adequate precision, rugged	When other noise it is difficult to hear
Float	x		Precise, suitable for permanent installation	Need straight and vertical well, metal floats corrode, floats sometimes stick, care to avoid slipping over pulley
Pneumatic	x	x	Fast, simple, precise	Need to keep fittings airtight, for recorder need Bourdon-tubes of different ranges, surcharge in deep wells
Electric	x		Rugged, simple, precise	Sometimes shorting of circuit - mainly between probe and well casing, care to avoid slipping where measuring wheel used.
Pressure Transducer	x	x	High-speed response	Needs frequent attention, costly, not robust.
Mercury			Suitable for free-flowing well	Not suitable where head below ground surface

to water. A steel tape weighted at the lower section and coated with carpenters chalk is lowered into the water. The depth to the water is then equal to the reading on the tape at the measuring point minus the depth to the clearly indicated point of immersion on the tape.

5.2 FOX-WHISTLE SOUNDER

The fox-whistle sounder is made from an ordinary fox-whistle soldered across the end of a short length of metal tube of identical diameter.

The sounder is lowered down the well and when it strikes the water the air in the sounder is compressed and escapes through the fox-whistle emitting a whistle audible to the operator who then notes the depth to the water level on the tape.

5.3 FLOAT-TYPE INDICATOR AND RECORDER

Both the float-type water-level indicator and recorder include a float sufficiently small to travel down a well. For the indicator the float is connected to a thread which moves over a measuring wheel thereby indicating the depth, whereas for the recorder the thread passes over a pulley in the recorder and the float is balanced by a counterweight. Any movement of water is transmitted via this system to a stylus which traces the fluctuations of water level on a chart moving at constant speed. Gears are used to alter the depth or time scale.

5.4 PNEUMATIC INDICATOR AND RECORDER

The pneumatic water level indicator is simple. It consists of a 0.5 cm semi-flexible plastic tube, a source of pressurized air and a pressure gauge. The tube is strapped to the pump column at regular intervals and immersed in the water at a known depth below ground level. Air is forced down the tube until there is no further increase in pressure on the gauge, indicating that water has been completely driven from the airline. By multiplying this pressure (in kilograms per square centimetre) by the conversion factor 10.0, the head of water (in metres) above the bottom of the airline is determined. This process is repeated throughout the pumping test.

The pneumatic-type water-level recorder records the water level continuously. This is made possible by a rotometer and air purge system which regulates an even flow of air through the airline regardless of back-pressure changes through water-level fluctuations to a Bourdon-type tube under changes of pressure activates a pen which traces the head of water on a moving chart.

5.5 ELECTRIC PROBE

Water level can be detected electrically by fluid conductivity, self-potential, capacitance and inductance. The two common types depend on fluid conductivity; they are the single-electrode type in which only one electrode makes contact with the water and the circuit is completed through ground, and the two-electrode type in which the circuit is completed by two electrodes making contact with water. Both types include a spool and carriage on which are installed the electrical components. Current is supplied to the circuit by a battery through a variable resistance which can be adjusted for the desired deflection depending on the salinity of the water.

5.6 PRESSURE TRANSDUCER

A pressure transducer converts pressure changes to electrical changes; either by converting the output from the transducer to a proportional voltage or current, or into a proportional frequency. Several types are manufactured commercially specifically for the measurement of water-level fluctuations in wells.

5.7 MERCURY GAUGE

A mercury gauge can be used to measure the artesian pressure of a flowing well. It consist simply of a transparent and graduated U-tube partly filled with mercury. This tube is connected to the otherwise sealed top of the well. To calculate the head in metres of water above the measuring point, the difference in the arms of the U-tube, in centimeters of mercury, is multiplied by 0.13596.

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POTENTIAL AND FLOW FIELDS FOR MULTIPLE GROUND WATER WELLS IN A CONFINED AQUIFER [1]

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SYNOPSIS

An analytical method is used to determine the well discharge, streamlines and equipotentials for wells located in confined aquifers of arbitrary shape. The well discharge is found from the potential function, which is derived from Laplace's equation and the boundary conditions. A modified Gram-Schmidt method is used to help satisfy the boundary conditions. The lateral boundaries of the aquifer can include nonconducting segments. Data from the Ames Aquifer, Ames, Iowa, provide a check on the method. Calculation of steady state drawdown curves of tube wells in semiconfined aquifers is reported. Theory of seepage flow to kanats (underground water tunnel systems) is noted. The methods of analyses reported in this paper enable calculations of well discharge and other hydraulic quantities when aquifer characteristics are known. Such hydraulic information is basic for effective development of irrigation water, especially in countries where increased agricultural production is essential.

1. INTRODUCTION

This paper summarizes some recent work done at Iowa State University, U.S.A., on the development of groundwater from confined aquifers. Some unpublished and some published results will be cited.

The simplest problem of confined flow to a well is that for a well located in the center of a circular aquifer. This problem is considered in most elementary textbooks on groundwater hydrology and it can be readily solved by analytical methods (1). More advanced textbooks (2, 3) contain analytical solutions for aquifers where a single well is located other than at the center. Polubarinova-Kochina (4) provides an analytical solution for horizontal flow to a well at the center of an elliptically shaped confined aquifer. These solutions have important theoretical significance and are practical insofar as the theoretical geometries approximate actual geometries.

The principle reason that real pumped aquifers have been resistant to mathematical analysis is that a single solution to Laplace's equation must be found that will satisfy all the boundary conditions imposed by the natur-

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al aquifer. Powers et al. (5) developed a modified Gram-Schmidt process to satisfy the boundary conditions of such problems as exactly as the boundary conditions are known, and this process, which is described in detail in Kirkham and Powers (6) has been applied to a number of groundwater flow problems (5, 7, 8, 9, 10, 11, 12) in research at Iowa State University.

2. SINGLE WELL SYSTEMS

Fig. 1(a) shows the cross-sectional view of a confined aquifer taken of elliptical shape and chosen for its simplicity in developing our method.

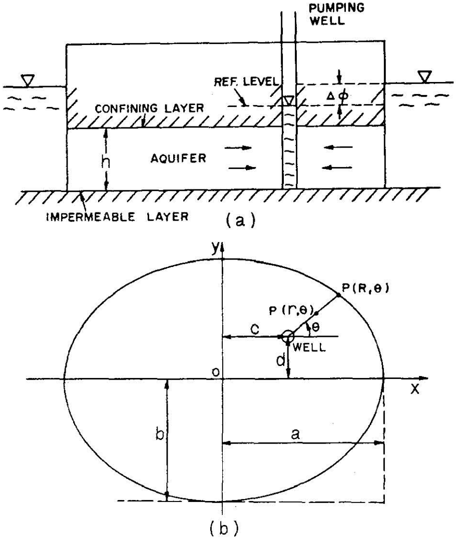

Fig. 1. Geometrical representation of the flow region: Part a, cross sectional view; part b, plan view.

Physically our problem can correspond to a hypothetical ellipse-shaped island with a confined aquifer of the same shape fed by the water surrounding the island. The problem can also correspond to a pumped well in a confined aquifer which has an ellipse-shaped equipotential somewhere in the field that surrounds the well.

Fig. 1(b) shows the plan view of the aquifer and well. The level of hydraulic head is assumed to be constant at the perimeter of the ellipse. This level is at height $\Delta\phi$ above the level of the water in the well. The major and minor axes of the ellipse are a and b, the coordinates of the well center with respect to the ellipse center are (c, d); r and θ are polar coordinates with R the value or r at the ellipse boundary; r_w (not shown) is the radius of the well; h is the aquifer thickness.

Fig. 2 shows the flow net for a confined elliptical aquifer when the well is located on the major axis. In this case the dimensions are a = 1, b = 0.5, c = 0.5, d = 0, and the radius r_w of the well is given by r_w = 0.02. The length units are arbitrary.

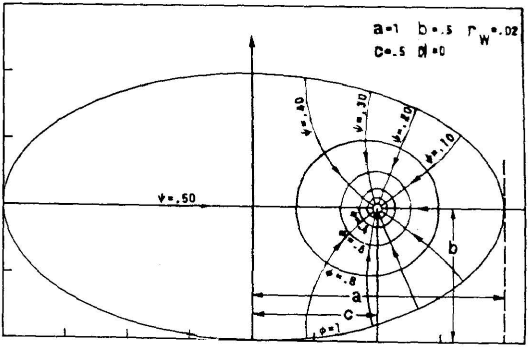

Fig. 2. Flow net for a confined elliptical aquifer with the well located on the major axis of the ellipse, where a = 1, b = 0.5, c = 0.5 and r_w = 0.02.

We have computed well discharges for a number of geometries where a, b, c, d, and r_w differ. Table 1 gives values of the dimensionless discharge, $Q/Kh\Delta\phi$, for a number of cases corresponding to Fig. 2. Here K, dimensions L/T, denotes the hydraulic conductivity of the flow medium and Q denotes the actual well discharge (L^3/T).

Table 1. $Q/Kh\Delta\phi$ values for a confined elliptical aquifer, with the well located on the major axis of the ellipse.

r_w/a	c/a	b/a = 1.0	b/a = 0.5	b/a = 0.2	b/a = 0.1
		$Q/Kh_{\Delta\phi}$			
1/100	1/2	1.455	1.607	2.039	2.585
	3/4	1.663	1,790	2.249	2.889
	7/8	1.985	2.096	2,561	3,33
1/400	1/2	1.102	1,186	1.406	1.648
	3/4	1.216	1,283	1,503	1.768
	7/8	1.381	1,433	1.638	1.93
1/4000	1/2	0.785	0.827	0.928	1.028
	3/4	0.841	0.873	0.969	1,073
	7/8	0.917	0.940	1,026	1,132
1/40,000	1/2	0.609	0.644	0.693	0.747
	3/4	0.643	0.661	0.715	0.771
	7/8	0.687	0.699	0.744	0.801

Fig. 3 illustrates a flow net when the well is at an arbitrary location within the elliptical aquifer. Actual geometry is indicated on the figure. Values of the normalized stream functions and equipotentials are shown.

Having learned how to mathematically calculate the well discharge and the flow nets (12), for a single well in an ellipse-shaped aquifer, we attacked the problem for a single well in an irregularly shaped aquifer. Fig. 4 shows the flow net. In this net we see from the way the streamlines and equipotentials intersect at right angles (orthogonality) that the theory has provided correct results. This figure was computed for a constant hydraulic head over an irregularly-shaped boundary. Our method enables the computation of the flow field when the hydraulic head may have values that change from point to point over the external boundary. The external boundary may be chosen arbitrarily. For the procedure of the com-

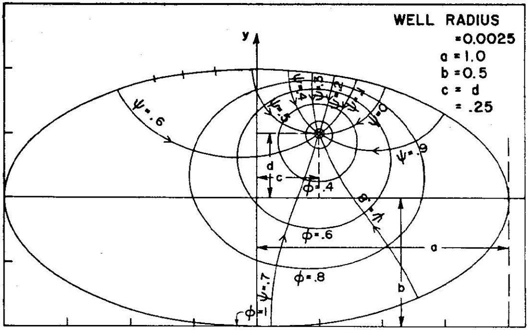

Fig. 3. Flow net for a confined elliptical aquifer, with the well not located in the center of the ellipse nor on an axis, with $a = 1$, $b = 0.5$, $c = d = 0.25$ and $r_w = 0.0025$.

putation, see van der Ploeg (13), and Selim and Kirkham (10, 11).

3. MULTIPLE WELL SYSTEMS

Muskat (2) gives equation for computing flow for systems of wells in a confined aquifer of circular shape. In our method we can compute the flow for multiple wells in an irregularly shaped aquifer. Fig. 5 is an example for two wells. The flow fields become more complex for more wells, but they can be computed.

4. WELL SYSTEMS WHERE PARTS OF THE OUTER BOUNDARY OF THE AQUIFER ARE NONCONDUCTIVE

To develop our method for predicting well discharge and computing flow nets when part of the external boundary of the flow system is nonconductive, we worked first with circular outer boundaries and Fig. 6 shows the flow net when one-fourth of the boundary is impervious, and Fig. 7, when one-half is impervious.

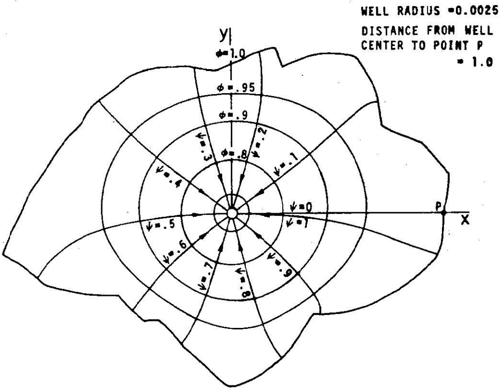

Fig. 4. Flow net for an irregularly shaped confined aquifer. The distance of the well center to the point P is taken as 1, the well radius r_w = 0.0025 times this unit distance.

A comment may be made for the well discharge for Figs. 6 and 7 and for a circular aquifer with no portion of the boundary impervious. We let Q(0), Q(1/4), and Q(1/2) be the well discharges when none of the boundary, 1/4, and 1/2 of the boundary is impervious. Then for a well radius of 0.0025 units and external boundary radius of 1 unit, we find the following proportionality:

Q(0):Q(1/4):Q(1/2) as 1.049 : 1.020 : 0.938

Without presenting other calculated flow nets for further idealized aquifers where part of the external boundary is impervious we go directly to a field situation.

5. A FIELD TEST

Fig. 8 shows a flow net for the Ames Aquifer, Ames, Iowa, U.S.A., when one of the wells (No. 9) alone was pumped during an experimental pumping

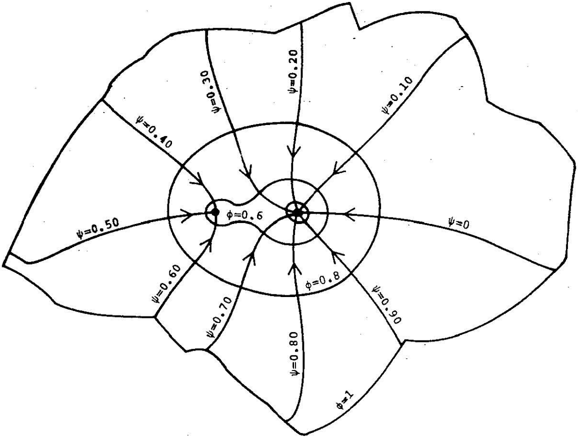

Fig. 5. Flow net for an irregularly shaped confined horizontal aquifer pumped by two wells of unequal strength and unequal radius. Distance between the wells is 0.3 length units. The left well is of radius 0.0025 and the right of radius 0.01. Discharge of the left well is one-half that of the right well.

test. The flow net was experimentally calculated from the measured equipotential values along the boundaries and from knowledge (14) of a barrier portion of the aquifer at the upstream end. Squaw Creek and the Skunk River partially surround the area as shown. The highest hydraulic heads are 880 feet (268.22 meters) above sea level at the upper end of the aquifer and about 870 feet (265.18 meters) above sea level at the lower end of the aquifer. The drawdown level of the water in the pumped well was at a height 865.3 feet (263.44 meters) above sea level which represented a 9.2 foot (2.80 meters) drawdown from a non-pumped condition at the well. The flow net was theoretically calculated with the barrier being taken into account. The well radius including gravel pack was 2.5 feet (.762 meter) and the pumping rate was 1280 gallons per minute (.08367 cubic meters per second).

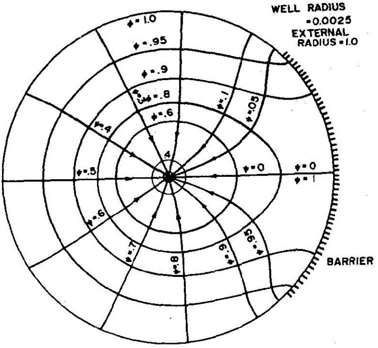

Fig. 6. Flow net for a confined circular aquifer, of which one-fourth of the outer boundary is impervious.

Using a measured average transmissibility of 200,000 gal/day/ft (.02876 m^3/sec/m) (14,15), we calculated the discharge of the well at the 9.2 foot (2.80 meter) level of drawdown to be 1163 gallons per minute (.07339 cubic meters per second). The difference between this calculated value and the measured value of 1280 gallons per minute (.08367 cubic meters per second) was about 10 percent.

Further pumping test data are being analyzed for this aquifer and, in particular, for more than one well being pumped. We would hope that our computational methods might be field tested in India and in other parts of the world.

6. WELLS IN SEMICONFINED AQUIFERS

Although our title refers to confined aquifers, we wish to mention some additional work on semiconfined aquifers. Wells in semiconfined aquifers often penetrate through water-bearing strata of different hydraulic conductivities. These wells are used to draw down the water tables for drainage purposes and at the same time, the wells can provide water for irrigation when the water is not too saline. Khan et al. (16) and Khan and Kirkham (17) have calculated the water table shape and flow nets for a

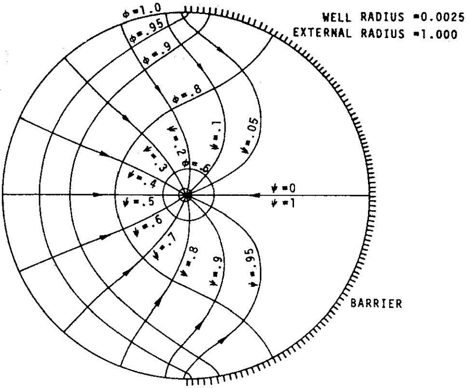

Fig. 7. Flow net for a confined circular aquifer, of which one-half of the outer boundary is impervious.

number of average steady state recharge conditions and for two and three layer aquifers. The study was made with the many tube wells of the India-Pakistan region in mind. This work has been published and details are available in the above cited references.

7. KANATS

A student of the senior author has noted that some of our recent drainage theory applies to the seepage of water through sloping hillsides to kanats. Kanats (18) connect the bottoms of shafts found over the high central valleys of Iran. They conduct water from the mountains to the valleys for irrigation. Kanats are dug by hand over long periods of time and can reach a maximum depth of about 500 feet (152.40 meters). Tolman (18) cites that a kanat at Mazandaran is 10,500 feet (3200.4 meters) long and 42 feet (12.80 meters) below the surface. Vertical shafts, about 60 to the mile (37 to the kilometer),-connect the tunnels with water-bearing strata below the valley floors. Some of the water flows directly through these shafts to the subterranean tunnels, some of the water seeps directly to the subterranean tunnels through the porous medium, which is generally

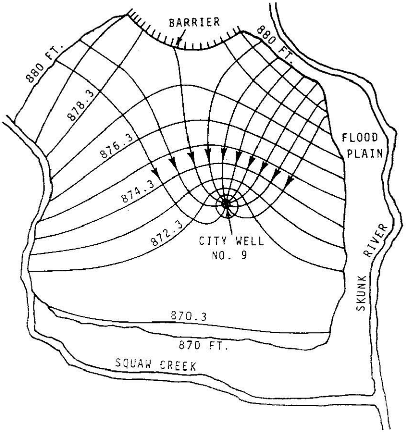

Fig. 8. Flow net for the Ames Aquifer, when the aquifer is pumped by well No. 9.

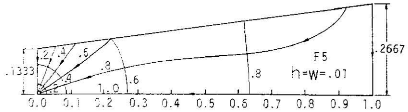

Fig. 9. Semisection of a flow net showing seepage from a hillside to an underground tunnel, as in a kanat.

conglomerates cemented by silts and calcareous material. It is the latter seepage water to which our recently developed theory might apply, and flow to a subterranean tunnel is shown in Fig. 9. The figure is taken from Powell (19) and could apply to a subterranean kanat system having a tunnel of semicircular shape with the tunnel bottom on an impermeable layer. The semicircular shape and impermeable layer are used for convenience of calculations. When the tunnel is of circular shape and is not near an impermeable barrier, the problem is more difficult but we know how to solve it.

8. CONCLUSION

For homogeneous horizontal confined aquifers we have presented sample flow nets and well discharges calculated by use of a recent analytical method. The method applies to both single and multiple wells.

With this method, knowledge of the hydraulic head at arbitrary points around the well enables a computation of the flow net and well discharge. The calculations can also be carried out when some of the aquifer boundary is nonconducting. However, then the shape and extent of the nonconducting part of the boundary must be known. For any aquifer, the transmissibility must be known to make calculations. The recent work presented differs from earlier work in that irregularly shaped boundaries with irregular hydraulic head distributions can be dealt with analytically.

Acknowledgment. Work supported in part by U.S. Department of Interior Office of Water Resources Grant B-019-IA.

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SPACING OF WATER-WELLS IN DECCAN TRAPS

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SYNOPSIS

Groundwater investigations in a hard rock area are beset with many uncertainties. Much of the present exploration is based on broad principles. Deccan Trap being an impervious rock the chances of seepage of water in the sub-surface are controlled by the fissures, joints, inter-trappean soft rock if present, weathered portion of the trap rock, gap between two successive flows, etc. These features are too broad and often do not help effectively in selecting sites for individual wells.

Since lately, and particularly during the last few years of drought, there has been a spurt in the activity of sinking wells and drilling tube wells in the country. If all these wells were to be properly logged and tested and the data studied, much light will be thrown on the behaviour of groundwater in this rock. But such data is not yet available.

Government agencies have provided large financial credit facilities for digging new wells and for re-vitalising old wells. The stipulations require that the yield expected from each well will be 150 cubic metres per day (in December) for new wells and 120 cubic metres per day (in December) for revitalised wells. Also, no new wells should be made within 300 m. of the existing wells. The recharge of rain water to groundwater is taken at nine per-cent. These guide lines are intended to ensure a full development and also a feasibility of economic return from the agricultural produce to repay the loan in the stipulated time. No specific data is available to determine a rational spacing between wells. Some experiments were carried out on pumping of wells and observing the effects on adjoin -ing wells. These indicate that in two wells 50 metres apart, there is no interference. It is, therefore, suggested that the safe spacing of wells may be about 150 metres (500 ft.)

1. INTRODUCTION

The general principles of the behaviour of rainwater on the surface and in the sub-surface are known. Deccan Trap when fresh and unaltered has no porosity and hence is impermeable to water. The chances of percolation, storage, underground movement etc. of water are primarily controlled by fractures and joints in the trap, inter-trappean soft rock material or gaps or weathered portion of the trap flows, both on the surface and in the sub-surface.

These principles are so broad that they do not help effectively in selecting sites for individual wells. Where exactly are these fissures ? What is their size and how they are oriented ? Are there any features, which pass from flow to flow in depth ? How can we diagnose these from the surface ?

The distinction between hard rock and soft rock is indeed very spectacular in the field of geohydrology. Porosity is a basic character of soft rocks while the lack of it is that of hard rocks, exceptions apart.

Geophysical methods do come in handy especially the seismic and the electrical but, here again, the effectiveness for siting individual wells is very limited, quite apart from the additional expenditure such methods call for. During the last drought these methods were almost conspicuous by their absence. Only a few agencies were able to employ them in a very few, almost exceptional cases.

In a country-wise analysis in India it is noticed that hard rock areas have not been studied to the extent possible and desirable. The study group of the Planning Commission Government of India (1) in their report (1971), have high-lighted this fact on the basis of their observation that not more than 17 per cent of the hard rock areas had been studied systematically. The situation has not materially changed since.

Consequently we have to remain content with the old ideas which have been prevailing and guiding our exploration in these rocks.

During the last 10 years there has been a spurt in exploration for groundwater. Most of the states in India have started separate wings for groundwater. The drilling operations for water have been intensified countrywise. Our potential of drilling tube-wells has increased manifold of what it was a decade back. Even so there is not much augmentation of water, commensurate with drilling effort. The hard rock areas still remain substantially at the mercy of rains.

The spurt during the last few years in digging wells and boring tube wells has now piled up voluminous data. If the results obtained from these could be studied carefully much scientific data will be available to improve upon our age-old tenets. It will be possible, for instance, to correlate petrological characters of hard rocks and their susceptibility to weathering, establish relation between topography and the extent of weathering, etc. Seismic methods which have so far been only indicative in a broad measure can be used to verify the results of these wells and tube-wells, to perfect, or at least improve upon, our ability to predict sources of groundwater more reliably.

These wells and tube-wells can be subjected to detailed pump-tests to ascertain the rate of percolation or recharge, specific yield of formation, total quantity of water available in any localised area, the aquifer performance etc. In the absence of this statistical data and its interpretation the exploration trend will continue to be based upon approximation, rather than upon precision derived from such a study.

2. WELL SPACING

The present knowledge of the behaviour of the Deccan Trap with regard to groundwater is thus inadequate. The geohydrological literature in India contains very few studies dealing with the results of elaborate pumping tests to determine the aquifer characters, total resource of water, the safe rate of withdrawal from wells etc.

The question of drinking water in a drought-prone area is quite serious in as much as water must be obtained at any cost. Normal considerations of economics do not apply to this activity. Against this the wells or tube-wells for irrigation needs must be economically viable and be able to repay the investment on their construction.

Special funds have been made available through - Governmental agencies, as loans to cultivators for development of groundwater by sinking new wells or by revitalising existing ones. The guidlines for these loans are generally as follows :-

i) The minimum discharge (December) expected from the wells dug will be 150 cubic metres per day (approximately 16 acre feet per year) in the case of new wells and 120 cubic metres per day in the case of revitalised wells.

ii) The new wells to be dug should not be less than 300 m. (1000 ft.) away from the existing wells. The recharge of rain water to the groundwater is here presumed to be nine per-cent.

The recharge of nine per cent is indeed an arbitrary figure. Das (2) has evaluated this at 10 per cent. It is obvious that the recharge to groundwater will depend upon several factors such as the total rain-fall, intensity of the rain-fall, number of rainy days, ambient temperatures, topographic gradient, susceptibility of rock to weathering, etc.

The amount of recharge will naturally vary from place to place and to some extent from year to year. In course of time, when more data is available, it would be possible to evaluate this factor more accurately for any given area. Till such time, it is necessary to assume a reasonable figure for being made applicable to the vast area of the Deccan Trap (512000 sq.km.) For this the figure of nine per cent is as good a guess as any other.

Regarding the requirement of the discharge of 150 cubic metres per day the figure is arrived at from the possibilities of the financial returns from the agricultural products raised by the well. While by economics this may be a reasonable proposition, the figure appears to be on a higher side for the general potentiality of the deccan Trap aquifers. However, this remains to be seen. The results of these wells, vis-a-vis, the crops raised by the beneficiary of the loan, will prove, in course of time, whether or not the figure is justified. Both these factors do not materially affect the plans or their execution for siting wells for farmers who wish to obtain the readily available loans.

The third factor, namely, the spacing of wells - not less than 300 m. (1000 ft.) away-needs some consideration from the hydrogeological point of view. This stipulation appears to have been arrived at on the basis of a general principle that the input of water in the formation and its output through wells, should be equal. This means that whatever the amount of water recharged to the underground source, the same may be withdrawn from that area. The stipulation of spacing is probably derived from the considerations of recharge to the basin, water requirement for the average cropping pattern, cropping intensity and the average area irrigated by the well.

The water requirement of the average crop is a factor that will vary from place to place in the region, the cropping intensity will depend largely upon the availability of groundwater. As more and more groundwater will become available the intensity of crops will increase leading to a possibility of over development, unless controlled at the proper time. The average area irrigated by a well will also vary according to the availability of water in the different parts of the year and according to the nature of the crop, availability of fertilizers, pesticides etc. It is conceivable that even if the annual draft of the well is less than 150 cubic metre per day there may be adequate returns from the agricultural produce from that well to repay the loan. This again will depend upon the nature and market value of the crop raised. The cash crop may be able to repay the loan even if the well yielded less than 150 cubic metres per day.

The principle of the input and output of water to be equal, also needs examination on geological grounds. In Deccan Trap the groundwater is usually on the move along the hydraulic gradient unless stopped or held up by the rocks for want of fissures for its passage. A total absence of such fissures is observed only in rare cases. Consequently even if we assume the recharge to be nine per cent, over the whole area of a valley, the groundwater, moving progressively in the direction of the surface flow of streams or rivers, is enriching the topographically lower parts. Thus the rainfall remaining the same all over the area, the enrichment of groundwater is accentuated towards the lower parts of a valley or basin.

If we consider a river basin in, say, three parts, the upper third A, the middle third B and the lower third C, and assume the rainfall to be same over the whole basin and therefore the rate of recharge also the same, the part A will be progressively yielding its groundwater to part B, the latter being topographically lower. The Part B, in its turn, will yield its groundwater to part C. If the groundwater in part C is not affluxed by any underground barrier, like a fissure-free flow, this water will also continue to flow to the lower parts, possibly into the next lower valley or emerge as springs. Thus at any given time the distribution of the ground water in any valley will be uneven in its different parts. This being so the principle of input equal to output will not hold good. It is a common observation that longitudinally in a valley less water is available for withdrawal from the upper parts and maximum from the lower parts. The accumulation of groundwater is not equal in relation to the recharge.

In a flat valley the situation will be different in that the groundwater will tend to stagnate in flatter or level portions. Further, this distribution will also be erratic if the sub-surface trap flows vary in their hydro-dynamic properties.

If, therefore, we intend to tap all the groundwater (contributed by the annual recharge), the intensity of wells will have to be the lowest in the upper reaches and highest in the lower reaches of the valley. The problem in development of groundwater is that, as the exploration advances and more wells are sunk, the topographically lower parts may have more water than can be gainfully utilised by the arable land in the vicinity while the topographically higher parts will remain undeveloped or under-developed. Apart from such an eventual disparity, between the available groundwater and the available land above, the formula of uniform spacing of 300 m will not hold true. The area will admit more points of penetration (wells) for maximum withdrawal, if the rate of withdrawal of 150 cubic metres per day is to be maintained.

The assessment of groundwater in the Deccan Trap is indeed a difficult task. It is certainly not amenable to mathemetical calculations with our present scanty data. However, in the light of the above considerations, it is desirable to approach this problem from another angle as well.

Since the groundwater in the Deccan Trap is not uniformly distributed in the sub-surface and since it is not always possible to know exactly where it is, it is desirable to presume in the earlier stages of development, that it is more or less uniformly distributed in the subsurface until it is proved otherwise by actual experience of dug wells or tube wells. It requires efforts to determine which parts are water-bearing and which are not.

If it is intended to exploit all the available groundwater (replenished annually) then we may put two adjacent wells in such a manner that they do not interfere in the production of each other for maximum withdrawal.

Many observers have noticed that in most areas of Deccan Traps two wells separated by varying distances of 150 m. 100 m. or even 50 m do not interfere with each others production. Presumably there is a barrier between the two wells and that they draw from different branches of sub-surface flows or water bodies. However, such sporadic observation cannot, by themselves, justify formulation of a policy for such an important issue as spacing of wells for large scale development.

Some data has been collected in the past one or two years. In a typical area where well density is fairly high, that is, the distances between the wells is less than 300 m. pumping tests were carried out by the AFPRO Groundwater Investigation Team, Maharashtra. The effects of pumping were observed in terms of depression of water table or water surface, in adjoining wells.

The results indicate that no interference is observed between two wells which are only 50 m apart. Out of 14 wells which were within 50 m., only five (36%) showed interference. Out of 31 wells within 100 m., 11 wells (about 35%) were affected; out of 57 wells within 150 m., only 14 wells (24%) were affected; out of 74 wells within 200 m., only 17 wells (23%) were affected. Beyond 200 metres there was no interference.

It may be mentioned here that the area of the experiment is in Ahmednagar district which has an average of 450 to 500 mm. of annual rainfall and is classed as a chronically drought affected area. The groundwater resources are comparatively poor, in relation to other areas of the Deccan. The results also bring out a significant point of departure between 100 to 150 m. distance. Here the percentage of interference is reduced from 35 to 24.

These results can be interpreted otherwise to show that out of 14 wells within a distance of 50 m. nine wells (64%) were not affected. Similarly out of 31 wells within a distance of 100 m. 20 wells (65%) were not affected. Out of 57 wells within a distance of 150 m. 43 well (75%) were not affected. In the next group within 200 m. out of 74 wells, 57 wells (77 %) were not affected. It is significant that within the range of 150 m. 75 per cent of wells were not affected while within the next range of 100 m, 65 per cent of the wells were unaffected. Therefore, it would be safe to presume that in this area there will be no material interference between wells 100 m apart.

In their normal operations, it is expected that adjacent wells will be under pumping simultaneously for a significant part of a day. Therefore the adjacent wells, if separated by a distance of 200 m. that is twice the distance of interference, there will be no significant adverse effects upon production of both the wells. It would be safe to site new well at a distance of 200 m. from the existing wells. The intereference in these cases will be only marginal, about half way between them, if the peak period of withdrawal coincide.

The need to reduce this stipulation of 300 m. is significant from yet another vital aspect, namely changes taking place in our agricultural pattern. The land ceiling introduced recently has fractionated the arable land between more owners than before. Every new owner wishes to have a well within his newly acquired land. In many cases such fractionation does not admit of putting a new well, more than 300 m. away. Consequently, there is a possibility that some portions of the land, which was cultivated under the previous owner, will not be cultivated under new ownership for want of source of water. If this distance is reduced to 150 m. (500 feet) there will be a relief in this direction and the object of stepping up of agricultural production will be accomplished. It may be added here that Adyalkar and Mani (3) have assumed valves of 300 to 500 feet within which there is no interference in wells in trappean acquifers.

3. AUGMENTATION OF RECHARGE.

It is significant that simultaneously steps are being taken to increase the recharge by contour bunding, nala bunding, percolation tanks, dams, weirs, etc. By these means local groundwater sources are augmented (4) and more recharge to groundwater can be expected in course of time. Such areas will admit of closer pattern of wells as recharge to groundwater increases.

The purpose of this paper is to stimulate discussion on the problem of spacing of wells in the Deccan Trap. Admittedly pump tests described above leave much to be desired. But in the absence of any other criteria even such tests can afford a much needed guidance. The problem is vast and of multiple dimensions. Solutions will emerge as more and more scientific data become available and are subjected to scrutiny, Till then the criterion of interference should hold good.

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SESSION IV

SEA WATER INTRUSION
ARTIFICIAL RECHARGE

1. Analysis of Fresh-Salt Water Interface in Aquifer During Salt Water Draining — IV. 1–8

Satoru Sugio and Toshihiko Ueda

2. Salt Water Intrusion due to Pumping by Two Drains in a Coastal Aquifer — IV. 9–17

G. C. Mishra, M. R. Madhav and K. Subramanya

3. Characteristics of Salt-Fresh Water Interface due to Hydraulic Structures — IV. 19–30

V. C. Kulandaiswamy, R. Sakthivadivel, S. Seetharaman and K. Venugopal

4. Skimming of Fresh Water Afloat upon Salt Water — IV. 31–42

B. M. Sahni

5. Study of Ground Water Mounds under Spreading Areas — IV. 43–54

J. T. Panikar and A. L. Mathur

6. The main Trends of Hydrogeological Studies in Artificial Recharge of Ground Water Resources (based on the practice of the USSR) — IV. 55–65

N. I. Plotnikov

ANALYSIS OF FRESH-SALT WATER INTERFACE IN AQUIFER DURING SALT WATER DRAINING

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SYNOPSIS

Controlling the depth of the fresh and salt water interface in the aquifer is very important in case of the fresh water reservoir on coastal aquifer.

The paper deals with the mathematical theory of lowering the interface by means of continuous pumping up salt water from a series of the circular conduits burried at equal depth and space. This steady and two-dimensional seepage flow toward the conduits is analyzed by the theory of complex potential. The theoretical solutions of the interface are checked with an experiment by means of Hele-Shaw model using viscous fluids. The relationship among the height of the interface, discharge rate and other boundary conditions are obtained in dimensionless form by the electronic computer.

1. INTRODUCTION

The depth of the fresh and salt water interface in the aquifer is strictly connected with the difference of water level between the reservoir and the sea. Even if the fresh water level becomes lower than the sea level, the situation, where the interface does not exist in the ground and salt water directly penetrates into the fresh water reservoir, must be avoided in order to conserve fresh water. So the various counterplans have been considered and theoretically investigated by many research workers. Here, we examine one method in which a series of circular conduits burried at equal depth and space drain salt water in the aquifer without sucking in fresh water. In consequence of draining salt water, the interface will be fixed in the ground.

This steady and two-dimensional seepage flow toward the conduits is analyzed by using the potential theory. Our analysis is characterized by the treatment of the sink. Formally the seepage flow toward the conduits have been analyzed on the assumption that the position of the sink is the center of the conduits. The exact solution, therefore, is derived from considering the difference of these positions.

2. BOUNDARY CONDITIONS

We'll consider the flow of salt water in a homogeneous isotropoc porous medium as shown in Fig.1. An interface between salt water and stationary fresh water is assumed as an abrupt interface that completely separates the regions occupied by each fluids. In case that the sea level is lower than the fresh water level, the interface is located at deep place even if the pumping from conduits is absent. Therefore the boundary conditions in this analysis is treated in case that the sea level is higher than the fresh water level. Since the equal spaced conduits are located at equal depth and the line FD and ED are the vertical stream one, the region DFEAD can be treated as a typical section for the analysis.

The velocity potential in the salt water region and the boundary condition on the interface are

$$\Phi = k\left\{\frac{p}{\rho_s g} + y\right\} \qquad \text{(1)}$$

$$p_s = p_f = \rho_f g\,(H_T + d - y)\,, \quad \text{(on the interface)} \qquad \text{(2)}$$

where ρ_s and ρ_f are the specific weights of salt water and fresh water respectively.

Substituting Eq.2 into Eq.1, the velocity potential of salt water on the interface are expressed as

$$\left.\begin{aligned}\Phi_{sf} &= k(1-\varepsilon)(H_T + d) + k\varepsilon y\\ \varepsilon &= (\rho_s - \rho_f)/\rho_s\end{aligned}\right\} \qquad \text{(3)}$$

Therefore, by using the Zhukovsky function $\theta = W + ik\varepsilon z$, the curved interface in the physical plane (z plane) can be mapped as a side EF of the polygon in θ plane as shown in Fig.2.

3. MAPPING THE COMPLEX PLANE W, z AND θ

Separating the Zhukovsky function into its real and imaginary parts

$$\left.\begin{aligned}\theta_1 &= \Phi - k\varepsilon y\\ \theta_2 &= \Psi + k\varepsilon x\end{aligned}\right\} \qquad \text{(4)}$$

In drawing the polygon in Fig.2, the variation of θ_1 on the line EA is an exercise.

$$\partial\theta_1/\partial y = k\{(\partial p/\partial y)/\rho_s g + 1 - \varepsilon\} \qquad \text{(5)}$$

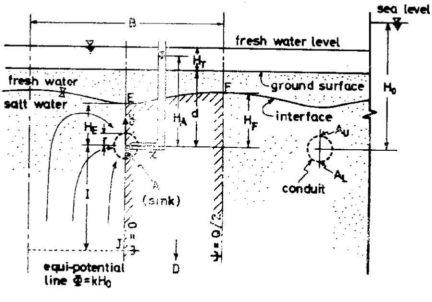

Fig.1 Physical plane (z plane)

In the vicinity of a stagnation point E, the hydrostatic pressure distribution [i.e., $(\partial p/\partial y)/\rho_s g = -1$] can be approximately valid and Eq.5 becomes $\partial\theta_1/\partial y < 0$. In the vicinity of the sink A, however, $\partial\Phi/\partial y$ $(= k\{\partial p/\partial y/\rho_s g + 1\})$ exceeds $k\varepsilon$ and Eq.5 becomes $\partial\theta_1/\partial y > 0$. So the existence of a singular point C that denotes $\partial\theta_1/\partial y = 0$ must be considered between the points E and A.

Now the inside of the polygon DFECAD is mapped onto the upper half of the t plane by using the Schwarz-Christoffel transformation, and similarly the t plane is mapped onto a quater of the ζ plane.

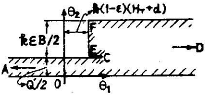

Fig.2 θ plane

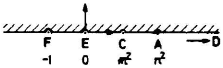

Fig.3 t plane

$$\theta = M\int\frac{t-m^2}{\sqrt{t+1}\,\sqrt{t}\,(t-n^2)}dt \quad \text{: (Fig.2→Fig.3)} \qquad \cdots(6)$$

$$t = -\sin^2\zeta \quad \text{: (Fig.3→Fig.4)} \qquad \cdots(7)$$

Integrating these equations, next equation is obtained.

$$\theta = 2iM\left[\zeta - \frac{1}{G}\tan^{-1}\left(\sqrt{n^2+1}\cdot\tan\zeta/n\right)\right] + C \quad \text{------}(8)$$

where $G = n\sqrt{n^2+1}/(n^2-m^2)$

Here, the complex potential of the flow toward two point sinks, that are symmetric each other with respect to the ξ axis as shown in Fig.4, can be expressed

$$W = \frac{q}{2\pi}\cdot\ln\{\sin(\zeta+ia)\cdot\sin(\zeta-ia)\} + \mu \quad \cdots(9)$$

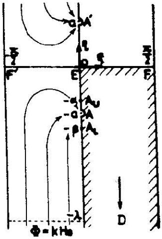

Fig.4 ζ plane

Table 1 Boundary conditions

Point	Φ	Ψ	x	y	X	Y
A_U	kH_A	q/2	0	r	0	$-\frac{\sqrt{n^2+1}}{n}\tanh\alpha$
A_L	kH_A	0	0	-r	0	$-\frac{\sqrt{n^2+1}}{n}\tanh\beta$
J	kH_0	0	0	-I	0	$-\frac{\sqrt{n^2+1}}{n}\tanh\lambda$
E	$k(1-\varepsilon)(H_T+d)+k\varepsilon H_E$	q/2	0	H_E	0	0
F	$k(1-\varepsilon)(H_T+d)+k\varepsilon H_F$	q/2	B/2	H_F	∞	0
E~F	$k(1-\varepsilon)(H_T+d)+k\varepsilon y$	q/2	x	y	$\frac{\sqrt{n^2+1}}{n}\tan\xi$	0

where $a = \sinh^{-1} n$, q is the magnitude of the sink and is equivalent to the discharge rate of salt water from one conduit.

Separating Eqs. 8 and 9 into its real and imaginary parts by substituting $\zeta = \xi + i\eta$, $M = M_1 + iM_2$ and $\mu = \mu_1 + i\mu_2$ into these equations, and evaluating these constant values from the boundary conditions at the point E, next equations are obtained.

$$\left.\begin{array}{l} M_1 = G q/2\pi \ , \quad M_2 = 0 \ , \quad C = k(1-\varepsilon)(H_T + d) \\ \mu_1 = k\varepsilon H_E + k(1-\varepsilon)(H_T + d) - (q/4\pi)\cdot \ln n^4 \ , \quad \mu_2 = q/2 \end{array}\right\} \quad \cdots\cdots (10)$$

Rewriting their real and imaginary part of Eqs. 8 and 9,

$$\Phi - k\varepsilon y = \frac{q}{4\pi}\cdot \ln \frac{X^2 + (1+Y)^2}{X^2 + (1-Y)^2} - \frac{q}{\pi} G\eta + k(1-\varepsilon)(H_T + d) \quad \cdots (11)$$

$$\Psi + k\varepsilon x = \frac{q}{2\pi}\cdot \tan^{-1} \frac{2X}{X^2 + Y^2 - 1} + \frac{q}{\pi} G\xi \quad \cdots\cdots (12)$$

$$\Phi = \frac{q}{4\pi}\ln\left[\{\sin^2\xi + \sinh^2(\eta + a)\}\{\sin^2\xi + \sinh^2(\eta - a)\}/n^4\right] + k\varepsilon H_E + k(1-\varepsilon)(H_T + d) \quad \cdots (13)$$

$$\Psi = \frac{q}{2\pi}\left[\tan^{-1}\{\cot\xi\cdot\tanh(\eta + a)\} + \tan^{-1}\{\cot\xi\cdot\tanh(\eta - a)\} + \pi\right] \quad \cdots\cdots (14)$$

where $X = \dfrac{G\cdot\sin 2\xi}{\cos 2\xi + \cosh 2\eta}$, $Y = \dfrac{G\cdot\sinh 2\eta}{\cos 2\xi + \cosh 2\eta}$

The velocity potential obtained from Eq.13 will become greater than that of the sea ($\Phi = kH_0$) at a certain depth. Then, we'll let I denote the depth from x-axis to the place that have a value of $\Phi = kH_0$ and assume that the velocity potential in Eq.13 have a value of kH_0 in $y < -I$ in the same manner as the analysis on the seepage flow toward wells.

4. DETERMINATION OF DISCHARGE RATE AND DEPTH OF INTERFACE

The typical boundary conditions are shown in Table-1. Substituting the boundary conditions at points A_U, A_L, F and J into Eqs. 11, 12 and 13, equations for the discharge rate and the highest and the lowest position of the interface are obtained.

$$\left\{\begin{array}{ll} q = \dfrac{k\varepsilon B}{G - 1} = \dfrac{k\varepsilon B(n^2 - m^2)}{n\sqrt{n^2+1} - (n^2 - m^2)} & \cdots\cdots (15) \\ k\varepsilon H_E = kH_A - \dfrac{q}{2\pi}\ln\left|\sinh(a-\beta)\cdot\sinh(a+\beta)/n^2\right| - k(1-\varepsilon)(H_T + d) & \cdots\cdots (16) \\ k\varepsilon H_F = \dfrac{q}{2\pi}\ln\left|(1+n^2)/n^2\right| + k\varepsilon H_E & \cdots\cdots (17) \end{array}\right.$$

where

$$\frac{q}{\pi}\lambda G = kH_0 + k\varepsilon I - k(1-\varepsilon)(H_T + d) - \frac{q}{2\pi}\ln\left|\frac{n - \sqrt{n^2+1}\cdot\tanh\lambda}{n + \sqrt{n^2+1}\cdot\tanh\lambda}\right|$$

$$\frac{q}{2\pi}\ln\left|\sinh(\lambda - a)\cdot\sinh(\lambda + a)/n^2\right| = kH_0 - k\varepsilon H_E - k(1-\varepsilon)(H_T + d)$$

$$2n^2 = \sinh^2\alpha + \sinh^2\beta$$

$$\frac{q}{2\pi}\cdot\ln\left|\frac{n - \sqrt{n^2+1}\cdot\tanh\beta}{n + \sqrt{n^2+1}\cdot\tanh\beta}\right| = kH_A + k\varepsilon r - \frac{q}{\pi}G\beta - k(1-\varepsilon)(H_T + d)$$

$$\frac{q}{2\pi}\left|\frac{n-\sqrt{n^2+1}\cdot\tanh\alpha}{n+\sqrt{n^2+1}\cdot\tanh\alpha}\right| = kH_A - k\varepsilon r - \frac{q}{\pi}G\alpha - k(1-\varepsilon)(H_T+d)$$

Again, substituting the boundary condition of the interface into Eqs. 11, 12 and 13, the shape of the interface is expressed

$$\left.\begin{aligned} x &= \frac{q}{2\pi k\varepsilon}\left\{2G\xi + \tan^{-1}\frac{2n\sqrt{n^2+1}\cdot\tan\xi}{(n^2+1)\cdot\tan^2\xi - n^2} - \pi\right\} \\ y &= \frac{q}{2\pi k\varepsilon}\cdot \ln\left\{(\sin\xi/n)^2+1\right\} + H_E \end{aligned}\right\} \quad \cdots\cdots (18)$$

5. HELE-SHAW MODEL TEST

The theoretical solutions are checked with the experimental results obtained by the Hele-Shaw model test. An illustration of the apparatus is shown in Fig.5. Viscous fluids used in this model test are a motor oil ($\rho = 0.880$ g/cm^3, $\nu = 2.103$ cm^2/sec at 20°C) and a solution of carboxymethyl cellulose ($\rho = 1.004$ g/cm^3, $\nu = 0.269$ cm^2/sec at 20°C) in-stead of fresh water and salt water respectively.

The experimental results of the interface are shown in Fig.6 with the numerical results. This experimental work required high accuracy in the measurement of the water level, especially the sea level and the hydraulic head in the conduit as described in section 6.

Considering this fact and the scale of this apparatus, it seems to be quite all right to consider that the difference between the experimental results and the theoretical one are caused by the error in the measurement of the water level. Therefore, it can be considered that the theoretical results agree well with the experimental results.

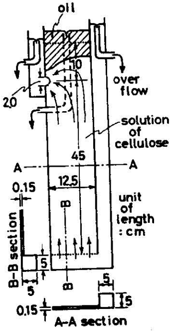

Fig.5 Hele-Shaw model

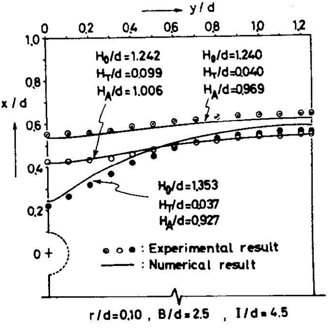

Fig.6 Experimental result

6. NUMERICAL RESULTS

Some of the calculated results are shown in Figs. 7, 8 and 9. Fig.7 shows that the calculated results in case of $H_o/d = 1.2$, $H_T/d = 0$ and $H_o/d = 1.4$, $H_T/d = 0$ are approximately the same with that in case of $H_o/d = 1.4$, $H_T/d = 0.2$ and $H_o/d = 1.6$, $H_T/d = 0.2$ respectively. From these results, we hope that the general tendencies of this flow will be clarified by the results in case of $H_T/d = 0$. Then Figs. 8 and 9 are obtained under the condition of $H_T/d = 0$.

In addition, the values $H_A/d = 0.975$ and 1.17 in these figures are derived from $H_A/d = (1-\varepsilon)(H_T/d + 1)$.

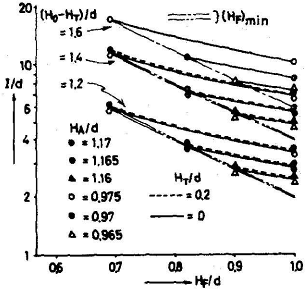

Fig.7 Numerical result of $I/d \sim H_F/d$

Again, we'll discuss a few points from the result of Fig.7 under the condition of $I/d = const$. The highest position of the interface (H_F) is inclined to lower rapidly as the hydraulic head in the conduits (H_A) reduces and to become higher rapidly as the difference of the water level between the sea and the reservoir increases. In this figure, dot-dash-lines show minimum values of H_F in each boundary conditions. Therefore, the left zone of these lines indicates the theoretical solutions with sucking fresh water into the conduits.

Fig.8 shows the change of $(H_F)_{min}$ according to the difference of the boundary conditions. From this result it is obvious that

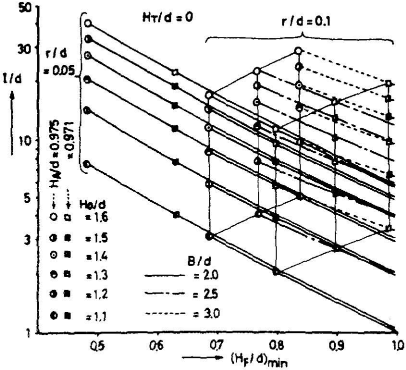

Fig.8 Numerical result of $I/d \sim (H_F/d)_{min}$

$(H_F)_{min}$ is inclined to become higher as both the space and the radius of conduit become larger under the conditions of $H_A/d = const$. Therefore, the conduits with smaller radius and narrower space are favorable to control the depth of the interface.

Fig.9 shows the discharge rate of salt water from the conduits. In this figure, the upper end of each lines indicates the states of $(H_F/d)_{min}=1$. The circular signs denote the values of H_A/d in case that H_F is minimum. Then these signs excepting this case shift up in each lines. From this figure, it is clear that the discharge rate is inclined to increase as the hydraulic head in conduits reduces and the distance between the conduit and the place of $\Phi = kH_0$ shortens.

7. CONCLUSION

The counterplan of penetrating salt water into the fresh water reservoir by means of pumping salt water by a series of circular conduits has been investigated mathematically under the condition of without sucking fresh water into the conduits.

The following properties have become evident after studying the numerical results. The possibility of this counterplan is confirmed even if the sea level is higher than the fresh water level. The depth of the fresh and salt water interface in the aquifer is inclined to become higher rapidly as both the hydraulic head in the conduits and the difference of the water

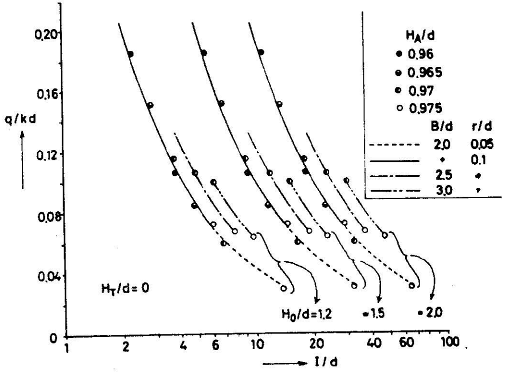

Fig.9 Numerical result of $I/d \sim q/kd$

level between the sea and the reservoir decrease, and also to become higher as both the space and the radius of the conduit increase in size in the condition of $H_A/d = const$. The discharge rate is inclined to increase as the hydraulic head in conduits reduce. Then, the conduits with smaller radius and narrower space are favorable to control the depth of the interface.

SALT-WATER INTRUSION DUE TO PUMPING BY TWO DRAINS IN A COASTAL AQUIFER

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SYNOPSIS

Salt-water intrusion due to pumping of fresh-water is of general interest in the coastal area ground water development. The problems are generally complicated and only a few cases are amenable to exact solution. The important works done in this area are indicated in this paper. The present paper investigates a two dimensional flow condition arising due to pumping of fresh-water from two drains spaced at a certain interval in an aquifer bounded by salt-water at its lower surface and an impervious boundary at its top. The two drains are located at the impervious boundary and are treated as line sinks of equal strength. The analysis is carried out by using the inversion of hodograph and Schwarz-Christoffel transformation. Results showing the influence of the spacing of the drains and the fresh-water pumping rate on the upconing of salt-water are presented.

1. INTRODUCTION

The salt-water intrusion problem into fresh water in coastal aquifers during the exploitation of natural resources is always an inadvertment result. This problem has therefore demanded for quite some time extensive investigation and control effort.

The flow of salt and fresh water in porous media is a particular case of seepage of two fluids flow. Extensive work on seepage of two fluids flow has been carried out by many schools of engineering like Chemical and Petroleum Engineering. It is beyond the scope of this paper to present them. Only the work done pertaining to ground water technology is mentioned below.

The salt-water intrusion problem in ground water technology can be broadly classified into two classes. The first category consists of the salt-water intrusion due to pumping of water by constructing well, drain or ditch with appropriate sheet piling. The salt-water in these processes trys to enter the system and thereby compels to restrict the pumping rate. Solutions to this category of problems are given by BRUINGTON(5), BEAR and DAGAN(2), MUSKAT(7). In the 2nd category fresh water discharges into sea, from the near by reservoir or due to recharging of fresh-water. Here the salt-water intrusion developing in the process controls the quantity of seepage discharged into sea and other flow characteristics like exit gradient and uplift pressure. These types of problem have been analysed by BEAR and DAGAN(4) and HENRY(6).

The flow of salt and fresh water in porous media is invariably an unconfined flow system, the interface of salt and fresh water being unknown. These types of problem are therefore generally solved either by hodograph method or by using Zhukovsky's function (SAKIYAMA and TOSHIHIKO(9)). The former method has been used widely solving the problems of category 1 and 2 mentioned earlier. The development and solutions to various problems of salt and fresh water flow connected with ground water technology can be found in the books of BEAR(4), BEAR, ZASLAVSKY and IRMAY(3), MUSKAT(7) and POLUBARINOVA-KOCHINA(8).

The two dimensional flow situation arising due to pumping of fresh-water by a single line sinks in finite and semi infinite thick aquifer bounded by stationary salt-water at the lower surface and an impervious boundary at the top has been analysed by BEAR and DAGAN(2), who have used hodograph and Schwarz-Christoffel transformation technique to arrive at the solution. They have analysed both cases i.e. case in which the critical state occurs and the case in which the critical state does not occur.

ACKERMAN and SHEN(1) have analysed the case of several line drains in an aquifer whose upper boundary is an equipotential line by using Zhukovsky's function.

The present study investigates the case of two line sinks bounded by an impervious boundary at the top and by the stationary salt-water at the bottom surface.

2. ANALYSIS

Fig. 1(a) demonstrates the flow domain. It is assumed that upconing takes place mid way between the two line sinks and the pumping rate is such that the point of inflection in the interface has vanished. With these assumptions the hodograph for half of the flow domain is shown in Fig. 1(b). It is well known that the immobile salt and fresh water interface will transform into a circle in the hodograph plane as shown in the figure. Fig. 1(c) shows the inverse of the hodograph.

The conformal mapping of the inverse hodograph plane to the auxiliary t plane is given by

$$\frac{dz}{dw} = K \int_0^t \frac{dt}{t^{1-\frac{3}{2}}\,(t-1)^{3/2}} + \frac{i}{K} \qquad (1)$$

$$= \frac{2M}{(-1)^{1/2}} \left[\frac{-(t)^{1/2}}{(1-t)^{1/2}} + \sin^{-1}(t)^{1/2} \right] + \frac{i}{K} \qquad (2)$$

in which,

$$K = \left(\frac{\rho_2}{\rho_1} - 1\right) k;$$

ρ_2 and ρ_1 = densities of salt and fresh water respectively;

k = co-efficient of permeability of the porous medium and

M = constant to be evaluated.

As t passes around a semi circle of radius tending to ∞ the corresponding change in $\frac{dz}{dw}$ at ∞ is equal to $\frac{i}{K}$. Making use of this condition and substituting $t = re^{i\theta}$, $dt = re^{i\theta}\, i d\theta$ and integrating

$$\frac{i}{K} = M \int_0^{\pi} \frac{(re^{i\theta})^{1/2}\; re^{i\theta} i d\theta}{(re^{i\theta} - 1)^{3/2}} = Mi\pi \qquad (3)$$
$$r \to \infty$$

Therefore

$$M = \frac{1}{K\pi} \qquad (4)$$

The complex potential w, where $w = \phi + i\psi$, for half of the flow domain is shown in Fig. 1(a), in which ψ is the stream function and ϕ is the velocity potential function defined as

$$\phi = -k\left(\frac{p}{\gamma_w} + y\right) + C \qquad (5)$$

In the above expression

C = a constant;

p = pressure;

y = co-ordinate

and γ_w = unit weight of fresh water.

The mapping of the complex potential plane w to the upper half of the t plane is given by

$$\frac{dw}{dt} = \frac{A}{t-b} \qquad (6)$$

As t passes around a semicircle of small radius at point B, the corresponding change in w is equal to - iq. Substitution of $t-b = re^{i\theta}$ in equation (6) and integration lead to

$$A = -\frac{q}{\pi} \qquad (7)$$

The relationship between z and t planes is found as follows:

For $0 \leq t \leq 1$

$$\frac{dz}{dt} = \frac{dz}{dw}\frac{dw}{dt}$$

$$= \frac{-2}{K\pi(-1)^{1/2}} \quad \frac{q}{\pi} \left[\frac{-(t)^{1/2}}{(1-t)^{1/2}} + \operatorname{Sin}^{-1}(t)^{1/2} \right] \quad \frac{1}{t-b}$$

$$+ \frac{1}{K} \quad \frac{-q}{\pi} \quad \frac{1}{t-b} \tag{8}$$

Replacing $\frac{1}{t-b}$ by $-\frac{1}{b} \sum_{n=o}^{\infty} A_n t^n$, in which $A_n = \frac{1}{b^n}$, and integrating

$$Z = \int_o^t \frac{2q}{K\pi^2 (-1)^{1/2}} \left[- \frac{1}{b} \sum_{n=o}^{\infty} A_n \, t^{n+\frac{3}{2}-1} (1-t)^{1/2-1} \right] dt$$

$$+ \int_o^t \frac{2q}{K\pi^2 (-1)^{1/2}} \quad \frac{\operatorname{Sin}^{-1}(t)^{1/2}}{b-t} \, dt$$

$$+ \int_o^t \frac{1}{K} \quad \frac{q}{\pi} \quad \frac{dt}{b-t} + Z_D \tag{9}$$

At $t = 1$, $Z = 0$.

Therefore,

$$0 = \frac{-2q}{bK\pi^2 (-1)^{1/2}} \sum_{n=o}^{\infty} A_n \, B\,(n+3/2,\, 1/2)$$

$$+ \int_o^1 \frac{2q}{K\pi^2 (-1)^{1/2}} \quad \frac{\operatorname{Sin}^{-1}(t)^{1/2}}{b-t} \, dt$$

$$- \frac{1}{K} \quad \frac{q}{\pi} \log(b-t)$$

$$+ \frac{1}{K} \quad \frac{q}{\pi} \log b + Z_D \tag{10}$$

in which, $B(n + 3/2, 1/2)$ is complete beta function.

The integration $\int_o^1 \frac{\operatorname{Sin}^{-1}(t)^{1/2}\, dt}{b-t}$ is to be evaluated numerically.

For $1 \leq t \leq b$, $\frac{dz}{dw}$ is given by

$$\frac{dz}{dw} = M \int_b^t \frac{t^{1/2}\, dt}{(t-1)^{3/2}} \tag{11}$$

Integrating by parts

$$\frac{dz}{dw} = M\left[\frac{-2(t)^{1/2}}{(t-1)^{1/2}} + \frac{2(b)^{1/2}}{(b-1)^{1/2}} + \log\frac{t + t^{1/2}(t-1)^{1/2}}{t - t^{1/2}(t-1)^{1/2}} - \log\frac{b + b^{1/2}(b-1)^{1/2}}{b - b^{1/2}(b-1)^{1/2}}\right] \tag{12}$$

The relationship between z and t is given by

$$Z = \int_b^t \frac{AM}{(t-b)}\left[\frac{-2(t)^{1/2}}{(t-1)^{1/2}} + \frac{2(b)^{1/2}}{(b-1)^{1/2}} + \log\frac{t + t^{1/2}(t-1)^{1/2}}{t - t^{1/2}(t-1)^{1/2}} - \log\frac{b + b^{1/2}(b-1)^{1/2}}{b - b^{1/2}(b-1)^{1/2}}\right] dt + Z_B \tag{13}$$

At $t = 1$, $Z = 0$.

Therefore,

$$0 = \int_b^1 \frac{AM}{(t-b)}\left[\frac{-2(t)^{1/2}}{(t-1)^{1/2}} + \frac{2(b)^{1/2}}{(b-1)^{1/2}} \log\frac{t + t^{1/2}(t-1)^{1/2}}{t - t^{1/2}(t-1)^{1/2}} - \log\frac{b + b^{1/2}(b-1)^{1/2}}{b - b^{1/2}(b-1)^{1/2}}\right] dt + Z_B \tag{14}$$

The integrals appearing in equation (14) are to be carried out numerically after substituting $t = 1 + T^2$. At $t = b$, the value of the function under integral sign can be obtained by applying LaHospital rule. From equations 10 and 14 the two unknowns b and Z_D can be obtained for known values of Z_B, q and K.

The expression for the interface is obtained as follows:

For $-\infty < t \leq 0$,

$$\frac{dz}{dw} = M\left[\frac{-2(t)^{1/2}}{(t-1)^{1/2}} + \log\left\{\frac{1}{2} - t - (t^2 - t)^{1/2}\right\} - \log 1/2\right] + \frac{i}{K} \tag{15}$$

and

$$Z = \int_0^t \frac{AM}{b-t}\left[\frac{-2(t)^{1/2}}{(t-1)^{1/2}} + \log\left\{1/2 - t - (t^2 - t)^{1/2}\right\} - \log 1/2\right] dt - \frac{i}{K} A \log\left(\frac{b-t}{b}\right) + Z_D \tag{16}$$

For any value of $t < 0$, the real part of equation (16) gives the x co-ordinate and the imaginary part the corresponding y co-ordinate along the interface.

3. RESULT AND DISCUSSION

The influences of the spacing of drains and pumping rate on Z_D were analysed in the present study.

Because of the implicit nature of equation 14, a certain value is assigned to the parameter b and Z_B is obtained for known value of K and q. For the assumed value of K, q and b, the corresponding Z_D is found out from equation 10. While computing the results only the positive values of the square roots appearing in equation 10 and 14 are considered.

The variation of $\frac{-KZ_D}{q}$ with $\frac{-KZ_B}{q}$ is shown in Figure 2. As seen from the figure, the variation is linear for all practical purposes. Figure 2 indicates that $Z_D \rightarrow 0$ at finite value of $\frac{-KZ_B}{q}$. For example, for $\frac{-KZ_B}{q}$ =.2, $\frac{-KZ_D}{q}$ = 0.056. Thus at $\frac{-KZ_B}{q} \simeq 0.18$, the salt-water will reach the impervious boundary and the flow domain will be divided into two parts. The salt-water at this stage would not have entered into the drains, and the pumping rate of fresh water can be still increased. However, this situation causes a different flow domain for which different analysis is needed. Hence the minimum value of $\frac{-KZ_B}{q} \simeq 0.18$ as given by the present analysis can be considered to give the safe value of pumping rate.

The location of the interface (not presented here) can be obtained from equation 16.

4. CONCLUSIONS

Using hodograph and Schwarz-Christoffel transformation an analysis is presented for the problem of two dimensional flow into two drains in a coastal aquifer. The variation of the upconing, spacing and discharge rate has been studied as a variation of $\frac{-KZ_B}{q}$ vs $-\frac{KZ_D}{q}$. The minimum value of $\frac{-KZ_B}{q}$ for which $Z_D = 0$, is found to be 0.18 and this represents, the condition of maximum pumping rate for which the assumed flow domain is valid.

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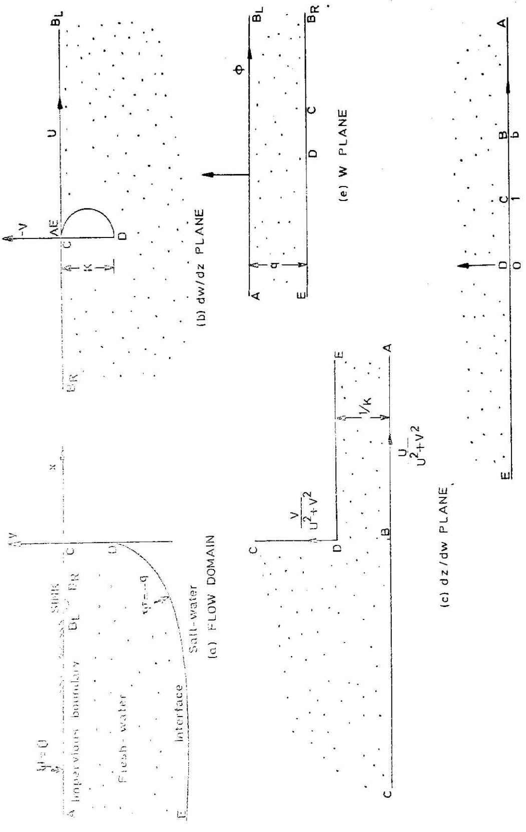

FIG.1 STEPS OF CONFORMAL MAPPING

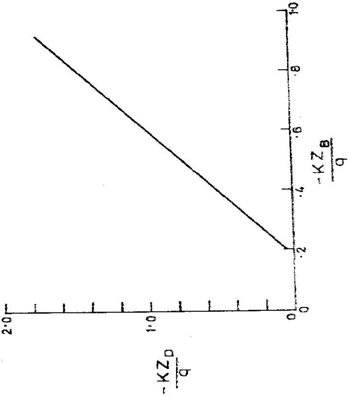

FIG. 2 INFLUENCE OF THE SPACING OF DRAINS ON Z_D

CHARACTERISTICS OF SALT-FRESH WATER INTERFACE DUE TO HYDRAULIC STRUCTURES

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SYNOPSIS

The characteristics of salt-fresh water interface under a hydraulic structure are discussed. Analytical solutions for the salt-fresh water interface have been developed for the case of steady flow of fresh water about a sheet pile with and without penetration into the ground. The theoretical solution for the interface for flow about a pile without penetration has been verified experimentally in a Hele-Shaw apparatus and good agreement is noticed. Equations obtained for the sheet pile penetrating into the ground have been solved numerically using a digital computer and the characteristics of the interface for different depths of penetration and head causing flow have been investigated.

INTRODUCTION

In recent years, competition for economically exploiting water resources has brought about an awareness that one of the principal problems confronting hydrologists is to quantitatively appraise the available ground water resources. In ever increasing numbers, engineers are being called upon to estimate how much ground water is available for development and what will be the consequences of its exploitation. The excessive pumping of ground water from an aquifer over and above its safe yield may lower the ground water table progressively leading to serious consequences such as sea water intrusion in the coastal aquifers. There are many methods available for prevention and control of sea water intrusion. One such is to artificially recharge the aquifer by ponding fresh water behind a hydraulic structure and allowing it to infiltrate into underground reservoirs.

Ponding of fresh water behind a hydraulic structure will alter the interface of salt and fresh water. The present investigation is concerned with a study of the characteristics of salt and fresh water interface underneath a hydraulic structure. Hydraulic structure referred herein pertains to a single sheet pile with or without penetration into the ground. It is assumed that before constructing the hydraulic structure, the salt and

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fresh water interface is in a horizontal plane extending to infinity. The objectives of this investigation are:

i) To develop theoretical solutions for salt-fresh water interface for steady flow about a pile with and without penetration into the ground and to study its characteristics.

ii) To verify the theoretical solutions in a Heleshaw apparatus.

2. THEORETICAL SOLUTION

2.1 Formulation of the Problem

The analytical development of the problem is based on the assumption of steady flow around the pile and stationary interface between salt and fresh water. Consider the case where a stratum of salt water A'D'D''A'' (vide Fig.1.(a)) lies at some depth under a hydraulic structure. The salt water overlies a stratum of impervious rock. The whole stratum ADD''A''A is assumed to be homogeneous and isotropic. It is also assumed that after the construction of the hydraulic structure, the salt water is squeezed so that the left part of the line A'D' sinks and the right part rises. The problem under investigation is to predict the interface under steady state flow conditions. We consider both salt and fresh water as incompressible having densities ρ_2 and ρ_1 respectively where $\rho_2 > \rho_1$. We assume that the permeability coefficients of the two regions are k_2 and k_1 respectively. Let φ_1, u_1, v_1 be the potential and velocities in the x and y directions in region I and φ_2, u_2, v_2 be the corresponding quantities in region II (vide Fig.1.a.). It can be shown (3) that in the case of steady flow, the fluid in region II must remain at rest and that the interface must satisfy the condition $\varphi - \mu y$ = constant where $\mu = \left(\frac{\rho_2}{\rho_1} - 1\right) k_1$, a condition exactly analogous to that prevailing at a phreatic surface.

Let us consider a hydraulic structure consisting of a single sheet pile impounding water to a depth H_1 on upstream side, and H_2 on the downstream side. The difference in water surface elevations between the two sides is denoted as H. (vide Fig.1.a). Along the boundaries of the reservoirs AB and CD the velocity potential has constant values. We put, $\varphi_1 = -k_1 H/2$ on AB and $\varphi_2 = k_1 H/2$ on CD. The depths of fresh water at $x = -\infty$ and $x = +\infty$ are h' and h'' respectively. Along the interface, $\varphi - \mu y$ = constant which can be written in the form $y = \varphi/\mu + c$; for $x = -\infty$, $-h' = -k_1 H/2\mu + c$ and for $x = +\infty$, $-h'' = k_1 H/2\mu + c$. From this, $c = -\left(\frac{h' + h''}{2}\right)$. For steady flow in a finite region one has to satisfy the condition of equal area occupied initially and finally by the salt water. In this particular case, these areas are finite and hence the above condition is replaced as: $\frac{h' + h''}{2} = h_1$ where h_1 is the initial depth of freshwater. Then $c = -h_1$.

$$h' = h_1 + \frac{k_1 H}{2\mu} \qquad (1)$$

$$h'' = h_1 - \frac{k_1 H}{2\mu} \qquad (2)$$

Depending on the depth of fresh and salt water, their densities and the depth of ponding the interface may take four possible forms as shown in Fig.1.(b).

2.2 Analysis of Flow About a Pile Without Penetration into the Ground:

Using hodograph and conformal transformations, the original Z-plane is transformed successively into the ζ-plane as shown in Figs.2(a) to 2(e). Assuming $\varphi_1 = -k_1 H/2$ along AB, $\varphi_2 = k_1 H/2$ along BC, and $\psi = 0$ along A'DC', the complex potential is represented in Fig.2(f). Connecting Figs. 2(e) and 2(f) by s-c transformation, we have,

$$W = A_1 \int \frac{d\zeta}{(\zeta-1)^{\frac{1}{2}}(\zeta+1)^{\frac{1}{2}}} + B_1 = A_1 \operatorname{Sin}^{-1}\zeta + B_1 \qquad (3)$$

Substituting the boundary conditions: $\zeta \to 1 \quad W = \frac{k_1 H}{2}$

$\zeta \to -1 \quad W = \frac{-k_1 H}{2}$

We obtain: $$W = \frac{k_1 H}{\pi} \sin^{-1}\zeta \qquad (4)$$

The Z-plane in Fig.2(a) is first transformed into the hodograph plane in Fig.2(b). From the hodograph plane the complex conjugate plane is sketched as in Fig.2(c). By inverse transformation, we obtain the t-plane as in Fig.2(d) where $t_{Im} = 1/\mu$. Using S-C transformation, the t-plane is transformed into the ζ-plane as in Fig.2(e). The mapping function connecting the t-plane and ζ-plane is:

$$\frac{dz}{dw} = C_1 \int \frac{(\zeta-a)d\zeta}{(1-\zeta^2)^{3/2}} + D_1 = C_1 \frac{1-a\zeta}{(1-\zeta^2)^{\frac{1}{2}}} + D_1 \qquad (5)$$

Using the boundary conditions:

Along ADC	$\lvert\zeta\rvert < 1$	$I_m\ dz/dw = 1/\mu$
Along BA & CB	$\lvert\zeta\rvert > 1$	Real $dz/dw = 0$
A & B	$\zeta = \infty$	$dz/dw = 0$

$C_1 = 1/\mu a$ and $D_1 = i/\mu$

$$\therefore \frac{dz}{dw} = -\frac{1}{\mu a}\left[\frac{1-a\zeta}{(1-\zeta^2)^{\frac{1}{2}}}\right] + i/\mu \qquad (6)$$

Now $\frac{dz}{d\varsigma} = \frac{dz}{dw}\frac{dw}{d\varsigma} = \frac{k_1H}{\pi\mu a}\frac{a\varsigma-1}{(1-\varsigma^2)} + i\frac{k_1H}{\pi\mu}\frac{1}{\sqrt{1-\varsigma^2}}$

Integrating, we get,

$$z = \frac{k_1H}{\pi\mu a}\left[\frac{1-a}{2}\ln(1-\varsigma) - \frac{(1+a)}{2}\ln(1+\varsigma)\right] + i\frac{k_1H}{\pi\mu}\sin^{-1}\varsigma + D_2 \tag{7}$$

Using $\varsigma \to 1$, $-h'' = [k_1H/2a\mu](1-a)$

$\varsigma \to -1$, $-h' = [k_1H/2a\mu](1+a)$

'a' is determined as: $-k_1H/2\mu h_1$

Substituting for 'a' in eqn.(7), we have

$$z = \frac{k_1H}{2\pi\mu}\left[\frac{-2\mu h_1 + k_1H}{k_1H}\ln(1-\varsigma) + \frac{2\mu h_1 - k_1H}{k_1H}\ln(1+\varsigma)\right] + i\frac{k_1H}{\pi\mu}\sin^{-1}\varsigma + D_2 \tag{8}$$

At the point D, $\varsigma = \varsigma_D$, hence eqn.(8) can be written as:

$$-ih_1 = \frac{k_1H}{2\pi\mu}\left[\frac{2\mu h_1 - k_1H}{k_1H}\ln(1+\varsigma_D) - \frac{2\mu h_1 + k_1H}{k_1H}\ln(1-\varsigma_D)\right] + i\frac{k_1H}{\pi\mu}\sin^{-1}\varsigma_D + (D_{2Re} - ih_1) \tag{9}$$

From eqn.(9), equating imaginaries, we get $\varsigma_D = 0$; equating reals it is seen that $D_{2Re} = 0$. Hence eqn.(9) can now be written as:

$$z = \frac{k_1H}{2\pi\mu}\left[\frac{2\mu h_1 - k_1H}{k_1H}\ln(1+\varsigma) - \frac{2\mu h_1 + k_1H}{k_1H}\ln(1-\varsigma)\right] + i\frac{k_1H}{\pi\mu}\sin^{-1}\varsigma - ih_1 \tag{10}$$

Hence the coordinates of the interface are obtained as: $(-1 < \varsigma < 1)$

$$X_0 = \frac{h_1}{\pi}\ln\left(\frac{1+\varsigma}{1-\varsigma}\right) - \frac{k_1H}{\pi\mu}\ln\sqrt{1-\varsigma^2} \tag{11}$$

$$Y_0 = \frac{k_1H}{\pi\mu}\sin^{-1}\varsigma - h_1 \tag{12}$$

2.3 Analysis of Flow About a Pile With Penetration into the Ground:

The transformation planes are shown in Figs.3(a) to 3(f). Applying S-C transformation to Figs. 3(e) and 3(f), we obtain:

$$W = A_3\, S_n^{-1}\left[\sqrt{\frac{(\alpha+1)(1-\varsigma)}{2(\alpha-\varsigma)}}, \sqrt{\frac{2(\alpha+\beta)}{(\alpha+1)(\beta+1)}}\right] + B_3 \tag{13}$$

Substituting the appropriate boundary conditions, we get,

$$W = \frac{-k_1H}{K}S_n^{-1}\left[\sqrt{\frac{(\alpha+1)(1-\varsigma)}{2(\alpha-\varsigma)}}, \sqrt{\frac{2(\alpha+\beta)}{(\alpha+1)(\beta+1)}}\right] + \frac{k_1H}{2} + i\frac{k_1HK'}{K} \tag{14}$$

Where K is the complete elliptic integral of the first kind with modulus $k = \sqrt{\frac{2(\alpha+\beta)}{(\alpha+1)(\beta+1)}}$, and K' is the complete elliptic integral of the first kind with complementary modulus. Again using the S-C transformation to Figs.3(d) and 3(e) and applying the appropriate boundary conditions, we get,

$$\frac{dz}{dw} = -1/\mu a \left[\frac{(1-a\zeta)}{(1-\zeta^2)^{\frac{1}{2}}} \right] + i/\mu \tag{15}$$

Differentiating eqn.(13),

$$\frac{dw}{d\zeta} = \frac{-k_1 H}{K\mu a} \quad \frac{(1-a\zeta)}{(1-\zeta^2)\sqrt{(\alpha-\zeta)(\beta+\zeta)}} \tag{16}$$

Using eqns.(15) and (16) and upon integration we get:

$$\begin{aligned} Z = \frac{k_1 H}{K\mu a} \Bigg[&- \frac{1+a}{2\sqrt{(\alpha+1)(\beta-1)}} \ell n \left\{ \frac{2X'^2+(\alpha-\beta+2)(1+\zeta)+2X'\sqrt{(\alpha-\zeta)(\beta+\zeta)}}{(1+\zeta)} \right\} \\ &+ \frac{(1-a)}{2Y'} \ell n \left\{ \frac{2Y'^2-(\alpha-\beta-2)(1-\zeta)+2Y'\sqrt{(\alpha-\zeta)(\beta+\zeta)}}{(1-\zeta)} \right\} \Bigg] \\ &- \frac{i}{\mu} \frac{k_1 H}{K} S_n^{-1} \left[\sqrt{\frac{(\alpha+1)(1-\zeta)}{2(\alpha-\zeta)}} , \sqrt{\frac{2(\alpha+\beta)}{(\alpha+1)(\beta+1)}} \right] + C_2 \end{aligned} \tag{17}$$

Where $X' = \sqrt{(\alpha+1)(\beta-1)}$, $Y' = \sqrt{(\alpha-1)(\beta+1)}$

$$\text{As } \zeta \to 1, \; h'' = \frac{k_1 H\pi}{2K\mu Y'} \left[\frac{1-a}{a} \right] \tag{18}$$

$$\text{As } \zeta \to -1, \; h' = \frac{k_1 H\pi}{2K\mu X'} \left[\frac{1+a}{a} \right] \tag{19}$$

From eqns.(18) and (19),

$$a = \frac{k_1 H\pi}{K\mu(h'X'+h''Y')} \tag{20}$$

Substituting for 'a' in eqn.(17),

$$\begin{aligned} Z = &\frac{-h'}{\pi} \ell n \; \frac{2X'^2+(\alpha-\beta+2)(1-\zeta)+2X'\sqrt{(\alpha-\zeta)(\beta+\zeta)}}{(1+\zeta)} \\ &+ \frac{h''}{\pi} \ell n \; \frac{2Y'^2-(\alpha-\beta-2)(1-\zeta)+2Y'\sqrt{(\alpha-\zeta)(\beta+\zeta)}}{(1-\zeta)} \\ &- i/\mu \; \frac{k_1 H}{K} \; S_n^{-1} \left[\sqrt{\frac{(\alpha+1)(1-\zeta)}{2(\alpha-\zeta)}} , \sqrt{\frac{2(\alpha+\beta)}{(\alpha+1)(\beta+1)}} \right] + C_2 \end{aligned} \tag{21}$$

Now at $\zeta = \alpha$, $Z = 0$ and equating imaginaries,

$$\frac{k_1 H}{2K\mu Y'} \left[\frac{K\mu(h'X'+h''Y')-k_1 H\pi}{k_1 H} \right] + I_m \; C_2 = 0 \tag{22}$$

At $\zeta = -\beta$, $Z=0$ and equating imaginaries,

$$\frac{k_1 H}{2K\mu X'} \left[\frac{K\mu(h'X'+h''Y')+k_1 H\pi}{k_1 H} \right] - \frac{k_1 H}{\mu} + I_m \; C_2 = 0 \tag{23}$$

Combining eqns.(22) and (23), we have

$$\frac{K\mu(h'X'+h''Y')+k_1Hx}{2K\mu X'} - \frac{K\mu(h'X'+h''Y')-k_1Hx}{2K\mu Y'} - \frac{k_1H}{\mu} = 0 \tag{24}$$

At $\zeta = \infty$, $Z = -id$, and equating imaginaries

$$d = \frac{h'}{\pi}\tan^{-1}\left[2\sqrt{\frac{(\alpha+1)(\beta-1)}{(\alpha-\beta+2)}}\right] - \frac{h''}{\pi}\tan^{-1}\left[2\sqrt{\frac{(\alpha-1)(\beta+1)}{(\alpha-\beta-2)}}\right] + h_1 \tag{25}$$

At $\zeta = \zeta_D$, $Z = -ih_1$. Substituting in eqn.(21)

$$\begin{aligned} Z = {} & \frac{-h'}{\pi}\ln\frac{2X'^2+(\alpha-\beta+2)(1+\zeta_D)+2X'\sqrt{(\alpha-\zeta_D)(\beta+\zeta_D)}}{(1+\zeta_D)} \\ & + \frac{h''}{\pi}\ln\frac{2Y'^2-(\alpha-\beta-2)(1-\zeta_D)+2Y'\sqrt{(\alpha-\zeta_D)(\beta+\zeta_D)}}{(1-\zeta_D)} \\ & - i/\mu\ \frac{k_1H}{K}\,S_n^{-1}\left[\sqrt{\frac{(\alpha+1)(1-\zeta_D)}{2(\alpha-\zeta_D)}},\ \sqrt{\frac{2(\alpha+\beta)}{(\alpha+1)(\beta+1)}}\right] \\ & -ih'' + \text{Real } C_2 \end{aligned} \tag{26}$$

Equating the imaginaries, we get

$$\zeta_D = \frac{X'-Y'}{X'+Y'} \tag{27}$$

Equating the reals, we get

$$\begin{aligned} \text{Real } C_2 = {} & \frac{h'}{\pi}\ln\frac{2X'^2+(\alpha-\beta+2)(1+\zeta_D)+2X'\sqrt{(\alpha-\zeta_D)(\beta+\zeta_D)}}{(1+\zeta_D)} \\ & - \frac{h''}{\pi}\ln\frac{2Y'^2-(\alpha-\beta-2)(1-\zeta_D)+2Y'\sqrt{(\alpha-\zeta_D)(\beta+\zeta_D)}}{(1-\zeta_D)} \end{aligned} \tag{28}$$

Hence the coordinates for the interface are given by: $(-1 < \zeta < 1)$

$$\begin{aligned} X_0 = {} & \frac{-h'}{\pi}\ln\frac{2X'^2+(\alpha-\beta+2)(1+\zeta)+2X'\sqrt{(\alpha-\zeta)(\beta+\zeta)}}{(1+\zeta)} \\ & + \frac{h''}{\pi}\ln\frac{2Y'^2-(\alpha-\beta-2)(1-\zeta)+2Y'\sqrt{(\alpha-\zeta)(\beta+\zeta)}}{(1-\zeta)} + \text{Real } C_2 \end{aligned} \tag{29}$$

$$Y_0 = \frac{-k_1H}{K\mu}\,S_n^{-1}\left[\sqrt{\frac{(\alpha+1)(1-\zeta)}{2(\alpha-\zeta)}},\ \sqrt{\frac{2(\alpha+\beta)}{(\alpha+1)(\beta+1)}}\right] - h'' \tag{30}$$

To compute X_0, Y_0, for $-1 < \zeta < 1$, values of α and β are to be obtained from eqns.(24) and (25).

3. EXPERIMENTAL INVESTIGATIONS

3.1 Equipment

The experimental investigations were carried out in a Heleshaw Apparatus to verify the analytical solutions for the salt-fresh water interface derived in the earlier section. The Heleshaw Apparatus consists mainly of two perspex sheets which can be adjusted to provide a spacing upto a maximum of 12mm, inlet-outlet arrangements, overhead tank and a gear pump. Photo 1 shows a view of the Heleshaw Apparatus. A detailed description of the equipment is furnished in (4).

3.2 Procedure

Keeping the gap between the parallel plates adjusted to 4mm, a pile was provided using rubber of size 6cm x 0.5cm, to a depth of 22cm, from the top at a distance 10cm away from the centre of the Heleshaw Apparatus. Salt water was simulated by diluted glycerine with a specific gravity of 1.22 at 32°C and fresh water was simulated by lubricating oil 30 with a specific gravity 0.895 at 32°C. Suitable arrangements were made to keep constant levels on the upstream and downstream sides of the pile. Salt water depth was varied from 2.45 - 14.1 cm. Head causing flow was varied from 2- 4.7 cm. In each of the runs, steady state was maintained and the coordinates were measured from the graph sheet provided on the perspex sheet. A typical interface obtained is shown in photo 1.

4. DISCUSSION OF RESULTS

i) Comparison of Theoretical and Experimental Results:

A typical plot of the experimentally determined profile of the interface and the corresponding theoretical curve computed from eqns.(11) and (12) shown in Fig.4, indicate good agreement thus proving the validity of the theoretical solution. Only two of the four theoretical shapes(vide Fig.1(b) were verified experimentally as the other two required a large range of head to simulate it in the Heleshaw Apparatus which was not possible with the existing set up.

ii) Comparison of Theoretical Solutions for zero Penetration Depth:

The case of a pile with zero penetration depth considered in section (2) is a limiting case of the problem considered in section (3). To verify the theoretical solution derived in section (3) a plot of the interface using equations (11) and (12) and numerical solutions of eqns.(29)and (30) for zero penetration depth was prepared as shown in Fig.5. It is noticed that the two curves agree closely thus proving the correctness of the analytical solution. The same problem has also been investigated by Polubarinova - Kochina (3). The following are the equations for the interface:

$$X_0 = \frac{k_1 H}{\pi \mu} \ln \left[2\sqrt{(1-\varsigma^2)} \right] + \frac{h_1}{\pi} \ln \frac{1+\varsigma}{1-\varsigma} \qquad (31)$$

$$Y_0 = \frac{k_1 H}{\pi \mu} \sin^{-1} \varsigma \ - h_1 \qquad (32)$$

A plot of these equations also presented in Fig.5. show that the interface does not pass through the origin and hence is not symmetrical about the origin as is assumed in the analytical formulation.

iii) Effect of Depth of Penetration and Head Causing Flow on the Shape of the Interface:

The effect of depth of penetration and head causing flow on the shape of the interface shown in Figs.(6) and (7) indicate that the head causing flow has a greater influence than that of depth of penetration of the pile. The characteristics of these curves will facilitate location of water wells on the downstream side of the pile to conserve fresh water flow and also to effectively prevent sea water intrusion.

5. CONCLUSIONS

i) Theoretical solutions for salt-fresh water interface are obtained for steady flow of fresh water about a pile with and without penetration. It has been shown that the solutions for zero penetration depth is a limiting case of the solutions for finite depth of penetration of the pile.

ii) Experimentally determined salt-fresh water interface in a Heleshaw Apparatus show good agreement with the theoretical solutions.

iii) Head causing flow has a marked effect on the salt-fresh water interface than that of depth of penetration of pile.

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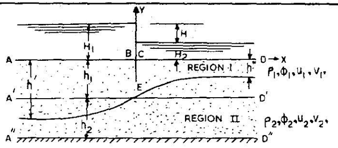

FIG. 1(a) DEFINITION SKETCH

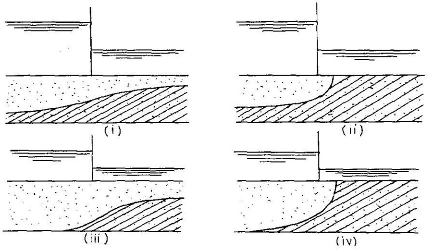

FIG. 1(b) TYPES OF INTERFACE

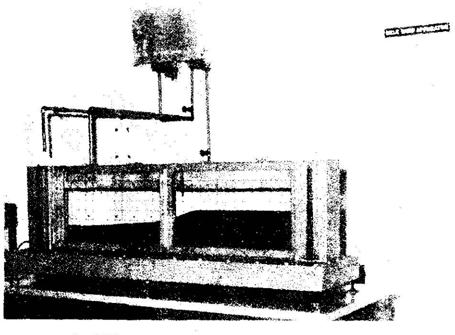

PHOTO. 1 EXPERIMENTAL SETUP

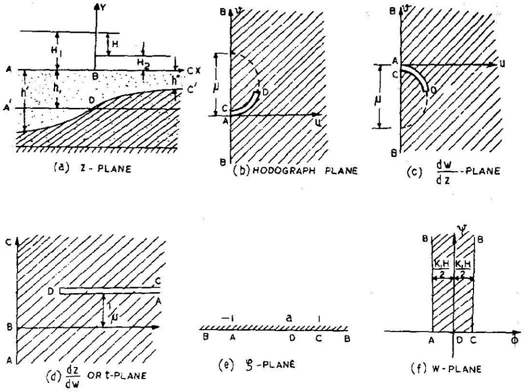

FIG. 2 FLOW ABOUT A PILE WITHOUT PENETRATION

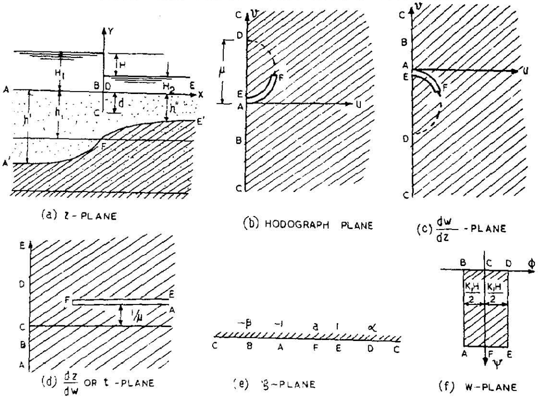

FIG. 3 FLOW ABOUT A PILE WITH PENETRATION

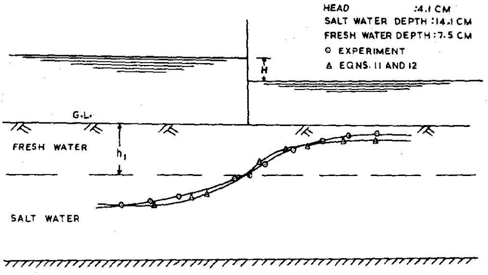

FIG. 4 COMPARISON OF THEORY AND EXPERIMENT

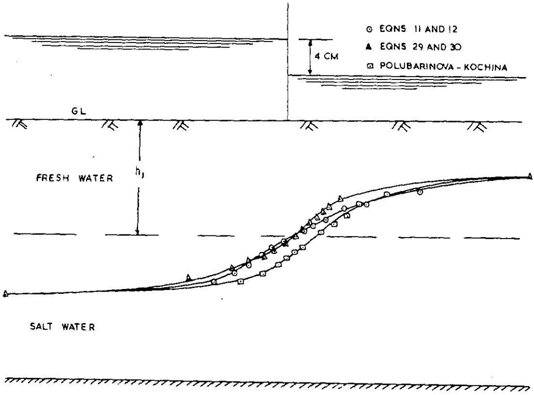

FIG. 5 COMPARISON OF SOLUTIONS FOR ZERO PENETRATION DEPTH

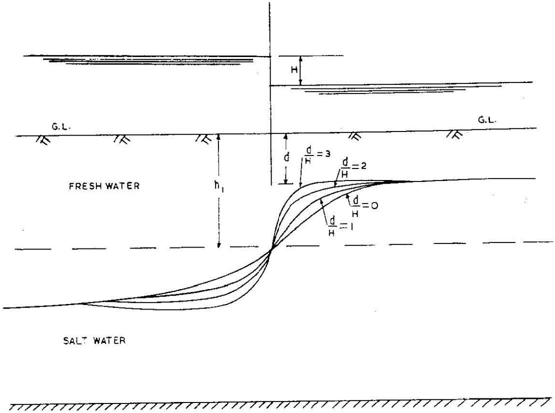

FIG. 6 EFFECT OF DEPTH OF PENETRATION

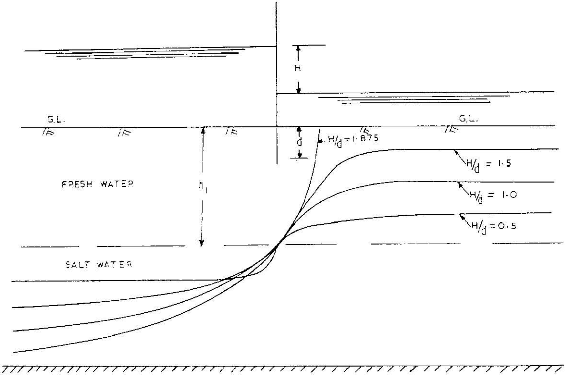

FIG. 7 EFFECT OF HEAD CAUSING FLOW

SKIMMING OF FRESH WATER AFLOAT UPON SALT WATER

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Synopsis:

In many coastal as well as some inland aquifers fresh water is underlain by saline water. In such areas when fresh water is "skimmed" using a partially penetrating well, the reduced head towards the well causes an upconing of the fresh water-saline water interface beneath the well. In order to predict the maximum uncontaminated fresh water discharge, it is necessary to know the hydrodynamics of the upconing of brine in response to pumping. Theoretical analysis as well as laboratory model studies were made to study this phenomenon. A mathematical model using finite difference iterative technique is also presented which is developed for evaluating the performance of skimming wells under a wide range of field conditions. Practical utility of the results in deciding the optimum fresh water production from skimming wells is discussed.

1. Introduction

Ground water is needed for irrigation and other purposes in many places where the supply of surface water is inadequate. In many coastal and several inland areas, including some of the world's most important agricultural lands, fresh water in the aquifer is underlain by saline water. Fresh water and salt water are separated by a zone of dispersion with density decreasing with elevation. It is not economical to instal wells for pumping fresh water from aquifers in which the fresh water zone has only a very small thickness. The concern of this study, therefore, has been those aquifers in which the dispersed layer is only a small fraction of the total thickness of the fresh water zone. In such cases the intermediate "layer" can be considered, for all practical purposes, as a boundary surface and is, therefore, referred to as an "interface" in this study. When it is desired to pump fresh water, the well should be so installed as to "skim" the fresh water from above the saline water with a minimum of mixing, either within the well or within the aquifer itself. When such a skimming well is pumped, the reduced head towards the well causes an upconing of the interface under the well.

The objective of this study was to understand the physics of the phenomenon of coning below a fresh water skimming well and to develop a mathematical model to predict the extent of coning and the maximum uncontaminated fresh water production that can be obtained under given aquifer conditions.

2. Theoretical Background

A skimming well partially penetrating the fresh water zone is shown in Fig. 1. It is assumed that the well has been pumped until it reaches a steady state. The interface mounds beneath the well to a height such that it will be in hydrodynamic equilibrium. The brine will then be static and the flow will take place only in the fresh water zone. The location and shape of the interface at any point is a function only of the fresh water velocity along the interface at that point.

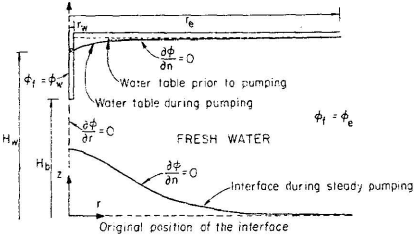

Figure 1. Salt water coning below a fresh water well.

Taking the original position of the interface prior to pumping as datum it can be shown that the elevation of the apex of the cone is given by

$$\zeta_{(r=0)} = \frac{\rho_f}{\Delta\rho}\left[\phi_e - (\phi_{if})_{r=0}\right] \tag{1}$$

where ϕ_{if} is the fresh water potential at the interface. In order to obtain the potential distribution analytically, one needs to solve the flow equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{2}$$

with the boundary conditions

$$\left.\begin{array}{lll} \phi = \phi_e & \text{at} & r = r_e \\ \phi = \phi_w & \text{at} & r = r_w,\ H_b \le z \le H_w \\ \frac{\partial\phi}{\partial r} = 0 & \text{at} & r = 0,\ \zeta_{(r=0)} \le z \le H_b \\ \frac{\partial\phi}{\partial n} = 0 & \text{along the free surface} & \\ \frac{\partial\phi}{\partial n} = 0 & \text{along the interface} & \end{array}\right\} \tag{3}$$

where $\partial/\partial n$ denotes partial derivative with respect to distance along the normal drawn to the surface. Thus, in order to calculate the position of the interface by an exact analytic method, equation (2) must be solved for the potentialdistribution using the boundary conditions (3). However, these boundary conditions in turn depend on the position of the interface. Therefore, it does not seem possible to obtain an exact analytic solution to the problem. It is useful to analyze the two approximate analytic approaches employed by Muskat (1) and Wang (2) to solve the coning problem.

In Muskat's approach the effect of mounding of interface on potential distribution is neglected. This is equivalent to assuming that the lower boundary of the aquifer was an impermeable bed instead of an interface. The potential distribution is then computed using his formula for a well partially penetrating a confined aquifer saturated with only one fluid. Further, as the well discharge is increased, the drawdown is increased and therefore, a greater cone height is expected. Muskat predicted that the cone should become unstable long before it reached the bottom of the well.

Wang's approach assumes that the maximum safe yield, is for a drawdown corresponding to which the apex of the brine-cone just reaches the bottom of the well. The critical drawdown was obtained by using the Ghyben-Herzberg relation corresponding to a cone height equal to the height of the well bottom above the original position of the interface. The discharge corresponding to this critical drawdown was computed by using a simpler version of Muskat's formula.

3. Model Studies of Coning Problem

Both physical as well as mathematical models were employed in this study. It was assumed in developing these models that the aquifer is nondeformable, isotropic and homogeneous; fluids are incompressible; steady state and isothermal conditions prevail; thickness of the dispersion zone is negligible compared to fresh water thickness; flow occurs only in the fresh water zone, it is radially symmetrical and obeys Darcy's law.

3.1 Physical model.

The physical model used in this study was designed (1) to understand the physics of coning problems, (2) to check the validity of the existing theoretical models, and (3) to check the validity of the numerical model. It was, therefore, not necessary to simulate an actual field situation in the model. It was sufficient to apply the existing theories to a few hypothetical prototypes and compare the results with those obtained by studying identical situations experimentally with the help of a physical model.

A 15° pie-shape model was designed. Spherical glass beads were used to represent the aquifer material and tap water and Soltrol 'C' were used to simulate salt water and fresh water respectively. The design of the model, reasons for choice of material and fluids, experimental set up and procedure are described in detail in a technical report by Sahni (3).

3.2 Mathematical model.

Considering a differential element of fluid in the frame of reference of cylindrical coordinate system (r, θ, z), the mass balance consideration for the element combined with Darcy's law gives the following flow equation

$$\frac{\partial}{\partial r}\left(K_r A_r \frac{\partial H}{\partial r}\right)\cdot \Delta r + \frac{\partial}{\partial z}\left(K_z A_z \frac{\partial H}{\partial z}\right)\cdot \Delta z = 0 \qquad (4)$$

where K_r and K_z are the conductivities in r and z directions, A_r and A_z the average area of cross section considered normal to the flow in these directions and H is the hydraulic head.

In order to solve equation (4) with given boundary conditions by a finite difference method it is necessary to write the flow equation in a discretized form. The entire flow region in a vertical plane is divided into a convenient grid system. For each of the blocks a flow equation is written in the discretized form. Thus the original problem of solving the complex second order non-linear equation is reduced to one of solving a set of simultaneous linear algebraic equations. A typical central block of the grid system used is shown in figure 2 together with its four adjoining grid blocks. Indices i and j denote,

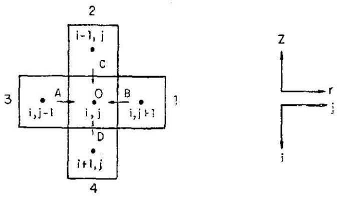

Figure 2. A typical central grid block.

respectively, the number of row and column in which a particular grid element lies. With reference to this grid system writing the flow equation in finite difference form and rearranging the terms, yields

$$A H_{i,j-1} + B H_{i,j+1} + C H_{i-1,j} + D H_{i+1,j} + E H_{i,j} = 0 \qquad (5)$$

where A, B, C, and D are the flow coefficients for the grid block (i,j) for the flow across the boundaries between adjacent block as shown in figure 2, and

$$E = -(A + B + C + D) \qquad (6)$$

Starting from Darcy's equation in differential form and considering the radial flow between the grid blocks 3 and 0, that between the block 0 and 1, it can be shown that

$$A = 2\pi (K \Delta z)_{i,j-\frac{1}{2}} / \ln\left[r_{i,j} / r_{i,j-1}\right] \qquad (7)$$

and

$$B = 2\pi (K\Delta Z)_{i,j+\frac{1}{2}} / \ln \left[r_{i,j+1} / r_{i,j} \right] \tag{8}$$

Likewise, considering the vertical flow between grid blocks 2 and 0 and that between blocks 0 and 4 it can be shown that

$$C = 2\pi K_{i-\frac{1}{2},j} \left(r^2_{i,j+\frac{1}{2}} - r^2_{i,j-\frac{1}{2}} \right) / \left(\Delta Z_{i-1,j} + \Delta Z_{i,j} \right) \tag{9}$$

and

$$D = 2\pi K_{i+\frac{1}{2},j} \left(r^2_{i,j+\frac{1}{2}} - r^2_{i,j-\frac{1}{2}} \right) / \left(\Delta Z_{i+1,j} + \Delta Z_{i,j} \right) \tag{10}$$

Coefficient E then is automatically defined by equation (6). Thus every term of the flow equation is now defined in the finite difference form.

A computer programme for finite difference scheme using iterative procedure has been written in Fortran IV language. The programme is written such as to include the flow above the water table. The procedure is outlined below.

(a) To start with a case is considered where there is no flow so that thefree surface and the interface are flat initially. Each grid is assigned a conductivity value equal to saturated conductivity. Flow coefficients are calculated for all active grid blocks and are stored in the matrix "T". This matrix is then solved for head distribution. From these head values the first approximate location of the interface and free surface are computed for each grid column and the discharge value obtained. This completes the first cycle.

(b) Next, all the grids lying below the position of the interface in the first iteration are made "hydraulically dead" by setting conductivity equal to zero in these grids. Using the most recently calculated pressure values, a new conductivity value for each grid above the interface is computed from formula given by Brooks and Corey (4). With these conductivity values new flow coefficients are calculated for the usable grids and stored in matrix T. All the subsequent steps in the first iteration are repeated in second cycle.

(c) This iteration process is repeated until the solution converges. The difference between discharge computed in two successive iterations called DIFQ was used as the criterion for convergence. The iteration was stopped if DIFQ $\leq$ EPS, a number so chosen that the discharge computation did not change by more than 0.005%.

For given aquifer conditions and well geometry, a 'critical discharge', that is, the maximum uncontaminated fresh water production can be predicted by studying the effect of various drawdowns by suitably changing the input data. For any assumed drawdown greater than the critical drawdown, the solution blows up. The later situation physically implies that the cone has become unstable and the well starts producing brine. The discharge computed for a simulated situation corresponding to critical drawdown is then the maximum fresh water that can be obtained without entrainment of brine under given aquifer conditions and well geometry.

The validity of this model was checked by comparing the results obtained therefrom with the experimental results. The comparisons showed very good agreement with regard to location of free surface and interface for critical conditions. Also, the critical discharge computed was only about 7% in error as compared to experimental values.

4. Results and Discussion

4.1 Verification of existing analytic solutions.

In order to check the validity of the two existing analytic solutions, results of the analysis of four different cases made by using these solutions are presented here. These cases were also studied experimentally with the help of the physical model. Figure 3 shows a comparison of the critical discharge obtained by the three methods. The comparison brings out the following points:

(a) Wang's theory always overestimates the critical discharge. This departure from the experimental results is more conspicuous at smaller values of α than at larger values. This can be explained as follows. In a case of shallow-well penetration, the cone beneath the well is relatively steep. The vertical flow

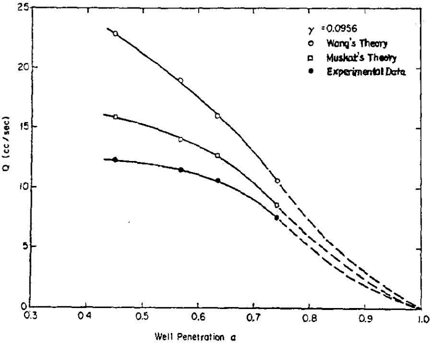

Figure 3. Comparison of theoretical critical discharge with the experimental results.

components, especially in the immediate vicinity of the well, cannot be neglected in this case and the Ghyben-Herzberg approximation is no longer valid. The resistance to flow due to much stronger convergence to-ward the well affects the potential field in the flow region. The experiments have shown beyond doubt that the cone does become unstable before it can rise to the bottom of the well. Therefore, one would expect that the value of the critical drawdown used in Wang's analysis should always be greater than the actual critical drawdown which in turn results in an overestimation of critical discharge by Wang's formula.

(b) As explained earlier, in Muskat's approach the perturbation in the potential field due the rise of the interface is not taken into account. Further, these formulae also assume a uniform flux density at all points on the surface of a well partially penetrating into the aquifer. This is not true in the rigorous sense. However, Muskat's analysis does consider the important physical phenomenon of the instability of the rising cone beneath the well. This explains why the critical discharge calculated by this method is in much better agreement with the experimental results than are the results obtained from Wang's formula as illustrated by figure 3.

Thus, both methods of analysis of the coning problem, namely those given by Wang and Muskat, are only approximate ones and both have their own limitations. Nevertheless, the results show that Muskat's analysis is more realistic than Wang's analysis.

4.2 Application of the mathematical model.

In order to derive a more meaningful inference from the results of computer simulation, all the important variables are transformed into dimensionless parameters and the results are expressed in the form of inter-relationships among these parameters. The following dimensionless groups are selected:

1. well-penetration

$$\alpha = \frac{PW}{H_e} \tag{11}$$

2. well-slimness

$$\gamma = \frac{r_w}{H_e} \tag{12}$$

3. effective radius of influence

$$\dot{L} = \frac{r_e}{H_e} \tag{13}$$

4. drawdown at the well

$$\dot{D} = \rho_f (H_e - H_w)/\Delta\rho H_e \tag{14}$$

5. dimensionless discharge

$$\dot{Q} = \rho_f Q/\Delta\rho H_e^2 K \tag{15}$$

The inter-relationships of these dimensionless variables are presented in figures 4 and 5. The following inferences are drawn from the data shown in these figures:

(a) For a given fresh-water thickness, the maximum permissible drawdown increases rapidly, especially for greater fresh-water thickness, as the well penetration decreases. At very small penetrations, the critical drawdown approaches infinity.

(b) For a given $\dot{L}$ the dimensionless critical discharge rapidly increases as well penetration decreases, especially at the very shallow penetrations.

(c) The Q versus α curves show maxima in the neighborhood of 10% to 20% penetration and at these points the curves tend to become flat. For the same cases Wang's analysis shows maxima at well penetrations of about 33% to 41%. Also, in the latter case the curves have inflexion at the maxima and pass through the origin. The results of the present study physically imply that for given aquifer and well conditions, there exists a well penetration such that the critical conditions can not occur at shallower penetrations than this value.

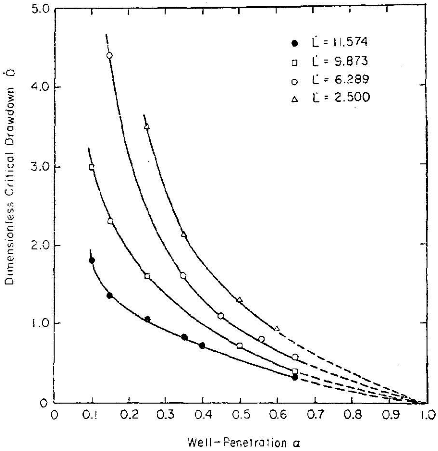

Figure 4. Variation of critical drawdown with well penetration.

(d) Other conditions being equal, the critical conditions occur at greater production rates for smaller $\dot{L}$. It is, therefore, possible to get more production of fresh water without getting salt water in the wells when a battery of larger number of wells is used.

5. Conclusions

The phenomenon of salt-water coning below a fresh-water skimming well was studied both theoretically and experimentally. The experimental part of the

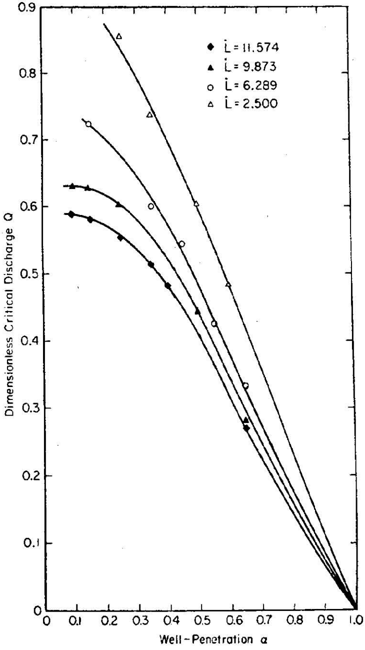

Figure 5. Dimensionless critical discharge versus well penetration.

study gave a better insight into the physics of the phenomenon and helped check the validity of the existing analytic solutions and of the mathematical model developed in this study. The following conclusions are drawn from the results of this work:

(a) Regardless of the well geometry and the aquifer and fluid properties, the highest stable cone can never rise as high as the bottom of the well-screen. The phenomenon of instability of the rising cone is confirmed beyond doubt with the help of both the laboratory model as well as computer simulation.

(b) Both Wang's theory as well as Muskat's theory are based on assumptions some of which are questionable. However, Muskat's analysis is more realistic than Wang's in the sense that the former does consider the phenomenon of instability.

(c) The mathematical model presented in this study takes into consideration the non-linearity of the boundary conditions. The results of this model showed better agreement than Muskat's analysis with the experimental results. The mathematical model presented in this study, therefore, has practical utility in studying the performance of fresh water skimming wells. Further, this model is sufficiently general in nature and can easily be modified to make it applicable to many field problems.

It is suggested that while applying the results presented in this paper to a field situation, their limitations pointed out earlier should be taken into account. The results are intended to indicate the conditions under which a stable cone can exist and the critical conditions beyond which the cone becomes unstable. It has been shown that the maximum uncontaminated fresh water production occurs as the critical conditions are approached. However, it is not intended to suggest that this alone should be the objective in using skimming wells to pump fresh water. Sometimes it may be more economical and practical to pump at lower rates. While making a decision with regard to the rate of pumping various factors such as water quality in the aquifer, the tolerable limits of salinity, amount of recharge available in the area, and economics of the operation must also be taken into account.

Further study is required to analyze several aspects of phenomenon of coning. The utility of nomograms presented in this paper can be increased by studying a wider range of possible situations with the help of the mathematical model developed in this study. It would be worthwhile to study the effect of dispersion on the maximum fresh water production predicted in this work. Finally, the mathematical model should be further generalized by considering the effects of heterogeneity and anisotropy with regard to to the hydraulic properties of the aquifer.

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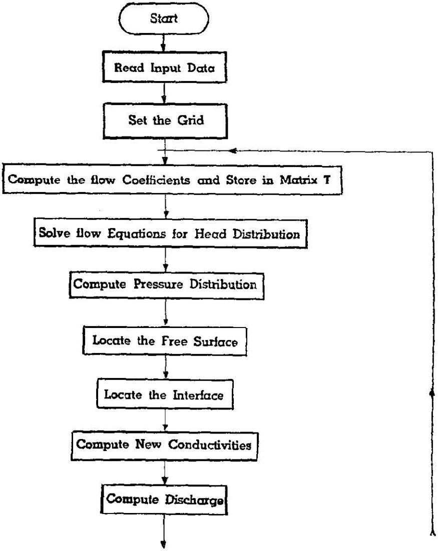
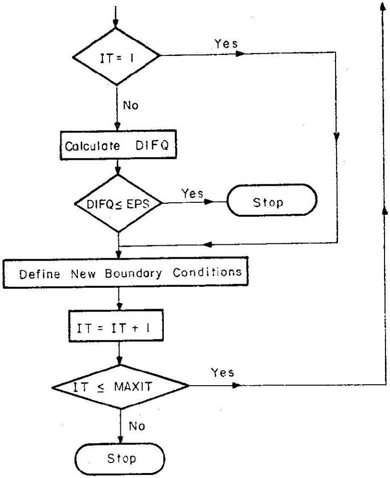
FLOW CHART



DR. B. M. SAHNI

FLOW CHART (cont'd)



Nomenclature

- ρ_f = fresh water density
- $\Delta\rho$ = density contrast between fresh water and salt water
- ϕ_f = fresh water potential
- ϕ_w = potential in the well
- ϕ_e = potential at a distance greater than the radius of influence (r_e)
- PW = depth of penetration of the well
- IT = iteration number in process
- MAXIT = maximum number of iterations desired

STUDY OF GROUND WATER MOUNDS UNDER SPREADING AREAS

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SYNOPSIS

The technique of artificial recharge which is widely used as a means of ground water conservation assumes significant role in the general field of ground water management. A common practice for artificial recharge is by spreading during which ground water mounds are formed. A rational approach towards the solution of the problem of ground water recharge calls for an evaluation of the formation, dissipation and equilibrium position of the mounds below spreading areas. The present study deals with the study of ground water mounds under transient and steady conditions both for rising and dissipating mounds. Analytical work involves the solution of the basic Boussinesq's equation for unsteady flow after linearisation for the given boundary conditions. Theoretical results for a wide range of parameters have been obtained by computations on a digital computer. Experimental studies have been carried out on a Sand Model and a Viscous Flow Model to determine the rising and stable mound configurations. Results highlight the limitations of the linearised solutions and of the sand model and the versatality of the viscous flow model in handling transient free surface flows.

1. INTRODUCTION

A major field of investigation in ground water relates to the artificial recharge which has long been recognised as a means of conserving the water resources of a region with improved quality as an incidental aim. The source of recharge may be storm run off, river water, water used for cooling, industrial waste water. Attempts have been made to recharge ground water artificially through surface spreading by means of basins, furrows or flooded areas. In the course of spreading typical profiles, termed as mounds, are formed. The mounds may be two or three dimensional depending on the shape of the spreading ground. Qualitative evaluation of the formation, equilibrium position and the dissipation of mounds is a prerequisite to a rational approach towards the solution of the problem of ground water mounds. The need for an understanding of the mechanism of ground water mounds under spreading areas cannot therefore be over emphasised.

In view of its immense practical utility, the study of ground water mounds has been the subject of several investigations in the past. Boussinesq (1,2) gave the classical presentation of unsteady two dimensional flow with a free surface. The resulting equation was non-linear and difficult to solve. Attempts have since been made to obtain solutions for different boundary conditions (3,4,5) with a view to evolving optimum designs and relative benefits. The present study deals with the mechanism of mound formation with particular reference to the determination of the configuration of the mound under steady and transient conditions both for the rising and dissipating mounds, time to attain equilibrium conditions and allied criteria. The study is in addition oriented towards the verification of the mathematical model through a comparison of the theoretical results with those of the experimental investigation carried out on a sand model and viscous flow model.

2. ANALYTICAL INVESTIGATIONS

It is well known that the application of the principle of conservation of mass through Dupuit's assumptions leads to the following equation for unsteady free surface flow:

$$\frac{\partial y}{\partial t} = -\frac{k}{m}\left[i_b \frac{\partial y}{\partial x} \pm \left(\frac{\partial y}{\partial x}\right)^2 \pm y \cdot \frac{\partial^2 y}{\partial x^2}\right] \text{------} (1)$$

where, k and m are the permeability and porosity of the medium,

i_b the slope of the impervious bed

y the ordinate of the free surface $= f(x,t)$

Neglecting the higher order terms like $i_b \frac{\partial y}{\partial x}$ and $\left(\frac{\partial y}{\partial x}\right)^2$ and replacing the variable y in the third term by a constant 'a_0', being the depth of the initial free surface, the equation reduces to

$$\frac{\partial y}{\partial t} = \frac{k a_0}{m} \cdot \frac{\partial^2 y}{\partial x^2} \text{------} (2)$$

The initial boundary conditions for the case of rising mound are

$$\left.\begin{array}{ll} \text{i)} & y(x,t) = 0, \text{ for } t = 0 \text{ and for all values of } x \\ \text{ii)} & y(x,t) = 0, \text{ for } x = L_d \text{ for all values of } t, 0 < L_d < \infty \\ \text{iii)} & y(x,t) = f(x) \text{ for } t = \infty \\ & \qquad = (q/ka_0 - i_b)(L_d - x) \end{array}\right\} (3)$$

which is a linear approximation for the stable mound as indicated in the definition sketch (Fig.1).

With $\frac{k a_0 t}{m} = j$ and $\beta = \frac{(2n-1)\pi}{2L_d}$

the general solution of Eqn.2 for the conditions defined by Eqn.3 is (5)

$$y = \left(\frac{q}{Ka_0} - i_b\right)(L_d - x) - \frac{8\left(\frac{q}{Ka_0} - i_b\right)L_d}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos \frac{(2n-1)}{2L_d} x \; e^{-\beta^2 j} \text{ ----------- } (4)$$

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where q = rate of spreading

L_d = distance to lateral control

Extensive computations of Eqn.4 have been carried out on a digital computer CDC-3600 to determine the configurations of the rising mound for various values of q, L_d, a_o and k.

3. EXPERIMENTAL INVESTIGATION

Experimental studies have been carried out on a sand model and viscous flow model.

a) Sand Model: The set up consists of a water tight box 135 x 30 x 45 cm with glass plate in front. One end was completely closed by a plank and at the other end a 100 AST mesh in wooden frame at 120 cms from the left end was placed, to simulate the lateral control. A wooden plank 30 x 15 cms served as an adjustable weir at the lateral control end and to simulate the initial water table. Sand passing through ISS No.25 and retained on ISS No.40 with an average grain diameter of 0.5 mm was filled in the chamber to represent the medium. Twenty five piezometric connections were provided along 5 rows at five elevations. Outflow measurements as a function of time were obtained by the record of rise of water level in a tank with time with the help of an automatic recorder. The spreading ground was simulated by a channel 30 x 5 cms and 4 cms deep having clearly spaced holes for uniform distribution of water. This was fed by a constant head tank.

b) Viscous Flow Model : The analogy between ground water flow and laminar flow of a viscous fluid between two clearly spaced parallel plates is particularly useful in the study of problems of free surface flows through porous media. The model mounted on a wooden stand (Fig.2, Plate 1) comprises of two glass plates 120 x 60 x 0.62 cms placed at 2mm apart through spacers and clamps on all the sides. Spreading ground was simulated by two glass plates 10 x 7 x 0.6 cm clamped at the top left corner of the glass plates.

The spreading ground was supplied by a constant head tank with in turn was fed by a 20 litre storage tank. Necessary overflow arrangements and a T-arrangement to collect and measure the recharge rate were provided. A variable depth lateral control and collecting tank were provided at the right end. The fluid employed for the investigation was clean lubrication oil 'veedol' whose viscosity-temperature characteristics were established by redwood viscometer.

Experimental investigations were carried out on both models for different values of the various parameters of the problem. The free surfaces were recorded by means of a 35-mm camera at desired intervals. The coordinates of the stable mound were recorded and the outflows as a function of time were measured during rising and dissipation.

4. RESULTS AND DISCUSSIONS

Computations of the transient free surfaces of rising mound by Eqn.4 have been made for different values of the parameters q, a_0, L_d and k. A typical plot of the theoretical configurations of the rising mound is presented in Fig.3. It may be observed that the rise of mound is rapid in the initial stages and the rise is slow after a while. This may be attributed to the fact that water goes into storage initially resulting in small outflow which increases with time as the medium gets more and more saturated.

Fig.4 shows a plot of the configuration of the stable mound for different flow rates. Shown by dotted lines on this figure are those obtained by sand model. It may be seen that the theoretical and experimental results fall closely. The difference in the configuration predicted by theory and sand model may be due to the assumptions inherent in the theoretical analysis and to the possible errors in permeability measurements. While sand model is useful in predicting the rising mound configuration, in spite of its questionable accuracy because of the capillary rise, it cannot be conveniently used for studies on the dissipation of mounds.

A typical plot of the configuration of the rising mound by viscous flow model is presented in Fig.5 and by photographic records in Plate 2. From a comparison of the experimental and theoretical free surfaces for different values of the parameters, it was observed (6) that the general character of the mounds from both the methods is the same but the two results are not in close agreement. This deviation may be attributed to the fact that in linearising the basic equation the variable 'y' is replaced by the constant a_0. The errors inherent in this technique will generally be small if the rise of mound y_{max} is small compared to the value of initial water table depth. But in the present investigation the rise is nearly the same order as a_0 or higher and hence the discrepancy. Further studies would be necessary to fix up a limit for the ratio $\frac{y_{max}}{a_0}$ to obtain acceptable results with the linearised solution.

An insight into the development of the stable mound can be obtained by means of the time-outflow graph presented in Fig.6. One of the curves for q = 190 cc/min corresponds to the experiment for which the transient free surfaces are presented in Fig.5. It may be observed that the time to attain steady state for this case is 12 mins beyond which the outflow remains constant and equal to the inflow. This tallies with the time recorded in Fig.5 for the stable mound. A point of interest is to note that the time for the establishment of a stable mound appears to be independent of q_0 for a given L_d as can be seen in Fig.6 and also of a_0 as per observations recorded in Ref.6. However, further work for a wider range of parameters would be necessary before this aspect is conclusively proved.

5. CONCLUSIONS

Although the general character of the mound predicted by theoretical and viscous flow model results are the same, the discrepancy between the two may be due to the assumptions inherent in the formulation of the basic nonlinear equation and in the linearisation of the same. Further studies would be necessary to suggest the limit of the ratio of rise to initial water table depth within which the linearised solution would be valid. Viscous flow models are superior to sand models in as much as the former can be used to study both the rising and dissipating mound and the later has limitation in respect of the development of a capillary rise which makes the identification of the free surface difficult.

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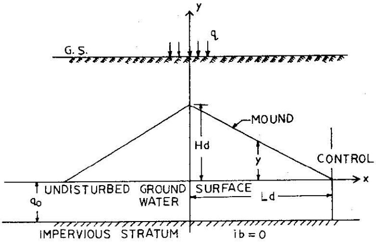

FIG. 1. DEFINITION SKETCH OF THE PROBLEM

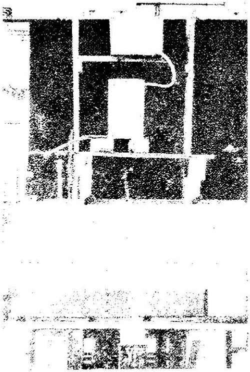

PLATE-1
VISCOUS FLOW MODEL

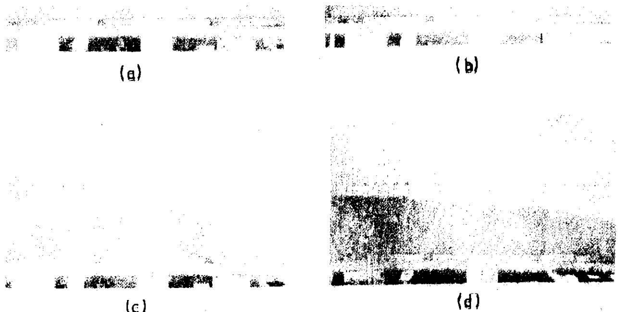

PLATE - 2

TRANSIENT FREE SURFACES (VISCOUS FLOW MODEL)

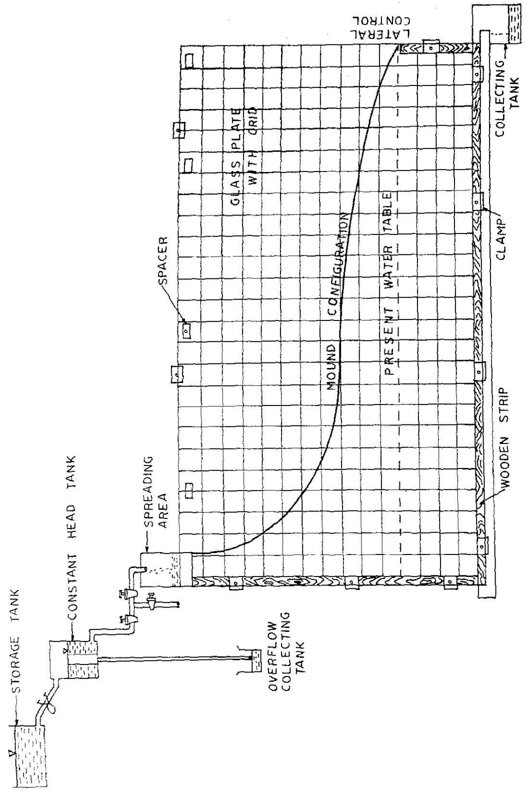

FIG. 2. VISCOUS FLOW MODEL

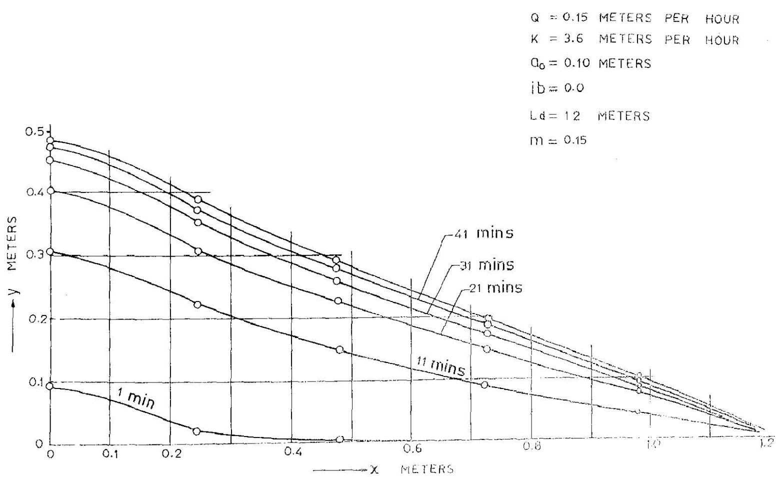

FIG. 3. RISING MOUND CONFIGURATION (THEORETICAL)

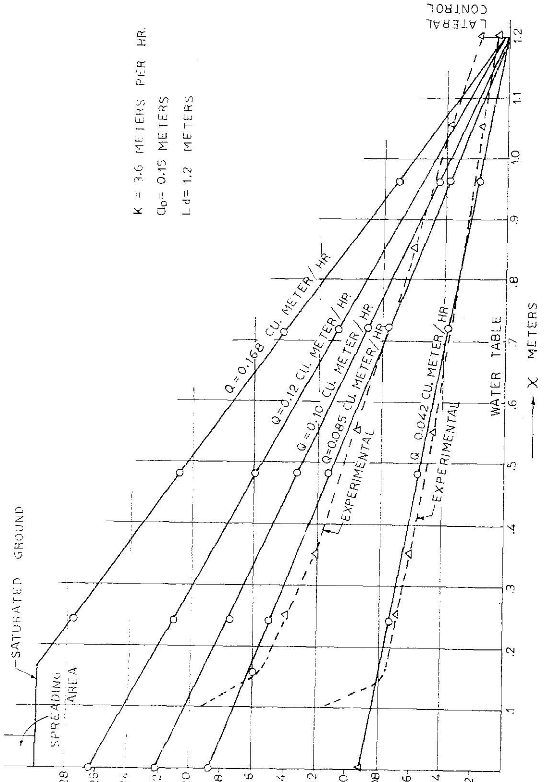

FIG.4. PLOT OF STABLE MOUNDS FOR DIFFERENT FLOW RATES

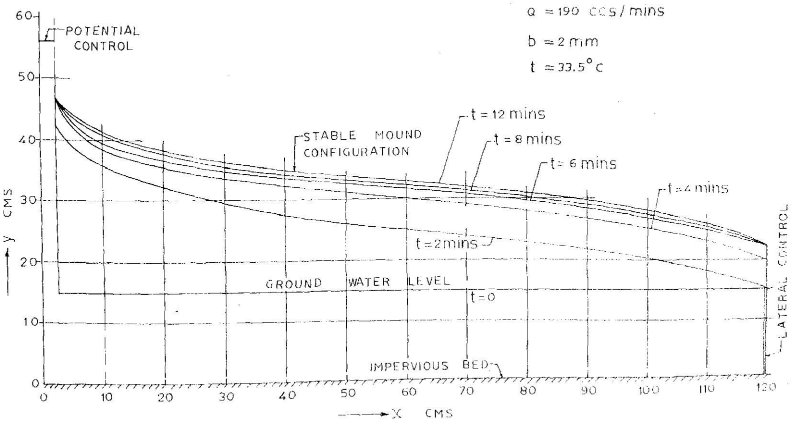

FIG. 5. RISING MOUND CONFIGURATION (VISCOUS FLOW MODEL)

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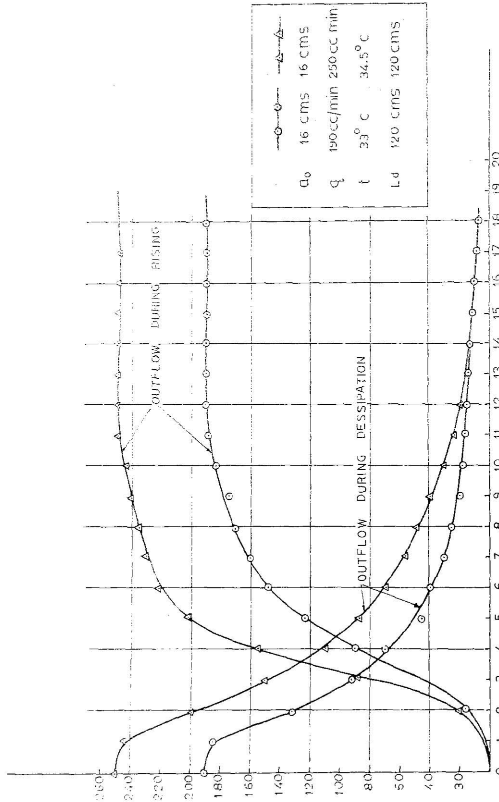

FIG. 6. TIME-OUTFLOW GRAPH

THE MAIN TRENDS OF HYDROGEOLOGICAL STUDIES IN ARTIFICIAL RECHARGE OF GROUND WATER RESOURCES (based on the practice of the USSR)

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SYNOPSIS

The problem of artificial recharge of fresh ground water resources in the present period of intensive development of the productive forces in many countries of the world including the USSR is very important and aquires actual significance.

In practice the artificial recharge of fresh ground water resources in the USSR is conducted in two main directions: a) artificial recharge of exploitation ground water resources immediately in the area under the influence of working large water intakes, and b) accumulation of the surface flow in natural storage reservoirs of new areas followed by building of captation constructions for the exploitation of artificial ground-water resources.

The hydrogeological investigations in the first trend represent a part of the exploitation stage of ground water exploration and are carried out on the sites of working water intakes for the purpose of preventing the ground water depletion or in connection with the additional water demand.

As the practice of the USSR shows these investigations are rather effective in an economical respect and defined by a number of natural factors.

The hydrogeological investigations in the second trend, in fact, have to be carried out in new poorly-studied regions. Therefore it is necessary to do much work which should answer, in its scope, the requirements of the adopted in the USSR exploration by stages of ground water.

At present, the problem of artificial recharge of fresh ground-water resources is very important and urgent in developed and developing countries. This problem is also of importance to the Soviet Union.

Fresh ground water is used for irrigation almost at the same scale in the USSR. The largest areas with irrigation by ground water are the Steppe Krimea, the Chu Valley in Kirgiziya, the Ararat Valley in Armenia, and the Kura-Araks lowland in Azerbaidjhan.

In Central Asia republics, the ground water of the so-called Kara-Su (spring flow in the peripheral parts of alluvial fans of the piedmont zone) is used for irrigation.

The prediction estimate of perspective utilization of fresh ground-water resources for the whole country and individual republics, fulfilled in the USSR, has shown that lack of ground water will be felt in a number of large regions and republics in 1980-2000 (N.I.Plotnikov, 1973).

Therefore the problem of public water supply in these regions will be solved by integrated use of surface and ground water and by artificial recharge of ground water.

The problem of artificial regeneration of fresh ground water in the upper part of the earth's crust consists of three closely interconnected large sections:

(1) the geological and hydrogeological substantiation (including the characterization of the recharge source);

(2) the techniques of regeneration, physical-chemical and biological principles; and

(3) technological means.

In the given report, only some aspects of the first section of the problem are discussed as the other two must be described separately.

In the USSR, the artificial regeneration of ground water resources is being carried out in two main directions: (a) recharge of exploitation ground water resources within the area of influence of large water intakes, and (b) artificial storage of surface water in natural underground reservoirs and further construction of water intakes for withdrawing the artificially created ground water resources.

In producing water-intake areas, ground water is recharged either for preventing ground water resources from depletion in the productive aquifer or for increase of the productivity of the water intake due to additional demands for water.

As the practice of the USSR shows, artificial recharge of exploitation ground water resources in producing water-intake areas is economically efficient and governed by the following principal factors:

(a) geological and hydrogeological conditions;

(b) existence of a high-quality recharge source in the vicinity of the water intake;

(c) the lithological composition, thickness and seepage properties of the rocks of the zone of aeration;

(d) the regime of the development of water supply;

(e) additional demand for water;

(f) the technical condition of the water intake and possibility of increasing withdrawal; and

(g) economic calculations determining the profitability of artificial recharge.

The hydrodynamic regime of the ground water flow (unconfined or confined) in the water-intake area plays an important role in selection of artificial recharge methods. At present, the world practice has enough methods for artificial recharge under unconfined flow conditions and some of them may be used - spreading basins or ditches, flooding, and recharge wells (Usenko, 1972; Favorin, 1967, Plotnikov, 1973).

The experience of artificial recharge under confined conditions is small so far and practically a single method has been developed, it is the recharge well method (Grigoryev, 1964).

Thus principal factors and conditions of artificial recharge of exploitation ground-water resources are clearly defined for producing water-intake areas.

The hydrogeological investigations for substantiation of artificial recharge of exploitation ground water resources in producing water-intake areas are carried out at the final stage of investigations - the development exploration of ground water (Plotnikov, 1971).

The geologo-economic and hydrogeological conditions and the direction of hydrogeological investigations are somewhat different in areas where ground water resources are generated by surface-water storage in natural underground reservoirs. Such areas are poorly studied in hydrogeological respect and have no producing water intakes, and here for substantiation of artificial storage it is necessary:

(a) to estimate the general structural, hydrogeological and hydrological conditions of the area under investigation and separate it into promising smaller areas;

(b) to register water users and demands for water;

(c) to conduct necessary hydrogeological investigations of a new favourable area for surface-water storage, including exploration for location of a water intake;

(d) to select and study a storage source and the method of surface-water delivery underground; and

(e) to estimate the profitability of expenditure on artificial ground-water storage; compare the estimate with other alternatives of water supply for the given project.

As is seen from the above, the storage problem is more complicated and calls for a large body of hydrogeological investigations which are similar to the stages of exploration for ground water accepted in the USSR - search, preliminary, detailed and development exploration. So, the artificial creation of fresh ground water in the Karakumy desert by storing surface runoff in spring involved a large complex of geological prospecting and test hydrogeological operations (Glazunov and Rogovskaya, 1968; Kunin and Leshchinsky, 1960).

Let us consider the hydrogeological investigations in both principle directions of artificial regeneration of ground water distinguished above.

A most rational complex of hydrogeological and hydrological investigations for substantiation of the artificial regeneration of the exploitation resources of unconfined ground water in producing water-intake areas that should be a part of the development exploration is presented in Table 1.

Table 1

A list of principal types of hydrogeological and hydrological investigations in producing water-intake areas for the purpose of artificial regeneration of unconfined ground water resources

Item No	Principal types of investigations	Purpose
1	Generalization and analysis of hydrogeological and hydrological data of the exploration and regime of exploitation ground water resources for the producing water-intake area	For drawing up a plan of development exploration for ground water in connection with artificial recharge
2	Qualitative and quantitative study of the source of artificial recharge of exploitation resources (special hydrological studies)	For working out recommendations for preparation of the development of the source and techniques of artificial recharge
3	Detailed studies of the seepage properties of the rocks of the zone of aeration and compilation of a transmissibility map for the water-intake area	For selecting the location of structures and method of infiltration recharge
4	Detailed hydrogeological and engineering geological investigations directly in the areas of planned structures in connection with artificial recharge of exploitation resources	For substantiation of designing engineering structures

5	Drilling test-production recharge wells and water-absorption tests	For artificial recharge with injection wells
6	Drilling control and additional observation wells and installation of other observation points (hydrometric posts, etc.) in the water-intake area	For the organization of stationary observations of the ground water regime
7	Test-production operations for artificial recharge of exploitation ground water resources as applied to the conditions of the recommended scheme and preliminary hydrodynamic computations	For testing the selected scheme of artificial recharge and recommendations for artificial recharge techniques
8	Office processing of the materials, compiling a report including the geologo-hydrogeological substantiation of artificial recharge	For working out a plan for artificial recharge
9	The hydrogeological supervision by the author of realizing the artificial recharge project in the water-intake area	For securing strict fulfilment of hydrogeological recommendations in artificial recharge
10	Organization and conduct of stationary observations of the ground water regime in a water-intake area	For studying the regime and conditions of prolonged development of ground water and further generalization of the experience obtained

The main hydrogeological, hydrological and engineering-geological investigations, enumerated in Table 1, are generally carried out using common techniques and their content is well known from the experience. The most important investigations are:

(1) studying the seepage, physico-chemical and biological properties of the rocks of the zone of aeration in the area of the water-intake influence;

(2) thorough hydrological and sanitary-bacteriological preparation of surface water prior to spreading; and

(3) studying the conditions of clogging the screen and rocks of the zone of aeration in spreading areas and also their self-cleaning capability

The results of these investigations are usually used in selection of the techniques and artificial recharge regime.

In the water-intake areas, tapping the confined aquifers, artificial recharge may be effected by a single method - the recharge wells - where surface water is injected.

The experience of the oil industry of the USSR in water injection in the course of oil field development may be used in artificial recharge. The methods of water preparation for injection and hydrodynamic computations for interfering wells with the pattern injection - withdrawal have been successfully developed.

For substantiation of this pattern in a water-intake area it is essential to carry out the following operations: (a) drilling and equipment of injection wells; (b) preparation and the method of delivery of the recharge source; (c) observation well drilling; and (d) injection tests.

As the oil-industry experience shows the effective artificial regeneration of exploitation resources of confined ground water may be effected if the following main tasks are solved: (a) selection of an optimal (for given conditions) design of screen for injection wells; (b) thorough preparation of the recharge source; and (c) working out the physico-chemical and biological principles of injection.

The oil-industry data show that for a number of reasons, in the course of development, injection wells become less productive.

Therefore conduct special tests is advisable at some projects. The compatibility of the ground water and the recharge water is also important.

The complex of investigations for surface water storage in natural underground reservoirs (the second direction of artificial recharge) is presented in Table 2.

Table 2

Main types of hydrogeological investigations for substantiation of surface -water storage in natural underground reservoirs and water-intake construc -tion

Item No	Types of investigations	Purpose
1	Estimation Generalization and analysis of hydrogeological materials of subdivision of the area under study in keeping with storage conditions and promising areas; preliminary characterization of the recharge source (in office)	For general estimation of prospects and substantiation of setting up of search operations in keeping with the demands for water
2	Search (1) Combined geologo-hydrogeological and geophysical survey in promising areas for storage and withdrawal of ground water (the scale of the survey is determined for each region depending on the availability of data)	For selecting areas and distribution of drilling volumes
	(2) Drilling hydrogeological search-mapping and test-contour wells	
	(3) Complex of geophysical logging investigations of hydrogeological wells	
	(4) Preliminary study of the seepage properties of rocks in promising areas for storage and withdrawal	
	(5) Preliminary study of the water to be stored	
	(6) Office processing of the materials, preliminary qualitative and geologo-economical estimation of operations for artificial recharge, comparison with other alternatives of water supply	

		All the complex of investigations is carried out for substantiation of setting up subsequent explo-ratory work, selection of the methods and techniques of artificial recharge, compiling a technological and economical report
3	Preliminary exploration (main stage of work) of the selected area for storage and construction of water intake	
	(1) Detailed (instrumental) combined geologo-hydrogeological and geophysical survey of the areas of planned structures (underground reservoir and water intake)	For substantiation of distribution of drilling volumes
	(2) Qualitative and quantitative study of the storage source for artificial recharge	For selecting the method of preparation of the storage source
	(3) Selective drilling of test injection wells, test-production wells (for withdrawal) and observation wells	For studying geological and hydrogeological conditions
	(4) Geophysical logging in hydrogeological wells	
	(5) Tests-injection, pumping and operations using the pattern injection - withdrawal	For studying the seepage properties of rocks and estimating artificially created ground water reserves
	(6) Engineering - geological investigations in the areas of structures planned	
	(7) Office processing of the materials and quantitative estimation of artificially created ground water resources in keeping with the pattern injection - withdrawal	

		All the complex of the investigations is carried out for substantiation of the detailed exploration of the area and geologo-economical evaluation of the storage project (practically for substantiation of design-ing and construction)
4	Detailed exploration of areas for injection and water intake (1) Completion of drilling development wells for recharge and withdrawal in keeping with the approved plan	The stage of detailed exploration is advisable to combine with the time of construction of all planned structures, and thorough hydrogeological supervision should be effected
	(2) Drilling a network of observation wells	
	(3) Test and development operations (on a production scale) for injection and tentative development	
	(4) A complex of stationary observations of the ground water regime and the recharge source in the process of long-term development	For subsequent generalization of development, experience and improving exploration methods

As the above shows, the hydrogeological investigations for artificial ground water storage are rather complicated. The lack of experience calls for fulfillment of a large body of stage operations to hold one of the most important principles of ground water exploration - the principle of successive approximations of the project under investigation (Plotnikov, 1973).

For the same reason, a number of points of the list presented call for refinement and are given as recommendations. So, the complex of operations for the stage of detailed exploration for areas of injection or spreading and water-intake construction is recommended to combine with the stage of construction and test development. Such a combination is rational and allows to reduce the total period of exploration of the project and preparation of it for long-term development.

In this connection, it is necessary to emphasize that the preliminary exploration of the storage project should be the main stage of the whole complex of investigations. The investigation results of this stage should essentially form the basis of planning and substantiating allocations for construction.

The studies for substantiating artificial recharge of exploitation ground water are a new direction in modern hydrogeological investigations and the relevant experience is small in the Soviet Union. Therefore the given recommendations for artificial recharge should be specified and refined depending on the complexity and availability of hydrogeological data for the area under investigation.

The techniques of certain types of hydrogeological investigations have been worked out poorly so far, particularly in artificial storage, in areas where there are no water intakes (the methods of studying the seepage properties of the rocks of the zone of aeration, injection methods, and prediction of clogging the bottom sediments of spreading structures).

Therefore it is advisable to carry out further theoretical and experimental investigations for working out and improving the methods of hydrogeological studies for substantiation of artificial recharge of ground water.

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