ELEMENTARY MATHEMATICS

Book VI for Form VI

S. BALAKRISHNA AIYAR

Price Re. 1

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BY

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PREFACE

The special features of the series are :

1. The Departmental Syllabus (the 1929 S.S.L.C. syllabus) is followed as the basis.

2. Instead of treating each topic separately in big individual chapters, the subject matter is as far as possible divided into study units. Each unit forms the basis of specific instruction in the fundamental principles and their varied applications without disturbing the general unity of the subject of Mathematics. This makes it possible for the teacher and the taught to have definite estimates of the ground covered and of the progress achieved in successive stages.

3. Useful introductions and experimental investigations leading the learner by induction to the discovery and enunciation of rules, principles and procedures, are followed by a variety of problems, questions and examples.

4. Exercises have not been classified as oral (mental) and written, as the capacity to do calculations mentally differs from pupil to pupil. They have therefore been so graded from the simplest to the relatively most complex types as to provide a scale which can be followed mentally as far as individual capacities of the pupils admit.

5. The treatment is a balanced blend of the logical and the psychological, retaining the good and popular features of the traditional methods and combining with them the accepted procedures characteristic of the new teaching, with graphic and practical work in all its aspects, historical notes, variety in methods on the basis of a

natural correlation between arithmetic, algebra and geometry.

6. The problems and examples are numerous and refer chiefly to Indian life and relevant situations in the other sciences.

7. The typography is so well set and arranged as to make the books easily readable and convenient to use.

Thanks are due to The Open Court Publishing Company, Chicago, for their permission to reproduce on page 48 of this book the portrait of Pythagoras taken from their *Philosophical Portrait Series*.

Suggestions for improvement and notices of corrections will be gratefully received.

S. B.

CONTENTS

1. EXAMINATION PAPERS, SERIES B

[BASED ON THE SUBJECT-MATTER OF BOOK V]

Paper I-A

1. What is the difference between *cost price* and *outlay* in problems relating to trade?

2. A number of measurements of lengths of straight lines are increased by the same amount. What effect has this on the average of those measurements?

3. State compactly, using brackets, the result of increasing a quantity Q by x per cent.

4. Write down the value of a number (in index notation) whose digits from the units place to the left are a, b, c, d.

5. Factorize $c(x-y) + 2d(x-y)$.

6. Find k from the following equations:

(i) $ka^2 = a^{10}$, (ii) $p^8 = \frac{p^{13}}{k}$, (iii) $k(x-a) = (a-x)$.

7. Point out which of the following equalities are correct:

(i) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$,

(ii) $\sqrt{a^2b^2} = ab$,

(iii) $\sqrt{a \div b} = \sqrt{a} \div \sqrt{b}$.

8. Two triangles are of equal area and stand on bases of 4·8 and 6·4 inches. If the height of the first is 3·6 in., find the altitude of the second.

Paper I-B

9. The sum of the interior angles of a figure bounded by n straight lines is $2n - 4$ right angles. Find the number of sides, if the sum of the interior angles is 16 right angles.

10. Find the difference between $3x^2 - 5x + 2$ and $2x^2 + 6x - 1$ (two answers).

11. Given that $4(a-b) = x$, $-5(a+b) = y$, express $a^2 - b^2$ in terms of x and y.

12. Write down the co-ordinates of the point where the graph of $y = -2x + 3\frac{1}{4}$ cuts the y-axis.

13. After paying income-tax at 6 ps. in the rupee on his income, spending $\frac{4}{9}$ of the remainder and lending Rs.75 to a friend, a man found he had only $\frac{5}{18}$ of his income left. What was his income ?

14. The capacities for work of two brothers are as 3 : 5. How long will they take to do a job separately which they can do together in $5\frac{5}{8}$ days ?

15. A mirasdar bought a field, but he had to sell it at a loss of 6 per cent. If he had sold it for Rs. 810 more, he would have realized $7\frac{1}{2}$ per cent profit. What did the field cost him ?

16. A moneylender auctioned away a mortgaged property for Rs. 7840 in full satisfaction of a loan advanced by him 2 years ago at 12 per cent C. I. What was the sum borrowed from him ?

17. A boy hires a bicycle by paying 8 as. for the use of the bicycle plus 2 as. for every mile that he rides.

(i) Express the cost of hiring in terms of the number of miles ridden.

(ii) Draw a graph representing the relation between the cost and the distance.

(iii) From the graph, find how many miles he can ride for Re.1-4-0.

Paper II-A

1. Show that a pure decimal which is a perfect square cannot contain an odd number of digits.

2. There are a number of rice bags containing different quantities of rice. x measures of rice are taken away from each of them. What is the effect of this on their average weight ?

3. Distinguish between $2x^2$ and $(2x)^2$. Find the value of each, when $x = 2$.

4. Work out the product $(a + b)(c - d)$. Check your answer using particular values for a, b, c, d.

5. Examine the correctness of the following statements :

(i) All squares are rectangles.

(ii) The diagonals of a rectangle cut each other at right angles.

6. A pair of adjacent angles of a parallelogram are in the ratio of 5 : 4. Find the angles of the parallelogram.

7. A number contains n digits. How many digits will its square contain ?

8. If $a = \frac{b+c}{b-c}$, express the relation (i) with b as the subject, (ii) with c as the subject, of the algebraic sentence.

Paper II-B

9. Two straight roads cross at right angles. A man walks across the country in such a way that twice his distance from one road is three times his distance from the other. Draw a diagram to illustrate his motion.

10. A debt of £65 is paid in francs at $9\frac{3}{4}d$ a franc, at a time when 25 francs are worth £1. What does the creditor gain or lose?

11. What should be added to $5x^2-8x+9$ in order that $x-3$ may be a factor of the sum?

12. Simplify: (i) $(a+b)^2+(a-b)^2$; (ii) $(a+b)^2-(a-b)^2$.

13. One of the diagonals of a quadrilateral field is 324 yards and the distances of the other two corners from it are 92 and 108 yards respectively. Find the area of the field in acres, to two decimal places.

14. The national debt of a country was reduced every year by 12 per cent of what it was at the beginning of that year. If at the beginning of 1923 it was £60500000, what was it at the beginning of 1925?

15. I borrowed Rs.4000 at 5 per cent per annum and Rs.5000 at $6\frac{1}{4}$ per cent on the same day. I was able to clear both the debts by paying Rs.10025 later. For how long was I indebted?

16. A boatman rowed a certain distance up and down a stream in 3 hours. He could row at 3 miles per hour in still water, and the stream flowed at 2 miles per hour. What was the distance?

17. Fifty-five per cent of the coolies working on an estate are men, thirty per cent women and the rest boys. The individual wages for the three classes are Rs.17-8-0, Rs.11-4-0 and Rs.4-2-0 per month. Find, to the nearest integer in each case, what percentage of the total wages is paid to (i) men, (ii) women, (iii) boys.

Paper III-A

1. By what powers of 10 should you multiply 0·0765 and 142·049 to reduce them to standard form?

2. Write down the formula for amount at compound interest. Indicate the quantities for which the letters in the formula stand.

3. Explain with reference to suitable examples the difference between *coefficient* and *index*.

4. If $F=\frac{9}{5}C+32$, express C in terms of F.

5. Simplify (i) $(-3b)^4\times(+5b)^3$, (ii) $(+2a)^4\div(-a)^3$.

6. State, in a compact form using brackets, the result of diminishing a given quantity P by r per cent.

7. In what respects do a square and a rectangle (i) agree with each other, (ii) differ from each other?

8. One of the angles of a parallelogram is p degrees. What are its other angles?

Paper III-B

9. At a certain place, the height in inches of the barometer at noon during a certain week in June was as given in the following table. Calculate the average pressure-reading for the week, with the least possible work.

Height	30·13	30·10	31·1	30·45	29·92	29·80	29·75
Days	1	2	3	4	5	6	7

10. If $e = 2{\cdot}7182818285$, find $\frac{1}{e}$ to 5 significant figures.

11. Two sides of a triangle are 4·8 and 6·4 cm., and its area is 12·8 sq. cm. Construct the triangle. How many such triangles can you draw?

12. Divide $7x^2 + 10x - 9$ by $x - 4$, and write down the result in the form $\frac{D}{d} = Q + \frac{R}{d}$.

13. Show that $a^2 - b^2 = (a + b)(a - b)$. Use it to simplify $775 \times 825 - 721 \times 679$.

14. The populations of two adjoining villages were equal. After the first had decreased by 20 per cent and the second increased by 15 per cent, the total was 19695. What was the population of each village at first?

15. A taxi can be hired for an initial charge of 10 as. and an additional charge of 4 as. for every mile it is run. If Rs. H is the total charge for m miles, write down the formula connecting H and m. Draw a graph of the relation, and use it to find the distance that can be travelled for Rs. 2-6-0, and the charges for 9 miles.

16. A playground 112 yd. long and 63 yd. broad is exchanged for a square one of equal area. Find the length of a side of the latter ground.

Paper IV-A

1. Bring out the difference in meaning between *remainder* and *difference* with respect to two given numbers.

2. The area of the square on a line is k times the area on $\frac{1}{3}$ of the line. What is k?

3. Mention separately the terms of $3x^3 - 4x^2 + 5x - 9$. Can you set down the expression with its terms occurring in any order without altering its value? If so, write it down in three other different ways.

4. The distance between two telegraph posts by the side of a railway is 66 yd., and the time taken by a train to pass this interval is 3 seconds. Find the speed of the train in miles per hour.

5. Give instances of zeroes that are significant and zeroes that are not significant in pure decimal numbers.

6. Given that $\frac{2824 \cdot 825}{74 \cdot 3375} = 38$, write down the values of $\frac{28248 \cdot 25}{7433 \cdot 75}$ and $\frac{2824825}{743375}$.

7. Two triangles have equal altitudes, but their bases are in the ratio of $m : n$. Compare their areas.

8. A studio charges a fixed sum of Rs. s for a sitting and Rs. c extra for each copy of a photograph. The pupils of Form VI of a school arrange for a sitting, buy n copies of the photo and pay on the whole Rs. r to the studio. Express r in terms of s, c and n. Hence find the value of c, when $s = 20$, $n = 76$, and $r = 210$.

Paper IV-B

9. If the length and breadth of a room are both increased by 10 per cent, by how much per cent is the area increased?

10. Draw a diagram to illustrate the identity $(x+a)(x+b) = x^2 + ax + bx + ab$. Supply the necessary particulars in it to bring out the equality.

11. A cotton merchant bought 100 bales of cotton at Rs. 131-4-0 per bale and sold them at 6 as. per lb. His customer failed and paid only 12 as. per rupee. If the weight of each bale was 500 lb., find his gain.

12. From the formula $T = 2\pi\sqrt{\frac{l}{g}}$, find T, if $\pi = 3 \cdot 1\dot{4}$, $l = 28$, $g = 32 \cdot 2$.

13. Use the expansions of $(a+b)^2$ and $(a-b)^2$ to simplify $988^2 + 1012^2$.

14. The number of times the circumference of a circle contains the diameter is denoted by π. If the value of $\pi = 3 \cdot 1415963$, to how many significant figures will it be true to take it for certain purposes as $3\frac{1}{7}$; as $\frac{355}{113}$?

15. The cost price of an article is Rs. 60. What should be fixed as the credit price if it is to be paid at the end of 6 months, interest being calculated at 5 per cent per annum?

16. The breadth of a rectangular field is 40 ft. shorter than the length. If each side is increased by 20 ft., the area is increased by 7600 sq. ft. Find the length and breadth of the field.

17. A contractor employs 78 men and 25 women and pays them daily wages amounting to Rs. 116-4-0. If 5 men together receive Re. 1 more than 7 women, what are the daily wages of each man and woman ?

18. A man who can walk at $3\frac{1}{2}$ miles per hour gives a start of 51 minutes to another man who can walk at 3 miles per hour. In what time will the first man overtake the second ? Illustrate your solution graphically.

Paper V-A

1. The income of a manufacturing firm in a certain year was 2060000 dollars, to the nearest thousand dollars. Point out the significant digits in the amount.

2. What number is represented by $3x^2+7x+\frac{6}{x}+\frac{9}{x^2}$ when x is made to stand for 10 ?

3. Explain the meanings of the following terms with the help of examples : (i) factor, (ii) multiple, (iii) power.

4. Fill up the blanks within brackets with suitable expressions :

$$\text{(i) } a-b=-\tfrac{1}{2}(\quad), \text{ (ii) } \frac{4}{3x-5}=\frac{-2}{(\quad)}.$$

5. Draw a diagram to illustrate the identity $(a-b)^2=a^2-2ab+b^2$. Enter in it the necessary particulars to bring out the equality.

6. From the sum of $-3x^3+x-7$ and $2x^2+5x-4$, subtract $2x^3-5x^2+7x-1$.

7. Show, by means of suitable diagrams, how you would cut a parallelogram into two parts and refit them to form a rectangle.

8. Taking the formula $\frac{1}{v}-\frac{1}{u}=\frac{2}{r}$, express (i) v in terms of u and r, and (ii) u in terms of v and r.

Paper V-B

9. A bookseller bought books at 11 for 10 shillings, and sold them at 10 for 11 shillings. What profit did he make on every dozen books ?

10. The length of a tropical year is 365·242218 days. Express this as a compound quantity (in days, minutes and seconds) to the nearest second.

11. Solve the equation : $\frac{2}{3}(x-5)-(4x+3)=\frac{3}{5}(5-2x)+44.$

12. In a triangle ABC, $\angle A - \angle B = 10°$, $\angle B - \angle C = 4°$. Find the three angles separately.

13. A can plough 4 acres of land in 18 hours, and B, 3 acres in 12 hours. If they both work together, how long will they take to plough 17 acres ?

14. Assuming that 15 cm. = 5·9 in., draw a graph to convert inches to centimetres and vice versa. From the graph, obtain (i) the number of inches in 4 cm. and 10 cm. ; (ii) number of cm. in 2·8 in. and 4·5 in.

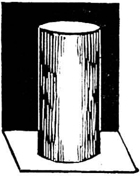
15. Construct a triangle ABC, given BC = 6·2 cm., CA = 5·4 cm., $\angle C = 68°$. If the figure represents a field on a scale of 1 cm. = 1 furlong, find the area in acres.

16. A manufacturer makes an article and sells it to an agent at a profit of 10 per cent. The agent sells it to a shopkeeper, securing a profit of 21 per cent above the cost to the manufacturer. What was the agent's profit per cent ?

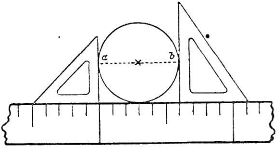
2. CIRCUMFERENCE OF A CIRCLE (1)

1. Introductory remarks

An uncut round pencil, the ordinary round ruler, the stone-roller for rolling tennis courts are examples of solids having their shape like that of the solid represented in figure (i) below. It is called a cylinder. Its two flat ends are *circular*. So also are the faces of some of our coins, the rupee, half rupee and quarter anna pieces.



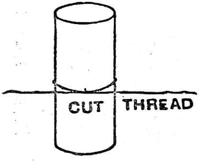
(i)



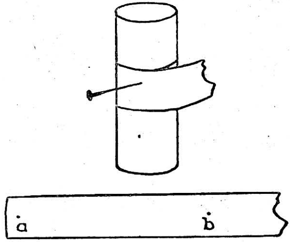
(ii)

Figure (ii) shows a method of finding the diameter of a plane face of a cylinder. It is supported by two set-squares, all resting on a graduated ruler. The diameter $ab = 0·62$ in. approximately.

Figure (iii) shows a method of finding the 'circumference' of the cylinder (i. e. of the flat end surface) with the help of a piece of thread. The thread is wound round once and cut. The piece cut out has the length of the circumference.



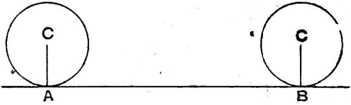
(iii)



(iv)

Figure (iv) shows an alternative method. A thin piece of paper is wrapped once round, and a pin-prick is made—to pierce the two layers of paper. If now the bit of paper is opened out, you get two punctures *a* and *b*, the distance between which is the length of the circumference required.

The figure below suggests a method of finding the circumference of a flat face of a coin or circular disc, by rolling it com-



pletely once round along a straight line. Write down a description of how it is done.

2. Circumference of a circle

Of two circles of different radii, that which has the longer diameter has the longer circumference. Let us now find out whether there is a definite relationship between the diameters and circumferences of circles.

Experiment. Draw circles of diameters 1, 2, 3, . . . inches and measure their circumferences. Set down your measurements in a tabular form, as below:

Circle No.	Diameter	Circumference	Circumference ÷ diameter
(1)	1 inch	...	...
(2)	2 inches	...	...
(3)	3 inches	...	...
(4)	4 inches	...	...

Calculate in each case the number of times the circumference is of the diameter, to two places of decimals. Compare the results. What do you notice?

If the measurements are very carefully taken, the quotients will be found to be nearly equal to one another. To neutralize errors of measurement, take the average of all the results. A good value of this average will be found to be 3·14. You then have the important relation:

The length of the circumference of a circle

= 3·14 × the diameter.

The circumference of a circle is proportional to the diameter.

Again $3{\cdot}14 = 3\frac{14}{100} = 3\frac{7}{50} = 3\frac{1}{7}$ nearly.

If c, d and r denote the lengths of the circumference, diameter, and radius of a circle (units understood), then

$$c = 3{\cdot}14d = 3{\cdot}14 \times 2r \qquad \ldots\ldots\ldots\ldots \quad (1)$$

$$c = 3\frac{1}{7}d = \frac{22}{7}d = \frac{22}{7} \times 2r \qquad \ldots\ldots\ldots\ldots \quad (2)$$

This ratio $c : d$, namely 3·14, is very important in mathematics, and is denoted by the Greek letter π (read *pi*).

Hence $c = \pi d = 2\pi r.$

The correct value of π is a never-ending decimal fraction. A large number of its digits will be required in calculations demanding great accuracy, as in astronomy.

Various devices are employed to remember the successive digits. *The Scientific American* (21–3–1914) gives the following pair of lines:

See I have a rhyme assisting
My feeble brain its tasks sometimes resisting.

Here the number of letters in each successive word gives the digits in order for the value of π, viz. 3·141592653599.

3. Exercises *(oral as far as possible)*

1. Find the circumferences of circles whose diameters are :

(i) 21 in., 44 ft., 147 yd. (iii) 28 cm., 100 m.
(ii) 5 in., 20 ft., 32 yd. (iv) a in., b ft., m miles.

2. Calculate the circumferences of circles of radii 7 in., 14 ft., 8 yd., 125 m.

3. The length of the minute-hand of a clock is 3·5 in. What distance does the free tip travel in one day ?

4. The radius of the earth is 3956·4 miles. Find its circumference to 3 significant figures, taking $\pi = 3·14159$.

5. A circular enclosure in a zoological garden has a diameter of 280 ft. Find the cost of putting up a railing round it at Rs. 6-4-0 per yard.

6. Assuming the orbit of the earth to be circular and of radius 92 million miles, find the distance travelled by the earth round the sun in one year.

7. A man rows across a circular pond 588 yards in diameter. His son runs round the edge of the pond to meet him. How much farther has he to travel than the father ?

8. The diameter of a wheel of a locomotive is 4·75 ft. How far does it go in 120 revolutions ?

9. The length of a curve is measured by rolling a coin along it. If the diameter of the coin is 1·6 in., and it is found to turn $14\frac{1}{2}$ times, what is the length of the curve ?

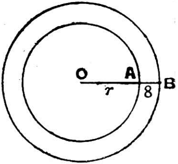
10. The diameter of a wheel is 2 ft. 4 in. How many times will it revolve in going over a mile ? Deduce from your result the number of times that wheels of diameters 1 ft. 2 in., 4 ft. 8 in., $3\frac{1}{2}$ times 2 ft. 4 in., turn in going over a mile.

11. The breadth of a circular race course is 33 yards. The inner boundary of the course is 11 furlongs. How long does a horse take to run round the outer boundary, if it gallops at the rate of 16 miles per hour ?

3. CIRCUMFERENCE OF A CIRCLE (2)

1. We shall now turn to problems and exercises in which the circumferences of circles are known and we are to find the diameters or radii.

Example. At the annual sports in a school, a mile race was run on a perfectly circular course, the track being 8 yd. wide. Four times round the course on the inner edge of the track was an exact mile. What was the length of the outer edge of the track?



The figure is a sketch of the course. Let r yd. be the radius of the inner bounding circle.

Then 4 times $2 \times \frac{22}{7} \times r = 1760$, whence $r = 70$.

Thus the inner radius of the course = 70 yd.

∴ the outer radius = 78 yd.

∴ the length of the outer edge of the track $= 2 \times \frac{22}{7} \times 78$ yd.
$= 490\frac{2}{7}$ yd.

2. Exercises (*oral as far as possible*)

1. Calculate the diameters of circles whose circumferences are: 44 in., 29 yd. 1 ft., 6·6 metres, 12 in., 35 ft., 37·2 metres. Use π = (i) $3\frac{1}{7}$, (ii) 3·14.

2. Calculate the radii of circles whose circumferences are: 11 in., 14 yd. 2 ft., 121 cm., 28 ft., 10 chains 50 links, 100 yards. Use π = (i) $3\frac{1}{7}$, (ii) 3·1416.

3. Taking $\pi = 3{\cdot}14159$, find the diameter, to the nearest foot, of a circular lawn whose boundary is 1000 ft.

4. Calculate the value of $\frac{1}{3{\cdot}1416}$ correct to the nearest thousandth. Use your result to find the radius of a circle whose circumference is 43·975 in.

5. The inner edge of the wall of a circular well measures 31 ft. 3 in. Find the diameter of the well.

6. The water edge of a circular tank is 6 furlongs. Calculate the diameter of the water surface.

7. The circumferences of two concentric circles are 99 and 77 cm. respectively. Find the breadth of the ring between the two circumferences.

8. If 198 boys are arranged in a circle, what will be its radius, allowing an average of 23 in. for each boy to stand in?

9. The wheel of a cart is observed to make 240 revolutions in going over 4 furlongs. Find its radius.

10. The circumference of the earth is 40000 kilometres Taking the ratio of the circumference of a circle to the diameter as $\frac{355}{113}$, find the radius of the earth.

11. The length of a quarter of a meridian of the earth (from the equator to a pole) is 10 million metres. If 1 metre = 39·3708 inches, find the diameter of the earth to the nearest mile. Take π = 3·1416.

4. SIMILAR FIGURES

1. Introduction

A photo of a man and its enlargement have the *same shape*. If a diagram is seen through a magnifying glass (microscope), an enlarged view is obtained; the image is *like* the original as regards *shape*. A large wall map of India and a small map of India in an atlas look alike in outline; they have the same shape, although of different sizes.

Figures that have the same shape are called **similar figures.**

2. Conditions of similarity

The floor of a room is a rectangle 40 ft. by 30 ft. To make a plan, if the length is represented by 4 inches, the breadth should be 3 inches. If the length is represented by 8 inches (twice 4 inches), the breadth should be 6 inches (twice 3 inches). If the first plan is looked at through a magnifying glass of double power, the size of the image will be that of the second plan.

If the length of a plan is doubled, but the breadth more than doubled, you would make the enlargement broader than it should be, and it would become out of shape. Thus it is important that *the proportionality of sides be maintained.*

If a wire is bent into an angle ABC, with $\angle$B = 35°, AB = 2 cm. and BC 1·5 cm., and seen through a lens which magnifies three times, AB will appear to be 6 cm. long, BC 4·5 cm. long, but $\angle$ABC will still be 35° and not 35 × 3 degrees.

You now have the following principle:

Two figures are similar, if the sides of the one in order are proportional to the corresponding sides of the other, but the angle between every pair of sides of the one is equal to the angle between the corresponding pair of sides of the other.

3. Exercises *(oral as far as possible)*

1. Draw a rectangle ABCD 4 cm. by $2\frac{1}{2}$ cm. Enlarge it to double the dimensions.

2. Draw a rectangle 12 cm. by 8 cm. Reduce it to two-thirds the dimensions.

3. In two plans of the same rectangular field, the lengths are in the ratio of 3 : 7. What is the ratio of (i) their breadths, (ii) their perimeters ?

4. There are two plans of a field drawn to scale in the ratio of 5 : 2. If the perimeter of the first is 35 inches, what is the perimeter of the second ?

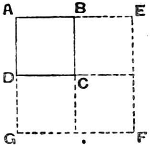
5. Show that all squares are similar figures. Are all rhombuses similar figures ? Give a reason for your answer.

6. In a plan of the floor of a room, the length and breadth are 5·2″ and 3·6″ respectively. If another plan of the same floor is prepared, (i) what should be the breadth if the length is to be 6·5″, (ii) what should be the length if the breadth is to be 3″ ?

7. In a photograph of a building, a window measures 2·5″ by 1·3″. If the height of the window is 8 feet, what is its breadth ?

8. An oil-painting 6 ft. by $2\frac{1}{2}$ ft. is photographed on a scale which gives a photo of breadth $7\frac{1}{2}$ inches. What is the height of the photograph ?

4. Areas of similar figures (*squares and rectangles*)



(1) The figure shows a square ABCD of side 1 cm., and another square AEFG of side 2 cm. AE = 2 AB.

The area of ABCD = 1 sq. cm., and that of AEFG = 2 × 2 or 4 sq. cm.

Thus, (i) if the side of a square is doubled, the area is increased 4 times, and, in general,

(ii) *if the side of a square is n times that of another square, its area will be $n \times n$ or n^2 times the area of the other.*

(2) If a rectangle is l inches by b inches, its area is $l \times b$ sq. inches. If the sides are increased to nl and nb inches, the rectangle remains similar to itself. The new area is $nl \times nb$ or $n^2 l \times b$ sq. inches.

Thus, *if the sides of a rectangle are changed in the ratio $1 : n$, the area is changed in the ratio $1 : n^2$.*

5. Exercises (*oral as far as possible*)

1. Two squares X and Y are such that the side of X is (i) $\frac{1}{2}$, (ii) $\frac{2}{3}$, (iii) $\frac{a}{b}$ times the side of Y. Compare their areas.

2. Two similar rectangles M and N are such that their lengths (or breadths) are in the ratio (i) 2 : 1, (ii) 5 : 3, (iii) $x : 1$, (iv) $1 : y$, (v) $p : q$. Compare their areas.

5. LINES AND PLANES

1. The following important definitions and relations about lines and planes should be carefully thought over and remembered.

(i) *If a straight line is perpendicular to a plane at any point on it, it should be so to at least two straight lines on the plane passing through the point in two different directions.*

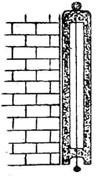
(ii) *The distance of a point from a plane is the length of the perpendicular from the point to the plane.*

(iii) *Two planes are said to be parallel if they do not meet each other however far they may be extended all round.*

(iv) *A straight line is said to be parallel to a plane, if it does not meet the plane however far they are extended.*

(v) *If a straight line l meets a plane P at a point A, and B is the foot of the perpendicular from any point of l to the plane P, then the angle between the line l and the plane P is the angle between l and AB.*

2. Vertical lines and planes



If a piece of thread is attached to a weight at one end and hangs by the other end A freely and at rest, the downward direction of the thread points to the centre of the earth. This is the *vertical direction* from A or at A.

A string carrying a weight at one end is used by masons to test whether rods, posts and edges of walls are truly upright or vertical. It is called a *plumb-line* or *plumb.*

The **vertical direction** *at any point is the direction of the plumb-line hanging freely at rest from that point.*

Hang a plumb-line at rest and hold a piece of cardboard touching it all along its length. Then the face of the cardboard in contact with the plumb is a *vertical plane.*

Any plane containing a vertical line is a **vertical plane.**

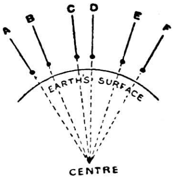
3. Horizontal lines and planes

Planes may slope in various directions in space.

A plane through a point A perpendicular to the vertical direction at A is called a **horizontal plane** *through A.*

Any line drawn on a horizontal plane is a **horizontal line.**

Note. If plumb-lines are hung from a number of points A, B, C, . . . near the earth's surface, all of them point towards the centre of the earth. Hence the vertical lines indicated by them converge to that centre. Now the radius of the earth is nearly 4000 miles. If, then, two of the points C and D are very near each other, the verticals through them *appear* parallel, because they tend to meet at a point 4000 miles away. For example, the wires by which the electric lights in a room hang, and the corner lines along which the walls of a room meet, may for practical purposes be regarded as parallel lines.



Again, in a circle of small radius, say 1 cm., the circumference is well curved. If the radius becomes larger and larger, the circumference becomes less and less curved. Because of the great radius of the earth, the curvature of its surface is not felt over small portions of it. For example, the surface of water at rest in a tub or in a small pond may be regarded as plane for practical purposes. Such a surface then may be regarded as perpendicular to the vertical at any point on it, and hence as a *horizontal plane.*

4. Exercises (*oral as far as possible*)

1. How many vertical lines can be made to pass through any given point? How many horizontal lines?

2. How many vertical lines can be drawn on a wall of an ordinary room? How many horizontal lines?

3. How many vertical planes may be supposed to pass through a given point? How many horizontal planes?

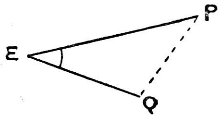
4. Place a cube on the horizontal top of a table. Point out the edges and faces (i) that are vertical; (ii) that are horizontal.

5. A wire bent in the form of a right angle is held with one arm vertical. What is the direction of the other arm? If the wire is rotated about the vertical arm kept fixed, what kind of surface will the other arm describe?

6. Take a cuboid and hold it so that four of its edges are horizontal and the remaining edges inclined to the horizontal.

5. Angular distance

Suppose P, Q are the positions of any two objects, and E, that of the eye of the observer. Join EP, EQ.



Then we say that

(i) PQ *subtends* the angle PEQ at E.

(ii) Angle PEQ is the *angular* distance between P and Q as observed from E.

Thus, the angular distance between two stars S_1 and S_2 is the angle between ES_1, ES_2, E being the observer's eye or position of observation.

6. Angles of elevation and depression

Mark a point H on a pillar or a wall on a level with your eye E. Step back a few yards and hold a thin straight rod with one end close to your eye E, and the rod itself pointing to H. The rod is now horizontal. Without changing the position of the eye end of the rod, raise the rod to any point P *vertically above* H.

The angle through which the rod has been raised or *elevated* is called the **angle of elevation** of the point P for the position of observation E.

If, on the other hand, the rod is lowered or *depressed* to a point Q *vertically below* H, then the angle through which this is done is called the **angle of depression** of Q for the position of observation E.

Note. The words *elevation* and *depression* are often used by themselves to denote *angle of elevation* and *angle of depression* respectively.

7. Clinometer

The angles of elevation and depression can easily be observed with a simple device (instrument).

Take a protractor graduated as shown in the figure below and attach a plumb-line to its centre O. If it is held with AB horizontal and its plane vertical, the plumb-line through O will lie opposite the 0° mark, Fig. (*a*).

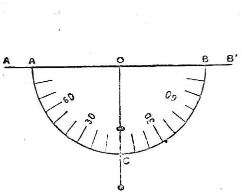


Fig. (*a*)

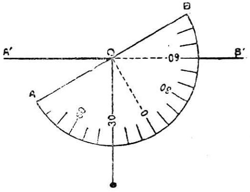


Fig. (*b*)

If the protractor is turned so that AB slopes as in Fig. (*b*), the plumb-line is still vertical. If A′B′ is the horizontal through O, the amount of slope of OB above the horizontal may be read by the size of the angle between 0° line and the plumb-line.

Such an instrument is called a **clinometer**, which is a contraction of *inclinometer* or measurer of inclinations. Get some practice in measuring with a clinometer the angles of elevation and depression of objects around you.

Example. A telegraph post is 20 feet high. A man 5 feet tall stands at a distance of 15 feet from the post. What will be the angle of elevation of the top as observed by him?

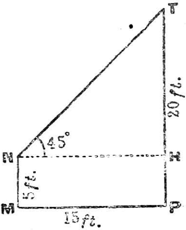
Take a scale, say 1 inch = 10 feet. Draw a horizontal line 1·5 in. long to represent the distance from the man to the post.

At M draw MN perpendicular to MP and 0·5 in. long to represent the height of the man.

At P draw PT perpendicular to MP and 2 in. long, to represent the height of the post.

Join NT, and draw NH, the horizontal line through N.

The angle HNT is the angle of elevation of T as seen from N (the eye of the observer). It is 45°.



8. Exercises

Note. Draw a rough sketch for each problem before proceeding to construct the diagram accurately.

1. A man stands at a distance of 60 ft. from a steeple which is known to be 120 ft. high. What will be the angle of elevation of the top of the steeple from his eye-level ?

2. A boy 3 ft. 6 in. tall stands at a distance of 16 ft. from a telegraph post 24 ft. high. What will be the angle of elevation of the top of the post as observed by the boy ?

3. A man 5 feet high stands at the top of a tower 95 feet high, and observes a milestone on the ground at a distance of 250 feet from the foot of the tower. What will be the angle of depression of the milestone as observed by him ?

4. A boat is known to be at a distance of $1\frac{1}{2}$ furlongs from the seashore. What will be the angle of depression of the boat as seen from the top of a lighthouse on the shore 220 feet high ? The height of the observer may be neglected.

5. A flagstaff 15 feet high stands on the terrace of a building 40 feet high. Find the angles of elevation of the foot and the top of the flagstaff from a point on the ground at a distance of 45 feet from the point on the ground directly below the flagstaff.

6. Show that the angle of elevation of a point B as seen from another point A is equal to the angle of depression of A as seen from B.

7. A man 6 feet high, stands in the sun and his shadow measures 7 feet 6 inches. What is the altitude of the sun at that instant ?

N.B. The angle of elevation of the sun is called its angular height or altitude.

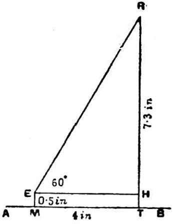
6. MEASUREMENT OF HEIGHTS AND DISTANCES (1)

1. You have already learnt that the heights of objects above eye-level, depths below eye-level, and the angles of elevation and depression, are intimately connected with the position of the observer with reference to those objects. We can make use of this intimate relationship to find the heights and distances of such objects.

Example 1. A man, standing at a distance of 20 ft. from the foot of a tree, finds the angle of elevation of the top of the tree to be 60°. Find the height of the tree, supposing the man's height (height of his eye above ground level) is 5 ft.

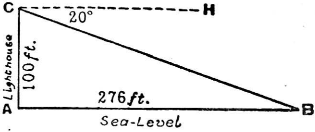
Let the scale of representation be 1 in. = 5 ft.

Draw a straight line AB to represent the horizontal ground. Take ME = 1″, to show the height of the man's eye. Take MT = 4″, to show the distance between the man and the tree TR. Through E draw EH parallel to AB. This line is the horizontal through E. At E make ∠HER = 60°, and let ER meet TR at R. Then TR gives the height of the tree.



Example 2. From the top of a lighthouse 100 feet above sea-level, a buoy floating on the sea has a depression of 20°. How far is the buoy from the foot of the lighthouse ?

Choose a convenient scale. Draw AB to represent the water-surface. At A draw a perpendicular AC to represent the lighthouse 100 ft. high. At C make an angle HCB of 20° with the horizontal CH ; let CB meet AB at B.



Then AB gives the required distance, namely 276 ft.

2. Exercises

1. The top of a telegraph post is connected to a peg on the ground by a straight wire. If the distance of the peg from the foot of the post is 16 feet, and the wire makes an angle of 36° with the horizontal, find the height of the post.

2. The angle of elevation of the top of a tower from a point on the horizontal ground at a distance of 120 feet from the foot of the tower is 50°. Find the height of the tower.

3. The line joining the tops of two vertical poles fixed on level horizontal ground makes an angle of 20° with the horizontal. If the distance between the posts is 60 feet, find the difference between the heights of the poles.

4. A man 5 feet 6 inches tall, stands at a distance of 34 feet from the foot of an upright coco-nut tree and observes the angle of elevation of the top to be 45°. Find the height of the tree.

5. A flagstaff stands on the top of a tower. To find the height of the flagstaff, a scout stands at a distance of 100 feet from the foot of the tower, and observes the angles of elevation of the top and foot of the flagstaff to be 60° and 40° respectively. What is the height that will be obtained by the scout using the observations?

6. I observed a balloon rising vertically from a point 2500 yards in front of me. At a certain moment, its angle of elevation was 60°; 10 minutes later, its elevation had increased to 70°. If the balloon rose at a uniform rate, what was the rate of ascent in feet per second?

7. From the top of a cliff 300 feet high, a balloon was observed to rise vertically from a point on the ground, level with the foot of the cliff at a distance of 2 furlongs. The angle of elevation at the first observation was 42°, and at the second observation 12 minutes later, it was 50°. Find the rate at which the balloon was ascending.

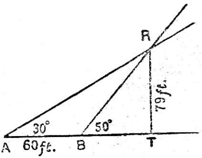
8. From the top of a mountain the angle of elevation of the top of a neighbouring mountain is 20°. If the difference in their heights is 500 feet, find the distance between the points in their bases vertically below their peaks.

9. A person on the top of a tower 80 feet high looks down on a milestone on a horizontal road running from the base of the tower. If the angle of depression of the milestone is 60°, find its distance from the tower.

7. MEASUREMENT OF HEIGHTS AND DISTANCES (2)

1. In the previous lesson, the heights and distances were obtained from the construction of right-angled triangles. There are objects like mounds and hills whose heights may be calculated by using data which do not involve distances from points vertically below their tops, as these cannot often be reached.

Example. A person wishing to find the height of the central tower of a big building stands at some distance away and finds the angle of elevation of the top to be 30°. Then he goes 60 feet directly towards the tower and again finds the angle of elevation to be 50°. Find from a suitable diagram the height as obtained by him.



(Reduced)

Choose any convenient scale, e.g. 1 cm. = 10 ft.

Draw AT to represent the horizontal ground. Mark AB = 6 cm. to represent 60 ft.

At A and B make angles TAR, TBR equal to 30° and 50° respectively. Let AR, BR meet at R. Then R will represent the top of the tower.

From R draw RT perpendicular to AT.

Then TR represents the tower. Its height is 79 ft.

2. Exercises

Note. For every question, draw a free sketch and enter in it the given measurements. Then reconstruct the figure to scale.

1. The elevation of the top of a tower from a certain point on a horizontal road through its foot is 30°. On walking 150 feet towards the tower, the elevation increases to 45°. What is the height of the tower ?

2. The angles of elevation of the top of a church steeple from two points 30 ft. apart and in a line with the base of the steeple, are 45° and 60° respectively. Find the height of the steeple.

3. From a window in the first floor of a building 25 ft. from the ground, the angle of elevation of an airplane was noted to be 48°, and simultaneously its elevation from another window in the fifth floor 75 ft. from the ground was observed to be 32°. Determine the height of the airplane at the instant of observation.

4. From the deck of a ship, the elevation of the light of a lighthouse is 45°, and from the mast-head it is 41°. If the height of the mast is 50 ft., find the height of the lighthouse above the level of the deck of the ship.

5. From the top of a mast of a ship 30 ft. high, the depression of a buoy was observed to be 25°. From the foot of the mast it was 8°. Find the distance of the buoy from the ship.

6. An observation tower 50 feet in height stands on a hillock. An observer on a level with the foot of the hillock, finds the elevations of the top and foot of the tower to be 45° and 30° respectively. Find the height of the hillock.

7. From the top of a hill the angles of depression of two milestones due east on level ground were observed to be respectively 25 and 30 degrees. What was the height of the hill?

8. Two observers in an airplane simultaneously observed the depressions of two milestones on a level ground below them to be respectively 40 and 45 degrees. How high was the machine flying then ?

8. CONSTRUCTION OF TRIANGLES: AMBIGUOUS CASE, ETC.

1. You have already learnt to construct triangles with the following combinations of data : (i) one angle and the containing sides, (ii) one side and two angles, and (iii) three sides.

Two more combinations of any three elements remain to be considered, namely (*a*) any two sides and the angle opposite to one of them, (*b*) the three angles.

2. To construct a triangle given two sides and the angle opposite to one of them.

To copy a $\triangle ABC$, suppose you take the measurements AB = 2·5 cm., AC = 1·7 cm. and $\angle B = 36°$ [Fig. (i)].

Then draw AB = 2·5 cm. [Fig. (ii)]. At B make $\angle ABX = 36°$. It now remains to put in the side AC. To do so :

With centre A and radius 1·7 cm., draw an arc of a circle cutting BX. You get two intersections C, C′.

Here $\triangle ABC$ is an exact copy of the $\triangle$ given. Take a tracing of the given $\triangle$ and place it on the copy to see if it fits.

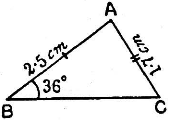


Fig. (i)

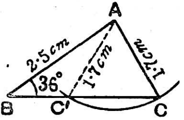
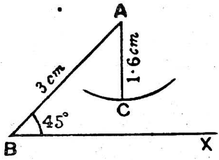


Fig. (ii)

Now you notice that if AC′ be drawn, you get a $\triangle ABC'$ satisfying the data used. Suppose that, instead of being asked to copy $\triangle ABC$, you are required to construct a $\triangle$ given AB = 2·5 cm., $\angle B = 36°$, AC = 1·7 cm. You can offer ABC or ABC′ as satisfying the data. You cannot definitely say which of them is meant by the examiner. Hence there are two alternative solutions. The construction of such $\triangle$s is referred to as that of the **ambiguous case.**



If the data noted above are given as AB = 3 cm., $\angle B = 45°$ and AC = 1·6 cm., the construction circle fails to reach BX, and you do not get a triangle at all.

In the figure on the left, what is the minimum length for AC in order that AC reaches BX at all? How many triangles will you get then? Draw the figure in that case.

How many triangles will you get if the data are

(i) AB = 3 cm., $\angle B = 45°$, AC = 3 cm.

(ii) AB = 3 cm., $\angle B = 45°$, AC = 4 cm. ?

Draw figures to illustrate each case.

Note that, in the ambiguous case, one of the triangles is acute-angled and the other obtuse-angled.

3. Exercises

1. Construct a triangle ABC, given that

(i) $\angle A = 34^\circ$, AB = 3·9 cm., BC = 2·3 cm.
(ii) $\angle A = 34^\circ$, AB = 3·9 cm., BC = 3·9 cm.
(iii) $\angle A = 34^\circ$, AB = 3·9 cm., BC = 4·5 cm.
(iv) $\angle A = 34^\circ$, AB = 3·9 cm., BC = 1·5 cm.
(v) $\angle B = 70^\circ$, BA = 1·6″, AC = 1·9″.
(vi) $\angle C = 44^\circ$, BC = 2·15″, AB = 1·45″.
(vii) $\angle A = 55^\circ$, AB = 1·6″, BC = 1·4″.
(viii) $\angle B = 49^\circ$, AB = 4 cm., AC = 2·5 cm.
(ix) $\angle B = 128^\circ$, AB = 1·5″ AC = 1·9″.

Point out the cases in which there are two solutions.

2. Two sides of a triangle are 3″ and 2·4″. One angle of the triangle is 30°. How many triangles can you describe? Find the third side in each case.

4. Construction of a triangle when its three angles are given

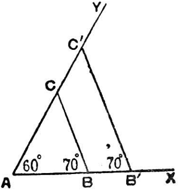
Consider the construction of a triangle ABC, given $\angle A = 60^\circ$, $\angle B = 70^\circ$, $\angle C = 50^\circ$.

Draw one of the given angles, say $\angle A = 60^\circ$.

Since nothing is known about the lengths of its arms, we shall take the point B somewhere on the arm AX of angle A.

At B make $\angle ABC = 70^\circ$, making BC meet AC at C.

Then $\angle C = 180^\circ - \angle A - \angle B = 50^\circ$. Thus the third condition also is satisfied, so that Δ ABC constructed satisfies all the three given conditions.



Since the point B may be marked anywhere on AX, we can get any number of triangles having its angles respectively equal to 60°, 70° and 50° (data). All these triangles are equiangular to one another, but are of different sizes.

The reason for the indefiniteness of the construction is this. If, in a triangle ABC, $\angle$s A and B are given, the third angle need not be separately given as an additional datum, since its value can at once be deduced as $180^\circ - A - B$. The value of the third angle is not thus an independent third condition, and we have to proceed with two independent conditions only.

9. PERCENTAGES : DISCOUNT (I)

1. In trade, catalogues are prepared giving the prices at which various articles are sold. In shops you often find tickets attached to the articles with their prices noted on them. These prices are referred to as the *catalogue prices* or *marked prices*.

The trader fixes these prices with a view to realizing the cost price and a certain rate of profit. Since the trader may not be able to sell the articles as soon as he buys them, he often adds to the profit in view the interest at any desired rate on the money invested.

2. Reduction, discount

Tradesmen are often prepared to sell goods at prices which are less than the catalogue or marked prices. If customers buy the goods in large quantities, some reductions are allowed. If any article is shop-soiled or otherwise damaged, it is sold at less than the marked price. If a large stock of things remains unsold at the end of the year, they may be disposed of at reduced prices to make room for fresh and new stock. Again, to induce customers to buy readily, or to pay down cash instead of delaying payments, the traders may allow a certain deduction on the advertised prices.

The reductions or deductions that are allowed for the various reasons above described are called **discounts.**

Discounts are deducted from the catalogue or marked prices. They are generally indicated as so much per cent on such prices.

Example. If a bookseller fixes his prices so as to include a profit of 30 per cent on the cost price, and offers a reduction of 10 per cent on the published price for cash payments, find his gain per cent. If the cost price of 34 copies is Rs. 50, what is his average gain per copy when all of them are sold for cash ?

(1) Let the cost price of a set of books be Rs. 100.

Then the selling price = Rs. (100 + 30) = Rs. 130.

On this price, a discount of 10 per cent is allowed.

∴ the net selling price = Rs. 130 × $\frac{9}{10}$ = Rs. 117.

∴ the gain on Rs. 100 = Rs. 17.

i. e. the gain per cent = 17.

(2) The gain on 34 books costing Rs. 50

= Rs. 50 × $\frac{17}{100}$ = Rs. $\frac{17}{2}$.

∴ the gain per book = Rs. $\frac{17}{2}$ ÷ 34 = 4 as

3. Exercises

1. A tradesman fixed his catalogue prices at 20 per cent higher than the cost. What is the catalogue price of an article which cost him (i) Rs. 25, (ii) Rs. x ?

2. The marked price of a Star of India trunk is Rs. 25. A discount of (i) Rs. 2-8-0, (ii) $6\frac{1}{4}$ per cent, is allowed for ready cash payment. What has the purchaser to pay ?

3. The publishers of a certain book priced at Rs. 3-8-0 allow a reduction of $12\frac{1}{2}$ per cent for cash payment. What is the net sale price ? Find the profit per cent to a retail bookseller who buys on these terms, if he sells the book at Rs. 3-12-0.

4. Two shopkeepers marked the same make of violin at the same price. One allowed a discount of 1a. 6ps. in the rupee off the marked price, while the other allowed 12 per cent off. Who offered the better bargain and by how much per cent ?

5. A school co-operative society got its English books at a discount of 10 per cent, and Tamil books at a discount of 20 per cent. A wholesale agent offered to supply the same at a uniform discount of 15 per cent. If the English books formed a third of the whole requirements, would the latter alternative be more advantageous or not ? By what per cent ?

6. A trader buys a clock for Rs.12 and labels the selling price as Rs. 15-12-0. What percentage of discount can he allow a purchaser, if he wants to realize a net profit of 25 per cent ?

7. A tradesman fixed his prices at a profit of 25 per cent on the cost prices. He allowed a discount of $17\frac{1}{2}$ per cent. Find his actual profit per cent.

8. A stationery merchant fixes the price of paper at 20 per cent above cost, and sells away 60 per cent of the stock. He cleared off what remained, allowing a discount of $12\frac{1}{2}$ per cent on the advertised price. Find his profit per cent on the whole transaction.

9. A firm of cabinet makers marked their furniture at a uniform rate of $33\frac{1}{3}$ per cent above cost of making. On 75 per cent of the stock bought from them they allowed a discount of 10 per cent, and on the remaining part a discount of 15 per cent. What percentage of profit did they make by the transaction ?

10. The cost of making a harmonium is 40 per cent of the price at which its sale is announced. A reduction of 15 per cent is allowed for cash payment. How much does the maker of the instrument gain when cash is paid down for it ?

11. If Rs.M is the marked price of an article, Rs. S the price at which it is sold after deducting a discount of D per cent, show that $M - S = \frac{1}{100}$ of MD. Find D, if $M = 64$, $S = 56$.

12. The catalogue price of an article is Rs. C. A discount of D per cent is allowed for cash payment, which works out as Rs. P. Write down the relation between C, D, P.

13. A soap factory advertise soaps at a per cent above actual cost of manufacture, but allow a discount of b per cent to wholesale purchasers. What net profit per cent do they make?

10. PERCENTAGES: DISCOUNT (2)

1. We shall now pass on to problems on discount in which the sale prices and cost prices are the chief things to be calculated on the basis of known data.

Example. A motor merchant bought a car for Rs. 3200. He wanted to make a profit of 35 per cent on his outlay, after allowing a discount of 10 per cent on his advertised price. At what price did he advertise the car?

(1) The cost price of the car = Rs. 3200.

There should be a gain of 35 per cent on this, i.e. of Rs. 1120.

$\therefore$ the selling price = Rs. (3200 + 1120) = Rs. 4320.

This is the net selling price after allowing a discount of 10 per cent on the advertised price.

$\therefore$ if the net selling price = Rs. 90, the advertised price should be Rs. 100.

$\therefore$ if the net selling price = Rs. 4320, the advertised price = Rs. $4320 \times \frac{100}{90}$ = Rs. 4800.

(2) The method of the simple equation may be used.

Let Rs. x be the advertised price.

The net selling price = Rs. $x \times \frac{90}{100}$ = Rs. $\frac{9x}{10}$. This gives a gain of 35 per cent.

$\therefore$ the cost price = Rs. $\frac{9x}{10} \times \frac{100}{135}$ = Rs. $\frac{2}{3}x$.

Thus $\frac{2}{3}x = 3200$, $\therefore x = 4800$.

$\therefore$ the advertised price = Rs. 4800.

2. Exercises

1. A crockery dealer said that he had reduced the prices of articles by 15 per cent. Find the sale price of a tea-set which was originally selling at Rs. 85.

2. At a clearance sale it was announced that the prices of a set of gramophone records were reduced 25 per cent. Find the original price of a record whose reduced price was Rs. 2-4-0.

3. I was allowed a discount of $7\frac{1}{2}$ per cent for cash, and bought a trunk at a net price of Rs. 11-9-0. What would I have paid if no discount had been allowed?

4. A tradesman marks his goods at such a price that he can deduct 10 per cent for cash, and yet make 15 per cent profit. What is the catalogue price of an article which cost him Rs. 54 ?

5. At what per cent above the cost price should a shopkeeper mark his goods, so that he may allow a reduction of $12\frac{1}{2}$ per cent and still gain 20 per cent ?

6. A silk merchant allows his customers a discount of 10 per cent off the marked price. On silk marked Rs. 5-10-0 per yard, he makes a profit of $12\frac{1}{2}$ per cent on what it cost him. What was the cost price per yard ?

7. A man marks his goods at $33\frac{1}{3}$ per cent above the cost price, but allows a discount of 10 per cent on the marked price for cash. Find the cost price of an article for which he receives 12 shillings.

8. A merchant marks his goods at 50 per cent above cost price. If he allows 20 per cent off the bill for cash, what percentage profit does he make? If a customer buys goods for Rs. 240, what did they cost the merchant ?

9. A merchant marks his goods at 25 per cent above the cost price. If, in a clearance sale, he deducts 10 per cent from the price so marked, find his net profit per cent.

Find also the cost price to the merchant of an article which he sells at Rs. 10-2-0.

10. At what per cent above the cost should a manufacturer mark his goods so that after allowing a discount of 1 anna in the rupee for cash down, he may realize a profit of 10 per cent ?

11. A manufacturer of water-pumps fixed the sale prices at 40 per cent above cost. 60 per cent of the output in a certain year was sold at the prices so marked, and the remainder was cleared at a discount of 10 per cent. If the net profits came to Rs. 25800, find the actual cost of making the pumps.

12. A cloth merchant had to clear his goods. He sold silks at a reduction of $37\frac{1}{2}$ per cent, cotton goods at a reduction of 15 per cent, and woollen goods at a reduction of 20 per cent, on the prices marked. He thus realized Rs. 500, Rs. 1020, Rs. 400 respectively. If the marked prices were 25 per cent above the cost prices, find his gain or loss per cent on the clearance as a whole.

11. PERCENTAGES : MIXTURES (1)

1. You have already done problems on mixtures in which the principles of percentages were not involved. In this lesson we shall take problems involving percentages.

Example. A vessel contains coco-nut oil and gingili oil mixed in the ratio of 3 : 1. Another mixture of the oils contains $37\frac{1}{2}$ per cent coco-nut oil. If equal quantities of the mixtures are poured into the same vessel, find the percentages of the oils in the new mixture.

For convenience, suppose 100 palams of each mixture are taken and put together.

In the first mixture, 75 palams are coco-nut oil and 25 palams gingili oil.

In the second mixture, $37\frac{1}{2}$ palams are coco-nut oil and $62\frac{1}{2}$ palams gingili oil.

$\therefore$ in the final mixture of 200 palams, you have $(75 + 37\frac{1}{2})$ or $112\frac{1}{2}$ palams of coco-nut oil.

This is $\frac{112\frac{1}{2} \times 100}{200}$ or $56\frac{1}{4}$ per cent of the whole.

Similarly the quantity of gingili oil is $\frac{(25 + 62\frac{1}{2}) \times 100}{200}$ or $43\frac{3}{4}$ per cent of the whole.

2. Exercises

1. In a mixture of two oils, there are 3 seers of the first for every 5 seers of the second oil. Express the parts as percentages of the whole mixture.

2. Analysis of a sample of milk showed that it contained 12 per cent of water and 3·5 per cent of some unknown solid. How much of each of these would there be in 40 gallons of such a milk?

3. To m seers of ghee, n seers of coco-nut oil are added. What per cent of the mixture is (i) ghee, (ii) oil?

4. An alloy is prepared with x oz. of copper, y oz. of tin and z oz. of zinc. Find the percentage of each metal in the alloy.

5. Two dilutions of milk were made thus: 1 ollock of water with $5\frac{1}{2}$ ollocks of milk, and $2\frac{1}{2}$ ollocks of water with 14 ollocks of milk. Which of the dilutions was richer in milk and by what per cent?

6. Sovereign gold contains pure gold and copper melted together in the ratio 22 : 2. Express the quantities of the two metals as percentages.

7. A measure of a mixture of ghee and oil contains 85 per cent of pure ghee. If $\frac{1}{2}$ ollock of oil is added to it, what is the percentage of ghee in the mixture?

8. To a given quantity of ghee, 10 per cent of groundnut oil is added. What percentage of the mixture is oil? Answer to the nearest tenth.

9. Two samples of a dilute acid contain 25 and 40 per cent of water respectively. If quantities in the ratio of 3 : 2 of the two samples are mixed together, find the percentage of water in the new mixture.

10. From a bottle containing 20 ounces of strong nitric acid, two ounces are drawn off and the bottle filled with water. Find the percentage of acid in the dilution. On another occasion, 4 ounces of the dilute acid were drawn off and the bottle again filled with water. Obtain the strength of the dilute acid now.

12. PERCENTAGES : MIXTURES (2)

1. An important class of problems on mixtures relate to the strength of solutions used in medicine, in the laboratory, etc.

A mixture of alcohol and water is said to be 30 per cent strong, if the quantity of alcohol in it is 30 per cent of the whole.

Example. How much pure spirit should be added to a gallon of dilute spirit 20 per cent strong to raise it to 30 per cent strength ?

One gallon of the dilute spirit contains 20 per cent of spirit, i.e. $\frac{1}{5}$ gallon of spirit and $\frac{4}{5}$ gallon of water.

Suppose x gallons of spirit are added to raise the strength. The whole quantity is then $1 + x$ gallons. It contains $\frac{1}{5} + x$ gallons of spirit.

Hence $\frac{1}{5} + x = 30$ per cent of $(1 + x) = \frac{3}{10}(1 + x) = \frac{3}{10} + \frac{3}{10}x$.

Transposing, $x - \frac{3x}{10} = \frac{3}{10} - \frac{1}{5}$. $\therefore \frac{7x}{10} = \frac{1}{10}$, or $x = \frac{1}{7}$.

Thus the quantity of spirit to be added is $\frac{1}{7}$ of a gallon.

Verification. $1\frac{1}{7}$ gallons of dilute spirit contains $\frac{1}{5} + \frac{1}{7}$ or $\frac{12}{35}$ gallons of pure spirit.

Also 30 per cent of $1\frac{1}{7}$ gallons $= \frac{3}{10} \times \frac{8}{7}$ or $\frac{12}{35}$ gallons. The results agree.

2. Exercises

1. A lump of silver alloy contains 75 per cent pure silver. How much pure silver should be melted with it to raise the proportion of silver to 90 per cent ?

2. A lump of ore weighing 24 lb. contains 10 per cent gold and the rest is copper. How much copper should be separated from the lump so that the remainder may contain 25 per cent gold ?

3. I have a gallon of a 20 per cent solution of ammonia. How much water should I add to reduce it to a 10 per cent solution ?

4. A jar contains 4 oz. of dilute sulphuric acid, 95 per cent strong. How much water should you add to it to reduce the strength by 20 per cent ?

5. 12 ounces of a mixture contain wine and water in the ratio 5 : 1. How many ounces of water should be added to convert it into a mixture having the ratio changed to 3 : 1 ?

6. I bought 6 sovereigns for making a jewel and melted them with some pure gold. The resulting alloy was found to contain 94·5 per cent of pure gold. If sovereign gold is known to contain 91·66 per cent of pure gold, find the quantity of pure gold added.

7. Two vessels contain mixtures of wine and water. In one there is twice as much wine as water, and in the other three times as much water as wine. Find how much should be drawn off from each vessel to fill a third vessel which can hold 15 gallons, in order that its contents may be half wine and half water.

8. A photographer has two bottles of a diluted developer, 10 per cent and 50 per cent strong respectively. How much should be taken from each bottle to make 4 oz. of a developer 25 per cent strong ?

13. PERCENTAGES : MIXTURES (3)

1. The problems in this lesson refer to profit and loss in trade as applied to mixtures and adulterations.

Example 1. A tradesman bought wine at 5*s*. per gallon and mixed it with water. By selling the mixture at 4*s*. per gallon, he gained $12\frac{1}{2}$ per cent on his outlay. How much water did each gallon of the mixture contain ?

Suppose each gallon of the mixture contained x gallons of water. The quantity of wine in a gallon would be $1-x$ gallons.

Then the C. P. of $1-x$ gallons of wine $=(1-x)\times 5s.$

The S. P. of 1 gallon of the mixture = the S. P. of $1-x$ gallons of wine = 4*s*.

Since the gain $=12\frac{1}{2}$ per cent or $\frac{1}{8}$ of the C. P., the S. P. is $\frac{9}{8}$ times the C. P.

$\therefore\ (1-x)\times 5\times\frac{9}{8}=4$, whence $1-x=\frac{32}{45}$, $\therefore\ x=\frac{13}{45}$.

Example 2. In what proportion must two kinds of paddy at Re. 1-14-0 and Re. 1-6-0 per kalam be mixed, so that the mixture may be sold at Re.1-14-0 per kalam, and a gain of 20 per cent may be cleared ?

Suppose the two kinds of paddy are mixed in the ratio $x : y$.

The cost prices of a kalam of the two kinds are 30 as. and 22 as.

$\therefore$ the C.P. of x kalams of the first kind and y kalams of the second kind $= (30x + 22y)$ as.

The S.P. of $x + y$ kalams of the mixture $= (x + y) \times 30$ as.

The S.P. is 20 per cent higher than the C.P.

$\therefore 30\ (x + y) = \frac{6}{5}\ (30x + 22y)$

$\therefore 30x + 30y = 36x + \frac{132}{5}y$

Whence $6x = \frac{18}{5}y$, $\therefore x = \frac{3}{5}y$, which gives $x : y = 3 : 5$.

Thus the two varieties of paddy should be mixed together in proportion to 3 and 5.

Note. Suppose that the two varieties are mixed in the ratio $1 : x$. Work out the solution.

2. Exercises

1. A trader bought two sorts of ghee costing respectively Rs. 3-4-0 and Rs. 2-14-0 per viss. Two viss of the first sort were mixed with three of the second, and the mixture was sold at Rs. 3-2-0 a viss. Find his gain or loss per cent.

2. The cost price of a 36-gallon cask of wine was £11-16-3. Of this, $6\frac{1}{2}$ gallons were lost by leakage. The merchant added $4\frac{1}{2}$ gallons of water to the remaining quantity, and sold it at 7*s*. 6*d*. per gallon. What profit per cent did he make?

3. A tradesman bought 80 lb. of coffee seeds at 4*s*. 9*d*. per lb., and 60 lb. of another sort at 3*s*. 6*d*. per lb. He mixed the two sorts, and sold 50 lb. of the mixture at 5*s*. 6*d*. per lb. and the remaining part at 5*s*. 4*d*. per lb. Find, correct to the nearest tenth, the percentage of gain on the transaction.

4. A milkman bought pure milk at 3 measures per rupee, added two parts of water to every five parts of milk, and sold it at $3\frac{1}{2}$ measures per rupee. Find his gain per cent. If instead, he supplied pure milk at $2\frac{1}{2}$ measures per rupee, how far would his gain per cent be affected ?

5. A dealer has two sorts of tea, one of which he could sell at 1*s*. 8*d*. per lb. and make 25 per cent on his outlay, and the other at 2*s*. 6*d*. per lb. and make $12\frac{1}{2}$ per cent on his outlay. What profit per cent will he make if he mixes them in equal quantities and sells the mixture at 1*s*. 11*d*. per lb. ?

6. How much water should be added to 60 gallons of wine worth 8*s*. per gallon to reduce the price per gallon to 6*s*. 6*d*. ?

7. A person buys milk at 1 a. 6 ps. per seer, and after adding some water to it sells the mixture at 1 a. 9 ps. per seer. If a profit of 30 per cent is made, find the amount of water used.

8. A milkman bought milk at 10 as. per measure for Rs. 20, added some water to it and supplied the diluted milk at a marriage at 12 as. per measure. He got Rs. 10 profit. How much water did he add?

9. A wine merchant bought 180 gallons of spirit at 10*s*. per gallon. 18 gallons of it were spilt. He added water to what remained and sold the mixture at 11*s*. per gallon, thereby gaining £3-10-0. How much water did he add?

10. A grocer bought 60 lb. of tea at 2*s*. 6*d*. per lb. He mixed with it a coarser sort at 1*s*. 9*d*. per lb. If he sold the mixture at 3*s*. per lb., and gained 25 per cent, find the quantity of coarser tea that he used.

11. A merchant had two kinds of wine worth Rs. 5-7-0 and Rs. 4-2-0 per gallon respectively. How many gallons of each sort must be taken to get a mixture of 16 gallons worth Rs. 76-8-0?

12. In what ratio should teas at Re. 1-2-0 and 14 as. per lb. be mixed so that the mixture may cost 15 as. per lb.?

13. In what proportion should coffee seeds costing Rs. 2-4-0 and Rs. 2-12-0 per viss be mixed so that the mixture may be worth Rs. 2-7-0 per viss?

14. A merchant bought two sorts of ghee at Rs. 13-0-0 and Rs. 13-8-0 per maund respectively. How many maunds of the inferior sort should be mixed with 26 maunds of the superior sort so that by selling the mixture at Re. 1-12-0 per viss, he might make a profit of 5 per cent on his outlay?

15. A merchant mixes two kinds of coffee seeds which cost him Rs. 2 and Rs. 2-12-0 per viss respectively, and sells the mixture at Rs. 3 per viss, gaining thereby 20 per cent on his outlay. Find the ratio in which the two kinds are mixed. Also calculate the quantity of each kind in one viss of the mixture.

14. MISCELLANEOUS EXERCISES

1. A shopkeeper having marked his goods at 20 per cent above the cost price, allows his customers a discount of $12\frac{1}{2}$ per cent. Find the cost price of an article for which the customer pays Rs. 62-5-6.

2. How much per cent must a trader add to the cost price of his goods in order that he might make a profit of 10 per cent over his cost price, after giving a discount of 10 per cent off his marked price?

3. At what per cent above the cost must a merchant mark his wares so that even after allowing a discount of 25 per cent for cash he may make a profit of 25 per cent? What should be the catalogue price of goods costing Rs. 840 to the merchant, and what his selling price?

4. A manufacturer sells his goods at 60 per cent over the cost of manufacture to a retail dealer, allowing him 5 per cent off this price for cash. The dealer puts his selling price up by 40 per cent over his net outlay, but allows his customers 10 per cent discount for cash. By how much per cent is this selling price greater than the cost of manufacture?

* 5. The catalogue price of a printing machine in England was £240. An Indian printer purchased it at a discount of $33\frac{1}{3}$ per cent on that price. He had to pay import duty at 35 per cent on the price he had paid. For what sum should he sell it in India, if he wanted to realize a profit of 25 per cent? Take £1 = Rs. 13-8-0.

6. An ore was known to contain 20 per cent of gold, but it was possible to extract only 85 per cent of the metal. How much metal could be got from 100 tons of the ore?

7. Two pint bottles contained respectively 8 and 12 oz. of water. I filled them up with pure nitric acid. Compare their percentage strengths. What would be the strength if the contents were put together (mixed up)? One pint = 20 oz.

8. A shopkeeper practises two kinds of fraud: (1) he adulterates articles to the extent of 8 per cent, (2) he uses a false balance which shows up 37 palams as one viss. Which of the two methods defrauds the customer to the greater extent? If a customer is given 1 maund of ghee by the first method, to what extent is he deceived?

9. A mining company in a certain year collected 16000 tons of silver ore and extracted 3500 oz. of silver. Next year, owing to increased facilities and improved methods, 60 per cent more of the ore than in the first year was collected, and each ton yielded 12 per cent more metal. How much silver was extracted in the second year?

10. A man had two kinds of sugar costing respectively 8 as. and 7 as. per viss. In what ratio would he mix them so that by selling the mixture at 10 as. per viss he might gain 40 per cent?

11. A milk pot contains 3 parts pure milk and 1 part water. How much of the milk so diluted should be drawn off and the pot filled with water, so that the resulting mixture may be half milk and half water?

12. Two bottles contain 24 ounces of water and wine respectively. One ounce of liquid is taken from each bottle and poured into the other. Find the percentage strength of wine in each bottle. Find the percentage strength after the operation has been repeated once again.

13. A bottle contains 20 oz. of concentrated sulphuric acid. Two ounces are taken out and the bottle filled with water. Again two ounces are taken from the dilute acid and the bottle again filled with water. Find the percentage strength of the final dilution.

14. Three equal glasses are filled with a mixture of spirit and water. The proportion of spirit to water in them are 2 : 3, 3 : 4, 4 : 5 respectively. The contents of the three glasses are then poured into the same vessel. What would be the proportion of spirit to water in it ?

15. A chemist has two bottles containing dilute nitric acid. The liquid in the first bottle contains 25 per cent of the acid and that in the second 37·5 per cent. How much of each should be taken to make a mixture containing 5 oz. of the acid and 9 oz. of water ?

15. CONSTRUCTION OF QUADRILATERALS

1. As in the case of triangles, you can describe quadrilaterals when the necessary and sufficient data are known.

Before you attempt to draw any figure accurately to scale, try to get a mental picture of it, then draw a rough sketch of the same and enter in it all the measurements known. This is very important.

Experiment 1. Take four rods and hinge them end to end to form a quadrilateral. Obviously, its shape and size may be varied by taking hold, say, of a pair of opposite vertices and pressing them towards each other, or pulling them away from each other, or straining them in any other way.

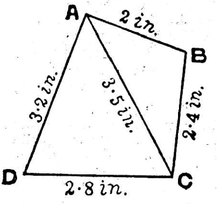
If you now join a pair of opposite corners by a diagonal rod, the quadrilateral frame becomes *rigid*, and no change of size or shape is now possible. This shows that only one quadrilateral can be constructed with four sides and a diagonal of specified lengths.

Experiment 2. In a quadrilateral frame, if two rods meeting at a vertex are immovably riveted, the frame will become rigid, unchangeable. This is equivalent to the fixing in size of the angle at that vertex. You see then that a specific quadrilateral is indicated by giving the sizes of the four sides and the angle at a vertex.

You thus see that by a proper choice of data any quadrilateral in view may be copied or constructed. In each of the above cases the data refer to *five* elements.

A similar conclusion may be reached as follows : A quadrilateral is divided into two triangles by a diagonal. To construct one of the triangles, three conditions are necessary and sufficient. To add on to it the other triangle two more conditions are necessary and sufficient. Thus you require five conditions altogether.

Example 1. Construct a quadrilateral ABCD, given AB = 2″, BC = 2·4″, CD = 2·8″, AD = 3·2″, AC = 3·5″.



An examination of the figure suggests the following procedure :

The three sides of the triangle ABC are known. Construct it. Again, on AC construct the triangle ADC with the help of the other two data. The quadrilateral will be complete.

Example 2. Construct a quadrilateral ABCD, having given AB = 2″, BC = 2·4″, CD = 2·8″, DA = 3·2″, angle A = 85°.

Draw a rough sketch and enter the given measurements in it.

If BD is joined, you see at once that the sides AB, AD and ∠A will enable you to draw △ABD, and the sides BC and DC will enable you to complete the figure.

Draw a correct figure, and write down the steps of the construction you have followed.

2. Exercises

1. Construct a quadrilateral ABCD, having given :

(*a*) AB = 6 cm., BC = 4·8 cm., CD = 4 cm., DA = 8 cm., AC = 8 cm.

(*b*) AB = BC = 4″, AD = CD = 3″, BD = 5·5″.

2. The diagonals AC, BD of a quadrilateral ABCD are 2·8 and 2·1 inches respectively. The sides BC, CD, DA are 1·0, 1·7, 2·5 inches respectively. Describe the quadrilateral.

3. Construct a quadrilateral EFGH, having given :

(*a*) EF = 2·3 cm., FG = 2·2 cm., ∠EFG = 118°, ∠FEH = 80°, ∠FGH = 116°.

(*b*) EH = 2″, EF = 1·1″, FG = 1·2″, ∠HEF = 95°, ∠EFG = 112°.

4. Construct a quadrilateral PQRS, having given :

(*a*) PQ = 1·8″, QS = 2·6″, SR = 3·4″, ∠PQS = 40°, ∠QSR = 50°.

(*b*) PQ = 2·6″, QS = 2·7″, QR = 1·5″, ∠PQS = 60°, ∠SQR = 38°.

(*c*) PS = 6·6 cm., PQ = 6·8 cm., PR = 8·5 cm., ∠SPQ = 62°, ∠QPR = 28°.

5. Construct a quadrilateral ABCD, having given,

(*a*) BD = 2·7″, ∠ADB = 50°, ∠ABD = 30° ∠CBD = 60°, ∠CDB = 40°.

(*b*) AB = 2·7″, ∠BCA = 55°, ∠ABC = 64°, ∠ACD = 30°, ∠DAC = 50°.

6. The distance between Calcutta and Bombay is 1050 miles, between Calcutta and Delhi 840 miles, between Delhi and Madras 1150 miles, between Delhi and Bombay 780 miles, between Bombay and Madras 660 miles. Draw a map to indicate the relative positions of the four towns. How far is Calcutta from Madras, on this plan ?

7. A house site is in the form of a quadrilateral and the following measurements are taken to prepare a plan of it : AC = 180 yards, BD = 200 yards, AB = 120 yards, ∠BAC = 30°, ∠ABD = 45°. Draw the plan, and obtain the length of the fence that should be put up to enclose the site.

8. A plan of a rice field is in the form of a quadrilateral ABCD in which AB = 4 cm., BC = 8 cm., AD = 3·5 cm., AC = 10 cm., ∠A = 120°. Reproduce the plan. Find the distance between the corner stones at D and C, if the scale of the plan is 1 cm. = 10 yards.

16. AREA OF A QUADRILATERAL

1. You learnt in Book V, lesson 75, how to find the area of any quadrilateral.

The area of a quadrilateral = ½ (*a diagonal*) × (*the sum of the distances of the other vertices from the diagonal*).

If in a quadrilateral ABCD, AC = d, DE = p_1, BF = p_2, then the area of ABCD = $\frac{1}{2}d\,(p_1 + p_2)$, the corresponding units being understood.

A number of exercises are added for additional practice.

2. Exercises

1. Construct a quadrilateral ABCD in which AB = 2″, BC = 3·5″, CD = 3″, DA = 2·8″, BD = 4″. Draw the perpendiculars from B and D on AC, and measure their lengths. Calculate the area of the quadrilateral.

2. The sides of a quadrilateral plot of grass taken in order are 8, 8, 7, 5 feet respectively, and the angle contained by the first two sides is 60°. Find the area of the plot.

3. ABCD is a plan of a garden with a well W in the middle. The distances of the well from the corners A, B, C, D are respectively 48, 60, 72, 54 yards. Also $\angle AOB = 90°$, $\angle BOC = 84°$, $\angle COD = 135°$. Draw a plan of the garden, and obtain its area.

4. In a field ABCD of the shape of a quadrilateral, AB = BC = CD = 60 yd., AD = 80 yd. and $\angle DAB$ = a right angle. Find the area of the field.

5. ABCD is a quadrilateral field in which AB = 200 yd., BC = 400 yd., AD = 175 yd., AC = 500 yd., and $\angle A = 120°$. Draw a plan of the field to a convenient size, and find the area in acres.

6. PQRS is a quadrilateral field in which the diagonals PR, QS are at right angles. PQ, QR, RS, PR are 600, 700, 800 and 1000 links respectively. Draw a plan of the field and find its area in acres.

7. Draw a quadrilateral ABCD, the corners A, B, C, D having the co-ordinates (14, 12), (12, 2), (4, 11), (1, 15). Suppose it represents a field (representative fraction $1 \div 1440$), find the area of the field in acres.

8. ABCD is a quadrilateral plot of ground in which AB = 13 ft., BC = 14 ft., CD = 24 ft.; the perpendicular from A on DC is 9 ft., and divides it into two equal parts. Find the area.

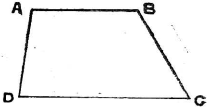
9. Make a rough sketch and find the area of the field ABCD from the following measurements taken in links:

BM, the perpendicular from B on AC = 400.
DN, the perpendicular from D on AC = 300.
AM = 300, AN = 400, AC = 625.

17. TRAPEZIUM

1. The Trapezium

The figure on the right is a quadrilateral ABCD in which the sides AB, DC are parallel. The other two sides are not parallel.



A quadrilateral which has only one pair of opposite sides parallel is called a **trapezium.**

Take a triangle ABC, and draw DE parallel to BC cutting the other two sides at D, E. Then the part DECB is a trapezium.

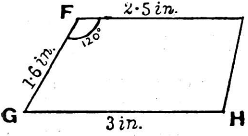
2. Construction of a trapezium

A trapezium can be constructed if the necessary and sufficient number of data are available.

Draw a trapezium and try to get copies of it, choosing in different ways the measurements of sides, angles and other elements. How many measurements are required to get an exact copy of a given trapezium?

You have already learnt that five independent conditions are necessary and sufficient to construct a quadrilateral in general. The trapezium is a particular case of a quadrilateral, having as a condition that two of its opposite sides are parallel. It therefore follows that four more conditions are necessary for constructing a trapezium in view.

Example 1. Draw a trapezium EFGH, having given: EF is parallel to HG, EF = 2·5 in., ∠ EFG = 120°, FG = 1·6 in., GH = 3 in.

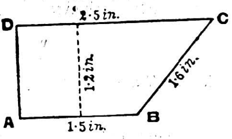


Draw a straight line EF = 2·5 in. At F make ∠ EFG = 120°.

Make FG = 1·6 in. Through G draw GH parallel to FE, making GH = 3 in.

Join EH. Then EFGH is the required trapezium.

Example 2. Construct a trapezium ABCD, having given AB = 1·5 in., BC = 1·6 in., CD = 2·5 in., the distance between the parallel sides AB, DC = 1·2 in.



Take a straight line AB = 1·5 in.

Draw a parallel to AB at a distance of 1·2 in. from it.

With B as centre and radius 1·6 in., draw an arc cutting this parallel at C.

Measure off CD = 2·5 in., and join DA. Then ABCD is a trapezium satisfying the given measurements.

Note. Since the arc with B as centre and radius 1·6 in. will cut the parallel at two points (C, C′), two trapeziums may be described. The figure for the alternative case is left to the student.

3. Exercises

Note. As in the construction of triangles and quadrilaterals, the student is advised to begin with a rough sketch of the required figure with the given measurements entered in it, and then to proceed to construct the correct figure to scale

1. Construct a trapezium ABCD, given that AB is parallel to DC, and

(i) AB = 5 cm., AD = 3·2 cm., DC = 6 cm., ∠ BAD = 120°.

(ii) AB = 2·6 in., AC = 2·9 in.; ∠ BAC = 35°, ∠ CAD = 44°.

2. Construct a trapezium PQRS, given that PQ is parallel to SR, and

(i) PQ = 2·4″, PR = 3·0″, PS = 1·7″, QR = 1·5″.

(ii) PQ = 2·9″, QS = 2·1″, SR = 4″, QR = 2·6″.

(iii) PR = 7·8 cm., SR = 5·1 cm., ∠ QPR = 35°, ∠ QRP = 59°.

3. Construct a trapezium PQRS, given that PQ = 1·5 in., QR = 1·7 in., RS = 2·5 in., and the distance between the parallel sides PQ, SR = 1·2 in.

4. Draw a trapezium PQRS in which PQ is parallel to SR. Measure the lengths of the four sides to the nearest tenth of an inch. Using these measurements obtain a copy of the trapezium.

5. ABCD is a trapezium in which ∠ A = 98°, ∠ B = 112°, AB = 2·2 in., and the distance between the parallel sides AB, CD = 1·4 in. Construct the figure.

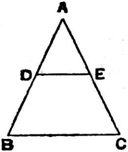
If the figure represents a field to a scale of 1 in. = 100 ft., find the cost of fencing it at 2 as. a running foot.

6. Two persons stand at A and B on a bank of a straight river of uniform breadth, and observe two trees P, Q close to the water's edge on the opposite bank. The following measurements are taken: AB = 80 ft., ∠ BAP = 110°, ∠ ABP = 45°, ∠ PAQ = 60°. Obtain the distance PQ, using the above measurements to draw a suitable diagram.

18. ISOSCELES TRAPEZIUM

1. Introduction

There is an interesting particular case of a trapezium which we shall now take up.

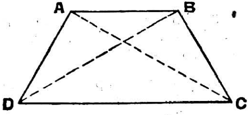


In an isosceles triangle ABC having AB = AC, if a straight line DE drawn parallel to BC meets AB, AC at D, E respectively, then it is easily shown by measurement (or by argument) that BD = EC. The quadrilateral DBCE will then be a trapezium having its non-parallel (oblique) sides equal.

A trapezium in which the non-parallel sides are equal is called an **isosceles trapezium.**

Draw an isosceles trapezium ABCD. Measure its angles and compare their sizes. What do you notice about them ?

Repeat the experiment with a number of isosceles trapeziums of various shapes and sizes. Do you observe the same relations ? State your inference in general terms.



Draw the diagonals in each case ; measure and compare their lengths. What do you observe ? Make a general statement.

Your inferences from these experiments can be stated thus :

(*i*) *In an isosceles trapezium, the angles at the ends of each of the parallel sides are equal to each other.*

(*ii*) *The oblique sides are equally inclined to either of the equal sides.*

Note. The above two statements refer to the same property.

(*iii*) *The diagonals of an isosceles trapezium are equal.*

2. Construction of an isosceles trapezium

Example. Describe an isosceles trapezium PQRS, having given: PQ is parallel to SR, SR = 5 cm., SQ = 4·4 cm., the distance between the parallel sides = 3·2 cm.

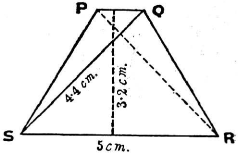
The figure on page 41 is a rough sketch showing the given measurements.

Draw a straight line SR = 5 cm. Draw a parallel to SR at a distance of 3·2 cm. from it.

With S as centre and radius = 4·4 cm., draw an arc to cut this parallel at Q.

Since SQ = PR, with R as centre and radius equal to 4·4 cm. draw an arc to cut the parallel at P.

Join QR, PS, as in the figure. Then PQRS will be the required trapezium.



3. Exercises

1. Cut out from paper a few isosceles trapeziums, and examine their symmetry. Make a statement about the symmetry of an isosceles trapezium.

[An isosceles trapezium is symmetrical about the straight line joining the middle points of the parallel sides.]

2. Draw two straight lines AB, CD, so as to have a common right-bisector. Join AC, BD. What kind of figure is ABDC? Give reasons for your answer.

3. Describe an isosceles trapezium ABCD, given that:

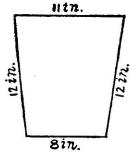
(i) AB is parallel to DC, AB = 5 cm., DC = 3·5 cm., AD = BC = 1·5 cm.

(ii) AB = 4·2 in., DC = 2·8 in.; the distance between the parallel sides AB, DC = 1·2 in.

(iii) AB = 2·5 in., CD = 4 in., the inclination of each of the oblique sides to DC = 60°.

4. Construct an isosceles trapezium EFGH, given that FH = 2·2 in., HG = 2·5 in., the distance between the parallel sides EF, HG = 1·6 in.

5. A street drain has a cross-section in the shape of an isosceles trapezium. The bottom is 1 foot broad, the sides slope at 30° to the vertical, and the depth of the drain is 2 feet. Find its breadth at the top.



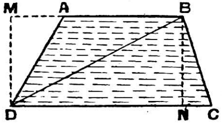
6. The figure on the left gives a rough sketch of the axial cross-section of a zinc bucket. Draw an enlarged diagram to scale and find the depth of the bucket.

7. A railway embankment has a cross-section in the form of an isosceles trapezium PQRS. The following data are available: PQ = 24 ft., QR = PS = 3 ft., RS = 30 ft. Draw an accurate representation of the cross-section.

19. AREA OF A TRAPEZIUM

1. Area of a trapezium

You have already learnt how to find the area of a quadrilateral. The trapezium is a quadrilateral, and its area may therefore be easily found by following the method available for any quadrilateral. Since, however, the trapezium is a special kind of quadrilateral, we can obtain a simple formula for its area.



Let ABCD be a trapezium, AB being parallel to DC.

Draw the diagonal BD, dividing the figure into two triangles ABD, BDC. Draw their respective altitudes DM, BN.

Since AB is parallel to DC, DM = BN. Let each be h (units understood).

Then, the area of the trapezium ABCD

$= \text{area of } \triangle ABD + \text{area of } \triangle BDC$

$= \frac{1}{2} AB \times h + \frac{1}{2} DC \times h$

$= \frac{1}{2} (AB + DC) \times h$

Expressed in words, this formula stands thus :

The area of a trapezium = ½ *(the sum of the parallel sides)* × *(the perpendicular distance between them).*

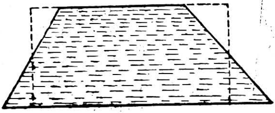
If the lengths of the two parallel sides be a and b units of length and the distance between them is h units, then

The area of the trapezium

$= \frac{1}{2} (a + b) \times h$ *units of area.*

2. Exercises

1. Calculate the areas of the trapeziums constructed in question 1, 2, 5, 3, 4, of lesson 17.



2. The figure on the right indicates how a trapezium can be converted

into a rectangle of the same area. Explain how this is actually done. Show that the formula for the area of a trapezium can be derived by using such a conversion.

3. Cut out two trapeziums of the same shape and size. Show how they can be pasted side by side so as to form a parallelogram. Use this arrangement to derive the formula for the area of a trapezium.

4. The parallel sides of an isosceles trapezium are respectively 2 in. and 1·5 in., and the distance between them is 1 inch. Construct the trapezium. If it represents a lawn on the scale 1 inch = 5 chains, find the area of the lawn in acres and cents, to the nearest cent.

5. The area of a lawn in the form of a trapezium is $4\frac{1}{2}$ acres. The distance between the parallel sides is 120 yards, and one of them is 220 yards. Find the length of the other side.

6. How many square inches of metal are required to make a tray having a rectangular bottom and top and of the following description: The adjacent sides of the base measure 9 in. and 15 in., and the corresponding sides of the top measure 10 in. and 16 in.; the width of the strip making the sides is 3 in.?

[*S. S. L. C.* 1914

20. SIMPLE FACTORIZATION (1)

1. You know how to express a given number as a product of two or more factors where possible. We shall now proceed to learn how to factorize expressions.

Now, $5(a + b) = 5a + 5b$. The coefficient 5 is distributed to a and b, and is common to both $5a$ and $5b$. In the reverse process of factorizing $5a + 5b$, the common coefficient (factor) 5 is taken out and the other factors put into brackets. The following are additional examples:

$$7x + 14y = 7 \times x + 7 \times 2y = 7\,(x + 2y)$$
$$5m - 10n = 5 \times m - 5 \times 2n = 5\,(m - 2n)$$
$$-\,8ax - 12ay = (-\,4a) \times (2x) + (-\,4a) \times (3y)$$
$$= -\,4a\,(2x + 3y)$$
$$ax - ay + 2az = a\,(x - y + 2z).$$

The following special cases are to be noted:

$$4x + 4 = 4 \times x + 4 \times 1 = 4\,(x + 1)$$
$$a - ab = a \times 1 - a \times b = a\,(1 - b)$$
$$p^2 + p = p \times p + p \times 1 = p\,(p + 1)$$

2. Exercises *(oral as far as possible)*

1. State the factor common to the terms in each of the following sets:

(i) $5x$, $5y$
(ii) $10x$, $15y$
(iii) $6x$, $12y$, $18z$
(iv) ap, aq, $2ar$
(v) $8ab$, $12ac$, $16ad$
(vi) $12a^2$, $16ab$, $20ac$
(vii) $14a^2b$, $21ab^2$
(viii) $18a^2bc$, $24ab^2c$, $36abc^2$
(ix) $-7a$, $-7b$
(x) $-18m$, $-45m$
(xi) $6x$, $-12y$, $18z$
(xii) px, py, $-pz$
(xiii) $12xy$, $-16xz$, $-20xw$
(xiv) b, b^2, b^3
(xv) $4b$, $-6b^2$, $8b^3$
(xvi) $12xy$, $-18x^2y$, $-24xy^2$

2. Factorize:

(i) $5x + 5y$
(ii) $7x + 14y$
(iii) $5x - 5y$
(iv) $7x - 14y$
(v) $-5a + 10b$
(vi) $-5p - 15q$
(vii) $ap + aq$
(viii) $sr - 3st$
(ix) $-2sr + 3st$
(x) $3x + 3$
(xi) $3x - 3$
(xii) $-3x + 3$
(xiii) $-3z - 3$
(xiv) $5y + 15$
(xv) $4a - 8$
(xvi) $x^2 + x$
(xvii) $x^2 - x$
(xviii) $-x^2 - 2x$
(xix) $m + mn$
(xx) $xy - y$
(xxi) $6by + 6bz$
(xxii) $x^2y - xy^2$
(xxiii) $3s^2 - 6sx$
(xxiv) $a^3 - a^2b$
(xxv) $25a^3 - 10ab^2$
(xxvi) $16x^2y - 36xy^2$
(xxvii) $-18x^2y^3 + 27x^3y^2$

3. Resolve into factors:

(i) $x + xy + xz$
(ii) $b + 2b^2 + 3b^3$
(iii) $4b + 6b^2 + 8b^3$
(iv) $12m^3 - 16m^2 - 20m$
(v) $-72a^3 + 48a^2b - 36ac$
(vi) $x^3 + x^2y + xy^2$
(vii) $x^3 - 2x^2y + 3xy^2$
(viii) $20x^2y - 30xy^2 + 35y^3$
(ix) $6a^3b^2 + a^2b^3 - 7a^2b^2$
(x) $-24p^3q^3 + 32p^2q^4 - 48p^4q^3$

4. The area of the four walls of a room is given by $2lh + 2bh$. Put it in a simplified (factorized) form.

5. The formula for amount at simple interest is $A = P + \frac{Pnr}{100}$. Put the right-hand member in factors.

6. The area of the total surface of a closed cylindrical drum is given by $2\pi r^2 + 2\pi rl$. Express this as a product of two factors.

3. The same principles are followed in more complicated cases, where the common factors are however explicitly available.

Example. Resolve into factors: $a(x + y) + 2b(x + y)$.

Here the binomial $x + y$ is common to both the terms $a(x+y)$ and $2b(x + y)$.

Hence $a(x + y) + 2b(x + y) = (x + y)(a + 2b)$

4. **Exercises** *(oral as far as possible)*

1. Factorize :

   (i) $a(p+q)+b(p+q)$
   (ii) $x(a+b)+2y(a+b)$
   (iii) $2m(p+r)-3n(p+r)+5s(p+r)$
   (iv) $-3a(x+y+z)+5b(x+y+z)-15c(x+y+z)$
   (v) $16x^2(2a+3b-4c)-24xy(2a+3b-4c)$
   $-32y^2(2a+3b-4c)$

2. Resolve into factors :

   (i) $(a+b)p-(a+b)r+(a+b)t$
   (ii) $(x+y+z)a^2+(x+y+z)b^2-(x+y+z)c^2$
   (iii) $-15a^2b(a^2+b^2+c^2)-20ab^2(a^2+b^2+c^2)$
   (iv) $36x^2yz(1+b+b^2)-48xy^2z(1+b+b^2)$
   $+60xyz^2(1+b+b^2)$

21. SIMPLE FACTORIZATION (2)

1. Method of convenient grouping

Consider the multiplication shown below. An inspection shows that the first two terms of the product are obtained by multiplying $a+b$ by c and therefore form $c(a+b)$, and the next two terms are obtained by multiplying $a+b$ by d and therefore form $d(a+b)$.

$$\begin{array}{l} a+b \\ c+d \\ \hline ac+bc \\ \qquad\quad ad+bd \\ \hline ac+bc+ad+bd \end{array}$$

Thus, if we want to factorize $ac+bc+ad+bd$, we proceed as follows :

$$\begin{aligned} ac+bc+ad+bd &= (ac+bc)+(ad+bd) \\ &= c(a+b)+d(a+b) \\ &= (a+b)(c+d) \end{aligned}$$

Example. Factorize $ab-10a+3b-30$. Here the terms are to be grouped into two groups of two terms each, so as to give a common factor of the two groups. Thus

$$\begin{aligned} ab-10a+3b-30 &= (ab-10a)+(3b-30) \\ &= a(b-10)+3(b-10) \\ &= (b-10)(a+3) \end{aligned}$$

Alternative grouping :

$$\begin{aligned} ab-10a+3b-30 &= (ab+3b)-(10a+30) \\ &= b(a+3)-10(a+3)=(b-10)(a+3) \end{aligned}$$

which is the same as before.

2. Exercises *(oral as far as possible)*

1. Factorize :

(i) $a(x+y)+x+y$	(iv) $p+q-5(p+q)$
(ii) $c(a+b)+2a+2b$	(v) $a^2-ab-(a-b)$
(iii) $c(a+b)-a-b$	(vi) $-ab+b^2-(b-a)$

2. Resolve into factors :

(i) $ac+ad+bc+bd$	(xi) $ab-2b+3a-6$
(ii) $ac+ad-bc-bd$	(xii) $1+a+c+ac$
(iii) $pm+pn-qm-qn$	(xiii) $2-2m+n-mn$
(iv) $ac-ad+bc-bd$	(xiv) x^3+x^2+x+1
(v) $ac-ad-bc+bd$	(xv) x^3+2x^2+x+2
(vi) $a^2+ac+ab+bc$	(xvi) x^3-x^2+x-1
(vii) $a^2-ab+ac-bc$	(xvii) $a+b+ab+b^2$
(viii) $a^2+ab-ac-bc$	(xviii) $a-b+ab-b^2$
(ix) $a^2-ab-ac+bc$	(xix) $xy-y-xz+z$
(x) $ab+a+b+1$	(xx) $pq+2q-3p-6$

3. Express as the product of two factors :

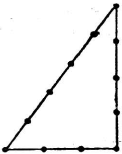
(i) $6ac+3bc+2ad+bd$	(iii) $6ac+3bc-2ad-bd$
(ii) $6ac-3bc+2ad-bd$	(iv) $6ac-3bc-2ad+bd$

22. THE THEOREM OF PYTHAGORAS (1)

1. Introduction

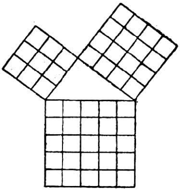
In ancient Egypt, the surveyors, known as rope-stretchers, constructed a right-angled triangle thus : They took a string having knots at equal intervals and tied the ends together. This endless loop consisted of 12 intervals and was stretched so as to form a triangle having 3, 4, 5 intervals for the sides. The angle opposite the longest side was found to be a right angle.

The *Suluva-Sutras* (an Indian work of the fifth century B.C.), says that right-angled triangles can be obtained by stretching strings of the following lengths as described above, viz. (3, 4, 5), (5, 12, 13), (8, 15, 17), . . . units.



Is there any relation between the numbers of each of these sets, responsible for the supply of right-angled triangles ? By putting them in various combinations it was discovered that the square of the largest of them was in every case equal to the sum of the squares of the other two. Thus $5^2=4^2+3^2$, $13^2=12^2+5^2$, $17^2=15^2+8^2$, and so on.

Now a number and its square (a, a^2) may be made to represent the side and area of a geometric square. This suggests that if squares be constructed on the three sides of the triangle formed as in the above figure, the square on the side which contains 5 intervals will be equal in area to the sum of the squares on the sides containing 4 and 3 intervals.



The figure on the right confirms this.

Now draw the triangles corresponding to the sets (5, 12, 13), (8, 15, 17); describe squares on their sides and see if results similar to the above are obtained.

You now see that in certain right-angled triangles, *the square on the hypotenuse = the sum of the squares on the other two sides.*

Is this interesting property true of any right-angled triangle?

2. The general investigation

Draw a right-angled triangle, and measure the sides accurately. Square the measures and test to see whether the greatest of these squares = the sum of the other two.

Repeat the experiment with a number of right-angled triangles of various sizes and shapes.

Since the measurements are only true as far as you are able to see, there may be slight differences from exact equalities. So, within the limits of error to which practical measurements are subject, the property is true of all right-angled triangles. Thus

In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

3. Pythagoras (*historical note*)

The truth of this theorem for special triangles was known to the ancient Egyptians, Hindus and Chinese. But it was Pythagoras who stated it in its general form and hence it is known as the **Theorem of Pythagoras.**

So great was his joy at the discovery that he is said to have sacrificed a hundred oxen (hecatomb) to the Muses.

Pythagoras is a most interesting personality in the history of mathematics. He was a Greek and a pupil of the famous Thales. He went to Egypt to study, and after many years returned to Southern Italy and founded a brotherhood known as the *Pythagorean School*, in which mathematics was the chief subject studied. On account of the secrecy that shrouded their activities, the people rose against them. Pythagoras himself fled and was assassinated.

PYTHAGORAS

Pythagoras was a man of various parts. He was a musician and an astronomer of high rank. He did much for the science of arithmetic also. He attributed mysterious properties to numbers, and held that 'the order and beauty of the Universe have their origin in numbers'. He took great interest in those parts of geometry and arithmetic which were intimately connected with one another.

4. Exercises

1. Calculate (to two decimal places) the length of the hypotenuse of a right-angled triangle in which the sides containing the right angle are:

(i) 2 in., 3 in. (iii) 3·5 cm., 4·2 cm.
(ii) 5 in., 4 in. (iv) 5·2 cm., 6.4 cm.

Verify by drawing the figures to scale.

2. Calculate (to two places of decimals) the third sides of right-angled triangles in which

(*a*) hypotenuse = 5 in., one side = 3·5 in.
(*b*) hypotenuse = 4·3 in., one side = 2·8 in.
(*c*) hypotenuse = 8·4 cm., one side = 6·8 cm.

Verify by drawing suitable figures.

3. Draw an acute-angled triangle and measure the sides. Compare the square on each side with the sum of the squares on the other two sides. What do you notice?

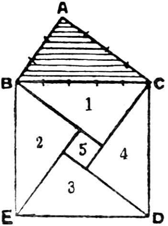
4. Draw an obtuse-angled triangle and measure the sides. Compare the square on each side with the sum of the squares on the other two sides. State what you observe?

5. Explain how you can find out without drawing figures whether a triangle is right-angled or not, if you are given the lengths of the three sides.

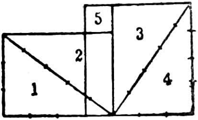
23. THE THEOREM OF PYTHAGORAS (2)

1. There are various ways in which the squares on the two sides of a right-angled triangle can be cut up and arranged so as to overlap exactly the square on the hypotenuse and *vice versa*. Such demonstrations of the theorem are usually referred to as *dissection proofs*.

2. Bhaskaracharya's dissection



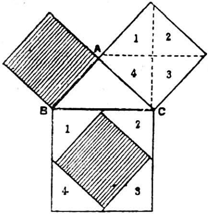
Let ABC be a right-angled triangle, $\angle A = 90°$. Construct the square on the hypotenuse. Draw, inside the square, the triangle ABC four times as indicated by 1, 2, 3, 4. An inner square is formed, whose side = AC − AB.



Now cut out the square BCDE and divide it into the five parts and rearrange them as shown on the left. Then you get a figure of two squares side by side, which can easily be verified as the squares on the two sides AC and AB.

Bhaskara after indicating this procedure simply says *pasya* (behold). Historians of mathematics think that the demonstration of Pythagoras was more or less like this.

3. Perigal's dissection

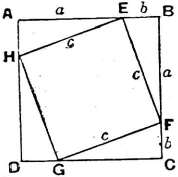


Construct the three squares on the three sides of a right-angled triangle. Through the centre of the square on AC draw two straight lines, one parallel and the other perpendicular to the hypotenuse BC. The square gets divided into 4 congruent quadrilaterals. Cut out the two squares on AC and AB; the pieces can be arranged to cover the square on BC as shown in the figure.

4. Exercises

1. Draw a number of right-angled triangles of various shapes and sizes, and verify the truth of the theorem by following the methods of dissection of Bhaskara and Perigal.

2. Draw a square ABCD. From the sides mark off AE = BF = CG = DH. Join the points E, F, G, H to form the figure EFGH. It is easy to show by measurement or otherwise that EFGH is a square.



Let $AE = BF = CG = DH = a$; $EB = FC = GD = HA = b$; $EF = FG = GH = HE = c$. Show that the area of $ABCD = (a+b)^2$ and $c^2 + 2ab$.

Hence show that $c^2 = a^2 + b^2$.

Taking the triangle AEH, $c^2 = a^2 + b^2$ means that the square on its hypotenuse = the sum of the squares on the other two sides.

If you vary the sizes of a and b, c will also vary, and you thus see that the theorem is true of all right-angled triangles.

3. Draw a right-angled triangle and the squares on the sides on a piece of cardboard of uniform thickness. Cut out the three squares and place them on the scales of a balance. The square on the hypotenuse in one scale-pan will balance the other two in the other scale-pan. What is the inference?

4. Repeat the experiment with an acute-angled triangle and an obtuse-angled triangle, and describe the nature of the relationships.

24. THE THEOREM OF PYTHAGORAS (3) : PROBLEMS

1. A number of interesting problems can be solved with the help of the Pythagorean Theorem.

Example. A bamboo ladder 30 ft. long stands upright, close to a wall. How far should its foot be drawn away from the wall, so that the top may come down by 6 feet ?

Let AC be the final position of the ladder. Then AB = (30 – 6) or 24 ft. Let x ft. be the distance BC of the foot of the ladder from the base of the wall.

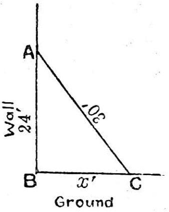
Then in the triangle ABC, angle ABC = a right angle.

$\therefore AC^2 = AB^2 + BC^2$ i.e. $30^2 = 24^2 + x^2$

Whence $x^2 = 30^2 - 24^2 = 324$.

$\therefore x = \sqrt{324} = 18$.

Hence the foot of the ladder should be drawn 18 ft. away from the foot of the wall.



2. Exercises

1. Two persons start from a place and travel due north and east respectively at 3 and 4 miles per hour. How far will they be apart in 1 hour ; in $2\frac{1}{2}$ hours ?

2. Two vessels start from a harbour and sail south and east at the rate of 25 and 20 miles per hour. How many miles will they be from each other at the end of 3 hours ?

3. A ladder 25 feet long leans against a vertical wall, with its top reaching up to a height of 20 feet. How far is the foot of the ladder from the wall ?

4. A wire stay 45 feet long reaches from a point 4 feet below the top of a flagstaff, to a point on the ground 10 feet from its foot. Calculate the height of the flagstaff. Check by drawing a figure.

5. Two vertical posts are 10 and 17 feet high and 25 feet apart. Find the distance between their tops.

6. A man on one side of a brook found that he could just rest a ladder 20 ft. long against a branch of a tree on the other bank. If the branch was known to be 12 ft. above the ground, how wide was the brook ?

7. A ladder 16 ft. long leans against a vertical wall, with its lower end 6 ft. from the base of the wall. The ladder slips and the top descends vertically one foot. Calculate how much farther the lower end has receded from the wall. Verify by drawing a figure.

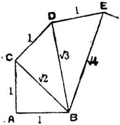
8. A ladder is placed in such a way as to reach a window 20 feet high on one side of a lane. If it is turned over to the opposite side, its top reaches a window 15 feet high. If the ladder is 25 feet long, find the width of the lane.

9. A cow is tethered to a peg at a distance of 32 feet from a straight fence with a rope 45 feet long. What length of the fence is within her reach? Verify by drawing a figure.

25. THE THEOREM OF PYTHAGORAS (4): SQUARE ROOTS

1. The square roots of natural numbers can be obtained to two places of decimals by the following geometrical method.

Draw two straight lines AB, AC perpendicular to each other, each 1 in. long.



Join BC. Then $BC^2 = AB^2 + AC^2 = 2$.

$\therefore BC = \sqrt{2}$. On measuring with a foot rule, you see that BC = 1·41 in.

Hence $\sqrt{2} = 1{\cdot}41$ nearly.

At C draw CD perpendicular to BC and 1 in. long.

In the right-angled $\triangle BCD$,

$$BD^2 = BC^2 + CD^2 = 2 + 1 = 3$$

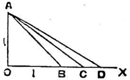
$$\therefore BD = \sqrt{3}$$

BD measures 1·73 in.

$$\therefore \sqrt{3} = 1{\cdot}73 \text{ nearly.}$$

The construction may be continued, and the values of $\sqrt{4}$, $\sqrt{5}$ .... may be obtained.

The above constructions may be more compactly arranged as in the figure on the right.



AOX is a right angle. OA = OB = 1. Then $AB = \sqrt{2}$.

Mark off OC = AB. Then $AC = \sqrt{3}$. Give a reason.

Mark off OD = AC. Then $AD = \sqrt{4}$. Give a reason.

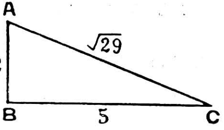
2. The above method is tedious for finding the square root of big numbers like 50. Further, the error that might be introduced in the successive constructions would accumulate and make the final result useless.

There are interesting alternative methods, however some of which are illustrated below.

Example 1. To find geometrically the square root of 29.

Here we try to express 29 as the sum of a number of perfect square numbers. Thus

$29 = 25 + 4 = 5^2 + 2^2$.



Draw a right angle ABC, having AB = 2 in., BC = 5 in. Join AC.

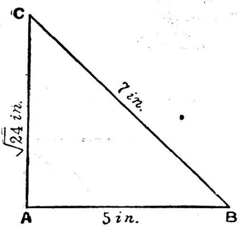
Then $AC^2 = AB^2 + BC^2 = 4 + 25 = 29$.

$\therefore AC = \sqrt{29}$.

By measurement, AC = 5·38 in. $\therefore \sqrt{29} = 5{\cdot}38$ nearly.

Example 2. Obtain geometrically the square root of 75.

Here $75 = 7^2 + 5^2 + 1^2$. Draw a right-angled triangle of sides 7 and 5 units. Then the hypotenuse will be $\sqrt{7^2 + 5^2}$ or $\sqrt{74}$ units. Construct another right-angled triangle, whose sides are $\sqrt{74}$ and 1. The hypotenuse of this will give $\sqrt{75}$. Draw a figure and complete the solution.



Example 3. Find geometrically the square root of 34.

Here you can express 24 as the difference of two squares. Thus $24 = 49 - 25 = 7^2 - 5^2$. Construct a right-angled triangle ABC having AB = 5 in. and the hypotenuse BC = 7 in. Then AC will give $\sqrt{7^2 - 5^2}$ or $\sqrt{24}$.

3. Exercises

1. Obtain geometrically the square roots of 34, 53, 61, 73, by expressing each as the sum of two square numbers.

2. Find geometrically the square roots of 11, 12, 15, 20, 32, by putting them as the difference of two squares.

3. Use the following relations to find the square roots of numbers on the left side of each equality.

(i) $14 = 9 + 4 + 1$ (iii) $32 = 5^2 + 4^2 - 3^2$
(ii) $50 = 25 + 16 + 9$ (iv) $23 = 6^2 - 3^2 - 2^2$

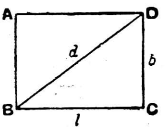
4. Show how to construct squares whose areas are 10 sq. in., 29 sq. cm., 53 sq. ft., 14 sq. yd., 32 sq. m.

5. Show that the three sides of a right-angled triangle can be represented by

(i) $l^2 + m^2,\ l^2 - m^2,\ 2lm,$ (ii) $a^2 + 1,\ a^2 - 1,\ 2a.$

26. THE THEOREM OF PYTHAGORAS (5): FURTHER APPLICATIONS

1. Rectangle



Let l, b, d represent the length, breadth and diagonal of a rectangle ABCD.

In the right-angled triangle BCD, $BD^2 = BC^2 + CD^2$.

Hence, $d^2 = l^2 + b^2$,

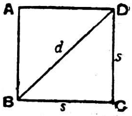
$\therefore d = \sqrt{l^2 + b^2}$

Again, $l^2 = d^2 - b^2$, and $b^2 = d^2 - l^2$

Whence $l = \sqrt{d^2 - b^2}$ and $b = \sqrt{d^2 - l^2}$

2. Square

Let s and d be the side and diagonal of a square ABCD. Then in the right-angled triangle BCD, $BD^2 = BC^2 + CD^2$.



Hence $d^2 = s^2 + s^2 = 2s^2$.

Whence $\mathbf{d} = s\sqrt{2}$.

Also $s = \dfrac{d}{\sqrt{2}} = \frac{1}{2}d\sqrt{2}$.

3. Exercises (*oral as far as possible*)

1. Find the length of the diagonal of a rectangle whose length and breadth are :

(i) 4 in., 3 in. (iii) 8 in., 6·2 in.
(ii) 13 in., 12 in. (iv) 4·2 in., 2·8 in.

2. Find the length of the diagonals of squares whose sides are 2″, 3·4 cm., $4\frac{1}{2}$ ft.

3. Find the length of the side of a square whose diagonal is 10 inches.

4. The sides of a rectangular field are 300 and 150 ft. respectively. A farmer has to go from one corner to the opposite corner. What distance will he save by walking direct along a diagonal, instead of walking along the boundary?

5. The distance along the diagonal of a square field is 100 yards. What is the length of fence required to surround the field?

6. The area of a square field is 40 acres. Find its diagonal.

7. According to the *Suluva-Sutras*, the diagonal of a unit-square $= 1 + \frac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 \times 34}$. Find how far this agrees with the value of $\sqrt{2}$.

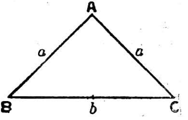
(Convert to decimals and compare.)

8. A rectangular field is 325 yards by 195 yards. A man walks from one corner to the opposite corner at 65 yards per minute. What time is saved by going along the diagonal instead of along the boundary?

4. Isosceles right-angled triangle

ABC is a right-angled triangle in which AB = AC, $\angle$ A = a right angle.

Let AB = AC = a, BC = b (units understood).



Then $BC^2 = BA^2 + AC^2$.

$\therefore\ b^2 = a^2 + a^2 = 2a^2$,

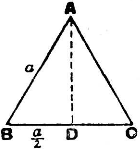
whence

$$b = a\sqrt{2},\quad a = \frac{b}{\sqrt{2}} = \tfrac{1}{2}b\sqrt{2}.$$

Compare the results with the relations between the side and diagonal of a square in section 2 above.

5. Equilateral triangle

Let ABC be an equilateral triangle, side a (units understood). Draw AD perpendicular to BC.



Then $BD = DC = \frac{a}{2}$.

In the right-angled triangle ABD,

$$AD^2 = AB^2 - BD^2$$

$$= a^2 - \frac{a^2}{4} = \frac{3a^2}{4}.$$

Hence the altitude $AD = \sqrt{\frac{3}{4}a^2} = \frac{a}{2}\sqrt{3} = 0{\cdot}866a$ nearly.

Again, the area of the triangle ABC

$= \frac{1}{2}$ (the base BC) $\times$ (the altitude AD)

$= \frac{1}{2}\, a \times a\frac{\sqrt{3}}{2} = \frac{a^2}{4}\sqrt{3}$ units of area.

Expressed in words, the *area of an equilateral triangle* $=$ *$\sqrt{3}$ times the square on half the base.*

6. Exercises

1. Find the altitude and area of an isosceles triangle whose base is 16 inches and the equal sides each 17 inches long.

2. The perimeter of an isosceles triangle is 306 cm., and each of the equal sides is $\frac{5}{8}$ of the base. Calculate the area.

3. Find the altitudes and areas of equilateral triangles whose sides are (i) 10 cm., (ii) 14·6 inches.

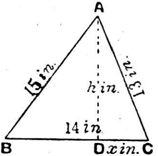
4. Find the sides of equilateral triangles whose areas are 25 sq. in., 362 sq. ft., 1943·737 sq. ft., 5 acres.

5. A municipal garden is in the form of an equilateral triangle, and to fence it round a sum of Rs. 225 was spent at 12 as. a foot. Find the distance from one corner of the enclosure to the opposite fence.

6. What must be the side of an equilateral triangle so that its area may be equal to that of a square whose diagonal measures 120 ft. ?

7. General formula for the area of a triangle

We can now develop a general formula for the area of a triangle when the lengths of its sides are known.



Example. Find the area of a triangle whose sides are 13, 14 and 15 inches long.

Take any side, say BC (= 14 inches), as base and construct the $\triangle ABC$ on it having $AB = 15$ in., and $AC = 13$ in.

Draw AD perpendicular to BC. We can calculate the altitude AD thus:

Let $AD = h$, $DC = x$, in inches. Then $BD = 14 - x$ inches.

From the $\triangle ADC$, $h^2 = 13^2 - x^2 = 169 - x^2$ . . . . (1)

,, ,, $\triangle ADB$, $h^2 = 15^2 - (14 - x)^2 = 225 - 196 + 28x - x^2$

$= 29 + 28x - x^2$ . . . . (2)

From (1) and (2), $169 - x^2 = 29 + 28x - x^2$.

$\therefore 28x = 140$, $x = 5$.

Using this value in (1), $h^2 = 169 - 25 = 144$. $\therefore h = 12$.

Thus the altitude $AD = 12$ inches.

$\therefore$ the area of the $\triangle ABC = \frac{1}{2} \times 14 \times 12$ or 84 sq. in.

✓ *If the sides of a triangle ABC are taken as a, b, c, and $2s = a + b + c$ (units understood), then the formula for the area of the triangle is*

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}.$$

If $a = 13$, $b = 14$, $c = 15$, then $2s = 42$,

$\therefore s = 21$, $s - a = 8$, $s - b = 7$, $s - b = 6$.

$\therefore \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21 \times 8 \times 7 \times 6} = 84$.

Thus the area of a triangle whose sides are 13, 14, 15 inches is 84 sq. inches, as before.

8. Exercises

1. The sides of a triangular grass plot 25, 39, and 56 feet respectively are bounded by three fences. Find the distance from the fence 56 feet long, to the opposite corner.

2. The roof of a house 21 feet broad has unequal slopes on the two sides. If their lengths are 13 feet and 20 feet, find the height of the ridge above the eaves which are of the same height from the ground.

3. The sides of a triangular field are 102, 104, 106 yards. A partition is effected by putting up a fence from the corner opposite to the side 104 yards long perpendicular to that side. Find the areas of the two parts.

4. Find the areas of triangles whose three sides are :
   (i) 7, 24, 25 feet, in sq. yd.
   (ii) 18, 20, 22 feet, in sq. ft. to 3 dec. places
   (iii) 20, 493, 507 yards, in sq. yd.
   (iv) 191, 245, 310 feet in acres
   (v) 15, 14, 13 feet in square links

5. The three sides of a triangular field are 800, 500 and 1200 links. By some mistake the third side was put down as 500 links instead of 1200 links. What error would the mistake occasion in the computed area ?

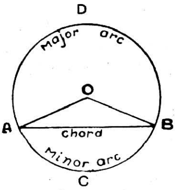
6. The sides of a triangular piece of land are 1400, 1760, 3000 links. If it is let at Rs.32 per acre, find the total rent.

7. A triangular area whose sides measure 375, 300 and 225 yards is sold for £8500. Find the price per acre.

27. ARCS AND CHORDS OF CIRCLES

1. Introduction

AB is a chord of a circle, centre O. It divides the circumference into two arcs, one longer than, and the other shorter than, half the circumference.



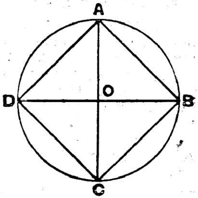
The shorter arc ACB is called the *minor* arc, and the longer arc ADB is called the *major* arc.

Join OA, OB. The chord AB, as well as the arc ACB, *subtends* the (obtuse) angle AOB at the centre O.

The arc ADB subtends the reflex angle AOB at the centre O.

2. Equal chords in a circle

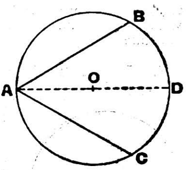
Experiment 1. Cut out a circle and fold it twice over, getting 4 quadrants. Draw the chords AB, BC, CD, DA. Compare their lengths. Also compare the lengths of the minor arcs cut off by these chords. Also compare the angles which these chords subtend at the centre O. State what you observe.

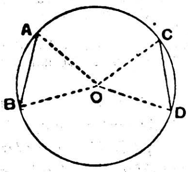


Experiment 2. Draw a circle, centre O, and place in it two equal chords AB, AC. Draw the diameter through A.

Cut out the circle and fold it along AO. Observe where B and C fall with respect to each other. What do you infer regarding (*a*) the lengths of the arcs AB, AC, (*b*) the angles AOB, AOC?

Repeat the experiment with chords AB, AC unequal.





Experiment 3. Draw a circle, centre O. Place in it two equal chords AB, CD. Join OA, OB, OC, OD. Measure the angles AOB, COD and compare them. What do you notice?

Cut out the figure and show how you would fold it to make AOB coincide with COD.

Repeat the experiments with AB, CD unequal. Do you get similar results?

The above experiments lead us to the following inferences:

In any circle (i) *if two chords are equal, the arcs they cut off are equal, the major arc to the major, the minor arc to the minor*; (ii) *if two chords are equal, the angles which they subtend at the centre are equal.*

3. Equal arcs of a circle

Cut out a circle and mark on its circumference two equal arcs. This may be done by using a piece of tracing paper.

By folding or by measurement, compare the chords which join their ends, and the angles which they subtend at the centre of the circle.

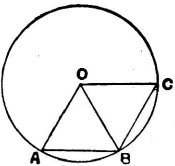
The following inferences can easily be drawn:

In any circle (i) *if two arcs are equal, the chords which join their ends are also equal*; (ii) *if two arcs are equal, the angles which they subtend at the centre are also equal.*

Note on the use of the protractor. In measuring or constructing angles with a semicircular protractor, we do not directly get at the angles at the centre. In fact the degrees are not marked at that centre. It is the equal arc-bits on the curved edge that we are using for the purpose. The reason why we can do so is now clear, viz. the equal arc-bits subtend equal angles at the centre of the instrument.

4. Equal angles at the centre

Experiment. Draw a circle, centre O. Place in it three radii OA, OB, OC, so that $\angle AOB = \angle BOC$. Join AB, BC.



Cut out the figure and fold it along OB. Observe where OA, OC fall with respect to each other. What do you infer about the lengths of the arcs AB, BC? Also about the lengths of the chords AB, BC?

Repeat the experiment with the angles AOB, BOC unequal. Do similar inferences follow?

You may now record your inferences as follows:

In any circle (i) *if two angles at the centre are equal, the arcs on which they stand are equal*; (ii) *if two angles at the centre are equal, the chords on which they stand are also equal.*

5. Exercises (*oral as far as possible*)

1. In a circle, centre O, two arcs AB, CD subtend angles AOB, COD at O. What is the relation between these angles, if (i) arc AB = twice arc CD, (ii) arc CD = $3\frac{1}{2}$ times arc AB, (iii) arc AB : arc CD = $m : n$?

2. With reference to the figure in the last question, state the relation between arcs AB, CD, if (i) $\angle AOB = 3$ times $\angle COD$, (ii) $\angle COD = \frac{5}{7}$ of $\angle AOB$, (iii) $\angle AOB : \angle COD = p : q$.

3. If a chord of a circle be twice as long as another chord, will the arcs they cut off, and the angles they subtend at the centre, bear the same relation? Test with suitable figures.

4. What fractional parts of the circumference of a circle subtend the following angles at the centre: (*a*) 90°, 60°, 45°, 30°, 36°, 72°; (*b*) 100°, 120°, 135°, x°?

5. Calculate the length of each arc in the last question, if the radius is (i) 7 cm., (ii) 3·5 inches. Take $\pi = \frac{22}{7}$.

6. One degree on the equator of the earth is 69·1 miles. What is the equatorial circumference of the earth?

7. What is the length of a '*degree of arc*' on the circumference of a circle whose diameter is 2520 inches?

8. If the equator of the earth is 24856 miles long, how many miles on it correspond to a degree?

9. A is a point on the circumference of a circle, centre O. A point P starts from A and moves along the circumference with uniform speed. How does the angle AOP change?

10. The radius of a circle is 9·4 cm. Find the length of the arc which subtends an angle of 22° 15′ at the centre.

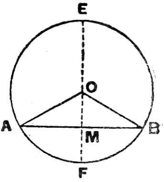
11. How would you use your protractor to divide a circular disk into 12 equal sectors? Draw a neat figure to illustrate.

12. The radius of a circle is 28 cm. What angle is subtended at the centre by an arc 16·5 cm. long?

28. CHORDS IN A CIRCLE

1. Recall to your mind the symmetry of a circle and the symmetric properties of isosceles triangles.

Suppose AB is a chord in a circle centre O. Join OA, OB. Then OAB is an isosceles triangle. If OM is perpendicular from the vertex O to the base AB, then AM = MB.



Verify by measurement, or by cutting out the circle and folding about OM.

Draw another chord CD through M, and compare the parts CM, MD. Are they equal? Note that CD is not perpendicular to OM.

The following inference is obvious:

In a circle, if a perpendicular is drawn from the centre to a chord, the chord is bisected at the foot of the perpendicular.

In the above figure, let OM be produced both ways to form the diameter EF. Then AB is a chord perpendicular to the diameter EF. You see then that

If a chord of a circle is perpendicular to a diameter, it is bisected by the diameter.

2. The converse property

You know that, in an isosceles triangle, the straight line joining the vertex to the middle point of the base is perpendicular to the base.

Draw a circle centre O, and place in it a chord PQ. Let N be the middle point of PQ. Join ON.

Since OPQ is an isosceles triangle and N the middle point of the base, ON is perpendicular to PQ. Thus

If in a circle, the middle point of a chord is joined to the centre, the joining line is perpendicular to the chord.

3. The right bisector of a chord in a circle

You know that, in an isosceles triangle, the right bisector of the base passes through the vertex.

If AB is a chord of a circle, centre O, then the triangle OAB is isosceles, and hence

The right bisector of the chord will pass through the centre.

4. Exercises

1. Draw a circle and place in it a number of chords parallel to one another. Mark their middle points. What do you observe about their positions ?

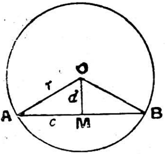
2. Mark two points A and B, 2·8 inches apart. Draw a circle of radius 1·8 inches to pass through A and B.

3. Draw a circle and place in it two chords AB, CD, which are not parallel. Construct their right bisectors. Where do these bisectors intersect ?

4. The centre of a circle is accidentally wiped out. Describe a construction for marking it.

5. Suppose you are given a figure of an arc of a circle. How will you complete the circle ?

5. Distance of a chord of a circle from the centre



Let AB be a chord of a circle, centre O. Draw OM perpendicular to AB. Then AM = MB.

Let r = the radius of the circle,

$2c$ = the length of AB,

and d = the distance OM from the centre (units understood).

Then, in the right-angled triangle OAM, $OM^2 = OA^2 - AM^2$.

$\therefore\ d^2 = r^2 - c^2$, whence $d = \sqrt{r^2 - c^2}$ . . . (1)

Also $c^2 = r^2 - d^2$, whence $c = \sqrt{r^2 - d^2}$ . . . (2)

If in (1) $r = 5$, $c = 3$ in inches, then $d = \sqrt{25 - 9}$ or 4 inches.

If in (2) $r = 5$, $d = 4$ in inches, then $c = \sqrt{25 - 16}$ or 3 inches.

Thus in a given circle, if the length of a chord is given, you can calculate its distance from the centre, and *vice versa*.

6. Exercises

1. Draw a circle of radius 3 inches, and a chord of length 4 inches. Calculate the distance of the chord from the centre, to the nearest hundredth of an inch. Verify by measurement.

2. In a circle of radius 5 cm., draw a chord at a distance of 3·2 cm. from the centre. Calculate the length of the chord to the nearest tenth of a cm. Check by measurement.

3. Two chords of a circle are equal in length. Compare their distances from the centre.

4. Two chords of a circle are at equal distances from the centre. Compare their lengths.

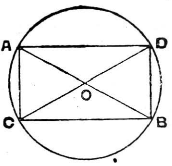
5. Draw two chords in a circle which are of unequal lengths. Which of them is nearer the centre, the longer or the shorter ?

6. Draw two chords in a circle which are at different distances from the centre. Which of them is the longer : the chord nearer to the centre, or the one further from the centre ?

29. ANGLES IN CIRCLES

1. Angle in a semicircle

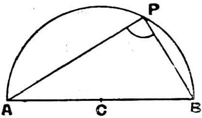
Experiment 1. Draw two equal straight lines AB, CD bisecting each other at O. If the ends be joined in order, you get a rectangle ACBD. Explain why.



The circle with O as centre and OA as radius passes through B, C, D.

AB is a diameter of the circle, so that ADB is a semicircle ; also ADB is a right angle. Point out three more semicircles and the corresponding right angles.

Experiment 2. Draw a semicircle on a straight line AB as diameter. Take any point P on the arc boundary and join AP, BP. The angle APB is called *an angle in the semicircle* APB. Measure it.



Repeat the experiment with a number of semicircles of various sizes. What is the size of all the angles in them ?

Your inference can be recorded thus :

The angle in a semicircle is a right angle.

2. Exercises

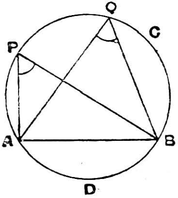
1. To draw a perpendicular to a given straight line AB from a given point P outside it.

Join P to any point Q in AB. On PQ draw a semicircle cutting AB at M. Join PM. Then PM is the required perpendicular. Justify this construction.

2. Draw a triangle ABC, having its sides 7, 9, 10 cm. long. Construct the perpendiculars from the vertices to the opposite sides.

3. Angles in a segment

Draw a circle and place in it a chord AB. If AB is not a diameter, it divides the circle into two unequal parts ACB, ADB. Each part is a *segment* of the circle.



Take any point P on the arc ACB, and join PA, PB. The angle APB is called an **angle in the segment** ACB.

Mark a few more points Q, R, S, . . . on the arc ACB. Measure and compare the angles AQB, ARB, ASB, etc. What do you notice?

Repeat the experiment with segments of circles of various sizes and shapes. Do you get similar results in all cases? Make a statement of your inference.

Your inference may now be recorded thus:

The angles in the same segment of a circle are equal to one another.

30. REGULAR HEXAGON

1. Regular polygons

A *rectangle* has all its angles equal, but all its sides are not equal to one another.

A *rhombus* has all its sides equal, but all its angles are not equal to one another.

A *square* is an improvement on the above two figures: it has all its sides equal to one another, and all its angles also equal to one another. It is a *regular quadrilateral*.

A polygon whose sides are equal and whose angles are also equal is a **regular polygon.**

An equilateral triangle is another example of a regular figure : it has all its sides equal and (consequently) all its angles equal.

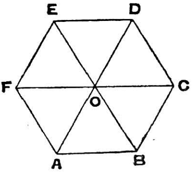
We shall now proceed to study the properties of a regular hexagon.

2. Regular Hexagon

A hexagon has six sides, and six interior angles. The sum of the six interior angles $=720$ degrees. (How ?)

If the hexagon is regular, then all its angles will be equal. Hence

Each interior angle of a regular hexagon
$= (\frac{1}{6}$ **of 720 or) 120** *degrees*

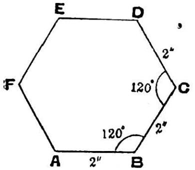


Experiment. Cut out six equilateral triangles of the same size, and arrange them as shown in the figure. It is easy to see that all the sides of the hexagon ABCDEF so built up are equal, and that each angle is *twice* an angle of an equilateral triangle and therefore 120°.

Hence the figure is a regular hexagon.

3. Construction of a regular hexagon

Example. Construct a regular hexagon on a base 2 inches long.



Draw a straight line AB = 2 in.

At B, draw angle ABC = 120°. Make BC = 2 in.

At C, draw angle BCD = 120°. Make CD = 2 in.

Continue until you reach the point F. Join AF. Measure AF. It will be equal to 2 in.

Measure the angles EFA, FAB. Each will be 120°.

You have now constructed a hexagon with each side = 2 in., and each angle = 120°. It is the regular hexagon required.

4. Exercises

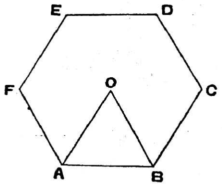
1. Construct regular hexagons whose sides are (i) 1·5 in., (ii) 4 cm., (iii) 3·4 cm. long.

2. Construct a regular hexagon carefully, and cut it out. Examine the symmetry of the figure by folding it in suitable ways. Describe the symmetry.

[A regular hexagon is symmetrical about (i) each diagonal, (ii) each of the lines joining the middle points of pairs of opposite sides.]

3. How many diagonals has a regular hexagon? Draw the diagonals. What do you notice regarding their intersections?

4. Draw a circle of radii 2 in. Place in it 6 radii OA, OB, . . . OF, equally inclined to one another. Draw the hexagon ABCDEF. Measure its sides and compare them. What do you notice? What is the relation between the sides of the hexagon and the radius of the circle?



[The sides of a regular hexagon inscribed in a circle are equal to the radius of the circle.]

Measure the angles of the hexagon and compare the results. What do you notice? What kind of hexagon is ABCDEF?

5. Make use of the observations of the last question to devise a method of constructing a regular hexagon of side 2·5 in. long.

5. Area of a regular hexagon

The preceding experiments and examples show that a regular hexagon can be divided into 6 equal equilateral triangles by drawing its diagonals.

Thus, in the above figure, the area of the regular hexagon = 6 times the area of one of these equilateral triangles, e.g. $\triangle OAB$.

If now $AB = a$ inches,

the area of $\triangle$ OAB $= \frac{\sqrt{3}}{4} a^2$ sq. in.

$\therefore$ the area of the hexagon $= 6 \times \frac{\sqrt{3}}{4} a^2$ sq. in.

$= \frac{3\sqrt{3}}{2} a^2$ sq. in.

6. Exercises

1. Find the areas of regular hexagons whose sides are (i) 2 in., (ii) 3·5 cm.

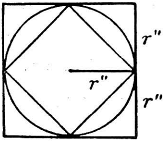
2. Calculate to three decimal places the area of a regular hexagon, each of whose sides is 10 ft.

3. Find the area of a regular hexagon whose perimeter is 1000 yd.

31. AREA OF A CIRCLE (I)

1. Introduction

The figure shows a circle enclosed in a square. There is also a square inscribed in the circle.

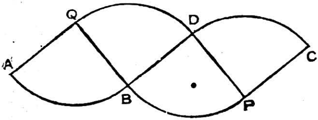


The diameter of the circle is $2r$ inches and this is also the side of the outer square.

Obviously the area of the circle is less than that of the outer square ($4r^2$ sq. in.) and greater than that of the inner square ($2r^2$ sq. in.).

We shall now try to get a more exact expression for the area of a circle.

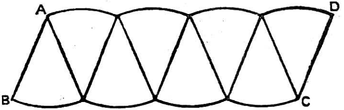
Prepare a number of circles of the same radius from paper. They all have the same area.



(1) Take one of the circles and fold it twice over to get 4 quadrants. Cut them out separately and paste them on a cardboard as in the figure on the left.

Then AB = QD = BP = DC = $\frac{1}{4}$ circumference, so that the boundary QDC = ABP = $\frac{1}{2}$ the circumference. The boundaries AQ, CP are the radii of the circle.

(2) Take another of the circles and fold it into 8 equal parts. Cut out the parts and paste them as in the figure on the right. Here also you notice that AB = CD = the radius of the circle, and the boundaries AD, BC are each $\frac{1}{2}$ the circumference.



Repeat the exercise with another of the circles dividing it into 16 equal parts, and note the lengths of the four boundaries as in the above two cases.

The experiments may be continued as far as practicable and convenient. Compare the various figures obtained. The two curved boundaries AD, BC appear to be straightening out, as it were. The corners at A, B, C, D are becoming more and more like right corners. Hence it appears that if the process be continued far enough, the figure would become practically indistinguishable from a rectangle; the area of the circle will ultimately be equal to the area of such a rectangle.

The length of the rectangle = $\frac{1}{2}$ the circumference of the circle, the breadth = the radius of the circle. Hence

The area of a circle = $\frac{1}{2}$ the circumference $\times$ the radius.

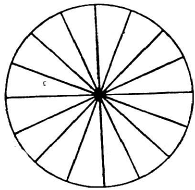
If r be the radius of a circle, then,

Area of a circle $= \frac{1}{2} \times 2\pi r \times r = \pi \times r \times r$

$= \pi r^2$ ($= 3\frac{1}{7} r^2$ approx.).

2. Alternative method

The area of a circle may also be obtained by another interesting method.



Take a circle and place in it a number of radii equally inclined to one another. The circle is then divided into a number of equal parts. The larger the number of radii, the leaner each part becomes. The arc boundary of each part is then so short that it may be regarded *practically* as a straight bit, so that each part is practically an isosceles triangle with its altitude practically equal to the radius of the circle.

Hence *ultimately*, the area of the circle

= the sum of the areas of these triangles

= $\frac{1}{2}$ the sum of their bases × the radius of the circle

= $\frac{1}{2}$ × circumference of the $\odot$ × its radius.

3. Exercises

1. Find the areas of circles whose radii are 7 cm., $10\frac{1}{2}$ in., 28 ft., 70 metres.

2. Calculate the areas of circles whose diameters are 14 in., 1 ft. 9 in., 5·6 yd., 11·2 metres.

3. The hour and minute hands of a clock are 3·5 and 4·9 inches long respectively. What areas are swept over by them in 12 hours?

4. A circular plate of silver is 21 inches in diameter. If it costs Rs. 108, how much is it per square inch?

5. Find the cost of turfing a circular plot of ground, diameter 120 yards, at 4 as. per square yard.

6. A cow is tethered to a peg with a rope 28 yards long at the centre of a football ground 110 yards long and 72 yards broad. Find the area of the ground beyond the reach of the cow.

7. A field is 42 yards by 56 yards. Half a dozen tents, each standing on a circular base of diameter 4 yards, are pitched on it. Find the surface of the field not under cover.

8. From a rectangular piece of silver plate 30 in. by 25 in., circular disks 5 in. in diameter are cut out for making the bases of tumblers. How many pieces can be cut out? How much of the plate is wasted?

9. For making quarter-anna token coins (1 inch diameter), circular pieces of the correct size are cut out from a piece of card-board 3 ft. by $1\frac{1}{2}$ ft. What part of the cardboard will be wasted after cutting out as many pieces as possible?

10. The cross-section of an underground drain is a rectangle (4 ft. by 6 ft.) with a semicircle on top of it. Find the area of the section.

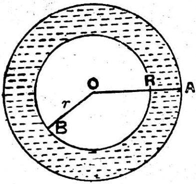
11. From a square sheet of metal whose side is 20 in., a dozen circular disks of radius $4\frac{1}{2}$ in. are cut out. By what fraction is the weight of the plate reduced?

12. Copper can stand a pull of 15 tons per square inch. What is the greatest pull a copper wire $\frac{1}{8}$ inch in diameter can stand without breaking?

13. Assuming the atmospheric pressure on a square centimetre to be 1072·73 grams, find the air pressure on the face of a circular disk whose diameter is 2·1 cm. Take $\pi = 3·14$.

32. AREA OF A CIRCULAR RING

1. Concentric circles; circular ring



The figure on the left shows two concentric circles, centre O. The shaded part enclosed between the two circumferences forms a *circular ring.*

Let the radii be OA = R inches, OB = r inches.

The area of the ring

= area of the whole circle − area of the inner circle

$= \pi R^2 - \pi r^2$ sq. in. $= \pi (R^2 - r^2)$ sq. in.

$= \pi (R + r)(R - r)$ sq. in.

2. Exercises

1. Find the areas of the circular rings enclosed between the concentric circles, whose radii are :

(i) 4 in., 2 in., (ii) 6·2 cm., 4·8 cm.

2. Find the area of the cross-section of a water pipe whose external and internal radii are respectively 9 in. and 7 in.

3. A target is 36 in. in diameter, and the outer ring is 6 in. wide. Find the space occupied by the ring.

4. A gravel path 5 ft. 3 in. broad surrounds a grass plot of diameter 70 yards. Find the area of the path.

5. A circular flower-bed 44 yd. in diameter is surrounded by a path 9 feet wide. Find the cost of metalling it at 6 as. per sq. yd.

6. A circular park has a row of coco-nut trees planted close to the boundary. If the space set apart for the purpose is a ring whose external and internal diameters are 320 and 315 yd. respectively, and if each coco-nut tree is allowed an area of 144 sq. ft., find the number of trees.

7. A circular mirror of diameter 1 ft. 2 in. has a wooden frame 2 in. wide. If the mirror costs 2 as. per sq. in. and the frame 7 pies per sq. in., find the total cost of the mirror.

8. Show that the area of the ring enclosed between two concentric circles of diameters D and d inches is $\frac{\pi}{4}(D + d)(D - d)$ sq. in.

9. A circular running-track has an internal diameter d yd., and the width of the track is w yd. Show that the area of the track is $\pi w\,(d + w)$ sq. yd.

10. A road runs round a circular park whose inner and outer boundaries measure 300 and 500 yards. Find the cost of gravelling the road at 6 as. per square yard.

33. AREA OF A CIRCLE (2)

1. The principal elements of a circle which are intimately related to one another are its radius, diameter, circumference and area. Each of these can be expressed in terms of the others, and evaluated if the necessary data are available.

Example 1. Find the radius of a circle, to the nearest tenth of an inch, if its area is 10 sq. inches. Take $\pi = 3{\cdot}14$.

If r in. be the radius, then $3{\cdot}14 \times r^2 = 10$.

$\therefore r^2 = 10 \div 3{\cdot}14$, or $r = \sqrt{10 \div 3{\cdot}14} \simeq 1{\cdot}78$.

Example 2. Express the circumference of a circle in terms of its area.

With the usual notation, $\pi r^2 = A$.

$$\therefore r^2 = \frac{A}{\pi}, \text{ whence } r = \sqrt{\frac{A}{\pi}}.$$

$$\therefore c = 2\pi r = 2\pi \times \sqrt{\frac{A}{\pi}} = 2\pi \times \frac{\sqrt{A}}{\sqrt{\pi}} = 2\sqrt{\pi A}.$$

2. Exercises

1. Calculate the radii of circles whose areas are :

   (i) 44 sq. in. (iii) 33 sq. ft.
   (ii) 110 sq. cm. (iv) 50 sq. yd.

2. Calculate the diameters of circles whose areas are :

   (i) 20 sq. in. (iii) 100 sq. cm.
   (ii) 35 sq. ft. (iv) 44·5 sq. m.

3. The areas of two concentric circles are 616 and 1232 sq. in. respectively. Find the breadth of the ring between them.

4. A cow is tethered by a rope to a peg and she can just graze over 1 acre of grassland. What is the length of the rope to the nearest foot ?

5. A circular pond has to be dug. Find the length of rope with which the area can be marked, if it is to occupy half an acre.

6. The radii of two circles are 8 and 6 ft. respectively. Find the radius of the circle whose area is equal to (i) the sum of the areas of the two circles, (ii) the difference of the areas of the two circles.

7. A circular tank occupies 3 acres. What is its greatest width ?

8. Express the radius of a circle (r) in terms of its area (A).

9. Compare the radii of two circles whose areas are as 1 : 4, 9 : 25, 2 : 3, $a^2 : b^2$, $p : q$.

10. A circle has a diameter of 10 in. Find the radius of a co centric circle whose circumference divides the given circle into two parts of equal area.

11. A circular grass plot is 40 ft. in radius. It is surrounded by a gravel path of equal area. Find the width of the path.

34. AREA OF A SECTOR OF A CIRCLE

I. Introduction

1. Cut out a circular piece of paper and fold it into four equal parts. Open out the circle and flatten it down. The circle is divided into four equal sectors. (i) What is the angle of each sector ? What fraction is it of the complete angle (360°) at the centre of the circle ? (ii) What fraction of the area of the circle is the area of each sector ? Compare these two fractions.

Repeat the experiment by folding circular pieces of paper into 8, 16, ..., 6, 12,... equal parts.

2. Describe a circle and draw in it two pairs of radii OA, OB and OC, OD, so that ∠AOB = 2∠COD. Compare the areas of the two sectors.

How many times will the area of sector AOB be of the area of sector COD if (i) ∠AOB = 5 times ∠COD (ii) n times ∠COD, (iii) if ∠AOB : ∠COD = $m : n$?

These experiments lead to the following inferences :

(i) *The area of a sector of a circle is proportional to the angle of the sector.*

(ii) $$\frac{\textit{The area of a sector}}{\textit{The area of the circle}} = \frac{\textit{The angle of the sector}}{360°}.$$

Hence *the area of a sector of a circle whose angle is* $x°$

$$= \frac{x}{360} \text{ of the area of the circle}$$

$$= \frac{x}{360} \times \pi r^2, \text{ if } r \text{ is the radius of the circle.}$$

As a particular case, if the angle of the sector is 60° and the radius of the circle = 2 inches, the area of the

sector $= \frac{60}{360} \times \frac{22}{7} \times 4$ or $2\frac{2}{21}$ square inches.

2. Exercises

1. Find the areas of the sectors of circles whose angles and bounding radii are: (*a*) 30°, 7 cm., (*b*) 48°, 14 in., (*c*) 72°, 35 ft.

2. Two diameters of a circle of radius 10 inches, cross each other at an angle of 48°. Find the areas of the sectors formed.

3. Find the area of a Japanese folding fan which opens out into a sector, whose angle is 120° and whose bounding radii are 10 inches long.

4. The minute hand of a clock is 6 inches long. Find the area it sweeps out in (i) 15 minutes, (ii) 35 minutes, (iii) between 1-10 a.m. and 2-55 a.m.

5. The diameter of a rupee is $1\frac{1}{4}$ inches. If three rupees are placed on a board so that each coin touches the other two, find the area of the uncovered space between them.

6. The side of an equilateral triangle is $2a$ inches. With each vertex as centre, a circle of radius a inches is described. Find the area enclosed between the three circles. What fraction of the area of the triangle is it?

3. Exercises

N.B. In the following questions obtain the elements wherever necessary by measurement from accurately drawn figures.

1. The arc of a sector of a circle is 10 cm. and the radius of the circle is 6 cm. Find the area of the sector.

2. Calculate the areas of the following sectors of circles:

   (*a*) radius of the circle 10 ft., arc of sector 14 ft.

   (*b*) ,, 50 ft., ,, 16 ft.

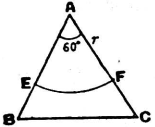
3. What fraction of the area of a circle is that of a sector whose arc is equal in length to the radius of the circle?

4. An arc of a circle of radius 21 cm. subtends an angle of 55° at the centre of the circle. Obtain the areas of the two sectors into which the circle is divided by the radii to the end-points of the arc.

5. A chord of a sector is 6 inches, the radius of the circle itself is 9 inches. Find the area of the sector.

4. When the areas of sectors are known, we can compute some of their elements when the necessary data are available.

Example. The area of a field in the form of an equilateral triangle is half an acre. What must be the length of the rope tying a cow to one of the corners of the field so as to enable her to graze over half the field ?



Let ABC be a rough sketch of the field, with the sector AEF as the part available for the cow to graze over. Let r yards be the length of the rope required.

The area of the sector AEF

$= \frac{1}{2}$ of half an acre $= 1210$ sq. yd. . . . (1)

Again the area of the sector

$= \frac{60}{360}$ of the area of a circle of radius r yards $= \frac{1}{6} \times \frac{22}{7} r^2$ sq. yd. . . . . . . (2)

$\therefore$ from (1) and (2) $\frac{1}{6} \times \frac{22}{7} r^2 = 1210$

$\therefore r^2 = \dfrac{1210 \times 7 \times 6}{22} = 2310.$ $\therefore r = 48{\cdot}06$ (approx.).

$\therefore$ the length of the rope required $= 48{\cdot}06$ yards (approx.).

5. Exercises

1. The area of a sector of radius 20 inches is 20 square inches. Find the angle of the sector.

2. The area of a sector is 108 sq. in., and the bounding radii are 18 inches. Find the length of the arc of the sector.

3. The area of a plot of ground in the form of a sector of a circle is 231 sq. yd., and the angle of the sector is 60°. Find the radius.

4. The area of a sector is 72 sq. in., and the area of the complete circle is $254\frac{4}{7}$ sq. in. Find the arc of the sector.

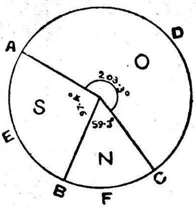
35. AREAS OF SECTORS : PROPORTIONATE REPRESENTATION

1. The areas of rectangles of the same breadth are proportional to their lengths. This property has enabled you to represent graphically the relative sizes of certain magnitudes by rectangles of the same breadth and of lengths proportional to the magnitudes.

In the last lesson, you learnt that the areas of sectors of a given circle are proportional to the arcs on which they stand, or to the angles at the centre of the circle contained by their bounding radii. In this respect the sectors resemble rectangles of constant breadths. Hence it appears that sectors of a circle may be used to represent relative magnitudes.

Example. A certain salt contains sodium, nitrogen and hydrogen in the proportion 23 : 14 : 48. Represent the composition by a circle divided into sectors of suitable sizes.

Take a circle to represent a definite quantity of the salt. It can be divided into 3 sectors whose areas are proportional to 23, 14 and 48. The angles of the sectors should therefore be proportional to the same numbers.



Since 23 + 14 + 48 = 85, these angles are respectively $\frac{23}{85}$ of 360°, $\frac{14}{85}$ of 360°, $\frac{48}{85}$ of 360°, i.e. 97·4°, 59·3°, 203·3° approximately.

Hence in the circle you have taken, draw three radii containing in succession angles 97·4°, 59·3°, and 203·3°. The three sectors thus obtained represent proportionately the quantities of the three elements in the salt.

2. Exercises

1. The land and water areas of the surface of the earth are approximately in the ratio 1 : 3. Represent the relative sizes by two sectors in a circle.

2. The numbers of Hindu, Mohammedan and Christian boys in a school are as 5 : 2 : 3. Draw a circle and divide it into three sectors whose areas are proportional to the three sections of boys.

3. The areas of three districts are respectively 240, 360 and 180 square miles. Obtain a sector representation of the relative sizes of the districts.

4. The populations of British India and the Feudatory States are respectively 244221377 and 70864995. Express these to the nearest crore. Draw a circle and divide it into two sectors with areas proportional to the two populations.

5. Draw a circle of 3 inches radius, and divide it into six sectors whose areas are proportional to the areas of the six continents given in the adjoining tabular statement.

(1) Indicate in your figure the values of the angles of the sectors, to the nearest degree.

(2) Enter neatly the names of the continents in the corresponding sectors.

[S.S.L.C.

Names of continents	Areas in millions of square miles
Asia ...	17·0
Europe ...	3·8
Africa ...	11·4
Australia ...	3·0
N. America ...	8·0
S. America ...	6·8

Names of oceans	Area
Pacific ...	68
Atlantic ...	35
Indian ...	25
Arctic ...	5·5
Others ...	11·5

6. The areas of the oceans of the world are given on the left in millions of square miles. Draw a circle of 2·5 inches radius and divide it into sectors whose areas are proportional to those of the oceans. Enter in each the name of the ocean it represents.

7. The lengths of the four seasons of an ordinary year are : Spring 92 days 21 hr., Summer 93 days 14 hr., Autumn 89 days 19 hr., and Winter 89 days, to the nearest hour. Draw a circle and represent these periods by sectors of proportionate areas.

36. STATISTICAL GRAPHS : ARBITRARY ORIGIN

1. The usefulness of a graph often depends on its size. Graphs cramped into a small space in some corner, or close to an edge of the graph paper, are inferior in value and attractiveness to those that are conveniently big and occupy the central portion of the paper.

Consider the following illustrations.

Example. The temperature of a patient taken every three hours was found to vary as shown in the table :

Monday	4 p.m.	100°·2
,,	7 ,,	101°·4
,,	10 ,,	102°·4
Tuesday	1 a.m.	103°·6
,,	4 ,,	102°·7
,,	7 ,,	100°·0
,,	1 p.m.	100°·1
,,	4 ,,	100°·3

Draw a curve indicating these changes, and from it deduce what was probably his temperature at midnight on Monday, and at 8-30 a.m. on Tuesday morning. [*S.S.L.C.* 1912

Here we note that the temperatures are all above 100°. Hence we may start marking the temperature scale on the vertical axis with 100°.

Again, the range of temperature is from 100° to 103°·6, and we have thus to represent a maximum rise of 3°·6 only. This gives us freedom to choose a fairly big scale for temperature.

2. Exercises

1. The height in inches of the barometer at noon in a certain week in June at a certain place is given in the following table :

Height	30·13	30·10	30·10	30·23	29·96	29·83
Days of the week ...	1	2	3	4	.6	7

Draw a graph, and from it find the probable pressure on 5 June.

2. Draw a graph to represent the variations in temperature (degrees Fahrenheit) given by the following record:

2 a.m.	4 a.m.	6 a.m.	8 a.m.	10 a.m.	2 p.m.
42°·2	40°·8	38°·8	40°·8	43°·8	48°·7

What was the probable temperature at 12 noon? When was the temperature the lowest?

3. The following table gives seven readings of the thermometer on a certain day. Draw a graph by plotting them, and use the graph to find the probable temperature at 1-20 p.m. and 3-10 p.m. [*M.U.*

Time	11 a.m.	11-30 a.m.	12 noon	1 p.m.	2 p.m.	2-30 p.m.	3 p.m.
Temperature in degrees ...	80·0	82·7	85·5	88·6	89·4	89·0	87·8

4. The maximum temperature in the shade at Trivandrum from 1 December to 16 December of a certain year is given (in degrees Fahrenheit) in the following table:

Date	1	2	3	4	5	6	7	9	11	13	14	16
Temperature ...	83·0	82·8	84·0	83·8	85·8	83·9	85·6	83·7	84·2	85·2	82·8	78·4

Draw a graph to represent the variation in temperature and from the graph make estimates of the temperatures on the 8th, 12th and 15th. [*Travancore S.S.L.C.* 1924

5. The following table gives the water level in the Red Hills lake on the 15th of every month:—

1911		1911 (cont.)	
April ...	42·1 ft.	November ...	39·1 ft.
May ...	40·5 ,,	December ...	44·0 ,,
June ...	39·0 ,,	**1912**	
July ...	37·9 ,,		
August ...	36·9 ,,	January ...	44·3 ,,
September ...	36·6 ,,	February ...	43·4 ,,
October ...	37·3 ,,	March ...	43·0 ,,

Exhibit the variations in level by a smoothly drawn graph, and find approximately when the level was (i) highest, (ii) lowest. [*S.S.L.C.* 1914

6. The thermometer readings at the Bangalore Observatory at 8 a.m. during a fortnight in December 1915 were as follows :

1st	67°·2	6th	64°·0	11th	58°·7
2nd	63°·5	7th	64°·3	12th	64°·2
3rd	64°·0	8th	67°·1	13th	64°·0
4th	61°·0	9th	67°·9	14th	61°·7
5th	63°·8	10th	66°·0		

Represent the above on ruled paper and determine *graphically* the approximate temperature at 2 a.m. on the 8th and at 2 p.m. on the 12th December. Find also the average daily reading of the thermometer for the fortnight. [*Mysore S.S.L.C.*

7. The following table gives the number of educational institutions under the management of Local Fund and Municipal Boards for a decade. The figures for 1914–15 and 1918–19 are not available. Draw a graph representing the variations in the number, and by interpolation supply the probable figures.

Year	1911–12	1912–13	1913–14	1914–15	1915–16	1916–17	1917–18	1918–19	1919–20	1920–21
Number of institutions in thousands.	29·28	33·53	36·16	...	39·04	41·00	42·47	...	45·07	49·37

8. The following table gives the power (P lb.) to be applied in order to move the weight (W lb.) in a certain machine :

Weight W	50	100	200	300	400	500
Power P	106	122	152	184	216	246

Draw a graph showing the relation between P and W, and find to the nearest integer the value of P, when W = 350, and of W, when P = 150. [*S.S.L.C. 1916*

9. The following table gives the population, in thousands, of two towns A and B at the beginning of each of the years specified :

Year	1835	1845	1855	1865	1875	1880	1890
A	24·4	26	29·5	34	40	43	50
B	34	36	38·4	41·1	43	34·8	46·7

Draw graphs in the same diagram to represent the above, and estimate therefrom the population of each of the towns at the beginning of 1870. In which year was the population approximately the same in both the towns, and what was it ?

10. The average height of a man and a woman and their corresponding weights are given in the following table:

Height	5′0″	5′2″	5′4″	5′6″	5′8″	5′9″	5′10″	6′0″
Average weight of a man in pounds	...	126	139	145	155	162	169	178
Average weight of a woman in pounds	103	112	126	139	152	156	...	...

Draw graphs in one and the same figure showing the relation between the average height and weight for each sex. Find from your graphs the difference between the average height of a man and a woman who weigh 10 stone each, and the difference in weight if they are both 5′6½″ in height.

11. Two boys A and B had their heights measured on the first days of six consecutive years and the measurements are given below:

Year	1916	1917	1918	1919	1920	1921
	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
A's height	3 0	3 4½	3 7¾	3 10¾	4 1	4 2¾
B's height	3 2	3 3½	3 5¾	3 8½	3 11¾	4 4

Draw, in one diagram, graphs to illustrate their growth, and find the dates when they were of the same height.

37. GRAPHS: MISCELLANEOUS EXERCISES

1. Construct a graph showing the relation between salaries in rupees per month and their equivalents in pounds sterling per year. From it deduce how many pounds a year are equivalent to Rs. 40 a month, and how many rupees a month are equivalent to £60 a year.

2. Construct a graph which will enable you to convert speeds given in miles per hour into feet per second, and *vice versa*.

From the graph, find (i) the speed in feet per second equivalent to 25 and 40 miles per hour, and (ii) the speed in miles per hour equivalent to 55 feet per second. Verify by calculation. (You may use the relation 15 miles per hour = 22 feet per second.)

3. A pressure of 30 pounds per square inch is approximately equal to one of 2·2 kilograms (metric unit of weight) per square centimetre. Construct a graph from which equivalent values of pressures in the two systems (British and Metric) can be read off. What are the equivalents of (i) 24 lb., 36 lb. per square inch, (ii) 3 kilograms, 1·8 kilograms per square centimetre ?

4. The heights of an individual at different ages are given by the following table :

Age in years ...	3	5	10	12	15	20	25	30
Height in inches ...	38	43	52	56	63	68	71	72

Draw a graph showing the relation between the age and height of the individual. Find the probable height when he was 18 years old.

5. D is the distance in yards at which a train running on the level can be stopped when running at a speed of V miles per hour. The following table gives a set of corresponding values :

V	20	30	40	50	60
D	50	100	170	260	370

Draw a graph showing the relation between D and V. Read the distances at which trains moving at 35 and 55 miles per hour can be brought to a dead stop.

6. The following table gives the velocity acquired by a body in falling from rest through different heights :

Fall ...	1	4	25	100	400	ft.
Velocity ...	8	16	40	80	160	ft. per second.

Draw a graph to represent the relation between the velocity of the body and the fall. Read off (i) the velocity acquired during a fall of 375 feet, (ii) the fall necessary in order that a velocity of 98 ft. per second may be acquired.

7. In an insurance society the premium £P to insure £100 for persons of different ages is given approximately by the following table:

Age	22	23	25	30	35	40	45	50	55
P	1·8	1·9	2·0	2·3	2·7	3·1	3·6	4·4	5·5

Illustrate the above statistics graphically, and state approximately the *premium* payable by persons aged 28, 37 and 49.

[*Mysore S.S.L.C.*

8. A test tube filled with a solution at 20°C is heated over a spirit lamp. The temperatures at various times are given by the following table:

Time in minutes	0	5	10	15	20	25	30	35	40
Temp. in degrees	20	25	29·4	33·4	37	40·2	43·2	46	48·3

Draw a time and temperature curve.

9. A man aged A years can expect to live for E years more as indicated in the following table:

A	0	4	8	12	16	20	24	28	32	36
E	41·4	51·0	49·1	46·0	42·6	39·4	36·4	33·5	30·7	28·0

Draw a graph showing the relation between A and E. From it find for how many years more a person aged 10, aged 25, aged 19 years can expect to live.

10. The following table gives the force of P lb. to be applied to a machine in order that it may move a weight of W lb.:

P	80	100	125	140	160	190	250
W	120	180	225	270	330	420	600

Use the above table to draw a graph showing the powers required to lift different weights. Read off the power required to lift a weight of 250 lb. and the weight that can be moved by exerting a power of 180 lb.

11. The following table gives the temperatures (T° C.) at a depth (D metres) below the surface of the earth determined in a certain place. Draw a graph to bring out the relationship.

D	10	41	72	103	134	165	196	227	258	289
T	12·8	13·8	15·0	15·3	16·3	16·7	17·2	18·0	18·8	19·6

38. SURFACE OF A PRISM

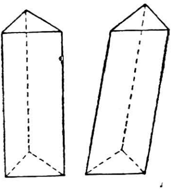
1. Prisms

A box stands on a rectangular base and has a rectangular top exactly equal to the base. The four vertical edges (corresponding to the height) are perpendicular to the base and top. All the four lateral (side) faces are rectangles. Such a solid is an example of a *prism*.

An uncut 6-faced pencil has 6 equal rectangles for the lengthwise faces, and two equal hexagons for the end faces. It is another prism. The six lengthwise edges are here perpendicular to the end faces.

We may also have solids in which the sides (lateral faces) are not rectangles, but oblique parallelograms.

A prism is a solid bounded by sides which are parallelograms and end faces which are two congruent figures parallel to each other.



Right prism Oblique prism

If the edges are perpendicular to the end faces, the solid is a **right prism**. If otherwise, it is an **oblique prism**.

The prism is a *triangular*, *quadrilateral*, *pentagonal*, *hexagonal* . . . *prism*, according as the end faces are respectively *triangles*, *quadrilaterals*, *pentagons*, *hexagons*, . . .

The perpendicular distance between the end faces is called the *length* or *height* of the prism.

We shall now proceed to find the surface areas of right prisms.

2. To find the surface area of a right prism

The following statements are obvious :

(i) The area of the lateral surface of a right prism
= the sum of the areas of the rectangles composing it
= the perimeter of the base × height of the prism.

(ii) The total surface of a right prism = the area of the lateral surface + the total area of the two ends (top and bottom).

If, now, h is the length of a prism, p the perimeter of either end face, and A the area of the end face (corresponding units being understood), then the

Total surface of a prism = 2A + ph units of area.

Example. The base of a right prism is a trapezium, in which the parallel sides are 4 and 6 inches, the oblique sides 2·5 and 3·2 inches, and the distance between the parallel sides works out as $2\frac{1}{2}$ inches. If the length of the prism is 28 inches, find the total surface area of the solid.

Here the area of each end face
$= \frac{1}{2}(4+6) \times 2\frac{1}{2}$ sq. in. $= 12\frac{1}{2}$ sq. in.

The area of the four lateral faces
= perimeter of either end × length of prism
$= 2(4+6+2{\cdot}5+3{\cdot}2) \times 28$ sq. in.
$= 439{\cdot}6$ sq. in.

∴ the total surface of the prism
$= 2 \times 12\frac{1}{2}$ sq. in. $+ 439{\cdot}6$ sq. in. $= 464{\cdot}6$ sq. in.

3. Exercises

1. Draw freehand sketches of right prisms having triangular, quadrilateral and hexagonal end faces.

2. Find (i) the lateral surface, and (ii) total surface of right prisms of the following descriptions :

(*a*) base a rectangle 3·2 in. by 2·4 in., height 14 in.

(*b*) base a right-angled triangle with sides containing the right angle 4 in. and 3 in. respectively, and height 1 foot.

(*c*) base a trapezium, having the parallel sides 5 and 8·4 cm., distance between 4·2 cm., length 2·5 cm., oblique sides 6·4 and 7·2 cm.

(*d*) base a regular hexagon of side 3 in., height 15 in.

(*e*) base a triangle of sides 6, 5 and 4·8 inches, length 10 in.

3. Find the total surface of a prism whose base is an equilateral triangle of side 1·5 in., and whose height is 3 ft. 6 in.

4. Each end face of a log of wood in the shape of a right prism is a right-angled triangle, hypotenuse 10 in. and another side 6 in. Find the total surface area, if the length of the log is 4 ft.

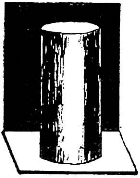
5. The cross-section of a vertical pillar is a hexagon of side 12 in. The height of the pillar is 28 ft. Find the area of its exposed surface, and the cost of painting it at 1 a. 6 ps. per sq. ft.

6. A stone pillar has a hexagonal cross-section of side 10 in. The length is 14 ft. Find the cost of polishing it all over at 4 as. 6 ps. a sq. ft.

39. SURFACE OF A CYLINDER

1. Cylinder

A solid cylinder has three separate surfaces or faces, namely, the two flat ends and the curved surface along the length.



The flat end-surfaces are two equal circles, having their planes parallel. A cylinder is thus a special case of a prism. In this lesson, we shall study the properties of right circular cylinders only.

If a cylinder is made to stand on one of its ends, that end is referred to as the *base*.

Take a cylinder and prepare a paper-tube into which the cylinder will just fit. Pull out the tube, cut it along a straight line AB perpendicular to the circular edges and flatten it out. It is a rectangle, of which the length is the length (height) of the cylinder, and the breadth is the circumference of the base (or either flat end).

2. Area of the curved surface of a cylinder

The above experiment shows that the area of the curved surface of a cylinder = the area of rectangular piece of paper which will just form a tube to cover that surface. Thus

If c *inches be the circumference of the base and* l *inches the length of the cylinder, the area of the curved surface is* $c \times l$ *square inches.*

Again, if d in. be the diameter of either end, then the circumference $= \pi d$ in.

The area of the curved surface

$= \pi d \times l$ or πdl sq. in.

$= 2\pi rl$ sq. in., r inches being the radius of the cross-section ($d = 2r$).

3. Total surface of a cylinder

The total surface of a cylinder is made up of the curved surface and the two end surfaces. Using the notation of the previous section,

The area of the total surface of a cylinder

= area of the curved surface + *twice* the area of an end surface

$= 2\pi rl + 2\pi r^2$ sq. in. $= 2\pi r\,(l + r)$ sq. in.

Example. Find the total surface area of a cylindrical roller whose length is $3\frac{1}{2}$ ft., and the diameter of the cross-section 14 in.

The circumference of the cross-section $= 3\frac{1}{7} \times \frac{14}{12}$ ft. $= \frac{11}{3}$ ft.

∴ the area of the curved surface $= \frac{11}{3} \times \frac{7}{2}$ sq. ft. $= 12\frac{5}{6}$ sq. ft.

The area of either end $= \frac{22}{7} \times \frac{14}{12} \times \frac{14}{12}$ sq. ft. $= 4\frac{5}{18}$ sq. ft.

∴ the total surface of the cylinder $= 12\frac{5}{6}$ + twice $4\frac{5}{18}$ sq. ft.

$= 21\frac{7}{18}$ sq. ft.

4. Exercises

1. Find the area of the curved surfaces of the following cylinders:

(i) Circumference of base = 9 in., length = 16 in.

(ii) Circumference of base = 14·5 cm., height = 8 cm.

(iii) Diameter of base = 14 in., length = 2 ft. 6 in.
(iv) Radius of base = $15\frac{3}{4}$ in., height = $16\frac{3}{4}$ in.

2. A right circular cylindrical log of teakwood is 12 ft. long and has a diameter of 14 in. Find its curved surface.

3. Measure the length and thickness of a new round uncut pencil, and calculate the amount of paper required just to cover its curved surface.

4. A cylindrical pillar is 12 ft. high. It stands on a base of radius 1 ft. 2 in. Find the cost of painting its exposed surface at 2 as. 6 ps. per sq. ft.

5. A well is to be dug 5 ft. in diameter on the inside, and 36 ft. in depth. How much money would be required for plastering the inside wall at 2 as. a sq. foot ?

6. A cylindrical stone roller is 6 ft. long and $3\frac{1}{2}$ ft. in diameter. How many square yards of gravelled ground is compressed by it in 60 revolutions ?

7. Find the total surface of each of the following cylinders :

(i) Diameter of base = 7 in., height = 14 in.
(ii) Radius of base = $5\frac{1}{2}$ in., height = 15·75 in.
(iii) Circumference of base = 21 cm., height = 30 cm.
(iv) Circumference of cross-section = 7·1 in., length = 12·8 in.

8. A rectangle 9 in. by 5 in. rotates about (i) a lengthwise side, (ii) a breadthwise side. State the length of the cylinder so generated and the radius of the ends. Obtain the total surface of each cylinder.

9. A square sheet of paper rotates about one of its sides. Find the area of the surface of the solid generated.

10. A cylindrical drum 7 in. in diameter and 15 in. long is to be made. Find the area of the metal sheet which will be required for the purpose.

11. A cylindrical gas tank is 14 ft. in diameter and 10 ft. high. How many iron sheets each 4 ft. by $2\frac{1}{2}$ ft. will be required for its curved surface ?

12. The radii of the bases of two cylinders are as 2 : 5, and their heights bear the same ratio. Compare the areas of their (i) curved surfaces, (ii) total surfaces.

If the total surface area of the first cylinder is 92 sq. in., what is the total surface area of the second ?

5. Surface of a cylinder : inverse problems

The elements of a cylinder which enter into the calculation of the surface area are the length (height) and circumference of either end (or radius or diameter).

We shall now consider some inverse problems in which these elements are to be calculated, when sufficient data are available.

Example. The curved surface of a cylinder is 121 square inches. If its height is 10 inches, calculate the diameter of its base.

If d in. be the diameter of the base, then the curved surface

$$= \frac{22}{7} \times d \times 10 \text{ sq. in.}$$

Hence $\frac{22}{7} \times d \times 10 = 121.$

$$\therefore d = \frac{121 \times 7}{22 \times 10} = 3\tfrac{17}{21}.$$

Thus the diameter of the base $= 3\frac{17}{21}$ in.

6. Exercises

1. A sheet of paper (14 in. by 8 in.) is curved into a cylindrical tube. In how many ways can this be done? Find the cross-section of the tube in each case.

2. What is the ratio between the height of a cylinder and the diameter of its base, when the curved surface is equal to the end surfaces together in area?

3. The curved surface of a cylinder is 118·1 sq. in., and the ratio of its radius to height is 4 : 15. Find separately the radius and the height.

40. VOLUME (1)

1. Preliminary remarks

The extent of space occupied by any object is called its *size* or **volume.** As in the case of measurement of lengths and areas, you require units for measuring volume.

In the English system, a cube whose side is 1 inch (an *inch-cube*) is used as a unit of volume. The volume of an object occupying a space equivalent to that of an inch-cube is a **cubic inch.**

2. Volume of a cuboid and a cube

A rectangular block 5 inches long, 4 inches broad and 3 inches high can be built up with $5 \times 4 \times 3$ inch-cubes, and its volume is therefore $5 \times 4 \times 3$ or 60 cubic inches. Verify by actually building up such a block.

Similarly, the volume of a cuboid 10 in. by 8 in. by 6 in. = $10 \times 8 \times 6$ or 480 cubic inches.

In general, *the volume of a rectangular block is obtained by multiplying together the measures of its length, breadth and height.*

In symbols, *if* **l**, **b**, **h** *inches are respectively the length, breadth and height of a cuboid, its volume* $= \mathbf{l} \times \mathbf{b} \times \mathbf{h}$ *cubic inches.*

If this be denoted by **V** cubic inches, then

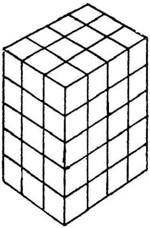
$$V = l \times b \times h = lbh.$$

Corollary. In a cube. the length, breadth and height are the same. Hence taking $l = b = h$, the above formula becomes:

The volume of a cube of side **l inch** $= l \times l \times l$ **or** l^3 *cubic inches.*

3. Alternative approach to volume

In the adjoining figure, you have a block 4 in. long, 3 in. broad and 5 in. high. It stands on a base 4×3 sq. in. in area. Its volume can therefore be written as $(4 \times 3) \times 5$ cubic inches.



In general terms:

The volume of a cuboid = the area of its base $\times$ its height.

If V, A and h, respectively denote the volume, area of base and height of a rectangular block, the corresponding units being understood, then

$$V = A \times h = Ah$$

4. Units of volume

Additional units of volume are a cubic foot, a cubic yard, and so on. The meanings are obvious.

1 cubic foot = $12 \times 12 \times 12$ or 1728 cubic inches

1 cubic yard = $3 \times 3 \times 3$ or 27 cubic feet

If the dimensions of a cuboid are given as compound quantities (in mixed units), they are reduced to the same denomination before the measures are multiplied to find the volume.

Example. Find the cost of a teakwood beam 28 ft. by 1 ft. 8 in. by 1 ft. 6 in., at Rs. 3-8-0 per cubic foot.

The volume of the beam $= 28 \times 1\frac{2}{3} \times 1\frac{1}{2}$ cubic ft.

$= 28 \times \frac{5}{3} \times \frac{3}{2}$ cubic ft.

∴ the cost required (at Rs. $3\frac{1}{2}$ a cubic ft.)

$=$ Rs. $\frac{7}{2} \times 28 \times \frac{5}{3} \times \frac{3}{2} =$ Rs. 245.

5. Exercises (*oral as far as possible*)

1. Calculate the volumes of cuboids (rectangular blocks) whose dimensions are:

 (i) 7 in., 5 in., 4 in.

 (ii) 5 ft. 6 in., 4 ft. 9 in., 3 ft. 3 in.

 (iii) 3 yd. 2 ft., 2 yd. 1 ft. 6 in., 4 yd.

2. From the formulæ $V = lbh$ and $V = Ah$ (units understood), find V when

 (i) $l = 3$ ft. 6 in., $b = 1$ ft. 8 in., $h = 9$ in.

 (ii) $A = 3$ sq. ft., $h = 2$ ft. $10\frac{1}{2}$ in.

3. Find the volume of a stack of 17600 bricks, if each brick has a volume of $82\frac{1}{2}$ cubic in.

4. If the volume of a brick is $121\frac{1}{2}$ cubic in., how many bricks will make a pile having a volume of 3 cubic yards?

5. Find the cost of a rectangular block of iron 2 ft. by 1 ft. 6 in. by 1 ft. at Rs. 22-8-0 per cubic foot. Answer to the nearest rupee.

6. Obtain the cost of digging a trench for the foundation of a wall 80 ft. long, 4 ft. broad and 5 ft. 6 in. deep at 5 as. 4 ps. per cubic foot.

7. The cross-section of a beam is 10 sq. in. If its length is 15 ft., how much does it cost at Rs. 5-4-0 per cubic ft.?

8. Atmospheric air contains 75·54 per cent of nitrogen, 23·33 per cent of oxygen and the remaining part is carbonic acid gas. A room is 30 ft. by 24 ft. by 16 ft. How much of each gas is there in the room?

9. A country cart can carry 500 bricks, and they occupy a space 6 ft. long, 2 ft. 6 in. broad and 2 ft. 1 in. deep. Find the size of a brick.

10. Teakwood is sold on the assumption that 50 cubic ft. correspond to a ton. If a ton of teakwood costs Rs. 240, what is the cost of a beam 24 ft. long and whose cross-section is a square of side 1 ft. 3 in.?

11. The lengthwise wall of a room is 32 ft. long, 18 ft. high and 1 ft. 6 in. thick. It has 2 doorways, each 6 ft. by $4\frac{1}{2}$ ft. and 4 windows, each 4 ft. 6 in. by 2 ft. 6 in. Find the space occupied by the brickwork of the wall.

12. A road 5 furlongs long and 12 ft. broad was covered with metal to an average depth of 3 inches, and the cost came to Rs. 1800. Find the cost, at the same rate, of repairing a road 1 mile long, 15 ft. broad, covering it with metal to a depth of 4 in.

13. A water tank is 15 ft. by 8 ft. by 10 ft. How many bucketsful of water can it hold, if each bucketful is 1200 cubic in.?

14. Water is distributed to a town of 50000 inhabitants from a reservoir consisting of 3 compartments, with vertical sides, each 200 ft. long, 100 ft. broad and 12 ft. deep. If the allowance is 15 gallons per head per day, how many days' supply will the reservoir hold? (1 gallon = 277·274 cubic in.)

15. A brick of ordinary size is $8\frac{1}{2}$ in. long, 4 in. broad and $2\frac{1}{2}$ in. thick. When built into a wall, a space of 9 in. by $4\frac{1}{2}$ in. by 3 in. is taken up by each brick because of the mortar round it. What is the increase per cent in the space occupied by a brick?

16. A block of timber is 36 ft. long, 24 in. broad and 12 in. thick. Into how many rafters 12 ft. by 4 in. by 3 in. can you cut it?

17. A bathing pond is 120 ft. by 30 ft. A number of persons bathing in it got out, and the level of the water sank by 6 in. If each person displaced 2 cubic ft. of water, how many bathed?

18. A tub is 10 ft. by 8 ft. by 6 ft. Bricks whose dimensions are 9 in. by 6 in. by $1\frac{1}{2}$ in. are dropped into it and the water rises 4 in. Find the number of bricks.

19. How many bricks each 9″ by $4\frac{1}{2}$″ by 2″ will be required to build a wall 15 ft. by 8 ft. by $1\frac{1}{2}$ ft., leaving $\frac{1}{10}$ of the whole space for mud? What will the bricks cost at Rs.8 per thousand?

20. Find the number of bricks required to build a wall 30 ft. long, 12 ft. high, 1 ft. 6 in. thick, given that the bricks are 9″ × $4\frac{1}{2}$″ × 2″, and chunam takes up $\frac{1}{8}$ of the whole space.

21. A tank 5 ft. long, 4 ft. broad and 6 ft. high is half full of water. If a brick absorbs $\frac{1}{16}$ of its volume of water, how many bricks 6 in. long, 3 in. broad and 2 in. thick can be put into the tank so that the water may just reach the top of the tank?

22. A tub 6 ft. by 4 ft. contains paper pulp to a depth of 2 ft. 9 in. How many sheets of paper 40 in. by 15 in. by $\frac{1}{300}$ in. can be made from it, supposing that the pulp is reduced to half the thickness on squeezing out the water?

23. Each of the dimensions of a rectangular block is doubled. How many times is the volume increased?

24. The base-area of a rectangular block is doubled while its height is increased 3 times. How is the volume of the block changed?

25. Compare the volumes of two cuboids of which $l_1 : l_2 = 3 : 2$, $b_1 : b_2 = 5 : 4$, $h_1 : h_2 = 9 : 7$.

26. Compare the volumes of two cubes whose edges are in the ratio (i) 2 : 1, (ii) 3 : 5, (iii) $a : b$.

41. VOLUME (2)

1. The formula for the volume of a cuboid involves four quantities, symbolically denoted by V, l, b, h, whose meanings you know. If any three of them are known, the fourth can be calculated at once. The following transformations of $V = lbh$ will be helpful according to requirements.

$$l = \frac{V}{bh},\ b = \frac{V}{lh},\ h = \frac{V}{lb}.$$

Read these transformed formulæ in words.

Example. Teakwood is purchased on the supposition that 50 cubic feet weigh a ton. Find the thickness of a beam of teakwood of length 20 ft. and breadth 2 ft., weighing $1\frac{1}{2}$ tons.

The weight of the beam $= 1\frac{1}{2}$ tons

$\therefore$ its volume $= 50 \times 1\frac{1}{2}$ or 75 c. ft.

$\therefore$ the thickness $= \left[\dfrac{\text{volume}}{\text{length} \times \text{breadth}}\right]$

$= \dfrac{75}{20 \times 2}$ or $\frac{15}{8}$ ft. $= 1$ ft. $10\frac{1}{2}$ in.

2. Exercises (*oral as far as possible*)

1. From the formula $V = lbh$ conveniently transformed, find
   (i) l, when $V = 36$, $b = 2$, $h = 2\frac{1}{4}$
   (ii) b, when $V = 108$, $l = 16{\cdot}4$, $h = 2{\cdot}5$
   (iii) h, when $V = 212$, $l = 16$, $b = 2{\cdot}5$

2. The cost of a beam of wood at Rs. 2-8-0 per cubic ft. is Rs. 225. Its length and breadth are respectively 18 ft. and 2 ft. 6 in. Find its thickness.

3. A tank 242 yd. long and 200 yd. broad was dug up, and 169400 cubic yd. of earth were removed. Find the depth.

4. A beam of timber, of square section, is 10 ft. long, and contains 3 cubic ft. Find its thickness to the nearest tenth of an inch.

5. A pit 60 ft. long, 20 ft. broad was dug and the earth spread over 24000 sq. ft. of ground to raise the level by 6 in. How deep was the pit dug?

6. An embankment whose volume is calculated to be 72000 cubic ft. is made with earth dug out of a trench of rectangular section 12 ft. broad and 3 ft. deep. What length of trench was dug?

7. 1 cubic ft. = 6·25 gallons. How many inches of rain stand on an acre of ground, if its volume is judged to be 16000 gallons?

8. A rectangular bath is 14 ft. long, 9 ft. broad and 4 ft. deep. How much deeper must it be made to hold 182 gallons of water more?

9. A swimming bath is 100 yd. long and 36 yd. broad. 150 men whose average size is $2\frac{1}{4}$ cubic ft. jump in. Find the rise of level of the water surface.

10. Find the area of the cross-section of a vessel which can hold 10 cubic ft. of water to a depth of 2 ft. 6 in. If a stone slab 1 ft. 6 in. by 1 ft. 6 in. is immersed in the water, the level of the water is raised 1 inch. Find the thickness of the slab.

11. A rectangular tub 8 ft. by 6 ft. by 4 ft. contains some water. Rectangular bricks 8 in. by 8 in. by 2 in. are dropped into it causing 1728 cubic in. of the water to flow over. What was the original depth of water in the cistern?

12. A terrace is 40 ft. by 24 ft. On a certain day 3 inches of rain are recorded. If the whole of the water falling on the terrace drains into a pit 12 ft. long, 6 ft. broad, what will be the depth of the water in the pit?

13. A kerosene tin is 9 in. by 9 in. by 15 in. If it contains 4 gallons of oil, find the interval at the top not occupied by the oil. Take 1 gallon = 277·274 cubic in. How many gallons can the tin hold?

14. A cartload of manure measures 12 ft. by 2 ft. by 1 ft. 9 in. Sixty cartloads are spread evenly over 4 acres of land. What is the thickness of the spread?

15. A cubic foot of gold is hammered into sheets whose total area is 6 acres. Find the thickness of the sheet as a decimal fraction of an inch, correct to two significant figures.

16 A tub 15 ft. by 2 ft. by 9 in. contained paper pulp. When boiled, it dried down to 0·6 of itself. The pulp yielded 3375 sheets of paper 27″ by 17″. How thick was each sheet?

17. A cistern 4 ft. by 2 ft. by 8 in. contained paper pulp to the full. It was boiled down to 75 per cent of its volume. The pulp then gave 160 sheets of paper each 27″ by 17″. Find the thickness of each sheet.

18. The catchment area of a tank extends over 4 square miles. If in the course of a week $3\frac{1}{2}$ inches of rain fell on it and drained into a tank raising the level of water in it by 2 ft. 6 in., find the area of the tank.

19. A reservoir 250 yd. by 170 yd. by 20 ft. drains an area of 1·5 square miles. How many inches of rainfall will be required to fill the reservoir, assuming that only $\frac{1}{3}$ of the rain-water drains into it?

20. The dimensions of a room are l ft., b ft. and h ft. respectively; p pounds of paint are used up in painting the four walls and the ceiling. If 1 pound of paint is a cubic inches in volume, find the thickness of the coat of paint.

21. A silver plate l in. by b in. by t in. is beaten into a thinner sheet of dimensions 24 in. by 16 in. Find an expression for its thickness. What will it be if $l = 24$, $b = 20$, $t = \frac{3}{8}$?

42. VOLUME (3)

1. There are a number of problems concerning the quantity of material used up in making boxes, the quantity of earth dug up to make trenches round gardens, etc.

Example. The internal dimensions of a closed rectangular box are 18, 16, and 12 inches. The thickness of the planks is $\frac{1}{2}$ inch. Find the quantity of wood used in making the box.

Obviously the required result is the measure of the space occupied by the box as a whole *less* the internal capacity.

Now the external dimensions of the box are 19, 17 and 13 inches.

∴ the external volume $= 19 \times 17 \times 13$ cubic in.

The internal volume (size of the hollow space) $= 18 \times 16 \times 12$ cubic in.

∴ the quantity of wood used $= (19 \times 17 \times 13 - 18 \times 16 \times 12)$ or 743 cubic in.

Note. If the weight of the box when empty be given, you can calculate the weight of a cubic foot of wood, and vice versa.

2. Exercises

1. Find the capacity of a closed box of wood half an inch thick, if the external dimensions are 28 in., 22 in. and 8 in.

2. A closed rectangular box is 2 ft. 6 in., 16 in., and 12 in. on the outside. The plank it is made of is 0·8 in. thick. Find the capacity of the box.

3. The internal dimensions of a record box are 22·3, 12·8 and 9·2 inches respectively. It receives a metal casing 0·08 inch thick. Find the cost of the metal used, if a cubic inch of it costs 10 as.

4. The external length, breadth and height of a closed rectangular wooden box are 18 in., 10 in. and 6 in. respectively. The thickness of the wood is half an inch. When the box is empty, it weighs 15 lb. Find the weight of a cubic inch of wood.

5. The box in the last question weighs 100 lb. when filled with sand. Find the weight of a cubic foot of sand.

6. A garden is 110 yd. by 90 yd. All round it on the inside a pathway is laid to a breadth of 3 yd. How many cubic feet of metal will be required to spread on the path to a thickness of $4\frac{1}{2}$ inches ?

7. A stone pillar is 16 ft. high and stands on a rectangular base 2 ft. by 1 ft. 8 in. Brickwork to a thickness of 6 inches is built round it. Find the quantity of metal added on.

8. A rectangular field is 300 yd. long by 200 yd. broad. A moat 10 yd. wide and 6 ft. deep is dug round it and the earth is spread evenly over the field. Find the rise in the level of the field, assuming that the earth dug out does not increase in volume.

9. In the last question, what will be the rise in the level, supposing that the mud has increased by $\frac{1}{8}$th of its volume when dug out and loosened thereby ?

10. The external dimensions of a closed rectangular cistern are l, b, h inches. If the thickness of the metal sheet is t inches, find expressions for (i) the internal volume, (ii) the external volume, (iii) the volume of the metal used.

11. A box without a lid is made of wood 1 inch thick. The external dimensions are 2 ft. 10 in., 2 ft. 5 in., 1 ft. 7 in. respectively. Find the volume of the wood used. What does it cost to make the box at Re. 1-12-0 per cubic ft. ?

43. VOLUME COMBINED WITH MOTION

1. A number of interesting problems refer to moving fluids, e.g. the quantity of water discharged through pipes in given intervals, or drained into the sea by rivers. The speed of the current is influenced by various factors like the area of the cross-section, the height of the source, and so on. Such complications are not introduced into the problems which we shall now take up. The method will involve simple combinations of volume and speed.

Example. A river is 25 ft. deep and 480 ft. wide on the average. It is flowing full at 3 miles per hour. How many tons of water per minute run past any place on its bank?

The cross-section of the river across any place = 480 × 25 sq. ft.

Again, in 1 hour the water flows 3 × 5280 ft.

∴ in 1 min. it will flow $\frac{3 \times 5280}{60}$ or 264 ft.

In other words, in 1 min. a stretch of water 264 ft. long and of cross-section 480 × 25 sq. ft. will flow past any place on the bank.

This quantity = 480 × 25 × 264 cubic ft.

But a cubic ft. of water weighs $62\frac{1}{2}$ lb. or $\frac{125}{2 \times 2240}$ tons.

∴ the weight of water required $= \frac{480 \times 25 \times 264 \times 125}{2 \times 2240}$ tons

$= 883928\frac{4}{7}$ tons.

2. Exercises

1. The bore of a water pipe is $3\frac{1}{4}$ sq. in., and the pipe discharges 5200 gallons in a given time. How many gallons can be got in the same time from a pipe whose bore is $2\frac{1}{2}$ sq. in.? What should be the bore, if the quantity of water to be got is 2800 gallons?

2. The cross-section of a water main is 48 sq. in. Find the quantity of water which flows through it in 6 hours, if the flow is at the rate of 4 miles an hour.

3. Water from a tank flows out through a pipe of square bore 4 in. by 4 in. at the rate of 72 ft. per minute. What quantity of water flows out in 1 hr. 20 min.?

4. A water tank supplies a college hostel and is on a base 40 ft. square and of height 16 ft. Water stands to a depth of $13\frac{1}{2}$ ft., and is drawn off for use by pipes carrying it at 40 cubic ft. per minute. If the water at the bottom to a depth of $1\frac{1}{2}$ feet contains

sediment and cannot be used, find for how long the pipes might be kept open.

5. In a recent flood, 1200 sq. miles of a district were under water to an average depth of 2 ft. 6 in. Making an allowance of 5 per cent of the quantity for evaporation, how long will it take for the flood to drain off into the sea, if the discharge is 250 million gallons per hour? (1 gal. = $277\frac{1}{4}$ cubic in.)

6. A cistern whose internal dimensions are 24 ft. by 20 ft. by 12 ft. is full of water. The water is allowed to run out of it at the rate of 3 cubic ft. per minute. Find the area of the inner surface of the cistern that will be exposed in half an hour.

44. VOLUME AND WEIGHT

1. In this lesson we shall take up problems relating to volumes and weights of bodies. It is to be remembered that a cubic foot of water weighs 1000 ounces.

Example. The external length, breadth and depth of a rectangular closed metal vessel are 14, 10 and 9 in. respectively and the thickness of the metal, half an inch. When the vessel is empty it weighs 1500 ounces, and when filled with water 2041·6 ounces. Find the weight of a cubic foot of water.

The external measurements are 14, 10, 9 inches.

∴ the internal measurements are 13, 9, 8 inches.

The internal volume (capacity) of the vessel = $13 \times 9 \times 8$ or 936 cubic in.

The weight of this volume of water = (2041·6 − 1500) or 541·6 oz.

∴ the weight of a cubic foot ($12 \times 12 \times 12$ cubic in.) of water

$$= \frac{541 \cdot 6 \text{ oz.}}{936} \times 12 \times 12 \times 12 \text{ or } 1000 \text{ oz. nearly}$$

2. Exercises

1. One ollock of water weighs nearly 6 palams. What is the approximate weight of a measure; of a markal?

2. A sample of sea water weighs 1·026 times an equal quantity of fresh water. What is the weight of a cubic foot of it?

3. The weight of iron is 7·73 times that of an equal quantity of water. Find the weight of a cubic foot of iron.

4. A cubic foot of water weighs $62\frac{1}{2}$ lb. The weight of a certain kind of oil is $\frac{4}{5}$ that of an equal quantity of water. Find the weight of a cubic foot of the oil.

5. Find the weight of a gallon of fresh water, taking 1 gallon = 0·16 cubic ft.

6. The weight of a cubic foot of brass is 551·25 lb. How many cubic feet are contained in a block weighing 6454·65 lb.?

7. Marble is 2·7 times as heavy as water. Find the weight of a marble block 3 ft. by $2\frac{1}{2}$ ft. by 1 ft. 8 in.

8. Find the weight of a log of wood, which is 18 ft. long and of cross-section 2·4 sq. ft., if a cubic foot of the wood weighs 40 lb.

9. A copper plate is $\frac{1}{4}$ in. thick. Find the weight of a square foot of it, if 1 cubic foot of copper weighs 550 lb.

10. An iron bar is 15 ft. long, $2\frac{1}{2}$ in. broad and 1 in. thick. Find its weight, if a cubic foot of iron weighs 480 lb.

11. The rainfall over a certain place in the course of a week was 6·5 inches. Express this in tons per acre, to the nearest ton.

12. Calculate the weight of water collected on an area of 2 acres due to a rainfall of $2\frac{1}{2}$ inches.

13. A square foot of a sheet of copper weighs 5 lb. Find its thickness, given that a cubic foot of copper weighs 550 lb.

14. A block of lead weighing 1000 lb. is beaten into a plate 0·5 inch thick. Find its area in square yards, if it is given that a cubic foot of lead weighs 710 lb.

15. A square foot of steel 1 in. thick weighs 6 lb. How much does a cubic foot of steel weigh?

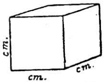
16. A vessel displaces 24000 tons of fresh water. How many tons of salt (sea) water would it displace? See question 2 above.

17. The external dimensions of a closed box are 6 ft. by 5 ft. by $3\frac{1}{2}$ ft. The thickness of the planks is $1\frac{1}{2}$ in. Find the weight of the box, if a cubic foot of wood weighs $15\frac{1}{2}$ lb

18. An open box made of wood 1 inch thick is externally 28 in. long, 25 in. broad and 15 in. high, and weighs 50 lb. Find the weight of a closed box of the same wood and of the same internal dimensions, the plank used being $\frac{1}{2}$ inch thick.

45. METRIC VOLUME AND CAPACITY

1. Volume



A cube whose edges are 1 cm. each is called a *centimetre-cube*, and the volume of a body equivalent to that of a centimetre-cube is a **cubic centimetre** (c.c.).

Similar meanings are to be attached to a cubic metre, etc. The relations among the metric units of volume are:

1000 c. mm. = 1 c. cm.
1000 c. cm. = 1 c. dm.
1000 c. dm. = 1 c. m., etc.

2. Capacity

The principal unit of capacity is a **litre.** It is equal to the volume of **1 cubic decimetre,** i.e. of a cube of side 10 cm.

1 litre = $1\frac{3}{4}$ pints nearly = 1·76077 pints, more accurately.
1 gallon = 4·543456 litres.

3. Exercises *(oral as far as possible)*

1. How many cm. cubes are required to build a rectangular block 12 cm. by 8 cm. by 7 cm.? What is the volume of the block in c.c.?

2. Calculate the volume of a cuboid whose dimensions are (i) 10, 8, 6 cm., (ii) 12·4, 9·5, 5·4 cm.

3. Atmospheric air contains 75·54 per cent of nitrogen, 23·33 per cent of oxygen and the remaining part is carbonic acid gas (chiefly). A room is 10 metres long, 7 metres broad and 5 metres high. How much of each gas is there in the room?

4. There are 120 ink-wells in an examination hall. Each well can hold 12·6 c.c. of ink. If they are filled from a jar containing 2·4 litres of ink, how much ink will still be left in the jar?

5. Write down 3·9655428938 c.c. correct to the nearest c.m., c.dm., c.c., c.mm.

6. Show that a litre is approximately 61·42 c.in. Take 1 metre = 39·371 inches.

7. Find the number of gallons of water required to fill a cistern whose internal dimensions are 73 cm., 77 cm. and 1[illegible] cm., given that 100 litres = 22 gallons.

8. Assuming a metre to be $39\frac{3}{8}$ in., find to the nearest integer the number of litres in 1 cubic foot.

9. Find the dimensions of a tank which is 2·56 metres deep on the average and which holds 3000 litres, the length of the tank being 3 times its width.

10. A closed box is made from wood 1 cm. thick. If the external measurements are 30 cm., 24 cm. and 16 cm., find (i) the internal capacity, (ii) the quantity of wood used.

46. METRIC WEIGHT

1. The chief unit of weight in this system is the **gramme** (gram, gm.). It is the weight of 1 c.c. of water at 4°C.

The prefixes in the following table of weights have the usual meanings.

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 **gram** (g., gm.)
10 grams	= 1 decagram (Dg.)
10 decagrams	= 1 hectogram (Hg.)
10 hectograms	= 1 kilogram (Kg.)

Of these, the milligram, centigram, gram and kilogram are most commonly used, specially in scientific measurements.

Example. If a piece of iron plate 2·1 metres long, 1·13 metres broad, 2·3 metres thick, weighs 425 kilograms, find to the nearest gram the weight of a square metre of sheet iron 2 mm. thick.

The volume of the iron plate $= 2{\cdot}1 \times 1{\cdot}13 \times 2{\cdot}3$ c.m.

This weighs 425 Kg.

$\therefore$ the weight of a c.m. of the iron $= \frac{425}{2{\cdot}1 \times 1{\cdot}13 \times 2{\cdot}3}$ Kg.

The volume of the square plate $= 1 \times 1 \times 0{\cdot}002$ c.m.

$\therefore$ the weight of this $= \frac{425}{2{\cdot}1 \times 1{\cdot}13 \times 2{\cdot}3} \times (1 \times 1 \times 0{\cdot}002)$ Kg.

$= 174$ gm.

2. Exercises

1. Find the weight of 42·7 c.c. of aluminium, if the metal is 2·6 times as heavy as water.

2. Taking an acre to be equal to 4047 sq. m., find the weight in Kg. of a rainfall of 78 mm. over an acre.

3. Stone is 2·5 times as heavy as water. What is the weight to the nearest Kg. of a block of stone 1·75 m. long, 1·12 m. broad and 0·8 m. thick ?

4. A rectangular trough is 20 cm. by 12 cm. by 8 cm. Find to the nearest 0·01 Kg. the weight of mercury it will hold, if mercury is 13·6 times as heavy as water.

5. An iron plate whose dimensions are 140 cm., 97·5 cm. and 20 mm. weighs 212·94 Kg. Compare the weight of iron with that of water.

6. A brass rod has a cross-section l mm. by b mm. Find the weight of a c. c. of brass, if s metres of the rod weigh g grams.

7. If 125 c. c. of water leak out from a cask every minute, find by how many kilograms the weight of the cask will be lessened in 3 hours 20 minutes ?

8. An iron bar of square cross-section and 8 metres long weighs 1000 kilograms. Find the number of the largest cubical pieces that can be cut out from the bar, and the weight of each.

Iron is 7·4 times as heavy as water.

47. MISCELLANEOUS EXERCISES

1. A rectangular box is 6 ft. 6 in. by 5 ft. 9 in. by 4 ft. 3 in. If it is filled with cubic blocks of wood of the same size, find the largest possible dimensions of each block.

2. When water freezes, its volume expands 10 per cent. How many c. in. of ice can be obtained from 1000 c. in. of water, and how much water will yield 1000 c. in. of ice?

3. A pond whose area is 4 acres is frozen over with ice to the uniform thickness of 6 inches. If a cubic foot of ice weighs 896 oz. (avoir.), what is the weight of ice in the pond?

4. A wooden granary is built of dimensions 10 ft. by 8 ft. by 5 ft. How many measures of paddy would it hold, if each measure is 100 c. in.?

5. The earth obtained from digging a canal 3 miles long, 12 ft. broad and 8 ft. deep was spread over 32 cawnies of land. Find the rise in the level.

6. An ounce of gold is beaten into a rectangular sheet 8 yd. by 5 yd. How many such sheets will make a pile 1 in. thick? One cubic foot of gold may be taken to weigh 1200 lb.

7. 80 pits, each 4 ft. long, 4 ft. broad and 3 ft. deep are dug in a garden 640 ft. long, 480 ft. broad. If the earth dug up is spread over the remaining part, find the rise in the level of the garden.

8. A c. in. of aluminium weighs 0·092 lb. and a c. in. of copper 0·31 lb. An alloy of the two metals is made so as to weigh 0·276 lb. to the c. in. Find the ratio by volume of the two metals in the alloy.

9. A tennis court is 78 ft. long, 36 ft. broad. The surface of the court together with a border of 2 ft. breadth all round is to be raised 6 inches. How many cartloads of gravel should be purchased, if each cartload contains 20 c.ft. of gravel?

10. A town has a population of 50000 inhabitants. It is supplied with water from a reservoir 2 sq. miles in area. If each inhabitant is allowed 20 gallons per day, what should be the depth of water in the reservoir, if the people can depend upon it for a year's supply if rains fail? *Note*: A bottom layer of water to a depth of 5 ft. is unfit for use; 1 gallon = 277·274 c.in.

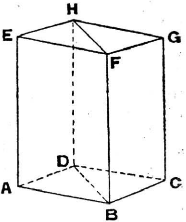
11. The internal dimensions of a closed wooden box are 16, 12 and 9 cm. It is made of wood 1 cm. thick. The box is filled with sulphur which weighs 2 gm. per c.c. Find the total weight of the box with the contents, if the density of wood is 0·86 gm. per c.c.

48. VOLUME OF A PRISM

1. You have already learnt how to find the surface-area of any right prism. We shall now take up the volume of such solids. The method of finding it has important practical applications, as you will find presently.

2. Volume of a triangular prism

Take a cuboid (rectangular prism), and divide it into triangular prisms by cutting along the plane BDHF.



The volume of each of these prisms

$= \frac{1}{2}$ the volume of the cuboid
$= \frac{1}{2} \times$ base of cuboid $\times$ its height.

Also, the area of each triangular base is half the area of the rectangular base of the original cuboid. Hence

The volume of a triangular prism
= the area of its base $\times$ *its height.*

The same may also be stated as *the area of an end-face* $\times$ *the length of the prism.*

Note 1. In the above instance, the end-face is a right-angled triangle. If either end-face of a triangular prism is not a right-angled triangle, we can divide it into two such triangles by a perpendicular from a vertex to the opposite side. If A_1, A_2 are the areas of these two parts, and h the height of the prism, then the volume of the whole prism

$$= A_1h + A_2h = (A_1 + A_2)h$$
$$= \text{the area of the base} \times \text{height.}$$

Note 2. If the end-faces of right prisms are figures of four or more sides, they can be divided into triangular parts by means of properly chosen diagonals. We can thus regard any such prism as built up of triangular prisms. The volume is then easily seen to be the product of the area of either end-face and the length.

Example. The cross-section of a bund is a trapezium of which the breadths at the top and bottom are respectively 20 and 26 ft. and the height is 3 ft. If the length of the bund is 220 yards, find the quantity of earthwork in it.

The area of the cross-section

$= \frac{1}{2}(20 + 26) \times 3$ sq. ft. $= 69$ sq. ft.

The quantity of earthwork

$= 69 \times (220 \times 3)$ c. ft. $= 45540$ c. ft.

3. Exercises

1. Find the volumes of the following right prisms :
   (i) Area of base $= 4\frac{1}{2}$ sq. in., height $= 16$ in.
   (ii) Area of base $= 2{\cdot}5$ sq. ft., height $= 14$ ft.

2. The end-faces of a right prism are right-angled triangles of which the sides containing the right angles are 5 in. and 12 in. respectively. If the length of the prism is 1 ft. 3 in., find its volume.

3. A wedge is in the shape of a prism whose base is an isosceles triangle, base 1·2 in. and altitude 3·5 in. If the length of the prism is 8 in., find its volume.

4. The base of a hollow prism is an equilateral triangle of side 4 in., and its length 10 in. Find its capacity.

5. A swimming bath is 70 yards long and 12 yards wide at the top. The depth at one end is 4 ft. and uniformly increases to 7 ft. at the other end. Find how many cubic feet of water the bath can contain.

6. A canal is 32 ft. wide at the top and 16 ft. wide at the bottom, and 10 ft. deep. How many cubic feet of earth should have been dug out to make a canal of such a section and one mile long?

7. A railway embankment is to be 16 ft. high and 1 furlong long. The top is 32 ft. wide, and the sides have a slope of 30 degrees to the vertical. How many cubic feet of earth should be obtained to make the embankment?

8. The section of a canal is 32 ft. wide at the top, 14 ft. wide at the bottom and 8 ft. deep. How many cubic yards of earth have been excavated in a mile of the canal?

9. The front elevation of a lecture hall of a college is a pentagon. The breadth of the hall is 40 feet, the (vertical) walls are 22 feet high, and the sloping sides of the roof are 27 ft. broad. If the length of the hall is 60 ft., find the air-space inside the hall. How many persons can be allowed into the hall without injury to health, if each person requires 120 c. ft. of air-space?

10. A vessel in the shape of a prism on a hexagonal base whose side is 4 in. is filled with liquid. Find (to three places of decimals of an inch) how much the surface of the liquid will be lowered if half a pint is drawn off?

11. How many cubic inches of space are occupied by a new uncut pencil, whose length is 7 in. and ends are regular hexagons of side 0·12 in.? If the black-lead inside is 0·08 in. thick, what decimal fraction of the whole pencil is the wood? Answer to two decimal places.

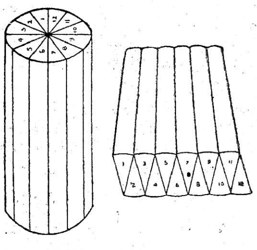
49. VOLUME OF A CYLINDER (1)

1. Introduction

You have learnt in the lesson on the surface area of a cylinder that the solid may be regarded as a special case of a right prism. The volume of a cylinder can therefore be deduced from that of a prism. There is however an independent and interesting method which we shall now take up.

2. Volume of a cylinder

You have learnt in lesson 31 a method of finding the area of a circle by cutting it up into a number of equal sectors and rearranging them to form a pattern; this pattern is observed to become more and more like a rectangle when the number of sectors becomes larger and larger.



A similar method may be adopted to find the volume of a cylinder: Take a cylinder and cut it up along its length into a large number of thin slices whose ends are equal sectors. Arrange the slices to touch each other as in the figure. You have then built up a *prism* which will become more and more like a rectangular block (cuboid) when the number of slices is made larger and larger, the slices themselves becoming thinner and thinner in consequence.

You can easily derive the following conclusion:

The volume of a cylinder
= the volume of the prism of rectangular section (into which it can be transformed)
= the area of the base of the prism × the height
= the area of the base of the cylinder × the height.

If now r inches be the radius of the base of the cylinder, h inches the height, then the volume of the cylinder

$$= \pi r^2 \times h \text{ cubic inches} = \pi r^2 h \text{ c. inches.}$$

Note. It follows from the above experiment that if a rectangular tub and a cylindrical tub have equal base-areas and equal heights, their capacities are also equal. For example, they will hold the same quantity of water.

Example 1. The radius of the cross-section of a stone roller is 1 ft. 9 in., and the length is 4 ft. Find its volume.

The area of the cross-section

$$= \frac{22}{7} \times (1\tfrac{3}{4})^2 \text{ sq. ft.}$$

∴ the volume of the roller

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 4 \text{ or } 38\tfrac{1}{2} \text{ c. ft.}$$

Example 2. A right circular cylindrical log of teakwood is 12 ft. long, and of diameter 14 in. Find its volume and curved surface.

If it is planed down to the least amount necessary to form a prism whose cross-section is a regular hexagon, what would be the volume of the prism in cubic feet, given $\pi = \frac{22}{7}$, and $\sqrt{3} = 1{\cdot}73$.

What percentage (correct to an integer) of the whole log would be wasted in planing down? [*S.S.L.C.* 1924

The volume of the log

$$= \frac{22}{7} \times \frac{7}{12} \times \frac{7}{12} \times 12 \text{ or } 12\tfrac{5}{6} \text{ cubic feet.}$$

The area of the curved surface

$$= 2 \times \frac{22}{7} \times \frac{7}{12} \times 12 \text{ or } 44 \text{ sq. ft.}$$

Again, the area of the hexagonal cross-section

$$= 6 \times \frac{\sqrt{3}}{4} \times \frac{7}{12} \times \frac{7}{12} \text{ sq. ft.}$$

∴ the volume of the hexagonal prism

$$= 6 \times \frac{1{\cdot}73}{4} \times \frac{7}{12} \times \frac{7}{12} \times 12 \text{ or } 10\tfrac{3}{5} \text{ c. ft. nearly.}$$

∴ the wastage in planing down

$$= (12\tfrac{5}{6} - 10\tfrac{3}{5}) \text{ c. ft.} = 2\tfrac{7}{30} \text{ c. ft.}$$

The total volume of cylinder $= 12\tfrac{5}{6}$ c. ft.

∴ the percentage of waste

$$= (2\tfrac{7}{30} \div 12\tfrac{5}{6}) \times 100 = 17 \text{ nearly.}$$

3. Exercises

1. Find the volumes of the following cylindrical bodies :

(i) A stone roller, 3 ft. 6 in. long, and with area of cross-section $4\frac{3}{4}$ sq. ft.

(ii) A pencil of length 5·25 in. and having its ends of radius 0·2 in.

2. Find the volumes of the following cylinders :

(i) radius of base 2 ft., height 5 ft. 3 in.

(ii) radius of base 5·2 cm., length 1·35 metre.

(iii) diameter of base 7 cm., length 6 dm. 5 cm.

3. A drum of paint has an internal diameter of 18 in. and is 2 ft. high. Find the quantity of paint that it can contain.

4. Find the cost of digging a well $3\frac{1}{2}$ ft. across and 42 ft. deep, at the rate of Rs.15 per 100 c. ft.

5. A laboratory tank has a cylindrical base of diameter 3 ft. 6 in. How much water should be let into it to occupy a depth of 1 ft. 8 in. ?

6. A cylindrical jar of base 6 sq. in. is half full of water. A lump of lead is dropped into it, and the water level rises $1\frac{1}{2}$ in. Find the size of the lump.

7. A roof is supported by 6 pillars, each of which has a diameter of base 1 ft. 6 in., height 24 ft. Obtain the cost of erecting them at Rs. 7 per cubic yard.

8. A barometer tube is 135 cm. long, and has a bore of 12 mm. How much mercury will it hold ? What will it cost to fill it with mercury costing Rs.5 per kilogram ? Mercury is 13·5 times as heavy as water.

9. Copper weighs 550 lb. a cubic foot. Find the weight of 4000 miles of copper wire cable $\frac{1}{8}$ inch thick.

10. A tunnel 2 fur. 120 yd. long, and the cross-section of which is a semicircle of diameter 4 yd. is cut through a mountain range. Find the quantity of rock removed.

11. A cylindrical oil tank is 20 ft. high and 14 ft. in diameter. Find its capacity in gallons.

12. A rectangular tank has internal dimensions 8 ft. by 6 ft. by 4 ft. $1\frac{1}{2}$ in. and is full of water. How many bucketfuls of water can be obtained from it, if the bucket is cylindrical with radius of base 6 in. and depth 12 in. ?

13. How many times can a cylindrical tumbler 4 in. high and 3 in. in diameter be filled from a cylindrical drum 40 in. high and 30 in. across ?

14. The trunk of a tree is a right circular cylinder, 3 ft. in diameter and 20 ft. high. Find the volume of the timber that remains when the trunk is trimmed just enough to reduce it to a right prism on a square base.

15. A cylindrical log is shaped into a post of hexagonal cross-section. What percentage of the wood is wasted?

16. Water flows at the rate of 4·96 ft. per second through a cylindrical water main of 11 in. in diameter. Calculate the quantity of water supplied, in gallons per minute. Take 1 c. ft. = $6\frac{1}{4}$ gallons.

50. VOLUME OF A CYLINDER (2)

1. In the last lesson we considered direct problems on the volume of a cylinder. We shall now study certain inverse problems in which one or more of the simpler elements of the cylinder have to be calculated.

Example. There is a copper cylindrical block 1 ft. across and 2 ft. high. What length of wire $\frac{1}{4}$ in. thick can be got by drawing it?

You know that the volume of a cylinder is area of section × length.

Now the thickness (diameter) of the wire $= \frac{1}{4} \times \frac{1}{12}$ or $\frac{1}{48}$ ft.
Hence the radius of the cross-section $= \frac{1}{96}$ ft.

∴ the area of the cross-section $= \frac{22}{7} \times \frac{1}{96} \times \frac{1}{96}$ sq. ft.

But the volume of the wire = the volume of the given block

$= \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 2$ c. ft.

∴ the length of the wire

$$= \frac{\text{its volume}}{\text{its cross-section}} = \frac{\frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 2}{\frac{22}{7} \times \frac{1}{96} \times \frac{1}{96}} \text{ ft.}$$

= 4608 ft. on reduction.

2. Exercises

1. A cubic inch of gold is drawn into a wire 3000 ft. long. Find the thickness of the wire, to the nearest thousandth of an inch.

2. The diameter of the base (inside) of a cylindrical tank is 5 ft. 3 in. It contains 4800 gallons of water. Obtain the necessary information and find the depth of the water.

3. If a mile length of copper wire weighs 1 cwt., find the area of its cross-section, copper being 8·96 times as heavy as water, and 1 c.ft. of water weighs 1000 oz.

4. The weight of copper wire 400 metres long is 10·32 kilograms. Find its cross-section.

5. A glass tube of uniform section is 6 in. long and contains 0·5 gm. of mercury. If mercury is $13\frac{1}{2}$ times as heavy as water, calculate the cross-section of the tube. 1 in. = 2·5 cm.

6. A cylindrical pipe through which water runs with a speed of 4 miles an hour lets off 121 c. ft. of water in 7 min. Find the diameter of the pipe.

51. CYLINDRICAL PIPES

1. In this lesson we shall take up problems relating to cylindrical pipes or tubes.

Example 1. Find the total surface of a hollow cylinder (cylindrical tube) open at both ends, if the external diameter is 10 cm., the thickness of the material 2 cm., and length of the tube 16 cm.

The internal diameter of the tube = 10 cm. − twice 2 cm.
= 6 cm.

The external curved surface $= \frac{22}{7} \times 10 \times 16$ sq. cm.
$= 502\frac{6}{7}$ sq. cm.

The internal curved surface $= \frac{22}{7} \times 6 \times 16$ sq. cm.
$= 301\frac{5}{7}$ sq. cm.

The areas of the two circular rings that form the ends of the tube $= 2[\frac{22}{7} \times 5^2 - \frac{22}{7} \times 3^2]$ sq. cm.

$= 100\frac{4}{7}$ sq. cm.

∴ the whole surface

$= (502\frac{6}{7} + 301\frac{5}{7} + 100\frac{4}{7})$ or $905\frac{1}{7}$ sq. cm.

Example 2. Find the weight of an iron tube whose external and internal radii are $3\frac{1}{2}$ and $2\frac{1}{4}$ inches and whose length is 14 ft.; given a cubic ft. of iron weighs 512 lb.

The external radius of the tube $= \frac{7}{2}$ in. $= \frac{7}{24}$ ft.

If it is a solid cylinder,

its volume would be $= \frac{22}{7} \times \frac{7}{24} \times \frac{7}{24} \times 14$ c. ft.

Again, the internal radius of the tube $= \frac{9}{4}$ in. $= \frac{9}{48}$ ft.

∴ the cubical size of the bore $= \frac{22}{7} \times \frac{9}{48} \times \frac{9}{48} \times 14$ c. ft.

∴ the quantity of material (iron) used to make the tube

$= (\frac{22}{7} \times \frac{7}{24} \times \frac{7}{24} \times 14) - (\frac{22}{7} \times \frac{9}{48} \times \frac{9}{48} \times 14)$ c. ft.

$= \frac{22}{7} \times 14 \times \frac{1}{12} \times \frac{1}{12} [\frac{7}{2} \times \frac{7}{2} - \frac{9}{4} \times \frac{9}{4}]$ c. ft.

$= \frac{22}{7} \times 14 \times \frac{1}{12} \times \frac{1}{12} \times \frac{115}{16}$ c. ft.

Now the weight of a c. ft. iron = 512 lb.

$\therefore$ the weight of the tube $= \frac{22}{7} \times 14 \times \frac{1}{12} \times \frac{1}{12} \times \frac{115}{16} \times 512$ lb.

$= 1124\frac{4}{9}$ lb.

Example 3. Obtain a formula for the volume of a hollow cylinder of height h in., outer radius R in. and inner radius r in.

The volume of a solid cylinder of height h in. and radius R in. is $\pi R^2 h$ c. in.

The volume of a solid cylinder of height h in. and radius r in. is $\pi r^2 h$ c. in.

$\therefore$ the volume of the hollow cylinder required

$= \pi R^2 h - \pi r^2 h$ c. in.

$= \pi h\,(R^2 - r^2)$ c. in. $= \pi h\,(R + r)\,(R - r)$ c. in.

2. Exercises

1. Calculate the quantity of material used in making cylindrical tubes of the following descriptions:

(i) length 10 in., inner diameter 2 in., thickness of the plate $\frac{1}{4}$ in.

(ii) length 12 cm., external diameter 6 cm., thickness of the plate 0·8 cm.

(iii) length 14 ft., internal diameter 4 in., external diameter 5·4 in.

2. A cylindrical pillar of stone of height 16 ft. is plastered all round with cement. If the internal and external diameters are 1 ft. 10 in. and 2 ft. respectively, find the quantity of cement used.

3. A well 30 ft. deep is enclosed by a wall 1 ft. thick. The internal diameter is 8 ft. Find the volume of the brickwork.

4. There is a cylindrical beam 20 ft. long and 1 ft. 8 in. across. If the diameter is reduced by 4 in., by how much is the volume reduced?

5. Find the volume of material required to make a metal disk of diameter 16 in. and thickness 3 in. with a hole at the centre of diameter 4 in.

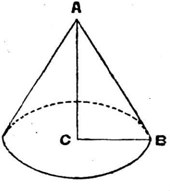
✓6. The internal and external diameters of a water main are respectively 1 ft. 8 in. and 2 ft. respectively. Find the quantity of metal used to make a pipe of length 45 ft.

✓7. A cast iron pipe is 2 ft. long; the internal and external diameters are respectively 8 in. and 10 in. Find the volume of the iron in it. What does it weigh, if a c. in. of iron is 0·26 lb.?

52. RIGHT CIRCULAR CONE

1. Introduction

If a piece of paper in the form of a right-angled triangle ABC (angle $C = 90°$) is taken and rotated about the side AC kept fixed, its plane will sweep through space resembling in shape the figure shown on the right.



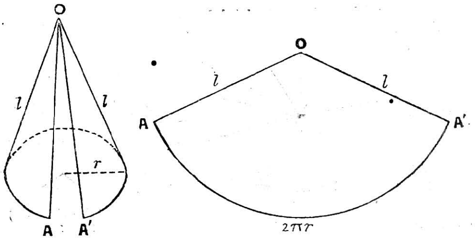
Of such a solid—called a **right circular cone**—AC is the axis of rotation. The side CB will sweep out a circle of which C is the centre and CB is the radius.

The hypotenuse AB will generate the curved surface, and is called the **generating line** of the surface.

The length AC is the **height** (sometimes called the *vertical height*) of the cone.

The length AB is the **slant height** or **slant side**.

Experiment. Take a cone, and wrap a piece of paper round it to cover the curved surface. Cut this covering



along a generating line, and spread it out flat. Study the figure so obtained.

You note that it is a *sector of a circle.*

The arc-boundary of this sector = the circumference of the base of the cone.

The radius of the sector = the slant height of the cone.

These are important observations and should be carefully remembered.

Let AC = h, BC = r, AB = l (units understood).

Then in the right-angled triangle ABC,

$$AB^2 = AC^2 + CB^2$$

Hence $l^2 = h^2 + r^2$, or $l = \sqrt{h^2 + r^2}$

Similarly, it is easy to deduce that

$$r = \sqrt{l^2 - h^2} \text{ and } h = \sqrt{l^2 - r^2}.$$

Example. A semicircular sheet of paper of diameter 10 inches is converted into a hollow cone by bringing together the two radii. Find (i) the diameter of the base of the cone, (ii) the vertical height of the cone.

Now, the circumference of the base of the cone obtained from the semicircle

= the arc-boundary of the semicircle

= 5π inches.

$\therefore$ the diameter of the base of the cone

= its circumference $\div$ π inches

= $5\pi \div \pi$ or 5 inches . . . . . (i)

Again, the radius of the base = $\frac{5}{2}$ in.

And the generating line = 10 in.

$\therefore$ the height of the cone = $\sqrt{10^2 - \frac{25}{4}}$ in.

2. Exercises

1. Find the altitudes and radii of bases of the conical surfaces which can be formed from the following sectors :

   (i) radius = 13 cm., length of arc = 31 cm.

   (ii) radius = 17 in., length of arc = 50·27 in.

2. Find the radius of the base and vertical height (altitude) of cones into which the following sectors can be converted :

(i)	radius = 5 in.,	angle at centre	= 90°
(ii)	,, = 2 ft.,	,,	= 60°
(iii)	,, = 14 cm.,	,,	= 150°
(iv)	,, = 10·5 in.,	,,	= 240°

3. A quadrant of a ⊙ of radius 4 in. is folded into the form of a cone. Find the altitude and the radius of its base.

4. A piece of paper just covers the curved surface of a cone of height 11·4 cm. and the radius of its base is 7·6 cm. The paper cone is cut along a line of slope and spread out flat. Find the angle of the sector so formed.

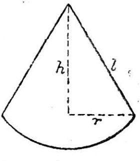
5. A certain boy took a circular piece of cardboard having a radius of 17·5 inches, and made an angle of 144° at the centre. He then cut out and removed from it the sector-piece containing the angle 144°, and made the bounding radii of the remaining piece coincide. What shape did he get? Calculate its height. [*S.S.L.C.* *1919*

53. SURFACE OF A CONE

1. Curved surface of a cone

In the previous lesson, you have learnt that the curved surface of a cone could be covered exactly by a suitably chosen sector of a circle. The area of the surface then is just that of such a sector, and can be obtained, if the necessary data to determine the latter are available.

Let h = the vertical height of the cone,
l = the slant height,
r = the radius of the base (the units of length being understood).



The area of the curved surface of the cone
= the area of the sector, of which the circumference of the base of the cone is the arc, and the slant height is the radius

$$= \frac{1}{2} \times 2\pi r \times l = \pi rl \quad \ldots\ldots (1)$$

Again, $l^2 = r^2 + h^2$, so that $l = \sqrt{r^2 + h^2}$.

Hence the result (1) becomes

$$\pi r \sqrt{r^2 + h^2}$$

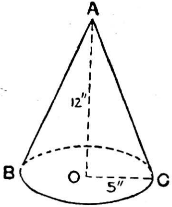
Thus *the curved surface of a cone*

$$= \pi rl = \pi r\sqrt{r^2 + h^2} \quad \ldots\ldots (2)$$

2. Total surface of a cone

The total surface of a cone is made up of the curved surface and the flat circular base.

Hence *the area of the total surface of a cone* $= \pi rl + \pi r^2 = \pi r(l + r)$ units of area.



Example. A right-angled triangle of which the sides are 5 in. and 12 in. long is made to turn round on its longer side kept fixed. Find the surface of the solid formed.

The slant height AC $= \sqrt{12^2 + 5^2}$ in.
$= 13$ in.

$\therefore$ the curved surface of the cone
$= \frac{22}{7} \times 5 \times 13$ sq. in.
$= 204\frac{2}{7}$ sq. in.

Also, the total surface
$= \frac{22}{7} \times 5 \times 13 + \frac{22}{7} \times 5 \times 5$ sq. in.
$= \frac{22}{7} \times 5\,(13 + 5)$ sq. in.
$= 282\frac{6}{7}$ sq. in.

3. Exercises

1. Find the curved surface and the total surface of the following cones :

(i)	slant side = 15 in.,	radius of base	5 in.
(ii)	,, = 20 cm.,	,,	7·5 cm.
(iii)	vertical height = 16 ft.,	,,	$4\frac{1}{2}$ ft.
(iv)	,, = 4·4 cm.,	,,	3·3 cm.

2. A tent having a conical shape is to be 10 ft. high and have radius of base 6 ft. Find the amount of canvas required to make it, neglecting seams, etc.

3. Find the area of the surface of a cone whose section along the axis is an isosceles triangle of base 2·8 inches and height 4 inches.

4. Calculate the total surface of a cone, whose axial section is an equilateral triangle of side 5 inches.

5. A right-angled triangle of which the sides are 12 and 16 cm. in length is made to turn about the shorter side. Find the total surface of the solid so formed.

6. A right-angled triangle whose sides are 4 in. and 3 in. respectively is made to rotate round its hypotenuse. Find the surface of the double cone so formed.

7. The interior of a church is in the form of a cylinder of 15 ft. radius and 12 ft. altitude. It is surmounted by a cone whose height is 15 ft. from the level of the top of the wall. What will it cost to paint the inner surface at Rs.3-8-0 per 100 sq. ft.? Neglect doorways, etc.

8. A circus tent is in the form of a cylinder surmounted by a cone. The cylindrical portion of the tent is of diameter 96 ft., the height of the top of the tent is 36 ft. from the ground; and the height of the cylindrical part 16 ft. Find the total area of canvas required for the tent. Answer to the nearest square foot. Take $\pi = 3{\cdot}1416$. [*S.S.L.C. 1921*

54. VOLUME OF A CONE

I. Introduction

Experiment. Obtain a hollow cylinder and a hollow cone—both right circular—and having equal circular bases and the same height. Fill the cylinder with water, and pour it into the cone. How many times are you able to fill the cone ?

Reverse the experiment and find out how many times you can fill the cylinder from cones full of water. These experiments lead to the following inferences :

The volume of a cylinder is 3 times the volume of a cone of the same base and the same height.

The volume of a cone is $\frac{1}{3}$ the volume of a cylinder of the same base and the same height.

If r be the radius of the base and h the height of both the solids, units being understood, then

The volume of the cone

$= \frac{1}{3} \times$ volume of the corresponding cylinder

$= \frac{1}{3} \pi r^2 h$ (units of volume)

If l (units of length) be the slant height,

$$\text{then } h = \sqrt{l^2 - r^2}.$$

The volume of the cone $= \frac{1}{3} \pi r^2 \sqrt{l^2 - r^2}$ (units of volume).

Example. If a substance weighs 4 tolas per cubic inch, find the weight of a solid right circular cone of the substance, of which the height is 20 inches, and the diameter of the base 14·6 inches.

The volume of the cone

$= \frac{1}{3} \times \frac{22}{7} \times (7{\cdot}3)^2 \times 20$ c. in.

$\therefore$ the weight of the cone at 4 tolas per c. in.

$= \frac{1}{3} \times \frac{22}{7} \times (7{\cdot}3)^2 \times 20 \times 4$ tolas

$= 4466{\cdot}2$ tolas

2. Exercises

1. Find the volumes of the following right circular cones :
   (i) radius of base 2 in., vertical height = 3 in.
   (ii) ,, 3·5 cm., altitude = 7 cm.
   (iii) ,, 12·4 in., ,, = 6·2 in.

2. Find the capacity of a conical vessel, given diameter of base = 1 ft. 2 in., and depth = $2\frac{2}{3}$ ft.

3. The base of a cone made of cast iron has a diameter 7·5 in. The vertical height = 11·48 in. If the weight of iron is 0·26 lb. per cubic inch, find the weight of the cone.

4. The sides of a right-angled triangle are 3 in. and 4 in. respectively. Find the volume of the double cone formed by the revolution of the triangle round the hypotenuse.

5. The section of a right circular cone by a plane through its vertex perpendicular to the base is an equilateral triangle, each side of which is 12 ft. Find the volume of the cone.

55. THE RIGHT CIRCULAR CONE: INVERSE PROBLEMS

1. The elements that enter directly into the calculation of the surface and volume of a cone, or are connected with them, are the altitude, the radius of the base and the slant height. If, then, sufficient data are available, we can calculate the elements themselves. Problems involving such calculations are appended.

Example 1. A certain boy took a circular piece of cardboard having a radius of 17·5 in. and made an angle of 144° at the centre. He then cut out and removed from it the sector containing the angle of 144° and made the bounding radii of the remaining piece to coincide. Calculate the height of the cone so formed.

The length of the arc of the sector subtending an angle of (360° − 144° or) 216° at the centre $= \frac{216}{360} \times 2 \times \frac{22}{7} \times 17{\cdot}5$ in.

$= 66$ in.

This is the circumference of the base of the cone.

∴ the radius of the base $= 66 \div \frac{44}{7}$ or 10·5 in.

Also the slant height = 17·5 in.

∴ the height of the cone $= \sqrt{17{\cdot}5^2 - 10{\cdot}5^2}$ in.

$= \sqrt{28 \times 7}$ or 14 in.

Example 2. A right circular cone made of iron weighs 62·8 tons and its height is 12 ft. Assuming that a cubic foot of iron weighs 448 lb., find the radius of the base.

Now the weight of the cone = 62·8 × 2240 lb.

The weight of a cubic foot of iron = 448 lb.

∴ the volume of the cone $= \frac{62{\cdot}8 \times 2240}{448}$ or 314 c. ft.

If the radius of the base $= r$ ft., then $\frac{1}{3}\pi r^2 \times 12 = 314$.

$$\therefore r^2 = \frac{314 \times 3}{3\cdot14 \times 12} \text{ (taking } \pi = 3\cdot14) = 25.$$

Whence $r = 5$.

Thus the radius of the base required is 5 ft.

2. Exercises

1. An open conical filter is filled with liquid to a depth of 5 in., and the area of the free surface of the liquid is $34\frac{4}{7}$ sq. in. Find the surface of the liquid in contact with the filter.

2. A sheet of paper is in the form of a sector of a circle, radius $=4\cdot5$ in. and angle $120°$. It is folded into a conical shape. Calculate the volume of the cone.

3. A thin piece of zinc sheet in the form of a semicircle 8·5 in. in radius, is bent to form the curved surface of a cone. How many cubic inches of liquid will the cone hold?

4. The slant height of a cone is 17 in., and the area of its curved surface is 448 8 sq. in. Find its volume.

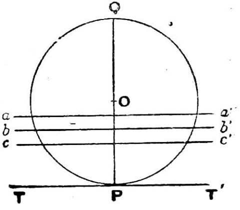
5. A cylindrical wooden rod $\frac{1}{2}$ in. in diameter is sharpened at one end into a cone of slant height one inch. How much of the wood has been cut away?

6. A conical tent is required to accommodate 5 people; each person must have 16 sq. ft. of space on the ground, and 100 c.ft. of air to breathe. Calculate the vertical height, slant height and the width of the tent.

56. TANGENTS TO A CIRCLE

1. Introduction

Draw a circle, centre O, and a number of lines aa', bb', cc', . . . perpendicular to a diameter PQ, cutting the circumference at a, a'; b, b'; c, c'; . . . .



Each of these cutting lines is a *secant* of the circle. These secants are parallel to one another, and may be regarded as positions of a secant moving parallel to itself, away from the centre O. The two points at which this moving secant cuts the circle are equidistant from the diameter PQ, and approach nearer and nearer to each other, until they coincide at P. Then TPT', the ultimate position reached by the secant, has only one point P in common with the circle. It cannot cut the circle at any other point however far it may be produced both ways. It is said *to touch* the circle at P. On that account it is called a *tangent* to the circle.

A **tangent** *to a circle is a straight line which meets the circle at one point only, and does not meet it again at any other point however far it is produced both ways.*

You notice again that the moving secant in the above experiment remains always perpendicular to the diameter PQ. Thus

A tangent to a circle at a point on it is perpendicular to the radius of the circle drawn from the centre to that point.

The point itself is called the **point of contact** of the tangent with the circle.

2. Exercises

1. Draw a circle of radius 1·4″. Mark a point A on the circumference. Draw the tangent to the circle at A.

2. Draw a circle, centre O. By trial and adjustment of a straight ruler, draw a straight line AB 'touching' the circle at P. Draw PX perpendicular to AB. Does this pass through the centre? Make a statement about the inference you can draw from this experiment.

3. What is the distance of a tangent to a circle from the centre of the circle?

4. Draw a straight line AB. From a point O outside it, draw OM perpendicular to AB. Draw the circle (O, OM). How is AB related to this circle?

5. Draw a straight line AB. Take a point P on it. Draw a circle to touch AB at P. How many circles can you draw touching AB at P? Where will their centres lie?

6. Mark a point P on a straight line ll'. Draw a circle of radius 1·5 in. to touch ll' at P. How many such circles can you draw?

7. Draw a straight line HK. Draw a circle of 1″ radius to touch HK. How many such circles can you draw? Where will all their centres lie?

8. Draw a diameter AB of a circle and the tangents to the circle at A, B. What do you know about these tangents?

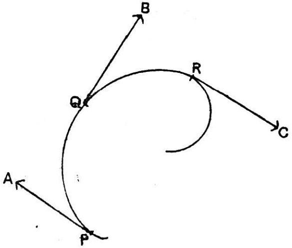
9. What is the distance between a pair of parallel tangents to a circle?

10. A cartwheel rolls along a straight track on a level road. What is the path of the centre of its hub?

3. Note on the direction of a curve

An essential difference between a straight line and a curve is that a straight line always has a constant direction, but a curve changes its direction from point to point.

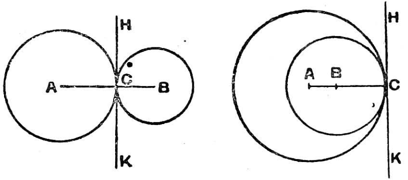
The direction of a curve at any point on it is the direction of the tangent to the curve at that point. In the figure, the direction of the curve at P is indicated by the tangent PA; that at Q by the tangent QB, and so on.



57. CIRCLES TOUCHING ONE ANOTHER

1. Introduction

1. Draw a straight line AB ; mark any point C on it.



Draw the circles (A, AC), (B, BC). Also draw HK perpendicular to AB through C. Since ACH, BCH are right angles, HK is the tangent to each of the circles at C. It is the common tangent to the circles at C.

If two circles have a common point and have a common tangent at that point, they are said to touch each other at that point.

Note that the distance between the centres of the two circles is the *sum* of their radii in the first figure, and the *difference* of their radii in the second figure.

2. By trial and adjustment, judging by eye, draw a pair of circles 'touching' each other. Join their centres. Does the line joining the centres pass through the point of contact of the circles?

You are able to draw the following inferences:

1. *If two circles touch each other externally, the distance between their centres is the sum of their radii.*

2. *If one circle touches another internally, the distance between their centres is the difference of their radii.*

3. *If two circles touch each other, the straight line joining their centres passes through the point of contact.*

2. Exercises

1. AB is a straight line 5 inches long. Pairs of circles with the following radii are described with A, B as centres: (i) 2″, 3·2″; (ii) 2·2″; 2·8″; (iii) 1·5″; 3·3″. Which of these pairs, if any, will touch each other? Draw figures to illustrate.

2. AB is a straight line 2 cm. long. Pairs of circles with the following radii are described with A, B as centres: (i) 5·3 cm., 2·8 cm.; (ii) 4·8 cm., 6·4 cm.; (iii) 3·6 cm., 5·6 cm. Which of these pairs, if any, will touch each other? Draw figures to illustrate.

3. Draw a circle, centre C, of radius 2″. Take a point D at a distance of 3·8″ from C. Draw the circles that have their centres at D and touch the given circle.

4. Draw a circle, centre O, radius 6·2 cm. Take a point P at a distance of 3 cm. from O. Draw the circle whose centre is at P, and which will touch the given circle.

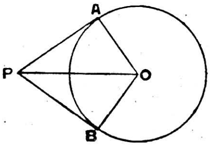
5. A, B, C are three circles having radii 1·2 inches long, and touching each other in pairs. What are the lengths of the sides of the triangle ABC?

6. Draw three circles of radii 2, 2·5, 3 inches that touch each other in pairs.

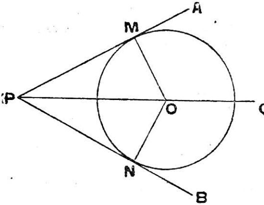
58. TANGENTS TO A CIRCLE FROM AN EXTERNAL POINT

1. Preliminary exercises

1. Draw a circle, centre O. Place any two radii OA, OB inclined to each other. Draw the two tangents to the circle at A, B. Let them meet at P. Measure the lengths PA, PB and compare. What do you notice? Measure the angles APO, BPO, and compare. What do you notice?



Describe the symmetry of the figure. You notice that PA, PB are two tangents to the circle from P.
Compare the angles AOP, BOP.



2. Draw any angle APB. Draw PQ the bisector of the angle. On PQ take any point O; draw OM, ON perpendicular to PA, PB. What do you know about the lengths of OM, ON?

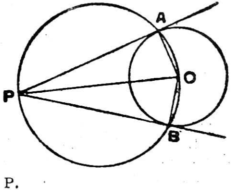
With O as centre and OM or ON as radius, draw a circle. How are PA, PB related to this circle? Give a reason for your answer.

As in the last exercise, you get two tangents PM, PN from P to the circle.

2. To draw the two tangents to a circle from an external point

Let O be the centre of the circle, P the external point.

If P be joined to a point X on the circumference, PX will be a tangent at X only if PXO is a right angle. In other words, the position of X should be such that PO subtends a right angle at it. It should therefore be on the circle on PO as diameter. This consideration gives the following method of drawing a pair of tangents to the circle from P.



Join PO. On PO as diameter draw a circle, cutting the given circle at A, B. Join PA, PB. These will be the required tangents.

Note. The points of contact, A and B, divide the circumference into two arcs, the smaller of which is *convex* to P, and the larger *concave* to P.

3. Properties of tangents to a circle

Experiment 1. Draw a circle, centre C. Take a point O outside the circle, and construct the tangents OP, OQ. Join CP, CQ.

Measure and compare (i) the lengths of the tangents ; (ii) the angles COP, COQ; (iii) the angles OCP, OCQ. State your inferences.

Experiment 2. Cut out the figure of the last experiment, and fold it along CO.

Discuss the symmetry of the figure, as brought out by the folding.

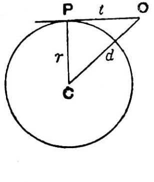
The experiments lead to the following conclusions :

1. *The two tangents that can be drawn to a circle from an external point are equal in length.*

2. *The straight line joining the external point to the centre of the circle bisects the angle between the tangents ; it also bisects the angle between the radii to the points of contact of the tangents.*

4. Length of a tangent

OP is a tangent from an external point O to a circle, centre C.



Let r = the radius of the circle,
d = the distance of O from C,
t = the length of the tangent OP.

Then in the right-angled triangle OPC,

$OC^2 = CP^2 + PO^2$ (Pythagoras's Theorem), i.e. $d^2 = r^2 + t^2$.

$\therefore t^2 = d^2 - r^2$

whence $t = \sqrt{d^2 - r^2}$

5. Exercises

1. Draw a circle, centre C and radius 3 cm. Take a point P 5 cm. from C, and draw a pair of tangents from P to the circle. Measure their lengths. Verify by calculation.

2. The radius of a circle is 5″. From a point 13″ distant from the centre, a tangent is drawn to the circle. What is its length?

3. Draw a circle of radius 2 inches. Mark a point at a distance of 4·2 inches from the centre. Draw a tangent from this point to the circle. Calculate its length to two places of decimals. Verify the result by measurement.

4. C is the centre of a circle of radius 3·3 cm. What should be the distance of a point P from C, so that the tangents drawn from P to the circle may each be 5 cm. long?

5. Draw two radii OA, OB in a circle, so that angle AOB = 120°. Draw the two tangents to the circle at A, B; let them meet at C. Find the ∠ACB (i) by measurement, (ii) by calculation.

6. Draw a circle of radius 1·5 inches. Construct a pair of tangents to the circle so as to include between them an angle of 90°, 30°, 120°. Draw a separate figure in each case.

59. ABSOLUTE AND RELATIVE ERRORS

1. Introduction

In buying cloth, you state your requirements in yards (usually), and an error of one or two inches would not matter much in several yards of cloth. If however a person deals with costly material, e.g. platinum wire, the error of an inch will be a serious matter.

In purchasing one pound of tea, the buyer would consider it a great deficiency if the packet was 2 ounces below weight. The same deficiency would not matter at all if the purchase was of a ton of coal.

A difference of Rs. 5 in the pay of a poor clerk will be a large item, but is a thing treated lightly in the revenue of a large district.

In expressing a magnitude in an approximate form, you should consider the error caused with reference to

(i) the actual value of the error, also called *absolute error*, and

(ii) the value with reference to (or judged in comparison with) the whole magnitude, also called *relative error*.

Relative error is expressed as a fraction of the whole magnitude. Compactly stated

$$\text{Relative error} = \frac{\text{absolute error}}{\text{actual value}}$$

If 10, 100, and 1000 are written as 9, 99, and 999 respectively, the absolute errors are 1 in each case. But the relative errors are $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$.

2. Importance of relative error

In measuring a distance, for example, errors tend to increase with the length you are measuring. Thus the distance between two points marked in your note-book may be in error by a small fraction of an inch, the diameter of the earth by a few miles, the distance of the sun from the earth by thousands of miles, the distances of stars by millions of miles. The differences in the relative error in the above cases may however be small ; for example, they may all prove to be more or less a hundredth part of the original magnitudes. It is the relative errors that are often more important than absolute errors in approximate calculations.

3. We shall now work out a few examples.

Example 1. Given 1 Kg. = 0·001968 cwt., express the relation to the nearest thousandth of a cwt. Obtain the absolute and relative errors.

Now 1 Kg. = 0·001968 cwt. = 0·002 cwt. correct to a thousandth.

The absolute error

= 0·002 — 0·001968 cwt. in excess

= 0·000032 cwt. in excess

The relative error $= \frac{0 \cdot 000032}{0 \cdot 001968} = \frac{32}{1968} = \frac{2}{123} \doteq \frac{1}{61}$.

Example 2. The distance of the moon from the earth is 240 thousand miles to the nearest thousand miles. Determine the maximum absolute and relative errors.

The actual distance lies between (240 + 0·5) and (240 – 0·5) thousand miles. Hence the maximum absolute error is 0·5 thousand miles, i.e. 500 miles.

You know that the relative error $= \frac{\text{absolute error}}{\text{actual value}}$. The maximum value of this is obtained by dividing the maximum absolute error by the smallest value of the distance.

Hence the maximum relative error $= \frac{500}{235000} = \frac{1}{470}$.

4. Exercises

1. What are the absolute errors if, for convenience, the numbers within brackets are taken instead of the numbers before them :

(i) 102 (100)	(v) $19\frac{5}{12}$ (20)
(ii) 99 (100)	(vi) $52\frac{1}{3}$ (50)
(iii) Rs. 3-2-8 (Rs. 3)	(vii) 62·5 (60)
(iv) £19-15-0 (£20)	(viii) 95·55 (100)

2. Taking $\pi = 3{\cdot}1415963$ as the correct value, show that the relative error is about 0·0005 if it is taken as 3·14, and about 0·0004 if it is taken as $3\frac{1}{7}$.

3. Calculate the maximum absolute and relative errors in the following cases :

(i) The distance between Egmore and Kumbakonam is 186 miles to the nearest mile.

(ii) The length of a football field is 120 yd. to the nearest yard.

4. A piece of gold weighs 435·7 centigrams. If the error may be as large as 0·1 centigram (i.e. 1 milligram), what is the maximum relative error ?

60. PERCENTAGE ERROR

1. Introduction

When an experiment is conducted by different persons, the results often differ from one another. That result which is subject to the least error is the most reliable and useful. The comparison is, of course, made on the basis of relative errors.

Example. A certain important quantity, which we shall call g, is known to be 981 cm. in the metric scale and 32·2 ft. in the English scale. One boy A measures it as 975 cm., and another boy B as 31·8 ft. Which of the two results is the more accurate ?

Here the absolute errors are 6 cm. and 0·4 ft.

The relative error of A $= \frac{6}{981}$; that of B is $\frac{0{\cdot}4}{32{\cdot}2}$.

These fractions can be compared by reducing them to percentages.

$$\frac{6}{981} = \frac{6 \times 100}{981}\,\% \simeq 0{\cdot}6\,\%, \quad \frac{0{\cdot}4}{32{\cdot}2} = \frac{0{\cdot}4 \times 100}{32{\cdot}2}\,\% \simeq 1{\cdot}2\,\%.$$

The first boy's error is less, and hence his result is the more accurate.

2. Exercises

1. Find the percentage errors if, for convenience, the numbers enclosed within brackets are taken for the numbers before them:

(i) 19 (20)
(ii) 105 (100)
(iii) 95·8 (100)
(iv) 845·5 (850)
(v) Rs. 4-15-0 (Rs. 5)
(vi) 2 tons 560 lb. (3 tons)
(vii) 20 yd. 2 ft. (20 yd.)
(viii) 2·54 cm. (2·5 cm.)

2. Find the percentage errors in the following cases:

(i) 1 maund = 24·64 lb. taken as 25 lb.
(ii) 1 cm. = 0·394 in. taken as 0·4 in. nearly
(iii) 1 metre = 39·37011 inches, taken as 40 inches
(iv) 1 metre = 1·0936 yd., taken as 1 metre = 1·09 yd.
(v) 1 kilogram = 0·001968 lb. expressed as 0·0020 lb. correct to two significant figures
(vi) 1 year = 365·2422 days taken as $365\frac{1}{4}$ days.

61. CERTAIN USEFUL CASES OF APPROXIMATION

1. In scientific work, we frequently meet with quantities which undergo very slight changes; e.g. the length of an iron rod when heated. Certain types of problems involving such quantities can be solved with the help of a few principles of approximation.

If $x = 0{\cdot}002$, then $x^2 = 0{\cdot}000004$, $x^3 = 0{\cdot}000000008$. Note how rapidly the powers of x are decreasing in size.

Again, if $x = 0{\cdot}0002$, $y = 0{\cdot}00003$, then $xy = 0{\cdot}000000006$. This shows that if x and y are small, then the product xy is far smaller than either of them.

Example 1. Consider the equality $(1 + x)^2 = 1 + 2x + x^2$. If x is very small compared with 1, (e.g. if $x = 0{\cdot}0004$)

then
$$(1 + x)^2 = 1 + 2 \times 0{\cdot}0004 + (0{\cdot}0004)^2$$
$$= 1 + 0{\cdot}0008 + 0{\cdot}00000016$$
$$= 1{\cdot}00080016 \;.\;.\;.\;.\;. \qquad (1)$$

Now in x^2, i.e. $(0{\cdot}0004)^2$, the first significant figure occurs in the 7th decimal place. Its omission does not affect the result (1) earlier than in the 7th place.

Hence if we write $(1 + x)^2 = 1 + 2x = 1{\cdot}0008$, the approximation agrees with the actual value (1) to 6 decimal figures.

Thus if x is very small as compared with 1,
$$(1 + x)^2 = 1 + 2x \text{ approximately.}$$
Similarly it can be proved that
$$(1 - x)^2 = 1 - 2x \text{ approximately.}$$

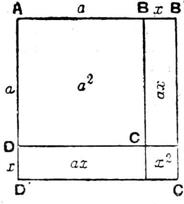
Example 2. A square piece of metal, side a in., expands when heated into a square of side $(a + x)$ in. Find the increase in the area.

The area at first $= a^2$ sq. in.
The area after expansion $= (a + x)^2$ sq. in.
$= a^2 + 2ax + x^2$ sq. in.

Let ABCD be the original square plate, which after expansion becomes AB′C′D′. The areas of the four parts are marked in them. If BB′ is very small, then the square CC′ is so small when compared with the whole area that neglecting it will not cause much error in the final result.

Thus if x is small as compared with a, then x^2 is much smaller as compared with $a^2 + 2ax$.

A good approximation to the new area $(a + x)^2$ is then $a^2 + 2ax$.



2. Exercises

1. When x and y are very small compared with 1, obtain the following approximations :

(i) $(1 + x)(1 + y) = 1 + x + y$
(ii) $(1 + x)(1 - y) = 1 + x - y$
(iii) $(1 - x)(1 + y) = 1 - x + y$
(iv) $(1 - x)(1 - y) = 1 - x - y$

2. Find approximate values for the following, and state to how many places the approximations agree with the actual values :

(i) $(1{\cdot}0002)^2$
(ii) $(1{\cdot}00012)^2$
(iii) $(0{\cdot}998)^2$
(iv) $(0{\cdot}9992)^2$
(v) $0{\cdot}9997 \times 1{\cdot}0005$
(vi) $1{\cdot}0003 \times 0{\cdot}9995$
(vii) $0{\cdot}9993 \times 0{\cdot}9998$
(viii) $1{\cdot}0008 \times 0{\cdot}9988$

3. Find the value of $(8{\cdot}00012)^2$ correct to 4 decimal places.

4. A boy had to calculate the value of $(a + x)^2$ when $a = 10$, $x = 0{\cdot}004$. He found out the value of $a^2 + 2ax$ and took it as sufficient for his purpose. How far does this agree with the correct value ?

5. Show that $(1 + x)(1 - x) \simeq 1$, if x is a very small proper fraction.

6. Show that, when x, y, z are very small fractions, $(1 + x)(1 + y)(1 + z) \simeq 1 + x + y + z$.

7. Find an approximation to 4 decimal places to each of the products:

(i) $1{\cdot}0003 \times 1{\cdot}0004 \times 1{\cdot}0012$

(ii) $1{\cdot}0003 \times 1{\cdot}0007 \times 1{\cdot}0014$

8. In taking $(1{\cdot}00015)^3$ as $1 + 3 \times (0{\cdot}00015)$, find to how many decimal places the result agrees with the actual value.

9. Prove that when x is small,

(i) $(1 + x)^3 \simeq 1 + 3x$

(ii) $(1 - x)^3 \simeq 1 - 3x$

10. If the length and breadth of a rectangle are l and b feet respectively, and they are each increased x per cent, x being small, show that the area is increased $2x$ per cent approximately.

11. If x and y are very small when compared with 1, show how to obtain the following approximations :

(i) $\dfrac{1}{1 + x} = 1 - x$ (ii) $\dfrac{1}{1 - x} = 1 + x$

(iii) $\dfrac{1 - x}{1 + x} = (1 - x)(1 - x) \doteq 1 - 2x$

(iv) $\dfrac{1 + x}{1 - x} = (1 + x)(1 + x) = 1 + 2x$

(v) $\dfrac{1 - x}{1 + y} = (1 - x)(1 - y) = 1 - x - y$

(vi) $\dfrac{1 + x}{1 - y} = (1 + x)(1 + y) = 1 + x + y$

12. Establish the following approximations :

(i) $\dfrac{1}{1{\cdot}015} = 0{\cdot}985$ (ii) $\dfrac{1}{1{\cdot}00025} = 0{\cdot}99975$

13. Find approximately the values of :

(i) $\dfrac{1}{(1{\cdot}002)^2}$ (ii) $\dfrac{1}{(1{\cdot}008)^3}$

62. QUESTIONS ON PRINCIPLES

1. Describe the various practical methods of finding the length of the circumference of a circle. Note down the defects of each method.

2. State the ratio of (i) the circumference of a circle to its diameter, (ii) the radius of a circle to its circumference.

3. Define *similar figures*. What tests would you adopt to determine whether two given figures are similar to each other ?

4. Two squares have their sides in the ratio $s : t$. Write down the ratio of (i) their perimeters, (ii) their areas.

5. The sides of a rectangle are changed in the ratio $1 : x$. In what ratio is (i) its perimeter, (ii) its area, altered ?

6. The areas of two squares are as P : Q. What is the ratio of their sides ?

7. What is meant by the vertical direction at any point on or near the earth's surface ?

8. How many horizontal lines can be drawn through a point in space ? Where do these lines lie ?

9. Describe, with the help of a suitable diagram, how you can find the height of a distant tower.

10. Describe how you will find the distance between two distant inapproachable objects? Draw figures to illustrate your method.

11. What do you understand by the *ambiguous case* in the construction of triangles ? Give three examples in illustration.

12. Write down some of the circumstances in which merchants offer discounts in transactions ? On what sums are discounts calculated ?

13. What do you understand by saying that a dilute acid is 45 per cent strong ?

14. If m seers of ghee and n seers of coco-nut oil are mixed together, what percentage of the mixture is each liquid ?

15. How much per cent of a gold sovereign is pure gold ?

16. How many conditions are necessary and sufficient for the construction of a quadrilateral ?

17. You are required to prepare a copy of a quadrilateral ABCD. Give three alternative sets of necessary and sufficient measurements which you would take to prepare a copy.

N.B. The measurements should involve different combinations of the elements of the quadrilateral.

18. Write down a formula for finding the area of a quadrilateral. State the quantities for which the letters involved in it stand.

19. Define (i) a trapezium, (ii) an isosceles trapezium. Write down two fundamental properties of an isosceles trapezium.

20. Write down a formula for the area of a trapezium. State the quantities for which the letters involved in it stand.

21. Write out the products :

(i) $(a + b)(c + d)$ (iii) $(a + b)(c - d)$

(ii) $(a - b)(c - d)$ (iv) $(a - b)(c + d)$

22. Write down the theorem of Pythagoras.

23. The three sides of a ΔABC ($\angle A = 90°$) are a, b, c inches long. Express each side in terms of the other two.

24. In how many different ways can you find practically the square root of 8? Draw a diagram to illustrate each method.

25. Write down the formula for finding:

(i) the diagonal of a rectangle when the lengths of the sides are given;

(ii) the diagonal of a square in terms of its side;

(iii) the hypotenuse of a right-angled isosceles triangle;

(iv) the altitude of an equilateral triangle in terms of its base;

(v) the area of an equilateral triangle in terms of one of the sides.

26. Write down the formula for the area of a triangle in terms of its three sides.

27. Two chords of a circle are equal. Write down your inference about (i) the arcs they cut off, (ii) the angles they subtend at the centre.

28. Two arcs of a circle are equal. Write down your inference about (i) the corresponding chords, (ii) the angles they subtend at the centre.

29. Write down an important geometrical theorem connected with (i) the perpendicular from the centre of a circle to a chord of the circle (not passing through the centre);

(ii) the straight line joining the centre of a circle to the middle point of a chord of the circle;

(iii) the right bisector of a chord of a circle (which is not a diameter).

30. A chord $2c$ inches long is placed in a circle of radius r inches; its distance from the centre is d inches. Express

(i) r in terms of c and d;

(ii) d in terms of c and r;

(iii) c in terms of d and r.

31. What do you understand by an angle in a segment of a circle?

32. Make a definite statement about a geometric property of

(i) an angle in a semicircle;

(ii) a number of angles in the same segment of a circle.

33. What is a regular polygon? Explain the need for each condition you mention in the answer.

34. Give (i) in right angles, (ii) in degrees, the size of each interior angle of a regular hexagon.

35. Draw a figure to show how you can build up a regular hexagon with equilateral triangles.

36. Write down a formula for the area of a regular hexagon.

37. Write down a formula for (i) the lateral surface of a prism, (ii) the total surface of a prism. Define the letters that occur in them.

38. Write down the formula for the area of a circle in terms of (i) its radius, (ii) its diameter.

39. Given that $\pi r^2 = A$, express r in terms of A.

40. Given that $\pi r^2 = A$, $C = 2\pi r$, obtain (i) A in terms of C, (ii) C in terms of A.

41. Two circles have their radii in the ratio $a : b$. Write down the ratio of (i) their circumferences, (ii) their areas.

42. The areas of two circles are in the ratio M : N. Write down the ratio of (i) their radii, (ii) their diameters, (iii) their circumferences.

43. In a circle of radius r inches, a sector whose central angle is $x°$ is marked off. Write down a formula for the area of the sector.

44. State the formulae for the curved surface of a cylinder and the total surface of a cylinder. Explain the meanings of the symbols employed in them.

45. What is the formula for the volume of (i) a cuboid, (ii) a cube, in terms of their edges?

Derive the formula $V = Ah$, for the volume of a rectangular block.

46. A rectangular block has its dimensions increased in the ratio (i) 1 : 3, (ii) $x : y$. In what ratio is its volume changed?

47. Two cubes have their edges in the ratio $h : k$. Write down the ratio (i) of their total surfaces, (ii) of their volumes.

48. Write down the formula for the volume of a right circular cylinder of length h inches and radius of cross-section r inches.

49. State the cubic contents of the material used in making a cylindrical tube of external and internal radii of section R and r inches respectively and length l inches.

50. A right circular cone has the following measurements for its elements :—radius of base $= r$ inches, axial (vertical) height $= h$ inches, slant height $= l$ inches.

(i) Express l in terms of h and r.
(ii) ,, h in terms of l and r.
(iii) ,, r in terms of l and h.

51. With reference to the cone of the last question, write down the formula for

(i) the area of the curved surface ;
(ii) the area of the total surface ;
(iii) the volume of the cone.

52. Define a *tangent* to a circle. How many tangents can be drawn to a circle (i) at a point on its circumference, (ii) from a point outside the circle ?

53. Obtain a formula for the length of a tangent to a circle drawn from an external point. On what does the length of the tangent depend ?

54. When are two circles said to touch each other ? What is the relation between the radii of two circles and the distance between their centres in order that they may touch each other ?

55. Explain with the help of examples the meaning of (i) absolute error, (ii) relative error, (iii) percentage error.

63. EXAMINATION PAPERS, SERIES C

[BASED ON THE SUBJECT-MATTER OF BOOK VI]

Paper I–A

1. The formula for the total surface of a cube is $S = 6a^2$. Express a in terms of S. Find a to two places of decimals if $S = 2{\cdot}25$ (units understood).

2. Write down the formulae for (i) the area of a sector of a circle, (ii) the volume of a right circular cone. State the quantities for which the letters employed in each formula stand.

3. Two circles whose radii are a and b units long have their centres d units apart. Write down the relation between a, b and d in order that the two circles may cut each other in two distinct points.

4. A circular running-track is of internal diameter d yards and the width of the track is w yards. Show that the area of the track is $\pi w\ (d + w)$ sq. yd.

5. If h in. is the height of an equilateral triangle, show that its side $= \frac{2h}{\sqrt{3}}$ in. $= 1 \cdot 15h$ in. approximately.

6. Two chords of a circle are equal. Write down your inferences about (i) the arcs they cut off, (ii) the angles they subtend at the centre.

7. An alloy is prepared with x oz. of copper, y oz. of tin and z oz. of zinc. Obtain the percentage of each metal in the alloy.

8. A regular hexagon is inscribed in a circle of radius r inches. What fraction of the area of the circle is that of the hexagon ?

9. Explain with the help of a diagram how you would measure the height of a tower standing on level ground, but not accessible.

Paper I–B

10. If a piece of iron plate 2·1 metres long, 1·13 metres broad and 2·3 cm. thick weighs 425 kilograms, find to the nearest gram the weight of a square metre of sheet iron 2 mm. thick.

11. Two kinds of tea worth Re.1-4-0 and 12as. per lb. respectively are mixed. The mixture is sold at Re. 1-2-0 per lb., giving a profit of 26 per cent. In what proportion are the teas mixed ?

12. A tradesman marks the price of his goods 12 per cent above the cost price and allows his customers 5 per cent discount. Find the cost price of the goods sold by him, if his personal profits amount to Rs.5000 after paying Rs.1400 as wages and rent.

13. At some school sports the mile race is run on a perfectly circular course, the track being 8 yards wide. Four times round the course on the inner edge is an exact mile. What is the length of the outer track ?

14. The foot of a ladder 30 ft. long, is 14 ft. from a wall and its top reaches the upper edge of a circular window in the wall. When the foot is drawn away to a distance of 17 ft. from the wall, the top reaches the lower edge of the window. What is the diameter of the window ?

15. A circus tent is in the form of a cylinder surmounted by a cone. The cylindrical portion of the tent is of diameter 96 ft., the height of the top of the tent is 36 ft. from the ground, and the height of the cylindrical part is 16 ft. Find the total area of the canvas required for the tent. Answer to the nearest square foot. Take $\pi = 3 \cdot 1416$. [*S.S.L.C. 1921*

Paper II–A

1. The lengths and breadths of two rectangles are respectively l_1, l_2 and b_1, b_2. Write down the relation between these dimensions in order that the rectangles may be similar.

2. The angle of a sector is $\frac{2}{3}$ of a right angle. What fraction of the area of the circle is the area of the sector?

3. Write down the formula for the area of a triangle in terms of the sides. Use it to find the area of a triangle whose sides are 15, 12, 9 inches respectively.

4. Two concentric circles have their radii a and b inches long. What is the area of the ring enclosed between the two? Find it, if $a = 2{\cdot}5$, $b = 1{\cdot}6$.

5. Two cylinders have cross-sections whose areas are in the ratio $A_1 : A_2$, and their heights are as $h_1 : h_2$. State the ratio of their volumes.

6. When x is very small when compared with unity, show that $1 - x$ is an approximation to the fraction $1 \div (1 + x)$.

7. Two arcs of a circle are equal. Write down your inference about (i) the corresponding chords, (ii) the angles they subtend at the centre.

8. Assuming a metre to be $39\frac{3}{8}$ inches, find the nearest whole number of litres in a cubic foot.

9. A triangle can be described if the length of a side and the angles at its ends are known. Explain how this principle is made use of in finding the height of a tower.

Paper II–B

10. The breadth of a circular race course is 33 yards. The inner boundary of the course is 11 furlongs. How long does a horse take to run round the outer boundary, if it gallops at the rate of 16 miles an hour?

11. A vessel is filled with a liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

12. A canal is 32 ft. wide at the top and 16 ft. wide at the bottom, and 10 ft. deep. If water is flowing full in the canal at 8 miles per hour, find the quantity of water that will flow past any cross-section per minute. Answer in cubic feet.

13. A bicycle agent allows 25 per cent discount on his advertised prices, and makes a profit of 20 per cent on his outlay. What is the advertised price of a machine on which he gains £3?

14. A cistern in the form of a rectangular block, whose base is 2 ft. 6 in. long and 1 ft. 4 in. broad, is partly filled with water. If a rectangular block of metal 1 ft. 6 in. long, 1 ft. broad and 10 in. thick is wholly immersed in the water, through what height will the water-level be raised?

15. A cylindrical granary with a conical top is 14 ft. in diameter on the inside. The wall is 11 ft. high, and the total height to the top is 14 ft. It was full of grain. Half of it was sold at Rs.5-12-0 a bag, and the other half at Rs.5-4-0 a bag. Find the amount realized, given that a bag of grain measures 8·25 c. ft.

[*S.S.L.C. 1918*

Paper III–A

1. The areas of two circles are respectively 4 and 8 acres. Write down the ratio of their diameters.

2. Write down the formula for the total surface of a cylinder. Indicate the magnitudes for which the letters used in it stand.

3. O is the centre of a circle of radius r, and P is a point outside the circle at a distance d from the centre (units understood). Find the length of either tangent drawn from P to the circle.

4. Given that $\pi r^2 = A$ and $2\pi r = C$, obtain the relation between A and C.

5. AB is a diameter of a circle whose centre is C. The circle is folded so that the points A and B are brought together at the centre C, thus leaving two creases. Show that the points A, B and the four ends of the two creases are the corners of a regular hexagon.

6. Obtain by a geometrical construction the square root of 11 to two places of decimals. Check by calculation.

7. What is the percentage of error in taking $\pi = 3\frac{1}{7}$ instead of $\pi = 3{\cdot}14159$?

8. Find the cost of the smallest square sheet of metal costing 2*s*. 6*d*. per square foot, from which a circular piece of metal 20 sq. ft. can be cut out.

9. Describe, with the aid of a diagram, a method of finding the distance between two objects which are not accessible.

Paper III–B

10. An inch is 2·54 cm. and a kilogram is 2·2 lb. Find the pressure of the atmosphere in grams per square centimetre, supposing it to be 15 lb. per sq. in.

11. A milkwoman buys milk, and adds water to it. She sells the mixture at the same price per seer as she paid for the pure milk, and makes a profit of 25 per cent. What percentage of water does the dilute milk contain?

12. A tradesman marks his goods 12 per cent above his cost, and for cash payment allows a discount of Re.1-5-0 on a bill of Rs. 21. Find his net profit per cent.

13. ABCD is a quadrilateral field in which the diagonals AC, BD are at right angles. AB, BC, CD and AC are 600, 700, 800 and 1000 links respectively. Draw the quadrilateral to scale, and hence find the area of the field in acres. [*S.S.L.C. 1925*

14. A boy fills a cylindrical bucket, which measures internally 12 inches in diameter and 16 inches in height, with moist sand. He empties the bucket and builds a conical mound with the sand on a base whose base is 18 inches. Find the height and volume of the mound, and the area of its curved surface. [*S.S.L.C. 1923*

15. Construct a graph which will enable you to convert speeds given in miles per hour into yards per minute, and vice versa. Use it to obtain (i) the equivalent to 40 m.p.h. in yards per minute, (ii) the equivalent in m.p.h. to 66 yd. per min.

64. SPECIMEN PAPERS ISSUED BY THE DEPARTMENT

SECONDARY SCHOOL LEAVING CERTIFICATE
PUBLIC EXAMINATION

ELEMENTARY MATHEMATICS

TIME : $2\frac{1}{2}$ HOURS

Part–A **Marks**

1. If a and b are two numbers will their difference be affected if both are increased or decreased by the same number? 1 + 1

2. Draw a diagram to illustrate the equality
$(a + b)(c + d) = ac + ad + bc + bd.$ 2

Enter the necessary particulars in the diagram to bring out the equality.

3. Say whether the value of a common fraction is altered or not when *both* its numerator and denominator are

(i) increased, (iii) multiplied, $\frac{1}{2} + \frac{1}{2}$
(ii) decreased, (iv) divided, $\frac{1}{2} + \frac{1}{2}$

by the same number.

4. Given $a : b = 3 : 5$, $b : c = 15 : 7$, express the proportion $a : b : c$, using the smallest whole numbers for the terms. 3

5. If C is the cost price of an article, S the selling price (units understood), and the loss is 15 per cent, by what fraction should you multiply S to obtain C? $1\frac{1}{2}$

6. The speed of a train is given as 60 miles per hour. Express it (i) in yards per minute, (ii) in feet per second. $1\frac{1}{2}$

7. State the formulae used in the calculation of

(i) Simple Interest, 1

(ii) Area of a trapezium, 1

(iii) Volume of a cone. 1

In each formula, state the quantities for which the letters stand.

8. The average age of m boys is x years and the average of n of them is y years. What is the average age of the remaining number? 3

9. (i) Given $v = u + at$, express t in terms of u, v and a. $2\frac{1}{2}$

(ii) From the formula $A = \frac{22}{7} r^2$, obtain r in terms of A. 2

10. The diameters of two solid cylindrical rollers of equal volumes are d_1, d_2; their lengths are h_1, h_2 (units understood). State the relation between d_1, d_2, h_1, h_2, in its simplest form. 3

11. Express symbolically

(i) Angle X is the complement of angle Y.

(ii) A sq. ft. is the area of a pathway p ft. wide which surrounds (on the outside) a lawn l ft. long, b ft. wide. 2

12. You are required to prepare a copy of a triangle ABC.

Give three alternative sets of necessary and sufficient measurements which you would take to prepare a copy. 3

[*N.B.* The measurements should involve different combinations of the elements of the triangle.]

Part–B

13. A train runs up an incline at 24 miles per hour, and returns at 36 miles per hour. What is the average speed of the train? 6

14. A dealer in motor cars bought a car for Rs.3200. He wanted to make a profit of 35 per cent on his outlay, after allowing a discount of 10 per cent on his advertised price. At what price should he advertise the car? 9

15. The population of a district according to the censuses of 1901, 1911, 1921 was respectively 115000, 128800, 144256. Find the law of increase. If that law continued to operate, what would have been the population in 1931? 3

16. How many yards of fencing would be required to enclose a square field of area 18·225 acres? 8

17. Find the discharge per minute in gallons from a full cylindrical pipe of 6 inches diameter, the mean velocity being 12 inches per second. (1 gallon = 277·25 cubic inches.) 10

18. Explain with the help of a diagram how you would measure the height of a tower standing on level ground, but not accessible. 6

19. A jar contains 4 ounces of dilute sulphuric acid 40 per cent strong. How much water should be added to it to reduce the strength by 8 per cent? 9

20. The temperature of a substance is C° on the Centigrade scale, and F° on the Fahrenheit scale. The relation between F and C is given by $F = \frac{9}{5}C + 32$.

Draw a graph from which you can read equivalent temperatures on the two scales. Use it to find the temperature on the Fahrenheit scale corresponding to 87°C, and the equivalent on the Centigrade scale for 156° F. 2 + 2

Secondary School Leaving Certificate Public Examination 1934

ELEMENTARY MATHEMATICS

TIME: $2\frac{1}{2}$ HOURS

Part–A

Marks

1. a, b, c are three integers, c divides $a + b$ without remainder; will c divide a or b without remainder? State whether 337 and 663 have common factors or not. 1 + 1

2. Equal numbers are added to the numerator and denominator of a proper fraction. State whether the fraction is increased or decreased by the equal additions. 1 + 1

Write down the vulgar fraction equivalent to the decimal $0{\cdot}0\dot{4}5\dot{0}$.

3. If the first of three numbers be to the second in the ratio of 2 : 5 and the second to the third in the ratio of 15 : 2, what is the ratio of the first number to the third? 3

4. Give rules for rapid mental calculation in the following cases :

(i) The cost of 8250 bricks at Rs.5-10-8 per thousand.

(ii) Simple interest on Rs.192 for 7 months at $6\frac{1}{4}$ per cent per annum. 2 + 2

5. A train going 30 miles per hour passes a telegraph post in 4 seconds. What is the length of the train ? 3

6 Draw a diagram to illustrate the identity

$$(a + b)(a + c) = ab + ac + b^2 + bc$$ 3

7. The average cost of 20 horses is 30 guineas, and 8 horses cost £30 each. What is the average cost of the remainder ? 3

8. S is the silver price of one gold sovereign. If the price of silver has risen 5 per cent, by what fraction should you multiply S to obtain the fall in the silver price of one sovereign ? 3

9. What are the formulae used in the calculation of :

(i) the area of the four walls of a rectangular room, 1

(ii) the volume of a right cylinder, 1

(iii) the curved surface of a right circular cone ? 1

10. (i) Given $ax - b = px - q$, find x in terms of a, b, p, q. 2

(ii) From the formula $t = \frac{44}{7}\sqrt{\frac{l}{g}}$, express g in terms of l and t. 2

11. (i) State the number of degrees in an angle of a regular hexagon. 1

(ii) Express the area of a pathway a feet wide which surrounds (on the inside) a semicircular lawn of diameter d feet, in terms of a and d. 2

12. The area of a square field is $\frac{1}{10}$ acre. Draw a plan of the field taking one centimetre to represent 11 ft., and measure the length of a diagonal. 4 + 3 + 2

Part–B

13. A boat goes down a river a certain distance and returns up the river to the starting point. If the speed of the boat in still water is 6 miles per hour and the rate of the current is 2 miles per hour, find the average speed of the boat. 11

14. A man buys a cycle. When he has ridden a certain distance, he reckons that the cycle has cost him $2\frac{1}{2}$ annas per mile. He rides 252 miles farther and finds that it has cost him 2 annas a mile. How much farther must he ride to reduce the cost to $1\frac{1}{2}$ annas per mile ? 11

15. A certain sum of money lent out at compound interest amounts to Rs. 551-4 in two years and to Rs. 578-13 in three years. Find the rate of interest and the sum lent. 5+7

16. A gallon of spirit weighs $\frac{1}{4}$ less than a gallon of water. How much water must be added to 4 gallons of the spirit till a gallon of the mixture weighs $\frac{1}{20}$ more than a gallon of spirit? 11

17. A merchant sells all his articles so as to gain 33 per cent on his outlay, after allowing a discount of 5 per cent on his advertised prices.

Draw a graph from which you can read the cost price and the advertised price of any article. Use it to find the cost price of an article whose advertised price is Rs. 350, and the advertised price of an article whose cost price is Rs. 280. 9+2+2

SECONDARY SCHOOL LEAVING CERTIFICATE PUBLIC EXAMINATION 1935

TIME: $2\frac{1}{2}$ HOURS

Part–A **Marks**

1. Will the values of the fractions $\frac{2}{3}$ and $\frac{5}{4}$ be increased or decreased when they are squared? 1+1

2. (i) How many feet per second, (ii) how many yards per minute, will be equivalent to x miles per hour? 1+1

3. (i) Draw a diagram to illustrate the equality

$$(a + b)\ (x + y) = ax + ay + bx + by.$$ 2

Enter the necessary particulars in the diagram to bring out the equality.

(ii) What simpler form will the equality take if $a = x$, $b = y$? 1

4. (i) For what purpose is the formula $V = \pi r^2 h$ used? What quantities are represented by V, r, h? 1

Express r in terms of V and h. 2

(ii) What formulae are used in the calculation of (*a*) amount at simple interest, (*b*) total surface of a cone? $1\frac{1}{2}+2$
In each formula, state the quantities for which the letters stand.

5. The length of a rectangle is doubled, while its breadth is changed to $\frac{2}{3}$ of itself.

(i) State whether the area is increased or decreased. 2

(ii) If the original area is 240 square yards, what is the new area? 2

6. In a 100 yards race, A gives B a start of 10 yards and yet beats him by 5 yards. State the ratio of the speeds of A and B in its simplest form. 3

7. A cricketer's average score for ten matches was 52 runs. In the last of those ten matches his score was zero. If, instead of that, he had scored 100 runs in that last match, what would have been his average score? $2\frac{1}{2}$

8. A bankrupt could pay his creditors 2 shillings in the pound. Find the loss per cent the creditors suffered on that account. $1\frac{1}{2}$

9. (i) A sum of money lent at simple interest doubles itself in ten years. How many times itself will it become in twenty years? $1\frac{1}{2}$

(ii) Also, answer the question if compound interest is allowed. 2

10. Aryabhatta (a Hindu mathematician) took $\pi = 3\frac{17}{120}$. To how many significant figures will its decimal equivalent agree with the correct value $\pi = 3{\cdot}1415963\ldots$? $1\frac{1}{2}+1\frac{1}{2}$

11. OA, OB are radii of a circle. If the angle between them is 100 degrees, at what angle will the tangents to the circle at A and B meet? 2

12. The side of a square sheet of metal is 7 inches. The greatest possible circular piece is cut out of it. What is the area of the remaining piece? Take $\pi = 3\frac{1}{7}$. 2

Part–B

13. On a certain Monday the sun rose at 6–33–20 a.m., and set at 5–52–45 p.m. The following Friday the sun rose at 6–36–38 a.m., and set at 5–57–14 p.m.

(i) Were the days lengthening or shortening during the period? 5

(ii) What was the average rate of change per day? Give your answer correct to a tenth of a second. 2

14. The sum of two angles of a triangle is 110 degrees; their difference is 30 degrees

(i) Find the three angles. 3

(ii) What kind of triangle is it? 1

(iii) If it is known that one of its three sides is 2 inches long, show that two triangles of the kind can be drawn. Give reasons. 2

(iv) Draw a neat sketch of each triangle, and enter the known measurements in it. 2

15. Given that 8 kilometres = 5 miles, draw a graph with the help of which you can convert miles into kilometres, and vice versa. 6

Use the graph to find the equivalent of 2·4 miles in kilometres (correct to one decimal place), and the equivalent of 6·7 kilometres in miles (correct to one decimal place). $1\frac{1}{2}$

Verify your answers by calculation. $1\frac{1}{2}$

16. Every year the value of a machine goes down by 20 per cent of its value at the beginning of the year. A person bought a machine, and after three years sold it for Rs.3360. At what price did he purchase it? 8

17. A train is due to start from a station in 2 minutes time. A cylindrical boiler in it is 6 feet long and 3 feet in diameter (internally). Water is let into it through a cylindrical pipe of diameter 2 inches. The velocity of flow is 12 miles per hour. Find (i) the quantity of water let into the boiler per minute; (ii) how soon before the train starts the boiler will be filled. Take $\pi = 3\frac{1}{7}$. 5+6

18. The length of a rectangular field is four times its breadth. The rent of the field, at Rs. 6-4-0 per acre, comes to Rs. 250. What will be the cost of putting up a fence round it at a cost of Rs.15-12-0 per 100 feet? 7+4

19. A dish contains a 6 per cent solution of salt and water (a sample of sea-water, of which 6 per cent by weight is salt). What percentage of evaporation must take place to convert what remains into an 8 per cent solution? 11

65. IMPORTANT FORMULAE

A. Pure and applied arithmetic

1. *Trade: Profit and loss.* If Rs.C is the cost price, Rs.S the selling price, and r the rate of profit or loss per cent, then

(i) Profit or loss $= \frac{r}{100} C$

(ii) $S = \frac{100 \pm r}{100} C$ (iii) $C = \frac{100}{100 \pm r} S$

2. *Simple Interest*

If Rs.P is the principal, Rs.I the interest for n years at r per cent per annum, then

$$I = \frac{Pnr}{100}$$

If Rs.A is the corresponding amount, then

$$A = P + \frac{Pnr}{100} = P\left(1 + \frac{nr}{100}\right)$$

3. *Compound Interest*

If Rs.P is the principal, Rs.A the amount in n years at r per cent per annum, then

$$A = P\left(1 + \frac{r}{100}\right)^n = P\left(\frac{100 + r}{100}\right)^n$$

4. *Average*

If A is the average value of n numbers $a_1, a_2, \ldots a_n$, then

$$A = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

B. Mensuration (*lengths*)

1. *Circle*

If r in. is the radius, d in. the diameter, c in. the circumference, then

$$c = 2\pi r = \pi d$$

2. *Rectangle*

If l in. is the length, b in. the breadth, p in. the perimeter, then

$$p = 2(l + b)$$

If d in. is the length of either diagonal, then

$$d = \sqrt{l^2 + b^2}$$

3. *Square*

If s in. is the side, p in. the perimeter, then

$$p = 4s$$

If d in. is the diagonal, then

$$d = s\sqrt{2} = \sqrt{2} \times s \simeq 1{\cdot}414 \times s$$

4. *Right-angled isosceles triangle*

If a in. is the length of either of the equal sides, h in. the hypotenuse, then

$$h = a\sqrt{2} = \sqrt{2} \times a$$

5. *Equilateral triangle*

If a in. is the length of a side, h in. the altitude, then

$$h = \frac{\sqrt{3}}{2} a \simeq 0{\cdot}866\, a$$

C. Mensuration (*areas*)

1. *Rectangle*

If l in. is the length and b in. the breadth and A sq. in. the area, then

$$A = l \times b = lb$$

2. *Parallelogram*

If l in. is the length of a side, h in. the distance between it and the opposite side, A sq. in. the area, then

$$A = l \times h = lh$$

[Area = base × height]

3. *Quadrilateral*

If d in. is the length of a diagonal, p_1 in., p_2 in. the perpendiculars on it from the (other) two vertices, A sq. in. the area, then

$$A = \tfrac{1}{2}d\,(p_1 + p_2)$$

4. *Rhombus*

If d_1 in. and d_2 in. be the lengths of the diagonals and A sq. in. the area, then

$$A = \tfrac{1}{2}d_1\, d_2$$

5. *Trapezium*

If a in. and b in., are the lengths of the parallel sides, h in. the distance between them, A sq. in. the area, then

$$A = \tfrac{1}{2}\,(a + b)\,h = \tfrac{1}{2}h\,(a + b)$$

6. *Triangle*

If a in. is the length of any side, h in. the corresponding altitude (height), A sq. in. the area, then

$$A = \tfrac{1}{2}\,ah$$

If a, b, c in. are the lengths of the sides, $2s = a + b + c$, then

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

7. *Equilateral triangle*

If a in. is the length of a side, A sq. in. the area, then

$$A = \frac{\sqrt{3}}{4} a^2 \simeq 0{\cdot}433a^2$$

8. *Regular hexagon*

If a in. is the side, A sq. in. the area, then

$$A = 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$$

9. *Circle*

If r in. is the radius, d in. the diameter, A sq. in. the area, then

$$A = \pi r^2 = \frac{\pi}{4} d^2$$

10. *Circular ring*

If R in. and r in. are the external and internal radii, A sq. in. the area, then

$$A = \pi (R^2 - r^2)$$

11. *Sector*

If l in. is the length of the arc, r in. the radius, A sq. in. the area, then

$$A = \tfrac{1}{2} lr$$

If $x°$ is the angle of the sector, then

$$A = \frac{x}{360} \text{ of the area of the circle}$$

$$= \frac{x}{360} \times \pi r^2$$

12. *Cuboid*

If l in., b in., h in. are the length, breadth and height, S sq. in. the total surface, then

$$S = 2(lb + bh + hl)$$

13. *Cube*

If l in. is the length of an edge, S sq. in. the total surface, then

$$S = 6l^2$$

14. *Walls of a room*

If l ft., b ft., h ft. are the dimensions, W sq. ft. the area of the four walls, then

$$W = 2h(l + b) = 2(l + b)h$$

15. *Right prism*

If p in. is the perimeter of the base (cross-section), h in. the height, L sq. in. the lateral surface, then

$$L = ph$$

16. *Cylinder* (right circular)

If r in. is the radius of cross-section, h in. the height, A sq. in. the area of the curved surface, then

$$A = 2\pi rh$$

$$\text{The total surface area} = 2\pi rh + 2\pi r^2$$
$$= 2\pi r(h + r)$$

17. *Cone* (right circular)

If h in. is the vertical (axial) height, l in. the slant height, r in. the radius of the base, c in. the circumference of the base, the area (A sq. in.) of the curved surface is given by

$$A = \tfrac{1}{2} cl$$
$$= \pi rl = \pi r\sqrt{h^2 + r^2}$$

The total surface (A sq. in.) is given by

$$A = \pi r^2 + \pi rl = \pi r\,(r + l)$$
$$= \pi r\,(r + \sqrt{r^2 + h^2})$$

D. Mensuration (*volumes*)

1. *Cuboid*

If l in., b in., h in. are the dimensions, V c. in. the volume, then

$$V = l \times b \times h = lbh$$

If A sq. in. = area of the face l in. by b in., then

$$V = Ah$$

2. *Cube*

If l in. is the length of any edge, V c.in. the volume, then

$$V = l^3$$

3. *Right prism*

If A sq. in. is the area of the base (cross-section), h in. the height, V c.in. the volume, then

$$V = Ah$$

4. *Cylinder* (right circular)

If r in. is the radius of the base (cross-section), h in. the height, V c. in., the volume, then

$$V = \pi r^2 h$$

5. *Cylindrical tube*

If R in. and r in. are the external and internal radii of the cross-section, h in. the height, the quantity of material V c.in. (volume) in the tube is given by

$$V = \pi h \,(R^2 - r^2)$$

6. *Cone* (right circular)

If r in. is the radius of the base, h in. the vertical height, V c.in. the volume, then

$$V = \tfrac{1}{3}\,\pi\, r^2 h$$

E. Algebra (generalized arithmetic)

1. $ap + aq = a\,(p + q)$
   $ap - aq = a\,(p - q)$
   $p\,(x + y + z) = px + py + pz$
2. $(a + b)\,(c + d) = ac + ad + bc + bd$
   $(a + b)\,(a + c) = a^2 + ab + ac + bc$
   $(a - b)\,(a - c) = a^2 - ab - ac + bc$
3. $(a + b)^2 = a^2 + 2ab + b^2$
   $(a - b)^2 = a^2 - 2ab + b^2$
4. $(a + b)\,(a - b) = a^2 - b^2$
   $x^2 - y^2 = (x + y)\,(x - y)$

ANSWERS

1. **Paper I.** **2.** Increased by the same amount. **3.** $\frac{100+x}{100}Q$ **4.** $10^3d+10^2c+10b+a$. **5.** $(x-y)(c+2d)$ **6.** (i) a^8, (ii) p^5, (iii) -1. **7.** (i) Not correct, (ii) correct, (iii) correct. **8.** 2·7 in **9.** 10. **10.** $x^2-11x+3$; $-x^2+11x-3$. **11.** $-\frac{xy}{20}$. **12.** $(0, 3\frac{3}{4})$. **13.** Rs. 288. **14.** 15 days 9 days. **15.** Rs. 6000. **16.** Rs. 6250. **17.** (i) $h=8+2m$ (h is the cost of hiring in annas, and m the number of miles ridden); (iii) 6 miles.

Paper II. **2.** Decreased by x measures. **3.** 8, 16. **5.** (i) Correct, (ii) not correct. **6.** 100°, 80°, 100° and 80°. **7.** $2n-1$ or $2n$ digits. **8.** (i) $b=\frac{c(a+1)}{a-1}$, (ii) $c=\frac{b(a-1)}{a+1}$. **10.** £1 loss. **11.** -30 to be added. **12.** (i) $2(a^2+b^2)$, (ii) $4ab$. **13.** 6·69 acres. **14.** 46851200. **15.** 2 years. **16.** $2\frac{1}{2}$ miles. **17.** (i) 71%, (ii) 25%, (iii) 5%.

Paper III. **1.** $+2$ and -2. **2.** $A=P\left(1+\frac{r}{100}\right)^n$. **4.** $C=\frac{5}{9}(F-32)$. **5.** (i) 101256^7, (ii) $-16a$. **6.** $\left(\frac{100-r}{100}\right)P$. **8.** $p°$, $180°-p°$, $180°-p°$. **9.** 30·18 in. **10.** $0\cdot3678\dot{8}$. **12.** $7x+38+\frac{143}{x-4}$. **13.** 149816. **14.** 10100. **15.** 16 $H=10+4m$; 7 miles; Rs. 2-4-0. **16.** 84 yd.

Paper IV. **2.** 9. **3.** $+3x^3, -4x^2, +5x, -9$; $-9+5x-4x^2+3x^3$, etc. **4.** 45 m.p.h. **6.** 3 8; 3·8. **7.** $m:n$. **8.** $r=s+nc$, $c=2\frac{1}{2}$. **9.** 21%. **11.** Rs.937-8-0. **12.** 5 86. **13.** 200288. **14.** 3 significant figures, 6 significant figures. **15.** Rs. 61-8-0. **16.** 200 ft., 160 ft. **17.** 20 as., 2 as. **18.** 5 hr. 6 min.

Paper V. **2.** 370·69. **4.** (i) $-(2b-2a)$, (ii) $-\left(\frac{3x-5}{2}\right)$. **6.** $-5x^3+7x^2-x-10$. **8.** (i) $v=\frac{ur}{r+2u}$, (ii) $u=\frac{vr}{r-2v}$. **9.** 2s. $3\frac{1}{2}d.$, nearly. **10.** 365 days 348 min. 48 sec. **11.** -25 **12.** 68°, 58°, 54°. **13.** 36 hr. **16.** 10%.

2—3. **1.** (i) 66 in., $138\frac{2}{7}$ ft., 462 yd., (ii) $15\frac{5}{7}$ in., $62\frac{6}{7}$ ft., $100\frac{4}{7}$yd., (iii) 88 cm., $314\frac{2}{7}$ m., (iv) $\frac{22a}{7}$ in., $\frac{22b}{7}$ ft., $\frac{22m}{7}$ miles. **2.** 44 in., 88 ft., $50\frac{2}{7}$ yd., $785\frac{5}{7}$ m. **3.** 528 in. **4.** 24900 miles. **5.** Rs. 1833-5-4. **6.** $578\frac{2}{7}$ miles. **7.** 336 yd. **8.** 2·72 fur. **9.** 6·1 ft. nearly. **10.** 720, 1440, 360, 206 nearly. **11.** 5·6 min. nearly.

3—2. **1.** 14″, 28′, 2·1 m., 3·1″, 11·1′, 11·8 m., etc. **2.** $3\frac{1}{2}$″, 14′, $38\frac{1}{2}$ cm., 9′, 31·8 yd., etc. **3.** 318 ft. **4.** 0·318, 14 in. nearly. **5.** 9·94 ft. **6.** 420 yd. **7.** $3\frac{1}{2}$ cm. **8.** $724\frac{1}{2}$ in. **9.** $1\frac{3}{4}$ ft. **10.** 6366·2 Km. **11.** 7912 miles.

4—3. **3.** (i) 3 : 7, (ii) 3 : 7. **4.** 14 in. **6.** (i) 4·5, (ii) $4\frac{1}{3}$. **7.** 4′2″ nearly. **8.** 18″.

4—5. **1.** (i) 1 : 4, (ii) 4 : 9, (iii) $a^2 = b^2$. **2.** (i) 4 : 1, (ii) 15 : 9, (iii) $x^2 = 1$, (iv) $1 = y^2$, (v) $p^2 = q^2$.

9—3. **1.** (i) Rs. 30, (ii) Rs. $\frac{6x}{5}$. **2.** (i) Rs. 22-8-0, (ii) Rs. 23-7-0. **3.** Rs. 3-1-0, 22·45%. **4.** The latter, $2\frac{5}{8}$%. **5.** The former more advantageous by 1·6%. **6.** $4\frac{16}{21}$%. **7.** $3\frac{1}{8}$% **8.** 14%. **9.** $18\frac{1}{3}$%. **10.** $112\frac{1}{2}$%. **11.** $12\frac{1}{2}$%. **12.** $\frac{100 - D}{100}C = P$. **13.** $\frac{100(a - b) - ab}{100}$ %.

10—2. **1.** Rs. 72-4-0. **2.** Rs. 3. **3.** Rs. $12\frac{1}{2}$. **4.** Rs. 69. **5.** $37\frac{1}{7}$%. **6.** Rs. $4\frac{1}{2}$. **7.** 10*s*. **8.** 20%, Rs. 200. **9.** $12\frac{1}{2}$%, Rs. 9. **10.** $17\frac{1}{3}$%. **11.** Rs. 75000. **12.** $4\frac{1}{6}$%.

11—2. **1.** $62\frac{1}{2}$%. **2.** 4·8 gal., 1·4 gal. **3.** (i) $\frac{100m}{m + n}$, (ii) $\frac{100n}{m + n}$. **4.** $\frac{100x}{x + y + z}$, $\frac{100y}{x + y + z}$, $\frac{100z}{x + y + z}$. **5.** Second richer by 0·23%. **6.** $91\frac{2}{3}$%, $8\frac{1}{3}$%. **7.** 80%. **8.** 9·1%. **9.** 31%. **10.** 90%, 72%.

12—2. **1.** 150. **2.** 14·8. **3.** 100 gal. **4.** 1 oz. **5.** $1\frac{1}{3}$ oz. **6.** $3\frac{27}{275}$. **7.** 9 gal. from first vessel ; 6 gal. from second. **8.** $2\frac{1}{2}$ oz. from first bottle ; $1\frac{1}{2}$ oz. from second.

13—3. **1.** 3·3% gain. **2.** 7·94%. **3.** 28·0%. **4.** 20% ; unaffected. **5.** 7·8 %. **6.** $13\frac{11}{13}$ gal **7.** $\frac{4}{35}$ sr. of water to every seer of milk. **8.** 8 measures. **9.** 8 gal. **10.** $9\frac{3}{13}$ lb. **11.** 8 gal. of each kind. **12.** 1 : 3. **13.** 5 : 3. **14.** 13 md. **15.** 1 : 2 ; $\frac{1}{3}$ and $\frac{2}{3}$ viss.

14. **1.** Rs. 59-6-0. **2.** $22\frac{2}{9}$%. **3.** $66\frac{2}{3}$% ; Rs. 1400. **4.** 91·52%. **5.** Rs. 3645. **6.** 17 tons. **7.** 40 : 60 ; 50%. **8.** Second method ; $\frac{2}{37}$ md. **9.** 6272 oz. **10.** 1 : 6. **11.** $\frac{1}{3}$. **12.** 95·8%, 4·2%, 92%, 8%. **13.** 81%. **14.** 401 : 544. **15.** 2 from the first and 12 from the second.

19—4. 4 acres 38 cents. **5.** 143 yd. **6.** 285 sq. in.

20—2. **2.** (i) $5(x+y)$, (ii) $7(x+2y)$, (iii) $5(x-y)$, (iv) $7(x-2y)$, (v) $-5a(1-2b)$, (vi) $-5(p+3q)$, (vii) $a(p+q)$, (viii) $s(r-3t)$, (ix) $-s(2r-3t)$, (x) $3(x+1)$, (xi) $3(x-1)$, (xii) $-3(x-1)$, (xiii) $-3(z+1)$, (xiv) $5(y+3)$, (xv) $4(a-2)$, (xvi) $x(x+1)$, (xvii) $x(x-1)$, (xviii) $-x(x+2)$, (xix) $m(1+n)$, (xx) $y(x-1)$, (xxi) $6b(y+z)$, (xxii) $xy(x-y)$, (xxiii) $3s(s-2x)$, (xxiv) $a^2(a-b)$, (xxv) $5a(5a^2-2b^2)$, (xxvi) $4xy(4x-9y)$, (xxvii) $-9x^2y^2(2y-3x)$. **3.** (i) $x(1+y+z)$, (ii) $b(1+2b+3b^2)$, (iii) $2b(2+3b+4b^2)$, (iv) $4m(3m^2-4m-5)$, (v) $-12a(6a^2-4ab+3c)$, (vi) $x(x^2+xy+y^2)$, (vii) $x(x^2-2xy+3y^2)$, (viii) $5y(4x^2-6xy+7y^2)$, (ix) $a^2b^2\times(6a+b-7)$, (x) $-8p^2q^2(3pq-4q^2+6p^2q)$. **4.** $2h(l+b)$.

5. $A = P\left(1+\frac{nr}{100}\right)$. **6.** $2\pi r(r+l)$.

20—4. **1.** (i) $(p+q)(a+b)$, (ii) $(a+b)(x+2y)$, (iii) $(p+r)\times(2m-3n+5s)$, (iv) $(x+y+z)(-3a+5b-15c)$, (v) $8(2a+3b-4c)\times(2x^2-3xy-4y^2)$. **2.** (i) $(a+b)(p-r+t)$, (ii) $(x+y+z)\times(a^2+b^2-c^2)$, (iii) $-5ab(a^2+b^2+c^2)(3a-4b)$, (iv) $12xyz(1+b+b^2)\times(3x-4y+5z)$.

21—2. **1.** (i) $(x+y)(a+1)$, (ii) $(a+b)(c+2)$, (iii) $(a+b)\times(c-1)$, (iv) $-4(p+q)$, (v) $(a-b)(a-1)$, (vi) $(b-a)(b-1)$. **2.** (i) $(c+d)(a+b)$, (ii) $(c+d)(a-b)$, (iii) $(m+n)(p-q)$, (iv) $(c-d)(a+b)$, (v) $(c-d)(a-b)$, (vi) $(a+c)(a+b)$, (vii) $(a-b)(a+c)$, (viii) $(a+b)(a-c)$, (ix) $(a-b)(a-c)$, (x) $(b+1)(a+1)$, (xi) $(a-2)(3+b)$, (xii) $(1+a)(1+c)$, (xiii) $(1-m)(2+n)$, (xiv) $(x+1)(x^2+1)$, (xv) $(x+2)(x^2+1)$, (xvi) $(x-1)(x^2+1)$, (xvii) $(a+b)(1+b)$, (xviii) $(a-b)(1+b)$, (xix) $(x-1)(y-2)$, (xx) $(p+2)(q-3)$. **3.** (i) $(3c+2d)(2a+b)$, (ii) $(2a-b)(3c+d)$, (iii) $(2a+b)(3c-d)$, (iv) $(2a-b)(3c-d)$.

22—4. **1.** (i) 3·61 in., (ii) 6·40 in., (iii) 5·47 cm., (iv) 8·24 cm. **2.** (a) 3·57 in., (b) 3·26 in., (c) 4·90 cm.

24—2. **1.** 5 miles; $12\frac{1}{2}$ miles. **2.** 96·1 miles. **3.** 15 ft. **4.** 48 ft. nearly. **5.** 26 ft. nearly. **6.** 16 ft. **7.** 6·5 ft. nearly. **8.** 35 ft. **9.** 63·3 ft.

26—6. **1.** 15 in., 120 sq. in. **2.** 1734 sq. in. **3.** (i) 8·66 cm. 43·3 sq. cm., (ii) 12·63 cm., 92·20 sq. in. **4.** 7·6 in., 28·9 ft., 67 ft., 236·4 yd. **5.** 26 ft. nearly. **6.** 129 ft. nearly.

26—8. **1.** 15 ft. **2.** 5 ft. **3.** 2160 sq. yd., 2520 sq. yd. **4.** (i) $3\frac{1}{9}$ sq. yd., (ii) 169·692 sq. ft., (iii) 3570 sq. yd., (iv) 0·54 acre, (v) 192·84 sq. lk. **5.** 25%. **6.** Rs. 236-8-8. **7.** £1218-19-3.

30—6. **1.** (i) 10·38 sq. in., (ii) 31·78 sq. in. **2.** 259·808 sq. ft. **3.** 72083 sq. yd.

31—3. **1.** 154 sq. cm., $346\frac{1}{2}$ sq. in., 2464 sq. ft., 15400 sq. m. **2.** 154 sq. in., $346\frac{1}{2}$ sq. in., 24·64 sq. yd., 98 sq. metres. **3.** $38\frac{1}{2}$ sq. in., $905\frac{13}{25}$ sq. in. **4.** 5 as. nearly. **5.** Rs. 2828-9-2. **6.** 5456 sq. yd. **7.** $2276\frac{4}{7}$ sq. yd. **8.** 30 pieces; $\frac{3}{14}$ of the plate. **9.** $\frac{3}{14}$. **10.** $30\frac{2}{7}$ sq. ft. **11.** 0·36 nearly. **12.** 0·74 tons. **13.** 3713·63 gm. nearly.

32—2. **1.** (i) $37\frac{5}{7}$ sq. in., (ii) 48·4 sq. cm. **2.** $100\frac{4}{7}$ sq. in. **3.** $905\frac{1}{7}$ sq. in. **4.** $3551\frac{5}{8}$ sq. ft. **5.** Rs. 166-2-10. **6.** 155. **7.** Rs. 22-14-8. **10.** Rs. 4772-11-8.

33—2. **1.** (i) 37·4 in., (ii) 6 cm., (iii) 3·24 ft., (iv) 3·99 yd. **2.** (i) 5·04 in., (ii) 6·68 ft., (iii) 11·28 cm., (iv) 7·52 m. **3.** 1·47 in. **4.** 118 ft. **5.** 8·3 ft. **6.** (i) 10 ft., (ii) 5·3 ft. **7.** 135·9 yd. **8.** $r=\sqrt{\frac{7A}{22}}$. **9.** 1 : 2, 3 : 5, $\sqrt{2}:\sqrt{3}$, $a:b$, $\sqrt{p}:\sqrt{q}$. **10.** 3·54 in. **11.** 16·56 ft.

34—2. **1.** (*a*) $12\frac{5}{8}$ sq. cm., (*b*) $82\frac{2}{15}$ sq. in., (*c*) 770 sq. ft. **2.** $36\frac{2}{3}$ sq. in., $157\frac{1}{7}$ sq. in. **3.** $104\frac{16}{21}$ sq. in. **4.** (i) $28\frac{2}{7}$ sq. in., (ii) 11 sq. in., (iii) 198 sq. in. **5.** 0·063 sq. in. **6.** $0·161a^2$; 0·093.

34—3. **1.** 30 sq. cm. **2.** (*a*) 70 sq. ft., (*b*) 400 sq. ft. **3.** $\frac{7}{44}$. **4.** $211\frac{3}{4}$ sq. cm., $1174\frac{1}{4}$ sq. cm. **5.** 27·6 sq. in.

34—5. **1.** $5\frac{8}{11}°$. **2.** 12 in. **3.** 63 ft. **4.** 16 in.

38—3. **2.** (*a*) 156·8 sq. in., 172·2 sq. in., (*b*) 144 sq. in., 156 sq. in., (*d*) 270 sq. in., 316·71 sq. in., (*e*) 158 sq. in. **3.** 384·78 sq. in. **5.** 148 sq. ft., Rs. 15-12-0. **6.** Rs. 19-11-0.

39—4. **1.** (i) 144 sq. in., (ii) 116 sq. in., (iii) 1320 sq. in., (iv) $1658\frac{1}{4}$ sq. in. **2.** 44 sq. ft. **4.** Rs. 13-12-0. **5.** Rs. 70-11-5. **6.** 3960 sq. ft. **7.** (i) 385 sq. in., (ii) 734·64 sq. in., (iii) 700 sq. in., (iv) 205 sq. in. nearly. **8.** (i) 9 in., 5 in., (ii) 5 in., 9 in.; 440 sq. in.; 792 sq. in. **10.** 368·5 sq. in. **11.** 44 sheets. **12.** (i) Curved surfaces as 4 : 25; 575 sq. in. (ii) 4 : 25. 575 sq. in.

39—6. **1.** Two ways; 15·6 sq. in. **2.** $r=h$ (1 : 1). **3.** 2·24 in., 8·40 in.

40—5. **1.** (i) 140 c. in., (ii) 84·9 c. ft., (iii) 110 c. yd. **2.** (i) $4\frac{3}{8}$ c. ft., (ii) $8\frac{5}{8}$ c. ft. **3.** 1452000 c. in. **4.** 384. **5.** Rs. 68. **6.** Rs. 586-10-8. **7.** Rs. 5-7-6. **8.** 8702·2 c. ft., 2687·6 c. ft., 129·2 c. ft. **9.** 108 c. in. **10.** Rs. 180. **11.** $715\frac{1}{2}$ c. ft. **12.** Rs. 4800. **13.** 1728. **14.** 30 days nearly. **15.** 43%. **16.** 72. **17.** 900. **18.** 569. **19.** 3456; Rs. 27-10-4. **20.** 10080. **21.** 3072. **22.** 28152. **23.** 8 times. **24.** 6 times. **25.** 135 : 56. **26.** (i) 8 : 1, (ii) 27 : 125, (iii) $a^3:b^3$.

41—2. **1.** (i) $7\frac{1}{5}$, (ii) $2\frac{36}{41}$, (iii) $8\frac{3}{10}$. **2.** 2 ft. **3.** $3\frac{1}{2}$ yd. **4.** $6\frac{1}{2}$ in. nearly. **5.** 10 ft. **6.** 2000 ft. **7.** 0·71″. **8.** 2·8 in. **9.** $\frac{1}{8}$ in. **10.** $1\frac{7}{9}$ in. **11.** $3\frac{3}{4}$ ft. **12.** $3\frac{1}{3}$ ft. **13.** 1·3 in.; 4·4 gallons nearly. **14.** 0·17 in. **15.** 0·000046 in. **16.** 0·016 in. **17.** 0·1 in. **18.** 29·87 sq. fur. **19.** 6·59 in. **20.** $\frac{pa}{2(hl+bh+lb)}$ in. **21.** $\frac{lbt}{384}$ in., 0·47 in.

42—2. **1.** 3969 c. in. **2.** 4253·2 c. in. **3.** Rs. 61-8-9. **4.** $\frac{1}{21}$ lb. **5.** 192 lb. **6.** $3928\frac{1}{7}$ c. ft. **7.** $74\frac{2}{3}$ c. ft. **8.** 1·04 ft. **9.** 1·17 ft. **10.** (i) $(l-2t)(b-2t)(h-2t)$ c. in., (ii) lbh c. in., (iii) $lbh-(l-2t)(b-2t)(h-2t)$ c. in. **11.** 3182 c. in.; Rs. 3-3-7.

43—2. **1.** 4000 gal.; $1\frac{3}{4}$ sq. in. **2.** 387840 c. ft. **3.** 640 c. ft. **4.** 8 hr. **5.** $2\frac{2}{3}$ months nearly. **6.** $16\frac{1}{2}$ sq. ft.

44—2. **1.** 48 pal.; 192 pal. **2.** 1026 oz. **3.** 7730 oz. **4.** 50 lb. **5.** 160 oz. **6.** 11·71 c. ft. **7.** 2109·4 lb. nearly. **8.** 1728 lb. **9.** 11·46 lb. **10.** 125 lb. **11.** 658 tons nearly. **12.** 506·42 tons nearly. **13.** 0·11 in. **14.** 3·76 sq. yd. **15.** 72 lb. **16.** 24624 tons. **17.** 251·6 lb. **18.** 31·67 lb.

45—3. **1.** 672 cm. cubes, 672 c.c. **2.** (i) 480 c.c., (ii) 636·12 c.c. **3.** 264390, 81655, 3955 litres. **4.** 888 c.c. **7.** 18·6 gal. nearly. **8.** 28 litres. **9.** 187·5 cm. by 62·5 cm. **10.** (i) 8624 c.c., (ii) 2896 c.c.

46—2. **1.** 111·02 gm. **2.** 295431 Kg. **3.** 3920 Kg. **4.** 26·11 Kg. **5.** 7·8 times. **6.** $\frac{g}{slb}$. **7.** 25 Kg. **8.** 61; 162578 gm.

47. **1.** 3 in. **2.** 1100 c. in. **3.** 2178 tons. **4.** 7112 measures. **5.** $\frac{33}{40}$ ft. (= 9·9 in.) **6.** 576000. **7.** ·015 in. **8.** 17 : 92. **9.** 82. **10.** 6·05 ft. **11.** 4353·84 gm.

48—3. **1.** (i) 72 c. in., (ii) 35 c. in. **2.** 450 c. in. **3.** 33·6 c. in. **4.** 69·2 c. in. **5.** 41580 c. ft. **6.** 1257200 c. ft. **7.** 432960 c. ft. **8.** 971520 c. ft. **9.** 74400 c. ft., 620 persons. **10.** 0·348 in. **11.** 0·26 c. in.; 0·12.

49—3. **1.** (i) 16·625 c. ft., (ii) 0·66 c. in. **2.** (i) 66 c. ft., (ii) 80208·8 c. c., (iii) 2502·5 c. c. **4.** Rs. 60-10-0 nearly. **5.** $16\frac{1}{24}$ c. ft. **6.** 9 c. in. **7.** Rs. 66. **8.** Rs. 103. **9.** 1800·6 lb. **10.** 3520 c. yd. **11.** 19250 gal. **12.** 252 buckets. **13.** 1000 tumblers. **14.** 51·4 c. ft. **15.** 18% nearly. **16.** 1249 gal.

50—2. **1.** 0·006 in. **2.** 2·2 ft. **3.** 0·0055 sq. in. **4.** 0·0028 sq. **5.** 0·0025 sq. in. **6.** 0·4 in.

51—2. **1.** (i) 17·7 c. in., (ii) 156·89 c. in., (iii) 1737·12 c. in., **2.** 8·032 c. ft. **3.** 848·6 c. ft. **4.** $15\frac{5}{7}$ c. ft. **5.** $565\frac{5}{7}$ c. in. **6.** 3·93 c. ft. **7.** $678\frac{6}{7}$ c. in.; 176·5 lb.

52—2. **1.** (i) $r = 4·9$ cm., $h = 12$ cm., (ii) $r = 8$ cm., $h = 15$ cm. **2.** (i) $r = 1·4$ in., $h = 4·33$ in., (ii) $r = 4$ in., $h = 23·6$., (iii) $r = 5·83$ cm., $h = 13·05$ cm., (iv) $r = 7$ in., $h = 7·7$ in. **3.** $r = 1$ in., $h = 3·87$ in. **4.** 200° nearly. **5.** 14 in.

53—3. **1.** (i) $235\frac{5}{7}$ sq. in., $314\frac{2}{7}$ sq. in., (ii) $471\frac{3}{7}$ sq. in., $648\frac{3}{14}$ sq. cm., (iii) 234·77 sq. ft., 298·41 sq. ft., (iv) 57·04 sq. cm., 91·26 sq. cm. **2.** 212·15 sq. ft. **3.** 18·128 sq. in. **4.** 58·93 sq. in. **5.** $1810\frac{2}{7}$ sq. cm. **6.** 52·8 sq. in. **7.** Rs. 74-8-0. **8.** 12667 sq. ft.

54—2. **1.** (i) $12\frac{4}{7}$ c. in., (ii) $141\frac{1}{6}$ c.c., (iii) 998 c. in. $\frac{77}{360}$ c. ft. nearly. **2.** 1478·2 c. in. 43·97 lb. **3.** 30·2 c. in. **4.** 391·92 c. ft.

55—2. **1.** 61·0 sq. in. **2.** 20·0 c. in. **3.** 139·3 c. in. **4.** 1091·8 c. in. **5.** 0·127 c. in. **6.** $h = 18·75$ ft., $r = 5·05$ ft., $l = 19·4$ ft.

58—5. **1.** 4 cm. **2.** 12 in. **3.** 3·69 in. **4.** 6 cm. nearly. **5.** 60°.

59—4. **1.** (i) 2 less, (ii) 1 more, (iii) Re. $\frac{1}{6}$ less, (iv) £ $\frac{1}{4}$ more, (v) $\frac{7}{12}$ less, (vi) $2\frac{1}{3}$ less, (vii) 2·5 less, (viii) 4·45 more. **3.** (i) $\frac{1}{371}$, (ii) $\frac{1}{239}$. **4.** $\frac{1}{4357}$.

60—2. **1.** (i) $5\frac{8}{19}$, (ii) $4\frac{16}{21}$, (iii) 4·4, (iv) 0·5, (v) $1\frac{21}{79}$, (vi) $33\frac{1}{3}$, (vii) $3\frac{7}{31}$, (viii) 1·6. **2.** (i) 1·46, (ii) 1·52, (iii) 1·6, (iv) 0·33, (v) 1·63, (vi) 0·0021.

61—2. **2.** (i) 1·0004, (ii) 1·00024, (iii) 0·006, (iv) 0·9984, (v) 1·0002, (vi) 0·9998, (vii) 0·9991, (viii) 0·9996. **3.** 64·0019. **7.** (i) 1·0019, (ii) 1·0024. **13.** (i) 0·006, (ii) 0·076.

63—**Paper I.** **1.** $a = \sqrt{\frac{5}{6}}$; 0·61. **2.** (i) $A = \frac{1}{2} lr$, (ii) $V = \frac{1}{3}\pi r^2 h$. **3.** $a + b > d$. **6.** (i) Equal, (ii) Equal. **7.** $\frac{100x}{x + y + z}$, $\frac{100y}{x + y + z}$, $\frac{100z}{x + y + z}$. **8.** $3\sqrt{3} \div 2\pi$. **10.** 15·573 Kg. **11.** 2 : 5. **12.** Rs. 19000. **13.** $490\frac{2}{7}$ yd. **14.** 1·9 ft. **15.** 12667 sq. ft.

Paper II. **1.** $l_1 : l_2 = b_1 : b_2$. **2.** $\frac{1}{6}$. **3.** 54 sq. in. **4.** $\frac{22}{7}(a^2 - b^2)$ sq. in., 11·6 sq. in. **5.** $A_1h_1 : A_2h_2$. **7.** Equal, equal. **8.** 28. **10.** 42 sec. **11.** $\frac{1}{5}$. **12.** 168960 c. ft. **13.** £24. **14.** $4\frac{1}{2}$ in. **15.** Rs.1232.

Paper III. **1.** $1 : \sqrt{2}$. **2.** $2\pi r^2 + 2\pi rh$. **3.** $\sqrt{d^2 - r^2}$. **5.** $2A - Cr$. **7.** 0·043 per cent. **8.** £3-3-8. **10.** 1056·8. **11.** 20 per cent. **12.** 5 per cent. **13.** 4·9 acres. **14.** 656 sq. in.

64—Specimen Paper : **1.** No, no. **3.** (i) Yes, (ii) yes, (iii) no. (iv) no. **4.** 9 : 15 : 7. **5.** $C = \frac{20}{3}S$. **6.** (i) 1760, (ii) 88. **8.** $\frac{mx - ny}{m - n}$ yr. **9.** (i) $t = \frac{v - u}{a}$, (ii) $r = \sqrt{\frac{7A}{22}}$. **10.** $d_1{}^2h_1 = d_2{}^2h_2$. **11.** (i) $X^\circ + Y^\circ = 90^\circ$, (ii) $2p(l + b + 2p)$. **13.** 28·8 miles per hour. **14.** Rs.4800. **15.** 12 per cent ; 161567. **16.** 1188 yd. **17.** 73·456 gal. per minute. **19.** 1 oz.

S.S.L.C. 1934 : **1.** c may or may not divide a or b without remainder ; 337 and 663 have no common factor except unity. **2.** Increased ; $\frac{5}{111}$. **3.** 3 : 1. **4.** (i) Add 8 times the sum to $\frac{1}{4}$ of it ; (ii) interest on Re.1 per month is 1 pie. Hence interest on Rs.192 for 1 month is Re.1, etc. **5.** 176 ft. **7.** £32-10-0. **8.** $\frac{1}{21}$. **9.** (i) $A = 2H(L + B)$, (ii) $V = \pi r^2 h$,. (iii) πrl. **10.** (i) $x = \frac{b - q}{a - p}$, (ii) $g = \frac{1936l}{49t}$. **11.** (i) 120°, (ii) $\frac{11}{7}a(d - a)$ sq. ft. **12.** 8·5 cm. **13.** $5\frac{1}{3}$ m.p.h. **14.** 420 miles. **15.** Rs. 500. **16.** $\frac{12}{17}$ gal.

S.S.L.C. 1935 : **1.** Decreased ; increased. **2.** (i) $\frac{22}{15}x$ ft. per sec., (ii) $\frac{88}{3}x$ yd. per min. **3.** (ii) $\sqrt{d^2 - r^2}$. **4.** (i) volume of cylinder, $r = \sqrt{\frac{V}{\pi h}}$, (ii) (a) $A = P\left(1 + \frac{nr}{100}\right)$, (b) $\pi r^2 + \pi rl$. **5.** (i) Increased, (ii) 320 sq. yd. **6.** 20 : 17. **7.** 62. **8.** 90%. **9.** (i) 3 times, (ii) 4 times. **10.** 4 sig. fig. **11.** 80°. **12.** $10\frac{1}{2}$ sq. in. **13.** (i) Lengthening, (ii) 17·8 secs. **14.** (i) 70°, 40°, 70°, (ii) isosceles. **15.** 3·8 Km., 4·2 miles. **16.** Rs. 6562-8-0. **17.** (i) $23\frac{1}{21}$ c. ft., (ii) $\frac{7}{44}$ min. before starting. **18.** Rs. 1039-8-0. **19.** 25% solution (26·6% of water).

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