H. Gavin Delin. et Sculp. Edin.

Pondere, mensurâ, numeris mihi pervius orbis;
Hinc decus et vires, hinc pleno copia cornu.

Serfojee Rajah 1827

THE

UNIVERSAL ACCOUNTANT

AND

COMPLETE MERCHANT.

IN TWO VOLUMES.

By WILLIAM GORDON,
of the ACADEMY, GLASGOW.

The third Edition, corrected, and revised by the AUTHOR.

VOLUME I.

Quid munus reipublicæ majus meliusve afferre possumus, quam si juventutem bene erudiamus? CICERO.

EDINBURGH:
Printed for ALEXANDER DONALDSON; and sold at his shop, corner of *Arundel street*, in the *Strand*, *London*; and at *Edinburgh*.
MDCCLXX.

TO

HIS GRACE,

ALEXANDER,

DUKE of GORDON.

MY LORD,

I HAVE the honour, once more, to approach your Grace, with a work originally intended to create, in the British youth, an ambition to excel in the practical arts of trade, which have ever been considered as the best support of civil policy.

The protection of arts, and of those who contribute to improve them, as it is the distinguished province of the great, so, it seems to be, of your Grace, peculiarly characteristical: The whole nation, my Lord, early remarked your virtues, and, to open a larger field for their exertion, have called you into the British senate.

Whilst your Grace's public character, as a patriot, as a liberal encourager and promoter of every useful and ornamental art, command our respect and admiration; your amiable private virtues, as a husband, father, friend, gain our strongest attachment and esteem.

Allow me, therefore, to congratulate my country, in having, at so early a period of life, a friend and benefactor so illustrious; and allow me, my Lord, at the same time, to express, on this occasion, my sincere gratitude, and particular attachment to your Grace, for the singular obligations, which your goodness to me and my family has laid me under.

I have the honour to be, with the utmost submission and respect,

May it please your GRACE,

Your GRACE's most obedient,

Most obliged, and

Most humble servant,

WILLIAM GORDON.

To the PUBLIC.

WHOEVER confiders the numberlefs tranfactions that diverfify the bufinefs of a merchant, muft readily fee, that a perfect knowledge in figures and accounts, a facility and concifenefs of operation, and fome idea of trade in general, are neceffary pre-requifites to fit a gentleman with propriety for the counting-houfe, without which, he can be little more than a fpectator, during all the courfe of his apprenticefhip, however important and extenfive the bufinefs, and however great the fkill and addrefs, with which he may fee it conducted.

The plan of education ufually followed in fchools, hath been by almoft conftant experience found to be far from anfwering the defign; nor have former publications contributed fo much as could have been wifhed to fupply that defect, the method of computation in all of them being too tedious and formal for bufinefs, and the plan of accountantfhip fo defective and confined,

and

and in many inſtances ſo confuſed and prolix, as to be altogether impracticable.

To remove this difficulty, which is one of the greateſt in the purſuit of buſineſs; to render mercantile computations intelligible, eaſy, and conciſe; to illuſtrate the method of arranging and adjuſting accounts of buſineſs, in a train of appoſite examples in every branch of trade, according to the practice of the moſt ingenious and experienced, were the prevailing motive of this undertaking: And the countenance with which two former impreſſions have been diſtinguiſhed, gives the author reaſon to conclude, that the execution hath been in ſome meaſure adequate to the deſign, that it will be of ſome importance in forming the Britiſh merchant, and enable many to purſue with pleaſure, what otherwiſe, perhaps, they might have but little reliſhed.

The method of conducting youth, previous to the counting-houſe, is ſo fully explained in the Eſſay on Education, and the obſervations in every rule ſo numerous and applicate, that there is no room for directions in this place. It may not however be improper to adviſe the reader, to make himſelf fully maſter of the elements, before he proceed to the application, otherwiſe difficulties may occur, which will require frequent retroſpection, render ſtudy diſagreeable, and greatly retard his progreſs.

This edition will appear with advantages greatly ſuperior to the former. The Author's practice and converſation have enabled him to make many ſenſible additions, alterations and improvements, in almoſt every chapter of both volumes; having had it conſtantly in view, to merit, as far as poſſible, that favour wherewith the public hath honoured his work.

GLASGOW, February 12,
1770.

ADVERTISEMENT.

At the Academy, in the Trongate, Glasgow.

Youth are boarded and instructed in Languages, ancient and modern; Writing, Figuring, Geography, Geometry, Algebra, and Merchants Accounts; with whatever may be judged proper to qualify them for the Counting-house, the Army, the Navy, or mechanical Employments,

By the Author, and proper Masters.

CON-

CONTENTS.

The UNIVERSAL ACCOUNTANT.

PART I.

The elements of ARITMETIC.

PART II.

Of FRACTIONS.

I. Of vulgar fractions.

II. Of decimal fractions.

PART III.

The elements of ALGEBRA.

PART

CONTENTS.

PART IV.

The application of arithmetic to the businefs of the merchant, the banker, cuftomhoufe, infurance-office, &c.

Varieties in proportion.

7. How

C O N T E N T S.

ESSAY

E S S A Y

O N

The EDUCATION of a YOUNG GENTLEMAN intended for the COUNTING-HOUSE.

IT is a truth, which the ingenious writers of all ages have acknowledged, and conſtant experience has confirmed, that commerce contributes to the proſperity of ſtates, communities, and individuals, in proportion to the wiſdom of the laws and regulations upon which it is eſtabliſhed, the privileges by which it is encouraged, and the judgement and addreſs wherewith it is conducted. Wiſe inſtitutions, and well-concerted bounties for promoting the intereſt of trade, are the happy effects of good government; and ſuch is the peculiar importance of an extenſive and well-regulated commerce to theſe kingdoms, that it is hoped it will ever be the object of our public care. But the beſt regulations, and the greateſt privileges, will ſignify little, unleſs they be rendered practical, operative, and uſeful, by the ſkill and addreſs of the judicious and induſtrious merchant. It is he who employs the poor, rewards the ingenious, encourages the induſtrious, interchanges the produce and manufactures of one country for thoſe of another; binds and links together, in one chain of intereſt, the univerſality of the human ſpecies; and thus becomes a bleſſing to mankind, a credit to his country, a ſource of affluence to all around him, his family, and himſelf. What extent of knowledge, what abilities muſt it require, to fit a man for ſo great and valuable purpofes? And yet it is certain, that there is not another clafs of men in the Britifh community, who labour under greater difadvantages, in point of education, than that of the commercial profeffion?

A few years are fpent at the grammar-fchool, and perhaps a few more at the univerfity; but fo little time is allotted for the grammar-fchool ftudies, that few, very few, can carry from thence the knowledge or the judgement prerequifite to univerfity-ftudies; by which means a number of years is fpent, and a confiderable expenfe laid out, to very little purpofe. Add to this, the low opinion that is generally entertained of the ufe of thofe ftudies among men of bufinefs; which, when it happens to be difcovered by their children, deftroys that emulation and ambition to excel, that ought to fupport them in the elements of learning; and, in fine, induces them to confider the whole as a formal drudgery impofed on them by cuftom, which continues only for four years.

At a certain age, not after certain acquifitions, a teacher of figures and accounts is applied to; and, in this cafe, the cheapeft market is often reckoned the beft. When the round of this teacher's form is once finifhed, the ftudent is then turned over to the counting-houfe; where, if he is found qualified for nothing higher, which is too often the cafe, he will be employed, during the time of his apprenticefhip, in copying letters, going meffages, and waiting on the poft-office.

The bufinefs of the counting-houfe is of fuch importance, and every moment fo precious to the mafter, that had he talents for communicating, he hath no time for attending to the inftruction of an apprentice;

tice; who, on the other hand, hath been ſo little accuſtomed to think, that his improvement by ſelf-application will be very inconſiderable. Beſides, his time of life, and conſtant habit of indulgence, render him more ſuſceptible of pleaſurable impreſſions, than of improvement in buſineſs; the more eſpecially when he was not previouſly prepared to underſtand it. Wherefore it is not at all ſurpriſing, if many, who, having no foundation in knowledge to qualify them for the purpoſes of the counting-houſe, profit little from the expenſe and the time of an apprenticeſhip, and from ſeeing the moſt extenſive buſineſs conducted with all the ſkill and addreſs of the moſt accompliſhed merchant. The conſequence is, no doubt, fatal to numbers; and the public intereſt, as well as private, muſt ſuffer greatly by every inſtance of this nature. It muſt indeed be acknowledged, that there have been, and ſtill are, gentlemen, who deſtitute of all previous mercantile inſtruction, without money and without friends, by the uncommon ſtrength of natural abilities, ſupported only by their own indefatigable induſtry and application, and perhaps favoured with an extraordinary ſeries of fortunate events, have acquired great eſtates. But ſuch inſtances are rare, and rather to be admired than imitated. For we have likewiſe ſeen many go through all the forms mentioned above, ſet out with large capitals, though perhaps without any other mercantile accompliſhment but an adventurous ſpirit, who have ſhone in the commercial world, while their capitals laſted, as meteors do in the natural; but, like them, ſoon deſtroyed themſelves, and involved in their ruin all ſuch who were unhappy enough to lie within the

 ſphere

ſphere of their influence *. Commerce is not a game of chance, but a ſcience; in which he who is moſt ſkilled bids faireſt for ſucceſs; whereas the man who ſhoots at random, and leaves the direction to fortune, may go miſerably wide of the mark. Parents ought by no means to truſt the future proſpects of their children in the world to a foundation ſo weak or uncertain; and, indeed, it is not reaſonable to expect, that the moſt ſubſtantial character in the Britiſh community can be formed from an education which is common even to the meaneſt citizen.

There never was a time when the neceſſity of a reformation in this particular was greater, or promiſed more ample rewards.——When Britain, by the force of her arms, hath opened, in all quarters of the world, a paſſage for an unlimited commerce, which the wiſdom of her councils hath eſtabliſhed and ſecured by a glorious peace;—when unanimity, formerly unknown, freedom, peace, and proſperity, will give new vigour to the polite arts;—when our neighbours, the French, who have long been our rivals in commerce, will ſtrain every nerve to recover by their trade what they have loſt by the war;—when the low intereſt of money, and the extent of our dominions abroad, will induce many people of fortune to ſtrike into trade, by which means the ſtores abroad will be multiplied, and many more hands employed;—it is hoped a few thoughts on the education of a merchant, will neither be unſeaſonable nor unacceptable.

* Novimus novitios quoſdam, qui cum ſe mercaturæ vix dederunt, in magnis mercimoniis ſe implicantes, rem ſuam male geſſiſſe; et profecto imperitos mercatores, multis captionibus ſuppoſitos, multorumque inſidiis expoſitos, experientia videmus. *St. de mercat.*

To

To be able to read the Engliſh language with ſome eaſe and accuracy, is certainly prerequiſite to every other ſtudy; and it is with pleaſure that we ſee daily improvements made in this particular. Men of education have not been aſhamed of late to take upon them the direction of children in reading Engliſh, which, but a few years ago, was committed to people of very little knowledge. This is a reformation, which, as it was very much wanted, ought to be particularly encouraged and promoted; although, at the ſame time, the purpoſes of it ſhould by no means be extended, eſpecially by thoſe of rank and fortune, beyond its real bounds. It is imagined by ſome, who have reaped little benefit from three or four years attendance at a grammar-ſchool, that the new method of teaching Engliſh will anſwer all the purpoſes intended by the ſtudy of dead languages to a man of buſineſs. But this opinion is ill-founded. The ſtudy of the Engliſh language is not yet carried to a proper extent; and if it was, it would ſtill fall ſhort of the purpoſes of a liberal education. There is no buſineſs whatever that requires a greater correſpondence, or a diction more pointed and conciſe, than that of the merchant; and it would require a ſingular ſtrength of genius to write even correctly in the Engliſh language, unleſs a foundation in the Greek and Latin languages had been previouſly laid. The arts and ſciences, by theſe means, are laid open to us, the moſt ingenious of all ages become our companions and acquaintances, whom we may upon all occaſions with freedom conſult.

The mind muſt be prepared and opened by degrees; and before we know the grammar which reſpects the genius of our own language, we muſt go

back

back to the ſource for the principles of which it is compoſed. The Roman language never arrived at its greateſt perfection till it called in the aſſiſtance of the Greek; and ours would have been void of force and harmony without the aid of both. Beſides, no period of life is ſo apt for proper impreſſions, as the years allotted for the grammar-ſchool; and no leſſons furniſh more excellent examples of correct writing and regular living, than what are contained in the claſſics, if they are properly attended to, and judiciouſly improved. It is here, where youth are furniſhed with the firſt opportunity of paſſing a proper judgement on what they read, with regard to language, thoughts, reflections, principles, and facts, without which the knowledge of words would be very inſignificant. How apt are young people, unleſs the knowledge of true criticiſm be properly laid, to admire and imitate the bright more than the ſolid, the marvellous more than the true, and what is external and adventitious more than perſonal merit and good ſenſe? And is it not of ſome importance, that youth ſhould be ſet to rights in particulars ſo eſſential? It is here where the taſte for writing and living may be in ſome meaſure formed, the judgement rectified, the firſt principles of honour and equity inſtilled, the love of virtue and abhorrence of vice excited in the mind, provided the grammar-ſchool ſtudies be properly directed, and carefully purſued. *Quare ergo liberalibus ſtudiis filios erudimus? non quia virtutem dare poſſunt, ſed quia animum ad accipiendam virtutem præparant. Quemadmodum prima illa, ut antiqui vocabant, literatura, per quam pueris elementa traduntur, non docet liberales artes, ſed mox percipiendis locum parat; ſic liberales artes non perducunt animum ad virtutem, ſed expediunt.*

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The ſtudy of rhetoric and compoſition ought by no means to be neglected by a young gentleman intended for the counting-room. This will give him an opportunity of reducing to practice, what formerly he had been only taught to reliſh. It will not only teach, but accuſtom him to range his thoughts, arguments, and proofs, in a proper order, and to clothe them in that dreſs which circumſtances render moſt natural. By this means he will not only be able to read the works of the beſt authors with taſte and propriety, but be taught to obſerve the elegance, juſtneſs, force, and delicacy of the turns and expreſſions, and ſtill more, the truth and ſolidity of the thoughts. Hereby will the connection, diſpoſition, force, and gradation of the different proofs of a diſcourſe be obvious and familiar to him, while at the ſame time he is led by degrees to ſpeak and write with that freedom and elegance, which in any other way will be found very difficult to attain.

But to ſpeak or write well, however neceſſary it may be, is not the only object of mercantile inſtruction. It will be of little conſequence to have the underſtanding improved, if the heart be totally neglected. Man was made by nature for ſociety, but the merchant both by nature and practice; who, if he is not qualified or not diſpoſed to act his part well, like a bad performer in a concert of muſic, will deſtroy the harmony, and render the whole diſagreeable. Therefore, to tune his mind to virtue and morality, to teach him to blend ſelf-love with benevolence, to moderate his paſſions, and to ſubject his actions to the teſt of reaſon, he muſt have recourſe to philoſophy.

The principles of law and government ought likewiſe to conſtitute a part of the mercantile plan of inſtruction;

ſtruction; by which we are taught to whom obedience is due, for what it is paid, and in what degree it may be juſtly required: more particularly in Britain, where we profeſs to obey the prince according to the laws; and indeed we ourſelves are ſecondary legiſlators, ſince we give conſent, by repreſentatives, to all the laws by which we are bound, and have a right to petition the great council of the nation, when we find they are deliberating upon any act, which we think will be detrimental to the intereſt of the community, with reſpect to commerce, or any other privilege whatever.

When a young man hath been thus accuſtomed to application, reaſon, and reflection; when his taſte hath been formed, and his judgement confirmed; the ſtudy of thoſe ſciences which more immediately reſpect the counting-houſe, will become eaſy and agreeable; but it is neceſſary his teachers ſhould keep up the ſame ſpirit and dignity in their inſtructions with which his earlier ſtudies were animated, otherwiſe the deſign of the whole may be in danger of being fruſtrated.

The firſt care of a ſcholar, who is put under the tuition of a new maſter, is, to obſerve, to ſtudy, and to ſound him; and it generelly holds, that the proficiency of the one, and the authority of the other, are both in proportion to the judgement which the ſcholar forms of his maſter's prudence and abilities; for which reaſon, parents cannot be too ſtrict in their inquiries concerning the temper, qualifications, and character of a maſter, before they truſt him with ſo important a charge as the happineſs and proſperity of their children during the whole courſe of their lives.

Writing, the elements of arithmetic, and the

2 French

French language, ſhould, I think, be the firſt objects of inſtruction, when a young man is ſent to an academy, to be prepared for the counting-houſe; and theſe ought to be taught at particular hours on the ſame day. It is neceſſary that a young man commence the ſtudy of the French language early, that he may be able not only to tranſlate, but ſpeak and write the language with eaſe, before he enters the counting-houſe.

Writing is a prerequiſite to every other ſtep; and therefore no time ſhould be loſt in making him as ſoon and as much maſter of the pen as poſſible. To teach arithmetic well, which is another leading ſtep, requires more ſkill and knowledge than perhaps is attended to. It is, of all other ſciences, the moſt neceſſary to the mercantile profeſſion; and it is not a little ſurpriſing that it ſhould by ſo many be ſo ſhamefully neglected. Before arithmetic is applied to computations in buſineſs, the powers, properties, and relations of numbers ſhould be particularly taught and explained. Every rule ſhould be demonſtrated, exemplified, and illuſtrated in an eaſy and intelligible manner; and the examples ſo multiplied and diverſified, that the learner may be thoroughly grounded, and have a reaſon always ready for what he doth; all the various compendiums which ſerve to abbreviate operations, ſhould be diſtinctly ſhown and demonſtrated that facility and diſpatch may be equally familiar. When he hath thus become maſter of the capital rules in vulgar and decimal arithmetic, involution and evolution, he ought then to be introduced to geometry and algebra, which of all other ſtudies contribute moſt to invigorate the mind, to free it from prejudice, credulity, and ſuperſtition, and to accuſtom it to attention, and to cloſe and demonſtrative reaſoning. In the courſe of theſe ſtudies, he ſhould be taught a new demonſtration of all his arithmetical rules; and the whole theory ought to be reduced to practice, in the menſuration of ſurfaces and ſolids, heights and diſtances, and in conſtructing the inſtruments he hath occaſion to uſe.——When practice is thus joined to demonſtration, the ſtudy of the ſciences becomes eaſy, entertaining, and inſtructive: whereas, were a young man to hear nothing elſe but demonſtration, he would ſoon be wearied of that kind of ſtudy, and conſider it as very dry and inſipid: but when he ſees the uſe of mathematics, in laying down plans and maps of countries, ſelling land by meaſure, aſcertaining the price of labour, and determining the quantity of liquors for a regulation of their price and duty, he muſt be convinced of their influence, and admire their excellency. To complete his mathematical courſe, he ſhould be made acquainted with navigation and geography. The firſt, after ſuch a general acquaintance with the mathematics, will require no great ſtudy: but to the laſt more time and reading will be abſolutely neceſſary.

The ſolution of a few problems on the globe, and three or four ſtudied harangues, will come far ſhort of anſwering the deſign. A teacher who conſiders the extent of geography neceſſary to a merchant, muſt ſee that the knowledge of the globes is no more than the elements of what he ſhould be inſtructed in. He muſt be made acquainted with the uſe of maps, the ſituation, extent, produce, manufactures, commerce, ports, politics, and regulations, with reſpect to trade, of all the nations in the world, not only by public lectures, but by private reading and converſation.

tion. This will not be the work of a few days or a month; and thoſe who allot no more time for geography, know very little of the ſubject. Half an hour every day for ſix months together ſpent in private inſtruction and examination, will perhaps be found little enough for a ſtudy ſo extenſive and important.

When the foundation is thus properly laid by ſuch a mathematical courſe as I have been deſcribing, communicated in that demonſtrative and practical manner, which will join ſcience with judgement, and conviction with experience; the counting-houſe muſt begin to open, and the *arcana mercatorum* be expoſed to view. Arithmetic muſt again be reſumed, and the former theory reduced to practice, in all the caſes which can occur to the merchant, the banker, the cuſtom-houſe, and inſurance office; to which every obſervation ought to be joined, which will ſerve to illuſtrate the uſe of the different examples in that particular branch of buſineſs to which they may be applicable. A proper courſe of reading at this period, which might be wonderfully improved by the converſation of a good maſter, upon the ſubjects of inſurance, factorage, exchange, and ſuch other branches of buſineſs, will be of ſingular uſe, not only to form the mind to buſineſs, but, when he comes to act for himſelf, to prevent many tedious and expenſive pleas, which an ignorance in the practical arts of negotiating them is frequently apt to create.

To this courſe of reading, an epiſtolary correſpondence among the ſtudents themſelves might, with great propriety, be added; as it would give them the practice of folding letters in a quick and dexterous manner, accuſtom them to digeſt well whatever they read, and improve their diction, under the correction of an accurate maſter, to that clear, pointed, and conciſe manner of writing which ought peculiarly to diſtinguiſh a merchant. Fictitious differences among merchants might likewiſe be ſubmitted to their judgement, ſometimes to two in the way of arbitration, and again to a jury of fifteen; whilſt one would aſſume the character of the plaintiff, and another that of the defendant, and each give in ſuch memorials or repreſentations, according to the nature of the facts condeſcended on, as he thinks moſt proper to ſupport the cauſe, the patronage of which was aſſigned him. Thus will youth be accuſtomed to think, write, and act like men before they come upon the real ſtage of action; and their appearance in real life, will have nothing of that awkward and ſtupid manner which is generally obſerved in young men for ſome time after they enter the counting-houſe.

When a young man hath thus attained to a proper accuracy and diſpatch in figuring, and ſome idea of the different branches of buſineſs with which every kind of computation is connected; it is time then to introduce the young merchant to book-keeping, which is the laſt, but not the leaſt important branch of education previous to the counting-houſe. It is become a proverb in Holland, that the man who fails, did not underſtand accounts. And indeed, however much a merchant, who is concerned in an extenſive trade, may be employed in matters of a higher nature, and upon that account be neceſſitated to make uſe of the aſſiſtance of others in keeping his books, he ought certainly to be capable of keeping them himſelf; otherwiſe he never can be a judge,

whether

whether juſtice is done him in that eſſential particular or not; neither can he have that idea of his own buſineſs, which is indiſpenſably neceſſary to the proſperity of his trade.

This happy method of arranging and adjuſting a merchant's tranſactions, muſt, like other ſciences, be communicated in a rational and demonſtrative manner, and not mechanically by rules depending on the memory only. The principles upon which the ſcience is founded, muſt likewiſe be reduced to practice by proper examples in foreign and domeſtic tranſactions: ſuch as, buying, ſelling, importing and exporting for proper, company, and commiſſion account; drawing on, remitting to; freighting and hiring out veſſels for different parts of the world; making inſurance and underwriting; and the various other articles that may be ſuppoſed to diverſify the buſineſs of the practical counting-houſe. The nature of all theſe tranſactions, and the manner of negotiating them, ought to be particularly explained as they occur; the forms of invoices and bills of ſales, together with the nature of all intermediate accounts, which may be made uſe of to anſwer particular purpoſes, ought to be laid open; and the forms of all ſuch writs as may be ſuppoſed to have been connected with the tranſactions in the waſte-book, ſhould be rendered ſo familiar, that the young merchant may be able to make them out at once without the aſſiſtance of copies.

As the following work is intended to be a complete courſe of mercantile computations and accountantſhip, to ſay more on the method of communicating them would be unneceſſary. Only I would beg leave to hint, that there are many things, the knowledge of of which is better inculcated by public lectures, private reading and converſation, than in the ordinary method of teaching, when, perhaps, there may be two or more claſſes to direct. The national commerce in general; the trade of the place where we live; the laws, cuſtoms, and uſages relative to the buſineſs of a merchant, the penalties to which he is liable, and the privileges to which he is entitled; the duties, impoſts, and other charges laid upon the Britiſh produce in other countries, with all the known maxims that relate to the proſperity of trade; will open a wide field for improvement in matters of real uſe to the maſter as well as the ſtudent.

When the education of a young gentleman is thus conducted, from his earlieſt years, in a manner calculated to engage his mind in the love of uſeful knowledge; to improve his underſtanding; to form his taſte, and ripen his judgement; to fix him in the habit of thinking, ſteadineſs, and attention; to promote his addreſs and penetration, and raiſe his ambition to excel in his particular province; will not the tranſition to the counting-houſe be extremely eaſy and agreeable? His knowledge will be ſo particular, and his morals ſo ſecured, that he will be proof againſt the arts of the deceitful, the ſnares of the diſingenuous, and the temptations of the wicked. He will, in a ſhort time, be ſo expert in every part of the buſineſs of the practical counting-houſe, and be able to form ſuch a judgement of every thing he ſees tranſacted, that when he comes to act for himſelf, every advantage in trade will lie open to him: his knowledge, ſkill, and addreſs will carry him through all obſtacles to his advancement; his talents will ſupply the place of a large capital; and when the beaten track

of

of busineſs becomes leſs advantageous, by being in too many hands, he will ſtrike out new paths for himſelf, and thus bring a balance of wealth, not only to himſelf, but to the community with which he is connected, by branches of trade unknown before.

How few are there, even among parents, who, perhaps, have felt the loſs of a proper education in their own practice, that conſider the extent of knowledge requiſite to make a young gentleman appear with dignity in the commercial life? and how few are there among thoſe who profeſs to qualify young gentlemen for the counting-houſe, that have knowledge in any degree proportionable to their credit? The reaſon is obvious: In every other article of expenſe, conſidered as communities or individuals, we are generally profuſe: but in that which relates to education, we are ſhamefully narrow. This falſe parſimony, this miſtaken frugality, prevents men of genius and education from appearing as teachers, becauſe their talents will turn out to much more account, in almoſt any other profeſſion whatever; and if circumſtances ſhould have rendered it neceſſary for a man of ſome abilities to turn his mind this way, he is obliged to divide his ſtudies among ſo many different ſciences and his time among ſo many different claſſes, to ſecure to himſelf a bare ſubſiſtence, that he hath neither the leiſure, the means, nor the opportunity of that reading or converſation, which is abſolutely neceſſary to his practice, in inſtructing youth in the moſt difficult and important branch of Britiſh literature. And if this is the caſe with the ableſt teachers, what can be expected of thoſe who became teachers becauſe they were really qualified for nothing elſe?

For

For the inſtruction of youth in every other ſcience, we have not only excellent inſtitutions, but eminent maſters, whoſe abilities are inquired into and approved of, before they are admitted to the important truſt: but in this caſe, great pretenſions, which are generally taken upon the teacher's word, and low prices for the articles of education in his ſcheme, are credentials ſufficient to procure him buſineſs, though neither the teacher nor the ſtudents reap much advantage from it.

The art of managing and forming the mind is perhaps of all others the moſt intricate and extraordinary, and certainly the moſt important; which, that it may be ſufficiently ſtudied, ought to be properly rewarded. It is no doubt the buſineſs of magiſtrates, to intereſt themſelves in the education of youth, ſince they are the nurſery of the ſtate, by whom it is renewed and perpetuated, and upon whom the national proſperity, as well as the national exiſtence depends. If part of the public revenues were employed in erecting academies for training up youth to buſineſs, eſpecially in trading cities, where every maſter ſhould have a ſalary proportioned to the difficulty of his department; if the moſt intelligent merchants were appointed as ſuperintendants of theſe academies, who would take care that none ſhould be admitted as ſtudents, whoſe proficiency in the languages, rhetoric, and philoſophy was not previouſly inquired into, nor any ſuffered to proſecute the ſtudies prerequiſite to the counting-houſe, whoſe genius was not in ſome meaſure turned to act with dignity in the mercantile profeſſion; if theſe gentlemen would inquire often into the morals and proficiency of the ſtudents, converſe frequently with the maſters on the ſubject of

 trade,

trade, and admit the ſtudents according to their ſeniority in letters to ſuch converſations, and, in ſhort, take every other method of encouraging both maſters and ſtudents to induſtry and attention, that they might go through the tedious, the difficult taſk with alacrity and ſpirit; if parents, at the ſame time, would ſet that value upon education which they ſometimes do upon trifles, and be but as careful in having the minds of their children adorned with virtue and good ſenſe, as they are in ſetting off every thing which relates to their bodies, we would then ſee a reformation indeed. Were this to be the caſe, our youth would be long acquainted with the arts of gaining before they would learn how to ſpend money, and they would not be grown old in debauchery and riot, before they were initiated into buſineſs. Were this to be the caſe, we would ſoon ſee a ſpirit of induſtry, knowledge, humanity, and good ſenſe diffuſe itſelf among all ranks and denominations, whilſt idleneſs and folly with all their miſchievous train, would be baniſhed the ſtreets. In one word, our teachers would be men of underſtanding, our young men would be ſenators, and our " merchants would be princes."

THE UNIVERSAL ACCOUNTANT.

PART I.

The ELEMENTS of ARITHMETIC.

INTRODUCTION.

ARITHMETIC is the art of reckoning by numbers, which, from the various combinations of theſe ten Arabian characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, teacheth to calculate with expedition, exactneſs and eaſe.

To render operations more ſhort and expreſſive, with great propriety have been introduced the following

CHARACTERS

The ſign		ſignifies		read
	+		Addition,	More.
	−		Subtraction,	Leſs.
	×		Multiplication,	Into.
	÷		Diviſion,	By.
	=		Equality,	Equal to.
	∷ :		Proportion,	As to. So is.
	⌐		Majority,	Greater than.
	¬		Minority,	Leſs than.
	$\sqrt{}^2$		Extraction of,	Square-root.
	$\sqrt{}^3$		Extraction of,	Cube-root.

It is not my preſent purpoſe, nor would it be material, to trace this uſeful art back to its original, or carry it through all the different ſteps of its improvement. The Lombards, no doubt, imported it into Britain; and, for its after improvements, we have been equally obliged to the productions and the practice of the ingenious.

CHAP. I. NUMERATION.

NUMERATION teacheth to read or write any number known, or proposed.

RULE I.

To read any number, divide it by commas into classes of three characters each, beginning at the right-hand; over the right-hand figure of the third class, make a point; over that of the fifth, two points; over that of the seventh, three points, &c. The number to the left hand of the first comma, express by thousands; that which hath over it the first point, by millions; that which hath over it the second, by billions; that which hath over it the third, by trillions, &c. The left-hand character of each class is expressed by hundreds, the middle one by tens, and the right-hand one by units.

RULE II.

To write down any proposed number, reverse the former rule; and, if any intermediate place is wanting, fill it up with the cipher (0). Upon these principles is effected the following table.

NUMERATION-TABLE.

Trillions.	1	1	⁝
Hundreds of Thouf. of Bill.	1	9	
Tens of Thouf. of Billions.	0	8	
Thoufands of Billions.	0	7	
Hundreds of Billions.	0	6	
Tens of Billions.	0	5	
Billions.	0	4	:
Hundreds of Thouf. of Mill.	1	3	
Tens of Thouf. of Millions.	0	2	
Thoufands of Millions.	0	1	
Hundreds of Millions.	0	9	
Tens of Millions.	0	8	
Millions.	0	7	.
Hundreds of Thoufands.	1	6	
Tens of Thoufands.	0	5	
Thoufands.	0	4	
Hundreds	0	3	
Tens.	0	2	
Units.	0	1	

OBSERVATION.

It is obvious from the table, that all numbers increaſe in a decuple proportion; and, conſequently, that, in a ſeries of numbers, every figure hath a local, as well as ſimple value: hence ciphers, though they have no ſimple value, when annexed to ſignificant figures, remove thoſe figures ſo many ſteps from the units place, and increaſe their value accordingly.

CHAP. II. ADDITION of INTEGERS.

ADDITION teacheth to find a ſum which ſhall be equal to ſeveral homogeneous ones given.

RULE.

Place the numbers, each of the ſame local value under one another ſucceſſively; then, beginning with the loweſt, or units place, find the ſum thereof by collecting them all together, and, of that ſum, or total, write down under the column of units what belongs to that name, and carry the number of tens to be added with the next column, being the denomination to which, in the very nature of numbers, it belongs: of the ſum or total of the column of tens, write likewiſe down, under that column, the units place, whoſe value will be tens, and carry the tens to be added with their homogeneous column, which is hundreds. Proceed thus through the whole, and take down the ſum of the laſt column all together, there being no other to which it can be carried.

EXAMPLES.

(1.)	(2.)	(3.)	(4.)
75468	74367	478561	59374
37546	43243	37456	38
18764	67548	5937	473
34653	32764	5478	89
21687	13487	5684	9
85464	54728		45
273582	286137		

Illuſtration

Illuſtration of the firſt example

$3+7=10+4=14+4=18+6=24+8=32$; of which total, 2 belongs to the units column, and is accordingly taken down there ; the 3 in its local value is 30, or 3 tens ; therefore, $3+6+8+5+6+4+6=38$ tens, or 380 ; of which total, 8 belongs to the place of tens, where it is taken down, and the 3, or 300, carried to be added with the column of hundreds ; thus, $3+4+6+6+7+5+4=35$, or 3500 ; therefore 5 falls to be noted down in the place of hundreds, whilſt the 3 remains to be added with the column of thouſands, to which it belongs ; thus, $3+5+1+4+8+7+5=33$, or, according to its local value, 33,000 ; of which total, the 3 to the right hand is taken down in the place of thouſands, and the other 3 carried to the column of tens of thouſands ; thus, $3+8+2+3+1+3+7=27$, or, in its local value, 270,000 ; of which ſum the 7 is taken down in its homogeneous column, and the 2 in the place of hundreds of thouſands, to which it naturally belongs. The whole taken together becomes 273,582.

OBSERVATIONS.

1. Had the units place of any of the ſums of the columns been 0, it is plain that 0 would have been taken down in that place.

2. Had any of the ſums conſiſted but of one place only, there would have been nothing to carry to the next column.

3. The reaſon of this manner of operation is ſufficiently demonſtrated in the illuſtration, being a plain deduction from the nature of numbers ; and, as all the parts of any thing whatever muſt be equal to the whole, the ſum or total, thus found with ſufficient accuracy, muſt be equal to the ſeveral given numbers taken together.

4. Operations in this rule may be proved by dividing the numbers into two or three different claſſes, finding the ſums of theſe ſeverally, and collecting their totals again into one ; which, if the operations were right, will agree with that total which was taken at once. Men of buſineſs prove their ſummations by adding firſt upwards and then downwards.

EXAMPLES.

EXAMPLES.

7458		54856
5978		34785
4567		67857
———	18003	34158
8452		51213
3456		———
3978	15886	242869 upwards.
———	———	242869 downwards.
33889=	33889	

5. Expedition, as well as exactneſs in calculating, depends much upon improving the memory; and, as addition occurs more frequently in buſineſs than almoſt any other rule in arithmetic, both diſpatch and accuracy are abſolutely neceſſary: Wherefore, when one is ſufficiently accuſtomed to add the figures one by one, he ſhould be gradually led, to take them two by two, three by three, by either or more, as appears convenient; which will not only promote diſpatch, but be leſs liable to errour. Thus, for inſtance, the firſt four figures in the units place, of the firſt example, to one who hath been ever ſo little accuſtomed to addition, will at once preſent the ſum of 18, and the other two figures 14=32, *&c.*

CHAP. III. SUBTRACTION of INTEGERS.

SUBTRACTION finds the difference, called the *remainder*, betwixt a leſſer number, called the *ſubtrahend*, and a greater, called the *minuend*, and is the converſe of addition.

RULE.

Place the numbers, each of the ſame local value under one another reſpectively, the ſubtrahend in a line directly under the minuend. Then, beginning at the units place, if the figure in the ſubtrahend be equal to that correſponding in the minuend, write down a cipher for the difference; if leſs, take down the figure which repreſents the difference; but, if greater, increaſe it by 10, and then take down the difference; remembering at the ſame time, that however

ever oft the minuend muſt be ſo increaſed, the next place in the ſubtrahend muſt be likewiſe increaſed by unity. Proceed thus till the whole remainder is taken down and completed.

EXAMPLES in INTEGERS.

	(1.)	(2.)	(3.)	(4.)
Minuend,	8467548	546317	47856478	5741[illegible]63
Subtrahend,	3254138	257168	39764789	31[illegible]4585
Remainder,	5213410	289149		

Illuſtration of the ſecond example.

Beginning with the units place, becauſe 8 is more than 7, increaſe 7 with 10, and 17 becomes the minuend; therefore, $17-8=9$, or $10+7-8=9$; becauſe the minuend was thus increaſed by 10, the next figure in the ſubtrahend muſt be increaſed by 1, which in effect is 10, and then it will be $6+1=7$; but ſtill the correſponding figure in the minuend is leſs, and therefore the ſame increaſe muſt be repeated, and then it will be $10+1-7=4$. In the ſame manner, and for the ſame reaſon, the next figure in the ſubtrahend muſt be increaſed by 1, and it will become 2; the correſpondent figure to which in the minuend is 3, and their difference, without any increaſe, is 1; which is noted down, and nothing carred to the next figure in the ſubtrahend, *&c.*

OBSERVATIONS.

1. When all the figures in the minuend are greater than, or ſome of them equal to their correſpondents in the ſubtrahend, it will be obvious, that the difference of the figures put down as correſpondents, muſt, when taken as one ſum, be the difference or remainder required; for as all the parts of any number taken together are equal to the whole, ſo the difference of all the parts of any two numbers make together the difference of the wholes.

3. When any figure in the ſubtrahend is greater than its correſpondent one in the minuend, the latter, before ſubtraction, is encreaſed by 10; and, for that reaſon, the next ſubtrahend figure is encreaſed by 1: becauſe, from the nature of numbers, 10 in any place is equal to 1, in the next place to the left; therefore an equal number increaſes both factors

and the difference muſt accordingly be equal; for the ſame difference will alway, exiſt betwixt 9 and 17, as betwixt 19 and 27, or betwixt 29 and 37, *ad infinitum.*

3. If one number is to be ſubtracted from ſeveral, ſeveral from one, or ſeveral from ſeveral, it is plain, that they muſt be reduced to two factors before ſubtraction, by addition.

4. The accuracy of operations in this rule may always be proved by adding the remainder to the ſubtrahend whoſe ſum, when the operation is right, will be equal to the minuend; becauſe the ſubtrahend and remainder are the parts of the minuend, which is conſidered as the whole.

CHAP. IV. MULTIPLICATION of INTEGERS.

MULTIPLICATION ſerveth inſtead of many additions; and, from two numbers given, called the *multiplier* and *multiplicand,* findeth a third, called the *product,* which ſhall repeat the multiplicand ſo oft as the multiplier contains unity. For the more expeditious management of this rule, it will be neceſſary to commit to memory the following

TABLE OF MULTIPLICATION.

2× 2= 4	3× 3= 9	4× 5=20	5× 8=40	6×12=72	8×12= 96
2× 3= 6	3× 4=12	4× 6=24	5× 9=45	7× 7=49	9× 9= 81
2× 4= 8	3× 5=15	4× 7=28	5×10=50	7× 8=56	9×10= 90
2× 5=10	3× 6=18	4× 8=32	5×11=55	7× 9=63	9×11= 99
2× 6=12	3× 7=21	4× 9=36	5×12=60	7×10=70	9×12=108
2× 7=14	3× 8=24	4×10=40	6× 6=36	7×11=77	10× 0=100
2× 8=16	3× 9=27	4×11=44	6× 7=42	7×12=84	10×11=110
2× 9=18	3×10=30	4×12=48	6× 8=48	8× 8=64	10×12=120
2×10=20	3×11=33	5× 5=25	6× 9=54	8× 9=72	11×11=121
2×11=22	3×12=36	5× 6=30	6×10=60	8×10=80	11×12=123
2×12=24	4× 4=16	5× 7=35	6×11=66	8×11=88	12×12=144

RULE.

When the multiplier conſiſts of any number within the bounds of the table, the product is found at once, by multiplying every figure, or place of the multiplicand into the multiplier, one after another, beginning with the units place; and the ſeveral products

are

are wrote down as the ſeveral ſums in addition; but, when the multiplier exceeds the bounds of the table, the product of every particular digit muſt be taken by itſelf, the firſt figure of every particular product placed directly below it's reſpective multiplier, to anſwer the local value thereof, and the ſum of theſe ſeveral products will be the product required.

EXAMPLES.

(1.)		(2.)	(3.)
875467543	Multiplicand.	437546754	478456789 78
8	Multiplier.	12	11
7003740344	Product.	5250561048	

Illuſtration of the firſt example.

Beginning with the units place, by the table, 8×3=24, of which product 4 is taken down in its own place: then 8×4=32+2, in the ſecond place of the laſt product, =34, in its local value =340, whereof 4 falls to be noted down in its own place, *viz.* the place of tens: again, 8×5=40+3=43; 3 is taken down in its own place, and 4 is reſerved to be carried to the product of the ſucceeding figure; then 8×7+4=60; here 0 is noted: 8×6+6=54, 4 is noted: 8×4+5=37, note 7; 8×5+3=43, note 3; 8×7+4=60, note 0; 8×8+6=70, which is taken down all together, becauſe there is no new product to which the figure in the higheſt place could be carried.

Example (4.)		(5.)	(6.)	
745678		549356	67540573	
345		678	30405	
3728390	3 above, 3 left, X, 1 right, 3 below		337702865	3 above, 3 left, X, 1 right, 3 below
2982712			270162292	
2237034			202621719	
257258910			2053571122065	

OBSERVATIONS.

1. When the multiplier conſiſts of any number within the bounds of the table, the reaſon of the operation will be plain

plain from what hath been said in addition, since multiplication is only a repetition of that rule, so oft as the multiplier contains unity; to be satisfied of which any one may make the experiment at pleasure, by finding the sum of any multiplicand repeated as oft as the multiplier contains unity.

2. When the multiplier exceeds the bounds of the table, the products are taken partially, and the sum of these products must as certainly be the whole product, as it is true that the whole is equal to all its parts taken together. The reason of placing the first figure of every particular multiplier's product below its respective multiplying figure, will appear from this consideration, that as each figure hath a simple and local value, both these values must be retained in the product: for instance, in the multiplier of the 4th example, 4, from the place in which it stands, is really 40, and consequently the first figure of its product is not 2, but 20; for which reason it must stand in the place of tens. For the same reason, 4 in the multiplier of the 6th example is 400; and therefore 2, the first figure of its product, is 200, and for that reason stands in the place of hundreds.

3. Operations in this rule may be proved by shifting the factors, the reason of which is obvious; but more expeditiously by casting out the 9s. For an example, take an illustration of the proof of example 4.

*7+4=11, exceeding 9 by 2, and 2+5+6=13, excess 4+7=11, excess 2+8=10, excess 1, which is noted on the right side of the cross. The same is done by the multiplier thus, 3+4+5=12, excess 3, noted on the left side of the cross then 3×1=3, noted, as it does not exceed 9, on the top of the cross. If it had been 9, 0 would have been noted; if more than 9 or 9s the excess. After the same manner are the 9s cast out of the sum of the products, and the last excess is found to be 3, which is set at the bottom of the cross, and proves the operation to be right, being equal to the figure at the top. For because in whatever place any figure stands, taken in its simple value, according to the place in which it stands, it will be equal to what remains, after all the 9s contained in its value are taken away; it follows, that the sum of all the figures of which any number consists, considered simply as so many units, is equal to the remainder, after all the 9s are taken out of that number, which can be found in the real value of each figure of which it consists. Hence, if this sum be less than 9, it is equal to what remains when all the 9s possible are taken out of that number. But if this sum is equal to, or exceeds 9, the remainder, when the 9s are taken out, will be equal to what remains when the 9s are taken out of the given number; becauſe the number of 9s in any number muſt be equal to the number of 9s which is contained in the ſeveral products, and in the ſum of the exceſs of 9s in thoſe parts.

CONTRACTIONS IN MULTIPLICATION.

1. When there are ciphers on the right of either or both factors, they may be neglected in the operation, but annexed to the ſum of the products.

EXAMPLES.

(1.)

```
   7415678000
       123000
   ----------
  22247034
 88988136
 ---------------
 912128394000000
```

```
    3
 2  X  6
    3
```

(2.)

```
   53400
     560
  ------
  3204
 2670
 ---------
 299040000
```

```
    6
 2  X  3
    6
```

Hence, to multiply by 1 and any number of ciphers annexed to it, is only to annex thoſe ciphers to the multiplicand.

2. When unity is in the place of tens of the multiplier, the product may be found in one line, by adding the product of that place in the multiplication; and the ſame method may be extended by practice, to 2 or 3 in the place of tens.

EXAMPLES.

(1.)

```
  874675432
         15
 -----------
 13120131480
```

```
    6
 6  X  1
    6
```

(2.)

```
 84670000
       23
 ----------
 1947410000
```

```
    8
 5  X  7
    8
```

3. It will be found convenient, in applicate queſtions, to work by the component parts of the multiplier, which, for any ſmall number, will be found in the table: but if the multiplier be ſuch a number, for which no component parts can

can be exactly found, the neareſt component parts muſt be taken, and the multiplicand being added ſo often to the laſt product, as the product of the component parts comes ſhort of the given multiplier, or ſo often ſubtracted from it, as the product of the parts exceeds the given multiplier, the ſum in the one caſe; and remainder in the other, will give the true product.

EXAMPLES.

(1.)

54537543 by 56
7
———————
381762801
8
———————
3054102408

Proof: 2 X 0 (0 above, 0 below)

(2.)

3741500 by 74
8
———————
29932000
9
———————
269388000 Product of 72
7483000 Product of 2
——————— ——
276871000 74

Proof: 2 X 2 (4 above, 4 below)

The proof is taken by caſting the nines out of the given multiplier, and not the artificial ones.

4. If the given multiplier is a number exceeding the bounds of the table, multiply by as many tens as the multiplier conſiſts of places ſave one, the laſt product by the firſt figure on the left hand, the next in order by the ſucceeding place, &c. the ſum of the products of theſe places gives that required.

EXAMPLES.

(1.)

8×436843246 by 578
10
———————
7×4368432460
10
———————
43684324600
5
———————
218421623000=the product of 500.
30579027220=the product of 70.
3494745968=the product of 8.
——————— ———
252495396188=the product of 578.

(2.)

74563000 by 6002
10
———————
745630
10
———————
7456300
10
———————
74563000
6
———————
447378000000
149126000
———————
447527126000

5. If

5. If the multiplier be any number near 100, 1000, 10000, &c increase the multiplicand by as many ciphers as there are figures in the multiplier, and subtract the multiplicand from itself thus increased as often as the multiplier wants units of that by which the multiplicand was increased.

EXAMPLES.

(1.) Multiply 8754687 by 999
999 is 1 short of 1000
Therefore 8754687000
8754687
―――――
8745932313

(2.) 4378 into 9998
43780000
8756 = 2×4378
―――――
43771244 = 9998×4378

6. When the multiplier can be parted into periods which are multiples of one another, the operation may be contracted in the following manner.

EXAMPLES.

(1.)

5697487
96488
―――――
45579896 = 8×5697487
273479376 = 6×45579896, because 60×8=480
546958752 = 2×273479376, because 200×480=96000
―――――
549739125656 = 5697487×96488

Note, One number is said to be the multiple of another, when it contains it a certain number of times without any remainder.

Or in a reversed order, thus:

(2.) 5742135 into 52575
52575
―――――
28710675 = 50000×5742135 for 50000
143553375 = 500×28710675 for 2500
430660125 = 3×143553375 for 75
――――― ―――
301892747625 = 5742135× 52575

7. If the multiplier be a repetend of the same figure, multiply by one of the repeating figures; and the figures of that product

duct added, as if they had been wrote down in as many products as the multiplier repeated the ſame figure, give the product required.

EXAMPLES.

```
     (1.)                     (2.)
  5478 6789                  54018
      22222                   3333
  ----------                ------
  1095713578                162054
  --------------          ---------
  12174473565158          180041994
```

8. When the repeating figure is a high digit, collect the product of as many ones as there are digits in the multiplier, from the multiplicand, according to the rule in the laſt contraction, which product being multiplied into the repetend, will give the true product.

```
Example.      784325634 into 7777777
         -----------------
           871472839519374 Products collected for 1111111.
                         7
         -----------------
          6100309876635618 Product of 7777777.
```

There is another contraction for finding the product of any ſeries of repeating figures, more elegant than any of the preceding, but it will come in more properly in the next chapter.

CHAP. V. DIVISION of whole numbers.

DIVISION findeth how oft one number is contained in another, and is a compendious method of ſubtraction, in the ſame ſenſe that multiplication is a compendious method of addition.

RULE.

Place the dividing number, called the Diviſor, on the left of the dividend, or number to be divided, and on the right of the dividend place the quotient, as in in the examples following. The factors being thus placed, point off so many places from the right of the dividend, as are equal to, or not exceeding the product of the divisor, into any one of the 9 digits, and this is called a *dividual*: in which, having considered how often the divisor is contained, note the number of times in the quotient, then subtract the product of that quotient-figure, after it is multiplied into the divisor, from the dividual, and to the remainder affix the next place in the dividend for a new dividual, with which proceed as before; and if the divisor is not once contained in any dividual, increase the quotient with a cipher, before any new place is taken down to the right of the dividual; if any thing remains after all the places are taken down from the dividend, it is called the *remainder*, which, with the divisor, expresseth some parts of unity, the number whereof is ascertained by the remainder, and the quality by the divisor.

EXAMPLES.

(1.)	(2.)
Divisor. Dividend. Quotient.	
12) 475678 ($39639\frac{10}{12}$	15) 7846735 ($523115\frac{10}{15}$
36	75
115	34
108	30
76	46
72	45
47	17
36	15
118	23
108	15
(10) Remainder.	85
	75
	(10)

Illustration of Example first.

The divisor 12 is found in the first dividual 47, three times, which is noted in the quotient, and being multiplied into the divisor, presents a product of 36, which is brought down below its own dividual 47, and subtracted therefrom, by which means we discover a remainder of 11: To this remainder, the next figure in the dividend being affixed, we are presented with a new dividual of 115, which contains the divisor 9 times; consequently 9 is noted in the quotient, and multiplied into the divisor; the product 108, being subtracted from its dividual 115, leaves a remainder of 7, to be increased by the next figure in the dividend, *viz.* 6; with which, and the remaining part of the dividend, we proceed as before, and at last there remains 10; which being taken up with the divisor, and noted after the integral part of the quotient, expresseth ten twelfth parts of one.

OBSERVATIONS.

1. Since, by the above method of division, the dividend is taken into as many dividuals as possible, and the quotient taken out of the first dividual as near as possible, the defect made the foundation of the succeeding dividual; and this operation being repeated so oft as there were places in the dividend to bring down, or quotient-figures to note, it will be plain, if there hath been no errour in the operation, that all the parts of the dividend have been added, and the number of times the divisor is contained in those parts hath been separately found: And since all the parts taken together are equal to the whole, it must follow, that however often the divisor is contained in those parts which constitute the dividend, so often must the divisor be contained in the whole dividend.

2. The best proof of operations in this rule is made by multiplying the quotient into the divisor, whose product added to the remainder, if any, must be exactly equal to the dividend.

A proof less certain, but much more expeditious, may be made by casting out the nines, as in multiplication, considering the integral part of the quotient as a multiplicand, the divisor as a multiplier, and the dividend—the remainder as a product. The proof by the nines, in either case, can only be applied to integers or decimals; so that, upon the whole, the best and most general proof of multiplication is division, and, *vice versa*, of division, multiplication.

Example (3.) (4.)

375)5478989($14610\frac{239}{375}$ 25476)88350768(3468

375	375		76428	
1728	73289		119227	0
1500	102270		101904	3 X 6
	43830			0
2289			173236	
2250	5478989	Proof by multiplication.	152856	
398			203808	Proof by casting out the nines.
375			203808	
239			0	

CONTRACTIONS IN DIVISION.

1. In dividing by unity, the quotient will be found just equal to the dividend; therefore in dividing by 1, and any number of ciphers, if as many places are cut off from the right of the dividend, as there are ciphers to the right of 1 in the divisor, the number to the left of the separating point in the dividend will be the integral part of the quotient, and that to the right will be the remainder, or fractional part.

EXAMPLES.

	Quot.	Rem.		Quot.	Rem.		Quot.	Rem.
1000)	4578.	567	1000)	545.	78	10000)	597.	8451

2. For the same reason, when there are ciphers to the right of any divisor, an equal number of ciphers or figures may be cut off from the right of the dividend, and the remaining figures to the left being divided by the significant figures in the divisor, will quote the integral part, and the figures on the right of the point, annexed to the last remainder, will give the fractional part.

EXAMPLES.

```
(1.)
25|000)35784675|000(1431387
       25
       ---
       107
       100
       ---
         78
         75
         ---
          34
          25
          ---
           96
           75
           ---
           217
           200
           ---
            175
            175
            ---
              0
```

```
(2.)
35|00)37645|67(1075 2067/3500
      35
      ---
       264
       245
       ---
        195
        175
        ---
       (2067)
```

Note, As an equal number of ciphers was cut off from both factors, there was nothing to constitute a remainder, and therefore there is no fractional part.

3. When the divisor consists but of one or two figures, the operation may be performed by a mental multiplication and subtraction; in which case, no part of the work needs to be noted but the quotient, and it may stand as in any of the subjoined

EXAMPLES.

(1.)

$$5)47856743$$
$$9571348\tfrac{3}{5}$$

(2.)

$$\frac{8007456}{70}=114392\tfrac{16}{70}$$

(3.)

$$75645\div25=3025\tfrac{20}{25}$$

4. When the divisor is a composite number, divide by its component parts continually, and the last quotient gives the integral part of the answer. For the fractional part, multiply the last remainder by the last divisor but one, and to the product add the remainder belonging to that divisor; multiply this sum by the next preceding divisor, to which add its correspondent

refpondent remainder; and thus proceed, till you have multiplied by the firft divifor, and added in the firft remainder.

Example. Divide 7456785 by 75

$$
\begin{array}{r|l}
75=3\times5\times5 & 7456785 \\
5 & 1491357 \\
 & 3)298271 : 2 \\
 & 99423 : 2
\end{array}
$$

The remainder is found by faying $2\times5+2\times5=\frac{66}{75}$

Or the remainders may be valued, as in the following

EXAMPLE.

Divide 37841 by 48

$$
\begin{array}{l}
48=8\times6\ )\ 37841 \\
\quad 8\ )\ 6306\frac{5}{6} \\
\quad\quad 788 \text{ Rem. } 2\frac{5}{6}=\frac{17}{48}
\end{array}
$$

For 2 when reduced to 6ths $=\frac{12}{6}$ and $\frac{12}{6}+\frac{5}{6}=\frac{17}{6}$, but $1\times17=17$.

$8\times 6=48$.

The laft method will be found to be moft convenient for practice, but it will be beft underftood when we are come to reduction of vulgar fractions.

5. When both factors are commenfurable, it will fhorten the divifion confiderably, to abridge both factors according to the following method.

57456 by 56

$$
\begin{array}{r|l}
4)56 & 57456 \\
2)14 & 14364 \\
7 & 7182 \\
 & 1026 \text{ Quotient.}
\end{array}
\qquad\qquad
\begin{array}{r|l}
11)88 & 47415665 \\
8 & 4310515 \\
 & 538814\frac{3}{8}
\end{array}
$$

This and the preceding contraction are founded on the fame principles, *viz.* That if equal quantities be divided by equal quantities, the quotients will be equal; and though they may at firft view feem to be one and the fame, they will be differently applied, as may be obferved in the fequel.

Supplement

Supplement to contractions in multiplication.

1. The shortest method of multiplication, when the multiplier is any even part of 100, 1000, &c. is by division: for if the multiplicand is encreased by a number of ciphers equal to the places in the multiplier, and a part of that product taken for the proportion the multiplier bears to 1 and the same number of ciphers annexed to it, the quotient will be the true product.

Multiply 74185 into 125
125 is of 1000 $\frac{1}{8}$ wherefore
8) 74185000

9273125 Product

Multiply 4759345 into $333\frac{1}{3}$
3) 4759345000

$1586448333\frac{1}{3}$ Product.

2. To multiply by a whole number and a fraction, find the product of the integral part as before, and take parts of the multiplicand for the fraction.

Multiply 3475 by $5\frac{1}{2}$	54789	6758	Note $\frac{1}{2}$ of $3=\frac{3}{4}$ of 1.
$5\frac{1}{2}$	$4\frac{2}{3}$	$3\frac{3}{4}$	
17375	219156	4)20274	
$1737\frac{1}{2}$	$18293\frac{1}{3}$	$5068\frac{1}{2}$	
	$18293\frac{1}{3}$		
$19112\frac{1}{2}$	$255742\frac{2}{3}$	$25342\frac{1}{2}$	

3. The digit 9 hath a property peculiar to itself, that whatever other digit, with any number of ciphers annexed, is divided by it, the quotient will consist wholly of such digits, and so many 9ths of an unit over; hence the following method of multiplying by repetends,

EXAMPLES.

(1.)	(2.)	(3.)
575 by 666	4745 by 777	3987 by 5555
6000	7000	50000
9)3450000	9)33215000	9)189350000
383333	3690555	22150000
383 Subtract.	3690 Subtract.	2215
382950 Product.	3686865 Product.	22147785

This last contraction will be demonstrated immediately after division of decimals, where it will be better understood.

CHAP.

CHAP. VI.

ESSAY ON MONEY, WEIGHTS, and MEASURES.

IN the preceding chapters we have endeavoured, with as much perſpicuity and conciſeneſs as poſſible, to ſhow the properties and combinations of abſtract or pure numbers, as far as concerned the fundamental rules of arithmetic, in a method that leads gradually to eaſe, diſpatch, and certainty in calculation. But, before we can ſhow the uſe of theſe rules in matters of buſineſs, it will be proper to give ſome account of the monies, weights, and meaſures, which the Britiſh merchant hath occaſion to be acquainted with; as a proper knowledge of theſe is not only intimately connected with, but a *ſine qua non* in the mercantile buſineſs.

I. OF MONEY.

IN the firſt ages of commerce, there was little occaſion for computation, as one commodity was bartered for another by the bulk; a cuſtom which, even at this day, prevails among the ſavage unpoliſhed nations of Chili on the South ſea, in the land of Jeſſo on the Pacific ocean, and other barbarous countries: but by degrees, as improvements were made in the world, ſomething new was added daily to the conveniencies of life; and as ſuch a method of bartering commodities was found to be difficult and inconvenient, it was agreed among mankind to make choice of one commodity, which being in general and conſtant eſteem, an equivalent quantity of it might always remove the difficulty of bartering in kind. To determine therefore this ſubſtance that ſhould be in univerſal and conſtant eſteem, they made choice of gold and ſilver, not only becauſe they were diviſible and portable, but becauſe they were more valuable than other metals. Since there was a conſiderable difference in the nature of theſe two metals, and gold was more precious than ſilver, both on account of its rarity and intrinſic worth: beſides, the expenſe in working gold far exceeds the charge which attends the working ſilver, as appears by the tax paid upon each to the ſovereign lords of mints, that upon gold being only 5 of the hundred, and that on ſilver 20: it was therefore juſt to aſcribe a greater value to gold. And becauſe the baſer metal ought to be given in greater meaſure, that what was wanting in quality might be made up in quantity; it was likewiſe was likewiſe found neceſſary to fix a proportion between them by ſome certain and determinate rule; whence it is, that, in the practice of commerce, though formerly the proportion of gold to ſilver was ſettled as ten to one, yet the matter is ſo ſettled at preſent, throughout the greateſt part of Europe, that 1 ounce of gold is worth about 15 ounces of ſilver. When this ſubſtance was agreed upon at firſt to be a common equivalent for any of the conveniences of life, the particular quantity of it to be given as the value of any thing elſe, was determined by the bulk and weight only: but afterwards, to ſave the trouble of proving this weight upon every occaſion, it was coined into certain forms by public authority, and impreſſed with a mark of diſtinction, expreſſing the quantity each piece contained, ſo that it ſhould always have the ſame determined value, and be every where the ſame both for matter and weight.

In Britain, as in all other trading places, the current money or ſpecie is either gold, ſilver, or copper. The ſtandard of gold coin is 22 carats of fine gold, mixed with 2 carats of alloy, in the pound weight Troy; and the ſtandard of ſilver coin is 11 ounces and 2 pennyweights of fine ſilver, mixed with 18 pennyweights of copper. Theſe maſſes being thus proportioned, and reſpectively divided into pieces of a certain weight, upon which the current ſtamp, authoriſed by the prince, is impreſſed, conſtitute the ſeveral coins we meet with in Britain; the value whereof is determined by an imaginary piece, called the *pound Sterling*, by which we buy and ſell, and keep all accounts.

The diviſion of the pound Sterling is as follows.

Farthings, marked -	qrs.
4 = 1 penny, - -	d.
48 = 12 = 1 ſhilling.	s.
960=240=20=1 pound, -	L.

When ſhillings and pence are wrote together, they are often in figures diſtinguiſhed thus, 4\|5, *i. e.* 4 s and 5 d 10\|, *i. e.* 10 s and \|10, *i. e.* 10 d.

The coins uſed in Britain, with their value.

GOLD COINS.		L.	s.	d.
The Guinea,	=	1	1	0
Half guinea,	=	0	10	6
Quarter guinea	=	0	5	3
SILVER COINS.				
The Crown,	=	0	5	0
Half-Crown,	=	0	2	6
Shilling,	=	0	1	0
Sixpence,	=	0	0	6
COPRER COINS.				
The Halfpenny,	=	0	0	$\frac{1}{2}$
Farthing,	=	0	0	$\frac{1}{4}$

There

There are ſome gold pieces bearing the ſtamp of other countries which are likewiſe current in Britain, namely, the Moidore, =L 1, 7 s and the Joannes, =L 3, 12 s and the half and quarter ditto.

The pound of gold Troy, including the alloy, is divided into 44 $\frac{1}{2}$ parts, which are ſtamped into guineas, and into 89 parts, when ſtamped into half-guineas.

OBSERVATION.

If an exact proportion between gold and ſilver is not maintained, and fixed unalterably, according to ſome univerſal rule adopted by the generality of the European nations, the conſequence may be dangerous to a kingdom in the affair of money.

Suppoſe that in ſome particular kingdom a money-ſyſtem prevails, that ſhall raiſe the gold above its real value, and that in this regulation, inſtead of the common proportion of 1 to 15, that now obtains, an ounce of gold is allowed to be equivalent to 16 ounces of ſilver, ſince ſuch an alteration would raiſe the gold $6\frac{2}{3}$ *per cent.* above its value, and reduce the ſilver to juſt ſo much below its worth; it is evident that this increaſe of the current price of gold would naturally cauſe the ſilver to be exported out of the kingdom; and as gold would be imported in its ſtead, and increaſe greatly, the nation muſt unavoidably loſe $6\frac{2}{3}$ *per cent.* of all the ſilver that would be thus exported.

On the other hand, ſhould the ſilver money be raiſed above its value, ſo that 14 ounces ſhould be deemed equivalent to one ounce of gold, while the proportion ſhould ſtand thus, the ſilver money would not only continue in the kingdom, but alſo increaſe greatly, and the gold coin would be exported in the ſame proportion; by which means the nation would ſuſtain a loſs of $7\frac{1}{7}$ *per cent.* Moreover, from theſe variations two abſurdities would follow: the one is, that both the prince and the people would loſe of that part of their monied property $6\frac{2}{3}$ *per cent.* ſhould the above diſproportion fall upon the gold coin; and $7\frac{1}{7}$ ſhould it fall upon the ſilver. The other inconvenience would be, that there would be no ſpecie to circulate in the kingdom, but either gold only, or ſilver only, according as the one or the other of theſe metals ſhould happen to be eſtimated above its true proportion. To maintain an exact proportion therefore between gold and ſilver, are eſſential points of good conduct, with regard to the preſervation

ſervation of money, that are by no means to be neglected. There is another conſideration, however, that hath a ſurpriſing effect on the money-matters of a kingdom, and that is the balance of trade; which when it is againſt a nation, its money muſt be carried to foreign nations, to pay for the exceſs of goods imported above thoſe which have been exported; but if it is in its favour, the money will not only continue in it, but alſo increaſe and multiply.

The Laws of ENGLAND *relating to* MONEY.

By 20th *Edw.* I. merchants are prohibited from trafficking with money, and importing clipped coin, under the pain of forfeiture.

9th *Edw*, III. *c.* 1. Gold or ſilver plate, or coin, not to be exported without licence, under the pain of forfeiture. Search to be made for falſe coin imported.

Money not to be impaired in weight or alloy, 25th *Ed.* III. *c.* 13.

No coin to be current but the King's own, and any perſon may refuſe foreign coin, 27th *Edw.* III. *c.* 14.

Foreign coin not to be current in England, but to be melted down, 17th *Rich.* II. *c.* 1.

Coin or plate found in the cuſtody of perſons ready to paſs the ſeas, or in any ſhip, to be forfeited to the King, 2d *Hen.* IV. *c.* 5.

Treaſon to clip or file money, 3d *Hen.* V. *c.* 6.

Gold to be received in payment by the King's weight, 9th *Hen.* V. *c.* 11.

The mint-maſter to keep to his alloy, and to receive ſilver at the true value, on pain of double damages, 2d *Hen.* VI. *c.* 12.

Coins of gold and ſilver to continue current, notwithſtanding they may be cracked or worn, but not if they are clipped; monies clipped to be exchanged at the mint. Coin tranſported to Ireland above 6 s 8 d or Iriſh coin imported above 3 s 4 d to be forfeited. A circle to be made round the outſide of money, 14th and 15th *Hen.* VIII. *c.* 12.

Counterfeiting, impairing, &c. of coin, or foreign coin made current, is made high treaſon by 14th *Eliz. c.* 3. and 4. and by 18th *Eliz. c.* 1 and 7.

Silver coin melted down to be forfeited, and double value, 13th and 14th *Ch.* II. *c.* 31.

Gold and ſilver delivered into the mint to be aſſayed, coined, and delivered out, according to the order and time of bringing in, 18th *Ch.* II. *c.* 5.

Buying or ſelling clippings or filings, L 500 penalty. Perſons melting coin to be impriſoned ſix months, beſides forfeiture, *&c.* Perſons apprehending money coiners, *&c.* to have L 40 reward; and guilty perſons diſcovering two others, to be pardoned, 6th and 7th *Will.* III. *c.* 17.

Perſons bringing plate to the mint to be coined, not to pay for coinage, but to have the ſame weight of money delivered out. Perſons keeping public houſes to have no manufactored plate but ſpoons. Molten ſilver or bullion not to be ſhipped off, without a certificate from the lord mayor that oath hath been made that it is foreign bullion, under the penalty of L. 200; and officers may ſeize the bullion as forfeited. Gold or ſilver not exceeding L. 200,000, may be exported with a licence. Guineas not to go for more than 22 *s.* 7th and 8th *Will.* III. *c.* 19.

Hammered ſilver coin brought to the mint, to be received at 5 *s.* 4 *d. per* ounce; receivers of taxes, *&c* to receive money at 5 *s* 8*d per* ounce, to be delivered back to the bringers in; and receivers, *&c.* to be paid into the exchequer, with an allowance of the deficiency in recoining. Silver-plate, *&c.* to contain 11 ounces and 10 pennyweights of fine ſilver in every pound, and to be marked with the two initial letters of the worker's name, on pain of forfeiture. Plate received at 5 *s.* 4 *d. per* ounce to be melted down, 8th and 9th *Will.* III. *c.* 7. and 8.

It is made high treaſon, to make any ſtamp, die, mold, *&c.* for coining, excepting by perſons employed in the mint, *&c.* Conveying ſuch out of the mint, the ſame. Colouring metal reſembling coin like gold or ſilver, or marking it on the edges, is likewiſe high treaſon: and mixing blanched copper with ſilver, to make it heavier, and look like gold, or receiving or paying counterfeit milled money, is felony, 8th and 9th *Will.* III. *c* 26.

Hammered ſilver coin may be refuſed in payment, as not being the lawful coin of this kingdom, 9th, *Will.* III. *c.* 2.

Any perſon may cut, break, or deface pieces of ſilver money, ſuſpected to be counterfeit, or diminiſhed otherwiſe than by wearing; but if they ſhould, upon trial, appear to be lawful money, *&c.* to ſtand to the loſs, 9th and 10th *Will.* III. *c.* 21.

No perſon to make or coin any farthings or halfpence, or pieces to go for ſuch, of copper, under the penalty of L. 5 for every pound weight, 9th and 10th *Will.* *c.* 33.

On

On a ſcarcity of ſilver coin, for remedy guineas were ſunk to 21 *s*. by proclamation 3d *Geo.* I.

Perſons counterfeiting broad pieces of gold, or uttering them knowingly, to be guilty of treaſon, 6th *Geo.* II. *c.* 26.

Waſhing, gilding, or altering the impreſſion of any real or counterfeit ſhilling, or ſixpence, or braſs-money, to make the one paſs for a guinea, or half-guinea, or the other for a ſhilling or ſixpence, is high treaſon. Knowingly uttering falſe money, for the firſt offence ſix months impriſonment, for the ſecond two years impriſonment, and for the third felony without benefit of clergy If any perſon, knowingly uttering falſe money, ſhall have about him any other falſe money, he ſhall ſuffer one year's impriſonment; and coiners of halfpence or farthings, two years impriſonment, *&c.* 15th *Geo* II. *c* 28.

Quarter-guineas were ordered to be coined in the 1ſt *Geo.* III. of which ſome had been ſtruck in the reign of *Geo* I. but were become ſo rare, that they were ſcarcely to be met with.

II. OF WEIGHTS.

As the ſecurity of commerce depends much on the juſtneſs of weights, moſt nations have taken care to prevent their being falſified. The ſtandard of weights in Britain is kept in the exchequer, by a particular officer, called the *clerk* or *comptroller of the market*. By the 27th chapter of *Magna Charta*, the weights are to be the ſame all over England; but as commerce flouriſhed, and introduced a greater variety of commodities, it was found convenient to vary the original weight, and likewiſe invent others better calculated for diſpatch in buſineſs, which hath introduced a diverſity of weights, in almoſt every different county or province. The firſt of all the weights uſed in Britain was a grain of wheat, picked out of the middle of the ear, which, being well dried, became the leaſt denomination of Troy weight, now uſed for gold, ſilver, jewels, ſeeds, liquors, bread, and medicines.

TABLE OF TROY WEIGHT.

Grains.				
24 =	1 pennyweight,			dwt.
480 =	20	1 ounce,		.
5760 =	240 =	12 =	1 pound,	lb.

 APOTHE-

APOTHECARIES WEIGHT.

Is deduced from Troy; but convenience taught them to vary the division, for compounding their medicines, according to the following

TABLE.

Grains.				
20 =	1 scruple,			
60 =	3 =	1 dram,		
480 =	24 =	8 =	1 ounce,	
5760 =	288 =	96 =	12 =	1 pound, lb.

AVOIRDUPOISE WEIGHT.

Was rather introduced by chance, and confirmed by custom, than fixed by any law. The Troy weight was in practice found to be too small for coarse and heavy goods, such as grocery wares, pitch, tar, rosin, wax, tallow, flax, hemp, &c. copper, tin, iron, lead, steel, fish, flesh, butter, cheese, salt, &c.; for which, and other such goods, it was thought proper to allow a greater weight than the law had provided, which in this weight exceeds the Troy by $\frac{1}{6}$, one pound Avoirdupoise being equal to 1 lb. 2 oz. 11 dwt. 15$\frac{1}{2}$ grains Troy. In lead, they give only 19$\frac{1}{2}$ cwt. to the ton or fodder.

TABLE of Avoirdupoise Greater Weight.

lb.			
28 =	1 quarter, qr.		
112 =	4 =	1 hundred weight, cwt.	
2240 =	80 =	20 =	1 tun.

Ditto Lesser Weight.

Drops.			
16 =	1 ounce.		
256 =	16 =	1 pound.	
3584 =	224 =	14 =	1 stone.

After the union, when the weights in Scotland were attempted to be reduced to English standard, it was found that the Scotch Troy pound was equal to 7600 grains; and the English Avoirdupoise to 7000: hence, the Scotch, Paris, or Amsterdam pound will be to the pound Avoirdupoise as 38 to 35. Besides the Scotch Troy weight, commonly known by the name of *Dutch weight*, whereof a table is subjoined, there is another weight derived from it, called *Tron weight*, which, in different places, consists of a heavier or lighter pound,

pound, according as custom hath established it. The pound Tron weight runs from 20 to 24 ounces generally, and in some places, but rarely, falls as low as 19 ounces.

TABLE of Scotch Troy, or Dutch Weight.

Grains.
36= 1 drop.
576= 16= 1 oz.
9216= 256= 16= 1 lb.
147456=4096=256=16=1 stone.

Note, The Tron weight is divided in the same manner as the Troy in the adjacent table, excepting the pound into ounces, of which there is no certain regulation.

Wool weight is founded on, and derived from Avoirdupoise, the pound in both being the same, but the greater weights different.

TABLE OF WOOL WEIGHT.

Pounds.

7= 1 clove.
14= 2= 1 stone.
28= 4= 2= 1 todd.
182= 26= 13= 6½= 1 wey.
364= 52= 26= 13 = 2= 1 sack.
4368=624=312=156=24=12= 1 last.

Note 1. Some few authors make 8 pounds in the clove.

2. Woolstaplers generally purchase their wool by the todd, but sell it again, when sorted and stapled, by the pack, consisting of 240 pounds.

The REFINERS WEIGHTS.

Blanks.
24= 1 perrot.
480= 20= 1 mite.
9600=400=20=1 grain.

Note, What they denominate *carats* are the $\frac{1}{24}$ of a lb, an oz., or any other weight.

The Weights for Merchandizes used in Holland.

24 grains = 1 drachm: 3 drachms, or 72 grains, = 1 gross; 30 grains = 1 engel; 10 engels, or 4 gross and 2 grains, = 1 loot;

1 loot; 16 loots, or 8 ounces, = 1 mark; 2 marks = 1 pound; 8 pounds = 1 ſtone; 165 pounds = 1 waggon, or wage; 400 pounds = 1 load; 15 pounds = 1 liſpond; 20 liſponds = 1 ſchippond.

DUTCH WEIGHTS for GOLD and SILVER.

32 aces = 1 engel, 20 engels = 1 ounce, 8 ounces = 1 mark, for groſs gold.

24 parts = 1 grain, 12 grains = 1 carat, 24 carats = 1 mark, for fine gold.

Note, The mark weights are about 1 *per cent.* lighter than the Troy weight of London.

WEIGHTS for MERCHANDISE uſed in HAMBURG.

2 loots = 1 ounce, 16 ounces = 1 pound, 10 pounds = 1 ſtone of wool or feathers, 14 pounds = 1 diſpound, 20 pounds = 1 ſtone of flax, 8 diſpound = 1 center = 120 pounds Avoirdupoiſe of London. 16 pounds of feathers or wool is a diſpound, and 20 diſpounds = 1 ſchippound of the ſame. 16 diſpounds = 1 tun of butter or tallow.

100 pounds of Hamburg = 98 of Amſterdam = $103\frac{1}{2}$ of Antwerp = $107\frac{1}{4}$ of London. See the table in the compariſon of weights and meaſures.

III. MEASURES.

The ſame neceſſity that introduced money and weights, may juſtly be ſuppoſed to have introduced meaſures; as the moſt certain method of aſcertaining quantities bought and ſold. The common meaſures uſed in Britain are thoſe which follow.

I. CLOTH MEASURE, which is of four kinds.

1. The yard = 4 quarters = 16 nails = 36 inches; by which are meaſured and ſold all kinds of Engliſh woolen cloths, linen, wrought ſilks, tape, &c.

2. The Engliſh ell = $1\frac{1}{4}$ yard = 20 nails = 45 inches, by which is meaſured chiefly a ſpecies of fine linen called *Holland*.

3. The Flemiſh ell = $\frac{3}{4}$ yard = 12 nails = 27 inches, chiefly applied to tapeſtry.

4. The

4. The Scotch ell=$1\frac{1}{36}$ yard=37 inches, by which green linen, and most of the private or house manufactures in the country, are bought and sold.

II. CORN MEASURES, which are of two kinds.

1. English, in which the pint is found to weigh 1 pound Troy, and the several denominations are as follow.

TABLE of ENGLISH DRY MEASURE.

Pints.

8= 1 gallon.
16= 2= 1 peck.
64= 8= 4= 1 bushel.
256= 32= 16= 4= 1 comb.
512= 64= 32= 8= 2= 1 quarter.
2560=320=160=40=10= 5=1 wey.
5120=640=320=80=20=10=2= 1 last.

Corn, salt, coals, lead-ore, and other dry goods, are measured according to this table. Corn is generally sold in England by the quarter, 5 whereof are reckoned to the ton of freight. The ton of wheat weighs between 2200 and 2500 pounds Avoirdupoise; of rye, between 2100 and 2240 pounds ditto; and of barley, between 1700 and 1800

By the standard in his Majesty's exchequer, every round bushel with a plain and even bottom, $18\frac{1}{2}$ inches wide throughout, and 8 inches deep, is esteemed a legal corn bushel, and will contain $2150\frac{21}{50}$ cubic inches; consequently the corn-gallon contains $268\frac{4}{5}$ cubic inches.

2. Scotch, in which the boll of meal weighs 8, and amongst the farmers in the north, frequently 9 stone Dutch, or Scotch Troy weight; is divided as follows.

Lippies.

4= 1 peck.
16= 4= 1 firlot.
64= 16= 4= 1 boll.
1024=256=64=16=1 chalder.

Note, The wheat firlot in Scotland contains $21\frac{1}{4}$ pints Scotch measure, and the bear or barley firlot 31; hence the Scotch wheat firlot is to the English corn bushel as 100 to $99\frac{21}{500}$.

3. Liquid

3. Liquid meaſure was originally raiſed from Troy weight, as is evident from ſeveral ſtatutes, enacting, that 8 pounds Troy of wheat, properly prepared, ſhould weigh one gallon of wine-meaſure, the diviſions and multiples whereof ſhould form the other meaſures, and be the common ſtandard throughout the whole kingdom; yet the invention of a new weight introduced likewiſe a new gallon adjuſted thereto, exceeding the former in the proportion of Avoirdupoiſe weight to Troy, which ſerves to proportion the ſeveral denominations of ale and beer meaſure. The ſealed gallon at Guildhall, which is the ſtandard for wines, ſpirits, mead, perry, cyder, vinegar, honey, oil, *&c.* is ſuppoſed to contain 231 cubic inches, on which ſuppoſition, the other meaſures raiſed therefrom will contain proportionally; yet, by actual experiment, made in 1688, in preſence of the Lord Mayor and commiſſioners of exciſe, this gallon was only found to contain 224 cubic inches; notwithſtanding it was agreed to continue the computation, upon the ſuppoſition of 231 cubic inches to the gallon, as before.

TABLE of WINE MEASURE.

Solid inches.

$231 = 1$ gallon.
$4158 = 18 = 1$ runlet.
$7276\frac{1}{2} = 31\frac{1}{2} = 1\frac{3}{4} = 1$ barrel.
$9702 = 42 = 2\frac{1}{3} = 1\frac{1}{2} = 1$ terce,
$14553 = 63 = 3\frac{1}{3} = 2 = 1\frac{1}{2} = 1$ hogſhead, (*hhd.*)
$19279 = 84 = 4\frac{2}{3} = 2\frac{2}{3} = 2 = 1\frac{1}{2} = 1$ puncheon.
$29106 = 126 = 7 = 4 = 3 = 2\frac{1}{4} = 1\frac{1}{2} = 1$ butt.
$58212 = 252 = 14 = 8 = 6 = 4\frac{1}{2} = 3 = 2 = 1$ tun.

Ale and beer meaſure, as was formerly obſerved, is deduced from the Avoirdupoiſe weight, and therefore the gallon muſt be much larger than the gallon in wine-meaſure. The ſtandard ale-quart, kept in the exchequer, hath been found by experiment to contain juſt $70\frac{1}{2}$ cubic inches; conſequently the ale-gallon muſt contain 282 cubic inches. Hence

ALE-MEASURE TABLE.

Cubic inches.

282= 1 gallon.
2256= 8=1 firkin.
4512=16=2=1 kilderkin.
9024=32=4=2=1 barrel.
13536=48=6=3=1½=1 hhd.

TABLE of BEER-MEASURE.

Solid inches.

282= 1 gallon.
2538= 9= 1 firkin.
5076= 18= 2=1 kilderkin.
10152= 36= 4=2=1 barrel.
15228= 54= 6=3=1½=1 hogſhead.
30456=108=12=6=3 =2=1 butt.

Note, This diſtinction, or difference betwixt ale and beer meaſure, is only uſed in London; for, in all other places, the following table of beer or ale, whether ſtrong or ſmall, is to be obſerved according to a ſtatute of exciſe, made in the year 1689.

Note, in meaſuring ſoap and herrings, 8 gallons is conſidered as a firkin.

Cubic inches.

35¼= 1 pint.
282 = 8= 1 gallon.
2397 = 68= 8½=1 firkin.
4794 =136=17 =2=1 kilderkin.
9588 =272=34 =4=2=1 barrel.
14382 =408=51 =6=3=1½ =1 hogſhead.

In Scotland, the exciſe and breweries uſe the Engliſh meaſure; but retailers and victuallers in the country uſe the Scotch pint, of 103⅖ ſolid inches, whoſe diviſions and multiples are as follow.

4 gills = 1 mutchkin, 2 mutchkins = 1 chopin, 2 chopins = 1 pint, 2 pints = 1 quart, and 4 quarts = 1 gallon.

4. Long meaſure, among other improvements, took its riſe from wheat, three grains of which, properly prepared, were, in length, made the meaſure of an inch, as in the

TABLE.

Grains.						
3=	1 inch.					
36 =	12=	1 foot.				
108=	36=	3 =	1 yard.			
594=	198=	$16\frac{1}{2}$=	$5\frac{1}{2}$=	1 pole.		
23760=	7920=	660 =	220 =	40=1 furlong.		
190080=	63360=	5280 =	1760 =	320=	8=1 mile.	

Note, 4 poles, or 22 yards, is the length of Gunter's chain, conſiſting of 100 links. each link = $7\frac{92}{100}$ inches. But the chain for ſurveying in Scotland ſhould be 74 feet.

Note. 3 miles = 1 league, and 20 leagues = 1 degree, by common reckonings; but a degree of a great circle, meaſured upon the ſurface of the earth, has been found, by the beſt geographers, to be equal to $69\frac{1}{2}$ Engliſh miles = 25 French leagues.

5. Square meaſure was founded upon long meaſure, and is differently divided in England and in Scotland.

In Engliſh ſquare meaſure, 144 ſquare inches = 1 foot ſquare; 9 feet ſquare = 1 ſquare yard; $30\frac{1}{4}$ ſquare yards = 1 pole; 40 poles = 1 rood; and 4 roods = 1 acre.

Though the ſtatute pole be $16\frac{1}{2}$ feet, in meaſuring fens and woodlands, they uſe a pole of 18 feet, and for foreſts 21 feet. 40 poles in length and 4 in breadth, or 220 yards in length, and 22 in breath, make a ſtatute acre. The French acre, or arpens, is $26\frac{1}{8}$ yards in breadth, and $261\frac{1}{8}$ yards in length, and is to our ſtatute acre as 19 to 16.

Maſons meaſure their hewn work by the Engliſh foot, painters and plaſterers by the yard. Glaziers reckon only 8 inches to their lineal foot, and 64 to their ſquare foot.

The ſquare meaſure uſed in Scotland is thus divided.

36 ſquare ells=1 fall, 40 falls=1 rood, and 4 roods=1 acre.

The Scotch acre by ſtatute is to the Engliſh as 100,000 to 78694.

In Scotland, ſlaters, maſons, and paviers uſe the ſquare ell and the fall in meaſuring their work, and the land-ſurveyors, the fall, the rood, and the acre.

There are ſome commodities ſold by the dozen, of which we reckon 12 = 1 dozen, 12 dozen = 1 ſmall groſs, and 12 ſmall groſs = 1 great groſs.

Paper

Paper is ſold by the following denominations, in which 24 ſheets = 1 quire, 20 quires = 1 ream, and 10 reams = 1 bale.

Parchment thus: 12 ſkins = 1 dozen, 15 dozen = 1 roll.

Yarn thus: 120 threads = 1 cut, 2 cuts = 1 heer, 6 heers = 1 hank, and 4 hanks = 1 ſpindle.

In glaſs, 5 pounds = 1 ſtone, and 24 ſtone = 1 ſeam.

The freight of bale goods is often determined by the tunnage, in which 40 ſolid feet are reckoned = 1 tun.

Having thus given a brief and ſuccinct account of the meaſures uſed in Great Britain, for the benefit of the dealer in foreign ſpirits, *&c.* I ſhall add Mr. Poſtlethwayt's account of the different meaſures and veſſels uſed in moſt parts of the world.

Of Measures for Wine and Vinegar.

" The veſſels for containing wine and brandy have different names, according to the quantities they contain, and the countries where they are made uſe of.

The veſſel called in Germany *woeder*, made uſe of for keeping the wines that grow upon the Rhine and the Moſelle, do ordinarily contain 14 aams of Amſterdam, but ſometimes they contain more and ſometimes leſs.

The aam of Amſterdam is a meaſure of 4 anckers, reckoning the ancker of 2 ſteckans.

The ſteckan contains 16 mingles, each of which makes two pints.

The verge, or verteel, of the wines upon the Rhine and the Moſelle, *&c.* is reckoned but 6 mingles, that of brandy is counted $6\frac{1}{6}$ mingles, as we ſhall ſee hereafter.

The hogſhead of Bourdeaux, according to the juſt meaſure, ſhould contain $12\frac{1}{2}$ ſteckans, or 200 mingles of wine and lee; and 12 ſteckans, or 192 mingles clear wine; ſo that the tun of Bourdeaux, conſiſting of 4 hogſheads, contains 50 ſteckans, or 800 mingles, wine and lee, and 48 ſteckans, or 768 mingles clear wine.

The tun of Bayonne, and other places thereabout, is reckoned 240 ſteckans, meaſure of Amſterdam, there being likewiſe 4 hogſheads to a tun.

In England, and eſpecially at London, they reckon the hogſhead 63, and the tun 252 gallons. The ſaid gallon weighs $7\frac{1}{2}$ pounds weight of London; ſo that the 63 gallons, or the hogſhead, ſhould weigh $472\frac{1}{2}$ pounds, and the tun

1890 pounds weight of London. The ſaid gallon is ſaid to contain 4 Paris pints.

The hogſhead of Bourdeaux ſhould contain 110 pots with the lee, and 100 pots clear wine, meaſure of the ſaid place; ſo that the ſaid pot of Bourdeaux contains about 2 mingles of Amſterdam.

The Bourdeaux tun of wine ſhould weigh, with the hogſhead, 2000 pounds weight; and in marine terms in freighting of ſhips, by a tun is meant 2000 pounds weight; ſo that when it is ſaid any ſhip is of ſo many tuns, it is to be underſtood that the ſhip can carry ſo many times 2000 pounds weight: though in Holland, Flanders, and other northern countries, they only talk of laſts, containing 2 tuns each, or 4000 pounds weight.

The Rheniſh and Moſelle wines are ordinarily ſold at Amſterdam, the former at ſo many florins of about 20 each, current money, and the latter ſo many rixdollars, of 50 ſtivers each, current money, for the aam of 20 verges or verteels, the verge being, at that rate, 6 mingles, as already ſaid.

French, Spaniſh, and Portugal wines are ſold at ſo many pound groſs the tun of 4 hogſheads, and there is ordinarily 1 *per cent.* rebate for payment in ready money, both buyer and ſeller paying brockerage, each 6 ſtivers *per* tun.

The muid of Paris contains 150 quarts, or 300 pints with the lee, and 280 pints clear wine, meaſure of Paris.

There are all over France a great many veſſels for keeping of wine, different from one another, according to the cuſtom of the ſeveral provinces where they are made uſe of; of which, though there be ſcarce any poſſibility to give an exact account we ſhall here ſet down the regular fractions of the muid of Paris, 3 of which make the tun of France; and, as we have occaſion to ſpeak of the meaſures of the other provinces of France, we ſhall give as diſtinct an account as we can of their contents.

The pint of Paris is a meaſure pretty well known all over the world; 2 of thoſe pints make 1 quart, 4 quarts 1 ſextier, and 36 ſextiers 1 muid of Paris: 3 of which (as is already ſaid) make 1 tun of France.

The meaſure they make uſe of in Provence is called a *millerole*; that of Thoulouſe ſhould weigh 130 pounds, and ought to contain 66 Paris Pints, which is about 100 pints of Amſterdam.

At

At Montpelier, and ſeveral other places of Languedoc, their muid contains 18 ſextiers, and the ſextier 32 pots; ſo that the muid, which makes but 35 ſteckans, or 560 mingles of Amſterdam, makes 756 pots of Montpelier; by which it appears that the pot of Montpelier is $\frac{1}{18}$ leſs than the mingle of Amſterdam However, you muſt here take notice that the caſks of Montpelier are not all of an equal meaſure, ſome being bigger than others; and in ſeveral places of Provence, as well as High and Low Languedoc, they frequently tranſport wines, oils, and other ſuch goods, in veſſels made of goat-ſkins.

The butts and pipes of Seville, Malaga, Alicant, Liſbon, Port a Port, Canaries, and iſles of Fagel, &c. are likewiſe of different ſizes; for the tun of Malaga, conſiſting of 2 butts or pipes, (which they call *perſemyn* at Amſterdam), is reckoned only 36 or 37 ſteckans; and thoſe of other places are reckoned at 25 or 26 ſteckans the butt or pipe.

As for the wine at Hamburg and Lubeck, it is ſold at ſo many rixdollars, of 48 ſtivers, or 3 marks lubs *per* tun.

Of BRANDY.

French, Spaniſh, and Portugueſe brandy, is ordinarily put into big caſks, which ſome call *pipes*, others *butts*, others *pieces*, *viz.* according to the cuſtom of the places, there being no poſitive meaſure regulated for that liquor.

In France it is ordinarily put into great caſks, which they call at Bourdeaux *pieces*, at Rochelle, Nantes, Cognac, Montguion, the iſle of Rhé, &c. *pipes*, which (as we have already ſaid) contain ſome more than others, there being ſome which hold at Amſterdam from 60 to 90 verges, or verteels; and they reduce thoſe meaſures into hogſheads, by reckoning as under, for

1 hogſhead makes
- 27 verges of Cognac, Montguion, Rochelle, and the iſle of Rhé.
- 29 of Nantes, and other places in Britany and Anjou.
- 32 of Bourdeaux, and other places in Guienne.
- 32 of Bayonne, and places thereabouts.
- 30 of Amſterdam, and other places of Holland.
- 30 of Hamburg and Lubeck.
- 27 of Embden.

In Provence and Languedoc, they ſell it at ſo much the quintal, or 100 weight with the caſk.

At

At Bruges they call the verges *ſeſters*, of 16 ſtoups to a ſeſter, which they ſell at ſo much a ſtoup.

At London, and generally through all England, they count only by gallons, as we have ſaid already.

The mingle of brandy weighs at Amſterdam, 2 pounds 4 ounces; and the verge, or verteel, about 14 pounds; at which rate, the 30 verges muſt weigh about 420 pounds.

At Bourdeaux, though pieces of brandy contain from 50 to 90 verges, they reckon but 32 to the hogſhead; the verge is ſomething leſs than $3\frac{1}{2}$ pots.

You muſt know that whatever there is at Bourdeaux in a piece of brandy more than 50 verges, is called by the farmers of the King's duty *exces*, or an exceſs, and pays ſo much *per* verge, beſides the duties of ſortie, or exportation (as they call it) for the 50 verges.

Thoſe that make brandy ſeldom or never put it in ſmall barrels, or tierces, except it be deſigned for ſome particular places in America, or elſewhere, where thoſe ſmall meaſures are advantageouſly ſold to people, who, perhaps, would not be able to buy a pipe at a time; for a piece of brandy that contains perhaps $1\frac{1}{2}$ of an ordinary piece, coſts but very little more of freight and carriage than one that contains $\frac{1}{2}$ or $\frac{1}{3}$ leſs.

At Hamburg it is likewiſe ſold at ſo many pounds groſs, of $7\frac{1}{2}$ marks lubs *per* pound groſs, or at ſo many rixdollars in Banco; but at Lubeck it is paid in current money there being no bank.

At Bremen, Copenhagen, and Embden, it is alſo ſold at ſo many rixdollars; and in this laſt place the hogſhead is counted but 27 verges.

At London it is ſold by the tun of 252 gallons; and, in ſhort, in every country according to the cuſtom of the country, which muſt always be ſtrictly inquired into by the dealers for their government.

Of Measures for Oil of Olives.

The oil of olives is ordinarily kept in butts or pipes, containing from 20 to 25 ſteckans, at 16 mingles a ſteckan; and there go 717 mingles, or 1434 pints of Amſterdam, to the tun of oil. They reckon at Genoa that the barrel of oil of olives weighs $187\frac{1}{2}$ pounds nett, of their weights, which make 125 pounds of Amſterdam; and 14 barrels make 717 mingles of the ſaid place, or thereabout.

At

At Leghorn, the barrel of oil of olives weighs 85 pounds of their weight, which is a little more than 59 pounds of Amſterdam.

In Provence they ſell it by the meaſure of that country, called *millerole*, containing 66 Paris pints, which make about 100 pints of Amſterdam; and, in ſome places of that country and of Lower Languedoc, they put it in certain veſſels made of goat-ſkins, as they do the wine.

In Spain and Portugal it is put in butts and pipes, to be carried over ſeas, and ſometimes in great earthen veſſels called *jars*.

Of Measures for Fish Oil.

Coarſe fiſh oil is ordinarily kept in barrels, containing from 15 to 20 ſteckans each.

Of Measures for Honey.

Honey is kept in many different ſorts of veſſels of wood and earth, and ſold in ſome places by meaſure, and in other places by weight.

At Amſterdam they ſell it at ſo many pounds groſs *per* ton conſiſting of 6 tierces or aams, or by ſo many florins the barrel, or the 100 weight. The Bourdeaux and Bayonne honey is ſold at Amſterdam from 30 to 40 l. groſs the tun.

Of Round Measures for Grain, &c.

As the great diverſity of meaſures of capacity renders it very troubleſome for merchants to calculate the quantities thereof, it will be very neceſſary to give an account of thoſe that are uſed in the principal places of Europe for trade.

The laſt is of ſeveral ſorts, but all comprehended in theſe two, *viz.* the ſea laſt, and that uſed by land.

A laſt is reckoned at ſea both with regard to meaſure and weight, according to the nature of the goods.

In meaſure, there are allowed to a laſt of goods 2 tuns, or 8 hogſheads of wine, 5 pieces of brandy, or prunes, 12 barrels of herrings or peaſe, 13 barrels of pitch, 4 pipes or butts of oil of olives, and ſeven quarters or barrels of fiſh oil.

By weight, there is generally allowed to the laſt 4000 lb. but, as wool is bulky, they reckon only 2000 lb. to the laſt thereof,

thereof, and 3600 lb. of almonds and ſo likewiſe they make ſome abatements of ſeveral other ſorts of goods in proportion to their bulk.

The land-laſt is not the ſame in all places, there being ſome difference introduced by cuſtom in the ſeveral countries of Europe.

Of the MEASURES of CAPACITY of AMSTERDAM and HOLLAND, &c.

The laſt of Amſterdam contains 27 muds, and each mud 4 ſcheppels.

Or, otherwiſe, the laſt of Amſterdam contains 36 ſacks, and the ſack 3 ſcheppels.

So that the mud is $\frac{1}{3}$ of the ſcheppel, and the ſcheppel is only $\frac{3}{4}$ of the mud.

A laſt of wheat commonly weighs between 4200 and 4800 lb. rye between 4000 and 4200 lb. and barley between 3200 and 3400 lb.

But thoſe commodities are ſo much ſubject to alteration, by their humidity, &c. that there is but little certainty in their weight.

The laſt of Amſterdam makes 19 ſextiers of Paris, or 38 buſhels of Bourdeaux; and 3 laſts make 4 muds of Rouen.

The laſt of Munickendam, Edam, Purmeran, and ſeveral other places of North Holland, is reckoned equal to that of Amſterdam.

But that of Hoorn and Enchuyſen, being likewiſe towns in North Holland, is of 22 muds, or 44 ſacks, of 2 ſcheppels each; and ſo is that of Muyden, Naerden, and Weeſoop, ſmall towns in the neighbourhood of Amſterdam.

At Haerlem they reckon 38 ſacks to the laſt, their ſacks conſiſting of 3 ſcheppels, 4 of which make 1 hoedt of Rotterdam, and 14 of thoſe ſacks make 1 hoedt of Delft.

The laſt of Alkmaer, in North Holland, contains 26 ſacks.

They reckon 44 ſacks to the laſt of Leyden, and 8 ſcheppels to the ſack.

The laſt of Rotterdam, Delft, and Schiedam, is compoſed of 29 ſacks, and the ſack of 3 ſcheppels, of which $10\frac{2}{3}$ make 1 hoedt; where it is to be obſerved, that the laſt of thoſe places is 2 *per cent.* more than that of Amſterdam.

At Tergow they reckon 28 ſacks to the laſt, 3 ſcheppels to the ſack, and 32 ſcheppels to the hoedt.

Of the LAST of UTRECHT.

At Utrecht they reckon 25 muds, or ſacks, to the laſt, 10½ of which ſacks make 1 hoedt of Rotterdam.

The laſt of Amersfort is compoſed of 64 ſcheppels.

That of Montfoort, Yſſelſteid, Vianen, *&c.* is greater than that of Rotterdam; it is compoſed of 18 muds, and the mud of 2 ſacks.

Of the LAST of FRIESLAND.

The laſt of Leeuwarden, Haerlingen, and other towns of Weſt Frieſland, is compoſed of 33 muds.

And that of Groningen in Eaſt Frieſland is of the ſame meaſure.

Of the LAST of GUELDERLAND, and county of CLEVES.

The laſt of Nimeguen, Arnham, and Dreſburg, is compoſed of 22 mouvers, and the mouver of 4 ſcheppels, 8 of which mouvers make 1 hoedt of Rotterdam.

At Thiel they reckon 33 ſcheppels to the laſt.

At Burenande 68 ſcheppels.

At Haerderwick they reckon 11 muds to 10 of Amſterdam

Of the LAST of OVER-YSSEL.

The laſt of Campen is of 25 muds for corn, 9 of which make 1 hoedt of Rotterdam.

And 9 muids of Zwoll make likewiſe 1 hoedt of Rotterdam.

The laſt of Deventer contains 36 muids of 4 ſcheppels each.

Of the LAST of ZELAND.

The laſt of Middleburg is compoſed of 4½ ſacks of 2 ſcheppels each, or a little more; and that of Fleſſing, Zirickzee, the Brill, and ſome other places, is ſomewhat different from it, the ſack being there reckoned 2½ ſcheppels.

Of the LAST of BRABANT.

The laſt of Antwerp is compoſed of 38 verteels, of which 37½ make 1 laſt of Amſterdam.

Their verteel is compoſed of 4 mukens, and 32 verteels made the ſack for oats.

At Bruſſels they reckon 25 ſacks equal to the laſt of Amſterdam.

At Malines they reckon 28 verteels equal to the laſt of Amſterdam.

The laſt of Louvain is compoſed of 37 muds, and each mud of 8 halſters.

At Breda and Steenbergue they reckon $33\frac{1}{2}$ verteels to the corn laſt, and 29 for oats; and 13 verteels make 8 ſacks, or 1 hoedt of Rotterdam.

At Bergen-op-zoom, they allow 34 verteels to the laſt of corn, and $28\frac{1}{4}$ for oats.

That of Bois-le-duc is compoſed of $20\frac{1}{2}$ mouvers, 8 of which make 1 hoedt of Amſterdam.

Of the LAST of ſeveral towns in FLANDERS.

The laſt of Ghent is compoſed of 56 halſters for corn, and of 38 for oats. Their mud is compoſed of 6 ſacks, each ſack of 2 halſters.

At Bruges, the laſt is compoſed $17\frac{1}{2}$ hoedts for corn, and $14\frac{1}{2}$ for oats, equal to the laſt of Amſterdam.

At St. Omers, the laſt is reckoned $22\frac{1}{2}$ raziers, the razier conſiſting of 2 ſcheppels.

At Dixmude, they reckon $30\frac{1}{2}$ raziers to the laſt of wheat, and 24 for oats.

At L'Iſle, they reckon 41 raziers to the laſt of wheat, and 30 for oats.

At Gravelin, they reckon $22\frac{1}{2}$ raziers to the laſt of corn, and $18\frac{3}{4}$ for oats.

Eighteen raziers of Dunkirk are equal to one hoedt of Rotterdam.

Of the LAST of LIEGE.

The laſt of Liege is compoſed of 96 ſextiers, of 8 muds each: they reckon the corn laſt of Tongres 15 muds, and that for oats but 14.

Of the LAST of GREAT BRITAIN and IRELAND.

The laſt of London conſiſts of $10\frac{1}{2}$ quarters, or barrels, compoſed of 8 buſhels each, and the buſhel of 4 gallons.

The buſhel weighs between 56 and 60 lb. and 10 buſhels of England make about 1 laſt of Amſterdam.

In Scotland they reckon 38 buſhels to the laſt, and 18 gallons to the buſhel; and in Ireland the ſame thing.

Of

Of the Last of Dantzick.

At Dantzick they reckon 36 ſcheppels to the laſt, which is equal to 58 ſcheppels of Amſterdam.

They likewiſe reckon 16 ſchippondts to the laſt, and 340 lb. to the ſchippondt, which makes 5440 lb. to the laſt; but they give only 15 ſchippondts, or 5100 lb. weight, the laſt of oats.

They likewiſe divide their laſt at Dantzick into 16 ſextiers, meaſure of Paris, or 20 buſhels of Bourdeaux.

They buy and ſell their corn at Dantzick, as every other thing, by Poliſh florins and groſs.

Of the Last of Riga.

At Riga they reckon 46 loopen to be equal to the laſt of Amſterdam; and they buy and ſell it by rixdollars of 3 florins, or 90 Poliſh groſs.

Of the Last of Koningsberg.

Six laſts of that place are equal to 7 of Amſterdam.

Of the Last of Copenhagen.

They have there ſeveral laſts, which differ from one another conſiderably, according to the different ſorts of grain, or other commodities that are meaſured by them. Ricard makes mention of three ſeveral ſorts of laſts uſual in Copenhagen, viz. of 42 barrels, of 80 ſcheppels, and of 96 ſcheppels.

Of the Last of Stockholm.

At Stockholm they reckon 23 barrels to the Last.

Of the Last of Hamburg, Bremen, and Embden.

The laſt of Hamburg conſiſts of 90 ſcheppels.

At Bremen they reckon 40 ſcheppels to the laſt; and 8 laſts of Bremen have held out to 7 laſts, 18 muds, and 1 ſcheppel at Amſterdam.

At Embden they reckon 15½ barrels to the laſt.

Of the Muid, *&c.* of France.

The principal meaſure made uſe of for grain, *&c.* at Paris, and moſt other places of the kingdom, is called *muid*.

 The

The muid contains 12 ſextiers, and the ſextier 12 buſhels.

The ſextier of good wheat weighs between 244 and 248 lb. marc weight.

They divide the ſextier of oats into 24 buſhels, which again are ſubdivided into ſeveral ſmaller meaſures.

Nineteen ſextiers of Paris are reckoned equal to 1 laſt of Amſterdam.

The muid of Rouen contains 12 ſextiers, which are equal to 14 of Paris: it ought to weigh about 3360 lb. marc weight, and makes 28 buſhels of Bourdeaux.

Four muids of Rouen are reckoned equal to 3 laſts of Amſterdam.

The ſextier of corn weighs 210 lb. weight of Rouen, and is divided into two mines, and the mine into 4 buſhels.

The muid of Orleans ought to weigh 600 lb. and is compoſed of 12 mines, equal to $2\frac{1}{2}$ ſextiers of Paris, or 5 buſhels of Bourdeaux.

The meaſure made uſe of at Lyons, called *aſnée*, is divided into 6 buſhels, equal to $1\frac{1}{3}$ ſextier, meaſure of Paris, or $2\frac{1}{2}$ buſhels of Bourdeaux.

Eight buſhels of Rouen make 1 ſextier of Paris, and 2 buſhels of Bourdeaux.

The aſnée of Macon makes $1\frac{2}{3}$ ſextiers of Paris, or $3\frac{1}{3}$ buſhels of Bourdeaux.

The 5 buſhels of Avignon make 3 ſextiers of Paris, and 6 buſhels of Bourdeaux.

The ſextier of Montpelier is compoſed of 2 emines, and the emine of 2 quarters. The ſextier, weighing between 90 and 95 lb. weight of that town, being between 75 and 80 lb. marc weight; ſo that 100 ſextiers make 1 laſt 22 muds of Amſterdam.

The ſextier of Caſtres is compoſed of 2 emines, and the emine of 16 buſhels. The ſextier weighs about 200 lb. weight of that place, which is about 170 lb. marc weight; ſo that it may be reckoned that 1001 ſextiers of Caſtres make 4 laſts of Amſterdam.

The ſextier of Abbeville is compoſed of 16 buſhels, and is equal to that of Paris.

The ſextier of Amiens weighs from 50 to 52 lb. and 5 ſextiers.

The ſextier of Bologne weighs 270 lb. ſmall weight; and 8 ſextiers of that place render 5 of Paris.

The ſextier of Calais weighs 260 lb. and 12 of them render 13 of Paris.

Which

Which ſextier of Paris renders

At	St Valery	1 ſextier.
	Dieppe	18 mines.
	Havre de Grace	5¼ buſhels.
	Amboiſe	14 buſhels.
	Saumur	1 buſhel.
	Tours	14 buſhels.
	Blois	20 buſhels.
	Aubeterre	5 buſhels.
	Barbeſieux	5 buſhels.
	Periguex	5 buſhels.

The ſextier of Arles weighs only 93 lb. marc weight, and the load is 360 lb. weight of that country.

The load of Beaucaire is 2 *per cent.* greater than that of Arles.

The load of Marſeilles is compoſed of four emines, and weighs 300 lb. weight of Marſeilles, or thereabout, which make 243 lb. marc weight; 100 lb. of which make 123½ lb. weight of Marſeilles; ſo that the emine weighs 75 lb. weight of Marſeilles.

The load of St. Giles's is 18 or 20 *per cent.* greater than that of Arles

The load of Tarſeon is 2 *per cent.* leſs than that of Arles.

The load of Toulon is compoſed of 3 ſextiers of that place, and the ſextier contains 1½ emines, 3 of which make 2 ſextiers of Paris; or, otherwiſe, they reckon that the buſhel weighs 31 lb. and that 7¾ buſhels make one ſextier of Paris.

The tun of Auray in Britanny is reckoned 2200 lb.

That of Audierne 2300 lb.

That of Breſt is 2240 lb.

That of Hennebon 2950 lb.

Port Lewis the ſame.

Quinpercorentin the ſame

The tun of Nantes is compoſed of 10 ſextiers, and the ſextier of 16 buſhels: it weighs between 2200 and 2250 lb. the meaſure being heaped, and 18 or 2c *per cent.* leſs, if otherwiſe.

The tun of Rennes weighs 2400 lb.

That of St. Malo the ſame.

The tun of Brieux 2600 lb.

That of Rochelle and Maron 42 buſhels.

Of

OF SPAIN.

At Seville they reckon 4 cahies to a laſt, each cahy conſiſting of 12 anegras.

The fanegue of Cadiz weighs 93¾ lb. weight of Marſeilles, 3⅓ lb. of which make the load of 300 lb. weight of Marſeilles aforeſaid, or 243 lb. marc weight.

OF PORTUGAL.

At Liſbon they reckon 4 alguiers to the fanegue, 15 fanegues to the muid, and 4 muids to the laſt of Amſterdam.

OF ITALY.

Grain is ſold at Genoa by the mine.

Two ſacks of wheat, at Leghorn, make 288 lb. weight of Marſeilles.

Corn is ſold at Venice by the ſextier, or ſtaro, which is the ordinary meaſure, 2 of which make a load of Marſeilles.

Of the chief MEASURES of CONSTANTINOPLE, and of the EAST INDIES in general.

There being but about 3 *per cent.* difference betwixt the aunes of Amſterdam and picos of Conſtantinople, 100 aunes of Amſterdam make 103 picos of Conſtantinople; 100 picos of Conſtantinople make 97 aunes of Amſterdam.

MEASURES of FORT ST GEORGE, or MADRASS.

GRAIN MEASURES.

1 meaſure weighs about . - -	2 lb. 10 oz. Avoir.
8 ditto is 1 mercal - . -	21 —
3200 ditto is 400 ditto, or 1 garſe	8400 —

1 Madraſs rupee weighs 7 dwts. 11 gr. Troy, and is better than Engliſh ſtandard 14 dwts. 10 gr. in 1 lb.: it is country-touch 9⅞ China touch 98¾.

LIQUID and DRY MEASURES.

1 meaſure is equal to 1½ pint Engliſh of 423 cubic inches.
8 ditto are equal to 1 mercal of 3384 cubic inches.
400 mercals are equal to 1 garſe of 1,353,600 cubic inches.
1 covid is equal to 18$\frac{6}{10}$ inches.

N. B.

N. B. One meaſure weighs about 2 lb. 8 oz. Avoirdupoiſe.

Eight ditto weigh about 21 lb. or 22 lb.

3200 ditto is 400 mercals, or 1 garſe, which weighs 8400 lb. which is $3\frac{3}{4}$ tuns, or 100 Bengal baazar maunds of 82 lb. 2 oz. 2 dr. each.

Bengal Measures.

One meaſure is five ſeer.

Eight ditto are forty ſeer.

The covid (in cloth meaſure) is nine inches.

Of Malacca Measures.

A Malacca quoing is 3200 chupas, or 800 cantins, equal to 5000 Dutch pounds, or 5475 lb. Engliſh, or Canton peculs, (according to the Dutch calculation of 125 lb. to a pecul), 40 peculs.

A laſt is 2000 chupas, 500 cantins, 3000 Dutch pounds, 24 peculs, 3285 lb. Engliſh.

Anjengo Measure.

One Anjengo covid is eighteen inches Engliſh.

Callicutt and Tellicherry Measure.

One covid is eighteen inches Engliſh; and the Callicutt guz, made uſe of in meaſuring timber, is equal to $28\frac{2}{10}$ inches Engliſh.

They likewiſe, ſometimes at Callicut, meaſure their timber by the covid and borrebl; twelve borrebls is one covid when the timber is ſawed, and 24 borrebls is one covid when unſawed: the price generally is one Callicutt fanam *per* ſolid covid

Carwar Measure.

One covid is eighteen inches Engliſh.

Surat Measures,

Are the larger and leſſer covid, *viz.*

One covid of 36 inches, and one covid 27 inches.

By the latter all things are ſold, except broad cloth, velvet and ſatin, which are ſold by the large covid, or Engliſh yard.

Gombroon

GOMBROON LONG MEASURE.

93 guz are equal to 100 yards Engliſh.

MOCHA MEASURES.

Rice, and other grain, are ſold by the kalla and tomand; forty kallas is one tomand, and weighs about 165 lb. but the governour's cuſtom (of half a kalla *per* tomand upon all grain ſold) being deducted, and the intolerable cheat in the meaſuring, together with the pilferage from the water-ſide home, being allowed for, the Bengal maund will not come out above nineteen kallas; whereas one bag, or Bengal maund, ought to hold out more than a tomand; but, for the forgoing reaſons, two Bengal maunds ſeldom come out above thirty-eight kallas, and rarely that.

Oil is ſold by the kudda, noosfia, and vakia.

Sixteen vakias is one noosfia.

Four noosfias, or meaſures, one cuddy poiſe, about 18 lb.

Of late years the price has been from three to five nooſfias *per* Mocha dollar; and, computing the dupper of two Bengal factory maunds to hold out about 67 or 68 meaſures each, at which rate, the noosfia, or meaſure, weighs about $2\frac{1}{4}$.

Cotton is ſold by the hearf, and nine hearfs is $11\frac{1}{2}$ Mocha dollars: it generally ſells from 30 to 40 hearfs *per* bahar.

LONG MEASURE.

The guz is twenty-five inches Engliſh.

The covid is nineteen inches Engliſh.

CHINA.

CANTON MEASURE.

Ten punts are one covid in piece-goods, equal to $14\frac{5}{8}$ inches."——Thus far from Poſtlethwayt's dictionary.

6. Time is a mode of duration, marked and aſcertained by certain unerring periods and meaſures, whereof the apparent motion and revolution of the ſun ſeems to be the principal. Hence the interval of time elapſed between the centre of the ſun's appearance on the meridian, and its return after one revolution to the ſame meridian again, hath been concluded on by all nations to be one day. Again, from the inſtant that the ſun is in the vernal equinox, or firſt degree of

of Aries, till it revolve round the ecliptic to the ſame point again, hath been found, from repeated obſervations, to contain 365 days and near $\frac{1}{4}$, and the time of this revolution is called a *tropical year*; which, as it did not amount to an exact number of entire days, but the fraction in 4 years would come little ſhort of a day; therefore every fourth year, called *biſſextile*, or leap-year, was made to conſiſt of 366 days, and the common years of 365 days.

The year, being thus eſtabliſhed, was divided as follows.

Seconds.
60= 1 minute.
3600= 60= 1 hour.
86400= 1440= 24= 1 day. h. min. ſec.
31556937=525949=8765=365 + 5+48+57=1 trop. year.

In almanacks, the year is divided into 12 calendar months, the names of which, and days they reſpectively contain, are immediately ſubjoined.

Months.	Days.	Months.	Days.	Months.	Days.
January,	31	May,	31	September,	30
February,	28	June,	30	October,	31
In leap year	29	July,	31	November,	30
March,	31	Auguſt,	31	December,	31
April,	30				

For ſome particular buſineſs, ſuch as payment of wages in the royal navy, they uſe the following table.

7 days = 1 week, 4 weeks = 1 month, and 13 months = 1 year.

CHAP. VII.

ADDITION OF APPLICATE NUMBERS.

RULE.

WHEN the numbers to be added are of one denomination, they muſt be placed and added as before: but when the denominations are different, like, or homogenial denominations, muſt ſtand in one column, ſo that there muſt be as many colums as there are denominations given, decreaſing from the left hand to the right, as in the ſubſequent examples. Then, beginning with the loweſt denomination, find its ſum as in whole numbers, out of which carry to be added with the next column, the units belonging thereto, and not what remains in its proper place; proceed through the whole in this manner, till you come to the integers, or higheſt place, which are added as before.

EXAMPLES IN MONEY.

L.	s.	d.	
57456	15	6½	
6478	19	7¾	
5745	17	11½	
6785	14	10¼	
598	11	11½	
678	10	10½	
57	11	10½	
98	14	9¼	
5	0	8½	
6	9	0½	
7	11	9½	
77919	19	1	Upwards.
77919	19	1	Downwards.

I am indebted as follows; to how much will it amount?

	L.	s.	d.
To A.	74568	19	11½
To B.	54789	18	10¾
To C.	4900	17	11½
To D.	578	18	6¾
To E.	489	14	7½
To F.	584	15	9¼
To G.	674	11	8½
To H.	495	0	9½
To I.	55	0	c½

ILLUSTRATION.

In the place of farthings expreſſed by the fractions on the right hand, in which ½ is reckoned 2, I find 24, which is juſt 6 d.; as there is nothing over to be noted, I carry the 6 d. to the column of pence, in which, including the 6 d. I find 97 pence, or 8 s 1 d. wherefore the penny falls to be noted down in the column of pence, and the 8 s. carried to be added in with its proper column; in the units place of the column of ſhillings, the 8 s. included, I find 49; wherefore, as in integers, I note 9, and carry 4 to be added with the 10's place, in which I find 13 tens = 6 twenties, and 1 ten, which is noted before the 9, and both together make 19 s.; the

the L. 6 is carried to be added in with the units place of the pounds, which are added as integers.

OBSERVATION.

1. If the reaſon of addition of integers hath been properly attended to, the reaſon of the laſt operation will be pretty evident. For ſince 12 pence is equal to 1 ſhilling, it is plain that 97 pence is equal to 8 ſhillings, and 1 penny; wherefore, as it would be not only inconvenient, but really abſurd, to write ſhillings in the place of pence, no more falls to be noted in the place of pence, but the penny which is over the ſhillings, being truly a part of the next column, &c.

2. From the addition of the laſt example, it will appear very neceſſary that the foregoing tables be committed to memory, otherwiſe it will be impoſſible to add at all.

3. It will be found very convenient, in adding thoſe denominations, in whoſe units place you cannot ſtop by ten, as you did in the place of ſhillings of the laſt example, to make a table of the nature of thoſe ſubjoined, which is effected by multiplying the number in queſtion by 2, 3, 4, 5, &c. and commit theſe ſeveral products to memory, eſpecially in ſuch caſes as more frequently occur in practice.

4. Though the method of performing addition is perhaps as eaſily diſcovered as any other rule in arithmetic; yet, to add with accuracy, and at the ſame time with diſpatch, requires a conſiderable practice: I would therefore adviſe the young arithmetician, to take frequent exerciſes by himſelf of this kind, beginning at firſt but with a few lines, and increaſing that number as he becomes more expert. He ſhould at firſt add ſlowly figure by figure, and repeat the ſame column again and again, till he can take in 2, 3, or 4 figures at once: and by thus accuſtoming himſelf to addition, he will be able to perform in a few minutes, with abſolute certainty, what would otherwiſe take him up for hours.

TABLE of grains Troy.

Grains.		*dwt.*
24	=	1
48	=	2
72	=	3
96	=	4
120	—	5

TABLE of pounds Avoirdupoiſe.

Pounds.		*quarters.*
28	=	1
56	=	2
84	=	3
112	=	4
140	=	5

Grains Troy continued.

Grains.		*dwt.*
144	=	6
168	=	7
192	=	8
216	=	9
240	=	10
264	=	11
288	=	12

Pounds Avoirdupoise continued.

Pounds.		*quarters.*
168	=	6
196	=	7
224	=	8
252	=	9
280	=	10
308	=	11
336	=	12
364	=	13

The uſe of theſe tables is obvious.

I ſhall now give ſome examples of weights and meaſures. Thoſe in which there may appear no difficulty, ſhall be left for the learner's practice; ſuch as require illuſtration ſhall be added.

Troy Weight.

Pounds.	*oz.*	*dwts.*	*gr.*
472	11	19	22
534	10	14	21
647	11	18	21
348	10	17	18
246	11	19	12
347	11	13	19
618	5	15	17
947	9	17	11
64	8	8	9
9	7	11	15
5	4	10	16

Dutch Weight.

Stones.	*lb.*	*oz.*	*dr.*	*gr.*
475	15	14	13	34
568	14	13	15	23
647	13	12	11	32
954	12	11	13	31
876	9	14	15	29
678	10	11	12	28
578	11	10	13	27
485	14	13	15	24
657	11	11	7	28
6	4	0	6	0
5	9	9	0	19
5937	1	13	15	25

Apothecaries Weight.

Pounds.	℥	ʒ	℈	*gr.*
57	11	7	2	19
66	10	4	1	18
57	8	6	2	17
8	9	7	0	5
9	11	4	1	18
8	10	0	2	0
5	11	5	1	13
0	6	0	2	0
9	3	4	1	16
5	2	2	2	12
3	1	3	1	15

ILLUSTRATION.

In the example of Dutch weight, I find in the column of grains, whereof 36 = 1 drop, the ſum of 275; wherefore, diſcovering at once from my memory, or a mental diviſion, that 252 grains = 7 drops, the remaining 23 is noted down, and 7 carried to be added in with the drops in the next column, &c.

Note, One that is not much verſant in addition, had better add the denominations, conſiſting of two places, at twice, till he can do that very quickly; a little practice will enable him afterwards to add up two places of any denomination at once with the greateſt eaſe.

Avoirdupoiſe weight.

Tuns.	*cwt.*	*qu.*	*lbs.*
8745	19	2	27
4856	18	3	26
5479	14	1	25
3568	17	2	24
594	16	1	23
55	11	2	21

Wool weight.

Laſts.	*ſa.*	*w.*	*t.*	*ſto.*	*c.*	*lbs.*
87	11	1	$6\frac{1}{4}$	1	1	6
43	5	1	$3\frac{1}{2}$	1	1	4
6	4	1	$2\frac{1}{4}$	1	1	3
8	6	1	$3\frac{1}{2}$	1	1	5
17	2	1	$5\frac{1}{4}$	1	1	3
2	0	0	0	0	0	4
165	8	0	$5\frac{1}{4}$	1	0	4

Cloth meaſure.

Yds.	*qrs.*	*nls.*	*parts.*
45	3	3	15
30	2	1	12
67	1	2	13
8	3	1	14
9	2	2	11
7	2	3	12
8	1	1	10
9	2	0	9

Engliſh ſquare meaſure.

Acres.	*r.*	*p.*	*yds.*	*feet.*	*inch.*
373	3	37	$27\frac{1}{4}$	6	115
485	2	38	$22\frac{1}{2}$	7	116
574	2	19	29	8	114
648	1	18	$18\frac{1}{2}$	5	125
741	2	17	$19\frac{3}{4}$	4	130
874	3	14	$24\frac{1}{2}$	3	134
878	1	18	30	5	116
6	3	17	15	6	117
4583	1	24	10	5	103

Note, In the above example of ſquare meaſure, I find the ſum of ſquare yards, including what I carried, to be $191\frac{1}{2}$, I then conſider that $6\times30=180+\frac{6}{4}=181\frac{1}{2}$, and the remainder eaſily occurs to be 10.

Scotch

Scotch square measure.

Acre.	*rood.*	*fall.*	*ell.*
42	1	38	10
57	3	39	35
43	2	27	24
8	3	15	27
65	2	24	17
37	3	31	32
45	2	15	18

Long measure.

Miles.	*fur.*	*yds.*	*feet.*	*inch.*
3467	5	219	2	10
4567	7	184	1	11
5678	6	62	2	8
78967	4	9	0	9
56789	3	84	2	11
24608	2	147	1	10
35791	1	210	2	6
209871	0	80	0	5

In the example of long measure, the perches are neglected; and as there is no other intermediate denomination betwixt yards and furlongs, I carry at 220.

Wine-measure.

Tun	*hhd.*	*gall.*	*qu.*	*pints*
436	3	61	2	1
678	2	61	3	0
569	1	48	1	1
456	0	29	3	1
789	1	36	2	0
987	2	54	1	1
672	3	46	3	1
4591	1	24	1	1

Ale and beer-measure.

Hhd.	*bar.*	*kil.*	*fir.*	*gal.*
176	1	1	1	$8\frac{1}{4}$
374	$1\frac{1}{4}$	1	1	$7\frac{1}{2}$
842	1	0	1	6
374	$1\frac{1}{4}$	1	0	$5\frac{3}{4}$
516	$0\frac{1}{2}$	1	0	$5\frac{1}{2}$
637	1	1	1	$6\frac{3}{4}$
591	1	1	1	8

Scotch dry measure.

Ch.	*b.*	*f.*	*p.*	*l.*
875	15	2	1	1
43	14	3	3	3
87	13	2	2	2
6	15	3	2	2
67	14	2	3	2

English dry measure.

Q.	*b.*	*p.*	*g.*
875	7	3	1
43	4	2	1
67	5	3	1
54	4	2	1
475	5	3	1

CHAP. VIII.

SUBTRACTION of applicate Numbers.

RULE.

PLACE the numbers, or denominations, homogeneal under homogeneal, and borrow according to the division of the integer, as illustrated in some of the following examples.

	Money.			Troy weight.				Apothecaries weight.				
	L.	*s.*	*d.*	*lbs.*	*oz.*	*dwt.*	*gr.*	*lbs.*	℥	ʒ	℈	*gr.*
From	54	13	$11\frac{1}{4}$	87	7	13	21	83	7	4	1	13
Take	35	15	$8\frac{1}{2}$	43	10	15	17	42	5	7	2	18
Rests	18	18	$2\frac{3}{4}$	43	08	18	4	41	1	4	1	15

ILLUSTRATION.

In the example of money, I cannot take $\frac{1}{2}$ from $\frac{1}{4}$, and therefore I take it from an unit of the next higher denomination, and to the remainder add the given $\frac{1}{4}$; thus 4 farthings $-2+1=\frac{3}{4}$. I replace the unit I had thus borrowed, by adding it to 8 d. whence I had taken it. And indeed, it would have answered the end to have taken the difference betwixt $8\frac{1}{2}$ and $11\frac{1}{4}$ d. at once, if it could upon all occasions have been recommended to practice, as it would have brought out the same remainder.

Subtraction is so easy an operation, and the memory is so little burthened with it, that a farther illustration would be needless, and therefore I shall only subjoin a few examples for practice.

Dry measure.					Long measure.					Cloth measure.		
Last.	*wey.*	*qu.*	*b.*	*p.*	*M.*	*f.*	*p.*	*y.*	*feet.*	*Yds.*	*qrs.*	*nails.*
87	1	2	4	2	47	5	34	$3\frac{1}{2}$	1	57	1	0
43	1	3	5	3	13	6	37	$4\frac{3}{4}$	2	14	3	1

Avoirdupoise weight.				Wine measure.				Ale measure.				
Tun.	*cwt.*	*qrs.*	*lbs.*	*Tuns.*	*p.*	*hhds.*	*gal.*	*Bar.*	*f.*	*gal.*	*q.*	*p.*
75	7	3	17	75	1	0	35	43	1	5	2	0
69	14	3	19	68	1	1	50	29	1	7	3	1

Yarn.

Sp.	*ha.*	*he.*	*c.*	*th.*
572	1	1	1	100
69	1	5	1	115

Time.

Y.	*d.*	*h.*	*m.*	*f.*
7415	199	14	50	40
897	270¼	19	50	52

Questions for practice in addition and subtraction.

1. A merchant, in balancing his books, finds he hath in ready money, L. 456, 17 s; in goods, L. 1749 : 19 : 6; his stock in a company-trade was L. 199 : 19 : 6½; due him in open accounts, L 2977 : 19 : 7¾; in bills, L. 647, 16 s; in ships and houses, L 1976 : 14 : 7½; and in consignments, L. 479 : 19 : 7. He owes to A, L. 1456 : 18 : 7½; to B, L. 99 : 19 : 11 : to C, L. 497 : 17 : 10; and to the bank, L. 490. What is his nett stock?

	L.	*s.*	*d.*
Cash,	456	17	0
Goods,	1749	19	6
Company,	199	19	6½
Accounts,	2977	19	7¾
Bills,	647	16	0
Ships, &c.	1976	14	7½
Consign.	479	19	7
Gross stock,	8489	5	10¾
Sum of debt	2544	16	4½
Nett stock,	5944	9	6¼ Answer.

	L.	*s.*	*d.*
He owed to A,	1458	18	7½
To B,	99	19	11
To C,	497	17	10
To the Bank,	490	0	0
Sum of his debt	2544	16	4½

2. A merchant hath a bill to pay of L. 500, for which he had prepared in cash, L. 197, 19 s he hath a bill on Edin-

 burgh

burgh for L 120; for how much muſt he draw on the bank to retire the bill of L 500?

Anſwer, L 182, 1 s.

3. A farmer paid of yearly rent for his poſſeſſion L 156 17 s. 8 d.; at the expiration of three years, when he was called to ſettle accounts with the landlord, he could produce receipts only for L 376 : 19 : 7½. How much muſt he pay to even the account?

Anſwer, L 93 : 13 : 4½.

4. Bought 8 hogſheads of raiſins, each weighing groſs 5 cwt 1 quarter, and 11 pounds; upon each hogſhead whereof I am allowed a deduction of 3 quarters and 21 lbs. What will be the nett weight?

Anſwer, 35 cwt. 1 quarter, 4 pounds.

5. A merchant ſent his clerk to a fair, where he bought linen to the amount of L 105 : 12 : 11; ſtockings to the amount of L 184 : 16 : 11; he recovered of accounts due the merchant to the amount of L 64, 10 s; and got payment of a bill for L 129, 19 s; he paid ſome few demands, amounting in all to L 19 : 19 : 11; his account of petty charges came to L 1 : 14 : 6; and he gave back to his maſter L 27 : 11 : 10. How much money had he got from the merchant before he ſet out?

Anſwer, L 135 : 7 : 1.

6. A caſtle was built in the year 1459; how old is it in the year 1769?

7 Received from Jamaica 103 tuns, 13 cwt. of logwood, by the happy Janet, of which I ſold to A, 15 tuns, 10 cwt. 3 qrs 17 lb.; to B, the double of what I ſold to A; and to C, as much as I had ſold to A and B together. How much have I on hand?

CHAP. IX.

MULTIPLICATION of APPLICATE NUMBERS.

RULE.

WHEN theſe numbers are of different denominations, beginning the multiplication at the right

right hand, carry, as in addition, one from each denomination to another, for as many as make an unit of the next ſuperiour order, and place the remainder under its proper denomination. A few examples will render this extremely plain.

Queſt. 1. What coſt 8 pieces of broadcloth, at L. 5 : 17 : 8 *per* piece?

L.	5	17	8	
			8	
L.	47	1	4	Anſwer.

2. What coſt 72 bags of cotton at L. 7 : 14 : 8 *per* bag?

L.	7	14	8	
			8	
L.	61	17	4	
			9	
L.	556	16	0	Anſwer.

3. What coſt 34 pieces of luteſtrings, at L 9 : 18 : 8 *per* piece?

L.	9	18	8 × 2	
			8	
	79	9	4	
			4	
	317	17	4	price of 32 pieces.
	19	17	4	price of 2 pieces.
	337	14	8	price of 34 pieces. Anſ.

4. What

4. What coſt 76 cwt. of ſhip biſcuit, at 13 s. 6 d. per cwt?

	s.	*d.*	
	13	6×4	
		12	
L. 8	2	0	price of 12
		6	
48	12	0	price of 6×12=72
2	14	0	price of 4× 1= 4
51	6	0	price of 76 Anſwer.

5. Sold 174 ingots of ſilver, each ingot weighing 15 lb. 11 oz. 19 dwt. 21 gr.; what is the weight of the whole?

lb.	℥.	*dwt.*	*gr.*	
15	11	19	21	
			10	
159	11	18	18	weight of 10.
			10	
1599	11	7	12	ditto of 10×10=100
1119	11	11	6	ditto of 7×10= 70
63	11	19	12	ditto of 4× 1= 4
2783	10	18	6	ditto of 174 Anſwer.

6. Bought 574 pounds of tobacco, at 7¾ d. *per* pound; what coſt the whole?

L. 0	0	7¾	
		10	
0	6	5½	value of 10 pounds.
		10	
3	4	7	ditto of 10×10=100
		5	
16	2	11	ditto 5×100=500
2	5	2½	ditto of 7× 10= 70
0	2	7	ditto of 4× 1= 4
L. 18	10	8½	ditto of 574 Anſwer.

These

Theſe examples are ſufficient for exhibiting all the neceſſary varieties in multiplication of mixed numbers, the more eſpecially as the learner will find, in the rules of practice, methods that are in general much more expeditious, and leſs burthenſome to the memory, for ſolving all queſtions of this kind; though in ſome caſes, where a ſingle multiplication, or perhaps two, are only neceſſary, this may be uſed with great propriety.

CHAP. X.

DIVISION of Applicate Numbers.

RULE.

IN dividing different denominations, the remainder of the integral part muſt be brought to the quality of the next inferiour denomination; and, if any of that denomination was given, it muſt be added to the product: then find how oft the diviſor is contained therein, and the quotient will be of that denomination: if any thing ſtill remain, proceed in the ſame way, as illuſtrated in the following examples.

Queſt. 1. L. 34, 16 s. is to be divided among 5 men equally. What will fall to each?

5)34 : 16(6 : 19 : $2\frac{2}{5}$
30
—
4×20+16=96
5
—
46
45
—
1×12=12
10
—
2

Note, After a little practice, the laſt queſtion, or any other where the diviſor is ſmall, may be expeditiouſly done by a mental multiplication and ſubtraction. When the diviſor is great, the memory may be helped in the more burthenſome part of the work, by uſing a piece of waſte paper, or ſtill better by abbreviating the terms when poſſible.

The laſt example reſumed. 5)34 : 16 : 0

6 : 19 : $2\frac{2}{5}$

Note, When L. 34 is divided, the remaining L. 4, 16 s. will readily occur to be =96 s.; which, being partially divided, quotes 19 s. with 1 remaining = 12 d. of which $\frac{1}{5}$= $2\frac{2}{5}$.

2. Bought 48 pieces of cloth for L. 256, 18 s. what did it coſt *per* piece?

8)256	18	0
6)32	2	3
5	7	$0\frac{1}{2}$ Anſwer.

3. Bought 375 pieces of Iriſh linen, for L. 701 : 15 : 3 freight and other charges came to L. 4 : 17 : 11. What did it coſt *per* piece?

L. 701	15	3
4	17	11
5)706	13	2
5)141	6	$7\frac{3}{5}$
5)28	5	4
3)5	13	$0\frac{4}{5}$
1	17	$8\frac{1}{5}$.

Note, It will be ſufficiently exact to take the quotient of the fractions that is neareſt the truth; and, if you are above the juſt quotient at one time, be below it at another time, as in the laſt example.

4. If

4. If 60 gallons of water fall into a ciſtern that will contain 200 gallons, in the ſpace of an hour, and by a pipe in the ſame ciſtern there run out 45 gallons in the ſame time; how long will it take to be full in this caſe?

60—45=15)200

Hours 13, 20 minutes. Anſwer.

5. There is a legacy of L. 4753, 19s. to be divided among A, B, C, D, and E, in ſuch a manner, that for every ſhilling E takes up, D ſhall have two, C 3, B 4, and A 5; what will fall to each?

1+2+3+4+5=15)4753 19 0

316	18	$7\frac{1}{5}$	E's ſhare.
633	17	$2\frac{2}{5}$	D's ſhare =2 E's ſhare.
950	15	$9\frac{3}{5}$	C's ſhare =D+E.
1267	14	$4\frac{4}{5}$	B's ſhare =C+E.
1584	13	0	A's ſhare =B+E.
4753	19	0	Proof.

6. A gentleman, on his death-bed, leaving his wife pregnant, and an eſtate of L. 6666 : 13 : 4, ordered by his teſtament, that, if his wife bore a ſon, $\frac{2}{3}$ of the fortune ſhould go to that ſon, and the other $\frac{1}{3}$ to the widow: but, if ſhe bore a daughter, the widow ſhould enjoy $\frac{2}{3}$, and the daughter the remainder: ſhe had a ſon and a daughter at the ſame time: In this caſe, how will the fortune be divided?

By the teſtament the ſon was ſecured in double the widow's ſhare, and the widow in double the daughter's; therefore, the daughter's ſhare muſt be to the widow's as 1 to 2, and the widow's to the ſon's as 2 to 4.

Upon theſe principles it will be divided thus:

1+2+4=7)6666 13 4

952	7	$7\frac{3}{7}$	the daughter's ſhare.
		2	
1904	15	$2\frac{6}{7}$	the widow's ſhare.
		2	
3809	10	$5\frac{5}{7}$	the ſon's ſhare.
6666	13	4	as before.

7. A

7. A privateer takes a prize. value L. 20,000: the crew conſiſted of the captain, the lieutenant, the maſter, maſter's mate, ſurgeon, ſurgeon's mate. purſer, 4 midſhipmen, and 100 men: by the ſhip's regulations, the private men ſhared equally, a midſhipman had as much again as a private man each mate the double of a midſhipman's ſhare, the ſurgeon and purſer drew each the double of a mate's ſhare, the lieutenant and maſter had each as much as the ſurgeon and purſer, and the captain as much as both lieutenant and maſter: According to theſe regulations, how will the prize be divided?

100+8+8+16+32+32=196)20,000

L. 10204	1	3=100 × - -	102	0	9¾	to each p. man.
					2	
816	6	6= 4 × - -	204	1	7½	to each midſhip.
					2	
816	6	6= 2 × - -	408	3	3	to each mate.
					2	
1632	13	0= 2 × - -	816	6	6	to the ſurgeon and purſer each.
					2	
3265	6	0= 2 × - -	1632	13	0	to the lieutenant and maſter each.
					2	
3265	6	0= 1 × - -	3265	6	0	to the captain.
0	0	9 loſt with the remainder.				
20000	0	0 as before.				

The

The Application of Multiplication and Division to ſquare and ſolid meaſures.

Queſt. 1. What is the ſquare contents of a room, 18 feet 8 inches by 14 feet 6 inches?

2)18 8
  7
———
130 8
  2
———
261 4 = 18 8 × 14
 9 4 = 18 8 ÷ 2
———
270 8 Anſwer.

2. What is the ſquare contents of a room, 12 feet 7 inches long and 11 feet 5 inches broad?

3)12 7
 11 5
———
138 5 = 11 × 12 7
4) 4 $2\frac{1}{3}$ = 12 7 ÷ 3 for 4 inches.
 1 $0\frac{7}{12}$ = 4 $2\frac{1}{3}$ ÷ 4 for 1 inch.
———
143 $7\frac{11}{12}$ Anſwer.

3. What is the ſolid content of a box, 5 feet 6 inches thick, 4 feet 4 inches broad, and 7 feet 3 inches long.

7 3
4 4
———
29 0 = 7 3 × 4
 2 5 = 7 3 ÷ 3 for 4 inches.
———
31 5 ſquare content.
 5 6
———
157 1 = 31 5 × 5
 15 $8\frac{1}{2}$ = 31 5 ÷ 2 for 6 inches.
———
172 $9\frac{1}{2}$ ſolid content.

4. There is a box 6 feet 6 inches long, 7 feet 9 inches broad, at one end, and 3 feet 7 inches at the other, and 4 feet 6 inches thick; what is its solid content?

7	9	one way.
3	7	the other way.
2)11	4	sum of the breadths.
5	8	mean breadth.
6	6	
34	0	
2	10	
36	10	square content
4	6	
147	4	
18	5	
165	9	solid content.

CHAP. XI. REDUCTION.

REDUCTION converts one denomination, or species, into another, without altering the value.

RULE I.

To reduce numbers of a higher denomination to numbers of the same kind of an inferior denomination, multiply by as many of the inferior denomination as makes one of the greater.

RULE II.

To reduce numbers of a lower denomination to numbers equivalent of a higher denomination, divide by as many of the inferior as makes one of the greater.

EXAMPLES.

(1.)

Reduce *L* 760 to farthings.

```
   20
-----
15200 shillings.
   12
-----
182400 pence.
     4
------
729600 farthings.
```

(2.)

Reduce 729600 farthings to pounds.

```
 4)729600
---------
12)182400 pence.
---------
20)15200 shillings.
---------
     760 pounds.
```

(3.)

```
   lb.  oz.  dwt.
In 74   11   15 Troy, how many grains?
   12
-----
  899 ounces.
   20
-----
17995 dwts.
   24
------
431880 gr.
```

(4.)

In 431880 grains, how many pounds Troy?

```
 4)431880
---------
 6)107970
---------
20)17995 dwts.
---------
12)899 15 oz.
---------
   74 11 15 Anſ.
```

More examples of this kind of reduction would be unneceſſary, as the reaſon and manner of the operation muſt be obvious.

RULE III.

To reduce one ſpecies into its equivalent of another, when the one is no even part of the other,—— multiply the given number by the value of an unit of the ſame ſpecies expreſſed in the loweſt name mentioned in the queſtion, and divide that product by the value of an unit of that ſpecies which is required, expreſſed in the ſame name.

Queſt.

Quest. 1. I have ſent 345 bank-notes. each 20 s. to be ex-changed for guineas; how many guineas will compenſate the notes?

```
      345
       20
   -------
   3)6900
   -------
   7)2300
   -------
```

328 guineas and 12 s or $\frac{4}{7}$ of a guinea.

2. I want to exhange for guineas, 360 five-pound notes, 150 ten-pound notes, and 27 ten-ſhilling notes; what number of guineas ought I to receive?

```
360× 5=1800
150×10=1500
27÷ 2=   13   10
       ---------
21)3313   10
    157   15
   ---------
```

Guineas 3155 15 ſhillings. Anſwer.

3. I have 1478 moidores, worth 27 s. each, which I would exchange for piſtoles at 16 s. 6 d. each; how many piſtoles ſhould I have in return?

Firſt, 27 s. = 54 ſixpences, the loweſt name mentioned, and 16 s. 6d. =33 ſixpences,

But 54 and 33 are each commenſurable by 3, and quote 18 and 11.

Therefore, 1478 moidores.

```
         18
     -------
   11)26604
     -------
```

2418$\frac{6}{11}$ piſtoles. Anſwer.

4. How many French crowns, at 31$\frac{1}{2}$ d. may I have in exchange for 37542 guilders, at 21$\frac{1}{4}$ d.?

```
                       37542
21×4+1 =                  85 farthings in a guilder.
                      ------
                      187710
                     300336
                     -------
31×4+2=126\          3191070
       ---  )        -------
        21 )          531845 the quotients of both factors by 6,
        --            ------
         7          177281⅔ ditto.—Ditto by 3.
                    -------
                    25325 3/7 French crowns.   Answer.
```

5. In 87547 ducats, at 3s. 9d. how many Hamburg marks, at 1s. 6d.?

```
   s.  d.                              1  6
   3   9                                  4
   4                                   ----
   ---                                3)6 threepences.
   3)15 threepences.                   ----
   ---                                    2 ninepences.
      5 ninepences        87547
                              5 ninepences.
                          -----
                        2)437735
                         ------
                          218867½ marks of Hamburg.
```

After a little practice, multipliers and divisors may be abridged mentally, without any formal multiplication or division: for, in the last instance, it will easily occur that there are 5 times 9 pence in 3s. 9d. and 2 ninepences in 1s. 6d.

6. If I had 4754 livres in France, which I rate at 10½d. each, how many milrees of Portugal, at 5s. 3d. would they answer for?

```
10½d. is contained in 5s. 3d. just 6 times.
        Therefore 6)4754
                    ----
                    792 2/6 milrees.   Answer.
```

7. In

7. In L. 4754, 19 s. how many guineas, crowns, shillings, and ſixpences, of each an equal number.

```
                    4754   19
                      20
                    ----------
                    95099
                        2
                    -----
42+10+2+1=55 )      190198
          --        ------
          11        38039⅗ abridged by 5.
                    ------
                     3458 8/55 of each ſpecies.
```

8. If I had guineas 90 ſcore, and crowns juſt 92; In place of 30 hundred pounds, what money would be due?

```
        90×21= 1890 l.
        92÷ 4=   23
        Take L. 1913 in the guineas and crowns.
           From 3000
                ----
      Remains  1087 due.
```

In the country, particularly in the north of Scotland, they keep their accounts in Scotch money, the pound of which is $\frac{1}{12}$ of a pound Sterling, and divided in the ſame manner. In making bargains, and ſometimes in granting bonds, they expreſs their money in merks, one of which is 13 s. 4 d. or $\frac{2}{3}$ of their pound. Hence, to reduce pounds Scots to merks, multiply by 3, and divide by 2; and to reduce merks to pounds, multiply by 2, and divide by 3. Again, ſince the merk is to the pound Scotch as 2 to 3, and the pound Scotch to the Engliſh as 1 to 12, the Scotch merk will be to the pound Sterling as 1 to 18.

Theſe diſtinction are never uſed among merchants,

9. I have a bond for 5000 merks Scotch, a bill for L. 559 Scotch, and 24 crown-pieces; what are theſe worth in Sterling?

2)5000

	2)5000 merks			
	9)2500			
L. 559 Scots.				
	277	15	$6\frac{2}{3}$	value of the bond.
12 =	46	11	8	ditto of the bill.
24 ÷ 4 =	6	0	0	ditto of the crowns.
	330	7	$2\frac{2}{3}$	value of the whole.

10. Two merchants, A and B, had been long in a company-trade; A's ſhare of the concern was to B's as 4 to 1: When circumſtances rendered it neceſſary for them to wind up and ſeparate, the ſtate of their affairs was as follows. Their caſh and other effects, by an inventory, amounted to L. 5000; bills and open accounts, in Britain, to L. 1300; Holland was indebted to them in nett proceeds of tobacco, for 7485 guilders, at $21\frac{1}{8}$ d.; and Dunkirk in crowns 7456, at $31\frac{1}{4}$ d.; they were due in Britain L. 3754, and in Hamburg 7315 marks, at 1 s. 7 d.: Required their nett ſtock, and a partition thereof, according to each partner's original input.

They had effects, *per* inventory, valued at - -	L. 5000					
Bills and open accounts, in Britain, for - - -	1300	0	0			
7485 guilders, at $21\frac{1}{8}$ d. =	658	16	$8\frac{1}{2}$			
7456 crowns, at $31\frac{1}{4}$ =	970	16	8	L.		
				7929	13	$4\frac{1}{2}$
They were due in Britain	3754	0	0			
In Hamburg 7315 marks, at 19 d.	579	2	0	4333	2	
Nett ſtock	4 + 1 = 5)			3596	11	$4\frac{1}{2}$
B draws				719	6	$3\frac{3}{10}$,
A draws				2877	5	$1\frac{2}{10}$

11. In 1000 ale-gallons Scotch meaſure, how many Engliſh.

103 ſolid inches in a Scotch pint.
8

282)824000

2922 nearly.

12. Suppoſe

12. Suppofe the diftance betwixt Glafgow and London 330 miles, how oft will a coach-wheel turn round, whofe circumference is 6 yards, in driving that diftance?

$$\frac{\overset{\text{Miles. Fur.}}{330 \times 8 \times 220}}{6} = 96800 \text{ times. Anfwer.}$$

13. In 25¾ French acres, how many Englifh?

$$\begin{array}{r} 52\frac{3}{4} \\ 4 \\ \hline 103 \\ 19 \\ \hline 4)1957 \\ \hline 4)\ 489\frac{1}{4} \\ \hline 4)\ 122\frac{5}{16} \\ \hline 30\frac{37}{64} \end{array}$$

$30\frac{37}{64}$ or $30\frac{1}{2}$ nearly.

14. Conftantinople lies 1500 geographical miles S. E. of London, what is the diftance in ftatute miles?

$$\begin{array}{r} 1500 \quad 69\frac{1}{2} \\ \hline 103500 \\ 750 \\ \hline 6,0)10425,0 \\ \hline 1737\frac{1}{2} \end{array}$$

$1737\frac{1}{2}$ Anfwer.

15. Quebec lies 300 geographical miles N. W. of Bofton, what is the diftance in French leagues?

$$\begin{array}{r} 300 \\ 5 \\ \hline 12)1500 \\ \hline 125 \end{array} \qquad 5)\frac{25}{60}\left(\frac{5}{12}\right.$$

125 Anfwer.

Queftions

Questions of this nature might be multiplied to any length but it is presumed there is a sufficient variety in the foregoing for the improvement of the ingenious.

CHAP. XII. OF NUMBERS.

NUMBER in general may be defined, A collection or assemblage of several units, or several things of the same kind. Sir Isaac Newton conceives number to consist, not in a multitude of units, as Euclid defines it, but in the abstract ratio of a quantity of any kind to another quantity of the same kind, which is accounted as unity; and, in this view, he divides number into three different kinds, integer, fraction, and surd.

Integers, whole or natural numbers, are all the various assemblages of unity, or the ideas we have of several multitudes.

Fractions are the divisions or subdivisions of unity, to which they refer, as a part doth to the whole.

Rational numbers are such as are commensurable with unity, or whereof unity is an aliquot part.

Surd, or irrational numbers, are such as are incommensurable with unity.

Even numbers are those which may be divided into two equal parts, without any remainder, as the numbers 6, 8, 10, 12, *&c.* which have likewise these properties, that the sum difference and product of even numbers, will each be an even number.

Uneven numbers, or such numbers as cannot be divided into two equal parts without a fraction, have these properties, that the sum or difference of two uneven numbers will be an even number, but the product will be an uneven number. If an even number be added to an uneven number, or if the one is subtracted from the other, the sum in the one case and the remainder in the other will be uneven; but if they are multiplied, the product will be even.

A prime number is such as hath no measure but itself and unity, and which of consequence can be the product of no other numbers, as 2, 3, 5, 7, 11. Numbers prime to one another are such as have no measure common to both but unity, as 12 and 19; for though both 3 and 4, or 2 and 6, will measure 12, yet neither of these will measure 19.

Compoſite numbers are ſuch as are diviſible by ſome num-ber beſides unity, as 8 diviſible by 4, and by 2.

Numbers compoſite to one another have ſome meaſure common to each beſides unity, as 12 and 15 diviſible by 3.

A number that can divide ſeveral numbers exactly, is cal-led a *common meaſure*, as in the laſt inſtance 3 is a common meaſure to 12 and 15.

Prob. 1. To find the greateſt common meaſure to any numbers propoſed, add all the propoſed numbers, excepting the leaſt, which is reſerved for a diviſor. If there is no re-mainder, after the ſum of the greater numbers hath been di-vided by the leaſt, then that diviſor is the common meaſure; but, if there is a remainder, it becomes a new diviſor, and the laſt diviſor becomes a new dividend, *&c.* the laſt diviſor, which left no remainder, is the common meaſure.

What is the greateſt common meaſure of 8, 16, 24, and 32?

```
    16
    24
    32
  ----
  8)72
  ----
     9
```

9 without a remainder: So 8 is the greateſt common meaſure for $\frac{8}{8}=1$, $\frac{16}{8}=2$, $\frac{24}{8}$ 3, and $\frac{32}{8}=4$.

What is the greateſt common meaſure of 12, 16, 24, 36, and 52?

```
      16
      24
      36
      52
   -----
  12)128
   -----
      10 rem. 8|12
                |  1 rem. 4|8
                           |2 and 0 remains.
```

Conſequently 4 is the greateſt common meaſure.

Note, If only two numbers were propoſed, the greater is divided by the leſſer, *&c.* Hence any number not exceed-ing the leaſt of ſeveral numbers that can meaſure their ſum, will meaſure them all ſeverally.

Prob. 2. To find the leaſt common multiple of ſeveral numbers. Write the numbers in a line, and divide them by 2, 3, or any other numbers which will meaſure two or three of them exactly; place the quotients with the numbers undivided below, and divide them continually till their quotes be 1: Then the product of all the diviſors multiplied continually, is the leaſt common multiple required, which, if they were primes to one another, will be the continual product of the given numbers.

What is the leaſt common multiple of 6, 12, 24, 36?

6	6 .	12 .	24 .	36
2	1 .	2 .	4 .	6
2		1 .	2 .	3
3			1 .	3
				1

$6 \times 2 \times 2 \times 3 = 72$

There are other problems which might have got a place here, but theſe I reckon ſufficient for my purpoſe.

THE

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UNIVERSAL ACCOUNTANT.

PART II.

OF FRACTIONS.

I. OF VULGAR FRACTIONS.

INTRODUCTION.

A VULGAR FRACTION is a part, or parts of an integer arising from division, and stands to an unit in the relation that a part doth to the whole.—Of those parts the numerator expresseth the number, as $\frac{3}{4}$ and the denominator the quality, as

Hence the denominator supposeth the integer to be divided into 4 equal parts; for instance, 1 yard into 4 quarters: and the numerator ascertaineth the number of those parts to be 3; and the fraction is accordingly read three fourths, or quarters of 1.

Since the denominator represents all the parts into which the integer is divided, and the numerator the number of those parts expressed by the fraction, it must follow, that, if the numerator be less, equal to, or greater than the denominator, the quantity represented by the fraction must be less equal to, or greater than the integer accordingly. Hence, if the numerator is less than the denominator, the fraction is called *proper*, and represents something less than the integer, as $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, &c.

If the numerator is equal to, or greater than the denominator, the fraction is called *improper*, and represents something equal to, or greater than the integer, as $\frac{4}{4}=1$, $\frac{5}{4}=1\frac{1}{4}$, &c.

Since an integer may be divided into any number of parts, each of these parts may be again subdivided, and each of these subdivisions again, *ad infinitum*; as a pound is divided into 20 shillings, each of these shillings into 12 pence, and each of these pence into 4 farthings. Hence, 3 farthings would be expressed fractionally $\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ and such fraction would be called a *compound fraction*.

Since any of the denominations, or parts of an integer, can be expressed fractionally, a fraction annexed to an integer will express the same thing as its equivalent denomination. Hence L. 4, 10 s. since 10 s. represents 10 of 20 parts of a pound, may be wrote $4\frac{10}{20}$ and so called a *mixed number*.

Observation 1. Those fractions are equal to each other, whose numerators have the same relation to their denominators, $\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\frac{14}{28}=\frac{1}{2}$. For as all fractions arise from remainders in division, when the divisor can no longer measure the dividend, so every fraction may be considered as the two given terms of a division, the numerator as the dividend, and denominator as the divisor: Consequently, if the numerator and denominator of a fraction be either multiplied or divided both by the same number, the products or quotients will still remain in the same proportion, and the numerator of the new fraction bear the same relation to its denominator as it did in its former state.

2. Fractions having a common denominator, are greater or less as their numerators, as $\frac{4}{5}$ represents a greater part of a quantity than $\frac{3}{5}$.

3. Of fractions whose numerators are equal, that which hath the least denominator represents the greatest part, as $\frac{3}{4}$ of a yard represents 3 quarters, and $\frac{3}{16}$ only 3 nails.

4. If two fractions are equal, the numerators multiplied into each other's denominator respectively, will make the products equal.——Suppose $\frac{2}{4}=\frac{1}{2}$, then $2\times2=4$, and $4\times1=4$.

CHAP. I.

REDUCTION of VULGAR FRACTIONS.

Prob. 1. To express a whole number fractionally.

The given integer will be the numerator, and unity the denominator: Thus $\frac{5}{1}$, $\frac{9}{1}$ &c.; because to divide 5 by 1, the quotient will be 5, &c.

Prob. 2. To reduce a mixed number to an improper fraction.

T

To the product of the integer and denominator multiplied add the numerator; the ſum ſhall be the numerator of the improper fraction, whoſe denominator ſhall be that of the fraction given.

Exam. $4\frac{4}{5}=4\times5+4=\frac{24}{5}$; and $5\frac{5}{6}=5\times6+5=\frac{35}{6}$.

Obſerv. The reaſon of this operation is evident; for the multiplication of the integer into the denominator + the numerator, expreſſes in the product all the parts contained in both, and the ſame denominator being again applied, the quality of thoſe parts is the ſame.

Prob. 3. To reduce an improper fraction to a whole, or mixed number.

Divide the numerator by the denominator, and to that quotient annex the remainder, if any, with the diviſor for the fractional part.

Exam. $\frac{24}{5}=4\frac{4}{5}$, and $\frac{35}{6}=5\frac{5}{6}$.

This is the reverſe, and conſequently an additional proof of the former problem.

Cor. Hence it will be obvious, that, to reduce an integer to an improper fraction of an aſſigned denominator, we have only to multiply the integer into the aſſigned denominator, and the product will be the numerator required. For inſtance to change 8 into a fraction whoſe denominator is 7 $8\times7=\frac{56}{7}$ &c.

Prob. 4. To reduce a compound fraction to its equivalent ſimple one.

The continued product of all the numerators will be the numerator, and the continued product of all the denominators will be the denominator required.

Ex. $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{3}{7}=\frac{3\times4\times3}{4\times5\times7}\frac{36}{140}$, and $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{8}=\frac{1\times3\times5}{2\times4\times8}=\frac{15}{64}$

Obſervation. The continued multiplication of the numerators and denominators reſpectively, brings each to the quality of the loweſt name: for, to expreſs 9d. as the fraction of a pound, it would be $\frac{9}{240}$, becauſe there are 240 pence in a pound; if we conſider it as a compound fraction, as it really is, it will be expreſſed $\frac{9}{12}$ of $\frac{1}{20}$, which, by the rule, $=\frac{9}{240}$ as before.

Cor. Hence all the known ſubdiviſions of an integer may be expreſſed in compound or ſimple fractions at pleaſure.

Prob.

Prob. 5. To reduce fractions having different denominators to other equivalent fractions, having a common denominator.

The continued product of each numerator into all the denominators but its own, will give correſpondent numerators, and the continued product of the denominators will give a common denominator.

Exam. 1. $\frac{3}{5}\ \frac{1}{2}\ \frac{4}{7}$, thus $\left.\begin{array}{l}3\times2\times7=42\\1\times5\times7=35\\4\times2\times5=40\end{array}\right\}$ Numerators.

Therefore $\frac{3}{5}=\frac{42}{70}$ $\quad 5\times2\times7=70$ Common denominator.

$\frac{1}{2}=\frac{35}{70}$

$\frac{4}{7}=\frac{40}{70}$, as required.

Exam. 2. $\frac{3}{5}$, $\frac{5}{6}$, and $\frac{1}{4}$ of $\frac{1}{2}$. Firſt, $\frac{1}{4}$ of $\frac{1}{2}=\frac{1}{8}$ in Ex. 2. Then $\frac{3}{5}\ \frac{5}{6}\ \frac{1}{8}$, the given fractions, which by the rule will produce $\frac{144}{240}=\frac{3}{5}$, $\frac{200}{240}=\frac{5}{6}$, and $\frac{30}{240}=\frac{1}{8}$, or $\frac{1}{4}$ of $\frac{1}{2}$.

Exam. 3. $\frac{5}{9}$, $\frac{6}{10}$, and $6\frac{3}{5}$. When $6\frac{3}{5}$ is made an improper fraction, $\frac{33}{5}$, $\frac{5}{9}$ by the rule will be $=\frac{250}{450}$, $\frac{6}{10}=\frac{270}{450}$, and $\frac{33}{5}=\frac{2970}{450}$.

Obſ. The reaſon of this problem is obvious; for, ſince the numerator and denominator of each fraction is equally multiplied, *viz.* by the denominators of all the other fractions, conſequently the fractions produced muſt be equivalent.

Prob. 6. To reduce a given fraction to another equivalent, having an aſſigned denominator when poſſible.

This is only an abbreviation of the laſt problem in poſſible caſes, and may be done by multiplying the numerator by the aſſigned denominator, and dividing that product by the old denominator, the quotient, if there is no remainder, will be the numerator required.

Exam. $\frac{2}{3}$ to a fraction whoſe denominator is 6. Thus, $\frac{2\times6}{3}=\frac{4}{6}$

Note, In addition of money, the different reckonings by Scotch and Engliſh will occaſion fractions of different denominators, to be annexed in the column of pence, as 4ths, 3ds, and 6ths; but theſe are very expeditiouſly added, by conſidering 12 as the common denominator; in which caſe, $\frac{1}{4}=\frac{3}{12}$, $\frac{1}{2}=\frac{6}{12}$, $\frac{1}{3}=\frac{4}{12}$, &c.

Prob. 7. To reduce a fraction to lower terms when poſſible.

Divide

Divide both numerator and denominator by any number that will meaſure both without a remainder; but, if nothing but unity is the ſame meaſure of both, the fraction is already in its loweſt terms.

Hence even numbers may be divided by 2 continually, while poſſible. Numbers with ciphers may have an equal number cut off from each, and afterwards divided when poſſible.

Numbers ending in 5s, or 5 and cipher, may be divided by 5.

By a little practice, a common meaſure will readily occur for diviſible numbers.

Exam. $5|\frac{365}{455}=\frac{73}{91}$. $2|\frac{26}{78}=\frac{13}{36}$. $3|\frac{1500}{1800}=\frac{5}{6}$. and $9|\frac{18}{27}=\frac{2}{3}$.

Prob. 8. To bring a fraction of a higher denomination to an equivalent fraction of a lower.

Reduce the numerator to the name required for the numerator of the new fraction, the denominator will be the ſame as before.

Exam. $\frac{5}{7}$ l. to the fraction of a ſixpence; thus,

$$\overset{s.}{5}\times\overset{6ds.}{20}\times 2=\frac{200}{7}$$

$\frac{3}{5}$ of a guinea to the fraction of a farthing, $\overset{s.}{3}\times\overset{d.}{21}\times\overset{q.}{12}\times 4=\frac{3024}{5}$

Prob. 9. To reduce the known parts of a relative unit to the equivalent fraction of that unit.

This was formerly taken notice of in the corollary of prob. 4. and it is only reſumed here for further illuſtration, to thoſe who may find it ſtill neceſſary.

Reduce all the given parts to the loweſt mentioned, for a numerator, and the integer into the ſame name for a denominator.

Exam. 5s. $7\frac{1}{2}$d. to be expreſſed fractionally, $\frac{5\times 12+7\times 2+1=135}{20\times 12\times 2=480}$

Prob. 10. To value fractions in the known parts of the integer.

This is the converſe of the laſt problem, and hath been exemplified in diviſion, but ſtill it may not be improper to give the rule.

Multiply the numerator by the parts of the next inferiour denomination, and divide the product by the denominator; the

the quotient shews the parts of that denomination, and the remainder becomes a new numerator, which must be valued as before, &c. till the fraction is brought to the lowest known name of the integer.

Exam. Value $\frac{135}{480}$ l.

```
       135
        20
      ----s. d.
480)2700(5 7½ as above.
      2400
      ----
       300
        12
      ----
      3600
      3360
      ----
       240
         4
      ----
       960
       960
      ----
         0
```

Value $\frac{7}{12}$ of cwt.

```
      7
      4
   -----Qr. lb. oz.
12)28 (2    9   5⅓
     24
    ---
      4
     28
    ---
    112
    108
    ---
      4
     16
    ---
     64
     60
    ---
      4
```

CHAP. II.

ADDITION of VULGAR FRACTIONS.

RULE.

REDUCE all the given fractions to simple fractions of the same integer and denominator, if not so already; then the sum of the numerators with the common denominator, will be the fractional sum required, which may be reduced to a mixed number, valued or expressed in shorter terms, as seems most expedient, or the case will admit.

EXAMPLES.

Add $\frac{3}{7}+\frac{4}{7}+\frac{5}{7}$ $3+4+5=\frac{12}{7}=1\frac{5}{7}$. Here 7 is a common denominator.

Add $\frac{3}{5}$, $\frac{1}{2}$, and $\frac{2}{3}$.

$$3\times2\times3=18$$
$$1\times5\times3=15$$
$$2\times5\times2=20$$
$$\overline{53} \text{ numerator.} \quad =1\frac{23}{30}$$
$$5\times2\times3=30 \text{ denominator.}$$

Add $\frac{3}{5}$ s. to $\frac{3}{5}$ l. First, $\frac{3}{5}$ s $=\frac{3}{5}$ of $\frac{1}{20}=\frac{3}{100}$.
Then $3\times100+5\times3=\frac{315}{500}=\frac{63}{100}$.
And $5\times100=500$.

Add L. 4 : 15 : 6, and L. 3, $\frac{7}{8}$. First, 15 s. 6 d. $=\frac{31}{40}$
Therefore, $7\times40=280$

$$31\times8=248$$
$$\frac{528}{8\times40=320}=1\frac{208}{320}$$
$$3$$
$$4$$
$$\overline{8\frac{208}{320}}$$

Obs. If reduction is well underſtood and remembered, the addition of fractions will be very eaſy; the reaſon of which will be obvious, if we conſider that the given fractions being ſuch, or reduced to ſuch a ſtate, that all the numerators repreſent things of the ſame denomination, both abſolute and relative; their ſum muſt therefore be a number of ſuch parts as the common denominator expreſſes of the ſame common integer.

CHAP. III.

SUBTRACTION OF VULGAR FRACTIONS.

RULE.

REDUCE the given fractions to ſimple ones of the ſame integer and denominator, as in addition,

 and

and the difference betwixt the numerators, with the common denominator, will be the fractional difference required.

EXAMPLES.

(1.)

$\frac{4}{5} - \frac{3}{5} = \frac{1}{5}$

(2.)

$\frac{5}{6} - \frac{3}{4}$ $\begin{array}{l} 5\times4=20 \\ 6\times3=18 \\ \hline \quad\;\; 2 \\ \hline 6\times4=24 \end{array} = \frac{1}{12}$

(3.)

$\frac{3\,l.}{4} - \frac{3\,s.}{4} = \frac{57}{80}$

For $\frac{60}{80} - \frac{3}{80} = \frac{57}{80}$

(4.)

L. $4\frac{1}{2}$ — *L.* $3\frac{1}{6} = 1\frac{2}{3}$. For $4\frac{3}{6} - 3\frac{1}{6} = 1\frac{1}{3}$, *&c.*

The reason of this rule will be evident from the last observation.

CHAP. IV.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.

REDUCE the given fractions to their simple expression, if not so before, and to improper fractions, if mixed numbers, then will the product of the numerators and denominators respectively give the product required.

EXAMPLES.

(1.)

$\frac{3}{5}\times\frac{5}{6}=\frac{15}{30}$, or $\frac{1}{2}$.

(2.)

$5\frac{5}{6}\times3\frac{3}{4}=\frac{35}{6}\times\frac{15}{4}=\frac{525}{14}$:

(3.)

$\frac{3}{4}$ of $\frac{5}{6}\times\frac{1}{4}$ of $\frac{1}{3}=\frac{15}{24}\times\frac{1}{12}=\frac{15}{288}$.

(4.)

$\frac{3}{4}$ s. $\times\frac{1}{2}$ l. $=\frac{3}{80}\times\frac{1}{2}=\frac{3}{100}$

OBSER-

OBSERVATION.

The operations in this rule are ſo ſimple, that more examples will be unneceſſary.

It is worth while however to obſerve, that multiplication of fractions at firſt ſight, would ſeem to contradict the definition of multiplication given in the firſt part. But this diſficulty will evaniſh, if we conſider that the more any integral number is increaſed, the farther is the figure in the higheſt place removed from unity, and the more any part of an integer is decreaſed, the farther will its value alſo be removed from its relative unit; conſequently, as it is the nature of integers to increaſe, and of fractions to decreaſe, the purpoſe of multiplication is equally anſwered in both caſes. But, to be plainer ſtill, we had inſtances in the application of multiplication and diviſion to ſquare and ſolid meaſure, that a part of the multiplicand was taken when we wanted to multiply by a part of a foot; for this reaſon, To multiply by 1, would give the multiplicand for the product, and to multiply by any part of 1, can give but that part of the multiplicand for the product which the fraction expreſſeth.

CHAP. V.

DIVISION OF VULGAR FRACTIONS.

RULE.

PREPARE the fractions as in multiplication, then place the denominator of the diviſor above the ſeparating line, and the numerator below it, work as in multiplication, and the products ſo found will be the quotient.

EXAMPLES.

(1.) Divide $\frac{3}{5}$ by $\frac{1}{3}$. Thus, $\frac{1}{3})\frac{3}{5}(\frac{9}{5}$.

(2.) $\frac{4}{7}$ by $\frac{1}{4}$ of $\frac{2}{5}$. $\frac{20}{2})\frac{4}{7}(\frac{80}{14}$.

(3.) $4\frac{5}{6}$ by $2\frac{2}{5}$. $\frac{5}{12})\frac{29}{6}(\frac{145}{72}$.

(4.) $4\frac{3}{7}$ by $1\frac{3}{4}$ of $\frac{1}{2}$. $\frac{8}{11})\frac{31}{7}(\frac{248}{77}$.

Obs. 1. Mixed numbers may be divided by mixed numbers, with as much brevity, and perhaps more intelligibly, especially in applicate cases, by the following rule.

Reduce the divisor to an improper fraction, whose numerator will be a new divisor, the quotient arising therefrom multiplied into the old numerator, will give the fractional part of the quotient; and, being multiplied into the denominator, will give the quotient due to unity.

An example will illustrate this.

Suppose L. 478 : 19 : 6 were to be divided among 3 men, who were to have equal shares, and a boy who was to get only $\frac{1}{3}$ of what any one of the men got; what would fall to each?

First, $3\times3+1=\frac{10}{3}$)	478	19	6	
	47	17	$11\frac{2}{5}$	to the boy, for the
			3	old numerator was 1.
	143	13	$10\frac{1}{5}$	to each man for the
	143	13	$10\frac{1}{5}$	denominator is 3.
	143	13	$10\frac{1}{5}$	
	478	19	6	as before.

Obs 2. The reason of division of fractions in general will be pretty obvious, if we consider, that, were the divisor and dividend reduced to one common denominator, the dividend would contain the divisor as oft as its numerator did that of the divisor; for, having one denominator, they are in the same state with respect to one another, as whole numbers. Now, though no mention is made in the rule of a common denominator, yet the operation is manifestly the same, as that by which fractions are reduced to a common denominator. Hence we have this general consequence, that if any two numbers be divided the one by the other, the quotient of the one by the other is the reciprocal of the quotient of that one by the other.—The method in observation 1. for dividing by mixed numbers in applicate cases, is so plain, that it needs no demonstration. For since all the parts are equal to the whole, the divisor, I mean the numerator by which we divided, expresseth all the parts contained in it when a mixed number, of which the quotient represents so many as the dividend containeth, considered as a mixed number; the quotient therefore, when multiplied by the numerator

merator of the fractional part of the divisor, in its first state, must undoubtedly give the quotient due to that fraction; and when multiplied by the denominator, must give the quotient due to unity; the sum of all the units, and the fractional part being for proof always equal to the dividend.

Application of vulgar fractions to questions.

1. A merchant had bonds for L. 774 : 13 : $4\frac{1}{3}$, bills amounting to L. 299 : 19 : $7\frac{5}{6}$, goods valued at L. 785 : 15 : $5\frac{4}{5}$, and open accounts for L. 570 : 19 : $6\frac{3}{4}$; his debts amounted to L. 999 : 19 : $9\frac{5}{8}$; how much was he worth?

First, *L.*	774	13	$4\frac{1}{3}=\frac{2}{6}$	
	299	19	$7\frac{5}{6}=\frac{5}{6}$	
	785	15	$5\frac{4}{5}=\frac{16}{20}$	
	570	19	$6\frac{3}{4}=\frac{15}{20}$	
From	2331	8	0	$\frac{43}{60}$
Take	999	19	9	$\frac{5}{8}$
Nett stock,	1331	8	3	$\frac{11}{120}$

$\frac{5}{6}+\frac{2}{6}=1\frac{1}{6}=1\frac{10}{60}$.

$\frac{16}{20}+\frac{15}{20}=1\frac{11}{20}=1\frac{33}{60}$.

Therefore I carry 2, and note $\frac{43}{60}$ for the fraction.

To one versant in additions of this kind, the summation of such fractions as can be brought to one assigned denominator, is as easy as any other.

2. What will $17\frac{3}{4}$ yards of cloths come to at 15 s. 8 d. *per* yard.

First, $17\frac{3}{4}=\frac{71}{4}$, and 15 s. 8 d. $=\frac{138}{240}$. Then, $\frac{71}{4}\times\frac{[illegible]}{240}=\frac{13348}{960}=$ *L.* 13 : 18 : 1.

3. Bought $17\frac{3}{4}$ yards of cloth for L. 13 : 18 : 1, what did it cost *per* yard?

First, $17\frac{3}{4}=\frac{71}{4}$, and L. 13 : 18 : 1 $=\frac{13348}{960}$. But, $\frac{4}{1})\frac{13348}{960}(\frac{53392}{68160}=$ 15 s. 8 d.

4. What is the solid content of a bale 5 feet 6 inches long, 4 feet 5 inches broad, and 3 feet 3 inches thick.

	f.	*i.*	
First,	5	$6=\frac{11}{2}$	least terms.
	4	$5=\frac{53}{12}$	
	3	$3=\frac{13}{4}$	least terms.

Then $\frac{11\times53\times13}{2\times12\times4}=\frac{7579}{96}=$ 77 *f.* $10\frac{7}{8}$ *i.*

5. What

5. What is the tunnage of a bale 77 feet $10\frac{7}{8}$ inches ſolid, at 40 *per* tun?

$$\begin{matrix} & f. & i. \\ \frac{1}{40}) & 77 & 10\frac{7}{8} = \frac{7579}{96}\ (= \frac{7579}{3840} = 1\frac{3739}{3840}. \end{matrix}$$

6. Divide L. 578, 19 s. among A, B, C, and D; let A, B, and C have equal ſhares, and each $\frac{1}{3}$ more than D.

	L.	s.	d.	
$3\frac{3}{4} = \frac{15}{4}$)	578	19	0	
	38	11	$11\frac{1}{5}$	
			3	
	115	15	$9\frac{3}{5}$	to D.
	154	7	$8\frac{4}{5}$	to A.
	154	7	$8\frac{4}{5}$	to B.
	154	7	$8\frac{4}{5}$	to C.
	578	19	0	Proof.

II. OF DECIMAL FRACTIONS.

INTRODUCTION.

IN decimal fractions, an unit is ſuppoſed to be divided into 10 equal parts, each of theſe into 10 other equal parts, and each of theſe into 10 other equal parts, if neceſſary, and ſo on *ad infinitum*.

A decimal fraction is diſtinguiſhed from an integer by a comma or point prefixed to it, whoſe denominator, though ſeldom or never expreſſed, is eaſily known by the diſtance of the firſt figure to the right hand, from the ſeparating point, counted as 1, at the ſame diſtance in the numeration-table, from the unit's place. Hence .5 is expreſſed $\frac{5}{10}$, .75 as $\frac{75}{100}$, .05 as $\frac{5}{100}$, .005 as $\frac{5}{1000}$, *&c.* Hence it will be obvious, that, in a ſeries of decimal parts, the decreaſe will be in proportion as the increaſe in a ſeries of whole numbers, and

and that, in consequence, ciphers immediately after the point diminish the value of the figures succeeding them, in the same proportion, as the figures considered as integers would be increased by the same number of ciphers annexed.

CHAP. I.

REDUCTION OF DECIMALS.

Prob. 1. To reduce a vulgar fraction to a decimal.

Divide the numerator by the denominator; and as ciphers must be annexed to the numerator before the divison, the quotient must consist of as many places as the numerator had ciphers annexed to it, which will be the decimal required.

EXAMPLES.

(1.) Reduce $\frac{4}{5}$ to a decimal. 5) $\frac{40}{.8}$.

(2.) Red. $\frac{3}{8}$ to a decimal. 8) $.\frac{3000}{375}$.

(3.) Reduce $\frac{1}{20}$ to a decimal. 20) $.\frac{100}{05}$.

(4.) Reduce $\frac{1}{80}$ to a decimal. 80) $.\frac{100}{0125}$.

Note, It will often happen, that, in the divison, there will continually be a remainder, and the quotient repeat the same figure, or figures, *ad infinitum*; in which case, it will be unnecessary to carry on the divison farther, when you have once got the repetend, which may be single, repeating always the same figure; or compound, always repeating or circulating the same figures. A single repetend may be marked or distinguished with a point above it, and compounds with a point above the first and last figures of the circulation.

EXAMPLES of SINGLE REPETENDS.

(1.)

Reduce $\frac{1}{3}$ to a decimal.

$$\frac{3)\ 10}{.\dot{3}333}$$ *ad infinitum.*

(2.)

Reduce $\frac{5}{6}$ to a decimal.

$$\frac{6)\ 50}{.8\dot{3}3333}$$ *ad infinitum.*

(3.)

(3.)

Reduce $\frac{7}{12}$ to a decimal. $12)\ 70$

$.58\dot{3}333$, &c.

(4.)

Reduce $\frac{5}{36}$ to a decimal. $36)\ 50$

$.13\dot{8}88$, &c.

EXAMPLES OF COMPOUND REPETENDS.

(1.)

Reduce $\frac{2}{11}$ to a decimal. $11)\ 20$

$.\dot{1}\dot{8}1818$ *ad infinitum.*

(2.)

Reduce $\frac{5}{27}$ to a decimal. $27)\ 50$

$.\dot{1}8\dot{5}185$, *etc.*

OBSERVATION.

The reaſon of this reduction is manifeſt: for, to add ciphers to the numerator, is in effect the very ſame thing as to multiply by 10, 100, 1000, *&c.*; conſequently the quotient will be ſo many 10's, 100's, 1000's, *&c.* as the places contained in it repreſent; the numerator and denominator being increaſed in the ſame proportion: hence, if the quotient is found without a remainder, it will be equal to the vulgar fraction, from which it was taken; and if it terminated in any repeating figure or figures, it will be ſhort of the fraction from which it was taken, by a fraction of which the remainder is the numerator, and the diviſor the denominator, in the laſt place of the qotient; that is, the remainder 1, in the firſt example of ſingle repetends, $=\frac{1}{3}$ of $\frac{1}{10000}$, and the remainder 2, in the ſecond example, $=\frac{2}{6}$ of $\frac{1}{1000000}$. If therefore, in any decimal reduction, the diviſion is carried on to 4 or 5 places, the remainder will be too minute to cauſe any ſenſible errour in calculations of buſineſs; though, at the ſame time, for the ſatisfaction of the curious, particular rules are given here for the management of circulating decimals.

Prob. 2. To reduce any of the known parts of an integer to a decimal.

This

This can be done by reducing the given parts to their equivalent vulgar fraction, and thence to a decimal, as above; but much more expeditiouſly by the following

RULE.

Annex ciphers to the given number of the loweſt denomination, and divide it by ſuch a number of that, as is equal to an unit of the next ſuperiour; to that quotient prefix the given number of units of the next ſuperiour, and divide by as many of that as will make an unit of the next ſuperiour; proceed in the ſame manner throughout the whole, and the laſt quotient is the decimal required.

EXAMPLES.

(1.)

Reduce 7s. 6d. to a decimal.

12)60
2)75
·375

(2.)

Reduce 4s. $6\frac{1}{2}$d. to a decimal.

2)10
12)65
2)45416
.227083

Note 1. The reaſon of this manner of reduction is the ſame with the foregoing, as it is only dividing by the component parts of the denominator of a vulgar fraction equivalent to the given parts.

2. In reducing to decimals, it is unneceſſary to mind any ciphers on the right of the diviſor, as ciphers muſt be annexed to the remainders to bring out the quotient.

3. When in any diviſion the quotient is repeated, the next diviſion is carried on, not by ciphers, but by that repetend as in the ſecond example, and that quotient will always terminate in a repetend.

By the laſt rule, it will likewiſe be found that

s. d. L. oz. dwt. gr. lb.
10 $8\frac{1}{3}$= .53472; that 11 18 20 Troy =.9951388; that
in. feet. c. q. lb. tuns.
$4\frac{1}{8}$=.34375, and that 19 3 14=.99375, &c.

OBSERVATION.

After this manner are all decimal tables constructed, and they who think such tables contribute any thing to dispatch, may easily construct them. But I cannot help thinking that any decimal parts may be found as quickly, as they can be picked out of a table, if not more so, by one much versant in figuring; and with this further advantage, that, by calculating them upon all occasions, one gains a certain practice which renders figuring easy and familiar, at the same time that he avoids the meanness of seeing with the eyes f others.

Prob. 3. To value decimals in the known parts of the integer.

Case 1. If the decimal was finite, multiply by as many of the next inferiour denomination as make one of the integer, and point off a number of places from the right of the product, equal to those in the given decimal; which will be a new decimal, to be multiplied by as many of the next inferiour denomination, as made one of the former; proceed thus till the decimal places become ciphers, or no denomination be left for a multiplier, and the figures to the left hand of the point give the value required.

EXAMPLES.

(1.)

Value .375 *l.*

	s.	d.
375	=7	6
20		
7.500		
12		
6.000		

(2)

Value .99375 tuns.

	cwt.	q.	lb.
.99375	=19	3	14
20			
19.87500			
4			
3.50000			
28			
14.00000			

Note, This is only the converse of the last problem.

Case

Case 2. If the given decimal terminate in a repetend, carry 1 for every 9 in the product of the first figure of the multiplication.

EXAMPLES.

L. *s.* *d.*	*cwt. q. lb.*	lb. *Troy, oz.*
$.8\dot{3}3$=16 8	$.\dot{6}66$ tun=13 1 $9\frac{1}{3}$	$.8\dot{3}3$ = 10
20	20	12
16.660	13.33	10.00
12	4	
8.00	1.33	
	28	
	9.33	

Note, In multiplying, the ciphers to the right need not be noted, but they must be remembered in pointing the product.

OBSERVATIONS.

1. The reason of carrying at 9 in multiplying repetends, will be pretty obvious, if we consider, that a series of 9's infinitely continued is equal to one in the left hand place; for it is evident that .9, or $\frac{9}{10}$, is only $\frac{1}{10}$ short of 1; that .99, or $\frac{99}{100}$, wants $\frac{1}{100}$ of 1, and that .999, or $\frac{999}{1000}$, is only $\frac{1}{1000}$ of 1; so if the series was carried on to infinity, the difference between that series of nines and an unit would be equal to unity divided by infinity, which would quote nothing.

2. The first three places in decimals, will give the value exact enough in business; and as, in mercantile calculations, the value of the decimal of a pound Sterling is more generally wanted than any other, it may be found by inspection thus

Double the figure next to the point, make shillings; if the next figure be 5, or above, add one to the shillings; the figure in the second place, if below 5, or the surplus above 5, added to the third figure, make farthings; but, if their sum be 25, or under, 25, one must be taken off, to make them farthings; and if their sum be 26, or above, two must be taken off.

As the last part of this rule differs from the rules commonly given, it will perhaps be necessary to give some illustration of it.

The difference between a pound in thousand parts, and a pound in farthings, is 40: consequently, as 1000 is to 40, so is any other number of decimal parts to what must be taken off to make them farthings; but 1000:40::500:20, so is 250 to 10, so is 25 to 1; by which it is plain that one taken from 25 1000 parts, make them 24 farthings exactly.

As the rules commonly given for finding the value by inspection, give an answer frequently not precisely the same with the other rule of multiplying by 20, 12, &c. and dividing by 1000, it seems necessary to show that both rules give the same answer precisely as the margin.

.507	.776
20	20
10.140	15.520
12	12
1.68	6.24
4	4
2.72	0.96

In the first example, there is seven thousand parts, from which I take one, conform to the rule, and there remains 6 farthings, or 1 penny 2 farthings; and to find the .72 that is over, I say, as 25 to 1, so 7 to .28, which must be taken from the 7, as in the margin.

7
.28
6.72

In the second example there is 26 over the shillings, from which take 2, conform to the rule, and there remains 24 farthings, or 6 pence; and to find the 96, I say, as 25 to one, so is 26 to 1.04, which must be subtracted from the 26.

26
1.04
24.96

CHAP. II. ADDITION OF DECIMALS.

Case 1. HAVING placed the numbers to be added, whether pure decimals or mixed numbers, successively below one another, in such a manner as the several points may be in one column, tenths under tenths, hundreds under hundreds, &c.; if the given decimals are finite, add them as integers, and mark the separating point in the sum directly under the points of the given decimals, or point off as many for decimals as were in any of the given numbers which had most places.

EXAMPLES.

75.436	$59\frac{1}{4}$	= 59.25
47.324	$67\frac{1}{2}$	= 67.5
3.21	$48\frac{3}{4}$	= 48.75
6.7547	$8\frac{1}{8}$	= 8.125
.307	$9\frac{1}{20}$	= 9.05
.005		
133.0367	$192\frac{27}{40}$	= 192.675

Case

Case 2. When all, or any of the decimals, repeat a single digit, make the repetends conterminous, and add 1 to the sum of the first, or right hand column, for every nine that is contained in it.

EXAMPLES.

$475\frac{2}{3}$ = 475.6666	L. 59	7	$7\frac{3}{4}$	=	L. 59.3822916
$397\frac{1}{6}$ = 397.1666	57	17	5	=	57.8708334
$475\frac{5}{6}$ = 475.8333	57	13	4	=	57.6666666
$99\frac{1}{9}$ = 99.1111	25	6	8	=	25.3333333
$8\frac{5}{9}$ = 8.5555	45	13	4	=	45.6666666
$1456\frac{1}{3}$ = 1456.3	245	18	$4\frac{3}{4}$	=	245.9197916

OBSERVATION.

The finite value of a pure circulate is a fraction whose numerator is the repetend, and denominator a number of as many places of nines, with a number of ciphers on the right, equal to the places betwixt the point and repetend. Hence, where the conterminous repetends of several circulates are added, their sum is a numerator to the common denominator; and if one for every nine in the sum is added thereto, it is reduced to the finite expression.

Case 3. If the decimals are compound repetends, from the place where all the repetends begin together, continue each decimal to a number of places, equal to the least common multiple of those several numbers, which represent the places of figures in the said repetends; then add, and to the last place add as many units as there are tens in the place, where the repetends all begin together, and the figures in these two places are the first and last of the repetend.

EXAMPLES.

175.3724	14.472956	121.47237
84.5634	12.307243	80.27553
126.4526	9.020763	64.90834
79.3279	11.912375	80.07444
105.7105	8.457813	9.83333
571.4270	56.171152	356.56405

OBSER-

OBSERVATION.

Any repetend whatever is to the ſame number complete, as 10 to 9, if of one place; as 100 to 99, if of two places; and as 1000 to 999, if of three places, &c. Hence any number multiplied by 1, with as many ciphers as it contains figures, and the product thereof divided by as many nines, will give the ſame number perpetually circulating; but to add 1 for every ten contained in the ſum, is the ſame as to multiply that ſum by 10, and divide by 9.

CHAP. III.

SUBTRACTION OF DECIMALS.

Caſe 1. PLACE the numbers homogeneous under homogeneous, ſo as point may be under point; then, when the decimals are finite, ſubtract as in integers, and let the point in the remainder ſtand directly under thoſe of the factors.

EXAMPLES.

From	*L.* 74	19	6 = 74.975		*L.* 171	13	6 = 171.675
Take	18	11	9 = 18.5875		97	18	9 = 97.9375
	56	7	9 = 56.3875		73	14	9 = 73.7375

Caſe 2. If a ſingle digit is repeated, borrow 9 in the firſt repeating place when neceſſary.

EXAMPLES.

From	*L.* 7849	6	8 = 7849.$\dot{3}$33	*L.* 17	11	7 = 17.5791$\dot{6}$
Take	6979	13	4 = 6979.$\dot{6}$66	14	16	8 = 14.8$\dot{3}$333
	869	13	4 = 869.666	2	14	11 = 2.7458$\dot{3}$

Caſe 3. If the decimals be compound repetends, order them as in addition of compound repetends; and if it is neceſſary to borrow one, where both repetends begin together, add one to the right-hand place of the ſubtrahend, and that figure

figure in the remainder, where both repetends begin together, will be the firſt, and the right-hand figure the laſt of the repetend.

EXAMPLES.

47.4178178	47.8540060	153.92749
15.5656565	40.9259259	142.85353
31.8521612	6.9280801	11.07395

CHAP. IV.

MULTIPLICATION OF DECIMALS.

Caſe I. WHEN the decimals are finite, find the product as in integers, from which point off as many for places as were in both factors; if the whole product doth not count ſo far, ſupply that defect by prefixing ciphers.

EXAMPLES.

368.5	672.5	.246
2.75	.365	.125
18425	33625	1230
25795	40350	492
7370	20175	246
10.13375	245.4625	.030750

OBSERVATION.

If we conceive the two given numbers as fractions, it will be plain that the numerators and denominators are multiplied together reſpectively, ſince as many places are taken from the product, as there are in the denominators of both factors; which likewiſe accounts for prefixing ciphers, when there are otherwiſe not ſo many places in the product as were in both factors.

CON-

CONTRACTIONS.

All the contractions in the first part, which regard multiplication, may be applied here, but the following seem peculiarly adapted to multiplication of decimals.

1. To multiply by 10, 100, 1000, &c. remove the decimal point so many steps further to the right, as there are ciphers in the multiplier.

As, 47.565×100=4756.5, and .45×1000=450, &c.

2. When the places of decimals run far in both factors, the work may be contracted to as few places of decimals as may be thought sufficient for the purpose, by the following rule.

Set the units place in the multiplier directly under that figure of the decimal part in the multiplicand, whose place you would preserve in the product; invert all the other figures of the multiplier, and, in multiplying, begin with the figure of the multiplicand, which stands over the figure wherewith you are then multiplying, and set down the first figure of every particular product directly under each other, remembering at the same time to add the increase which would arise from the multiplication of the two next right-hand figures of the multiplicand, to the first figure of every product; that is, if the product of the next right-hand figure, with as many units added to it as there are tens in the product of the second right-hand figure, be any number betwixt 5 and 15, carry 1: if 15, or any number below 25, carry 2, and so in proportion.

EXAMPLES.

Multiply 54.321711 into 3.12321, and preserve 4 decimal places in the product.

	54321711
	12321.3
543217×3 =	1629651
54321×1+1 =	54322
5432×2 =	10864
543×3+1 =	1630
54×2+1 =	109
5×1 =	5
	1696581

Multiply 231.3121 into 21.32, and save 3 decimal places.

231.3121
23.12
4626242
231312
69394
4626
4931.574

Case 2. If the multiplicand terminate in a single repetend, and the multiplier is only a single digit, carry at 9 in the first figure of the multiplication on the right hand.

EXAMPLES.

10.7016	9.305	476.05	74.86
5	7	.08	.6
53.5083	65.138	38.0844	44.920

3. If the multiplier consists of several digits, or figures, make the products conterminous before addition.

EXAMPLES.

748.65	158.83
.634	123
299462	47650
2245966	1906000
44919333	
474.64762	19536.50

Case 3. If the multiplier be a repetend, multiply the product found, as before, by 10, and divide that product by 9, which will quote the true product, if the division is continued, till the quotient terminate in a single or compound repetend.

EXAMPLES.

724.35	251.43	48.754
5.04	8.74	2.13
9)289.740	9)100.572	9)14.6263
32.193	11.1746	1.625148
3621.75	176001	4875444
	201144	97508888
Product 3653.943	Product 2198.6156	104.009481

Case 4. If the multiplicand be a compound repetend, and the multiplier only a single digit, to the product of the first figure on the right hand add as many units as there are tens in the product of the left hand place of the repetend.

EXAMPLES.

```
582.347
      8
-------
4658.778
```

```
5924.378
    ·03
--------
177.73135
```

Case 5. If there are no repetends in the multiplicand, and the multiplier be a compound repetend, add the product found, as in finite decimals, to itself in this manner. Set the first left-hand figure so many places forward, as exceeds the number of places in the repetend by one, and the rest of the figures in order after it, and proceed thus till the highest figure of the product stand directly below, or be removed beyond the lowest figure in the first position. Then, beginning with the lowest figure of the product in its first position, add it with all the figures that are below it, and do the same by the rest in their order; then point off as many places for a repetend as the multiplier consists of.

EXAMPLES.

```
235.01
  3.26
------
141006
 47002
70503
---------
766.1326
   7661326
        7661, &c.
---------
766.8994
```

```
42710.36
  .20403
--------
12813108
17084144
8542072
-----------
8714.1947508
     87141947, &c.
          871
-----------
8714.2818938
```

All

All these cases of circulating decimals might be effected as intelligibly, though not so quickly, by managing them as vulgar fractions.

CHAP. V.

DIVISION of DECIMALS.

Case 1. WHEN the decimals are finite, the quotient is found, as in integers, and in all cases pointed or valued, by the following rules.

1. If the places in the divisor and dividend are equal, the quotient is integral.

2. If the divisor hath most places, annex ciphers to the dividend, to make them equal, and the quotient will still be integral.

3. If the dividend hath most places, point off places for the excess in the quotient.

4. If the whole quotient is not equal to the excess, prefix ciphers for the defect.

Note, If, after the quotient is qualified, there be a remainder, the division may be continued at pleasure.

EXAMPLES.

(1.)

```
24.35)78345.15(3217
      7305
      ----
       5295
       4870
       ----
        4251
        2435
       -----
        18165
        17045
        -----
       (1120)
```

(2.)

```
.4725)113.4
4725)113.4000(240
      9450
     -----
      18900
      18900
     ------
      ....0
```

(3.)

```
2.87)67.05627(23.364
     574
     ---
      965
      861
      ---
      1046
       861
      ----
       1852
       1722
       ----
        1307
        1148
        ----
        (159)
```

(4.)

```
.543).0020091(.0037
      1629
      ----
      3801
      3801
      ----
       (0)
```

ILLUSTRATION.

In the first example, the places in each factor are two, and the quotient is integral.

In the second example, because the divisor consisted of four places, and the dividend but of one, three ciphers were annexed to the dividend, which made the places in both equal, and the quotient was accordingly integral.

In the third example, because there were five places in the dividend, and but two in the divisor, the excess, 3, was pointed off for decimals.

In the fourth example. as there were seven places in the dividend, and three in the divisor, the quotient required four places of decimals; and as there were but two places in the quotient, two ciphers were prefixed to make up the deficiency.

OBSERVATION.

The rationale of valuing or qualifying the quotient will appear, if we consider, that the product of the quotient and divisor is equal to the dividend, and consequently the places of the divisor and quotient, counted together, will always be equal to the dividend; or, which is the same thing, the number of places in the quotient must be equal to the difference of the places in the divisor and dividend.

CONTRACTIONS.

1. In dividing by 10, 100, 1000, &c. the quotient is found

found by removing the decimal point in the dividend ſo many ſteps towards the left hand, as there were ciphers in the diviſor. Hence,

$$\frac{34.5}{10}=3.45, \text{ and } \frac{34.5}{100}=.345.$$

2. The work of diviſion may be contracted in the ſame manner as multiplication, by the following

RULE.

Having conſidered in what place the firſt figure of the quotient ought to ſtand, and ſo found its value, or denomination, take as many of the left-hand figures as you intend to have figures in the quotient, for the firſt diviſor, and then take as many figures of the dividend as will anſwer them; in dividing, omit, or point off one figure at each operation, at the ſame time judging as exactly as poſſible what would be the increaſe, ariſing from the figure, or figures, ſo omitted.

EXAMPLES.

```
384.672158)14169.2066238510(36.8345
           11540 16474
           -----------
           26290 4188.
           23080 3295.
           -----------
            3210 0893..
            3077 3772..
            -----------
             132 7121...
             115 4016...
             -----------
              17 3105....
              15 3869....
              -----------
               1 9236.....
               1 9234.....
               -----------
```

```
9.365407)87.076326(9.297655
         84 288663
         ---------
          2 787663
          1 873081
          --------
            914582
            842886
            ------
             71696
             65558
             -----
              6138
              5619
              ----
               519
               468
               ---
                51
                47
                --
               (4)
```

All the other contractions proposed in the division of integers may be very properly applied here.

Case 2. When the dividend contains a single repetend, and the divisor is either a single terminate digit, or any number of terminate digits, the quotient will either repeat a single digit or compound repetend, commencing generally where the repetend is first taken down.

EXAMPLES.

$8)79.2\dot{6}$	$6)3076.\dot{1}\ldots$	$7)51.\dot{2}$
$9.908\dot{3}$	$512.6\dot{8}5\dot{1}$	$7.31746\dot{0}$

2. When the divisor is only a single repetend, and the dividend either a terminate number, or contains a repetend, multiply the dividend by 9, and point off the same number of integral places, which were first given, the product thus qualified will be a new dividend.

Exam. 378.45 by $\dot{6}$.	Or thus, 572.4 by $.\dot{8}$
9	57.24 subtract $\frac{1}{10}$
$.\dot{6})340.605$	$.\dot{8})515.16$
567.675	643.95

3. If the divisor hath terminate numbers joined to the repetend, subtract the terminate part of the divisor from the whole divisor,—prepare the dividend as before, and work by the new factors.

Exam.

```
Exam. 48.6 \ 8567.28               3.3 ] 728.5
      48    ) 856.728              3   |  72.85
      ----  | -------              --- | ------
      4.38 /  7710.552(1760.4      3.0 ] 655.65
              438                        ------
              ----                       2185.5
              3330
              3066
              ----
               2645
              2628
              ----
               1752
               1752
               ----
```

4. When both factors, or the divisor only, consists of compound repetends, set the divisor and dividend under themselves, so many places forward, as there are places in the repetend of the divisor; subtract them, and the remainders will be respectively a new divisor and dividend.

EXAMPLES.

```
111.98 \ 243.306               587645 \ 411351.9
    11  )    243                 58.7  )   411.3
------  | ------               -------  | --------
111.87 /  243.063(2.172        587058. /  410940.6(7
          223 74                          410940 6
          ------                          --------
           19 323                           (0)
           11 187
           ------
            8 1360"
            7 8309
            ------
             30510
             22374
             -----
             8136"
```

APPLI-

APPLICATE QUESTIONS in DECIMAL FRACTIONS.

1. What is the decimal difference betwixt L. 1 and 13 s. 4 d.?

s. d.		
		1.000
13 4	=	0.666
		.333 Answer.

2. What is the square content of a room, 15 feet 6 inches by 14 feet 9 inches?

$\frac{1}{2}$) 14.75
$15\frac{1}{2}$

221.25
7.375

228.625 = 228 f. $7\frac{1}{2}$ i.

3. What is the solid content of a box 5 feet 4 inches long, 4 feet 10 inches broad, and 3 feet 8 inches thick

4.8333
$5\frac{1}{3}$

24.1666
1.6111

25.7777
$5\frac{2}{3}$

77.3333
8.5925
8.5925

94.5185 = 94 f. 6 i.

4. What

4. What is the tunnage of a bale 9 feet 9 inches long, 7 feet 3 inches broad, and 5 feet 6 inches thick?

$$\begin{array}{r} 9.75 \\ 7\frac{1}{4} \\ \hline 68.25 \\ 2.4375 \\ \hline 70.6875 \\ 5.5 \\ \hline 353\ 4375 \\ 3534\ 375 \\ \hline 4.0)388.78125 \text{ solid content.} \\ \hline 9.71953125 \text{ tunnage.} \end{array}$$

5. A piece of cloth, consisting of $25\frac{1}{2}$ yards, is valued at £ 23 : 17 : $5\frac{1}{2}$; how must one yard be valued at that rate?

$$\begin{array}{r|l} 25.5 & 23.872916 \\ \hline 5.1 & 4.774583 \\ \hline & .9361927 = 18\text{ s } 8\frac{1}{2}\text{ d.} \end{array}$$

6. What cost 22 cwt. 3 qrs. of sugar, at L 3 : 17 : 6 *per* cwt.?

$$\begin{array}{rl} 3.875 & \\ 22\frac{3}{4} & \\ \hline 7750 & \\ 7750 & \\ \hline 85.25 & = 22 \text{ cwt} \\ 1.9375 & = \frac{1}{2} \\ .96875 & = \frac{1}{4} \\ \hline 88.15625 & = 22\frac{3}{4} = \text{L } 88 : 3 : 1\frac{1}{2} \end{array}$$

7. Divide L. 875 : 14 : 6, among A, B, and C, whereof let A have $\frac{3}{4}$, B $\frac{3}{8}$, and C

$\frac{3}{4}$ = ·75
$\frac{3}{8}$ = ·375
$\frac{3}{5}$ = .6

5) 1.725 — 875·725
·345 — 175·145
.069 — 35·029(507·6666× { ·75 = 380.75 = A's ſhare. / ·375 = 90·375 = B's ſhare. / .6 = 304.6 = C's ſhare.
34 5 — 875·725 Proof.
529
483
460
414
.46 *ad infinitum.*

lb. ℥. dwt.
8. What will 46 8 16 amount to at L. 7 : 8 : 6 *per* lb.?

40)46.73 = the weight.
7·4 = L. 7, 8 s,

18.6933
327·1333
For 6 ds. 1.1683

346.995 = L. 346 : 19 : 10$\frac{3}{4}$

Demonſtration of the laſt contraction after diviſion of integers, promiſed in this place.

The ſum of 1, 10, 100, 1000, &c. continued to any number of terms = 1, the firſt term, as often repeated as the number of terms to be added: Thus, 1+10=11, and 1+10+100=111; and 1+10+100+1000=1111, as is obvious from the nature of numerical notation. Therefore, if any number,

number, as N, is multiplied by a ſeries of equal figures, as fff, &c. the firſt product by f, in the place of units, will be fN, or $1fN$; the next product by f, in the place of tens, will be $10fN$, the next $100fN$, &c.; and the total product of the whole multiplication will be $111fN$, according to the number of places in the multiplier. If 1, 10, 100, 1000, is divided decimally by 9, the quotient figures will be 1111, &c. continued to the number of periods in the dividend, thus $\frac{1}{9}=.1111$, and $\frac{10}{9}=1.1111$, and $\frac{100}{9}=11.1111$, ſtill leaving 1 of a remainder, and the quotient continually repeating 1. Therefore, $\frac{fN}{9}=.1111\,fN$; and, if we compare the two expreſſions, $111\,fN$ and $1111\,fN$, &c. we will find they may be made equal, by placing the decimal point after the third figure, in the laſt, thus, 111.111, &c. and then ſubtracting 111 from the decimal parts, the laſt, as well as the firſt, will be equal to the total product required. That is, $\frac{fN}{9}-.1111\,fN=111\,fN=T$, the total product.

For let the multiplicand $N=$ 784
The multiplier f = 333

Therefore fN = 2352

$111\,fN=\frac{fN}{9}=$ 261.3333, &c.

$111\,fN$ the difference deduced from the right hand = 2613

.261072$=T$, the total product.

Q. E. D.

Note, The figure 3 repeating equally in both on the right hand, and making no difference, is not to be ſubtracted.

THE
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PART III.

THE ELEMENTS OF ALGEBRA.

INTRODUCTION.

ALGEBRA is the art of reckoning by ſymbols, or a ſhort and conciſe method of reaſoning on the various relations of numbers and quantities to each other, by the help of letters, or ſuch other characters, as the algebraiſt thinks moſt convenient to adopt.

By algebra are reſolved, in a method of reaſoning which is clear and connected, and founded on ſelf-evident principles, many obſcure queſtions in arithmetic and geometry, and from thence theorems deduced, which hold univerſally with reſpect to problems of the ſame nature. Hence it is called by ſome *the new analyſis*, by others *ſpecious arithmetic*, or *numerical geometry*, becauſe it is equally connected with both, and extends not only to abſtract numbers, but alſo to lines, ſurfaces, and ſolids. It is not intended in this place to conſider algebra in its full extent; what is preciſely neceſſary for conducting the arithmetician ſcientifically through the whole of mercantile computations, ſhall be reduced to a few plain and intelligible rules; by the help of which we ſhall be able to demonſtrate the reaſon of the moſt complicated operation that can occur in arithmetic, with greater perſpecuity and elegance, than from principles purely arithmetical. At the ſame time, in the courſe of this work, we will not make uſe of algebraic demonſtrations, but where we cannot otherwiſe do juſtice to the ſubject, that the reader may have an alternative, if his time ſhould not permit him to ſtudy this part of the treatiſe.

CHAP.

CHAP. I.

NOTATION OF ALGEBRA.

ANY ſimple number is repreſented by a ſingle letter, and, for diſtinction's ſake, numbers that are known, and given in a queſtion, are generally expreſſed by conſonants, and numbers that are required are expreſſed by ſome of the ſmall vowels. Compound quantities are repreſented by two or more letters joined together; the ſum of two or more numbers is expreſſed by the ſign + connecting two or more letters together, as $a+b$ means a and b added together; $a+e+b$ means the ſum of a and e and b added together. The difference of two numbers is noted by the ſign —: thus $a-b$ ſignifies the difference of a and b, or what b wants cf being equal to a. The product of two or more letters is expreſſed by connecting them together in the form of a word, as $ab=a\times b$; $abc=a\times b\times c$; that is, the continual product of a, b, and c. The quotient of two numbers by diviſion is expreſſed like a vulgar fraction, with one letter repreſenting the dividend above the line, aed another below it repreſenting the diviſor, as $\frac{a}{b}$ ſignifies the quotient ariſing from a divided by b, or $a\div b$. The ſquare of any number or quantity is repreſented by a letter doubled, as aa, or its power noted above it thus a^2, each of which notations expreſs the ſquare of a, or a ſquare. The cube of any number or quantity is repreſented by tripling the letter, or noting its power, as aaa, or a^3 the biquadrate thus, $aaaa$, or a^4, &c. The ſquare root of any number is thus noted $\sqrt{2}$, the cube root $\sqrt{3}$, the biquadrate $\sqrt{4}$, &c.; as $\sqrt{2}\ 186$ is to be underſtood the ſquare root of 186; $\sqrt{3}\ 27c$ is to be underſtood the cube root of $27\ c$.

OBSERVATION.

The characters + and — in algebra import ſomething more than ſimple addition and ſubtraction, as in arithmetic: + is an affirmative or poſitive ſign, expreſſive of real exiſtence; and when it is put before any quantity, it is to be underſtood that that quantity is of a real and poſitive nature; whereas — is the ſign of negation, or negative exiſtence, denoting the quantity to which it is prefixed to be leſs than nothing.

Though no real body or ſubſtance can properly be ſuppoſed fed to be lefs than nothing, yet quantities, whereby the different degrees of qualities are eftimated, may eafily be conceived to pafs from an exiftence through nothing into negation. Thus a merchant's ftock was at firft 1000 *l.* incumbered only with 50 *l.* of debt, then his neat ftock was 1000—50; but in the courfe of trade he met with misfortunes, which raifed his debts to 1000 *l.* and his fubject when collected together amounted to no more than 1000 *l.* and in this cafe he is juft worth nothing: unwilling, however, to give up his trade, he ftruggles hard to preferve his credit for another year, and then, upon examining his books, finds he was ftill 1000 *l.* in debt, and no fubject to pay it, and in this cafe he is 1000 *l.* worfe than he was the preceeding year, and at that time he was worth nothing, confequently he will be at this time 1000 *l.* worfe than nothing. Wherefore, as affirmative and negative quantities are contrary in their nature, they are likewife contrary in their effects; a confideration which, if properly attended to, will obviate all the difficulties that may occur in the application of thefe figns; for the refult of working by affirmative quantities is, in every operation, obvious, and therefore like operations, where the quantities are negative, may be known by the rule of contraries.

Where there is no fign prefixed to a quantity, the quantity is always underftood to be affirmative, and if there is no numercial coefficient before it, unity muft always be underftood: thus $3a$ fignifies $+3a$, and a fignifies $+1a$; but the fign of a negative quantity is never omitted, nor indeed the fign of an affirmative one, excepting when fuch an affirmative quantity is confidered by itfelf, or happens to be the firft in a feries of quantities fucceeding one another.

CHAP. II.

ADDITION OF ALGEBRAIC QUANTITIES.

1. ADD homogeneous quantities according to their number.

EXAMPLES.

(1.) $a+a+a=3a$

(2.) $27a+3a=30a$

(3.) $4b+3b+5b=12b$

2. Heterogeneous

2. Heterogoneous or different quantities, expreſſed by different letters, are added by connecting them with the ſign +. Thus a and b and c are added $a+b+c$, &c.

3. When the quantities to be added have like ſigns, the ſame ſign is to be retained in the ſum.

EXAMPLES.

(1.)	(2.)	(3.)
$+3a-4b$	$15c+5b-4a$	$43a+56$
$+7a-5b$	$16c+3b-3a$	$37a+13$
$10a-9b$	$31c+8b-7a$	$80a+69$

In this caſe it is plain, that one affirmative increaſed by the acceſſion of another affirmative, muſt make a greater affirmative, in the ſame ſenſe that a ſtock of 1000 *l.* increaſed by a legacy of 500 *l.* would in this new ſtate be worth 1500 *l.*; and that one negative increaſed by the addition of another negative muſt make a greater negative, in the ſame ſenſe that the loſs of 1000 *l.* followed by another loſs of 500 *l.* would increaſe the loſs to 1500 *l.*

4. If the quantities to be added have contrary ſigns, the one affirmative and the other negative, the ſign of the greater quantity muſt be prefixed to their difference.

EXAMPLES.

$+86$	$+2a$	$+37d$	$+a$
-36	$-9a$	$-14d$	$-a$
$+50$	$-7a$	$+23d$	0

In this caſe it is plain, that the ſum of the quantities muſt be found by ſubtracting the leſſer coefficient from the greater and prefixing the ſign of the greater quantity; for let $+37a$ repreſent 37 *l.* which a man hath in his pocket, and $-19a$, 19 *l.* which he muſt pay out of it, he will after all have $+18a$: but reverſe the caſe, and ſuppoſe he hath only $+19a$ to pay $-37a$, he will then have $-18a$ or 18 l. ſhort of his ſum.

From theſe four caſes may be deduced one general rule for addition of algebra, *viz.* Add like ſigns, and retain the ſame ſign in the ſum; but ſubtract contrary ſigns, and keep the ſign of the greater.

EXAM-

EXAMPLES.

$4a-8b$	$4b+3c$	$3a+b$	$b-c$
$a+9b$	$2b-c$	$a+2b$	$b-2c$
$5a+b$	$6b+2c$	$4a+3b$	$2b-3c$

CHAP. III.

SUBTRACTION of ALGEBRAIC QUANTITIES.

1. HOmogeneous quantities are ſubtracted by taking the difference of their coefficients, as

$$5a-3a=2a. \quad 76-6=70 \text{ and } 8b-b=7b$$

2. Heterogeneous quantities admit of no other ſubtraction but the ſign $-$, as the difference betwixt a and b would ſtand $a-b$, upon the ſuppoſition that a is the greateſt quantity.

3. If quantities have like ſigns, their difference retains the ſame ſign.

4. If a greater quantity muſt be taken from a leſſer, to the difference betwixt the quantities prefix the contrary ſign. Thus if it were required to ſubſtract $+5b$ from $+2b$, the remainder would be $-3b$. For ſuppoſe a man owes me 2 *l.* or $2b$, and I owe another man 5 *l.* or $5b$; it is plain that the difference betwixt theſe ſums is a debt upon me, for which I have nothing equivalent, and therefore I am 3 *l.* or $3b$ worſe than nothing: for had the ſums been equal, I ſhould have had nothing in reverſion, but then the firſt 3 *l.* or $3b$ I ever got ſhould have been my own; but whilſt I am 3 *l.* or $3b$ in debt, I muſt get 3 *l.* or $3b$ before I am even with the world, and therefore in every acceptation in this caſe the remainder is a negative quantity. Beſides, for a proof of ſubtraction, $+5b-3b$ being added together will be found $=+2b$.

5. When the quantities to be ſubtracted have different ſigns, add their coefficients, and to that ſum prefix the ſign of the minuend, which will expreſs the difference. From $+5a$ if it was required to ſubtract $-3a$, the remainder would be $+8a$: but if it had been required to take $+5a$ from $-3a$ the remainder would be $-8a$. Hence to ſubtract a negative is the ſame thing as to add an affirmative, and to ſubtract an affirmative

affirmative is the ſame thing as to add a negative: For in the firſt inſtance, if I am in poſſeſſion of $5a$, and indebted in $3a$, ſhould that debt be forgiven, I am certainly $3a$ better than when it ſtood againſt me; and in like manner, if I am indebted in $5a$, repreſented by $-5a$, and have only $+3a$ to pay it, ſhould theſe $3a$ be loſt or taken from me, I am certainly $3a$ worſe than I was in my former ſituation.

EXAMPLES of COMPOUND QUANTITIES.

From	$12x+6a-4b-12c \quad -7e-5f$
Take	$2x-3a+4b-5c+6d-7e$
Rem.	$10x+9a-8b-7c-6d-5f$
	$12x+6a+4b-12c \quad -7e-5f$

If never a member of the ſubtrahend is found to be of the ſame denomination with any member of the minuend, change the ſign of every member of the ſubtrahend, and then add it to the other; as if $5c-6d$ was to be ſubtracted from $3a-4b$, the remainder would be $3a-4b-5c+6d$.

CHAP. IV.

MULTIPLICATION of ALGEBRAIC QUANTITIES.

IN multiplying quantities, it muſt be obſerved, that like ſigns, whether affirmative or negative, produce +, but different ſigns produce −.

EXAMPLES.

$+b\times+c=+bc \qquad -b\times-c=+bc \qquad -b\times+c=-bc$

OBSERVATIONS.

1. That $+b$ into $+c$ muſt produce $+bc$, is ſelf-evident: for inſtead of the letters let us take $+4$ and $+3$ for a multiplicand and multiplier, the product in this caſe muſt be $4+4+4=+12$; wherefore $+3$ into $+4$ muſt likewiſe give $+12$.

2. $-b$ into $-c=+bc$. For aſſume the ſame figures 3 and 4 with negative ſigns, then multiply -4 into 3,0 and -3 ſucceſſively, and the products will be an arithmetical progreſ-

sion; but the two first products are -12 and 0, and the third product will be $+12$.

3. $+b$ into $-c=-bc$. For if the product were $+bc$, it would be the same with the product of $+b$ into $+c$, which would be absurd. But further, suppose a man 10 *l.* in debt, and without any subject to pay it, his worth may be expressed by -10, *i. e.* 10 worse than nothing; the double of -10 would be -20, the triple would be -30, &c. according to any multiplication whatever. Therefore $-10 \times +2\ +3$, &c. by any multiplication whatever must give a negative product.

EXAMPLES.

(1.)

$$\begin{array}{r} 4a+3c \\ a-b \\ \hline -4ba-3bc \\ +3ac+4aa \\ \hline 4aa+3ac-4ba-3bc \end{array}$$

(2.)

$$\begin{array}{r} a-b \\ a-b \\ \hline -ab+bb \\ aa-ab \\ \hline aa-2ab+bb \end{array}$$

(3.)

$$\begin{array}{r} a+b \\ a-b \\ \hline -ab-bb \\ +aa+ab \\ \hline +aa\ .\ .\ .-bb \end{array}$$

(4.)

$$\begin{array}{r} a+b \\ a+b \\ \hline +ab+bb \\ +aa+ab \\ \hline +aa+2ab+bb \end{array}$$

(5.)

$$\begin{array}{l} 6x-7a-8b \\ 2x-3a+4b \\ \hline 12xx-14ax-16bx+21aa+24ab-32bb \\ \quad\ -18ax+24bx \qquad\quad -28ab \\ \hline 12xx-32ax+8bx+21aa-4ab-32bb \end{array}$$

A dash over two or more quantities imports, that all those quantities are to be considered as constituting one compound quantity.

OBSER-

OBSERVATION.

From this rule there may be very naturally deduced the following theorems.

1. *Exam.* $a+b$ into $a-b$ produceth $aa-bb$. Hence the ſum and difference of any two numbers being multiplied together will give the difference of their ſquares, and *vice verſa*.

For $4\times4=16$
$2\times2=4$

Diff. 2 — 12 the difference of the ſquares.
Sum 6

Product 12 of their ſum and difference.

2. *Exam.* 4 $a+b$ into $a+b$ produceth $aa+2ab+bb$. Hence, if a number be reſolved into any two parts whatever, the ſquare of the whole will be equal to the ſquare of each part, and the double rectangle, or product of the multiplication of thoſe parts added together. Thus, if the number 10 be reſolved into 7 and 3, 100 the ſquare of 10, the whole will be equal to $7\times7+3\times3+7\times3\times2$, which is the ſquare of each part, and double the product of the parts. *Eucl. book* 2. *prop*. 4.

3. *Exam.* 2 $a-b$ into $a-b$ produceth $aa-2ab+bb$. Hence, if from the ſum of the ſquares of any two numbers be ſubtracted the double product of thoſe two numbers, there will remain the ſquare of their difference. Thus in the numbers 7 and 3, $7\times7+3\times3=58$ and $7\times3\times2=42$ but $58-42=16=7-3\times7-3$, *i. e.* 4×4. *Eucl.* 2. 7.

CHAP. V.

DIVISION OF ALGEBRAIC QUANTITIES.

1. TO divide ſimple algebraic quantities, divide the coefficient of the dividend by that of the diviſor, when poſſible, and then put down after the quotient all the quantities in the dividend that are not in the diviſor; and for the ſign to be prefixed to the quotient obſerve the rules in multiplication, ſince the quotient ought to contain a quantity, which being multiplied into the diviſor, will produce the dividend.

EXAMPLES.

(1.)

$+4b)-8bc(-2c$
$+4b$

$-8bc$ Proof.

(2.)

$4a)24abc(6bc$
$4a$

$24abc$ Proof.

(3.)

$3b)18bc(6c$
$3b$

$18bc$ Proof.

(4.)

$4a)12abc(3bc$
$4a$

$12abc$ Proof.

2. When the coefficients are incommenſurable, or the quantities heterogeneous, place the diviſor below the dividend in the manner of a vulgar fraction; as

$$\frac{b}{a} \qquad \frac{17b}{9a} \qquad \frac{5ab}{7a} \qquad \frac{9xm}{7xm}$$

CHAP VI.

ALGEBRAIC FRACTIONS.

FRactions in algebra are managed in the ſame manner as in arithmetic, in their reduction, addition, ſubtraction, multiplication, and diviſion; with this difference only, that the operations in the one are numerical, and in the other algebraical.

EXAMPLES.

1. Reduce $\frac{4ab}{6bc}$ to their loweſt terms,

Here $2b$ is a common meaſure for both numerator and denominator, and therefore when they are both divided by $2b$, the quotient will be $\frac{2a}{3c}=\frac{4ab}{6bc}$, but expreſſed in lower terms. Hence it follows, that whenever a common letter or factor is to be found in every member both of the numerator and denominator, it may be cancelled every where without affecting the value of the fraction; thus the fraction $\frac{ac+\text{[illegible]}}{cd+ce}$ expunging c, becomes $\frac{a+b}{d+e}$; but if there be any one member wherein the factor is not concerned, it muſt not be expunged at all.

2. Reduce

2. Reduce $\frac{a}{2}$ $\frac{b}{3}$ $\frac{c}{4}$ to one common denominator.

$$\left.\begin{array}{l} a\times3\times4=12a \\ b\times2\times4=\ \ 8b \\ c\times3\times2=\ \ 6c \end{array}\right\}\text{ Numerators.}$$

$$2\times3\times4=24 \quad \text{Common denominator.}$$

3. Add $\frac{2b}{3a}$ and $\frac{3b}{4a}$

$$3a\times3b=9ab$$
$$4a\times2b=8ab$$
$$3a\times4a=\frac{17ab}{12aa}=\frac{17b}{12a}$$

4. Subtract $\frac{a}{2d}$ from $\frac{4a}{5a}$

$$a\times5d=5da$$
$$4a\times2d=8da$$
$$2d\times5d=\frac{3da}{10dd}=\frac{3a}{10d}$$

5. Multiply $\frac{4a}{9}$ into $\frac{4ba}{7d}$

$$4a\times4ba=\frac{16baa}{63d}$$
$$9\times7d=$$

6. Divide $\frac{a}{4d}$ by $\frac{4b}{c}$

$$\frac{c}{4b}\Big)\frac{a}{4d}\Big(\frac{ca}{16bd}$$

CHAP. VII.

INVOLUTION NUMERICAL and ALGEBRAICAL.

INvolution is the raising of a quantity or number to any assigned power, and is performed like multiplication, the multiplier always continuing the same. Hence, when a quantity or number is said to be raised to a certain power, it is multiplied into itself as often as the power to which it is raised expresseth.

Thus when 4 is multiplied into itself, it becomes 16, which is the square or 2d power of 4; when the square 16 is multiplied into 4, it will produce 64, the cube or 3d power; and when 64 is multiplied into 4, it will produce 256, the biquadrate or 4th power; and that multiplied into 4 will produce 1024, the sursolid or 5th power, &c.

In like manner, $a\times a=aa$ or a^2; $a^2\times a=aaa$ or a^3; $a^3\times a=a^4$; and $a^4\times a=aaaaa$ or a^5, &c. To express the power by a digit is is no doubt the best method of notation, as it is shortest, and the power discovered by its index at once, without the trouble of numbering the repetitions, which in the other case is unavoidable.

Examples of roots raised to the 3d power.

$$\begin{array}{l} 4a \\ 4a \\ \hline 16a^2 \text{ square.} \\ 4 \\ \hline 64a^3 \text{ cube} \end{array} \qquad \begin{array}{l} 5b \\ 5b \\ \hline 25b^2 \\ 5b \\ \hline 125b^3 \end{array}$$

$$\begin{array}{l} a+b \text{ a binomial root.} \\ a+b \\ \hline a^2+2ba+b^2 \\ a+b \\ \hline a+3ba^2+3bba+b^3 \end{array} \qquad \begin{array}{l} a-b \text{ a residual root.} \\ a-b \\ \hline a^2-2ba+b^2 \\ a-b \\ \hline a^3-3ba^2+3bba^3 \end{array}$$

CHAP. VIII.

EVOLUTION OF ALGEBRAIC QUANTITIES.

EVolution reduces any given power back to its original quantity when possible. To do which it must be observed, that some quantities are rational and have roots; but there are others surd or irrational, whose just root cannot be extracted. In the first species of quantities, consider what quantity involved or multiplied into itself as often as the index represents will produce a quantity equal to that given, and the quantity found in this manner will be that required. Thus, suppose it was required to extract the root of a^2, it would be $a \times a = a^2$, consequently a is the root required. In the same manner a is found to be the root of a^3, for $a \times a \times a = a^3$. In the same manner $81d^2$ will be found to proceed from $9d$, for $9d \times 9d = 81d^2$.

Thus likewise will a be found to be the cube root of a^3, for $a \times a \times a = a^3$, $4a$ the cube root of $64a^3$, and a the biquadrate root of a^4.

If

If the ſquare root of $a^2+2ab+b^2$ was required, it would be found to be $a+b$; for $a+b \times a+b = a^2+2ab+b^2$. In the ſame manner will $a-b$ be found to be the ſquare root of $aa-2ab+b^2$.

In this manner likewiſe may be found the roots of higher powers, as $a^3+3ba^2+3bba+b^3$, whoſe root is $a+b$.

In ſurd or irrational quantities, the roots are ſignified or denoted by the radical ſign prefixed thus, $\sqrt[2]{g}$. $\sqrt[2]{12\ B+c}$, $\sqrt[2]{a^2-G}$ is the ſquare root of thoſe quantities to which the ſign $\sqrt[2]{\ }$ is prefixed; and $\sqrt[3]{B+H}$ $\sqrt[3]{24S+a3}$ is the cube root of the quantities to which the ſign $\sqrt[3]{\ }$ is prefixed.

CHAP. IX.

OF SURD QUANTITIES.

1. IF the given ſurd quantities are heterogeneous, they are added by connecting them with the ſign +, and ſubtracted with the ſign —; as $\sqrt[2]{B}+\sqrt[2]{C}$ expreſſeth the ſum of theſe two ſurd quantities, $\sqrt[3]{G}-\sqrt[3]{ab}$ expreſſeth the difference of the ſurd quantities on each ſide of the negative ſign.

2. When they are homogeneous, they are added and ſubtracted according to their number; as $4\sqrt[2]{B}+8\sqrt[2]{B}=12\sqrt[2]{B}$, and $9\sqrt[3]{G}-3\sqrt[3]{G}=6\sqrt[3]{G}$, &c.

3. Surds are ſometimes multiplied and divided by the ſigns × and ÷ connecting them, $\sqrt[2]{b}\times\sqrt[2]{c}$, or $\sqrt[2]{b}\div\sqrt[2]{c}$; and ſometimes by multiplying and dividing the quantities themſelves, when it can be done, and prefixing the radical ſign to the product or quotient, as $\sqrt[2]{ab}\times2\sqrt[2]{12b}=\sqrt[2]{12ab^2}$; or $\sqrt[2]{b}\times\sqrt[2]{a}=\sqrt[2]{ab}$, and $\sqrt[2]{12ba}\div\sqrt[2]{4a}=\sqrt[2]{3b}$; and $\sqrt{21bb}\div\sqrt{7b}=\sqrt{3b}$, &c.

4. Surds are involved merely by removing the radical ſign; thus to ſquare $\sqrt[2]{G}$, take away the ſign $\sqrt[2]{\ }$, and there is left $G=$ the ſquare of $\sqrt[2]{G}$. To cube $\sqrt[3]{G}$, take away the radical ſign $\sqrt[3]{\ }$, and there will remain $G=$ to the cube of $\sqrt[3]{G}$; and this holds good in all caſes, whatever power is repreſented by the radical ſign.

CHAP. X.

RULES OF EQUATION.

BY equation is to be underſtood in general one expreſſion of a number or quantity made equal to another; thus, $4+6=10$, and $7-3=4$; or $9\times6=54$; or $12\div4=3$. And in the ſame manner may theſe quantities $a+b=G$, or $a-b=D$, or $ab=P$, or $\frac{a}{b}=Q$, &c.

There is no queſtion can occur in numbers or quantities, but ſome equation will ariſe from the very terms of the queſtion, if they are intelligibly put: for inſtance, ſuppoſe a number was required, whereof $\frac{1}{2}$ and $\frac{1}{4}$ being added, the ſum would be 18. Let this number be repreſented by a, then will $\frac{1}{2}a+\frac{1}{4}a=18$. Wherefore the equation here is $\frac{3}{4}a=18$, and from this equation the number is required. Again, let the queſtion be to find two numbers, the ſum whereof is 24, and difference 12. In this caſe, one equation will be $a+e=24$, and another $a-e=12$; and ſo there are two equations given, and two numbers required. Hence ariſes this general

RULE.

When the number of equations given in any queſtion are juſt equal to the number of quantities required, and the one equation no way inconſiſtent with the other, the queſtion is truly limited, and capable of a ſolution; but when the number of equations given are not ſo many as the numbers required, the queſtion is ambiguous, and admits of various anſwers. If the number of equations given exceed the number of quantities required, and if any one of theſe equations is found inconſiſtent with the other, the queſtion is impoſſible, and can have no rational anſwer.

Rules for reſolving ſimple equations.

1. To letter a queſtion, or to repreſent quantities by proper characters.

As this is in a great meaſure arbitrary, or matter of choice, we need only obſerve, that in general given quantities are repreſented either by conſonants or abſolute numbers, as they

 lie

lie in the queſtion; and quantities required, by ſome of the ſmall vowels, one or more as there is occaſion.

2. To ſtate a queſtion, or, in other words, to raiſe an equation.

The nature of every queſtion muſt be well conſidered; given and known quantities diſtinguiſhed from ſuch as are required; numbers are to be added, ſubtracted, multiplied, divided, or involved exactly, according to the conditions of the queſtion; and then one ſide of an equation compared with the other, which being done, the equation is truly ſtated. To make this ſtill plainer, by an example: Suppoſe it was required to find a number to which $\frac{1}{4}$ of itſelf being added, 5 ſubtracted from the ſum, the remainder multiplied by 10, and the product divided by 4, gives 25 in the quotient.

For the unknown number put *a*, and for the given quotient put $\mathcal{Q}$. The other members, being but ſmall, may be taken abſolutely as they lie, or repreſented by letters; but in this inſtance, let them be taken abſolutely as they lie. Then by conſidering the queſtion, the following equation will occur, $a+\frac{1}{4}a-5\times10\div4=\mathcal{Q}$; which is ſtating the queſtion at large. And the ſame may be abridged in the manner which the ſigns of addition, ſubtraction, multiplication, and diviſion require; thus, $a+\frac{1}{4}a=\frac{5}{4}a$, by ſubtracting 5, it becomes $\frac{5}{4}a-5$, which, multiplied into 10, it becomes $\frac{50}{4}a-50$; and this being divided by 4, it becomes $\frac{12\frac{1}{2}a-50}{4}=\mathcal{Q}$. And thus will the equation be ſtated and properly abridged.

3. To reduce an equation.

After the queſtion hath been properly ſtated and abridged, the equation muſt be reduced, by bringing the unknown quantity *a*, and its coefficients, to one ſide, and the known quantities to the other: for this purpoſe, obſerve the following caſes.

Caſe 1. When $a-b=G$, reduce by addition, ſince equal quantities added to equal quantities give equal ſums; *b* added to both ſides gives $a=G+b$.

Caſe 2. When $a+b=G$, reduce by ſubtraction, ſince equal quantities taken from equal quantities give equal remainders; thus, by ſubtracting *b*, it will be $a=G-b$.

Caſe 3 When $\frac{a}{b}=G$, reduce by multiplication, ſince equal quantities multiplied by equal quantities give equal products; by multiplying both ſides by b, it will be $a=Gb$.

Caſe 4. When $ab=G$, reduce by diviſion, ſince equal quantities divided by equal quantities give equal quotients; therefore, dividing both ſides by b we have $a=\frac{G}{b}$.

Caſe 5. When $\sqrt{a}=G$, reduce by involution; for ſince equal roots have equal ſquares, cubes, biquadrates, *&c.*; wherefore by ſquaring, cubing, *&c.* both ſides, we will have $a=GG$, or $a=G^2$.

Caſe 6. When $a^2=G$, reduce by evolution; for ſince equal quares, cubes, biquadrates, *&c.* have equal roots, by extracting the root of each, we will have $a=\sqrt{G}$.

By theſe ſix caſes, founded on their reſpective axioms, all equations whatever may be reduced; but to ſet the application in a more practical point of view, let us reſume the equation $\frac{12\frac{1}{2}a-50}{4}=Q$.

Since one ſide of the equation, where a the unknown quantity lies, muſt be divided by 4, in order to remove the fraction, multiply both ſides by the denominator 4; and then you will have $12\frac{1}{2}a-50=4Q$; for not to divide $12\frac{1}{2}a-50$ by 4, is the ſame thing as to multiply Q by 4.

Then ſeeing 50 is a known number, affected with the ſign —, remove it by caſe 1. to the known ſide of the equation, and then we will have $12\frac{1}{2}a=4Q+50$; and, in this caſe likewiſe, not to ſubtract 50 from $12\frac{1}{2}a$, is the ſame thing as to add it. But ſtill there is a fraction upon one ſide; wherefore multiply both ſides of the equation by the denominator 2, and we will have $25a=8Q+100$. Laſtly, dividing both ſides by the coefficient of a, *viz.* 25, we will have $a=\frac{8Q+100}{25}$, by which means the equation is not only reduced, but reſolved; for the unknown quantity a, by itſelf alone is found equal to ſuch as are known, namely $8Q=200=8\times25$, for Q was $=25$; then $\frac{200+100}{25}=\frac{300}{25}=12$, the number required, or the value of a.

CHAP.

CHAP. XI.

PROBLEMS producing SIMPLE EQUATIONS.

A Simple equation is that wherein the unknown number or quantity a is found equal to known numbers, by multiplication or division only, without the extraction of roots, as in the above example; or, if $\frac{a}{4}$ 12, then $a=4\times 12=48$; or if $4a=12$, then, by division, $a=\frac{12}{4}=3$, and it will be the same in letters for $\frac{a}{b}=g$, therefore, $a=bg$; and $ab=g$, therefore $a=\frac{g}{b}$, &c.

Prob. 1. A man being asked how many bank-notes he had in his pocket-book, made answer, If to what I have you add as many, $\frac{1}{2}$ as many, and $7\frac{1}{2}$ more, I would then have 130; required the number he had?

In this problem there is only one number required, for which put a, and there are two given, *viz.* 130, for which put S, and $7\frac{1}{2}$, for which put B.

Then according to the conditions of the question, $a+a+\frac{1}{2}a+B=S$; which, being properly abridged, will be $2\frac{1}{2}a+B=S$.

Then remove B to the known side of the equation by subtraction and it will be $2\frac{1}{2}a=S-B$.

Take away the fraction by multiplication, and it will be $5a=2S-2B$. But by division $a=\frac{2S-2B}{5}$; wherefore $2=S$ 260, from which take $2B=15$, and there will remain 245, which being divided by 5, quotes 49, the answer.

Prob. 2. A man speaking of his money, said, that with one half of his money he could buy a certain horse, and had he 30 *l.* more, he could purchase three horses of the same value; how many pounds had he?

Here it appears that $\frac{1}{2}a$ is the value of the horse, and $\frac{3}{2}a$ the value of 3 such horses, which, by the question, is $=a+30$. Therefore, to remove the fraction by multiplication, it will be $2a+60=3a$, and by subtracting $2a$ from both sides, $60=a$, the answer required.

Prob 3. A gentleman being aſked how many horſes he kept, made anſwer, For want of room in my own ſtable, I muſt put 5 horſes in my neighbour's ſtable; but I am now building a ſtable twice as large, and then I can accommodate my own horſes and 5 of my neighbour's: You may find out the number of horſes I keep.

Here it is plain that $a-5=$ the number the ſtable contains, and $2a-10=$ the number a ſtable twice as large will contain, which being 5 more than the number the gentleman had, the equation will ſtand $a+5=2a-10$; then, by adding 10 to both ſides, $a+15=2a$, and by ſubtracting a, we have $15=a$ the anſwer.

Prob 4. A man meeting ſome beggars, found, that by giving 4 *d.* to each, he would be 4 *d.* ſhort of ſerving them all, and by giving 3 d. to each, he ſhould have 4 *d.* over; required the number of beggars, and how many pence the man had in his pocket.

Here two numbers are required, which let be repreſented by $a=$ the number of beggars, and $e=$ the number of pence

By the queſtion it appears, that $\frac{e+4}{a}=4$, and $\frac{e-4}{a}=3$, the two given equations. By the firſt $e+4=4a$, and $e=4a-4$; but by the ſecond $e-4=3a$, and $e=3a+4$; therefore, by comparing both equations of e, it will be $4a-4=3a+4$, and $4a=3a+8$, and $a=8$, and $e=28$. Anſwer.

Prob. 5. Two perſons, *A* and *B*, were talking of their ages; ſays *A* to *B*, 7 years ago I was juſt three times as old as you were, and 7 years hence I ſhall be twice as old as you will be. I demand their preſent ages.

Let a and e repreſent the preſent ages of *A* and *B*, that is, let $a=A$ and $e=B$; then their ages 7 years ago were $a-7$ and $e-7$, and their ages 7 years hence will be $a+7$ and $e+7$; whence, and from the conditions of the problem, may be derived the two following fundamental equations:

$$a-7=\overline{e-7}\times3=3e-21, \text{ and}$$

$$a+7=\overline{e+7}\times2=2e+14$$

From the former of theſe two equations, *viz.* $a-7=3e-21$, we have $a=3e-14$; and from the ſecond equation $a+7=2e+14$, we have $a=2e+7$; therefore $3e-14=2e+7$, ſince both are equal to a; whence $e=21$, and $2e+7$, or $a=49$.

A's

A's age therefore was 49 years, and *B*'s 21: For 7 years before *A*'s age was 42 and *B*'s 14, and $42=14\times3$; but on the other hand, 7 years after *A*'s age would be 56 and *B*'s 28; but $56=28\times2$.

Prob. 6. A certain company at a tavern found, that when they came to pay their reckoning, if there had been 3 more in company to the ſame reckoning, they might have paid 1 ſhilling apiece leſs than they did; but had there been 2 fewer in company, they muſt have paid 1 ſhilling apiece more than they did: what did each pay, and how many were there in company?

Repreſent the number in company by a, and the number of ſhillings every one actually paid by e; now, if 4 perſons were to pay 5 ſhillings apiece, the whole reckoning would be 4×5: therefore if a perſons are to pay e ſhillings apiece, the whole reckoning will be ae ſhillings. This being premiſed, ſuppoſe them now to be 3 more in company, then will the number of perſons be $a+3$; and to find what every particular perſon ought to pay in this caſe, the whole reckoning ae muſt be divided by $a+3$, and the quotient will be $\frac{ae}{a+3}=$ every one's particular reckoning: but, according to the problem, every one's particular reckoning in this caſe would have been one ſhilling leſs than it actually was, *i. e.* $e-1$; therefore $\frac{ae}{a+3}=e-1$. In like manner, the ſecond condition of the problem gives us this equation, *viz.* $\frac{ae}{a-2}=e+1$. The firſt of theſe equations, *viz.* $\frac{ae}{a+3}=e-1$, being reduced, gives $a=3e-3$; and the ſecond equation, *viz.* $\frac{ae}{a-2}=e+1$, being likewiſe reduced, gives $a=2e+2$; therefore $3e-3=2e+2$ and $e=5$; wherefore $2e+2$, or $a=5\times2+2=12$.

Hence there were 12 in company, their reckoning came to $5\times12=60$ ſhillings.

Prob. 7. One lays out 2 ſhillings and 6 pence on apples and pears, buying his apples at 4 and his pears at 5 a penny, and afterwards accommodates his neighbour with half his apples and $\frac{1}{3}$ of his pears for 13 pence, which was the price he bought them at; I demand how many he bought of each ſort?

Put a for the number of apples, and e for the number of pears.

Then

Then if 4 apples cost 1 penny, a will cost $\frac{a}{4}$; and for the same reason e will cost $\frac{e}{5}$ pence, and then we will have $\frac{a}{4}+\frac{e}{5}=30$ for one fundamental equation. Again, the price of $\frac{a}{2}$ half of his apples, will be $\frac{a}{8}$, and the price of $\frac{e}{3}=\frac{1}{3}$ of his pears will be $\frac{e}{15}$; hence we will have $\frac{a}{8}+\frac{e}{15}=13$ for another fundamental equation.

Equ. 1. $5a+4e=600$
2. $15a+8e=1560$

Subtract the second equation from thrice the first, and it will be

Equ. 3. $4e=240$
4. $e=60$

Now, substitute 60 for e, that is, 240 for $4e$ in the first equation $5a+4e=600$, and we will have $5a+240=600$; whence equation 5. $a=72$: therefore the number of apples was 72, and the number of pears 60: for $\frac{72}{2}=36$ apples at the assigned price $=9$ *d.* and $\frac{60}{3}=20$ pears at 5 for a penny $=4$ *d.*; but $9+4=13$.

Prob. 8. Three men had each a certain sum of money; *A* and *B* together had 16 *l.*; *B* and *C* together had 27 *l.*; *A* and *C* together had 25 *l.*; how much had each?

Here three numbers are required, a, e, y, and *per* question $a+e=16$, and consequently $e=16-a$; also *per* question $e+y=27$, consequently $e=27-y$. By comparing these equations of e, it will be $16-a=27-y$, and $16-a+y=27$, and $y-a=27-16=11$, and $y=11+a$. *Lastly*, *per* question $a+y=25$, and $y=25-a$. By comparing these two equations of y, it will be $=11+a=25-a$, and $11+2a=25$, $2a=14$, $a=7$, $e=9$, and $y=18$.

CHAP. XII.

OF DIRECT PROPORTION.

BY Euclid, four numbers or quantities are proportional, when the first hath the same ratio to the second, as the third hath to the fourth; or, in other words, when the first contains the second, or is contained therein, so oft as the third

third contains the fourth, or is contained therein. Thus 3 : 9 :: 6 : 18, and 6 : 4 :: 12 : 8, becaufe in the firft, 3 is as often found in 9 as 6 is found in 18; and in the fecond, 6 contains 4 as often as 12 contains 8. From this idea of proportion it follows, *firft*, That if four numbers are proportional, and expreffed by thefe four letters, A : B :: C : D, the quotient of the firft divided by the fecond, is equal to the quotient of the third divided by the fourth: and this expreffion is univerfally true $\frac{A}{B}=\frac{C}{D}$, as may be obferved in any rank of four proportionals whatever: thus, if 4 : 5 :: 12 : 15, then $\frac{4}{5}=\frac{12}{15}$; if 6 : 2 :: 12 : 4, then $\frac{6}{2}=\frac{12}{4}$, *&c.* *Secondly*, It is alfo true, that the quotient of the fecond divided by the firft is equal to the quotient of the fourth divided by the third, *i. e.* $\frac{B}{A}=\frac{D}{C}$, for $\frac{5}{4}=\frac{15}{12}$ and $\frac{2}{6}=\frac{4}{12}$, *&c.* *Thirdly*, When it happens in any four numbers or quantities that the quotients are equal in this manner, thefe four numbers or quantities muft be proportional: thus $\frac{A}{B}=\frac{C}{D}$, therefore A : B :: C : D. *Fourthly*, If four numbers are proportional, the product of the extremes will be equal to the product of the means: for if A : B :: C : D, then as before $\frac{A}{B}=\frac{C}{D}$, multiply each fide of the equation by B, then $A=\frac{BC}{D}$; multiply each fide by D, and then AD=BC, the product of the extremes = the product of the means. *Fifthly*, If in any four numbers the product of the extremes is equal to the product of the means, thefe four numbers are proportional: for if AD=BC, then by divifion $A=\frac{BC}{D}$, and again by divifion $\frac{A}{B}=\frac{C}{D}$; therefore A : B :: C : D. *Sixthly*, If three numbers are proportional, the product of the extremes is equal to the fquare of the means: for if A : B :: B : C, it will be AC=BB by the fourth; and if AC=BB then by the fifth it will be A : B :: B : C.

Prob. 1. Three numbers being given, to find a fourth direct proportional. Let the number fought be a, and the three given numbers B, C, D. Then B : C :: D : a; therefore the product of the extremes will be equal to the product of the means Ba=CD. Divide both fides of the equation by B, and then $a=\frac{CD}{B}$, the product of the fecond and third divided by the firft.

Prob.

Prob. 2. Two numbers being given, to find a third proportional, put BC the two given numbers, and a the third proportional, then B:C::C:a; and as before Ba=CC; dividing both ſides by B we have $a=\frac{CC}{B}$, the ſquare of the ſecond divided by the firſt.

CHAP. XIII.

Alternation, Inverſion, Compoſition, and Diviſion of Proportion.

WHat Euclid demonſtrates, *book* 5. concerning proportional quantities, as far as relates to numbers and quantities that are commenſurable, may be demonſtrated algebraically from what hath been ſaid in the laſt chapter. *Firſt*, If A:B::C:D, then by alternation it will be A:C::B:D becauſe in the laſt as well as the firſt the product of the extremes is equal to the product of the means, *viz.* AD=CB. *Secondly*, It will be by inverſion D:C::B:A; for in this alſo DA=CB. *Thirdly*, It will be by compoſition D+C:C::B+A:A; for ſince AD=CB, add AC to both, and then AD+AC=CB+AC. But the firſt of theſe is the product of D+C×A, and the ſecond is the product of B+A×C. *Fourthly*, It will alſo be by diviſion, D—C:C::B—A:A; for AD=CB, and by ſubtracting AC, it will be AD—AC=CB—AC. But the firſt is the product of D—C×A, and the ſecond is the product of B—A×C. In like manner we may demonſtrate proportion converted alternately compounded or divided, *&c.* by the ſame pinciples.

CHAP. XIV.

COMPOUND PROPORTION.

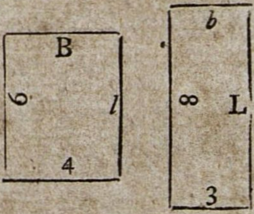
IF A:B::C:D and E:D::F:a, then it will be as AE: the product of the firſt two antecedents, :B the firſt conſequent; ::CF the product of the ſecond two antecedents, :a the ſecond conſequent. For by the firſt A:B::C:D, hence AD=BC; and by the ſecond, E:D::F:a, hence Ea=DF: therefore the products of thoſe equal quantities will be equal; that is, $AD\times Ea$, or $ADEa=BC\times DF$, or $BCDF$: divide both ſides by D, and then $AEa=BCF$; therefore $AE:B::CF:a$. *Q. E. D.*

If therefore five numbers are given, $\begin{matrix}A:B::C,\\ E\quad\ F\end{matrix}$ to find *a*, the ſixth direct proportional or anſwer required, it is plain, that *a* is found by multiplying the antecedents together, then multiplying the product of the ſecond two antecedents by B, the ſecond term of the proportion, and dividing by the product of the two firſt: for ſince $AE:B::CF:a$, it will be $AEa=BCF$ and $a=\frac{BCF}{AE}$; and however many antecedents there may be, the operation will ſtill be the ſame, which might be eaſily demonſtrated from the ſame principles.

CHAP. XV

RECIPROCAL, or INVERSE PROPORTION.

IF two rectangles, as thoſe in the margin, are equal the one to the other, but differing in length and breadth; or, which is the ſame thing, if any two products are equal the one to the other, as $8\times3=6\times4$, in letters $Lb=lB$: for, as the length of the longeſt rectangle 8, or L, is to the breadth of the ſhorteſt, 4, or *b*; ſo reciprocally is the length of the ſhorteſt 6, or *l*, to the breadth of the longeſt, 3, or *b*: Hence as $L:B::l:B$. Here he firſt antecedent, and the laſt conſequent, are found in one and the ſame rectangle, or product, as are the firſt conſequent and laſt antecedent found in the other. If this proportion is inverted in ſuch a manner, as to take the firſt antecedent and firſt conſequent in one number, and ſecond antecedent and ſecond conſequent in another, inſtead of direct proportion, we will then have inverſe proportion, and no longer will the product of the extremes be equal to that of the means; but, inſtead thereof, the product of the firſt and ſecond terms ſhall be equal to that of the third and fourth. Thus directly $L:B::l:b$, but inverſely, $L:b::l:B$. Therefore, to find a fourth



fourth inverſe proportional, we muſt always multiply the two firſt terms, and divide their product by the laſt. For as $L:b::l:B$, and $Lb=lB$, and $\frac{LB}{l}=B$. Hence it is obvious, that the inverſe rule of proportion may be made direct, or the direct rule of proportion may be made inverſe, only by changing the order of the terms.

CHAP. XVI.

To turn Equations into Analogies; or The application of Algebra to the varieties in Proportion.

Prob. 1. TWO merchants, A and B, had each a certain ſtock; A's ſtock was to B's as 2 to 3. By trading A gained 3000 *l.* and B 3500 *l.* and then A's ſtock was to B's as 4 to 5. Required their reſpective ſtocks at firſt?

For 3 put r, and for 4 put R; let A's gain be repreſented by $g=3000$, and B's by $G=3500$, 4 in the ſecond proportion by q, and 5 by Q, and let the numbers required be a and e.

Firſt then $a:e::r:R$, *per* queſtion, and by multiplying extremes and means, $aR=er$, and dividing both by r; $e=\frac{aR}{r}$, to both add their reſpective gain, and we have, *per* queſtion, $a+g:\frac{aR}{r}+G::q:Q$, and by multiplying extremes and means, $aQr+gqr=aRq+Gqr$, and by tranſpoſition $gQr-Gqr=aRq-aQr$; and dividing by the coefficients of a, it will be $a=\frac{gQr-Gqr}{qR-Qr}$. Therefore, to find the value of a by the rule of proportion, take the diviſor $qR-Qr$ for your firſt number, the coefficients of r, $gQ-Gq$, and let r be the third; ſo ſhall the equation be reſolved in this analogy, 2 : 1000 : 2 :: 1000 *l.* A's ſtock at firſt, and $e=1500$ *l.*

Prob. 2. If A works a garden in 18 days $=t$, and B does it in 28 days $=T$, in what time will they do it together?

Here time and motion being reciprocally proportional, it will be as t, A's time, is to $G=$ the garden; ſo inverſely is T, B's time, to what he can work in the ſame time; that is, as $t:G::$ ſo inverſely $T:tG$, in which A's work is G, and B's work is $\frac{tG}{T}$. Wherefore $G+\frac{tG}{T}$, both their works in equal time

time $:t$, the time in which they work $::G$, the work of the garden $:a$, the time required : that is $G+\frac{tG}{T}:t::G:a$; therefore, by multiplying extremes and means, $Ga+\frac{tGa}{T}=tG$; and $TGa+tGa=TtG$; and $a=\frac{TG}{TG+tG}=\frac{Tt}{T+t}$

By which procefs it appears, that the queftion is to be refolved by making the product of both given times the dividend, and the fum of both the times the divifor. Hence it will be $18 : 28::49:10\frac{22}{23}$ inverfely ; or directly $46 : 18:: 28: 10\frac{22}{23}$, anfwer.

Prob. 3. Three workmen could finifh a certain piece of work as follows, *viz.* A once in three weeks, B thrice in eight weeks, and C five times in twelve weeks ; it is required to find in what time they would finifh it working together ?

Here the powers of the agents A, B, C are fuch as in the times 3, 8, 12, refpectively produce effects, and it is required in what time they will produce the effect $1=a$. It will be $a=\frac{1}{\frac{1}{3}+\frac{3}{8}+\frac{5}{12}}=\frac{8}{9}$ of a week $=6$ days, $5\frac{1}{2}$ hours, *&c.*

Prob. 4. How many ancres of brandy, at 30 *s.* each, may I have in barter for 48 yards of cloth, at 10 *s. per* yard ?

To find by algebra how this queftion, and all of the fame nature, are to be ftated by the rule of proportion. Let N reprefent the given quantity of any commodity, here 48, and V its given value $=10$ *s. per* yard : let P reprefent the value of the other commodity for which the firft is to be exchanged, and a the quantity required. Then it is plain that $NV=Pa$, that it is the quantity of the firft $\times$ by its refpective value $=$ to the quantity of the fecond $\times$ its value. Therefore, $a=\frac{NV}{P}$, and directly $P:N::V:a$, or inverfely, $V:N::P:a$. In numbers, as $10 : 48 ::$ fo inverfely $30 : 16$, the anfwer. And in this manner the arithmetician may examine, and prove the truth of the rule of fellowfhip, alligation, and other arithmetical operations.

Prob. 5.

There is a cup of gold, that weighs ten ounces and fourfcore,
A filver cup, of the fame fize, weighs fifty and no more ;
To caft a cup of fixty fix, mix'd metal in that mould,
How many ounces muft I mix of filver and of gold ?

$a+e=66=M$, and $e=M-a$. $G=90$ $S=50$ $M=66$; then, as $G:1::a:\frac{a}{G}$ Mould, the part of the mould filled with gold, and as $S:1::M-a:\frac{M-a}{S}$ Mould, the part of the mould filled with silver; which two parts added must be equal to one whole mould; that is, $\frac{a}{G}+\frac{M-a}{S}=1$ mould. This equation multiplied and brought out of fractions will give $aS-aG+MG=sG$, and by division and transposition, $a=\frac{MG-SG}{G-S}$, which gives this analogy for a 1st, $G-S:M-S::G:a$
40 : 16 :: 90:36,

and for e $\left\{\begin{array}{l} G-S:G-M::S:e \\ 40:24 :: 50:0. \end{array}\right.$

CHAP. XVII.

ARITHMETICAL PROGRESSION.

NUmbers in arithmetical progression are such as increase or decrease by the same common difference, as in the margin.
$\left\{\begin{array}{l} 1, 2, 3, 4, 5, 6, 7, 8, 9, \&c. \\ 3, 7, 11, 15, 19, 23, 27, 31, \&c. \\ 20, 18, 16, 14, 12, 10, 8, \&c. \end{array}\right.$
In which it is easy to observe, that any three numbers in this progression being equally distant from each other, or exceeding each other equally, the sum of the extremes will be still equal to double the mean or middle term. For let f represent the first, s the second, t the third, let d represent the common difference, then $f+d=s$, and $f+2d=t$, and therefore the three numbers will stand f. $f+d$ $f+2d$, the sum of the two extremes, will be $2f+2d=$ double $f+d$, the mean. Also, if four numbers are in arithmetical progression, the sum of the extremes will be = the sum of the means. For then the terms will be $f, f+d, f+2d, f+3d$, the sum of the extremes, $2f+3d=$ the sum of the means.

In any number of terms in this progression, the sum of the extremes is equal to the sum of any two means equally distant from the extremes, as 2 4 6 8 10 12, $2+12=4+10=6+8$. For the difference of the first and second being the same with the difference of the third and fourth, the numbers will stand thus: F:F+dT:T+d, in which F+T+d=F+d+T. And hence the sum of any series of numbers in arithmetical progression is equal to the sum of the first and last added together and multiplied by ½ the number of terms: For the sum of of this ſeries being compoſed of the ſum of the extremes and means, it is evident the number of extremes and means will be equal to ½ the number of terms; alſo the difference betwixt the firſt and laſt term will be equal to the common difference of each two ſubſequent terms × the number of terms —1; as 4, 7, 10, 13, 16, 19, in which $19-4=3\times5$. From this it follows, that if the number of terms is expreſſed by *N*, the common difference by *D*, the firſt term *F*, the laſt term *L*, the total ſum of the ſeries *S*; and if any three of theſe are given, the other two may be eaſily found, without a continued addition of the terms, as appears in the following examples.

1. A nobleman gives a penny to a beggar the firſt day of the year, twopence the ſecond day, threepence the third, and a penny more every day to the year's end; required how much he gives the laſt day of the year, and what is the total amount of the whole?

Here is given $F=1$ penny, D alſo $=1$, and $N=365$, the days of the year. To find L and S. Firſt, $L-F=D\times N-1$, and $L=D\times N-1+F=365$ pence given the laſt day of the year. Then $L+F\times=\frac{N}{2}=S$, that is $365+1=366\times\frac{N}{2}$, or 182 $=66795$ pence the ſum of the whole ſeries =L. 278 : 6 : 3.

Suppoſe 500 eggs laid in a ſtraight line, at one yard's diſtance from each other, and the firſt egg one yard's diſtance from a baſket, into which a man muſt carry them one by one; required the number of yards he muſt travel?

Here $F=2$ yards, D alſo $=2$, $N=500$; conſequently, as before, $L=DN-1+F=2\times499$, or $998+2=1000$ yards, and $S=L+F\times\frac{N}{2}$, that is, $1000+2=1002\times250=250500$ yards in all.

CHAP. XVIII.

GEOMETRICAL PROGRESSION.

NUmbers in geometrical progreſſion are ſuch as either increaſe or decreaſe proportionally by the ſame common ratio, as 1. 2. 4. 8. 16. 32. 64. *&c.* where the common ratio is (2), each ſubſequent term being double the preceding, or 1. 3. 9. 27. 81. 243. *&c.* where the ratio is (3), each ſubſequent term being triple the preceding, and the ſame of any other ſeries, whether increaſing or decreaſing. In any ſeries of of this kind, it is evident from what has been ſaid of proportional numbers in general, *firſt*, That of any three numbers of this progreſſion, the product of the extremes is equal to the ſquare of the means. *Secondly*, That of four numbers of this progreſſion, the product of the two extremes is equal to the product of the two means. And, *thirdly*, That in any ſeries exceeding four terms, the product of the means is equal to the product of the extremes, equally diſtant from theſe means; as in 1. 2. 4. 8. 16. 32. where $1\times32=2\times16=4\times8$; the reaſon of which is, becauſe theſe extremes and means are proportional by the hypotheſis, and conſequently by the general theorem their products muſt be equal. It is alſo evident, that in any ſeries of this kind, as the firſt antecedent is to the firſt conſequent, ſo is the ſum of all the antecedents to the ſum of all the conſequents; as in 2 : 4 : 8 : 16 : 32 : 64, whereas $2:4::2+4+8+16+32=64:4+8+16+32+64=124$; for it is plain, that as oft as the firſt antecedent is contained in the firſt conſequent, ſo oft is the ſum of all the antecedents contained in the ſum of all the conſequents. Hence, if *F* repreſents the firſt term of any progreſſion, *L* the laſt, *R* the common ratio, *N* the number of terms, *S* the ſum of the whole ſeries, if any three of theſe are given, the other two may be found without a continued multiplication, diviſion, or addition of the terms, as appears by the following problem.

Problem. In any ſeries of numbers in geometrical continued progreſſion, the firſt term *F*, the number of terms *N*, and the common ratio *R* being given, to find the laſt term *L*, and the ſum of the whole ſeries *S*, let this common queſtion be propoſed. A man bought a horſe with 7 nails in each ſhoe, 28 in all, and, by agreement, was to pay 1 farthing for the firſt, 2 for the ſecond, 4 for the third, 8 for the fourth, in double proportion till all the nails were paid for; required the price of the horſe? Here $F=1$, $R=2$, $N=28$, required *L* the laſt nail, and *S* the ſum of the whole ſeries. Since *F* repreſents the firſt term, *FR* will be the ſecond, *FRR* or FR^2 the third, FR^3 the fourth, FR^4 the fifth, FR^5 the ſixth, and ſo $FR^{27}=$ the twenty eighth and laſt term; that is, if the ratio 2 is involved to the 27th power, being leſs by 1 than $N=28$, and this multiplied by $F=1$, (which in this queſtion neither multiplies nor divides), you will have *L* the laſt term. Now, the ratio 2 may be briefly raiſed to the 27th power, thus,

thus, $R^2=4$, $R^2 \times R^2=R^4=16$, $R^4 \times R^4=R^8=256$, $R^8 \times R^8 =R^{16}=65536$, $R^{16} \times R^8=R^{24}=16777216$, $L^{24} \times R^3=R^{27}=134217728=L$. the laſt term or number of farthings to be paid for the laſt nail. Then $S-L=$ the ſum of all the antecedents, and $S-F$ the ſum of all the conſequents; and as $F:FR::S-L:S-F$, and by multiplying extremes and means $FS-FF=FRS-FRL$, and by diviſion $S-F=RS-RL$, and by tranſpoſition $RS-S=RL-F$, conſequently $S=\frac{RL-F}{R-1}$, in words as follows.

Multiply the laſt term by the ratio, ſubtract the firſt term, divide the remainder by the ratio—unity, the quotient gives the ſum of the whole ſeries. In the queſtion propoſed, the laſt term was found 134217728, this × by 2=268435456, from which ſubtract 1, remains 268435455. This divided by the ratio—1, gives the total number of farthings paid for the horſe 268435455=279620 l. 5 s. 3 d. 3 qrs.

Example 2. An European having taught an Indian prince to play at cheſs, was deſired to aſk his reward. In order to give the Indian an idea of numbers increaſing in geometrical progreſſion, the European told him, that there being ſixty-four little ſquares in the cheſs-board, for the firſt he demanded only 1 ſquare inch of ground, 2 for the ſecond, 4 for the third, 8 for the fourth, *&c* and ſo forward in double geometrical progreſſion. This ſeeming at firſt a very moderate demand, was readily complied with; but the prince ſoon found it was more ground than the whole world could afford. Required the number of ſquare inches it would amount to?

Firſt, to raiſe the ratio 2 to the 63d power, $R \times R=R^2=4$, $R^2 \times R^2=R^4=16$, $R^4 \times R^2=R^6=64$, $R^6 \times R^6=R^{12}=4096$, $R^{12} \times R^{12}=R^{24}= 16777216$, $R^{24} \times R^6=R^{30}=1073741824$, $R^{30} \times R^3=R^{33}=8589934592$; then $R^{33} \times R^{30}= R^{63} = 9223372036854775808$, which is the laſt term or number of ſquare inches of ground for the 64th ſquare of the cheſsboard. This multiplied by R^2, gives in all 18446744073709551615 ſquare inches. This divided by 144 ſquare inches in 1 ſquare foot, gives 1281024129927441 ſquare feet. This divided by 36000000, the number of ſquare feet in a ſquare mile, reckoning 6000 feet one geographical mile, gives 35584003 ſquare miles, nearly equal to $\frac{1}{4}$ of the globe, including land and water.

CHAP.

CHAP XIX.

EXTRACTION of the SQUARE ROOT.

SInce the ſquare of any number is the product of that number multiplied into itſelf, as 2×2=4, and 3×3=9, and 9×9=81, &c. ſo reverſely, the root of a ſquare number is that number, which being multiplied into itſelf, produceth the ſquare; hence the ſquare root of 4 is 2, the ſquare root of 9 is 3, and the ſquare root of 81 is 9.

TABLE OF POWERS.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81
Cubes	1	8	27	64	125	216	343	512	729
Biquad.	1	16	81	256	625	1296	2401	4096	6561
Surſolids	1	32	243	1024	3125	7776	16807	32768	59049
Sq. cubes	1	64	729	4096	15625	46656	117649	262144	531441

Prob. 1. *To extract the ſquare root of a whole number.*

1. Draw a crooked line on the right of the given ſquare number, as in diviſion, where the figures of the root, or the quotient expreſſing the root muſt be placed.

2. Put a point over the unit's place, and omitting one, point every other figure towards the left hand, ſo ſhall the given ſquare be pointed into ſeveral periods of 2 figures each, the number of which periods diſcover the number of places of which the root muſt conſiſt.

3. The root of the firſt period to the left hand is found by the above table, or without any other table than that of multiplication; wherefore, place the neareſt root in the quotient, and ſubtract its ſquare from the firſt period; to the remainder annex another period, and double the quotient for a new diviſor; then conſider how often the new diviſor is contained

in all the figures excepting the unit's place, and quote the number of times; then affixing this new quotient-figure to its own divisor, multiply the divisor thus increased by the new quotient-figure, and subtract the product from the whole dividend as before; bring down another period, which must be managed exactly as the last, and proceed thus till all the periods be brought down; if at last there be no remainder, then will the quotient express the true root; but should there be something still remaining, annex periods of ciphers, and carry on the division decimally, till the root be brought sufficiently near the truth, according to the exactitude required in the solution of any question depending thereon.

EXAMPLES.

1. Required the square root of $\dot{3}2\dot{1}4\dot{8}9$ (567×567=321489 Pr.

25=5×5

―――

6 annexed to 5×2= 106)714 = 7 and 14 annexed.

636 = 6 × 106

―――

7 annexed to 2×56= 1127)7889 = 78 with 89 annexed.

7889

―――

0

ILLUSTRATION.

First, the unit's place 9 is pointed, omitting 8, 4 is pointed, and omitting 1 we point 2: hence as there are three points, we conclude there will be three places in the root. Then, since 5 is the nearest root of 32, for six would be too much, as 6×6=36, we place 5 in the quotient, which being squared and set under 32, there remains 7, to which the period 14 is annexed. Then we double 5 for a divisor, which becomes 10; then we will say 10 out of 71, (for the unit's place is not regarded), and that can be found 7 times; yet, as 7 cannot be found once in 4, we take it but 6 times, which 6 being quoted and annexed to the divisor 10, we multiply 106 by 6, and the product is 636; the difference betwixt 714 and 636 is 78, to which the period 89 being affixed, we have a new dividual or resolvend of 7889; the former part of the quotient being multiplied by 2, becomes 112, which is found in 788, 7 times, and this being quoted and annexed to the divisor, makes the whole divisor 1127, which exactly corresponds to the number taken, for there is no remainder.

Exam. 2. What is the square root of 6968?

```
          6968·(83.4745, &c.
          64
         ----
    163) 568
         489
        -----
   1664) 7900
         6656
        ------
  16687) 124400
         116809
        --------
 166944) 759100
         667776
        --------
1669485) 9132400
         8347425
        ---------
         (784975)
```

DEMONSTRATION.

The reason of this method is founded in geometry, *Eucl. book* 2. *prop.* 4. and in algebraic multiplication. Let G represent any given number whose square root is required; let R represent the first member of that root found by the table, and e the second member, or part required to complete it: then $R+e$ is the complete root, and $RR+2Re+ee=G$ the given number, and by subtracting from both sides $2\,Re+ee=G-RR$, and dividing by the coefficients of e, it is $e=\frac{G-RR}{2R+e}$; which in words is the very method above explained: for, from the given number G subtract the square of the first member of the root represented by R, and divide the remainder by $2R$, or double the root, annexing the quotient figure or second member of the root, represented by e, to the said double root or divisor, by which the value of e is ascertained, the second member required; and for a third member y, the method

method is plainly the ſame; for then $R+e$ is the root, and is regarded as the firſt member, $2R+2e$ the double root, and $2R+2e+y$ the diviſor for finding y, as will be evident from the following operation.

$G=$ 55225 (2 . . $=R$
$2R=4$ . . $RR=4$ . . . .

$+e=3.$ 152 . . . . 3. $=e$
$2R+2e=46.$ 129 . .

$+y=5$ 2325 . . 5 $=y$
$2R+2e+y=465$

2325 —————
235$=R+e+y.$
(0)

Exam. 3. What is the ſquare root of 43623?

43623 (208. 861. &c.
4

408) 3623
3264

4168) 35900
33344

41766) 255600
250596

417721) 500400
417721

82679, &c.

Note 1. If the root of a mixed number is propoſed to be extracted, make the number of decimal places even, that a point may fall upon the unit's place of the integral part of the propoſed number, and point off as many places for decimals in the root as there were periods in the fractional part of the ſquare.

Exam. 4. What is the ſquare root of 751417.574560 ?

```
         751417.574560(866.843
         64
         ----
166)     1114
          996
         -----
1726)     11817
          10356
          ------
17328)     146157
           138624
           -------
173364)     753345
            693456
            -------
1733683)     5988960
             5201049
             -------
    Rem.      787911
```

Note 2. To extract the root of a vulgar fraction ;——reduce it to its loweſt terms, if it is not ſo already, then will the root of the numerator and the root of the denominator, ſeverally extracted, be the fractional root required.——Thus $\frac{9}{16}=\frac{3}{4}$ and $\frac{49}{64}=\frac{7}{8}$, &c.

Note 3. To extract the root of any pure decimal, make the places of an even number, and then the operation will be the ſame as before.

What is the ſquare root of .5625 ?

```
      .5625(.75
       49
      -----
145)   725
       725
      -----
        0
```

What

What is the ſquare root of .1250?

```
   1250(.353
   9
   ------
 65)350
    325
   ------
703)2500
    2109
    ------
     391
```

APPLICATION OF THE SQUARE ROOT.

Prob. 1. To find a geometrical mean between any two numbers.

Multiply the given numbers into one another, and the ſquare root of their product is the geometrical mean required.

Required a geometrical mean betwixt 9 and 16?

```
   16
    9
   ---
  144(12 Anſwer
  1
  ---
22)44
   44
   ---
    0
```

Prob. 2. To find the ſide of a ſquare, whoſe area ſhall be equal to that of any given ſurface whatever.——The ſquare root of the given content will be the ſide of the ſquare required.

Exam. There is a piece of ground in the form of a parallelogram, whoſe longeſt ſide is 134 chains, and ſhorteſt 80 chains, to be exchanged for a ſquare piece of ground cut out of a large field, of the ſame area; required the ſide of the ſquare?

```
                                134
                                 80
                               -----
Area of the parallelogram      10720(103.53 Answer.
                                1
                              -------
                              203)0720
                                   609
                              ---------
                              2065)11100
                                   10325
                                 -----------
                                 20703)77500
                                       62109
                                       -----
                                      (15391)
```

A maltster hath a kiln which he finds too little for his business, its diameter being only 15 feet; the diameter of another is required, which will hold double the quantity of the old one?

```
15×15=225 square of the diameter.
        2
     -----
      450(21.2 Answer.
      4
     ----
     41)50
        41
     -------
     422)900
         844
         ---
         (56)
```

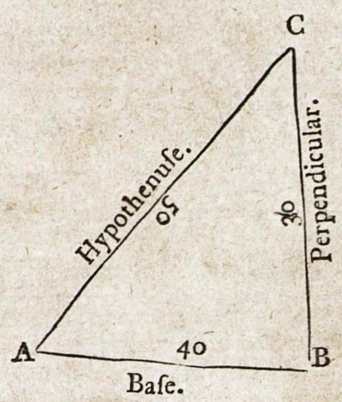
A maltster hath a kiln which he finds too large for his business, its diameter being 21.2 feet; the diameter of another which will hold half the quantity is required?

21.2×21.2=450 nearly.

Then $\frac{450}{2} = 225(15.$ Answer.

Prob.

Prob. 3. Having any two ſides of a right angled triangle given to find the third, the ſum of the ſquares made upon the baſe and perpendicular, is equal to the ſquare made upon the hypothenuſe, *per* 47. *b*. 1. *Euc*. *Elem*.



Therefore to find A C, extract the ſquare root of the ſum of the ſquares of A B, and B C.

To find A B, or B C, the ſquare root of the difference of the ſquares of the other two will give that required.

E X A M P L E S.

A B = 40
B C = 30 to and A C

40×40 = 1600
30×30 = 900

2500(50 = A C.

A C = 50
A B = 40 and B C

50×50 = 2500
40×40 = 1600

900(30 = B C.

A travels north 50 miles, and B travels eaſt 60 miles; how many miles are they diſtant?

$$50\times50=2500$$
$$60\times60=3600$$

```
         6100 (78 miles, Ans.
         49
    148)1200
        1184
          16
```

The ſemidiameter of the earth being 3984.58 miles, and the perpendicular height of a mountain 3 miles; how far will it be ſeen at ſea, or on plain ground, ſuppoſing the eye of the ſpectator to be on the ſurface of the ground or water?

```
3984.58 ſemidiameter
     3  height.

3987.58×3987.58=15900794.2564
3984.58×3984.58=15876877.7764

                    23916.4800(154.64 miles.
                    1
                   25)139
                      125
                  304)1416
                      1216
                  3086)20048
                       18516
                 30924)153200
                       123696
                       (29504)
```

Prob. 4. The diameter and capacity of one circle being given, to find the diameter of another circle, that ſhall have any proportional capacity, greater or leſſer than the circle whoſe diameter and capacity is given.

Since

Since the areas, or capacities of circles are to one another as the ſquares of their diameters, *per prop.* 2. *b.* 12. of *Euc. Elem.* it will be, as the ſquare of the diameter of any circle is to its area or capacity, ſo is the ſquare of the diameter of any other circle to its area, and *vice verſa.*

Exam. 1. If the diameter of a circle be 6, and its area 28.2742; what will be the area of another circle whoſe diameter is 8?

36:28.2742::64
9 16

9)452.3872

50.2652

Exam. 2. If the diameter of a circle, whoſe area is 50.2652, be 8, what is the diameter of that circle whoſe area is 28.2742?

50.2652:64::28.2742:36, the ſquare of 6, the anſwer.

CHAP. XX.

The EXTRACTION of the CUBE ROOT.

AS in a ſquare, which is the ſecond power, every ſecond figure from the right hand is to be pointed, ſo, in a cube, which is the third power, every third figure, beginning with the unit's place, is to be pointed in the like manner. Then find, by the table of cubes, the neareſt cube to the firſt pointed period of the reſolvend, the cube root of which place in the quotient to the right hand, and ſubtract the cube thereof from the firſt pointed period, bringing down to the remainder the ſecond period.

Secondly, For a diviſor to this reſolvend, take thrice the ſquare of your firſt quotient-figure, either annexing, or ſuppoſing tobe annnexed thereunto, a competent number of ciphers, according to the number of points in the given reſolvend; by which find the ſecond quotient-figure of the required root.

Thirdly, To your diviſor, which was thrice the ſquare of your firſt quotient-figure, annex thrice the ſame quotient-figure, multiplied by the ſecond, and alſo the ſquare of the ſecond; and in this obſerve, to advance the figures annexed one ſtep further, to the right hand; and then multiply the diviſor thus increaſed by your ſecond quotient-figure, and ſubtract the product from the reſolvend.

Fourthly, To your remainder bring down your next period, and find a new diviſor, as you did the firſt; by taking thrice the ſquare of the two figures, of which the root or quotient now conſiſts, and, having divided and noted down your third quotient-figure, annex to your diviſor triple the two firſt figures, and the ſquare of the third, obſerving, as before, to advance the annexed figures a ſtep to the right hand. Continue this method till all the periods of the reſolvend are taken down, and then, if the propoſed number be a perfect cube, there will be no remainder; but, if ſomething ſhould remain, annex ciphers, three at a time, and carry on the extraction, decimally, to any propoſed degree of exactneſs, as in the extraction of the ſquare root.

EXAMPLES.

1. Required the cube root of 13824?

$$\dot{1}38\dot{2}4(24 \text{ the root required.}$$

$2\times2\times2=8$

5824 For $24\times24\times24=13824$.

$2\times2\times3=12..$
$2\times3\times4=\ 24.$
$4\times4\ =\ 16$

Diviſor $1456\times4=5824$

(0)

2. Required the cube root of 14886936?

```
    14886936(246
     8
    ----
 1456)6886
      5824
    -------
177156)1062936
       1062936
    ---------
        (0)
```

Illustration of the last example.

Having pointed the proposed cube, I find the root of the first period to be 2, which is noted in the quotient, and its cube being subtracted from the first period, there remains 6; to which another period being annexed, there is a new resolvend of 6886. To find a divisor for this resolvend, upon a piece of waste paper, I take the triple square of the quotient-figure, 2=12, which I consider as 1200, and note 4 in the quotient. Then, 3×2×4=24

And 4×4= 16

Divisor 1456 completed.

The divisor being multiplied into 4, and the product subtracted from the last resolvend, the remainder, with another period annexed to it, constitutes the new resolvend 1062936. Then on a piece of waste paper, as before.

24×24×3=1728 which I consider as 172800, and
3×24×6= 432 note 6 in the quotient.
Then 6×6= 36

Divisor 177156 completed.

Which being multiplied into 6, and the product subtracted from the resolvend, leaves no remainder, and the root is 246; which, being involved, will be found equal to the given cube.

3. Required the cube root of 27407028375?

```
      27407028375(3015
      27
   ---------------
   270901)407028
          270901
   ---------------
  27225475)136127375
           136127375
           ---------
              (0)
```

DEMONSTRATION.

Let G represent any given cube, the root whereof, a, is required; let r represent the nearest root to the first period of the pointed cube, and e what r wants of the just root.

Then $r+e=a$

$rr+2re+ee=aa$

$r^3+3rre+3ree+e^3=a^3=G$

By subtracting r^3. $3rre+3ree+e^3=G-r^3$
and dividing by the coefficients of $e=\frac{G-r^3}{3r^2+3re+e^2}$

Which being expressed in words, we have this rule. From the given cube, subtract the cube of r. *i. e.* the cube nearest to the first period: divide the remainder by thrice the square of r, the first quotient figure, annexing thereto thrice the said first figure multiplied by the second, and also the square of the second; which in effect is the rule given above.

APPLICATION of the CUBE ROOT.

The cube root is of singular use in mathematics; but as mercantile computations ingross our present attention, we shall only give a few instances, and therewith conclude this part.

Prob. 1. To reduce any solid to a cube. The cube root of the solid content of any figure is the side of a cube, whose solidity shall be equal to that of the figure given.

Ex.

Ex. Suppoſe a box or cheſt, whoſe length is 24, breadth 9, and depth 8 inches; required the ſide of a cube whoſe ſolid content ſhall be equal thereto?

```
 Length 24
 Breadth 9
        ---
        216
 Depth    8
        ---
        1728(12 Answer.
        1
       ---
     364)728
         728
         ---
           0
```

Prob. 2. Having the dimenſions of a ſolid given, to determine the dimenſions of a ſimiliar, or like ſolid, that ſhall be any number of times greater or leſs than that given.—Cube the given dimenſions, and multiply their ſeveral cubes by the difference, if the ſolid required is greater than the given one; or divide the ſeveral cubes by the difference, if the required ſolid be leſs than that given.

Ex. What will be the ſide of a cube that ſhall contain four times as much as that cube whoſe ſide is 12?

```
 12×12×12=1728
             4
          ----
          6912(19.04
          1
          ----
      651)5912
          5859
          --------
 10852816)53000000
          43511264
          --------
           9488736
```

Prob.

Prob. 3. To determine the dimenſions of a ſimilar ſolid, by having the dimenſions or capacity of a ſimiliar ſolid given.

Ex. If a ſhip of 100 tuns be 44 feet long at the keel, of what length ſhall the keel of that ſhip be, whoſe burthen is 220 tuns?

$$\sqrt[3]{\frac{44\times44\times44\times220}{100}}=57.225. \text{ Anſwer.}$$

2. Let the length of a ſhip's keel be 125 feet, and the breadth of the midſhip beam 25 feet, and the depth of the hold 15 feet; I demand the dimenſions of another ſhip of the ſame figure, that ſhall carry three times the burthen of the given ſhip? The cube roots of the cubes of the reſpective given dimenſions, multiplied by 3, will produce the ſeveral dimenſions required; *viz.*

The length of the keel - -	215.9 feet.	Anſwer.
Breadth of the midſhip beam	36 feet.	
Depth in the hold - - -	21.6 feet.	

3. Suppoſe the length of a ſhip's keel be 76.95 feet, the breadth of the midſhip beam 29.58 feet, and the depth in her hold 14.877 feet; required the dimenſions of another ſhip of the ſame mould, that will carry only 72 tuns burthen?

The ratio of the given dimenſions is 5, for $\frac{360}{72}=5$; wherefore, if the given dimenſions are ſeverally divided by the cube root, the ratio, the quotients will be the dimenſions required; *viz.*

Length 45
Breadth 17.3
Depth 8.7

4. Required the ſide of a cube that will be equal in capacity to a veſſel which contains 1728 ſolid inches?

$$\sqrt[3]{1728}=12$$

5. Suppoſe a cannon-ball, 3 inches diameter, to weigh 54 ounces Avoirdupois; what muſt the diameter of that mould be, in which another ball weighing 27 pounds may be caſt?

$$54 : 3\times3\times3 :: 432$$

Abridged $2 : 1 :: 432 : \sqrt[3]{216}=6$ inches, Anſwer.

6. Suppoſe a carpenter is employed to make a cubical veſſel, of equal length, breadth, and depth, which ſhall juſt contain 100 Scots gallons of wort; required the dimenſions thereof?

100 gallons.
872 ſolid inches in a Scots gallon.

$\sqrt[3]{87200}=44.34$ inches, the length, breadth, and depth required.

7. Required to make a girnal, which ſhall contain 1500 buſhels of malt, whoſe length ſhall be double the breadth, and the breadth and depth equal?

Firſt, 2150 ſolid inches in a buſhel.
1500 buſhels.

3225000 the ſolid content in inches.

2dly, Put a for the required length, the breadth and depth will be each $\frac{1}{2}a$; therefore by multiplying length, breadth, and depth together, we have $a\times\frac{1}{2}a\times\frac{1}{2}a=\frac{1}{4}aaa=S$ the ſolid content, and $aaa=4\ S$. If therefore we multiply the ſolid content by 4, and extract the cube root of the product, we have the required length, $\frac{1}{2}$ whereof gives the breadth, to which the depth is equal.

Hence 3225000
4

$\sqrt[3]{12900000}=234.54$ inches = the length.
$\frac{1}{2}=117.27$ inches = the breadth.

8. Suppoſe a ſhip 150 tuns burden, and the dimenſions as follow: 65 feet by the keel, 18.5 by the beam, and as much depth in the hold, by which a carpenter is to proportion the dimenſions of another ſhip of 1200 tuns burden; required the length of her keel, and other dimenſions in proportion?

Firſt,

First, $150 : 65 \times 65 \times 65 :: 1200 : \sqrt[3]{2197000} = 130$ the length.

2dly, For the breadth and depth.

$65 : 18.5 :: 130$

Abridged $13 : 3.7 :: 130 : 37$ the breadth.

RECAPITULATION of the principles of Arithmetic and Algebra.

AXIOM I.

Since whole numbers increaſe, and decimals decreaſe in a decuple proportion, 10 is the univerſal ratio of any ſeries of numbers whatever; and the reaſon for carrying at 10 in addition and multiplication is ſelf-evident, ſince 10 in any place to the right is equal to 1 in the next place to the left. Hence alſo the reaſon for carrying according to the ſubdiviſions of any integer, when ſeveral denominations are to be added.

AXIOM. II.

If two numbers are equally increaſed, their difference is always the ſame. Hence the reaſon of borrowing 10 in one place to the right, and paying it back by carrying 1 to the next place. Hence likewiſe the reaſon will be evident, for placing the firſt figure to the right of the product of every particular multiplier directly below its own multiplier.

AXIOM III.

The multiplicand will be increaſed or diminiſhed, in proportion to the multiplier, when the ſame multiplicand is uſed. Hence the reaſon why the multiplicand is encreaſed, when it is multiplied by any thing greater than unity, and decreaſed when it is multiplied by a fraction.

AXIOM IV.

The dividend will be increaſed or diminiſhed in proportion to the diviſor, when the ſame dividend is uſed. Hence to divide by any thing greater than unity, will quote a number leſs than the dividend; and, on the contrary, to divide by any thing leſs than unity, will quote a number greater than the dividend.

AXIOM V.

The whole is equal to all its parts taken together. Hence one ſum may be made equal to ſeveral by addition, and ſubtraction may be proved by adding the difference to the leaſt given ſum.

AXIOM VI.

If equal quantities are added to equal quantities, the ſums will be equal; and if equal quantities are taken from equal quantities, the remainders will be equal. Hence the reaſon of the reduction of equations by addition and ſubtraction.

AXIOM VII.

If equal quantities are either multiplied or divided by other equal quantities, the products and quotients will reſpectively be equal. Hence the reaſon of reducing equations by multiplication and diviſion, and of abridging commenſurable terms, and cancelling equal quantities and numbers.

AXIOM VIII.

To multiply any quantity or number by other quantities or numbers continually, is the ſame as to multiply by the product of theſe other numbers. Hence the reaſon of multiplying by component parts, *&c.*

AXIOM IX.

To divide one number or quantity, by other numbers or quantities continually, is the ſame as to divide that one quantity or number by the product of the reſt. Hence the reaſon of dividing by component parts, *&c.*

AXIOM X.

If four numbers or quantities are proportional, the rectangle or product of the extremes will be equal to the product of the means; and, *vice verſa*, if the product of the extremes be equal to that of the means, the numbers or quantities are proportional.

AXIOM XI.

The quotient of any two ſucceeding powers, when the next higher is divided by the next lower, exhibits the root of these powers. On the contrary, if any power is multiplied by the root of that power, the product will be the next higher power of the root; and if a higher power is divided by the root, the quotient will exhibit the next lower power. Again, if a proportional part of a higher power is divided by a proportional part of the next lower power, the quotient will exhibit a proportional part of the root. Hence the first figure or figures in the root of any power being raised to the power next lower than that whose root is wanted, and that power multiplied by a number expressing the proportion which the given power bears to its root, produces a proportional divisor, whose ratio, compared with the dividual, is a proportional part of the root, which being annexed to the former part of the root, and raised to the full power of the given number, will be either the whole, or a proportional part of the given power, discoverable by subtraction, *&c.* Hence we have a general rule for extracting the root of any power whatever.

THE UNIVERSAL ACCOUNTANT.

PART. IV.

The Application of Arithmetic to the Buſineſs of the Merchant, the Banker, Cuſtom-houſe, Inſurance-office, &c.

CHAP. I.

Simple PROPORTION, or RULE of THREE.

PRoportion may be defined in general, The identity, ſimilitude, or equality of ratios, as ratio is the relation, or habitude of two numbers, which determines the value of the one from the value of the other; for inſtance, the ratio of 4 and 8 is 2, and the ratio of 8 and 16 is likewiſe 2; hence, the relation betwixt 4 and 8, and 8 and 16 being the ſame, theſe four numbers are ſaid to be in proportion.

The rule of proportion, or rule of three, finds a fourth proportional to three numbers given, one of which ſhall have the ſame ratio to that fourth, which exiſts betwixt the remaining two, as was demonſtrated in the algebraic part. But to ſpeak in applicate terms:

Proportion is that rule, by which the value, quantity, or number of one ſpecies of things is proportioned to the value, quantity, or number of another ſpecies of things, according to ſome fixed ſtipulation, or known concluſion. For inſtance, if I purchaſe 4 yards of cloth for 10 ſhillings, and then agree to take the piece of 16 yards at the ſame price *per* yard; it is plain that the thing required here, is to proportion the price of 16 yards to the price ſtipulated for 4 yards, by ſtill preſerving the ſame ratio betwixt 16 yards and the price thereof,

of, as betwixt 4 yards and the ſtipulated price of 10 ſhillings,
yds. s. yds.
Thus, 4 : 10 :: 16:40; in which 16 hath the ſame ratio to 50 that 4 hath to 10, and 10 the ſame ratio to 40 that 4 hath to 16.

All queſtions in this rule are either in a direct or reciprocal proportion.

1. Direct, when the firſt bears the ſame ratio to the ſecond, as the third doth to the fourth; in which caſe, the greater the ſecond term is in reſpect to the firſt, the greater will the fourth term be in reſpect to the third, and the contrary. 4 : 10 :: 16 : 40. Here, becauſe 10 is greater than 4, 40 is proportionally greater than 16; and 40 : 16 :: 10 : 4. Here, becauſe 16 is leſs than 40, 4 is of conſequence leſs than 10 in the ſame proportion. Hence we have this corollary for proving all operations in direct proportion, that the product of the extremes will always be equal to that of the means: for it is plain, that $4 \times 40 = 10 \times 16$
and that $40 \times 4 = 16 \times 10$

2. Reciprocal, when the third bears the ſame ratio to the firſt, as the ſecond doth to the fourth; in which caſe the leſs the third term is in reſpect to the firſt, the greater will the fourth term be in reſpect to the ſecond, and *vice verſa*. For inſtance: Suppoſe 8 men could do a certain piece of work in 4 days, and it were required to know in what time 16 men could do it; upon the leaſt conſideration it would occur, that 16 hands would do more work than 8, and conſequently require leſs, time wherefore, as 8 : 4 :: 16 : 2. In which 16 bears the ſame proportion to 8 that 4 doth to 2; and by ſhifting the ſuppoſition, 16 : 2 :: 8 : 4. Hence, when the terms are in reciprocal proportion, the product of the two firſt terms will always be equal to the product of the two laſt,
For $8 \times 4 = 16 \times 2$, and $16 \times 2 = 8 \times 4$.

From the foregoing conſiderations are deduced the following rules.

1. For ſtating, or ranking the numbers in a proportional order. Make that number the third term upon which the demand lies; that number the firſt term which is of the ſame kind, or ſignifies the ſame thing, with that term which was made the third; then will the remaining one, which is to poſſeſs the ſecond place, be of the ſame kind, or ſignify the ſame thing, with the fourth, or number required.

2. For finding a fourth proportional. If the terms are in direct

direct proportion, that is, if more require more, or lefs require lefs, the product of the two laft divided by the firft will quote the anfwer, or 4th proportional.——But if the terms are in reciprocal proportion, that is, if more require lefs, or lefs require more, the product of the two firft divided by the laft will quote the anfwer.

E X A M P L E S.

1. Bought 1420 yards of ozenbrigs, at 12 *s.* the fcore, or 20 yards; what will be the charge of the whole?

yds. *s.* *yds.* *L.* *L* *s.*
Firft 20 : .6=12 :: 1420 : 42.6=42·12

For 1420
.6
———
2|0)85.20
———
42.6

And 20×42.6=.6×1420

2. It is computed that 6 men would build a wall in 40 days, but the proprietor would have it finifhed in 10 days; how many men, according to that computation, muft be hired for building the wall?

days *m.* *days* *m.*
Firft 40 : 6 :: 10 : 24

For $\frac{40\times6}{10}$=24 men.

and 40×6=10×24.

Iluftration of the laft two Examples.

In the firft example, the number upon which the demand lies is 1420 yards, and therefore by the rule it ftands in the third place; the correfpondent number to 1420 yards muft be that one of the other two which implies yards, or fome denomination of that integer, which in this cafe is found to be 20 yards, and therefore by the rule adopted for the firft term: but to prove that we are fo far right, we have ftill another check, namely, that the remaining term for the fecond place muft be money, becaufe by the queftion the anfwer muft be money.

money: here we find it is 12 ſhillings, and therefore we may conclude that the terms are properly ſtated. Next, we conſider that the 3d term, 1420 yards, contains a greater quantity than the firſt term, 20 yards, and conſequently requires a greater price; therefore we conclude the terms to be in direct proportion, and find the anſwer, by dividing the product of the two laſt terms by the firſt.

In the ſecond example the demand lies upon ten days for the 3d term, to which 40 days correſpond for the firſt; and 6 men muſt be the ſecond, becauſe it correſponds with what is required. It will likewiſe be obvious, that to finiſh any work in ten days, will require more men than it would do to finiſh it in 40 days, and therefore we conclude that the terms are in reciprocal proportion.

OBSERVATION.

When the terms are mixed numbers, or of different denominations, they may be made homogeneal by reduction, vulgar fractions, or decimals; and the operations thereafter abbreviated, when poſſible, in any of thoſe methods propoſed in the firſt part, which may ſeem moſt adapted to the purpoſe; or by other methods which judgement and experience may dictate, equally, and perhaps ſtill better calculated for diſpatch.

To give a more particular idea of my meaning, I ſhall vary the work of the next queſtion, by different methods of operation, and afterwards give the ſolution of others in that method which bids faireſt for diſpatch, which, next to accuracy, ought to be the principal object attended to by an accountant; and though ſome figures that are not neceſſary to the operation, ſuch as the ſtating of the queſtions, for the ſake of illuſtration, may be introduced, no figure ſhall be omitted, for the ſake of an affected brevity, which I myſelf have occaſion to uſe in the operation.

Queſt. 3. If for $5\frac{3}{4}$ yards of velvet I get *L.* 4, 12 *s.* what may I reckon $84\frac{1}{2}$ yards worth, which is all that I have of the kind?

1. By

1. By reduction.

Yds.		*L. s.*		*Yds.*		*L. s.*
$5\frac{1}{2}$	:	4·12	::	$84\frac{2}{4}$	:	67 12
23 qrs.		20		338 qrs.		
		92 s.		92		

$$\begin{array}{r} 676 \\ 3042 \\ \hline 23)\ 31096 \\ \hline 2|0)\ 135|2\ s. \\ \hline L.\ 67\text{·}12 \end{array}$$

2. By vulgar fractions.

$$\overset{Yds.}{\frac{23}{4}} : \overset{L.}{\frac{92}{20}} \text{ or } \overset{L.}{\frac{23}{5}} :: \overset{Yds.}{\frac{169}{2}} : \overset{L.}{\frac{15548}{230}}$$

For
$$\frac{\frac{23}{5} \times \frac{169}{2}}{\frac{23}{5}} = \frac{15548}{230} = L.6712$$

3. By decimals.

	Yds.		*L.*		*Yds.*
	5·75	:	4.6	::	84.5

Abridge the dividing and multiplying terms equally by 5, and they will stand.

	1.15	:	4.6	::	16.9
Again	.23	:	4.6	::	3.38
by 23	.01	:	.2		.2

$$\begin{array}{r} .01)\ .676 \\ \hline 67.6 \end{array}$$

4. By

4. By component parts.

Yds.	L. s.		Yds.		L. s.
5	4·12	::	$84\frac{1}{2}$	:	67·12
	7				
	32 4				
	12				
	386 8 into 84				
	2 6 into $\frac{1}{8}$				
23)	388 14				
	16 18				
	4				
	L. 67 12 *Answer.*				

Many of these figures might have been omitted but for illustration.

5. By multiplying the number of yards $84\frac{1}{2}$ into the price of 1 yard, $\frac{1}{4}$ of which is found by dividing the given price 92 *s.* by the number of quarters in the first term 23, and the quotient will be 4 *s.* which multiplied by 4, produces 16 *s.* then $16 \times 84\frac{1}{2} = 1352$, as formerly.

6. Or take $\frac{1}{4}$ for 4 *s.* of 338 quarters, and you will have *L.* 67, 12 *s.* as before.

7. Or multiply 84.5 by ·8, the decimal of 16 *s.* and it will produce *L* 67, 12 *s.*

Quest. 4. Bought 36 pipes of wine for *L.* 1221 : 19 : 10, how may I sell it *per* pipe to save one for my own use, and lose nothing on the purchase?

Note, When one pipe is deduced, there remains 35, to be valued at the given price. Therefore,

Pipes.	L. s d.		Pipe		L. s. d.
35	: 1221-19-10	::	1	:	$34\text{-}18\text{-}3\frac{1}{4}$
For	5) 1221 19 10				
	7) 244 7 $11\frac{3}{4}$				
	34 18 $3\frac{1}{4}$				

Quest.

Queſt. 5. The perpendicular height of my ſtaff from the ground is 4 feet 3 inches, and it caſts a ſhadow of 5 feet 6 inches; what is the height of a ſteeple which caſts a ſhadow of 573 feet 9 inches at the ſame time of the day?

By reducing the inches to decimals, which is done mentally, and dividing the two firſt terms by 5, the terms will ſtand thus:

Sha.	*H.*	*Sha.*	*H.*
1.1 :	.85 ::	573·75 :	443·4$\frac{2}{9}$

For 573·75
.85
———
286875
459000
———
1.1) 487.6875
——— Feet. Inch.
443·352=443 · 4$\frac{2}{9}$ *Anſwer.*

Queſt. 6. What will 245 days ſalary amount to at 85 guineas *per annum*?

Becauſe the third term is days, the firſt term will be 365 days=1 year, and that term and the third being each divided by 5, the terms will ſtand abridged thus:

Days.	*L.*	*Days.*	*L.*	*s.*	*d.*
73 :	89.25 ::	49 :	57	18	1$\frac{1}{6}$

For 89.25
7
———
624·75
7
———
73) 4373·25
———
59·907=*L.* 59-18-1$\frac{1}{6}$

Queſt. 7. How much printed paper $\frac{3}{4}$ of a yard broad will line a room 70 yards in circumference, and 6 yards high?

	Br.		*Length.*		*Br.*		*Length.*
Reciprocally, as	6	:	70	::	.75	:	560
For			70				70
			6		or		24
			5) 420.00				3) 1680
			5) 8400				560
			3) 1680				
			560 *Answer.*				

Or abridged.

Thus,	Thus,	Or thus,
14	14	70=6×4÷3
6	2	8
.15) 84	.05) 28	560
560	560	

Quest. 8. What may one ſave at the year's end, who hath *L.* 456, 15*s.* *per annum*, and ſpends only *L* 4, 13*s*, 4 d. *per* week?

First find what he ſpends a-year.

Week.		*L. s. d.*		*Weeks.*		*L. s. d.*
1	:	4 - 13 - 4	::	52	:	242 - 13 - 4

For 4,,13,,4 × 10

5) 46 13 4
× 5
233 6 8
9 6 8
He ſpends annually 242 13 4

or 3|52 × $4\frac{2}{3}$
208
17 6 8
17 6 8
242 13 4

	L.	*s.*	*d.*
He hath *L.*	456	15	0
He ſpends	242	13	4
He ſaves	214	1	8

Quest. 9. *A* can ditch 40 roods in 20 days, *B* can do it in 30; in what time will they do it together?

This

This requires two operations.

	Days.	*R.*	*Days.*	*R.*
First, for what *A* can do in *B*'s time,	20 :	40 ::	30 :	60

R. R. D. R. D.

For what time they will take together, 60×40 : 30 :: 40 : 12 *Ans.*

Quest. 10. If ten men or 14 women can drink a butt of beer in 87 days, in what time would 4 men and 6 women drink it?

M. W. M. W.
First, 10 : 14 :: 4 : 5.6
W. Days. W. W.
Then, 14 : 87 :: 5.6+6 reciprocally : 105 days.

For 87
14
———
11.6) 1218.0
———
105 days. *Answer.*

Quest. 11. When the bushel of wheat sold at 10*s.* the fourpence loaf weighed $4\frac{1}{2}$ *lb.* what should the sixpence loaf weigh, when the bushel of wheat sells at 15*s.*?

s. lb. s.
First, 10 : 4.5 :: 15 reciprocally : 3
d. lb. d. lb
And, 4 : 3 :: 6 : $4\frac{1}{2}$

These numbers are so simple, that the operation may be entirely mental.

Quest. 12. There is a case 6 feet 9 inches long, 5 feet 3 inches broad, and 7 feet 6 inches thick, what freight will it pay at *L* 6, 10*s.* *per* tun?

```
                 6.75
                  5¼
              ---------
                33.75
                 1.6875
              ---------
                35.4375
                    7½
              ---------
               248.0625
                17.71875
Feet.  L.     ---------
 40  : 6½ :: 265.78125 : 43.1895
                    6½
            -----------
           1594.68750
            132.890625
          -------------
       40) 1727.578125
          -------------
            43.1895=L. 43 3 9
```

Quest. 13. A merchant, in balancing his books, finds he is due *L.* 575, 17 *s.* and that his whole subject taken together goes no higher than *L.* 487, 18 *s.* 6 *d.*; how much may he offer his creditors of the pound?

```
  Debt.    Subject.    D.  Sub.
575.85 : 487.925 :: 1 : 16 11⅝
                                  s.  d.
       For 575.85)487.925(.847=16 11⅝ Answer.
                  460680
                  ------
                   272450
                   230340
                  -------
                    421100
                    403095
                   -------
                     18005
```

Quest. 14. A merchant wants a piece of ground before his land or tenement paved with stones, 3 feet by 2; the ground is 30 yards by 4; required the number of stones?

Feet.

```
                    30
                     4
Feet.             ----
 3                 120
 2                   9
 --  St.          ----
 6 : 1 ::         1080 : 180 Stones.
        For 6)    1080
                  ----
                   180 Anſwer.
```

Queſt. 15. If I lend a friend *L.* 200 for 6 months, how long ought I to retain *L.* 500 of his at another time to indemnify myſelf?

L.	*m.*	*L.*	*m.*
200 :	6 ::	500 :	$2\frac{2}{5}$ *Anſwer.*

Queſt. 16. When *L.* 36 valued rent pays 4 *s.* 10 *d.* of ceſs; what will *L.* 70 pay?

```
L.    s. d.     L.    s. d.
36 :  4 10  ::  70 :  9 4 7/9
         7
     -----
     33 10
         5
     -----
6)  169  2
     -----
3)   28  2 1/3
     -----
   s. 9 4 7/9 Anſwer.
```

The following queſtions are propoſed and anſwered, but the young practitioner is referred, for the method of ſolution, to the application of the foregoing.

Queſt. 17. Suppoſe a purchaſe is made of 5 pieces Dutch holland, each meaſuring 56 Flemiſh ells, at 3 *s.* 2 *d. per* ell Flemiſh, what will be gained upon the whole, if it is ſold at 5 *s.* 8 *d. per* ell Engliſh? *Anſwer L.* 3, 5 *s.* 4 *d.*

Queſt. 18. When wheat was at 12 *s.* the buſhel, the 6 *d.* loaf weighed 1 lb. 4 oz. what ought it to weigh when the wheat falls to 9 *s.* 6 *d.*? *Anſwer* 1 *lb.* 8 *oz.* 4 *pwts.* 3 *gr.*

Queſt.

Quest. 19. There is an iſland 134 miles in circumference, in which at the ſame inſtant, and from the ſame point, *A* and *B* ſet out back to back, to travel round it. *A* travels 11 miles every 2 days, and *B* 17 every three days After what time, and how many miles travel to each, will they meet ? *Anſwer*, *A* will travel 66 miles, *B* 68, and they will meet in 12 days ?

Queſt. 20. If the longeſt end of the beam of a balance be 36 inches, and the ſhorteſt 27 ; how much ſuſpended on the ſhorteſt end will equiponderate 84 *lb.* on the longeſt end ? *Anſwer* 112 *lb.*

CHAP. II.

COMPOUND PROPORTION.

IN this rule five numbers are given to find a ſixth proportional, which may be anſwered by two ſucceſſive operations in the laſt rule, but much more expeditiouſly as follows.

Of the given numbers three are conditional, or ſuppoſed, and the other two move the queſtion ; therefore, of the three conditional terms, let that which is the principal cauſe of gain or loſs, increaſe or decreaſe, action or paſſion, be put for the firſt term ; that number which denotes diſtance of time or place be put for the ſecond term, and the remaining number which will denote action, paſſion, gain or loſs, be put for the third term, then place the other two terms which move the queſtion in the ſame order with the preceding.

Rule 1. If the term ſought be of the ſame name with the firſt or ſecond, multiply the 1ſt, 2d, and laſt terms continually for a dividend, and the other two for a diviſor, the quotient ariſing therefrom will be the 6th proportional.

Rule 2. But if the term ſought, be of the ſame name with the 3d, the continued product of the three laſt terms, divided by the product of the firſt two, will quote the ſixth proportional.

Queſt. 1. If 8 men receive *L.* 4, 16 *s* for 6 days work ; how many men may be paid with *L.* 19, 4*s*. for 16 days work ?

men. days. L. days. L.
8 : 6 :: 4.8 : 16 : 19.2

ILLU-

ILLUSTRATION.

If 8 men for 6 days work receive *L.* 4, 16 *s.* Theſe are the conditional or ſuppoſed terms in the queſtion, and therefore poſſeſs the three firſt places; 8 men, as being the cauſe of action, or gain, make the firſt term; 6 days, as being the ſpace of time, make the ſecond, and the money which is gained in that time becomes the third term: then, becauſe days are put before money in the conditional terms, 16 days ſtand befor *L.* 19.2 in the terms which move the queſtion. When the terms are compared, it occurs at once that a number of men is demanded, and therefore the queſtion is wrought by the firſt rule, and may be previouſly abridged as follows.

	men.	*d.*		*L.*	*d.*	
State reſumed	8 :	6	::	4.8 :	16 :	19.2
	1	1		.8	2)	24
				.1		
						12 *Anſwer.*

Here it is neceſſary only to divide 24 by 2 after the terms are abridged, whereas otherwiſe the proceſs would have been,

$$\frac{8\times6\times19\,2}{6\times4.8}=12$$

Queſt. 2 If the intereſt of *L.* 100 for one year is *L.* 5, what will be the intereſt of *L.* 5780 in 120 days?

L.	*days.*	*int.*	*L.*	*days.*
100 :	365 :	*L.* 5 ::	5780 :	120
	73	1	120	

73) 6936.00

L. 95 0 3¼

Here the term ſought was of the ſame kind with the third, and the anſwer found by the ſecond rule.

OBSERVATION.

If we put N= the given number of men in the firſt queſtion, t=6 days, their time, S=*L.* 4, 16 *s.* their wages, T=16 days, the time propoſed in the queſtion, P=*L.* 19, 4 *s.* the ſum propoſed, and a= the number of men required.

If we ſtate it twice, it will be

1. $S:N::P:\frac{PN}{S}$ the number of men in equal times.

2. $t:\frac{PN}{S}::T:a.$

By

By multiplying extremes and means, it will be

$$ta=\frac{PNT}{S}, \text{ and } Sta=PNT$$

and by diviſion

$$a=\frac{PNT}{St}$$ according to the rule, and ſo of any other.

Queſt. 3. An undertaker contracted to finiſh 500 yards of turnpike in 30 days, and for that purpoſe hired 60 men; but, at the expiration of 20 days, he found he had only got the length of 260 yards; how many men muſt be added to finiſh the work in the ſtipulated time?

m.		*d.*		*yds.*		*d.*		*yds.*
60	:	20	::	260	:	10	:	240

500
260
———
240

By abridging the terms, and cancelling an equal number of ciphers.

	m.		*d.*		*y.*		*d.*		*yds.*
It will be	6	:	1	::	13	:	1	:	240

Then $\frac{6\times1\times240 \ men.}{13}=110$ from which deduce the men he had, *viz.*
60
———
50 to be added.

All queſtions in this rule may be readily proved by multiplying the diviſor into the quotient, the product of which muſt always be equal to the dividend; or by ſhifting the conditions of the queſtion thus: If 103 men in 10 days can finiſh 240 yards, how many yards may 60 men finiſh in 20 days, &c.

CHAP III.

RULE OF CONJUNCTION.

THe rule of conjunction joins together ſeveral ſtatings in the rule of proportion into one, and by the relation that ſeveral antecedents have to their conſequents, the proportion between the firſt antecedent and the laſt conſequent is diſcovered, as well as the proportion between the others in their ſeveral reſpects.

To diſpoſe this rule aright, the antecedents muſt be ranged in the left hand column, and the conſequents in the right hand one,

The firſt antecedent and the laſt conſequent, whoſe antecedent is ſought, muſt be of the like ſpecies; ſo muſt the ſecond conſequent and the third antecedent; and this order muſt be continued throughout the whole.

The terms being thus diſpoſed, the diviſor is found by multiplying all the antecedents into one another continually, and the dividend by multiplying all the conſequents in the ſame manner, the quotient ariſing from theſe two factors gives the antecedent required.

This rule may be ſo abridged by cancelling equal quantities and abbreviating commenſurables, that the whole operation may be performed with very little trouble.

EXAMPLES.

1. Suppoſe 100 *lb.* of Amſterdam=100 *lb.* of Paris, 100 *lb.* of Paris = 150 *lb.* of Genoa, 100 *lb.* of Genoa = 70 *lb.* of Leipſic, 100 *lb.* of Leipſic = 160 *lb.* of Milan, how many *lbs.* of Milan will equiponderate 548 *lb.* of Amſterdam?

Antecedents. *lb.*	Conſequents. *lb.*	Abridged. Ant.	Con.
100 of Amſterdam	=100 of Paris.	5	3
100 of Paris	=150 of Genoa.	5	2
100 of Genoa	= 70 of Leipſic.		7
100 of Leipſic	=160 of Milan.		548
How many of Milan	=548 of Amſterdam. ?		

Then $\dfrac{3\times2\times7\times548}{5\times55}=920\tfrac{16}{25}$ *lb.* of Milan=548 *lb.* of Amſterd.

ILLUSTRATION.

The two 100's on both ſides cancel each other, and let the laſt cipher of the three remaining antecedents be cancelled, 100 *lb.* of Paris, 100 *lb.* of Genoa, and 100 *lb.* of Leipſic, which is dividing them by 10; and to preſerve the equality on the ſide of the conſequents, cancel alſo the laſt ciphers in 150, 70, and 160; after which divide one of the remaining 10's on the antecedent ſide by 5, and the 15 on the conſequent-ſide by 5, and the quotients will be 2 on the ſide of the antecedents, and 3 on that of the conſequents: then 2 will meaſure 2 on the antecedent-ſide, and 16 on the conſequent-ſide

ſide; as it will do 10 and 8, and the quotients thereafter will be 5 and 4; which being again repeated for the remaining 10 and 4 on both ſides, leaves another 5 on the antecedent-ſide, and 2 on the conſequent-ſide. And as there is no further room for abridging, by reaſon of the odd numbers 5 and 5 on the one ſide, and 7 and 3 on the other, the operation is made, and the anſwer found as above.

The uſe of this rule may be extended to all queſtions in proportion whatever, whether ſimple or compound, integral or fractional?

Exam. 2. If 12 yards of cloth coſt *L.* 10, 10 *s.* what will 20 yards coſt?

Ant.	Con.	Abridged.	
12 yds.	*L.* 10.5	3	10.5
What will	20 yds coſt?		5

Then $\frac{10.5 \times 5}{3} = L.\ 17\ 10.$

Exam. 3. If *L.* 100 in 12 months gain *L.* 5 intereſt, what will *L.* 500 gain in 6 months?

Ant.	Con.	Abridged.	
L. 100 principal	*L.* 5 intereſt.	2	5
in 12 months.	6 months.		5

What intereſt will 500 principal gain?

Then $\frac{5 \times 5}{2} = L.\ 12\ 10$

Exam. 4. If 18 roods of ditching be done by 3 men in 16 days, when the day is 15 hours long, how much may be done by 8 men in 4 days, when the day is 9 hours long?

Ant.	Con.
3 men.	18 roods.
16 days.	4 days.
15 hours long.	9 hours long.

How many roods may 8 men do?

Each ſide being properly abridged, it will be

$$\frac{6 \times 3 \times 2}{5} = 7\tfrac{1}{5}$$

Exam.

Exam. 5. If 12 *cwt.* be carried 100 miles for 6 guineas, how many *cwt.* may be carried 150 miles for 12 guineas?

Ant.	Con.
First, 12 *cwt.*	6 guineas.
100 miles.	150 miles.
For 12 guineas how many *cwt.*?	

Abridge both fides, and it will be $\frac{2\times2\times12}{3}=16$ *cwt.*

Exam. 6. If 12 men build a wall 30 feet long, 6 feet high, and 5 feet thick, when the day is 10 hours long, in how many days will 60 men build a wall 300 feet long, 8 feet high, and 6 feet thick, when the day is but eight hours long?

The terms being properly arranged and abridged, it will be $2\times30=60$.

Exam. 7. If 24 men dig a ditch 724 yards long, 7 feet deep, 3 feet broad at the bottom, and 5 feet broad at the top, in 236 days, when they work 13 hours and 24 minutes each day; in how many days will 16 men dig a ditch 975 yards long, 9 feet deep, 7 feet at the top, and 5 at the bottom, when they work 12 hours and 36 minutes every day?

The terms being properly arranged and abridged, it will be

$$\frac{3\times59\times201\times975}{2\times62\times181\times7}=1520 \text{ days.}$$

Exam. 8. If 18 men build a wall 40 feet long, 3 feet thick, and 16 feet high, in 12 days, how many men muft be employed to build a wall 360 feet long, 8 feet thick, and 10 feet high, in 60 days?

The terms being properly aranged and abridged, it will be $2\times9\times3=54$ men. *Anfwer.*

All queftions performed in this manner may be quickly proved by reverfing the queftion: thus, If 54 men build a wall 360 feet long, 8 feet thick, and 10 feet high, in 60 days, how many men, *&c.*? The anfwer will be found to be 18.

CHAP. IV.

RULES OF PRACTICE.

These compendiums in proportion, which are diſtinguiſhed by the name of the *rules of practice*, becauſe they were invented occaſionally by merchants in expediting practice, comprehend a great part of the calculations uſed in counting-houſes, particularly when an unit is the firſt term in the proportion; and it is certain, when any proceſs is ſhort and unperplexed, one is leſs liable to errour than when he hath to do with heavy multiplications and diviſions. In order to aſſiſt the young practitioner, I have inſerted a table of aliquot parts, and given the method of inventing it.

Table of the aliquot parts of a pound.

s.	*d.*	*L.*	*s.*	*d.*	*L.*
10	0	$=\frac{1}{2}$	1	8	$=\frac{1}{12}$
6	8	$=\frac{1}{3}$	1	0	$=\frac{1}{20}$
5	0	$=\frac{1}{4}$	0	8	$=\frac{1}{30}$
4	0	$=\frac{1}{5}$	0	6	$=\frac{1}{40}$
3	4	$=\frac{1}{6}$	0	4	$=\frac{1}{60}$
2	6	$=\frac{1}{8}$	0	3	$=\frac{1}{80}$
2	0	$=\frac{1}{10}$	0	2	$=\frac{1}{120}$

This table, and any other of the ſame kind, may be effected by reducing the given parts or denominations to the loweſt name mentioned in them for a diviſor, and an unit of the integer to the ſame name for a dividend, the quotient is the fractional part in the loweſt terms; for inſtance 6 *s* 8 *d.*=80 pence, and 1 *l.*=240 pence; but $\frac{240}{80}=3$, conſequently 6 *s.* 8*d.*=$\frac{1}{3}$ of 1 *l.* and ſo of any other.

Caſe 1. When the price of the integer is any aliquot part of a pound contained in the table, the anſwer is found by one ſingle diviſion.

Caſe 1. At 10 *s.* what coſt 375 yards?

2) 375

L. 187 10 for $\frac{1}{2}$ of 1 *l.* remaining = 10 s.

Exam.

Exam. 2. At 6 *s.* 8 *d.* what cost 545 *lb.*?

3) 545

L. 181 13 4; here the remainder is 2×6 *s.* 8*d.*

Exam. 3. At 5 *s.* what cost 475 yards?

4) 475

L. 118 15; for $\frac{3}{4}$ = 15 *s*

Exam. 4. At 4 *s.* what cost 274 yards?

5) 274

L. 54 16; for $\frac{4}{5}$ = 16 *s.*

Exam. 5. At 2 *s.* 6 *d.* what cost 875 *lb.*?

8) 875

L. 109 7 6; for 3×2 *s.* 6 *d.* = 7 *s.* 6 *d.*

Exam. 6. At 2 *s.* what cost 7425 yards?

10) 7425

L. 742 10; for 5×2 *s.* = 10 s.

Exam. 7. At 1 *s.* 8 *d.* what cost 4167 yards?

12) 4167

L. 347 5; for 3×1 *s.* 8 *d.* = 5 *s.*

Exam. 8. At 6 *d.* what cost 4578 yards?

4|0) 457|8

L. 114 9; for 18 sixpences = 9 s.

Exam. 9. At 4 *d.* what cost 975 yards?

60) 975

L. 16 5; for 15 fourpences = 5 *s.*

Exam. 10. At 3 *d.* what cost 6875 yards?

8|0) 687|5

L. 85 18 9; for $\frac{75}{4}$ = 18 *s.* and the remaining 3×3=9 *d.*

Exam.

Exam. 11. At 2 *d.* what coſt 3745 ounces?

12|0) 374|5 •

L. 31 4 2; for $\frac{25}{6}$ = 4 *s.* 2 *d.*

Caſe 2. When the given price is any even part of a pound leſs than 2 *d* it will be an even part of ſome of thoſe mentioned in the table, and the anſwer is found by dividing and ſubdividing accordingly. The remainders may be valued as above, or carried on decimally.

Exam. 1. At 1 farthing what coſt 3600 yards?

12) 3600

80) 300 threepences; for 12 *qrs.* = 3 *d.*

L. 3 15 by the table.

Exam. 2. At $1\frac{1}{4}$ *d.* what 504 yards?

$1\frac{1}{4}$ *d.* = $\frac{1}{4}$ of 5 *d.* therefore 4) 504

6) 126 fivepences.

8) 21 half-crowns.

L. 2 12 6 by the table

Caſe 3. If the given price is no aliquot part of a pound, it will be compoſed of aliquot parts, which either may be divided for ſeverally as before, or, when the remaining part is any even part of the foregoing, divide the quotient for it, and the ſum of the quotients will give the anſwer. Sometimes we can, with great propriety, divide for the neareſt aliquot part above the given price, and when the value of the difference is ſubtracted from that quotient, the remainder will be the anſwer. A few examples will be the beſt illuſtration.

Exam. 1 At $2\frac{1}{2}$ *d.* what coſt 8754 *lbs.*?

120) 8754

$\frac{1}{2}$ *d.* is ^{1} of 2 *d.* 4) 72 19 at 2 *d.*

18 4 9 at $\frac{1}{2}$ *d.*

L. 91 3 9

By