

OBSERVATIONS

ON

REVERSIONARY PAYMENTS,

*&c. &c. &c.*

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VOL. II.

*Original Copy - 1827*  
~~Widows and Old Age~~  
O B S E R V A T I O N S

O N

REVERSIONARY PAYMENTS;

O N

SCHEMES for providing ANNUITIES for  
WIDOWS, and for Persons in OLD AGE;

O N

THE METHOD OF CALCULATING THE VALUES OF  
ASSURANCES ON LIVES;

A N D O N

THE NATIONAL DEBT.

A L S O,

ESSAYS on different Subjects in the Doctrine of LIFE-  
ANNUITIES and POLITICAL ARITHMETIC;

A Collection of NEW TABLES, and a POSTSCRIPT on the  
POPULATION of the KINGDOM.

By RICHARD PRICE, D.D. F.R.S.

TO WHICH ARE ADDED,

ALGEBRAICAL NOTES, the SOLUTION of several NEW PROBLEMS in the  
DOCTRINE OF ANNUITIES,

And a GENERAL INTRODUCTION.

By WILLIAM MORGAN, F.R.S.

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FIFTH EDITION.

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L O N D O N:

PRINTED FOR T. CADELL, IN THE STRAND.

M.DCC.XCII.



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## GENERAL INTRODUCTION:

CONTAINING

*An Account of the NEW TABLES  
of the Duration of Human Life at  
Chester, Warrington, the Kingdom  
of Sweden, Stockholm, London,  
&c. inserted in the following Col-  
lection of Tables.*

I HAVE, in the preceding Volume, p. 352, and in the Postscript to the Second Essay, p. 308, given an account of the improvement which was made in the former edition of this work, of the Table of Observations for *Northampton*, and of my reasons for wishing to discard the tables of the values of single and joint lives, founded on Mr. *De Moivre's* hypothesis, and substituting in their room the tables in the following collection.—I was farther enabled to improve this work, in that edition of it, by inserting tables

VOL. II. Part I.      A      formed

formed from a register of mortality established near twenty years ago at CHESTER.—This register was formed on the plan proposed in the preceding Volume, p. 367; and, therefore, is more comprehensive than any register of the same kind that has been hitherto established.

Chester is a healthy town, of moderate size, where the births had, for many years, a little exceeded the burials; and the register to which I refer had the particular advantage of being under the direction of Dr. *Haygarth*, its founder\* as well as conductor. As it gives an accurate account of the distempers of which all the inhabitants die in every season, and at every age, it contains much physical instruction; but my views lead me only to take notice of that part of it which gives the law according to which human life wastes in all its different stages, both among males and females.

A summary of this part of the register is given in the introduction to the CHESTER tables, in the following collection of tables.

\* This able and ingenious physician has given another proof of his zeal to render his professional character as useful as possible, by instituting a plan, which he has been carrying on at *Chester*, for preventing the spread of the small-pox by infection, and thus gradually exterminating it.

Concerning

Concerning these tables it is necessary I should make the following observations.

The table for females must be considered as particularly correct, because the number of females *born* and *buried* in *Chester* are very nearly equal.—On the contrary; the number of males *born* being about an 8th greater than the number *buried*, it follows that, in the table of decrements for males, the numbers of the living, and consequently the probabilities of living at every age, for at least 10 or 15 of the first years of life, must be given too low.

The expectation of a female at birth is, according to these tables, nearly  $33\frac{1}{4}$  years; and of a male  $28\frac{1}{8}$ . The number of females, therefore, at *Chester*, is to the number of males as  $33\frac{1}{4}$  to  $28\frac{1}{8}$ , or in the proportion of 8000 to 6771, which is the proportion discovered by a survey in 1774, when the females in this city were found to be 8016, and the males 6697 (*a*).

(*a*) It appeared from this survey (made with great care under the direction of Dr. *Haygarth*), that in 1774 there were in the ten parishes of *Chester*, including the suburbs,

Families.	Inhabitants.	Males.	Females.
3428.	14713	6697	8016
Married.	Widowers.	Widows.	Under 15.
4881	258	736	4486
Above 70.	Recovered of the small-pox in 1774.	Dead of the small-pox in 1774.	
625	1183	202	
Ill of the small-pox in Jan. 1775.		Not had the small-pox in Jan. 1775.	
19		1060	

These tables are farther confirmed by the proportion which they give of the number of males and females living under 15 to the whole number. This proportion is by the tables nearly that of 4486 to 14,888, and the *actual* numbers found by the enumeration in 1774, were 4486 and 14,713.

In like manner; the number of the living above 70 was, by the same survey, found to be 625; and the tables give this number nearly the same.

The expectation at birth, taking males and females together, is at *Chester*, by the tables, near 31; and therefore one in 31 ought to die annually. But the quotient arising from dividing the number of inhabitants (14,713) by 409 (the medium of annual burials from 1772 and 1781), will shew that in reality no more than one in 36 die annually.—The reason of this difference is, first, that the births exceed the burials; and that, consequently, a table which takes the burials for its *radix*, must give the expectations of life too low.—A second reason is, the emigration of males from *Chester*; in consequence of which, though more males than females are born, and though males are also more short-lived; yet fewer die at *Chester*, many dying in the army, navy, militia, &c. The effect of the first of these causes will be particularly exemplified hereafter, in the case of the kingdom of SWEDEN.

Observa-



Observations similar to these may be made on the tables in the following collection, formed from a register of mortality at *Warrington* in *Lancashire*, founded and conducted by the ingenious Dr. *Aikin*, (then the physician there, but now physician at *Yarmouth* in *Norfolk*) to whose kindness and communicativeness, as well as to Dr. *Haygarth's*, I have been much obliged. See Tables 39th and 40th.

The expectation of a male just born, at *WARRINGTON*, is, by these tables,  $20\frac{1}{2}$ ; of a female  $25\frac{1}{3}$ ; and of males and females taken together,  $23\frac{1}{10}$  nearly.

In the beginning of 1781 Dr. *Aikin* procured an enumeration of the houses and inhabitants in *Warrington* and its vicinity, consisting of the town of *Warrington*, the township as far as the lays are collected, *Poulton*, *Fearnhead*, and *Woolston*. The number of houses, including 74 uninhabited, was 2000; of inhabitants 9501, or  $4\frac{5}{1000}$  to a house.——The number of inhabitants divided by 302 (the annual average of burials for 9 years from 1773 to 1781) gives  $31\frac{2}{3}$ , but divided by 321, the annual average of burials for five years from 1777 to 1781 (which, in this case, seems the fairest average) gives  $29\frac{3}{5}$ . There is, therefore, in this town, a greater difference between the proportions dying annually, as determined by enumeration and by calculation from the register, than there is at *Chester*; and the



reason is, that the two causes just mentioned operate more here. The births in particular (the annual average of which for the 5 years just mentioned was 411) exceed the burials much more at *Warrington*; and therefore the burials are much more below the true average, and the probabilities of living exhibited by the table of decrements, much more below the true probabilities. Every one must be struck with the difference, in respect of longevity, which these tables exhibit between the inhabitants of *Warrington* and *Chester*; and it will appear more remarkable when it is considered, that about an 8th or 9th of the inhabitants included in the *Warrington* bills, are inhabitants of the country for a mile or two round *Warrington*.——*Chester* appears, indeed, to be an extraordinary exception to the hurtful effects of towns on the duration of life. The probabilities of living in it, though lower than in country parishes, are considerably higher than in any other city where observations have been made. I am not qualified to explain the causes which give it this distinction. A probable account of them has been given by Dr. *Haygarth*, in a paper printed at *Chester*, and containing *Observations on the Population and Diseases of Chester in 1774 (a)*.

It is farther observable, that these tables agree in exhibiting, in a striking light, the difference between the probabilities of living among males and females. But this difference  
will

will appear more evidently from the Tables for *Sweden*, of which I am next to give an account.

There are two sorts of *data* for forming tables of the probabilities of the duration of human life at every age. One is furnished by registers of mortality shewing the numbers dying at all ages. The other, by the proportions of deaths at all ages to the numbers living at those ages discovered by surveys or enumerations.—Tables formed from the former of these *data*, are correct only when there is no considerable fluctuation among the inhabitants of a place, and the births and burials are equal. When there are more removals *from* than *to* a place, and the births exceed the burials, as is almost always the case in country parishes and villages, tables so formed give the probabilities of living too low. When the contrary happens, as is generally the case in towns, they give the probabilities of living too high. But tables formed from the latter of these *data*, are subject to no errors. They must be correct, whatever the fluctuations are in a place, and how great soever the inequalities may be between the births and burials.—I know of no observations extant which furnish the means of forming *such* tables, except those published by the late Mr. *Wargentin* in the *Memoirs* of the Academy

of Sciences at STOCKHOLM, in 1776; an abstract of which I have given in an Essay at the end of this volume; and a continuation of which, from 1763 to 1776, Mr. *Wargentin* with the greatest goodness, communicated to me some time before his death. These observations are more curious than any that have been yet published, and leave us little to wish for on this subject, except that similar observations were made in other kingdoms under the direction of men equally able and ingenious with Mr. *Wargentin*.—It is from the result of all these observations taken together, that I have constructed Tables 42d, 43d, &c. in the following collection.

The tables for SWEDEN at large, compared with those for STOCKHOLM the capital, confirm, in a very striking manner, all that I have said in the 1st Essay, Vol. I. and other parts of this work, of the difference between the duration of life in great towns, and in the country.—They likewise furnish the most indisputable evidence for the shorter duration of the lives of males than of females; and it deserves particular notice, that the tables for *Sweden* at large differ, in this respect, but little from the tables formed from Dr. *Haygarth's* Observations at *Chester*. These observations give sufficient *data* for calculating, with some correctness, distinct tables of the values of lives among males and females, taken separately



parately and conjunctly; but I have preferred for this purpose the SWEDEN observations, because (as has been just observed) more correct *in their nature*; and because also (being made on the inhabitants of a whole kingdom for 21 years, and the enumeration which gives them their chief value having been repeated at *seven* different periods) they are much more to be depended on, and must give a juster valuation of lives among mankind at large, including all town and country inhabitants.

I have, for my own satisfaction, constructed tables for SWEDEN and STOCKHOLM from the former of the *data* I have mentioned (or the numbers dying every year in every stage of life, as given by Mr. *Wargentin*); but being afraid of crowding this volume too much with tables, I have not inserted them. The reader, if he chuses to make such tables for himself, is furnished with sufficient means of doing it in the first Essay at the end of this volume: and he will find, on comparing them with Tables 42d, &c. all the errors exemplified arising from the common methods of constructing tables of observation. In particular; he will find that though it appears from the tables for *Sweden* in the following collection, that the *true* expectation of a child just born in that kingdom, taking males and females together, is  $35\frac{1}{10}$ ; yet,

yet, a table formed from the numbers dying in every stage of life in the method described in the last Essay in the former Volume, will, (in consequence of the births exceeding the burials near a *third* of the burials) give this expectation only 25 years and three quarters; in connexion with which, he will also find, that in all the first stages of life it gives the probabilities of living much too low.

I must add, that such a table formed for *Stockholm*, and compared with the correct table (or Table 44th), will exhibit all the errors in the common tables for *London*, described in the Essay just referred to (a).

For

(a) In a table thus constructed (that is, on the supposition that all who die at *Stockholm* were born there) the numbers in the column of the living will be,

	Males.	Females.
at age 0	10,000	10,000
1	7,082	7,260
2	6,522	6,648
5	5,699	5,809
10	5,302	5,422
15	5,108	5,290
19	4,915	5,180
20	4,865	5,145
25	4,480	4,854
30	3,958	4,449
40	2,807	3,498
50	1,796	2,629
60	1,036	1,918
70	478	1,171
80	138	412
85	53	179
90	15	39

Totals, including the num- bers omitted — — }	242,100	285,367
		These



For instance. According to the correct table, the expectation of a male at birth in *Stockholm* is only  $14\frac{1}{4}$ ; and of a female 18. But in a table formed from the deaths only,

These totals divided by 10,000, and the quotients diminished by half unity, give 23.71 the expectation of a male at birth in *Stockholm*, and 28 the expectation of a female. The expectation, therefore, at birth of males and females conjointly, is, by this table, 25.85 (or  $25\frac{1}{2}$ ) which agrees almost exactly with the expectation at birth by a table formed in the same manner for *London*. See the former volume, p. 337; and Table 13th, in the following collection. — It deserves particular notice, that there is a like agreement between these tables at every age between birth and the utmost extent of life, as will sufficiently appear from the following comparison.

EXPECTATIONS of males and females conjointly, by a table of observations constructed from the bills, on the supposition that all who *die* were *born*

at STOCKHOLM.			at LONDON.
Age 10	$36\frac{4}{5}$	—	37
20	29	—	$29\frac{1}{3}$
30	$23\frac{1}{3}$	—	$24\frac{1}{10}$
40	$19\frac{1}{2}$	—	$19\frac{1}{2}$
50	$15\frac{4}{5}$	—	$15\frac{9}{10}$
60	$11\frac{3}{5}$	—	$11\frac{7}{10}$
70	$7\frac{3}{4}$	—	8

With these expectations compare the true expectations at *Stockholm*, deduced from Table 44th.

Age 10	$33\frac{2}{3}$
20	$26\frac{2}{5}$
30	$22\frac{1}{5}$
40	$17\frac{1}{3}$
50	$13\frac{1}{2}$
60	$9\frac{1}{2}$
70	$5\frac{4}{5}$

in

in the same manner with Table 13th for *London*, the former expectation comes out no less than  $23\frac{3}{4}$ , and the latter 28.—Again. The correct table makes 62 hundredths die annually of the males living between birth and five years of age; one in  $3\frac{1}{2}$  of the males living between 5 and 10; one in 65, between 8 and 16; one in 69, between 10 and 20; one in 40, between 20 and 30; one in  $29\frac{1}{2}$  between 30 and 40; one in 22, between 40 and 50; one in 16, between 50 and 60; one in 11, between 60 and 70; and one in 7 between 70 and 80. But the other table, would make only 43 hundredths die between birth and five years of age; one in 70, between 5 and 10; one in 120, between 8 and 16; one in 117, between 10 and 20; one in 30, between 20 and 30; one in 30, between 30 and 40; one in 23, between 40 and 50; one in  $18\frac{1}{2}$ , between 50 and 60; one in  $13\frac{1}{2}$ , between 60 and 70; and one in 9 between 70 and 80.

Of FEMALES, the *correct* table makes fifty-nine hundredths die annually of the living between birth and five years of age; one in  $3\frac{2}{5}$  of the living between 5 and 10; one in 90, between 8 and 16; one in 107, between 10 and 20; one in 68, between 20 and 30; one in 41, between 30 and 40; one in 30, between 40 and 50; one in  $24\frac{1}{2}$ , between 50 and 60; one in 15, between 60 and 70; and one in

$7\frac{1}{2}$ , between 70 and 80. But the other table would make only forty-two hundredths (a) of females die between birth and five years of age; one in 72, between 5 and 10; one in 180, between 8 and 16; one in 191, between 10 and 20; one in 70, between 20 and 30; one in 42 between 30 and 40; one in 35, between 40 and 50; one in 32, between 50 and 60; one in 21, between 60 and 70; and one in  $10\frac{1}{2}$ , between 70 and 80.

Farther. The correct table makes the number of inhabitants (taking males and females together) dying annually at *Stockholm*, to be nearly a 16th and a half. The other would make it a 26th part of the inhabitants; whereas, the number actually dying is nearly a 19th. — The former table gives this proportion too great, because, in consequence of giving the true order in which a given number born will die, it gives only the expectation at *birth* in *Stockholm* (b); and therefore, cannot include the expecta-

(a) Compare the last note with the correct Table, or Table 44th.

(b) And this too on the supposition, that the probabilities of living, at every particular age, among the inhabitants born in *Stockholm*, are the same that they are among the whole body of inhabitants at that age, consisting of *natives* and *foreigners*; whereas the truth is, that the mortality of great towns falls more on the newcomers, than on those who have been seasoned to it by having lived in it some time.

tion



tion at *entrance* of those who begin their residence in *Stockholm* after infancy.——The other must give this proportion too little, for the reasons explained in the preceding Volume, p. 337, &c.

In order to make a table constructed for *Stockholm* in the manner mentioned in the note p. 10, a just representation of the inhabitants, the numbers of the living (the decrements continuing the same) should be diminished at *every* age by a number equal to the annual average of new-comers at and after that age. After this diminution, the table will exhibit the same probabilities of life at every age with Table 44th; and if the sum of the remaining numbers is divided by the sum of the decrements, the quotient lessened by half unity will agreeably to the rule in p. 341 of the former Volume, give the number which I have called the *expectation at entrance*, and consequently the true proportion of inhabitants dying annually.——But there being no observations which make a subtraction of this kind at every particular age practicable; it is necessary to be satisfied with such a subtraction at the beginning of mature life as that directed in the preceding Volume, p. 339, &c. The *Stockholm* observations happily give a proof of the necessity and use of this subtraction, by informing us of the *true* probabilities of living at *Stockholm*, as exhibited in table 44th; and at the same time furnishing us with the means

means of constructing a table (like the 13th for *London*) of the probabilities of living in this town, on the supposition that all who die were born there. Let therefore, (since the excess of the burials above the births is nearly the same (*a*) in both cities) the correction be applied to this last table which has been applied to Table 13th for *London*. That is; let it be supposed that one quarter of all males and females who die at *Stockholm*, begin their residence in their 20th year; and in conformity to this supposition, let 2500, or a quarter of the *radix*, be subtracted from all the numbers living at every age before 20, preserving the decrements the same. The result will be a table which, when compared with Table 44th, will appear to exhibit more nearly the true probabilities of living in all the stages of life. By giving them, however, too high, it will appear that the correction (*b*) has not been sufficient; and that, consequently, the expectation at entrance will come out, though much *nearer*, yet still *above* the truth.

I have

(*a*) In nine years before 1764, the births at *Stockholm*, exclusive of the still-born, were 7,907, and the burials 11,344.

(*a*) After this correction, the numbers in the note p. 10, will be

Males



I have thought it worth while to make these observations, in order to shew, from an unquestionable fact, what judgment ought to be formed of the tables for *London* in the following collection; and it seems impossible not to be convinced by them that though these tables give the probabilities of the duration of life in *London* (and consequently the values of life-annuities) strikingly lower than in other situations, yet they do not give them low enough; and that, in particular, the number by which the annual deaths ought to be multiplied to find the number of inhabitants, and which Table 14th determines to be  $20\frac{3}{4}$ , is not probably so much as 20.

Males at age		Living	Females at age		Living
0	7,500		0	7,500	
1	4,582		1	4,760	
2	4,022		2	4,148	
5	3,199		5	3,309	
10	2,802		10	2,909	
15	2,608		15	2,790	
19	2,415		19	2,680	
20	4,865		20	5,145	
25	4,480		25	4,854	
&c.	&c.		&c.	&c.	
Totals (including the numbers omitted) after deducting 5000 - -		187,100			230,367

Therefore the expectation at entrance of *males* is  $18\frac{71}{100}$ , of *females* is  $23\frac{93}{100}$ ; of both conjointly  $20\frac{37}{100}$ ; but these expectations are really (as appears from the observations) 16.80—20.93, and 18.89 respectively.

In

In short. From the agreement in almost every particular between the *London* and *Stockholm* bills, and between two tables formed on the same principles from the deaths *only* in both towns, it seems a necessary conclusion that, since one of these tables (even after the correction explained in the fourth essay) gives certainly too favourable a representation of human life, the other must do the same.

The following fact has some tendency to confirm this conclusion.

It appears from the midwifery reports of the general *Westminster* Infirmary, that of 1618 married men, and 1618 married women, examined by Dr. *Bland* the physician to this Infirmary, only 329 of the *men* and 495 of the *women*, had been born in *London* (*a*); that is, a *fifth* of the men, and somewhat more than a *quarter* of the women. But the correction I have been considering implies, that a number equal to *half* of all turned of 20 in *London*, are natives of *London*; and therefore, if we may judge at all from this fact, it must be an insufficient correction.

(a) See Dr. *Bland's* account in the Philosophical Transactions, Vol. 71st, Part II. p. 370.—Of the whole number (3236) four-sevenths, or 1870, were born in the different counties of *England* and *Wales*; 209 in *Scotland*; 280 in *Ireland*; and 53 were foreigners.

T A B L E I.

The present Value of 1*l.* to be received at the End of any Number of Years, not exceeding 100; discounting at the Rates of 3, 3½, 4, 4½, 5, and 6 *per cent.* Compound Interest.

Yrs.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
1	,970874	,966184	,961538	,956938	,952381	,942396
2	,942596	,933511	,924556	,915730	,907029	,889996
3	,915142	,901943	,888996	,876297	,863838	,839619
4	,888487	,871442	,854804	,838561	,822702	,792094
5	,862609	,841973	,821927	,802451	,783526	,747258
6	,837484	,813501	,790315	,767896	,746215	,704961
7	,813092	,785991	,759918	,734828	,710681	,665057
8	,789409	,759412	,730690	,703185	,676839	,627412
9	,766417	,733731	,702587	,672904	,644609	,591898
10	,744094	,708919	,675564	,643928	,613913	,558395
11	,722421	,684946	,649581	,616199	,584679	,526788
12	,701380	,661783	,624597	,589664	,556837	,496969
13	,680951	,639404	,600574	,564172	,530321	,468839
14	,661118	,617782	,577475	,539973	,505068	,442301
15	,641862	,596891	,555265	,516720	,481017	,417265
16	,623167	,576706	,533908	,494469	,458112	,393646
17	,605016	,557204	,513373	,473176	,436297	,371364
18	,587395	,538361	,493628	,452800	,415521	,350344
19	,570286	,520156	,474642	,433302	,395734	,330513
20	,553676	,502566	,456387	,414643	,376889	,311805
21	,537549	,485571	,438834	,396787	,358942	,294155
22	,521893	,469151	,421955	,379701	,341850	,277505
23	,506692	,453286	,405726	,363350	,325571	,261797
24	,491934	,437957	,390121	,347703	,310068	,246979
25	,477606	,423147	,375117	,332731	,295303	,232999
26	,463695	,408838	,360689	,318402	,281241	,219810
27	,450189	,395012	,346817	,304691	,267848	,207368
28	,437077	,381654	,333477	,291571	,255094	,195630
29	,424346	,368748	,320651	,279015	,242946	,184557
30	,411987	,356278	,308319	,267000	,231377	,174110
31	,399987	,344230	,296460	,255502	,220359	,164255
32	,388337	,332590	,285058	,244500	,209866	,154957



# T A B L E S.

19

## T A B L E I. continued.

Yrs.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
33	,377026	,321343	,274094	,233971	,199873	,146186
34	,366045	,310476	,263552	,223896	,190355	,137912
35	,355383	,299977	,253415	,214254	,181290	,130105
36	,345032	,289833	,243669	,205028	,172657	,122741
37	,334983	,280032	,234297	,196199	,164436	,115793
38	,325226	,270562	,225285	,187750	,156605	,109239
39	,315754	,261413	,216621	,179665	,149148	,103056
40	,306557	,252572	,208289	,171929	,142046	,097222
41	,297628	,244031	,200278	,164525	,135282	,091719
42	,288959	,235779	,192575	,157440	,128840	,086527
43	,280543	,227806	,185168	,150661	,122704	,081630
44	,272372	,220102	,178046	,144173	,116861	,077009
45	,264439	,212659	,171198	,137964	,111297	,072650
46	,256737	,205468	,164614	,132023	,105997	,068538
47	,249259	,198520	,158283	,126338	,100949	,064658
48	,241999	,191806	,152195	,120898	,096142	,060998
49	,234950	,185320	,146341	,115692	,091564	,057546
50	,228107	,179053	,140713	,110710	,087204	,054288
51	,221463	,172998	,135301	,105942	,083051	,051215
52	,215013	,167148	,130097	,101380	,079096	,048316
53	,208750	,161496	,125093	,097014	,075330	,045582
54	,202670	,156035	,120282	,092837	,071743	,043001
55	,196767	,150758	,115656	,088839	,068326	,040567
56	,191036	,145660	,111207	,085013	,065073	,038271
57	,185472	,140734	,106930	,081353	,061974	,036105
58	,180070	,135975	,102817	,077849	,059023	,034061
59	,174825	,131377	,098863	,074497	,056212	,032133
60	,169733	,126934	,095060	,071289	,053536	,030314
61	,164789	,122642	,091404	,068219	,050986	,028598
62	,159990	,118495	,087889	,065281	,048558	,026980
63	,155330	,114487	,084508	,062470	,046246	,025453
64	,150806	,110616	,081258	,059780	,044044	,024012
65	,146413	,106875	,078133	,057206	,041946	,022653
66	,142149	,103261	,075128	,054743	,039949	,021370
67	,138009	,099769	,072238	,052385	,038047	,020161
68	,133989	,096395	,069460	,050129	,036235	,019020
69	,130086	,093136	,066788	,047971	,034509	,017943
70	,126297	,089986	,064219	,045905	,032866	,016927

T A B L E I. continued.

Yrs.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
71	,122619	,086943	,061749	,043928	,031301	,015969
72	,119047	,084003	,059374	,042037	,029811	,015065
73	,115580	,081162	,057091	,040226	,028391	,014213
74	,112214	,078428	,054895	,038494	,027039	,013408
75	,108945	,075766	,052784	,036836	,025752	,012649
76	,105772	,073204	,050754	,035250	,024525	,011933
77	,102691	,070728	,048801	,033732	,023357	,011258
78	,099700	,068336	,046924	,032282	,022245	,010620
79	,096796	,066026	,045120	,030890	,021186	,010019
80	,093977	,063793	,043384	,029559	,020177	,009452
81	,091240	,061636	,041716	,028287	,019216	,008917
82	,088582	,059551	,040111	,027069	,018301	,008412
83	,086002	,057538	,038569	,025903	,017430	,007936
84	,083497	,055592	,037085	,024787	,016600	,007487
85	,081065	,053712	,035659	,023720	,015809	,007063
86	,078704	,051896	,034287	,022699	,015056	,006663
87	,076412	,050141	,032969	,021721	,014339	,006286
88	,074186	,048445	,031701	,020786	,013657	,005930
89	,072026	,046807	,030481	,019891	,013006	,005595
90	,069928	,045224	,029309	,019034	,012387	,005278
91	,067891	,043695	,028182	,018215	,011797	,004979
92	,065914	,042217	,027098	,017430	,011235	,004697
93	,063994	,040789	,026056	,016680	,010700	,004432
94	,062130	,039410	,025053	,015961	,010191	,004181
95	,060320	,038077	,024090	,015274	,009705	,003944
96	,058563	,036790	,023163	,014616	,009243	,003721
97	,056858	,035546	,022272	,013987	,008803	,003510
98	,055202	,034344	,021416	,013385	,008384	,003312
99	,053594	,033182	,020592	,012808	,007985	,003124
100	,052033	,032060	,019800	,012257	,007604	,002947



# T A B L E S.

21

## T A B L E II.

The present Value of an Annuity of One Pound for any Number of Years not exceeding 100, at the several Rates of 3, 3½, 4, 5, and 6l. per cent.

Year	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
1	<del>0.708</del>	.9662	.9615	.9523	.9433
2	1.9134	1.8997	1.8861	1.8594	1.8333
3	2.8286	2.8016	2.7751	2.7232	2.6730
4	3.7170	3.6731	3.6299	3.5459	3.4651
5	4.5797	4.5151	4.4518	4.3294	4.2123
6	5.4171	5.3286	5.2421	5.0756	4.9173
7	6.2302	6.1145	6.0020	5.7863	5.5823
8	7.0196	6.8740	6.7327	6.4632	6.2097
9	7.7861	7.6077	7.4353	7.1078	6.8016
10	8.5302	8.3166	8.1109	7.7217	7.3600
11	9.2526	9.0015	8.7605	8.3064	7.8868
12	9.9540	9.6633	9.3850	8.8632	8.3838
13	10.6349	10.3027	9.9856	9.3935	8.8526
14	11.2960	10.9205	10.5631	9.8986	9.2949
15	11.9379	11.5174	11.1184	10.3796	9.7122
16	12.5611	12.0941	11.6523	10.8377	10.1058
17	13.1661	12.6513	12.1656	11.2740	10.4772
18	13.7535	13.1897	12.6593	11.6895	10.8276
19	14.3238	13.7098	13.1339	12.0853	11.1581
20	14.8774	14.2124	13.5903	12.4622	11.4699
21	15.4150	14.6980	14.0291	12.8211	11.7640
22	15.9369	15.1671	14.4511	13.1630	12.0415
23	16.4436	15.6204	14.8568	13.4885	12.3033
24	16.9355	16.0584	15.2469	13.7986	12.5503
25	17.4131	16.4815	15.6220	14.0939	12.7833

T A B L E II. continued.

Year	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
26	17.8768	16.8904	15.9827	14.3751	13.0031
27	18.3270	17.2854	16.3295	14.6430	13.2105
28	18.7641	17.6670	16.6630	14.8981	13.4061
29	19.1884	18.0358	16.9837	15.1410	13.5907
30	19.6004	18.3920	17.2920	15.3724	13.7648
31	20.0004	18.7363	17.5884	15.5928	13.9290
32	20.3887	19.0689	17.8735	15.8026	14.0840
33	20.7657	19.3902	18.1476	16.0025	14.2302
34	21.1318	19.7007	18.4111	16.1929	14.3681
35	21.4872	20.0007	18.6646	16.3741	14.4982
36	21.8322	20.2905	18.9082	16.5468	14.6209
37	22.1672	20.5705	19.1425	16.7112	14.7367
38	22.4924	20.8411	19.3678	16.8678	14.8460
39	22.8082	21.1025	19.5844	17.0170	14.9490
40	23.1147	21.3551	19.7927	17.1590	15.0462
41	23.4124	21.5991	19.9930	17.2943	15.1380
42	23.7013	21.8349	20.1856	17.4232	15.2245
43	23.9819	22.0627	20.3707	17.5459	15.3061
44	24.2542	22.2828	20.5488	17.6627	15.3831
45	24.5187	22.4955	20.7200	17.7740	15.4558
46	24.7754	22.7009	20.8846	17.8800	15.5243
47	25.0247	22.8994	21.0429	17.9810	15.5890
48	25.2667	23.0912	21.1951	18.0771	15.6500
49	25.5016	23.2766	21.3414	18.1687	15.7075
50	25.7297	23.4556	21.4821	18.2559	15.7618
51	25.9512	23.6286	21.6174	18.3389	15.8130
52	26.1662	23.7958	21.7475	18.4180	15.8613
53	26.3749	23.9573	21.8726	18.4934	15.9069
54	26.5776	24.1133	21.9929	18.5651	15.9499
55	26.7744	24.2641	22.1086	18.6334	15.9905

T A B L E II. continued.

Year	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
56	26.9654	24.4097	22.2198	18.6985	16.0288
57	27.1509	24.5504	22.3267	18.7605	16.0649
58	27.3310	24.6864	22.4295	18.8195	16.0989
59	27.5058	24.8178	22.5284	18.8757	16.1311
60	27.6755	24.9447	22.6234	18.9292	16.1614
61	27.8403	25.0674	22.7148	18.9802	16.1900
62	28.0003	25.1859	22.8027	19.0288	16.2170
63	28.1556	25.3004	22.8872	19.0750	16.2424
64	28.3064	25.4110	22.9685	19.1191	16.2664
65	28.4528	25.5178	23.0466	19.1610	16.2891
66	28.5950	25.6211	23.1218	19.2010	16.3104
67	28.7330	25.7209	23.1940	19.2390	16.3306
68	28.8670	25.8173	23.2635	19.2753	16.3496
69	28.9971	25.9104	23.3302	19.3098	16.3676
70	29.1234	26.0004	23.3945	19.3426	16.3845
71	29.2460	26.0873	23.4562	19.3739	16.4005
72	29.3650	26.1713	23.5156	19.4037	16.4155
73	29.4806	26.2525	23.5727	19.4321	16.4297
74	29.5928	26.3309	23.6276	19.4592	16.4431
75	29.7018	26.4067	23.6804	19.4849	16.4558
76	29.8076	26.4799	23.7311	19.5094	16.4677
77	29.9102	26.5506	23.7799	19.5328	16.4790
78	30.0099	26.6190	23.8268	19.5550	16.4896
79	30.1067	26.6850	23.8720	19.5762	16.4996
80	30.2007	26.7488	23.9153	19.5964	16.5091
81	30.2920	26.8104	23.9571	19.6156	16.5180
82	30.3805	26.8700	23.9972	19.6339	16.5264
83	30.4665	26.9275	24.0357	19.6514	16.5343
84	30.5500	26.9831	24.0728	19.6680	16.5418
85	30.6311	27.0368	24.1085	19.6838	16.5489

T A B L E II. continued.

Year	3 per Ct.	$3\frac{1}{2}$ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
86	30.7098	27.0887	24.1428	19.6988	16.5556
87	30.7862	27.1388	24.1757	19.7132	16.5618
88	30.8604	27.1873	24.2074	19.7268	16.5678
89	30.9324	27.2341	24.2379	19.7398	16.5734
90	31.0024	27.2793	24.2672	19.7522	16.5787
91	31.0703	27.3230	24.2954	19.7640	16.5836
92	31.1362	27.3652	24.3225	19.7752	16.5883
93	31.2002	27.4060	24.3486	19.7859	16.5928
94	31.2623	27.4454	24.3736	19.7961	16.5969
95	31.3226	27.4835	24.3977	19.8058	16.6009
96	31.3812	27.5203	24.4209	19.8151	16.6046
97	31.4380	27.5558	24.4431	19.8239	16.6081
98	31.4932	27.5902	24.4646	19.8323	16.6114
99	31.5468	27.6234	24.4852	19.8403	16.6145
100	31.5989	27.6554	24.5050	19.8479	16.6175
Perpetuity.	33.3333	28.5714	25.0000	20.0000	16.6666



# T A B L E S.

25

## T A B L E III.

Shewing the Sum to which 1*l*. Principal will increase at Compound Interest in any Number of Years not exceeding a hundred.

Yrs.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
1	1.030,000	1.035,000	1.040,000	1.050,000	1.060,000
2	1.060,900	1.071,225	1.081,600	1.102,500	1.123,600
3	1.092,727	1.108,717	1.124,864	1.157,625	1.191,016
4	1.125,508	1.147,523	1.169,858	1.215,506	1.262,476
5	1.159,274	1.187,686	1.216,652	1.276,281	1.338,225
6	1.194,052	1.229,255	1.265,319	1.340,095	1.418,519
7	1.229,873	1.272,279	1.315,931	1.407,100	1.503,630
8	1.266,770	1.316,809	1.368,569	1.477,455	1.593,848
9	1.304,773	1.362,897	1.423,311	1.551,328	1.689,478
10	1.343,916	1.410,598	1.480,244	1.628,894	1.790,847
11	1.384,233	1.459,969	1.539,454	1.710,339	1.898,298
12	1.425,760	1.511,068	1.601,032	1.795,856	2.012,196
13	1.468,533	1.563,956	1.665,073	1.885,649	2.132,928
14	1.512,589	1.618,694	1.731,676	1.979,931	2.260,903
15	1.557,967	1.675,348	1.800,943	2.078,928	2.396,558
16	1.604,706	1.733,986	1.872,981	2.182,874	2.540,351
17	1.652,847	1.794,675	1.947,900	2.292,018	2.692,772
18	1.702,433	1.857,489	2.025,816	2.406,619	2.854,339
19	1.753,506	1.922,501	2.106,849	2.526,950	3.025,599
20	1.806,111	1.989,788	2.191,123	2.653,297	3.207,135
21	1.860,294	2.059,431	2.278,768	2.785,962	3.399,563
22	1.916,103	2.131,511	2.369,918	2.925,260	3.603,537
23	1.973,586	2.206,114	2.464,715	3.071,523	3.819,749
24	2.032,794	2.283,328	2.563,304	3.225,099	4.048,934
25	2.093,777	2.363,244	2.665,836	3.386,354	4.291,870
26	2.156,591	2.445,958	2.772,469	3.555,672	4.549,382
27	2.221,289	2.531,567	2.883,368	3.733,456	4.822,345
28	2.287,927	2.620,171	2.998,703	3.920,129	5.111,686
29	2.356,565	2.711,877	3.118,651	4.116,135	5.418,387
30	2.427,262	2.806,793	3.243,397	4.321,942	5.743,491
31	2.500,080	2.905,031	3.373,133	4.538,039	6.088,100
32	2.575,082	3.006,707	3.508,058	4.764,941	6.453,386
33	2.652,335	3.111,942	3.648,381	5.003,188	6.840,589
34	2.731,905	3.220,860	3.794,316	5.253,347	7.251,025
35	2.813,862	3.333,590	3.946,088	5.516,015	7.686,086
36	2.898,278	3.450,266	4.103,932	5.791,816	8.147,252

T A B L E III. continued.

Yrs	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
37	2.985,226	3.571,025	4.268,089	6.081,406	8.636,087
38	3.074,783	3.696,011	4.438,813	6.385,477	9.154,252
39	3.167,026	3.825,371	4.616,365	6.704,751	9.703,507
40	3.262,037	3.959,259	4.801,020	7.039,988	10.285,717
41	3.359,898	4.097,833	4.993,061	7.391,988	10.902,861
42	3.460,695	4.241,257	5.192,783	7.761,587	11.557,032
43	3.564,516	4.389,702	5.400,495	8.149,666	12.250,454
44	3.671,452	4.543,341	5.616,515	8.557,150	12.985,481
45	3.781,595	4.702,358	5.841,175	8.985,007	13.764,610
46	3.895,043	4.866,941	6.074,822	9.434,258	14.590,487
47	4.011,895	5.037,284	6.317,815	9.905,971	15.465,916
48	4.132,251	5.213,588	6.570,528	10.401,269	16.393,871
49	4.256,219	5.396,064	6.833,349	10.921,333	17.377,504
50	4.383,906	5.584,926	7.106,683	11.467,399	18.420,154
51	4.515,423	5.780,399	7.390,950	12.040,769	19.525,363
52	4.650,885	5.982,713	7.686,588	12.642,808	20.696,885
53	4.790,412	6.192,108	7.994,052	13.274,948	21.938,698
54	4.934,124	6.408,832	8.313,814	13.938,696	23.255,020
55	5.082,148	6.633,141	8.646,366	14.635,630	24.650,321
56	5.234,613	6.865,301	8.992,221	15.367,412	26.129,340
57	5.391,651	7.105,586	9.351,910	16.135,783	27.697,101
58	5.553,400	7.354,282	9.725,986	16.942,572	29.358,927
59	5.720,003	7.611,682	10.115,026	17.789,700	31.120,463
60	5.891,603	7.878,090	10.519,627	18.679,185	32.987,690
61	6.068,351	8.153,824	10.940,412	19.613,145	34.966,952
62	6.250,401	8.439,207	11.378,029	20.593,802	37.064,969
63	6.437,913	8.734,580	11.833,150	21.623,492	39.288,867
64	6.631,051	9.040,290	12.306,476	22.704,667	41.646,199
65	6.829,982	9.356,700	12.798,735	23.839,900	44.144,971
66	7.034,882	9.684,185	13.310,684	25.031,895	46.793,669
67	7.245,928	10.023,131	13.843,112	26.283,490	49.601,290
68	7.463,306	10.373,941	14.396,836	27.597,664	52.577,367
69	7.687,205	10.737,029	14.972,709	28.977,548	55.732,009
70	7.917,821	11.112,825	15.571,618	30.426,425	59.075,930
71	8.155,356	11.501,774	16.194,483	31.947,746	62.620,485
72	8.400,017	11.904,336	16.842,262	33.545,134	66.377,715
73	8.652,017	12.320,988	17.515,952	35.222,390	70.360,378
74	8.911,578	12.752,222	18.216,591	36.983,510	74.582,000
75	9.178,925	13.198,550	18.945,254	38.832,685	79.056,920
76	9.454,293	13.660,499	19.703,064	40.774,310	83.809,336
77	9.737,922	14.138,617	20.491,187	42.813,036	88.828,356

# T A B L E S.

27

## T A B L E III. continued.

rs.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
78	10.030,059	14.633,468	21.310,834	44.953,688	94.158,057
79	10.330,961	15.145,640	22.163,268	47.201,372	99.807,541
80	10.640,890	15.675,737	23.049,799	49.561,441	105.795,993
81	10.960,117	16.224,388	23.971,791	52.039,513	112.143,753
82	11.288,920	16.792,241	24.930,662	54.641,488	118.872,378
83	11.627,588	17.379,970	25.927,889	57.373,563	126.004,720
84	11.976,416	17.988,269	26.965,004	60.242,241	133.565,004
85	12.335,708	18.617,858	28.043,604	63.254,353	141.578,904
86	12.705,779	19.269,483	29.165,349	66.417,071	150.073,638
87	13.086,953	19.943,915	30.331,963	69.737,924	159.078,057
88	13.479,561	20.641,952	31.545,241	73.224,820	168.622,740
89	13.883,948	21.364,421	32.807,051	76.886,061	178.740,104
90	14.300,467	22.112,175	34.119,333	80.730,365	189.464,511
91	14.729,481	22.886,102	35.484,106	84.766,883	200.832,381
92	15.171,365	23.687,115	36.903,470	89.005,227	212.882,324
93	15.626,506	24.516,164	38.379,609	93.455,488	225.655,264
94	16.095,301	25.374,230	39.914,794	98.128,263	239.194,580
95	16.578,160	26.262,328	41.511,385	103.034,676	253.546,254
96	17.075,505	27.181,510	43.171,841	108.186,410	268.759,030
97	17.587,770	28.132,802	44.898,715	113.595,730	284.884,572
98	18.115,403	29.117,513	46.694,663	119.275,517	301.977,646
99	18.658,866	30.136,626	48.562,450	125.239,293	320.096,305
100	19.218,631	31.191,407	50.504,948	131.501,257	339.302,083

T A B L E



## T A B L E IV.

Shewing the Sum to which 1*l.* *per ann.* will increase at Compound Interest in any Number of Years not exceeding a hundred.

Yrs.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
1	1.000,000	1.000,000	1.000,000	1.000,000	1.000,000
2	2.030,000	2.035,000	2.040,000	2.050,000	2.060,000
3	3.090,900	3.106,225	3.121,600	3.152,500	3.183,600
4	4.183,627	4.214,942	4.246,464	4.310,125	4.374,616
5	5.309,135	5.362,465	5.416,322	5.525,631	5.637,092
6	6.468,409	6.550,152	6.632,975	6.801,912	6.975,318
7	7.662,462	7.779,407	7.898,294	8.142,008	8.393,837
8	8.892,336	9.051,686	9.214,226	9.549,108	9.897,467
9	10.159,106	10.368,495	10.582,795	11.026,564	11.491,315
10	11.463,879	11.731,393	12.006,107	12.577,892	13.180,794
11	12.807,795	13.141,991	13.486,351	14.206,787	14.971,642
12	14.192,029	14.601,961	15.025,805	15.917,126	16.869,941
13	15.617,790	16.113,030	16.626,837	17.712,982	18.882,137
14	17.086,324	17.676,986	18.291,911	19.598,631	21.015,065
15	18.598,913	19.295,680	20.023,587	21.578,563	23.275,969
16	20.156,881	20.971,029	21.824,531	23.657,491	25.672,528
17	21.761,587	22.705,015	23.697,512	25.840,366	28.212,879
18	23.414,435	24.499,691	25.645,412	28.132,384	30.905,652
19	25.116,868	26.357,180	27.671,229	30.539,003	33.759,991
20	26.870,374	28.279,681	29.778,078	33.065,954	36.785,591
21	28.676,485	30.269,470	31.969,201	35.719,251	39.992,726
22	30.536,780	32.328,902	34.247,969	38.505,214	43.392,290
23	32.452,883	34.460,413	36.617,888	41.430,475	46.995,827
24	34.426,470	36.666,528	39.082,604	44.501,998	50.815,577
25	36.459,264	38.949,856	41.645,908	47.727,098	54.864,512
26	38.553,042	41.313,101	44.311,744	51.113,453	59.156,382
27	40.709,633	43.759,060	47.084,214	54.669,126	63.705,765
28	42.930,922	46.290,627	49.967,582	58.402,582	68.528,111
29	45.218,850	48.910,799	52.966,286	62.322,711	73.639,798
30	47.575,415	51.622,677	56.084,937	66.438,847	79.058,186
31	50.002,678	54.429,470	59.328,335	70.760,789	84.801,677
32	52.502,758	57.334,502	62.701,468	75.298,829	90.889,778
33	55.077,841	60.341,210	66.209,527	80.063,770	97.343,164
34	57.730,176	63.453,152	69.857,908	85.066,959	104.183,754
35	60.462,081	66.674,012	73.652,224	90.320,307	111.434,779
36	63.275,944	70.007,603	77.598,313	95.836,322	119.120,866



# T A B L E S.

29

## T A B L E IV. continued.

Yrs.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
37	66.174,222	73.457,869	81.702,246	101.628,138	127.268,118
38	69.159,449	77.028,894	85.970,336	107.709,545	135.904,205
39	72.234,232	80.724,906	90.409,149	114.095,023	145.058,458
40	75.401,259	84.550,277	95.025,515	120.799,774	154.761,965
41	78.663,297	88.509,537	99.826,536	127.839,762	165.047,683
42	82.023,196	92.607,371	104.819,597	135.231,751	175.950,544
43	85.483,892	96.848,629	110.012,381	142.993,338	187.507,577
44	89.048,409	101.238,331	115.412,876	151.143,005	199.758,031
45	92.719,861	105.781,672	121.029,392	159.700,155	212.743,513
46	96.501,457	110.484,031	126.870,567	168.685,163	226.508,124
47	100.396,500	115.330,972	132.945,390	178.119,421	241.098,612
48	104.408,395	120.388,256	139.263,206	188.025,392	256.564,528
49	108.540,647	125.601,845	145.833,734	198.426,662	272.958,400
50	112.796,867	130.997,910	152.667,083	209.347,995	290.335,904
51	117.180,773	136.582,837	159.773,767	220.815,395	308.756,058
52	121.696,196	142.363,236	167.164,717	232.856,165	328.281,422
53	126.347,082	148.345,949	174.851,306	245.498,973	348.978,307
54	131.137,494	154.538,057	182.845,358	258.773,922	370.917,006
55	136.071,619	160.946,889	191.159,173	272.712,618	394.172,026
56	141.153,768	167.580,039	199.805,539	287.348,249	418.822,348
57	146.388,381	174.445,332	208.797,761	302.715,661	444.951,689
58	151.780,032	181.550,918	218.149,672	318.851,444	472.648,790
59	157.333,433	188.905,200	227.875,658	335.794,017	502.007,717
60	163.053,436	196.516,882	237.990,685	353.583,717	533.128,180
61	168.945,039	204.394,973	248.510,312	372.262,903	566.115,871
62	175.013,391	212.548,797	259.450,725	391.876,048	601.082,824
63	181.263,792	220.988,005	270.828,754	412.469,851	638.147,793
64	187.701,706	229.722,585	282.661,904	434.093,343	677.436,661
65	194.332,757	238.762,876	294.968,380	456.798,011	719.082,860
66	201.162,740	248.119,577	307.767,115	480.637,911	763.227,832
67	208.197,622	257.803,762	321.077,800	505.669,807	810.021,502
68	215.443,551	267.826,894	334.920,912	531.953,297	859.622,792
69	222.906,858	278.200,835	349.317,748	559.550,962	912.200,160
70	230.594,063	288.937,864	364.290,458	588.528,510	967.932,169
71	238.511,885	300.050,689	379.862,077	618.954,936	1027.008,099
72	246.667,242	311.552,463	396.056,560	650.902,683	1089.628,585
73	255.067,259	323.456,800	412.898,822	684.447,817	1156.006,300
74	263.719,277	335.777,788	430.414,775	719.670,208	1226.366,679
75	272.630,855	348.530,010	448.631,366	756.653,718	1300.948,679
76	281.809,781	361.728,561	467.576,621	795.486,404	1380.005,600
77	291.264,074	375.389,060	487.279,686	836.260,724	1463.805,936

T A B L E IV. continued.

Yrs.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
78	301.001,996	389.527,677	507.770,873	879.073,760	1552.634,292
79	311.032,056	404.161,146	529.081,708	924.027,448	1646.792,350
80	321.363,018	419.306,786	551.244,976	971.228,821	1746.599,801
81	332.003,909	434.982,524	574.294,775	1020.790,262	1852.395,884
82	342.964,026	451.206,912	598.266,566	1072.829,775	1964.539,637
83	354.252,947	467.999,154	623.197,229	1127.471,264	2083.412,016
84	365.880,535	485.379,125	649.125,118	1184.841,827	2209.416,737
85	377.856,951	503.367,394	676.090,123	1245.087,068	2342.981,741
86	390.192,660	521.985,253	704.133,728	1308.341,422	2484.560,645
87	402.898,440	541.254,737	733.299,077	1374.758,493	2634.634,284
88	415.985,393	561.198,652	763.631,040	1444.496,418	2793.712,341
89	429.464,955	581.840,605	795.176,282	1517.721,238	2962.335,082
90	443.348,903	603.205,027	827.983,333	1594.607,300	3141.075,187
91	457.649,370	625.317,202	862.102,667	1675.337,665	3330.539,657
92	472.378,851	648.203,305	897.586,773	1760.104,549	3531.372,080
93	487.550,217	671.890,420	934.490,244	1849.109,776	3744.254,465
94	503.176,723	696.406,585	972.869,854	1942.565,265	3969.909,669
95	519.272,025	721.780,815	1012.784,648	2040.693,528	4209.104,249
96	535.850,186	748.043,144	1054.296,034	2143.728,205	4462.650,504
97	552.925,692	775.224,654	1097.467,875	2251.914,615	4731.409,534
98	570.513,462	803.357,517	1142.366,590	2365.510,346	5016.294,106
99	588.628,866	832.475,030	1189.061,254	2484.785,863	5318.271,755
100	607.287,732	862.611,656	1237.623,704	2610.025,156	5638.368,058

## CONSTRUCTION of the four preceding Tables.

THESE Tables may be met with in most of the books which treat of compound interest and annuities; but there has been, in this work, so much occasion for referring to them, that it was necessary to save the reader the trouble of turning to other books for them.

The 1st, 2d, 3d, &c. numbers in the first table, are the quotients of unity divided by the 1st, 2d, 3d, &c. powers respectively of 1*l.* increased by its interest for a year; that is,  $\frac{1}{r}$ ,  $\frac{1}{r^2}$ ,  $\frac{1}{r^3}$ , &c. *r* signifying 1*l.* increased by its interest for a year; or 1.03, 1.035, 1.04, 1.045, 1.05, 1.06, as the interest is 3,  $3\frac{1}{2}$ , 4,  $4\frac{1}{2}$ , 5, or 6 *per cent.*

The 2d, 3d, 4th, &c. numbers in the second table, are the *sums* of the 1st and 2d; of the 1st, 2d, and 3d; of the 1st, 2d, 3d, and 4th, &c. &c. numbers respectively in the first Table.

The numbers in the 3d Table are the powers of 1*l.* increased by its interest for a year; that is, *r*, *r*<sup>2</sup>, *r*<sup>3</sup>, &c.

The 2d, 3d, 4th, &c. numbers in the 4th Table, are the *sums* of the 1st and 2d; of the 1st, 2d, and 3d; of the 1st, 2d, 3d, and 4th, &c. numbers in the 3d Table, with unity added.

N. B. At the close of this collection there is a continuation of these Tables for the interests of 2,  $2\frac{1}{2}$ , 7, 8, 9, and 10 *per cent.*



USES of the preceding Tables.

Question I. To what *sum* or *annuity* will any given *sum* or *annuity* increase in a given number of years, at a given rate of compound interest?

Ans. Multiply the number in Table 3d under the given rate and opposite to the given number of years, by the given *sum* or *annuity*, and the product will be the answer.

EXAMPLE. The product of 40*l.* into 2.0258 (that is, 1.81.032) is the sum to which 40*l.* principal will increase in 18 years, reckoning interest at 4 *per cent.*; and the same product is likewise the *annuity* to which an *annuity* of 40*l.* will increase in the same time, reckoning the same interest.

Quest. II. To what sum will a given *annuity* amount at a given rate of compound interest for a given number of years?

Ans. Multiply the number in the *fourth* Table under the rate and opposite to the given number of years, by the given *annuity*, and the *product* will be the answer.

EXAMPLE. The product of 40*l.* into 25.6454 (that is, 1.1025.826) is the sum to which 40*l.* *per ann.* will amount in 18 years, reckoning interest at 4 *per cent.*

Quest.



Quest. III. In what number of years will a given *sum* or *annuity* increase to another given *sum* or *annuity* in consequence of being improved at a given rate of interest?

Ans. Divide the latter *sum* or *annuity* by the former. Find the quotient (or the number nearest to it) in the *third* Table, under the given rate, and the years opposite to it will be the answer.

EXAMPLE. The quotient of 1025.826*l.* divided by 40, is 25.6454, which number, under 4 *per cent.* in the third Table, is opposite to 18 years; which, therefore, is the number of years in which 40*l.* will increase to 1025.826*l.* if improved at 4 *per cent.* compound interest.

Quest. IV. In what time will a given *annuity* amount to a given *sum* at a given rate of interest?

Ans. Divide the given *sum* by the given *annuity*. Find the quotient (or the number nearest to it) in the *fourth* Table under the given rate, and the number of years corresponding to it will be the answer.

EXAMPLE. A person owes 1000*l.* and resolves to appropriate 10*l.* *per ann.* of his income towards discharging it. In what time will such an appropriation, interest being at 4 *per cent.* amount to a sum equal to the debt?—1000*l.* divided by 10*l.* gives 100*l.* The number in the *fourth*

Table, under 4 *per cent.* and nearest to this quotient, is 99.8265, which corresponds to 41 years; and this, therefore, is the time in which such an appropriation would sink the debt. In like manner, it may be found that an appropriation of a million *per ann.* would, in the same time, sink a public debt of a *hundred millions*, carrying 4 *per cent.* interest; and, in 56 years a debt of *two hundred millions*; and in 82 years, a debt of *six hundred millions*.

Quest. V. In what time will a *given principal* be annihilated by taking out of it, at the end of a year, a given sum; and after that, the same sum annually, together with its growing interests?

Ans. In the same time in which an equal annuity would amount to the *given principal*.

A person, therefore, possessed of 1000 *l.* capital, bearing interest at 4 *per cent.* would, by Quest. IV. reduce it to nothing in 41 years, by taking out of it 10 *l.* at the beginning of the first year, and as much more every following year as would be necessary, together with the interest of the remaining capital, to make his annual income constantly 50 *l.*

## T A B L E V.

Shewing the Probabilities of the Duration of Life, as deduced by Dr. *Halley* from Observations on the Bills of Mortality of BRESLAW.

Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.
1	1000	145	31	523	8	61	232	10
2	855	57	32	515	8	62	222	10
3	798	30	33	507	8	63	212	10
4	760	28	34	499	9	64	202	10
5	732	22	35	490	9	65	192	10
6	710	18	36	481	9	66	182	10
7	692	12	37	472	9	67	172	10
8	680	10	38	463	9	68	162	10
9	670	9	39	454	9	69	152	10
10	661	8	40	445	9	70	142	11
11	653	7	41	436	9	71	131	11
12	646	6	42	427	10	72	120	11
13	640	6	43	417	10	73	109	11
14	634	6	44	407	10	74	98	10
15	628	6	45	397	10	75	88	10
16	622	6	46	387	10	76	78	10
17	616	6	47	377	10	77	68	10
18	610	6	48	367	10	78	58	9
19	604	6	49	357	11	79	49	8
20	598	6	50	346	11	80	41	7
21	592	6	51	335	11	81	34	6
22	586	7	52	324	11	82	28	5
23	579	6	53	313	11	83	23	4
24	573	6	54	302	10	84	19	4
25	567	7	55	292	10	85	15	4
26	560	7	56	282	10	86	11	3
27	553	7	57	272	10	87	8	3
28	546	7	58	262	10	88	5	2
29	539	8	59	252	10	89	3	2
30	531	8	60	242	10	90	1	1

## T A B L E VI.

Shewing the Probabilities of the Duration of Human Life at all Ages, formed from the Register of Mortality at *Northampton*, for 46 Years from 1735 to 1780.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decrem.
0	11650	1340	31	4310	75	65	632	80
3 months	10310	554	32	4235	75	66	1552	80
6 months	9756	553	33	4160	75	67	1472	80
9 months	9203	553	34	4085	75	68	1392	80
1 Year	8650	1367	35	4010	75	69	1312	80
2 Years	7283	502	36	3935	75	70	1232	80
3	6781	335	37	3860	75	71	1152	80
4	6446	197	38	3785	75	72	1072	80
5	6249	184	39	3710	75	73	992	80
6	6065	140	40	3635	76	74	912	80
7	5925	110	41	3559	77	75	832	80
8	5815	80	42	3482	78	76	752	77
9	5735	60	43	3404	78	77	675	73
10	5675	52	44	3326	78	78	602	68
11	5623	50	45	3248	78	79	534	65
12	5573	50	46	3170	78	80	469	63
13	5523	50	47	3092	78	81	406	60
14	5473	50	48	3014	78	82	346	57
15	5423	50	49	2936	79	83	289	55
16	5373	53	50	2857	81	84	234	48
17	5320	58	51	2776	82	85	186	41
18	5262	63	52	2694	82	86	145	34
19	5199	67	53	2612	82	87	111	28
20	5132	72	54	2530	82	88	83	21
21	5060	75	55	2448	82	89	62	16
22	4985	75	56	2366	82	90	46	12
23	4910	75	57	2284	82	91	34	10
24	4835	75	58	2202	82	92	24	8
25	4760	75	59	2120	82	93	16	7
26	4685	75	60	2038	82	94	9	5
27	4610	75	61	1956	82	95	4	3
28	4535	75	62	1874	81	96	1	1
29	4460	75	63	1793	81			
30	4385	75	64	1712	80	Total	299198	11650

N. B.



N. B. The decrements in this Table for the four quarters of the first year of life, are given nearly in conformity to the *Chester* register of mortality (see Table 41st in this collection); and the same is true of the decrements at 3 and 4 years of age, the *Northampton* register affording no direction at these ages, because it gives only the totals of deaths under two years of age, and between two and five. Many more observations on the method I have pursued in forming this Table, may be found in the Postscript to the Second Essay in the preceding Volume, p. 308, &c. and in the Fourth Essay, p. 352, &c.

It is proper to add, that it has been taken to be the foundation and guide of the business transacted by the Society in CHATHAM-PLACE, for *Equitable Assurances on Lives and Survivorships*; and that the Tables of this Society, which will be given hereafter, together with the Tables of the values of single and joint lives from Table XVIII. to Table XXXII. have been all calculated from it.

## T A B L E VII.

Shewing the EXPECTATIONS of Human Life at every Age, deduced from the *Northampton* Table of Observations.

Ages.	Expectat.	Ages.	Expectat.	Ages.	Expectat.	Ages.	Expectat.
0	25.18	25	30.85	50	17.99	75	6.54
1	32.74	26	30.33	51	17.50	76	6.18
2	37.79	27	29.82	52	17.02	77	5.83
3	39.55	28	29.30	53	16.54	78	5.48
4	40.58	29	28.79	54	16.06	79	5.11
5	40.84	30	28.27	55	15.58	80	4.75
6	41.07	31	27.76	56	15.10	81	4.41
7	41.03	32	27.24	57	14.63	82	4.09
8	40.79	33	26.72	58	14.15	83	3.80
9	40.36	34	26.20	59	13.68	84	3.58
10	39.78	35	25.68	60	13.21	85	3.37
11	39.14	36	25.16	61	12.75	86	3.19
12	38.49	37	24.64	62	12.28	87	3.01
13	37.83	38	24.12	63	11.81	88	2.86
14	37.17	39	23.60	64	11.35	89	2.66
15	36.51	40	23.08	65	10.88	90	2.41
16	35.85	41	22.56	66	10.42	91	2.09
17	35.20	42	22.04	67	9.96	92	1.75
18	34.58	43	21.54	68	9.50	93	1.37
19	33.99	44	21.03	69	9.05	94	1.05
20	33.43	45	20.52	70	8.60	95	0.75
21	32.90	46	20.02	71	8.17	96	0.50
22	32.39	47	19.51	72	7.74		
23	31.88	48	19.00	73	7.33		
24	31.36	49	18.49	74	6.92		

## T A B L E VIII.

Shewing the Probabilities of Life at NOR-  
WICH. See page 353, Vol. I.

Ag <sup>cs</sup> .	Persons living.	Decr. of Life	Ages.	Persons living.	Decr. of Life.	Ages.	Persons living	Decr. of Life
0	1185	320	32	392	6	63	174	9
1	865	160	33	386	6	64	165	9
2	705	60	34	380	6	65	156	9
3	645	32	35	374	6	66	147	9
4	613	23	36	368	6	67	138	9
5	590	20	37	362	6	68	129	9
6	570	16	38	356	6	69	120	9
7	554	13	39	350	7	70	111	9
8	541	11	40	343	6	71	102	8
9	530	9	41	337	6	72	94	8
10	521	7	42	331	6	73	86	8
11	514	6	43	325	7	74	78	8
12	508	6	44	318	7	75	70	8
13	502	5	45	311	7	76	62	7
14	497	5	46	304	7	77	55	7
15	492	5	47	297	7	78	48	6
16	487	5	48	290	7	79	42	5
17	482	5	49	283	7	80	37	5
18	477	5	50	276	7	81	32	4
19	472	5	51	269	7	82	28	4
20	467	6	52	262	7	83	24	4
21	461	6	53	255	8	84	20	3
22	455	6	54	247	8	85	17	3
23	449	6	55	239	8	86	14	3
24	443	6	56	231	8	87	11	2
25	437	6	57	223	8	88	9	2
26	431	7	58	215	8	89	7	2
27	424	7	59	207	8	90	5	2
28	417	7	60	199	8	91	3	2
29	410	6	61	191	8	92	1	1
30	404	6	62	183	9	93	1	1
31	398	6						

## T A B L E IX.

Shewing the Probability of the Duration of Life in LONDON, deduced by Mr. *Simpson* from Observations on the Bills of Mortality in LONDON for 10 Years, from 1728 to 1737.

Ages.	Persons living.	Decre. of Life.	Ages.	Persons living.	Decre. of Life.	Ages.	Persons living.	Decre. of Life.
0	1000	320	27	321	6 <sup>e</sup>	54	135	6
1	680	133	28	215	7	55	129	6
2	547	51	29	308	7	56	123	6
3	496	27	30	301	7	57	117	5
4	469	17	31	294	7	58	112	5
5	452	12	32	287	7	59	107	5
6	440	10	33	280	7	60	102	5
7	430	8	34	273	7	61	97	5
8	422	7	35	266	7	62	92	5
9	415	5	36	259	7	63	87	5
10	410	5	37	252	7	64	82	5
11	405	5	38	245	8	65	77	5
12	400	5	39	237	8	66	72	5
13	395	5	40	229	7	67	67	5
14	390	5	41	222	8	68	62	4
15	385	5	42	214	8	69	58	4
16	380	5	43	206	7	70	54	4
17	375	5	44	199	7	71	50	4
18	370	5	45	192	7	72	46	4
19	365	5	46	185	7	73	42	3
20	360	5	47	178	7	74	39	3
21	355	5	48	171	6	75	36	3
22	350	5	49	165	6	76	33	3
23	345	6	50	159	6	77	30	3
24	339	6	51	153	6	78	27	2
25	333	6	52	147	6	79	25	
26	327	6	53	141	6			

TABLE



# T A B L E S.

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## T A B L E X.

Shewing the *Expectations* of life in LONDON,  
according to the preceding Table. See  
*Mr. Simpson's Select Exercises*, p. 255.

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
0	19.2	27	25.1	54	14.5
1	27.0	28	24.6	55	14.2
2	32.0	29	24.1	56	13.8
3	34.0	30	23.6	57	13.4
4	35.6	31	23.1	58	13.1
5	36.0	32	22.7	59	12.7
6	36.0	33	22.3	60	12.4
7	35.8	34	21.9	61	12.0
8	35.6	35	21.5	62	11.6
9	35.2	36	21.1	63	11.2
10	34.8	37	20.7	64	10.8
11	34.3	38	20.3	65	10.5
12	33.7	39	19.9	66	10.1
13	33.1	40	19.6	67	9.8
14	32.5	41	19.2	68	9.4
15	31.9	42	18.8	69	9.1
16	31.3	43	18.5	70	8.8
17	30.7	44	18.1	71	8.4
18	30.1	45	17.8	72	8.1
19	29.5	46	17.4	73	7.8
20	28.9	47	17.0	74	7.5
21	28.3	48	16.7	75	7.2
22	27.7	49	16.3	76	6.8
23	27.2	50	16.0	77	6.4
24	26.6	51	15.6	78	6.0
25	26.1	52	15.2	79	5.5
26	25.6	53	14.9	80	5.0

## T A B L E II.

Shewing the Value of an Annuity on *One* Life, according to the Probabilities of Life in LONDON.  
See Mr. *Simpson's Select Exercises*, p. 260.

Age.	Yrs. purchase at 3 per Cent.	Yrs. purchase at 4 per Cent.	Yrs. purchase at 5 per Cent.	Age.	Yrs. purchase at 3 per Cent.	Yrs. purchase at 4 per Cent.	Yrs. purchase at 5 per Cent.	Age.	Yrs. purchase at 3 per Cent.	Yrs. purchase at 4 per Cent.	Yrs. purchase at 5 per Cent.
6	18.8	16.2	14.1	31	14.8	12.9	11.4	56	10.1	9.1	8.4
7	18.9	16.3	14.2	32	14.6	12.7	11.3	57	9.9	8.9	8.2
8	19.0	16.4	14.3	33	14.4	12.6	11.2	58	9.6	8.7	8.1
9	19.0	16.4	14.3	34	14.2	12.4	11.0	59	9.4	8.6	8.0
10	19.0	16.4	14.3	35	14.1	12.3	10.9	60	9.2	8.4	7.9
11	19.0	16.4	14.3	36	13.9	12.1	10.8	61	8.9	8.2	7.7
12	18.9	16.3	14.2	37	13.7	11.9	10.6	62	8.7	8.1	7.6
13	18.7	16.2	14.1	38	13.5	11.8	10.5	63	8.5	7.9	7.4
14	18.5	16.0	14.0	39	13.3	11.6	10.4	64	8.3	7.7	7.3
15	18.3	15.8	13.9	40	13.2	11.5	10.3	65	8.0	7.5	7.1
16	18.1	15.6	13.7	41	13.0	11.4	10.2	66	7.8	7.3	6.9
17	17.9	15.4	13.5	42	12.8	11.2	10.1	67	7.6	7.1	6.7
18	17.6	15.2	13.4	43	12.6	11.1	10.0	68	7.4	6.9	6.6
19	17.4	15.0	13.2	44	12.5	11.0	9.9	69	7.1	6.7	6.4
20	17.2	14.8	13.0	45	12.3	10.8	9.8	70	6.9	6.5	6.2
21	17.0	14.7	12.9	46	12.1	10.7	9.7	71	6.7	6.3	6.0
22	16.8	14.5	12.7	47	11.9	10.5	9.5	72	6.5	6.1	5.8
23	16.5	14.3	12.6	48	11.8	10.4	9.4	73	6.2	5.9	5.6
24	16.3	14.1	12.4	49	11.6	10.2	9.3	74	5.9	5.6	5.4
25	16.1	14.0	12.3	50	11.4	10.1	9.2	75	5.6	5.4	5.2
26	15.9	13.8	12.1	51	11.2	9.9	9.0				
27	15.6	13.6	12.0	52	11.0	9.8	8.9				
28	15.4	13.4	11.8	53	10.7	9.6	8.8				
29	15.2	13.2	11.7	54	10.5	9.4	8.6				
30	15.0	13.1	11.6	55	10.3	9.3	8.5				

# T A B L E S.

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## T A B L E XII.

Shewing the Value of an Annuity on the joint Continuance of Two Lives according to the Probabilities of Life in LONDON. See Mr. *Simpson's Select Exercises*, p. 266.

Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
10	10	14.7	13.0	11.6	20	20	12.8	11.3	10.1
	15	14.3	12.7	11.3		25	12.2	10.8	9.7
	20	13.8	12.2	10.8		30	11.6	10.3	9.2
	25	13.1	11.6	10.2		35	10.9	9.8	8.8
	30	12.3	10.9	9.7		40	10.2	9.2	8.4
	35	11.5	10.2	9.1		45	9.5	8.6	7.9
	40	10.7	9.6	8.6		50	8.8	8.0	7.4
	45	10.0	9.0	8.1		55	8.1	7.5	6.9
	50	9.3	8.4	7.6		60	7.4	6.9	6.4
	55	8.6	7.8	7.1		65	6.7	6.3	5.9
	60	7.8	7.2	6.6		70	6.0	5.7	5.4
	65	6.9	6.5	6.1		75	5.2	5.0	4.8
	70	6.1	5.8	5.5	25	25	11.8	10.5	9.4
	75	5.3	5.1	4.9		30	11.3	10.1	9.0
15	15	13.9	12.3	11.0		35	10.7	9.6	8.6
	20	13.3	11.8	10.5		40	10.0	9.1	8.2
	25	12.6	11.2	10.1		45	9.4	8.5	7.8
	30	11.9	10.6	9.5		50	8.7	7.9	7.3
	35	11.2	10.0	9.0		55	8.0	7.4	6.8
	40	10.4	9.4	8.5		60	7.3	6.8	6.3
	45	9.6	8.8	8.0		65	6.6	6.2	5.8
	50	8.9	8.2	7.5		70	5.9	5.6	5.3
	55	8.2	7.6	7.0		75	5.1	4.9	4.7
	60	7.5	7.0	6.5	30	30	10.8	9.6	8.6
	65	6.8	6.4	6.0		35	10.3	9.2	8.3
	70	6.0	5.7	5.4		40	9.7	8.8	8.0
	75	5.2	5.0	4.8					

T A B L E XII. continued.

Age of the youngst.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Age of the youngst.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
30	45	9.1	8.3	7.6	45	65	6.3	5.8	5.4
	50	8.5	7.8	7.2		70	5.6	5.3	5.0
	55	7.9	7.3	6.7		75	4.9	4.7	4.5
	60	7.2	6.7	6.2	50	50	7.6	6.8	6.2
	65	6.5	6.1	5.7		55	7.2	6.5	6.0
	70	5.8	5.5	5.2		60	6.7	6.1	5.7
	75	5.1	4.9	4.7		65	6.2	5.7	5.3
35	35	9.9	8.8	8.0		70	5.5	5.2	4.9
	40	9.4	8.5	7.7		75	4.8	4.6	4.4
	45	8.9	8.1	7.4	55	55	6.9	6.2	5.7
	50	8.3	7.6	7.0		60	6.5	5.9	5.5
	55	7.7	7.1	6.6		65	6.0	5.6	5.2
	60	7.1	6.5	6.1		70	5.4	5.1	4.8
	65	6.4	6.0	5.6		75	4.7	4.5	4.3
	70	5.7	5.4	5.1	60	60	6.1	5.6	5.2
	75	5.0	4.8	4.6		65	5.7	5.3	4.9
40	40	9.1	8.1	7.3		70	5.2	4.9	4.6
	45	8.7	7.8	7.1		75	4.6	4.4	4.2
	50	8.2	7.4	6.8	65	65	5.4	5.0	4.7
	55	7.6	6.9	6.4		70	4.9	4.6	4.4
	60	7.0	6.4	6.0		75	4.4	4.2	4.0
	65	6.4	5.9	5.5	70	70	4.6	4.4	4.2
	70	5.7	5.4	5.1		75	4.2	4.0	3.9
	75	5.0	4.8	4.6		75	3.8	3.7	3.6
45	45	8.3	7.4	6.7					
	50	7.9	7.1	6.5					
	55	7.4	6.7	6.2					
	60	6.8	6.3	5.8					



## T A B L E XIII.

Shewing the Probabilities of Life in LONDON, on the Supposition that all who die in LONDON were born there. Formed from the Bills, for 10 Years, from 1759 to 1768. See Vol. I. p. 343, &c.

Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.
0	1000	240	31	404	9	62	132	7
1	760	99	32	395	9	63	125	7
2	661	42	33	386	9	64	118	7
3	619	29	34	377	9	65	111	7
4	590	21	35	368	9	66	104	7
5	569	13	36	359	9	67	97	7
6	556	10	37	350	9	68	90	7
7	546	7	38	341	9	69	83	7
8	539	5	39	332	10	70	76	6
9	534	4	40	322	10	71	70	6
10	530	4	41	312	10	72	64	6
11	526	4	42	302	10	73	58	5
12	522	4	43	292	10	74	53	5
13	518	3	44	282	10	75	48	5
14	515	3	45	272	10	76	43	5
15	512	3	46	262	10	77	38	5
16	509	3	47	252	10	78	33	4
17	506	3	48	242	9	79	29	4
18	503	4	49	233	9	80	25	3
19	499	5	50	224	9	81	22	3
20	494	7	51	215	9	82	19	3
21	487	8	52	206	8	83	16	3
22	479	8	53	198	8	84	13	2
23	471	8	54	190	7	85	11	2
24	463	8	55	183	7	86	9	2
25	455	8	56	176	7	87	7	2
26	447	8	57	169	7	88	5	1
27	439	8	58	162	7	89	4	1
28	431	9	59	155	8	90	3	1
29	422	9	60	147	8			
30	413	9	61	139	7			

## T A B L E XIV.

Shewing the true Probabilities of Life in  
London till the Age of 19. See Vol. I.  
p. 347, &c.

Age.	Persons liv- ing.	Decrements of Life.
0	750	240
1	510	99
2	411	42
3	369	29
4	340	21
5	319	13
6	306	10
7	296	7
8	289	5
9	284	4
10	280	4
11	276	4
12	272	3
13	269	3
14	266	3
15	263	3
16	260	3
17	257	4
18	253	4
19	249	5
20	494	

# T A B L E S.

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## T A B L E X V.

Shewing the *true* Probabilities of Life in LONDON  
for all Ages, formed from the Bills for 10 Years,  
from 1759 to 1768. See Vol. I. p. 341. &c.

Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.
0	1518	486	31	404	9	62	132	7
1	1032	200	32	395	9	63	125	7
2	832	85	33	386	9	64	118	7
3	747	59	34	377	9	65	111	7
4	688	42	35	368	9	66	104	7
5	646	23	36	359	9	67	97	7
6	623	20	37	350	9	68	90	7
7	603	14	38	341	9	69	83	7
8	589	12	39	332	10	70	76	6
9	577	10	40	322	10	71	70	6
10	567	9	41	312	10	72	64	6
11	558	9	42	302	10	73	58	5
12	549	8	43	292	10	74	53	5
13	541	7	44	282	10	75	48	5
14	534	6	45	272	10	76	43	5
15	528	6	46	262	10	77	38	5
16	522	7	47	252	10	78	33	4
17	515	7	48	242	9	79	29	4
18	508	7	49	233	9	80	25	3
19	501	7	50	224	9	81	22	3
20	494	7	51	215	9	82	19	3
21	487	8	52	206	8	83	16	3
22	479	8	53	198	8	84	13	2
23	471	8	54	190	7	85	11	2
24	463	8	55	183	7	86	9	2
25	455	8	56	176	7	87	7	2
26	447	8	57	169	7	88	5	1
27	439	8	58	162	7	89	4	1
28	431	9	59	155	8	90	3	1
29	422	9	60	147	8			
30	413	9	61	139	7			

## T A B L E S.

## TABLE XVI.

Shewing the Probabilities of the Duration of Human Life in LONDON, and formed from the Bills for ten Years, from 1771 to 1780.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decr.
0	28452	9018	34	7949	190	68	1831	130
1	19434	3000	35	7759	190	69	1701	130
2	16434	1536	36	7569	190	70	1571	130
3	14898	1200	37	7379	190	71	1441	120
4	13698	800	38	7189	190	72	1321	120
5	12898	500	39	6999	200	73	1201	120
6	12398	318	40	6799	210	74	1081	110
7	12080	210	41	6589	210	75	971	110
8	11870	160	42	6379	210	76	861	100
9	11710	130	43	6169	210	77	761	100
10	11580	130	44	5959	210	78	661	90
11	11450	130	45	5749	200	79	571	80
12	11320	130	46	5549	200	80	491	70
13	11190	130	47	5349	200	81	421	60
14	11060	130	48	5149	200	82	361	52
15	10930	130	49	4949	193	83	309	48
16	10800	130	50	4756	190	84	261	44
17	10670	130	51	4556	190	85	217	40
18	10540	135	52	4376	180	86	177	35
19	10405	135	53	4196	180	87	142	30
20	10270	140	54	4016	180	88	112	25
21	10130	150	55	3836	170	89	87	20
22	9980	155	56	3666	170	90	67	15
23	9825	155	57	3496	165	91	52	12
24	9670	160	58	3331	160	92	40	10
25	9510	160	59	3171	160	93	30	8
26	9350	160	60	3011	160	94	22	7
27	9190	170	61	2851	150	95	15	6
28	9020	170	62	2701	150	96	9	5
29	8850	171	63	2551	150	97	4	3
30	8679	180	64	2401	150	98	1	1
31	8499	180	65	2251	140			
32	8319	180	66	2111	140	Tot. 572781		28452
33	8139	190	67	1971	140			



## REMARKS on the preceding Table.

According to this Table, the numbers dying in every deced of life from 20 to old age, are the very numbers given by the bills. For instance. The sum of the decrements in the Table between 20 and 30, between 30 and 40, between 40 and 50, between 50 and 60, between 60 and 70, between 70 and 80, between 80 and 90, and above 90, are 1591, 1880, 2043, 1745, 1440, 1080, 423, and 68, respectively; and these are the average numbers which, according to the bills, have died annually in *London*, in these several divisions of life, from 1771 to 1780. The sum of all these numbers is 10,270, which, therefore, agreeably to the directions in the 4th Essay, p. 339, &c. is given in the Table as the number of the living at the age of 20.

The *proportions* of the decrements *before* 20, are likewise exactly the same with those given by the bills. For instance. The number (*deducting the abortive and still-born*) given by the bills as having died annually under two years of age from 1771 to 1780, is 7000; and the numbers given as having died between 2 and 5, between 5 and 10, and between 10 and 20, are 2060, 768, and 763. These decrements, according to the Table, are 12018, 3535, 1318, and 1310: which numbers are in the same proportion to one another with the former numbers; and the

numbers of the living corresponding to these decrements are so adjusted, as to make the number dying annually between 8 and 16, as *small* as is consistent with any degree of credibility; that is, they have been so adjusted as to make this last number only an 86th part of the whole number living, which is a smaller proportion than Mr. *Wales* says have for 20 years died of children of the same ages in *Christ's-Hospital*, though near a third reside in the country. See the note p. 343, Vol. I.

It should be observed here, that the number living *at* 20, and the proportions of the decrements *before* 20, and the probabilities of living in one division of life being obtained or assumed, all the numbers in the second column of this Table, are so far determined as to render it not possible to fall into any material error in fixing them.—It is necessary to add, that though the particular decrements under two years of age, between 2 and 5, &c. are given by the bills too small; this affords no reason for concluding that their *proportions* are not given right. On the contrary; the reasons mentioned in the note p. 354, Vol. I. seem to prove they may be depended on.

The account now given shews, that most *probably* the preceding Table exhibits the probabilities of living considerably too high before the age of 20; and it does this *certainly* from 20 to 35 or 40, for the reasons explained

explained in p. 339, 340, &c. Vol. I.; and in old age it gives the probabilities of living rather higher than they are in situations the most healthful. We may, therefore, safely conclude that it exhibits the state of human life in *London* as upon the whole more favourable than it is. According to this Table, however, one half of all born in *London* die in the first four years; and the expectation of a child at birth is only 19.8—It is farther observable, that for all ages after 20, it agrees so nearly with Table 9th formed from the bills from 1728 to 1737, and with Table 15th formed from the bills from 1759 to 1768, as to demonstrate that, for the last 50 years, there has been no change in the state of *London* which has greatly affected its influence on the duration of human life. This will appear from the following comparison.

Expectations of Life at	By Table 9th	By Table 15th	By Table 16th
20	28.9	29.3	29.6
25	26.1	26.6	26.7
30	23.6	24.1	24.1
35	21.5	21.7	21.6
40	19.6	19.5	19.3
45	17.8	17.6	17.4
50	16.0	15.9	15.5
55	14.2	13.9	13.6
60	12.4	11.7	11.7
65	10.5	9.7	9.8
70	8.8	8.0	7.9
75			
80			



It cannot but be reckoned remarkable, that the duration of human life in *London* should come out by the bills so nearly the same at the three periods for which the Table mentioned in this comparison were formed. A small difference, indeed, appears from the age of 20 to 30 in favour of *London* in its present state; but it must not be depended on as a reason for concluding that *London* is now less prejudicial to health than it was; for Mr. *Simpson*, in forming Table 9th, did not take, as I have done, the decrements of life between 20 and 30 exactly from the bills, but extended his corrections very properly to this division of life as well as those preceding it; and had I done the same, the expectations for 20 and 25, deduced from Tables 15th and 16th, would have been less than they are.—With respect to all ages before 20, nothing certain can be collected from these Tables. The last makes, indeed, one half of the children born to survive 4 years of age, whereas the other Tables make one half live only to three years of age; but it should be recollected, that this difference has been occasioned by the act of parliament passed in 1767, and mentioned in the notes, p. 251, 354, Vol. I. requiring all parish children to be sent into the country for six years. If only a thousand burials of infants under two years of age, and born in *London*, have by this act been taken out of the bills, which used to be, and ought to



to be, included in them, it will follow that one half of the children born in *London* do not live to three years of age; and a table constructed in the manner of the last table, would have shewn this as well as the other tables.—Mr. *Howlett* tells us, that this deficiency amounts to 2100; and were this true, it would follow that *London* is now more fatal to children than ever it was. But I have learnt not to rely on Mr. *Howlett*'s accounts. See the note in Vol. I. p. 251.

This Table would have been very nearly the same, had it been formed from the bills for the last *five* years from 1777 to 1781, instead of being formed as it is from the bills for ten years from 1771 to 1780.

## T A B L E XVII.

Shewing the Value of an Annuity on a single Life at every Age, according to the Probabilities of the Duration of Human Life at NORTH-AMPTON. See Table VI. p. 36.

Ages.	Value at 8 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.	Value at 7 per Ct.	Value at 8 per Ct.
Birth		10.327	8.863			
$\frac{1}{2}$ year		13.008	11.274			
1	16.021	13.465	11.563	10.107	8.963	8.046
2	18.599	15.633	13.420	11.724	10.391	9.321
3	19.575	16.462	14.135	12.348	10.941	9.812
4	20.210	17.010	14.613	12.769	11.315	10.147
5	20.473	17.248	14.827	12.962	11.489	10.304
6	20.727	17.482	15.041	13.156	11.666	10.466
7	20.853	17.611	15.166	13.275	11.777	10.570
8	20.885	17.662	15.226	13.337	11.840	10.631
9	20.812	17.625	15.210	13.335	11.846	10.641
10	20.663	17.523	15.139	13.285	11.809	10.614
11	20.480	17.393	15.043	13.212	11.752	10.569
12	20.283	17.251	14.937	13.130	11.687	10.517
13	20.081	17.103	14.826	13.044	11.618	10.461
14	19.872	16.950	14.710	12.953	11.545	10.401
15	19.657	16.791	14.588	12.857	11.467	10.337
16	19.435	16.625	14.460	12.755	11.384	10.268
17	19.218	16.462	14.334	12.655	11.302	10.200
18	19.013	16.309	14.217	12.562	11.226	10.137
19	18.820	16.167	14.108	12.477	11.157	10.081
20	18.638	16.033	14.007	12.398	11.094	10.030
21	18.470	15.912	13.917	12.329	11.042	9.986
22	18.311	15.797	13.833	12.265	10.993	9.947
23	18.148	15.680	13.746	12.200	10.942	9.907
24	17.983	15.560	13.658	12.132	10.890	9.865
25	17.814	15.438	13.567	12.063	10.836	9.823
26	17.642	15.312	13.473	11.992	10.780	9.778
27	17.467	15.184	13.377	11.917	10.723	9.732
28	17.289	15.053	13.278	11.841	10.663	9.685
29	17.107	14.918	13.177	11.763	10.602	9.635
30	16.922	14.781	13.072	11.682	10.539	9.584
31	16.732	14.639	12.965	11.598	10.473	9.531
32	16.540	14.495	12.854	11.512	10.404	9.476
33	16.343	14.347	12.740	11.423	10.333	9.418
34	16.142	14.195	12.623	11.331	10.260	9.359

T A B L E XVII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per C.	Value at 7 per Ct.	Value at 8 per Ct.
35	15.938	14.039	12.502	11.236	10.183	9.296
36	15.729	13.880	12.377	11.137	10.104	9.231
37	15.515	13.716	12.249	11.035	10.021	9.164
38	15.298	13.548	12.116	10.929	9.935	9.093
39	15.075	13.375	11.979	10.819	9.845	9.019
40	14.848	13.197	11.837	10.705	9.752	8.941
41	14.620	13.018	11.695	10.589	9.657	8.863
42	14.391	12.837	11.551	10.473	9.562	8.783
43	14.162	12.657	11.407	10.356	9.466	8.703
44	13.929	12.472	11.258	10.235	9.366	8.620
45	13.692	12.283	11.105	10.110	9.262	8.533
46	13.450	12.089	10.947	9.980	9.154	8.443
47	13.203	11.890	10.784	9.846	9.042	8.348
48	12.951	11.685	10.616	9.707	8.925	8.249
49	12.693	11.475	10.443	9.563	8.804	8.146
50	12.436	11.264	10.269	9.417	8.681	8.041
51	12.183	11.057	10.097	9.273	8.559	7.937
52	11.930	10.849	9.925	9.129	8.437	7.833
53	11.674	10.637	9.748	8.980	8.311	7.725
54	11.414	10.421	9.567	8.827	8.181	7.614
55	11.150	10.201	9.382	8.670	8.047	7.499
56	10.882	9.977	9.193	8.509	7.909	7.379
57	10.611	9.749	8.999	8.343	7.766	7.256
58	10.337	9.516	8.801	8.173	7.619	7.128
59	10.058	9.280	8.599	7.999	7.468	6.996
60	9.777	9.039	8.392	7.820	7.312	6.860
61	9.493	8.795	8.181	7.637	7.152	6.719
62	9.205	8.547	7.966	7.449	6.988	6.574
63	8.910	8.291	7.742	7.253	6.815	6.421
64	8.611	8.030	7.514	7.052	6.637	6.262
65	8.304	7.761	7.276	6.841	6.449	6.095
66	7.994	7.488	7.034	6.625	6.256	5.922
67	7.682	7.211	6.787	6.405	6.058	5.743
68	7.367	6.930	6.536	6.179	5.855	5.559
69	7.051	6.647	6.281	5.949	5.646	5.370
70	6.734	6.361	6.023	5.716	5.434	5.176
71	6.418	6.075	5.764	5.479	5.213	4.978
72	6.103	5.790	5.504	5.241	5.000	4.778
73	5.794	5.507	5.245	5.004	4.781	4.576
74	5.491	5.230	4.990	4.769	4.565	4.375

T A B L E XVII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.	Value at 7 per Ct.	Value at 8 per Ct.
75	5.199	4.962	4.744	4.542	4.354	4.180
76	4.925	4.710	4.511	4.326	4.154	3.994
77	4.652	4.457	4.277	4.109	3.952	3.806
78	4.372	4.197	4.035	3.884	3.742	3.609
79	4.077	3.921	3.776	3.641	3.514	3.394
80	3.781	3.643	3.515	3.394	3.281	3.174
81	3.499	3.377	3.263	3.156	3.055	2.960
82	3.229	3.122	3.020	2.925	2.836	2.751
83	2.982	2.887	2.797	2.713	2.632	2.557
84	2.793	2.708	2.627	2.551	2.479	2.410
85	2.620	2.543	2.471	2.402	2.337	2.275
86	2.462	2.393	2.328	2.266	2.207	2.151
87	2.312	2.251	2.193	2.138	2.085	2.035
88	2.185	2.131	2.080	2.031	1.984	1.939
89	2.013	1.967	1.924	1.882	1.842	1.803
90	1.794	1.758	1.723	1.689	1.656	1.625
91	1.501	1.474	1.447	1.422	1.398	1.374
92	1.190	1.171	1.153	1.136	1.118	1.102
93	0.839	0.827	0.816	0.806	0.795	0.785
94	0.536	0.530	0.524	0.518	0.512	0.507
95	0.242	0.240	0.238	0.236	0.234	0.232
96	0.000	0.000	0.000	0.000	0.000	0.000



The values of annuities in the preceding Table (and in all the other Tables in this collection), suppose the payments to be made *yearly*, and to begin at the end of a year; except in the single instance of an annuity on a life aged *half* a year, the value of which is given in the preceding Table, on the suppositions that the first payment is to be a half-yearly one made at the end of half a year, and ~~that~~ all the subsequent payments are yearly ones.

If all the payments are to be *half-yearly* payments, and to be made at the end of every *half* year from the time of purchase, their value will be increased about *one fifth* of a year's purchase. When the *tabular* value (that is, the value of an annuity to commence at the end of a year, and payable yearly) is greater than 11 or 12 years purchase, this addition will give somewhat *more*, and when less it will give somewhat *less* than the value of the same annuity payable *half-yearly*; but in no instance will the error exceed a 20th of a year's purchase.

## T A B L E XVIII.

Shewing the Value of an Annuity on the *joint Continuance* of Two Lives, having the same common Age, according to the *Northampton* Table of Observations. See Table VI. p. 36.

Difference of Age 0.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1- 1	9.491	8.252	7.287	6.515
2- 2	12.789	11.107	9.793	8.741
3- 3	14.196	12.325	10.862	9.689
4- 4	15.181	13.185	11.621	10.365
5- 5	15.638	13.591	11.984	10.691
6- 6	16.099	14.005	12.358	11.031
7- 7	16.375	14.224	12.596	11.251
8- 8	16.510	14.399	12.731	11.382
9- 9	16.483	14.396	12.744	11.404
10-10	16.339	14.277	12.665	11.345
11-11	16.142	14.133	12.546	11.249
12-12	15.925	13.966	12.411	11.139
13-13	15.702	13.789	12.268	11.023
14-14	15.470	13.604	12.118	10.899
15-15	15.229	13.411	11.960	10.767
16-16	14.979	13.212	11.793	10.626
17-17	14.737	13.019	11.630	10.489
18-18	14.516	12.841	11.483	10.365
19-19	14.316	12.679	11.351	10.255
20-20	14.133	12.535	11.232	10.156

T A B L E XVIII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
21-21	13.974	12.409	11.131	10.074
22-22	13.830	12.293	11.042	10.002
23-23	13.683	12.179	10.951	9.928
24-24	13.534	12.062	10.858	9.853
25-25	13.383	11.944	10.764	9.776
26-26	13.230	11.822	10.667	9.697
27-27	13.074	11.699	10.567	9.616
28-28	12.915	11.573	10.466	9.533
29-29	12.754	11.445	10.362	9.448
30-30	12.589	11.313	10.255	9.360
31-31	12.422	11.179	10.146	9.270
32-32	12.252	11.042	10.034	9.178
33-33	12.079	10.902	9.919	9.082
34-34	11.902	10.759	9.801	8.984
35-35	11.722	10.612	9.680	8.883
36-36	11.539	10.462	9.555	8.778
37-37	11.351	10.307	9.427	8.670
38-38	11.160	10.149	9.294	8.558
39-39	10.964	9.986	9.158	8.442
40-40	10.764	9.820	9.016	8.322
41-41	10.565	9.654	8.876	8.202
42-42	10.369	9.491	8.737	8.083
43-43	10.175	9.326	8.599	7.965
44-44	9.978	9.160	8.457	7.843
45-45	9.776	8.990	8.312	7.718
46-46	9.571	8.815	8.162	7.589
47-47	9.362	8.637	8.008	7.455
48-48	9.149	8.453	7.849	7.316



T A B L E XVIII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
49-49	8.931	8.266	7.686	7.173
50-50	8.714	8.081	7.522	7.030
51-51	8.507	7.900	7.366	6.893
52-52	8.304	7.723	7.213	6.758
53-53	8.099	7.544	7.056	6.620
54-54	7.891	7.362	6.897	6.480
55-55	7.681	7.179	6.735	6.336
56-56	7.470	6.993	6.571	6.190
57-57	7.256	6.805	6.404	6.041
58-58	7.041	6.614	6.234	5.890
59-59	6.824	6.421	6.062	5.735
60-60	6.606	6.226	5.888	5.579
61-61	6.387	6.030	5.712	5.420
62-62	6.166	5.831	5.533	5.259
63-63	5.938	5.626	5.347	5.089
64-64	5.709	5.417	5.158	4.917
65-65	5.471	5.201	4.960	4.736
66-66	5.231	4.982	4.759	4.551
67-67	4.990	4.760	4.555	4.363
68-68	4.747	4.537	4.348	4.171
69-69	4.504	4.312	4.140	3.977
70-70	4.261	4.087	3.930	3.781
71-71	4.020	3.862	3.719	3.584
72-72	3.781	3.639	3.510	3.387
73-73	3.548	3.421	3.304	3.193
74-74	3.324	3.211	3.105	3.005
75-75	3.114	3.015	2.917	2.827
76-76	2.920	2.833	2.750	2.668



## T A B L E S.

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T A B L E XVIII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
77-77	2.741	2.656	2.583	2.511
78-78	2.550	2.470	2.410	2.346
79-79	2.338	2.271	2.217	2.161
80-80	2.122	2.068	2.018	1.969
81-81	1.917	1.869	1.827	1.786
82-82	1.719	1.681	1.642	1.606
83-83	1.538	1.510	1.472	1.441
84-84	1.416	1.387	1.357	1.330
85-85	1.309	1.339	1.256	1.232
86-86	1.218	1.195	1.171	1.149
87-87	1.141	1.124	1.098	1.078
88-88	1.103	1.030	1.063	1.044
89-89	1.036	1.015	1.001	0.984
90-90	0.938	0.922	0.909	0.895
91-91	0.769	0.756	0.748	0.737
92-92	0.591	0.583	0.576	0.569
93-93	0.369	0.365	0.361	0.357
94-94	0.203	0.201	0.199	0.197
95-95	0.060	0.060	0.059	0.058
96-96	0.000	0.000	0.000	0.000

T A B L E

## T A B L E XIX.

Shewing the Value of an Annuity on the *joint* Continuance of Two Lives, according to the *Northampton* Table of Observations. See Table VI. p. 36.

Difference of Age *five* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1- 6	12.347	10.741	9.479	8.467
2- 7	14.461	12.581	11.100	9.911
3- 8	15.300	13.319	11.755	10.498
4- 9	15.809	13.775	12.165	10.869
5-10	15.974	13.933	12.315	11.010
6-11	16.110	14.068	12.447	11.136
7-12	16.137	14.111	12.498	11.192
8-13	16.089	14.089	12.492	11.197
9-14	15.957	13.992	12.421	11.144
10-15	15.762	13.841	12.302	11.048
11-16	15.538	13.664	12.158	10.929
12-17	15.308	13.480	12.009	10.805
13-18	15.086	13.303	11.864	10.685
14-19	14.870	13.130	11.723	10.568
15-20	14.660	12.961	11.585	10.453
16-21	14.457	12.799	11.452	10.342
17-22	14.265	12.646	11.327	10.239
18-23	14.082	12.500	11.209	10.140
19-24	13.908	12.361	11.096	10.048
20-25	13.741	12.229	10.989	9.960
21-26	13.584	12.105	10.890	9.879
22-27	13.433	11.987	10.796	9.803

T A B L E XIX. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
23-28	13.280	11.866	10.699	9.724
24-29	13.124	11.743	10.600	9.643
25-30	12.966	11.618	10.499	9.561
26-31	12.805	11.489	10.396	9.476
27-32	12.641	11.359	10.289	9.389
28-33	12.474	11.225	10.181	9.299
29-34	12.304	11.088	10.069	9.207
30-35	12.131	10.948	9.954	9.112
31-36	11.955	10.805	9.837	9.014
32-37	11.775	10.659	9.716	8.913
33-38	11.592	10.508	9.591	8.808
34-39	11.404	10.354	9.463	8.701
35-40	11.213	10.196	9.331	8.589
36-41	11.021	10.037	9.198	8.476
37-42	10.828	9.877	9.062	8.362
38-43	10.635	9.716	8.927	8.246
39-44	10.437	9.550	8.787	8.127
40-45	10.236	9.381	8.643	8.003
41-46	10.033	9.210	8.497	7.878
42-47	9.829	9.037	8.350	7.751
43-48	9.624	8.862	8.200	7.621
44-49	9.414	8.683	8.046	7.488
45-50	9.204	8.503	7.891	7.353
46-51	8.997	8.326	7.737	7.219
47-52	8.790	8.147	7.582	7.084
48-53	8.579	7.965	7.424	6.945
49-54	8.366	7.780	7.262	6.802
50-55	8.152	7.593	7.098	6.658

T A B L E XIX. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
51-56	7.941	7.409	6.936	6.515
52-57	7.730	7.225	6.774	6.371
53-58	7.518	7.039	6.609	6.225
54-59	7.304	6.850	6.442	6.076
55-60	7.088	6.659	6.272	5.924
56-61	6.870	6.465	6.100	5.770
57-62	6.651	6.270	5.925	5.613
58-63	6.427	6.070	5.744	5.450
59-64	6.201	5.867	5.561	5.284
60-65	5.970	5.658	5.372	5.112
61-66	5.737	5.447	5.180	4.938
62-67	5.503	5.285	4.986	4.760
63-68	5.265	5.017	4.786	4.576
64-69	5.025	4.798	4.585	4.390
65-70	4.783	4.573	4.378	4.199
66-71	4.540	4.349	4.169	4.005
67-72	4.298	4.124	3.960	3.811
68-73	4.059	3.901	3.752	3.616
69-74	3.825	3.683	3.547	3.423
70-75	3.599	3.471	3.347	3.236
71-76	3.386	3.270	3.159	3.059
72-77	3.176	3.070	2.971	2.882
73-78	2.963	2.869	2.780	2.701
74-79	2.743	2.659	2.580	2.511
75-80	2.526	2.448	2.381	2.323
76-81	2.325	2.258	2.195	2.147
77-82	2.131	2.077	2.013	1.975
78-83	1.947	1.899	1.838	1.810



# T A B L E S.

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T A B L E XIX. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
79-84	1.793	1.751	1.750	1.672
80-85	1.645	1.608	1.573	1.539
81-86	1.511	1.478	1.447	1.417
82-87	1.385	1.356	1.329	1.303
83-88	1.234	1.259	1.235	1.212
84-89	1.185	1.164	1.145	1.124
85-90	1.074	1.054	1.038	1.021
86-91	0.921	0.902	0.892	0.879
87-92	0.756	0.738	0.734	0.725
88-93	0.562	0.554	0.547	0.541
89-94	0.377	0.373	0.369	0.365
90-95	0.179	0.177	0.175	0.174
91-96	0.000	0.000	0.000	0.000

## T A B L E XX.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *ten* Years.

Ages.	Value at 3 per Cent	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-11	12.346	10.782	9.544	8.547
2-12	14.239	12.438	11.010	9.857
3-13	14.895	13.019	11.528	10.324
4-14	15.287	13.374	11.850	10.617
5-15	15.391	13.479	11.954	10.716
6-16	15.486	13.578	12.052	10.812
7-17	15.490	13.599	12.083	10.849
8-18	15.436	13.569	12.070	10.847
9-19	15.316	13.482	12.006	10.799
10-20	15.151	13.355	11.906	10.719
11-21	14.974	13.217	11.797	10.631
12-22	14.795	13.078	11.686	10.541
13-23	14.612	12.934	11.570	10.446
14-24	14.424	12.784	11.450	10.348
15-25	14.230	12.630	11.324	10.244
16-26	14.030	12.470	11.193	10.135
17-27	13.832	12.311	11.063	10.027
18-28	13.642	12.158	10.939	9.924
19-29	13.461	12.013	10.820	9.826
20-30	13.286	11.873	10.707	9.732
21-31	13.121	11.742	10.600	9.644

## T A B L E S.

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T A B L E XX. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
22-32	12.661	11.615	10.498	9.561
23-33	12.798	11.485	10.393	9.474
24-34	12.632	11.352	10.285	9.386
25-35	12.463	11.217	10.175	9.295
26-36	12.291	11.078	10.062	9.201
27-37	12.116	10.936	9.946	9.105
28-38	11.937	10.791	9.826	9.005
29-39	11.755	10.642	9.703	8.902
30-40	11.568	10.490	9.576	8.795
31-41	11.382	10.336	9.448	8.688
32-42	11.195	10.182	9.320	8.580
33-43	11.007	10.027	9.190	8.471
34-44	10.817	9.869	9.058	8.358
35-45	10.622	9.706	8.921	8.242
36-46	10.424	9.540	8.781	8.122
37-47	10.221	9.370	8.636	7.998
38-48	10.014	9.195	8.487	7.870
39-49	9.803	9.015	8.333	7.737
40-50	9.590	8.834	8.177	7.602
41-51	9.383	8.658	8.025	7.470
42-52	9.179	8.483	7.875	7.340
43-53	8.975	8.308	7.724	7.208
44-54	8.767	8.130	7.569	7.073
45-55	8.557	7.948	7.411	6.935
46-56	8.344	7.763	7.249	6.793
47-57	8.127	7.574	7.084	6.648
48-58	7.907	7.382	6.915	6.498
49-59	7.684	7.186	6.742	6.344



T A B L E XX. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
50-60	7.461	6.989	6.568	6.189
51-61	7.240	6.795	6.395	6.035
52-62	7.021	6.600	6.222	5.880
53-63	6.795	6.399	6.042	5.719
54-64	6.568	6.196	5.860	5.555
55-65	6.334	5.986	5.671	5.384
56-66	6.098	5.774	5.479	5.209
57-67	5.860	5.559	5.283	5.031
58-68	5.621	5.341	5.084	4.849
59-69	5.380	5.121	4.883	4.665
60-70	5.139	4.900	4.680	4.478
61-71	4.898	4.679	4.476	4.289
62-72	4.659	4.458	4.272	4.099
63-73	4.420	4.236	4.066	3.908
64-74	4.186	4.019	3.864	3.719
65-75	3.958	3.806	3.665	3.533
66-76	3.743	3.606	3.477	3.357
67-77	3.529	3.405	3.289	3.180
68-78	3.310	3.199	3.095	2.996
69-79	3.077	2.979	2.887	2.799
70-80	2.843	2.757	2.675	2.598
71-81	2.618	2.542	2.470	2.402
72-82	2.401	2.334	2.271	2.211
73-83	2.199	2.141	2.085	2.032
74-84	2.043	1.991	1.941	1.894
75-85	1.903	1.856	1.811	1.769
76-86	1.781	1.739	1.699	1.661
77-87	1.670	1.633	1.597	1.562



T A B L E XX. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
78-88	1.580	1.546	1.514	1.483
79-89	1.456	1.427	1.400	1.373
80-90	1.302	1.278	1.255	1.234
81-91	1.096	1.078	1.061	1.044
82-92	0.877	0.864	0.852	0.840
83-93	0.622	0.614	0.606	0.599
84-94	0.498	0.493	0.398	0.394
85-95	0.189	0.187	0.185	0.183
86-96	0.000	0.000	0.000	0.000

## T A B L E XXI.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *fifteen* Years.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-16	11.864	10.406	9.243	8.301
2-17	13.659	11.981	10.642	9.555
3-18	14.277	12.531	11.134	9.998
4-19	14.657	12.876	11.447	10.284
5-20	14.776	12.993	11.561	10.391
6-21	14.904	13.121	11.685	10.510
7-22	14.950	13.178	11.748	10.576
8-23	14.929	13.178	11.761	10.597
9-24	14.834	13.112	11.715	10.566
10-25	14.683	12.998	11.627	10.497
11-26	14.508	12.861	11.519	10.410
12-27	14.323	12.715	11.402	10.314
13-28	14.132	12.564	11.280	10.215
14-29	13.936	12.408	11.153	10.110
15-30	13.734	12.246	11.021	10.001
16-31	13.527	12.078	10.883	9.886
17-32	13.320	11.911	10.746	9.771
18-33	13.121	11.750	10.613	9.660
19-34	12.930	11.595	10.486	9.554
20-35	12.744	11.445	10.363	9.451
21-36	12.567	11.302	10.246	9.354
22-37	12.394	11.163	10.132	9.260
23-38	12.218	11.020	10.015	9.163
24-39	12.038	10.874	9.895	9.063
25-40	11.854	10.725	9.771	8.960

## T A B L E S.

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T A B L E XXI. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
26-41	11.670	10.574	9.647	8.855
27-42	11.486	10.423	9.522	8.751
28-43	11.302	10.272	9.396	8.645
29-44	11.114	10.117	9.267	8.536
30-45	10.923	9.959	9.135	8.424
31-46	10.728	9.797	8.998	8.309
32-47	10.530	9.631	8.858	8.189
33-48	10.327	9.461	8.714	8.066
34-49	10.120	9.286	8.565	7.938
35-50	9.912	9.110	8.415	7.809
36-51	9.707	8.937	8.267	7.681
37-52	9.503	8.763	8.119	7.553
38-53	9.296	8.586	7.966	7.421
39-54	9.085	8.406	7.810	7.286
40-55	8.870	8.221	7.651	7.146
41-56	8.655	8.035	7.489	7.005
42-57	8.439	7.848	7.326	6.862
43-58	8.222	7.660	7.162	6.718
44-59	8.003	7.469	6.994	6.570
45-60	7.781	7.274	6.822	6.418
46-61	7.556	7.076	6.648	6.263
47-62	7.328	6.875	6.469	6.104
48-63	7.093	6.667	6.283	5.937
49-64	6.854	6.454	6.093	5.767
50-65	6.611	6.236	5.897	5.590
51-66	6.369	6.019	5.701	5.412
52-67	6.127	5.801	5.504	5.233
53-68	5.884	5.580	5.303	5.050
54-69	5.638	5.357	5.100	4.864



T A B L E XXI. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
55-70	5.391	5.132	4.893	4.674
56-71	5.145	4.905	4.685	4.482
57-72	4.899	4.679	4.477	4.289
58-73	4.656	4.455	4.269	4.096
59-74	4.418	4.234	4.064	3.906
60-75	4.189	4.021	3.866	3.721
61-76	3.974	3.821	3.679	3.546
62-77	3.760	3.621	3.492	3.371
63-78	3.538	3.414	3.297	3.188
64-79	3.303	3.192	3.088	2.990
65-80	3.063	2.965	2.873	2.786
66-81	2.833	2.746	2.664	2.587
67-82	2.610	2.533	2.461	2.393
68-83	2.403	2.336	2.272	2.211
69-84	2.244	2.183	2.126	2.071
70-85	2.097	2.042	1.991	1.941
71-86	1.963	1.914	1.867	1.823
72-87	1.838	1.794	1.753	1.713
73-88	1.736	1.697	1.660	1.625
74-89	1.603	1.570	1.538	1.508
75-90	1.440	1.413	1.387	1.361
76-91	1.221	1.200	1.180	1.160
77-92	0.985	0.970	0.955	0.942
78-93	0.706	0.697	0.688	0.679
79-94	0.458	0.453	0.448	0.443
80-95	0.210	0.208	0.206	0.204
81-96	0.000	0.000	0.000	0.000



## T A B L E XXII.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton Table of Observations*, p. 36.

Difference of Age *twenty Years*.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-21	11.413	10.053	8.961	8.070
2-22	13.172	11.605	10.344	9.313
3-23	13.794	12.161	10.843	9.764
4-24	14.178	12.511	11.163	10.057
5-25	14.301	12.633	11.281	10.170
6-26	14.420	12.754	11.400	10.285
7-27	14.451	12.798	11.452	10.341
8-28	14.417	12.786	11.455	10.354
9-29	14.310	12.710	11.401	10.315
10-30	14.150	12.586	11.304	10.239
11-31	13.965	12.441	11.188	10.144
12-32	13.770	12.286	11.062	10.042
13-33	13.570	12.125	10.932	9.934
14-34	13.363	11.959	10.796	9.822
15-35	13.151	11.787	10.655	9.703
16-36	12.932	11.609	10.507	9.579
17-37	12.714	11.430	10.358	9.454
18-38	12.502	11.257	10.214	9.333
19-39	12.297	11.089	10.074	9.215
20-40	12.096	10.924	9.937	9.100
21-41	11.906	10.768	9.809	8.992
22-42	11.723	10.619	9.685	8.889
23-43	11.540	10.470	9.562	8.785
24-44	11.354	10.317	9.435	8.679

T A B L E XXII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
25-45	11.164	10.160	9.304	8.569
26-46	10.970	10.000	9.170	8.455
27-47	10.773	9.836	9.032	8.338
28-48	10.572	9.667	8.890	8.217
29-49	10.366	9.495	8.744	8.092
30-50	10.160	9.321	8.596	7.966
31-51	9.957	9.151	8.451	7.841
32-52	9.756	8.980	8.306	7.716
33-53	9.550	8.806	8.157	7.588
34-54	9.342	8.629	8.005	7.457
35-55	9.131	8.448	7.849	7.322
36-56	8.916	8.264	7.690	7.183
37-57	8.699	8.076	7.527	7.041
38-58	8.477	7.884	7.360	6.894
39-59	8.253	7.689	7.189	6.744
40-60	8.025	7.490	7.015	6.590
41-61	7.796	7.290	6.838	6.434
42-62	7.567	7.088	6.660	6.276
43-63	7.332	6.881	6.477	6.112
44-64	7.095	6.671	6.289	5.944
45-65	6.850	6.453	6.094	5.769
46-66	6.602	6.230	5.894	5.588
47-67	6.351	6.004	5.690	5.403
48-68	6.096	5.774	5.481	5.213
49-69	5.839	5.541	5.268	5.019
50-70	5.582	5.306	5.054	4.822
51-71	5.328	5.074	4.841	4.626
52-72	5.077	4.845	4.630	4.430

T A B L E XXII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
53-73	4.829	4.614	4.417	4.234
54-74	4.585	4.389	4.208	4.040
55-75	4.350	4.171	4.006	3.852
56-76	4.129	3.966	3.815	3.674
57-77	3.908	3.761	3.623	3.494
58-78	3.682	3.549	3.424	3.308
59-79	3.440	3.322	3.210	3.105
60-80	3.197	3.092	2.992	2.899
61-81	2.964	2.870	2.782	2.699
62-82	2.739	2.656	2.578	2.504
63-83	2.530	2.457	2.387	2.321
64-84	2.371	2.305	2.242	2.182
65-85	2.223	2.163	2.107	2.053
66-86	2.080	2.035	1.984	1.936
67-87	1.963	1.915	1.870	1.826
68-88	1.860	1.817	1.777	1.737
69-89	1.722	1.685	1.650	1.616
70-90	1.545	1.515	1.486	1.459
71-91	1.303	1.280	1.259	1.238
72-92	1.044	1.028	1.012	0.997
73-93	0.743	0.733	0.723	0.714
74-94	0.480	0.474	0.469	0.464
75-95	0.219	0.217	0.215	0.213
76-96	0.000	0.000	0.000	0.000



## T A B L E XXIII.

Shewing the Value of an Annuity on the *joint* Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *twenty-five* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-26	11.037	9.770	8.742	7.897
2-27	12.722	11.264	10.080	9.104
3-28	13.307	11.790	10.555	9.537
4-29	13.661	12.116	10.855	9.813
5-30	13.762	12.220	10.959	9.913
6-31	13.859	12.322	11.062	10.015
7-32	13.871	12.350	11.100	10.060
8-33	13.820	12.323	11.090	10.061
9-34	13.698	12.234	11.024	10.012
10-35	13.525	12.098	10.916	9.925
11-36	13.328	11.941	10.788	9.820
12-37	13.120	11.773	10.651	9.707
13-38	12.906	11.600	10.509	9.588
14-39	12.686	11.420	10.360	9.464
15-40	12.459	11.234	10.205	9.333
16-41	12.229	11.044	10.046	9.198
17-42	12.002	10.856	9.889	9.065
18-43	11.785	10.677	9.739	8.938
19-44	11.574	10.502	9.592	8.814
20-45	11.367	10.330	9.448	8.692
21-46	11.167	10.165	9.310	8.574
22-47	10.969	10.001	9.173	8.458
23-48	10.768	9.833	9.031	8.338



# T A B L E S.

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TABLE XXIII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
24-49	10.562	9.661	8.886	8.214
25-50	10.356	9.488	8.739	8.089
26-51	10.154	9.318	8.595	7.966
27-52	9.952	9.148	8.451	7.842
28-53	9.748	8.975	8.304	7.716
29-54	9.540	8.799	8.153	7.586
30-55	9.329	8.619	7.999	7.453
31-56	9.115	8.436	7.841	7.316
32-57	8.897	8.250	7.680	7.175
33-58	8.677	8.060	7.515	7.031
34-59	8.454	7.866	7.346	6.884
35-60	8.227	7.669	7.174	6.732
36-61	7.997	7.469	6.998	6.577
37-62	7.765	7.265	6.819	6.418
38-63	7.525	7.053	6.631	6.252
39-64	7.281	6.838	6.440	6.081
40-65	7.030	6.614	6.240	5.901
41-66	6.776	6.388	6.037	5.718
42-67	6.522	6.159	5.831	5.532
43-68	6.266	5.929	5.622	5.343
44-69	6.008	5.696	5.411	5.150
45-70	5.749	5.460	5.195	4.953
46-71	5.488	5.222	4.978	4.753
47-72	5.228	4.983	4.758	4.551
48-73	4.970	4.746	4.539	4.348
49-74	4.716	4.511	4.322	4.146
50-75	4.472	4.285	4.112	3.951
51-76	4.245	4.074	3.916	3.768

T A B L E XXIII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
52-77	4.019	3.864	3.720	3.586
53-78	3.787	3.648	3.518	3.396
54-79	3.540	3.416	3.299	3.189
55-80	3.291	3.180	3.076	2.978
56-81	3.051	2.953	2.861	2.774
57-82	2.820	2.733	2.651	2.574
58-83	2.608	2.530	2.457	2.388
59-84	2.446	2.376	2.310	2.247
60-85	2.297	2.234	2.174	2.118
61-86	2.162	2.105	2.051	2.000
62-87	2.036	1.985	1.937	1.891
63-88	1.932	1.886	1.843	1.802
64-89	1.790	1.751	1.714	1.678
65-90	1.606	1.575	1.544	1.515
66-91	1.354	1.330	1.307	1.285
67-92	1.083	1.067	1.050	1.035
68-93	0.770	0.760	0.750	0.740
69-94	0.497	0.491	0.485	0.480
70-95	0.227	0.224	0.222	0.220
71-96	0.000	0.000	0.000	0.000

## T A B L E S.

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## T A B L E XXIV.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *thirty* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-31	10.605	9.438	8.483	7.691
2-32	12.203	10.865	9.767	8.855
3-33	12.743	11.355	10.213	9.263
4-34	13.061	11.651	10.488	9.518
5-35	13.136	11.732	10.572	9.602
6-36	13.207	11.812	10.656	9.687
7-37	13.195	11.819	10.676	9.715
8-38	13.122	11.772	10.648	9.701
9-39	12.981	11.665	10.565	9.637
10-40	12.791	11.513	10.442	9.537
11-41	12.580	11.342	10.302	9.420
12-42	12.363	11.165	10.156	9.298
13-43	12.144	10.985	10.007	9.173
14-44	11.918	10.799	9.852	9.042
15-45	11.687	10.607	9.690	8.905
16-46	11.448	10.408	9.522	8.762
17-47	11.210	10.208	9.353	8.617
18-48	10.975	10.011	9.186	8.473
19-49	10.746	9.818	9.021	8.332
20-50	10.523	9.630	8.861	8.195
21-51	10.313	9.454	8.712	8.067
22-52	10.111	9.284	8.568	7.944



T A B L E XXIV. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
23-53	9.905	9.111	8.421	7.818
24-54	9.696	8.934	8.270	7.688
25-55	9.484	8.754	8.116	7.555
26-56	9.269	8.570	7.958	7.419
27-57	9.051	8.383	7.797	7.279
28-58	8.830	8.193	7.632	7.135
29-59	8.605	7.999	7.464	6.988
30-60	8.378	7.802	7.292	6.837
31-61	8.147	7.601	7.116	6.682
32-62	7.914	7.397	6.937	6.524
33-63	7.673	7.186	6.750	6.359
34-64	7.429	6.971	6.559	6.189
35-65	7.177	6.747	6.360	6.010
36-66	6.922	6.520	6.156	5.827
37-67	6.663	6.288	5.948	5.639
38-68	6.401	6.052	5.735	5.446
39-69	6.137	5.813	5.518	5.249
40-70	5.871	5.571	5.298	5.047
41-71	5.605	5.329	5.076	4.844
42-72	5.341	5.087	4.854	4.640
43-73	5.081	4.848	4.634	4.436
44-74	4.826	4.613	4.417	4.235
45-75	4.580	4.386	4.206	4.040
46-76	4.348	4.171	4.006	3.853
47-77	4.115	3.954	3.805	3.666
48-78	3.875	3.731	3.596	3.469
49-79	3.619	3.490	3.369	3.256
50-80	3.362	3.247	3.140	3.039



T A B L E XXIV. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
51-81	3.117	3.015	2.920	2.829
52-82	2.882	2.792	2.707	2.627
53-83	2.665	2.585	2.510	2.438
54-84	2.501	2.428	2.360	2.295
55-85	2.349	2.284	2.222	2.164
56-86	2.211	2.153	2.097	2.044
57-87	2.082	2.030	1.980	1.932
58-88	1.975	1.928	1.883	1.841
59-89	1.828	1.788	1.750	1.713
60-90	1.641	1.608	1.577	1.547
61-91	1.382	1.358	1.334	1.311
62-92	1.105	1.088	1.071	1.055
63-93	0.785	0.774	0.764	0.754
64-94	0.506	0.500	0.494	0.489
65-95	0.230	0.228	0.226	0.224
66-96	0.000	0.000	0.000	0.000

## T A B L E XXV.

Shewing the Value of an Annuity on the *joint Continuance* of Two Lives, according to the *Northampton Table of Observations*, p. 36.

Difference of Age *thirty-five Years*.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-36	10.104	9.047	8.173	7.442
2-37	11.600	10.392	9.390	8.551
3-38	12.087	10.838	9.800	8.928
4-39	12.362	11.097	10.043	9.157
5-40	12.405	11.150	10.102	9.219
6-41	12.446	11.203	10.163	9.283
7-42	12.412	11.190	10.165	9.296
8-43	12.325	11.130	10.124	9.270
9-44	12.174	11.012	10.031	9.197
10-45	11.976	10.851	9.900	9.088
11-46	11.756	10.697	9.774	8.962
12-47	11.525	10.481	9.592	8.827
13-48	11.288	10.284	9.425	8.686
14-49	11.045	10.080	9.252	8.538
15-50	10.799	9.872	9.076	8.386
16-51	10.554	9.665	8.899	8.234
17-52	10.313	9.461	8.724	8.083
18-53	10.076	9.260	8.552	7.934
19-54	9.845	9.063	8.383	7.788
20-55	9.617	8.869	8.216	7.643
21-56	9.394	8.679	8.053	7.502
22-57	9.174	8.491	7.891	7.362
23-58	8.951	8.299	7.725	7.218
24-59	8.725	8.104	7.556	7.070
25-60	8.495	7.906	7.383	6.919
26-61	8.263	7.704	7.207	6.764
27-62	8.028	7.499	7.027	6.605
28-63	7.785	7.286	6.839	6.439

# T A B L E S.

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T A B L E XXV. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
29-64	7.539	7.069	6.648	6.268
30-65	7.286	6.844	6.447	6.089
31-66	7.028	6.615	6.243	5.905
32-67	6.768	6.382	6.033	5.717
33-68	6.504	6.146	5.820	5.524
34-69	6.239	5.906	5.603	5.326
35-70	5.971	5.663	5.382	5.125
36-71	5.703	5.419	5.159	4.920
37-72	5.435	5.174	4.934	4.714
38-73	5.169	4.930	4.710	4.507
39-74	4.908	4.690	4.488	4.301
40-75	4.656	4.457	4.272	4.101
41-76	4.420	4.238	4.069	3.912
42-77	4.184	4.019	3.865	3.722
43-78	3.942	3.794	3.655	3.525
44-79	3.685	3.552	3.428	3.312
45-80	3.426	3.308	3.197	3.093
46-81	3.176	3.072	2.973	2.881
47-82	2.936	2.843	2.756	2.673
48-83	2.714	2.632	2.554	2.481
49-84	2.544	2.470	2.400	2.334
50-85	2.388	2.322	2.258	2.198
51-86	2.248	2.188	2.131	2.077
52-87	2.117	2.063	2.012	1.963
53-88	2.008	1.960	1.914	1.870
54-89	1.858	1.817	1.778	1.740
55-90	1.666	1.633	1.601	1.570
56-91	1.402	1.377	1.353	1.330
57-92	1.120	1.102	1.085	1.069
58-93	0.794	0.784	0.773	0.763
59-94	0.511	0.505	0.499	0.494
60-95	0.233	0.230	0.228	0.226
61-96	0.000	0.000	0.000	0.000



## T A B L E XXVI.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *forty* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-41	9.523	8.585	7.800	7.135
2-42	10.907	9.839	8.942	8.182
3-43	11.343	10.242	9.315	8.528
4-44	11.578	10.468	9.531	8.733
5-45	11.597	10.500	9.571	8.778
6-46	11.610	10.528	9.609	8.823
7-47	11.550	10.491	9.589	8.815
8-48	11.435	10.404	9.524	8.767
9-49	11.260	10.263	9.409	8.673
10-50	11.044	10.085	9.260	8.548
11-51	10.816	9.894	9.100	8.411
12-52	10.582	9.698	8.934	8.270
13-53	10.344	9.497	8.763	8.123
14-54	10.100	9.290	8.586	7.970
15-55	9.851	9.077	8.403	7.812
16-56	9.595	8.858	8.214	7.648
17-57	9.340	8.639	8.024	7.481
18-58	9.089	8.422	7.835	7.316
19-59	8.841	8.207	7.648	7.153
20-60	8.597	7.995	7.463	6.990
21-61	8.357	7.787	7.281	6.830
22-62	8.119	7.580	7.100	6.670
23-63	7.874	7.365	6.910	6.503
24-64	7.626	7.147	6.717	6.331
25-65	7.370	6.920	6.515	6.151
26-66	7.110	6.689	6.309	5.966
27-67	6.847	6.454	6.098	5.776



T A B L E XXVI. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
28-68	6.581	6.215	5.883	5.581
29-69	6.313	5.973	5.664	5.383
30-70	6.043	5.729	5.442	5.180
31-71	5.772	5.483	5.218	4.974
32-72	5.502	5.236	4.992	4.767
33-73	5.235	4.991	4.766	4.559
34-74	4.973	4.749	4.543	4.353
35-75	4.720	4.516	4.327	4.152
36-76	4.481	4.295	4.123	3.962
37-77	4.242	4.073	3.916	3.770
38-78	3.996	3.844	3.702	3.570
39-79	3.734	3.598	3.471	3.352
40-80	3.469	3.349	3.236	3.130
41-81	3.216	3.109	3.009	2.914
42-82	2.973	2.878	2.789	2.705
43-83	2.750	2.666	2.587	2.511
44-84	2.581	2.505	2.433	2.365
45-85	2.424	2.356	2.291	2.230
46-86	2.282	2.221	2.162	2.107
47-87	2.148	2.093	2.041	1.991
48-88	2.036	1.987	1.941	1.895
49-89	1.882	1.840	1.800	1.761
50-90	1.685	1.651	1.619	1.590
51-91	1.417	1.391	1.367	1.343
52-92	1.130	1.113	1.095	1.079
53-93	0.801	0.790	0.780	0.770
54-94	0.515	0.509	0.503	0.498
55-95	0.234	0.232	0.230	0.228
56-96	0.000	0.000	0.000	0.000

## T A B L E XXVII.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *forty-five* Years.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-46	8.888	8.071	7.379	6.787
2-47	10.147	9.221	8.435	7.760
3-48	10.515	9.566	8.759	8.063
4-49	10.697	9.744	8.932	8.230
5-50	10.679	9.742	8.941	8.248
6-51	10.664	9.745	8.956	8.271
7-52	10.586	9.690	8.919	8.248
8-53	10.458	9.591	8.841	8.188
9-54	10.276	9.442	8.718	8.085
10-55	10.055	9.256	8.560	7.951
11-56	9.814	9.052	8.386	7.801
12-57	9.566	8.839	8.203	7.643
13-58	9.312	8.622	8.015	7.479
14-59	9.053	8.399	7.821	7.310
15-60	8.790	8.170	7.622	7.135
16-61	8.521	7.935	7.416	6.953
17-62	8.252	7.700	7.208	6.770
18-63	7.981	7.462	6.998	6.583
19-64	7.714	7.226	6.789	6.396
20-65	7.444	6.986	6.576	6.205
21-66	8.177	6.749	6.364	6.015
22-67	6.911	6.512	6.151	5.824
23-68	6.643	6.271	5.934	5.628
24-69	6.372	6.027	5.713	5.427
25-70	6.099	5.780	5.489	5.223

T A B L E XXVII. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
26-71	5.826	5.532	5.263	5.016
27-72	5.554	5.283	5.035	4.807
28-73	5.284	5.036	4.808	4.597
29-74	5.019	4.792	4.583	4.390
30-75	4.764	4.557	4.365	4.188
31-76	4.523	4.335	4.160	3.997
32-77	4.282	4.111	3.952	3.804
33-78	4.035	3.881	3.737	3.602
34-79	3.771	3.633	3.505	3.384
35-80	3.506	3.383	3.268	3.160
36-81	3.251	3.142	3.040	2.944
37-82	3.005	2.909	2.818	2.733
38-83	2.779	2.694	2.613	2.537
39-84	2.607	2.530	2.457	2.388
40-85	2.448	2.379	2.313	2.251
41-86	2.304	2.241	2.182	2.126
42-87	2.168	2.113	2.060	2.009
43-88	2.055	2.006	1.959	1.914
44-89	1.901	1.859	1.818	1.779
45-90	1.702	1.668	1.635	1.604
46-91	1.431	1.405	1.380	1.356
47-92	1.140	1.122	1.105	1.089
48-93	0.808	0.797	0.786	0.776
49-94	0.519	0.512	0.507	0.501
50-95	0.235	0.233	0.231	0.229
51-96	0.000	0.000	0.000	0.000



## T A B L E XXVIII.

Shewing the Value of an Annuity on the *joint* Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *fifty* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-51	8.171	7.479	6.855	6.370
2-52	9.300	8.520	7.848	7.264
3-53	9.611	8.815	8.128	7.529
4-54	9.751	8.957	8.269	7.668
5-55	9.707	8.931	8.256	7.665
6-56	9.659	8.902	8.241	7.662
7-57	9.549	8.817	8.176	7.612
8-58	9.395	8.691	8.073	7.527
9-59	9.191	8.519	7.927	7.403
10-60	8.952	8.314	7.750	7.250
11-61	8.696	8.092	7.557	7.081
12-62	8.433	7.863	7.357	6.905
13-63	8.161	7.625	7.147	6.719
14-64	7.884	7.381	6.931	6.527
15-65	7.597	7.127	6.705	6.325
16-66	7.304	6.866	6.472	6.115
17-67	7.012	6.604	6.236	5.903
18-68	6.721	6.343	6.001	5.689
19-69	6.434	6.084	5.766	5.476
20-70	6.149	5.826	5.532	5.262
21-71	5.870	5.572	5.300	5.050
22-72	5.595	5.321	5.070	4.840
23-73	5.323	5.072	4.841	4.628

T A B L E XXVIII, continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
24-74	5.056	4.827	4.615	4.419
25-75	4.799	4.589	4.396	4.216
26-76	4.556	4.365	4.188	4.024
27-77	4.313	4.140	3.979	3.829
28-78	4.064	3.908	3.762	3.626
29-79	3.798 <sup>o</sup>	3.659	3.528	3.406
30-80	3.530	3.406	3.290	3.181
31-81	3.274	3.164	3.060	2.963
32-82	3.027	2.929	2.838	2.751
33-83	2.800	2.713	2.632	2.555
34-84	2.627	2.549	2.476	2.406
35-85	2.468	2.398	2.331	2.268
36-86	2.323	2.260	2.200	2.143
37-87	2.187 <sup>o</sup>	2.130	2.077	2.026
38-88	2.072	2.022	1.974	1.929
39-89	1.915	1.872	1.832	1.792
40-90	1.713	1.679	1.646	1.614
41-91	1.439	1.413	1.388	1.364
42-92	1.146	1.128	1.111	1.094
43-93	0.811	0.800	0.790	0.779
44-94	0.521	0.515	0.509	0.503
45-95	0.236	0.234	0.232	0.230
46-96	0.000	0.000	0.000	0.000

## T A B L E XXIX.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *fifty-five* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-56	7.412	6.843	6.346	5.911
2-57	8.392	7.756	7.199	6.709
3-58	8.630	7.986	7.421	6.922
4-59	8.712	8.075	7.514	7.017
5-60	8.629	8.011	7.466	6.982
6-61	8.542	7.944	7.415	6.945
7-62	8.400	7.828	7.319	6.865
8-63	8.214	7.669	7.184	6.750
9-64	7.984	7.470	7.010	6.598
10-65	7.718	7.236	6.803	6.414
11-66	7.437	6.987	6.581	6.215
12-67	7.149	6.730	6.351	6.009
13-68	6.857	6.468	6.116	5.796
14-69	6.562	6.202	5.876	5.578
15-70	6.264	5.933	5.631	5.355
16-71	5.964	5.660	5.382	5.127
17-72	5.667	5.389	5.133	4.899
18-73	5.378	5.123	4.889	4.673
19-74	5.098	4.866	4.651	4.453
20-75	4.831	4.619	4.424	4.242
21-76	4.583	4.391	4.212	4.046
22-77	4.339	4.164	4.001	3.850
23-78	4.087	3.930	3.783	3.646



T A B L E XXIX. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at. 6 per Ct.
24-79	3.820	3.679	3.548	3.424
25-80	3.550	3.425	3.308	3.198
26-81	3.292	3.181	3.077	2.979
27-82	3.043	2.945	2.853	2.765
28-83	2.815	2.728	2.646	2.568
29-84	2.641	2.563	2.489	2.418
30-85	2.481	2.411	2.344	2.280
31-86	2.336	2.272	2.212	2.154
32-87	2.198	2.142	2.088	2.036
33-88	2.083	2.033	1.985	1.939
34-89	1.925	1.882	1.841	1.802
35-90	1.723	1.688	1.654	1.622
36-91	1.446	1.420	1.395	1.371
37-92	1.152	1.134	1.116	1.099
38-93	0.815	0.804	0.793	0.783
39-94	0.523	0.517	0.511	0.505
40-95	0.237	0.235	0.233	0.231
41-96	0.000	0.000	0.000	0.000

## TABLE XXX.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *sixty* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-61	6.571	6.123	5.725	5.372
2-62	7.391	6.894	6.452	6.059
3-63	7.545	7.048	6.605	6.209
4-64	7.562	7.076	6.641	6.251
5-65	7.429	6.963	6.546	6.171
6-66	7.290	6.846	6.447	6.087
7-67	7.104	6.684	6.306	5.963
8-68	6.884	6.490	6.134	5.811
9-69	6.628	6.262	5.929	5.626
10-70	6.347	6.008	5.700	5.418
11-71	6.056	5.744	5.460	5.199
12-72	5.763	5.478	5.216	4.976
13-73	5.473	5.212	4.972	4.751
14-74	5.188	4.950	4.731	4.528
15-75	4.911	4.695	4.495	4.310
16-76	4.649	4.452	4.270	4.101
17-77	4.388	4.210	4.045	3.892
18-78	4.123	3.964	3.815	3.677
19-79	3.846	3.704	3.571	3.447
20-80	3.569	3.443	3.325	3.214
21-81	3.307	3.195	3.091	2.992
22-82	3.057	2.958	2.865	2.777

T A B L E XXX. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
23-83	2.828	2.740	2.657	2.579
24-84	2.653	2.574	2.499	2.429
25-85	2.492	2.421	2.354	2.290
26-86	2.348	2.282	2.221	2.163
27-87	2.208	2.151	2.096	2.044
28-88	2.091	2.041	1.992	1.946
29-89	1.933	1.889	1.848	1.808
30-90	1.729	1.694	1.660	1.628
31-91	1.451	1.425	1.400	1.376
32-92	1.155	1.137	1.119	1.102
33-93	0.817	0.806	0.795	0.785
34-94	0.524	0.518	0.512	0.506
35-95	0.238	0.235	0.233	0.231
36-96	0.000	0.000	0.000	0.000



## T A B L E XXXI.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton Table of Observations*, p. 36.

Difference of Age *sixty-five* Years.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-66	5.633	5.295	4.996	4.728
2-67	6.266	5.896	5.569	5.276
3-68	6.330	5.965	5.641	5.352
4-69	6.277	5.924	5.611	5.332
5-70	6.102	5.768	5.472	5.209
6-71	5.925	5.610	5.331	5.084
7-72	5.714	5.418	5.157	4.929
8-73	5.480	5.204	4.963	4.752
9-74	5.225	4.969	4.747	4.556
10-75	4.962	4.725	4.522	4.350
11-76	4.707	4.487	4.301	4.148
12-77	4.449	4.368	4.195	3.943
13-78	4.185	4.022	3.871	3.729
14-79	3.904	3.759	3.624	3.497
15-80	3.621	3.492	3.372	3.259
16-81	3.348	3.235	3.128	3.028
17-82	3.087	2.987	2.893	2.804
18-83	2.849	2.760	2.677	2.598
19-84	2.668	2.589	2.513	2.442
20-85	2.503	2.431	2.364	2.299
21-86	2.354	2.290	2.229	2.171
22-87	2.216	2.158	2.104	2.051
23-88	2.099	2.048	1.999	1.953
24-89	1.939	1.895	1.854	1.814
25-90	1.734	1.699	1.665	1.633
26-91	1.455	1.429	1.404	1.379
27-92	1.158	1.140	1.122	1.105
28-93	0.819	0.808	0.797	0.786
29-94	0.525	0.519	0.513	0.507
30-95	0.238	0.236	0.234	0.231
31-96	0.000	0.000	0.000	0.000

## T A B L E XXXII.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *seventy* Years.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-71	4.611	4.380	4.169	3.976
2-72	5.061	4.814	4.588	4.380
3-73	5.051	4.811	4.591	4.389
4-74	4.953	4.726	4.516	4.323
5-75	4.768	4.557	4.362	4.181
6-76	4.599	4.403	4.221	4.053
7-77	4.402	4.222	4.055	3.899
8-78	4.180	4.016	3.864	3.722
9-79	3.921	3.775	3.638	3.510
10-80	3.647	3.517	3.395	3.281
11-81	3.380	3.264	3.156	3.054
12-82	3.122	3.020	2.924	2.833
13-83	2.884	2.794	2.709	2.628
14-84	2.703	2.622	2.545	2.472
15-85	2.535	2.462	2.393	2.327
16-86	2.380	2.315	2.253	2.194
17-87	2.235	2.177	2.121	2.069
18-88	2.112	2.061	2.012	1.965
19-89	1.948	1.904	1.862	1.822
20-90	1.739	1.704	1.670	1.638
21-91	1.459	1.432	1.407	1.382
22-92	1.160	1.142	1.124	1.107
23-93	0.820	0.809	0.798	0.788
24-94	0.526	0.520	0.514	0.508
25-95	0.238	0.236	0.234	0.232
26-96	0.000	0.000	0.000	0.000

*Directions for using the preceding Tables of the  
Values of Two joint Lives.*

IF the two lives have the same common age, or their difference of age is five years, or any multiple of five years, the value of their joint continuance is expressed in the Tables, and may be found by inspection.

If their difference of age is any number of years between 1 and 5, 5 and 10, 10 and 15, &c. the required value may be easily found by the following rule.

“ Find, in the preceding Tables, the value of two joint lives, whose difference of age is that multiple of 5 which is greater than, but at the same time nearest to the difference of age between the proposed lives; and the oldest of which is of the same age with the oldest of the proposed lives.—Find also, in the preceding Tables, the value of two joint lives whose difference of age is five years less than the multiple of 5 just mentioned; and the oldest of which is, in like manner, of the same age with the oldest of the proposed lives; and the 1st, 2d, 3d, or 4th arithmetical mean between the least and the greatest of these two values will be the value sought, according as one of the proposed lives is one year, 2 years, 3 years, or 4 years younger than the other.”

EXAMPLE.



## EXAMPLE.

Let the value be required of two joint lives aged 15 and 18, reckoning interest at 3 per cent.

That multiple of 5 which is greater than the difference between these ages, but comes nearest to it, is 5.—The value of two joint lives, whose difference of age is 5 years, and the oldest of which is of the same age with the oldest of the two proposed lives; that is, the value of two joint lives aged 18 and 13, is by Table 19th, 15.086. The value of two joint lives whose difference of age is 5 years less, and one of which is also 18; that is, the value of two joint lives aged 18 and 18, is, by Table 18th, 14.516.—These, then, being the values of two joint lives aged 18 and 13, and of two joint lives aged 18 and 18, it is obvious that the value of two joint lives, aged 18 and 15, must be the *third* of four arithmetical means between 14.516 and 15.086.

N. B. The 1st, 2d, 3d, or 4th arithmetical mean between the least and greatest of any two values, is the least increased by 1, 2, 3, or 4 fifths of the difference between them.

In the present instance, the difference between the two values is .570; its fifth part

is .114; and 14.516 increased by thrice this fifth part, makes 14.858, the required value of two joint lives aged 18 and 15.

### EXAMPLE II.

Let the value be required of two joint lives aged 31 and 45, reckoning interest at 3 *per cent.*

That multiple of 5 which is the next greater number to 14 (the difference of age between 45 and 31), is 15. The value of two joint lives, whose difference of age is this number, and the oldest of which is of the same age with the oldest of the proposed lives; that is, the value of two joint lives aged 45 and 30, is, by Table 21st, 10.923.

The value of two joint lives, whose difference of age is 5 years less than 15, and the oldest of which is, in like manner, of the same age with the oldest of the proposed lives; that is, the value of two joint lives aged 45 and 35, is, by Table 20th, 10.622.

These then being the values of two joint lives aged 45 and 30, and of two joint lives aged 45 and 35, it follows that the value of two joint lives aged 45 and 31, must be the 4th of 4 arithmetical means between the least and the greatest of these two values. That is; it is 10.622 (the least) increased by four-fifths of .301 (the difference),

rence), or by .240, which makes 10.862 the required value of two joint lives aged 45 and 31.

In the same manner may the values not specified in the Tables be found universally for any of the four rates of interest. And that they are sufficiently correct, will appear from the following comparison.

*Values of two joint Lives by the Rule just explained, reckoning interest at 3 per cent. compared with the correct Values.*

Ages.	Value by Rule.	Correct Value.
18 and 14	14.972	14.978
18 and 15	14.858	14.864
18 and 16	14.744	14.744
18 and 17	14.630	14.626

Ages.	Value by Rule.	Correct Value.
45 and 31	10.862	10.869
45 and 32	10.802	10.811
45 and 33	10.742	10.751
45 and 34	10.682	10.688

Ages.	Value by Rule.	Correct Value.
66 and 27	7.092	7.095
66 and 28	7.076	7.080
66 and 29	7.060	7.063
66 and 30	7.044	7.046

In the higher rates of interest the agreement is greater.

I have been enabled to make this comparison by the Tables in the office for



*Equitable* Assurances, where, in order to lay the foundation of accuracy in conducting the business of the office, it has been thought necessary to compute minutely to four places of decimals the values by the *Northampton* Observations, at 3 per cent. of two joint lives for every possible difference of age.

The values of any two joint lives being given, the values of the longest of any two single lives are obtained by the following rule.

“ From the sum of the values of the  
 “ single lives subtract the value of their  
 “ joint continuance. The remainder will  
 “ be the value of the longest of the two  
 “ lives.”

In the former editions of this work, I gave a table of these values; but it is so easy to compute them by this rule, that it is by no means worth while to swell this volume with any such table.

EXAMPLE. Let it be required to find the value of the longest of two lives aged 10 and 15, interest being at 4 per cent.

The value of a life aged 10, is, by Table 17th, 17.523. The value of a life aged 15, is 16.791. The sum of these two values is 34.314. The value of the joint continuance of these two lives is (by Table 19th) 13.992, which subtracted from 34.314, leaves 20.322, the value sought.

In

In the First Volume, p. 173, I signified my intention to insert, in this collection, the tables of the office just mentioned for *Equitable Assurances*. Some of these tables have been already inserted; namely, Table 6th, and the columns shewing the values at 3 *per cent.* in all the Tables from the 17th to the last Table.—The values of single and joint lives have been calculated in the office for this rate of interest, because it is the interest by which it regulates all its demands. The values, in the preceding Tables, for the *other* rates of interest, have been calculated with much labour for this work, in order to set aside all occasion for having recourse to Mr. *De Moivre's* hypothesis. See Vol. I. p. 308, &c.—The remaining Tables of this office are those that follow.

## T A B L E XXXIII.

Shewing the Value of an Annuity on a single Life, for 1, 2, 3, 5, and 7 Years, reckoning the Probabilities of living at every Age as they are given in Table VI. and Interest at 3 *per cent*.

Ages.	One Year.	Two Years.	Three Years	Five Years.	Seven Years.
10	.962	1.887	2.778	4.459	6.015
15	.962	1.886	2.774	4.443	5.971
20	.957	1.873	2.748	4.385	5.880
25	.956	1.868	2.740	4.367	5.849
30	.954	1.864	2.733	4.349	5.816
35	.953	1.860	2.724	4.328	5.777
40	.951	1.853	2.710	4.294	5.716
45	.948	1.845	2.694	4.256	5.646
50	.943	1.832	2.669	4.195	5.538
55	.938	1.818	2.641	4.128	5.420
60	.932	1.798	2.604	4.041	5.266
65	.923	1.773	2.554	3.919	5.045



## T A B L E XXXIV.

Shewing the Value of an Assurance of 100*l*. on a single Life, for 1, 5, or 7 Years, or the whole Duration of Life; reckoning the Probabilities of living as they are in the NORTHAMPTON Table of Observations (or Table VI.), and interest at 3 *per cent*.

N. B. With respect to the values in this Table, and also in those that follow to Table XXXVI. it must be remembered, that the values in *annual* payments suppose, that the first payment is made at the time of purchasing; and also that a purchaser is allowed his option either to pay the value of the Assurance in the *annual* payments, or in the *single* payments specified in the Table; and that whichever of these he chuses, he is excluded the other.

Ages.	1 Year. Pre- mium.	5 Years.		7 Years.		Whole durat. of life.	
		Single Premium	Annual Premium	Single Premium	Annual Premium	Single Premium	Annual Premium
8	1.336	4.632	1.004	6.052	.973	36.256	1.657
10	.890	4.069	.878	5.566	.890	36.903	1.704
15	.895	4.893	1.058	7.129	1.146	39.832	1.928
20	1.362	6.636	1.447	9.048	1.471	42.801	2.179
25	1.530	7.216	1.578	9.817	1.603	45.201	2.403
30	1.661	7.833	1.718	10.656	1.747	47.801	2.667
35	1.816	8.566	1.884	11.714	1.930	50.666	2.991
40	2.030	9.748	2.155	13.290	2.206	53.841	3.397
45	2.332	11.025	2.451	15.166	2.540	57.208	3.894
50	2.753	13.111	2.943	17.848	3.031	60.866	4.530
55	3.252	15.341	3.478	20.870	3.600	64.612	5.318
60	3.906	18.254	4.196	24.733	4.355	68.610	6.366
65	4.759	22.450	5.260	30.541	5.542	72.899	7.835

From these values of Assurances of 100*l.* the values of Assurances of any other sum may be easily collected.

This Office makes assurances for any number of months, or years, of any sums not exceeding 2000*l.* \* on one life; and its tables contain the values for all the intermediate years omitted in this and the two following Tables.

It may be necessary here to add, for the information of those who may not be conversant with decimal arithmetic, that in every value the number on the left hand of the point expresses so many pounds, and that allowing 2*s.* for every unit in the *first* figure on the right hand of the point, 2½*d.* for every unit in the *second* figure, and one farthing for every unit in the *third*, will give very nearly the shillings and pence to be added to the pounds in each value.—Thus; 1.336 in the preceding Table is 1*l.* 6*s.* 9*d.*—4.632 is 4*l.* 12*s.* 8*d.*—1.004 is 1*l.* 0*s.* 1*d.*—6.052 is 6*l.* 1*s.* 1*d.*; and .973 is 19*s.* 6¼*d.* See the note in Vol. I. p. 14.

There is one remark more necessary to be here attended to; but which I cannot make without some reluctance. In giving an account of this Society, in Vol. I. p. 176, I have recommended, for reasons there mentioned, that in transacting the business of the Society, an addition of 3 or 4 *per cent.* should be made to all the calculated values.

\* The Society has lately extended its assurances to the sum of 5000*l.* Ed.

But the Society, having lately thought proper to increase its expences of management, and fearing the effect of too great and sudden a reduction, has carried this addition as high as 15 *per cent.* \* This, when added to the other advantages which the Society enjoys (and particularly that derived from estimating the improvement of the money it receives at 3 *per cent.*) would, without doubt, be a very *exorbitant*, were it intended to be a *permanent* charge. But this is not the case. Even this charge leaves a reduction in the payments of above a *quarter*; and should the Society find that, notwithstanding this reduction, it continues still to prosper, as there is every reason to think it will, farther reductions may be expected: And, perhaps, in time it may find itself capable of reducing the payments for Assurances even *BELOW* those in the preceding Table. Nothing renders this improbable, but the difficulty of keeping out bad lives, and preventing fraudulent assurances; for a comparison of the *Northampton* Table of decrements with the Tables which will be given presently for *CHESTER*, the parish of *HOLY-CROSS*, and for the kingdom of *SWEDEN*, will shew, that were the Society to take the *premiums* in the preceding Table without any addition, it would still be governing itself by probabilities of living much below those among mankind in general.

\* This addition to the premiums of the Society has been discontinued since the 1st of January, 1786.



## T A B L E XXXV.

Shewing the Value of an Assurance of 100*l*.  
on two joint Lives, according to the  
NORTHAMPTON Table of Observations,  
reckoning interest at 3 *per cent*.

Ages.	Single Premium.	Annual Premium.	Ages.	Single Premium.	Annual Premium.
10	10 49.498	2.855	20	20 55.923	3.695
	15 51.177	3.053		25 57.065	3.871
	20 52.958	3.279		30 58.390	4.087
	25 54.319	3.463		35 59.968	4.363
	30 55.873	3.688		40 61.856	4.723
	35 57.693	3.972		45 63.979	5.173
	40 59.832	4.339		50 66.438	5.766
	45 62.206	4.794		55 69.077	6.506
	50 64.919	5.390		60 72.049	7.508
	55 67.801	6.133		65 75.406	8.930
15	60 71.012	7.135	25	25 58.106	4.040
	65 74.606	8.557		30 59.322	4.248
	15 52.731	3.249		35 60.786	4.515
	20 54.388	3.473		40 62.559	4.867
	25 55.641	3.653		45 64.571	5.308
	30 57.083	3.874		50 66.923	5.893
	35 58.783	4.154		55 69.461	6.625
	40 60.799	4.517		60 72.343	7.619
	45 63.047	4.969		65 75.621	9.035
	50 65.634	5.563	30	30 60.418	4.446
20	55 68.395	6.303		35 61.754	4.703
	60 71.485	7.302		40 63.392	5.044
	65 74.960	8.719		45 65.271	5.474

T A B L E XXXV. continued.

Ages.			Single Premium.	Annual Premium.	Ages.			Single Premium.	Annual Premium.
30	50		67.495	6.048	45	45		68.611	6.367
	55		69.915	6.769		50		70.278	6.887
	60		72.685	7.751		55		72.164	7.551
	65		75.866	9.156		60		74.424	8.476
35		35	62.944	4.947	50	65		77.134	9.825
		40	64.428	5.275		50		71.705	7.381
		45	66.149	5.692		55		73.344	8.014
		50	68.217	6.252		60		75.357	8.907
		55	70.492	6.958		65		77.831	10.226
		60	73.125	7.925	55	55		74.713	8.606
		65	76.181	9.316		60		76.443	9.451
40		40	65.736	5.588		65		78.637	10.721
		45	67.274	5.988	60	60		77.846	10.235
		50	69.154	6.530		65		79.699	11.434
		55	71.250	7.218		65	65	81.152	12.541
		60	73.713	8.168					
		65	76.612	9.541					

## T A B L E XXXVI.

Shewing the Value of 100*l.* depending on the Contingency of one Life surviving another, according to the NORTHAMPTON Table of Observations, reckoning Interest at 3 per cent.

Pollitor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.	Pollitor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.
10	10	24.749	1.427	5.723	20	35	34.176	1.759	7.570
	15	24.198	1.444	6.213		40	22.692	1.733	8.246
	20	23.498	1.455	6.738		45	21.058	1.703	9.059
	25	22.531	1.437	7.197		50	19.294	1.674	10.085
	30	21.468	1.417	7.746		55	17.410	1.640	11.356
	35	20.317	1.399	8.422		60	15.381	1.603	13.029
	40	19.070	1.383	9.272		65	13.206	1.564	15.341
	45	17.696	1.364	10.314		70	10.892	1.523	18.634
	50	16.214	1.346	11.652					
	55	14.631	1.324	13.362		10	31.789	2.027	5.316
15	60	12.925	1.299	15.671	25	15	31.093	2.042	5.729
	65	11.098	1.273	18.935		20	30.254	2.052	6.178
	70	9.553	1.246	23.651		25	29.053	2.020	6.557
						30	27.683	1.982	6.998
	10	26.979	1.609	5.505		35	26.198	1.946	7.540
	15	26.365	1.625	5.954		40	24.590	1.913	8.215
	20	25.602	1.635	6.435		45	22.819	1.876	9.027
	25	24.549	1.612	6.849		50	20.907	1.841	10.055
	30	23.391	1.588	7.340		55	18.866	1.799	11.329
	35	22.136	1.564	7.944		60	16.667	1.755	13.004
20	40	20.778	1.544	8.698	30	65	14.310	1.710	15.313
	45	19.281	1.520	9.617		70	11.803	1.662	18.595
	50	17.666	1.497	10.791					
	55	15.941	1.469	12.271		10	34.404	2.271	5.282
	60	14.083	1.439	14.264		15	33.694	2.287	5.689
	65	12.092	1.407	17.086		20	32.843	2.299	6.136
	70	9.973	1.373	21.219		25	31.640	2.266	6.526
						30	30.209	2.223	6.974
	10	29.461	1.824	5.345		35	28.589	2.177	7.510
	15	28.786	1.838	5.760		40	26.834	2.135	8.183
25	20	27.961	1.848	6.207	35	45	24.901	2.088	8.995
	25	26.811	1.819	6.582		50	22.815	2.044	10.025
	30	25.546	1.788	7.027		55	20.588	1.993	11.307



# T A B L E S.

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T A B L E XXXVI. continued.

Possessor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.	Possessor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.
30	60	18.188	1.939	12.997	45	40	36.775	3.273	7.974
	65	15.616	1.885	15.330		45	34.306	3.183	8.762
	70	12.880	1.829	18.642		50	31.432	3.080	9.727
35	10	37.375	2.573	5.236	50	55	28.364	2.968	10.940
	15	36.647	2.590	5.632		60	25.057	2.854	12.552
	20	35.794	2.604	6.073		65	21.514	2.740	14.797
	25	34.588	2.569	6.464		70	17.744	2.629	18.012
	30	33.166	2.526	6.924	55	10	48.705	4.044	5.064
	35	31.472	2.474	7.466		15	47.968	4.066	5.415
	40	29.540	2.419	8.128		20	47.144	4.091	5.809
	45	27.413	2.359	8.930		25	46.017	4.052	6.170
	50	25.116	2.302	9.952		30	44.680	4.004	6.608
	55	22.664	2.237	11.227		35	43.101	3.950	7.153
	60	20.022	2.170	12.917		40	41.208	3.891	7.838
	65	17.191	2.102	15.255		45	38.846	3.807	8.657
	70	14.179	2.034	18.590		50	35.853	3.691	9.634
	10	40.763	2.956	5.178		55	32.353	3.535	10.791
40	15	40.023	2.974	5.560		60	28.581	3.378	12.338
	20	39.164	2.991	5.986		65	24.540	3.224	14.491
	25	37.969	2.954	6.371		70	20.239	3.075	17.570
	30	36.560	2.909	6.830	60	10	53.170	4.810	5.012
	35	34.888	2.857	7.384		15	52.454	4.834	5.349
	40	32.868	2.794	8.048		20	51.668	4.867	5.727
	45	30.501	2.715	8.825		25	50.596	4.826	6.074
	50	27.946	2.639	9.821		30	49.329	4.776	6.497
	55	25.218	2.555	11.064		35	47.829	4.721	7.027
	60	22.278	2.468	12.714		40	46.034	4.664	7.702
	65	19.128	2.382	15.005		45	43.800	4.583	8.530
	70	15.776	2.296	18.274		50	40.993	4.479	9.569
	10	44.511	3.430	5.124		55	37.357	4.303	10.771
45	15	43.766	3.450	5.491		60	33.002	4.080	12.272
	20	42.921	3.471	5.903		65	28.336	3.863	14.383
	25	41.753	3.433	6.278		70	23.370	3.656	17.409
	30	40.369	3.386	6.730	60	10	58.087	5.836	4.960
	35	38.735	3.333	7.287		15	57.403	5.863	5.282

T A B L E XXXVI. continued.

Possessor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.	Possessor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.
	20	56.669	5.905	5.643		20	62.203	7.367	5.556
	25	55.675	5.863	5.975		25	61.311	7.325	5.871
	30	54.499	5.811	6.379		30	60.251	7.271	6.253
	35	53.103	5.755	6.887		35	58.990	7.213	6.734
	40	51.437	5.699	7.539		40	57.484	7.159	7.353
60	45	49.367	5.622	8.352	65	45	55.620	7.085	8.130
	50	46.777	5.529	9.402		50	53.293	7.002	9.149
	55	43.439	5.371	10.695		55	50.302	6.858	10.446
	60	38.923	5.117	12.274		60	46.279	6.640	12.156
	65	33.419	4.795	14.315		65	40.576	6.270	14.321
	70	27.563	4.490	17.283		70	33.466	5.787	17.150
65	10	63.510	7.285	4.906					
	15	62.870	7.313	5.213					

## EXPLANATION.

THE annual premium in this Table is supposed to be payable during the joint continuance of the lives of the *possessor* and *expectant*; and the first payment is supposed to be made at the time of purchasing the Assurance.

The *equivalent annuity* signifies that annuity to which either the *single* premium specified in the Table, or the *annual* premium, will entitle an *expectant* during his survivorship, should such an annuity be preferred to a gross sum payable on survivorship.—Thus; the payment of either  $l.34.588$  ( $34l. 11s. 10d.$ ) in hand, or of  $l.2.569$  ( $2l. 11s. 5d.$ ) annually, during the joint lives of a wife aged 25 and a husband aged 35, the first payment to be made immediately, will, according to this Table, entitle the wife, should she survive the husband, either to  $100l.$  payable to her when she becomes a widow, or to an annuity payable during her life, after becoming a widow, of  $l.6.464$  ( $6l. 9s. 4d.$ )—If she is 35 (or of the same age with her husband) a single payment of  $l.31.472$ , or an annual payment of  $l.2.474$  will, by the Table, entitle her either to  $100l.$  payable on her survivorship, or to an annuity for her life of  $l.7.466$  after survivorship.

Any payments greater or less will entitle to gross sums or annuities proportionably greater or less.

It



It is necessary to repeat here the observation made at the end of Table 34th, p. 104. that these are the exact premiums according to the *Northampton* Table of Observations, reckoning interest at 3 per cent. The *Equitable Society* adds to these premiums a charge of 15 per cent. \*; and in this case, there is a reason which makes the addition less improper than in any other; I mean, the increase of value which the longer duration of the lives of *females* gives to all assurances depending on *their* survivorship; and which the Society, for want of proper observations, have not yet had the means of calculating. These means, however, will, I think, be furnished by some of the following Tables.

\* See Note, p. 105.

TABLE

## T A B L E XXXVII.

Shewing the Values of *three equal* joint Lives,  
 according to the *Northampton* Table of  
 Observations, reckoning Interest at 4 *per*  
*cent.*

Common Age.	Value at 4 per Ct.	Common Age.	Value at 4 per Ct.	Common Age.	Value at 4 per Ct.
1	5.309	25	9.796	49	6.482
2	8.251	26	9.685	50	6.317
3	9.632	27	9.572	51	6.161
4	10.661	28	9.457	52	6.011
5	11.170	29	9.340	53	5.859
6	11.707	30	9.221	54	5.705
7	12.058	31	9.099	55	5.550
8	12.266	32	8.975	56	5.393
9	12.298	33	8.848	57	5.235
10	12.200	34	8.718	58	5.076
11	12.043	35	8.585	59	4.916
12	11.865	36	8.448	60	4.755
13	11.678	37	8.309	61	4.593
14	11.481	38	8.165	62	4.432
15	11.274	39	8.017	63	4.263
16	11.056	40	7.865	64	4.093
17	10.845	41	7.714	65	3.914
18	10.656	42	7.567	66	3.733
19	10.490	43	7.423	67	3.550
20	10.342	44	7.276	68	3.366
21	10.222	45	7.126	69	3.181
22	10.118	46	6.972	70	2.995
23	10.012	47	6.813	71	2.810
24	9.905	48	6.650	72	2.627

T A B L E XXXVII. continued.

Common Age.	Value at 4 per Ct.	Common Age.	Value at 4 per Ct.	Common Age.	Value at 4 per Ct.
73	2.448	81	1.245	89	0.614
74	2.277	82	1.092	90	0.563
75	2.119	83	0.949	91	0.452
76	1.985	84	0.860	92	0.337
77	1.855	85	0.782	93	0.185
78	1.720	86	0.716	94	0.085
79	1.563	87	0.662	95	0.015
80	1.400	88	0.646		

T A B L E



## T A B L E XXXVIII.

Shewing the Values of THREE joint Lives, whose Differences of Age are 10 and 20 Years, according to the *Northampton* Table of Observations, reckoning Interest at 4 per cent.

Differences of Age 10 and 20 Years.

Ages.			Value at 4 per Ct.	Ages.			Value at 4 per Ct.
1	11	21	8.627	23	33	43	8.586
2	12	22	9.914	24	34	44	8.451
3	13	23	10.344	25	35	45	8.313
4	14	24	10.598	26	36	46	8.171
5	15	25	10.655	27	37	47	8.027
6	16	26	10.708	28	38	48	7.878
7	17	27	10.700	29	39	49	7.725
8	18	28	10.654	30	40	50	7.571
9	19	29	10.562	31	41	51	7.420
10	20	30	10.438	32	42	52	7.272
11	21	31	10.305	33	43	53	7.123
12	22	32	10.170	34	44	54	6.971
13	23	33	10.031	35	45	55	6.816
14	24	34	9.887	36	46	56	6.658
15	25	35	9.738	37	47	57	6.497
16	26	36	9.584	38	48	58	6.332
17	27	37	9.429	39	49	59	6.164
18	28	38	9.278	40	50	60	5.994
19	29	39	9.131	41	51	61	5.827
20	30	40	8.986	42	52	62	5.662
21	31	41	8.850	43	53	63	5.494
22	32	42	8.718	44	54	64	5.322

T A B L E XXXVIII. continued.

Ages.			Value at 4 per Ct.	Ages.			Value at 4 per Ct.
45	55	65	5.145	61	71	81	2.224
46	56	66	4.965	62	72	82	2.044
47	57	67	4.782	63	73	83	1.875
48	58	68	4.597	64	74	84	1.743
49	59	69	4.408	65	75	85	1.623
50	60	70	4.219	66	76	86	1.519
51	61	71	4.032	67	77	87	1.425
52	62	72	3.847	68	78	88	1.350
53	63	73	3.660	69	79	89	1.248
54	64	74	3.477	70	80	90	1.122
55	65	75	3.298	71	81	91	0.951
56	66	76	3.128	72	82	92	0.767
57	67	77	2.959	73	83	93	0.548
58	68	78	2.785	74	84	94	0.362
59	69	79	2.598	75	85	95	0.169
60	70	80	2.408				

REMARKS

## REMARKS on the two preceding Tables.

THESE Tables contain the exact values of *three* joint lives having either the same common age, or whose differences of age are 10 and 20 years, according to the *Northampton* Table of Observations, or Table VI. interest being at  $\frac{4}{100}$  per cent.

In order to find the values nearly of three joint lives, having *other* differences of age, the following rules should be observed.

If the age of the youngest of the three lives is between 10 and 50, and the difference of age between the youngest and oldest not more than *eight* years, take the *third* of the sum of the three ages for a common age; and the value in the last Table but one, corresponding to that common age, will be the value sought,

## EXAMPLE.

Let the value be required of three joint lives whose ages are 15, 16, and 23.

The sum of the ages is 54, the third part of which is 18, and the value (in Table 37th) corresponding to this age, is 10.656, the value required.

Within the limits I have mentioned this rule is tolerably correct. But these limits are so narrow as to render it of little use;



## 118 *Remarks on the two preceding Table.*

and, therefore, till some person will undertake to finish what has been begun in the two preceding Tables, it will be necessary to make use of the following *general* and very easy rule given by Mr. *Simpson*, for finding the values of any *three* from the values given of any *two* joint lives.

“ Let A be the youngest, and C the oldest of  
 “ the three proposed lives. Take the value of  
 “ the *two* joint lives B and C, and find the  
 “ age of a *single* life D of the same value.  
 “ Then find the value of the joint lives A  
 “ and D, which will be the answer.”

EXAMPLE. Let the three given ages be 20, 30, and 40; and let the rate of interest be 4 *per cent*. The value of the two oldest joint lives B and C will (by Table XX.) be 10.490, answering in Table XVII. to a single life D of 54 years, wanting  $\frac{60}{220}$  (*a*) of a year. And the value of the joint lives A and D, which (by the rule in p. 75, and by Tables XXIV. and XXV.) (*b*) is 9.085, will be the value sought.

(*a*) The value (in Table XVII.) which is *nearest* to but *less* than 10.490, is 10.421; which is the value of a single life aged 54. This value subtracted from 10.490 leaves 69, the *numerator* of this fraction. The *denominator* is the difference between 10.421 and 10.641, the last being the value of a life one year younger.

(*b*) The value deduced from the Tables (by the rule in p. 75) of two joint lives aged 20 and 54, is 9.038.—The value of two joint lives aged 20 and 55, is (by Table XXV.) 8.869; and of two joint lives aged 20 and 50, is (by Table XXIV.) 9.630. A fifth part of the difference between these values (that is, .153) multiplied by the fraction  $\frac{60}{220}$ , gives .047, which added to 9.038 makes 9.085, the value deduced from Tables XXIV. and XXV. of two joint lives, one aged 20 and the other wanting  $\frac{60}{220}$  of a year of 54.—This shews the proper method of calculation in every case; but the difference will be little, if, for the sake of more expedition, D is always taken for that age, whether greater or less, which answers most nearly to the value of the joint lives B and C, without regarding the fraction;

The

The following comparison will shew how near this rule comes to correctness.

Values of *three joint Lives*.

Ages.	Correct Value at 4 per cent. by Table 38th.	Value by Rule.	Ages.	Correct Value at 4 per cent. by Table 37th.	Value by Rule.
10-20-30	10.438	10.563	10-10-10	12.200	12.244
15-25-35	9.738	9.840	15-15-15	11.274	11.376
20-30-40	8.986	9.085	20-20-20	10.342	10.504
25-35-45	8.313	8.395	25-25-25	9.796	9.937
30-40-50	7.571	7.651	30-30-30	9.221	9.351
35-45-55	6.816	6.886	35-35-35	8.585	8.701
40-50-60	5.994	6.046	40-40-40	7.865	7.984
45-55-65	5.145	5.169	45-45-45	7.126	7.249
50-60-70	4.219	4.238	50-50-50	6.317	6.432
55-65-75	3.298	3.292	55-55-55	5.550	5.636
			60-60-60	4.755	4.816
			65-65-65	3.914	3.942
			70-70-70	2.995	3.000
			75-75-75	2.119	2.110

My principal design in calculating the two preceding Tables has been, to enable myself to make this comparison; and it may be inferred from it, that Mr. *Simpson's* rule gives the values of three joint lives generally within a *ninth* or *tenth*, and sometimes within less than a 20th of a year's purchase.

It may be also observed, that when the oldest of the three ages does not exceed 75, and the youngest is not less than 10, the error falls always on the side of excess; and, consequently, that if .05 (that is, a 20th of a year's purchase) is deducted from the value by the rule, the true value will be obtained, in *some* cases, almost exactly; and, in *most* cases, much more nearly. The

The value of three joint lives being known, the value of the longest of any three lives is to be computed by the following rule.

“ From the sum of the values of all the single lives, subtract the sum of the values of all the joint lives combined two and two. Then to the remainder add the value of the three joint lives; and this last sum will be the value of the longest of the three lives.”—See Mr. *Simpson's Doctrine of Annuities, &c.* p. 23. —or Mr. *Dodson's Mathematical Repository*, Vol. II. p. 244.

EXAMPLE. The sum of the values of three single lives whose ages are 10, 20, and 30, is, by Table XVII. (reckoning interest at 4 per cent.) 48.338. The value of two joint lives whose ages are 10 and 20, is 13.355; of two joint lives whose ages are 10 and 30, is 12.586; of two joint lives whose ages are 20 and 30, is 11.873, by Tables XX. and XXII. And the sum of these three values is 37.814. This sum subtracted from 48.388, leaves 10.524, which remainder added to 10.485 (the value just found of the three joint lives) gives 20.009 the value of the *longest* of the three lives.

The value of three lives at the same ages by the Tables that follow shewing the values of single and joint lives among mankind at large according to observations in *Sweden*, is 21.870.

TABLE



## T A B L E XXXIX.

Shewing the Probability of the Duration of Human Life at all Ages among Males and Females, at *Warrington in Lancashire*; formed from a Register of Mortality kept there by Mr. *Aikin*, for Nine years, from 1773 to 1781.—See the Introduction p. 4, &c.

According to his Register there were born at *Warrington* from 1773 to 1781.

	Males.	Females.	Total.
1780	1777	3557	
Died in the same time, including 14 males and 5 females who died at ages unknown - - -	1287	1432	2719

Marriages in the same time 778, or 86 annually.

			Males.	Females.
Died between birth and 1 month	-	99	65	
From 1 to 2 months	—	37	25	
2 to 3	—	26	19	
3 to 6	—	48	57	
6 to 9	—	62	67	
9 to 12	—	70	80	
From birth to 1 year	—	342	313	
From 1 to 2 years	—	182	210	
2 to 3	—	87	94	
3 to 4	—	53	51	
4 to 5	—	32	32	
5 to 6	—	22	21	
6 to 7	—	11	9	
7 to 8	—	7	9	
8 to 9	—	3	10	
9 to 10	—	4	7	
10 to 14	—	21	18	
From birth to 14 years	—	764	774	Of

TABLE XXXIX. continued.

		Unknown.	Bachelors.	Husbands.	Widowers.	Total.
Of males turned of 14 died from	14 to 17	0	16	0	0	16
	17 to 20	0	21	1	0	22
	20 25	1	16	13	0	30
	25 30	5	14	15	1	35
	30 35	3	5	23	3	34
	35 40	5	3	28	2	38
	40 45	3	1	25	3	32
	45 50	2	0	21	3	26
	50 60	12	6	48	10	76
	60 70	21	6	39	25	91
	70 80	11	5	28	36	80
	80 90	4	0	10	11	25
Above 90		0	0	0	4	4
Total		67	93	251	98	509
Died under 14						764
Total						1273

# T A B L E S.

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T A B L E XXXIX. continued.

		Un- known.	Maids.	Wives.	Wi- dows.	Total.
Of females turned of 14 died from	14 to 17	0	16	0	0	16
	17 to 20	0	20	1	0	21
	20 to 25	1	21	10	2	34
	25 to 30	7	12	22	2	43
	30 to 35	4	3	29	3	39
	35 to 40	8	8	28	4	48
	40 to 45	9	11	27	1	48
	45 to 50	4	8	20	3	35
	50 to 60	13	7	52	22	94
	60 to 70	16	5	38	55	114
	70 to 80	12	12	22	61	107
	80 to 90	4	2	7	32	45
	Above 90	1	0	1	7	9
	Total	79	125	257	192	653
Died under 14					—	774
Total					—	1427

From



From these *data* the following Table has been formed.

Age.	M A L E S.		F E M A L E S.	
	Living.	Decrements.	Living.	Decrements.
0	1273	162	1427	109
3 months		48		57
6 months		62	6	67
9 months		70	6	80
1 year	931	182	1114	210
2 years	749	87	904	94
3	662	53	810	51
4	609	32	759	32
5	577	22	727	21
6	555	11	706	9
7	544	7	697	9
8	537	3	688	10
9	534	4	678	7
10	530	5	671	5
11	525	5	666	5
12	520	5	661	4
13	515	6	657	4
14	509	5	653	5
15	504	5	648	5
16	499	6	643	6
17	493	7	637	7
18	486	8	630	7
19	478	7	623	7
20	471	6	616	7
21	465	6	609	6
22	459	6	603	7
23	453	6	596	7
24	447	6	589	7
25	441	7	582	8
26	434	7	574	8
27	427	7	566	9

T A B L E XXXIX. continued.

Ages.	M A L E S.		F E M A L E S.	
	Living.	Decrements.	Living.	Decrements.
28	420	7	557	9
29	413	7	548	9
30	406	6	539	8
31	400	7	531	8
32	393	7	523	7
33	386	7	516	8
34	379	7	508	8
35	372	7	500	9
36	365	8	491	9
37	357	8	482	10
38	349	8	472	10
39	341	7	462	10
40	334	7	452	10
41	327	7	442	10
42	320	6	432	10
43	314	6	422	9
44	308	6	413	9
45	302	6	404	8
46	296	5	396	7
47	291	5	389	6
48	286	5	383	7
49	281	5	376	7
50	276	6	369	8
51	270	6	361	8
52	264	7	353	9
53	257	7	344	9
54	250	8	335	10
55	242	8	325	10
56	234	8	315	10
57	226	8	305	10
58	218	9	295	10
59	209	9	285	10
60	200	9	275	11

T A B L E XXXIX. continued.

Age.	M A L E S.		F E M A L E S.	
	Living.	Decrement:	Living.	Decrements.
61	191	9	264	11
62	182	9	253	11
63	173	9	242	11
64	164	9	231	12
65	155	10	219	12
66	145	9	207	12
67	136	9	195	12
68	127	9	183	11
69	118	9	172	11
70	109	9	161	11
71	100	9	150	11
72	91	9	139	11
73	82	9	128	11
74	73	8	117	11
75	65	8	106	11
76	57	8	95	11
77	49	7	84	10
78	42	7	74	10
79	35	6	64	10
80	29	25	54	45
Above 90	4	4	9	9
Totals —	27010	1273	36681	1427

It appears from this Table, and from the register on which it is grounded, that though the probabilities of living among females are higher than among males, and a smaller number is born, yet more die. The reason must be, that more males emigrate, and that many of them die in the army, the navy,



navy, and the militia. To this also it is owing, that more *wives* die at WARRINGTON than husbands.

It is proper to add, that in consequence of this greater emigration, the preceding Tables gives the proportion of the expectations of life among *males* to those among females lower than it really is. But at the same time it should be remembered, that it does this only for the ages *before* which, and *during* which, the emigration happens. After these ages, (that is, probably after the age of 40 or 50) the correctness of the table cannot be affected by this cause.

See the remarks in the general introduction to these Tables, p. 4, &c.

TABLE

## T A B L E XL.

Shewing the Probability of the Duration of Human Life, at all Ages, among Males and Females; formed from a Register kept by Dr. *Haygarth*, at CHESTER, for Ten Years, from 1772 to 1781.

According to this Register there were born at CHESTER in ten years from 1772 to 1781.

	Males.	Females.
	2192	2115
There were buried at CHESTER during the same time, including 24 whose ages were unknown — — —	1939	2151

Marriages 1500, or 150 annually.

Died between birth and 1 month	115	80
from 1 to 2 months	67	51
2 to 3 —	38	30

Died from birth to 3 months	220	161
from 3 to 6 months	75	64
6 to 9 —	76	69
9 to 1 year —	67	74

Died from birth to 1 year —	438	368
from 1 to 2 years —	180	181
2 to 3 —	107	127
3 to 4 —	67	77
4 to 5 —	34	53
5 to 10 —	91	75
10 to 15 —	28	34
15 to 20 —	48	53

Died in all under 20 years of age 993 968

Of

T A B L E XL. continued.

		Batchelors.	Huf- bands.	Wi- dowers.	Total.
Of males turned of 20 died be- tween - - }	20 and 25	50	8	0	58
	25 and 30	30	31	1	62
	30 35	19	29	1	49
	35 40	16	38	5	59
	40 45	12	53	6	71
	45 50	9	61	7	77
	50 55	11	54	14	79
	55 60	10	49	13	72
	60 65	13	63	29	105
	65 70	7	40	17	64
	70 75	10	49	40	99
	75 80	3	29	27	59
	80 81	1	9	8	18
	81 82	2	1	6	9
	82 83	0	4	5	9
	83 84	0	1	2	3
	84 85	1	2	2	5
	85 86	0	4	1	5
	86 87	0	3	5	8
	87 88	0	1	2	3
	88 89	0	2	2	4
	89 90	0	0	2	2
	90 91	0	2	2	4
	91 92	0	0	1	1
	92 93	0	2	0	2
	93 94	0	0	1	1
	94 —	0	1	1	2
	97 —	0	0	1	1
	99 —	0	0	1	1
	106 —	1	0	1	2
Died in all of males above 20		195	536	203	934
Under 20		—	—	—	993
Total		—	—	—	1927

T A B L E XL. continued.

		Maid.	Wives.	Widows.	Total.
Off female turned of 20 died be- tween - - }	20 and 25	38	13	2	53
	25 and 30	28	49	3	80
	30 35	21	40	4	65
	35 40	7	58	6	71
	40 45	11	54	9	74
	45 50	14	46	16	76
	50 55	16	34	21	71
	55 60	13	32	24	69
	60 65	26	53	37	116
	65 70	9	28	46	83
	70 75	19	37	86	142
	75 80	18	20	70	108
	80 81	3	3	29	35
	81 82	1	0	12	13
	82 83	1	2	15	18
	83 84	1	0	10	11
	84 85	4	6	15	25
	85 86	2	0	8	10
	86 87	1	0	8	9
	87 88	1	0	6	7
	88 89	0	2	6	8
	90	0	0	6	6
	91	0	0	3	3
	92	1	0	4	5
	94	0	0	2	2
	95	0	0	1	1
	96	0	0	1	1
	97	0	1	0	1
	98	0	1	3	4
	99	0	0	1	1
	101	0	0	1	1
	102	0	0	1	1
	103	0	0	1	1
Died in all of females <i>above</i> 20		235	479	456	1171
<i>Under</i> 20		—	—	—	968
Total		—	—	—	2139



Of 22 females above the age of 80 who died at *Chester* in 1772, the register specifies no more than that 4 of them were maids, and 14 of them widows who died between 80 and 90; and that the remaining 4 were widows who died above 90.—Of the 4 who had never been married, one has been supposed to die at each of the ages 81, 83, 84, and 85.—Of the 18 widows, 2 have been supposed to die at each of the ages between 80 and 88; two at 91; one at 92; and one at 93.—It was proper to make some distribution of this kind; but it is of little consequence whether it is right or wrong. In every other instance the numbers dying at every age have been taken just as the register has given them; and the following Table has been formed from them.

T A B L E XL. continued.

Age.	M A L E S.		F E M A L E S.	
	Living.	Decrements.	Living.	Decrements.
0	1927	220	2139	161
3 months	—	75	—	64
6 months	—	76	—	69
9 months	—	67	—	74
1 year	1489	180	1771	181
2 years	1309	107	1580	127
3	1202	67	1463	77
4	1135	34	1386	53
5	1101	30	1333	30
6	1071	24	1303	10
7	1047	18	1285	11
8	1029	11	1274	9
9	1018	8	1265	7
10	1010	6	1258	6
11	1004	5	1252	6
12	999	5	1246	7
13	994	6	1239	7
14	988	6	1232	8
15	982	7	1224	9
16	975	9	1215	10
17	966	10	1205	11
18	956	11	1194	12
19	945	11	1182	11
20	934	11	1171	10
21	923	11	1161	10
22	912	12	1151	10
23	900	12	1141	11
24	888	12	1130	12
25	876	13	1118	16
26	863	13	1102	16
27	850	13	1086	16
28	837	12	1070	16
29	825	11	1054	16

## T A B L E S.

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T A B L E XL. continued.

Age.	M A L E S.		F E M A L E S	
	Living	Decrements.	Living.	Decrements
30	814	10	1038	13
31	804	9	1025	13
32	795	10	1012	13
33	785	10	999	13
34	775	10	986	13
35	765	11	973	14
36	754	11	959	14
37	743	12	945	14
38	731	12	931	14
39	719	13	917	15
40	706	13	902	15
41	693	14	887	15
42	679	14	872	15
43	665	15	857	14
44	650	15	843	15
45	635	15	828	15
46	620	15	813	15
47	605	15	798	15
48	590	16	783	16
49	574	16	767	15
50	558	16	752	15
51	542	16	737	14
52	526	16	723	14
53	510	16	709	14
54	494	15	695	14
55	479	14	681	13
56	465	14	668	13
57	451	14	655	13
58	437	14	642	15
59	423	16	627	15
60	407	19	612	20
61	388	22	592	25
62	366	22	567	25
63	344	22	542	25

## T A B L E S.

T A B L E XL. continued

Age.	M A L E S.		F E M A L E S.	
	Living.	Decrements.	Living.	Decrements.
64	322	20	517	21
65	302	16	496	17
66	286	13	479	15
67	273	11	464	15
68	262	11	449	16
69	251	13	433	20
70	238	16	413	25
71	222	22	388	30
72	200	22	358	30
73	178	21	328	30
74	157	18	298	27
75	139	15	271	23
76	124	12	248	22
77	112	11	226	21
78	101	11	205	21
79	90	10	184	21
80	80	10	163	21
81	70	10	142	21
82	60	9	121	21
83	51	8	100	21
84	43	7	79	18
85	36	6	61	12
86	30	5	49	8
87	25	4	41	6
88	21	4	35	4
89	17	3	31	4
90	14	3	27	4
91	11	3	23	4
92	8	3	19	4
93	5	2	15	4
94	3	2	11	4
95	1	1	7	3
96			4	3
97			1	1



In this and the last Table there are several irregularities in the decrease of the probabilities of the duration of life, which would not have taken place, had the observations been made on a larger body of people, or for a longer period of years; but they do not much affect the correctness of the expectations and values of lives deducible from these Tables, except at the extremity of life after the age of 80 or 85. According to the *Chester* register, the whole number of males that died at every age for ten years between 80 and 85, was 44—22 died between 85 and 90, and 14 above 90. This register also makes 102 the number of females that died between 80 and 85, and 34 and 27 the numbers that died between 85 and 90, and *above* 90. The preceding Table, from the age of 80 to 97, is formed just as it would have been formed had the register given only this information without particularizing the numbers dying in every single year of life after 80. It will be easily seen that this was necessary. The deaths at the extreme ages beyond 96 or 97, bear so small a proportion to the rest, that there is no occasion for including them in a Table of Observations; nor is it possible to do it properly.

It should be further considered, that the remark at the end of the Table for *Warrington* is applicable to this Table.

COMPARISON of the Duration of the Lives of  
*Males and Females*, according to the preceding  
Table.

Ages.	Expectations of Males.	Expectations of Females.
Birth	28.13	33.27
5	43.20	47.44
10	41.92	45.17
15	38.05	41.36
20	34.86	38.10
25	32.00	34.78
30	29.25	32.27
35	25.97	29.26
40	22.92	26.37
45	20.20	23.50
50	17.64	20.62
55	15.14	17.52
60	12.36	14.20
65	10.79	11.94
70	8.05	8.81
75	7.00	7.14
80	5.43	5.20
85	4.25	4.81
90	2.50	3.46

ABSTRACT of the Rev. Mr. GORSUCH's  
*Observations and Register in the Parish of*  
 HOLY CROSS, near SHREWSBURY.

In 1755 the number of inhabitants in this parish was 1049.

In 1760 the families were 235—the inhabitants 1048, of whom two were males, and 13 females above 80.

In 1765 the families were 249—the inhabitants 1096.

		In 1770.	1775.	1780.
Families	- -	240	—	246
Inhabitants	- -	1046	1057	1113
Males under 10	- -	126	—	155
Females under 10	-	122	—	135
Males from 70 to 80	-	20	20	11
Females from 70 to 80	-	24	21	19
Males above 80	- -	6	9	4
Females above 80	- -	11	7	5

The increase in 1765 was occasioned by the removal of four numerous families into four great houses in the Parish, which for many years before had been almost uninhabited.

In 1767 several houses were pulled down to open a way to a new stone bridge over the *Severn*, and 38 persons went out of the parish.

In

In 1774 a fire destroyed 48 houses, mostly thatched; but the sufferers provided themselves with lodgings in the parish, and only 24 left it.—The vacant ground was covered with little tenements fit for poor people, and so commodious as to draw into the parish a greater number of persons than had resided there before.—See a further account of this parish in Vol. I. p. 261.

BIRTHS for 30 years,		{ Males - 565 }	} 1098
from 1750 to 1780		{ Females 533 }	
BURIALS	- -	{ Males - 458 }	} 966
		{ Females 508 }	

The births have exceeded the burials in the proportion of 15 to 13; and this ought to have increased the inhabitants in 30 years to at least 1200; but it appears that it has occasioned little or no increase; and, consequently, that the excess of the births has been but just sufficient to supply the loss produced by emigrations to the navy and army, and settlements in towns.



# T A B L E S.

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From 1750 to 1760.		From 1760 to 1780.		
	Males and Females.	Males.	Females.	Total.
Died under 1 month	17	22	25	64
1 year	27	42	44	113
2 years	15	23	23	61
From 2 to 5	28	33	38	99
5 10	23	16	21	60
10 15	6	4	4	14
15 20	7	6	8	21
20 25	18	4	7	29
25 30	11	10	7	28
30 35	9	4	8	21
35 40	11	16	9	36
40 45	13	16	11	40
45 50	8	9	10	27
50 55	10	16	17	43
55 60	13	12	12	37
60 65	13	22	20	55
65 70	15	11	13	39
70 75	10	17	29	56
75 80	10	15	14	39
80 85	15	22	20	57
85 90	8	1	5	14
90 95	1	0	9	10
95 96	1	0	1	2
96 101	0	0	1	1
	289	321	356	966

It is obvious, that these observations do not give sufficient *data* for forming distinct tables of the probabilities of living among males

males and females: And it is also obvious, that the numbers dying in every period of five years after 10, are much more irregular than they would have been had these observations been made for a greater number of years, or on a larger body of inhabitants. In constructing, therefore, the following Table, the decrements of life have been taken as the register gives them for both sexes in every period of ten years after the age of ten. And in this way the register exhibits with remarkable regularity and consistency the progress of human mortality from birth to old age, representing human life in conformity to other observations, as particularly weak in the first month, (though much less so than in towns) and from that age as growing gradually stronger, till at 10 it acquires its greatest strength, which it afterwards loses, but more slowly till 50, and after 50 more rapidly, till at 70 or 75 it is brought back to all the weakness of the first month.

## TABLE XLI.

Shewing the Probabilities of the Duration of Human Life at all Ages, as deduced from the Rev. Mr. GORSUCH's Observations, during a Period of 30 Years, in the Parish of HOLY CROSS, near SHREWSBURY. See Vol. I. p. 261.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decr.
0	966	64	21	529	5	44	395	7
Under 1 } month }	—	113	22	524	5	45	388	7
			23	519	6	46	381	7
			24	513	6	47	374	7
1 year	789	61	25	507	6	48	367	7
2	728	44	26	501	6	49	360	7
3	684	30	27	495	6	50	353	7
4	654	25	28	489	6	51	346	7
5	629	20	29	483	6	52	339	7
6	609	16	30	477	5	53	332	8
7	593	12	31	472	5	54	324	8
8	581	7	32	467	5	55	316	8
9	574	5	33	462	6	56	308	8
10	569	4	34	456	6	57	300	9
11	565	3	35	450	6	58	291	9
12	562	3	36	444	6	59	282	9
13	559	3	37	438	6	60	273	9
14	556	3	38	432	6	61	264	9
15	553	3	39	426	6	62	255	9
16	550	4	40	420	6	63	246	9
17	546	4	41	414	6	64	237	9
18	542	4	42	408	6	65	228	9
19	538	4	43	402	7	66	219	10
20	534	5						

T A B L E XLI. continued.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decr.
67	209	10	77	111	9	87	28	6
68	199	10	78	102	9	88	22	5
69	189	10	79	93	9	89	17	4
70	179	10	80	84	9	90	13	3
71	169	10	81	75	8	91	10	2
72	159	10	82	67	8	92	8	2
73	149	10	83	59	8	93	6	2
74	139	10	84	51	8	94	4	2
75	129	9	85	43	8	95	2	1
76	120	9	86	35	7	96	1	1

EXPECTA-



## EXPECTATIONS of Life by the preceding Table.

Age.	Expectation.
Birth	33.93
5	46.30
10	46.00
15	42.25
20	38.66
25	35.58
30	32.66
35	29.43
40	26.40
45	23.35
50	20.40
55	17.47
60	14.86
65	12.30
70	10.00
75	7.87
80	5.75

The proportion of the living under ten years of age to the living at ten and upwards, is, by this Table, as 6807 to 26452, or as 10 to 39; but the real proportion appears from the survey to be greater: And it is evident, that the excess of the births above the burials, and the emigrations from the parish after ten, must make it considerably greater; and it should not be forgotten, that these also are circumstances which must render the probabilities and expectations of life, as given by the Table, less than they *really* are.

T A B L E

## T A B L E XLII.

Shewing the Probabilities of the Duration of Human Life among Males and Females, deduced from Observations of the Proportions of the Living to the Numbers who have died at all Ages for 21 Years, from 1755 to 1776, in the Kingdom of SWEDEN.

## PRELIMINARY OBSERVATIONS.

According to the medium of seven different enumerations in 1757, 1760, 1763, 1766, 1769, 1772, and 1775, there were living in the kingdom of SWEDEN :

Ages.	Males.	Females.
Under 1 year	33882	33640
Between 1 and 3 years	62155	63005
3 5	62696	63551
5 10	121871	122460
10 15	117879	118419
15 20	103093	105845
20 25	91907	102306
25 30	82919	93315
30 35	78615	87129
35 40	70390	77077
40 45	63961	70405
45 50	52083	59580
50 55	44908	52689
55 60	36253	44211
60 65	30772	39416
65 70	21170	29610
70 75	14610	21776
75 80	8224	12515
80 85	4036	6418
85 90	1522	2492
Above 90	486	869
Total —	1,103,432	1,206,728
And females —	1,206,728	
Total of males and females	2,310,160	

# T A B L E S.

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Fencible men between 15 and 55 } 587,876 } or a *quarter* nearly  
 Males and females } 1,201,909 } or a little more than  
 under the age of 25 } } *half* the inhabitants

Of these numbers there died annually in SWEDEN during twenty-one years from 1755 to 1776,

Ages.	Males.	Females.
Under 1 Yr.	9664 or 1 of 3.5*	8355 or 1 of 4.0*
Between 1 } and 3 Yrs. }	3592 or 1 of 17.3	3531 or 1 of 17.8
3 and 5	1816 or 1 of 34.5	1774 or 1 of 35.8
5 10	1789 1 of 68.1	1672 1 of 73.2
10 15	898 1 of 131.2	802 1 of 147.6
15 20	741 1 of 139.1	714 1 of 148.2
20 25	874 1 of 105.1	776 1 of 131.8
25 30	879 1 of 94.3	872 1 of 106.9
30 35	955 1 of 82.3	1058 1 of 82.3
35 40	907 1 of 77.6	901 1 of 85.5
40 45	1119 1 of 57.1	1129 1 of 62.3
45 50	1077 1 of 48.3	958 1 of 62.2
50 55	1233 1 of 36.4	1127 1 of 46.7
55 60	1180 1 of 30.7	1163 1 of 38.0
60 65	1383 1 of 22.2	1597 1 of 24.6
65 70	1328 1 of 15.9	1510 1 of 19.6
70 75	1360 1 of 10.7	1935 1 of 11.2
75 80	1023 1 of 8.0	1527 1 of 8.2
80 85	784 1 of 5.1	1230 1 of 5.2
85 90	383 1 of 4.0	609 1 of 4.1
Above 90	195 1 of 2.5	339 1 of 2.6
Of all ages	33180 1 of 33.25	33579 1 of 35.94

\* It should be considered, that this is a *higher* proportion than that of the number that dies under one year of age to the number born in a year. The latter number is equal to the former increased by the number living at one time under one year. See the note, p. 158.

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K

The



The enumerations and deaths for the first 9 years from 1755 to 1763 included the whole kingdom of *Sweden*, consisting of 26 principalities or provinces.---In 1764 there was a suspension of all the observations. In 1765 they were taken up again; but in this and the following years, the enumeration of one of the provinces was omitted, together with the registration of the deaths in that province.—In the three years from 1767 to 1770 three provinces were omitted, in the enumerations and registers.---In the three years from 1770 to 1773, there was also an omission of three provinces, together with the city of STOCKHOLM. And in the remaining three years (to 1776) four out of the 15 diocesses in *Sweden* were omitted. But these omissions will produce no incorrectness in the tables of the decrements and values of lives formed from the preceding *data*.

I have formed tables from the enumerations and deaths in the first nine years, comprehending all *Sweden*; but there is no other difference between them and the following Tables, except that the latter give the probabilities of the duration of life a little lower than the former; and the reason of this is, that the mortality of the years 1771, 1772, and 1773, exceeded greatly the mortality of the other years (*a*).

(*a*) It is also owing to this that the proportions of annual deaths to the living at all ages, as here given, are somewhat greater than those in the Second Essay at the end of this work.

In



In the healthiest of the seven ternaries of years into which these observations have been divided (that is, in the three years (b) from 1765 to 1767) only one in  $36\frac{1}{2}$  of males, and 1 in  $39\frac{2}{3}$  of females, died. The average proportion for the whole period of 21 years is 1 in  $33\frac{1}{4}$  of males; and 1 in  $35\frac{2}{3}$  of females. But, in the sickly years just mentioned, there died 1 in 27 of males, and 1 in 29 of females.—The number of the living in the following Tables, at the end of one year of age, is the *difference* between the number born in *Sweden* in a year,

(b) The whole number of males living in these years was 1,182,848; of females 1,290,068. I have said that one of the 26 provinces of SWEDEN was omitted in the observations for these three years. The addition of this province will make the inhabitants of SWEDEN in 1766 above two millions and a half. In 1757 they were 2,323,195. They increased, therefore, at the rate of near 200,000 in *nine* years. But it appears that this increase had not been of long continuance; for had it been so, a table formed from the decrements as given by the registers, and by taking the medium of annual deaths from 1755 to 1763 for the *radix*, would have given the probabilities of living much too small (and much less than those in the following Table) through the whole duration of life; whereas it does this only in the first stages of life. From 45 to 60 it gives them nearly equal; and after 60 it gives them greater, which is a plain proof that about the beginning of this century *Sweden* was decreasing.—To the same purpose it appears from the enumerations, that while the numbers living in the *first* stages of life were increasing fast, the numbers in the *last* stages were decreasing.

and the number of deaths under one year of age (exclusive in both cases of still-borns) accommodated to 10,000 as a radix.

The decrements among *males* in the following Table, increase regularly through every period of life from 10 to 75. But among *females* this increase is interrupted for a few years after 35, and again for a few years after 45.—This cannot be an accidental irregularity, the numbers being too great, and the period for which the observations have been made, too long, to admit of such an irregularity.—Probably, therefore, it must be accounted for in the following manner.—From the age of 30 to 35, the number of married, and consequently of child-bearing women, is greater than at any other ages; and this raises the decrements in that division of life. After 35, this number is diminished, and the decrements fall. Between 40 and 45 the critical periods come on, and the decrements are raised again; but after 45 the number of deaths arising from hence becoming less, the decrements become also less, but continue afterwards to increase with increasing years, till they become greatest at 74 or 75.—It is, however, remarkable that notwithstanding the peculiar dangers to which the lives of females are subject from the causes just mentioned,  
there

there are no ages at which a *smaller* proportion of them does not die than of males, except the ages in which the number of deliveries is greatest; and that even *then* the probabilities of living among them are nearly equal to those among males.

T A B L E XLII. continued.

M A L E S.				F E M A L E S.			
Born 10,282—282 born dead				10,277—217 born dead			
Ages.	Living.	Decr.	Expectat.	Living.	Decr.	Expect.	
Born alive	10,000	2300	33.20	10,000	2090	35.70	
1 year	7,700	500	42.45	7,910	518	44.00	
2	7,200	337	43.83	7,392	350	46.05	
3	6,863	240	44.96	7,042	250	47.31	
4	6,623	150	45.57	6,792	135	48.04	
5	6,473	125	45.62	6,657	120	48.00	
6	6,348	105	45.50	6,537	105	47.87	
7	6,243	90	45.26	6,432	85	47.64	
8	6,153	75	44.91	6,347	70	47.28	
9	6,078	65	44.46	6,277	60	46.80	
10	6,013	55	43.94	6,217	52	46.25	
11	5,958	45	43.26	6,165	46	45.55	
12	5,913	45	42.58	6,119	40	44.85	
13	5,868	40	41.91	6,079	35	44.15	
14	5,828	40	41.24	6,044	35	43.46	
15	5,788	39	40.56	6,009	35	42.76	
16	5,749	39	39.83	5,974	40	42.04	
17	5,710	39	39.11	5,934	40	41.31	
18	5,671	44	38.39	5,894	42	40.59	
19	5,627	44	37.67	5,852	43	39.87	
20	5,583	50	36.95	5,809	43	39.15	
21	5,533	50	36.28	5,766	43	38.43	
22	5,483	50	35.62	5,723	43	37.72	
23	5,433	55	34.96	5,680	44	37.01	
24	5,378	55	34.30	5,636	45	36.29	
25	5,323	55	33.63	5,591	45	35.58	
26	5,268	55	32.98	5,546	50	34.90	
27	5,213	55	32.32	5,496	52	34.21	
28	5,158	55	31.66	5,444	55	33.53	
29	5,103	56	31.00	5,389	55	32.85	
30	5,049	59	30.34	5,334	60	32.17	



T A B L E XLII. continued.

M A L E S.				F E M A L E S.		
Ages.	Living.	Decr.	Expectat.	Living.	Decr.	Expectat.
31	4,988	60	29.69	5,274	60	31.54
32	4,928	60	29.04	5,214	65	30.91
33	4,868	60	28.39	5,149	65	30.28
34	4,808	60	27.74	5,084	65	29.66
35	4,748	60	27.09	5,019	60	29.03
36	4,688	60	26.43	4,959	56	28.26
37	4,628	60	25.76	4,903	56	27.50
38	4,568	60	25.09	4,847	56	26.74
39	4,508	60	24.42	4,791	58	25.97
40	4,448	65	23.75	4,733	65	25.21
41	4,383	72	23.15	4,668	75	24.68
42	4,311	80	22.54	4,593	76	24.75
43	4,231	80	21.93	4,517	76	23.62
44	4,151	80	21.32	4,441	75	23.10
45	4,071	80	20.71	4,366	72	22.57
46	3,991	80	20.12	4,294	67	21.91
47	3,911	80	19.52	4,227	65	21.24
48	3,831	80	18.92	4,162	65	20.58
49	3,751	85	18.32	4,097	70	19.92
50	3,666	95	17.72	4,027	75	19.26
51	3,571	95	17.17	3,952	80	18.64
52	3,476	95	16.63	3,872	85	18.01
53	3,381	95	16.08	3,787	85	17.39
54	3,286	95	15.53	3,702	85	16.77
55	3,191	95	14.98	3,617	85	16.15
56	3,096	95	14.43	3,532	85	15.53
57	3,001	100	13.87	3,447	90	14.92
58	2,901	100	13.33	3,357	90	14.31
59	2,801	100	12.79	3,267	100	13.69
60	2,701	105	12.24	3,167	110	13.08
61	2,596	110	11.72	3,057	118	12.56
62	2,486	115	11.21	2,939	120	12.04
63	2,371	115	10.73	2,819	120	11.52
64	2,256	115	10.26	2,699	120	11.01

T A B L E XLII. continued.

Ages.	M A L E S.			F E M A L E S.		
	Living.	Decr.	Expectat.	Living.	Decr.	Expect.
65	2,141	115	9.78	2,579	120	10.49
66	2,026	115	9.30	2,459	120	9.97
67	1,911	120	8.84	2,339	120	9.46
68	1,791	125	8.40	2,219	120	8.94
69	1,666	125	7.99	2,099	120	8.42
70	1,541	125	7.60	1,979	130	7.91
71	1,416	125	7.22	1,849	140	7.53
72	1,291	120	6.87	1,709	150	7.16
73	1,171	120	6.53	1,559	160	6.78
74	1,051	110	6.22	1,399	150	6.40
75	941	105	5.89	1,249	140	6.03
76	836	100	5.56	1,109	130	5.73
77	736	90	5.25	979	120	5.43
78	646	85	4.92	859	110	5.11
79	561	80	4.59	749	100	4.79
80	481	75	4.27	649	95	4.47
81	406	70	3.96	554	90	4.13
82	336	65	3.69	464	85	3.84
83	271	60	3.45	379	80	3.59
84	211	50	3.30	299	75	3.42
85	161	40	3.16	224	55	3.40
86	121	30	3.04	169	40	3.34
87	91	22	2.88	129	30	3.22
88	69	17	2.64	99	23	3.05
89	52	14	2.34	76	18	2.82
90	38	12	2.02	58	15	2.55
91	26	9		43	12	
92	17	7		31	10	
93	10	6		21	8	
94	4	3		13	6	
95	1	1		7	4	
96	0	0		3	2	
97	0	0		1	1	

# T A B L E S.

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## T A B L E XLIII.

Shewing the Probabilities of the Duration of Human Life among Males and Females, taken collectively, deduced from the preceding Table.

Born - 10,249 - 249 born dead				Age.	Living.	Dece.	Expect.
Age.	Living.	Dece.	Expect.				
Born alive	10000	2195	34.42	35	4884	59	28.03
1 year	7805	509	42.95	36	4825	58	27.31
2 years	7290	344	44.92	37	4767	58	26.68
3	6952	245	46.11	38	4709	58	26.01
4	6707	143	46.78	39	4651	60	25.33
5	6564	122	46.79	40	4591	65	24.66
6	6442	105	46.66	41	4526	73	24.05
7	6337	87	46.43	42	4453	78	23.44
8	6250	73	46.07	43	4375	78	22.83
9	6177	62	45.61	44	4297	78	22.22
10	6115	54	45.07	45	4219	76	21.61
11	6061	45	44.38	46	4143	74	20.98
12	6016	42	43.70	47	4069	72	20.35
13	5974	38	43.01	48	3997	73	19.72
14	5936	37	42.33	49	3924	78	19.09
15	5899	37	41.64	50	3846	85	18.46
16	5862	40	40.92	51	3761	87	17.87
17	5822	40	40.19	52	3674	90	17.29
18	5782	42	39.47	53	3584	90	16.70
19	5740	43	38.74	54	3494	91	16.12
20	5697	47	38.02	55	3403	91	15.53
21	5650	47	37.33	56	332	92	14.95
22	5603	48	36.64	57	3220	95	14.37
23	5555	48	35.96	58	3125	95	13.79
24	5507	50	35.27	59	3030	100	13.21
25	5457	50	34.58	60	2930	108	12.63
26	5407	52	33.91	61	2822	114	12.12
27	5355	54	33.23	62	2708	118	11.62
28	5301	55	32.56	63	2590	118	11.11
29	5246	55	31.88	64	2472	118	10.61
30	5191	59	31.21	65	2354	118	10.10
31	5132	60	30.57	66	2236	118	9.62
32	5072	62	29.94	67	2118	121	9.15
33	5010	63	29.30	68	1997	124	8.67
34	4947	63	28.67	69	1873	124	8.20
				70	1749	127	7.72

T A B L E XLIII. continued.

Age.	Living.	Decr.	Expectat.	Age.	Living.	Decr.	Expect.
71	1622	133	7.32	86	144	35	3.09
72	1489	135	6.89	87	109	27	2.92
73	1354	140	6.53	88	82	20	2.71
74	1214	130	6.23	89	62	15	2.43
75	1084	121	5.91	90	47	14	2.05
76	963	115	5.59	91	33	12	1.71
77	848	105	5.28	92	21	10	1.40
78	743	95	4.96	93	11	6	
79	648	90	4.61	94	5	3	
80	558	90	4.28	95	2	1	
81	468	84	4.01	96	1	1	
82	384	75	3.80				
83	309	65	3.57				
84	244	55	3.39				
85	189	45	3.23				

In forming this Table from the decrements of life among males and females in Table XLII. it is necessary to consider, that the proper decrements for a body of males and females taken collectively, are not the means between those for males and females in that Table; but the numbers dying in every period of life out of a given number living at the beginning of that period, supposed to consist of *equal* numbers of males and females.

For example. Table XLII. shews that of 2701 males living at 60 years of age, 560 will die in five years; and that of 3167 *females* living at the same age, 588 will die in the same time. From hence it may be easily deduced, that of 2930 persons (the number



number in this Table living at 60) consisting one half of males and one half of females, 576 will die in the same time. The number, therefore, living at 60 will at 65 be reduced to 2354; which number must again be supposed to consist one half of males and the other half of females, and the proper decrement for the next five years, deduced in the same manner from Table XLII. And it is in this method the whole of this Table has been constructed, which, therefore must exhibit more accurately than any other, the probabilities of living among the general mass of mankind, consisting of males and females taken collectively.

## T A B L E XLIV.

Shewing the Probabilities of the Duration of Human Life among Males and Females in STOCKHOLM, formed from the Proportions of the Living to the Numbers who have died in STOCKHOLM at all Ages for Nine Years from 1755 to 1763.

There were born alive in STOCK- } Males. Females.  
HOLM annually from 1755 to } 1335 1207  
1763 — — — }  
Still-born — — — 43 $\frac{1}{2}$  31

According to the medium of three different enumerations in 1757, 1760, and 1763, there were living in STOCKHOLM,

	Males.	Females.
Under 1 year	666	727
From 1 to 3 years	1239	1376
3 to 5	1185	1281
5 10	2662	2769
10 15	2971	2791
15 20	2780	2662
20 25	3293	4255
25 30	3371	4325
30 35	3533	4156
35 40	2763	3101
40 45	2528	2837
45 50	1668	1911
50 55	1402	1892
55 60	874	1340
60 65	705	1247
65 70	404	806
70 75	285	626
75 80	131	314
80 85	57	148
85 90	15	51
Above 90	8	27
Under 15	8723	8944
Between 15 and 55	21338	25139
Above 55	2479	4559
Of all ages	32540	38642

# T A B L E S.

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Of these numbers there died annually at STOCKHOLM during nine years from 1755 to 1763.

Ages.	MALES.	FEMALES.
Under 1 year, exclusive of still-born - - - }	565 or 1 of 1.17	505 or 1 of 1.44
Between 1 and 3 yrs.	179 or 1 of 6.93	187 or 1 of 7.37
3 5	89 1 of 13.27	81 1 of 15. 8
5 10	77 1 of 34. 5	71 1 of 38. 8
10 15	38 1 of 78. 8	24 1 of 114.7
15 20	37 1 of 59. 1	27 1 of 99. 8
20 25	74 1 of 44. 3	54 1 of 79. 3
25 30	101 1 of 33. 2	75 1 of 57. 9
30 35	119 1 of 29. 6	96 1 of 43. 3
35 40	104 1 of 26.56	79 1 of 39. 1
40 45	110 1 of 23. 0	92 1 of 31. 0
45 50	86 1 of 19. 4	69 1 of 27. 7
50 55	85 1 of 16. 4	75 1 of 25. 3
55 60	62 1 of 14. 1	56 1 of 24. 0
60 65	69 1 of 10.74	77 1 of 16.06
65 70	43 1 of 9.47	60 1 of 13.35
70 75	37 1 of 7.63	77 1 of 8.09
75 80	29 1 of 4.56	61 1 of 5.15
80 85	16 1 of 3.51	47 1 of 3.42
85 90	7 1 of 2.00	21 1 of 2.37
Above 90	3 1 of 2.66	12 1 of 2.31
Of all ages -	1930 1 of 16.86	1846 1 of 20.93

TABLE

From these *data* the following Table has been formed.

Ages.	MALES.		FEMALES.	
	Born - 10324—324 born dead		born 10235—225 born dead	
	Living.	Decr.	Living.	Decrements.
Born alive	10000	4232	10000	3885
1 year	5768*	800	6115*	900
2 years	4968	541	5215	530
3	4427	380	4685	350
4	4047	235	4335	200
5	3812	150	4135	155
6	3662	110	3980	115
7	3552	90	3865	90
8	3462	85	3775	75
9	3377	75	3700	60
10	3302	55	3640	45
11	3247	40	3595	30
12	3207	35	3565	25
13	3172	35	3540	25
14	3137	37	3515	30
15	3100	40	3485	30
16	3060	45	3455	30
17	3015	50	3425	35
18	2965	55	3390	35
19	2910	60	3355	40
20	2850	60	3315	40
21	2790	60	3275	40

\* The annual average of males born alive at *Stockholm* for 9 years from 1755 to 1763, was 1335. Of these 565 died annually under one year of age. The number, therefore, that lived to one year of age was 770; and 770 is the same part of 1335 that 5768 is of 10000.

In the same manner the number of females who lived to one year of age has been determined; after which, the totals living between 1 and 3, and between 3 and 5, and between 5 and 10, &c. &c. are always made to be in the same ratio to the number dying at those ages that they were found to be by observation.

In this method also the last Table, shewing the probabilities of life in the kingdom of SWEDEN at large, has been formed.



# T A B L E S.

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## T A B L E XVII. continued.

Age.	MALES.		FEMALES.	
	Living.	Decrements.	Living.	Decrements.
22	2730	60	3235	40
23	2670	60	3195	40
24	2610	65	3155	43
25	2545	70	3112	45
26	2475	70	3065	47
27	2405	70	3020	50
28	2335	70	2970	55
29	2265	70	2915	60
30	2195	70	2855	60
31	2125	70	2795	60
32	2055	70	2735	63
33	1985	65	2672	65
34	1920	65	2607	65
35	1855	65	2542	62
36	1790	65	2480	60
37	1725	65	2420	60
38	1660	60	2360	60
39	1600	60	2300	65
40	1540	60	2235	66
41	1480	60	2169	66
42	1420	60	2103	67
43	1360	60	2036	67
44	1300	60	1969	67
45	1240	60	1902	65
46	1190	57	1837	65
47	1133	55	1772	65
48	1078	55	1707	63
49	1023	55	1644	60
50	968	53	1584	60
51	915	50	1524	60
52	865	50	1464	55
53	815	50	1409	55
54	765	50	1354	53
55	715	45	1301	50
56	670	45	1251	50
57	625	45	1201	50
58	580	40	1151	50
59	540	40	1101	50
60	500	40	1051	55
61	460	40	996	60
62	420	38	936	60

TABLE XLIV. continued.

Age.	MALES.		FEMALES.	
	Living.	Decrements.	Living.	Decrements.
63	382	35	876	55
64	347	32	821	53
65	315	30	768	49
66	285	28	719	49
67	257	25	670	49
68	232	22	621	49
69	210	22	572	49
70	198	20	523	49
71	168	20	474	49
72	148	18	425	49
73	130	17	376	49
74	113	17	327	49
75	96	16	278	45
76	80	15	233	40
77	65	15	193	35
78	50	11	158	30
79	39	9	128	25
80	30	7	103	23
81	23	5	80	20
82	18	4	60	17
83	14	4	43	12
84	10	3	31	10
85	7	2	21	7
86	5	2	14	5
87	3	2	9	4
88	1	1	5	2
89	0	0	3	2
90	0	0	1	1
Total	147593	10000	185924	10000

COMPARISON of the Duration of the Lives of Males and Females, according to the preceding Table.

Ages.	Expectations of Males.	Expectations of Females.
Birth	14.25	18.10
5	31.05	37.12
10	30.00	36.89
15	26.74	33.43
20	23.85	30.01
25	21.40	26.80
30	19.42	23.98
35	17.58	21.62
40	15.61	19.25
45	13.78	17.17
50	11.95	15.12
55	10.30	12.89
60	8.69	10.45
65	7.39	8.39
70	5.81	6.16
75	4.09	4.39

From this comparison, and from Tables XL. and XLII. p. 136 and 150, it appears, that the difference between the duration of the lives of *males* and *females* is *least* in the kingdom of SWEDEN at large, *greater* at CHESTER, and *greatest* at STOCKHOLM, which seems to indicate that this is a difference not entirely natural.

## T A B L E XLV.

Shewing the Values of Annuities on Single Lives among Males and Females, according to the Probabilities of the Duration of Life in the Kingdom of SWEDEN. See Table XLII. page 150.

Ages.	MALES.		FEMALES.		Lives in general.	
	4 per Ct.	5 per Ct.	4 per Ct.	5 per Ct.	4 per Ct.	5 per Ct.
1	16.503	14.051	16.820	14.271	16.661	14.161
2	17.355	14.778	17.719	15.034	17.537	14.906
3	17.935	15.279	18.344	15.571	18.139	15.425
4	18.328	15.624	18.780	15.951	18.554	15.787
5	18.503	15.786	18.927	16.088	18.715	15.937
6	18.622	15.901	19.045	16.203	18.833	16.052
7	18.693	15.977	19.131	16.291	18.912	16.134
8	18.725	16.021	19.162	16.335	18.943	16.178
9	18.715	16.030	19.151	16.343	18.933	16.186
10	18.674	16.014	19.109	16.325	18.891	16.169
11	18.600	15.970	19.041	16.286	18.820	16.128
12	18.491	15.896	18.952	16.229	18.721	16.062
13	18.378	15.819	18.840	16.153	18.609	15.986
14	18.246	15.724	18.707	16.059	18.476	15.891
15	18.105	15.624	18.568	15.960	18.336	15.792
16	17.958	15.517	18.424	15.856	18.191	15.686
17	17.803	15.404	18.290	15.761	18.046	15.582
18	17.643	15.285	18.151	15.662	17.897	15.473
19	17.492	15.175	18.013	15.563	17.752	15.369
20	17.335	15.059	17.872	15.462	17.603	15.260
21	17.192	14.955	17.725	15.356	17.458	15.155
22	17.042	14.846	17.573	15.245	17.307	15.045
23	16.887	14.732	17.414	15.129	17.150	14.930
24	16.742	14.627	17.252	15.009	16.997	14.818
25	16.592	14.517	17.087	14.886	16.839	14.701
26	16.436	14.402	16.915	14.757	16.675	14.579
27	16.274	14.282	16.751	14.636	16.512	14.459
28	16.105	14.156	16.588	14.515	16.346	14.335
29	15.930	14.024	16.427	14.396	16.178	14.210
30	15.751	13.889	16.261	14.272	16.006	14.080
31	15.575	13.756	16.104	14.156	15.839	13.956
32	15.395	13.619	15.941	14.035	15.668	13.827
33	15.208	13.477	15.787	13.923	15.497	13.700
34	15.014	13.327	15.629	13.806	15.321	13.566
35	14.812	13.170	15.465	13.684	15.138	13.427



# T A B L E S.

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TABLE XLV. continued.

Ages.	MALES.		FEMALES.		Lives in general.	
	4 per Ct.	5 per Ct.	4 per Ct.	5 per Ct.	4 per Ct.	5 per Ct.
36	14.601	13.006	15.278	13.542	14.939	13.274
37	14.382	12.833	15.070	13.382	14.726	13.107
38	14.154	12.652	14.854	13.213	14.504	12.932
39	13.916	12.462	14.629	13.036	14.272	12.749
40	13.668	12.261	14.401	12.856	14.034	12.558
41	13.426	12.065	14.185	12.687	13.805	12.376
42	13.196	11.880	13.994	12.538	13.595	12.209
43	12.984	11.710	13.798	12.387	13.391	12.048
44	12.763	11.532	13.596	12.229	13.179	11.880
45	12.535	11.347	13.383	12.061	12.959	11.704
46	12.297	11.153	13.151	11.876	12.724	11.514
47	12.051	10.951	12.894	11.668	12.472	11.309
48	11.795	10.738	12.620	11.443	12.217	11.090
49	11.528	10.516	12.333	11.205	11.930	10.860
50	11.267	10.298	12.049	10.970	11.658	10.634
51	11.030	10.100	11.769	10.737	11.399	10.418
52	10.785	9.895	11.492	10.507	11.138	10.201
53	10.531	9.682	11.220	10.280	10.875	9.981
54	10.269	9.460	10.937	10.042	10.603	9.751
55	9.998	9.229	10.642	9.792	10.320	9.510
56	9.717	8.988	10.334	9.529	10.025	9.258
57	9.425	8.736	10.012	9.253	9.718	8.994
58	9.140	8.489	9.692	8.976	9.416	8.732
59	8.845	8.232	9.358	8.684	9.101	8.458
60	8.540	7.963	9.039	8.406	8.789	8.184
61	8.241	7.700	8.739	8.144	8.490	7.922
62	7.950	7.442	8.453	7.895	8.201	7.668
63	7.669	7.193	8.166	7.643	7.917	7.418
64	7.382	6.938	7.870	7.382	7.626	7.160
65	7.090	6.676	7.566	7.111	7.328	6.893
66	6.792	6.408	7.252	6.831	7.022	6.619
67	6.489	6.134	6.930	6.541	6.709	6.337
68	6.201	5.872	6.596	6.239	6.398	6.055
69	5.933	5.628	6.253	5.926	6.093	5.777
70	5.670	5.389	5.897	5.599	5.783	5.494
71	5.418	5.158	5.564	5.293	5.491	5.225
72	5.180	4.940	5.261	5.013	5.220	4.976
73	4.940	4.719	4.998	4.770	4.969	4.744
74	4.724	4.521	4.792	4.581	4.758	4.551
75	4.487	4.302	4.582	4.388	4.534	4.345
76	4.253	4.084	4.367	4.189	4.310	4.136
77	4.024	3.871	4.145	3.983	4.084	3.927

TABLE XLV. continued.

Ages.	M A L E S.		F E M A L E S.		Lives in general.	
	4 per Ct.	5 per Ct.	4 per Ct.	5 per Ct.	4 per Ct.	5 per Ct.
78	3.768	3.631	3.913	3.767	3.840	3.699
79	3.512	3.390	3.668	3.536	3.590	3.463
80	3.260	3.152	3.402	3.285	3.331	3.218
81	3.017	2.921	3.145	3.041	3.081	2.981
82	2.792	2.706	2.905	2.812	2.848	2.759
83	2.600	2.523	2.699	2.615	2.649	2.569
84	2.473	2.403	2.559	2.480	2.516	2.441
85	2.371	2.306	2.552	2.476	2.461	2.391
86	2.281	2.222	2.518	2.446	2.399	2.334
87	2.154	2.103	2.431	2.365	2.292	2.238
88	1.955	1.912	2.294	2.236	2.124	2.074
89	1.698	1.664	2.108	2.059	1.903	1.861
90	1.417	1.392	1.873	1.833	1.645	1.612
91	1.154	1.136	1.628	1.596	1.391	1.366
92	0.835	0.824	1.349	1.325	1.092	1.074
93	0.477	0.471	1.071	1.054	0.774	0.762
94	0.240	0.238	0.799	0.788	0.519	0.513
95	0.000	0.000	0.544	0.537		
96	0.000	0.000	0.320	0.317		

TABLE

## T A B L E XLVI.

Shewing the Values of Annuities on two *joint* Lives, according to the Probabilities (in Table XLIII.) of the Duration of Human Life among Males and Females collectively, reckoning Interest at 4 *per cent*.

INTEREST 4 *per cent*.

Differences of Age, 0, 6, 12, and 18 Years.

Ages.	Values.	Ages.	Values.	Ages.	Values.	Ages.	Values.
1- 1	12.252	1- 7	13.989	1-13	13.894	1-19	13.309
2- 2	13.583	2- 8	14.780	2-14	14.557	2-20	14.008
3- 3	14.558	3- 9	15.323	3-15	14.988	3-21	14.417
4- 4	15.267	4-10	15.685	4-16	15.259	4-22	14.671
5- 5	15.577	5-11	15.817	5-17	15.326	5-23	14.725
6- 6	15.820	6-12	15.887	6-18	15.354	6-24	14.740
7- 7	16.003	7-13	15.914	7-19	15.351	7-25	14.727
8- 8	16.109	8-14	15.888	8-20	15.310	8-26	14.673
9- 9	16.152	9-15	15.824	9-21	15.244	9-27	14.590
10-10	16.141	10-16	15.729	10-22	15.149	10-28	14.484
11-11	16.087	11-17	15.617	11-23	15.033	11-29	14.357
12-12	15.982	12-18	15.477	12-24	14.889	12-30	14.202
13-13	15.855	13-19	15.327	13-25	14.736	13-31	14.045
14-14	15.701	14-20	15.164	14-26	14.566	14-32	13.874
15-15	15.535	15-21	15.001	15-27	14.392	15-33	13.700
16-16	15.361	16-22	14.832	16-28	14.216	16-34	13.520
17-17	15.196	17-23	14.665	17-29	14.042	17-35	13.340
18-18	15.023	18-24	14.491	18-30	13.860	18-36	13.141
19-19	14.854	19-25	14.320	19-31	13.687	19-37	12.934
20-20	14.682	20-26	14.144	20-32	13.512	20-38	12.720
21-21	14.525	21-27	13.976	21-33	13.345	21-39	12.505
22-22	14.360	22-28	13.807	22-34	13.173	22-40	12.286
23-23	14.194	23-29	13.635	23-35	12.997	23-41	12.073
24-24	14.020	24-30	13.455	24-36	12.801	24-42	11.873
25-25	13.849	25-31	13.284	25-37	12.599	25-43	11.683
26-26	13.671	26-32	13.108	26-38	12.387	26-44	11.485
27-27	13.495	27-33	12.935	27-39	12.170	27-45	11.284
28-28	13.323	28-34	12.763	28-40	11.953	28-46	11.072
29-29	13.148	29-35	12.586	29-41	11.742	29-47	10.847
30-30	12.965	30-36	12.390	30-42	11.543	30-48	10.606
31-31	12.795	31-37	12.192	31-43	11.359	31-49	10.365
32-32	12.624	32-38	11.988	32-44	11.170	32-50	10.128



## TABLE XLVI continued.

INTEREST 4 *per cent.*

Ages.	Values.	Ages.	Values.	Ages.	Values.	Ages.	Values.
33-33	12.456	33-39	11.779	33-45	10.978	33-51	9.905
34-34	12.286	34-40	11.568	34-46	10.775	34-52	9.679
35-35	12.109	35-41	11.361	35-47	10.557	35-53	9.452
36-36	11.904	36-42	11.156	36-48	10.314	36-54	9.207
37-37	11.683	37-43	10.953	37-49	10.059	37-55	8.951
38-38	11.452	38-44	10.741	38-50	9.805	38-56	8.683
39-39	11.209	39-45	10.519	39-51	9.558	39-57	8.404
40-40	10.964	40-46	10.286	40-52	9.308	40-58	8.124
41-41	10.732	41-47	10.049	41-53	9.066	41-59	7.839
42-42	10.531	42-48	9.813	42-54	8.830	42-60	7.569
43-43	10.346	43-49	9.581	43-55	8.597	43-61	7.318
44-44	10.154	44-50	9.351	44-56	8.354	44-62	7.075
45-45	9.954	45-51	9.129	45-57	8.101	45-63	6.836
46-46	9.736	46-52	8.897	46-58	7.841	46-64	6.586
47-47	9.497	47-53	8.658	47-59	7.563	47-65	6.323
48-48	9.236	48-54	8.402	48-60	7.281	48-66	6.048
49-49	8.966	49-55	8.139	49-61	7.008	49-67	5.764
50-50	8.707	50-56	7.874	50-62	6.749	50-68	5.487
51-51	8.469	51-57	7.613	51-63	6.505	51-69	5.221
52-52	8.230	52-58	7.351	52-64	6.256	52-70	4.952
53-53	7.994	53-59	7.083	53-65	6.004	53-71	4.697
54-54	7.748	54-60	6.814	54-66	5.743	54-72	4.455
55-55	7.495	55-61	6.555	55-67	5.474	55-73	4.231
56-56	7.229	56-62	6.299	56-68	5.204	56-74	4.043
57-57	6.954	57-63	6.045	57-69	4.936	57-75	3.844
58-58	6.678	58-64	5.788	58-70	4.664	58-76	3.637
59-59	6.388	59-65	5.519	59-71	4.395	59-77	3.430
60-60	6.104	60-66	5.249	60-72	4.149	60-78	3.210
61-61	5.844	61-67	4.984	61-73	3.927	61-79	2.974
62-62	5.600	62-68	4.729	62-74	3.747	62-80	2.744
63-63	5.367	63-69	4.482	63-75	3.563	63-81	2.557
64-64	5.128	64-70	4.231	64-76	3.370	64-82	2.396
65-65	4.881	65-71	3.982	65-77	3.180	65-83	2.252
66-66	4.626	66-72	3.750	66-78	2.974	66-84	2.123
67-67	4.362	67-73	3.527	67-79	2.743	67-85	2.010
68-68	4.130	68-74	3.340	68-80	2.514	68-86	1.910
69-69	3.851	69-75	3.147	69-81	2.324	69-87	1.798
70-70	3.593	70-76	2.946	70-82	2.155	70-88	1.661
71-71	3.345	71-77	2.752	71-83	2.004	71-89	1.464
72-72	3.128	72-78	2.558	72-84	1.875	72-90	1.189
73-73	2.935	73-79	2.355	73-85	1.768	73-91	0.937



TABLE XLVI. continued.

INTEREST 4 *per cent.*

Ages.	Values.	Ages.	Values.	Ages.	Values.	Ages.	Values.
74-74	2.797	74-80	2.172	74-86	1.692	74-92	0.708
75-75	2.648	75-81	2.017	75-87	1.605	75-93	0.575
76-76	2.490	76-82	1.877	76-88	1.497	76-84	0.481
77-77	2.340	77-83	1.756	77-89	1.339	77-95	0.421
78-78	2. 70	78-84	1.639	78-90	1.097		
79-79	1.967	79-85	1.524	79-91	0.863		
80-80	1.758	80-86	1.416	80-92	0.638		
81-81	1.600	81-87	1.320	81-93	0.511		
82-82	1.472	82-88	1.225	82-94	0.427		
83-83	1.364	83-89	1.094	83-95	0.379		
84-84	1.276	84-90	0.902				
85-85	1.212	85-91	0.725				
86-86	1.172	86-92	0.556				
87-87	1.127	87-93	0.459				
88-88	1.071	88-94	0.396				
89-89	0.949	89-95	0.364				
90-90	0.718						
91-91	0.516						
92-92	0.326						
93-93	0.236						
94-94	0.190						
95-95	0.024						

## TABLE XLVII.

Shewing the Values of two *joint* Lives, according to the Probabilities (in Table XLIII.) of the Duration of Human Life among Males and Females collectively.

INTEREST 4 *per cent.*

Differences of Age 24, 30, 36, and 42 Years.

Ages.	Values.	Ages.	Values.	Ages.	Values.	Ages.	Values.
1-25	12.832	1-31	12.196	1-37	11.465	1-43	10.546
2-26	13.409	2-32	12.730	2-38	11.913	2-44	10.946
3-27	13.778	3-33	13.066	3-39	12.164	3-45	11.168
4-28	14.003	4-34	13.264	4-40	12.284	4-46	11.260
5-29	14.037	5-35	13.277	5-41	12.242	5-47	11.183
6-30	14.033	6-36	13.242	6-42	12.185	6-48	11.064
7-31	14.006	7-37	13.170	7-43	12.112	7-49	10.915
8-32	13.944	8-38	13.059	8-44	12.004	8-50	10.743
9-33	13.855	9-39	12.913	9-45	11.865	9-51	10.560
10-34	13.741	10-40	12.743	10-46	11.694	10-52	10.357
11-35	13.604	11-41	12.563	11-47	11.493	11-53	10.140
12-36	13.428	12-42	12.379	12-48	11.259	12-54	9.898
13-37	13.234	13-43	12.196	13-49	11.011	13-55	9.644
14-38	13.023	14-44	11.997	14-50	10.759	14-56	9.371
15-39	12.798	15-45	11.787	15-51	10.514	15-57	9.087
16-40	12.570	16-46	11.562	16-52	10.264	16-58	8.799
17-41	12.351	17-47	11.328	17-53	10.018	17-59	8.503
18-42	12.146	18-48	11.076	18-54	9.761	18-60	8.208
19-43	11.951	19-49	10.819	19-55	9.500	19-61	7.928
20-44	11.751	20-50	10.567	20-56	9.228	20-62	7.658
21-45	11.550	21-51	10.332	21-57	8.953	21-63	7.396
22-46	11.335	22-52	10.092	22-58	8.675	22-64	7.127
23-47	11.107	23-53	9.852	23-59	8.385	23-65	6.851
24-48	10.862	24-54	9.602	24-60	8.097	24-66	6.566
25-49	10.612	25-55	9.347	25-61	7.823	25-67	6.275
26-50	10.364	26-56	9.080	26-62	7.557	26-68	5.986
27-51	10.130	27-57	8.807	27-63	7.297	27-69	5.702
28-52	9.894	28-58	8.534	28-64	7.032	28-70	5.415
29-53	9.659	29-59	8.250	29-65	6.761	29-71	5.136
30-54	9.413	30-60	7.967	30-66	6.481	30-72	4.881
31-55	9.167	31-61	7.702	31-67	6.197	31-73	4.646
32-56	8.912	32-62	7.446	32-68	5.917	32-74	4.453
33-57	8.651	33-63	7.196	33-69	5.642	33-75	4.251
34-58	8.389	34-64	6.942	34-70	5.364	34-76	4.040
35-59	8.114	35-65	6.679	35-71	5.093	35-77	3.833
36-60	7.833	36-66	6.402	36-72	4.840	36-78	3.605

TABLE XLVII. continued.

INTEREST 4 per Cent.

Ages.	Values.	Ages.	Values.	Ages.	Values.	Ages.	Values.
37-61	7.561	37-67	6.115	37-73	4.603	37-79	3.352
38-62	7.296	38-68	5.828	38-74	4.405	38-80	3.098
39-63	7.033	39-69	5.543	39-75	4.195	39-81	2.889
40-64	6.763	40-70	5.254	40-76	3.975	40-82	2.710
41-65	6.492	41-71	4.977	41-77	3.762	41-83	2.553
42-66	6.225	42-72	4.730	42-78	3.539	42-84	2.418
43-67	5.957	43-73	4.507	43-79	3.295	43-85	2.305
44-68	5.689	44-74	4.322	44-80	3.052	44-86	2.203
45-69	5.426	45-75	4.128	45-81	2.854	45-87	2.083
46-70	5.153	46-76	3.921	46-82	2.684	46-88	1.933
47-71	4.884	47-77	3.715	47-83	2.533	47-89	1.708
48-72	4.633	48-78	3.489	48-84	2.396	48-90	1.385
49-73	4.398	49-79	3.238	49-85	2.277	49-91	1.090
50-74	4.205	50-80	2.990	50-86	2.171	50-92	0.818
51-75	4.008	51-81	2.792	51-87	2.050	51-93	0.662
52-76	3.803	52-82	2.623	52-88	1.901	52-94	0.551
53-77	3.605	53-83	2.475	53-89	1.681	53-95	0.468
54-78	3.389	54-84	2.344	54-90	1.366		
55-79	3.150	55-85	2.232	55-91	1.078		
56-80	2.909	56-86	2.130	56-92	0.810		
57-81	2.710	57-87	2.010	57-93	0.655		
58-82	2.539	58-88	1.864	58-94	0.546		
59-83	2.385	59-89	1.644	59-95	0.464		
60-84	2.248	60-90	1.333				
61-85	2.135	61-91	1.050				
62-86	2.037	62-92	0.789				
63-87	1.926	63-93	0.639				
64-88	1.790	64-94	0.533				
65-89	1.585	65-95	0.456				
66-90	1.290						
67-91	1.017						
68-92	0.764						
69-93	0.617						
70-94	0.514						
71-95	0.441						

THE directions given in p. 96, for using the tables of the values of joint lives deduced from the *Northampton* Observations, are applicable to the two last Tables, and may be easily adapted to them, by taking the differences of age in those directions at *six* years and its multiples, instead of *five* years and its multiples.

#### R E M A R K S.

The values of *joint* lives in these Tables have been computed for only one rate of interest; and of *single* lives in Table XLV. for only two rates of interest. The following rules will shew, that it would be a needless labour to compute these values (in strict conformity to the observations) for any other rates of interest.

*ACCOUNT of a method of deducing, from the correct values (according to any observations) of any single or joint lives at one rate of interest, the same values at other rates of interest.*

#### P R E L I M I N A R Y P R O B L E M S.

**PROBLEM I.** The *expectation* given of a single life by any table of observations, to find its value, supposing the decrements of life equal, at any given rate of interest.

SOLU-



SOLUTION, Find in Table II. the value of an annuity certain for a number of years equal to twice the expectation. Multiply this value by the PERPETUITY increased by unity, and divide the product by twice the expectation. The quotient subtracted from the perpetuity will be the value required.

## EXAMPLE.

The *expectation* of a male life aged ten, by the SWEDEN observations (See Table XLV.) is 43.94. Twice this expectation is 87.88. The value of an annuity certain for 87.88 years is, by Table II. (reckoning interest at 4 *per cent.*) 24.200. The product of 24.200 into 26 (the perpetuity increased by unity) is 629.2, which, divided by 87.88, gives 7.159. And this quotient subtracted from 25 (the perpetuity) gives 17.84 years purchase, the value of a life aged ten, deduced from the expectation of life at that age, according to the SWEDEN observations.

This is the rule by which Mr. *De Moivre* has calculated the table commonly used of the values of lives according to his hypothesis; and from this Table (the first of the two Tables at the end of this volume) the value required in this problem may be deduced more compendiously in the following

following manner, provided the expectation does not exceed 38.—“ Take the difference between twice the expectation and 86; and the value in the Table corresponding to that difference, if not less than 10, will be the value sought.”—Thus; twice the *expectation* of a female life aged 30 (that is, its *complement*) is, by Table XLII. 64.34. The difference between it and 86, is 21.66. And since the value corresponding to age 21 in Mr. *De Moivre's* valuation of lives (or in Table I. at the end of this volume) is (reckoning interest at 4 *per cent.*) 15.781; and the value corresponding to age 22 is 15.669; it is obvious, that the value corresponding to age 21.66 must be the *greatest* of these two values lessened by  $\frac{.66}{1.06}$  of the difference between it and the *least*. This difference is .112; and  $\frac{.66}{1.06}$  of it (or .112 multiplied by .66) is .074, which subtracted from 15.781, gives 15.707 the value sought of a life whose expectation is 32.17 (or whose complement is 64.34) on the supposition of an equal decrement of life.

### PROBLEM II.

Having the expectations given of any two lives by any table of observations, to deduce from thence the value of the joint lives at any

any rate of interest supposing an equal decrement of life.

**SOLUTION.** Find the difference between twice the expectation of the youngest life, and twice the expectation of the oldest life increased by unity and twice the perpetuity. Multiply this difference by the value of an annuity certain for a time equal to *twice* the expectation of the *oldest* life; and by twice the same expectation divide the product, reserving the quotient.

From twice the perpetuity subtract the reserved quotient, and multiply the remainder by the perpetuity increased by unity (*a*). This last product divided by twice the expectation of the *youngest* life, and then subtracted from the perpetuity, will be the required value.

### EXAMPLE.

Let the joint lives proposed be a female life aged 10, and a male life aged 15, and let the table of observations be the *Sweden*

(*a*) When twice the expectation of the youngest life is *greater* than twice the expectation of the oldest life increased by unity and twice the perpetuity, the reserved quotient instead of being *subtracted* from twice the perpetuity, must be added to it, and the *sum*, not the *difference*, multiplied by the perpetuity increased by unity.

Table

Table for lives in general, and the rate of interest 4 *per cent.* Twice the expectations of the two lives are 90.14 and 83.28. (See Table XLIII.)

Twice the expectation of the *oldest* life, increased by unity and twice the perpetuity, is 134.28, which lessened by 90.14 (twice the expectation of the *youngest* life) leaves 44.14 for the *reserved* remainder.——This remainder multiplied by 24.045 (the value of an annuity certain (a) for 83.28 years) and the product divided by 83.28 (twice the expectation of the *oldest* life) gives 12.744 the quotient to be *reserved*; which subtracted from double the perpetuity, and the remainder (or 37.255) multiplied by the perpetuity increased by unity (or by 26) gives 968.630, which divided by 90.14 (twice the expectation of the *youngest* life) and the quotient subtracted from the perpetuity, we have 14.254 for the required value.

(a) This value, when the number of years is a whole number with a fraction added (as will be commonly the case) may be best computed in the following manner.

In this example the number of years is 83.28.

The value of an annuity certain for 83 years is (by Table II. p. 21. 24.035.——The same value for 84 years is 24.072.——The difference between these two values is .037; which difference multiplied by .28 (the fractional part of the number of years) and the product (.0103) added to the *least* of the two values, will give 24.045 the value for 83.28 years.

This



This calculation may be made more easily by logarithms in the following manner.

Twice the expectation of the eldest is	}	83.28
Add twice the perpetuity increased by unity - -	}	51.00
Sum - -		134.28
SUBTRACT twice the expectation of the youngest -	}	90.14
Remainder - -		44.14
Log. of 44.14 is		1.644,832
Log. of 24.045 is	}	1.381,024
(See the note in the last page)	}	3.025,856
Sum - -		3.025,856
Subtract		1.920,540
Remainder -		1.105,316
	}	the number of which is 12.744 or the remainder to be reserved.

Twice the perpetuity is	}	50
Subtract		12.744
Remains -		37.255
Add Log. of the perpetuity increased by unity		1.571,184
Sum -		1.414,973
Subtract Log. of twice the expectation of the youngest	}	2.986,157
Sum -		1.954,917
Remainder -		1.031,240

The

The number of this last remainder is 10.745, which subtracted from 25 (the perpetuity) leaves 14.254, the value sought.— See the algebraical canon in Note (L) at the end of this volume.

### GENERAL RULE.

Call the *correct* value (supposed to be computed for any rate of interest) the FIRST value.

Call the value deduced (by the preceding problems) from the *expectations* at the same rate of interest, the SECOND value.

Call the value deduced from the *expectations* for any other rate of interest the THIRD value.

Then, the difference between the *first* and *second* values added to or subtracted from the *third* value, just as the *first* is greater or less than the *second*, will be the value at the rate of interest for which the THIRD value has been deduced from the expectations.

*The following examples will make this perfectly plain.*

### EXAMPLE I.

In the last Tables the correct values are given of two joint lives among mankind at large, without distinguishing between males and females, according to the SWEDEN observations, reckoning interest at 4 per cent.

Let

Let it be required to find from these values the values at 3 *per cent.*; and let the ages of the joint lives be supposed 10 and 10.

The correct value by Table XLVI. (reckoning interest at 4 *per cent.*) is 16.141. The expectation of a life aged 10 is (by Table XLVII.) 45.07.—The value deduced from this expectation at 4 *per cent.* by Prob. II. is 14.539.—The value deduced by the same problem from the same expectation at 3 *per cent.* is 16.808.—The difference between the *first* and *second* values, is 1.602, which, added to the *third* value (the first being *greater* than the second) makes 18.410 the value required.

### EXAMPLE II.

Let the values be required of two joint lives aged 50 and 60, at an interest of 3 *per cent.* from the correct value given at an interest of 4 *per cent.* according to the *Northampton* observations.

*First* or correct value at 4 *per cent.* by Table XX. is 6.989. The expectation of 50 is 17.99; of 60, is 13.21, by Table VII. p. 38. The *second* value, or the value deduced from these expectations at 4 *per cent.* is, by Prob. II. 7.182. The *third* value, or the value deduced from the same expectations at 3 *per cent.* is 7.704.—The

difference between the first and second is .193, which (since the *second* is greater than the *first*) must be subtracted from the *third*, and the remainder (or 7.511) will be the value required.—The exact value at 3 *per cent.* is, by Table XX. 7.460.

If the value is required at 5 *per cent.* the *third* value will be 6.732; and the difference subtracted from 6.732, will leave 6.539 the value at 5 *per cent.*

The *exact* value at this rate of interest, is (by Table XX.) 6.568.

### EXAMPLE III.

Let the value be required of a single male life aged 10, at 3 *per cent.* interest, from the correct value at 4 *per cent.* according to the *Sweden* observations.

*First*, or correct value at 4 *per cent.* (by Table XLV.) is 18.674. The expectation of a male life aged 10, is (by Table XLII.) 43.94.

The *second* value (or the value deduced from this expectation by Prob. I.) is 17.838.

The *third* value (or the value deduced from the same expectation at 3 *per cent.*) is 21.277.

The difference between the first and second is .836; which (since the first is greater than the second) must be added to the



the *third*; and the *sum*, (that is, 22.113) will be the value required.

The *third* value at 5 *per cent.* is 15.286; and the difference added to 15.286 makes 16.122 the value of a male life aged 10 at 5 *per cent.* according to the SWEDEN observations.——The exact value at 5 *per cent.* is (by Table 45th) 16.014.

Again. The difference between 16.014 (the correct value at 5 *per cent.*) and 15.286 (the value at the same interest deduced from the expectation) is .728; which, *added* (because the first value is greater than the second) to 13.235, (the value deduced at 6 *per cent.* from the expectation) gives 14.063, the value of the same life, reckoning interest at 6 *per cent.*

These deductions, in the case of single lives particularly, are so easy, and give the true values so nearly, that it will be scarcely ever necessary to calculate the *exact* values (according to any given observations) for more than *one* rate of interest.

If, for instance, the correct values are computed at 4 *per cent.* according to any observations, the values at 3,  $3\frac{1}{2}$ ,  $4\frac{1}{2}$ , 5, 6, 7, or 8 *per cent.* may be deduced from them by the preceding rules, as occasion may require, without much labour or any danger of considerable errors.——The following comparisons will shew in some measure how far these deductions may be depended on.

Value of Single Male Lives by the SWEDEN  
Table of Observations, p. 162.

Ages.	Values at 5 per cent. deduced from the correct values by Table XLV. at 4 per cent.	Correct values by Table XLV. at 5 per cent.
5	15.879	15.786
10	16.122	16.014
15	15.707	15.624
30	13.909	13.889
60	7.969	7.963
70	5.417	5.389

Values of two joint Lives by the NORTH-  
AMPTON Table of Observations.

Ages.	Values at 5 per cent. deduced from the correct values at 4 per cent. by Table 18th &c.	Correct values at 5 per cent. by Tables 18, 19, &c.	Values at 3 per cent. deduced from the correct values at 4 per cent. by Table 18th, &c.	Correct values at 3 per cent. by Table 18, 19, &c.
5-5	11.989	11.984	15.618	15.638
15-15	11.986	11.960	15.184	15.229
25-25	10.775	10.764	13.389	13.383
40-40	9.006	9.016	10.756	10.764
60-60	5.842	5.888	6.692	6.606
15-40	10.214	10.205	12.368	12.459
30-60	7.285	7.292	8.396	8.378
50-60	6.555	6.568	7.471	7.461

Values of Single Lives by the NORTHAMPTON Table of Observations.

Ages.	Values at 5 per cent. deduced from the correct values at 4 per cent. by Table 17.	Correct values at 5 per cent. by Table 17.	Values at 3 per cent. deduced from the correct values at 5 per cent. by Table 17.	Correct values at 3 per cent. by Table 17.
5	14.825	14.827	20.435	20.473
10	15.162	15.139	20.652	20.663
68	6.546	6.536	7.353	7.367

Ages.	Values at 4 per cent. deduced from the correct values at 5 per cent. by Table 17.	Correct values at 4 per cent. by Table 17.	Values at 5 per cent. deduced from the correct values at 3 per cent. by Table 17.	Correct values at 5 per cent.
5	17.239	17.248	14.850	14.827
10	17.500	17.523	15.173	15.139
68	6.920	6.930	6.560	6.536

It may be observed in these examples, that the *deduced* values are *sometimes* almost the same with the *correct* values; that *generally* they do not differ more than a 20th or 30th of a year's purchase; that in *joint* lives they differ less than in *single* lives; and that they come equally near to one another *whatever* the rates of interest are.

The following observation will shew the reason of the circumstance last mentioned.

The value *deduced* from the expectation coincides with the *correct* value when the rate of interest is little or nothing; and, consequently, the difference between the two



values becomes then little or nothing; and to this it is continually tending as the interest is diminished. On the contrary; the *increase* of value occasioned by the *decrease* of interest tends to make the difference greater. There is, therefore, in this case, the counter-action of two causes which always keep the difference nearly the same in all rates of interest.

The preceding rules seem to leave nothing wanting on this subject, except tables of the values of two joint lives *at any one rate of interest*, when the lives are either both male or both female lives. But the following rule for finding these values from the values in the two last Tables, will render the labour of composing such tables almost needless.

RULE for computing from the values of two joint lives in Tables XLVI. and XLVII. the values of two joint lives both male or both female.

- “ Find in that column of Table XLIII.
- “ which shews the expectations of *lives in*
- “ *general*, two ages whose expectations come
- “ nearest to the expectations of the two
- “ *male* or the two *female* lives proposed.
- “ From these expectations deduce, by the
- “ rule in p. 176, the value of two joint
- “ lives



“ lives at those ages ; and take the difference between this value and the *correct* value at those ages in Tables XLVI. and XLVII.

“ Deduce also, by the rule in p. 176, the value of the joint lives proposed, from the expectations in Table XLIII. of male and female lives. The difference just found *added* to this last value, if the value before deduced from the expectations of *lives in general* is *less* than the correct value, or *subtracted* from it if *greater*, will be nearly the correct value of the two joint lives proposed.”

## E X A M P L E.

Let the two proposed lives be both *female* lives, one aged 20 and the other aged 50.

The expectation of a *female* life aged 20 is, by Table XLII. 39.15. The expectation nearest to it, in Table XLIII. shewing the expectations of lives in general, is 39.47, corresponding to a life aged 18.—In like manner ; the expectation in the same Table nearest to the expectation of a female life aged 50, is 19.09, corresponding to age 49.—The value (deduced from these expectations) of two joint lives aged 18 and 49, is, by the rule in p. 176, 10.245. The *correct* value, taken from Table XLVII. is

M 4

10.851,

10.851, and the difference is .606, which difference *added* (since the former value is *less* than the latter) to 10.281 (the value of two joint female lives aged 50 and 20, deduced from the expectations by the rule in p. 176) makes 10.887, the *correct* value nearly of the joint *female* lives.

In order to find how near the values thus found come to the exact values, let the value of a *single female* life aged 20 (reckoning interest at 4 *per cent.*) be computed in the same manner from the correct values given in Table XLV. of the values of lives in general.

The expectation in Table XLIII. nearest to the expectation in Table XLII. of a *female* life aged 20, is 39.47, which, in Table XLIII. (shewing the expectation of lives in general) is the expectation of a life aged 18.——The value of a life aged 18, deduced from *this* expectation by the rule in p. 176, is 17.138. The correct value in that column of Table XLV. which shews the values of lives in general, is 17.897. The former value is the least, and the difference is .759.——The value deduced by the same rule from 39.15 (the expectation of a *female* life aged 20) is 17.083, and the difference just found *added* to this value, makes it 17.842, which is very nearly the same with 17.872, the correct value in Table XLV.——The value deduced in the  
same

same manner of a *male* life aged 20, is 17.363.  
The correct value (in Table XLV.) is  
17.335.

Value by *this Rule* of

A *female* life aged 50, is 12.000 — Correct value is 12.049  
aged 60, is 9.018 — Correct value is 9.039  
Of a *male* life aged 30, is 15.722 — Correct value is 15.751  
aged 70, is 5.702 — Correct value is 5.670

In calculating by this rule, when any other rate of interest than 4 *per cent.* is used; the values of the joint lives, at that rate of interest, (deduced from the expectations and from the values in Tables XLVI. and XLVII. at 4 *per cent.* by Prob. II. p. 176.) must be taken for the *correct* values. — It must likewise be remembered, that this Rule cannot be used when the youngest of the two joint lives is *less* than ten years of age. In other cases, the values found by this Rule will be right *generally* within a 30th or 40th of a year's purchase, and *never*, I believe, wrong more than a 15th or 20th of a year's purchase.

THE last Tables from p. 162, I reckon the most important in this collection, not only because the only ones that give the separate values of the lives of males and females, and because derived from observations in their nature more correct; but on account of their particular use in furnishing instruction to the numerous institutions for granting annuities to widows. Mr. *Wargentin* informs me, that even in *Sweden* several societies of this kind have become bankrupts for want of such instruction. I think it, therefore, necessary to add the following Table.

## T A B L E XLVIII.

Shewing the Value of an Annuity for the Life of a Wife after the Death of her Husband; deduced from the SWEDEN Observations on the separate Probabilities of the duration of Life among Males and Females.

The Annuity 10*l.*—Interest 4 *per cent.*

Wife's Age.	Huf- band's Age.	Value of the Annuity.		Wife's Age.	Huf- band's Age.	Value of the Annuity.	
		Single Payment.	Annual Payment.			Single Payment.	Annual Payment.
		£.	£.			£.	£.
16	16	30.63	1.87	20	20	31.90	2.03
	22	35.92	2.26		26	37.28	2.46
	28	42.08	2.76		32	43.60	3.00
	34	49.04	3.38		38	51.52	3.80
	40	58.54	4.31		44	61.21	4.80
	46	68.62	5.46		50	73.05	6.31
	52	81.60	7.24		56	86.44	8.36
	58	96.25	9.82		62	102.14	11.79



T A B L E XLVIII. continued.

Wife's Age.	Huf-band's Age.	Value of the Annuity.		Wife's Age.	Huf-band's Age.	Value of the Ann.	
		Single Payment.	Annual Payment.			Single Payment.	Annual Paym.
		£.	£.			£.	£.
24	24	32.32	2.15	42	42	34.62	3.00
	30	37.97	2.62		48	41.81	3.86
	36	44.51	3.22		54	51.63	5.25
	42	53.79	4.18		60	64.25	7.49
	48	63.90	5.38		66	77.69	10.75
	54	76.50	7.21		72	92.63	16.16
	60	91.55	10.06				
28	28	32.64	2.28	46	46	34.15	3.18
	34	38.25	2.77		52	42.54	4.29
	40	46.35	3.58		58	53.10	6.00
	46	55.16	4.57		64	65.65	8.65
	52	66.94	6.14		70	79.97	12.99
	58	80.54	8.45	50	50	33.42	3.44
	64	95.56	11.90		56	41.75	4.70
32	32	33.16	2.43		62	53.00	6.83
	38	39.52	3.04		68	65.62	10.11
	44	47.71	3.92	54	54	31.89	3.63
	50	58.13	5.22		60	41.23	5.27
	56	70.29	7.09		66	51.94	7.70
	62	84.95	10.05		72	64.82	11.88
	68	100.24	14.49				
36	36	33.74	2.61	58	58	30.14	3.92
	42	41.81	3.86		64	39.04	5.75
	48	49.64	4.38		70	50.28	8.87
	54	61.71	6.04				
	60	74.44	8.43				
	66	88.76	12.00				

REMARKS.

## R E M A R K S.

THE single payments in this table are the excesses multiplied by 10 of the values of female lives in Table XLV. above the values of the joint lives of males and females in Tables XLVI. and XLVII. And the annual payments are the quotients arising from dividing the single payments by the values of the joint lives increased by unity, agreeably to the rules in Vol I. p. 13, 14, and 15. The annual payments, therefore, suppose that the first is to be made immediately; and that they are to be continued during the joint duration of the lives of the wife and husband. And both the *annual* and *single* payments include the *whole value* of the annuity, and consequently suppose that if one is preferred the other is excused.

One circumstance a little curious appears in this Table. It shews, that the value in a single payment of an annuity during the survivorship of one life after another (when the difference of age is not very great) is less in the younger ages, and greatest in the middle ages. This is owing to the high probabilities of living in the younger ages, in consequence of which it happens that the survivorship is postponed to a period so late as to *sink* the value of the annuity more on that account than it is *raised* by the longer duration of the survivorship.

The

The values in this Table would have been (supposing the ages of husbands and wives equal or nearly equal) from an 8th to a 12th or 13th lower than they are, had they been computed from the *means* between the values of the lives of males and females in Table XLV.; that is, from the values of lives in the kingdom of *Sweden* taken in the gross, without distinguishing between males and females. There is, therefore, a deficiency to this amount in such values when deduced from the common Tables of single and joint lives.

In Vol. I. p. 124, an account has been given of an institution in the dutchy of *Oldenberg*, which provides annuities for widows, at prices specified in Tables correctly calculated by Mr. *Oeder*, from the values of single and joint lives according to Mr. *Susmilch's* Table of Mortality. Another institution of the same kind at *Hamburg*, has been described in p. 178 of the former Volume. And, lately, an account has been sent me, by Mr. *Oeder*, of a new institution for the same purpose, established in *Denmark* and *Norway*, under the sanction and guarantyskip of his *Danish* Majesty.

The Office for Equitable Assurances in *Chatham-Place*, *London*, includes also in its plan a like provision for widows. And these are all the annuity institutions, with  
which

which I am acquainted, that are guided in this instance by the lights derived from correct observations and mathematical principles. But hitherto it has not been possible for any of them in calculating the contributions necessary to support the annuities, to be governed by any regard to the longer duration of the lives of women. It has been just observed, that this renders the payments from an 8th to a 12th or 13th *too little* for such annuities, when deduced from any tables which give (as all Tables have hitherto done) only the values of lives in general, without any discrimination between males and females. But it will be of use here to shew, by the following comparisons, the particular differences between the payments for such annuities as determined *accurately* for a whole kingdom, and the payments required, without regarding the longer duration of the lives of females, by the Tables of the four institutions just mentioned.



COMPARISON of the Values, in the preceding Table, of a Life Annuity to a Wife after her Husband, with the Values of the same Annuity in the Tables of the *Danish* and *Oldenberg* Institutions, calculated on the Supposition of the Improvement of Money at an Interest of 4 per cent.

Annuity 10*l*.

Wife's Age.	Huf- band's Age.	Value of the Annuity.				
		By Table XLVIII.		By <i>Oldenberg</i> Tables.		By <i>Danish</i> Tables.
		Single Payment.	Annual Payment.	Single Payment.	Annual Payment.	Single Payment.
		£.	£.	£.	£.(a)	£.
20	20	31.90	2.03	29.82	2.11	
	26	37.28	2.46	34.34	2.60	35.74
	50	73.05	6.31	69.93	6.70	69.11
28	28	32.64	2.28	29.94	2.41	31.15
	34	38.25	2.77	36.30	2.84	35.50
	52	66.94	6.14	63.10	6.54	
42	42	34.62	3.00	30.72	3.34	30.00
	48	41.81	3.86	38.24	4.06	38.27
	60	64.25	7.49	55.84	7.18	57.00
35	35	33.55	2.55	31.36	2.74	31.45
	40	40.00	3.20	36.26	3.30	36.63
	60	76.09	8.59	67.44	8.36	68.49

(a) In the *Oldenberg*, and also in the *Hamburg* Tables, these are *half-yearly* payments which I have doubled, and reckoned equivalent to *yearly* payments beginning immediately, and which therefore are over-rated, as may be learnt from the observations in p. 28, Vol. I. The Table for *Denmark* gives only the single payment.

COMPA-

COMPARISON of the Values in Table XLVIII. of a Life Annuity for a Wife after her Husband, with the Values of the same Annuity in the Tables of the *Hamburg* and *Equitable Institutions*, calculated at an Interest of 3 per cent.

Annuity 10 *l.*—Interest 3 per cent.

Wife's Age.	Husband's Age.	Value of the Annuity.					
		By Sweden Table.		By Hamburg Tables.		By Equitable Society.	
		Single Payment.	Annual Payment.	Single Payment.	Annual Payment.	Single Payment.	Annual Payment.
		£. (a)	£. (a)	£.	£.	£. (b)	£. (b)
20	20	44.00	2.51	40.17	2.27	45.05	2.97
	26	50.62	3.01	47.47	2.85	49.82	3.40
	50	85.82	6.93	86.76	7.60	81.15	7.04
28	28	43.40	2.74	40.30	2.73	43.74	3.14
	34	50.40	3.33	48.08	3.52	49.14	3.67
	52	84.64	7.21	79.40	7.40	73.72	6.75
35	35	43.03	2.99	39.80	2.80	42.16	3.31
	40	50.44	3.70	45.81	3.54	47.25	3.86
	60	92.83	9.88	82.14	9.40	77.11	8.35

(a) In computing these payments, the values of lives at 3 per cent. according to the *Sweden* Tables, have been deduced from the values at 4 per cent. by the rules in p. 170, &c.

(b) These payments may be easily deduced, either from the Tables in this collection of the values of single and joint lives, according to the *Northampton* Observations, or from Table XXXVI.

For example. It appears from this last Table, that the annuity for a life aged 20 after another of the same age, to which either a single payment of 27.96 *l.* or an annual payment of 1.848 *l.* during the joint lives will entitle an expectant, is 6.207 *l.*; from whence it will follow, by the rule of proportion, that the annuity being 10 *l.* the single payment must be 45.05 *l.* and the annual payment 2.97 *l.*

From

From these comparisons it appears that, supposing interest at 4 *per cent.* and the *Sweden* Tables a proper standard (and till similar observations are made in other kingdoms they ought to be reckoned the properest) the payments required by the *Danish* establishment are somewhat too little. The same appears to be true of the *single* payments in the *Oldenburg* establishment; but the *annual* payments in this establishment appear to be more than the value (*a*).

In

(*a*) Agreeably to this observation, Mr. Oeder, in the examination mentioned in Vol. I. p. 126, found the single payments deduced from Mr. *Susmilch's* Table of mortality to be frequently too little, but the annual payments almost always too great. This is to be accounted for in the following manner:

The values of single and joint lives are greater by the *Sweden* Table of mortality, than by either Mr. *Susmilch's* or the *Northampton* Table; and had they been greater in the same proportion, the difference between them, that is, the value in one present payment of an annuity for the life of a woman after her husband, would have been nearly the same according to all the Tables; and consequently this difference, divided by the greater value of the joint lives according to the *Sweden* Table, would have given a less quotient; that is, a less value of the annuity in annual payments. But the value of the single female life being greater in *proportion* by the *Sweden* Table than that of the joint lives, the difference is increased, but not so much as to produce, when divided by the greater value of the joint lives, a quotient equal to that produced by dividing a smaller difference resulting from the other Tables by a smaller value of the joint lives.

The *Danish* establishment makes the annuities payable only during widowhood, and on this account makes an abatement in the contributions; but it is impossible to determine properly what this abatement ought to be.—It has, I have said, the advantage of being guarantied by the King of *Denmark* for all his dominions. It has also the following securities. All the military and naval, and other officers who receive their pay from the King's treasury, are obliged, when appointed, to give



In the HAMBURGH establishment it appears, that, if money is improved at no higher rate than 3 *per cent.* the *single* payments are almost always too low, but the *annual* payments sometimes too high. With respect to the EQUITABLE Society, it appears, that on the same supposition of no higher improvement of money than at an interest of three *per cent.* the *single* payments are generally too little, but the *annual* payments generally too high; and that when compared with the values at 4 *per cent.* and the difference of age is not very great, they are near a *third* or a *quarter* too high. It seems, therefore, that in those cases of survivorship where there was most reason to suspect, that the NORTHAMPTON Tables might give values unfavourable to the Society, it gives them sufficiently high; and that consequently, even in these cases, there is no reason for continuing that addition of 15 *per cent.* to all the values which has

up to this fund one month of their pay; and all subscribers are obliged at admission to contribute 10 *per cent.* more than the payments in the Tables.—I will add, that the calculations for this establishment, like those for the *Hamburgh* and *Oldenberg* establishments, have been made with such pains and ability from Mr. *Susmilch's* Table of mortality (in his *Gottliche Ordnung*, Vol. II. p. 319) by two of the first *Danish* mathematicians (Mr. *Lous*, Professor of Mathematicks and Navigation in the Academy of *Sea Cadets*; and Mr. *Bugge*, Professor of Astronomy in the University of *Copenhagen*; and both of them Fellows of the Royal *Danish* Academy of Sciences) that there is not the least danger of its sharing the same fate with a former *Danish* establishment described in Vol. I. p. 122.

been



been ordered by the Society.—Upon the whole; I cannot help thinking that this Society ought once more to lower its demands, and to content itself with the advantage it derives from computing by the *Northampton* Tables at so low an interest as 3 *per cent.* without making any additional charge, except, perhaps, such a small charge as that proposed in Vol. I. p. 176, towards bearing the expences of management \*.

In order to prove this more fully, I will here add a comparison, in a few instances, of the *premiums* (exclusive of the additional charge) required for assurances on single lives by this Society, with the values of the same assurances deduced from the SWEDEN Tables.

Values of the Assurance of 100*l.* on a Single Life.—Interest 3 *per cent.*

Age.	For one year by			For seven years by			For the whole Life by		
	Equit. Society Pay-ment.	Sweden Tables.		Equit. Society Annual Paymt.	Sweden Tables.		Equit. Society Annual Paymt.	Sweden Tables.	
		Male Payment.	Female Payment.		Male Annual Payment.	Female Annual Payment.		Male Annual Payment.	Female Annual Payment.
20	1.36	.87	.71	1.47	.92	.76	2.18	1.80	1.64
28	1.53	1.03	.98	1.68	1.13	1.11	2.55	2.20	2.03
35	1.81	1.22	1.16	1.93	1.32	1.21	3.06	2.85	2.44
44	2.27	1.87	1.64	2.46	2.00	1.60	3.78	3.65	3.24

It appears from hence, that without the charge of 15 *per cent.* and reckoning interest so low as 3 *per cent.* the *premiums* for

\* See note p. 105.

Affurances on Single Lives required by the Equitable Society are, in many cases, above a *third*, and, in general, above a *quarter* greater than the true values for *mankind at large*, deduced from the SWEDEN Observations. And yet such is the temptation to bad lives to seek admission, such the uncertainty what the rate of mortality in the Society may in the end prove, and such the necessity on these accounts (as has been before observed) of securing the permanency of the Society by erring rather on the side of *excess* than *defect*, that these premiums, were no addition made to them, could not reasonably be thought exorbitant.

In the last comparison there are two circumstances which may deserve the notice of this Society.

The price in annual payments of the assurance of a female life at 28 for seven years is, according to the *Swedish* Tables, almost equal to the price of the same assurance at 35. And at 44 the *annual* payment for *seven* years is less than the *single* payment for assuring only the first of these 7 years. These circumstances, instead of being, as they may seem, the effect of errors in the *Swedish* Tables, shew a correctness not to be found in any other tables. Females whose ages are between 27 and 36 consist chiefly of *child-bearing* women; and though, taking

ing the whole duration of marriage, the lives of *married* women may (agreeably to Mr. Muret's Observations in *Switzerland*, hereafter mentioned) be *less* hazardous than the lives of *single* women, yet at these ages they may be *more* so; and particularly in great towns and polished societies, where absurd customs, wrong management, and a pernicious delicacy, render an event dangerous which is naturally safe\*. According to Mr. *Susmilch's* observations in *Germany*, one birth in a hundred produces the death of the mother; but in *London* the proportion is much higher. This suggests the true reason of the first of the circumstances I have mentioned.—With respect to the other, it must be considered, that at 44 the critical period raises the value of the assurance of a female life; but recovering after this period particular firmness, an assurance for seven or eight years becomes less in *annual* value than an assurance for only one or two years. See p. 148.

In p. 171. of the preceding volume, an account has been given of the mortality among the persons assured by the Society for 12 years to 1780. I can now add, that during 14 years to *January* 1782, the number assured (exclusive of assurances on survivor-

\* In the *Equitable Society*, though established near 30 years, and assuring the lives of women at all ages, I do not know an instance (except two which happened last year) of a claim's having been produced by *child-birth*. ED.



ships for different sums not exceeding 2000*l*. on any single life) has been 12,391, and that of this number 9890 have been persons under 50 years of age, among whom the deaths have been fewer, in the proportion of 3 to 4, than those which should have happened according to the *Northampton* Table of Observations\*, and correspond best at every age to the mortality exhibited in the *Sweden* Table. Of the remaining assurances, 1997 have been on single lives between 50 and 60, among which the mortality, compared with that exhibited in the *Northampton* Table, has been as 9 to 10. There have been in the same period 504 assurances of persons between 60 and 70, and among them the mortality has been nearly equal to that in the *Northampton* Table.—This great success at the outset of the institution, has been particularly favourable to it, and must strengthen it for all future time; but it would be wrong to rely on the continu-

\* During the last 22 years, from Jan. 1768 to Jan. 1790, the number of assurances on single lives has been 30,998, of which number 24,083 have been on the lives of persons under 50 years of age, among whom the deaths have been fewer than those in the *Northampton* Table in the proportion of 3 to 5. Between the ages of 50 and 60 the number of assurances on single lives has been 5182, and compared with the *Northampton* Table the number of deaths has been as 3 to 4. Between 60 and 70 years of age the number of assurances on single lives has been 1733, and among them the decrements compared with those in the *Northampton* Table have been in the ratio of 7 to 6 nearly.—See a further account of this Society in the Introduction at the beginning of the 1st volume. ED.



ance of it. Seasons of uncommon mortality must come; and the increasing credit and numbers of the Society will, as I have before observed, increase the danger of the intrusion of bad lives.

## T A B L E XLIX.

Shewing the Probabilities of the Duration of Human Life at all Ages, in a Kingdom at large; deduced from Observations in the Kurmark of BRANDENBURGH; and formed on the Supposition that a *Third* of a Kingdom consists of Inhabitants of Towns, and *Two Thirds* of the Inhabitants of Country Parishes and Villages. See Mr. *Susmilch's* GOTTISCHE ORDNUNG, VOL. III. Tables p. 33.

Decrements of Life in the Kurmark of BRANDENBURGH.

Age.	A In <i>Berlin</i> . the capital.	B In the other Towns.	C In the Country Pa- rishes and Villages.	D $\frac{A+B+C}{6}$
Still-born	40	34	44	42
Under 1	254	194	187	199
1—5	185	196	138	156
0—5	479	424	369	397
5—10	40	61	59	56
10—15	10	17	24	20
15—20	16	17	22	20
0—20	545	519	474	493
20—25	34	18	28	27
25—30	46	25	25	29
30—35	37	24	26	28
35—40	49	40	32	36
40—45	36	31	33	33
45—50	37	42	36	37
50—55	38	47	40	41
55—60	42	58	55	53
20—60	319	285	275	284

# T A B L E S.

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T A B L E. XLIX. continued.

Age.	A In <i>Berlin</i> , the capital.	B In the other Towns.	C In the Country Pa- rishes and Villages.	D $\frac{A+B+4C}{6}$
60—65	31	46	63	55
65—70	32	56	61	55
70—75	2	35	58	49
75—80	23	32	34	32
80—85	11	16	22	19
85—90	7	8	8	8
90—95	3	2	3	3
95—100	2	1	1	1
Above 100	0	0	1	1
60—100	136	196	251	223
	1000	1000	1000	1000

From Column D the following Table has been formed.

Born 10,000—Still-born 42.					
Age.	Living.	Decre- ments.	Proportion dying annually.	Sum of all the Living.	Expecta- tions.
0	958	199	1 of 4 $\frac{3}{4}$	29877	30.68
1	759	70	1 of 11	28918	
2	689	38	1 of 18	28159	
3	651	26	1 of 25	27470	
4	625	22	1 of 28	26819	42.93
5	603	19	1 of 32	26194	
6	584	14	1 of 42	25591	
7	570	10	1 of 57	25007	
8	560	8	1 of 70	24437	42.14
9	552	5	1 of 110	23877	
10	547	4	1 of 137	23325	

T A B L E XLIX. continued.

Age.	Living.	Decrements.	Proportion dying annually.	Sum of all the Living.	Expectations.
11	543	4	1 of 136	22778	38.64
12	539	4	1 of 135	22235	
13	535	4	1 of 134	21696	
14	531	4	1 of 133	21161	
15	527	4	1 of 132	20630	
16	523	4	1 of 131	20103	
17	519	4	1 of 130	19580	34.52
18	515	4	1 of 129	19061	
19	511	4	1 of 128	18546	
20	507	5	1 of 101	18035	
21	502	5	1 of 100	17528	
22	497	5	1 of 99	17026	
23	492	5	1 of 98	16529	31.76
24	487	5	1 of 97	16037	
25	482	6	1 of 80	15550	
26	476	6	1 of 79	15068	
27	470	6	1 of 78	14592	
28	464	6	1 of 77	14122	
29	458	6	1 of 76	13658	28.70
30	452	6	1 of 75	13200	
31	446	6	1 of 74	12748	
32	440	6	1 of 73	12302	
33	434	6	1 of 72	11862	
34	428	6	1 of 71	11428	
35	422	7	1 of 60	11000	25.56
36	415	7	1 of 59	10578	
37	408	7	1 of 58	10163	
38	401	7	1 of 57	9755	
39	394	7	1 of 56	9354	
40	387	7	1 of 55	8960	22.65



# T A B L E S.

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## T A B L E XLIX. continued.

Age.	Living.	Decrements.	Proportion dying annually.	Sum of all the Living.	Expectations.
41	380	7	1 of 54	853	
42	373	7	1 of 53	8193	
43	366	7	1 of 52	7820	
44	359	7	1 of 51	7454	
45	352	7	1 of 50	7095	19.65
46	345	7	1 of 49	6743	
47	338	7	1 of 48	6398	
48	331	7	1 of 47	6060	
49	324	7	1 of 46	5729	
50	317	8	1 of 40	5405	16.55
51	309	8	1 of 39	5008	
52	301	8	1 of 38	4779	
53	293	9	1 of 32	4478	
54	284	9	1 of 31	4185	
55	275	10	1 of 27	3901	13.68
56	265	10	1 of 26	3626	
57	255	10	1 of 25	3361	
58	245	11	1 of 22	3106	
59	234	11	1 of 21	2861	
60	223	11	1 of 20	2627	11.28
61	212	11	1 of 19	2404	
62	201	11	1 of 18	2192	
63	190	11	1 of 17	1991	
64	179	11	1 of 16	1801	
65	168	11	1 of 15	1622	9.15
66	157	11	1 of 14	1454	
67	146	11	1 of 13	1297	
68	135	11	1 of 12	1151	
69	124	11	1 of 11	1016	
70	113	10	1 of 11	892	7.48

T A B L E XLIX. continued.

Age.	Living	Decrements.	Proportion dying annually.	Sum of all the Living.	Expectations.
71	103	10	1 of 10	779	6.17
72	93	10	1 of 9	676	
73	83	10	1 of 8	583	
74	73	9	1 of 8	500	
75	64	8	1 of 8	427	
76	56	7	1 of 8	363	
77	49	6	1 of 8	307	
78	43	6	1 of 7	258	
79	37	5	1 of 7	215	5.06
80	32	5	1 of 6	178	
81	27	4	1 of 7	146	
82	23	4	1 of 6	119	
83	19	3	1 of 6	96	
84	16	3	1 of 5	77	
85	13	2	1 of 6	61	
86	11	2	1 of 5	48	4.18
87	9	2	1 of 4	37	
88	7	1	1 of 7	28	
89	6	1	1 of 6	21	
90	5	1	1 of 5	15	
91	4	1	1 of 4	10	
93	3	1			
95	2	1			
100	1	1			

## R E M A R K S.

THIS Table is the same with that published in the last edition of Mr. *Susmilch's* *Gottliche*

*Gottliche Ordnung*, with the addition of the *Expectations*, and an alteration in the arrangement of the number of the still-born, which I have placed by itself, and deduced from the *whole* number born, in order to make the number *born alive* the *radix* of the Table.

This Table, it should be further observed, has been formed without any regard to the correction explained in the Fourth Essay in the former Volume of this work; and, on this account, (as far, as it has been deduced from the numbers dying at every age in the towns of *Brandenburg*) makes the probabilities of living too high in the first stages of life. But it should be likewise attended to, that on *another* account, it makes them in a much greater proportion too low. I mean, on account of the great excess of the births above the burials in the country parishes and villages. The effect of such an excess may be learnt from what is said in p. 9, &c. of the Introduction to these Tables.

There is another Table of the probabilities of living at every age in a kingdom at large, in the Second Volume of Mr. *Susmilch's Gottliche Ordnung*, p. 319, which has been made the *basis* of all the computations in GERMANY of the values of payments dependent on lives. This is the Table referred to in p. 189, and in the Note p. 193. It differs but little from this Table; and is  
liable

liable to the same objections. I must add, that the like is true of a table formed with the same view, and on the same principles, by Mr. FLORENCOURT, the ingenious author of a *Mathematical Treatise on Political Arithmetick*, published in *Germany*, in 1781.

Having occasion to mention these two writers, I cannot help adding with regret, that being ignorant of the *German* language, I have found myself incapable of profiting by their works in the manner I wish.

In Tables 12th, 13th, 20th, 21st, and 24th, at the end of the Second Volume of Mr. SUSMILCH's *Gottliche Ordnung*, the decrements of life at all ages are given separately for males and females in *BERLIN* for 14 years; in the parish of *St. Sulpice*, *PARIS*, for 30 years; and in several country parishes and villages in *BRANDENBURGH* for different periods of years. These decrements are so far from giving a just representation of human mortality, that a table of *observations* deduced from them would necessarily be very erroneous. They confirm, however, the difference in favour of females exhibited in the four preceding Tables; and therefore it will not be improper to insert a summary of them.



## DECREMENTS of Life.

Age.	In St. SULPICE Parish.		In BERLIN.		Country Parishes in BRANDEN- BURGH.	
	Males.	Females.	Males.	Females.	Males.	Females.
Still-born			360	253	45	39
I	5718	4615	2758	2370	420	383
I—5	5925	6093	843	847	276	246
5—10	1597	1536	211	215	120	110
10—20	789	749	196	205	87	72
20—30	1293	1337	709	493	126	97
*30—45	2207	2315	1052	796	166	168
45—60	2026	2442	1023	746	280	234
60—70	1768	2177	443	506	237	207
70—80	1453	3505	337	417	148	183
80—90	648	1673	114	160	68	48
90—95	28	101	11	29	8	8
95—100	19	72	9	22	2	1
Above 100	0	0	1	4	7	2
Totals	24071	24467	8067	7063	1990	1798
*30—40			725	582	102	124
40—50			652	445	151	103
50—60			698	515	193	175

The decrements in the country parishes in *Brandenburg* are too great in the first stages of life on account of the excess of the births above the burials, the former having been, in some of these parishes,  
more

more than *double* the latter. The decrements in *Berlin*, on the contrary, are too small, for reasons sufficiently explained in the course of this work; but in the parish of St. *Sulpice*, *Paris*, they are particularly erroneous, for the reasons mentioned in the Postscript to the First Essay, Vol. I. p. 291, 292.

T H E R E

THERE have been now given in this collection, tables of the duration and values of human life in great cities, in moderate towns, in country villages and parishes, and among the inhabitants of a whole kingdom, consisting of all *country* as well as *town* inhabitants. The accounts which have been given of the *data* from which they have been formed, and of the method of forming them, shew how far they are to be reckoned just representations of the duration and values of lives in the different situations I have mentioned. But there is one remark which is applicable to all of them; and that is, that having been formed from observations on whole bodies of people of all ages and conditions, they cannot give a correct representation of the duration and values of such lives as form a body of *state* annuitants, or of persons on whose lives annuities have been purchased to commence either immediately or at any given future year. The reason is obvious. Such a body of annuitants are likely to consist of a *selection* of the best lives from the common mass; the interest of every person who purchases an annuity on any life requiring that he should take care that it is a good life (*a*). Tables of mortality for such  
lives

(*a*) The following account of the life-annuities sold by our government, will, in some measure, prove the truth  
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lives have been published by Mr. *De Parcieux*, in *France*, from the lists of the *French Tontines*; and by Mr. *Kerſſeboom*, in *Holland*, from some registers of *Dutch annuitants*. That nothing on this subject may be wanting which I am able to furnish, I shall here insert these Tables, with the addition of the expectations of life for every fifth year, according to each of them.

of this observation.——There were granted in 1745, 22,500*l.* per ann. In *January*, 1782, they were reduced by deaths to 13,104*l.* which is a reduction of two-fifths in 36 years, and a slower decrease than the highest of the preceding Tables of mortality shew in the same time among bodies of people, all 30 years of age. The same is true of the annuities sold in 1746, which, in *Jan.* 1782, were reduced from 45,000*l.* (their original amount) to 24,400*l.* But the decrease has been slowest in the annuities granted in 1757, which, in *Jan.* 1782, had fallen from 33,750*l.* to 27,069*l.*; that is, only a fifth in 24 years.



# T A B L E S.

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## T A B L E L.

Shewing the Decrements and Expectations of Life among Bodies of Life-Annuitants, according to the Tables of Mortality published by Mr. *Kersseboom*, and by Mr. *De Parcieux* (a).

Age.	By Mr. KERSEBOOM.			By Mr. DE PARCIEUX.		
	Living.	Decr.	Expectat.	Living.	Decr.	Expectat.
0	1400	275	34.47	10000	2550	34.79
1	1125	50	41.77	7450	362	45.52
2	1075	45	42.69	7088	265	46.82
3	1030	37	43.53	6823	205	47.62
4	993	29	44.14	6618	150	48.09
5	964	27	44.45	6468	123	48.19
6	947	17		6345	102	
7	930	17		6243	91	
8	913	9		6154	81	
9	904	9		6073	69	
10	895	9	42.71	6004	58	46.76
11	886	8		5946	49	
12	878	8		5897	43	
13	870	7		5854	39	
14	863	7		5815	37	

(a) The copy here given of Mr. *De Parcieux*'s Table is not that published by Mr. *De Moivre* at the end of his Book on the *Doctrine of Chances*; and by Mr. *Ferguson* in his *Tables and Tracts*, &c. p. 289; but an improved copy published by Mr. *Florencourt* in Germany, at the end of his Treatise on *Political Arithmetick*.

A comparison of the expectations will shew a considerable difference between this Table and Mr. *Kersseboom*'s; and one reason of this difference may be, that Mr. *Kersseboom*'s Table has been formed partly from observations on the mortality of the inhabitants of some Dutch villages.

T A B L E L. continued.

Age.	By Mr. KERSEBOOM.			By Mr. DE PARCIEUX.		
	Living.	Decr.	Expectat.	Living.	Decr.	Expectat.
15	856	7	39.55	5778	38	43.46
16	849	7		5740	41	
17	842	7		5699	44	
18	835	9		5655	47	
19	826	9		5608	50	
20	817	9	36.31	5558	52	40.08
21	808	8		5506	53	
22	800	8		5453	54	
23	792	9		5399	55	
24	783	11		5344	56	
25	772	12	33.27	5288	57	37.01
26	760	13		5231	58	
27	747	12		5173	57	
28	735	12		5116	56	
29	723	12		5060	55	
30	711	12	30.92	5005	54	33.96
31	699	12		4951	54	
32	687	12		4897	53	
33	675	10		4844	52	
34	665	10		4792	52	
35	655	10	28.36	4740	52	30.73
36	645	10		4688	51	
37	635	10		4637	49	
38	625	10		4587	49	
39	615	10		4538	48	
40	605	9	25.49	4490	49	27.30
41	596	9		4441	49	
42	587	9		4392	50	

# T A B L E S.

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T A B L E L. continued.

Age.	By Mr. KERSEBOOM.			By Mr. DE PARCIEUX.		
	Living.	Decr.	Expectat.	Living.	Decr.	Expectat.
43	578	9		4342	51	
44	569	9		4291	52	
45	560	10	22.34	4239	53	23.77
46	550	10		4186	54	
47	540	10		4132	55	
48	530	12		4077	56	
49	518	11		4021	57	
50	507	12	19.41	3964	59	20.24
51	495	13		3905	62	
52	482	12		3843	66	
53	470	12		3777	70	
54	458	12		3707	76	
55	446	12	16.72	3631	81	16.88
56	434	13		3550	85	
57	421	13		3465	88	
58	408	13		3377	91	
59	395	13		3286	95	
60	382	13	14.10	3191	99	13.86
61	369	13		3092	102	
62	356	13		2990	105	
63	343	14		2885	107	
64	329	14		2778	109	
65	315	14	11.56	2669	110	11.07
66	301	14		2559	111	
67	287	14		2448	112	
68	273	14		2336	113	
69	259	14		2223	114	
70	245	14	9.15	2109	116	8.34
71	231	14		1993	119	

T A B L E L. continued.

Age.	By Mr. KERSSEBOOM.			By Mr. DE PARCIEUX.		
	Living.	Decr.	Expectat.	Living.	Decr.	Expectat.
72	217	14		1874	125	
73	203	14		1749	132	
74	189	14		1617	138	
75	175	15	6.81	1479	142	5.79
76	160	15		1337	139	
77	145	15		1198	134	
78	130	15		1064	128	
79	115	15		936	124	
80	100	13	5.05	812	115	4.73
81	87	12		697	107	
82	75	11		590	98	
83	64	9		492	88	
84	55	10		404	77	
85	45	9	3.38	327	66	3.45
86	36	8		261	55	
87	28	7		206	47	
88	21	6		159	42	
89	15	5		117	37	
90	10	3	2.47	80	30	1.79
91	7	2		50	22	
92	5	2		28	14	
93	3	1		14	8	
94	2	1		6	3	
95	1			3	2	
96	0.6			1	1	
97	0.5			0	0	
98	0.4					
99	0.2					
100	0.0					

TABLE



## T A B L E LI.

Shewing the Values of Single Lives according to the Probabilities of the Duration of Life in Mr. *De Parcieux's* Table of Mortality.—See Mr. *Florencourt's* Dissertations on Political Arithmetick, p. 288.

Interest 5 per cent.

Age.	Value.	Age.	Value.	Age.	Value.	Age.	Value.
0	11.083	26	15.040	52	10.926	78	3.953
1	14.620	27	14.969	53	10.673	79	3.719
2	15.135	28	14.893	54	10.418	80	3.501
3	15.509	29	14.810	55	10.168	81	3.283
4	15.750	30	14.722	56	9.930	82	3.072
5	15.924	31	14.627	57	9.682	83	2.868
6	16.041	32	14.527	58	9.431	84	2.668
7	16.118	33	14.421	59	9.177	85	2.461
8	16.169	34	14.306	60	8.923	86	2.237
9	16.204	35	14.189	61	8.669	87	1.976
10	16.210	36	14.065	62	8.413	88	1.688
11	16.194	37	13.930	63	8.155	89	1.409
12	16.145	38	13.786	64	7.893	90	1.164
13	16.077	39	13.632	65	7.626		
14	15.994	40	13.466	66	7.351		
15	15.901	41	13.296	67	7.069		
16	15.807	42	13.116	68	6.778		
17	15.716	43	12.931	69	6.479		
18	15.631	44	12.738	70	6.171		
19	15.550	45	12.539	71	5.856		
20	15.474	46	12.333	72	5.540		
21	15.401	47	12.119	73	5.232		
22	15.328	48	11.897	74	4.942		
23	15.256	49	11.666	75	4.674		
24	15.184	50	11.425	76	4.429		
25	15.112	51	11.178	77	4.190		

From the values in this Table at 5 *per cent.* the values at all other rates of interest may be easily found by the rule in p. 170. But I am very happy that, on this occasion, I can inform the public, that complete tables of the values of *single* lives, deduced with perfect correctness (from the copy of Mr. *De Parcieux's* Table of Mortality at the end of Mr. *De Moivre's* Doctrine of Chances) for every rate of interest from 2 to 10 *per cent.* and also of *two joint* lives at  $3\frac{1}{2}$  and  $4\frac{1}{2}$  *per cent.* have been published by Mr. *Masferes*, Curfitor Baron of the Exchequer, in a work on the principles of the doctrine of life-annuities.——To this work the ingenious author has added many calculations on the best means of redeeming the public debts; and I wish his name and abilities may be the means of engaging the attention of the kingdom effectually to this most important object.

IN p. 118, Vol. I. a scheme has been mentioned for providing for the Widows and Orphans of the Clergy within the Diocese of *Exeter*, and which the Reverend Mr. *Gandy* of *Plymouth*, had, with great public spirit, but without success, endeavoured to carry into execution.

Much time and pains were employed in computing the necessary tables for this scheme; and as it is possible that in some future time they may be still of use, I shall here insert the chief of them.

## T A B L E LII.

Shewing the Values in Annual Payments during the Joint Lives (first Payment to be made at Admission), and also in Single Payments, of a Life-Annuity of 10*l.* to be entered upon by a Wife at the Death of her Husband.

INTEREST reckoned at 4 *per cent.*

Husband's age.	Annual payment, supposing equal ages	Addition to annual payment for each year the age of the husband exceeds the wife's	Equivalent composition, or single payment	Addition to the single payment for each year the husband's age exceeds the wife's.
	£. s. d.	s. d.	£. s. d.	£. s. d.
25 } or less	2 : 15 : 0	0 : 10	35 : 6 : 0	0 : 14 : 0
26	2 : 15 : 0	0 : 10	35 : 5 : 0	0 : 14 : 0
27	2 : 15 : 6	0 : 10	35 : 4 : 0	0 : 14 : 0
28	2 : 16 : 0	0 : 10	35 : 3 : 0	0 : 14 : 0
29	2 : 16 : 6	0 : 10	35 : 2 : 0	0 : 14 : 0
30	2 : 17 : 6	0 : 10	35 : 0 : 0	0 : 14 : 0
31	2 : 18 : 0	0 : 10	34 : 18 : 0	0 : 14 : 0

T A B L E LII. continued.

Husband's age.	Annual payment, supposing equal ages.	Additional annual payments for each year the age of the husband exceeds the wife's	Single payment, supposing the annual excused.	Addition to the single payment for each year the age of the husband exceeds the wife's.
	£. s. d.	s. d.	£. s. d.	£. s. d.
32	2:18:6	0:11	34:16:0	0:14:6
33	2:19:0	0:11	34:14:0	0:15:0
34	2:19:6	1:0	34:12:0	0:15:6
35	3:0:0	1:0	34:10:0	0:16:0
36	3:0:0	1:0	34:8:0	0:16:0
37	3:0:6	1:1	34:5:0	0:16:6
38	3:1:0	1:1	34:2:0	0:17:0
39	3:1:6	1:2	33:18:0	0:17:6
40	3:2:0	1:2	33:14:0	0:18:0
41	3:2:6	1:3	33:10:0	0:18:6
42	3:3:0	1:4	33:6:0	0:19:0
43	3:3:6	1:5	33:2:0	0:19:6
44	3:4:0	1:6	32:17:0	1:0:0
45	3:4:6	1:7	32:12:0	1:1:0
46	3:5:0	1:8	32:6:0	1:1:6
47	3:5:6	1:9	32:0:0	1:2:0
48	3:6:0	1:10	31:14:0	1:2:6
49	3:6:6	1:11	31:8:0	1:3:0
50	3:7:0	2:0	31:2:0	1:3:6
51	3:7:6	2:1	30:16:0	1:4:0
52	3:8:0	2:2	30:9:0	1:5:0
53	3:8:6	2:3	30:1:0	1:6:0
54	3:9:0	2:4	29:12:0	1:7:0
55	3:10:0	2:6	29:3:0	1:8:0
56	3:10:6	2:7	28:14:0	1:9:0
57	3:11:0	2:9	28:4:0	1:10:0
58	3:11:6	2:11	27:14:0	1:11:0
59	3:12:0	3:1	27:4:0	1:12:0
60	3:12:6	3:3	26:13:0	1:13:0
61	3:13:0	3:5	26:2:0	1:14:0

In



In calculating this Table, the values of single and joint lives were taken from the two Tables at the end of this volume, which were then reckoned the best guides. But a comparison of these values, with those in Table XLVIII. p. 186, will shew they want correction; and, particularly, that though when the ages of husbands and wives are under 40, and nearly equal, the values in this Table are a little too high; yet, in other cases, they are below, and, in some cases, *much* below the proper values.

## T A B L E S.

## T A B L E LIII.

Shewing the Values of a Life-Annuity of 5 $\frac{1}{2}$ . payable to a Wife after her Husband, provided he lives *three* Years from the Time of purchasing; and of an additional Annuity of 5 $\frac{1}{2}$ . provided he lives *five* Years from the Time of purchasing.

INTEREST 4 *per Cent.*

Age.	Annual payment, supposing equal ages.	Additional annual payment for each year the age of the husband exceeds the wife's		Single payment, supposing the annual excluded.	Addition to the single payment for each year the age of the husband exceeds the wife's.	
		£. s. d.	s. d.		£. s. d.	£. s. d.
25 } or less }	2 : 2 : 6	0 :	9	25 : 13 : 6	0 :	11 : 10
26	2 : 2 : 6	0 :	9	25 : 13 : 6	0 :	11 : 10
27	2 : 2 : 6	0 :	9	25 : 13 : 6	0 :	11 : 10
28	2 : 2 : 6	0 :	9	25 : 13 : 6	0 :	11 : 10
29	2 : 2 : 6	0 :	9	25 : 13 : 6	0 :	11 : 10
30	2 : 2 : 6	0 :	9	25 : 13 : 6	0 :	11 : 10
31	2 : 2 : 6	0 :	9	25 : 9 : 6	0 :	12 : 1
32	2 : 2 : 6	0 :	9	25 : 5 : 6	0 :	12 : 5
33	2 : 3 : 0	0 :	10	25 : 2 : 6	0 :	12 : 9
34	2 : 3 : 0	0 :	10	24 : 18 : 6	0 :	13 : 0
35	2 : 3 : 0	0 :	11	24 : 15 : 0	0 :	13 : 4
36	2 : 3 : 6	0 :	11	24 : 10 : 6	0 :	13 : 9
37	2 : 3 : 6	1 :	0	24 : 5 : 6	0 :	14 : 1
38	2 : 3 : 6	1 :	0	24 : 1 : 0	0 :	14 : 6
39	2 : 4 : 0	1 :	1	23 : 16 : 6	0 :	14 : 10
40	2 : 4 : 0	1 :	1	23 : 11 : 6	0 :	15 : 2
41	2 : 4 : 0	1 :	2	23 : 5 : 6	0 :	15 : 7
42	2 : 4 : 0	1 :	2	22 : 19 : 6	0 :	15 : 11
43	2 : 4 : 0	1 :	3	22 : 13 : 6	0 :	16 : 4
44	2 : 4 : 0	1 :	4	22 : 7 : 6	0 :	16 : 10
45	2 : 4 : 0	1 :	4	22 : 11 : 6	0 :	17 : 3
46	2 : 4 : 0	1 :	5	21 : 13 : 0	0 :	17 : 9

TABLE LIII. continued.

Age.	Annual pay- ment, suppo- sing equal ages.	Additional an- nual payment for each year the age of the huf- band exceeds the wife's.	Single pay- ment, supposing the annual ex- cess.	Addition to the single payment for each year the age of the huf- band exceeds the wife's.
	£. s. d.	s. d.	£. s. d.	£. s. d.
47	2: 4: 0	1: 6	21: 4: 6	0: 18: 3
48	2: 4: 0	1: 7	20: 16: 6	0: 18: 10
49	2: 4: 0	1: 8	20: 8: 6	0: 19: 3
50	2: 3: 6	1: 9	20: 0: 6	0: 19: 9
51	2: 3: 6	1: 10	19: 11: 6	1: 0: 2
52	2: 3: 6	1: 11	19: 2: 6	1: 0: 8
53	2: 3: 6	2: 0	18: 15: 0	1: 1: 0
54	2: 3: 6	2: 0	18: 7: 0	1: 1: 6
55	2: 3: 0	2: 1	17: 18: 6	1: 1: 11
56	2: 3: 0	2: 2	17: 7: 6	1: 2: 5
57	2: 2: 6	2: 3	16: 16: 6	1: 2: 11
58	2: 2: 0	2: 5	16: 5: 6	1: 3: 6
59	2: 2: 0	2: 7	15: 14: 6	1: 4: 0
60	2: 1: 6	2: 8	15: 3: 6	1: 4: 6
61	2: 0: 6	2: 10	14: 8: 6	1: 5: 2
62	1: 19: 6	3: 0	13: 14: 0	1: 5: 10

This Table has been computed by the Rule in Quest. VII. Vol. I. p. 22, taking the probabilities of the duration of life as they are in Table V. p. 35; and the values of single and joint lives as they are in the two Tables at the end of this Volume. The correct and legitimate Table would be a Table computed by the same rule from the SWEDEN Tables in this collection.

TABLE

## T A B L E LIV.

Shewing the Values of 100*l.* payable to such Children, *under Age*, of a *married* Man, as shall happen to be living at the Time of his Decease, provided he leaves no Widow.

INTEREST 4 *per cent.*

Age:	Annual pay- ment during life.	Single pay- ment, suppo- sing the annual excused.	Age.	Annual pay- ment during life.	Single pay- ment, suppo- sing the annual excused.
	<i>£. s. d.</i>	<i>£. s. d.</i>		<i>£. s. d.</i>	<i>£. s. d.</i>
25	0:10:0	8: 0:0	47	1: 3:6	14:18:0
26	0:11:0	8:10:0	48	1: 4:6	15: 6:0
27	0:11:6	9: 1:0	49	1: 5:6	15:15:0
28	0:12:0	9:10:0	50	1: 6:6	16: 4:0
29	0:12:6	9:18:0	51	1: 7:6	16:12:0
30	0:13:6	10: 6:0	52	1: 8:6	17: 0:0
31	0:14:0	10:14:0	53	1: 9:6	17: 8:0
32	0:14:6	11: 0:0	54	1:11:0	17:16:0
33	0:15:0	11: 5:0	55	1:12:6	18: 4:0
34	0:15:6	11: 9:0	56	1:13:6	18:13:0
35	0:15:6	11:13:0	57	1:15:0	19: 3:0
36	0:16:0	11:19:0	58	1:16:6	19:13:0
37	0:17:0	12: 4:0	59	1:18:6	20: 3:0
38	0:17:6	12:10:0	60	2: 0:6	20:13:0
39	0:18:0	12:15:0	61	2: 2:6	21: 3:0
40	0:18:6	13: 0:0	62	2: 5:0	21:13:0
41	0:19:0	13: 5:0			
42	0:19:6	13:10:0			
43	1: 0:0	13:15:0			
44	1: 1:0	14: 0:0			
45	1: 1:6	14: 5:0			
46	1: 2:6	14:11:0			

METHOD



## METHOD OF CALCULATION.

LET the age be reckoned 35.—The value (interest being at 4 *per cent.*) of 100*l.* payable at the death of a person aged 35, provided he survives another person of the same age, is *l.* 14.55, by Mr. *Simpson's* Problem quoted in Question XII. Vol. I. p. 39, and by the correction explained in Vol. I. p. 34 and 62: deducing the values of the longest of the two lives from the two Tables at the end of this volume, by the rule in p. 79.

This gives the value sought for this Table, on the supposition that it is *certain*, that a married man will at his death leave children under age. If one *tenth* of those who die widowers leave either no children, or none under age, then this value must be diminished, on that account, one tenth. And if, besides, one in five of all who are left widowers marry a second time wives not older than themselves, one *half* at least of whom, (that is. one *tenth* of all that are left widowers) must be reckoned to die in a 2d or 3d marriage; then the same value must be diminished again another tenth; that is, a *fifth* in all; and this will make it *l.* 11.64, (or 11*l.* 13*s.* nearly) which is the value in a *single payment* given in the Table.—Divide *l.* 11.64 by 14.98 (the value

value increased by unity of a life aged 25 by Table I. at the end of this Volume) and the quotient will be .777 (or 15s. 6d.) which is the value in annual payments during the single life, the first payment to be made immediately.

In this Table no allowance has been made for the inequality of age between a man and his wife, and for the chances of survivorship being, on this and other accounts, much against him in marriage. The values in it, therefore, are probably much too high.

Had the value just determined been deduced from the *Sweden* Tables for males and females taken collectively, it would have been in the single payment 10*l.* 16*s.*; in the annual payment 13*s.* 7*d.*—Had the wife been reckoned 29 (the husband being 35), it would have been in the single payment 9*l.* 4*s.* 6*d.*; in the annual payment 11*s.* 7*d.*—A society, therefore, for relieving orphans on this plan, might safely adopt lower payments than those in this Table; nor would there be any danger from the admission of bad lives.

## T A B L E LV.

Shewing the present Value of an Annuity of 10*l.* for five Years; 20*l.* for the next succeeding five Years; and 30*l.* for the whole of Life after Ten Years; payable *quarterly*; and to commence at FIFTY-FIVE Years of Age.—See the Reference to this and the following Table in Vol. I. p. 144.

Age of the purchaser.	Value of the annuity in one present payment.		Value of the annuity in annual payments, to be continued till 55, the 1st payment to be made immediately.	
	£.	s.	£.	s.
20	38	: 6	2	: 4
21	40	: 7	2	: 7
22	42	: 8	2	: 10
23	44	: 9	2	: 13
24	46	: 11	2	: 16
25	48	: 13	3	: 0
26	51	: 3	3	: 4
27	53	: 14	3	: 8
28	56	: 6	3	: 13
29	58	: 18	3	: 18
30	61	: 11	4	: 4
31	64	: 16	4	: 11
32	68	: 1	4	: 18
33	71	: 7	5	: 5
34	74	: 13	5	: 13
35	78	: 0	6	: 1
36	81	: 16	6	: 11
37	85	: 12	7	: 2
38	89	: 9	7	: 13
39	94	: 0	8	: 6
40	98	: 11	9	: 0
41	103	: 16	10	: 0
42	109	: 0	11	: 0
43	114	: 4	12	: 3
44	121	: 0	13	: 13
45	128	: 8	15	: 9

## TABLE LVI.

Shewing the Values of an Annuity of 10*l.* for five Years; 20*l.* for the next succeeding Five Years; and 30*l.* for the whole of Life after Ten Years; payable *quarterly*, and to commence at Sixty Years of Age.—See Vol. I. p. 144.

Age of the purchaser.	Value of the annuity in one present payment.		Value of the annuity in annual payments, to be continued till the age of 60, the first payment to be made immediately.	
	£.	s.	£.	s.
20	22	: 13	1	: 5
21	23	: 18	1	: 6
22	25	: 3	1	: 8
23	26	: 8	1	: 10
24	27	: 13	1	: 12
25	28	: 19	1	: 14
26	30	: 10	1	: 16
27	32	: 2	1	: 18
28	33	: 13	2	: 0
29	35	: 4	2	: 3
30	36	: 18	2	: 6
31	38	: 12	2	: 9
32	40	: 8	2	: 12
33	42	: 5	2	: 15
34	44	: 2	2	: 19
35	46	: 0	3	: 3
36	48	: 10	3	: 8
37	51	: 0	3	: 13
38	53	: 10	3	: 19
39	56	: 5	4	: 5
40	59	: 0	4	: 12
41	61	: 10	5	: 0
42	64	: 10	5	: 8
43	68	: 0	5	: 18
44	72	: 10	6	: 14
45	77	: 0	7	: 10
46	81	: 10	8	: 4
47	86	: 0	9	: 0
48	90	: 10	9	: 16
49	96	: 0	11	: 0
50	102	: 0	12	: 10

These



These two last Tables have been calculated by the rules in Vol. I. p. 17, 18, &c.

The probabilities of the duration of life have been supposed *nearly* the same with those in the *Northampton* Table of mortality.

The interest of money has been reckoned at *3 per cent.*; and it must be further remembered, that the values in *each* of the 2d and 3d columns are the *whole* values.

*ACCOUNT of the Values of the Renewal of Leases, and of the Method of computing them.*

T A B L E LVII.

Shewing the Fines due on the Renewal of a Lease of 21 Years after 5, 7, 9, or 11 Years have elapsed.

Years unexpired.	Values of the Renewal.			
	At 4 per cent.	At 6 per cent.	At 8 per cent.	At 10 per cent.
16	$2\frac{38}{100}$ } Years purchase.	$1\frac{66}{100}$ } Years purchase.	$1\frac{16}{100}$ } Years purchase.	$\frac{32}{100}$ } Years purchase.
14	$3\frac{46}{100}$	$2\frac{47}{100}$	$1\frac{77}{100}$	$1\frac{28}{100}$
12	$4\frac{64}{100}$	$3\frac{38}{100}$	$2\frac{48}{100}$	$1\frac{83}{100}$
10	$5\frac{92}{100}$	$4\frac{2}{3}$	$3\frac{3}{10}$	$2\frac{1}{2}$

The value in every case of this kind is the difference between the value (in Tables II. and LIX.) of the whole term, and the value (in the same Tables) of the unexpired part of the term.

If leases are held by lives (the value of their renewal is the difference between the value of all the lives (including the life or lives to be added) and the value of the existing life or lives.—For example.

The value of the renewal of a lease held by two lives after one has dropped is (supposing the existing life a male life aged 50, and the life to be added a female life aged 20) the *difference* between 18.575 (the value by

by Table XLVII. and the rule in p. 79, of the longest of the two lives) and 11.267 the value by Table XLV. of a single male life aged 50. That is, 7.308, or  $7\frac{3}{4}\%$  of a year's purchase nearly, reckoning interest at 4 *per cent.*—Again, the value of the renewal of a lease held by *three* lives, after *one* has dropped, is (supposing the two existing lives aged 50 and 56, and the life to be added aged 20) the *difference* between 19.537 (the value of the longest of the three lives by the column for lives in general in Table XLV. and by Tables XLVI. and XLVII. and the rule in p. 97) and 13.809 (the value by the same Tables and the Rule in p. 79, of the longest of two lives aged 50 and 56). This difference is 5.728, or  $5\frac{3}{4}\%$  years purchase; which, therefore, is the fine due for such a renewal, reckoning interest at 4 *per cent.*

*N. B.* If the values of such renewals are wanted at any rates of interest higher or lower than those for which the values of single and joint lives are given in the preceding Tables, they must be deduced from the values given in the Tables by the Rules in p. 170.

It would be an *endless* labour to compute tables shewing the value of such renewals in all cases; and these directions render it an *unnecessary* labour.



Sometimes a right may be purchased to put in, on the first vacancy among the lives by which an estate is held, such a new life as the purchaser shall chuse.—In order to find the present value of such a right, it is necessary to assume some given value for the life to be nominated, and this assumed value multiplied by the difference between the value of the existing life, if there is but one (or the value of the joint continuance of the existing lives, if there are two or more), and the perpetuity; and the product, divided by the perpetuity, will give the answer.

## E X A M P L E.

Let there be but one existing life, and let it be a male life, its age 50, and consequently its value (by Table XLV. p. 162.) 10.298, reckoning interest at 5 *per cent.* — Let the life to succeed it be reckoned a life of the greatest possible value, that is, a female life aged 9, and consequently worth (by Table XLV.) 16.343 year's purchase at 5 *per cent.* — The difference between 20 (the perpetuity) and 10.298 multiplied by 16.343, is 158.54; which product, divided by 20, gives 7.927, the answer.

If there are two existing lives, one male and the other female, and both 50, the value of their joint continuance will be (by Table XLVI. p. 165) 8.707; the difference  
between



between which value and the perpetuity is 11.293, which multiplied by 16,342, and the product divided by the perpetuity, gives 7.114 the answer in this case, or the number of years purchase which ought to be paid for a right of renewing a lease now held by two lives both aged 50, by putting in the best life in the room of the first of the two lives that shall happen to drop.

The rule for finding the value is the same, if the right to be sold is the right of presentation to a church living at the death of the present incumbent.

The estate meant in these rules is the nett *surplus* rent after deducting all taxes and repairs.

## T A B L E LVIII.

The present Value of 1*l.* to be received at the End of any Number of years not exceeding 100, at the Rates of 2, 2½, 7, 8, 9, and 10 *per cent.* Compound Interest; being a Supplement to Table I. p. 18.

Years	2 per cent.	2½ per cent.	7 percent.	8 per cent.	9 per cent.	10 per cent.
1	.980392	.975609	.934579	.925925	.917431	.909090
2	.961168	.951814	.873438	.857338	.841679	.826446
3	.942322	.928599	.816297	.793832	.772183	.751314
4	.923845	.905950	.762895	.735029	.708425	.683013
5	.905730	.883854	.712986	.680583	.649931	.620921
6	.887971	.862296	.666342	.630169	.596267	.564473
7	.870560	.841265	.622749	.583490	.547034	.513558
8	.853490	.820746	.582009	.540268	.501866	.466507
9	.836755	.800728	.543933	.500248	.460427	.424097
10	.820348	.781198	.508349	.463193	.422410	.385543
11	.804263	.762144	.475092	.428882	.387532	.350493
12	.788493	.743555	.444011	.397113	.355534	.318630
13	.773032	.725420	.414964	.367697	.326178	.289664
14	.757875	.707727	.387817	.340461	.299246	.263331
15	.743014	.690465	.362446	.315241	.274538	.239392
16	.728445	.673624	.338734	.291890	.251869	.217629
17	.714162	.657195	.316574	.270268	.231073	.197844
18	.700159	.641165	.295863	.250249	.211993	.179858
19	.686430	.625527	.276508	.231712	.194489	.163508
20	.672971	.610270	.258419	.214548	.178430	.148643
21	.659775	.595386	.241513	.198655	.163698	.135130
22	.646839	.580864	.225713	.183940	.150181	.122845
23	.634155	.566697	.210946	.170315	.137781	.111678
24	.621721	.552875	.197146	.157699	.126404	.101525
25	.609530	.539390	.184249	.146017	.115967	.092296
26	.597579	.526234	.172195	.135201	.106392	.083905
27	.585862	.513399	.160930	.125186	.097607	.076277
28	.574374	.500877	.150402	.115913	.089548	.069343
29	.563112	.488661	.140562	.107327	.082154	.063039
30	.552070	.476742	.131367	.099377	.075371	.057308
31	.541245	.465114	.122773	.092016	.069147	.052098
32	.530633	.453770	.114741	.085200	.063438	.047362
33	.520228	.44270	.107234	.078888	.058200	.043056
34	.510028	.431905	.100219	.073045	.053394	.039142

# T A B L E S.

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TABLE LVIII. continued.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
35	.500027	.421371	.093662	.007634	.048986	.035584
36	.490223	.411093	.087535	.0062624	.044941	.032349
37	.480610	.401067	.081808	.057985	.041230	.029408
38	.471187	.391284	.076456	.053690	.037826	.026734
39	.461948	.381741	.071455	.049713	.034702	.024304
40	.452890	.372430	.066780	.046030	.031837	.022094
41	.443989	.363346	.062411	.042621	.029208	.020086
42	.435304	.354484	.058328	.039464	.026797	.018260
43	.426768	.345838	.054512	.036540	.024584	.016600
44	.418400	.337403	.050946	.033834	.022554	.015091
45	.410196	.329174	.047613	.031327	.020692	.013719
46	.402153	.321145	.044498	.029007	.018983	.012472
47	.394268	.313312	.041587	.026858	.017416	.011338
48	.386537	.305671	.038866	.024869	.015978	.010307
49	.378958	.298215	.036324	.023026	.014658	.009370
50	.371527	.290942	.033947	.021321	.013448	.008518
51	.364243	.283846	.031726	.019741	.012338	.007744
52	.357101	.276922	.029651	.018279	.011319	.007040
53	.350099	.270168	.027711	.016925	.010384	.006400
54	.343234	.263579	.025898	.015671	.009527	.005818
55	.336504	.257150	.024204	.014510	.008740	.005289
56	.329906	.250878	.022620	.013435	.008018	.004808
57	.323437	.244759	.021140	.012440	.007356	.004371
58	.317095	.238789	.019757	.011519	.006749	.003973
59	.310877	.232965	.018465	.010665	.006192	.003612
60	.304782	.227283	.017257	.009875	.005680	.003284
61	.298806	.221740	.016128	.009144	.005211	.002985
62	.292947	.216331	.015073	.008466	.004781	.002714
63	.287203	.211055	.014087	.007839	.004386	.002467
64	.281571	.205907	.013165	.007259	.004024	.002243
65	.276050	.200885	.012304	.006721	.003692	.002039
66	.270637	.195985	.011499	.006223	.003387	.001853
67	.265331	.191205	.010746	.005762	.003107	.001685
68	.260128	.186542	.010043	.005335	.002851	.001532
69	.255028	.181992	.009386	.004940	.002615	.001392
70	.250027	.177553	.008772	.004574	.002399	.001266
71	.245125	.173223	.008198	.004235	.002201	.001151
72	.240318	.168998	.007662	.003921	.002019	.001046
73	.235606	.164876	.007161	.003631	.001852	.000951
74	.230986	.160854	.006692	.003362	.001699	.000864
75	.226457	.156931	.006254	.003113	.001559	.000786



TABLE LVIII. continued.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
76	.222017	.153103	.005845	.002882	.001430	.000714
77	.217664	.149369	.005463	.002669	.001312	.000649
78	.213396	.145726	.005105	.002471	.001204	.000590
79	.209211	.142172	.004771	.002288	.001104	.000537
80	.205109	.138704	.004459	.002118	.001013	.000488
81	.201087	.135321	.004167	.001961	.000929	.000443
82	.197145	.132021	.003895	.001816	.000852	.000403
83	.193279	.128800	.003640	.001682	.000782	.000366
84	.189489	.125659	.003402	.001557	.000718	.000333
85	.185774	.122594	.003179	.001442	.000658	.000303
86	.182131	.119604	.002971	.001335	.000604	.000275
87	.178560	.116687	.002777	.001236	.000554	.000250
88	.175059	.113841	.002595	.001144	.000508	.000227
89	.171626	.111064	.002425	.001059	.000466	.000207
90	.168261	.108355	.002267	.000981	.000428	.000188
91	.164962	.105712	.002118	.000908	.000392	.000171
92	.161727	.103134	.001980	.000841	.000360	.000155
93	.158556	.100619	.001850	.000779	.000330	.000141
94	.155447	.098165	.001729	.000721	.000303	.000128
95	.152399	.095770	.001616	.000667	.000278	.000116
96	.149411	.093434	.001510	.000618	.000255	.000106
97	.146481	.091155	.001411	.000572	.000234	.000096
98	.143609	.088932	.001319	.000530	.000214	.000087
99	.140793	.086763	.001233	.000490	.000197	.000079
100	.138032	.084647	.001152	.000454	.000180	.000072



## T A B L E LIX.

The present Value of an Annuity of 1*l.* for any Number of Years not exceeding 100, at the several Rates of 2, 2 $\frac{1}{2}$ , 7, 8, 9, and 10 *per cent.* being a Supplement to Table II. p. 21.

Years	2 per cent.	$2\frac{1}{2}$ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	.9803	.9756	.9345	.9259	.9174	.9090
2	1.9415	1.9274	1.8080	1.7832	1.7591	1.7355
3	2.8838	2.8560	2.6243	2.5770	2.5312	2.4868
4	3.8077	3.7619	3.3872	3.3121	3.2397	3.1698
5	4.7134	4.6458	4.1001	3.9927	3.8896	3.7907
6	5.6014	5.5081	4.7665	4.6228	4.4859	4.3552
7	6.4719	6.3493	5.3892	5.2063	5.0329	4.8684
8	7.3254	7.1701	5.9712	5.7466	5.5348	5.3349
9	8.1622	7.9708	6.5152	6.2468	5.9952	5.7590
10	8.9825	8.7520	7.0235	6.7100	6.4176	6.1445
11	9.7868	9.5142	7.4986	7.1389	6.8051	6.4950
12	10.575	10.257	7.9426	7.5360	7.1607	6.8136
13	11.348	10.983	8.3576	7.9037	7.4869	7.1033
14	12.106	11.690	8.7454	8.2442	7.7861	7.3666
15	12.849	12.381	9.1079	8.5594	8.0606	7.6060
16	13.577	13.055	9.4466	8.8513	8.3125	7.8237
17	14.291	13.712	9.7632	9.1216	8.5436	8.0215
18	14.992	14.353	10.059	9.3718	8.7556	8.2014
19	15.678	14.978	10.335	9.6035	8.9501	8.3649
20	16.351	15.589	10.594	9.8181	9.1285	8.5135
21	17.011	16.184	10.835	10.016	9.2922	8.6486
22	17.658	16.765	11.061	10.200	9.4424	8.7715
23	18.292	17.332	11.272	10.371	9.5802	8.8832
24	18.913	17.884	11.469	10.528	9.7066	8.9847
25	19.523	18.424	11.653	10.674	9.8225	9.0770
26	20.121	18.950	11.825	10.809	9.9289	9.1609
27	20.706	19.464	11.986	10.935	10.026	9.2372
28	21.281	19.964	12.137	11.051	10.116	9.3065
29	21.844	20.453	12.277	11.158	10.198	9.3696
30	22.396	20.930	12.409	11.257	10.273	9.4269
31	22.937	21.395	12.531	11.349	10.342	9.4790
32	23.468	21.849	12.646	11.434	10.406	9.5263
33	23.988	22.291	12.753	11.513	10.464	9.5694
34	24.498	22.723	12.854	11.586	10.517	9.6085
35	24.998	23.145	12.947	11.654	10.566	9.6441
36	25.488	23.556	13.035	11.717	10.611	9.6765
37	25.969	23.957	13.117	11.775	10.652	9.7059

TABLE LIX. continued.

Years.	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
38	26.440	24.348	13.193	11.828	10.690	9.7326
39	26.902	24.730	13.264	11.878	10.725	9.7569
40	27.355	25.102	13.331	11.924	10.757	9.7790
41	27.799	25.466	13.394	11.967	10.786	9.7991
42	28.234	25.820	13.452	12.006	10.813	9.8173
43	28.661	26.166	13.506	12.043	10.837	9.8339
44	29.079	26.503	13.557	12.077	10.863	9.8490
45	29.490	26.833	13.605	12.108	10.881	9.8628
46	29.892	27.154	13.650	12.137	10.900	9.8752
47	30.286	27.467	13.691	12.164	10.917	9.8866
48	30.673	27.773	13.730	12.189	10.933	9.8969
49	31.052	28.071	13.766	12.212	10.948	9.9062
50	31.423	28.362	13.800	12.233	10.961	9.9148
51	31.787	28.646	13.832	12.253	10.974	9.9225
52	32.144	28.923	13.862	12.271	10.985	9.9295
53	32.495	29.193	13.889	12.288	10.995	9.9359
54	32.838	29.456	13.915	12.304	11.005	9.9418
55	33.174	29.713	13.939	12.318	11.013	9.9471
56	33.504	29.964	13.962	12.332	11.022	9.9519
57	33.828	30.209	13.983	12.344	11.029	9.9562
58	34.145	30.448	14.003	12.356	11.036	9.9602
59	34.456	30.681	14.021	12.366	11.042	9.9638
60	34.760	30.908	14.039	12.376	11.047	9.9671
61	35.059	31.130	14.055	12.385	11.053	9.9701
62	35.352	31.346	14.070	12.394	11.057	9.9728
63	35.639	31.557	14.084	12.402	11.062	9.9753
64	35.921	31.763	14.097	12.409	11.066	9.9775
65	36.197	31.964	14.109	12.415	11.070	9.9796
66	36.468	32.160	14.121	12.422	11.073	9.9814
67	36.733	32.351	14.132	12.427	11.076	9.9831
68	36.993	32.538	14.142	12.433	11.079	9.9846
69	37.248	32.720	14.151	12.438	11.082	9.9860
70	37.498	32.897	14.160	12.442	11.084	9.9873
71	37.743	33.071	14.168	12.447	11.086	9.9884
72	37.984	33.240	14.176	12.450	11.088	9.9895
73	38.219	33.404	14.183	12.454	11.090	9.9904
74	38.450	33.565	14.190	12.457	11.092	9.9913
75	38.677	33.722	14.196	12.461	11.093	9.9921
76	38.899	33.875	14.202	12.463	11.095	9.9928
77	39.116	34.025	14.207	12.466	11.096	9.9935
78	39.330	34.170	14.212	12.469	11.097	9.9940
79	39.539	34.313	14.217	12.471	11.098	9.9946

# T A B L E S.

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TABLE LIX. continued.

Years	2 percent.	2½ percent.	7 percent.	8 percent.	9 percent.	10 per cent.
80	39.744	34.451	14.222	12.473	11.099	9.9951
81	39.945	34.587	14.226	12.475	11.100	9.9955
82	40.142	34.719	14.230	12.477	11.101	9.9959
83	40.336	34.847	14.233	12.478	11.102	9.9963
84	40.525	34.973	14.237	12.480	11.103	9.9966
85	40.711	35.096	14.240	12.481	11.103	9.9969
86	40.893	35.215	14.243	12.483	11.104	9.9972
87	41.071	35.332	14.246	12.484	11.104	9.9974
88	41.247	35.446	14.248	12.485	11.105	9.9977
89	41.418	35.557	14.251	12.486	11.105	9.9979
90	41.586	35.665	14.253	12.487	11.106	9.9981
91	41.751	35.771	14.255	12.488	11.106	9.9982
92	41.913	35.874	14.257	12.489	11.107	9.9984
93	42.072	35.975	14.259	12.490	11.107	9.9985
94	42.227	36.073	14.261	12.490	11.107	9.9987
95	42.380	36.169	14.262	12.491	11.108	9.9988
96	42.529	36.262	14.264	12.492	11.108	9.9989
97	42.675	36.353	14.265	12.492	11.108	9.9990
98	42.819	36.442	14.266	12.493	11.108	9.9991
99	42.960	36.529	14.268	12.493	11.108	9.9992
100	43.098	36.614	14.269	12.494	11.109	9.9992
Perp.	50.000	40.000	14.286	12.500	11.111	10.000

TABLE



## T A B L E L X.

Shewing the Sum to which 1*l*. Principal will increase at 2, 2½, 7, 8, 9, and 10 per cent. Compound Interest, in any Number of Years not exceeding 100; being a Supplement to Table III.  
P. 25.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	1.02000	1.02500	1.07000	1.08000	1.09000	1.10000
2	1.04040	1.05062	1.14490	1.16640	1.18810	1.21000
3	1.06120	1.07689	1.22504	1.25971	1.29502	1.33100
4	1.08241	1.10381	1.31079	1.36048	1.41158	1.46410
5	1.10408	1.13140	1.40255	1.46932	1.53862	1.61051
6	1.12616	1.15969	1.50073	1.58687	1.67710	1.77156
7	1.14868	1.18868	1.60578	1.71382	1.82803	1.94871
8	1.17165	1.21840	1.71818	1.85093	1.99256	2.14358
9	1.19509	1.24886	1.83845	1.99900	2.17189	2.35794
10	1.21899	1.28008	1.96715	2.15892	2.36736	2.59374
11	1.24337	1.31208	2.10485	2.33163	2.58042	2.85311
12	1.26824	1.34488	2.25219	2.51817	2.81266	3.13842
13	1.29360	1.37851	2.40984	2.71962	3.06580	3.45227
14	1.31947	1.41297	2.57853	2.93719	3.34172	3.79749
15	1.34586	1.44829	2.75903	3.17216	3.64248	4.17724
16	1.37278	1.48450	2.95216	3.42594	3.97030	4.59497
17	1.40024	1.52161	3.15881	3.70001	4.32763	5.05447
18	1.42824	1.55965	3.37993	3.99601	4.71712	5.55991
19	1.45681	1.59865	3.61652	4.31570	5.14166	6.11590
20	1.48594	1.63861	3.86968	4.66095	5.60441	6.72749
21	1.51566	1.67958	4.14056	5.03383	6.10880	7.40024
22	1.54597	1.72157	4.43040	5.43654	6.65860	8.14027
23	1.57689	1.76461	4.74052	5.87146	7.25787	8.95430
24	1.60843	1.80872	5.07236	6.34118	7.91108	9.84973
25	1.64060	1.85394	5.42743	6.84847	8.62308	10.8347
26	1.67341	1.90029	5.80735	7.39635	9.39915	11.9181
27	1.70688	1.94780	6.21386	7.98806	10.2450	13.1099
28	1.74102	1.99649	6.64883	8.62710	11.1671	14.4209
29	1.77584	2.04640	7.11425	9.31727	12.1721	15.8630
30	1.81136	2.09756	7.61225	10.0626	13.2676	17.4494
31	1.84758	2.15000	8.14511	10.8676	14.4617	19.1943
32	1.88454	2.20375	8.71527	11.7370	15.7633	21.1137
33	1.92223	2.25885	9.32533	12.6760	17.1820	23.2251
34	1.96067	2.31532	9.97811	13.6901	18.7284	25.5476
35	1.99988	2.37320	10.6765	14.7853	20.4139	28.1024
36	2.03988	2.43253	11.4239	15.9681	22.2512	30.9126



# T A B L E S.

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TABLE LX. continued.

Years	2per cent.	2½per cent.	7per cent.	8per cent.	9per cent.	10 per cent.
37	2.08068	2.49334	12.2236	17.2456	24.2538	34.0039
38	2.12229	2.55568	13.0792	18.6252	26.4366	37.4043
39	2.16474	2.61957	13.9948	20.1152	28.8159	41.1447
40	2.20803	2.68506	14.9744	21.7245	31.4094	45.2592
41	2.25220	2.75219	16.0226	23.4624	34.2362	49.7851
42	2.29724	2.82099	17.1442	25.3394	37.3175	54.7636
43	2.34318	2.89152	18.3443	27.3666	40.6761	60.2400
44	2.39005	2.96380	19.6284	29.5559	44.3369	66.2640
45	2.43785	3.03790	21.0024	31.9204	48.3272	72.8904
46	2.48661	3.11385	22.4726	34.4740	52.6767	80.1795
47	2.53634	3.19169	24.0457	37.2320	57.4176	88.1974
48	2.58707	3.27148	25.7289	40.2105	62.5852	97.0172
49	2.63881	3.35327	27.5299	43.4274	68.2179	106.718
50	2.69158	3.43710	29.4570	46.9016	74.3575	117.390
51	2.74541	3.52303	31.5190	50.6537	81.0496	129.129
52	2.80032	3.61111	33.7253	54.7060	88.3441	142.042
53	2.85633	3.70139	36.0861	59.0825	96.2951	156.247
54	2.91346	3.79392	38.6121	63.8091	104.961	171.871
55	2.97173	3.88877	41.3150	68.9138	114.408	189.059
56	3.03116	3.98599	44.2070	74.4269	124.705	207.965
57	3.09178	4.08564	47.3015	80.3811	135.928	228.761
58	3.15362	4.18778	50.6126	86.8116	148.162	251.637
59	3.21669	4.29247	54.1555	93.7565	161.496	276.801
60	3.28103	4.39978	57.9464	101.257	176.031	304.481
61	3.34665	4.50978	62.0026	109.357	191.874	334.929
62	3.41358	4.62252	66.3428	118.106	209.142	368.422
63	3.48185	4.73809	70.9868	127.554	227.965	405.265
64	3.55149	4.85654	75.9559	137.759	248.482	445.791
65	3.62252	4.97795	81.2728	148.779	270.845	490.370
66	3.69497	5.10240	86.9619	160.682	295.222	539.407
67	3.76887	5.22996	93.0492	173.536	321.792	593.348
68	3.84425	5.36071	99.5627	187.419	350.753	652.683
69	3.92113	5.49473	106.532	202.413	382.321	717.951
70	3.99955	5.63210	113.989	218.606	416.730	789.746
71	4.07954	5.77290	121.968	236.094	454.235	868.721
72	4.16114	5.91722	130.506	254.982	495.117	955.593
73	4.24436	6.06515	139.641	275.381	539.677	1051.15
74	4.32925	6.21678	149.416	297.411	588.248	1156.26
75	4.41583	6.37220	159.876	321.204	641.190	1271.89
76	4.50415	6.53151	171.067	346.900	698.898	1399.08
77	4.59423	6.69480	183.042	374.652	761.798	1538.99

T A B L E L X continued.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
78	4.68611	6.86217	195.851	404.625	830.360	1692.89
79	4.77984	7.03372	209.564	436.995	905.093	1862.18
80	4.87543	7.20956	224.234	471.954	986.551	2048.40
81	4.97294	7.38980	239.930	509.711	1075.34	2253.24
82	5.07240	7.57455	256.725	550.488	1172.12	2478.56
83	5.17385	7.76391	274.696	594.527	1277.61	2726.42
84	5.27733	7.95801	293.925	642.089	1392.59	2999.06
85	5.38287	8.15696	314.500	693.455	1517.93	3298.96
86	5.49053	8.36088	336.515	748.933	1654.54	3628.86
87	5.60034	8.56991	360.071	808.847	1803.45	3991.75
88	5.71235	8.78415	385.276	873.555	1965.76	4390.92
89	5.82660	9.00376	412.245	943.439	2142.68	4830.02
90	5.94313	9.22885	441.102	1018.91	2335.52	5313.02
91	6.06199	9.45957	471.980	1100.42	2545.72	5844.32
92	6.18323	9.69606	505.018	1188.46	2774.83	6428.75
93	6.30690	9.93846	540.370	1283.53	3024.57	7071.63
94	6.43303	10.1869	578.196	1386.22	3296.78	7778.79
95	6.56169	10.4416	618.669	1497.12	3593.49	8556.67
96	6.69293	10.7026	661.976	1616.89	3916.91	9412.34
97	6.82679	10.9702	708.314	1746.24	4269.43	10353.58
98	6.96332	11.2444	757.897	1885.94	4653.68	11388.93
99	7.10259	11.5255	810.949	2036.81	5072.51	12527.82
100	7.24464	11.8137	867.716	2199.76	5529.04	13780.61

# T A B L E S.

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## T A B L E L X I.

Shewing the Sum to which an *Annuity* of 1*l.* will increase at 2, 2½, 7, 8, 9, and 10 *per cent.* Compound Interest, in any Number of Years not exceeding 100; being a Supplement to Table IV. p. 28.

Years	2per cent.	2½per cent.	7per cent.	8per cent.	9per cent.	10 per cent.
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.02000	2.02500	2.07000	2.08000	2.09000	2.10000
3	3.06040	3.07562	3.21490	3.24640	3.27810	3.31000
4	4.12160	4.15251	4.43994	4.50611	4.57312	4.64100
5	5.20404	5.25632	5.75073	5.86660	5.98471	6.10510
6	6.30812	6.38773	7.15329	7.33592	7.52333	7.71561
7	7.43428	7.54743	8.65402	8.92280	9.20043	9.48717
8	8.58296	8.73611	10.2598	10.6366	11.0284	11.4358
9	9.75462	9.95451	11.9779	12.4875	13.0210	13.5794
10	10.9497	11.2033	13.8164	14.4865	15.1929	15.9374
11	12.1687	12.4834	15.7835	16.6454	17.5602	18.5311
12	13.4120	13.7955	17.8884	18.9771	20.1407	21.3842
13	14.6803	15.1404	20.1405	21.4952	22.9533	24.5227
14	15.9739	16.5189	22.5504	24.2149	26.0191	27.9749
15	17.2934	17.9319	25.1290	27.1521	29.3609	31.7724
16	18.6392	19.3802	27.8880	30.3242	33.0033	35.9497
17	20.0120	20.8647	30.8402	33.7502	36.9737	40.5447
18	21.4123	22.3853	33.9990	37.4502	41.3013	45.5991
19	22.8405	23.9460	37.3789	41.4462	46.0184	51.1590
20	24.2973	25.5446	40.9954	45.7019	51.1601	57.2750
21	25.7833	27.1832	44.8651	50.4229	56.7645	64.0025
22	27.2989	28.8628	49.0057	55.4567	62.8733	71.4027
23	28.8449	30.5844	53.4361	60.8932	69.5319	79.5430
24	30.4218	32.3490	58.1766	66.7647	76.7898	88.4973
25	32.0302	34.1577	63.2490	73.1059	84.7008	98.3470
26	33.6709	36.0117	68.6764	79.9544	93.3239	109.181
27	35.3443	37.9120	74.4838	87.3507	102.723	121.009
28	37.0512	39.8598	80.6976	95.3386	112.966	134.209
29	38.7922	41.8562	87.3465	103.955	124.135	148.630
30	40.5680	43.9027	94.4607	113.283	136.307	164.494
31	42.3791	46.0002	102.073	123.345	149.575	181.943
32	44.2270	48.1502	110.218	134.213	164.036	201.137
33	46.1115	50.3540	118.933	145.950	179.800	222.251
34	48.0338	52.6128	128.258	158.626	196.982	245.476
35	49.9944	54.9282	138.236	172.316	215.710	271.024
36	51.9943	57.2014	148.913	187.102	236.124	299.126



TABLE LXI. continued.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
37	54.0342	59.7339	160.337	203.070	258.375	330.039
38	56.1149	62.2272	172.561	220.315	282.629	364.043
39	58.2372	64.7829	185.640	238.941	309.066	401.447
40	60.4019	67.4025	199.635	259.056	337.882	442.592
41	62.6100	70.0876	214.609	280.781	369.291	487.851
42	64.8622	72.8398	230.632	304.243	403.528	537.636
43	67.1594	75.6608	247.776	329.583	440.845	592.400
44	69.5026	78.5523	266.120	356.949	481.521	652.640
45	71.8927	81.5161	285.749	386.505	525.858	718.904
46	74.3305	84.5540	306.751	418.426	574.186	791.795
47	76.8171	87.6678	329.224	452.900	626.862	871.974
48	79.3535	90.8595	353.270	490.132	684.280	960.172
49	81.9405	94.1310	378.998	530.342	746.865	1057.18
50	84.5794	97.4843	406.528	573.770	815.083	1163.90
51	87.2709	100.921	435.985	620.671	889.441	1281.29
52	90.0164	104.444	467.504	671.325	970.490	1410.42
53	92.8167	108.055	501.230	726.031	1058.83	1552.47
54	95.6730	111.756	537.316	785.114	1155.13	1708.71
55	98.5865	115.550	575.928	848.923	1260.09	1880.59
56	101.558	119.439	617.243	917.837	1374.50	2069.65
57	104.589	123.425	661.450	992.264	1499.20	2277.61
58	107.681	127.511	708.752	1072.64	1635.13	2506.37
59	110.834	131.699	759.364	1159.45	1783.29	2758.01
60	114.051	135.991	813.520	1253.21	1944.79	3034.81
61	117.332	140.391	871.466	1354.47	2120.82	3339.29
62	120.679	144.901	933.469	1463.82	2312.69	3674.22
63	124.092	149.523	999.812	1581.93	2521.84	4042.65
64	127.574	154.261	1070.79	1709.48	2749.80	4447.91
65	131.126	159.118	1146.75	1847.24	2998.28	4893.70
66	134.748	164.096	1228.02	1996.02	3269.13	5384.07
67	138.443	169.198	1314.99	2156.71	3564.35	5923.48
68	142.212	174.428	1408.03	2330.24	3886.14	6516.83
69	146.056	179.789	1507.60	2517.66	4236.90	7169.51
70	149.977	185.284	1614.13	2720.08	4619.22	7887.46
71	153.977	190.916	1728.12	2938.68	5035.95	8677.21
72	158.057	196.689	1850.09	3174.78	5490.18	9545.93
73	162.218	202.606	1980.59	3429.76	5985.30	10501.53
74	166.462	208.671	2120.24	3705.14	6524.98	11552.68
75	170.791	214.888	2269.65	4002.55	7113.23	12708.95
76	175.207	221.260	2429.53	4323.76	7754.42	13980.81
77	179.711	227.792	2600.60	4670.66	8453.32	15379.93



# T A B L E S.

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T A B L E LXI. continued.

Years	2 per cent.	2½ percent.	7 percent.	8 percent.	9 percent.	10 percent
78	184.305	234.486	2783.64	5045.31	9215.12	16918.92
79	188.992	241.348	2979.49	5449.94	10045.4	18611.82
80	193.771	248.382	3189.06	5886.93	10950.5	20474.00
81	198.647	255.592	3413.29	6358.89	11937.1	22522.40
82	203.620	262.982	3653.22	6868.60	13012.4	24775.64
83	208.692	270.556	3909.95	7419.08	14184.5	27254.20
84	213.866	278.320	4184.65	8013.61	15462.2	29980.62
85	219.143	286.278	4478.57	8655.70	16854.8	32979.69
86	224.526	294.435	4793.07	9349.16	18372.7	36278.65
87	230.017	302.796	5129.59	10098.0	20027.2	39907.52
88	235.617	311.366	5489.66	10906.9	21830.7	43899.27
89	241.330	320.150	5874.93	11780.4	23796.5	48290.20
90	247.156	329.154	6287.18	12723.9	25939.1	53120.22
91	253.099	338.383	6728.28	13742.8	28274.7	58433.25
92	259.161	347.842	7200.26	14843.2	30820.4	64277.57
93	265.345	357.538	7705.28	16031.7	33595.2	70706.33
94	271.651	367.477	8245.65	17315.2	36619.8	77777.96
95	278.084	377.664	8823.85	18701.5	39916.6	85556.76
96	284.646	388.105	9442.52	20198.6	43510.1	94113.43
97	291.339	398.808	10104.5	21815.5	47427.0	103525.8
98	298.166	409.778	10812.8	23561.7	51696.4	113879.3
99	305.129	421.023	11570.7	25447.7	56350.1	125268.3
100	312.232	432.548	12381.6	27484.5	61422.6	137796.1

THE four *last* Tables are to be considered as continuations of the four *first* Tables; and they have been added to this collection, *partly* because it will be found sometimes necessary to determine the values and amounts of sums and annuities at the *higher* and *lower* rates of interest specified in them; but *chiefly* because they furnish with the means of determining easily these values and amounts for the most common *half-yearly* as well as *yearly* rates of interest; Mr. SMART, in his very useful and comprehensive Tables, having given these *half-yearly* values and amounts improperly.

It is very obvious, that the amount at any given *yearly* interest of any given annuity payable *half-yearly*, is the same with the amount of *half* that annuity at *half* the interest, and payable a *double* number of times. The amount, for instance, at 4 *per cent.* of an annuity of 10*l.* payable *yearly* for 30 years, is, by Table IV. p. 28, *l.* 560.849.

If it is payable *half-yearly*, its amount will be the same with the amount at 2 *per cent.* of an annuity of 5*l.* payable for 60 years, which, by Table LXI. is *l.* 570.257.  
 ———In like manner; the amount at 5 *per cent.* of an annuity of 50*l.* for 40 years, payable *half-yearly*, is the same with the amount at  $2\frac{1}{2}$  *per. cent.* of an annuity of 25*l.*  
 for

for 80 years, which, by Table LXI. appears to be  $\text{£} 6209.567$ . The amount at  $5 \text{ per cent.}$  of the same annuity, payable *yearly*, appears, by Table IV. to be  $\text{£} 6039.988$ .

Farther. The amount of  $10\text{l.}$  *principal* put out to *yearly* interest at  $4 \text{ per cent.}$  and forborne for 30 years, is (by Table III. p. 25)  $\text{£} 32.433$ . But if it is put out to  $4 \text{ per cent.}$  *half-yearly* interest, its amount will be the same with the amount of the same principal, bearing *half* the interest in double the time; that is, it will, in the present instance, be the same with the amount of  $10\text{l.}$  bearing  $2 \text{ per cent.}$  interest in 60 years, which, by the last Table but one, appears to be  $\text{£} 32.810$ .

These amounts can be thus determined from these Tables only, when the term for which they are wanted does not exceed 50 years, or 100 *half* years.

In order to find them for any longer term, the following method must be taken :

“ If the amount required is the amount  
 “ not of an *annuity*, but of a sum—find first  
 “ the *half-yearly* amount for 50 years; after  
 “ which find the *half-yearly* amount of that  
 “ amount for the remainder of the term, and  
 “ this last will be the amount desired.”

## E X A M P L E.

Let the amount be required, at  $4 \text{ per cent.}$  of  $10\text{l.}$  in 80 years, supposing the interest payable *half-yearly*.

Q 3

Ans.

Anf. The amount in 50 years, determined in the manner just described, is 72.446; and the amount of 72.446 in 30 years, determined in the same way, is 1.237.676, which is the amount required.

This amount, supposing the interest payable yearly, is 1.230.049.

But if the amount required is the amount of an *annuity* improved at any given rate of compound interest payable *half-yearly*, it will be necessary, after finding the sum which is the amount for 50 years, to find the yearly interest that sum will carry at the given rate; and the amount for the remainder of the term, of this interest increased by the annuity, added to the amount for 50 years, will be the amount required.

### EXAMPLE.

Let the amount be required, at 4 *per cent.* of 10*l.* *per ann.* in 80 years, supposing the annuity payable *half-yearly*.

Anf. The amount in 50 years (being the same with the amount of 5*l.* *per ann.* in 100 years, at 2 *per cent.*) is, by Table LVIII. 1.1561.116.——The yearly interest of 1.1561.116, at 4 *per cent.* is 1.62.446, which increased by 10*l.* makes 1.72.446; and the amount of 1.72.446 *per ann.* payable *half-yearly* in 30 years (or of 1.36.223 in 60 *half years*)



years) is  $\text{£} 4620.96$ , which added to  $\text{£} 1561.116$ , makes  $\text{£} 6182.076$  the amount required.

This amount, supposing the annuity payable yearly, is  $\text{£} 5982.665$ .

*N. B.* These amounts for any given term and rate of interest are the same with the *debts* bearing that interest, which will be gradually sunk in that term by any given annuity appropriated to the redemption of the debt.—It appears, therefore, from the last example, that a sinking fund of a million *per ann.* never diverted, would pay off, in 82 years, a public debt of 598 millions, bearing 4 *per cent.* interest, supposing it applied to that purpose *yearly*; but that if applied *half-yearly*, it would pay off, in the same time, a debt of 618 millions. See p. 34.

These examples shew the method of finding, by the preceding Tables, the values at any rate of interest of annuities payable for any given terms, supposing them payable *half-yearly*; and likewise the values of any sums payable at the end of any terms, supposing a *half-yearly* instead of an *yearly* discount allowed. But in such cases, these Tables will be of no use, if the terms exceed 50 years, or 100 *half-years*; and it will be necessary to have recourse to the *theorems* at the beginning of the *third* of the following Additional Essays, by which, with the help of logarithms, it is easy, in

all cases, to compute the difference between the values of annuities (including *life-annuities*) as they are payable yearly, half-yearly, or quarterly.

With respect to *life-annuities*, it may be proper to observe here particularly, that their values deduced from the *complements*, that is, from twice the *expectations* according to any given table of mortality (by the rules in p. 170, and p. 172), and payable *half-yearly* or *quarterly*, is the same with the values of *half* or a *quarter* the annuities at *half* or a *quarter* the yearly interest, deduced (by the same rules) from *double* or *quadruple* the complements; and that the difference between the *yearly* values and these *half-yearly* or *quarterly* values added to the *true* yearly values according to the same table of mortality, will give, with almost perfect correctness, the *half-yearly* or *quarterly* values according to that table.

#### EXAMPLE.

Let the *different* values be required of an annuity on a single life aged 50, according as it is payable *half-yearly* or *quarterly*, reckoning interest at 4 *per cent.* and the probabilities of the duration of life, as they are in Table XLII. p. 150,

Ans.

Ans. The *complement* (that is, twice the *expectation*) by Table XLII. of a life aged 50 is 36 92.—The value of an annuity payable yearly on a life at this age deduced at 4 *per cent.* from this *complement*, is by the rule in p. 170, 11.533.—The value deduced from *double* this complement (that is, of a life whose complement is supposed 73.84) at 2 *per cent.* is 23.466 (a), the half of which

(a) In computing in this case, by the rule here referred to, it is necessary to find the value at 2 *per cent.* of an annuity certain payable for 73.84 years. This value (by the First Theorem in the Third Additional

Essay) is  $50 - \frac{1}{.02 \times \overline{1.02}^{73.84}}$ . By logarithms it is

easy to find that  $\overline{1.02}^{73.84}$  is 4.3154; and, consequently, that this expression is 38.416, which multiplied, according to the rule in p. 170, by 51 (the perpetuity increased by unity), and the product divided by 73.84 will give 36.533, which quotient subtracted from 50, (the perpetuity) leaves 23.466.

In like manner; the value, by the same theorem, at 1 *per cent.* for an annuity certain for a number of years equal to four times the complement (that is, to 147.68

years) is  $100 - \frac{1}{.01 \times \overline{1.01}^{147.68}}$ , which is equal to 76.994; and the product of 76.994 into 101, divided by 147.68, gives 52.654, which, subtracted from 100, leaves 47.345, the quarter of which is 11.836.

It is necessary to add here, that in computing the yearly value of any life-annuity from the expectation by the rule in p. 170, the value of an annuity certain for a number of years equal to twice the expectation (or the complement) may be always taken from Table II. p. 21, when the complement is any *whole* number of years; and also,



which is 11.733.—The difference is .200. And this difference, added to 11.658 (the *true* value by Table XLIII. of an annuity payable yearly on the supposed life), makes 11.858; which is the true value of the annuity payable *half*-yearly.

The value of the same annuity deduced (by the rule in p. 170) from *quadruple* the complement at 1 *per cent.*; that is, the value at 1 *per cent.* of a life whose complement is supposed to be 147.68 years, is 47.345, the quarter of which is 11.836. The difference between this value and 11.533 is .303, which added as before to 11.658, makes 11.961, the true value of the annuity payable quarterly.

In the same way the values are to be computed (by the second rule in p. 172) of annuities payable *half*-yearly or *quarterly* on any two joint lives.

also, that when it is not any whole number of years, it may be taken for the correspondent arithmetical mean between the two nearest yearly values in the Table. Thus; in the example given above, the value at 4 *per cent.* of an annuity certain for 36 years, by Table II. p. 21, is 18.908. The value for 37 years is 19.142. The difference is .234; and this difference multiplied by .92 (the fractional part of the complement) and added to the least of these two values, gives 19.123 for the value of an annuity certain for 36.92 years.

The exact value by the first Theorem is 25 —

1

————, which is equal to 19.123, and the same  
 $.04 \times 1.04 \overline{) 36.92}$   
 with the former value

If



If the annuity is a life-annuity (a) secured on land, the value is to be computed by the directions in the Third Additional Essay.——If such an annuity is payable *half-yearly*, as is most common, its value, in the present instance, will be 11.858 (the *half-yearly* value just determined) increased by the quotient of 19.206 (the value of an annuity certain and payable *half-yearly* for a number of *half* years, equal to *four* times the *expectation*, or *twice* the *complement*; that is, 73.84) divided by *four* times the complement. This quotient is .130; and the value, therefore, is 11.989.

The following comparison will shew, in some measure, what additions should be made, at all ages, to the yearly values of life-annuities, on account of these different modes of payment.

(a) It should be remembered, that all the values of life-annuities in the preceding Tables suppose that, when the annuitant dies, nothing can be claimed for the time that has past since the last payment became due. If a payment proportioned to that time may be claimed; that is, if the annuity is payable to the last moment of life, it is called an annuity *secured on land*.

## T A B L E LXII.

Shewing the Additions to the Values of Life-Annuities on account of their being payable *half-yearly*, or *quarterly*, or *half-yearly* and secured on Land.

Interest 4 per cent.

Age	Yearly value by Table XLIII.	Half-yearly value.	Excess above yearly value.	Quarterly value.	Excess above yearly value.	Value secured by land and payable half-yearly.	Excess above yearly value.
10	18.891	19.018	.127	19.089	.198	19.085	.194
20	17.603	17.746	.143	17.819	.216	17.824	.221
30	16.006	16.168	.162	16.249	.243	16.259	.253
40	14.034	14.216	.182	14.307	.273	14.324	.290
50	11.658	11.858	.200	11.961	.300	11.989	.331
60	8.789	9.014	.225	9.119	.330	9.170	.381
70	5.783	6.019	.236	6.136	.353	6.204	.421
75	4.534	4.770	.236	4.892	.358	4.967	.433

Interest 5 per cent

Age	Yearly value by Table XLIII.	Half-yearly value.	Excess above yearly value.	Quarterly value.	Excess above yearly value.	Value secured by land and payable half-yearly.	Excess above yearly value.
10	16.169	16.278	.109	16.331	.162	16.332	.163
20	15.260	15.383	.123	15.445	.183	15.447	.187
30	14.080	14.223	.143	14.293	.213	14.299	.219
40	12.558	12.720	.162	12.801	.243	12.812	.254
50	10.634	10.820	.186	10.914	.280	10.933	.299
50	8.184	8.403	.219	8.498	.314	8.544	.360
70	5.494	5.719	.225	5.835	.341	5.891	.397
75	4.345	4.575	.230	4.692	.347	4.762	.417

These excesses are the same from whatever tables of mortality the yearly values are deduced.—They are also nearly the same (supposing equal *yearly* values) whether the yearly values are the values of single, or of joint lives, or of any number of lives.

A SUPPLE-



A  
SUPPLEMENT, (a)

CONTAINING

*Additional Observations on the Duration of Human Life in different Situations; and on the Population of the Kingdom.*

SINCE the first publication of this work, I have had the pleasure of reading an ingenious Memoir on the State of Population in the *Pais de Vaud*, a district of the province of *Bern* in *Switzerland*. The author of this memoir is Mr. *Muret*, the first minister at *Vevey*, a town in that district, and secretary to the Oeconomical Society there. It forms the first part of the *Bern Observa-*

(a) This supplement was an addition to this Treatise in the Second and Third Editions of it. I have in the present Edition added to it a *Postscript*, containing a review of the arguments for and against the increasing population of the kingdom.

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tions for the year 1766; and a good abstract of it may be found in the 69th article of a work entitled, *De re Rustica*, or the *Repository*. It contains an account of many facts which appear to me curious and important; and which confirm the observations I have made in the First and Fourth Essays in the First Volume of this Treatise.—Some of these facts I will here recite.

In the First Essay I have asserted, that there is a much greater difference between the probabilities of the duration of life in *great towns* and in *county parishes*, than is commonly suspected; and, as one proof of this, I have observed, that though in *London* the greatest part of the natives die under three years of age, in the country the greater part live to marry. Mr. *Muret's* Observations and Tables give a distinct demonstration of this, by shewing, that in the province of *Vaud*, the greater part of the inhabitants live many years beyond the age of maturity.—But to be a little more explicit.

The district of *Vaud*, in *Switzerland*, contains 112,951 inhabitants of all ages; 25,778 families; 38,328 married persons: and the annual medium of *births*, for 10 years before 1766, had been 3155; of *weddings*, 808; of *deaths*, 2504.—It appears, therefore, that the married are very nearly a *third* part of the inhabitants, that the number of persons



to a family is  $4\frac{1}{3}$ ; and that one in 45 of the inhabitants die annually. It may be further learnt (by dividing half the number of the married by the annual medium of weddings), that the *expectation* of marriage in this country is 23 years and  $\frac{1}{2}$ ; and (from the proportions of the births, weddings, and deaths) (*a*) that the greater part of those who are born live to marry. But of this fact there is, I have just intimated, a more particular and distinct proof.—From a Table given by Mr. *Muret*, of the rate of human mortality in this country (derived from registers kept in 43 parishes, of the ages at which the inhabitants die), it appears, that one *half* of all that are born live beyond 41 years of age.—The examination of this Table will, undoubtedly, be a gratification to the reader; and, therefore, I have chosen to make it a part of these additions. See p. 259. I have also given a Table which I have formed from a register in *Susmilch's* works, of the ages at which the inhabitants of a country parish in BRANDENBURGH died, during 50 years, ended at 1759.—And I have further thought proper to add, as contrasts to these Tables, two Tables exhibiting the probabilities of life at VIENNA and BERLIN. See p. 260, 261, and 262.

The following observations concerning these Tables should be attended to.

(*a*) See the note, p. 264, &c. Vol. I.

The

The Table for the country of VAUD, though it gives the probabilities of life in its first stages very high; and, at some ages, more than double to the probabilities of life in great cities; yet, certainly, gives them too low. For, first, it has just appeared, that in this country the births exceed considerably the deaths. The emigrations, likewise, from it are very numerous, as will be presently observed: And the necessary effect of these two causes is, to make the registers give the number of deaths in the first stages of life too great in comparison of the deaths in the last stages. A Table formed from such registers must give the probabilities of life too low, according to the observations in the Fourth Essay; and, in the introduction to the preceding Collection of Tables.

After 40, the probabilities of living in this country decrease very fast; and, after 65, they appear to be rather lower than is common. Mr. *Muret* has taken notice of this fact, and ascribes it to the particular prevalence of drunkenness in his country. He had, he says, once the curiosity to examine the register of deaths in one town, and to mark those whose deaths might be imputed to drunkenness; and he found the number so great, as to incline him to believe, that hard drinking kills more of mankind, than pleurisy and fevers, and all the most malignant distempers.

The

The former of these observations is applicable to the Table for the country parish in *Brandenburg*; for it appears from *Susmilch's* account (*a*), that the births there exceed the deaths more than in the country of *VAUD*; nor is it to be imagined, that there are not likewise many emigrations from it, particularly, to *BERLIN* and the King of *Prussia's* armies.

From the Tables for *VIENNA* and *LONDON*, compared with the Table for *BERLIN*, it appears, that the last of these towns, though much the smallest, has at some ages even a worse effect on the duration of life, than either of the former: And the reason, perhaps, may be, that the inhabitants there are much more crowded together. See p. 295, Vol. I. Between the ages of 30 and 35, and also between 42 and 52, there is an irregularity in the *BERLIN* Table, which, very probably, would not have appeared in it, had it been formed from the bills for a longer term of years.

From the age of 25 to 45, *VIENNA* appears, in the Tables, to be less unfavourable to life than *LONDON*; but it cannot be depended upon that this is the truth, for the *VIENNA* Table may give the probabilities of living at these ages higher, only because the recruits from the country come to it later,

(*a*) See the remarks on the Table in p. 207 in the preceding collection.

or in greater numbers, after 30 and 40, than in LONDON. A like effect would also arise from a greater number of migrations in old age from LONDON than from VIENNA.

In forming the Tables for VIENNA and BERLIN, I have applied the correction explained in the Fourth Essay, and demonstrated there to be necessary; and, in making this correction, I have supposed, agreeably to the proportion of the births to the burials, that a fifth of all who die in these cities, are persons who removed to them at 20 years of age.—Notwithstanding this correction, the Table for BERLIN gives the probabilities of life between 10 and 20 so high, and in such disproportion to the probabilities of life immediately after 20, as to exceed all the bounds of credibility. The true reason of this may be learnt from what has been said in p. 295, Vol. I. of the rapid increase of BERLIN.

TABLE



TABLE I. (a).

Shewing the Probabilities of Life in the District of VAUD, SWITZERLAND, formed from the Registers of 43 Parishes, given by Mr. Muret, in the First Part of the BERN Memoirs for the Year 1766.

Age.	Living	Decr.	Age.	Living	Decr.	Age.	Living	Decr.
0	1000	189	31	558	5	62	200	12
1	811	46	32	553	5	63	274	12
2	765	30	33	548	4	64	262	12
3	735	20	34	544	5			
4	715	14				65	250	14
			35	539	6	66	236	16
5	70	13	36	531	6	67	220	18
6	588	11	37	527	7	68	202	18
7	677	10	38	520	7	69	184	16
8	667	8	39	513	7			
9	659	6				70	168	15
			40	506	6	71	153	13
10	653	5	41	500	6	72	140	11
11	648	5	42	494	6	73	129	10
12	643	4	43	488	6	74	119	10
13	639	4	44	482	6			
14	635	4				75	109	11
			45	476	7	76	98	13
15	631	5	46	469	8	77	85	14
16	626	4	47	461	10	78	71	13
17	622	4	48	451	10	79	58	12
18	618	4	49	441	10			
19	614	4				80	46	10
			50	431	9	81	36	7
20	610	4	51	422	8	82	29	5
21	606	4	52	414	8	83	24	4
22	602	5	53	406	9	84	20	3
23	597	5	54	397	9			
24	592	5				85	17	3
			55	388	11	86	14	3
25	587	5	56	377	13	87	11	2
26	582	5	57	364	16	88	9	2
27	577	5	58	348	17	89	7	2
28	572	5	59	331	17			
29	567	4				90	5	1
			60	314	15			
30	563	5	61	299	3			

(a) All the Bills, from which this and the following Tables are formed, give the numbers dying under 1 as well as under 2 years; and, in the numbers dying under 1 are included, in the country parish in *Brandenburg* and at *Berlin*, all the still-borns. All the bills also give the numbers dying in every period of five years.

## TABLE II.

Shewing the Probabilities of Life in a Country Parish in  
BRANDENBURGH, formed from the Bills for 50 Years,  
from 1710 to 1759, as given by Mr. SUSMILCH, in  
his *Gottliche Ordnung*, p. 43.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decr.
0	1000	225	31	482	5	62	260	12
1	775	57	32	477	5	63	248	12
2	718	31	33	472	5	64	236	12
3	687	23	34	467	5	65	224	11
4	664	22	35	462	6	66	213	11
5	642	20	36	456	6	67	202	12
6	622	15	37	450	6	68	190	12
7	607	12	38	444	6	69	178	12
8	595	10	39	438	6	70	166	13
9	585	8	40	432	5	71	153	15
10	577	7	41	427	5	72	138	16
11	570	6	42	422	5	73	122	15
12	564	5	43	417	5	74	107	14
13	559	5	44	412	6	75	93	13
14	554	5	45	407	6	76	80	12
15	549	5	46	400	6	77	68	9
16	544	5	47	394	6	78	59	8
17	539	4	48	388	7	79	51	7
18	535	4	49	381	7	80	44	6
19	531	4	50	374	7	81	38	6
20	527	5	51	367	8	82	32	6
21	522	5	52	359	8	83	25	6
22	517	5	53	351	8	84	21	5
23	512	5	54	343	9	85	15	4
24	507	5	55	334	10	86	11	3
25	502	4	56	324	10	87	8	2
26	498	3	57	314	10	88	6	2
27	495	3	58	304	11	89	4	1
28	492	3	59	293	11	90	3	1
29	489	3	60	282	11	91	2	1
30	486	4	61	271	11	92	1	1

# SUPPLEMENT.

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## T A B L E III.

Shewing the Probabilities of Life at VIENNA, formed from the Bills for Eight Years, as given by Mr. SUSMILCH, in his *Gottliche Ordnung*, Page 32, Tables.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decr.
0	1495	682	31	364	6	62	129	6
1	813	107	32	358	5	63	123	7
2	706	61	33	353	6	64	116	7
3	645	46	34	347	7	65	109	8
4	599	33	35	340	8	66	101	8
5	566	30	36	332	8	67	93	8
6	536	20	37	324	8	68	85	7
7	516	11	38	316	9	69	78	7
8	505	9	39	307	9	70	71	6
9	496	7	40	298	8	71	65	5
10	489	6	41	290	7	72	60	5
11	483	5	42	283	6	73	55	4
12	478	5	43	277	6	74	51	4
13	473	6	44	271	7	75	47	5
14	467	6	45	264	8	76	42	5
15	461	6	46	256	9	77	37	5
16	455	7	47	247	9	78	32	5
17	448	6	48	238	9	79	27	4
18	442	6	49	229	9	80	23	3
19	436	6	50	220	8	81	20	2
20	430	5	51	212	7	82	19	2
21	425	5	52	205	7	83	16	2
22	420	5	53	198	7	84	14	2
23	415	6	54	191	7	85	12	2
24	409	6	55	184	8	86	10	2
25	403	6	56	176	8	87	8	2
26	397	6	57	168	9	88	6	2
27	391	7	58	159	8	89	4	1
28	381	7	59	151	8	90	3	1
29	377	7	60	143	7	91	2	1
30	370	6	61	136	7	92	1	1

## TABLE IV.

Shewing the Probabilities of Life at BERLIN, formed from the Bills for Four Years, from 1752 to 1755, given by Mr. SUSMILCH (a), in his *Gottliche Ordnung*, Vol. II. page 37, Tables.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decr.
0	427	54	33	361	7	65	112	6
1	903	15	34	354	7	66	106	7
2	752	61				67	99	7
3	691	73	35	347	8	68	92	6
4	618	45	36	339	9	69	86	6
			37	330	10			
5	573	21	38	320	10	70	80	6
6	552	15	39	310	10	71	74	6
7	536	13				72	68	6
8	513	9	40	300	10	73	62	5
9	514	7	41	290	9	74	57	5
			42	281	8			
10	507	5	43	274	7	75	52	5
11	502	4	44	266	7	76	47	5
12	498	4				77	42	5
13	494	4	45	259	7	78	37	5
14	490	4	46	252	7	79	32	4
			47	245	7			
15	486	4	48	238	7	80	28	4
16	482	5	49	231	7	81	24	3
17	477	5				82	21	2
18	472	5	50	224	7	83	19	2
19	467	6	51	217	7	84	17	2
			52	210	7			
20	461	6	53	203	8	85	15	2
21	455	6	54	195	8	86	13	2
22	449	6				87	11	2
23	443	7	55	187	8	88	9	2
24	436	8	56	179	8	89	7	1
			57	171	8			
25	428	9	58	163	9	90	6	1
26	421	9	59	154	9	91	5	1
27	412	9				92	4	1
28	403	9	60	145	8	93	3	1
29	394	9	61	137	7	94	2	1
			62	130	6			
30	385	9	63	124	6			
31	376	8	64	118	6			
32	368	7						

(a) This writer has also given the bills of the parish of St. Peter's at BERLIN, for 24 years; and a Table formed from them, agrees nearly with this.

The



THESE Tables exhibit, in a striking light, the *difference* between the duration of human life, in *great cities* and in the *country*. I will here lay some of the chief particulars of it before the reader, desiring him to take with him this consideration, that, for the reasons I have explained, they can be erroneous only by giving the difference much too *little*.

Proportion of Inhabitants dying annually in

Pais De Vaud.	Country Parish in Brandenburg.	Vienna.	Berlin.
1 in 45	1 in 45	1 in 19½	1 in 26½ (a)

Ages to which half the born live.

Pais De Vaud.	Country Parish in Brandenburg.	Vienna.	Berlin.
41	25½	2	2¾

Proportion of Inhabitants (b) who reach 80 Years of Age.

Pais De Vaud.	Country Parish in Brandenburg.	Vienna	Berlin.
1 in 21½	1 in 22½	1 in 41	1 in 37

The

The numbers *born* at BERLIN, during the 4 years abovementioned, were, *males*, 9219; *females*, 8743; or 21 to 20.

The numbers that died under 2 years of age, were, *males*, 3118; *females*, 2623; or 7 to 6.

The numbers that died upwards of 80 years of age, were, *males*, 135; *females*, 215; or 5 to 8.

The numbers that died between 91 and 105, were, *males*, 21; *females*, 55.

(a) See p. 295 Vol. I. This proportion, were there either no increase, or but a slow increase at BERLIN, would probably be found to be much the same with that in VIENNA and LONDON.

(b) It should be recollected here, that a considerable part of those who die turned of 80 years of age in great towns,

## The Probabilities of living one Year in

Odds.	Pais De Vaud.	Country Parish in Brandenburg.	Vienna.	Berlin.
At birth	$4\frac{1}{2}$ to 1	$3\frac{1}{2}$ to 1	$1\frac{1}{5}$ to 1	$1\frac{1}{4}$ to 1
Age 12	160 to 1	112 to 1	84 to 1	123 to 1
25	117 to 1	110 to 1	66 to 1	50 to 1
30	111 to 1	107 to 1	56 to 1	44 to 1
40	83 to 1	78 to 1	36 to 1	32 to 1
50	49 to 1	50 to 1	27 to 1	30 to 1
60	23 to 1	25 to 1	19 to 1	18 to 1

## EXPECTATIONS of Life.

	Pais De Vaud.	Country Parish in Brandenburg.	Vienna.	Berlin.
At birth	37 yrs	$32\frac{1}{2}$ years	$16\frac{1}{2}$ yrs	18 yrs
Age 12	$44\frac{1}{5}$	44	$35\frac{3}{4}$	$35\frac{1}{2}$
25	$34\frac{1}{4}$	$35\frac{1}{2}$	$28\frac{1}{3}$	$27\frac{1}{3}$
30	$31\frac{1}{4}$	$31\frac{1}{2}$	$25\frac{1}{2}$	$25\frac{1}{4}$
35	$27\frac{1}{2}$	28	$22\frac{1}{2}$	$22\frac{3}{4}$
40	24	25	$20\frac{1}{2}$	$20\frac{3}{4}$
45	$20\frac{1}{2}$	$21\frac{1}{2}$	$17\frac{1}{4}$	$18\frac{3}{4}$
50	$17\frac{1}{2}$	18	16	$16\frac{1}{3}$
55	$14\frac{1}{2}$	15	$13\frac{1}{2}$	14
60	12	$12\frac{1}{4}$	$11\frac{3}{4}$	$12\frac{1}{2}$

towns, are *emigrants* from the country, who came to them in full maturity, after escaping the weakness of infancy. And that also in general these *emigrants* consist of the more hearty and robust part of the kingdom. On both these accounts the number of inhabitants (including *aliens* as well as *natives*) attaining old age in great towns ought to be much greater than in the country. In *London*, *Vienna*, and *Berlin*, it ought to be nearly *double*; but we see, that, in reality, it is scarcely *half*. There are no observations from which the proportion of *natives* in great towns, who live to 80, can be deduced with correctness, except those made at *Stockholm*; and these prove, that of females one in a 100, and of males one in 300, live to 80.—See Vol. I. p. 273; and this Volume, p. 13; and Table XLIV, p. 158.

From

From this comparison (*a*) it appears with how much truth great cities have been called the *graves* of mankind. It must also convince all who will consider it, that, according to the observation at the end of the Fourth Essay in the former Volume, it is by no means strictly proper to consider our diseases as the original intention of nature. They are, without doubt, in general, our own creation. Were there a country, where the inhabitants led lives entirely natural and virtuous, few of them would die without measuring out the whole period of present existence allotted them; pain and distempers would be unknown among them; and death would come upon them like a sleep, in consequence of no other cause than gradual and unavoidable decay.—Let us then, instead of charging our Maker with our miseries, learn more to accuse and reproach *ourselves*.

The reasons of the baleful influence of great towns, as it has been now exhibited, are plainly,

(*a*) A more distinct and striking comparison of this kind may be drawn from the Tables for *London* and the parish of *Holy-Cross*; and from the Tables for *Stockholm* and *Sweden* at large in the preceding collection of Tables. See the Introduction to these Tables.

First,

First, The irregular modes of life, the luxuries, debaucheries, and pernicious customs, which prevail more in towns than in the country.

Secondly, The foulness of the air in towns, occasioned by uncleanness, smog, the perspiration and breath of the inhabitants, and putrid steams from drains, church-yards, kennels, and common-sewers.—It is, in particular, well known that air, spoiled by breathing, is rendered so noxious, as to kill, instantaneously, any animal that is put into it. There must be causes in nature (a) continually operating, which restore the air after being thus spoiled. But in towns it is, probably, consumed faster than it can be adequately restored; and the larger the town is, or the more the inhabitants are crowded together, the more this inconvenience must take place.

(a) A celebrated and excellent philosopher has for some time been employed in enquiring into these causes; and, among other curious and important facts, he has discovered, that one of these causes is the vegetation of plants, and the action of light upon them. See the Fourth and Fifth Volumes of Dr. *Priestley's* Experiments on Air; and an Oration on presenting him with a prize-médal, delivered by Sir *John Pringle*, President of the Royal Society.—See, likewise, *Experiments on Vegetables, discovering their Power of purifying common Air in Sunshine, &c.* by Dr. *Ingenbousz*, Counsellor of the Court, and Body Physician to their Imperial and Royal Majesties, F. R. S. &c.

But



But I must proceed to some more of Mr. Muret's observations. — At the end of the Fourth Essay in the former Volume, &c. I have given an account of several facts which prove the probabilities of life to be higher among females than males. Agreeably to this it appears, that in the district of VAUD, half the *females* don't die till the age of 46 and upwards, though half the *males* die under 36. This great difference is in some measure owing to the military and commercial emigrations among the males; but it appears undeniably, that their greater mortality contributes likewise to it. The number of *males* who died, for a course of years, in 39 parishes of this district, was 8170; of *females* 8167; of whom the numbers that died under one year of age were 1817 *males*, and 1305 *females*; and under 10 years of age, 3099 *males*, and 2598 *females*. In the *beginning* of life, therefore, and before any emigrations can take place, the rate of mortality among *males* appears to be much greater than among *females*: And this is rendered yet more certain, by the account Mr. Muret gives of the proportion of the deaths among males and females in the *first* year of life at VEVEY. In this town, he acquaints us, that for 20 years ending in 1764, there died in the first month, of *males* 135, to 89 *females*; and, in the first year, 225 to 162. — To the same effect it appears

appears, from a Table given by *Susmilch* (a), that in BERLIN 203 *males* die in the first month, and but 168 *females*; and in the first year, 489 to 395; and also, from a Table of *Struyck's*, that in HOLLAND, 396 *males* die in the first year, to 306 *females*.—What is most of all remarkable is, that these accounts shew, that both at VEVEY and BERLIN the *still-born males* are to the *still born females*, as 30 to 21, or nearly in the proportion given by the accounts referred to in Vol. I. p. 364.

The whole number of inhabitants at VEVEY in 1764, was 3350. Of these 1931 were females, and only 1419 males. Sixty-six were *widowers*, and 200 *widows*. The number of *bachelors*, above 16 years of age, was 529; and of *virgins*, above 14 years of age, 734. See Mr. *Muret's* Tables, p. 124.

Mr. *De Parcieux* at PARIS, and Mr. *Wargentin* in SWEDEN, have observed, that not only *women* live longer than *men*, but that *married* women live longer than *single* women. The registers examined by Mr. *Muret* confirm this; and it appears in some of them, that, of equal numbers of *single* and *married* women between 15 and 25, more of the former died than of the latter, in the proportion of 2 to 1. This is a difference so great, that it must, I suppose, have been in some degree accidental. The fact, how-

(a) See *Susmilch's* *Gottliche Ordnung*, Vol. II. p. 317, &c.

ever, in general, when understood with abatements for that part of female life which is most exposed to the dangers of child-bearing, is highly probable; for first, the women who marry are likely to be a select body, consisting of the more healthy and vigorous part of the sex. And secondly, it is reasonable to expect that in this, as well as in all other instances, the consequences of following nature must be favourable.

The facts recited here, and at the end of the Fourth Essay, prove (a), that there is a difference between the mortality of males and females.—I must however observe, that it may be doubted, whether this difference, so unfavourable to males, is *natural*; and the following facts will prove, that I have reason for such a doubt.

It appears, from several registers in *Susmilch's* works, that this difference is much less in the *country parishes* and *villages* of BRANDENBURG, than in the *towns*: And, agreeably to this, it appears likewise, from the accounts of the same curious writer, that the number of males in the country comes much nearer to the number of females.

In 1056 small *villages* in BRANDENBURG, the *males* and *females*, in 1748, were 106,234,

(a) This is put out of all doubt in the present Edition of this work, by the Tables in the preceding collection, deduced from the *Chester* and *Sweden* observations.

and

and 107,540, or to one another as 100 to 101 $\frac{1}{5}$ . In twenty small towns they were 9544, and 10,333; or as 100 to 108 $\frac{1}{4}$ . In BERLIN they were, exclusive of the garrison, 39,116 and 45,938: or as 100 to 117 $\frac{1}{2}$ .

At the time the accounts, mentioned in p. 276, Vol. I. were taken of the inhabitants in the province of NEW-JERSEY in AMERICA, they were distinguished particularly into *males* and *females* under and above 16.

In 1738, the number of  
Males under 16 was, 10639. Females 9700  
Males above 16 ——— 11631. Females 10725

In 1745, these numbers were,  
Males under 16 ——— 14523. Females 13754  
Males above 16 ——— 15087. Females 13704

The inference from these facts is very obvious. They seem to shew sufficiently, that human life in males is more brittle than in females, only in consequence of adventitious causes, or of some particular debility, that takes place in polished and luxurious societies, and especially in great towns (*a*).

(*a*) See on this subject the remark at the end of Table XLIV. p. 161.

It will not be amiss to insert here the following accounts of the mortality of *summer* compared with that of *winter*, that is of the four months, *June, July, August,* and *September*, compared with *December, January, February,* and *March*.

The deaths for 60 years at VEVEY in the former months, were to the deaths in the latter, as 2140 to 1697;  
or



or 5 to 4. (See Mr. Murel's Tables, p. 100). In LONDON and at PARIS, this proportion is nearly the same. At EDINBURGH, as 4 to 3. In 25 country towns and parishes mentioned by Dr. Short (*New Observations*, p. 142) as 50 to 41.—The sick admitted into the *Hôtel Dieu* at Paris, for 40 years, from 1724 to 1763, were, in the former months, 314,824; in the latter, 238,522, or as 4 to 3. See *Recherches sur la Population*, &c. par M. Meffance, p. 181.—It is remarkable that the births also in winter to those in summer, are, at VEVEY, as 5 to 4; in LONDON, as 8 to 7; in the country towns and parishes just mentioned as 7 to 6.

*Annual average of births and deaths in all SWEDEN for 13 years.—See the Memoirs of the Royal Academy of Sciences at Stockholm, published at Paris, 1772, p. 20, &c.*

	Births	Deaths
In the four summer months —	28080	18880
In the four winter months —	31327	20690
In April and May —	14078	12274
In October and November —	17178	8612

*Annual average of births and deaths in STOCKHOLM for five years. Ibid.*

	Births	Deaths
Summer ——— —	933	1515
Winter ——— —	870	1139
April and May ———	426	739
October and November ———	469	645

*Whole number of births and deaths at Gainborough for 20 years ended at 1771.*

	Births	Deaths
Summer ——— —	779	590
Winter ——— —	811	765
April and May ———	427	390
October and November ———	410	345

*Whole*

*Whole number of deaths at WARRINGTON in Lancashire,  
for eight years ended at 1780.*

		Deaths
Summer	— — —	692
Winter	— — —	968
<i>April and May</i>	— — —	508
<i>October and November</i>	— — —	280

*Whole number of deaths at MANCHESTER for nine years  
ended at 1780.*

		Births	Deaths
Summer	— — —	3308	1788
Winter	— — —	3608	2427
<i>April and May</i>	— — —	1956	1098
<i>October and November</i>	— — —	1736	1022

*Whole number of deaths at ECCLES near MANCHESTER,  
for five years ended at 1779.*

		Births	Deaths
Summer	— — —	440	415
Winter	— — —	521	455
<i>April and May</i>	— — —	314	226
<i>October and November</i>	— — —	212	234

The deaths at CHESTER, for the years 1772, 1773, and 1774, were, in summer 340; in winter, 478; in *April and May*, 185; in *October and November*, 274. And they were more numerous in *Autumn* than *Spring*, only because in one of these years the small pox carried off 90 persons in *October and November*.

*Of POPULATION; the general Causes which promote or obstruct it; and the present State of it in ENGLAND compared with its State formerly.*

FROM the proportion of the births to the deaths in the district of VAUD, as mentioned in p. 254, it follows, by the rule in the Note, Vol. I. p. 278, that the inhabitants ought to double their own number in 120 years. But the fact is, that so many migrate into foreign armies and with commercial views, that their increase is scarcely sensible. Mr. Muret, after observing this, enters into a general account of the causes which obstruct population in his country. Among these he insists particularly on LUXURY and the ENGROSSING OF FARMS. I wish his observations on these subjects were not applicable to the present state of this kingdom: But, perhaps, there is no kingdom in the world to which they are *so* applicable.—In consequence of the easy communication, lately created, between the different parts of the kingdom, the LONDON fashions and manners and pleasures, have been propagated every where; and almost every distant town and village now vies with the capital in all kinds of expensive dissipation

and amusement. This enervates and debilitates; and, together with our taxes, raises every where (a) the price of the means of subsistence, checks marriage, and brings on poverty, dependance, and venality.—With respect, particularly, to the custom of *engrossing farms*, Mr. Muret observes, with the highest reason, that a large tract of land, in the hands of one man, does not yield so great a return, as when in the hands of several, nor does it employ so many people; and, as a proof of this, he mentions two parishes in the district of VAUD, one of which (once a little village) having been bought by some rich men, was sunk into a single *demesne*; and the other (once a single *demesne*), having fallen into the hands of some peasants, was become a little village.—How many facts of the former kind can this country now furnish?—And there is reason to apprehend they will go on increasing.—The custom of engrossing farms eases *landlords* of the trouble attending the necessities of little tenants and the repairs of cottages.—A great farmer, by having it more in his power to

(a) The price of corn, in particular, has for some time been complained of by the poor as oppressively high, though far from being so high as it generally was at the end of the last century. This is a striking fact which implies that the *lower* part of the nation are now more distressed than ever. The consequence has been a reduction of their number; and this is the effect that must go on increasing, with increasing luxury and taxes.

speculate



speculate and command the markets, and by drawing to himself the profits which would have supported several farmers, is capable, with less culture, of paying a higher rent. Our superiors, therefore, find their account in this evil — But it is, indeed, erecting *private* benefit on *public* calamity; and, for the sake of a temporary advantage, giving up the nation to depopulation and distress. — We have, for many years, been feeling the truth of this observation (a).

Dr. *Davenant* (the best, while not venal, of all political writers), tells us, that at *Michaelmas*, in the year 1685, it appeared

(a) “ Those who contribute towards the destruction of small farms” (says a gentleman of great knowledge and experience in this way) “ can have very little reflexion. If they have, their feelings are not to be envied. Where this has been the practice, we see a vast number of families reduced to poverty and misery, the poor rates much increased, the small articles of provision greatly diminished in quantity and number, and consequently augmented in price.” — See *Hints to Gentlemen of Landed Property*, printed for Mr. *Dodley* in 1776, p. 223, &c. &c.; where the pernicious tendency of large farms seems abundantly proved. There are thousands of parishes, he says, which, since little farms have been swallowed up in greater, do not support so many cows as they did by 50 or 60 in a parish; and the inhabitants have decreased in proportion. — He concludes his observations on this subject with expressing “ his anxious wishes that the destructive practice of engrossing farms may be carried no farther, the stab already given by it to plenty and population having greatly affected the prosperity of this country.”

by a survey of the hearth-books (*a*), that the number of houses in all ENGLAND and WALES was 1,300,000, of which 554,631 were houses of only one chimney. See Dr. *Davenant's Works*, Vol. II. p. 203.—In his *Essay on Ways and Means*, &c. Vol. I. p. 33, he gives a particular account of the number of houses in every county, according to the *hearth-books* of Lady-day, 1690; and the sum total then was 1,319,215.—At the *restoration* it appeared by the same hearth-books, that the number of houses in the kingdom (*b*), was 1,230,000.—In the interval, therefore, between the *restoration* and the *revolution*; the people of ENGLAND had increased above 300,000; and “of SMALLER  
“ TENEMENTS, Dr. *Davenant* observes (*c*),  
“ there had been, from 1666 to 1688, about  
“ 70,000 new foundations laid.”—But what a reverse has taken place since?—In 1759 the number of houses in ENGLAND and WALES was 986,482; of which not more than 330,000 were houses having less than

(*a*) At this time there was a tax of two shillings on every *fire-hearth*; which was taken off at the REVOLUTION, because reckoned “not only a great oppression to  
“ the poorer sort, but a badge of slavery on the whole  
“ people, exposing every man’s house to be entered into  
“ and searched at pleasure by persons unknown to him.”  
*Preamble to the Act for taking away the revenue arising by  
hearth-money.* 1 William and Mary, Chap. 10.

(*b*) Continuation of *Rapin*, Vol. I. p. 53.

(*c*) Dr. *Davenant's Works*, Vol. I. p. 370.

seven windows ; and 282,429 were *cottages* not charged on account of poverty.—In 1765, notwithstanding the increase of buildings in LONDON, the number of houses was reduced to 980,692 (a) ; of which 276,149 were *cottages* not charged. According to these accounts then, our people have, since the year 1690, decreased near a *million and a half*.—And the waste has fallen principally on the inhabitants of cottages ; nor indeed could it fall any where more unhappily ; for, from cottages our navies and armies are supplied, and the lower people are the chief strength and security of every state (b).—

What

(a) See *Considerations on the Trade and Finances of this Kingdom*, p. 95, 97, 98, printed for Wilkie, 1766. See also Vol. I. p. 246, &c. of this Treatise ; and my *Appeal to the Public on the Subject of the National Debt*, p. 86, &c.—It deserves particular notice, with respect to the accounts here given of the number of houses in 1759 and 1765, that, being returns made by the surveyors of the house and window-duties throughout all ENGLAND and WALES, they are subject to no such deficiencies as those in the account of the number of houses in LONDON, taken by Mr. *Maitland* from the *parish* books, and mentioned in the Note, Vol. I. p. 246.—The reason is, that no landlord or tenant can ever consent that any *two or more houses* belonging to him, should be charged by the assessors of the window-tax as *single* houses ; because, in this case, he would be taxed too high, and pay more than the law required.—The number of houses, therefore, subject to the house and window-duty, given in the above returns, must probably be the full number of such houses in the kingdom.

(b) Cottagers are indisputably the most beneficial race of people we have : “ They are bred up in greater sim-



What renders this calamity more alarming is, that the inhabitants of the cottages thrown down in the country, fly to LONDON and other towns, there to be corrupted and perish (*a*).—I know I shall be here told that

"plicity, live more primitive lives, more free from vice  
 "and debauchery, than any other set of men of the lower  
 "class; and are best formed and enabled to sustain the  
 "hardships of war, and other laborious services. Great  
 "towns are destructive both to morals and health, and  
 "the greatest drains we have; for where many of the  
 "lower sort of people crowd together, as in *London*,  
 "*Norwich*, *Birmingham*, and other manufacturing towns,  
 "they are obliged to put up with bad accommodations,  
 "and an unwholesome and confined air, which breeds  
 "contagious distempers, debilitates their bodies, and  
 "shortens their lives.—Since, therefore, it is appar-  
 "ent that all such towns must cause a diminution or  
 "waste of people, we cannot be at a loss to trace the  
 "spring which feeds these channels. The country must  
 "be the place; and cottages and small farms the chief  
 "nurseries which support population."—*Hints to landed*  
*Gentlemen*, p. 243, 244.—In what follows a representation is made of the misery of cottagers in their present state, and proposals offered for better accommodating and encouraging them, which do honour to Mr. *Kent's* public spirit and humanity.

(*a*) Dr. *Davenant* says, from Mr. *King's* Observations, "that the supply of LONDON alone takes up above half the neat increase of the kingdom."—Is it then to be wondered at, that the supply of the waste in all the towns of the kingdom, added to that increase of luxury and taxes, and of the drain to our *armies*, and *navies*, and *foreign settlements*, which has taken place within these 70 years, should have so far exceeded the increase of the kingdom, as to produce the depopulation I have mentioned?—It has been asserted by political calculators, that no population can bear more than one soldier for every hundred souls.



that the *Revenue* thrives. But this is not a circumstance from which any encouragement can be drawn. It thrives, by a cause that is likely in time to destroy both itself and the kingdom; I mean, by an increase of luxury (*a*), producing such an increase of *consumption* and *importation* as secretly *accelerates* ruin, while *at present* (as far as the *Revenue* is concerned) it overbalances the effects of depopulation.—What remedies can be applied in such circumstances? —The answer is obvious.

fouls. This is saying a great deal too much; but were it true, the number of our soldiers and sailors, even in *peace*, would alone be sufficient to reduce us to nothing in a little time.

A flourishing commerce, though favourable to population in some respects, is, I think, on the whole, extremely unfavourable; and, while it flatters, may be destroying: particularly, by increasing luxury, the worst enemy of population, as well as of public virtue; and, by calling off too many persons from agriculture to unhealthy trades and the sea-service.—Suppose 100,000 soldiers and sailors, added to other burdens, to have been formerly the whole number the nation could bear without decreasing. In such circumstances, it is plain, that any causes which doubled or tripled that number, would depopulate with rapidity.

(*a*) For example. In LONDON, those who used to satisfy themselves with *one* house, or perhaps *half* a house, must now have *two* houses. Those who used to live plain, must now live high; and those who used to *walk*, must now be *carried*. This is the reason of the increase of consumption and of buildings in LONDON, and not an increase of the inhabitants, for the number of inhabitants is certainly (if any regard is due to the bills) less now than it was fifty years ago.

Enter immediately into a decisive enquiry into the state of population in the kingdom.—Promote agriculture.—Drive back the inhabitants of towns into the country.—Establish some regulations for preserving the lives of infants.—Discourage luxury, and celibacy, and the engrossing of farms.—Let there be entire liberty ; and maintain public peace by a government founded, not in *constraint*, but in the *respect* and the *hearts* of the people.—But above all things, if it be not now too late ; “ find out means of avoiding the miseries of an impending bankruptcy, and of easing the nation of that burden of debts and taxes under which it is sinking.”

I will here enter a little more minutely into the consideration of some of the heads now mentioned, and of the present compared with the former state of the body of the people in this kingdom.

One of the most obvious divisions of the state of mankind is, into the *wild* and the *civilized* state. In the former, man is a creature rude, ignorant, and savage ; running about in the woods ; and living by hunting, or on the spontaneous productions of the earth. In this state, the means of subsistence being scarce, and a large quantity of ground necessary to support a few, there can never be any considerable increase.—In the latter state,

state, man is a creature fixed on one spot, employing himself in cultivating the ground, and enjoying the advantages of science, arts, and civil government. Of this last state there are many different degrees or stages, from the most simple to the most refined and luxurious. The first or the simple stages of civilization, are those which favour most the increase and the happiness of mankind: For in these states, agriculture supplies plenty of the means of subsistence; the blessings of a natural and simple life are enjoyed; property is equally divided; the wants of men are few, and soon satisfied; and families are easily provided for.—On the contrary. In the refined states of civilization property is engrossed, and the natural equality of men subverted; artificial necessities without number are created; great towns propagate contagion and licentiousness; luxury and vice prevail; and, together with them, disease, poverty, venality, and oppression. And there is a limit at which, when the corruptions of civil society arrive, all liberty, virtue, and happiness must be lost, and complete ruin follow.—Our *American* colonies are at present, for the most part, in the first and the happiest of the states I have described; and they afford a very striking proof of the effects of the different stages of civilization on population. In the inland parts of NORTH-AMERICA, or the back settlements, where  
the



the modes of living are most simple, and almost every one occupies land for himself; there is an increase so rapid as to have hardly any parallel. Along the sea-coast, where trade has begun to introduce refinement and luxury, the inhabitants increase more slowly: And in the maritime towns (if I may judge from the bills of mortality at BOSTON, mentioned in p. 268, Vol. I.) they do not increase at all (a).

But to confine my thoughts to my own country.—Here, it is too evident that we are far advanced into that last and worst state of society, in which false refinement and luxury multiply wants, and debauch, enslave, and depopulate.—Among the evils of this state, and the causes of depopulation, I have mentioned the accumulation of property.

“Only revive, says Mr. *Susmilch*, the laws of *Licinius*, forbidding any Roman to hold more than seven *jugera* of land; or that of *Romulus*, which limited every Roman to two *jugera*, and you will soon convert a barren desert into a busy and crowded hive.”—The doubts of some ingenious men on this subject, have, indeed, greatly surprized me. I can scarcely think

(a) Along the sea-coast they double their own number in about 35 years; but in the back-settlements, in 15 years. See *Essay I.* Vol. I. p. 276; and p. 109 of *A Discourse on Christian Union*, by Dr. STYLES, now the worthy President of the College of YALE in CONNECTICUT.

of



of a more evident maxim, than that “the division of property promotes population.”—Let a tract of ground be supposed in the hands of a multitude of little proprietors and tenants, who maintain themselves and families by the produce of the ground they occupy, by sheep kept on a common, by poultry, hogs, &c.; and who, therefore, have little occasion to purchase any of the means of subsistence. If this land gets into the hands of a few great farmers, the consequence must be, that the little farmers will be converted into a body of men who earn their subsistence by working for others, and who will be under a necessity of going to market for all they want (*a*). And, subsistence in this way being difficult, families of children will become burdens, marriage will be avoided, and population will decline.—

(*a*) “Every speculative Englishman,” says Mr. *Kent*, “who travels through the *Austrian Netherlands*, is astonished at the great population of that country, and at the sight of the markets, which are plentiful beyond description. Upon enquiring into the internal state and regulation of the country, he finds that there are no large farms, no class of men who pass under the character of gentlemen farmers, acquiring large fortunes merely by superintending the business of farming; but that the whole country is divided into much smaller portions than land is with us, and occupied by a set of laborious people, who in general work for themselves, and live very much on a footing of equality.”—See *Hints to Gentlemen of Landed Property*, p. 217.

At

At the same time there will, perhaps, be more labour, because there will be more compulsion to it. More bread will be consumed, and, therefore, more corn grown; because there will be less ability of going to the price of other food. Parishes, likewise, will be more loaded, because the number of poor will be greater. And towns and manufactures will increase, because more will be driven to them in quest of places and employments.—This is the way in which the engrossing of farms naturally operates: And this is the way in which, for many years, it has been actually operating in this kingdom.

It deserves particular notice, that the observations now suggested shew, that the very causes which produce depopulation among us, may, for some time, promote tillage; and I will take this opportunity to add, that they will also account for the following fact.—In the year 1697, wheat was at 3 $\frac{1}{2}$  a quarter, and other grain proportionably dear. But there was no clamour, and the exportation went on. See a valuable and useful pamphlet, entitled, *Three Tracts on the Corn Trade*, page 100, 107, 145. At present, though the quantity of money (or of what passes for money) is doubled, when wheat is below this price, and in general before any grain, except oats, gets above  
the

the prices at which the law used to allow a bounty on exportation, there is an alarm, the poor are starving, and the exportation is prohibited. I referred to this fact in the Note, p. 274; and the true reason of it seems to be, that the high price of bread was not, at the time I have mentioned, of essential consequence to the lower people, because they could live more upon other food which was then cheap; and because also being more generally occupiers of land, they were less under a necessity of purchasing bread. Whereas now, being forced by greater difficulties, and the high price of all other food, to live principally or solely on bread, if that is not cheap, they are rendered incapable of maintaining themselves.

In confirmation of this account, I will beg leave to mention, that though during the whole last century, corn (wheat, rye, oats, and barley) was generally dearer than it has been, at an average, for 40 years to 1773; yet flesh-meat was about half its present price: And that, in an *Act of Parliament* of the 25th of Henry VIII. beef, veal, pork, and mutton are mentioned as the food of the poor, and their price limited to about a halfpenny a pound. Beef and pork, in particular, were sold in LONDON at two pounds and a half, and three pounds for a penny; at the same time that wheat  
was

was at 7*s.* and 8*s.* a quarter (*a*), and bore the same proportion to the price of flesh as it would bear now, were it at about 4*l.* a quarter. See *Chronicon Pretiosum*, p. 116.—

It

(*a*) Even so far back as the year 1463, the price of wheat was reckoned not too high at 6*s.* 8*d.* per quarter; nor that of barley at 3*s.* and rye at 4*s.*; for it was in that year enacted, that the *importation* of these three sorts of grain should not be allowed till they got above these prices. See Mr. *Anderson's Chronological Deduction of Commerce*, Vol. I. p. 280.

By a statute of 1 *Philip and Mary*, 1553, leave was given to *export* these three kinds of grain till they rose to these prices. *Ib.* p. 387.

By an ordinance in 1563, the exportation prices were fixed to 10*s.* per quarter for *wheat*; 8*s.* for *rye*, *pease*, and *beans*; and 6*s.* 8*d.* for *malt*.—And in 1593, to 1*l.* for *wheat*; 13*s.* 4*d.* *pease* and *beans*; and 12*s.* *barley* and *malt*. *Ib.* p. 401 and 442.

#### PRICES per QUARTER.

	Of Wheat.			Of Malt.			Of Oats.		
	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
In 1491,	0	14	8	0	0	0	0	00	0
1494,	0	4	0	0	0	0	0	00	0
1504,	0	5	8	0	0	0	0	00	0
1512,	0	6	2	0	4	0	0	2	0
1521,	1	0	0	0	0	0	0	00	0
From 1553 to—	1556,	0	8	0	5	0	0	00	0
Before harvest, in	1557,	2	13	4	2	4	0	00	0
After harvest, in	1557,	0	8	0	5	0	0	10	0
	1560,	0	8	0	5	0	0	5	0
Before harvest, in	1574,	2	16	0	0	0	0	00	0
After harvest, in	1574,	1	4	0	0	0	0	00	0
	1587,	3	4	0	0	0	0	00	0

A dearth



It appears, indeed, that our ancestors took great care to keep the price of flesh low for the poor; and this was one of the reasons of the many proclamations published by Queen Elizabeth,

	Of Wheat.			Of Malt.			Of Oats.		
	l.	s.	d.	l.	s.	d.	l.	s.	d.
A dearth occasion-	1594,	2	16	0	0	0	0	0	0
ed by excessive	1595,	2	13	4	1	0	0	0	0
exportation; &	1596,	4	0	0	1	6	8	0	0
in 1596 by great	1597,	5	4	0	2	6	4	0	0
rains									

AVERAGE PRICE,

From 1606 to —	1706,	1	18	6	1	2	0	0	0
From 1707 to —	1765,	1	12	6	1	1	9	0	0
From 1766 to —	1772,	2	3	6	0	0	0	1	9

See Bp. Fleetwood's *Chronicon Pretiosum*, from p. 113 to p. 124; and *Three Tracts on the Corn Trade*, p. 98, &c.

With these prices of corn let us compare the prices of flesh, at two or three different periods.

In 1512, the price of wheat was from 5s. 8d. to 6s. 8d. in Yorkshire. See the *Regulations and Establishment of the Household of Henry Algernon Percy, the fifth Earl of Northumberland, at his Castles of Wressill and Lekingfield, in Yorkshire*, begun Anno Dom. 1512, page 2, 4. Let us call the mean price 6s. 2d. The price of malt was 4s. and of oats 2s. We may therefore reckon, that the nominal price of grain at this time was about a seventh of its nominal price for the last 40 years.

The price of a fat ox at the same time, and in the same county, was 13s. 4d.; of a lean ox, 8s.; of a weather, 1s. 8d.; of a calf, 1s. 8d.; of a hog, 2s. lb. p. 5, 6, 7.—The nominal price of meat, therefore, was no more than about a 15th of its present price, and bore the same proportion to the price of corn that it would now bear, were it at half its present price.—A like inference may be drawn from comparing the following prices:

Wheat, in 1549, was about 12s. per quarter in LONDON. Malt, 10s. Barley, 9s. Rye, 6s. 6d. Oats, 4s.  
—A

*Elizabeth, James I. and Charles I.* against eating flesh in Lent and on fish days; and against the erection of new buildings in *London*, and the residence in it of the nobility and gentry.

—A middling ox, 1*l.* 18*s.* A weather, 3*s.* Butter, three farthings and a penny a pound. Cheese, a halfpenny a pound. See *Maitland's History of London*, page 143, 144.

“In 1574, there was a great dearth, and wheat was before harvest, at 2*l.* 16*s.* per quarter; and beef at *Lammas* so dear, as to be sold at twopence-halfpenny a pound.” See *Chronicon Pretiosum*, p. 123. That is, beef compared with wheat, was at least one half cheaper than it is now.

In 1445, wheat was at 4*s.* 6*d.* per quarter. In 1447, at 8*s.* In 1448, at 6*s.* 8*d.* In 1449, 5*s.*—A bullock, in 1445, 5*s.* A sheep, 2*s.* 5*d.* $\frac{1}{2}$ . A hog, 1*s.* 11*d.* $\frac{1}{2}$ .—Fine cloth for surplices, in 1446, 8*d.* per ell. Cloathing for a year, at the same period, of a common servant of husbandry, 3*s.* 4*d.* Of a chief carter and shepherd, 4*s.* Of a bailiff of husbandry, 5*s.* Ib. page 108, 109, 160.—Cloathing, therefore, at this time, seems to have been cheaper in comparison of the price of corn than even flesh.

The weight of silver coin *formerly*, to the weight of silver coin of the same denomination *now*, was from 1461 to 1509, as 62 to 37 $\frac{1}{2}$ . From 1509 to 1543, as 62 to 45. From 1552 to 1600, as 62 to 60. And from 1600 to the present time as 62 to 62. But nothing depends on this in the present enquiry; the object of which is, not the proportion of the prices of the different articles of subsistence *now* to their prices *formerly*, but the proportion TO ONE ANOTHER of their prices *now*, in comparison with the same proportion *formerly*. And this may be as well deduced from the *nominal* as from the *absolute* prices.—Thus. The price of bread now is nearly the same that it was 100 years ago; but, in *comparison* with the price of beef and mutton, it is at least one half cheaper.

The

The reason now assigned accounts farther for the great variations in the price of grain which used to take place formerly. These were such as could not be now endured; but, bread being then less a necessary article of subsistence, they were less felt and regarded.

I have taken for granted, in those observations, that the quantity of ground brought under tillage in this kingdom is now more than ever it was. This is generally believed; and, if true, the causes of it have been those I have mentioned, in conjunction with the encouragement given to the growth of corn by the bounty on exportation, and the increase of luxury occasioning an increase of horses, and rendering even the poor averse to all bread except that made of the (a) finest flour. But, perhaps, the fact may not be so certain as some think it. At least, there is reason to apprehend, that whatever the increase of tillage might have been for 50 or 60 years after the Revolution, it is now at an end.—I have lately received an account of a large common field in *Leicestershire*, which used to produce annually 800 quarters of corn, besides maintaining 200 cattle; but

(a) Bread made of *bran*, and even of *pease* and *beans*, was formerly not uncommon among the lower people. But no distresses could force them now to eat such bread, or even to live upon *rice*, though the food of a considerable part of the rest of mankind. See the *Earl of Northumberland's Household Book*, Preface, p. 13, &c.



which now, in consequence of being inclosed and getting into few hands, produces little or no corn; and maintains no more cattle than before, though the rents are considerably advanced.—This is only one instance among many of an evil that has been prevailing for some time, and which is the general effect of the laws for inclosing open fields.—In *Northamptonshire* and *Leicestershire*, inclosing has greatly prevailed; and most of the new-inclosed lordships, says a very sensible writer, “are turned into pasturage; in consequence of which, many lordships have not now 50 acres ploughed yearly, in which 1500, or at least 1000 were ploughed formerly; and scarce an ear of corn is now to be seen in some that bore hundreds of quarters.—And so severely are the effects of this felt, that worse wheat has been lately sold in these counties on an average, at 7s. and 7s. 6d. the *Winchester* bushel, for many months together, than used to be sold at 3s. 6d. and 4s. And 5s and 5s. 6d. has been given for malt that has been usually bought there at little more than half-a-crown.” See a pamphlet, entitled, *An Enquiry into the Reasons for and against inclosing Open Fields*, by the Rev. Mr. Addington. Published in 1772 for Mr. Buckland, Paternoster Row.—In the counties of *Northampton* and *Leicester*, says the same writer, p. 43,  
“the



“ the decrease of the inhabitants in almost  
 “ all the inclosed villages in which they  
 “ have no considerable manufacture, is ob-  
 “ vious to be remarked by every one who  
 “ knew their state 20 or 30 years ago, and  
 “ sees them now ; and that to a degree that  
 “ cannot but give every true friend to his  
 “ country the most sensible concern. The  
 “ ruin of former dwelling-houses, barns, sta-  
 “ bles, &c. shew every one who passes  
 “ through them that they were once better  
 “ inhabited. A hundred houses and families  
 “ have in some places, dwindled into eight  
 “ or ten.—The landholders, in most pa-  
 “ rishes that have been inclosed only 15  
 “ or 20 years, are very few in comparison  
 “ of the numbers who occupied them in  
 “ their open field state. It is no uncommon  
 “ thing to see four or five wealthy graziers  
 “ engrossing a large inclosed lordship, which  
 “ was before in the hands of 20 or 30  
 “ farmers, and as many smaller tenants or  
 “ proprietors. All these are hereby thrown  
 “ out of their livings with their families,  
 “ and many other families which were em-  
 “ ployed and supported by them.” *Ib.* p. 37.  
 See an account of *Norfolk*, in some respects  
 similar to this, in my *Appeal to the Public on*  
*the Subject of the National Debt*, p. 93, &c.  
 I can scarcely think of any thing that should  
 be more alarming than such accounts.—

How astonishing is it that our parliament, instead of applying any remedy to these evils, should chuse to promote them, by passing every year, bills almost without number, for new inclosures? (a)

The device, says Lord *Bacon*, (*Essays, civil and moral*, Sect. 20.) “ of King Henry VII. “ was profound and admirable, in making “ farms and houses of husbandry of a “ standard; that is, maintained with such “ a proportion of land to them, as may “ breed a subject in convenient plenty and “ no servile condition, and to keep the “ plough in the hands of the *owners* and “ not *hirelings*.”—“ Inclosures,” says the same great writer (in his *History of the Reign of Henry the Seventh*), “ began at “ that time (or in 1489) to be more frequent, whereby arable land was turned

(a) I have here in view inclosures of *open fields and lands* already improved. It is acknowledged by even the writers in defence of inclosures, that these diminish tillage, increase the monopolies of farms, raise the prices of provisions, and produce depopulation. Such inclosures, therefore, however gainful they may be at present to a few individuals, are undoubtedly pernicious.—On the contrary. Inclosures of *waste lands and commons* would be useful, if divided into small allotments, and given up to be occupied at moderate rents by the poor. But if, besides lessening the produce of fine wool, they bear hard on the poor by depriving them of a part of their subsistence, and only go towards increasing farms already too large, the advantages attending them may not much exceed the disadvantages.

“ into

“ into pasture, which was easily managed  
 “ by a few herdsmen. This bred a decay  
 “ of people. In remedying this inconve-  
 “ nience, the King’s wisdom and the Par-  
 “ liament’s was admirable. *Inclosures* they  
 “ would not forbid; and *tillage* they would  
 “ not compel; but they took a course to  
 “ take away *depopulating inclosures*, and *de-*  
 “ *populating pasturage* by consequence. The  
 “ ordinance was, that all houses of husban-  
 “ dry, with 20 acres of ground to them,  
 “ should be kept up for ever, together with  
 “ a competent proportion of land to be oc-  
 “ cupied with them, and in no wise to be  
 “ severed from them. By these means, the  
 “ houses being kept up, did, of necessity,  
 “ enforce a dweller; and the proportion of  
 “ land for occupation being also kept up,  
 “ did, of necessity, enforce that dweller not  
 “ to be a beggar (a).” The statute here  
 mentioned was renewed in King *Henry* the  
 Eighth’s time; and every person who con-  
 verted tillage into pasture subjected to a for-  
 feiture of half the land, till the offence was  
 removed. See Mr. *Anderson’s Chronological*  
*Deduction of Commerce*, Vol. I. page 347.  
 —In a law of the 25th of the same reign,  
 it is set forth, “ that many farms, and great  
 “ plenty of cattle, particularly sheep, had  
 “ been gathered into few hands, whereby

(a) See Lord Bacon’s Works, Vol. III. p. 437.



“ *the rents of lands had been increased, and*  
 “ *tillage very much decayed; churches and*  
 “ *towns pulled down; the price of provi-*  
 “ *sions excessively enhanced, and a mar-*  
 “ *vellous number of people rendered inca-*  
 “ *pable of maintaining themselves and fa-*  
 “ *milics; and, therefore, it was enacted,*  
 “ *that no person should keep above 2000*  
 “ *sheep, nor hold more than two farms.*”

*Ib.* p. 363.—In the 3d of *Edw. VI.* a bill was brought in for the benefit of the poor, for rebuilding decayed farm houses, and maintaining tillage *against too much inclosing*. *Parliamentary Hist.* Vol. III. p. 247.—In the year 1638, there was a special commission from *Charles I.* for enforcing the statute of the 30th of *Elizabeth*, by which no cottage was allowed in any country place, without at least four acres of land to it, to prevent the increase of the poor, by securing to them a maintenance; nor were any inmates allowed in any cottage, to secure the full cultivation of the land, by diffusing the people more over it. See *Rymer's Fæd.* 20. 256, and 340.—By an Act in *Cromwell's* time, no new house was to be built within ten miles of *LONDON*, unless there were four acres of land occupied by the tenant. *Parliamentary History*, Vol. XXI.

Such was the policy of former times.—*Modern* policy is, indeed, more favourable to the higher classes of people; and the consequence of it may in time prove, that the whole



whole kingdom will consist of only *gentry* and *beggars*, or of *grandees* and *slaves*.

I cannot conclude this Supplement without adding one farther observation which has struck me on the present subject.—As in former times the number of the occupiers of land was greater, and all had more opportunities of working for *themselves*, it is reasonable to conclude, that the number of people willing to work for *others*, must have been smaller, and the price of day-labour higher. This is now the case in our *American* colonies; and this likewise, upon enquiry, I find to have been the case in this country formerly. —The *nominal* price of day-labour is at present no more than about *four* times, or at most *five* times higher than it was in the year 1514. But the price of corn (*a*) is *seven* times, and of flesh-meat and raiment about *fifteen* times higher. See the Note, p. 286. — So far, therefore, has the price of labour been from advancing in proportion to the increase in the expences of living, that it does not appear that it bears now *half* the proportion to those expences that it did bear formerly (*b*).

Upon

(a) See *Chronicon Pretiosum*, Chap. V. From whence, compared with the account in Chap. IV. of the price of corn and other commodities, for the last 600 years, abundant evidence for what I have here observed, may be collected.

(b) “The balance at present is considerably against the labourer; and yet the landlord and tenant derive

Upon the whole. The circumstances of the lower ranks of men are altered in almost every respect for the worse. From little occupiers of land, they are reduced to the state of *day-labourers* and *hirelings*; and at the same time their subsistence in that state is become more difficult, in consequence of the cause just assigned; and also of *luxury*, which has extended its influence even to them, though starving, and rendered *tea, fine wheaten bread*, and other delicacies, necessary to them, which were formerly unknown among them.—Such a change cannot but draw after it important consequences. They are the lower people chiefly who pay the taxes of a state, fight its battles, carry on its commerce, and maintain its splendor. In every country, the higher ranks are a very small body, compared with them. Even in this country, where their numbers are probably much lessened, they are still more the majority than is commonly imagined; for, from the returns made by the surveyors of the house and window-duties, it appears, that near THREE-FOURTHS of all the houses in the kingdom are houses not having more than *seven* windows.

“ultimately no advantage from hence,—The great increase in the poor rates may be accounted for in a few words. The rise upon land and its produce, is at least 60 *per cent.*; the rise upon labour not above 20 *per cent.* The difference is of course against the working hands; and when their earnings are insufficient for the absolute necessities of life, they must inevitably fall upon the parish.”—*Hints to Gentlemen of Landed Property*, p. 273.

## P O S T S C R I P T,

C O N T A I N I N G

*A Review of the Controversy relating to the State of Population in England and Wales since the Revolution.*

THE observations, in the preceding Supplement, on the population of this kingdom, are the same with those which have been published in the former editions of this work. A more particular account of the evidence which seems to prove a progressive decrease in our population, has been given in an ESSAY on this subject first published at the end of Mr. Morgan's Treatise on the *Doctrine of Annuities and Assurances on Lives and Survivorships*, and since republished with the addition of an *Appendix*, containing remarks on Mr. EDEN's objections in his fifth letter to Lord CARLISLE. These publications have been lately followed by others on the same subject; particularly, Mr. Wales's *Enquiry into the present State of*  
the

*the Population of ENGLAND and WALES; and Mr. Howlett's Examination of Dr. Price's Essay on the Population of England; and a pamphlet entitled The Uncertainty of the present Population of this Kingdom, deduced from a candid Review of the Accounts lately given of it by Dr. PRICE on the one Hand, and Mr. EDEN, Mr. WALES, and Mr. HOWLETT, on the other.*

In the Preface to the ESSAY just mentioned, fearing that I might have expressed my conviction too strongly, I referred myself to the candour of the Public, and desired that my assertions might not be regarded any farther than they were supported by undeniable facts.—The prospect of an increasing depopulation is so discouraging, that nothing but the fairest overbalance of evidence should engage us to admit it. I thought such evidence did exist, and, therefore, stated it; believing that satisfaction ought never to be founded on imposition, and that by endeavouring to apprize the kingdom of its true state, I might be doing it an important service.—The ingenious Author of the pamphlet last mentioned, writes in the character of one who doubts, and wishes only to know how things are; but Mr. Wales and Mr. Howlett zealously maintain, in opposition to the arguments I have produced, that our population is increasing fast. My intention in this *Postscript*



is to give as fair and yet as brief an account as I can of the present state of this dispute, by reciting the evidence offered on both sides, and making such remarks upon it as shall appear to me necessary.

The principal evidence to prove that our population has declined, is taken from the comparison stated in page 276 of this Volume (but more particularly in the *ESSAY*), between the number of houses in the kingdom at different periods from the Revolution to the present time.

Houses in *England* }  
and *Wales* at Lady- } 1,319,215 { including  
day 1690 — — } { 554,641 hav-  
} { ing only one  
} { hearth.

	Charged and chargeable.	Excused for poverty.	Total.
Houses in 1750	729,048	.	.
in 1759 (a)	704,053	282,429	986,482
in 1761	704,543	276,149	980,692
in 1777	701,473	251,261	952,734

The number of houses at Lady-day 1690, is stated distinctly by Dr. *Davenant* for every county (see his Works, Vol. I. p. 38); and represented by him as an important instruction derived from the hearth-books then

(a) This year was the first in which an order was given to return the cottages excused for poverty.—The chargeable or uninhabited houses in this year, and in 1761 and 1777, were 24,904, 25,628, and 19,396 respectively. See the *Essay on the Population of England and Wales*, printed for Mr. *Cadell*, p. 10 and 12.

existing,

existing, and containing accounts fairly kept and stated. *Ib.* p. 136, 373.

The numbers for the subsequent years are given from the returns to the tax-office of the surveyors of the house and window-dues in every district in the kingdom, made by the order of government in those years.

A comparison of these numbers with those given by Dr. *Davenant*, affords an evidence which, as far as it can be trusted, is full and decisive.—I know of nothing which has been urged against Dr. *Davenant*'s account, except that by *houses* he meant *families*; but it has been observed, that the difference between the number of *families* and *houses* in the kingdom, is by no means considerable enough to account for the excess in Dr. *Davenant*'s total; and that, were the contrary true, it is evident he must have meant *houses*, because he has divided this total into two numbers (namely, 1,208,000 and 111,215) the first of which he supposes to be the number of houses having *ground* about them; and the second, the houses not having ground about them.

The principal objections which have been made to the other accounts are the following.

First; the cottages are included in them, and these being excused, and no account kept of them, the surveyors could not be correct in returning them.

This

This is certainly true. But it should be remembered, that the same objection holds against the returns of the cottages made from the hearth-tax; that if in any instance such returns have been made from *conjecture*, they are more likely to exceed the truth, than to fall short of it; and that it is quite incredible that these returns should be so deficient as not to give above two out of five of the true number; or that the cottages of the poor should be almost equal to all the other houses in the kingdom, which must be the case if there has been no decrease.—I have been, however, assured that in some districts, the returns of the cottages have been made from actual surveys, and may be depended on.—And, if in other districts, they have been made carelessly, or perhaps in some not at all, an allowance on this account of an omission of *half* the cottages would still leave the number of houses short of what it was formerly.

According to the returns, the decrease in the cottages has been much more considerable than in the other houses; and, in the interval between the two last returns, amounted to 24,888. Such an authority only as the returns of the cottages, gives no sufficient reason for believing this. But there are two facts which give it credibility. The first is, that acknowledged destruction of cottages which has been the  
confe-

consequence of the increase of large farms. And the other is, that decrease of the houses *charged* having seven windows or less, amounting to 24,651, which took place in the same interval of time. See the account of this decrease in the *Essay on the population of England and Wales*, p. 11.—To this nothing has been opposed but a strange objection of Mr. *Howlett's*, implying, that, on account of the distresses of the poor, it is not possible that these houses and the cottages should decrease together.

The same writer has endeavoured to discredit all the returns to the tax-office, by observing, in p. 60, that they have represented the number of houses as diminished (since 1755) in some places where it is known they have increased. He instances in *Thaxted* in *Essex*, consisting of 350 houses; two parishes in the same county and one in *Kent*, consisting between them of only 206 houses; and *Maidstone*, consisting of 1106 houses. He gives no other proof that these places have not decreased than a bare assertion; and if I may judge from his principal instance (or *Maidstone*), his account of the returns for these places deserves no regard. According to him, the return of the houses for this town in 1777 was 633, and less by 23 than in 1755: Whereas the number returned in that year of inhabited houses only paying the house and window-duties, and  
therefore



therefore exclusive of all the other houses (which were included in the general return for the county) was 727; as any one may know who can either enquire at the tax-office, or will consult the accounts printed by the House of Commons in 1781.

Mr. *Howlett*, after making this objection to the tax-office accounts, informs the public (p. 62), from the authority of some surveyor of the window duties, that *doubtless* there was no return at all of the cottages in 1777.—It is difficult to account for so gross an error. In the first session of the present parliament, Lord MAHON moved the House of Commons for an account of all the returns to the tax-office of the houses in the kingdom. In consequence of this motion, the general return for 1777 was, among other returns, laid by the commissioners of the tax-office before parliament. This return was afterwards printed, and it distinctly specifies the number of cottages, as well as of other houses, in *every* county; and it is the same with the return for 1777 which I have given at the beginning of this Postscript, but more at large in the Essay on the Population of *England* and *Wales*.

After finding Mr. *Howlett* so mistaken in this and some other instances (a), I might, I think, be excused were I to save myself

(a) See Vol. I. p. 255, and 258, 259, 260.

the trouble of taking any farther notice of him. There are, however, some other mistakes into which he has fallen, still more important and palpable, which in what follows it will be proper to mention.

In this argument, a great deal depends on the proportion of the houses *charged* and *chargeable* (and consequently entered in the books of the assessors) to the whole number of houses in the kingdom. The return in 1777 makes this proportion to be as 701,473 to 952,734, or as 3 to 4 nearly. See p. 299. A comparison of this proportion with the like proportion in a great variety of parishes and towns in different parts of the kingdom, ascertained by careful enumerations, would shew how far it deviates from truth, and what addition ought to be made to the excused houses, in order to obtain the whole number of houses.— I am not possessed of many such accounts. Those which I think most to be depended on are the following.

			Total of Houses.	Houses charged.
<i>Beccles in Suffolk</i>	—	—	468	297
<i>Bungay</i>	—	—	326	220
<i>Henham, Sotherton, Shipmeadow, Weston, and two other parishes in Suffolk</i>	—	—	135	106
			929	623
				<i>Wenhaston</i>

	Total of Houses.	Houses charged.
Brought over — —	929	623
<i>Wenbaston</i> (a) in <i>Suffolk</i> —	76	73
<i>Southwold, Aldeburgh, Orford,</i> and <i>Gorleston</i> , parishes in } <i>Suffolk</i> — — —	720	563
Remainder of the district in } <i>Suffolk</i> in which these pa- } rishes are — —	5906	4859
<i>Warrington</i> in <i>Lancashire</i> , } with its vicinity —	1941	558
	<hr/> 9572	<hr/> 6676

(a) Only 56 houses have been reckoned in this parish; but in the office accounts 73 houses are charged, in consequence of the division of several cottages deemed single houses, into two or three separate dwellings, holding so many families.—One of the excused houses in this parish (and also in *Bungay*) is an alms-house, and in this account reckoned but one house, though consisting of several apartments, and therefore capable of being reckoned 5 or 6 houses; and in all accounts of this kind it should be remembered, that some differences will arise, as a house or cottage containing two or more families, having no communication, is reckoned a single or two or more houses.

*Weston* parish consists only of 21 houses, *Shipmeadow* of 11, *Henham* of 15, and *Sotherton* of 24. It is not conceivable that any parishes should have been always so small; and yet there are multitudes of such parishes in *Suffolk, Norfolk, Northamptonshire, Sussex, Kent*, and some other counties, and some of them provided with large churches. In *Norfolk*, particularly, the dilapidated churches in some places, and their disproportionate size in others, prove that it must have been formerly more populous. Even *Norwich* itself bears evident marks of having been once a much more considerable city.

	Total of Houses.	Houses charged.
Brought over —	9572	6676
<i>Sandwich</i> in <i>Kent</i> (a) —	578	349
<i>Christleton</i> in <i>Lancashire</i> , by } an exact survey in 1780 }	102	72
<hr/>		
<i>First</i> totals —	10,252	7097
Add <i>Sudbury</i> division —	7740	4122
<hr/>		
<i>Second</i> totals —	17,992	11,219

Accounts collected by Mr. *Wales*. See his  
*Enquiry*, p. 39, 43, 47, &c.

	Total of Houses.	Houses charged.
The two divisions of <i>Ag-</i> <i>bridge</i> and <i>Morley</i> in the } <i>West-Riding</i> of <i>Yorkshire</i> }	21,929	12,832
Twenty-eight villages in } <i>Northamptonshire</i> — }	1024	706
<i>Westball, Wangford, Holton,</i> <i>Spexball, Swilland, Tud-</i> <i>denham, Westerfield, Wisset,</i> <i>Witnesham, Blythford, and</i> <i>Bramfield</i> , parishes in <i>Suf-</i> <i>folk</i> — — }	391	351
<hr/>		
	23,344	13,889

(a) According to an accurate account taken by Mr. *Boys* in 1776. The number of inhabitants was 2252, or 3 $\frac{1}{10}$  to a house; though *three* workhouses containing 33 persons, and *two* hospitals containing 21 persons, are reckoned as only five families.

*Ashill,*



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	Total of Houses.	Houses charged.
Brought over —	23,344	13,889
<i>Ashill, Clapton, Ilminster, and Wayford, in Somersetshire</i> }	388	134
<hr/>		
<i>Third totals</i> —	23,732	14,023
Add the <i>Second totals</i> —	17,992	11,219
<hr/>		
<i>Fourth total</i> —	41,724	25,242
<hr/>		

If we may judge from the first totals, which are those alone in which from my own enquiry I can confide, and which (including in them a town with its vicinity full of the poorest manufacturers, where the proportion of charged houses is lower than I have found it any where else) may not possibly be an improper guide in this case, the proportion of charged to the whole number of houses will be as 7097 to 10,252. And, since the charged and chargeable houses are known by the returns in 1777 to have been then 701,473, the whole number of houses in the kingdom will come out 1,013,000, or nearly a million, as I have reckoned it. If we add to these totals those for Sudbury and its neighbourhood, where also (because full of poor manufacturers) the proportion of charged houses is particularly low, the number of houses in the kingdom will come out 1,125,000. — If we judge

by the accounts Mr. *Wales* has collected, this number will come out 1,187,000.— If we judge by all these accounts taken together it will come out 1,159,000.

All these determinations shew a great diminution in the number of houses since the *Revolution*; nor (supposing Dr. *Davenant's* account right, or even not *very* wrong) is it possible to reckon it equal now to what it was then without contradicting all probability.

A confirmation of this might be derived from Mr. *Howlett's* accounts, could they be trusted. He has (in his *Examination of Dr. Price's Essay*, p. 139, &c.) given a list of towns and parishes in 20 different counties, in which the total of houses is 29,262 by *enumeration*, and 17,225 by the *returns* of the surveyors. The last of these totals includes in it only the *charged* houses; and it gives a proportion of these to all the houses in the kingdom, which makes their number 1,191,000. But the truth is, that Mr. *Howlett's* account of the returns of the surveyors cannot at all be depended on; and the following particulars will abundantly prove this.

The numbers returned for *Beccles*, *Bungay*, *Shipmeadow*, *Mettingham*, and *Homersfield* in *Suffolk*, were in 1780 (a), according to him, 169, 260, 7, 21, and 21 for these places respectively.—I am assured, on the

(a) There was no return in this year.

contrary, that the numbers (when the last general return was made in 1777) were 297, 220, 11, 27, and 23 returned as charged; and 171, 106, 0, 3, and 11, returned as *excused*.——Thenumbers returned for *Northampton, Maidstone, Chester, and Shrewsbury*, he makes to be 768, 623, 1227, and 967 respectively; whereas it appears, from the accounts printed by the House of Commons in 1781, that the numbers returned to the tax-office for these towns in 1777, were, 706, 727, 1244, and 904, exclusive of the *uninhabited*, and *excused* houses which were likewise returned, but included in the totals for the counties.

But Mr. *Howlett* has here fallen into a still greater mistake; for, through haste or inattention, he has taken the numbers in his list (being in reality only the number of houses *taxed* given very inaccurately) for the whole of the numbers (*a*) *returned*, including *uninhabited* and *excused* houses; and, arguing upon this mistake, he makes the houses in the kingdom 1,609,555; which is above a third more than, by computing in his own way,

(a) “The number of houses in Mr. *Howlett*’s list said to be returned for *Tenterden* in *Kent*, is 96, the total 198. A correspondent, on whose veracity I can depend, assures me that these 198 houses are all in the parish duplicate; and that the 96 are those which are “charged.”—*Uncertainty of the Population of this Kingdom*, p. 24.

he must have found them had he not fallen into this mistake (a).

It

(a) Mr. *Howlett*, in consequence of thus over-rating the number of houses, and allowing 5 and two-fifths to a house, makes the inhabitants of *England* and *Wales* to be near nine millions. The proportion of inhabitants to houses may be, in some measure, collected from the Table in p. 6th of the Essay on the *Population of England and Wales*, which has been reprinted with some additions at the end of the First Essay in the preceding Volume of this work. To the towns and parishes in that Table I will here add SANDWICH in KENT, where, by an accurate survey in 1776, the houses were found to be 578, and the inhabitants 2252, or  $3\frac{2}{3}$  to a house; and also EASTRY in the same county, where, in 1774, the houses were 141, and the inhabitants 656, or  $4\frac{2}{3}$  to a house. The total of houses in that Table, with these added, is 45,217; and of inhabitants 231,842, which makes 5 and an eighth to a house.

Mr. *Howlett* has inserted in his *Examination*, &c. p. 144, the houses and inhabitants in *Birmingham*, *Norwich*, *Manchester*, *Nottingham*, and *Liverpool*, just as I had given them in the Essay on the *Population of England*, &c. but with such additions as to bring out the allowance just mentioned 5 and two-fifths to a house. But had Mr. *Howlett* chosen to add to his own list the whole of my list in the Essay, as well as that part of it just mentioned which gives the highest allowance, he would have found (taking 4338 for the number of houses at *Manchester* and *Salford* in 1773, and not 4268 as he makes it) the total of houses to be 41,030, and of inhabitants 244,422; and consequently the allowance to a house not to be so much as five and one-fifth to a house.

Mr. *Howlett's* additions, with SANDWICH and EASTRY, and the additions which have been made (in the Table in the First Volume, p. 298) to the Table in the Essay on the *Population of England and Wales*, will make the total of houses 52,036, and of inhabitants 268,568, and the allowance 5 and a sixth,

It



It is necessary to observe, that the method here used of deducing the total of houses

It should be considered, that these totals, consisting chiefly of the houses and inhabitants in five of the most populous towns in the kingdom, give most probably a proportion of inhabitants to houses too high for the kingdom at large. If we throw out BIRMINGHAM and the town of MANCHESTER, the remainder will perhaps make a properer mixture of great and small towns and country parishes; and the totals (or 41,675 and 210,158) will give  $5\frac{1}{10}$  to a house. If LIVERPOOL is likewise thrown out, the totals will give less than 5 to a house.

In the Table just referred to I have given the number of houses and inhabitants at *Birmingham* from a survey in 1770; when the houses were 6025, and the inhabitants 30,804; of whom 15,363 were males, and 15,441 females. — I have lately been informed that, according to a very accurate survey of *Birmingham* in autumn 1782, the houses (exclusive of the hamlet of *Deretend*) were then 8125, of which 291 were uninhabited. From the same account I learn, that the annual average of burials at *Birmingham* (exclusive of *Deretend*) for four years to 1774, was 1116; and for six years to 1780, was 1342. — The number of inhabitants in 1770, divided by the *first* of these averages, makes the proportion dying annually at *Birmingham* to be one in  $27\frac{1}{5}$ ; which, being very nearly the same with the proportion dying annually at *Liverpool* and *Manchester*, cannot probably be far from right: and this number (or  $27\frac{1}{5}$ ) multiplied by the *second* average, makes the inhabitants in 1780 to be 37039. In order, however, to allow for the increase of *Birmingham*, and to be more sure of finding a number not less than the truth, let the burials in 1782 be reckoned 1500, and the proportion dying annually 1 in 28; and it will follow that the inhabitants were then 42,000, and the number of persons in a house 5., including about 700 in the workhouse and hospital. — I am sensible that this falls below the common estimates; but I pay no regard, in cases of this kind, to any estimates which are not derived from careful surveys.

houses in the kingdom from the proportion (ascertained by surveys) of the houses taxed to

The annual average of births at *Birmingham* was (according to the register) 1408 for 10 years to 1780. The excess of the births above the deaths is plainly owing to that over-proportion of people in the first stages of mature life, which always takes place in towns, in consequence of their being kept up or increased by an influx of people from other places. See the First of the following Essays. That this is the cause of the increase of *Birmingham* is undoubted, for the excess of the births cannot account for a 40th part of the increase; and before it became so rapid as it has been for some time, the burials exceeded the births, the annual average of the former having been, if the register deserves any regard, 708; and of the latter, 619. — The same register makes the annual medium of burials for 10 years to 1697 to have been 156, and of births, 150. But this only confirms an observation before made, that the registers in former times were very deficient; for it is not probable, that *Birmingham* was then so small a town; and an old account which I have seen of a survey in 1700 makes it to consist in that year of 2504 houses, and 15032 inhabitants. The register, therefore, did not then give above a third of the births and burials.

In Vol. I. p. 301, I have also given the number of houses and inhabitants at *Maidstone* in *Kent*, from a survey in 1781. I have since learnt, that another survey was made at *Maidstone* in *September* 1782; and as some instruction may be derived from it, I will here give the results just as I find them in a pamphlet published in this town by Mr. *Howlett*, and entitled, *Observations on the increased Population, Healthiness, &c. of the town of Maidstone.*

	Families.	Houses.	Inhabitants.	Males.	Females.	Male servants.
In the town —	1037	982	5028	2306	2722	145
In the country —	139	133	727	357	370	41
In the whole parish	1176	1115	5755	2663	3092	186

In

to the totals of houses in country towns and parishes, must be too favourable; because this

	Female servants.	Women above 70.	Men above 70.	Girls under 15.	Boy. under 15
In the town	- 325	161	96	847	776
In the country	- 40	9	10	165	144
In the whole parish	365	170	106	1012	920

Persons to a house <i>in</i> the town	—	—	5 $\frac{1}{16}$
In the parish <i>out</i> of the town	—	—	5 $\frac{3}{8}$
Persons to a family <i>in</i> the town	—	—	4 $\frac{5}{8}$
In the parish <i>out</i> of the town	—	—	5 $\frac{1}{4}$
Proportion of children under 15 to the total of inhabitants <i>in</i> the town	-	} as 100 to 309	
In the parish <i>out</i> of the town	—		as 100 to 235

In the town one in 17 of the women exceeds 70 years of age, and one in 24 of the men; but in the country only one in 41 of the women exceeds this age, and one in 36 of the men.

*Annual average (according to the register) in the whole parish for 20 years—*

Of births to 1702	130	Of marriages	29	Of burials	132
to 1722	120	—	30	—	118
to 1742	129	—	40	—	144
to 1762	143	—	46	—	140
to 1782	160	—	50	—	148

By a survey in 1695, the inhabitants were 3676.

From these particulars it seems to appear, that *Maidstone*, at the beginning of this century, was a *decreasing* town; but that lately it has been *increasing*, not by an excess of births, but, like other towns, by drawing supplies from other places. The ratio of the births to the burials, (if it can be depended on) and the great over-proportion of persons in mature life in the town, prove this.

The number of females in it turned of 70 is greater than the number of males, partly, because males are more



this proportion in *London, Southwark*, and all *Middlesex* (containing at least an 8th or

more short-lived, but chiefly in this instance because the males, after removing to the town, are taken off again to the navy, army, &c. And the proportion of both males and females turned of 70 in the country is smaller than in the town, because removals from thence are chiefly to the town; and these being also chiefly removals of females, the town is rendered, at every age, much fuller of females than of males.

It is farther observable, that the town, when compared with the country round it, appears to be particularly unfavourable to population, the proportion of children under 15 being much less there than in the country. — The same is remarkable in the country round *Manchester*. See the First of the following Additional Essays.

It seems, indeed, that the consumption of towns tends to promote the population of the country near them; and were they fed with people only from hence, they would not probably be so prejudicial as they are to population. But the fact is, that there are few towns which would not soon come to nothing, did they draw their supplies of people only from the adjacent country. So true is this of *London* in particular, that, notwithstanding this natural tendency of its consumption, there is scarcely a village or parish within ten or twelve miles of it, in which, if we may believe Mr. *Howlett's* extracts from the registers, the births do not fall considerably short of the burials. See his *Examination*, &c. p. 96, 97, &c.

In a note at the beginning of the First of the following Essays, it appears that the number of houses at *MANCHESTER*, exclusive of *Salford*, in 1773, was 3446, including 44 empty houses. My friend Dr. *Percival* has just informed me, that at the end of last year (1782) a new and very accurate enumeration of this town (exclusive of *Salford*) was completed, which made the houses then to be 4606. An addition, therefore, has been made to *MANCHESTER* of 1160 houses within the last ten years.



9th. of the kingdom) is, and, for obvious reasons, must be much higher than it is in the other districts of the kingdom. The returns in 1777 make the houses taxed in *London*, *Southwark*, and all *Middlesex* to be 77,008, and the total of houses 90,570; whereas the same returns for the whole kingdom make the former to be 701,473, and the latter 952,734,—I think it worth adding, that from a return for *London* and *Middlesex*, in 1780, and laid before parliament, it appears that the number of empty houses in this part of the kingdom had increased, between 1777 and 1780, from 3,381 to 6,810.

The evidence now insisted on, taken from the returns of the surveyors and assessors of the house and window-duties, is the only direct evidence comprehending the whole kingdom with which we are furnished on this subject; and it is so discouraging, that I do not wonder that the advocates for the increase of our population endeavour to discredit it; and I should certainly join them in this, were I less desirous to know things as they are, than to prove them what I wish them.—The care and attention of Mr. *Rose* (now one of the secretaries to the treasury, but lately the secretary of the tax-office), in collecting these returns, cannot, I believe, be doubted; and he who considers that they are founded upon old taxes, and made upon oath, will not be able easily to per-  
suade

suade himself that they can be very grossly deficient.

Mr. *Wales*, a writer whose abilities I respect and whose accounts I am not inclined to distrust, has collected several accounts of enumerations of houses *in or about* 1750 and 1780, which he thinks afford a presumptive proof of a general increase during that period. I will transcribe his summary of them, p. 48 (a).

	Houses in 1750.	Houses in 1780.
<i>North Riding in Yorkshire</i> —	1716	1985
Eight villages in the <i>West-Riding</i>	784	943
Seventeen villages in <i>Derbyshire</i> —	1001	1348
Twenty-seven villages in <i>North-</i> <i>amptonshire</i> —	1036	1024
Fourteen parishes in <i>Suffolk</i> (families)	653	704
Four parishes in <i>Suffex</i> —	144	223
Four villages in <i>Somersetshire</i> —	428	388

Mr. *Wales* has added an account taken from the returns (which in this instance he is willing to trust) of the surveyors for *Agbridge* and *Morley* divisions in the *West-Riding of Yorkshire*. From these returns it appears, that in 1761 the houses in these

(a) In p. 67, there is a comparison of enumerations at different periods of *Manchester*, *Liverpool*, *Birmingham*, *Leeds*, *Nottingham*, *Norwich*, and *Farnham*, which shews, what is well-known concerning the four first of these towns, that they have greatly increased.

divisions

divisions were 17,764; that in 1767, they were 20,526; and in 1779, 21,929.

I will add a similar account of a district in the county of *Suffolk*, where

In 1761	{ the houses <i>charged</i> were	5584
	{ the houses <i>excused</i> were	1391
		<hr/>
		6975
		<hr/>
In 1777	{ the houses <i>charged</i> were	6118
	{ the houses <i>excused</i> were	1521
		<hr/>
		7639

There has undoubtedly been an increase in *Yorkshire*, and perhaps also in *Derbyshire*; but he that will judge of it from the numbers in these accounts will be in danger of being misled: For I understand, that it is in part an *apparent* increase only, owing to the conversion of houses holding two or more families, and *formerly* charged as *single* houses, into apartments having no communication, and therefore *now* charged as so many separate houses.—The inducements to such conversions among the lower ranks of people have been so great since 1761, as to be irresistible. For first, their poverty has increased, and therefore they have found it more necessary to save every needless expence.—And secondly, in 1761 the window-duties were nearly doubled; and houses having 8 or 9 windows, *before* excused, were subjected to the payment of 1*s.* *per ann* for

for every window. In 1766 these duties were again increased, and houses having only *seven* windows were subjected to them. By dividing, therefore, single houses holding more than one family into several tenements having each of them few windows, the tax upon them might be either lessened or entirely avoided (a). The decrease of small farms has likewise contributed to this change, by causing many farm-houses to be turned into cottages for day-labourers.

Perhaps, these have been the only causes of the increase of the district in *Suffolk* just mentioned; and there is reason to believe that they have been the principal causes of the increase in *Agbridge* and *Morley* divisions in *Yorkshire*. For the returns shew an increase in these divisions equal to above a 6th of the whole number of houses in so short a time as six years, or from 1761 to 1767; but afterwards, or from 1767 to 1779, they do not shew *half* this increase in *double* the time. The first increase, therefore, was probably occasioned, as I have observed, by the alteration in the window-duties in 1761; nor, indeed, could it have any other cause than either this, or the de-

(a) In Mr. *Wales's* accounts of the increase of houses in the *North-Riding* of *Yorkshire*, and in *Derbyshire*, it appears that a great part of it proceeded from alterations in old houses; that is, perhaps, from such alterations as those here meant.



fertion of other parts of the kingdom ; for it was too great and too sudden to be accounted for by an excess of the births above the deaths, which is the only cause that can produce a general and permanent increase.

There is one more source of information on the subject of our population which is of particular importance ; I mean, a comparison of the births and burials and marriages at different periods. Such a comparison for the whole kingdom would decide the question I am discussing. But we are far from being furnished with the means of making it. It is, however, the evidence on which the advocates for a progressive increase in our population principally rely ; and I shall here give a fair representation of it, with such remarks as a regard to truth will render necessary.

Annual average of baptisms	Baptisms.	Burials.
and burials about or soon after the Revolution, in 33 parishes in ten counties, taken indiscriminately in different parts of England.—See Mr. <i>Wales's Enquiry</i> , p. 49. (a) —	1460	1518

(a) In Mr. *Wales's* list the average of burials corresponding to the births is not given for *Liverpool* and *Bowden* in *Lancashire*, and for *Lamborn*, *Shefford*, and *Wilford* in *Lerkshire* ; and, therefore, these places are not included in this account.

	Baptisms.	Burials.
Annual average in the same parishes for some years before 1780.— <i>Ib.</i> p. 50 }	4064	3537

Annual average of baptisms and burials about the year 1745 in 142 parishes in 21 counties taken indif- criminately.— <i>Ib.</i> p. 53. }	4712	4067
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Annual average in the same parishes between 1770 and 1780.— <i>Ib.</i> 57. }	7179	5689
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Annual average of births and  
burials in the Deaneries of  
*Melineth, Elvel, Buillt, Hay,*  
and *Brecon* in the diocese of  
St. *David's*.—*Ibid.* p. 65.

From 1700 to 1730 —	341	325
From 1730 to 1760 —	715	587
From 1760 to 1763 or 1764	727	580

Annual average in the other  
parts of the diocese

From 1700 to 1730 —	888	753
From 1730 to 1760 —	1111	921
From 1760 to 1763 or 1764	1302	1183

Annual average in the whole  
diocese of St. *David's*

From 1700 to 1730 —	1229	1078
From 1730 to 1760 —	1826	1508
From 1760 to 1763 or 1764	2029	1663

All

All these accounts have been extracted from the parish registers. The deficiencies in these registers, and the carelessness with which they are kept, have been often complained of. I wish, therefore, something had been said to establish their credit; or at least to shew, that they have been preserved entire, and that they were not *more* deficient formerly than they are now (a). Supposing them

(a) May it not be doubted whether at the *Revolution* the parish registers had recovered from the confusion into which all church affairs had been thrown in the times of the civil war and commonwealth?—The number of popish and protestant dissenters was then probably much greater than it is now.—But the observation most to the present purpose may be, that registers of mortality are of late origin, and have been for a course of years growing more and more into use and estimation. Among the Dissenters in *London* the registration of births was, some years ago, much neglected. At present it is more practised in consequence of notifications of the establishment of a public register, which have been read annually from the pulpit. And in the country I suspect, that people of all denominations are got so much more into the habit of reckoning it important, as sometimes to register in more than one place.

“ In 1538 *Henry the Eighth* gave orders that the incumbent of every parish should keep true and exact registers of all christenings, weddings, and funerals in his district. But this order, in many places, was little regarded till *Queen Elizabeth*, in 1558, gave another order for keeping them more exactly. Yet after all they were but remissly kept in many parishes, and often committed only to loose papers, by which means some were lost, some rotted away, and others were devoured. To remedy these evils, orders were given

them correct, they take in but a very inconsiderable part of the kingdom, and chiefly that very part which, it is well known, has increased, but the increase of which must have been, in some measure, occasioned by removals from other parts of the kingdom. The *second* of these accounts is the principal; and, if from the numbers in it are deducted the births and burials in *Manchester, Rochdale, and Warrington in Lancashire; and in Skel-*

“in 1559, that all registers should be kept in parchment-books only, and that all preceding ones which  
 “could be found, should be transcribed into new books.  
 “But no place in *England* slighted these orders so much  
 “as *London*; for, except in two or three years of great  
 “plagues, we find no bills in *London* till 1604.—But  
 “neither country nor city registers, where there has been,  
 “or still is any considerable body of dissenters, popish or  
 “protestant, are to be much relied on after 1644, when  
 “the division in the church first broke out. And even  
 “in places where there are no dissenters, registers are  
 “little to be regarded on account of several unhappy  
 “concurring circumstances, as the negligence or frequent  
 “absence of the register-keeper, and the ignorance, poverty, mistakes, and *prejudices* of several of  
 “the people.”—See the preface to the *New Observations on Town and Country Bills of Mortality*, by Dr. Short, p. 9, &c.

In *London* the bills did not include the distempers till 1629; nor the ages till 1728; and still it is well known that they are very defective.

Conclusions drawn from registers of burials, be they ever so exact, are rendered more uncertain than is commonly imagined, by epidemics, and the different degrees of healthiness or sickness of different years. This may be learnt in some measure from what is related of *SWE- DEN* in p. 146.

*field,*



*field, Wakefield, Hallifax, &c. in Yorkshire,* the remainder will be, in the first period, 1630 births *per ann.* and 1408 burials; and, in the second 2010 births *per ann.* and 1502 burials, which makes a small increase.

The *first* account overthrows itself by making the burials at the *Revolution* in eleven counties to exceed the births. These counties, therefore, if we are to judge from these extracts, must have been *then* decreasing. The increase which appears at present is almost entirely the increase of the towns just mentioned; and if they are struck out, the remainder in this *first* account, as well as the *second*, will be little; and that little will shew a decrease in *Somersetshire*, no increase in *Nottinghamshire*, and only a small increase even in *Yorkshire*.

Mr. *Wales's third* list shews an increase at the beginning of this century so rapid in the diocese of St. *David's* as in 30 years to double the inhabitants of five deaneries; but, in the other parts of the diocese, so much slower, as in the same time not to add a *quarter* to the inhabitants.—It deserves notice farther, that they represent the increase which took place in the *first* period as changed into a decrease in the *second* and *third* periods. This will appear upon considering, that had the increase in the *first* period been continued to the end of the *second*, the annual averages at the end of this

*second* period, (or which is nearly the same) the annual averages from 1760 to 1763, must have been much greater than they are; for they must have borne the same proportion to the averages of the *second* period that the mean between these averages and the averages of the *first* period bear to these last averages. That is, in the five deaneries, the average of burials about 1760 should have been to 587 as the mean between 587 and 325 (or as 456) is to 325. It should have been, therefore, 823 (or some number not very distant from this) instead of 580; which last number is so much too little as to be nearly equal to the annual burials about the middle of the *second* period; and, therefore, if not very wrong, proves a decrease must have taken place.

By the same reasoning it will appear, that in the whole diocese, if the increase in the *first* period had continued, the burials at the end of the *second*, or the beginning of the *third* period should have been nearly 1808, instead of 1663. The same conclusions may be deduced by computing from the births.

These are circumstances which give a suspicious appearance to this register evidence (a); but there is a third circumstance which destroys its credit.

At

(a) One plain reason of the inconsistencies in these accounts has been intimated, namely, that the births  
and

At the same time that, in the five deaneries, they shew an extravagant increase in the *first* period, they give the births and burials nearly equal, and therefore make it impossible there should have been any increase (*b*).—The like will be observed presently of the whole diocese.

That part of the kingdom where the parish registers give the strongest proofs of an increase is the diocese of *Chester*.—The following is a summary of the extracts from them as I have received it from a friend in the diocese.

	Births.	Burials.
In the archdeaconry of <i>Chester</i> — }	in 1717 7703	6380
	in 1779 16791	12573
In the whole diocese in 1717	10604	8755
	in 1779 21463	16080

There appears here an increase which has *doubled* the inhabitants in 62 years ; and

and burials in former periods are given by the extracts much *more* below the truth than in the latter periods. And as far as this is the case, they prove nothing.

(*b*) The births in the first period, in order to produce (in conformity to the extracts) a double number in 30 years, should have been more than double the burials ; that is, supposing the burials not too high, the births should have been about 700 ; and both the births and burials in the *second* period, instead of being 715 and 587, should have been double these numbers.

there is no reason to doubt out that this part of the kingdom (including in it some of the chief manufacturing towns in *Lancashire*, *Cheeshire*, and *Yorkshire*) has considerably increased. I cannot, however, trust my belief of this merely to these extracts (a); for they destroy their own authority by giving a proportion of the births to the burials, which is inconsistent with any such increase, as will appear from the following observations.

If the annual average of burials about 1717 is multiplied by 35 (a multiplier which, in the case of a large *country* district cannot be much too high), it will appear that the whole number of inhabitants in the diocese was then 306,000. The excess of the births above the burials was 1849, or the 166th part of the inhabitants; and this is an excess which, supposing the increase produced by it uniformly accelerated, without being once checked by sickly seasons

(a) The author of the pamphlet entitled, *The Uncertainty of the Population of the Kingdom*, mentions a very material circumstance relating to the registers of births kept in *Lancashire*, and some other northern counties.—“I am assured,” says he, “by the most authentic information, that, in consequence of the late multiplication of chapels, it is no uncommon thing for baptisms (and sometimes burials) to be entered, in some parishes in these counties, twice over; first in the chapel register, and afterwards, for greater security, in that of the mother church, p. 28.”



and emigrations (that is, supposing it a much greater increase from a given surplus of births than there is reason to expect), could not have doubled the inhabitants in less time than 115 years, as may be found by computing in the manner directed in the Note, Vol. I. p. 279. If, therefore, agreeably to the parish extracts, they were doubled in 62 years, it must have been the effect, not of the excess of the births above the burials (the only general cause of the increase of countries), but of an influx of people from other parts of the kingdom; and, therefore, proves no more than that one part of the kingdom has gained by taking away from other parts. And this may probably have happened in this diocese. The truth, however, more probably is, that the parish registers do not give us true information in consequence either of having been more deficient formerly, or not having been duly preserved. See the Notes in p. 321, &c.

This observation is applicable to all the other accounts which I have met with taken from parish registers.—In the diocese of St. David's there appears, by the extracts, to have been an addition (between 1715 and 1760) of *three fifths* to the inhabitants. But the excess of the births above the deaths will not account for more than a *third* of this increase; and as very probably more people leave WALES than flock into it, either (in conformity to the excess of the births) there

may have been no increase, or the register in the first period must have been so deficient as to give the births near a third less than the truth (*a*).

This argument holds equally with respect to the second of the accounts taken from Mr. *Wales*. And his first account carries, as before observed, impossibility on the face of it.

The following is a summary of Mr. *Howlett's* accounts, taken from p. 128 of his *Examination*, &c.

Annual average of births and burials for 20 years about the Revolution, compared with the annual average for the last 20 years, in 68 parishes in *Kent*, 43 in *Essex*, and 17 in *Surry*.

	Births.	Burials.
About the <i>Revolution</i> —	2993	3054
For the last 20 years —	3947	3983
In the same parishes, with the addition of 18 in <i>Sussex</i> , 15 in five southern counties, 29 in <i>Suffolk</i> , the city of <i>Norwich</i> , and five parishes in <i>Wales</i> .		

	Births.	Burials.
About the <i>Revolution</i> —	7553	7740
For the last 20 years —	10023 ( <i>b</i> )	10175
		To

(*a*) If the burials are supposed deficient, as certainly they ought, the births must have been proportionably more deficient than the third here reckoned.

(*b*) There are many errors in Mr. *Howlett's* numbers, but I have not discovered any that will materially affect the proportion of the totals here given

To these accounts Mr. *Howlett* has added (in p. 13.) a comparison of the births and burials for two periods of *five* years in 162 parishes in 26 counties; the first period beginning with 1758, 1760, or 761; and the second with 1773, 1775, or 1776.

	Annual average of births.	Annual average of burials.
In the first period	9527	9910
In the second period	1191	1060

This is all the register evidence which Mr. *Howlett* has produced, exclusive of Mr. *Wales's*, and that taken from the parish registers in the diocese of *Chester* already noticed. This evidence he has displayed with great pomp, and insisted upon as a full proof of an *astounding* increase in our population. But never before was an evidence offered so absurd and self-destructive. For it should be observed, that, according to these accounts, the deaths in the kingdom from the Revolution to the present time have exceeded

In a postscript he has added to the parishes abovementioned the births and burials in 17 others; and all together make the annual averages.

	Births.	Burials.
At the Revolution	8375	8493
At present	11195	11382

the

the births (*a*). Mr. *Howlett*, therefore, will, I hope, some time or other, inform us how the increase in which he triumphs has been produced.—But to be serious. An excess of deaths cannot exist long in any kingdom. The appearance of it, therefore, in

(*a*) It may be said, that the excess of burials in this and the other accounts before noticed, is occasioned by a great over-proportion of omissions in the registration of births. But what confidence can be placed in registers which admit of such defects? or how is it to be known that they were not much greater formerly, agreeably to the observations in the Note p. 321?

The omission of still-born and unbaptized infants scarcely deserves notice, because they contribute nothing to population, and are probably, in most places, omitted in the burials as well as the births. And with respect to other omissions, were we to reckon them a *tenth* of the births, and only *half* as much of the burials, still an excess of births would be left, which would be almost equally inadequate to the increase.

In short; let the registers of births be ever so deficient, the increase they shew must have taken place if they were not more deficient formerly than they have been lately: And yet, this increase could not take place unless they were deficient to a degree which is incredible, and which, were it credible, would render them unworthy of much notice.—The increase, for instance, which on this supposition must have taken place in the diocese of *Chester*, cannot be accounted for from the excess of births without reckoning the omissions in the registers of births equal in both periods to at least a *third* of the registered births, even though the registers of burials are reckoned correct and complete. This will appear to any one who will calculate in the manner explained in p. 326, &c. The supposition, therefore, must be wrong that the registers of births were not more deficient formerly than they have been lately.

The



in these extracts must be owing either to their being miserably erroneous ; or to their being taken mostly from *towns* ; for in these it seldom happens that an excess of deaths does not take place ; nor is there any worse cause or symptom of depopulation than their increase.

All the evidence taken from the parish registers has been now laid before the reader, as far as I am acquainted with it. I am informed that Mr. *Wales* and Mr. *Howlett* are proceeding with their enquiries (*a*) ; and I hope they will be able hereafter to offer to the public some more consistent and probable accounts. When, however, I consider the reason there is for believing that the

The effect which the omission only of baptisms among Dissenters may have, will appear from the following fact.—The number of baptisms at *Sandwich* in *Kent*, among Protestant Dissenters (exclusive of *Baptists*) was

From 1690 to 1699	—	120
From 1730 to 1739	—	58
From 1770 to 1779	—	13

The number of baptisms in the same town for the same periods respectively was, exclusive of Dissenters, 755, 744, and 758

(*a*) I have not sought for any accounts of this kind, not chusing to give trouble to obtain so indecisive and percarious an evidence. The following are all I can add from my own information to those already given.

Lincoln.

the parish registers were in former periods particularly defective, I cannot help doubting

	Annual births	Annual burials	Annual marriages
Lincolnshire— <i>Sawnderby</i> parish }			
10 years to 1690 }	7.3	7.5	2.5
to 1720	5.8	5.0	2.0
to 1770	7.1	5.0	1.4
Durham— <i>Staindrop</i> parish }			
10 years to 1745 }	37.6	28.5	7.0
to 1771	49.3	44.8	12.9
Kent— <i>Tenterden</i> parish }			
20 years to 1729 }	29.8	33.6	9.1
to 1769	34.5	34.0	11.9
<i>Sandwich</i> parish }			
10 years to 1629 }	148.3	159.6	41.3
to 1689	103.2	95.8	11.7
to 1739	74.4	70.4	16.3
to 1779	75.8	68.8	21.3
<i>Eastry</i> parish }			
10 years to 1629 }	20.1	12.1	6.4
to 1689	13.7	12.2	2.6
to 1739	17.3	13.0	4.2
to 1779	20.7	13.4	5.2
<i>Word</i> parish }			
10 years to 1739 }	7.6	4.9	1.2
to 1779	6.7	4.8	2.0
<i>Woodnesborough</i> parish }			
10 years to 1719 }	15.5	10.9	7.3
to 1779	14.8	12.4	4.1
<i>Ash</i> parish }			
20 years to 1578 }	27.7	25.7	6.6
to 1777	50.0	39.7	11.9
Cornwall— <i>Liskeard</i> parish }			
20 years to 1719 }	51.7	45.3	13.0
to 1769	48.3	45.3	12.8
Devonshire— <i>Okeford</i> parish }			
20 years to 1719 }	12.2	8.0	
to 1769	12.2	7.5	
Staffordshire— <i>Biddulph</i> 20 years }			
to 1719 }	20.3	15.6	4.3
to 1739	27.8	21.1	4.4
to 1769	38.9	21.1	6.1

whether

whether any examination of them is capable of furnishing with sufficient evidence to prove that our population has not decreased since the *Revolution*, I question even whether it can inform us properly of the proportion of births to deaths in the kingdom. This alone, could it be ascertained, would enable us to form some judgment of the present state of our population, and to determine, with some probability, whether it is increasing or decreasing. If we unite all the extracts before given, rejecting Mr. *Hewlett's*, this proportion will come out  $\frac{28}{100}$ . Were these extracts to be depended on, they would probably give this proportion too high for the kingdom at large, because taken chiefly from the register of the diocese of *Chester*, the most populous and flourishing part of the kingdom (a). We may, however, argue upon it, and reckon it the just proportion for

(a) Dr. *Short* has employed much time and pains in collecting extracts from the registers of a great variety of market-towns and country parishes and villages in different parts of the kingdom for two periods, the first extending from the reign of Queen *Elizabeth* to the middle of the last century; and the second from different years at the end of the last century to the middle of the present century: and from a comparison of these extracts it appears, that in the former period the births exceeded the burials in the proportion of 124 to 100: but that in the latter they exceeded them only in the proportion of 111 to 100

This,

for *England* and *Wales*, exclusive of *London* and its environs ; on which supposition, if we reckon the annual burials such as, in consequence of multiplying by 35, will make the inhabitants of *England*, exclusive of *London*, four millions and a half, the annual burials will be nearly 128,000, and the births 164,000, leaving an annual excess of 36,000 ; and this is an excess which would produce an increase in most other countries, notwithstanding the waste in their capitals, and all the other causes which usually check the increase of countries (a)  
But

This, were there sufficient evidence for it, would manifest too plainly an encumbered and declining population. It appears (as Dr. *Short* speaks) with no less evidence from the registers *than that the sun shines in a cloudless day at noon* ; and he concludes from it, that in consequence of the irregularities and debauchery occasioned since the Revolution, by increasing opulence and luxury, the kingdom has been for many years growing less healthy. But the truth is, that the registers (having certainly been more defective formerly than they are at present) cannot be trusted as a just foundation for any conclusions.— See Dr. *Short's New Observations*, Tables 1st, 2d, and 3d, and p. 80.—See likewise the Preface to his *History of the Comparative Increase and Decrease of Mankind* ; and the Tables at the end.

(a) The proportion of births to deaths in	}	130 to 100
all SWEDEN for 9 years to 1763 was	}	
In the kingdom of NAPLES for 5 years to	}	144 to 100
1777 — — —	}	
In all FRANCE for 5 years to 1774 as	}	117 to 100
928,918 to 793,931, or — —	}	
		Annual



But perhaps there are few kingdoms now existing in which most of these causes operate  
fo

Annual average of births, deaths, and marriages in  
*Breslaw, Glogaw, and the other towns of SILESIA* for  
four years to 1778.

Births.	Deaths.	Marriages.	Proportion of births to marriages.	Proportion of births to deaths.
10900	10935	2409	45 to 10	996 to 1000

Annual average of births, deaths, and marriages in the  
*country parishes and villages of SILESIA* for the same  
period.

Births.	Deaths.	Marriages.	Proportion of births to marriages.	Proportion of births to deaths.
53694	42894	11848	45 to 10	125 to 100

SILESIA appears from hence to consist of near two  
millions of inhabitants; of whom the inhabitants of towns  
are about a *sixth* part.

The following accounts (copied from the Tables at  
the end of the First Volume of Mr. *Susmilch's Gottliche  
Ordnung*, 3d Edition) will shew, in some measure, the  
usual progress of population in a country. They will  
also serve for a contrast to the inconsistent extracts which  
I have given from our parish registers; for it will appear  
that instead of shewing an increase too great for the sur-  
plus of births, they always (in consequence of sickly years  
and other causes) shew a much smaller increase than it  
was capable of producing.

In the old PRUSSIAN dominions and the provinces of  
*Brandenburg.*

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
4 years to 1701	66247	44680	18145	36 to 10	148 to 100
7 years to 1728	82934	60821	20726	40 to 10	136 to 100
6 years to 1756	102935	78865	24487	40 to 10	136 to 100
In					

so much as in this. Few kingdoms have been engaged within so short a period in so many desolating wars. Few kingdoms have had such armies and garrisons and settlements to maintain in so many distant regions, and

In the kingdom of *Prussia* and dukedom of *Lithuania*.

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
10 years to 1702	21963	14718	5908	37 to 10	150 to 100
5 years to 1716	21602	11984	4968	39 to 10	180 to 100
5 years to 1756	28392	19154	5599	50 to 10	148 to 100

*N. B.* In 1709 and 1710 a pestilence carried off 247,733 of the inhabitants of this country; and in 1736 and 1737 epidemics prevailed, which again checked its increase.

In the *Churmark* of BRANDENBURGH.

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
5 years to 1702	13433	7605	3597	37 to 10	176 to 100
4 years to 1756	23486	18840	6646	38 to 10	124 to 100

Duchy of POMERANIA.

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
6 years to 1702	6540	4647	1810	36 to 10	140 to 100
6 years to 1708	7455	4208	1875	39 to 10	177 to 100
6 years to 1726	8432	5627	2131	39 to 10	150 to 100
4 years to 1756	12767	9281	2957	43 to 10	137 to 100

In this instance the inhabitants appear to have been almost doubled in 56 years, no very bad epidemic having once interrupted the increase; but the three years immediately following the last period (to 1759) were years so sickly that the births were sunk to 10,229, and the burials raised to 15,068.

Neumark

and in such unhealthful climates. No kingdom ever supported such a navy, or carried on so extensive a foreign commerce, or wanted, on these accounts, such a supply of *men* for the sea-service : Nor was there ever a king-

Neumark of BRANDENBURG.

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
5 years to 1701	5433	3483	1436	37 to 10	155 to 100
5 years to 1726	7012	4254	1713	40 to 10	164 to 100
5 years to 1756	7978	5567	1891	42 to 10	143 to 100

Epidemics prevailed for 6 years from 1736 to 1741, which checked the increase.

Dukedom of MAGDEBURG.

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
5 years to 1702	6431	4103	1681	38 to 10	156 to 100
5 years to 1717	7590	5335	2076	36 to 10	142 to 100
5 years to 1756	8850	8069	2193	40 to 10	109 to 100

The years 1738, 1739, 1740, 1741, 1750, and 1751 were particularly sickly.

Duchy of HALBERSTADT.

Annual average.	Births.	Burials.	Marriages.	Births to marriages.	Births to burials.
4 years to 1692	2366	1478	604	39 to 10	160 to 100
5 years to 1746	2803	2052	712	39 to 10	136 to 100
6 years to 1756	2917	2621	778	37 to 10	111 to 100

Duchy of RAVENSBERG.

Annual average.	Births	Burials.	Marriages.	Births to marriages.	Births to burials.
5 years to 1692	3899	2552	964	40 to 10	152 to 100
4 years to 1756	5041	3814	1371	36 to 10	132 to 100

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a kingdom which consisted so much of people employed in trades and manufactures, which

## Dukedom of CLEVE and County of *Mark*.

Annual average	Births.	Burials.	Marriages.	Births to marriages.	Births to deaths.
4 years to 1701	6249	4132	1729	36 to 10	151 to 100
5 years to 1739	7358	5535	1741	42 to 10	134 to 100
4 years to 1756	7612	5567	1966	38 to 10	136 to 100

## AUSTRIAN MILANESE ;

Consisting in 1774, of 211,479 families, and 1,116,859 inhabitants ; and in 1769, of 1,101,723 inhabitants, of whom 9638 were priests, 5616 friars, and 7140 monks and nuns.

Annual average of	Births.	Burials	Marriages.	Births to marriages.	Births to deaths.
1769, 1773 and 1774	44030	40030	9619	45 to 10	110 to 100

N. B. The last of these years appears to have been particularly sickly ; for the burials exceeded the births, and were 9156 higher than the average of the years 1769 and 1773.

## DENMARK.

Annual average of	Births.	Burials.	Births to burials.
5 years to 1747	22996	18864	121 to 100
5 years to 1756	24298	21706	112 to 100

Epidemics prevailed in 1755, and 1756, which made the burials in those years nearly equal to the births.

The medium of these ten years is nearly 20,000 ; and, multiplying it by 35, will make the number of inhabitants then in *Denmark* 700,000.

## NORWAY.

Annual average of	Births.	Burials.	Births to burials.
5 years to 1747	17522	10955	160 to 100
14 years to 1756	19947	14661	136 to 100

Multiplying 16000 (the average of burials in *Norway* for four years to 1756) by 35, will make the number of inhabitants 560,000 in 1756. In



which shorten life, or whose metropolis was so large, or *half* so large, in comparison with the number of its inhabitants.—If we include in LONDON all the parishes and little towns near LONDON, where, almost universally, the burials exceed the births, it is moderate to reckon that the former exceeds the latter in this part of the kingdom about 10,000 annually; and that, consequently, LONDON demands a recruit of people every year equal to this number. Forty years ago there was this excess of burials within the bills only. This will make the annual surplus for the whole kingdom 26,000 which may probably be sufficient, or perhaps more than sufficient, to supply all the waste occasioned by sickly seasons, emigrations to the colonies, and the other causes I have mentioned.—But the truth is, that it cannot be reckoned with any degree of

In 1056 country parishes and villages in the *Churmark* of *Brandenburgh*, consisting (in 1748) of 106,204 males and 107,540 females.

Annual average of	Births.	Burials.	Marriages.	Births to marriages.	Births to burials
10 years to 1748	7099	5561	1966	36 to 10	127 to 100

In seven market-towns and 54 country-parishes in *England*, consisting (in 1740) of 10434 families and 46,650 inhabitants, according to Dr. *Short's New Observations*, p. 133.

Annual average.	Births.	Burials.	Marriages.	Births to marriages.	Births to burials.
In 1748	1575	1360	399	40 to 10	115 to 100

confidence, that there exists any such surplus.

Mr. King, in 1693, stated the births of the kingdom, exclusive of those in *London*, at 170,000, and the burials at 148,000, which makes the proportion of the former to the latter as 115 to 100. See Dr. *Davenant's Works*, Vol. II. p. 180. Mr. King deduced this from the assessments then imposed on births, marriages and burials; and he has shewn such sagacity in his other estimates, that I cannot help paying some regard to him in this. If he was right, the kingdom has probably been decreasing, such a surplus being incapable of supporting a population so encumbered as ours, and which ever since Mr. King's time has had such increasing demands upon it.

I cannot help taking this opportunity to observe, that there is reason to believe that poor countries (provided the ground supplies them with plenty of food, and the poverty of the inhabitants consists only in their wanting *conveniencies* and *elegancies*, in other countries deemed *necessaries*) increase faster than *rich* countries. The reason is obvious. The greatest enemies of population are the artificial wants, the accumulation of property, and the luxury and vices which are the constant attendants of opulence, and which prevent a regular and early union between the sexes. The inhabitants of poor countries are more simple,  
more

more healthy, and more virtuous; and, wanting little besides food, families are no burdens, and the prolific powers of nature have free scope to display themselves.—Perhaps IRELAND is one instance of this. If we may depend on an account in the Philosophical Transactions (Abridgement, Vol. III. p. 666.) the number of people in *Ireland*, in 1695, did not much exceed a million. At present they are, I suppose, about two millions.—According to an account published annually at *Dublin*, in *Watson's Almanack*, the houses in *Ireland*, in 1754, were 395,439. In 1767 they were increased to 424,046; and in 1777 to 448,426. But I have been informed that this account is of no authority, and deserves little credit. Nor can I learn that there are in *Ireland* any *documents* from which a judgment tolerably correct can be formed of the progress and present state of its population. It might have been expected, that the hearth-tax would have furnished such *documents*: But this is not the case; and all that is known with certainty is the yearly produce of the tax; the average of which for the last five years to 1781, having been 60,648*l*. makes the number of hearths that pay the tax (at 2*s*. per hearth) to be 606,480. It is supposed that a house may be allowed for every *two* hearths, and that a *third* of the houses are excused on account of inability



and, on these suppositions, the number of houses will exceed 400,000 (*a*); and, consequently, the inhabitants will be (as just reckoned) about two millions (*b*).

(*a*) In the year 1787 the following account was returned to the House of Commons of Ireland, of the number of houses in that kingdom paying hearth-money.

No. of Houses containing Hearths.	No. of Houses containing Hearths.	No. of Houses containing Hearths.	No. of Houses containing Hearths.
1 397,644	15 99	29 4	45 4
2 24,031	16 127	30 16	46 1
3 7,562	17 46	31 4	50 3
4 5,542	18 42	32 4	55 1
5 4,062	19 23	33 6	56 1
6 3,556	20 61	34 3	67 1
7 * 3,330	21 13	35 3	92 1
8 2,209	22 10	36 6	112 1
9 985	23 9	37 1	Houses exempted by law 23,975
10 772	24 20	39 1	
11 316	25 20	40 7	
12 295	26 10	41 3	
13 147	27 5	42 3	
14 139	28 8	44 2	

From this table it appears that the number of hearths (exclusive of those exempted by law) is 612,577; and therefore, on the supposition adopted in this postscript, the whole number of houses in Ireland will be 408,384.—But if the preceding accounts be accurate, their real number amounts to 474,234, and consequently the inhabitants will rather exceed two millions and a quarter. Ed.

(*b*) A survey of BELFAST was made in Jan. 1782, from which it appeared, that it consisted of 2026 houses, containing 13,105 inhabitants, 6133 of whom were males, and 6972 females.—Looms 388; and houses for selling beer and spirits 119, or a 17 part of all the houses.—On Jan. 1, 1757, the number of looms was 399, and the houses 1779, containing 8549 inhabitants, of whom 7993 were *Protestants*, and 556 *Papists*.

*Sweden,*



*Sweden, Norway (a)*, and the kingdom of *Naples*, are increasing fast; and also *RUSSIA*, if we may judge from the following facts.

In the viceroyalty of *Tweer* (in 1780) there died 4315 males; 3646 females; but there were born 11948 males, and 9013 females. The marriages were 6074.

In the eparchy of *Vologda* the deaths in the same year were 2688 males, and 2377 females. The births were 6517 males, and 5366 females. The marriages 3232.

In both these provinces, therefore, the births were considerably more than *double* the deaths; and the increase must be rapid.

At the beginning of the same year (1780) there were found in the district of *Moscow* 137,698 males, and 134,918 females; of whom died in the course of the year 2101 males and 1601 females, or the 65th part of the males, and 84th part of the females. But there were born in the course of the year 4546 males, and 4075 females, which added 5919 (or a 46th part) to the inhabitants; and the number of inhabitants actually counted at the end of the year was 140,143 males, and 137,392 females (b)

(a) See the Preliminary Observations to Table XLII. p. 146; and the Essay on the *Population of England*, p. 14.

(b) These accounts have been given by authority in *RUSSIA*; and were communicated to me by Mr. *Howard*; who with views of unparalleled humanity, travelled through that country in 1781.—To Mr. *Howard's* enquiries I likewise owe the account in the note p. 335 of *SILESIA*.

But there exists probably among mankind no such increase as that among the United States of NORTH-AMERICA, according to the account of it in Vol I. p. 276, &c.

The reflection on these facts must be mortifying to this country (the richest upon earth) if it be indeed true that our population is declining. But we must comfort ourselves by considering that in this case, *value* is of more consequence than *number*. Commerce, arts, and liberty, once placed the little state of *Athens* at the head of the world; and the same causes once raised this island to the same eminence.

To the direct evidence already stated of a decrease in our population, it is proper to add the following facts.

1st. The decrease of LONDON. This I must reckon certain, till some other satisfactory reason (*a*) can be given for a diminution since 1727, of more than 7000 *per ann.* in the registered burials, and near 2000 in the registered births.

(*a*) The new burying grounds (taken notice of in the Notes p. 255 and p. 260, Vol. I.) have been opened but lately; and therefore, cannot account for this diminution; nor do the burials in them amount to a number equal to it.

Annual medium of registered burials in LONDON.

For five years to 1722 inclusive	26,443
to 1727	26,747
to 1732	26,582
	Annual

Secondly. The decrease in the produce of the hereditary and temporary excise upon beer. This was almost the only excise that existed before the Revolution; and though the country was then poorer, it produced a *quarter* more than it has lately. This fact, together with the objections to the inference I have drawn from it, may be found distinctly stated in the Essay on the *Population of England*, &c. p. 18, &c. and p. 45, &c.

Thirdly. The growing distress among the lower orders of people, who are the majority of the nation, deserves to be parti-

For five years to 1737	26,848
to 1742	28,344
to 1748	23,884
to 1753	22,006
to 1758	20,875
to 1763	22,593
to 1768	23,319
to 1773	22,754
For four years to 1777	20,945
For three years to 1780	20,438
For two years to 1782	19,313

Annual medium of registered births in LONDON.

For five years to 1727	18,898
to 1768	16,291
to 1782	16,966

The decrease which this Table shews to have taken place lately in the excess of burials above the births, has been ascribed to an improved state of LONDON with respect to its influence on the health of its inhabitants; but the true reason is the fact referred to at the beginning of this note.

cularly



cularly attended to on this subject. The increase of the poor rates proves this fact; and it seems to be universally acknowledged. A people at their ease will increase; but increasing difficulties in procuring the means of subsistence, producing a forced industry, and an aversion to marriage, must depopulate.

The increased produce of the taxes on candles, leather, &c. the inclosures of waste lands, and the improvements in agriculture which have taken place lately, have been urged in opposition to these facts. But I am afraid they only prove that luxury has increased consumption more than it has lessened the number of our people.

Upon the whole. I beg it may be remembered, that my opinion, in this instance, is by no means a clear and decided conviction. I may probably be influenced too much by a desire to maintain an assertion once delivered.—Some time or other, perhaps, the Legislature will think this a point worth its attention. Much light may be thrown upon it, and the state of our population kept constantly in view, by only ordering exact registers to be kept of the births, burials, and marriages in the kingdom. This is done in other kingdoms. It has lately been done in *France*; and the result has been a discovery that the population of FRANCE exceeds all that had been conjectured

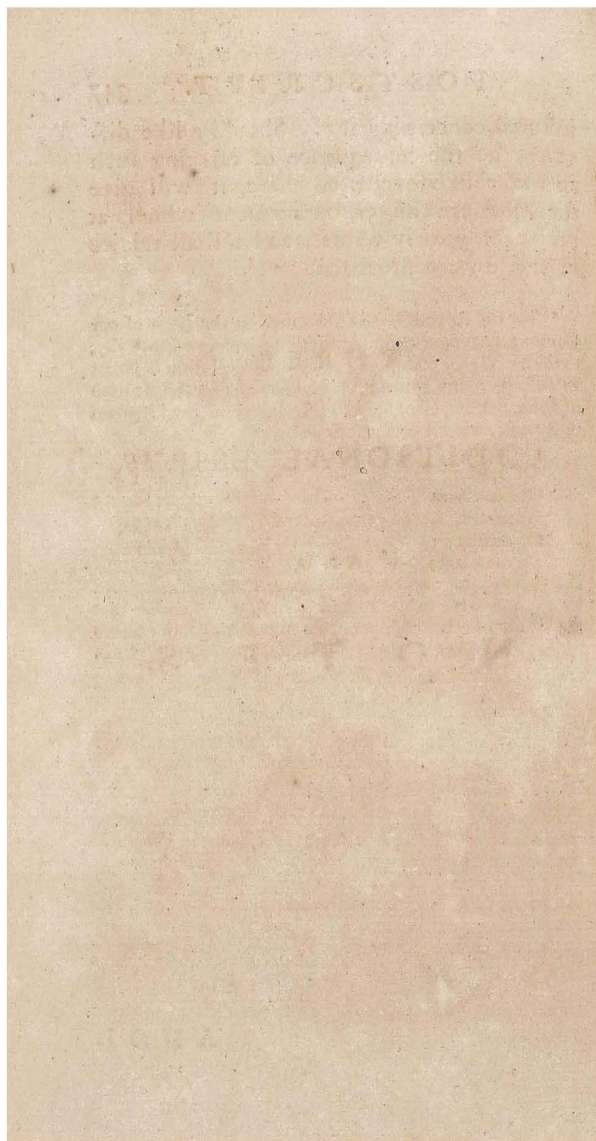


jectured concerning it \*. Should a like discovery be the consequence of carrying such an order into execution here, it will give the kingdom an encouragement which at present it greatly wants ; and I shall rejoice in my own confutation.

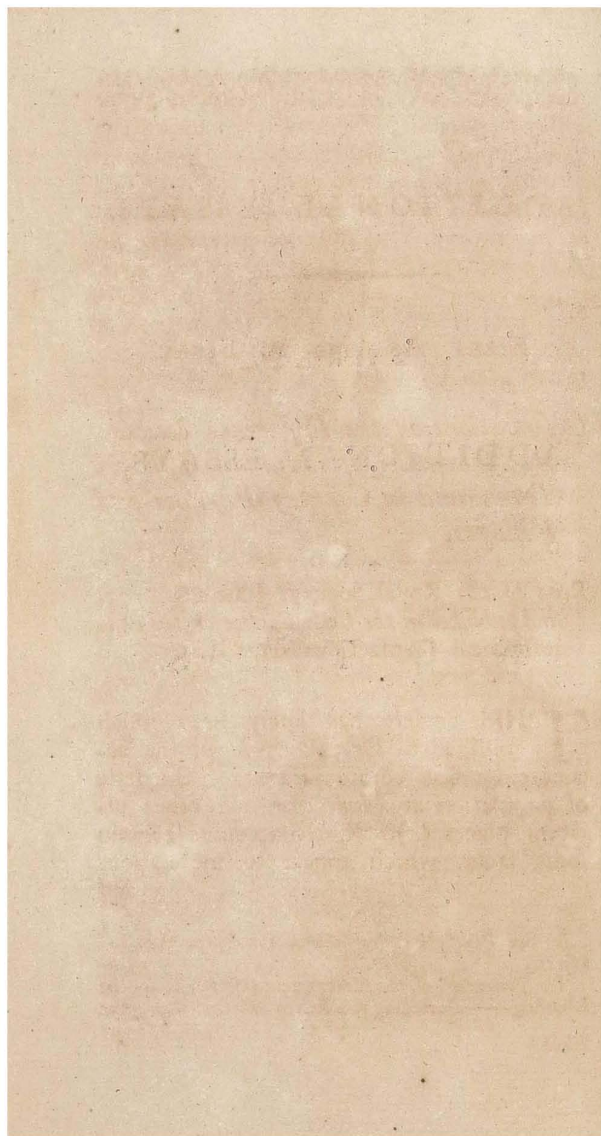
\* See the Appendix to a Discourse on the Love of our Country, delivered by the Author on November 4th, 1789, to the Society for commemorating the Revolution in Great Britain.—In this Appendix it is observed, that the medium of annual deaths, births, and marriages, in the kingdom of France, was

Of births for four years, to 1774	914,710
Of deaths — — —	793,931
Of marriages — — —	192,180
Of births, for six years, to 1780	958,419
Of deaths — — —	834,865
Of marriages — — —	228,170

If 834,865, the number of deaths to 1780, be multiplied by 35, agreeable to the rule in p. 326, it will appear that the whole number of inhabitants in this kingdom exceeds *twenty-nine millions.* Ed.



THREE  
ADDITIONAL ESSAYS,  
AND  
NOTES.







## ADDITIONAL ESSAYS.

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### FIRST ADDITIONAL ESSAY.

*Observations on the Difference between  
the Duration of Human Life in  
Towns and in Country Parishes and  
Villages.*

*Read to the Royal Society, June 22, 1775,  
and published in the 65th Volume of the Phi-  
losophical Transactions, Part II.*

THIS Society has lately been much obliged to Dr. *Percival*, for the accounts he has communicated of the state of population at *Manchester* and other adjacent places (a). These accounts contain some facts, which appear to me curious and

(a) See *Philosophical Transactions*, vol. 65, p. 322, and vol. 64, p. 57.

The particulars of the surveys here referred to are the following.—According to a survey executed with great care

and important. From the last in particular, there appears to be reason for concluding, that whereas a 28th part of the inhabitants die annually in the town of *Manchester*, not more than a 56th part die annually in the adjacent country. This implies a difference so great between the rates of human mortality in these different situations, that some, whose judgements I reverence, have thought it incredible. I will, there-

care there were, in the summer of 1773, in the town of

<i>Manchester,</i>			<i>Salford,</i>
3402	—	Houfes	866
5317	—	Families	1099
10548	—	Males	2248
11933	—	Females	2517
7724	—	Married	1775
432	—	Widowers	89
1064	—	Widows	149
7782	—	Under 15	1793
3252	—	Above 50	640
342	—	Male Lodgers	18
150	—	Female Lodgers	13
44	—	Empty Houfes	26

According to a survey in 1774 there were in the parish of *Manchester*, containing thirty-one townships, exclusive of the towns of *Manchester* and *Salford*.

Tenanted Houfes	—	2371	Under 15	—	5545
Families	—	2525	Above 50	—	1762
Inhabitants	—	13786	Above 60	—	470
Males	—	6942	Above 70	—	261
Females	—	6844	Above 80	—	87
Married	—	4319	Male Lodgers	—	68
Widowers	—	232	Female Lodgers	—	51
Widows	—	315	Empty Houfes	—	41

fore,

therefore, beg leave to offer the following observations on this subject.

In the first place, the evidence in this instance is such as seems to leave little room for doubt. From an accurate survey it appears, that the number of inhabitants in the town was 27,246 in the year 1773. The number of deaths the same year (and also the average for 1772, 1773, and 1774), was 973 (*a*); that is, a 28th part of the number of inhabitants. From an equally careful survey it appears, that the number of inhabitants in that part of the parish of *Manchester* which lies in the country, was 13,786. The number of deaths in 1772 was 246; that is, a 56th part of the number of inhabitants. The chief objection to this evidence is, that the number of deaths in that part of the parish which lies in the country is given only for one year; whereas the average of several years ought to be given.

(*a*) The numbers of burials in the town, including the addition of 50 every year for Dissenters, was, in

1772,	—	954
1773,	—	973
1774,	—	1008

Within the parish, but out of the town, there are 13 episcopal and dissenting chapels; and the number of burials in all these chapels, in 1772, was 246. The christenings were 401. The number of burials brought from the country into the town is not considerable; and it is, I am informed, pretty exactly balanced by the burials carried out of the town into the country.

But first, the number of deaths in 1772, in the town, was nearly the same with the medium for seven years; and from hence there arises a probability, that in the adjacent country, the number of deaths, in the same year, could not have been much lower than the medium. Secondly, supposing it lower, there is the highest probability, that it was not more than a 4th or 5th lower. Suppose then the true annual medium to be 300, instead of 246, and it will follow, that whereas a 28th part of the inhabitants die in the town annually, a 46th part die in the country; and this is a difference very considerable. But farther, I would observe, that the difference which this survey gives between the rate of mortality in the town of *Manchester* and the adjacent country, is confirmed by a variety of other accounts. It may be stated in general, that whereas in great towns, the proportion of inhabitants dying annually is from 1 in 19 to 1 in 22 or 23, and in moderate towns from 1 in 24 to 1 in 28 (a); in country parishes and villages, on the contrary, this proportion seldom exceeds 1 in 40 or 50. The

(a) The number dying annually in towns is seldom so low as 1 in 28, except in consequence of a rapid increase produced by an influx of people, at those periods of life when the fewest die. This is the case at *Manchester*. It is also the case at *Liverpool* and at *Berlin*; in the former of which towns, 1 in 27 dies annually; and in the latter, 1 in 26½ died from 1755 to 1759. See Vol. I. of this Treatise, Essay I. page 250—295.



proofs of this are numerous and unexceptionable; and I have elsewhere given a particular account of them. I will here only mention the following facts.

The number of inhabitants at *Stockholm* in 1763 was 72,979. The average of deaths for the six preceding years had been 3802 (a). One, therefore, in nineteen died there annually.

At *Rome*, an account is taken every year of the number of inhabitants; and, in the year 1771, it was 1,59,675. The average of deaths for ten years had been 7367. One, therefore, in  $21\frac{1}{2}$  died annually.

(a) See a Memoir by M. *Wargentin*, in the 15th volume of the *Collection Academique*, printed at *Paris*, 1772. From this memoir I learn, that in 1757, and 1760, and 1763, a survey was made of the inhabitants of *Sweden*, distinguishing, particularly, the numbers of both sexes living at every age; and that also, for nine years (or from 1755 to 1763), an exact Register was kept of the number of births and burials in each year, distinguishing the age and sex of every one that died. The result, as given by M. *Wargentin* in this Memoir, contains indeed a most curious account of the state of population in *Sweden*; and it is particularly to my present purpose to mention, that it shews, that though a 19th part of the inhabitants of *Stockholm* die every year, yet in the whole kingdom, taking all the towns and country together, not more than a 35th part die every year. In 1757, *Sweden* consisted of 1,101,595 males, and 1,221,600 females; in 1760, of 1,121,053 males, and 1,246,445 females; and in 1763, of 1,165,489 males, and 1,280,905 females. The annual average of births, from 1755 to 1763, was 46,223 males, and 44,017 females; of marriages, 21,219; of deaths, 34,088 males, and 35,037 females.

In *London* I have shewn, with an evidence which I think little short of demonstration, that at least 1 in  $20\frac{1}{4}$  of the inhabitants die annually (a). And, from a particular survey and a very accurate register of mortality at *Northampton*, it appears, that 1 in  $26\frac{1}{2}$  die there annually.

Let these facts be compared with the following. In 1767, a survey was made of the inhabitants of the island of *Madeira*, under the direction of Dr. *Thomas Heberden*, and their number was found to be 64,614. The average of burials for eight preceding years had been 1293. Only 1 in 50, therefore, of the inhabitants died annually (see *Philosophical Transactions*, vol. lvii. p. 461.)

The district of *Vaud*, in *Switzerland*, in 1766, contained 112,951 inhabitants. The average of deaths for ten preceding years had been 2504. Only 1 in 45, therefore, died annually (b).

The number of inhabitants in the parish of *Ackworth*, in the county of *York*, in 1757, was 603; and the average of deaths for ten years had been  $10\frac{7}{10}$ , or a 56th part. In 1767, the inhabitants were increased to

(a) See Volume I. of this work, Essay IV. page 267, &c.

(b) See M. *Muret's* Memoir on the State of Population, in the *Pays de Vaud*, printed at *Bern*, in 1766.

728; and the annual average of deaths was  $15 \frac{3}{10}$ , or nearly a 47th part. (a)

The reason of this striking difference between the rate of human mortality in towns and in country parishes and villages must be, first, the luxury and the irregular modes of life which prevail in towns; and, secondly, the foulness of the air. But it has been inquired, whether the migrations of people from the country to towns may not produce this difference, by lessening the proportion of inhabitants that die in the country, and increasing the same proportion in towns? In answer to this enquiry I would observe; first, that this difference being a difference of near a half, it is apparently much greater than can be accounted for by any such cause. But, secondly, it should be considered, that if migrations lessen the number of deaths, they also lessen the number of inhabitants; and that it depends intirely on the ages at which the inhabitants remove from a place, whether the effect of their removal shall be lowering or raising the proportion of the annual deaths to the number of inhabitants. In the present case, the truth appears to be, that the most common age of migration

(a) I owe this information concerning the parish of *Ackworth* to a curious Register kept there by Dr. *Lee*. I have taken the liberty to insert this register in the Post-script, together with the annual register and survey of *Rome* from 1762 to 1771.

from the country is such as raises this proportion in the country. This will be evident from the following considerations. The period of life in which persons remove from the country to settle in towns is chiefly the beginning of mature life, or from the age of 10 or 15 to 25 or 30. Towns, therefore, will be inhabited more by people in the firmest parts of life; and, on the other hand, the country will be inhabited more by people in the weakest parts of life; and the consequence of this is, that in the country, the inhabitants must die faster in proportion to their number than they otherwise would, and that in towns they must die more slowly. In particular, the number of children is always much greater in the country than in towns; and this is a circumstance which must be extremely unfavourable to the former: for it is well known, that there are no years of life, in which so many die as the first three or four years. Till the age of five, human life, like a fire beginning to burn, is very feeble; and in some situations more than half, and in others, a third or fourth of all that are born die before that age. After this, life grows less and less precarious till it acquires its utmost vigour at 10 or 12; and of the living at this age, not above 1 in 70 or 80 dies annually in the worst situations; and in the best situations, not  
above



above 1 in 150 or 160. After 15, life declines, and continues to do so more and more, till it becomes quite extinct in old age. If therefore, in any situation, the inhabitants consist more of persons in mature life, and yet die faster, it must be owing to some particular causes of mortality that operate there. This is the case in all towns where, any observations have been made. *Manchester*, in particular, is not only kept up, but increases fast, by removals to it of persons in the prime of life. The country round it increases likewise; but it is by an excess of the births above the deaths; that is, by accessions to it of children in the very feeblest part of life. This ought to raise the proportion of annual deaths to inhabitants in the country, much above the same proportion in the town; but, instead of this, it is near one-half lower.

It may be needless to add any thing to these observations.

In order, however, to put this matter out of all doubt, I will observe farther, that it appears in fact, from the accounts furnished by Dr. *Percival*, that the number of inhabitants in the period of life when mankind die fastest (a) (that is in the first and last stages

(a) In towns, about a fourth of the inhabitants die commonly between 14 and 51; a fifth or sixth die at 51 and upwards; and the remainder die under 15. In country

stages of life), is considerably less in the town of *Manchester* than in the adjacent country. The number of inhabitants in the town under 15 and above 50, is 13,467; in the country, 7305. And the whole number is, in the town, 27,246; in the country, 13,786. In the town, therefore, the inhabitants, in the first and last stages of life, do not make half the whole number; but in the country they make considerably more than half. At *Ackworth*, likewise, in *Yorkshire*, the inhabitants under 15 and above 50 are more than half the whole number; and the same is true at *Hale* near *Altringham*; at *Horwich*; at *Darwen*, near *Blackburn*, in *Lancashire*; and at *Cockey Moor* (a), near *Bolton*,

try parishes and villages about a fifth die between 14 and 51; about two-fifths at 51 and upwards; and the remainder under 15.

(a) I am much indebted to Dr. *Percival* for the following account of these places. The society belonging to the chapel at *Hale* is composed of 140 males, 136 females, 92 married persons, 8 widowers, 12 widows, 105 under 15, and 41 above 50. The deaths, during seven years, have been 28, and the births 68. Mr. *Evans's* congregation at *Horwich*, consists of 305 individuals; viz. 149 males, 156 females, 94 married persons, 9 widowers, 8 widows, 127 under 15 years of age, and 50 above 50. The births, for seven years, 101; the deaths 32. A 66th part, therefore, die annually in both these places. The Rev. Mr. *Smalley's* congregation at *Darwen*, consists of 1850 individuals; viz. 900 males, 950 females, 640 married persons, 30 widowers,  
48 wi-

*Bolton*, in the same county ; and yet in some of these places it appears, that not a 60th part of the inhabitants die annually.

At

48 widows, 737 persons under the age of 15, and 218 above 50. During the last seven years the births have amounted to 508, the deaths to 233. A 56th part, therefore die annually. Mr. *Barnes's* congregation at *Cockey Moor*, consists of 154 families and 711 individuals ; namely, 320 males, 391 females, 248 married persons, 10 widowers, 27 widows, 252 persons under the age of 15, and 99 above 50. Deaths in seven years 114 ; in which period the deaths were considerably increased by an uncommon fatality of the small pox. One person in 44 died annually. The Rev. Mr. *Mercer's* congregation at *Chowbent* in *Lancashire*, consists of 1160 persons ; viz. 554 males, 606 females, 173 males and 150 females under the age of ten, 83 males and 91 females above 50, 398 married persons, 26 widowers, and 43 widows. The baptisms during six years, wanting six weeks, have amounted to 293, and the deaths to 169. One person, therefore, in 41 died annually. These surveys were made in the year 1773.—In August 1774 the inhabitants of *Tattenhall* and *Waverton* (two parishes in the neighbourhood of *Chester*) were surveyed. The former consisted of 382 males and 399 females, of whom 462 were above 14 years of age. The latter contained 310 males and 322 females, of whom 406 were above 14 years of age.—At *Tattenhall* the annual average of christenings, for 10 years ending in 1773, had been 28 ; of burials, 13.—At *Waverton* the same average had been 19  $\frac{3}{4}$  and 8  $\frac{1}{10}$ .—In the former parish, therefore, a 60th part of the inhabitants, and in the latter a 75th part had died annually.—In 1775 the town and parish of *Ashton under Line* (distant 8 miles from *Manchester*, and consisting of manufacturers and farmers) were surveyed. The number of inhabitants was 5097, of whom 2534 were males, and 2513 females ; 1679 were married ; and their ages were, under five, 896—from 5 to

At *Stockholm*, in 1763, the inhabitants under the age of 5 were only a 12th; above 70, only a 46th part of the whole number. But in all *Sweden*, the number under 5 was a 7th; and above 70, near the 32d part of all the inhabitants: and yet 35 die in the town to 19 in the whole kingdom. This may be easily deduced from Table I. in the Postscript.

To the accounts which give the proportion of inhabitants to annual deaths so high as 50 or 60 to 1, it has been farther objected, that if true, it must follow, that in such situations half the inhabitants must live to 50 or 60 years of age. But were this a right inference, there would be nothing in it incredible. For though in most cities one-half die in the first two or three years after birth; yet, in many country si-

10, 764—from 10 to 20, 1011—from 20 to 50, 1882—from 50 to 70, 471—from 70 to 90, 73. Of these 2700 at least, or more than half, must have been under 15, and above 50.—See a communication of Dr. *Percival's* in the *Philosophical Transactions*, vol. 66, p. 160.

I will add here that, according to an accurate survey communicated to me by one of the gentlemen concerned in making it, of the township of *Leeds*, in *Yorkshire*, it consisted (in 1775) of 15,216 inhabitants in the town, and 1905 inhabitants in the villages and country near the town. The number of males was 8112; of females 9009; of whom 6309 were married; 724 were widows, and 417 widowers; 1333 were females, and 861 males above 20 who had never married; and 3765 were girls, and 3712 boys under 20.

tuations.



tuations, the greater part live to marry : and in the parish of *Ackworth*, particularly, it appears with undeniable evidence from the Register, that one-half of all born there live to the age of 46. It appears also, with equal evidence, from M. *Muret's* Tables in the *Bern* Memoirs for 1766, that in 43 parishes in the district of *Vaud*, one-half of all born there live beyond the age of 41. In truth, did all mankind lead natural and virtuous lives, that waste of the species which happens in infancy and childhood would not take place, and few would die except in old age. The inference, however, which I have mentioned, cannot be made with reason. It is just only in the particular case of an uniform decrease in the probabilities of living from birth to old age ; and this is a case that has never existed. In all other cases, there is not any necessary connexion between the proportion of inhabitants dying annually, and the age to which the greater part live. In most cities one-half, as I have just observed, of all that are born die before two or three years of age. But it cannot be imagined, that there is any place where so many as one-half or a third of the inhabitants die every year.

But to return to Dr. *Percival's* account of the town and parish of *Manchester*. It appears from this account, that the number of children under 15 compared with the  
number

number of inhabitants between 14 and 51, is greater in the country than in the town of *Manchester*, in the proportion of no less than 5 to 4 (a). It follows, therefore, that though in consequence of a constant influx of people to the town, it is more filled than the country with inhabitants in the most vigorous periods of life; yet one child in four less is born in the town than in the country. This is a remarkable circumstance, and the reasons of it must be the two following. First, the town inhabitants being less healthy, and dying faster, have not the same strength of constitution with the country inhabitants. Secondly, in the town a smaller proportion of the adult inhabitants marry; and they marry later than in the country. The survey fully proves this; for it appears, that though the number of inhabitants at the most common marrying ages, compared with the whole number of the living above the age of 14, is smaller in the country than the town; yet the proportion of the married to the living above 14, is very nearly the same in both situations.

(a) In the town the number of inhabitants between 14 and 51 is 13,779; and 9575 under 15. In the country the former number is 6481; and the latter, 5545. But the last number would have been only 4503, had the proportion of the inhabitants between 14 and 51 to the inhabitants under 15 been the same in both situations. It is owing to this, that the number of persons in a family in the country is  $5\frac{1}{2}$ ; but in the town only  $4\frac{3}{4}$ .

And

And there are more widows and widowers in the town than in the country in the proportion of near 16 to 11. We learn from hence, I think, clearly, in what manner towns operate in checking population, and preventing the increase of mankind.

Dr. *Percival* informs us, that the reverend and learned Dr. *Tucker* has been led, by some observations he has made at *Bristol*, to doubt whether the common opinion is right, with respect to the disproportion between the number of male and female births; and that he, therefore, wishes a farther inquiry may be made into this subject. This has induced me to collect the following facts, which, I think, will abundantly settle this point.

	Born Males.	Females.	Proportion.
In <i>London</i> for the last 110 } years, or from 1664 to } 1773	862293	817072	20 to 19
<i>Paris</i> , for 8 years (a),	79693	76481	25 to 24
<i>Leyden</i> , for 50 years (b),	46773	44933	26 to 25
<i>Vienna</i> , for 27 years, ending } 1746 (c),	67060	64893	31 to 30
<i>Berlin</i> , for 40 years, ending } 1761 (d),	71188	67431	20 to 19
<i>Kurmark of Brandenburg</i> , } for 9 years, ending 1759 } (e),	102425	96521	18 to 17

(a) See *Susm. Gottliche Ordnung* Tables, p. 16.

(b) *Ibid.* p. 17.

(c) *Ibid.* p. 13.

(d) *Ibid.* p. 12.

(e) *Ibid.* p. 3.

	Born Males.	Females.	Proportion.
Dukedom of <i>Magdeburgh</i> , for 38 years, ending 1759 (a),	153227	145985	21 to 20
All the <i>Prussian</i> towns, for a course of years, (b),	691826	659072	21 to 20
In a great number of coun- try parishes, for a course of years (c),	59067	56282	21 to 20
In the same country pa- rishes, for another pe- riod of years (d),	89530	84954	19 to 18
<i>Leeds, Manchester, Coventry</i> , &c. for a period of years (e),	108784	103449	20 to 19
In the same towns, for ano- ther period (f),	57084	54128	20 to 19
Total	2388950	2271201	20 to 19
<i>Sweden</i> , for 9 years, ending 1763,	416007	396124	20 to 19

Mr. *Derham*, in his *Physico-Theology*, p. 175, has stated the proportion of male to female births at 14 to 13, and this proportion has ever since been generally received as the true one; but it appears from this Table, that it ought to have been stated at 20 to 19. But though it appears that the number of males born is in this proportion greater than the number of females born, yet, in most places, the number of

(a) See *Susm. Gottliche Ordnung* Tables, p. 5.

(b) *Ibid.* p. 9.

(c) See Dr. *Short's* New Observations, p. 27. 31.

(d) *Ibid.* p. 30. (e) *Ibid.* p. 49. (f) *Ibid.*



males living has been found to be less than the number of females. The reason is, without doubt, that males are more short-lived than females; and this owing partly to the peculiar hazards to which males are subject, and their more irregular modes of life; but it is owing principally to some particular delicacy in the male constitution which renders it less durable: For there are many observations which prove, that the greater mortality of males takes place chiefly in the first and last stages of life. A few facts of this kind I will beg leave to mention, because I have just met with them.

In the parish of St. *Sulpice*, at *Paris*, during 30 years, 5 males under a year old died to 4 females. But under 10, only 13 males died to 12 females (see *Susmilch*. Tables, vol. II. p. 30.)

In *Stockholm*, during 9 years ending in 1763, the number of still-borns amounted to 666; of whom 390 were males, and 276 females; that is, 10 to 7. The number of the living in that city above the age of 80 was, in 1760, 332; of whom 248 were females, and 84 males, or near 3 to 1. In the whole kingdom of *Sweden*, including all town and country inhabitants, the number of still-borns, during the 9 years just mentioned, was 19,845; of whom 11,424 were males, and 8421 females, or near 4 to 3. The number of the living in the whole king-

kingdom consisted of more females than males, in the proportion of 10 to 9. It consisted of more females turned of 80 than males, in the proportion of 33 to 19; and of more females turned of 90 than males in the proportion of near 2 to 1. See a Memoir of M. *Wargentin's* in the *Memoires abreges de l'Academie Royale des Sciences de Stockholm*, printed at *Paris* in 1772, p. 21. Having now had occasion to refer again to this Memoir, I will just add, that it appears, that by the excess of the births above the deaths, *Sweden* gains every year an addition of above 20,000 inhabitants; and that in six years they increased from 2,323,195 to 2,446,394. I am afraid, were regulations established for a similar inquiry in this kingdom, we should be far from finding our state so encouraging. *London* alone is a gulph which probably swallows up an increase equal to almost the whole increase (a) of *Sweden*.

RICHARD PRICE.

## P O S T S C R I P T.

THE following Tables have been selected from several more of the same kind in M.

(a) This is meant on a supposition which, I think, not extravagant, that the annual supply of people in mature life from the country, to keep up *London* and its environs, is 10,000. In order to provide this supply there must be about double that number born in the country.

*Wargen-*

*Wargentim's* Memoir on the state of population in *Sweden*. I have inserted them here, because they fully verify most of the observations in the preceding paper, and contain more distinct and authentic information on the subject of human mortality than I have ever before met with.

## TABLE I.

Shewing the Rate of human Mortality in Sweden.

	Annual deaths, being the average of three years, 1761, 1762, & 1763.		Number of the living in 1763.		
	Males.	Females.		Males.	Females.
Still-born,	1324	988	Born,	47216	44892
Died under 1	11172	9850	Living under 1	36994	35453
Died betw <sup>n</sup> . 1 & 3	4393	4336	Living betw. 1 & 3	66059	67234
3—5	2206	2249	3—5	66454	67711
5—10	2151	2057	5—10	130019	130758
10—15	933	834	10—15	126696	128021
15—20	711	658	15—20	108312	109985
20—25	834	756	20—25	92299	105115
25—30	883	863	25—30	88056	101003
30—35	1020	1146	30—35	85936	95811
35—40	955	923	35—40	74826	81453
40—45	1180	1170	40—45	67448	74854
45—50	1099	938	45—50	52398	59551
50—55	1280	1113	50—55	47298	56646
55—60	1177	1097	55—60	37086	45537
60—65	1586	1721	60—65	34892	44925
65—70	1237	1566	65—70	20649	28964
70—75	1322	2041	70—75	15454	23159
75—80	1092	1695	75—80	8858	13556
80—85	917	1446	80—85	4620	7487
85—90	414	650	85—90	1508	2694
Above 90	215	379	Above 90	527	988
Total of annual deaths,	36777	37488	Total of living at all ages,	1165489	1280905



In this Table it is observable, that the number of the living, in every equal division of life from birth, decreases continually till all become extinct; and that though the males born are more than the females born, in the proportion of 20 to 19; yet the males living of all ages are less in number, in the proportion of 1,165,489 to 1,280,905, or nearly of 10 to 11; notwithstanding which, the males that die annually are to the females as 52 to 53.

TABLE II.

Shewing the Rate of human Mortality at *Stockholm*.

	Annual deaths, being the average of three years, 1761, 1762, & 1763.		Number of the living in 1763.		
	Males.	Femal.		Males.	Females.
Still-born,	54	43	Born,	1406	1340
Died under 1	567	489	Living under 1	684	733
Died betw. 1 & 3	161	170	Living betw. 1 & 3	1173	1348
3—5	80	79	3—5	1022	1106
5—10	71	72	5—10	2630	2774
10—15	49	24	10—15	3151	2918
15—20	53	30	15—20	3018	2865
20—25	91	64	20—25	3070	4056
25—30	121	78	25—30	3380	4251
30—35	14	102	30—35	3705	4234
35—40	118	96	35—40	3019	3288
40—45	140	115	40—45	2846	3130
45—50	101	84	45—50	1775	1984
50—55	105	91	50—55	1581	2129
55—60	61	54	55—60	853	1329
60—65	79	88	60—65	826	1383
65—70	41	54	65—70	370	778
70—75	33	77	70—75	260	574
75—80	28	59	75—80	128	324
80—85	18	45	80—85	58	127
85—90	7	20	85—90	16	51
Above 90	3	11	Above 90	10	22
Total of annual deaths,	2068	1902	Total of living at all ages,	33575	39404

In this Table it may be observed, that the number living at every age from birth decreases only till five. Between 5 and 10 *Stockholm* begins to receive recruits from the country, and they come in faster and faster till 35; after which age it appears, that more die than come in; and that the living in every subsequent period goes on decreasing continually till the end of life. It is farther observable, that this Table exhibits a greater difference than the former, between the mortality of males and females.

A comparison of these Tables will shew a striking contrast in other respects between the state of human mortality in the whole kingdom of *Sweden* and in its capital. In order to make this more obvious and unexceptionable, I will add the following Table, deduced from all M. *Wargentin's* Tables taken together.

TABLE III.

In all Sweden for nine years.			In Stockholm for 9 years		
	Males.	Females.	Males.	Females.	
Still-born,	1 in 36	1 in 47	1 in 32	1 in 43 $\frac{1}{2}$	
Died under 1 of all born,	1 in 4 $\frac{1}{3}$	1 in 4 $\frac{4}{5}$	1 in 2 $\frac{1}{3}$	1 in 2 $\frac{3}{10}$	
Died annually of the } living betw <sup>n</sup> . 1 and 3 }	1 in 17 $\frac{1}{3}$	1 in 17 $\frac{3}{4}$	1 in 7	1 in 7 $\frac{1}{3}$	
Between 3—5	1 in 34 $\frac{1}{2}$	1 in 36	1 in 13 $\frac{1}{3}$	1 in 16	
5—10	1 in 71	1 in 76	1 in 34 $\frac{1}{2}$	1 in 39	
10—15	1 in 149	1 in 161	1 in 79	1 in 114	
15—20	1 in 149	1 in 164	1 in 59	1 in 99	
20—25	1 in 108	1 in 139	1 in 44	1 in 79	
25—30	1 in 98	1 in 113	1 in 33	1 in 58	
30—35	1 in 85	1 in 84	1 in 31	1 in 43	
35—40	1 in 78	1 in 91	1 in 26 $\frac{1}{2}$	1 in 39	
40—45	1 in 56	1 in 63	1 in 23	1 in 31	
45—50	1 in 49	1 in 65	1 in 19 $\frac{1}{2}$	1 in 28	
50—55	1 in 37	1 in 50	1 in 16 $\frac{1}{2}$	1 in 25 $\frac{1}{2}$	
55—60	1 in 31	1 in 40	1 in 14	1 in 24	
60—65	1 in 23	1 in 26	1 in 11	1 in 16	
65—70	1 in 17	1 in 18 $\frac{1}{2}$	1 in 9 $\frac{1}{2}$	1 in 13 $\frac{1}{3}$	
70—75	1 in 11 $\frac{1}{3}$	1 in 11 $\frac{1}{2}$	1 in 7 $\frac{3}{10}$	1 in 8	
75—80	1 in 8	1 in 8	1 in 4 $\frac{1}{2}$	1 in 5	
80—85	1 in 5 $\frac{1}{4}$	1 in 5 $\frac{1}{2}$	1 in 3 $\frac{1}{2}$	1 in 3 $\frac{1}{2}$	
85—90	1 in 3 $\frac{1}{5}$	1 in 4	1 in 2	1 in 2 $\frac{1}{10}$	
Above 90	1 in 2 $\frac{1}{2}$	1 in 2 $\frac{1}{2}$	1 in 2 $\frac{3}{10}$	1 in 2 $\frac{3}{5}$	
Died of all living at all ages	1 in 33 $\frac{1}{2}$	1 in 36	1 in 17 $\frac{1}{10}$	1 in 21 $\frac{1}{4}$	



A general Bill of all the Christenings and Burials in the Parish of *Ackworth*, in the County of *York*, extracted from the Parish Register, for ten Years, from March 25, 1747, to March 25, 1757.

In ten years christened, Males 62. Females 65. Total, 127.								
In ten years buried, Males 58. Females 49. Total, 107.								
	Males	Fem.	Tot.		Males	Fem.	Tot.	
Whereof have died				And there have died of				
Under 2 years old,	6	11	17	Apoplexy,	0	1	1	
Between 2 and 5	1	2	3	Cancer,	1	0	1	
5—10	2	2	4	Cholic,	1	0	1	
10—20	1	2	3	Consumptions,	10	13	23	
20—30	6	2	8	Dropsy,	4	1	5	
30—40	2	3	5	Fevers,	23	12	35	
40—50	11	3	14	Infants,	6	7	13	
50—60	9	2	11	Lunacy,	0	1	1	
60—70	9	7	16	Old age	9	15	24	
70—80	9	8	17	Palsy,	1	0	1	
80—90	1	6	7	Quinsy,	0	1	1	
90—100	1	1	2	Small-pox,	1	0	1	
Of all, in 10 years,	58	49	107	Of the above dis- tempers, in 10 yrs. }	56	51	107	

In this parish there are { 160 Houses, 12 of which are uninhabited.  
603 Souls of the following ages, viz.

	Males	Fem.	Tot.		Males	Fem.	Tot.
Under 2 years old,	12	19	31	Between 40 and 50	40	22	62
Between 2 and 5	25	19	44	50—60	38	33	71
5—10	30	38	68	60—70	25	14	39
10—20	59	58	117	70—80	4	8	12
20—30		41	96	80—90	4	0	4
30—40	26	33	59	90—100	0	0	0
				Total of all ages,	318	285	603

A general Bill of all the Christenings and Burials in the Parish of *Ackworth*, in the County of *York*, for ten years, from March 25, 1757, to March 25, 1767.

In ten years christened, Males 104. Females 108. Total, 212.				In ten years buried, Males 79. Females, 77. Total 156.			
	Males	Fem.	Tot.		Males	Fem.	Tot.
Whereof have died				And there have died of			
Under 2 years old,	18	13	31	Apoplexy,	2	1	3
Between 2 and 5	9	7	16	Asthma, °	2	1	3
5 — 10	4	1	5	Cancer,	0	1	1
10 — 20	2	2	4	Casualties,	5	1	6
20 — 30	7	5	12	Childbed,	0	2	2
30 — 40	3	8	11	Chincough,	0	2	2
40 — 50	2	4	6	Consumptions,	23	15	38
50 — 60	11	3	14	Convulsions,	4	2	6
60 — 70	13	13	26	Diabetes.	1	0	1
70 — 80	7	14	21	Dropsy,	0	3	3
80 — 90	3	6	9	Dysentery	1	1	2
90 — 100	0	1	1	Fever,	12	11	23
Of all ages in 10 yrs.	79	77	156	Jaundice,	1	0	1
				Infants,	7	6	13
				Lunacy,	0	1	1
				Measles,	0	2	2
				Mortification,	2	1	3
				Old age,	11	19	30
				Palsey,	1	0	1
				Quinsy,	1	0	1
				Small-pox,	7	6	13
				Teeth,	0	1	1
				Of all the above dis- } orders, in 10 years }	80	76	156

In this parish there are { 184 Houses, 11 of which are uninhabited.				728 Souls of the following ages, viz.			
	Males	Fem.	Tot.		Male	Fem.	Tot.
Under 2 years old,	31	25	56	Between 40 and 50	31	38	69
Between 2 and 5	32	36	68	50 — 60	28	32	60
5 — 10	34	38	72	60 — 70	20	28	48
10 — 20	50	51	101	70 — 80	7	10	17
20 — 30	44	63	107	80 — 90	2	4	6
30 — 40	61	62	123	90 — 100	0	1	1
				Total of all ages,	339	389	728

In 1702 there were only eleven children baptized, six of whom are now living in the parish, and have resided here almost all the time.

## Account of the Inhabitants of Rome, from 1762 to 1771.

	1762	1763	1764	1765	1766	1767	1768	1769	1770	1771
Parish churches, - - - -	81	81	81	81	82	82	82	82	82	82
Families, - - - -	35739	35696	35453	35771	35894	36375	36409	36521	37449	37285
Bishops, - - - -	42	62	45	45	51	52	54	47	52	62
Priests, - - - -	2742	2699	2718	2617	2531	2652	2676	2819	3031	2925
Religious of sundry orders -	4381	4291	3588	4509	4258	4105	4310	4088	3792	3739
Nuns, - - - -	1725	1892	1661	1759	1684	1738	1709	1695	1692	1594
Collegians and scholars, - -	863	970	763	888	734	1153	907	1197	939	491
Cardinals courts or attendants,	812	791	765	544	827	588	491	592	72	665
Poor pensioners of the hospital,	1050	858	1271	1725	1903	2839	2010	1970	1426	1386
Prisoners, - - - -	339	240	336	402	370	390	251	405	446	402
Males of all ages, - - - -	90230	87396	88618	87295	88280	88577	88865	88415	86610	87547
Females of all ages, - - - -	67219	71423	73286	70890	69588	71183	69982	70491	71833	72128
Above 14 years of age, - - -	120696	123211	125391	120300	119661	122150	120820	121455	120385	119984
Under 14, - - - -	36762	35608	36508	37795	38207	37610	38027	37451	38058	39691
Nonconformists to the church } of Rome, - - - -	37	61	75	86	120	49	63	77	84	91
Blacks, - - - -	9	11	8	8	12	5	10	9	5	5
Devotees, - - - -	18	30	28	31	23	22	20	25	20	20
Births, - - - -	4989	5336	5420	4828	4962	4310	4595	4891	4967	4216
Deaths, - - - -	7149	6493	7361	8375	7722	7528	9574	6972	6646	5850
Total of inhabitants, - - -	157458	158819	161899	158095	157868	159760	158847	158906	158443	159675



## E S S A Y II.

*Proofs of the Insalubrity of marshy Situations. In a Letter to the Rev. Dr. Horsley, read to the Royal Society Jan. 13, 1774, and published in the Philosophical Transactions. Vol. 64, P. 96.*

DEAR SIR,

DR. Priestley's paper on the noxious effects of stagnant waters, read last Thursday to the Royal Society, brought to my remembrance a Table exhibiting the rate of mortality in a parish situated among marshes, which I had seen in Mr. MURET's Observations, published in the Memoirs for 1766 of the Oeconomical Society at Bern. I have since examined this Table, and found that it contains a full confirmation of Dr. Priestley's assertions. This parish is a part of the district of *Vaud*, belonging to the canton of *Bern*, in *Switzerland*, and contained 169 families, and 696 inhabitants. Mr. MURET's Table of the rate of mortality in it is formed from a register of the ages at which all died in it for 15 years. With this Table he has also given Tables  
from



from like registers of the rates of mortality in seven small towns; in 36 country parishes and villages; in 16 parishes situated in the *Alps*; in 12 corn parishes; and in 18 vintage parishes.—From comparing these Tables it appears that the probabilities of living are highest in the most hilly parts of the province, and lowest in the marshy parish just mentioned. The difference is indeed remarkable, as will appear from the following particulars. One half of all born in the mountains live to the age of 47. In the marshy parish, one half live only to the age of 25. In the hills one in 20 of all that are born live to 80. In the marshy parish, only one in 52 reaches this age. In the hills, a person aged 40 has a chance of 80 to 1, for living a year. In the marshy parish, his chance for living a year is not 30 to 1.—In the hills, persons aged 20, 30, and 40, have an even chance for living 41, 33, and 25 years respectively. In the fenny parish, persons, at these ages, have an even chance of living only 30, 23, and 15 years.

I am sensible that observations for only 15 years, in one small parish, do not afford as decisive and ample an authority, in the present case, as there is reason to wish for; and that, therefore, the perfect exactness of the particulars I have recited, cannot be depended on.—They are, however, sufficiently

ciently near the truth to demonstrate, in general, the unhealthfulness of a marshy situation, and as the register from whence they are derived is the only one, in such a situation, which I have ever met with, and Dr. *Alexander's* experiments may lead some to very wrong conclusions on this subject; I could not help thinking, that there would be no impropriety in sending you the account I have now given. If you think it of any importance, I shall be obliged to you for reading it to the Royal Society.

I cannot help taking this opportunity to add my wishes, that such registers of mortality as those published by Mr. *Muret*, were established in every part of this kingdom. We might then determine immediately every such question as that which has occasioned this letter; and know certainly what influence different airs and different situations have on the duration of life. Two ingenious physicians, Dr. *Percival* at *Manchester* (a), and Dr. *Haygarth* at *Chester*, have lately, with much zeal, promoted institutions of this kind; and a great deal of useful information may be expected from the accurate and compre-

(a) Dr. PERCIVAL has not succeeded at *Manchester*. But it has been seen, in the course of this work, that I have derived a great deal of information from Dr. *Haygarth's* register. Dec. 1781.

hensive registers of mortality, which, under their direction, have been established in these towns. But the instruction arising from these establishments cannot be complete, till they become universal.

I am, Sir,

Your most obedient

and humble Servant,

Newington-Green,  
Dec. 21, 1773.

RICHARD PRICE.

## E S S A Y III.

*Short and easy Theorems for finding, in all Cases, the Differences between the Values of Annuities payable Yearly, and of the same Annuities payable Half-yearly, Quarterly, or Momently. Communicated in a Letter to Sir John Pringle, Bart. P. R. S. and read to the Royal Society, Nov. 9, 1775, and published in the Philosophical Transactions, Vol. 66, Part I.*

THE values of annuities, as given in all the common Tables, suppose them paid yearly. But it is well known, that generally they are paid half-yearly, and sometimes quarterly: and that this is a circumstance which always adds to their value. The difference between the values of annuities, according as they are paid in these different ways, I have seen no where stated with accuracy; and therefore, I have thought that the following attempt to do this may be of some use.

Annuities



Annuities are of two sorts. They are either payable certainly or conditionally. Of the former sort are all annuities which are payable at fixed times, without depending on any contingency. Of the latter sort are all annuities on lives. I will first consider the first sort of annuities.

Let  $r$  denote the interest of 1  $\text{£}$ . for a year; and  $n$  the term or number of years during which any annuity is to be paid. Let  $p$  denote the value of the perpetuity, or the quotient arising from dividing 1  $\text{£}$ . by its interest for a year. Let  $y$  denote the value of an annuity for  $n$  years, supposing it to be paid yearly;  $b$  its value, payable half-yearly;  $q$  its value, payable quarterly; and  $m$  its value, payable momentarily.

THEOREM I.

$$y = p - \frac{1}{r \times 1 + r} \sqrt[n]{\phantom{x}}$$

THEOREM II.

$$b = p - \frac{1}{r \times 1 + \frac{r}{2}} \sqrt[2n]{\phantom{x}}$$

THEOREM

## THEOREM III.

$$q = P - \frac{1}{r \times 1 + \frac{r}{4}}^{4n}$$

## THEOREM IV.

$M = P - \frac{1}{r^N}$ . where  $N$  denotes the number which hath  $r^n$  for its hyperbolic logarithm, and  $r^n \times 0.43429448$  for its logarithm in *Brigg's* system.

## EXAMPLE.

Let the rate of interest be 4 *per cent.* and the term 5 years, and consequently  $r = 0.04$ .  $n = 5$ .  $P = 25$ .

$$\text{Then, } y = 4.4518$$

$$b = 4.4913$$

$$q = 4.5120$$

$$m = 4.5415$$

## EXAMPLE II.

Let the rate of interest be the same, and the term for which the annuity is payable 25 years.

Then,

$$\text{Then, } y = 15.6220$$

$$b = 15.7118$$

$$q = 15.7694$$

$$m = 15.801$$

## EXAMPLE III.

Interest being the same, let the term be 50 years.

$$\text{Then, } y = 21.4822$$

$$b = 21.5491$$

$$q = 21.582$$

$$m = 21.616$$

## EXAMPLE IV.

Interest being the same, let the term be 100 years.

$$\text{Then, } y = 24.505$$

$$b = 24.523$$

$$q = 24.532$$

$$m = 24.542$$

In the foregoing Theorems it may be observed, that the *ratio* to one another of the values of annuities payable yearly, half-yearly, quarterly, and momentarily, is greatest when  $n$  is least; that it decreases continually as  $n$  increases, till at last it vanishes when  $n$  becomes infinite or the annuity is a

perpetuity. Agreeably to this it appears, in the examples I have given, that the values in the first example differ more from one another in proportion than the values in the second example; and that these also differ more than the values in the third; and that in the last example all the values are nearly the same.

These values computed by Mr. *De Moivre's* rules in his Treatise on Life-annuities, p. 86 and 124, &c. come out greater when  $n$  exceeds and less when  $n$  falls short of 15 or 20 years. But those rules suppose the half-yearly and quarterly interests of money to be less than half or a quarter of the yearly interest. For instance; the value of an annuity of 1*l.* payable half-yearly and quarterly for 50 years is, according to Mr. *De Moivre's* rules, 21,699 and 21,772, or a 99th part and 74th part more than the value of the same annuity payable yearly, supposing money improved at 4 *per cent.* when the annuity is paid yearly; and at 1,98*l. per cent.* when it is paid half-yearly; and at 0,985*l. per cent.* when it is paid quarterly: That is, supposing money improved at a rate of half-yearly or quarterly interest, which, instead of being a half or a quarter of the yearly interest, is only that half-yearly or quarterly payment which, in consequence of being laid up and improved at  
compound



compound interest, will in a year amount to the sum that makes the yearly interest. It is obvious that this cannot be the proper method of computing these values. But not to insist on this; I will next state the different values of the second sort of annuities; or of *life-annuities*, according as they are supposed to be payable yearly, half-yearly, quarterly, or momentarily.

Let  $r$  as before be the interest of 1  $l.$  for a year;  $n$  the complement of a given life ( $a$ );  $y, b, q$ , and  $m$ , the values respectively of an annuity certain for  $n$  years payable yearly, half-yearly, quarterly, or momentarily;  $p$  the perpetuity;  $v$  the present value of an annuity on a life whose complement is  $n$ , payable yearly;  $h$  the value of the same annuity payable half-yearly; and  $q$  and  $m$  the values of the same annuity payable quarterly and momentarily.

( $a$ ) The complement of a life is, in Mr. *De Moivre's* hypothesis, the number of years it wants of 86. In all other cases, it is double the expectation of a life; that is, it is double the quotient (diminished by  $\frac{1}{2}$  unity) arising from dividing the sum of all the living in a Table of Observations from the age (inclusive) of the given life to the extremity of life, by the number of the living at that age. See Essay I. in the preceding volume.

$$\text{Then, } y = P - \frac{1+r}{nr} \times y.$$

$$H = P - \frac{1+\frac{r}{2}}{nr} \times b.$$

$$Q = P - \frac{1+\frac{r}{4}}{nr} \times q.$$

$$M = P - \frac{m}{nr}.$$

## EXAMPLE I.

Let the life be supposed of the age of 36. The complement of such a life is 50, according to Mr. *De Moivre's* hypothesis; and also very nearly, according to the *Breslaw* and the *Northampton* Tables of observations. Therefore,  $n$  will be 50. Let the rate of interest be 4 per cent. or  $r = 0,04$ .  $P = 25$ .  $y = 21,482$ .  $b = 21,549$ .  $q = 21,582$ .  $m = 21,616$ . See p. 385.

$$\text{Therefore, } y = 25 - \frac{1,04}{50 \times 0,04} \times 21,482 = 13,829$$

$$H = 25 - \frac{1,02}{50 \times 0,04} \times 21,549 = 14,010$$

$$Q = 25 - \frac{1,01}{50 \times 0,04} \times 21,582 = 14,101$$

$$M = 25 - \frac{21,616}{50 \times 0,04} = 14,191$$

EXAMPLE

## EXAMPLE II.

Let the life be supposed of the age of 61. The complement of this life is 25 by Mr. *De Moivre's* hypothesis and the *Nor-thampton* Table of observations. Therefore, interest supposed at 4 per cent.

$$Y = 25 - \frac{1,04}{25 \times 0,04} \times 15,622 = 8,753$$

$$H = 25 - \frac{1,02}{25 \times 0,04} \times 15,712 = 8,973$$

$$Q = 25 - \frac{1,01}{25 \times 0,04} \times 15,769 = 9,072$$

$$M = 25 - \frac{15,801}{25 \times 0,04} = 9,199$$

The different values, given by these theorems, (a) of life-annuities payable yearly, half-yearly, and quarterly, suppose nothing to be due to an annuitant for that year, half-year, or quarter, in which he shall happen to die. If, on the contrary, he is to be

(a) It is of no consequence that these theorems are founded on the *hypothesis* of an equal decrement of life; for taking equal *yearly* values, (or values nearly equal) the differences between them and *half yearly* and *quarterly* values are almost exactly the same, whether they are deduced from real observations, or from this hypothesis. — Even in the hypothesis itself it requires a considerable difference in the yearly value, to produce any material difference in the excess of the half-yearly and quarterly values.

entitled to such part of the annuity as shall be proportioned to the time which shall happen to intervene between his death and the time when the payment immediately preceding his death became due; or in other words, if the annuity is an annuity secured

by land,  $\frac{y}{2n}$  must be added to the first theorem in order to obtain the value of such an annuity payable yearly. And in like manner,  $\frac{b}{4n}$  must be added to the second theorem to obtain the value of the same annuity payable half-yearly: and  $\frac{g}{8n}$  to the third theorem, to obtain its value payable quarterly.

The value, therefore, in the first example, of an annuity payable yearly on a life aged 36 being 13,829; its value, if secured by land, or to be enjoyed to the last moment of life, will be  $13,829 + \frac{21,482}{100} = 14,043$ .

If secured by land and payable half-yearly, its value will be  $14,010 + \frac{21,549}{200} = 14,117$ .

If secured by land and payable quarterly, its value will be  $14,101 + \frac{21,582}{400} = 14,155$ .

The like values in the second example are 9,065, 9,130, and 9,151.

Life-annuities payable monthly or weekly may be considered as of the same value with annuities



annuities payable momentarily; and it is evident, that they must be enjoyed nearly to the last moment of life.

From these rules and examples it may be gathered, that the difference between the values of annuities on lives payable yearly, half-yearly, quarterly, and momentarily, increases continually with the ages; but, if not secured by land, this difference can never be so great as a quarter of a year's purchase in the case of annuities payable yearly and half-yearly; three-eighths of a year's purchase in the case of annuities payable yearly and quarterly; and half a year's purchase in the case of annuities payable yearly and momentarily.

Mr. *Simpson*, in his Treatise on the Doctrine of Life-Annuities, p. 78, and in his Select Exercises, p. 283, hath given a quarter of a year's purchase as the addition always to be made to the value of a life-annuity payable yearly, in order to obtain its value payable half-yearly; and three-eighths of a year's purchase, if its value payable quarterly is required. But it appears, that these are too large additions; and, whatever be the rate of interest or the number of lives, a fifth of a year's purchase will be generally more than a sufficient addition, if the value of the annuity is desired payable half-yearly; and three-tenths of a year's purchase, if the value of the annuity

is desired payable quarterly. Mr. *De Moivre's* rules, in p. 85 of his Book on Life-annuities, for finding the values of life-annuities payable half-yearly and quarterly from their values payable yearly, are still less correct; for they suppose the difference between these values the same, whether the annuities are life-annuities or annuities certain.

Mr. *Dodson*, in the first question in the third volume of his Mathematical Repository, hath given a rule for finding the value of an annuity secured by land and payable yearly, which coincides with that here given; and Mr. *De Moivre*, in p. 338 of his Treatise on the Doctrine of Chances, hath given a theorem for this purpose, which also brings out nearly the same answers. But Mr. *Simpson*, in Prob. I. p. 323 of his Select Exercises, makes the excess of the value of such an annuity above the value of an annuity payable yearly but not secured by land, double to the same excess derived from Mr. *Dodson's* and Mr. *De Moivre's* rules. The truth is, that Mr. *Dodson's* rule gives the exact value; and that Mr. *Simpson's* problem gives the value, not of an annuity secured by land and payable yearly, but of an annuity secured by land and payable momentarily; and also, that his method of solution implies a rate of interest somewhat less when the annuity is payable momentarily than when it is payable yearly.

But

But to prevent all perplexity on this subject, I will subjoin the following investigations, which will be easily understood by those who are acquainted with the common methods of calculating the values of life-annuities.

Let  $r$ , as before, be the interest of 1  $\text{£}$ . for a year. Then the present value of 1  $\text{£}$ . payable at the end of one year, two years, three years, &c. will be  $\frac{1}{1+r}$ ,  $\frac{1}{(1+r)^2}$ ,  $\frac{1}{(1+r)^3}$ , &c. respectively. And the present value of an annuity certain for  $n$  years payable yearly is the sum of this series continued to  $n$  terms (a), or  $\frac{1}{r} - \frac{1}{r \times (1+r)^n} = P - \frac{1}{r \times (1+r)^n} = y$ .

In like manner, the present value of half 1  $\text{£}$ . (that is, of 10s. =  $\text{£}$ . 0, 5) payable at the end of half a year, a year, a year and a half, &c. reckoning half-yearly interest at half

(a) In the postscript it will be proved, that the sum of  $n$  terms of the series  $\frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \frac{1}{a^4}$ , &c. is

$\frac{1}{a-1} - \frac{1}{a^n \times a-1}$ . Substitute  $1+r$  for  $a$ , and it will

appear, that the sum of  $n$  terms of the series  $\frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3}$ , &c. is  $\frac{1}{r} - \frac{1}{r \times (1+r)^n}$ .

the



the annual interest, is  $\frac{0,5}{1+\frac{r}{2}}, \frac{0,5}{1+\frac{r}{2}}^2, \frac{0,5}{1+\frac{r}{2}}^3, \&c.$

And the present value of an annuity certain payable half-yearly for  $n$  years, each payment to be half the yearly payment, is the sum of this series continued to  $2n$  terms; or,

$$\frac{0,5}{\frac{r}{2}} - \frac{0,5}{\frac{r}{2} \times 1 + \frac{r}{2}}^{2n} = \frac{1}{r} - \frac{1}{r \times 1 + \frac{r}{2}}^{2n} = P -$$

$$\frac{1}{r \times 1 + \frac{r}{2}}^{2n} = b.$$

By the same steps it will appear, that the present value of an annuity certain for  $n$  years to be received in quarterly payments, each a quarter of the annual payment, is

$$\frac{0,25}{\frac{r}{4}} - \frac{0,25}{\frac{r}{4} \times 1 + \frac{r}{4}}^{4n} = P - \frac{1}{r \times 1 + \frac{r}{4}}^{4n} = q. \text{ And}$$

also, that the present value of an annuity certain for  $n$  years, to be received in momentarily payments, each the same proportional part of the yearly payment that the moment is of the year, must be  $P - \frac{1}{r \times 1 + \frac{r}{1000, \&c. n}}^{1000, \&c. n}$

But, by the binomial theorem,

$$1 + \frac{r}{1000, \&c. n}^{1000, \&c. n} = 1 + rn + \frac{r^2 n^2}{2} + \frac{r^3 n^3}{2 \times 3} +$$



$\frac{r^4 n^4}{2 \times 3 \times 4}$ , &c. which series approximates indefinitely to the number of which  $rn$  is the hyperbolic logarithm, by Prob. 1. Sect. XI. Vol. II. of Mr. *Simpson's* Fluxions; or by Prop. 1. p. 40, of his Treatise on Trigonometry. Therefore,  $P = \frac{1}{r \times 1 + \frac{1}{1000, \&c. n}}$

$= P = \frac{1}{rN} = m$ , as explained before. See p. 384.

If the value of an annuity of 1*l.* for  $n$  years is required payable half-yearly, and the half-yearly interest of 1*l.* instead of being half the yearly interest (or  $\frac{r}{2}$ ), is sup-

posed to be  $1 + r^{\frac{1}{2}} - 1$ ; the answer will be  $\frac{0,5}{1 + r^{\frac{1}{2}}} + \frac{0,5}{1 + r} + \frac{0,5}{1 + r^{\frac{3}{2}}} + \frac{0,5}{1 + r^2}$ , &c. conti-

nued to  $2n$  terms  $= \frac{0,5}{1 + r^{\frac{1}{2}} - 1} - \frac{0,5}{1 + r^{\frac{1}{2}} \times 1 \times r^{\frac{1}{2}} - 1}$

$= 1 - \frac{1}{1 + r^{\frac{1}{2}}} \times \frac{1}{2 \times 1 + r^{\frac{1}{2}} - 2}$ ; which value is

to  $1 - \frac{1}{1 + r^{\frac{1}{2}}} \times \frac{1}{r}$  (the value of the same annuity payable yearly supposing the yearly interest

interest of 1*l.* to be  $r$ ) as  $\frac{\frac{1}{r}}{1+r} - 1$  to  $\frac{1}{r}$  ( $a$ ),

agreeably to Mr. *De Moivre's* deduction in his *Treatise on Life-annuities*, p. 125, 4th edit.

( $a$ ) In the same manner the value payable quarterly is

$$1 - \frac{1}{1+r} \times \frac{1}{4 \times 1+r} - 1 \quad \text{and the value payable mo-}$$

$$\text{mently} = 1 - \frac{1}{1+r} \times \frac{1}{1000, \&c. \times 1+r} - 1$$

Consequently the value of an annuity certain, payable quarterly or momentarily, is to the same value, payable

yearly, as  $\frac{1}{4 \times 1+r} - 1$ , or  $\frac{1}{N}$ , to  $\frac{1}{r}$  ( $N$  being the

hyperbolic logarithm of  $1+r$ ). Supposing, therefore, the interest to be 4 *per cent.* the value of an annuity payable yearly must be *invariably* increased in the ratio of 1.0101, or 1.0152 or 1.01986 to 1, according as it is payable either half-yearly, quarterly or momentarily. The difference, however, between the values of annuities payable yearly and at shorter intervals is known to be continually lessening in proportion to the length of the term, till at last, when the term is extended to a perpetuity, those values become the same, whether the payments are made yearly or momentarily. But such an equality can never take place according to Mr. *De Moivre's* rules; nay, if the term be extended only to 70 years, and interest be 6 *per cent.* an annuity payable quarterly will be worth more than even the *perpetuity* when the payments are made yearly. This appears to be very erroneous, and sufficient to prove the fallacy of Mr. *De Moivre's* method of solution.

Ed.

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This implying, in the case of annuities payable half-yearly, a smaller interest than half the yearly interest (for  $\sqrt[1]{1+r}^{\frac{1}{2}} - 1$  is less than  $\frac{r}{2}$ ) gives the difference between their value and the value of annuities payable yearly, greater than the truth.

But to return to the investigation of the theorems in the former part of this paper.

Let us again call  $p$  the perpetuity, and  $y$  the value of an annuity certain for  $r$  years and payable yearly; it is well known that the value of 1  $l.$  payable yearly on a life whose complement is  $n$  is (supposing an

$$\begin{aligned} &\text{equal decrement of life)} \quad \frac{n-1}{n \times 1+r} + \frac{n-2}{n \times 1+r^2} \\ &+ \frac{n-3}{n \times 1+r^3}, \text{ \&c. continued to } n \text{ terms (a)} \\ &= P - \frac{1+r}{nr} \times y = Y. \end{aligned}$$

In

(a) See Mr. *De Moivre's* Treatise on Life-annuities, p. 99, 4th edit. Or his *Doctrine of Chances*, p. 311, 3d edition. Or Mr. *Dodson's* Mathematical Repository, Vol. II. p. 137. Or Mr. *Simpson* on Annuities and Reversions, p. 14. In consulting these writers, care should be taken to remember, that they use  $r$  to denote the principal and interest of 1  $l.$  for a year; whereas it hath been most convenient for me in these observations to make  $r$  stand only for the interest. In these writers, therefore,  $r$  signifies the same with  $1+r$  in this paper; and  $r-1$  the same with  $r$ .

It

In like manner, supposing money improved at an half-yearly interest equal to half the yearly

It is said above, that the value of an annuity payable yearly on a life whose complement is  $n$ , is  $\frac{n-1}{n \times 1+r} +$

$\frac{n-2}{n \times 1+r}^2 + \frac{n-3}{n \times 1+r}^3$ , &c. continued to  $n$  terms.

This expression is equal to  $\frac{n}{n \times 1+r} + \frac{n}{n \times 1+r}^2 + \frac{n}{n \times 1+r}^3$ , &c.  $(n) - \frac{1}{n} \times \frac{1}{1+r} + \frac{2}{(1+r)^2} + \frac{3}{(1+r)^3}$ , &c.

$(n)$ . But  $\frac{n}{n \times 1+r} + \frac{n}{n \times 1+r}^2 + \frac{n}{n \times 1+r}^3$ , &c.  $(= \frac{1}{1+r} + \frac{1}{1+r}^2 + \frac{1}{1+r}^3$ , &c.)  $= \frac{1}{r} - \frac{1}{r \times 1+r}^n = y$

(see p. 393.) Also, by a theorem which will be demonstrated in the postscript, and putting  $a$  for any given quantity,

$\frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3}$ , &c. continued to  $n$  terms,  $= \frac{a}{a-1}^2$

$-\frac{n}{a^n} \times \frac{1}{a-1} - \frac{1}{a^n} \times \frac{a}{a-1}^2$ . Therefore, if  $1+r$  is

substituted for  $a$ , and  $y$  for  $\frac{1}{r} - \frac{1}{r \times 1+r}^n$ , the sum (mul-

tiplied by  $\frac{1}{n}$ ) of  $n$  terms of the series  $\frac{1}{1+r} + \frac{2}{1+r}^2 +$

$\frac{3}{1+r}^3$ , &c. will come out  $\frac{1+r}{nr} \times y - \frac{1}{r} \times \frac{1}{1+r}^n$ ; or

$\frac{1+r}{nr} \times y + y - \frac{1}{r}$ . Therefore, the series  $\frac{1}{n} \times \frac{1}{1+r} +$

$\frac{2}{1+r}^2$



yearly interest, or to  $\frac{r}{2}$ , the value of the same

annuity payable half-yearly, is  $\frac{1}{2} \times \frac{\overline{n - \frac{1}{2}}}{n \times 1 + \frac{r}{2}}$

$$+ \frac{\overline{n - 1}}{n \times 1 + \frac{r}{2}}^2 + \frac{\overline{n - \frac{3}{2}}}{n \times 1 + \frac{r}{2}}^3, \text{ \&c. continued to } 2n$$

$$\text{terms} = \frac{1}{2} \times \frac{\overline{n}}{n \times 1 + \frac{r}{2}} + \frac{\overline{n}}{n \times 1 + \frac{r}{2}}^2 + \frac{\overline{n}}{n \times 1 + \frac{r}{2}}^3,$$

$$\text{\&c. continued to } 2n \text{ terms} = \frac{1}{2} \times \frac{\overline{\frac{1}{2}}}{n \times 1 + \frac{r}{2}} +$$

$$\frac{\overline{1}}{n \times 1 + \frac{r}{2}}^2 + \frac{\overline{\frac{3}{2}}}{n \times 1 + \frac{r}{2}}^3, \text{ \&c. continued to } 2n$$

terms. But the sum of the first of these two

$$\text{series, or of } \frac{1}{2} \times \frac{\overline{n}}{n \times 1 + \frac{r}{2}} + \frac{\overline{n}}{n \times 1 + \frac{r}{2}}^2, \text{ \&c. } (= \frac{1}{2}$$

$$\frac{\overline{2}}{1+r}^2 + \frac{\overline{3}}{1+r}^3, \text{ \&c. continued to } n \text{ terms and sub-}$$

$$\text{tracted from the series } \frac{1}{1+r} + \frac{1}{1+r}^2 + \frac{1}{1+r}^3, \text{ \&c.}$$

continued to  $n$  terms; that is, the value of the life will

$$\text{be } y - \frac{1+r}{nr} \times y + y - \frac{1}{r} = \frac{1}{r} - \frac{1+r}{nr} \times y = p - \frac{1+r}{nr}$$

$$\times y = x.$$

×

$\times \frac{1}{1+\frac{r}{2}} + \frac{1}{1+\frac{r}{2}}^2$ , &c.) is  $b$ , see p. 393, &c.

And the sum of the second series is the same with half the sum of the series  $\frac{1}{2n} \times$

$\frac{1}{1+\frac{r}{2}} + \frac{2}{1+\frac{r}{2}}^2 + \frac{3}{1+\frac{r}{2}}^3$ , &c. ( $2n$ ). But by the

theorem mentioned in the last note, the sum of  $n$  terms of the series  $\frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3}$ , &c. is

$\frac{a}{a-1} - \frac{n}{a^n} \times \frac{1}{a-1} - \frac{1}{a^n} \times \frac{a}{a-1}^2$ . Therefore,

if  $1 + \frac{r}{2}$  is substituted for  $a$ ,  $2n$  for  $n$ , and

$b$  for  $\frac{1}{r} - \frac{1}{r \times 1 + \frac{r}{2}}^{2n}$ , the sum of the second

series (that is, of  $\frac{1}{2} \times \frac{1}{2n} \times \frac{1}{1+\frac{r}{2}} + \frac{2}{1+\frac{r}{2}}^2 +$

$\frac{3}{1+\frac{r}{2}}^3$ , &c. ( $2n$ ) will come out  $\frac{1+\frac{r}{2}}{nr} + b$

$\frac{1}{r} \times \frac{1}{1+\frac{r}{2}}^{2n}$ , or  $\frac{1+\frac{r}{2}}{nr} \times b + b - \frac{1}{r}$ . There-

fore,

fore, the second series subtracted from the

first, leaves  $\frac{1}{r} - \frac{1 + \frac{r}{2}}{nr} \times b = P - \frac{1 + \frac{r}{2}}{nr} \times b = H$ , agreeably to the second theorem in p. 388.

By reasoning in the same way it may be

easily found, that  $Q = P - \frac{1 + \frac{r}{4}}{nr} \times q$ ; and

$M = P - \frac{1 + \frac{r}{1000, \&c.}}{nr} \times m = P - \frac{m}{nr}$ , agreeably to the third and fourth theorems in p. 388.

These theorems, I have said, suppose that an annuitant is entitled to no payment for that year, half-year, or quarter, in which he dies. If, on the contrary, he is to be entitled when he dies, to such a part of the yearly, half-yearly, or quarterly payment as shall bear the same proportion to the said payments respectively, as the intermediate time between the last payment and his death bears to the whole year, half-year, or quarter; in this case, supposing the annuity payable yearly, it is evident, since there is the same chance for his dying in one half, of any year as in the other, that he will have an expectation of half a year's payment more than he would be otherwise entitled to. But the value of half 1/. to be paid at the death of a person

whose complement of life is  $n$ , is  $\frac{1}{2} \times$

$$\frac{1}{n \times 1+r} + \frac{1}{2} \times \frac{1}{n \times 1+r|^2} + \frac{1}{2} \times \frac{1}{n \times 1+r|^3}, \&c.$$

continued to  $n$  terms  $(a) = \frac{y}{2n}$ .

In like manner, a person who enjoys an annuity secured by land, payable half-yearly, will have an expectation of a quarter of a year's payment more than he could be otherwise intitled to; the value of which is

$$\frac{1}{4n} \times \frac{1}{1+\frac{r}{2}} + \frac{1}{1+\frac{r}{2}}^2 + \frac{1}{1+\frac{r}{2}}^3, \&c. \text{ continued}$$

to  $2n$  terms  $= \frac{b}{4n}$ . By the same reasoning it

will appear, that  $\frac{q}{8n}$  is the addition to be made to the value of an annuity payable quarterly, in order to obtain its value when secured by land.

## P O S T S C R I P T.

IN the note, p. 393, the expression  $\frac{1}{a-1}$

$— \frac{1}{a^n} \times \frac{1}{a-1}$  is given as the sum of  $n$  terms

of the series  $\frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \frac{1}{a^4}, \&c. \text{ to } \frac{1}{a^n},$

and the expression  $\frac{a}{a-1|^2} — \frac{n}{a^n} \times \frac{1}{a-1} — \frac{1}{a^n}$

(a) See page 393, &c.



$\times \frac{a}{a-1}^2$ , is given, in p. 398, as the sum of

$n$  terms of the series  $\frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3} + \frac{4}{a^4}$ , &c.

The following investigation of these theorems being very easy, will not, perhaps, be unacceptable to those who have studied this subject.

$$\text{Put } A = \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \frac{1}{a^4}, \&c. \frac{1}{a^n}. \quad B = \frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3} + \frac{4}{a^4}, \&c. \frac{n}{a^n}.$$

$$\begin{aligned} \text{Then } A \times a &= 1 + \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3}, \&c. \text{ to } \frac{1}{a^{n-1}}. \\ \text{and } A \times a - 1 &= \frac{1}{a^n} + \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3}, \&c. \text{ to } \frac{1}{a^{n-1}} \\ &+ \frac{1}{a^n} = A, \end{aligned}$$

$$\text{and } A \times a - A (= A \times \overline{a-1}) = 1 - \frac{1}{a^n}.$$

Therefore,  $A = \frac{1}{a-1} - \frac{1}{a^n} \times \frac{1}{a-1}$ , which is the first theorem.

$$\text{Again, } A \times a = 1 + \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3}, \&c. \text{ to } \frac{1}{a^{n-1}},$$

$$\text{and } B \times a = 1 + \frac{2}{a} + \frac{3}{a^2} + \frac{4}{a^3}, \&c. \text{ to } \frac{n}{a^{n-1}}.$$

$$\text{Therefore, } B \times a - A \times a = \frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3}, \&c.$$

$$\text{to } \frac{n-1}{a^{n-1}}.$$

To both sides of the last equation add  $\frac{n}{a^2}$ , and it will appear, that

$$B \times a - A \times a + \frac{n}{a^n} = \frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3} + \frac{4}{a^4}, \&c.$$

$$\text{to } \frac{n-1}{a^{n-1}} + \frac{n}{a^n} = B.$$

$$\text{Therefore, } B \times a - B = B \times a - 1 = A \times a - \frac{n}{a^n};$$

$$\text{and } B = \frac{A \times a}{a-1} - \frac{n}{a^{n+1}-a^n}.$$

For A, in this last equation, substitute its equal, or  $\frac{1}{a-1} - \frac{1}{a^n} \times \frac{1}{a-1}$ , and the resulting

$$\text{equation will be } \frac{a}{a-1} - \frac{n}{a^n} \times \frac{1}{a-1} - \frac{1}{a^n} \times \frac{a}{a-1} = B, \text{ which is the second theorem.}$$

When  $n$  is infinite, all but the first terms in both these theorems vanish; and therefore,  $\frac{1}{a-1}$  is the sum of the series

$$\frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3}, \&c. \text{ continued infinitely; and}$$

$\frac{a}{a-1}^2$  is the sum of the series  $\frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3}, \&c. \text{ continued infinitely.}$

By a like deduction, putting

$$C = \frac{1}{a} + \frac{2 \times 2}{a^2} + \frac{3 \times 3}{a^3} + \frac{4 \times 4}{a^4}, \&c. \text{ to } \frac{n^2}{a^n},$$

and

$$\text{and } D = \frac{1}{a} + \frac{2 \times 2 \times 2}{a^2} + \frac{3 \times 3 \times 3}{a^3} + \frac{4 \times 4 \times 4}{a^4}, \&c.$$

$$\text{to } \frac{n^3}{a^n}, \text{ it may be found that } c = \frac{A + 2B + 1}{a - 1} -$$

$$\frac{(n+1)^2}{a^{n+1} - a^n}, \text{ and } D = \frac{A + 3B + 3C + 1}{a - 1} - \frac{(n+1)^3}{a^{n+1} - a^n}.$$

And consequently, substituting the values of A and B, that

$$C = \frac{a^2 + a}{(a-1)^3} - \frac{n^2}{a^n} \times \frac{1}{a-1} - \frac{2an}{a^n} \times \frac{1}{(a-1)^2} - \frac{a^2 + a}{a^n} \times \frac{1}{(a-1)^3}.$$

And, substituting the values of A, B, C, that

$$D = \frac{a^3 + 4a^2 + a}{(a-1)^4} - \frac{n^3}{a^n} \times \frac{1}{a-1} - \frac{3an^2}{a^n} \times \frac{1}{(a-1)^2} - \frac{3a^2n + 3an}{a^n} \times \frac{1}{(a-1)^3} - \frac{a^3 + 4a^2 + a}{a^n} \times \frac{1}{(a-1)^4}. \quad \text{Or,}$$

since all but the first terms in these expressions vanish when  $n$  is infinite, that the sum of the series  $\frac{1}{a} + \frac{4}{a^2} + \frac{9}{a^3}, \&c.$  continued infinitely is  $\frac{a^2 + a}{(a-1)^3}$ ; and that the sum of the

series  $\frac{1}{a} + \frac{8}{a^2} + \frac{27}{a^3} + \frac{64}{a^4}, \&c.$  continued infinitely is  $\frac{a^3 + 4a^2 + a}{(a-1)^4}$ .

These are all the theorems necessary for calculating the values of annuities on single lives, and on any two or three joint lives,

upon the hypothesis of an equal decrement of life.

Supposing  $r$  the interest of 1  $l.$  for a year, the sum of  $n$  terms of the series  $\frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3}$ , &c. is the present value of an annuity certain for  $n$  years; and  $\frac{1}{1+r} + \frac{2}{(1+r)^2} + \frac{3}{(1+r)^3} + \frac{4}{(1+r)^4}$ , (continued to  $n$  terms) is the present value of an annuity certain beginning with 1  $l.$  and increasing to 2  $l.$  the second year, to 3  $l.$  the third year, &c.

If this last annuity is not an annuity certain for a given term, but a life-annuity, the value of it (supposing  $n$  the complement of the life,  $A$  the value of an annuity certain for  $n$  years,  $G$  the value of two equal joint lives whose common complement is  $n$ ,  $P$  the perpetuity, and  $p$  the value of 1  $l.$  to be received at the end of  $n$  years) will be  $A - G \times n + n \cdot p \cdot P - A \cdot P \times 1 + r$ .

#### E X A M P L E S.

*Let the term be forty-one years, and the rate of interest 4 per cent.*

The value of an annuity of 1  $l.$  certain for this term is 20  $l.$

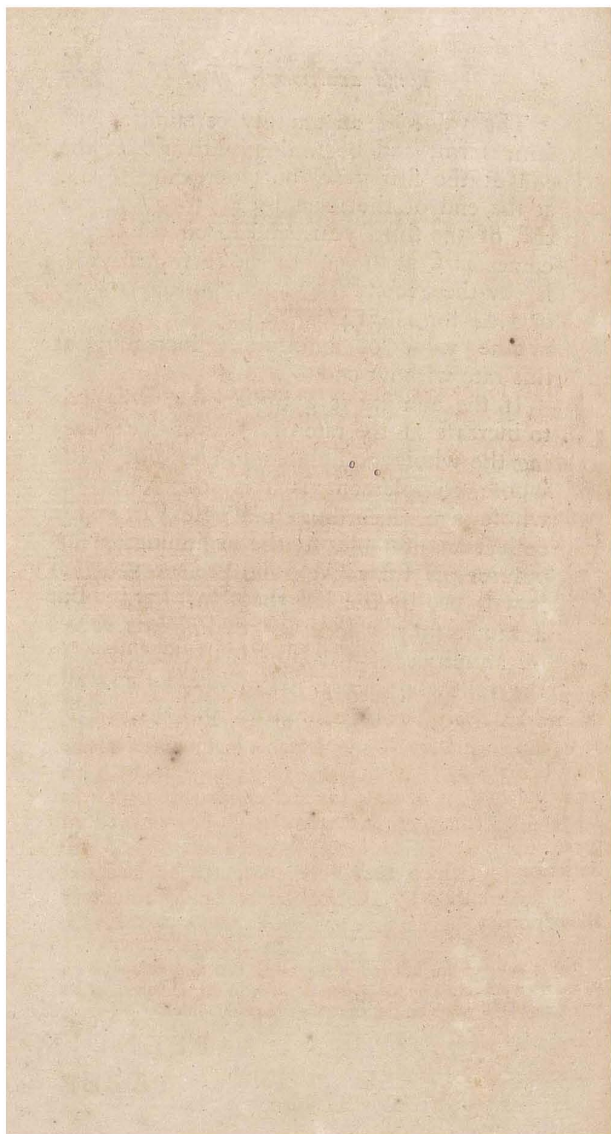
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The value of an annuity certain for the same term, and beginning with 1*l.* at the end of the first year, but increasing to 2*l.* at the end of the second year, to 3*l.* at the end of the third year, and so on till it becomes 41*l.* at the end of the forty-first year, is (by the Second Theorem, putting  $1 + r$ , or 1,04 for  $a$ ) 314*l.* 10*s.*

The value of an annuity increasing at this rate without end is 650*l.*

If the annuity is a life-annuity which is to increase at the rate of 1*l.* every year during the whole possible continuance of a life whose complement is forty-one years (or whose age, according to Table VI. in the collection of tables at the beginning of this volume, is forty-five), the present value of it will be, by the last theorem, 135*l.* But a much simpler rule for finding the values of annuities of this sort will be given in the following notes. See Note I; and also Mr. *Morgan* on Assurances, p. 119.



## A P P E N D I X I.

THE following tables were computed by Dr. *Price*, at the request of a committee of the House of Commons, and were intended to form the foundation of a plan for enabling the labouring poor to provide support for themselves in sickness and old age, by small weekly savings from their wages.—A bill for establishing a plan of this kind was formed and approved by the Commons in the year 1789, but, like Mr. *Dowdeswell's* bill for the same purpose in the year 1773, (a), it was rejected by the Lords. The importance, however, of these tables is not lessened by this circumstance, and it was the author's intention to have published them, had he lived to complete the present edition of this work. In order therefore to fulfil his intentions, as well as to preserve those valuable fruits of his labour from being lost, I have inserted them, together with his own explanations of their use and construction, in this Appendix; thinking that they may be rendered of great public service in some future time, should the Societies for which they were computed be hereafter established either by the legislature or by voluntary associations.

ED.

(a) A copy of this bill and of the tables that were computed for it, has been published by Mr. *Baron Maseres*, in the 2d volume of his very valuable Treatise on the Doctrine of Life-annuities.

TABLE I.

Shewing the Weekly Allowances, during Incapacities of Labour, produced by Sicknefs or Accidents, and the corresponding Weekly Contributions necessary to entitle Persons to those Allowances.

*N. B.* The Ages in this and the following Tables, are the Ages at Admission, and the Contributions at Admission are reckoned to continue invariable till they cease at Sixty-five.

Ages of Contributors at Admission.		Under 32	From 32 to 42	From 42 to 51	From 51 to 58	From 58 to 64			Bedlying Pay.	Walking Pay.
Weekly Contributions	Class	d.	d.	d.	d.	s. d.	Weekly Allowances.	Class	£.	s.
	I.	1	1 $\frac{1}{4}$	1 $\frac{1}{2}$	1 $\frac{3}{4}$	0 2		I.	0	4
	II.	1 $\frac{1}{2}$	1 $\frac{1}{8}$	2 $\frac{1}{4}$	2 $\frac{5}{8}$	0 3		II.	0	6
	III.	2	2 $\frac{1}{2}$	3	3 $\frac{1}{2}$	0 4		III.	0	8
	IV.	2 $\frac{1}{2}$	3 $\frac{1}{8}$	3 $\frac{3}{4}$	4 $\frac{3}{8}$	0 5		IV.	0	10
	V.	3	3 $\frac{3}{4}$	4 $\frac{1}{2}$	5 $\frac{1}{4}$	0 6		V.	0	12
	VI.	3 $\frac{1}{2}$	4 $\frac{3}{8}$	5 $\frac{1}{4}$	6 $\frac{1}{8}$	0 7		VI.	0	14
	VII.	4	5	6	7	0 8		VII.	0	16
	VIII.	4 $\frac{1}{2}$	5 $\frac{5}{8}$	6 $\frac{3}{4}$	7 $\frac{7}{8}$	0 9		VIII.	0	18
	IX.	5	6 $\frac{1}{4}$	7 $\frac{1}{2}$	8 $\frac{3}{4}$	0 10		IX.	1	0
	X.	5 $\frac{1}{2}$	6 $\frac{7}{8}$	8 $\frac{1}{4}$	9 $\frac{5}{8}$	0 11		X.	1	2
	XI.	6	7 $\frac{1}{2}$	9	10 $\frac{1}{2}$	1 0		XI.	1	4



SUPPOSITIONS *on which this TABLE is formed.*

*First,* That in societies consisting of persons under 32 years of age, a 48th part of them will be always in a state of incapacitation by illness and accidents; and therefore entitled to allowances proportioned to their contributions. Various reasons, and particularly the experience of friendly clubs, determine me to believe that the proportion of the sick to the well in such a society will not be so great as this, and consequently that a weekly allowance during sickness will be more than supported by weekly contributions not exceeding a 48th part of that allowance.

*Secondly,* It is supposed that from the age of 32 to 42 this proportion increases to one quarter more than a 48th part; from 43 to 51 to one half more; from 52 to 58 to three quarters more; and from 59 to 64 to double. The reason of assuming this rate of increase is, that the probability of the duration of human life decreases after 30 nearly in this manner, or so that a person of the age of 60 has but half the probability of living any given time that a person at 32 has, and consequently must be then doubly subject to the causes that produce sickness and mortality.

TABLE II.

Shewing the Weekly Allowances to Persons in Old Age after 65 and 70; and the corresponding Weekly Contributions in early Life necessary to support those Allowances.

Ages at Admission.		Class I.		Class II.		Class III.		Class IV.	
		s.	d.	s.	d.	s.	d.	s.	d.
Weekly Contributions till 65.	Under 21	0	1	0	1 $\frac{1}{2}$	0	2	0	2 $\frac{1}{2}$
	21 & 22	0	1 $\frac{1}{4}$	0	1 $\frac{7}{8}$	0	2 $\frac{1}{2}$	0	3 $\frac{1}{8}$
	23 & 24	0	1 $\frac{1}{2}$	0	2 $\frac{1}{4}$	0	3	0	3 $\frac{3}{4}$
	25 & 26	0	1 $\frac{3}{4}$	0	2 $\frac{5}{8}$	0	3 $\frac{1}{2}$	0	4 $\frac{3}{8}$
	27 & 28	0	2	0	3	0	4	0	5
	29 & 30	0	2 $\frac{1}{4}$	0	3 $\frac{3}{8}$	0	4 $\frac{1}{2}$	0	5 $\frac{5}{8}$
	31 & 32	0	2 $\frac{1}{2}$	0	3 $\frac{3}{4}$	0	5	0	6 $\frac{1}{4}$
	33	0	2 $\frac{3}{4}$	0	4 $\frac{1}{8}$	0	5 $\frac{1}{2}$	0	6 $\frac{7}{8}$
	34	0	3	0	4 $\frac{1}{2}$	0	6	0	7 $\frac{1}{2}$
	35	0	3 $\frac{1}{4}$	0	4 $\frac{7}{8}$	0	6 $\frac{1}{2}$	0	8 $\frac{1}{8}$
	36	0	3 $\frac{1}{2}$	0	5 $\frac{1}{4}$	0	7	0	8 $\frac{3}{4}$
	37	0	3 $\frac{3}{4}$	0	5 $\frac{5}{8}$	0	7 $\frac{1}{2}$	0	9 $\frac{3}{8}$
	38	0	4	0	6	0	8	0	10
	39	0	4 $\frac{1}{4}$	0	6 $\frac{3}{8}$	0	8 $\frac{1}{2}$	0	10 $\frac{5}{8}$
	40	0	4 $\frac{1}{2}$	0	6 $\frac{3}{4}$	0	9	0	11 $\frac{1}{4}$
	41	0	4 $\frac{3}{4}$	0	7 $\frac{1}{8}$	0	9 $\frac{1}{2}$	0	11 $\frac{7}{8}$
	42	0	5	0	7 $\frac{1}{2}$	0	10	1	0 $\frac{1}{2}$
	43	0	5 $\frac{1}{2}$	0	8 $\frac{1}{4}$	0	11	1	1 $\frac{3}{4}$
	44	0	6	0	9	1	0	1	3
	45	0	6 $\frac{1}{2}$	0	9 $\frac{3}{4}$	1	1	1	4 $\frac{1}{4}$
	46	0	7 $\frac{1}{2}$	0	11 $\frac{1}{4}$	1	3	1	6 $\frac{3}{4}$
	47	0	8 $\frac{1}{2}$	1	0 $\frac{3}{4}$	1	5	1	9 $\frac{1}{4}$
	48	0	9 $\frac{1}{2}$	1	2 $\frac{1}{4}$	1	7	1	11 $\frac{3}{4}$
	49	0	10 $\frac{1}{2}$	1	3 $\frac{3}{4}$	1	9	2	24 $\frac{3}{4}$
	(*) 50	0	11 $\frac{1}{2}$	1	5 $\frac{1}{4}$	1	11	2	4 $\frac{1}{4}$

TABLE II. continued.

Clafs V.		Clafs VI.		Clafs VII.		Clafs VIII.		Clafs IX.		Clafs X.		Clafs XI.	
s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.
0	3	0	$3\frac{1}{2}$	0	4	0	$4\frac{1}{2}$	0	5	0	$5\frac{1}{2}$	0	6
0	$3\frac{3}{4}$	0	$4\frac{3}{8}$	0	5	0	$5\frac{5}{8}$	0	$6\frac{1}{4}$	0	$6\frac{7}{8}$	0	$7\frac{1}{2}$
0	$4\frac{1}{2}$	0	$5\frac{1}{4}$	0	6	0	$6\frac{3}{4}$	0	$7\frac{1}{2}$	0	$8\frac{1}{4}$	0	9
0	$5\frac{1}{4}$	0	$6\frac{1}{8}$	0	7	0	$7\frac{7}{8}$	0	$8\frac{3}{4}$	0	$9\frac{5}{8}$	0	$10\frac{1}{2}$
0	6	0	7	0	8	0	9	0	10	0	11	1	0
0	$6\frac{3}{4}$	0	$7\frac{7}{8}$	0	9	0	$10\frac{1}{8}$	0	$11\frac{1}{4}$	1	$0\frac{3}{8}$	1	$1\frac{1}{2}$
0	$7\frac{1}{2}$	0	$8\frac{3}{4}$	0	10	0	$11\frac{1}{4}$	1	$0\frac{1}{2}$	1	$1\frac{1}{4}$	1	3
0	$8\frac{1}{4}$	0	$9\frac{5}{8}$	0	11	1	$0\frac{3}{8}$	1	$1\frac{3}{4}$	1	$3\frac{1}{8}$	1	$4\frac{1}{2}$
0	9	0	$10\frac{1}{2}$	1	0	1	$1\frac{1}{2}$	1	3	1	$4\frac{1}{2}$	1	6
0	$9\frac{3}{4}$	0	$11\frac{1}{8}$	1	1	1	$2\frac{5}{8}$	1	$4\frac{1}{4}$	1	$5\frac{9}{8}$	1	$7\frac{1}{2}$
0	$10\frac{1}{2}$	1	$0\frac{1}{4}$	1	2	1	$3\frac{3}{4}$	1	$5\frac{1}{2}$	1	$7\frac{1}{4}$	1	9
0	$11\frac{1}{4}$	1	$1\frac{1}{8}$	1	3	1	$4\frac{7}{8}$	1	$6\frac{3}{4}$	1	$8\frac{5}{8}$	1	$10\frac{1}{2}$
1	0	1	2	1	4	1	6	1	8	1	10	2	0
1	$0\frac{3}{4}$	1	$2\frac{7}{8}$	1	5	1	$7\frac{1}{8}$	1	$9\frac{1}{4}$	1	$11\frac{3}{8}$	2	$1\frac{1}{2}$
1	$1\frac{1}{2}$	1	$3\frac{3}{4}$	1	6	1	$8\frac{1}{4}$	1	$10\frac{1}{2}$	2	$0\frac{3}{4}$	2	3
1	$2\frac{1}{4}$	1	$4\frac{5}{8}$	1	7	1	$9\frac{3}{8}$	1	$11\frac{3}{4}$	2	$2\frac{1}{8}$	2	$4\frac{1}{2}$
1	3	1	$5\frac{1}{2}$	1	8	1	$10\frac{1}{2}$	2	1	2	$3\frac{1}{2}$	2	6
1	$4\frac{1}{2}$	1	$7\frac{1}{4}$	1	10	2	$0\frac{3}{4}$	2	$3\frac{1}{2}$	2	$6\frac{1}{4}$	2	9
1	6	1	9	2	0	2	3	2	6	2	9	3	0
1	$7\frac{1}{2}$	1	$10\frac{3}{4}$	2	2	2	$5\frac{1}{4}$	2	$8\frac{1}{2}$	2	$11\frac{3}{4}$	3	3
1	$10\frac{1}{2}$	2	$2\frac{1}{4}$	2	6	2	$9\frac{3}{4}$	3	$1\frac{1}{2}$	3	$5\frac{1}{4}$	3	9
2	$1\frac{1}{2}$	2	$5\frac{3}{4}$	2	10	3	$2\frac{1}{4}$	3	$6\frac{1}{2}$	3	$10\frac{3}{4}$	4	3
2	$4\frac{1}{2}$	2	$9\frac{1}{4}$	3	2	3	$6\frac{3}{4}$	3	$11\frac{1}{2}$	4	$4\frac{1}{4}$	4	9
2	$7\frac{1}{2}$	3	$0\frac{3}{4}$	3	6	3	$11\frac{1}{4}$	4	$4\frac{1}{2}$	4	$9\frac{3}{4}$	5	3
2	$10\frac{1}{2}$	4	$4\frac{1}{4}$	4	10	5	$3\frac{3}{4}$	5	$9\frac{1}{4}$	6	$3\frac{1}{4}$	6	9

TABLE II. continued.

		After 65.		After 70.			
Weekly Allowances for Life.	{ Class		s.	d.	£.	s.	d.
		I.	2	0	0	4	0
		II.	3	0	0	6	0
		III.	4	0	0	8	0
		IV.	5	0	0	10	0
		V.	6	0	0	12	0
		VI.	7	0	0	14	0
		VII.	8	0	0	16	0
		VIII.	9	0	0	18	0
		IX.	10	0	1	0	0
		X.	11	0	1	2	0
		XI.	12	0	1	4	0

(\*) The weekly contributions in the first class, which are equivalent to the weekly allowances after 65 and 70 in the same class, have been computed by Dr. Price for all the intermediate ages between 50 and 65, and are as follow;

Age.	Weekly Contribution.			Age.	Weekly Contribution.			Age.	Weekly Contribution.		
	£.	s.	d.		£.	s.	d.		£.	s.	d.
51	0	1	0 $\frac{1}{2}$	56	0	1	11 $\frac{3}{4}$	61	0	6	0
52	0	1	2	57	0	2	4 $\frac{1}{4}$	62	0	8	0 $\frac{1}{4}$
53	0	1	3 $\frac{3}{4}$	58	0	2	9 $\frac{1}{4}$	63	0	12	6
54	0	1	5 $\frac{3}{4}$	59	0	3	4	64	1	5	0
55	0	1	8 $\frac{3}{4}$	60	0	4	2 $\frac{1}{2}$				

From these sums the weekly contributions in the other ten classes may be easily obtained. But it will be seldom necessary to have recourse to them; for at a period of life so far advanced, the weekly contributions become so high in those classes as to render it almost impossible for the labouring poor to pay them.

Ed.

*Method*



*Method of calculating Table II.*

The rule for finding the value in a single present payment of an annuity payable for life to a person of a given age, should he survive any other given age, may be found in Volume I. Quest. 6. p. 17.

EXAMPLE.

Let the rate of interest be  $3\frac{1}{2}$  per cent. The table of the probabilities of the duration of human life, that for Northampton given in Vol. II. p. 36. and the tables of the values of lives that in Vol. II. p. 54. Also, let the given age be 20; and let the enquiry be what sum ought to be given for an annuity of 1*l.* payable weekly for life to a person of this age, provided he should survive 65.

The value by the table just referred to, at  $3\frac{1}{2}$  per cent. of an annuity payable weekly during a life aged 65, is (a) 8.332. The probability that a life at 20 will continue in being till it is 65, is (by the other table just referred to)  $\frac{1632}{5132}$ ; that is, it is the fraction whose numerator is the number of the living at 65, and whose denominator is the number living at 20. The value of 1*l.* payable at the end of a number of years, equal to the difference between the two ages 20 and 65, or at the end of 45 years, is (reckoning interest at  $3\frac{1}{2}$  per cent.) .2126 by Table I. Vol. II. p. 18.

£8.332 multiplied by  $\frac{1632}{5132}$  is = 2.648; and this

(a) The values of lives at  $3\frac{1}{2}$  per cent. are not given in this table; but the means between the two values at 3 and at 4 per cent. give them with sufficient exactness.

The value of a life-annuity payable weekly, is worth three-tenths of a year's purchase more than the value of the same annuity payable yearly; and therefore, in all these calculations, this addition is made to every tabular value.

product multiplied by .2126 makes £.5629 the value required.

The value being thus found, in a single payment of an annuity of 1*l.* payable *weekly* for the life of a person of a given age after another given age; the *equivalent* value, in weekly payments, dependant on the continuance of the given life till it reaches the age it is to survive, is found by dividing the value in a single payment, by the value of an annuity payable weekly on the given life, for a term of years equal to the difference between the age of the given life and the age it is to survive (*a*); which, in the present case, is for a term equal to the difference between 20 and 65, or 45 years. The value of a life aged 20 for this term is £17.072. And £.5629 (the value in a single payment just found) divided by 17.072 gives £.0329 the *annual* sum payable *weekly* due from a person aged 20, for an annuity of 1*l.* payable weekly during what may happen to remain of his life after 65. The payment *per week* equivalent to this *annual* sum is, plainly, the sum divided by the number of weeks in the year; that is, £.0329 divided by 52, which will give £.00063. In like manner, an annuity of 1*l.* payable weekly may be found to be equivalent to a payment *per week* of £.0192. Since, therefore, a weekly allowance of £.0192 after 65 is worth to a person aged 20, a payment or contribution *per week* till 65 of £.00063, any other weekly allowance will be worth as much more

(*a*) The value of a life for a term of years is found by subtracting the value of the life after the term from its whole value. Thus the value of an annuity on the whole continuance of a life aged 20, is (adding three-tenths to obtain the value of the annuity payable weekly) 17.635 year's purchase. Its value after a term of 45 years (that is, after 65) is (as shewn above) .5629 year's purchase. The difference (£17.072) is its value for 45 years. — See Quest. 6th. Vol. I.

or less than £.00063, as the allowance itself is more or less. The weekly allowance, therefore, after 65 being reckoned *two shillings* (or .01) the weekly contribution due for it, will be £.00328; for as .0192 is to 0.1 so is £.00063 to £.00328.

By the very same method of calculation it may be found that an allowance to a person now in his 21st year of *two shillings per week* for life after 70 years of age, is worth, in weekly contributions till he reaches 65 and subject to his death in the intermediate time, £.00171. Therefore, a weekly allowance of two shillings *per week* for life to a person in his 21st year after 65, and also an allowance of two shillings more to the same person after 70, is worth, in weekly contributions till he reaches 65 and subject to his death, £.00328 added to £.00171; that is, it is worth £.00499, which is nearly *one penny and  $\frac{2}{3}$  of a farthing*.

In this manner have all the values in the 2d Table been calculated.

The value of any weekly contribution for a given term of years, dependant on the continuance of any life during that term, is 52 times the weekly contribution multiplied by the value of an annuity payable weekly on that life for the given term.—Thus, supposing the life 20 years of age, and the weekly contribution *two pence*, 52. multiplied by .00833, and also by 17.072 (a) (that is, £7.397) will be the value in a single present payment of that contribution dependant on the continuance of the life till 65. And this, therefore, is the sum which, according to Table II. a person under 21, if a contributor in the first Class, ought to pay, in order to be excused all subsequent payments.

(a) See the Note in page 416.



TABLE III.

Shewing the Weekly Allowances during Sicknefs and Old Age, and the corresponding Weekly Contributions for supporting those Allowances; being Tables I. and II. combined.

Ages at Admission.		Clafs I.		Clafs II.		Clafs III.		Clafs IV.	
		s.	d.	s.	d.	s.	d.	s.	d.
Weekly Contributions till 65.	Under 21	0	2	0	3	0	4	0	5
	21 & 22	0	$2\frac{1}{4}$	0	$3\frac{3}{8}$	0	$4\frac{1}{2}$	0	$5\frac{5}{8}$
	23 & 24	0	$2\frac{1}{2}$	0	$3\frac{3}{4}$	0	5	0	$6\frac{1}{4}$
	25 & 26	0	$2\frac{3}{4}$	0	$4\frac{1}{8}$	0	$5\frac{1}{2}$	0	$6\frac{7}{8}$
	27 & 28	0	3	0	$4\frac{1}{2}$	0	6	0	$7\frac{1}{2}$
	29 & 30	0	$3\frac{1}{4}$	0	$4\frac{7}{8}$	0	$6\frac{1}{2}$	0	$8\frac{1}{8}$
	31	0	$3\frac{1}{2}$	0	$5\frac{1}{4}$	0	7	0	$8\frac{3}{4}$
	32	0	$3\frac{3}{4}$	0	$5\frac{5}{8}$	0	$7\frac{1}{2}$	0	$9\frac{3}{8}$
	33	0	4	0	6	0	8	0	10
	34	0	$4\frac{1}{4}$	0	$6\frac{3}{8}$	0	$8\frac{1}{2}$	0	$10\frac{5}{8}$
	35	0	$4\frac{1}{2}$	0	$6\frac{3}{4}$	0	9	0	$11\frac{1}{4}$
	36	0	$4\frac{3}{4}$	0	$7\frac{1}{8}$	0	$9\frac{1}{2}$	0	$11\frac{7}{8}$
	37	0	5	0	$7\frac{1}{2}$	0	10	1	$10\frac{1}{2}$
	38	0	$5\frac{1}{4}$	0	$7\frac{7}{8}$	0	$10\frac{1}{2}$	1	$11\frac{1}{8}$
	39	0	$5\frac{1}{2}$	0	$8\frac{1}{4}$	0	11	1	$14\frac{3}{4}$
	40	0	$5\frac{3}{4}$	0	$8\frac{5}{8}$	0	$11\frac{1}{2}$	1	$2\frac{3}{8}$
	41	0	6	0	9	1	0	1	3
	42	0	$6\frac{1}{2}$	0	$9\frac{3}{4}$	1	1	1	$4\frac{1}{4}$
	43	0	7	0	$10\frac{1}{2}$	1	2	1	$5\frac{1}{2}$
	44	0	$7\frac{1}{2}$	0	$11\frac{1}{4}$	1	3	1	$6\frac{3}{4}$
	45	0	8	1	0	1	4	1	8
	46	0	9	1	$1\frac{1}{2}$	1	6	1	$10\frac{1}{2}$
	47	0	10	1	3	1	8	2	1
	48	0	11	1	$4\frac{1}{2}$	1	10	2	$3\frac{1}{2}$
	49	1	0	1	6	2	0	2	6
	50	0	1	1	$7\frac{1}{2}$	2	2	2	$8\frac{1}{2}$



TABLE III. continued.

Clafs V.	Clafs VI.	Clafs VII.	Clafs VIII.	Clafs IX.	Clafs X.	Clafs XI.
<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>
0 6	0 7	0 8	0 9	0 10	0 11	1 0
0 6 $\frac{3}{4}$	0 7 $\frac{7}{8}$	0 9	0 10 $\frac{1}{8}$	0 11 $\frac{1}{4}$	1 0 $\frac{3}{8}$	1 1 $\frac{1}{2}$
0 7 $\frac{1}{2}$	0 8 $\frac{3}{4}$	0 10	0 11 $\frac{1}{4}$	1 0 $\frac{1}{2}$	1 1 $\frac{3}{4}$	1 3
0 8 $\frac{1}{4}$	0 9 $\frac{5}{8}$	0 11	1 0 $\frac{3}{8}$	1 1 $\frac{3}{4}$	1 3 $\frac{1}{8}$	1 4 $\frac{1}{2}$
0 9	0 10 $\frac{1}{2}$	1 0	1 1 $\frac{1}{2}$	1 3	1 4 $\frac{1}{2}$	1 6
0 9 $\frac{3}{4}$	0 11 $\frac{3}{8}$	1 1	1 2 $\frac{5}{8}$	1 4 $\frac{1}{4}$	1 5 $\frac{7}{8}$	1 7 $\frac{1}{2}$
0 10 $\frac{1}{2}$	1 0 $\frac{1}{4}$	1 2	1 3 $\frac{3}{4}$	1 5 $\frac{1}{2}$	1 7 $\frac{1}{4}$	1 9
0 11 $\frac{1}{4}$	1 1 $\frac{1}{8}$	1 3	1 4 $\frac{7}{8}$	1 6 $\frac{3}{4}$	1 8 $\frac{5}{8}$	1 10 $\frac{1}{2}$
1 0	1 2	1 4	1 6	1 8	1 10	2 0
1 0 $\frac{3}{4}$	1 2 $\frac{7}{8}$	1 5	1 7 $\frac{1}{8}$	1 9 $\frac{1}{4}$	1 11 $\frac{3}{8}$	2 1 $\frac{1}{2}$
1 1 $\frac{1}{2}$	1 3 $\frac{3}{4}$	1 6	1 8 $\frac{1}{4}$	1 10 $\frac{1}{2}$	2 0 $\frac{3}{4}$	2 3
1 2 $\frac{1}{4}$	1 4 $\frac{5}{8}$	1 7	1 9 $\frac{3}{8}$	1 11 $\frac{3}{4}$	2 2 $\frac{1}{8}$	2 4 $\frac{1}{2}$
1 3	1 5 $\frac{1}{2}$	1 8	1 10 $\frac{1}{2}$	2 1	2 3 $\frac{1}{2}$	2 6
1 3 $\frac{3}{4}$	1 6 $\frac{3}{8}$	1 9	1 11 $\frac{5}{8}$	2 2 $\frac{1}{4}$	2 4 $\frac{7}{8}$	2 7 $\frac{1}{2}$
1 4 $\frac{1}{2}$	1 7 $\frac{1}{4}$	1 10	2 0 $\frac{3}{4}$	2 3 $\frac{1}{2}$	2 6 $\frac{1}{4}$	2 9
1 5 $\frac{1}{4}$	1 8 $\frac{1}{8}$	1 11	2 1 $\frac{7}{8}$	2 4 $\frac{3}{4}$	2 7 $\frac{5}{8}$	2 10 $\frac{1}{2}$
1 6	1 9	2 0	2 3	2 6	2 9	3 0
1 7 $\frac{1}{2}$	1 10 $\frac{3}{4}$	2 2	2 5 $\frac{1}{4}$	2 8 $\frac{1}{2}$	2 11 $\frac{3}{4}$	3 3
1 9	2 0 $\frac{1}{2}$	2 4	2 7 $\frac{1}{2}$	2 11	3 2 $\frac{1}{2}$	3 6
1 10 $\frac{1}{2}$	2 2 $\frac{1}{4}$	2 6	2 9 $\frac{3}{4}$	3 1 $\frac{1}{2}$	3 5 $\frac{1}{4}$	3 9
2 0	2 4	2 8	3 0	3 4	3 8	4 0
2 3	2 7 $\frac{1}{2}$	3 0	3 4 $\frac{1}{2}$	3 9	4 1 $\frac{1}{2}$	4 6
2 6	2 11	3 4	3 9	4 2	4 7	5 0
2 9	3 2 $\frac{1}{2}$	3 8	4 1 $\frac{1}{2}$	4 7	5 0 $\frac{1}{2}$	5 6
3 0	3 6	4 0	4 6	5 0	5 6	6 0
3 3	3 9 $\frac{1}{2}$	4 4	4 10 $\frac{1}{2}$	5 5	5 11 $\frac{1}{2}$	6 6

## APPENDIX I.

TABLE III. continued.

Infants of Weekly Allowances during Sickness.	Clafs	1st Infance		2d Infance		After 65.		After 70.	
		Bedlying Pay.		Walking Pay.					
		£.	s.	£.	s.	£.	s.	£.	s.
	I.	0	4	0	2	0	2	0	4
	II.	0	6	0	3	0	3	0	6
	III.	0	8	0	4	0	4	0	8
	IV.	0	10	0	5	0	5	0	10
	V.	0	12	0	6	0	6	0	12
	VI.	0	14	0	7	0	7	0	14
	VII.	0	16	0	8	0	8	0	16
	VIII.	0	18	0	9	0	9	0	18
	IX.	1	0	0	10	0	10	1	0
	X.	1	2	0	11	0	11	1	2
	XI.	1	4	0	12	0	12	1	4

TABLE IV.

Shewing the Fines, or Composition Money, payable at Admission by Contributors in the First Class who have commenced their Contributions at Ages above 21, and who may prefer the Payment of a Fine to an Increase of Weekly Contribution, on Account of the Excess of their Ages above 21, as specified in Table III.

N. B. The Sums in the following Table are also the Sums payable, at Removals, to Contributors, who, at Admission, paid Fines in lieu of an Increase of Weekly Contribution.

Age at Admission or Removal.	Weekly Contribution 2d.	Sums payable	Year.	£.	s.	Age at Admission or Removal.	Weekly Contribution 2d.	Sums payable	Year.	£.	s.	Age at Admission or Removal.	Weekly Contribution 2d.	Sums payable	Year.	£.	s.
In 22d	0 9		In 31st	4 5		In 40th	9 17		In 49th	19 16		In 58th	35 10				
23d	0 18		32d	4 16		41st	10 7		50th	21 0		59th	38 6				
24th	1 6		33d	5 12		42d	11 0		51st	22 5		60th	42 0				
25th	1 15		34th	6 6		43d	11 16		52d	23 14		61st	46 0				
26th	2 3		35th	7 0		44th	12 12		53d	25 6		62d	50 0				
27th	2 12		36th	7 12		45th	13 14		54th	26 18		63d	54 0				
28th	3 0		37th	8 4		46th	15 0		55th	28 13		64th	58 0				
29th	3 8		38th	8 16		47th	16 10		56th	30 16		65th	62 0				
30th	3 16		39th	9 7		48th	18 0		57th	33 1							

EXPLANATION *and* USES of TABLE IV.

THIS Table implies that all persons *under* 21 years of age entitle themselves to the expectation of their different classes, as specified in the two last columns of Table III. without paying any fine; and also that should they remove before they get into their 22d year, no money is payable by the parish they leave on that account.

If advanced into their 22d year when they enter, and do not chuse the *increase* of weekly contribution specified in Table III., under that age, this Table shews the *fine* due from them in lieu of that increase, if they enter into the 1st Class. The fines to be paid in the other classes are in proportion to the weekly contributions in those classes, and are immediately obtained from the fines in this Table. Thus, in the 2d Class they will be 13 s. 6 d.—in the 3d Class 18 s.—in the 4th Class 1 l. 2 s. 6 d. and so on. In like manner the fines due from persons in their 23d, 24th, 25th, 26th, &c. years, when they enter in the first Class (that is, aged then 22, 23, 24, 25, &c.) in lieu of an increased weekly contribution, are the sums corresponding to their ages as specified in this Table; and the fines in the other classes will, as observed above, be in proportion to the weekly contributions in those classes. The sums payable at *removal* to persons who have entered under 21, but do not remove before they are turned of this age, are the same with these fines. For example:

A contributor who has entered in the first Class under 21, if he leaves the parish in which he entered in his 22d, 23d, 24th, 25th, &c. years, will be entitled, at his removal, to the sums in the Table opposite to these ages; that is, to 9 s.—18 s.—1 l. 6 s.—1 l. 15 s. &c. If he has entered in the 2d Class it may



be found from those sums that he will be entitled to 13s. 6d.—1l. 7s.—1l. 19s.—2l. 12s. 6d. &c.

If in the 3d Class to 18s.—1l. 16s.—2l. 12s.—3l. 10s. &c. according as he is in his 22d, 23d, 24th, 25th, &c. years respectively.

It may be a necessary observation, that it is of no consequence to a parish how many removals a contributor in any particular Class had made before he came to it, provided it receives with him the sum in the Table corresponding to his age and class. For example :

A contributor under 21 has entered in the 1st Class; that is, he has entitled himself, by taking upon him a contribution of 2d. *per week*, payable till he is 65, to an allowance, whenever he is sick or disabled, of *four shillings per week bedlying pay*, and two shillings *per week walking pay*; and also to an allowance for life after 65 of *two shillings per week*, and after 70 of *four shillings per week*. Let this person be supposed to remove to another parish in his 28th year. This Table shews that the parish he leaves ought to remit to the parish to which he removes 3l. Should he remove again, the second parish will be obliged to remit to a third parish the sum opposite to his age at that time; and the same is true of this third parish in case of a removal to a fourth parish; and so on.

Again: A contributor aged 22 (that is, in the 23d year of his age) has entered (let us suppose) in the 3d Class; that is, he has entitled himself, either by a weekly contribution, *without* a fine, of *four pence half-penny payable* till he is 65; (see Table III.) or *with* a fine and a weekly contribution of *four pence payable* till 65, to an allowance during sickness of *eight shillings per week bedlying pay*, and *four shillings per week walking pay*, and also to an allowance of *four shillings per week during life* after 65, and *eight shillings*  
I
per



*per* week after 70.—Such a contributor, should he remove in his 30th year, will, as appears by the Table, be entitled to twice 3*l.* 16*s.* or 7*l.* 12*s.* for the parish into which he removes; and should he remove again in his 40th year, he will be entitled to twice 9*l.* 17*s.* or 19*l.* 14*s.* for a *second* parish; and should he remove a third time in his 50th year, he will be entitled to twice 21*l.* or 42*l.* for a *third* parish.

METHOD of computing TABLE IV.

WHEN a contributor removes to a new parish he continues there the weekly contribution with which he first entered. But to this parish he will be the same with a new contributor entering at his age; and, therefore, this parish will be entitled either to a weekly contribution suitable to that age and class, as specified in Table III. or to such a sum as will be equivalent to the value of the *difference* between his contribution and the higher contribution due from a person in that class and at that age, supposing him not to have been before a contributor. If this compensation is not made, the parish left will be a gainer at the expence of the parish to which the contributor removes; and, consequently, while the one is benefited, the other will be injured.—In other words, the parish left by a contributor is a gainer by the removal; and having no right to that gain, without being liable to sustain the burden, a sum equivalent to it ought to be transferred to the parish into which the removal is made, in order to place it on the same footing with respect to such a contributor as if he had never before been a contributor. This equivalent is the value of the *difference* just mentioned; and it must be calculated by the following rule.

D d 4

Multiply

Multiply the difference between the contribution to be received by the parish to which a contributor removes, and the contribution due from a person in his class and at his age, when he removes (as specified in Table III.), by the value of an annuity, payable weekly, on a life at that age, for a number of years equal to the difference between his age at removal and 65 years of age. The product will be the equivalent sum payable at his removal.

#### EXAMPLE.

Let a person be supposed to have made himself a contributor in the *second* Class under 21 years of age, and afterwards at 28 or in his 29th year, to remove. In this case the contribution is *3d. per week*: but in Table III. it appears that in that Class the contribution due from one at that age, supposing him then to commence his contribution, is *four pence halfpenny per week*. The difference is *three halfpence per week*, which is the same with *six shillings and six pence per ann.*; and the value of this annuity, payable weekly by a person aged 28 (or in his 29th year) till he is 65, and subject to the contingency of his dying in the mean time, is (by the rule in Quest. 6th, p. 19. vol. I. and the Observations in vol. II. p. 40 and 41) *15.80 year's purchase*, reckoning interest, at  *$3\frac{1}{2}$  per cent.* and the probabilities and values of lives as given in Tables VII. and XVII. vol. II. This value multiplied by *£.325* gives *£5.135*, that is nearly *5l. 2s. 6d.* which is in due proportion to the sum specified in this Table for the 1st Class. In this manner have all the sums in this Table been computed; and it is evident that they express not only the sums payable in all cases at removals, but also the

the fines payable by persons who begin their contributions at a greater age than 21, supposing them excused an *increase* of weekly contribution on that account.

The three first Tables are necessary *data* for composing the *fourth* Table. But should *fines* only be admitted on account of excess of age, no other Table would be necessary besides the fourth; and this would give great simplicity to the scheme. Perhaps, however, it may be adviseable to give an option to contributors above age at entrance, either to pay the higher weekly contributions in Table III. or to compound by paying the fines in the 4th Table. In this case the following Tables will be necessary, which exhibit the sums payable at removals to contributors at any particular ages greater than 24 (*a*).

(*a*) These Tables also (like the preceding one) exhibit the sums payable *by* those persons who shall chuse on their entrance into the club or society, to begin with such contributions as are first paid by members of any particular age less than their own, and greater than 21 years.—Thus, if a person in his 24th year wishes to be admitted into the 1st Class with contributors of 22 years of age, by beginning with a contribution of  $2\frac{1}{2}d.$  he should pay 9*s.* for such admission.—If he is in his 40th year he should pay 9*l.* 3*s.*—if in his 50th year 20*l.* 10*s.* and so on. Again: If a person in his 29th year should chuse to be admitted into the 1st Class with contributors of 23 and 24 years of age, by beginning with a contribution of  $2\frac{1}{2}d.$  he should pay 1*l.* 15*s.* for such admission—if he is in 39th year he should pay 8*l.*—if he is in his 49th year 18*l.* 14*s.* and so on. The fines payable on admission into the other classes at those respective ages are in proportion to the weekly contributions, and are easily deduced from this Table. (See Note, p. 430.)  
ED.



TABLES, shewing the Sums payable at Removals, to Contributors who have begun their Contributions in the several Years of their Age, after the 21<sup>st</sup>, without Fines.

Table V. Clafs 1ft.				Table VI. Clafs 1ft.			Table VII. Clafs 1ft.			Table VIII. Clafs 1ft.			Table IX. Clafs 1ft.			Table X. Clafs 1ft.			
Weekly Contributions 2½d.				2½d.			2¾d.			3d.			3½d.			3½d.			
Age at Subscription 22 & 23.				24 & 25.			26 & 27.			28 & 29.			30 & 31.			32.			
Age at Removal.		Sums payable.		Sums payable.			Sums payable.			Sums payable.			Sums payable.			Sums payable.			
In their	Year	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
	24th	0	9	0															
	25th	0	18	0															
	26th	1	7	0	0	9	0												
	27th	1	15	0	0	18	0												
	28th	2	3	0	1	6	0	0	9	0									
	29th	2	11	0	1	15	0	0	18	0									
	30th	3	0	0	2	3	0	1	6	0	0	9	0						
	31st	3	10	0	2	11	0	1	14	0	1	0	0						
	32d	4	3	0	3	6	0	2	9	0	1	12	0	0	10	0			
	33d	4	16	0	4	0	0	3	4	0	2	8	0	1	12	0	0	12	0
	34th	5	10	0	4	14	0	4	0	0	3	3	0	2	7	0	1	11	0
	35th	6	4	0	5	8	0	4	15	0	3	17	0	3	2	0	2	7	0
	36th	6	17	0	6	2	0	5	7	0	4	10	0	3	15	0	3	0	0
	37th	7	10	0	6	15	0	6	0	0	5	3	0	4	8	0	3	13	0
	38th	8	2	0	7	7	0	6	13	0	5	17	0	5	2	0	4	7	0
	39th	8	13	0	8	0	0	7	5	0	6	10	0	5	16	0	5	1	0
	40th	9	3	0	8	10	0	7	16	0	7	4	0	6	10	0	5	16	0
	41st	9	14	0	9	0	0	8	6	0	7	17	0	7	3	0	6	9	0
	42d	10	5	0	9	10	0	8	17	0	8	10	0	7	16	0	7	2	0
	43d	11	2	0	10	10	0	9	17	0	9	4	0	8	11	0	7	18	0
	44th	12	0	0	11	10	0	10	17	0	10	4	0	9	11	0	8	18	0
	45th	13	0	0	12	10	0	11	17	0	11	4	0	10	11	0	9	18	0
	46th	14	8	0	13	10	0	12	17	0	12	10	0	11	18	0	11	5	0
	47th	15	16	0	15	5	0	14	13	0	14	2	0	13	10	0	12	18	0
	48th	17	10	0	17	0	0	16	8	0	16	0	0	15	9	0	14	18	0
	49th	19	5	0	18	14	0	18	2	0	17	12	0	17	1	0	16	10	0
	50th	20	10	0	20	0	0	19	10	0	19	0	0	18	9	0	17	18	0
	51st	21	15	0	21	5	0	20	15	0	20	5	0	19	15	0	19	5	0
	52d	23	6	0	22	16	0	22	6	0	21	10	0	21	0	0	20	11	0
	53d	24	17	0	24	8	0	23	18	0	23	10	0	23	1	0	22	12	0
	54th	26	10	0	26	0	0	25	11	0	25	5	0	24	16	0	24	8	0
	55th	28	5	0	27	17	0	27	8	0	27	0	0	26	12	0	26	6	0
	56th	30	9	0	30	0	0	29	12	0	29	10	0	29	2	0	28	15	0
	57th	32	14	0	32	7	0	32	0	0	31	14	0	31	7	0	31	0	0
	58th	35	6	0	35	0	0	34	13	0	34	10	0	34	3	0	33	17	0
	59th	38	0	0	37	14	0	37	8	0	37	0	0	36	14	0	36	10	0
	60th	41	0	0	40	14	0	40	9	0	40	10	0	40	4	0	39	15	0
	61st	45	16	0	45	10	0	45	5	0	45	0	0	44	15	0	44	14	0



TABLES, shewing the Sums payable at Removals, to Contributors who have begun their Contributions in the several Years of their Age, after the 21st, without Fines.

Table XI. Clafs 1st.				Table XII. Clafs 1st.				Table XIII. Clafs 1st.				Table XIV. Clafs 1st.				Table XV. Clafs 1st.				Table XVI. Clafs 1st.			
Weekly Contributions 3½d.				4d.				4½d.				4½d.				4¾d.				5d.			
Age at Subscription 33.				34.				35.				36.				37.				38.			
Age at Removal.		Sums payable.		Sums payable.		Sums payable.		Sums payable.		Sums payable.		Sums payable.		Sums payable.		Sums payable.		Sums payable.		Sums payable.			
Year		£.	s. d.	£.	s. d.	£.	s. d.	£.	s. d.	£.	s. d.	£.	s. d.	£.	s. d.	£.	s. d.	£.	s. d.	£.	s. d.		
In their		34th	0 15 0																				
	35th	1 10 0	0 15 0																				
	36th	2 5 0	1 10 0																				
	37th	3 0 0	2 5 0																				
	38th	3 15 0	3 0 0																				
	39th	4 7 0	3 13 0																				
	40th	5 0 0	4 6 0																				
	41st	5 14 0	5 0 0																				
	42d	6 6 0	5 14 0																				
	43d	7 4 0	6 11 0																				
	44th	8 7 0	7 14 0																				
	45th	9 7 0	8 14 0																				
	46th	10 15 0	10 3 0																				
	47th	12 7 0	11 15 0																				
	48th	14 5 0	13 14 0																				
	49th	16 0 0	15 9 0																				
	50th	17 10 0	17 0 0																				
	51st	18 15 0	18 5 0																				
	52d	20 0 0	19 10 0																				
	53d	22 0 0	21 11 0																				
	54th	24 0 0	23 11 0																				
	55th	26 0 0	25 12 0																				
	56th	28 10 0	28 2 0																				
	57th	30 10 0	30 8 0																				
	58th	33 10 0	33 4 0																				
	59th	36 5 0	36 0 0																				
	60th	39 10 0	39 5 0																				
	61st	44 9 0	44 5 0																				

TABLES, shewing the Sums payable at Removals, to Contributors who have begun their Contributions in the several Years of their Age, after the 21st, without Fines.

Table XVII. Clafs 1ft.			XVIII. Clafs 1ft.			Table XIX. Clafs 1ft.			Table XX. Clafs 1ft.			Table XXI. Clafs 1ft.			Table XXII. Clafs 1ft.		
Weekly Contr <sup>i</sup> bution 5¼ d.			5½ d.			5¾ d.			6 d.			6½ d.			7 d.		
Age at Subscription 39.			40.			41.			42.			43.			44.		
Age at Removal.		Sums payable.	Sums payable.			Sums payable.			Sums payable.			Sums payable.			Sums payable.		
	Year	£. s. d.	£. s. d.			£. s. d.			£. s. d.			£. s. d.			£. s. d.		
In their	40th	0 16 0															
	41ft	1 12 0	0 18 0														
	42d	2 6 0	1 16 0	1 0 0													
	43d	3 6 0	2 15 0	2 2 0	1 3 0												
	44th	4 10 0	3 17 0	3 2 0	2 10 0	1 5 0											
	45th	5 12 0	4 17 0	4 4 0	3 15 0	2 10 0											
	46th	7 0 0	6 12 0	6 0 0	5 7 0	4 0 0											
	47th	8 16 0	8 4 0	7 12 0	7 0 0	5 18 0	3 0 0										
	48th	10 14 0	10 6 0	9 15 0	9 0 0	8 0 0	6 15 0										
	49th	12 14 0	12 3 0	11 12 0	11 0 0	10 0 0	8 16 0										
	50th	14 10 0	14 0 0	13 10 0	13 0 0	11 12 0	10 10 0										
	51ft	15 15 0	15 5 0	14 15 0	14 4 0	13 4 0	12 3 0										
	52d	17 0 0	16 10 0	16 0 0	15 10 0	15 0 0	14 0 0										
	53d	19 6 0	18 18 0	18 8 0	17 19 0	17 0 0	16 0 0										
	54th	21 6 0	20 17 0	20 8 0	20 0 0	19 0 0	18 0 0										
	55th	23 7 0	23 0 0	22 12 0	22 4 0	21 5 0	20 10 0										
	56th	26 5 0	25 18 0	25 10 0	25 4 0	24 0 0	23 5 0										
57th	28 10 0	28 3 0	27 16 0	27 9 0	26 10 0	26 0 0											
58th	31 14 0	31 8 0	31 2 0	31 0 0	30 0 0	29 8 0											
59th	34 8 0	34 2 0	33 16 0	33 10 0	33 0 0	32 8 0											
60th	38 0 0	37 15 0	37 10 0	37 5 0	36 15 0	36 5 0											
61ft	43 0 0	42 15 0	42 10 0	42 5 0	41 15 0	41 5 0											





## \* N O T E.

In the original Tables the sums to be paid at removal have been computed for all the Eleven Classes at every age from 22 to 50; but I have only inserted the First Class for each age in these Tables, because the insertion of the other Ten Classes would have swelled the work without answering any essential purpose. If the sums payable at removal be known when the weekly contributions are  $2\frac{1}{4}d.$  in the 1st Column,  $2\frac{1}{2}d.$  in the 2d Column, and so on: the sums to be paid in those respective cases when the weekly contributions are  $3\frac{1}{8}d.$ ,  $3\frac{3}{4}d.$ , &c. are easily obtained by the common rule of proportion. Thus, if instead of  $2\frac{1}{4}d.$  in the 1st Column, the weekly contribution had been  $3\frac{1}{8}d.$  the sum to be paid on removal would have been a fourth proportional to  $2\frac{1}{4}d.$  9s. and  $3\frac{1}{8}d.$ ; that is, expressing these numbers in decimals, it would have been

$$= \frac{.45 \times .014062}{.009375} = .67497 = 13s. 6d. \text{ or more simply } = .45 \times \frac{3}{2}.$$

If the weekly contributions had been  $7\frac{1}{8}d.$  the sum to be paid on removal would have been  $\frac{.45 \times .032812}{.009375} = 1.575 = 1l. 11s. 6d. \text{ or}$

$.45 \times \frac{7}{2}$ . But if the contributions had been  $6\frac{3}{4}d.$   $11\frac{3}{4}d.$  or any other multiple of  $2\frac{1}{4}d.$  the sum to be paid would have been the same multiple of 9s. and therefore immediately ascertained.

ED.



## A P P E N D I X II.

CONTAINING

N O T E S.

Note (A). See Question III. Page II.

LET  $E$  be any given expectation of Life; and  $\frac{4E-x}{4E} \times p$  will be the number of persons alive at the end of  $x$  years, arising from  $p$  persons left annually as widows (or added annually to a town or society) at the age whose *expectation* is  $E$ . The *maximum*, therefore, is always  $pE$ —. In Mr. *De Moivre's Hypothesis*,  $E$  is always  $\frac{1}{2}$  the difference between the given age and 86. See the Note, page 2, and the latter end of the Note in page 37. Vol. I. See likewise the beginning of the First Essay, in Vol. I.; and Note (K), in the following Notes, where the investigation of this rule will be given.

It will not be amiss to give the following example of the application of this rule.

At the time of the commencement of the scheme among the ministers and professors in SCOTLAND for making provision for their widows, it was necessary, that a calculation should be made of the number of widows that would be upon the scheme at the end of every year till they came to  
a *maximum*,

a *maximum*, on the supposition that, (agreeably to what particular enquiry had shewn to have happened for many preceding years,) 20 new widows would be left every year (*a*). In order to make this calculation, let 4 of the 20 widows be supposed to be under 32 years of age when left; and let 28 be supposed their mean age. Let the same number be left between 32 and 39, and let 35 be their mean age; between 39 and 47, and 43 their mean age; between 47 and 57, and 52 their mean age; between 57 and the extremity of life, and 63 their mean age. The number in life together to which, in 10 years, 4 widows left annually at the age of 28 will grow, is, by the rule, (*E* being 29)

$$\frac{116-10}{116} \times 40, \text{ or } 36.55. \text{—The number alive at}$$

the end of 20 years, will be  $\frac{116-20}{116} \times 80, \text{ or } 66.2.$

At the end of 30 years, the number alive will be 89; of 40 years, 104.82; of 58 years 116.—— These numbers, found in the same way, for the 2d class, (*E* being 25.5,) at the end of 10, 20, 30, 40, and 51 years, will be 36.7—64.31—84.7—97.25—102.—— For the 3d Class, (*E* being 21.5) at the end of 10, 20, 30, 40, and 43 years, 35.34—61.4—78.13—85.6—86.—— For the 4th class, (*E* being 17) at the end of 10, 20, 30, and 34 years, 34.11—56.47—67—68.—— For the 5th class, (*E* being 11.5) at the end of 10, 20, and 23 years, 31.3—45.2—46.—— The whole number, therefore, consisting of all the classes, will come to a *maximum* nearly in 58 years; and the totals in life, at the end of 10, 20, 30, 40, 50, and 58 years, will be 173.37—293.58—364.83—401.67—418.

(*a*) For a term of 35 years and eight months, being from the commencement of the scheme to the year 1783, this number was  $19\frac{1}{16}$ .

These

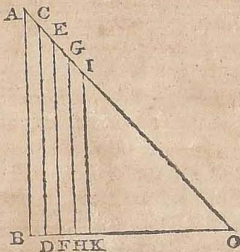
These determinations suppose none to marry. In 10 years, from 1757 to 1767, I have been informed, that but 9 widows married. Let us then suppose, that one widow of the first class marries every year; and let all that marry, be supposed to continue, one with another, 5 years in widowhood before they marry. On these suppositions, the foregoing totals will, at the end of the same periods of years, be 169.23—282—347.5—380.47—394.

These calculations are made from Mr. *De Moivre's* Hypothesis. Had they been made exactly from Dr. *Halley's* or the *Northampton Table*, the results would have been very nearly the same.

See more on this subject in note F ( $\alpha$ ).

LET

( $\alpha$ ) This theorem is deduced from a fluxional computation in note (K); but it may be demonstrated without having recourse to fluxions in the following manner:—Suppose at the time of admission there were 50 persons aged 36 years, or a number equal to their common complement. Suppose also the same number were added annually at the same age of 36. In the rectangled isosceles triangle ABO let AB (=BO) be = 50, or the complement of a life at 36  
 —CD (=DO) be = 49.  
 EF (=FO) be = 48, and so on. Hence BD will be = 1, BF = 2. BH = 3, &c. By the hypothesis of an equal decrement of life, it is evident that under the circumstances of this case the area ABCD, or  $\frac{2AB-BD}{2} \times BD$ , will express the number of annuitants at



the end of the first year; the area ABEF, or  $\frac{2AB-BF}{2} \times BF$ , the number of annuitants at the end of the 2d year; the area ABGH, or  $\frac{2AB-BH}{2} \times BH$ , the number of annuitants at

the end of the 3d year, and so on. If the number of years be  $x$ , the annuitants living will be  $\frac{2AB-x}{2} \times x$ , or

$\frac{4E-x}{2} \times x$ ; for  $AB$  being constantly  $= 50$ , or the com-

plement, will be twice the expectation, or  $2E$ .—As 50 or  $2E$ , (the numbers of persons admitted annually) is to

$\frac{4E-x}{2} \times x$ , (the number of annuitants at the end of  $x$

years) so is any other number ( $p$ ) to  $\frac{4E-x}{4E} \times px$ , the num-

ber of annuitants in the same time from ( $p$ ) persons admitted yearly at the age whose expectation is  $E$ ; and when  $x$  becomes equal to  $BO (= AB = 2E)$ , the number of annuitants will arrive at its *maximum*, and be constantly expressed by the area  $pE$ . Q. E. D. E D.



Note (B). Question VI. Page 21. Vol. I.

LET  $r$  signify the sum of 1*l.* and its interest, for one year. The value of a life, whose complement is  $n$ , being (by Mr. *De Moivre* on *Annuities*, 4th edition, page 14, and p. 100.)  $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4}$ , &c. the present value of the remainder of it after *two* years must be  $\frac{n-3}{nr^3} + \frac{n-4}{nr^4}$ , &c. which is equal to  $\frac{1}{r^2} \times \frac{n-2}{n} \times \frac{n-3}{n-2r} + \frac{n-4}{n-2r^2} + \frac{n-5}{n-2r^3}$ , &c.

Now  $\frac{1}{r^2}$  is the present value of 1*l.* due at the end of two years.  $\frac{n-2}{n}$  is the probability that a life, whose complement is  $n$ , shall continue two years, and  $\frac{n-3}{n-2r} + \frac{n-4}{n-2r^2} + \frac{n-5}{n-2r^3}$ , &c. is the value of a life two years older than the life whose complement is  $n$ . And, therefore, (since any number of years less than  $n$  may be substituted for two years) the first rule given in this Question is right ( $\beta$ ).

The

( $\beta$ ) The rules in this and the following Notes are demonstrated rather more satisfactorily, and with equal ease and perspicuity, from the real probabilities of life.

Let  $a$  represent the number of persons living in the table at the age of A, and  $b, c, d, e$ , &c. the number living at the end of the 1st, 2d, 3d, 4th, &c. years from the age of A. Now since the value of an annuity on the life of A is known to be  $= \frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3}$ , &c. the value of this annuity after

The same process, applied to joint lives, will demonstrate what is said in the *Scholium*.

two years on the same life will be  $= \frac{d}{ar^3} + \frac{e}{ar^4} + \frac{f}{ar^5}, \&c.$

$= \frac{1}{r^2} \times \frac{c}{a} \times \frac{d}{cr} + \frac{e}{cr^2} + \frac{f}{cr^3}, \&c.$  But  $\frac{c}{a}$  is the probabi-

lity that A lives two years, and the series  $\frac{d}{cr} + \frac{e}{cr^2} + \frac{f}{cr^3}, \&c.$

is the value of an annuity on a life two years older than A. The general rule therefore in the 6th Question is right; for the reasoning applied to this particular case will also apply to any other interval between the present time and the period at which the annuity is to commence. ED.

Note (C). See Question VII. P. 22. Vol. I.

LET the complements of any two assigned lives be  $n$  and  $m$ . The present value of the first possible payment of an annuity to be enjoyed by the life whose complement is  $n$ , provided *both* lives continue 7 years, and the life, whose complement is  $n$ , survives the other *after* that term, is the probability, that the life of the expectant shall continue 8 years, and the other life 7 years and then fail in the 8th year, multiplied by  $\frac{1}{r^8}$ , or by 1%. discounted for 8 years.—The probability that the life of the *expectant* shall continue 8 years is  $\frac{n-8}{n}$ . The probability that the *other* life shall continue 7 years is  $\frac{m-7}{m}$ . The probability that it shall continue 7 years, and fail in the 8th year, is  $\frac{m-7}{m} \times 1 - \frac{m-8}{m-7} = \frac{1}{m}$ . The probability, therefore, that the life of the *expectant* shall continue 8 years, and the other life continue 7 years and fail in the 8th, is  $\frac{n-8}{n} \times \frac{1}{m}$ ; and the present value of the first possible payment of the annuity supposed, is  $\frac{n-8}{nr^8} \times \frac{1}{m}$ .

See *The Doctrine of Annuities*, by Mr. Simpson, p. 6—15, or his *Select Exercises*, p. 315, &c.—In like manner, the present value of the 2d payment, at the end of the 9th year, may be found to be  $\frac{n-9}{nr^9} \times \frac{m-7}{m} \times 1 - \frac{m-9}{m-7}$ , or  $\frac{n-9}{nr^9} \times \frac{2}{m}$ . and the present value of all the possible payments,

$$\frac{1}{r^7} \times \frac{n-8}{nr} \times \frac{1}{m} + \frac{n-9}{nr^2} \times \frac{2}{m} + \frac{n-10}{nr^3} \times \frac{3}{m}, \&c.$$

But this series is equal to  $\frac{1}{r^7} \times \frac{n-7}{n} \times \frac{m-7}{m} \times$

$$\frac{n-8}{n-7r} \times \frac{1}{m-7} + \frac{n-9}{n-7r^2} \times \frac{2}{m-7} + \frac{n-10}{n-7r^3} \times \frac{3}{m-7},$$

E c 3

$\frac{3}{m-7}$ , &c. Now  $\frac{n-8}{n-7r} \times \frac{1}{m-7} + \frac{n-9}{n-7r^2} \times \frac{2}{m-7}$ , &c. is the value of an annuity for a life seven years older than the expectant, after another life seven years older than the life whose complement is  $m$ .  $\frac{n-7}{n} \times \frac{m-7}{m}$  is the probability that both the assigned lives shall continue 7 years. And  $\frac{1}{r^7}$  is the value of 1*l.* due at the end of 7 years. The rule, therefore, given for solving this question, is right.

This demonstration, as well as that in the last note, is, for the sake of more ease and clearness, applied to the hypothesis of an equal decrement of life. It does not, however, depend upon it, but may be applied to any table of observations ( $\gamma$ ).

( $\gamma$ ) Let  $a, b, c, d, e, f$ , &c. represent the same quantities as in the preceding Note. Let  $m$  represent the number of persons living at the age of B, and  $n, o, p, q, s$ , &c. the number living at the end of the 1st, 2d, 3d, &c. years from the age of B.—By reasoning in the same manner with Dr. Price in the solution of this question, and supposing the first payment of the annuity to become due at the end of the 4th year, its present value will be  $= \frac{e}{a} \times \frac{p-q}{mr^4} = \frac{dp}{amr^3} \times \frac{e}{dr} - \frac{eq}{apr}$  — the present value of the payment at the end

of the 5th year will be  $= \frac{f}{a} \times \frac{p-s}{mr^5} = \frac{dp}{amr^3} \times \frac{f}{dr^2} - \frac{sf}{dpr^2}$  — the present value of the payment at the end of the 6th year will be  $= \frac{g}{a} \times \frac{p-t}{mr^6} = \frac{dp}{amr^3} \times \frac{g}{dr^3} - \frac{gt}{dpr^3}$ , and so on.

Hence the whole value will be  $= \frac{dp}{amr^3} \times \frac{e}{dr} + \frac{f}{dr^2} + \frac{g}{dr^3}$ , &c.  
 $= \frac{dp}{amr^3} \times \frac{eq}{dpr} + \frac{sf}{dpr^2} + \frac{gt}{dpr^3} + \text{&c.}$  Let  $\dot{A}$  and  $\dot{A}\dot{B}$  denote the respective values of annuities on the single and joint lives of two persons 3 years older than A and B, and the general value will become  $= \frac{dp}{amr^3} \times \dot{A} - \dot{A}\dot{B}$ . — Q.E.D.

ED.

Note



Note (D). Question IX. Page 29. Vol. I.

**L**ET the complement of any two assigned lives be  $n$  and  $m$ , and the given term be *seven* years, as in Note (C). The probability that the former life (supposed to be the life in expectation) shall last 8 years, is, by Mr. *De Moivre's* Hypothesis,  $\frac{n-8}{n}$ ; and the probability that the latter life shall fail in 8 years, is  $\frac{8}{m}$ ; and the first payment of the annuity mentioned in this question, depends on the happening of *both* these events, the probability of which is  $\frac{n-8}{n} \times \frac{8}{m}$ .

The present value, therefore, of the first possible payment of the annuity is  $\frac{n-8}{nr^8} \times \frac{8}{m}$ .—In like manner, the present value of the *second* possible payment is  $\frac{n-9}{nr^9} \times \frac{9}{m}$ ; and of all the payments,

$\frac{n-8}{nr^8} \times \frac{8}{m} + \frac{n-9}{nr^9} \times \frac{9}{m} + \frac{n-10}{nr^{10}} \times \frac{10}{m}$ , &c. But

$\frac{n-8}{nr^8} \times \frac{8}{m} = \frac{n-8}{nr^8} \times \frac{1}{m} + \frac{n-8}{nr^8} \times \frac{7}{m}$ ; and  $\frac{n-9}{nr^9} \times \frac{9}{m} = \frac{n-9}{nr^9} \times \frac{2}{m} + \frac{n-9}{nr^9} \times \frac{7}{m}$ . The foregoing se-

ries, therefore, is equal to the two series's  $\frac{1}{r^7} \times$

$\frac{n-8}{nr} \times \frac{1}{m} + \frac{n-9}{nr^2} \times \frac{2}{m} + \frac{n-10}{nr^3} \times \frac{3}{m}$ , &c. and  $\frac{1}{r^7}$

E e 4 X

$$\begin{aligned} & \times \frac{n-8}{nr} \times \frac{7}{m} + \frac{n-9}{nr^2} \times \frac{7}{m} + \frac{n-10}{nr^3} \times \frac{7}{m}, \&c. \text{ or to} \\ & \frac{1}{r^7} \times \frac{n-7}{n} \times \frac{m-7}{m} \times \frac{n-8}{n-7r} \times \frac{1}{m-7} + \frac{n-9}{n-7r^2} \times \\ & \frac{2}{m-7} + \frac{n-10}{n-7r^3} \times \frac{3}{m-7}, \&c. + \frac{1}{r^7} \times \frac{7}{m} \times \frac{n-7}{n} \times \\ & \frac{n-8}{n-7r} + \frac{n-9}{n-7r^2} + \frac{n-10}{n-7r^3}, \&c. \text{ which is the very} \\ & \text{rule given for solving this question } (\delta), \text{ as will} \\ & \text{appear from Notes (B) and (C).} \end{aligned}$$

( $\delta$ ) Retaining the same symbols as in the two foregoing Notes, ( $\beta$ ) and ( $\gamma$ ) and supposing the first payment of the annuity to become due at the end of the 4th year, the present value of the several payments will be  $= \frac{e}{a} \times \frac{m-q}{mr^4} + \frac{f}{a} \times \frac{m-s}{mr^5} + \frac{g}{a} \times \frac{m-t}{mr^6} + \&c. = \frac{d}{ar^3} \times \frac{e}{dr} + \frac{f}{dr^2} + \frac{g}{dr^3} + \&c. - \frac{dp}{amr^3} \times \frac{eq}{dpr} + \frac{sf}{dpr^2} + \frac{gt}{dpr^3} + \&c. = \frac{d}{ar^3} \times \dot{A} - \frac{dp}{amr^3} \times \dot{A}\dot{B}. — Q. E. D.$

This rule, though it agrees in its result with the rules given by Dr. Price, is rather more concise, and may be thus expressed:—"Find by Quest. VI. the value of the annuity for the remainder of the life in expectation after the given time. Find also by the scholium to that question the value of the annuity for the remainder of the two joint lives after the given time. The latter subtracted from the former will be the value required."

Ed.

Note (E). See the Scholium to Quest. X.  
Page 34. Vol. I.

ACCORDING to the calculations, the time in which the first yearly payment of a reversionary *annuity* becomes due, is the end of the year in which the event happens that entitles to it, however little or much of the year may then happen to be unelapsed. And this, likewise, is the time when a reversionary *sum* becomes due. Those who know how the calculations of the values of reversions are instituted, must know this. But an annuity, the first payment of which is to be made at the same time with another payment of a sum in hand, sufficient to buy an equal annuity, is worth one year's purchase more than that sum. For instance. Reckoning interest at 4 *per cent.* and *r* being 1 *l.* increased by its interest for a year, or 1.04

$\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$ , &c. = 25 *l.* is the present value of an estate of 1 *l.* *per annum* for ever. That is, it is the value of it, supposing the first rent of it is to be paid a year hence. — If the first rent is to be received immediately, or at the same time with another payment of 25 *l.* it is worth one year's purchase more, or equivalent to 26 *l.* — I have not found, that any of the writers on annuities and reversions, have attended to this observation. It suggests a correction necessary to be applied to the common solutions of several important problems: particularly to the 21st and 22d in Mr. *Simpson's Treatise on Annuities*, and the 26th, 27th, 32d, 33d, and 40th problems in his *Select Exercises*; and to all other problems of the same kind in other writers. There  
can

can be no great occasion for being more explicit. It will not, however, be amiss to add the following demonstration. —  $\frac{1}{n}$  is the present probability that a life whose complement is  $n$  will fail in any one assignable year of its duration.  $S \times \frac{1}{nr} + \frac{1}{nr^2} + \frac{1}{nr^3}$ , &c. ( $n$ ), or the present value of 1*l.* per annum for  $n$  years, multiplied by  $\frac{S}{n}$ , is the present value of the sum or legacy denoted by  $S$ , payable at the failure of the given life. Therefore, ( $n$  being 56; the life 30; interest 4 per cent.  $r=1.04$ ; the sum 25*l.*) the value of the expectation, by Mr. De Moivre's hypothesis, is 9.919.

Further. The value of 1*l.* to be received at the end of a year, provided the life whose complement is  $n$  fails, is the probability of the failure of the life multiplied by 1*l.* discounted for a year, or

$$1 - \frac{n-1}{n} \times \frac{1}{r}. \text{ In like manner; the value of 1} \textit{l.}$$

to be received at the end of two years, if the same

life fails in 2 years, is  $1 - \frac{n-2}{n} \times \frac{1}{r^2}$ . And, there-

fore, the value of all the possible payments of an estate or annuity of 1*l.* for ever, to be entered

upon after the given life, is  $1 - \frac{n-1}{n} \times \frac{1}{r} + 1 -$

$$\frac{n-2}{n} \times \frac{1}{r^2} + 1 - \frac{n-3}{n} \times \frac{1}{r^3}, \text{ \&c. } (n) + \frac{1}{rn+1} +$$

$$\frac{1}{rn+2}$$



$\frac{1}{r^{n+2}}$ , &c. or  $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$ , &c. —  $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3}$ , &c. that is, the value of the life subtracted from the perpetuity; or, in this example, *l.* 14.684, (the value of a life at 30) subtracted from 25; that is, *l.* 10.316. But 10.316 is to 9.919, in the same ratio with 104 to 100, or 26 to 25, agreeably to the rule in the *Scholium* ( $\epsilon$ ).

(3) The difference between the values of reversionary *sums* and reversionary *estates* (which was first pointed out in this Note) does not depend on the hypothesis of an equal decrement, but may be as readily demonstrated from the real probabilities of life. Supposing *a, b, c, d, e*, &c. to represent the same quantities as in Note ( $\beta$ ), the value of the *sum* *S*, to be received on the death of *A*, will be properly expressed

$$\text{by the series } \frac{S}{a} \times \frac{a-b}{r} + \frac{b-c}{r^2} + \frac{c-d}{r^3} + \&c. = S \times \frac{\frac{1}{r} + \frac{b}{ar^2} + \frac{c}{ar^3} + \frac{d}{ar^4} + \&c.}{\frac{1}{r} + \frac{b}{ar^2} + \frac{c}{ar^3} + \frac{d}{ar^4} + \&c.} - S \times \frac{\frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3} + \&c.}{\frac{1}{r} + \frac{b}{ar^2} + \frac{c}{ar^3} + \frac{d}{ar^4} + \&c.}$$

$$= S \times \frac{A+1}{r} - A = \frac{S \cdot r-1}{r} \times P-A. \text{— (P denoting}$$

the perpetuity, and *A* the value of an annuity on the life of *A*).—But in the case of an *annuity* or *estate*, the value of the reversion of  $\pounds 1$  *per ann.* after the death of *A* will be

$$= \frac{a-b}{ar} + \frac{a-c}{ar^2} + \frac{a-d}{ar^3} + \&c. \text{— (t) } + \frac{1}{r^{t+1}} + \frac{1}{r^{t+2}} + \frac{1}{r^{t+3}} + \&c. \text{ (t denoting the number of years between the}$$

age of *A* and that of the last surviving life in the table of observations). The sum of these two series is easily found = *P*—*A*.—If *S* represent a sum equal to the perpetuity of

$\pounds 1$  *per ann.* or, in other words, if *S* be taken =  $\frac{1}{r-1}$ , it will appear that the value of the reversion of an *estate* is to the value of the reversion of an equivalent *sum* as *P*—*A* to  $\frac{P-A}{r}$ , or as *r* to 1, agreeable to what has been observed

above.

ED.

Note

Note (F). Quest. XIII. Page 44. Vol. I.

WHEN I here call 48 the mean age of all married men, and 40 the mean age of married women, I do not intend to suppose, that there are as many married persons who exceed these ages, as there are who fall short of them. It is likely that the latter are most numerous; and it is necessary that this should be the case, to render the supposition I make just.—If all marriages commenced at 33 for the man, and 25 for the woman, one half of them would be dissolved by the time the men were 50, and the women 42; for (by the *Hypothesis*, and also nearly by the *Breslaw*, *Norwich*, and *Northampton* tables) there is an equal chance for the joint continuance of two lives, whose ages are 25 and 33, *seventeen* years. Forty-two and fifty then would be properly the mean ages at which widowhood would commence: meaning by these “the ages on each side of which equal numbers are left widows and widowers.”—But, though in this case half the marriages of every year would be dissolved in 17 years, they would not be *all* dissolved in twice that time. So far would this be from happening, that about a 7th part would continue beyond twice 17 years; nor would it be *certain*, that they would be all dissolved till near the extremity of the possible extent of life. Though, therefore, an equal number of marriages would be dissolved, or an equal number of widows and widowers left *before* 50 and 42 and *afterwards*, yet the ages of the latter would, one with another, much more exceed 50 and 42, than the ages of the former (that is, of the widows and widowers left  
*before*

*before 50 and 42*) would fall short of them. And the number of marriages also in the world, among persons of greater ages than these, would be much fewer than among persons of lesser ages.—In other words: The period, at which the marriages that have been contracted are half dissolved, is not the period at which the number of marriages constantly existing is equally divided, but this period falls some years sooner; and the period I have in view falls in that part of the interval between these two periods, where the greater ages of the marriages on one side, are just enough to compensate (in such a calculation as that I have given) their deficiencies in number, compared with the number of marriages on the other side.

In short. Suppose 35 marriages every year, between persons 33 and 25 (*a*). In 12 years there would be half as many in the world, as could possibly arise from such a number of yearly weddings. In 17 years, half every set would be extinct. The *expectation* of every marriage would be 19 years, by prob. 21 of Mr. De Moivre's *Treatise on Annuities*, or by the note (K) in the following notes: That is, taking them all together, they would exist just as long as an equal number of *single* persons, supposed to be sure of living just 19 years, and no more: or, as long as an equal number of single persons, all 48 years of age, supposed to be subject to the common laws of mortality. One with another, then, they will be all extinct in 19 years; the marriages which continue beyond this term, though fewer in number, enjoying among them just as

(*a*) In the *Pais de Vaud*, Switzerland, the mean age at which women marry, is nearly the very age here mentioned: But it is shewn in the *Supplement*, that the expectation of marriage there is no less than 23 years and  $\frac{1}{2}$ ; so much higher are the probabilities of life in the *country* than in *towns*, or than they ought to be according to Mr. De Moivre's *Hypothesis*. See p. 254, Vol. II.



much *more* duration, as those that fall short of it enjoy *less*. *Widows*, then, at a medium, will commence widowhood at 44 (that is, 25 increased by 19) years of age, and *widowers* at 52. The values, therefore of the lives of the *former*, when they commence widowhood, will, one with another, be the same with the value of a life at 44; or, (reckoning interest at 4 per cent.) 12.5 years purchase, in one present payment, (the annuity to begin at the end of a year); and their *expectation* of life will be 21 years, or half the difference between 44 and 86. The value of the lives of the *latter* will be 10.92, and their *expectation* 17 years.—The whole number of marriages constantly existing, which would result from 35 supposed to commence annually, would be  $19 \times 35$ , or 665; and 53 years (the difference between 33 and 86) would be the time in which they would increase to this number.—The chance of survivorship would be the odds of 69 to 53, by prob. 18th, Mr. De Moivre on Annuities; that is, in 53 years, 35 relicts of these marriages would be left every year, and the number of *widows* would be to the number of *widowers*, as 69 to 53; or 19.8 *widows* would be left annually, and 15.2 *widowers*. The *maximum* of widows in life together, if none married, would be  $21 \times 19.8$ , or 416; and they would increase to this number in 114 years (or 61 years after the number of marriages had arrived at a *maximum*)—The *maximum* of *widowers* would be  $15.2 \times 17$ , or 258; and they would increase to this number in 106 years.

An easy method may be hence deduced of solving the question which occasions this note — If the number of the members of the establishment I have supposed is 665, and the mean ages at which marriage may be deemed to commence are 25 and 33, 19 8 widows will (it has just appeared) be left every year; and the values of their lives,

when



when they commence widowhood, will be, one with another,  $12\frac{1}{2}$  years purchase. An annuity of 20*l.* will, therefore, be worth, to each widow, 250*l.* and 19.8 such annuities must be worth 4950*l.* which, consequently, is the annual income necessary for the support of the establishment, the first payment to be received immediately: or *l.* 7.44 from each of the 665 members; which answers nearly to the determination in Vol. I. p. 44.

In the last Essay in Vol. I. p. 364, it has been shewn, that observations determine the chance of survivorship in favour of the wife in marriage, to be really so great as 3 to 2; and in some circumstances greater. I have also there observed, that in order to account for this from the difference of age between men and their wives, this difference must be at least 12 years, and the mean ages of all who marry annually must be supposed to be about 23 and 35. In this case, 19, as before, will nearly be the *expectation* of all marriages. The mean age at which widows and widowers will commence such will be 42 and 54. The number of annual marriages necessary to keep up 665 marriages constantly existing, will be 35. The number of widows left annually, by such a number of marriages, will be 21; and the values of their lives, at the time they commence widowhood, will be 12.85 years purchase by the first of the following Tables; and, therefore, the whole annual income necessary for the support of the supposed establishment, will be 5397*l.* or an annual payment, beginning immediately, of *l.* 8.11 from each member—The number of widows on such an establishment will, in 63 years, grow, if none marry, to 462; and the number of widowers to 224.—It may be depended on, that all this would happen as far as Dr. *Halley's* Table, or the Tables for *Norwich* and *Northampton*, exhibit the true state of human mortality.

Among

Among the ministers and professors in SCOTLAND, the number of married men being 667, or nearly that here mentioned, the number of annual weddings has, for many years, been at an average 30, and the number of widows left annually  $19\frac{1}{4}$ ; and, therefore, the chance of survivorship in favour of the wife, as 19.2 to 11.8, or 5 to 3. This is not more different from the results I have given, than might have been expected; and the chief reason of the difference is, that the *expectations* of *single* and *joint* lives among the ministers and their wives in SCOTLAND are greater than those given by Dr. Halley's, and the other tables of observation—These tables give the expectations of lives as they are among the bulk of mankind in moderate towns. The expectations of lives among the better sort of men, living mostly in country villages and parishes, are much greater. The fact is, that among the ministers in Scotland, the expectation of a *single* life, at the age of 27, is near 4 years greater; and, of *joint* lives, about three years greater, than the same expectations by Dr. Halley's Table. See the latter end of the last Essay in the former Volume.

I cannot help just mentioning another remark here.—It may be observed, that supposing no second marriages, and, at the same time, that the odds for the woman's surviving in marriage is 3 to 2, the number of *widows* in the world would be *double* the number of *widowers*. But it has been found, in fact, that the number of widows is, in some situations, five times the number of widowers. How this is to be accounted for, I have shewn in the Essay just referred to.

Note (G). Question XIV. Page 48. Vol. I.

LET  $r$  be 1%. increased by its interest for one year;  $t$  the given time or number of years for which the assurance is to be made;  $a, b, c, \&c.$  the *probabilities* taken out of a table of observations, that the person whose age is given shall live 1, 2, 3, &c. years; and  $P$  the probability that he

shall live  $t$  years. Then  $\frac{1-a}{r} + \frac{1-b}{r^2} + \frac{1-c}{r^3}, \&c.$

$(t-1) + \frac{1-P}{r^t} + \frac{1-P}{r^{t+1}} + \frac{1-P}{r^{t+2}}, \&c. = \frac{1}{r} + \frac{1}{r^2} +$

$\frac{1}{r^3}, \&c. (t) - \frac{a}{r} + \frac{b}{r^2} + \frac{c}{r^3}, \&c. (t-1) + \frac{P}{r^t} +$

$\frac{1-P}{r^t} \times \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}, \&c.$  will be the exact value

of an annuity to be entered upon at the failure of the given life, provided it happens in  $t$  years: And the rule is nothing but this value expressed in words. In a similar manner may be demonstrated the other rule for finding the values of assurances for a given time, on two joint lives, or the longest of two lives.



Note (H. Question XV. Page 56. Vol. I.

LET  $r$  signify as before;  $S$  the given sum to be assured;  $t$  the given time;  $N$  and  $n$  the number of the living in the table of observations, at the age of  $B$  and  $A$  respectively;  $A, B, C, \&c.$  and  $a, b, c, \&c.$  the number of the living in the table, at the end of  $1, 2, 3, \&c.$  years from the ages of  $B$  and  $A$ ;  $D, D, D, D, \&c.$  and  $d, d, d, d, \&c.$

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the decrements of life in the table, at the end of  $1, 2, 3, \&c.$  years from the same ages. Then, by reasoning in the same manner with Mr. *Simpson*, in p. 316, &c. *Select Exercises*, it will appear that  $S \times$

$$\frac{A \times d}{Nnr} + \frac{B \times d}{Nnr^2} + \frac{C \times d}{Nnr^3}, \&c. (t) + S \times \frac{Dd}{2Nnr} +$$

$$\frac{Dd}{2Nnr^2} + \frac{Dd}{2Nnr^3}, \&c. (t) = \frac{S}{n} \times \frac{Ad}{Nr} + \frac{Bd}{Nr^2} +$$

$$\frac{Cd}{Nr^3}, \&c. (t) + \frac{S}{2N} \times \frac{Dd}{nr} + \frac{Dd}{nr^2}, \&c. (t). \quad \text{This}$$

is the exact answer to Question XV. and the rule is as near an approximation to it as there is reason to desire.

In the same manner, retaining all the same symbols, it may be found, that the answer to Question XVI. is

$$S \times \frac{Dd}{2Nnr} + \frac{Dd}{Nnr^2} + \frac{D + D \times d}{Nnr^3} + \frac{D + D + D \times d}{Nnr^4}$$

$$(t), \&c. + S \times \frac{Dd}{2Nnr^2} + \frac{Dd}{2Nnr^3} + \frac{Dd}{2Nnr^4}, \&c.$$

( $t-1$ )



$$(t-1) = \frac{S}{nr} \times \frac{Dd}{Nr} + \frac{D+D \times d}{Nr^2} + \frac{D+D+D \times d}{Nr^3},$$

$$\&c. (t-1) + \frac{S}{2N} \times \frac{Dd}{nr} + \frac{Dd}{nr^2} + \frac{Dd}{nr^3}, \&c. (t).$$

$$\text{But } \frac{D}{Nr} + \frac{D+D}{Nr^2} + \frac{D+D+D}{Nr^3}, \&c. (t-1) \text{ is the}$$

same with the excess of the value of an annuity *certain* for a number of years less by one year than the given term, above the value of an annuity on the life of A, for the same number of years, from whence the reason of the rule for solving this question may be easily discovered (§).

(§) The solution of the 15th question may be deduced in a similar, but rather more accurate, manner from the first of the two rules given in Note (O); where the value of the reversion for  $t$  years is expressed by the two series  $\frac{S}{2ab} \times$

$$\frac{ca'}{r} + \frac{da''}{r^2} + \frac{ea'''}{r^3} \dots (t) + \frac{S}{2ab} \times \frac{ba'}{r} + \frac{ca''}{r^2} + \frac{da'''}{r^3} \dots (t).$$

If  $a$  denote the sum of the decrements of life from the age of A for  $t$  years divided by  $t$  (which may be called the *complement* of A's life for the given term), and B and  $\bar{B}$  the values of an annuity on the life of B for  $t$  and  $t-1$  years respectively, the sum of these two series may be found  $= \frac{S.a}{2a}$

$$\times B + \frac{\bar{B}+1}{r}.$$

In like manner, the solution of the 16th Question may be derived from the second of the two rules given in Note (O);—the series expressing the value of the reversion

$$\text{in this case being } \frac{S}{2ab} \times \frac{b-ca'}{r} + \frac{c-da'+a''}{r^2} \dots (t) + \frac{S}{2abr}$$

$$\times \frac{c-da'}{r} + \frac{d-ca'+a''}{r^2} \dots (t-1). \text{ Let } \beta \text{ denote the complement}$$

plement of B's life for  $t$  years,  $\bar{A}$  and  $\bar{A}^i$  the values of an annuity on the life of A for  $t$  and  $t-1$  years, and  $N$  and  $N^i$  the values of an annuity certain for those respective terms; then will the above series be found  $= \frac{S \cdot \beta}{2b} \times$

$$\bar{N} - \bar{A} + \frac{N^i - A^i}{r}.$$

It is to be observed, when the decrements of A's life for  $t$  years in the first of these rules, and the decrements of B's life in the second are equal, that the exact value of the reversion is obtained; and if the term do not exceed 10 or 12 years, the values are always so nearly true as not to require greater accuracy. This also is the case in general with regard to Dr. Price's rules; against which there can be no objection, excepting the application of Mr. De Moivre's hypothesis in one part of them, which it is best entirely to exclude from the doctrine of survivorships.

ED.

Note (I). Page 139 and 177. Vol. I.

SUPPOSING  $r$  to signify as in the last notes, and  $n$  to be the complement of a given life; the present value of 1*l.* 2*l.* 3*l.* &c. payable at the end of 1, 2, 3, &c. years to  $t$  years, but subject to failure when the life fails, is  $\frac{n-1}{nr} + \frac{n-2 \times 2}{nr^2} + \frac{n-3 \times 3}{nr^3}$ , &c. continued to  $t$  years; which expres-

sion is equal to  $n \times \frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3}$ , &c. ( $t$ )

$$= n \times \frac{\frac{n-1}{n^2r}} + \frac{\frac{n-2}{n^2r^2}} + \frac{\frac{n-3}{n^2r^3}} , \text{ \&c. } (t).$$

To find, therefore, the value of an annual payment dependent on a given life, to begin with 1*l.* and to increase at the rate of 1*l.* every year after the first, for a given term; find the value of an annuity on the given life for the given term; and also the value for the given term of an annuity on two joint lives both equal to the given life. The difference between these two values multiplied by the complement of the given life, will be the value sought.—If such a course of payment, instead of beginning at the end of a year, is to begin immediately, and to be made at the beginning of every year till  $t+1$  payments are made in  $t$  years; add to the preceding value the value increased by unity of an annuity on the given life for  $t$  years, found by Question VI, and the sum will be the value sought. And this value, divided by the

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present

present value of what may happen to remain of the given life after  $t$  years, found by the same question, will give the annuity to which such a series of increasing annual payments, beginning immediately, will entitle for the remainder of the given life after  $t$  years.

If such a course of payment is to begin at the end of a year, and to be continued during life (that is, if  $t = n$ ) it is obvious, that its value will be the complement of the life multiplied by the difference between the value of the life, and the value of two joint lives having the same common age with it; and that if it is not to commence till the end of a given number of years, its value will be the value for a life so many years (lessened by one) older than the given life, and multiplied by the value of 1*l.* payable at the end of a number of years less by one year than the given number of years, and also multiplied by the probability that the given life will exist for the same number of years. —Supposing, for instance, the given life 30 years of age, and such a course of payment to begin when it has completed its 56th year, the value would be the value of a life aged 55, diminished by the value of two joint lives both 55, and the remainder multiplied by the complement of a life aged 55, and also by the product of the probability that a life aged 30 will exist 25 years, into the value of 1*l.* payable at the end of 25 years. —The value thus computed will, in this case, come out 19*l.* nearly, in a single present payment, reckoning interest at 4 *per cent.* and taking the probabilities of the duration of life from the *Northampton Table of Observations*.

With



With the assistance of these rules, all that is said in Vol. I. p. 139 and p. 177, may be investigated. But more particular directions for computing the values of annuities of this sort may be found in *Mr. Morgan's Treatise on Life-Annuities and Assurances*, p. 119, &c.

Note (K). See Essay I. P. 230, 231. Vol. I.

THE *sum* of the probabilities that any given lives will attain to the end of the 1st, 2d, 3d, &c. years from the present time to the utmost extremity of life (for instance,  $\frac{4}{5} + \frac{4}{6} + \frac{4}{7}$ , &c. to  $\frac{1}{40} = 22\frac{1}{2}$  for lives of 40, by the *hypothesis*) may be called their *expectation*, or the number of payments due to them, as *yearly annuitants*. The sum of the probabilities that they will attain to the end of the 1st, 2d, 3d, &c. *half-years*, (or, in the particular case specified,  $\frac{2}{1} + \frac{2}{2} + \frac{2}{3} + \frac{2}{4}$ , &c. =  $2\frac{1}{2}$  *half-years*, or  $22\frac{3}{4}$  *years*) is their expectation as *half-yearly annuitants*. And the sums just mentioned of the probabilities of their attaining to the end of the 1st, 2d, 3d, &c. *moments* (equal in the same particular case to 23 years) is properly their *expectation of life*, or their *expectation* as annuitants secured by land.

Mr. De Moivre has omitted the demonstrations of the rules he has given for finding the *expectations* of lives, and only intimated in general, that he discovered them by a calculation deduced from the method of fluxions. See his *Treatise on Annuities*, page 66. It will, perhaps, be agreeable to some to see how easily they are deduced in this method, upon the hypothesis of an equal decrement of life.

Let  $x$  stand for a moment of time, and  $n$  the complement of any assigned life. Then  $\frac{n-x}{n}$ ,  $\frac{n-2x}{n}$ ,  $\frac{n-3x}{n}$ , &c. will be the *present* probabilities of its continuing to the end of the 1st, 2d, 3d, &c. moments; and  $\frac{n-x}{n}$  the probability of its continuing  
to

to the end of  $x$  time.  $\frac{n-x}{n} \times \dot{x}$  will therefore be the *fluxion* of the sum of the probabilities, or of an *area* representing this sum, whose *ordinates* are  $\frac{n-x}{n}$ , and *axis*  $x$ .—The *fluent* of this expression, or

$x - \frac{x^2}{2n}$ , is the sum itself for the time  $x$ ; and this, when  $x = n$ , becomes  $\frac{1}{2}n$ , and gives the *expectation* of the assigned life, or the sum of all the probabilities just mentioned, for its whole possible duration.—In

like manner: since  $\frac{(n-x)^2}{n^2}$  is the probability that two equal joint lives will continue  $x$  time,  $\frac{(n-x)^2}{n^2} \times \dot{x}$  will be the *fluxion* of the sum of the probabilities. The *fluent* is  $x - \frac{x^2}{n} + \frac{x^3}{3n^2}$ , which, when  $x = n$ , is  $\frac{n}{3}$ , or the *expectation* of two equal joint lives.—

Again: since  $\frac{n-x}{n} \times \frac{2x}{n}$  is the probability that there will be a survivor of two equal joint lives at the end of  $x$  time,  $\frac{n-x}{n} \times \frac{2x}{n} \times \dot{x}$  will be the *fluxion* of the sum of the probabilities; and the *fluent*, or  $\frac{x^2}{n} - \frac{2x^3}{3n^2}$  is (when  $x = n$ )  $\frac{1}{3}n$ , or the *expectation* of survivorship between two equal lives; which, therefore, appears to be equal to the *expectation* of their joint continuance. The *expectation* of two *unequal* joint lives, found in the same way, is  $\frac{m}{2} - \frac{m^2}{6n}$ ,  $m$  ( $n$ ) being the *complement* of the

( $n$ ) The *expectation* of two unequal joint lives is  $= \frac{m-x}{m}$

$\times \frac{n-x}{n} \times \dot{x}$ , whose *fluent* (when  $x = m$ ) is easily found  $=$

$$\frac{m}{2} - \frac{mm}{6n}$$

Ep.

oldest



oldest life, and  $n$  the *complement* of the youngest. The whole expectation of survivorship is  $\frac{n}{2} - \frac{m}{2} + \frac{m^2}{3n}$  (9). And the expectation of survivorship of the oldest will be to the expectation of survivorship of the youngest, as  $\frac{m^2}{6n}$  to  $\frac{n}{2} - \frac{m}{2} + \frac{m^2}{6n}$ . It is easy to apply this investigation to any number of joint lives, and to all cases of survivorship.

It may be observed, concerning the first of the fluents here given, that it expresses not only the expectation of a given life for the time  $x$ , and therefore its whole expectation when  $x = n$ , but likewise the number of persons alive, to which one person added annually to a society, at a given age, will increase in  $x$  time.—Thus: Suppose one

(9) The expectation of survivorship due to the oldest life is expressed by the *fluent*  $\frac{m-x}{m} \times \frac{x}{n} \times \dot{x}$ , whose *fluent*

(when  $x = m$ ) is  $\frac{mm}{6n}$ . The expectation of survivorship due

to the youngest life for  $m$  years is the *fluent* of  $\frac{n-x}{n} \times \frac{x}{m}$

$\times \dot{x}$ , which (when  $x = m$ ) is  $\frac{m}{2} - \frac{mm}{3n}$ . But this life has a

further expectation, after  $m$  years, expressed by the *fluent* of  $\frac{n-m-x}{n-m} \times \frac{n-m}{n} \times \dot{x}$ , which (when  $x = n-m$ ) will be

$\frac{n}{2} - m + \frac{mm}{2n}$ . The sum of these two *fluents*, or  $\frac{n-m}{2} + \frac{mm}{6n}$

will therefore be the whole expectation of survivorship due to the youngest life. And this expression added to  $\frac{mm}{6n}$

(which has been found above to be equal to the expectation of survivorship due to the oldest life) will give  $\frac{n}{2} - \frac{m}{2}$

$+ \frac{mm}{3n}$  for the whole expectation of survivorship due to both lives.

Ex.

annuitant



annuitant, whose age is 28, (and whose *complement* of life, therefore, is 58, or *expectation* of life 29) to come upon a society every year; the number of annuitants alive, deduced from hence, will, in  $x$  years, be  $x - \frac{x^2}{4 \times 29}$ , or  $\frac{4 \times 29 - x^2}{4 \times 29} \times x$ ; and, therefore, the number of annuitants alive, deduced in the same time from  $p$  annuitants left annually at the same age, will be  $\frac{4 \times 29 - x^2}{4 \times 29} \times px$ .—In like manner, the 2d fluent, or  $\frac{x^3}{3n^2} - \frac{x^3}{n} + x$ , gives the number of marriages in being together, that will, in  $x$  years, grow out of *one* yearly marriage, between persons of *equal* ages, whose complement of life is  $n$ . If they are of *unequal* ages, and the complement of the oldest life is  $m$ , and of the youngest  $n$ , this number will be  $\frac{x^3}{3nm} - \frac{n+m \times x^2}{2nm} + x$ . And if the number of years is required, in which any given number of yearly marriages, between men and women at given ages, will increase so far as to be in any given proportion to the greatest number that can possibly grow out of such marriages, this expression must be made equal to the *expectation* of the joint lives, or of each marriage, multiplied by the fraction expressing the given proportion; and the root of the equation will be the answer. Thus: it may be found, that one marriage every year, between persons 33 and 25 years of age, would in 10 years increase to 8.35; in 15 years, to 11.38; and in 53 years, to 19, or their greatest possible number; and, consequently, that 35 such yearly marriages would, in 10 years, increase to 292; in 15 years to 398; and in 53 years, to 665.—And if it is enquired in what number of years 35 such yearly marriages would increase to half the number in being together, possible to be

be derived from them, the value of  $x$ , in the cubic equation  $\frac{x^3}{3nm} - \frac{n+m \times x^2}{2nm} + x = \frac{m}{2} - \frac{m}{6n} \times \frac{r}{2}$ , must be found; which, in the present instance, is nearly 12.

I have, in some parts of this work, had occasion to make such deductions as these. See note (A), p. 431; and note (F), p. 444; and Questions III. and XIII. Vol. I.

Note (L). Vol. I. Essay II. Page 306.

**L**ET  $r$  signify 1%. increased by its interest for one year.

**V** the PERPETUITY.

$n$  the difference between the age of the youngest life, and 86; or its *complement*.

$m$  the complement of the oldest life.

**P** the value (in Table II. at the beginning of this volume) of an annuity certain for  $m$  years.

And the exact value of any two given joint lives, according to the hypothesis of an equal decrement

of life, will be  $V - \frac{V+1}{n} \times \frac{n-m-2V-1}{m} \times \frac{P}{+ 2V(x)}$ . Example:

Let

( $x$ ) This general rule is taken from Mr. *Simpson's* Doctrine of Annuities, and is easily demonstrated by the assistance of the Postscript to the third additional Essay in this work, p. 402: Vol. II.—The series expressing the value of an annuity on two joint lives, whose complements are  $n$  and  $m$ , is known to be =

$$\frac{m-1}{m} \times \frac{n-1}{nr} + \frac{m-2}{m} \times \frac{n-2}{nr^2} \dots (m) = \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$$

$$\dots (m) - \frac{m+n}{mn} \times \frac{1}{r} + \frac{2}{r^2} + \frac{3}{r^3} +, \&c. \dots (m) + \frac{1}{mn}$$

$\times \frac{1}{r} + \frac{4}{r^2} + \frac{9}{r^3} \dots (m)$ . By the rule in the Postscript just referred to, the first of these series may be found =

$$\frac{1}{r-1} - \frac{1}{r \cdot r-1} \dots \text{the second} = - \frac{m+n}{mn} \times \frac{r}{r-1} +$$

$$\frac{mm+n}{mn} \times \frac{1}{r \cdot r-1} + \frac{m+n}{mn} \times \frac{r}{r \cdot r-1}^2, \text{ and the third}$$

$$= \frac{r^2+r}{mn \cdot r-1} - \frac{mm}{mmr} \times \frac{1}{r-1} - \frac{2mr}{mmr} \times \frac{1}{r-1}^2 - \frac{r^2+r}{mmr}$$

$$\times \frac{1}{r-1}^3. \text{—Adding these different series together their sum}$$

will

Let the ages be 27 and 38; and the rate of interest 4 *per cent.* Then  $n = 59$ .  $m = 48$ .  $V = 25$   
 $P =$

$$\begin{aligned} \text{will be} &= \frac{1}{r-1} - \frac{m+n}{mn} \times \frac{r}{r-1}^2 + \frac{n-m}{mn} \times \frac{r}{r^m r-1}^2 + \\ &\frac{r^2+r}{mn \cdot r-1}^2 - \frac{r^2+r}{mn \cdot r^m r-1}^3 = \left( \text{since } \frac{r}{r-1} \text{ is } = V+1, \text{ and} \right. \\ &\frac{r^2+r}{r-1}^3 \text{ is } = \frac{r}{r-1}^2 + \frac{2r}{r-1}^3 \Big) V - \frac{m+n-1}{mn} \times \frac{V+1}{r-1} + \\ &\frac{n-m-1}{mn} \times \frac{V+1}{r^m r-1} + \frac{V+1 \cdot 2V}{mn \cdot r-1} - \frac{V+1 \cdot 2V}{mn \cdot r^m r-1}. \text{ But} \\ &\frac{1}{r-1} - \frac{1}{r^m r-1} \text{ is } = P; \text{ therefore the above expression} \\ &\text{becomes} = V - \frac{V+1}{n} \times \frac{n-m-2V-1}{n-m-2V-1} \times \frac{P}{m} + 2V. \\ &\text{Q. E. D.} \end{aligned}$$

If the annuity be payable *half-yearly*, and  $1 + \frac{r}{2}$  denote, as in the 3d additional Essay of this work (p. 383)  $\mathcal{L} 1$  increased by its interest for half a year, the series expressing the value of this annuity will be  $= \frac{1}{2} \times \frac{m-\frac{1}{2} \cdot n-\frac{1}{2}}{mn \cdot 1 + \frac{r}{2}} + \frac{1}{2} \times$

$$\frac{m-1 \cdot n-1}{mn \cdot 1 + \frac{r}{2}}^2 + \frac{1}{2} \times \frac{m-\frac{3}{2} \cdot n-\frac{3}{2}}{mn \cdot 1 + \frac{r}{2}}^3 \dots (zm). \text{ By proceed-}$$

ing in the same manner as in the foregoing theorem, and putting  $H$  to represent the value of an annuity certain, payable half yearly, for  $m$  years, the general rule in this case may be found  $= V - \frac{V+\frac{1}{2}}{n} \times \frac{n-m-\frac{1}{2}-2V}{n-m-\frac{1}{2}-2V} \times \frac{H}{m} + 2V.$

If the annuity be payable *quarterly*, and  $1 + \frac{r}{4}$  denote  $\mathcal{L} 1$  increased by its interest for three months, the series will be  $= \frac{1}{4} \times \frac{m-\frac{1}{4} \cdot n-\frac{1}{4}}{mn \cdot 1 + \frac{r}{4}} + \frac{1}{4} \times \frac{m-\frac{1}{2} \cdot n-\frac{1}{2}}{mn \cdot 1 + \frac{r}{4}}^2 + \frac{1}{4} \times \frac{m-\frac{3}{4} \cdot n-\frac{3}{4}}{mn \cdot 1 + \frac{r}{4}}^3$   
 $+ \&c.$



$P = 21.195. n - m - 2V - 1 = -40. n - m -$   
 $2V - 1 \times \frac{P}{m} + 2V = 50 - 17.660 = 32.340.$  And  
 $V - \frac{V+1}{n} \times n - m - 2V - 1 \times \frac{P}{m} + 2V = 25 - \frac{26}{39}$   
 $\times 32.340 = 10.748,$  the value of two joint lives  
 whose ages are 27 and 38.

+ &c. --- (4m), and its sum =  $V - \frac{V+\frac{1}{4}}{n} \times$   
 $n - m - \frac{1}{4} - 2V \times \frac{Q}{m} + 2V,$  Q representing the value of  
 an annuity certain payable *quarterly* for *m* years.—In like  
 manner, if the annuity be payable *momently*, and  $1 + \frac{r}{1000, \&c.}$   
 denote £1 increased by its interest for a moment, the general  
 rule for determining the value of the annuity will be =  $V -$   
 $\frac{1}{n} \times n - m - \frac{1}{1000, \&c.} - 2V \times \frac{M}{m} + 2V =$   
 $V - \frac{V}{n} \times n - m - 2V \times \frac{M}{m} + 2V;$  M representing the  
 value of an annuity certain payable *momently* for *m* years.

Supposing the ages of two lives to be 20 and 36 years, the  
 value of an annuity at 4 *per cent.* during their joint continu-  
 nuance, and payable either yearly, half yearly, quarterly,  
 or momentarily will, by Mr. De Moivre's hypothesis, be 11.227  
 ...11.427...11.565...or 11.629. If their ages be 36 and 61,  
 the values will be 7.448...7.673...7.793...or 7.901. If  
 both their ages be 36, the values will be 10.394...10.600...  
 10.703...or 10.808...and if both their ages be 61, the values  
 will be 6.144...6.374...6.517...or 6.602.

By comparing the values of the equal joint lives, given  
 above, with the values of the single lives of the same ages, com-  
 puted in the third additional Essay (p. 388 & 389), it appears  
 that the differences in the former between annuities payable  
 yearly and those which are payable half yearly, quarterly,  
 or momentarily, are greater than the differences in the latter;  
 and therefore that the addition to be made to an annuity  
 on the longest of two lives, in consequence of its being  
 payable at shorter intervals than a year, will be rather less  
 than the addition to be made on this account, either to the  
 single or the joint lives of the same ages. Ed.

Note

Note (M). Vol. I. Essay III. Page 324.

IT is plain that the purchaser of A's right, as stated in the first of the questions to which this note refers, cannot get into possession till the year when A and B shall be both dead; nor then, unless A happens to die *last*. Now, supposing the common complement of life  $n$ ; the probability that A and B shall be *both* dead at the end of the *first* year, and A die last, is  $1 - \frac{n-1}{n} \times 1 - \frac{n-1}{n}$

$$\times \frac{1}{2} = \frac{1}{2} - \frac{n-1}{2n} - \frac{n-1}{2n} + \frac{(n-1)^2}{2n^2}.$$

— In like manner, the probability that they shall be *both* dead at the end of the 2d, 3d, &c. years, and A survive is,  $\frac{1}{2} - \frac{n-2}{2n} - \frac{n-2}{2n} + \frac{(n-2)^2}{2n^2}$ ;  $\frac{1}{2} - \frac{n-3}{2n} -$

$$\frac{n-3}{2n} + \frac{(n-3)^2}{2n^2}, \text{ \&c.}$$

The *present* value, therefore, of the 1st, 2d, 3d, &c. rents of the reversionary

$$\text{estate is } \frac{1}{2r} - \frac{n-1}{2nr} - \frac{n-1}{2nr} + \frac{(n-1)^2}{2nr^2}, \frac{1}{2r^2} - \frac{n-2}{2nr^2} -$$

$$\frac{n-2}{2nr^2} + \frac{(n-2)^2}{2n^2r^2}, \frac{1}{2r^3} - \frac{n-3}{2nr^3} - \frac{n-3}{2nr^3} + \frac{(n-3)^2}{2n^2r^3}, \text{ \&c.}$$

Supposing  $r$  to signify *1l.* increased by its interest for a year; and the estate to be *1l. per annum*. And the *sum* of these terms continued *in infinitum* is the

value *required*.—But  $\frac{1}{2r} + \frac{1}{2r^2} + \frac{1}{2r^3}, \text{ \&c.}$  is *half*

$$\text{the perpetuity. And } \frac{n-1}{2nr} + \frac{n-1}{2nr} - \frac{(n-1)^2}{2n^2r}, \text{ \&c.}$$

$$\frac{n-2}{2nr^2} + \frac{n-2}{2nr^2} - \frac{(n-2)^2}{2n^2r^2} + \frac{n-3}{2nr^3} + \frac{n-3}{2nr^3} - \frac{(n-3)^2}{2n^2r^3}, \text{ \&c.}$$

is half the value of the *joint* lives, subtracted from *half* the sum of the values of the two *single* lives;

that is, *half* the value of the *longest* of the two lives.

A similar demonstration may be applied to the other question ( $\lambda$ ).

( $\lambda$ ) The purchaser of A's right, in the 2d Question, will get into possession in that year in which A either survives B, or dies after him. The value of his expectation in the

1st year will be  $= \frac{n-1}{nr} \times 1 - \frac{n-1}{n} + \frac{1}{2r} \times 1 - \frac{n-1}{n} \times$   
 $1 - \frac{n-1}{n} = \frac{1}{2r} - \frac{n-1}{2nr}$  .... In the 2d, 3d, 4th, &c. years,

his expectation depending on the same events will be worth

$\frac{1}{2r^2} - \frac{n-2}{2nr^2}, \frac{1}{2r^3} - \frac{n-3}{2nr^3}, \frac{1}{2r^4} - \frac{n-4}{2nr^4},$  &c. The

present value therefore of the 1st, 2d, 3d, &c. rents of the

reversionary estate is  $\frac{1}{2r} + \frac{1}{2r^2} + \frac{1}{2r^3} + \&c. - \frac{n-1}{2nr} -$

$\frac{n-2}{2nr^2} - \frac{n-3}{2nr^3} - \&c.$  — If instead of an *estate* the value

of a given *sum* were required it would, agreeable to the foregoing demonstrations, be expressed in the first case

by  $\frac{S \cdot r - 1}{2r} \times \overline{V} + BB - 2B$ , and in the 2d case by

$\frac{S \cdot r - 1}{2r} \times \overline{V} - BB$  ( $V$  denoting the perpetuity,  $B$  the value

of an annuity on the life of  $B$ , and  $BB$  the value of an annuity on two joint lives whose common age is that of  $B$ ).

The latter value therefore according to *De Moivre's* hypothesis, and in the particular case where the two lives

are equal, exceeds the former value by  $\frac{S \cdot r - 1}{r} \times \overline{B} - BB$ .

— That this is likewise true whatever be the decrements of life, or the ages of  $A$  and  $B$ , may be proved from the two Theorems in Note. (O): For by the 2d of these theorems the

value of  $S$  is  $= \frac{S}{2} \times \frac{\beta \cdot \overline{F} - AF}{b} - \frac{c \cdot \overline{P} - AP}{br} - \frac{r-1 \cdot \overline{B} - AB}{r}$ ,

and by the first it is  $= \frac{S}{2} \times \frac{\beta \cdot \overline{F} - AF}{b} - \frac{c \cdot \overline{P} - AP}{br} + \frac{r-1 \cdot \overline{B} - AB}{r}$ ;

from which it appears that the latter reversion exceeds the

former by  $\frac{S \cdot r - 1}{r} \times \overline{B} - AB$ , and consequently that the difference between them will be the same in all cases.

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Note



Note (N). Vol. I. Essay II. Page 320.

LET  $r$  be 1%. increased by its interest for one year.

Let  $S$  represent any given interval of time, or number of years, during which the decrements of life in a table of observations continue equal.

$a$  the number of the living in the table at the beginning of the first year of that interval.

$b$  the number of the living in the table at the beginning of the year immediately following the same interval.

$P$  the value of an annuity certain for  $S$  years.

$p$  the value of 1% due at the end of  $S$  years.

$Q$  the value, in Table I. immediately following this Note, of an annuity for the life of a person whose age wants  $S$  years of 86.

$N$  the value, in strict agreement with the given table of observations, of an annuity on the life of a person whose age is  $S$  years greater than the age at which the interval of equal decrements begins.

Then,

$Q + \frac{b}{a} \times \overline{P - Q}$  will be the value, according to the table of observations, of an annuity for  $S$  years, on a life of the same age with that at which the interval of equal decrements begins. And

$Q + \frac{b}{a} \times \overline{P - Q} + pN$  will be the value of an annuity on the whole duration of that life.

When  $S$  represents *one year*,  $Q$  vanishes, and the last expression becomes  $\frac{b}{ar} \times \overline{1 + N}$ ; which is the rule for finding, from the value given of any life, the value of a life one year younger ( $\mu$ ).

In

( $\mu$ ) The value of an annuity payable *half yearly* during any life (A), may be deduced from the value of the same annuity



In like manner, supposing G to signify the value of two given joint lives by any table of ob-

annuity during a life (B), one year younger than A, with nearly as much ease as the values of annuities payable *yearly* are deduced. Let  $b$  represent the number of persons living in the Table at the age of B, and  $c, d, e, f, \&c.$  the number living at the end of the 1st, 2d, 3d, &c. years from the age of B. Let  $r$  represent the interest of £1 for a year, and

$p = 1 + \frac{r}{2}$ ; then will the value of the annuity be =

$$\frac{b+c}{4bp} + \frac{c}{2bp^2} + \frac{c+d}{4bp^3} + \frac{d}{2bp^4} + \frac{d+e}{4bp^5} + \&c. \text{ which may be}$$

$$\text{found} = \frac{1}{4p} + \frac{c}{4b} \times \frac{1}{p} + \frac{2}{p^2} + \frac{1}{p^3} + \frac{d}{4b} \times \frac{1}{p^3} + \frac{2}{p^4} + \frac{1}{p^5}$$

$$+ \frac{e}{4b} \times \frac{1}{p^5} + \frac{2}{p^6} + \frac{1}{p^7}, \&c. \text{ From this series, if the age}$$

of B be very old, the value of the life annuity will be obtained with little difficulty; and having this, the value of an annuity on a life one year younger may be derived from it in the following manner:—Let  $a$  denote the number of persons living at the age of (A), who is one year younger than B; then, since the series expressing the value of an annuity on the life of the latter is found above to be =

$$\frac{b+c}{4bp} + \frac{c}{2bp^2} + \frac{c+d}{4bp^3} + \&c. \text{ the series expressing the value of}$$

$$\text{an annuity on the life of the former will be} = \frac{a+b}{4ap} + \frac{b}{2ap^2} +$$

$$\frac{b+c}{4ap^3} + \&c. = \frac{a+b}{4ap} + \frac{b}{ap^2} \times \frac{1}{2} + \frac{b+c}{4bp} + \frac{c}{2bp^2} + \frac{c+d}{4bp^3} + \&c.$$

Therefore if the value of the annuity on the life of B be called M, the value of the annuity on the life of A will be

$$= \frac{a+b}{4ap} + \frac{b}{ap^2} \times \frac{1}{2} + M.$$

From this Theorem a table may be computed of the values of annuities payable *half yearly* on lives of all ages; and by proceeding in the same manner a general Theorem may be obtained for computing a table of the values of annuities payable *quarterly*. But the labour of forming a table of this kind will be rendered unnecessary, if we are possessed of the values payable *yearly*: for I have found that the differences between annuities payable half yearly and yearly are the same, whether those values be derived from the real probabilities of life and the preceding Theorems, or from M. De Moivre's hypothesis, and the Theorems in the 3d additional Essay in this work (pag. 388. Vol. II.). Ed.

servations,  $a$  the living at the age of one of them,  $c$  the living at the age of the other, and  $b$  and  $d$  the numbers living at the two next younger ages,

$\frac{a \times c}{b \times d \times r} \times \overline{1+G}$  will be the value of two joint lives each one year younger than the former.

The method of calculating the values of lives from any given tables of observations, described at the end of the Second Essay in the preceding volume, is founded entirely on these Theorems; and a distinct explanation of them has been given by Mr. Morgan, in the Second Section of the Second Chapter of his book on the Doctrine of Life-Annuities and Assurances.

The expressions  $Q + \frac{b}{a} \times \overline{P-Q}$ , and  $Q + \frac{b}{a} \times \overline{P-Q} + pN$ , with their investigation, may be found in p. 341, 3d Edition of Mr. De Moivre's *Treatise of the Doctrine of Chances* (v). But it is necessary

(v) The Solution of this theorem may be deduced in a manner different from that of M. De Moivre. Let  $a$  be the number of persons dying annually in  $s$  years, while the decrements of life continue equal, then will the value of the annuity during this term be  $= \frac{a-sa}{ar} + \frac{a-2a}{ar^2} + \frac{a-3a}{ar^3} \dots + \frac{a-sa}{ar^s} = \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} \dots (s) - \frac{sa}{a} \times \frac{1}{sr} + \frac{2}{sr^2} + \frac{3}{sr^3} \dots (s)$ . But the first series is  $= P$ , and the second series is  $= -\frac{sa}{a} \times \overline{P-Q} =$  (since  $a-b$  is  $= sa$ )  $= -\frac{a-b}{a} \times \overline{P-Q}$ , and therefore the value of the annuity during the first  $s$  years will be  $= Q + \frac{b}{a} \times \overline{P-Q}$ . The value of the annuity after  $s$  years (supposing  $m, n, o, p, q$ , &c. to denote the number of persons living in the table at the end of  $s+1, s+2, s+3$ , &c. years

fary to observe, that the direction of Mr. *De Moivre* has given for finding the value of  $Q$  is wrong. In consequence of calculating agreeably to this direction, he gives the value of a life at the age of 42 by Dr. *Halley's* table, greater than the value of the same life by his own hypothesis; whereas, it is evident that the probabilities of living after 42,

years is  $= \frac{m}{ar^{s+1}} + \frac{n}{ar^{s+2}} + \frac{o}{ar^{s+3}} + \&c. = \frac{b}{ar^s} \times$   
 $\frac{m}{br} + \frac{n}{br^2} + \frac{o}{br^3} + \&c. = \frac{b}{a} \times pN$ . If this expression be added to the value of the annuity, found above, for the first  $s$  years, the whole value will be  $= Q + \frac{b}{a} \times \overline{P-Q} + pN$ .  
 Q. E. D.

It is necessary to observe that the series  $\frac{a-a}{ar} + \frac{a-2a}{ar^2} + \&c.$  supposes the annuity to be payable yearly, and therefore that  $\frac{sz}{a} \times \frac{1}{sr} + \frac{2}{sr^2} + \frac{3}{sr^3} + \&c.$  expresses the difference, multiplied into  $\frac{sz}{a}$ , between the values of an annuity certain for  $s$  years, and of an annuity payable yearly during the continuance of a life whose complement is  $s$ .---The latter of these values, denoted by  $Q$ , is given in the 1st Table at the end of this volume.—But M. *De Moivre* has deduced the

value of  $Q$  from the fluxional quantity  $\frac{\dot{z}}{n, r-1} - \frac{\dot{z}}{nr^2, r-1}$ ,

which, expressing the value of an annuity secured upon land, must necessarily be always greater than the series  $\frac{s-1}{sr} + \frac{s-2}{sr^2}$

+ &c.; for the one supposes the annuity to be payable to the last moment of existence, while the other makes no allowance for that part of the year which shall have elapsed between its commencement and the extinction of the life. This value of  $Q$  therefore is improperly applied to the foregoing Theorem; where the value of  $N$ , as well as the whole solution, is founded upon the principle of the annuity's being payable only at the conclusion of each year, provided the life shall continue so long. E<sub>n</sub>.

being all along less in Dr. *Halley's* table than in the hypothesis, the value of the life must be also less.—The mathematical reader may easily satisfy himself, that the value of *Q* ought to be taken, as I have directed, from Table I. at the end of this volume.

I cannot help adding here, that though the rules for finding from the value given of any single or joint lives, the value of any single or joint lives one year younger, are an obvious corollary from the two expressions just mentioned, yet it is probable that Mr. *De Moivre* did not attend to them, or consider the facility which they give to calculations of this kind; for if he had, he would not probably have insisted so much as he has on his hypothesis of an equal decrement of life; much less would he, in order to obtain an easy method of calculation, have had recourse to that Second Hypothesis, which, in the Second Essay in the preceding volume, has been shewn to be so very erroneous.

Mr. *Simpson* is, I believe, the first who has given these rules, in his Treatise on the Doctrine of Annuities and Reversions; but in his *Select Exercises*, p. 275, he has given a rule for approximating to the values of single lives, according to Dr. *Halley's* table, which must not be depended on, for I have found it half a year's purchase, and sometimes three-quarters of a year's purchase wrong.



## Note (O).

**I**N a note at the conclusion of the 3d Essay \*, Dr. Price refers to the end of this work for more accurate solutions of his 11th and 12th Questions, which had been investigated by myself, and published in the 78th vol. of the Philosophical Transactions.—With the view of fulfilling his intentions in this respect, I shall here, in an abridged manner, insert the solutions to which he refers.

**SOLUTION OF QUESTION XI.** Let  $a$  represent the number of persons living in the Table at the age of A, the younger of the two lives,  $a'$ ,  $a''$ ,  $a'''$ , &c. the decrements of life at the end of the 1st, 2d, 3d, &c. years from the age of A;  $b$  the number of persons living at the age of B, the older of the two lives, and  $c$ ,  $d$ ,  $e$ ,  $f$ , &c. the number of persons living at the end of the 1st, 2d, 3d, &c. years from the age of B. Then will the value of  $S$  (the given sum), depending on the contingency of B's surviving A, be expressed by  $\frac{S}{2ab} \times$

$$\frac{ca'}{r} + \frac{da''}{r^2} + \frac{ea'''}{r^3} + \&c. + \frac{S}{2ab} \times \frac{ba}{r} + \frac{ca''}{r^2} + \frac{da'''}{r^3} + \&c.$$

$$= \frac{S}{2r} \times \frac{\beta r \cdot F - AF - c \cdot P - AP}{b} + r - 1 \cdot B - AB; F \text{ denoting a life one year younger, and } P \text{ a life one year older than } B; AF, AP, AB, \text{ the values of the joint lives of } A \text{ and } F, A \text{ and } P, \text{ and } A \text{ and } B; \text{ and } \beta \text{ the number of persons living in the Table at the age of } F. \text{—Having now the value of the given sum payable on the contingency of } B's$$

\* Vol. I. page 326.

surviving A, the value of the same sum payable on the contingency of A's surviving B is easily obtained; by subtracting the value found above from the whole value of the Reversion after the extinction of the joint lives of A and B.

SOLUTION OF QUESTION XII. Retaining the same symbols as in the preceding solution, the value of the sum S will in this case be  $= \frac{S}{2ab} \times$

$$\frac{\overline{b-c.a'}}{r} + \frac{\overline{c-d.a''}}{r^2} + \frac{\overline{d-e.a'''}}{r^3} + \&c. + \frac{S}{abr} \times$$

$$\frac{\overline{a'.c-d}}{r} + \frac{\overline{a'+a''.d-e}}{r^2} + \frac{\overline{a'+a''+a'''.e-f}}{r^3} + \&c. =$$

$$\frac{S}{2r} \times \frac{\beta r.F - AF - c.P - AP}{b} - \frac{c}{r-1} \cdot \frac{B - AB}{b}.$$

When the value of the reversion is required, depending on the contingency of A's having died after B, the foregoing value is to be subtracted from the whole value of the Reversion after the extinction of *both* lives.

The solutions which are given of these questions in the 1st Volume of this work, have been taken from Mr. *Simpson's* Select Exercises, and are in some instances so incorrect as to be unfit for use, —more especially when one of the lives is very young and the other very old; in which case the results are often *one third*, and sometimes even *one half* wrong.—This inaccuracy arises from Mr. *Simpson's* having had recourse to Mr. *De Moivre's* hypothesis, by deducing his solutions from the *expectations* rather than from the *real probabilities* of life. When the ages of neither of the lives exceed 60, or fall short of 10 years, his rules are tolerably correct; but since the *exact* values may be obtained with so little difficulty, I think it can seldom be adviseable to have recourse to them.

The

The general rule derived from both the foregoing Theorems may be expressed in nearly the same words.—“ Let K represent a life one year younger, and C a life one year older than B. “ Multiply the difference between the values of “ the life of K, and of the joint lives of A and “ K into the number of persons living in the table “ at the age of K, and also into £ increased by “ its interest for a year. Multiply the difference “ between the values of the life of C, and of the “ joint lives of A and C, into the number of persons living in the table at the age of C. Subtract this from the former product; divide the remainder by the number of persons living in the table at the age of B, and reserve the quotient.—Again; multiply the difference between the values of the life of B, and of the joint lives of A and B, into the interest of £ 1 for a year—then, if the *sum* of this product and the reserved quotient in the 11th question, or their *difference* in the 12th, be divided by £ 1 increased by its interest for a year, and multiplied into half the given sum, this last product will be the value of the Reversion, when B the *expectant* is the oldest of the two lives.”—If B be the youngest, the value will be obtained in the same manner as in Mr. *Simpson's* rules,—by subtracting the value of A's expectation, found above, from the whole value of the Reversion after the *joint lives* of A and B in the former case, and after the *longest of their two lives* in the latter.

#### EXAMPLE I.

Let it be required to determine the value of £ 100 payable on the death of A aged 35 should B aged 75 be then living; computing at 4 *per cent.*  
and

and from the probabilities of life in the *Northampton* Table of Observations.—In this case the ages of K and C will be 74 and 76 years.—The value of an annuity on the life of K is 5.230, and on the joint lives of A and K, 4.737.—The difference between these two sums, or .493, multiplied into 912, the number of persons living at the age of K, and into 1.04, produces 467.6005.—The difference between 4.710 and 4.303, the respective values of annuities on the life of C, and the joint lives of A and C, is .407; which being multiplied into 752, the number of the living at the age of C, gives 306.064. This product subtracted from 467.6005, and 161.5365, (the remainder) divided by 832, the number of persons living at the age of B, quotes .185416 to be reserved.—Again; the values of annuities on the life of B, and the joint lives of A and B, are 4.962 and 4.516 respectively. Their difference, or .446, multiplied into .04 gives .01784; which being added to .185416, the reserved quotient, amounts to .203256. This sum divided by 1.04, and the quotient multiplied into 50, produces £9.772 for the value of the Reversion.—If A had been 75 and B 35 years of age, the foregoing value must have been deducted from 78.784, the whole value of the Reversion after the extinction of the joint lives of A and B (a), and the remainder, or £69.012, would have been the answer in this case.

(a) The *whole* values of the Reversions in these Examples are deduced from Quest. X. Vol. I. by substituting the *joint*, or the *longest of the two lives*, instead of the *single life* in that Rule.



EXAMPLE II.

Let it be required to determine the value of £100 payable on the death of B aged 75, should that happen *after* the death of A aged 35, computing at the same rate of interest, and from the same probabilities of life, as in the preceding Example. — This case belongs to the 12th Question, and as the ages are the same with those above, the reserved quotient, and the product to be subtracted from it will also be the same. — These having been found to be .185416 and .01784 respectively, their *difference* is .167576; which being divided by 1.04, and .161131, the quotient, multiplied into 50, will give £8.05655 for the value of the Reversion.

Supposing A to be 75 and B 35 years of age, the foregoing sum must be subtracted from 40.442 (a), the whole value of the reversion after the longest of the two lives of A and B and £32.385, the remainder, will be the value required. ED.

## Note (P).

**I**N the same note to which Dr. *Price* refers (\*) for more accurate solutions of his 11th and 12th questions (and which have been given in the preceding pages), a further reference is made to the end of this volume, for rules which give in all cases correct values of sums payable on any survivorships between any three lives. These rules have been deduced by myself; and when the above note was written, it was my intention to have submitted the whole of them to Dr. *Price*, in order that he might use his own discretion in the manner of inserting them. But this is no longer possible, and I am now induced for many reasons to withhold for the present the greater part of them from the public. Were those rules together with their demonstrations to be given (and the one would be very unsatisfactory without the other), I am apprehensive that my additions to this invaluable work would be much too long. I shall therefore insert here only such rules as have been already published in the 79th and 81st volumes of the *Philosophical Transactions*, to which the reader is referred for their demonstrations.

From the complicated nature of questions involving survivorships between three lives, it becomes necessary in their solution to have recourse to a great variety of symbols.—In order however to prevent repetition, the same symbols are uniformly made to denote the same quantities in all the following rules, and it may not be improper to begin with explaining them.

(\*) Vol. I. p. 326.

- A. } denote the value of an Annuity on the re-  
 B. } spective lives of A, B, and C.  
 C. }
- D. denotes the value of S on the contingency of C's surviving A (by Quest. XI. Note O).
- E. denotes the same value on the contingency of B's surviving A, found by the same Question.
- F. denotes the value of an annuity on a life one year younger than B.
- G. denotes the value of the absolute Reversion of S after the death of A (by Quest. X. Vol. I.)
- H. denotes the value of an annuity on a life one year younger than A.
- K. denotes the same value on a life one year younger than C.
- L. denotes the value of an annuity on the longest of the three lives A, B, and C.
- M. denotes the value of S, by the first Problem in this Note, on the contingency that A's life shall be the *first* that fails.
- N. denotes the value of an annuity on a life one year older than A.
- P. denotes the same value on a life one year older than B.
- Q. denotes the value of S, by the 8th Problem, on the contingency of A or B, being *either* of them the *first* that fails.
- R. denotes the value of S on the contingency of B's dying after A (by Quest. XII. Note O).
- S. denotes the given sum.
- T. denotes the value of an annuity on a life one year older than C.
- V. denotes the perpetuity.
- W. denotes the value of S on the contingency of C's dying after A (by Quest. XII. Note O).
- $x$  and  $a$ , denote the number of persons living in a table

a table of observations at the ages of H and A.

$\beta$  and  $b$ , denote the number of persons living at the ages of F and B.

$x$  and  $c$ , denote the number of persons living at the ages of K and C.

$s$ ,  $m$ , and  $d$ , denote the number of persons living at the end of the first year from the respective ages of A, B, and C.

$r$ , denotes the value of £1 increased by its interest for a year.

The combinations of two or three of the several letters, A, B, C, F, H, &c. denote the values of annuities on the *joint* continuance of two or three of those respective lives.

### P R O B L E M    I.

To determine the value of a given sum, payable if A should be the *first* that fails of the three lives A, B, and C.

### S O L U T I O N.

When B or C are the oldest of the three lives the value of the Reversion will be = S into  $\frac{x}{3c} \times$

$$\frac{\beta.FK - AFK}{b} + \frac{BK - ABK}{2} + \frac{\beta}{6b} \times \overline{FC - AFC} +$$

$$\frac{r-1}{3r} \times \overline{BC - ABC} - \frac{m.PC - APC}{6br} - \frac{d}{3cr} \times$$

$$\frac{BT - ABT}{2} + \frac{m.PT - APT}{b}.$$

When



When A is the oldest of the three lives the value will be = S into  $\frac{\beta}{3b} \times \frac{a \cdot \overline{HF} + \frac{1}{2} \overline{HFC}}{a} - \overline{AF} + \frac{1}{2} \overline{AFC}$   
 $+ \frac{1}{6} \times \frac{a \cdot \overline{HB} + 2 \overline{HBC}}{a} - \overline{AB} + 2 \overline{ABC} + \frac{1}{3r}$   
 $\times \frac{s \cdot \overline{BN} - \overline{BNC}}{d} - \overline{AB} - \overline{ABC} + \frac{m}{6br} \times$   
 $\frac{s \cdot \overline{PN} - \overline{PNC}}{a} - \overline{AP} - \overline{APC}.$

When the three lives are equal, the value will be =  $\frac{S}{2} \times \frac{r - 1 \cdot \overline{V} - \overline{CCC}}{r}.$

### PROBLEM II.

To determine the value of a given sum, payable if A should be the *second* that fails of the three lives A, B, and C.

#### SOLUTION.

When the ages are *unequal*, the value of the Reversion will be =  $D + E - 2M.$

When the ages are *equal*, its value will be =  $\frac{S}{3} \times \frac{r - 1}{r} \times \overline{V} - 3 \overline{CC} - 2 \overline{CCC}.$

### PROBLEM III.

To determine the value of a given sum, payable on the death of A, if his life should be the *last* that fails of the three lives A, B, and C.

#### SOLUTION

## SOLUTION.

The value of the Reversion in this case will be either  $G + M - \overline{D + E}$ , or  $\frac{S.r-1}{3r} \overline{V-L}$ , according as the ages of the lives are unequal or equal.

## PROBLEM IV.

To determine the value of a given sum, payable on the extinction of the lives of A and B, should they be the *first* that fail of the three lives A, B, and C.

## SOLUTION.

Let  $\Sigma$  denote the value of S on the contingency of C's surviving B (by Quest XI. Note O), and the general rule, when the lives are *unequal*, will be  $= \Sigma + \frac{S.x}{6a} \times \overline{HC - HBC} - \frac{S.x}{3} \times \frac{a.HK - HBK}{2a} + \overline{AK - ABK} - \frac{2S.r-1}{3r} \times \overline{AC - ABC} - \frac{S.s}{6ar} \times \overline{NC - NBC} + \frac{S.d}{3cr} \times \overline{AT - ABT} + \frac{s.NT - NBT}{2a}$ . ----- If the three lives be *equal*, the Rule becomes  $= \frac{S.r-1}{3r} \times \overline{V - 3CC - 2CCC}$ .

## PROBLEM V.

To find the value of a given sum, payable on the death of A, if his life should be the *first* or *second* that fails of the three lives A, B, and C.

## SOLUTION

SOLUTION.

The value of the Reversion, when the lives are *unequal*, will be  $= D + E - M$ .

When the lives are *equal*, it will be  $= \frac{S.r-1}{3r} \times \frac{2V - 3CC - CCC}{3r}$ .

PROBLEM VI.

To find the value of a given sum, payable on the death of A, should his life be the *second* or *third* that fails of the three lives A, B, and C.

SOLUTION.

If the lives be *unequal*, the value of the Reversion will be  $= G - M$ .----If the three lives be *equal*, its value will be  $= \frac{S.r-1}{3r} \times \frac{2V - 3C - CCC}{3r}$ .

PROBLEM VII.

To find the value of a given sum, payable on the death of A, should his life be the *first* or the *last* that fails of the three lives A, B, and C.

SOLUTION.

In this case the value of the Reversion will be  $= G - \frac{D+E}{3r} + 2M$ , if the lives be *unequal*, and  $= \frac{S.r-1}{3r} \times \frac{2V - 3C - 3CC + 2CCC}{3r}$ , if the lives be *equal*.





## SOLUTION.

When the lives are of *unequal* ages, the value of the Reversion will be  $= \frac{S. \overline{r-1}. \overline{V-AB}}{r} D + \Sigma - 2Q$  --- ( $\Sigma$  denoting the same value as in Prob. IV. and VIII.) When the ages of the three lives are *equal*, the value will be  $= \frac{2S. \overline{r-1}}{3r} \times \overline{V-3CC-2CCC}$ .

## PROBLEM. X.

To find the value of a given sum, payable on the decease of B or C; should either of them be the *last* that fails of the three lives A, B, and C.

## SOLUTION.

The value of the Reversion, when the lives are *unequal*, will be  $= \frac{S. \overline{r-1}}{r} \times \overline{BC-ABC} + R + W - M$ , and when the lives are all *equal*, it will be  $= \frac{2S. \overline{r-1}}{3r} \times \overline{V-L}$ .

## PROBLEM XI.

To determine the value of a given sum, payable on the contingency of C's surviving B, provided the life of A shall be then extinct.

## SOLUTION.

When either B or C are the oldest of the three lives, the value of the given sum will be = S into  $\frac{x}{6c} \times$

$$\frac{\beta \cdot \overline{FK} - \overline{FKC}}{b} - \overline{BK} - \overline{ABK} + \frac{\beta}{3b} \times \overline{FC} - \overline{AFC} \\ - \frac{r-1}{3r} \times \overline{BC} - \overline{ABC} - \frac{m}{2br} \times \overline{PC} - \overline{APC} + \\ \frac{d}{6cr} \times \overline{BT} - \overline{ABT} - \frac{m \cdot \overline{PT} - \overline{APT}}{b}.$$

When A is the oldest of the three lives, the value will be =  $\Sigma - \frac{S \cdot x}{3c} \times \frac{x \cdot \overline{HK} - \overline{HBK}}{a} + \frac{\overline{AK} - \overline{ABK}}{2} -$

$$\frac{S \cdot x}{6a} \times \overline{HC} - \overline{HBC} - \frac{r-1}{3r} \times \overline{AC} - \overline{ABC} + \frac{s}{6r} \\ \times \overline{NC} - \overline{NBC} + \frac{d}{3cr} \times \frac{\overline{AT} - \overline{ABT}}{2} + \frac{s \cdot \overline{NT} - \overline{NBT}}{a}$$

( $\Sigma$  denoting the same value as in Prob. IV. VIII. and IX.)-----When the three lives are *equal*, the

value of the Reversion will be =  $\frac{S \cdot r-1}{6r} \times$   
 $V - 3CC - 2CCC.$

In the further pursuit of these enquiries, I have discovered a very simple method of approximating to the values in the preceding Problems. But it would be improper to enter more fully into the subject at present, and therefore the publication of those rules must be postponed to another opportunity.—I shall only observe here, that the solutions of those cases which involve three lives, and even of

of those which involve two lives in the survivorship, being formerly deduced from an erroneous hypothesis, it was impossible to determine how far any approximations could be depended upon. By the assistance of the foregoing rules, which have been derived from the real probabilities of life, this point may now be ascertained with the greatest precision;—though perhaps it may not often be adviseable to have recourse to *approximations*, when the *exact values* can be obtained with so little additional trouble.

ED.

## TABLE I. (a)

Shewing the present Values of an Annuity of 1*l*. on a Single Life, according to Mr. *De Moivre's* hypotheſis. See Vol. I. p. 2.

Age.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
8	19,736	18,160	16,791	15,595	14,544	12,790
9	19,868	18,269	16,882	15,672	14,607	12,839
10	19,868	18,269	16,882	15,672	14,607	12,839
11	19,736	18,160	16,791	15,595	14,544	12,790
12	19,604	18,049	16,698	15,517	14,480	12,741
13	19,469	17,937	16,604	15,437	14,412	12,691
14	19,331	17,823	16,508	15,356	14,342	12,639
15	19,192	17,707	16,410	15,273	14,271	12,586
16	19,050	17,588	16,311	15,189	14,197	12,532
17	18,905	17,467	16,209	15,102	14,123	12,476
18	18,759	17,344	16,105	15,015	14,047	12,419
19	18,610	17,220	15,999	14,923	13,970	12,361
20	18,458	17,093	15,891	14,831	13,891	12,301
21	18,305	16,963	15,781	14,737	13,810	12,239
22	18,148	16,830	15,669	14,641	13,727	12,177
23	17,990	16,696	15,554	14,543	13,642	12,112
24	17,827	16,559	15,437	14,442	13,555	12,045
25	17,664	16,419	15,318	14,340	13,466	11,978
26	17,497	16,277	15,197	14,235	13,375	11,908
27	17,327	16,133	15,073	14,128	13,282	11,837
28	17,154	15,985	14,946	14,018	13,186	11,763
29	16,979	15,835	14,816	13,905	13,088	11,688
30	16,800	15,682	14,684	13,791	12,988	11,610
31	16,620	15,526	14,549	13,673	12,855	11,530
32	16,436	15,367	14,411	13,553	12,780	11,449
33	16,248	15,204	14,270	13,430	12,673	11,365

(a) This Table is the same with Mr. *De Moivre's* Table of the values of ſingle lives, published in his *Treatiſe on Life Annuities*, and carried as far as the age of 79, to three places of decimals, by Mr. *Dodſon* in his *Mathematical Repository*, Vol. II. p. 169.

TABLE



# APPENDIX II.

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## TABLE I. continued.

Age.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
34	16,057	15,039	14,126	13,304	12,562	11,278
35	15,864	14,871	13,979	13,175	12,449	11,189
36	15,666	14,699	13,829	13,044	12,333	11,098
37	15,465	14,524	13,676	12,909	12,214	11,003
38	15,260	14,345	13,519	12,771	12,091	10,907
39	15,053	14,163	13,359	12,630	11,966	10,807
40	14,842	13,978	13,196	12,485	11,837	10,704
41	14,626	13,789	13,028	12,337	11,705	10,599
42	14,407	13,596	12,858	12,185	11,570	10,490
43	14,185	13,399	12,683	12,029	11,431	10,378
44	13,958	13,199	12,504	11,870	11,288	10,263
45	13,728	12,993	12,322	11,707	11,142	10,144
46	13,493	12,784	12,135	11,540	10,992	10,021
47	13,254	12,571	11,944	11,368	10,837	9,895
48	13,012	12,354	11,748	11,192	10,679	9,765
49	12,764	12,131	11,548	11,012	10,515	9,630
50	12,511	11,904	11,344	10,827	10,348	9,492
51	12,255	11,673	11,135	10,638	10,176	9,349
52	11,994	11,437	10,921	10,443	9,999	9,201
53	11,729	11,195	10,702	10,243	9,817	9,049
54	11,457	10,950	10,478	10,039	9,630	8,891
55	11,183	10,698	10,248	9,829	9,437	8,729
56	10,902	10,443	10,014	9,614	9,239	8,561
57	10,616	10,181	9,773	9,393	9,036	8,387
58	10,325	9,913	9,527	9,166	8,826	8,208
59	10,029	9,640	9,275	8,933	8,611	8,023
60	9,727	9,361	9,017	8,694	8,389	7,831
61	9,419	9,076	8,753	8,449	8,161	7,633
62	9,107	8,786	8,482	8,197	7,926	7,428
63	8,787	8,488	8,205	7,938	7,684	7,216
64	8,462	8,185	7,921	7,672	7,435	6,997
65	8,132	7,875	7,631	7,399	7,179	6,770
66	7,794	7,558	7,333	7,119	6,915	6,535
67	7,450	7,234	7,027	6,831	6,643	6,292
68	7,099	6,902	6,714	6,534	6,362	6,040
69	6,743	6,565	6,394	6,230	6,073	5,779
70	6,378	6,219	6,065	5,918	5,775	5,508

TABLE I. Continued

Age.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
71	6,008	5,865	5,728	5,596	5,468	5,228
72	5,631	5,505	5,383	5,265	5,152	4,937
73	5,246	5,136	5,029	4,926	4,826	4,636
74	4,854	4,759	4,666	4,576	4,489	4,324
75	4,453	4,373	4,293	4,217	4,143	4,000
76	4,046	3,978	3,912	3,847	3,784	3,664
77	3,632	3,575	3,520	3,467	3,415	3,315
78	3,207	3,163	3,111	3,076	3,034	2,953
79	2,776	2,741	2,707	2,673	2,641	2,578
80	2,334	2,309	2,284	2,259	2,235	2,188
81	1,886	1,867	1,850	1,832	1,816	1,783
82	1,429	1,411	1,406	1,394	1,384	1,362
83	0,961	0,955	0,950	0,943	0,937	0,925
84	0,484	0,483	0,481	0,479	0,476	0,472
85	0,000	0,000	0,000	0,000	0,000	0,000

TABLE

TABLE II.

Shewing the Value of an Annuity on the joint continuance of Two Lives, according to Mr. De Moivre's Hypothesis; computed by the Rule in Note (L). See Vol. I. p. 2 and 3, and Essay II. p. 308, &c.

Age of the youngest,	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
10	10	15.206	13.342	11.855
	15	14.878	13.093	11.661
	20	14.503	12.808	11.430
	25	14.074	12.480	11.182
	30	13.585	12.102	10.884
	35	13.025	11.665	10.537
	40	12.381	11.156	10.128
	45	11.644	10.564	9.646
	50	10.796	9.871	9.074
	55	9.822	9.059	8.391
	60	8.704	8.105	7.572
15	65	7.417	6.980	6.585
	70	5.936	5.652	5.391
	15	14.574	12.860	11.478
	20	14.225	12.593	11.266
	25	13.822	12.281	11.022
	30	13.359	11.921	10.736
	35	12.824	11.501	10.402
	40	12.207	11.013	10.008
	45	11.496	10.440	9.541
	50	10.675	9.767	8.985
	55	9.727	8.975	8.318
	60	8.632	8.041	7.515
	65	7.377	6.934	6.544
	70	5.932	5.623	5.361

TABLE II. Continued

Age of the youngst.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
20	20	13.904	12.341	11.067
	25	13.531	12.051	10.840
	30	13.098	11.711	10.565
	35	12.594	11.314	10.278
	40	12.008	10.847	9.870
	45	11.325	10.297	9.420
	50	10.536	9.648	8.880
	55	9.617	8.879	8.233
	60	8.549	7.967	7.448
	65	7.308	6.882	6.495
	70	5.868	5.590	5.333
25	25	13.192	11.786	10.621
	30	12.794	11.468	10.367
	35	12.333	11.093	10.067
	40	11.770	10.655	9.708
	45	11.130	10.131	9.278
	50	10.374	9.509	8.761
	55	9.488	8.766	8.134
	60	8.452	7.880	7.371
	65	7.241	6.826	6.440
	70	5.826	5.551	5.294
30	30	12.434	11.182	10.133
	35	12.010	10.838	9.854
	40	11.502	10.428	9.514
	45	10.898	9.936	9.112
	50	10.183	9.345	8.620
	55	9.338	8.634	8.018
	60	8.338	7.779	7.280
	65	7.161	6.748	6.373
	70	5.777	5.505	5.254



# APPENDIX II.

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TABLE II. Continued.

Age of the youngst.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
35	35	11.632	10.530	9.600
	40	11.175	10.157	9.291
	45	10.622	9.702	8.913
	50	9.955	9.149	8.450
	55	9.156	8.476	7.879
	60	8.202	7.658	7.172
	65	7.066	6.662	6.294
	70	5.718	5.450	5.203
40	40	10.777	9.826	9.014
	45	10.283	9.418	8.671
	50	9.677	8.911	8.244
	55	8.936	8.283	7.710
	60	8.038	7.510	7.039
	65	6.951	6.556	6.198
	70	5.646	5.383	5.141
45	45	9.863	9.063	8.370
	50	9.331	8.619	7.987
	55	8.662	8.044	7.500
	60	7.831	7.332	6.875
	65	6.807	6.425	6.080
	70	5.556	5.300	5.063
50	50	8.892	8.235	7.660
	55	8.312	7.738	7.230
	60	7.568	7.091	6.664
	65	6.623	6.258	5.926
	70	5.442	5.193	4.964
55	55	7.849	7.332	6.873
	60	7.220	6.781	6.386
	65	6.379	6.036	5.724
	70	5.201	5.053	4.833

TABLE II. Continued.

Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
	60	6.737	6.351	6.001
60	65	6.043	5.730	5.444
	70	5.081	4.858	4.653
65	65	5.547	5.277	5.031
	70	4.773	4.571	4.385
70	70	4.270	4.104	3.952

THE END.

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