

Surgeon Major. 1827
~~Richard Price~~
OBSERVATIONS
ON
REVERSIONARY PAYMENTS;

2731

ON
SCHEMES for providing ANNUITIES for
WIDOWS, and for Persons in OLD AGE;

ON
THE METHOD OF CALCULATING THE VALUES OF
ASSURANCES ON LIVES;

AND ON
THE NATIONAL DEBT.

ALSO,
ESSAYS on different Subjects in the Doctrine of LIFE-
ANNUITIES and POLITICAL ARITHMETIC;

A Collection of NEW TABLES, and a POSTSCRIPT on the
POPULATION of the KINGDOM.

By RICHARD PRICE, D.D. F.R.S.

TO WHICH ARE ADDED,
ALGEBRAICAL NOTES, the SOLUTION of several NEW PROBLEMS in the
DOCTRINE of ANNUITIES,

And a GENERAL INTRODUCTION.

By WILLIAM MORGAN, F.R.S.

FIFTH EDITION.

LONDON:

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Miscellaneous

2731

OBSERVATIONS

ON

REVERSIONARY PAYMENTS,

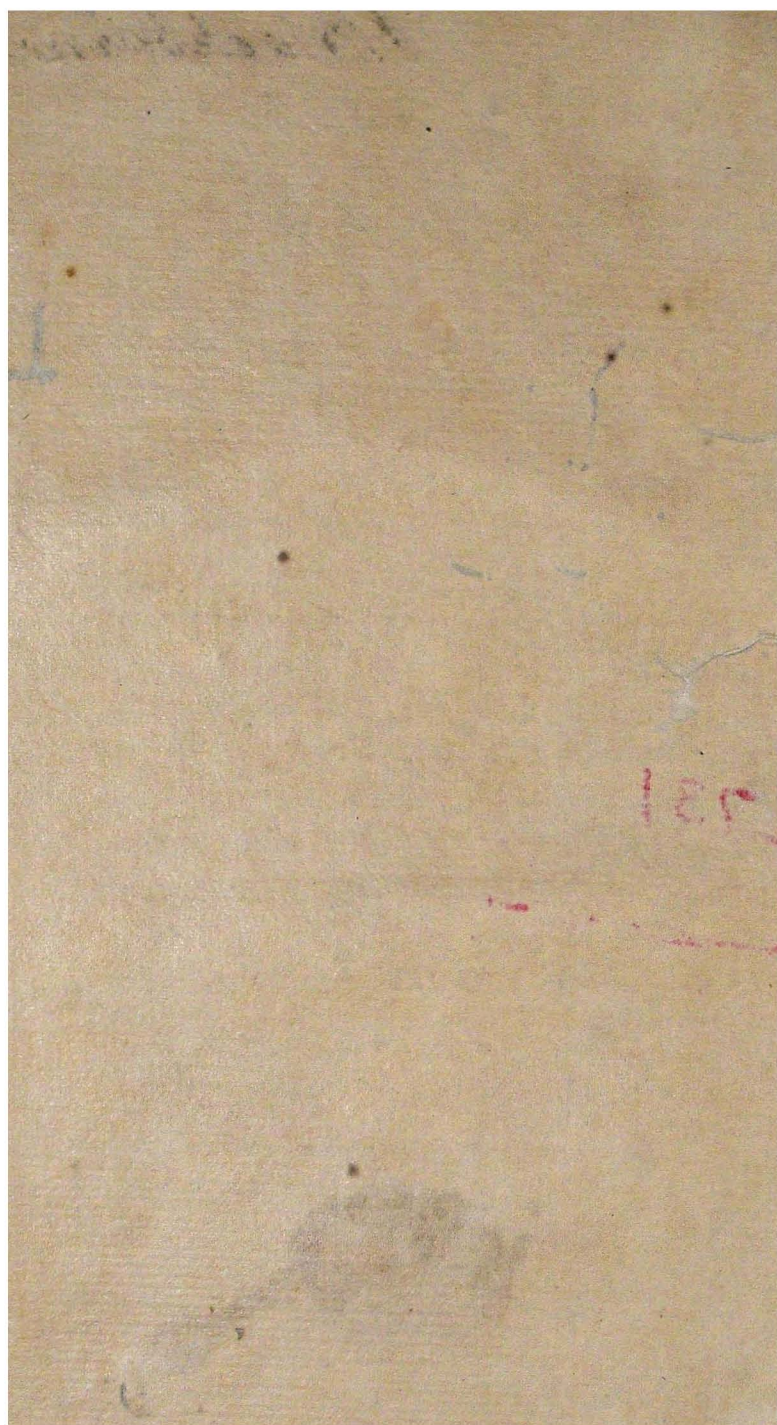
By W. G. W.

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VOL. I.

~~1876~~



1896

THE RIGHT HONOURABLE
THE
EARL OF SHELBURNE,
THIS WORK

Is, with all GRATITUDE and RESPECT,

INSCRIBED,

BY

His LORDSHIP'S

Most obliged, and

Most obedient humble Servant,

March 29th,
1783.

RICHARD PRICE,

TO

THE RIGHT HONOURABLE

THE

EARL OF CHELSEA

THIS WORK

BY

INSCRIBED

BY

THE

THE

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GENERAL INTRODUCTION.

THE following invaluable work is one of the fruits of a life which was uniformly devoted to studious enquiry, and to the promotion of the best interests of mankind. The motives that first led the author to engage in these labours were the most humane and honourable, and the success that attended them, he considered as the highest reward and gratification he could enjoy. Even in preparing his notes for the present edition, which unfortunately were never completed, he mentions his life as then drawing near its close, but that he had reason to reflect with satisfaction on the time which he had employed in those pursuits.

The different Prefaces which have been affixed to the former editions render it unnecessary

necessary to enter minutely into the origin and progress of this work. But it may not be improper to observe, that at the time of its first publication in 1769, the advice and instruction which it contained could not possibly have been communicated at a more seasonable opportunity.—The various Societies for the benefit of age and widows which, like the present Tontines, were then continually rising up to allure and to defeat the hopes of the ignorant and distressed, were become an object of serious alarm, and if the evil had not been effectually opposed, it is difficult to imagine the extent to which it might have proceeded. On the first appearance of this work the rage for establishing *new* Societies immediately subsided, a *partial* reformation took place in some of those which had been already formed, and in a short time the greater part of them, convinced of their mistakes, dissolved themselves. A few, indeed, persevered in an obstinate adherence to their original plans, but they have lately exhibited a melancholy proof of their own folly, and of the truth and justice of the admonitions which had been wasted upon them.

It

It was Dr. *Price*'s intention to have written a new Preface to the present edition, in which he meant to have continued his account of the state and progress of the few Societies which now remain, and also to have inserted whatever additions and remarks he might have thought necessary to the improvement of the following volumes.—But he died soon after the *first* of them was printed, and has left only a few detached hints and observations in regard to the plan which he was to have adopted.—Being anxious to exert every effort in my power towards fulfilling his intentions I shall endeavour, though in a much inferior manner, to pursue the method which he had pointed out for himself; but, not without lamenting the event which has obliged me to engage in so arduous an undertaking.

Besides the peculiar satisfaction which he derived from this work as having been instrumental in doing much good, Dr. *Price* was also accustomed to reflect on it with pleasure as it contained, in his opinion, many improvements in the doctrine of Annuities and in Political Arithmetic.—It was his wish therefore to have rendered it as complete

plete as possible; and the great number of tables and valuable observations with which he has enriched each edition, and particularly the *last*, are a proof of his zeal and success in accomplishing this purpose.—To the present edition a new table has been added of the values of two joint lives, computed under his direction from the probabilities of life at *Northampton*, reckoning interest of money at 6 *per cent.* and also three other tables of the values of a single life, communicated and computed by myself, from the same table of observations at the several rates of 6, 7, and 8 *per cent.* These I believe were all the *new* tables, excepting those in the first Appendix, which Dr. Price meant to have added to this edition:—nor did he seem to entertain the most remote idea of making any material alterations either in the arrangement or the matter of the second volume, which he did not live to correct.—When this edition was put to the press it was done in some haste, and his other engagements prevented him from attending to the correction of it before the two first chapters were printed off. Had not this been the case, he meant instead of the present

sent answers to the 11th and 12th questions in the first Chapter, to have substituted others from a paper communicated by myself to the *Royal Society* in the year 1788, which was honoured with their approbation, and published in the 78th Volume of their Transactions. But being disappointed in this intention, he reserved what he had to say on the subject for a Note in the second Appendix.—As far as relates to these questions I have endeavoured to supply the deficiency. I have also explained some of the notes in this Appendix which appeared to me to be too concise, and have deduced from the real probabilities of life other solutions of such Problems as had been derived from Mr. *De Moivre's* hypothesis,—intending by this means to give a further proof that it can seldom or ever be necessary to have recourse to such an expedient in the doctrine of Annuities.—In consequence of a reference made by Dr. *Price* in a note at the end of his third Essay (Vol. I.), I have inserted all the different rules respecting the values of reversions depending on survivorships between three lives, which I had communicated to the *Royal Society* in the year

1789 and 1791, and which were published in the 79th and 81st Volumes of the Philosophical Transactions.—These, together with a few notes in the second Volume (some of which had become indispensably necessary since the publication of the last edition), comprehend all that I have ventured to add to this work.—The text has been preserved invariably the same, and the additions have consequently been kept separate; so that, if any errors have been committed, they may be easily ascribed to their real author.

As all the solutions in the first Chapter, except those of the 11th and 12th questions, are strictly true (*) according to any table of the probabilities of life, I do not know that this part of the work admitted of any other improvement than the addition in those two questions of correct rules in lieu of the approximations which had been taken from Mr. *Simpson's* Select Exercises. But in regard to the second Chapter, which contains an account of the different Societies for the benefit of age and widows, the circumstances

(*) The *Solutions* of the 15th and 16th questions are true, and the *rules* deduced from them are sufficiently correct for any useful purpose.—See Note (H) 2d vol.

and even plans of some of these have undergone such considerable alterations since the year 1783, when the last edition was published, that it becomes necessary, agreeable to the author's own intention, to give a more accurate account of their present state.

The LONDON ANNUITY SOCIETY for the benefit of widows had been established about five years before the first publication of the Treatise on Reversionary Payments, and, like all the other institutions of the same kind, had been founded on such principles as must have inevitably produced its ruin. The premiums it required were almost *one half* too low for the annuities it promised, and nothing could have saved it but an immediate and entire alteration of its plan.—Happily, the advice and instruction which were offered to the members by Dr. Price were not wasted upon them. The premiums in a short time were increased, and the annuities reduced so far as to secure the permanence of the Society; and subsequent experience has proved the wisdom and success of this reformation.—In the year 1780 the annuities of those widows whose husbands had continued their payments 15 years were raised from £20 to £24; and

so

so favourable had events appeared to prove during the course of the ten following years, that it was resolved in 1790 to lay the accounts of the Society before Dr. *Price* and myself, in order that we might determine whether they were not capable of making still further additions to their annuities.

In January 1789 the Society consisted of 328 members, holding 377 policies, the number of widows was 85, and their annuities amounted to £1967; their income, including interest on £70,500 stock in the 3 *per cents*, general premiums, annual payments, &c. exceeded their claims and expenditure above £2000 a year, and their real *surplus* Stock, after discharging every probable demand from all the present and future widows, appeared by a minute investigation to be so considerable as to justify an addition to the annuities, provided a few alterations were made in their plan.—In consequence therefore of our joint opinion and advice those alterations have in a great measure been adopted, the Society, in addition to the annuity of £20 if a subscriber lives one year and £24 if he lives 15 years, engages to pay a further annuity of £1 for every year that a subscriber lives beyond

beyond 24 years, and there is every reason to believe that this addition may be made without in the least endangering its stability. —The friends and founders of this institution must derive much pleasure from beholding it in a state so prosperous, and also from reflecting that, by their candour and prudence in immediately receding from errors when pointed out to them, they have not only preserved the Society from ruin, but are likely to render it a permanent benefit to the public.

THE LAUDABLE SOCIETY *for the benefit of widows*, when first established in the year 1761, proceeded upon principles as irrational as those of the *London Annuity Society*, and therefore the ruinous consequence of its plan, if not speedily reformed, was equally necessary and inevitable. The same friendly admonitions were offered by *Dr. Price* to each of those institutions, and happy had it been for the one if in this instance it had imitated the prudence and discretion of the other. But the *Laudable Society*, though warned of its danger in every new publication of this work, and admonished to consent to such a reformation as might have
pre-

prevented the impending mischief, determined to persevere in error, and to reject all instruction however frequently repeated or strongly enforced. The more respectable part of its members were also continually urging the necessity of reforming their plan, but no arguments could prevail upon the majority, excepting reluctantly to consent to some feeble and trifling alterations which could have no other effect than by postponing to increase their calamity (*).—About four years ago, after every argument had proved ineffectual, the declining state of its affairs became so manifest as to awaken this infatuated Society to a slight sense of its danger, and it then raised its annual payments from 5 to 6 guineas, and lowered its Annuities to £10 if a member lived 3 years, £20 if he lived 7 years and £30 if he lived 13 years. By this measure all the former defects of the institution were said to be remedied and “the Scheme rendered of that public utility for which it was established(†).” But in the deplorable state of

(*) See Vol. I. p. 114.

(†) See the Deed of Settlement in 1788.

the Society at that time it was easy to foresee, that such alterations could not possibly produce any substantial benefit.—All therefore that has been said of its folly and danger in the first Volume of this work (*) has been since verified. Its number of members from 700 in 1772 and 550 in 1782 had sunk to 330 in the last year; at the same time the number of its annuitants had increased to 217, and its income by falling £1740 short of its expenditure, had rendered it necessary to sink so much of its capital in order to make up the deficiency. By persevering in this plan its stock, amounting to £42,200 in the 4 *per cent.* Bank Annuities, must in a few years have been exhausted, the payment of the annuities stopped, and the whole must have terminated in a melancholy scene of disappointment and distress.—The ruin which, though always certain, had in former years appeared too distant or perhaps imaginary to be regarded, seemed now to be hastening so fast as to excite a general alarm. It was therefore at length determined to examine the real state of the Society's

(*) See page 107 to page 115.

affairs, and to adopt such measures as might effectually save it from bankruptcy.—

On this occasion they were pleased to have recourse to my assistance, and I am sorry to observe that after a minute investigation, the finances of the Society were proved to be in a very unfavourable state.

—From the computations which were laid before the Directors it appeared that their whole stock could not even pay *four-fifths* of the claims of the present annuitants, and that there was a real deficiency in their accounts of near £30,000. Under these circumstances it was proposed and recommended that the annuities of the present widows be immediately reduced *one half*, and that those of all future claimants should be reduced *one fifth* (*). These measures

(*) This distinction between the two classes of annuitants may perhaps at first sight appear to be unreasonable. But the objection will be removed, if it be considered that many of the present widows, by having received more than their due, have been the chief cause of bringing the Society into its difficulties, and that the future claimants are certainly entitled to some allowance not only on this account, but because their husbands are also now paying *higher premiums* than those of the present annuitants had done.

have

have since been adopted, and it is indeed to be lamented that the distressed state of the Society should have rendered them so indispensibly necessary. Had the reformation commenced with the admonitions of Dr. Price, the reduction in the annuities would have been much more inconsiderable, and possibly, instead of affording a striking example to the public of the fatal consequences of an obstinate perseverance in error, they might by this time, like the *London Annuity Society*, have been enabled to increase their business and reputation by increasing their annuities. But the Society, by a late resolution, is for ever closed against the further admission of *new* members, and it is probable therefore that it may never again be necessary to make its concerns the object of public notice.

THE SCOTCH ESTABLISHMENT *for providing for the widows and orphans of the ministers and professors*, continues to promise all that can be expected from it. Since the former edition of this work an attempt has been made by Sir *Francis Hutchinson* and others to establish a similar institution for providing for the widows and orphans of the

the clergy in *Ireland*; but it has met with an opposition that has defeated it, and thus a very great public benefit has been prevented.

THE SCHEME described in page 119, Vol. I. *for providing for the nominees of East India commanders*, is likely to answer all the purposes for which it was established. The sums payable to nominees for the contributions there mentioned, have been raised to £600, the number of members who have died has been less than the calculations suppose on which the scheme has been founded, and there is every reason to expect that this Society will soon find itself capable either of increasing its claims still further, or of lowering the contributions required from its members.

Having been able to obtain no information, either from Dr. Price's papers or by any other means, respecting the progress of any of the *foreign schemes* mentioned in the first Volume from page 121 to page 127, I can add nothing to what has been there said concerning them.

THE LAUDABLE SOCIETY *for the benefit of age* is now almost the only one remaining of
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the numerous institutions of this kind which had been established before the first publication of this treatise. Excepting the *Amicable Society for the benefit of age*, which had reduced its annuities from £26 to £8, when the last edition was published in 1782, and which is probably now secure, I believe they have all either dissolved themselves or terminated in ruin. Unfortunately for the *Laudable Society*, it had persisted too long in error, like the other *Laudable Society for the benefit of widows*, before it had recourse to any effectual reformation, and in consequence of this obstinacy it has involved itself in similar difficulties. The number of its members from 1300 was reduced in 1785 to 204, of whom 152 were between 40 and 50 years of age, who of course must very soon become claimants,—the number of its annuitants amounted to 564, its stock which was originally £96,000 had diminished more than *one half*, and though the annuities, which at first were £44, had been gradually reduced till they were now only £8, its annual outgoings still exceeded its income above £2300, and this excess was continuing to increase so fast that in a few years its whole

stock would have been exhausted. Had the first admonitions of Dr. Price been immediately and properly regarded, the annuities need not have been reduced so low as £8. But by paying the early claimants so much more than their due, and obstinately persisting in this plan for so many years, the Society, when it adopted the measure of paying this reduced annuity, had become unable to perform it, and was obliged in the year 1786 even to lower its scanty allowance still further, by restraining the annuity to only *six guineas*. This was done in consequence of my having been employed at that time, by the desire of the Society, in examining the real state of its affairs, and recommending such a reduction. By going on to pay the annuity of £8 it appeared that in less than 18 years its ruin must certainly have been completed, and that it could not afford, with any prospect of security, to allow its present and future claimants a higher annuity than six guineas. An exception however was made in regard to the 52 members under 40 years of age, whose payments entitled them to an annuity of £8 even if they were then just entering into the Society,

Society. No alteration therefore has been made in those annuities, nor do I think that this exception will be attended with any danger.—In its present reduced state there is every reason to believe that the Society will be permanent; but it is not probable that the accession of *new* members will ever render it of sufficient importance to publish any account of its circumstances in future.

THE AMICABLE SOCIETY *for a perpetual Assurance Office* has been established ever since the year 1706, and seems to have withstood the trials of time and experience so long, that it is no wonder if its safety and permanence should be considered inviolable. From the nature of the institution this must be the case to a certain degree; but it is by no means perfectly secure.—There are evils to which it is exposed from imprudent measures which, by destroying its credit, may eventually destroy the institution itself.—In the sixth Section of the second Chapter of this work it was shewn that the Society was in danger of being too much encouraged by its apparently prosperous state in 1780, in consequence of an increase which in a few years had nearly tripled the number

of its shares; and it was also demonstrated there, that if no alteration in its plan took place, the dividend among nominees, though then at near £200 for each nominee, would in time fall below £150, the guarantied dividend, and produce the necessity of either entering into its capital or lowering this dividend.—I do not know that any attention has been paid to these observations, and if we may judge from the conduct of other Societies on similar occasions, it is not probable that they will be much regarded in the present circumstances of this Society.—When it was determined in the year 1770 that the claims on each number should not be less than £150, the members did not exceed 1120.—From that period the Society increased very rapidly, and in ten years had nearly attained its full complement of 2000. Hence it is evident that near *two thirds* of the present members have been admitted within these 20 years, and that the claims must now bear a much smaller proportion to the number of shares than they will do at the end of 20 or 30 years from this time.—If the whole annual contributions therefore be divided among the nominees of those that

now die in the year, the dividend must be much more considerable than it will be in future; for those nominees not only receive the sums to which they are entitled, but also the *surplus*, which should be yearly laid by and improved in order to form a fund which shall be sufficient to pay the claims when their number shall have arrived at its *maximum*. In the year 1786 the dividend upon each share amounted to £303, and in several of the preceding years to more than £200. But since the year 1787 it has fallen far short of this sum; nay, in one of those years the dividend from the annual contributions was so near the guarantied dividend that, with the view I suppose of preserving the credit of the Society, it was thought proper to sell out a part of its Stock in order to make an addition of £15 to each claim.—The mischievous tendency of this measure, if repeated always under similar circumstances, may be easily perceived; but it is possible, especially as the Society hath lately increased the number of its shares from 2000 to 4000, that in this and some following years, it may not be necessary to have recourse to it, and that the dividend

may

may again exceed £200. The time however, though perhaps distant, must inevitably arrive when this will not be the case. —The average number of annual deaths must go on increasing with the average age of the members for many years to come. The average of the dividends therefore must go on diminishing, till at last it gets below £100, which it appears to have often done before the year 1757.—Had the Society adopted the more rational plan which has been proposed in the first Volume of this Work (*) it might have always, for the same contributions, provided for claimants nearly double the sum of £100; but it does not seem at present disposed to profit by instruction.—On the contrary, it has lately rendered its plan more absurd and inequitable than ever. For when the number of its shares was extended to 4000, it was determined that every subscriber, when the existing number exceeded 2000, should be required to pay a contribution of £6. 4s. instead of £5 *per annum*. It must be needless to point out the folly and injustice of

(*) See page 150, &c.

such a distinction; and it is to be lamented that a Society which, with some improvements in its plan, is capable of doing much good, should chuse to adopt measures that tend to disgrace and injure it, rather than regard the advice and instruction which have been repeatedly offered to it in this treatise with no other view than that of promoting its credit and usefulness.

THE SOCIETY *for Equitable Assurances on Lives and Survivorships* is certainly the most extensive and important of all the institutions of the same kind in this Kingdom, and is the only one that has been uniformly guided in its practice by calculation. In the different editions of this work an account of the state and progress of the Society has been regularly given to the time in which those several editions were published, and this makes it necessary that the account should be continued to the present time.— In the year 1782 new Tables of premiums were computed agreeable to the probabilities of life in *Northampton* in lieu of the tables then in use which had been computed from the probabilities of life in *London*. But in order to prevent too great a reduction at one time

time, an addition of 15 *per cent.* was then charged upon those premiums; and with the view of making an adequate compensation to the existing members for their former payments, which had been so much higher than those to be required in future, an addition was made to their claims of £1. 10s *per cent.* for every premium they had paid previous to this reduction.—The consequence of these measures proved highly favourable to the Society; for its business increased so fast that in the year 1785 it was nearly doubled.—At this period it was determined to enter minutely into the state of the Society, by computing the value of each separate assurance, and ascertaining from thence the real surplus it possessed after discharging every possible demand upon it. The result of this laborious enquiry was a resolution to take off the 15 *per cent.* which had been charged upon the premiums in the year 1782, and to make a further addition to the claims of £1. *per cent.* for every payment that had been made prior to the first of January 1786.—By thus lowering its premiums and compensating its members without diminishing its capital, the Society
has

has not only preserved the credit which it derived from that capital, but has enlarged its usefulness to a greater degree than ever.

—The assurances which in 1782 were limited to £2000 on one and the same life have been extended, first to £3000 in 1784, and then in 1790 to £5000. The business has consequently been more than doubled since the year 1785, and events have proved so favourable to the Society that it has lately been determined to make another addition to the claims of £1. *per cent.* for every payment which has been made prior to the first of January 1792.—In consequence of these different additions many of the older members, when claimants, will be entitled to 50 and some of them even to 70 and 80 *per cent.* more than the sums which they had originally assured.—It is hardly possible to devise a more safe and equitable method than this of making the members to participate of the profits of the Society. Its operation is gradual, every person is benefited in proportion to the money which he has paid, the grand object of making the assurance is promoted in the most desirable manner by increasing that provision which it was designed

to secure for a surviving family, and if the Society perseveres in the same plan, it success will continue, and probably in a few years still further additions may be expected. Its present capital (which it is to be hoped will always be preserved sacred and inviolate) guards the Society against the effects of any extraordinary mortality. The number of its members is also now become so considerable that the decrements of life in each year are nearly the same, and the annual increase of the Society is continually rendering this uniformity more certain.

From the year 1768 to the year 1791 (a term of 23 years) the decrements of life in the Society to those in the *Northampton* table of observations have been

from 10 to 20 as 1 to 4

20 to 30 as 4 to 7

30 to 40 as 3 to 5

40 to 50 as 3 to 4

50 to 60 as 3 to 4

60 to 80 as 9 to 10.

Hence it appears from this comparative view that the Society was perfectly justified in taking off every additional charge on its premiums. The probabilities of life among
its

its members being so much higher than they are in the table of observations from which those premiums are computed, will generally produce such a balance in its favour, that together with the annual interest of its *surplus* Stock will be more than sufficient to discharge every reasonable expence of management, or even to secure it from the danger of any *temporary evil*. Provided therefore the same caution be continued in admitting none but good lives to be assured, and the same wise resolution be maintained of preserving the capital undivided, there can be no doubt but that the credit, the prosperity, and the usefulness of this Society will go on to improve in an accelerated progression.

In the first Volume of this work (*) an account is given of a plan for enabling the labouring poor to provide support for themselves in sickness and old age by small weekly savings from their wages. Since the first publication of this plan Dr. Price had be-

(*) Note page 140.

flowed great attention on the subject; and being furnished with more information than he then possessed, he was enabled to compute the valuable Tables which are inserted in the first Appendix.—These were intended to have served as the foundation of a plan to be established by the legislature, which should hold forth such particular encouragement to the poor as should induce them to give it the preference to those insufficient and erroneous plans which are now forming all over the Kingdom. Nothing could be more important than this design. It was founded on the best principles, and had the happiest tendency to assist the poor by encouraging industry and saving, and by raising them above the wretched necessity of depending upon the parish for subsistence when rendered incapable of providing for themselves. The bill for establishing this plan was brought into parliament about two years ago, and after having passed the house of Commons was rejected by the Lords.—But it is to be hoped that this want of success will not prevent the subject from being soon resumed: For its usefulness and importance, when properly understood, can hardly fail to engage

gage every branch of the legislature in its support. It is observed by Dr. Price (*) that the poor's rate in the year 1777 amounted to £1,556,804. Since that time it has increased near half a million; and the truth seems to be, that there is a growing distress among the poor occasioned most probably by the price of provisions having increased more in proportion than the price of labour.—From an impartial comparison of the one with the other, it will appear to be hardly possible now for a poor man, if he have a wife and family, to subsist on his wages.—This melancholy truth, were I disposed to enter further into the subject, might be proved in the most convincing manner from the valuable information communicated to Dr. Price by Mr. Pitt, the member for Dorsetshire, whose humanity has led him to make particular enquiries into the state of the poor of this Kingdom.—But it will be best to refer the detail of those proofs to the author himself; by whose benevolent example it is to be wished that others may be stimulated to the pursuit of similar en-

* Note page 141, Vol. 1.

quiries. Happy would it be for the state were those persons who are called of higher rank to interest themselves more in adopting proper measures for relieving the distress of the labouring poor. By the formation of rational plans for this purpose the many wretched schemes which are now continually rising up to delude their hopes would disappear, the indigent labourer and his family would look forward without anxiety to seasons of sickness and old age, and the more wealthy would enjoy the advantage of a reduction in the enormous expence with which the parishes are now loaded for the maintenance of their poor.—It is much to be lamented that the different parochial clubs which are established in this kingdom should in general be founded on such erroneous principles, and that a disposition so laudable in itself should not be properly directed and encouraged by the legislature. But of all the phantoms which are held up to entice and to deceive the multitude, none are more mischievous or deserve more severe reprehension than the TONTINES which have lately become so prevalent in every part of this country. By these, while the adventurer

turer is lured with the extravagant hope of making his fortune in a short period and at a small expence, the worst spirit of gambling and idle speculation is called forth, and those baneful effects which are produced by a State Lottery in *London* are extended to the remotest corners of the Kingdom. It is not only the person who can afford to subscribe sixpence or a shilling a week from his income that becomes the dupe of those bubbles; they are crowded in the poorer parts of the country with domestic servants whose wages do not exceed £ 3 or £ 4—a sum which even properly applied is hardly sufficient for their maintenance. This class of subscribers must necessarily either involve themselves in poverty and distress to complete their payments, or, which is more probable, they must find themselves unable to go on with them, and by this means lose all the money they have already advanced.—It is indeed no wonder, considering the ignorance which prevails on the subject, that so many should be captivated with the advantages which are promised them in these tontines, and it may not perhaps be improper to state a few facts which, if they produce no other effect,

will however tend to moderate the expectations and consequently to lessen the disappointment of the subscribers at the final division of their stock. The well known increase of money when improved at compound interest and the continual mortality of the human race (which is proved by the melancholy experience of every day) are the two principles upon which all tontines are founded, and from which they derive all their encouragement. But it is impossible to apply those principles more improperly than to the present purpose. In the short term of seven years the accumulation of money at simple and compound interest is much the same, and the decrements of life are so inconsiderable as to produce little or no effect in increasing this accumulation.--- A weekly payment of sixpence improved at *4 per cent. compound* interest for seven years will amount to 10*l.* 5*s.* 3*d.* but at *simple* interest it will amount to 10*l.* 3*s.* 10*d.* and at *no interest at all* to 9*l.* 2*s.*—The addition therefore to the principal from the mere operation of compound interest is so inconsiderable that were all the subscribers to live to the end of the term, each share would be increased by this means only 1*l.* 3*s.* 7*d.*—Again; let the

the number of subscribers to the tontine be 10,000, and let them consist of persons of all ages under 60 years.—According to the probabilities of life at *Northampton* (which however are much too low in the present case) 8647 of those persons will survive a term of seven years; so that if the whole 10,000 lived to make their last payment, and none of them died till just before the final distribution of the stock, the share of each survivor would be no more than 1*l.* 17*s.* 6*d.*—But it is to be observed that these lives will be continually dying from the time of the first subscription to the conclusion of the tontine, and that these deaths by lessening the weekly contributions will reduce the share of each survivor to *eleven guineas* nearly. When the expences of management are also deducted, and allowance is made for the loss which is likely to be sustained by investing the subscriptions in the Public Funds, where the improvement of money is always uncertain, but particularly so at the very high price which they now bear, it is more than probable that the shares will fall greatly below the sums which I have stated, and that the surviving members

will have the satisfaction to find at the end of seven years that they have barely received their principal, after having endangered the loss of the greatest part of it by dying in the mean time. The only source from which those tontines can derive any additional increase must be from the inability of some of the members to go on with their subscriptions. But this is an evil of the worst kind, and defeats the very end for which those plans are said to be principally intended. Instead therefore of relieving they will add to the miseries of the poor, and the only persons that will be benefited by them (excepting indeed the *treasurers* and *secretaries*) will be the more wealthy subscribers, whose shares will have acquired their chief increase from the spoils of the distressed.—I am sorry to see those schemes adopted and encouraged by so many respectable persons, who have contributed very much by their character and situation in life to spread the contagion; and by submitting to become *trustees* of the different societies, they have unfortunately given them a weight and credit which they would not otherwise have enjoyed.—In one of the most numerous of those tontines,
which

which consists of 43,000 members, it has been urged in its defence that it would have a tendency *to improve the morals of the people* by leading them to habits of saving. Although no doubt can be entertained of the good intentions of those who *patronize* this scheme, it must, I think, be acknowledged that the method they have chosen of reforming the poor is rather equivocal.—It is not likely that an indigent man, when allured to save by the same motives which stimulate a gambler to his ruin, should be *improved in his morals* by this means, or that he should be much prejudiced in favour of this disposition when his hopes and expectations are kept alive by a delusion. After looking forward during the term of seven years to the accumulation of an immense fortune (*), and finding himself at last barely in possession of the miserable pittance he has paid, it will be no wonder if the mortifying disappointment he feels should lead him to squander away a sum, too scanty for estab-

(*) One of the subscribers to the *Bristol Tontine* wrote to inform me that he expected to receive *One thousand pounds* at the final division of the Stock.

lishing him in trade or for any other valuable purpose, and that, instead of convincing him of the good effects of saving, it should confirm him in the habits of dissipation.

Considered in every point of view, a tontine for a short term of years is absurd in the highest degree. If it be extended to the whole duration of life, and include persons of all ages, a term of 80 or 90 years must elapse before all the lives become extinct, and the subscriptions will necessarily accumulate during this time to a large amount. But the probability is so remote that any particular member will be the survivor of thousands to enjoy this accumulation, and the value of money is so diminished when the payment of it is postponed for such a long succession of years, that, were the subject properly understood, this hopeless and unprofitable method of speculation which is now encouraged by the ignorance and credulity of the multitude, would be universally reprobated. If, instead of a periodical subscription during life with the view of appropriating the accumulation to the last survivor, the whole money be paid at once, and the interest be annually divided among the subscribers till they

they are reduced to one person who is to receive the principal, the prospect in this case is not more encouraging.—Suppose 1000 persons between 10 and 20 years of age to subscribe £100 each, and that the interest at 4 *per cent.*, or £4000, be annually divided among them. In the course of 20 years the dividend of each member will be raised from £4 to £5 *per annum*—in 37 years it will be doubled, or increased to £8 *per annum*—in 54 years it will be quadrupled or increased to £16 *per annum*—in 64 years it will be decupled or increased to £40 *per annum*—and in about 80 years the last survivor will come into possession of the whole principal.—The present value of £100,000 to be received at the end of 80 years is £4330, and as it is nearly 1000 to one against any individuals coming into possession of it, the value of each subscriber's expectation in this principal is only 4*l.* 6*s.* 7*d.*—It has lately been the custom to borrow money by the assistance of *tontines*, where the subscriber sinks his principal upon condition of having his annual interest increased. But I do not know that a worse method can be adopted; for it has the peculiar property of
being

being injurious both to the borrower and the lender.—If the annual interest be much increased it is highly injurious to the *borrower*; since it evidently appears from what has been already proved, that the addition to be allowed yearly on each share as an equivalent for the principal is altogether inconsiderable. But it is necessary to allure the subscribers by an immediate benefit, as well as by the prospect of future advantages, and therefore this addition is generally much greater than it ought to be; so that were the difference between the common interest and that which is allowed to the subscribers to be properly improved during the continuance of the tontine, it would in most cases amount to a sum which would be double or even triple of that principal for which it is given as an equivalent.—The subscriber on the other hand is no gainer. His property is deposited where he cannot dispose of it without considerable loss, or continue it without being flattered with hopes which can never be realised; for if he comes into possession of the whole dividend, it will be at the end of 80 years, when it is more than an equal chance that he does not live six months to enjoy

enjoy it.—I am much concerned to observe the legislature of this kingdom giving a sanction and encouragement to the very evil which I have been endeavouring to expose. Wretched indeed must be the condition of that country where the morals of the people are sacrificed to the interests of the revenue, and the public exigencies are provided for by the pernicious assistance of lotteries and tontines. When the spirit of idle speculation and gambling is thus cherished and maintained by those who ought to exert their power in suppressing it, what mischievous consequences may not be apprehended? The absurd schemes of every designing projector, encouraged by such flagrant examples, will succeed in imposing upon the credulity of the ignorant and distressed, who, being deceived in their first hopes, will be allured to go on by other schemes still more flattering and delusive, till their rage for adventure, increasing with disappointment, will extinguish every habit of honest industry, and render them at last totally corrupted and depraved.—Being convinced of the certainty of these and of worse consequences, I sincerely wish that the facts which I have stated may engage the attention

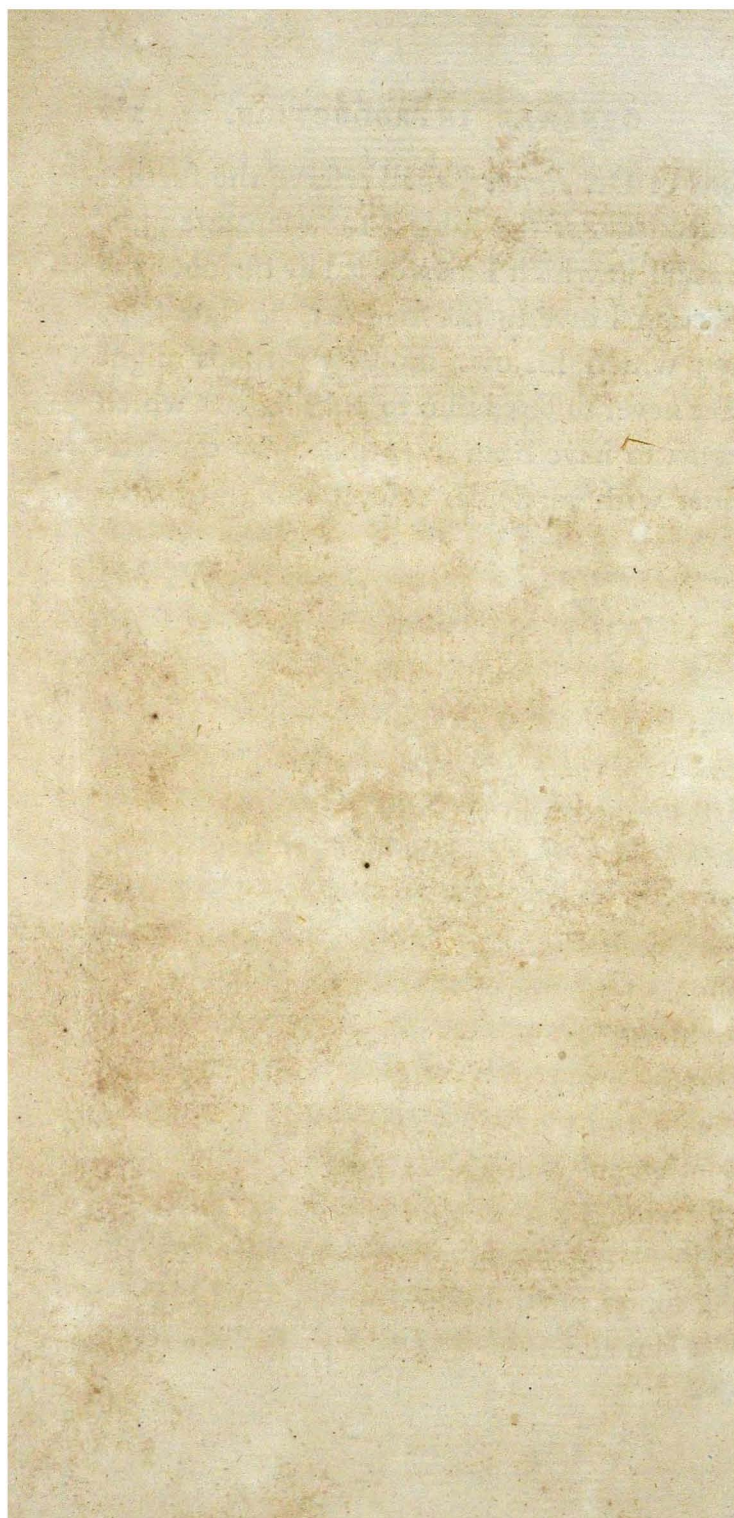
tion of those from whom the evil at present derives its chief strength and support; for unless the reformation begin with them, it will be of little use to attempt it by reason and argument.

P. S. It was Dr. *Price's* intention, as appears from some references in the first Volume, to have continued in this edition his account of the public revenue and expenditure to the beginning of the year 1791.—He had also proposed to have published the three plans for paying off the national debt which were communicated by him to Mr. *Pitt* in consequence of an application having been made to him for his opinion and advice on this subject.—One of these plans was adopted by the minister without the slightest acknowledgment, and now forms the foundation of an Act for reducing the public debt, which was established by the legislature in 1786, and has contributed more than any other measure to raise the credit of the present Administration.—Being apprehensive of rendering this introduction too long by attempting at this time to fulfil the intentions

tions of Dr. *Price*, I shall reserve the further prosecution of the subject for a separate publication, in which I may be led by the duty and affection I owe to his memory, to state some facts which his own modesty perhaps might have never suffered him to publish, but which ought to have been mentioned by the Minister with particular respect and gratitude.

CHATHAM-PLACE,
29th February 1792.

W. M.



P R E F A C E
TO THE
F I R S T E D I T I O N.

BEFORE the Reader enters upon this Work, it will not be improper to give him the following information concerning it.

. A few years ago, many gentlemen, of the first eminence in the law, formed themselves into a *Society*, for providing annuities for the widows of all such persons in judicial offices, barristers, civilians, and solicitors, as should chuse to become members. A plan was agreed upon and printed; but, some doubts happening to arise with respect to it, the directors resolved to ask the opinion and advice of three gentlemen, well known for their skill in calculation. This occasioned a further reference to me; and the issue was, that the plan being found to be insufficient, the whole design was laid aside.

About

About the same time, several other societies were formed with the same views; but all on plans alike improper and insufficient. Finding, therefore, that the public wanted information on this subject, I was led to undertake this work; imagining, that it might be soon finished, and that all I could say might be brought into a very narrow compass. But in this I have been much mistaken. A design, which I at first thought would give little trouble, has carried me far into a very wide field of enquiry: and engaged me in many calculations that have taken up much time and labour. I shall, however, be sufficiently rewarded for my labour, should it prove the means of preventing any part of that distress, which is likely to be hereafter produced by the societies now subsisting for the benefit of widows.—I have proved the inadequateness of their plans, by undeniable facts and mathematical demonstration.—I have, further, given an account of some of the best plans, which are consistent with a sufficient probability of permanency and success.—Should, therefore, any of these societies determine to reform themselves; or should any institutions of the same kind be hereafter

hereafter established, they will here find direction and assistance.

In Question VI. Chap. I. a general method is described of finding the values, in *single* and *annual* payments, of all life-annuities which are to begin after a given term of years; and, in the 4th Section of the 2d Chapter, the plans of the societies for granting such annuities are particularly considered, and proved to be extremely deficient.—Indeed, the general disposition which has lately shewn itself to encourage these societies, is a matter of the most serious concern; and ought, I think, to be taken under the notice of the Legislature. The leading persons among the *present* members, will be the *first* annuitants; and they are sure of being gainers: And the more insufficient the scheme is, on which a society is formed, the greater will be the gains of the first annuitants. The same principle, therefore, that has produced and kept up other *bubbles*, has a tendency to preserve and promote these; and, for this reason, it is to be feared, that, in the present case, no arguments will be attended with any effect. The consideration, that “the gain made by some in these societies,
d “will

“ will be so much plunder taken from
“ others,” ought immediately to engage all
to withdraw from them, who have any regard
to justice and humanity; but experience
proves, that this argument, when opposed to
private interest, is apt to be too feeble in its
influence.

It cannot be said with precision, how long
these societies may continue their payments
to annuitants, after beginning them. A con-
tinued increase, and a great proportion of
young members, may support them for a
longer time than I can foresee. But the
longer they are supported by such means, the
more mischief they must occasion.—So, a
tradesman, who sells cheaper than he buys,
may be kept up many years by increasing
business and credit; but he will be all the
while *accumulating* distress; and the longer
he goes on, the more extensive ruin he will
produce at last.

In the latter end of the first Chapter, I
have stated very particularly, the method of
computing the values of *assurances* on lives
and survivorships, in all cases where no more
than two lives are concerned: And, in the
third Essay, I have pointed out a considerable
error,

error, into which there is danger of falling in computing some of these values. The societies and offices for transacting business in this way, are very useful; and it is necessary that they should go upon the best principles, and possess all the information that can be given them.

But there is no part of this work in which the public is so much concerned, as the third Chapter. It will be there proved, that had the sums raised for public services since the REVOLUTION, been much greater than they have been, the increase of the public debts to their present state might have been prevented in the easiest manner, and at a trifling expence. A method, likewise, of reducing within due bounds these debts, heavy as they now are, will be proposed.—All competent judges will, I believe, see, that this method, being founded on the most perfect improvement that can be made of money, is the most expeditious and effectual that the natures of things admit of. Nor, in my opinion, if the nation is not yet too near the *limit* of its resources, can there be any good reason against carrying it into execution.—It is well known, to what prodigious sums, money,

improved for some time at *compound interest*, will increase (a). A state, if there is no misapplication of money, must necessarily make this improvement of any savings, which can be applied to the payment of its debts. It need never, therefore, be under any difficulties; for, with the *smallest* savings,

(a) A *penny*, so improved from our Saviour's birth, as to double itself every 14 years, or which is nearly the same, put out to 5 *per cent.* compound interest at our Saviour's birth, would, by this time, (that is, in 1773 years) have increased to more money than would be contained in 150 millions of globes, each equal to the earth in magnitude, and all solid gold (*).

(*) This computation is made on the supposition that a cubic inch of gold is worth about £ 35.—But since a cubic inch of water is known to weigh 254 grains nearly, and the specific gravities of *standard* gold and water are to each other as 18,888 to one, it will follow that a cubic inch of gold, according to the *Mint* price of 3*l.* 17*s.* 10½*d.* per oz. will amount to 38*l.* 1*cs.* 1½*d.* and consequently that a penny improved at compound interest so as to double itself once in 14 years, from the time of our Saviour's birth to the year 1773, would have accumulated only to 107 millions of such globes as are mentioned by Dr. Price,—but this is abundantly sufficient to prove the strength of his argument.—In the present year, it would have accumulated on the above supposition to 274 millions of those globes.

If instead of supposing money to double itself every 14 years, it be improved at 5 *per cent.* compound interest, the accumulation in 1791 years will amount only to 144 millions, and in 1773 years to no more than 60 millions of globes. The difference therefore between the supposition of money's doubling itself every 14 years and of its being improved at 5 *per cent.* compound interest, is far from being inconsiderable, especially in a long term of years. ED,

it may, in as little time as its interest can require, pay off the *largest* debts.

In the *first* Essay I have made many observations on the expectations of lives, the pernicious influence of great towns on health, and manners, and population; the increase of mankind; and other subjects in the doctrine of Annuities and Political Arithmetick.—

In the Last Essay I have stated carefully the proper method of forming Tables of the probabilities of human life, from given observations: And, at the close of this Essay, besides several new Tables, I have thought it necessary to give Mr. *Simpson's* Tables of the values and expectations of LONDON lives.

—I have also, in the Notes at the end of this work, given the Demonstrations of the Answers to the *Questions* in Chap. I. These Demonstrations I have chosen to keep out of sight in the body of the work, in order to avoid discouraging such readers as may be unacquainted with mathematics.

Upon the whole. A great part of the work now offered to the Public is, I believe,

new; and I am in hopes also, that it will be found to contain some improvements in those branches of philosophical enquiry, which are the subjects of it.

P R E F A C E

TO THE

T H I R D E D I T I O N.

THAT favourable reception of this Work, which has occasioned the present Edition of it, so soon after two former editions, is such a proof that it has been of some use to the public, as amply rewards me for the attention and labour which I have bestowed upon it. In revising it on the present occasion, I have been anxious about improving it as far as possible. Several additional facts and observations have been inserted in different places, particularly in the First Essay and the Postscript to it.—That

part of the Second Section, Chap. II. which treats of the *Scotch* establishment, has been new composed, and carefully accommodated to the more accurate information concerning it, with which I have been favoured.

The SUPPLEMENT is an addition which was made to the *second* edition.—The observations in it on the present state of our population have been a good deal enlarged.—This is a very serious and important subject. If, indeed, there has been that diminution of our people which the evidence I have produced seems to shew, it must alarm every one who wishes well to his country, and it ought to engage the immediate and vigorous attention of government.—Many differ from me on this point; and I wish I could find sufficient reason to believe as they do. Several great manufacturing towns have, I know, increased; but these are nothing to the whole kingdom; and even by their increase, our population may, on the whole, have lost more than it has gained.—In truth; it would have been strange if our numbers had not been declining; for I can scarcely think of any great cause of depopulation,

lation, which has not for the last 80 years been operating among us.

The prodigious traffic now carried on in Life-annuities, and the rage for forming and encouraging Annuity Schemes, which has for some time been spreading through the kingdom, has rendered the information which I have meant to convey in the following work particularly necessary. And I have had the pleasure to observe that it has been attended to. Several of the Annuity Societies in LONDON have been dissolved; and there is reason to hope, that those which still remain will not be able much longer to support themselves on their present plans, in opposition to the evidence of demonstration, and the calls of justice and humanity.—These *Bubbles*, however, are of little consequence, compared with that GRAND NATIONAL EVIL, which is the subject of the second chapter of this treatise. This is an evil on which I could not imagine, that any such efforts as mine would make any great impression. Perhaps, indeed, the united efforts of all the independent part of the kingdom would now be too weak

weak to save us from the distress with which it threatens us.

Much has been said for some time of a plan mentioned in PARLIAMENT, at the end of the last session, for paying off the NATIONAL DEBT. This raised some expectations; and I will beg leave here to give a brief account of it.

After providing for all the current services, there remains this year (1773) a *saving* or *overplus* of 1,200,000*l.* With this sum, and a profit of 150,000*l.* from a Lottery consisting of 60,000 tickets, a MILLION AND A HALF of the 3 *per cent.* annuities, purchased at 90, will be paid off.—When this was proposed to the House of Commons, it was at the same time announced, that it would be the COMMENCEMENT OF A PLAN FOR PAYING OFF THE NATIONAL DEBT; for, if no extraordinary services should call for any other application of the public surplusses, the same payment increased by the interest of former payments, is intended to be made every year while the peace lasts; And thus, reckoning compound interest

interest at 3 *per cent.* SEVENTEEN MILLIONS will be paid off during a peace of ten years.

On this plan I will take the liberty, with all the deference which becomes me to the station and abilities of the proposer of it, to offer the following remarks.

1st. It implies, that there is to be a *Lottery* every year during the whole continuance of peace.—Formerly, lotteries were expedients for procuring money on more advantageous terms, to which government had recourse, when pressed by the necessities of war. They are now, it seems, to be established as *permanent* resources never to be given up or suspended.—This must shock every person who is duly acquainted with the mischief occasioned by lotteries, particularly among the lower classes of people. The rage for gaming threatens the ruin of all that is virtuous and manly among us. It is increasing fast, and wants not to be fostered by government.

2dly. The *surplus* of the present year is in part the effect of some *extraordinary* savings in the last year, (1772) which cannot
be

be expected another year: And, I believe, that those who are best acquainted with this subject, must be sensible that there is no sufficient reason to expect, while the augmentation of the navy is continued, a constant *surplus* of so much as a MILLION *per ann.* I mean this on the supposition, that the produce of the *Sinking Fund* will continue what it is taken for this year, and what it has been the last three years, or 2,600,000*l.* But this is certainly more than can be depended on. The difficulties of the *East India Company*; that stagnation of credit which has lately distressed the public; and many other causes, may possibly occasion *Deficiencies*. Should there, however, be an *increase*, it will be owing, I am afraid, to a very bad cause: I mean, to an increase of our importations proceeding from luxury, and turning the balance of trade against us; and, consequently, draining the kingdom of its *specie*, and leaving it more and more to the precarious and dangerous support of paper-money. But,

3dly, Let the *surplus* of the public revenue prove what it will, there is too much probability that, even during the continuance of peace, some emergencies or other will be
often

often furnishing reasons or pretences for employing it in other ways than the payment of the public debts. This has been the case hitherto; and from the year 1730 to the present time, it has never happened, that we have gone on above three or four years together employing *surplusses* in discharging debts. Though in profound peace, there have been calls for a different application of them; nor can I imagine what reason there is for believing that our circumstances are so much changed for the better, that there will arise no such calls for ten years to come, should the peace last so long. But,

4thly, The most capital defect in this plan is, that its operation is to cease as soon as a war begins. That is; it is to cease at the very time when it would operate to most advantage, and make the quickest progress in redeeming the public debts. This has been demonstrated in the chapter on public credit in this Treatise, and in my *Appeal to the Public on the Subject of the National Debt*.

Is it then any wonder, that such a plan has had no effect on public credit?—Does it mean any more than that the *surplusses* of the
revenue

revenue shall be applied to the discharge of our debts, when there are no other uses for them?—And was there ever a time when this was not done? Is not this the very plan we have been pursuing these forty years, and to which we owe our present incumbrances? Certain it is, that nothing but a plan that shall go on operating uniformly in *war* as well as in *peace*, or the establishment of a permanent fund that shall never be diverted; that is, in other words, a return to the scheme adopted by the legislature in 1716; and which even now stands established by law, but which, through the unpardonable misconduct of men in power, has been defeated of its good effects: Nothing, I say, but this can do us any essential service; or, in our present circumstances, be much more than trifling with the difficulties and dangers of the public.—Establish such a fund—Consign it to a particular commission, acting under penalties, in such a manner as shall take it out of the hands of the *Treasury*, and form a check even on the *House of Commons* itself.—Supply from time to time all deficiencies just as if no such fund existed; and, by these and other measures, convince the kingdom that something

something effectual is meant, and that the public debts are indeed in the way to be extinguished.—LET THIS BE DONE; and we may soon see a new state of things; public credit may revive; and the kingdom enjoy at least a chance for being preserved.—By the confidence which such a measure would give in government security; but more especially, by the increasing sums which would be thrown annually into the public markets, and returned to the public creditors, the 3 *per cents.* would be soon raised to *par*, and in some time probably far above *par*. It is well known, what an effect *borrowing* every year has in sinking funds. Paying every year would certainly have an equal contrary effect. In a time of war, particularly, it would give such a demonstration to the public, that an *irrevocable* plan of redemption was at last established, as could not but produce the happiest effects. It would indeed in these circumstances be necessary to borrow an extraordinary sum annually equal to the appropriation. That is; supposing the fund to set out with a million *per ann.* it would be necessary to borrow so much more annually than would have been wanted

wanted had the fund been capable of being diverted. But this being done to convey a conviction with which the very power of borrowing was connected; and to preserve a *fund* on which the very being of the state depended; no bad consequences could follow. The annual charge on the public, occasioned by the war, would be even *less* than it must have otherwise been. For, let us suppose *ten millions* necessary to be borrowed every year to defray the expences of war, *nine millions* only of which would have been wanted, had not the million *surplus* been locked up.—Suppose farther, that the scheme, by keeping up public credit, and throwing money every year into the hands of lenders, enables government to borrow at *1 l. per cent.* less interest than would be otherwise required; that is, at *4* instead of *5 per cent.*—In these circumstances, there would arise a present saving to the kingdom of *50,000 l. per ann.*; for the interest of *ten millions* at *4 per cent.* is *50,000 l.* less than the interest of *nine millions.* at *5 per cent.* And such a saving, repeated every year of a war, would be an object of some importance to the kingdom.—Indeed, there may be no possibility

possibility of conceiving what important effects in this way, the establishment of such a scheme might produce. During its progress in discharging our debts, and before it could give any relief by the annihilation of taxes, it might *save* the kingdom, by preserving it from difficulties which would have sunk it. And every one must be sensible of this, who has considered what danger there is that a war, should it become unavoidable before our debts are put into any certain course of redemption, will either entirely overwhelm public credit, or so much weaken it, as to produce an impossibility of borrowing except on very exorbitant interest, and, consequently, of finding taxes sufficiently productive to pay such interest. The general apprehension now is, that the nation is overloaded; and that its debts will never be paid. This keeps the funds near 18 *per cent.* lower than they were in the last *peace*. In the next war such apprehensions will increase, and produce great danger. But should it be then seen, that a plan for redeeming our debts, the most efficacious possible, was going on; and, in consequence of being guarded in some such manner as I

have hinted, *would* not (or *could* not easily) be revoked; in these circumstances, all danger would be so far lessened, that it might be practicable to find new taxes which would support the expences of war during the operations of the scheme.

But I am got far beyond the limits I prescribed myself when I began this Preface.—As the national debt is a subject unspeakably interesting to this nation, I could not allow myself to omit any thing that appeared to me of consequence upon it; and the Reader of this Treatise will on this account, I hope, excuse me, if I have detained him here too long and too unprofitably. Much has been before said on this subject by writers of more consequence to no purpose; and we shall pursue the path we are in, till the edge of the precipice towards which we are advancing awakens us, and ruin becomes unavoidable.—The distress occasioned by the shock lately given to the bubble of paper-credit, is, I am afraid, a prelude to greater calamities, and a warning to prepare for them.

P R E F A C E

TO THE

F O U R T H E D I T I O N .

THIS work having been for some time out of print, I resolved about four years ago to prepare for the press a new edition of it, expecting that I should have only a few corrections and additions to make of no particular consequence. But in this expectation I have found myself greatly mistaken. Such a variety of new matter came in my way, and such means of improving this work were communicated to me, as have led me to bestow upon it more attention and labour than can be easily imagined, and to increase it from one to two volumes.

It is probable that nothing could have engaged me to undertake so much labour

had I foreseen it; but having begun, I could not avoid going on; and I was encouraged by the reflection on the favourable manner in which the former editions of this work had been received, and also by the hope that on *one* subject of human enquiry I should be able to produce a work more complete than any that the public has been yet furnished with.

The additions of most consequence in the present edition are the following.

There has been added to the *Second Chapter* an account of several *foreign Societies*; and a continuation of the history of such annuity societies as are still subsisting in *London*, to the time when that Chapter was printed off; that is, to the beginning of the year 1782.—The largest of these additions respect the *Amicable Corporation for Perpetual Assurances at Serjeant's Inn*; and the Society in *Chatham Place for Equitable Assurances on Lives and Survivorships*.—The former of these Societies, should they think proper to look into these observations, will, I am persuaded, find that they require their attention.—But it is the
Society

Society last mentioned, which, through the whole of this Treatise, I have had chiefly in view. Having for many years been concerned in advising this Society (the first of the kind in the world, and increasing fast), I have been anxious about giving it all the information and assistance possible.—The additional observations addressed to it in this edition will be found chiefly in this Volume from page 170 to 178; and in the Second Volume from p. 101 to p. 112, and from p. 189 to p. 199. And, from these observations, it may be learnt in particular, that though this Society, in consequence of having happily begun too high, has already found itself capable of making great abatements in its demands, it is still capable of making farther abatements.

In the former editions of this work, I had intimated that a publication of the tables by which its business is transacted would be proper, together with an account of the principles assumed, and the method taken in composing them. This is an information to which the public has a right, and which is now given it in different parts

of this work, and particularly in the Second Volume from p. 101 to p. 112.

Many corrections have been made and several additional notes inserted in the First Essay. Some of these have been occasioned by two late publications; one by Mr. *Wales*, master of the mathematical school at *Christ-Church*; and the other by Mr. *Howlet*. See, particularly, the notes in p. 243, 251, 254, and 255.—The Table in p. 298 is also now first inserted.

To the Second Essay a *Postscript* has been added; the principal intention of which is to point out the reasons for discarding the valuation of single and joint lives derived from Mr. *De Moivre's* hypothesis; and also to describe a method of computing these values from any given tables of mortality, which, while it leaves no possibility of any mistakes, renders such computations as easy and expeditious as their nature will allow.

The Third Essay is the same that it has been in all the former editions. But to the *Fourth* some additions have been made; particularly, the notes in p. 343, 345, and
346.

346. The two first of these notes have in view a remark of Mr. *Wales's*; and the design of the last is, to retract an assertion in the former editions concerning the duration of life in old age in great towns, and to shew the reason why a greater proportion of the inhabitants of *London* died formerly in old age than have died lately.

But the additions of most consequence are the Tables in the Second Volume, with the remarks explaining the construction, use, and application of them.—In the general introduction to these Tables, and in the remarks on Table XVI. p. 48. it is shewn (and, I think, undeniably) that the Tables of the values of lives deduced from the *London* bills of mortality err only by giving them too high; and that, with respect to the main body of the inhabitants, the unfavourableness of *London* to the duration of life continues much the same that it used to be.

Tables I. II. III. and IV. and Tables LVIII. LIX. LX. and LXI. are an abridgement of Mr. *Smart's* tables of compound interest, and contain all that is important in them.

The VIth Table, shewing the mean probabilities

babilities of the duration of life according to a register of mortality at NORTHAMPTON, has been inserted in all the former editions; but it is now given more correctly; and tables deduced from it have been added, of the *expectations* of life and the values of *single* lives and of any *two joint* lives at all ages, and for three rates of interest. The labour of computing these tables was undertaken in order to set aside all occasion for using the defective valuation of lives founded on Mr. *De Moivre's* hypothesis; but not having being able to finish these computations till a great part of this Treatise had been printed off, I have been obliged to continue the use of the old tables so far as to take from them many of the examples of the solutions of questions in the first and following chapters.

When tables of the values of *two joint* lives are given, the values of *three joint* lives may be deduced from them, with perfect ease, by Mr. *Simpson's* rule inserted in the Second Volume of this Treatise, p. 118.

This rule saves so much trouble, that I have thought it worth while to procure calculations of the XXXVIIth and XXXVIIIth tables,

Tables, on purpose to determine how far it may be depended on. The result is given in the remarks and comparisons from p. 117 to p. 120. And it seems to appear that it finds the values of *three joint* lives so nearly as to leave little room for wishing in this instance any greater degree of correctness.

The rules from p. 170 to p. 182 describe a method of deducing, with sufficient accuracy, the value of any life or number of lives at *all* rates of interest, from the correct value given at *one* rate of interest.—In computing, therefore, tables of the values of lives, according to any given observations, no more will hereafter be necessary than to compute them for *any one* rate of interest.

All improvements, however, of this kind would be of little consequence, were there no tables which state correctly the laws that govern human mortality in different situations. One principal part of my business in this work has been to frame such tables; and any one who will look over the collection of tables in the Second Volume, and particularly from p. 121 to p. 161, will see that I
have

have been furnished with the best means of doing this.

With respect to the Tables, in particular, deduced from the *Swedish* Observations, I cannot hesitate to pronounce that they exceed in correctness every thing of this kind which has been hitherto offered to the public; and that nothing is wanting to make our knowledge in this instance complete, but similar observations in other kingdoms.—By these Tables I have been enabled to state minutely the different rates of mortality at all ages among males and females; and to form tables of the values of single and joint lives for *each* sex, as well as for both sexes collectively; in consequence of which, I have been farther enabled to determine the increase of the values of annuities payable during survivorship, occasioned by the longer duration of lives among females; and thus to furnish a direction of some importance to the various societies in this kingdom and abroad for providing annuities for widows.

I must not in this place neglect to acknowledge the great obligation I am under to Mr. WARGENTIN of the Royal Academy of Sciences at STOCKHOLM, for the communication;

munications which have enabled me to make the additions to this Treatise last-mentioned. It will be found also, that I have been much indebted to Mr. OEDER of *Oldenburgh* for several useful communications.

In the observations and rules which follow the LVith Table, p. 226, I have given a general account of the method of computing the values of *presentations* to livings, and of the renewals of leases held either for terms certain or for lives.

The rules from p. 244 to p. 252, shew how to deduce from the values given of any annuities payable *yearly* their values when payable *half-yearly* or *quarterly*, or when secured by land and payable half-yearly. And the last Table in p. 252, exhibits the particular differences between these values for two rates of interest.

The collection of Tables is followed by a SUPPLEMENT which formed a part of the Third Edition of this work. In the present Edition several notes have been added to this Supplement; but the addition of most consequence is the POSTSCRIPT on the subject of the population of the kingdom.—

In

In the former Editions, and also in the publication entitled an *Essay on the Population of England from the Revolution, &c.* I gave an account of several facts which seemed to me to shew, that our population has declined. Great pains have been taken to prove this to be a mistake (a) In the *Postscript* just mentioned, I have entered a little farther into this controversy; and it will appear that though I still retain my former opinion, yet I wish to be considered as far from being decided in it, and therefore as

(a) Much has been said also about a mistake into which it is supposed I have fallen in estimating the quantity of gold coin in the kingdom. The truth, in this instance, is briefly this.——The third proclamation for calling in the gold coin brought in near *double* the sum that was expected. In consequence of this, an estimate which I had published in the First Tract on Civil Liberty, proved short of the truth about *three millions and a half*; and it appears now, that, exclusive of two millions purchased by the bank and melted into bars, the gold coin of the kingdom was (in 1773) about *sixteen millions*, instead of *twelve millions and a half*, as I had reckoned it. And this is the account I have given in the *last* edition, p. 74, of the Tract just mentioned, except that being then not informed of the coin purchased by the bank, I have not mentioned it.

open

Open to receive any evidence which can be produced to overthrow it.

Being willing to comprize in this Edition all that I have written on the duration of human life, and the values of life-annuities, I have inserted at the end of the Second Volume Three Essays on these subjects, which have been published in the *Philosophical Transactions of the Royal Society*.

A pretty copious Index closes the whole.

The additions I shall last mention are those which relate to public credit and the national debt; and I have chosen to mention them last, on account of their particular nature and importance.

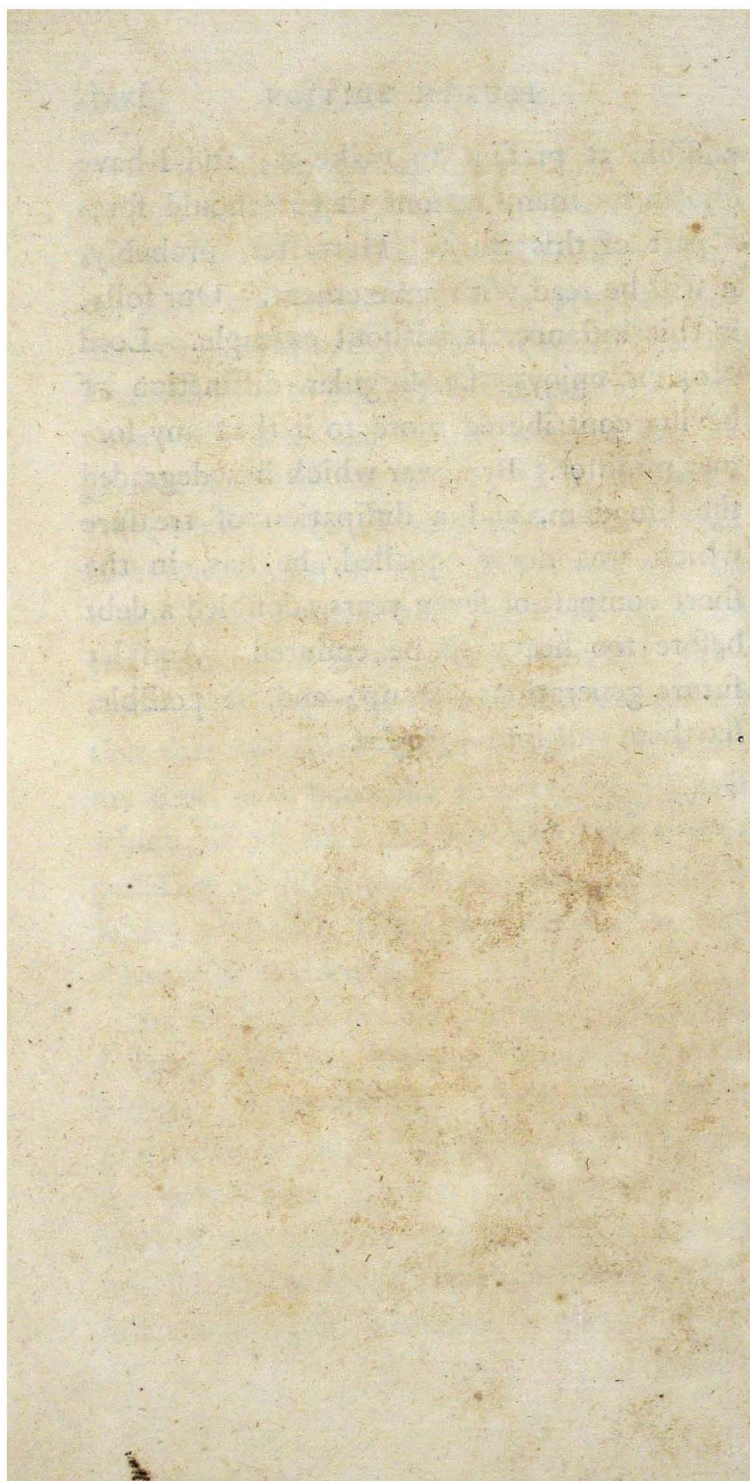
In the Preface to the Third Edition, (see p. lviii) I took notice of a plan announced in 1773 by Lord *North* to the House of Commons, for paying in the ten following years 17 millions of the public debt. It is necessary I should here just mention, that this plan was never afterwards heard of. — The remarks I have made upon it, were followed with a proposal for expediting a plan of redemption in such a manner,

as to cause an appropriation of a million *per ann.* to discharge, in forty years, a *hundred millions* of the public debts then bearing 3 *per cent.* interest. This proposal has not been continued in this Edition, because I intend soon to lay before the public a plan more efficient, and better adapted to the present state of our funds. I must, however, observe that, having now no hope that an efficient plan of redemption will ever be established, I think with regret of the time and attention I have bestowed on this subject. Nothing relieves me, but the reflection that the object about which I have lost my time, has been the removal of an evil which, if no such measures as I have proposed are adopted, must bring on a *catastrophe* which will make this country a warning and a terror to the world.

At the end of the chapter on public credit I have, in this Edition, inserted a brief history of the Sinking Fund; and also a particular account of the increase of the public debts from 1776 to 1783, and of the state of our finances at the time of signing the Preliminaries of peace in *January* last. This account is, I believe, as correct as it is possible

possible at present to make it; and I have chosen for many reasons that it should form a part of this work. Hereafter, probably, it will be read with amazement. Our folly, in this instance, is without example. Lord NORTH enjoys the singular distinction of having contributed more to it than any former minister. By a war which has degraded the kingdom, and a dissipation of treasure which was never equalled, he has, in the short compass of seven years, doubled a debt before too heavy to be endured. And let future generations rise up; and, if possible, let them call him—*Blessed*.

*Newington-Green,
March 29th, 1783.*



C O N T E N T S.

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ERRATUM.

In Page 205, Vol. I. Line 6, for "*Enquiries*," read "*Enemies*."



C H A P. I.

*Questions relating to Schemes for granting
Reversionary Annuities, and the
Value of Assurances on Lives.*

Q U E S T I O N I.

“ A SET of married men enter into a
“ society for securing annuities to
“ their widows. What sum of money,
“ in a single present payment, ought every
“ member to contribute, in order to entitle
“ his widow to an annuity of 30*l.* *per*
“ *ann.* for her life, estimating interest at
“ 4 *per cent* ?”

A N S W E R.

It is evident, that the value of such an expectation is different, according to the different ages of the purchasers, and the proportion of the age of the wife to that of the husband. Let us then suppose, that every person in such a society is of the same age with his wife, and that one with another all the members when they enter may be reck-

oned 40 years of age, as many entering above this age as below it. It has been demonstrated by Mr. *De Moivre* and Mr. *Simpson*, that “ the value of an annuity on the *joint continuance* of any two lives, subtracted from “ the value of an annuity on the life in expectation,” gives the true present value of annuity on what may happen to remain of the latter of the two lives after the other.

In the present case, the value of an annuity to be enjoyed during the *joint continuance* of two lives, each (a) 40, (b) is 9.826, according

(a) See Table II. at the end of this work.

(b) The values of *joint* lives and reversions, as deduced from the *Breslaw* observations, are not given in any part of this work from Mr. *De Moivre's* rules in his treatise on annuities on lives. For these rules are approximations, which give results so far from the truth, as to be, not only useless, but dangerous. In the second essay in the Appendix, a particular account of this will be given, and also of the method in which these values have been calculated.

Mr. *De Moivre* has calculated the values of *single* lives, on the supposition of an *equal decrement of life* thro' all its stages till the age of 86, which he considered as the utmost probable extent of life. Thus, let there be 56 persons alive at 30 years of age. It is supposed that one will die every year till, in 56 years, they will be all dead. The same will happen to 46 at 40, in 46 years. To 36 at 50, in 36 years, and so on for all other ages. The number of years which a given life wants of 86, he calls the *complement* of that life. Fifty-six, therefore, is the complement of 30; 46 of 40, and 36 of 50.

This hypothesis eases very much the labour of calculating the values of lives; and at most ages between 30 and 70 or 75, it is so conformable to Dr. *Halley's* Table of Observations, that I shall not, in these questions, think it necessary

ing to the probabilities of life in the Tables of Observations formed by Dr. *Halley*, from the bills of mortality of *Breslaw* in *Silesia*. The value of a single life 40 years of age, as given by Mr. *De Moivre*, agreeably to the same Table, is 13.20 (*a*); and the former subtracted from the latter, leaves 3.37, or the true number of years purchase, which ought to be paid for any given annuity, to be enjoyed by a

necessary to distinguish between the values of single lives as deduced from this Table, and the same values deduced from the hypothesis.

In order to avoid putting the reader to trouble, I have given this table among other tables in the Appendix. And I have also given two tables which I have formed from the bills of mortality at *Northampton* and *Norwich*. These last answer more nearly to Mr. *De Moivre*'s hypothesis than Dr. *Halley*'s table; and the difference between the values of *single* and *joint* lives by the *hypothesis*, and the same values computed strictly from the tables, is generally less in these tables than in Dr. *Halley*'s, as will be shewn in the last Essay. When, therefore, in the course of this work the values of *single* and *joint* lives are mentioned, as given agreeably to Dr. *Halley*'s table, it must be understood, that they are taken from the two tables in the last leaves of this work, and given in strict agreement only to the *hypothesis*; and that for this reason, they are in reality still more conformable to the *Northampton* and *Norwich* tables.

The inhabitants of *London*, as is well known, not living so long as the rest of mankind, the values of *single* and *joint* lives there, are considerably less than those just mentioned. And, therefore, whenever I have had *London* lives in view, I have given particular notice of it, and taken their values from Mr. *Simpson*, who has calculated them from the *London* tables of observation. See the Tables in the Appendix.

(*a*) See Table I. at the end of this work.

person 40 years of age, *provided* he survives another person of the same age, interest being reckoned at 4 *per cent. per annum*. The annuity, therefore, proposed in this Question being 30 *l.* the present value of it is 30 multiplied by 3.37, or 101 *l.* 2*s.*

By calculating from Mr. *Simpson's* Tables (a), formed from the bills of mortality of *London*, this value comes out 102 *l.*

The difference in the value of the reversion will be inconsiderable, whether the common age is taken a few years more or less than 40. Thus married men of 30 ought not, according to Dr. *Halley's* Table, to give two-fifths of a year's purchase more, for any given reversionary annuity for their wives, than married men of 50, provided they are of the same ages with their wives; and one quarter more, according to Mr. *Simpson's* Table. If the wives are younger (as is generally the case) there will indeed be a considerable difference; for the value now determined would be 120 *l.* according to the *Breslaw* Observations, supposing the two lives to be 40 and 33, or that wives are one with another seven years younger than their husbands; and 118 *l.* 10*s.* according to the *London* Observations.

(a) See Tables X. and XI. Appendix.

QUESTION II.

“ Supposing such a society as that describ-
 “ ed in the preceding Question, to be limit-
 “ ed to a certain number of members, and
 “ constantly kept up to that number, by the
 “ admission of new members as old ones are
 “ lost, in consequence of their own deaths,
 “ and the deaths of their wives: What is the
 “ number of annuitants which, in some time
 “ after its establishment, will come to be
 “ constantly upon it?”

ANSWER.

Since every marriage produces either a widow or widower; and since all marriages taken together would produce as many widows as widowers, were every man and his wife of the same age, and the chance equal which shall die first; it is evident, that the number of widows that have ever existed in the world, would, in this case, be equal to *half* the number of marriages. And what would take place in the world, must also, on the same suppositions, take place in this society.—In other words; every *other* person in such a society leaving a widow, there must arise from it a number of widows equal to half its own number.—But this does not determine what number, all living at one and the same time, the society may expect will

come to be constantly upon it. For if every widow lived no more than a year, the society would never have more annuitants upon it, than came on in a year. And on the contrary, if none ever died, the number of annuitants would go on increasing for ever.—'Tis, therefore, necessary, in order to answer the present enquiry, to determine how long the *duration of survivorship* between persons of equal ages will be, compared with the *duration of marriage*. And the truth is, that, supposing the probabilities of life to decrease uniformly (a), the former is equal to the latter; and consequently, that the number of *survivors*, or (which is the same supposing no second marriages) of *widows* and *widowers* alive together, which will arise from any given set of such marriages constantly kept up, will be equal to the whole number of marriages; or *half* of them (the number of widows in particular) equal to *half*

(a) That is, supposing that out of any given number alive at any age, the same number will die every year 'till all are dead. See the preceding note. That on this hypothesis, the duration of survivorship is equal to the duration of marriage, when the ages are equal; or, in other words, that the *expectation* of two joint lives, the ages being equal, is the same with the *expectation* of survivorship, may be learnt from the 18th and 20th problems of Mr. *De Moivre's* treatise on annuities; and a demonstration of it, together with a particular explanation of this subject, may be found at the beginning of the first Essay to which I must beg the reader to turn, if he is at any loss about the full meaning of what is here said.

the

the number of marriages.—Now, it appears that in most towns the decrease in the probabilities of life, is in fact nearly uniform. According to the *Breslaw*, the *Northampton* and *Norwich* Tables of Observation, almost the same numbers die every year from 20 years of age to 77 (*a*). After this, indeed, fewer die, and the rate of decrease in the probabilities of life is retarded. But this deviation from the hypothesis is inconsiderable; and its effect, in the present case, is to render the duration of survivorship *longer* than it would otherwise be. According to the *London* Table of Observations, the numbers dying every year begin to grow less at 50 years of age; and from hence to extreme old age, there is a constant retardation in the decrease of the probabilities of life (*b*). Upon the whole, therefore, it appears in answer to the present Question, “ that according to the *three* “ *former* Tables of Observations, and suppo- “ sing no widows to marry, the number “ enquired after is *somewhat greater* than “ half the number of the society; but, ac- “ cording to the *London* Table, a *good deal* “ *greater*.”

It must be carefully remembered, that this has been determined on the supposition, that

(*a*) See Tables V. VI. and VII. Appendix.

(*b*) The reason of this difference between the *London* and other Tables, will be given at the end of the fourth Essay.

husbands and their wives are of equal ages, and that in this case it becomes an equal chance which shall die first. In reality neither of these suppositions is just. Husbands in general are older than their wives; and, in equal ages, the mortality of males has been found to be greater than the mortality of females. For both these reasons, it is much more than an equal chance that the husband will die before his wife, or that the woman shall be the survivor of a marriage, and not the man. This will increase considerably the duration of survivorship on the part of the woman, and consequently the number enquired after in this Question. The marriage of widows will also diminish this number, and the operation of these causes will be different in different situations. But it is by no means to be expected (in the situation of the societies I have in view) that the diminution from the latter cause will be considerable enough to overbalance the operation of all the other causes which have been mentioned, and reduce the number under consideration so low, as half the number of marriages (*a*).

SCHOLIUM.

In *London* it appears, that there is a retardation of the decrease in the probabilities

(*a*) It will be observed hereafter, that this observation has been found to be true in fact.

of

of life, which renders the duration of survivorship between two lives of equal ages, considerably longer than their joint continuance. It seems worth observing, that this is the reason why, though the probabilities of life, and therefore the values of single and joint lives, are less in *London* than in other places, yet the values of reversions depending on survivorships, are in some cases greater there. It is proper to add, that this likewise is the reason why, in calculating the values of joint lives and reversions, the present value of an annuity payable yearly to the survivor of two equal lives, may come out equal to, or even greater than, the present value of a like annuity for the joint lives. As an annuity, during such survivorship, will probably not become payable for some years, and therefore the money given for it will have time to accumulate, it is manifest, that the value of it could never be equal to the value of an annuity on the joint lives, the payment of which begins immediately, were not the observation now made true.

QUESTION III.

“ Such a society as that described in the preceding Questions being supposed; in what time will the number of annuitants upon it come to a *maximum*?”

ANSWER.

A N S W E R.

In order to be more clear in answering this Question, I will first suppose the society to comprehend in it from its first establishment, *all* the married persons of *all* ages in any town or country, where the number of people continue constantly the same. In this case, the whole collective body of members will be, at their greatest age, at the time of the establishment of the society; and the number of members, together with the number of widows left every year, will, taking one year with another, admit of no increase or diminution. The number of widows in life together, derived from any given number coming on a society every year, will increase continually, 'till as many die off as are added every year; that is 'till they come to die off as fast as possible. But they cannot die off as fast as possible, 'till the whole collective body of widows are at their greatest age: or, 'till there is among them the greatest number possible of the oldest widows; and, therefore, not 'till there has been time for an accession to the oldest widows, from the youngest part of the widows that come on annually.

Let us, for the sake of greater precision, divide the whole medium of widows that come on every year, into different classes according to their different ages, and suppose some to be left at 56 years of age, some at 46,
some

some at 36, and some at 26. The widows, constantly in life together, derived from the first class, will come to their greatest age, and to a *maximum*, in 30 years, supposing, with Mr. *De Moivre*, 86 to be the utmost extent of life. The same will happen to the second class in 40 years, and to the third in 50 years (*a*). But the whole body, composed of these classes, will not come to a *maximum*, 'till the same happens to the fourth or youngest class; that is, not 'till the end of 60 years. After this, the affairs of the society will become *stationary*, and the number of annuitants upon it of all ages will keep always nearly the same.

• Such is the answer to this Question, supposing a society to begin with its complete number of members, consisting of married persons of all ages, in the same proportion to one another, with the proportions in which they exist in the world.—If it begins with its complete number of members, but at the same time admits none above a particular age: If, for instance, it begins with 200 members all under 50, and afterwards limits itself to this number, and keeps it up by admitting every year, at all ages between 26 and 50, new members as old ones drop off;

(*a*) In note (A), at the end of this treatise, a rule is given by which the numbers alive at the end of any particular number of years may be very easily determined.

in

in this case, the period necessary to bring on the *maximum* of annuitants will be just doubled. For, in the first place, the whole collective body of members will be 60 years in getting to their greatest age, as may easily appear from what has been just said. The annual medium of widows therefore, that will come on the society will increase continually for 60 years; it being evident, that the older any set of married men are, taken one with another, the faster they will leave widows. And after this annual medium is increased to a *maximum*, 60 years more will be necessary to bring to a *maximum* the number in life together, derived from such a *fixed* annual medium constantly coming on.—If such a society is any number of years in gaining its *maximum* of members, the time necessary to bring on the *maximum* of annuitants will be still further prolonged, and will be equal to twice 60 years with that number of years added.—Most of the societies for granting annuities to widows are of this kind; and, therefore, supposing them to gain their complete number of members in ten years, and for ever afterwards to preserve it, the number of annuitants upon them will go on increasing for 130 years—It is proper, however, to be remembered, that the increase will be quicker at first, and afterwards slower; and that, within 20 or 30 years of the end
of

of this term, it will be so slow as scarcely to be sensible, though still real.

All who will bestow due attention on this subject must see these decisions to be just; and a demonstration of them might be given, in a form more strictly mathematical, were it necessary.

QUESTION IV.

“ Suppose the members of such a society
 “ as that described in the preceding Question,
 “ to chuse making *annual payments during the continuance of marriage*, in lieu of
 “ the sum which the reversionary annuity for
 “ their widows is worth in *present money*:
 “ What ought these *annual payments* to be,
 “ estimating interest at 4 per cent?”

ANSWER.

This will be easily determined, by finding what annual payments, during two joint lives of given ages, are equivalent to the value of the reversionary annuity in *present money*.— Suppose, as in Question I. the two joint lives to be each 40, and the reversionary annuity 30 *l. per annum*. An annual payment during the continuance of two such lives is worth, according to Dr. Halley's Table of Observations, 9.82 (a) years purchase. The annual

(a) See Table II. in the last leaves of this work.

payment then ought to be such as being multiplied by 9.82, will produce (a) $\text{£} 101.1$, the present value of the annuity in one payment by Question I. Divide then $\text{£} 101.1$ by 9.82, and the *quotient*, or $\text{£} 10.3$ will be the answer.—This is very nearly the annual payment of all the members at an average, supposing equal numbers to offer themselves for admission of every age between 30 and 50. As much as some give less, others ought to give more, according to their excess of age. Thus, the annual payment of a married person, 30 years of age, ought to be $\text{£} 9.39$; and of a person 50 years of age $\text{£} 11.33$.—If the values of joint lives and of the reversionary annuity are taken agreeably to the *London Table of Observations*, these annual payments will be, for 30 years of age (b), $\text{£} 10.9$ —for 40, $\text{£} 12.5$,—for 50, $\text{£} 14.5$.

If

(a) Particular notice should be taken of the method of notation here used, because it will be carried through the whole of this work.—The figures on the right hand of the full-point signify the decimal parts of $\text{£} 1$. Thus; $\text{£} 101.1$, is 101 and the 10th of $\text{£} 1$, or $\text{£} 101$ and 2s.— $\text{£} 9.39$, is $\text{£} 9$, and 39th hundredths of $\text{£} 1$, or $\text{£} 9 : 7s. : 10d.$ — $\text{£} 11.33$, is $\text{£} 11$, and 33 hundredths of $\text{£} 1$, or $\text{£} 11 : 6s. : 7d.$ —In general; it should be remembered, that 2 shillings allowed for every unit in the first place of decimals, and two-pence half-penny for every unit in the second place of decimals, will give, nearly enough, the value of the decimal part of every such expression.

(b) The value of two joint lives of 30, taken from Table XI. Appendix, is 9.6. This subtracted from the value of the life in expectation, or from 13.1, by Table X. gives 3.5, the

If either the rate or interest is supposed lower, or wives are supposed younger than their husbands, the annual payments will be increased. But there is no occasion for pointing out particularly the difference. It may be easily found in any cases by the directions now given. There is, however, one observation which ought to be here carefully attended to.—This method of calculation supposes, that the first annual payment is not to be made 'till the end of a year. If it is to be made *immediately*, the value of the joint lives will be increased one year's purchase; and, therefore, in order to find in this case the annual payments required, the value in present money found by Quest. I. must be divided by the value of the joint lives increased by unity, and, in this way, the preceding values at 4 *per cent.* according to the *Breslaw* Observations, will be found to be *l.* 8.62—*l.* 9.35.—*l.* 10.07—According to the *London* Observations, *l.* 10,—*l.* 11.2,—*l.* 12.7,

the number of years purchase which an annuity for a life of 30 years of age, *after* another life of the same age, is worth. This remainder, multiplied by 30, gives 105*l.* the value in a single payment, supposing the reversionary annuity to be 30*l.* And 105*l.* divided by 9.6, gives *l.* 10.9, the value of the same annuity in annual payments, during the joint continuance of the two lives, according to the *London* observations.—By similar operations all the other values above given have been found.

QUESTION V.

“ A society may chuse to make abatements in these annual payments, and to require the remainder of the value of the reversionary annuity to be given, in fines or premiums, at the time of admission. It may, for instance, chuse to fix the annual payments of all the members to 5 guineas. What, in this case, would be the premium due at admission, the annuity being supposed 30 *l. per annum*, and interest being at 4 *per cent* ?”

ANSWER.

From the whole present value of the annuity in one payment, subtract the value of 5 guineas *per annum*, during the joint lives; and the remainder will be the answer.

Supposing the joint lives, both 40, the whole present value of the annuity in one payment is, according to the *Breslaw* Observations, *l. 101.1*, by Quest. I.—The value of 5 guineas *per annum*, or of *l. 5.25 per annum*, during two such joint lives, is *l. 5.25*, multiplied by the value of the joint lives; that is, *4.25*, multiplied by *9.82*, or *l. 51.55*; and this subtracted from *l. 101.1*, gives *l. 49.5*, the answer required for two lives at the age of 40. The answer found in the same way for two lives whose common age is 30, is *l. 46.5*,—and for two lives at 50, *50 l.*

Accord-

According to the *London Observations*, these values are, for two lives, at 30, *l.*54.6.—At 40, *l.*59.4.—At 50, *l.*63.3.

If the first of the annual payments is to be made immediately, the true answer will, in every instance, be the values found in the manner now directed, diminished by the annual payment; or, in the present case, 5 guineas less than the values specified.

The values, in *premiums* and *annual payments*, of any other reversionary annuity, will be as much greater or less than these, as the annuity itself is greater or less.

QUESTION VI.

“ A person 35 years of age wants to buy
 “ an annuity, for what may happen to re-
 “ main of his life after 50 years of age.
 “ What is the value of such an annuity in
 “ *ready money*, and also in *annual payments*,
 “ till he attains to the said age; that is, in
 “ annual payments for 15 years, subject in
 “ the mean time to failure, should his life
 “ fail?”

ANSWER.

The present value of such an annuity is the *present* value of a life at 50, in money to be received 15 years hence, and the payment of which depends on the contingency of the continuance of the given life 15 years. That is; it is equal to the value of a life at 50,

multiplied by the present value of 1*l.* to be received at the end of 15 years, and also by the probability that the given life will continue so long.—A life at 50, according to Mr. *De Moivre's* valuation of lives, and reckoning interest at 4 *per cent.* is worth 11.34 years purchase. The present value of 1*l.* to be received at the end of 15 years, is, by Table I, (see the last leaves of Vol. II.) 0.5553. And the probability that a life at 35, will continue 15 years, is, according to the *Breslaw* Observations $\frac{1}{4}\frac{4}{9}\frac{6}{10}$ (a). And these three values, multiplied by one another, give 1.4.44, or the number of years purchase that ought to be given for the annuity.—The annuity then being supposed 50*l.* its value in present money is 222*l.*

(a) The probability that a given life shall continue any number of years, or reach a *given age*, is (as is well known) the fraction, whose *numerator* is the number of the living in any Table of Observations opposite to the *given age* and *denominator*, the number opposite to the present age of the given life.—Thus, in the present instance; 346 is the number in Dr. *Halley's* Table opposite to 50, and 490 the number opposite to 35.— $\frac{3}{4}\frac{4}{9}\frac{6}{10}$. (or the odds of 17 to 7) is, therefore, the probability that a person whose age is 35 shall attain to 50, or live 15 years. In the same manner it will appear, that, according to the same Table, the probability that a person at this age shall live 25 years, is $\frac{2}{3}\frac{4}{9}\frac{6}{10}$; or nearly an even chance.

At *Northampton* and *Norwich* a person at the same age, has an even chance of living 26 years; but in *London*, scarcely 20 years. See Tables V. VI. VII. and VIII. at the beginning of Vol. II. I will add, though foreign to my present purpose, that a person at the same age has in these towns a better chance of living one year, than in *London*, in the proportion of 3 to 2.

In

In order to find this value in *annual payments*, while the given life is attaining 50, it is necessary to find the value of an annuity for 15 years, subject to failure on the extinction of the given life. And the value of such annuity is, evidently, the last value subtracted from the value of the given life; or, in the present instance, $l.4.44$, subtracted from $l.13.97$. (See Table I. at the end of Vol. II.) that is, $l.9.53$.— $222l.$ then, being the present value of an annuity of $50l.$ for the remainder of a life now 35, after attaining 50; and 9.53 being the number of years purchase, which ought to be given for an annual payment to last 15 years, if a life now 35 lasts so long, it follows that the value of the same annuity in annual payments, till this life attains 50, is $222l.$ divided by 9.53 ; or $l.23.3$.

This calculation supposes, that the first of the annual payments is not to be made till the end of a year. If the first payment is made immediately, the value will be, the *single payment* divided by the value of the life for the given term increased by unity; that is, in the present case, $222l.$ divided by 10.53 ; or $l.21.08$.

If the value of the annuity is required in a single payment, over and above any given annual payment; deduct the value of the annual payment from the whole value in a single present payment, and the remainder will

be the answer.—Thus, let 5 guineas, in the present instance, be the given annual payment for the assigned term; and let the enquiry be, how much more in present money the supposed annuity is worth. By what has been just said, 9.53, multiplied by 5 guineas, that is, 50*l.* is the value of the annual payment; and this sum deducted from 222*l.* leaves 172*l.* the answer.

If the annual payment begins immediately, its value is 10.53, multiplied by 5 guineas, and the answer comes out 1.166.75.

In this way may be found the value, in single and annual payments, of any other annuity, payable to an assigned life, after a given term of years, taking any valuation of lives or interest of money. But care must be taken to remember, that it is the title to the annuity that will commence at the end of the given term, and that the first payment is not to be made till a year afterwards; that is, in the case here specified, not till the end of 16 years.

S C H O L I U M.

The value of the *remainder* of two joint lives, after a given term of years, is likewise the value of 1*l.* due at the end of the given term multiplied by the value of two joint lives, each older by the given term than the given lives; and this product, multiplied by the probability, that the given joint lives shall
not

not fail in the given term; or (which is the same) by the product of the two probabilities, that the single lives shall each continue the given term. And the value of an annuity, on any given joint lives for a term of years beginning now, is this last value subtracted from the whole present value of the joint lives. Thus; the value of two joint lives, one 40 years of age, and the other 50, (see Table II. at the end of Vol. II.) is 8.91; which, multiplied by 0.6755, the value of 1*l*. due 10 years hence, and by $\frac{4}{5} \frac{1}{3} \frac{5}{4}$ (the probability that a life at 30 shall continue 10 years) and also by $\frac{3}{4} \frac{4}{5}$, (the probability that a life at 40 shall continue 10 years) give 3.92, the present value of the remainder of two joint lives, aged 30 and 40, after 10 years; and this value, subtracted from 10.43, (the value in Table II. *ibid.* of two joint lives, aged 30 and 40) leaves 6.51, their value for 10 years.

As the value of the longest of two lives is always the value of the *joint* lives, subtracted from the sum of the values of the two *single* lives; their value also for any *given term*, is the value of the *joint* lives for the given term, subtracted from the sum of the values of the *single* lives for the given term.

The truth of these rules may easily appear without particular proof. I have, however, pointed out the method of demonstrating them in a note (a) at the end of this work.

a See note (B) at the end of Vol. II.

By similar operations, may be found the values of 3 or more *joint* lives, or the longest of *three* or more lives, for a given term of years, or of what shall remain of them after a given term of years.

QUESTION VII.

“ The present value is required of an annuity to be enjoyed by one life, for what may happen to remain of it beyond another life, after a given term: that is, provided *both* lives continue, from the present time, to the end of a given term of years ? ”

ANSWER.

Find the value of the annuity for two lives greater, by the given term of years, than the given lives. Discount this value for the given term ; and then, multiply by the probability, that the two given lives shall *both* continue the given term ; and the product will be the answer.

EXAMPLE.

Let the two lives be each 30. The term seven years. The annuity 10%. Interest, 4 *per cent.*—The given lives, increased by 7 years, become each 37. The value of two joint lives each 37, is (by Table II. in the last leaves of the next Volume) 10.25.
The

The value of a single life at 37, is (by Table I. *ibid.*) 13.67. The former, subtracted from the latter, is 3.42, or the value of an annuity for the life of a person 37 years of age, after another of the same age, by Quest. I.—3.42 discounted for 7 years, (that is, multiplied by 0.76, the value of 1% due at the end of seven years by Table I. at the beginning of Vol. II.) is 2.6. —The probability that a single life at 30 shall continue 7 years, is (by the hypothesis explained page 2.) $\frac{49}{50}$ (a). The probability, therefore, that two such

(a) In this case, it is on some accounts best, as well as easiest, to take the probabilities of life from the hypothesis, rather than immediately from the Tables.—Fifty-six persons being supposed alive at 30, one will die every year, according to the hypothesis. At the end of seven years then, the number of the living will be 49, and $\frac{49}{50}$, or the odds of 7 to 1, is, by note, p. 18, the probability, that a life, aged 30, will continue 7 years; and this fraction, multiplied by itself, is the probability, that two lives of this age, shall *both* continue 7 years. In general, it must be remembered, that the probability, that any two or more events shall *all* happen, is the product arising from multiplying by one another, the probabilities of all the events taken separately. The probability, therefore, that any number of persons will *all* live any given time, is rightly found by multiplying into one another the probabilities that each of them will live that time.—It may further be of use to some, that I should observe here, that the difference between unity and the fraction expressing the probability that an event will happen, gives the probability that it will *not* happen. Thus; the probability, that a person 40 years of age will live 11 years, is, by the Breslaw Table (or Table V. beginning of Vol. II.) $\frac{33}{44}$. The probability, therefore, that he will *not* live 11 years, is $\frac{11}{44}$ subtracted from

such lives shall both continue 7 years, is $\frac{2401}{3136}$, or, in decimals 0.765. And 2.6, multiplied by 0.765, is 1.989, the number of years purchase which ought to be given for an annuity, to be enjoyed by a life now 30 years of age, after a life of the same age, provided both continue 7 years. The annuity then being 10*l.* its present value is 1.19.89.

By similar operations, it may be found, that supposing the term one year, and the ages and the rate of interest the same, the present value of the same reversionary annuity is 1.32.4; and that if the term is 15 years, the value is 1.9.7.

For two lives each 40, these values are 1.30.33.—1.17.44.—1.7.3. the term being 1, 7, or 15 years.

For two lives each 50, the same values for the same terms, are 1.28.2,—1.13.86,—1.4.34 (a).

These values, according to the *London Observations* and Mr. *Simpson's* Tables of the values of single and joint lives, are,

from unity or $\frac{110}{144}$. In like manner: The probability that two persons aged 30 shall *both* live 7 years, being 0.765, the probability that they will *not* both live so long, or that *one* or *other* of them will die in 7 years, is 0.765 subtracted from unity, or .235.

If any reader is unwilling to take these assertions for granted, he should consult the beginning of Mr. *De Moivre's*, or Mr. *Simpson's* Treatises on the Doctrine of Chances, where he will find them demonstrated.

(a) See Note (C) at the end of Vol. II.

For

For 2 lives at 30—*l.* 32.05—*l.* 18.62—*l.* 7.66.
 at 40—*l.* 30.7—*l.* 15.6—*l.* 5.45.
 at 50—*l.* 29.36—*l.* 12.33—*l.* 3.24.

QUESTION VIII.

“ Let the scheme of a society for granting
 “ annuities to widows, be, that if a member
 “ lives *a year* after admission, his widow shall
 “ be entitled to a life annuity of 20*l.* If
 “ *seven* years, to 10*l.* more, or 30*l.* in the
 “ whole. If *fifteen* years, to another addi-
 “ tional 10*l.* or 40*l.* in the whole. What
 “ ought to be the annual payments of the
 “ members for the ages of 30, 40, and 50,
 “ supposing them of the same ages with their
 “ wives, and allowing compound interest at
 “ 4 per cent ?”

ANSWER.

According to the *hypothesis*, explained
 p. 2; and, therefore, very nearly, according
 to the Tables of Observation for *Breslaw*,
Norwich and *Northampton*, or Tables V. VI.
 VII. at the beginning of Vol. II.

l. 8.44—*l.* 8.69—*l.* 9.05.

According to the *London* Observations.

l. 9.41—*l.* 10.17—*l.* 10.92.

These

These values are easily deduced from the values in the last Question. For example. The value of 10*l.* *per annum* for life to 40 after 40, provided the joint lives do not fail in *one* year, is, according to the *hypothesis*, 1.30.33. The value of 20*l.* *per annum*, in the same circumstances, is therefore 1.60.66.—In like manner, the value of 10*l.* after *seven* years, is, 1.17.44. And of 10*l.* after 15 years 1.7.3.—These values together make 1.85.4, or the value of the expectation, described in this Question, in a *single present payment*; which, divided by 9.82, the value (by Table II. at the end of Vol. II.) of two joint lives at 40, gives 1.8.69, the value of the same expectation in *annual payments*, during the joint lives.—In the same manner may be found the answer in all cases to any Questions of this kind.

These calculations suppose, that the annual payments do not begin till the end of a year. If they are to begin *immediately*, the true *annual payments* will be, as was before observed, the *single payments*, divided by the value of the joint lives increased by unity; and in the present case they will be, by the *hypothesis*,

$$1.7.75—1.7.9—1.8.07.$$

By the *London Observations*,

$$1.8.52—1.9.06—1.9.51.$$

By

By the method of calculation now explained may be easily found in all cases, supposing the annual payments previously settled, what the reversionary annuities are corresponding to them in value.—Thus, the annuities being the same with those mentioned in this Question, the *mean* annual payments for all ages between 30 and 50, are nearly 8*l.* according to the *highest* probabilities of life; 9*l.* according to the *lowest*; and 8 guineas the *medium* (*a*); interest being at 4 *per cent.* and the first payment to be made immediately.

If the mean annual payments, beginning immediately, are fixed to five guineas, the corresponding life annuities will be nearly (by the *hypothesis*) 12*l.* if the contributor lives a year, and 24*l.* if he lives seven years; or (by the *London Observations*) 12*l.* if he lives a year, and 20*l.* if he lives seven years (*b*).

It

(*a*) The value of this expectation, supposing married men 40 years of age, and their wives 30, is, in a *single* payment, 113*l.* In annual payments beginning immediately, *l.* 9.88, by the *hypothesis*. And 107*l.*—and *l.* 10.93, by the *London Observations*.

(*b*) If the annuities in expectation are 14*l.* provided a member lives a year, and 20*l.* provided he lives seven years, the proper *mean single* payments for all ages, taken one with another, under 50 or 52, is 50 guineas nearly, according to all the Tables of Observation, supposing equality of age between men and their wives. And the addition which ought to be made, on account of excess of age on the man's side, is, taking the nearest and the easiest

It is observable, that the difference in the values of the annuities, arising from difference of ages and the difference in the probabilities of life, is less in this Question than in Question 4th; and that, consequently, the plan proposed in it, is the safest, as well as the most equitable and encouraging, that a society can adopt.

It is necessary to remark here further, that *yearly* payments which begin immediately, are more advantageous than *half-yearly* payments which begin immediately. In an Essay published in the *Philosophical Transactions*, vol. 66. p. 109, and inserted in the 2d volume of this work, I have shewn that in the case of life annuities, *half-yearly* payments which begin at the end of half a year, are nearly a *fifth* of a year's purchase better than *yearly* payments which begin at the end of a year. And it is manifest, that *half-yearly* payments, which begin immediately, are no

easiest round sums, about a guinea and $\frac{1}{2}$ for every year as far as 17 years; or, in the annual payments, (supposed 5 guineas) $\frac{1}{2}$ a guinea *per annum* for five years excess, and $\frac{1}{2}$ a guinea more for every four years excess beyond five years, till the excess comes to be 17 years. And, I believe, that 60 guineas in *single payments*, and six guineas in *annual payments* beginning immediately, may very well be stated as the *lowest common* payments proper to be required, supposing all married men under 52, taken into a society, without enquiring into the difference of age between them and their wives, the annuities being all along supposed to be *life annuities*, and interest reckoned at 4 *per cent*.

more

more than half a year's purchase better than those which begin at the end of half a year. But *yearly* payments, which begin immediately, are a *whole year's* purchase better than the same payments to begin at the end of a year. The difference of value, therefore, between *yearly* and *half yearly* payments, supposing both to begin immediately, is three tenths of a year's purchase in favour of the former.—The whole of this subject may be seen accurately stated in the essay just referred to.

QUESTION IX.

“ The value is required of an annuity to
 “ be enjoyed for what may happen to re-
 “ main of one life after another, provided the
 “ life in expectation continues a given time?”

ANSWER.

Find by Question VI. the present value of the annuity for the remainder of the life in expectation, after the given time, and multiply this value by the probability, that the other life shall fail within that time. Find also by Question VII. the value of the reversion, provided *both* lives continue the given time. Add these values to one another, and the *sum* will be the answer in a single present payment.

EXAMPLE.

EXAMPLE.

An annuity of 10*l.* for the life of a person now 30, is to commence at the end of 11 years (*a*), if another person now 40, should be then dead; or, if this should not happen, at the end of any year beyond 11 years in which the former shall happen to survive the latter: What is the present value of such an annuity, reckoning interest at 4 *per cent.* and taking the probabilities of life as they are in *Dr. Halley's Table*, or Table V. at the beginning of Vol. II.?

The value of 10*l. per annum*, for the remainder of the life of a person now 30, after 11 years, found by *Quest. VI.* is 1.69.43.—The probability that a person 40 years of age shall live 11 years, is, by *Dr. Halley's Table*, $\frac{335}{445}$. The probability, therefore, that he will die in 11 years, is $\frac{335}{445}$ subtracted from unity (*b*), or $\frac{110}{445}$; which multiplied by 1.69.43, gives 1.17.16.—The value of the reversion, provided *both* live 11 years, found by *Quest. VII.* is 17*l.* And this value added to the

(*a*) That is, the title to the annuity is to commence at the end of 11 years, and the first payment to be made a year afterwards, in case the life in expectation should continue so long, and the other fail. But if *both* lives should continue the given term, the first payment is always to be made at the end of the year, in which the former life shall happen to survive the latter. See *Quest. VI.*

(*b*) See the Note, p. 23.

former,

former, makes $l. 34.16.$ the value required in a *single present payment*; which payment divided by $l. 11.43.$ (the value by Table II. at the end of Vol. II. of two joint lives, aged 30 and 40, with unity added) gives $3l. (a)$; or the value required in annual payments during the joint lives, the first payment to be made immediately.—If, every thing else being the same, the assigned term is 15 years, the value required will be $29l.$ in a *single payment*, and $l. 2.55.$ in *annual payments*.

QUESTION X.

“ What money in hand, and also in annual payments during life, ought a person of an assigned age to give for a sum of money, payable at his death to his heirs (*b*)?—
“ In other words, what money in hand, and in annual payments during life, ought a person of a given age to pay for an *assurance* of any given sum on his life?”

ANSWER.

Subtract the value of the life from the *perpetuity*. Multiply the remainder by the

(*a*) See the demonstration of this rule in Note (D) Vol. II.

(*b*) This Question is the same with Problem 16th, in Mr. *De Moivre's* Treatise on Annuities, and Problem 26th, in Mr. *Simpson's* Select Exercises; but the answers there given are right only when applied to reversionary *estates*, and therefore must be materially wrong when applied to reversionary *sums*, as will appear from the *Scho- lium* to this Question, and from Note (E) Vol. II.

product

product of the given sum into the interest of 100 *l.* for a year: and this last product, divided by 100 *l.* increased by its interest for a year, will give the answer in a *single present* payment. And this payment, divided by the value of the life, will give the answer in *annual* payments, during the continuance of the life.

Example. Let the life be 30. The sum 100 *l.* The rate of interest 4 *per cent.* And the valuation of lives, that in Table I. at the end of Vol. II. The perpetuity, therefore (a), is 25. The interest of 100 *l.* for a year is 4 *l.* 100 *l.* increased by its interest for a year, is 104 *l.* And the value of the life 14.68—The value of the life, subtracted from the perpetuity, gives 10.32, which, multiplied by the product of 100 *l.* into 4, or by 400, gives 4128. And this, divided by 104, gives *l.* 39.7, the value of 100 *l.* payable at the death of a person aged 30, in a *single present* payment.—And this payment, divided by 14.68, is *l.* 2.7, the same value in annual payments during the continuance of the life.

These values found in the same way agreeably to the valuation of lives for *London*, in Table X. at the beginning of Vol. II. are *l.* 45.76, and *l.* 3.49.—If the life is 36, and interest 4 *per cent.* these values are 43 *l.* and *l.* 3.1, by Table I. at the end

(a) That is; the value of the *fee-simple* of an estate found by dividing 100 *l.* by the rate of interest.

of

of Vol. II. and *L. 4.1*, by Mr. *Simpson's* valuation of lives for *London* in Table X.—If interest is reckoned at 3 *per cent.* the same values are, by *De Moivre's* valuation of lives, for 30 years of age, *L. 48.14*, and *2.86*—For 36 years of age, *L. 51.43*, and *1.3.28*.

It appears here, that difference of interest makes no considerable difference in the answers to Questions of this kind, except when the values are required in a single payment.

If the first of the annual payments is to be made immediately, the single payment is to be divided by the value of the life, with unity added to it, agreeably to what has been already observed; and the annual payments in this case (interest supposed at 4 *per cent.*) will be by Mr. *De Moivre's* valuation of lives (or Table I. at the end of the next Vol.) for a life at 30, *L. 2.53*—At 36, *L. 2.9*.

If the payments are half-yearly payments beginning immediately, the single payment must be divided by the value of the life increased by seven tenths, (see Quest. VIII.) And the half-yearly payments, for the age of 36, will be half *2.96*, or *1.48*. And half *1.48*, or *74*, is likewise nearly the proper quarterly payments.

Again; if an annual payment, beginning immediately, of *L. 2.9*, ought (reckoning interest at 4 *per cent.*) to purchase *100 l.* payable at the failure of a life now 36; *5 l.* by the rule of proportion, ought to purchase *172 l.* And in like manner, it may be found,

that the same annual contribution, in half-yearly or quarterly payments, beginning immediately, ought to purchase 170 *l.*—These sums, according to the *London Observations*, are 132 *l.* and 130 *l.* nearly.

The reason of mentioning these particulars will be seen in the next chapter.

SCHOLIUM.

If the reversion is not a *sum*, but an annuity for ever, or an *estate in fee-simple*, to be entered upon after a given life, its present value, *in a single payment*, will be “the value
“ of the life subtracted from the perpetuity,
“ and the remainder multiplied by the an-
“ nuity, or the annual rent of the estate.”—And the value, in *annual payments*, will be, as before, the single payment divided by the value of the life.—Universally. It ought to be remembered, that a reversionary *estate*, after any given life or lives, is worth as much more than a corresponding reversionary *sum*, as 100 *l.* increased by its interest for a year, is greater than 100 *l.*—Thus, the present values, in single and annual payments, of 4 *l.* *per annum* for ever, and of 100 *l.* in money after any assigned life, are to one another, (interest being at 4 *per cent.*) as 104 to 100, or 1.04 to 1.—The reason of this difference is, that the calculations suppose, that the reversionary *sum*, and the first yearly rent of the *estate*, or first payment of the annuity,
are

are to be received at the same time, after the extinction of the lives in possession. It is easy to see, that this is a circumstance which must make the latter of most value. But to prevent any doubts about it, I shall explain it more particularly in a note in the Appendix. (a)

QUESTION XI.

“ A person of a given age, having a yearly income which will fail with his life, wants to make provision for another person of a given age, in case the latter should happen to survive. What ought the former to give in a single payment, and also in annual payments during their joint lives for a given sum, payable at his death to the latter?”

It is manifest, that the value of the given sum in this case, must be less than in the case stated in the last Question ; because, here the payment of it is suspended on the contingency, that one life shall survive another ; whereas, in the other case, it is *certainly* to be paid at the failure of a given life.

ANSWER.

Find, by the solution of problem 32d, p. 297, Mr. *Simpson's* Select Exercises, the

(a) Vid. Appendix, note (E).

value of an estate, corresponding to the given sum, and depending on the given survivorship. Divide this value by 1 $\frac{1}{2}$. increased by its interest for a year, and the quotient will be the value of the given sum in a single present payment. And the single payment, divided by the value of the given joint lives, will be the answer in annual payments during the joint lives.

The solution I have referred to is as follows:

“ Find the value of an annuity on two
 “ equal joint lives, whereof the common age
 “ is equal to the age of the older of the two
 “ proposed lives; which value subtract from
 “ the perpetuity, and take half the remain-
 “ der. Then say, as the *expectation* of the
 “ duration of the younger of the two lives is
 “ to that of the elder, so is the said half re-
 “ mainder to a 4th proportional, which will
 “ be the number of years purchase to be gi-
 “ ven for the estate when the life in expec-
 “ tation is the oldest of the two. But if this
 “ life is the youngest, then add the number
 “ of years purchase just found to the value
 “ of the joint lives, and let the sum be sub-
 “ tracted from the perpetuity, and you will
 “ also have the answer in this case.” (a)

Let

(a) Mr. *Simpson* has given the following examples of this solution, adapted to *London* lives:— Example I.
 “ Suppose the age of the expectant to be 40, of the *pos-*
 “ *seffor* 30. The rate of interest 4 per cent. and the
 “ given

Let the life in expectation be 30; and the other life 40: The sum, 100*l*. Interest, 4 *per cent*. The valuation of lives, Mr. De Moivre's, or that in Table I. at the end of the next Vol.

The *expectation* of the first life, is 28; of the second life 23, by Mr. De Moivre's by-

“ given legacy 5000*l*. or 200*l*. *per annum*. Then the
“ value of two equal joint lives of 40, being 8.1, (see
“ Table XI.) and the perpetuity 25, the remainder or
“ difference will be here 16.9; whereof the half is 8.45.
“ Therefore, it will be as 23.6 to 19.6, so 8.45 to 7.02
“ years purchase, or 1.1404, the required value.”

Example II. “ Let the age of the *expectant* be 30, of
“ the *possessor* 40, and the rest as in the preceding exam-
“ ple. Here the value of the joint lives 30 and 40, will
“ be 8.8; which added to 7.02, (found above) the sum
“ will be 15.82; whence the answer, in this case, is
“ 9.18 years purchase, or 1836.”

I have shewn, that the values of reversionary *estates*, and reversionary *sums*, are not the same as is here supposed.—The rule gives the true value when applied to the former; but, when applied to the latter, the values given by it must be divided by 1*l*. increased by its interest for a year, as above directed.—The same observation is to be applied to Mr. Simpson's next Problem, or the 33d.

In these Examples 23.6 and 19.6, are the expectations, in Table IX, of 30 and 40, according to the *London Tables of Observation*; and the method of finding them for any age, and from any Tables of Observation, is explained at the beginning of the first Essay.

In Mr. De Moivre's *hypothesis*, the expectation of a life is always *half* the complement. See note, p. 2.—Sometimes the *complement* of a life is mentioned without any view to Mr. De Moivre's hypothesis, and it then means double the *expectation* of the life, whatever that may be, according to any Table of Observations.

pothefis. The value of the joint lives is 10.43. by Table II. at the end of the next volume. The value of two joint lives, both 40, is 9.82, by the same Table. The estate corresponding to 100*l.* is 4*l. per ann.* and the present value of such an estate to be entered upon by a person 30 years of age, provided he survives a person 40 years of age, is, by the rule just quoted, 1.33.32. And this value, divided by 1*l.* increased by its interest for a year, or by 1.04, is 1.32.03. the value in a *single present payment* of the sum of 100*l.* dependent on the given survivorship. And this single payment divided by 10.43, is 1.3.07, the required value in *annual payments*, during the joint lives, if the first payment is not to be made till the end of a year. But if the first payment is to be made immediately, the required value in *annual payments* will be 1.32.03, divided by 11.43, or 1.2.8.—These values, according to the *London Observations*, or Mr. *Simpson's* Tables founded upon them, are 1.35.30, in a *single payment*, and 1.3.6, in *annual payments*, beginning immediately.

Mr. *Simpson*, in the Problems following that here quoted, has given solutions of most other Questions, concerning the values of reversions depending on survivorships, where the whole duration of two or three lives is concerned. And I am acquainted with no other solutions of these Questions, which are applicable

applicable to all Tables of Observations, and which at the same time (proper regard being paid to the correction explained in the last Question) may be considered as sufficiently correct. (a)

QUESTION XII.

“ Suppose an institution for the relief of
 “ widows to extend its assistance likewise
 “ to the families of married men, provided
 “ they leave no widows. Suppose, for in-
 “ stance, that in this case children are to be
 “ entitled to 100*l*. What is such an expec-
 “ tation worth, in present payment, interest
 “ being at 4 *per cent*. ?”

ANSWER.

If 40 is the mean age at which members are admitted on such an institution, and 32 the mean age of their wives, the answer (supposing no subsequent marriages) is, by the 33d Problem in Mr. *Simpson*'s Select Exercises, p. 298, and the correction already explained, *l*. 13. 80, (b) taking the expectations and values agreeably to Mr. *De Moivre*'s hypothesis.

But

(a) See the third Essay.

(b) This Problem and its solution are given by Mr. *Simpson* in the following words: “ A. and his heirs are
 “ entitled to an estate of a given value, upon the decease
 “ of

But there is a reduction necessary, on account of the chance there is, that a widower may marry again. Suppose, therefore, one half of all widowers to marry a second and third time, and that two fifths of such widowers survive these subsequent marriages. In this case, $\frac{1}{2}$ added to $\frac{2}{5}$ of $\frac{1}{2}$, or $\frac{7}{10}$ of all who become widowers, will die without leaving widows, and therefore $\frac{7}{10}$ of $l. 13.8$, or $l. 9.66$, will be the answer. If only *one fourth* of

“ of B, provided B survives A ; to find the value of
 “ their expectation in *present* money.”—Solution. “ Find
 “ the value of an annuity on the longest of two equal
 “ lives, whereof the common age is that of the older of
 “ the lives A and B ; which value subtract from the
 “ perpetuity, and take half the remainder ; then it will
 “ be, as the expectation of duration of the younger of
 “ the lives A and B, is to that of the older, so is the
 “ said half remainder to the number of years purchase
 “ required, when the life of B is the *older of the two*. But
 “ if B be the *younger* ; then to the number thus found,
 “ add the value of an annuity on the longest of the lives
 “ A and B, and subtract the sum from the perpetuity,
 “ for the answer in this case.”

If the estate is *4l. per annum*, the age of B 40, and of A 32, *interest 4 per cent.* the answer by this rule comes out $l. 14.35$, which divided (as in the preceding Question) by 104, gives $l. 13.80$, the value, as above, of 100*l.* in money. If B is 30 and A 40, the same value is 20*l.*

N. B. The value of the longest of two lives is always the *difference* between the value of the *joint* lives, and the *sum* of the values of the two given *single* lives. Thus ; the value of a life at 40, is, by Table I, at the end of next vol. 13.2. The *sum* of the values of two such lives is 26.4. The value of two joint lives, whose common age is 40, is, by Table II, 9.82 ; and the difference is 16.58, or the value of the *longest* of two lives at 40.

all

all who become widowers marry again, and two fifths of these survive, the answer will be £. 11. 73.

This calculation supposes all marriages to leave children who survive their parents. If this is considered as uncertain, the values now determined must be diminished in the proportion of this uncertainty.—Thus; if one marriage in seven fails of leaving children (*a*) that survive their parents; these values will be reduced a *seventh* part, or to £. 8. 28, if *half*, and £. 10. 05, if a *quarter* of all widowers marry.

In this way may any other questions of the same kind be answered on any suppositions that may be thought most reasonable.

QUESTION XIII.

“ Let an establishment be supposed which
 “ takes in at once all the marriages in a
 “ country, or all marriages among persons
 “ of a particular profession within a given
 “ district, and subjects them for perpetuity
 “ to a certain equal and common tax, or annual
 “ payment, in order to provide life annuities
 “ for such widows as shall result from
 “ these marriages. What ought the tax to
 “ be, supposing the annuity 20*l.* and calculating
 “ at 4 *per cent.* from Mr. De Moivre’s
 “ valuation of lives ?”

(*a*) This for many years has been nearly the fact among the ministers and professors in *Scotland*.

ANSWER.

A N S W E R.

Since, at the commencement of such an establishment, all the oldest, as well as the youngest marriages, are to be entitled equally to the proposed benefit, a much greater number of annuitants will come immediately upon it, than would come upon any similar establishment, which limited itself in the admission of members to persons not exceeding a given age. This will check that accumulation of money, which should take place at first, in order to produce an income equal to the disbursements at the time when the number of annuitants comes to a *maximum*; and, therefore, will be a particular burden upon the establishment in its infancy. For this, some compensation must be provided; and the equitable method of providing it, is, by levying *fin*es at the beginning of the establishment, on every member *exceeding* a given age, proportioned to the number of years which he has lived beyond that age. But in the present question, it is supposed, that such *fin*es cannot be conveniently levied, or that every payment must be equal and common, whatever disparity there may be in the value of the expectations of different members. The *fin*es, therefore, must be reduced to one common one, answering as nearly as possible to the disadvantage I have mentioned, and payable

payable by every member at the time when the establishment begins. After this, the establishment will be the same with one that takes upon it all at the time they marry; and the tax or annual payment of every member adequate to its support, will be the annual payment during marriage, due from persons who marry at the mean age at which, upon an average, all marriages may be considered as commencing.—There are then two points to be here determined. The *finer* necessary to be paid at first, according to the account I have just given; and the *constant annual payment*, necessary to be made by every member, as an equivalent for the expectation provided by the establishment.—The *finer* to be paid at first are, for every particular member, the same with the difference between the value of the expectation to him at his present age, and what would have been its value to him had the scheme begun at the time he married? Or, they are, for the whole body of members, the difference between the value of the common expectation, to persons at the mean age of all married persons taken together as they exist in the world, and to persons at that age, which is to be deemed their mean age when they marry.

Thus; let 33 for the man, and 25 for the woman, be the mean ages of all that marry annually. Let also 48 be the mean age of all the married men in the world, and 40 of married

married women. (a)—Now, he that will calculate for these ages, in the manner directed in Quest. IV. will find, that the value in *annual payments* during marriage, and beginning immediately, of the expectation of an annuity of 20*l.* *per annum* by a person 25 years of age, after a life whose age is 33, is 76.64.—And that 78.04, is the value of the same expectation, the ages being 48 and 40.

The former, therefore, is the payment for perpetuity from every member of the establishment; and the value of the *difference* between it and the latter, or of 71.4. *per ann.* payable during two joint lives, whose ages are 40 and 48, that is, 714.2, is the *fine* necessary to be levied on every married member at the beginning of the establishment. (b)

It would be easy to extend the benefit of such an establishment, so far as to provide 100*l.* for the children of members, provided

(a) I must beg leave to refer to note (F) in the Appendix, for an explanation of what I mean by the mean ages of married men and women, and also for a confirmation of the answer I have given to this Question.

(b) An annuity for ever, the first payment of which is to be made immediately, is worth 26 years purchase, interest being at 4 *per cent.* 714.2 therefore is equivalent in value to 0.55*l.* or 11*s.* *per annum*, for ever. Add this to 76.64, and it will appear, that 77.19 *per annum*, beginning immediately, is the answer to this Question, supposing the value of the *fine* to be provided for in the perpetual annual payments.

they

they leave no widows ; and the necessary addition on this account to the perpetual annual payments, can scarcely, in the circumstances this question supposes, be much more than about 15*s.* payable during life, and excluding from all benefit such as happen to be widowers at the commencement of the establishment, and do not afterwards marry.

If, in such an establishment, all persons of a particular denomination, whether married men, widowers, or batchelors, are subjected alike to the taxes and fines ; they ought to be as much *less*, as the whole number of persons subjected to them, is *greater* than the number of marriages constantly existing.

In carrying these schemes into execution, there cannot be a more easy, or equitable way of raising the necessary fines, than by providing, that none shall be entitled to any expectation for a few of the first years. Thus ; an establishment, entitling widows to 20*l.* *per annum* for life, and consisting of 667 married members, and 344 unmarried, always kept up at an average, ought to begin with a capital of $l. 14.2$ multiplied by 667, or 9471*l.* besides one payment in hand of the constant annual payments. That is, (the proper annual payment of every member being in this case $\frac{6.67}{1000}$, multiplied by $l. 6.64$. or $l. 4.38$) it ought to begin with a capital
of

of 13.899*l.* over and above the payment of 1.4.38, at the *end* of every year for ever afterwards. (a)—The exclusion of all the first members from any benefit, unless they survive the first *two* years, or live to make *three* payments, would raise this capital nearly. And such an exclusion for *three* or *four* years, would be an advantage so considerable, that it would probably give security and stability to the scheme for all subsequent time.

In these observations, I have had in view some schemes which have been established in this kingdom; but more particularly, one established by act of parliament among the clergy in *Scotland*; of which I shall have occasion in the next chapter to take further notice.

I have chosen to calculate here only from Dr. *Halley's* Table, or Mr. *De Moivre's hypothesis* grounded upon it, because the *London* Table is, by no means, adapted to the cases in view.

It should be further remembered, that when the mean ages, at which marriages commence, are supposed to be 33 and 25,

(a) Or, supposing the value of 9471*l.* (the fine) provided for in the annual payments, it ought to receive every year, at the *beginning* of the year, a contribution from each member of 1.4.74.

all

all second and third marriages are included; and that it is to be expected, that almost all these marriages will begin after these ages; and likewise, that a considerable proportion of the first marriages will begin a much longer time *after* these mean ages, than any of the other first marriages will begin before them.—Probably, therefore, these mean ages should not be taken younger. One or two years, however, more or less, in every supposition I have made, will make no difference of any consequence.

QUESTION XIV.

“ A person of a given age has an estate depending on the continuance of his life for a given term. What ought he to give for having it *assured* to him for that term ?”

ANSWER.

From the value of an annuity certain for the given term, found by Table II. subtract the value of the life for the given term, found by Quest. VI. and *reserve* the remainder.—Multiply the value of 1*l.* due at the end of the given term, (found by Table I.) by the *perpetuity*, and also by the *probability*, that the given life shall fail in the given term. The *product* added to the *reserved* remainder, and the *sum* multiplied by the given annuity, will

will be the required value of the assurance in one present payment. (a)

EXAMPLE.

An estate or annuity of 10*l.* *for ever*, will be lost to the heirs of a person now 34, should his life fail in 11 years. What ought he to give for the *assurance* of it for this term?—That is; What is the present value of such an annuity to be entered upon at the failure of such a life, should that happen in 11 years.

The value of the life of a person whose age is 34 for 11 years, is, by Quest. VI. (reckoning interest at 4 *per cent.* and taking Mr. *De Moivre's* valuation of lives) 7.76; which, subtracted from 8.760, (the value of an annuity certain for 11 years) leaves 1*l.* the remainder to be reserved.

The value of 1*l.* to be received at the end of 11 years, is, 0.6496, by Table I. Vol. II. The probability that the life of a person, aged 34, shall fail in 11 years, is, by Dr. *Halley's* Table, $\frac{103}{199}$; and the perpetuity is 25. These numbers multiplied by one another, and 1 added to the product, make 4.34, which, multiplied by 10, (the given annuity) gives 43.4, the required value in a single present payment.

(a) See the demonstration in note (G) Appendix.

$l.43.4$, divided by 1.04 , gives $l.41.7$, the true value (by Scholium to Quest. X.) of the assurance of an *equivalent sum*, or of $250 l.$ for 11 years on the given life.

Again. 41.7 , divided by 8.76 , (the value of the given life for the given time with unity added to it) gives 4.76 , the same value in annual payments beginning immediately, for 11 years, (a) subject to failure should the life fail.

SCHOLIUM.

In a similar way may the price of assurances on any two joint lives, or the *longest* of two lives for any given terms, be calculated; the rule being as follows:

“ From the value of an annuity certain
 “ for the *given term*, subtract the value of
 “ the joint lives, or the longest of the two
 “ lives for the *given term*, found by Scholium to Quest. VI. and *reserve* the remainder.—Multiply the value of $1 l.$ to be received at the end of the given term by the perpetuity, and also by the probability that the *joint lives*, or the *longest of the two lives*, shall fail within the given term. This product added to the reserved *remainder*, and the *sum* multiplied by the annuity to be assured, will be the value of the assurance in a single present payment.”

(a) The last payment to be made at the end of the 11th year; or 12 payments in all.

EXAMPLE.

“ What is the value of 10 *l. per annum*, to
 “ be entered upon, should *either* of two
 “ persons, one 40 and the other 30 years of
 “ age, die in ten years, reckoning interest
 “ at 4 *per cent.* and calculating from Dr.
 “ *Halley's Table.*”

The value of two joint lives at these ages, for 10 years, (found by *Scholium* to *Quest. VI.*) is 6.51; which, subtracted from 8.111, (the value of an annuity certain for 10 years, at 4 *per cent.*) leaves 1.60, the remainder to be reserved.

The value of 1 *l.* to be received at the end of 10 years, is, .6755, by Table I. in the first collection of tables, vol. ii.

The probability, that the lives of one or other of two persons, aged 30 and 40, shall fail in 10 years, is, $\frac{185}{531}$ by Table V. (a) And the perpetuity 25. These numbers, multiplied by one another, and 1.60 added to the product, make 7.48, which, multiplied by 10, (the given annuity) gives *l.* 74.8, the answer in a single present payment.

£. 74.8, divided by 1.04, gives *l.* 71.92, the value of the assurance of an *equivalent sum*;

(a) The probability taken from the Table, that a person aged 30, shall live 10 years, is, $\frac{445}{531}$. That a person, aged 40, shall live 10 years, is, $\frac{346}{531}$. That they shall both live 10 years, is, $\frac{146}{531}$, multiplied by $\frac{445}{531}$, or $\frac{346}{531}$: That they shall *not both* live 10 years, or that *one or other* of them shall die in this time, is $\frac{385}{531}$, subtracted from unity, or $\frac{185}{531}$. See note p. 23.

or of 250*l.*—1.71.92, divided by 7.51, (the value of the two joint lives for 10 years with unity added) gives 9.57, the value of the same sum in annual payments beginning immediately, for 10 years, subject to failure should the joint lives fail.

EXAMPLE II.

“ What is the value of 10*l.* *per ann.* to be entered upon, should two persons, one 30, and the other 40, *both* die; that is, should the *longest* of the two lives fail in 10 years, reckoning interest at 4 *per cent.* and calculating from Dr. *Halley's* Table?”

The value of the *longest* of the two lives for 10 years, (that is, the value of the joint lives for 10 years, subtracted from the sum of the (a) values of the single lives for 10 years) is, 7.91; which, subtracted from 8.111, the value of an annuity certain for 10 years, leaves .20 the remainder to be reserved.—The value of 1*l.* to be received at the end of 10 years, is, 6755. The probability that the lives of two persons, aged 30 and 40, shall fail in 10 years, is, by Table V. $\frac{86}{531}$, multiplied by $\frac{99}{345}$ or $\frac{8514}{236295}$; and the perpetuity 25. These numbers, multiplied by one another, and .20 added to the product, make .740, which, multiplied by 10, (the

(a) See Scholium to Quest. VI.

given annuity) gives 7.4, the answer in a single payment.

7.4, divided by 1.04, gives 7.11, the value of the assurance of 250/.

R E M A R K I.

The values of single lives for given terms, when these terms are less than ten years, must, in answering these Questions, and also in answering the following Questions, be found true to at least 2 or 3 places of decimals. When they cannot be found to this exactness by any Tables, they must be calculated in the following manner:

“ Multiply the probability, taken out of
 “ the Table of Observations, that the life
 “ shall exist 1, 2, 3, &c. years, by the value
 “ of 1/ due at the end of 1, 2, 3, &c. years;
 “ and the sum of the products will be the
 “ value of the life for 1, 2, 3, &c. years.”

For Example. The probability, that a person whose age is 34, shall live a year, is, by Dr. *Halley's* Table, $\frac{420}{499}$. The probability at the same age, of living 2 years, is, $\frac{481}{499}$; 3 years, $\frac{472}{499} - \frac{420}{499}$ multiplied by .9615, (the value, by Table I. of 1/ due at the end of a year, interest being at 4 per cent.) is, .942; or the value of the life for one year— $\frac{481}{499}$, multiplied by .245, (the value of 1/ due at the end of 2 years) is, .891. And this added
 to

to the former product, gives 1.833; or the value of the life for 2 years.— $\frac{472}{99}$ multiplied by .8890, (the value of 1% due at the end of 3 years) is, .841; and this product, added to 1.833, makes 2.674, or the value of the given life for 3 years.

When the term exceeds 10 years, the rule in Quest. VI. will give these values with sufficient exactness; and it would do the same in all cases, were the values of lives given true to 3 or 4 places of decimals (*a*), and in strict agreement to the Tables of Observation used.

The remark now made is to be extended to the values of *joint* lives for given terms. For these values, like those of *single* lives cannot be found in solving these Questions with sufficient accuracy, (when the terms are small, and the values of lives are given only to one or two places of decimals) by any method, except the tedious one, of multiplying the probability that the 2 lives shall *both* continue, 1, 2, 3, &c. years, by the value of 1% due at the end of 1, 2, 3, &c. years, and taking the sum of the products in the manner just described.

(*a*) Such tables are given in the present edition of this treatise; and therefore this remark is now less necessary than it was. See the tables of the values of single and joint lives in the next volume, deduced from the register of mortality at Northampton.

REMARK II.

If the annuity is to be entered upon, in case of the failure within a given time of any life or lives, *at the end of that time*; and not *at the end of the year in which the failure may happen*; its present value will be the product arising from the continual multiplication by one another of the perpetuity increased by unity; the value of 1*l.* due at the end of the given time; the annuity, and the probability that the life, or lives, shall fail within the given time. And care should be taken not to confound these two sorts of Questions with one another.—Thus, the value in one payment of 10*l. per ann.* to be entered upon eleven years hence, in case a person aged 34 should not live so long, is 26, (the perpetuity increased by unity, interest being at 4 *per cent.*) multiplied by .6496, and by 10*l.* and also by $\frac{1}{4} \frac{0}{9} \frac{3}{9}$; or 34.8.—This value, divided by 1.04, is 33.5, the value of an equivalent sum, or of 250*l.* to be obtained on the same conditions.

The value of the *assurance* of any annuity on the whole continuance of any single life is, by Quest. X. the *excess* of the perpetuity above the value of the life, multiplied by the annuity. And in like manner; the value of the *assurance* of any annuity on the whole continuance of any two *joint* lives, or the *longest* of two lives, is the excess of the *perpetuity*

petuity above the value of the joint lives, or of the longest of two lives, multiplied by the annuity. This is very obvious; but no general method has been yet explained of finding the values of *assurances* on lives and survivorships for terms of years less than the whole continuance of the lives. For this reason, I have been here more explicit than I should otherwise have been; and, as such assurances are now much practised, and may be very useful if their values are rightly determined, I have thought proper to add the two following Questions, which, when joined to Question XI. and Mr. *Simpson's* 33d *Problem* given in the note, p. 39, will, I believe, exhaust this subject as far as two lives can be concerned.

QUESTION XV.

“ B, expectant, will lose a given sum,
 “ should he survive A, *within a given time*.
 “ What ought he to pay for the *assurance* of
 “ it?”—In other words: “ What ought he
 “ to pay for a given sum to be received at
 “ the death of A, should he happen to sur-
 “ vive him within a given time?”

ANSWER.

Divide the *sum* of the decrements of life in the Table of Observations from the age of A, for the given time, by the given time; and, by the *quotient*, divide the number of

the living in the Table at the age of A; and again, by this *second* quotient (*a*), divide the given sum, reserving the *third* quotient.

Find the value of an annuity on the life of B, for the given time. To this value add the *quotient*, that will arise from dividing the value of an annuity certain, for the given time, by twice the *complement* of the life of B; and the *sum*, multiplied by the *reserved quotient*, will be the required value in a single present (*b*) payment.

EXAMPLE.

Let the Table of Observations be Mr. *Simpson's* for *London* (see the Tables in the next volume). Let the rate of interest be 3 *per cent*. A, seven years of age. B, 30. The given time 14 years. The given sum 100*l*. —The sum of the *decrements*, for 14 years from the age of seven, is 73, which, divided by 14, gives 5.2. The number of the living at seven is 430, which, divided by 5.2, and 100*l*. divided by the quotient, gives *l*.1.21, the *quotient* to be *reserved*.

(*a*) When the age of A is under 60, and the term so large as to exceed the difference between it and 70, it will be best when the *London* Table is used, to divide the given sum, not by the second quotient here mentioned, but by the *complement*, or double the expectation of A.

(*b*) See the demonstration of this rule, and also of the rule that will be given for solving the next Question, in the Appendix, note (H).

The

The value of an annuity for 14 years on the life of B, is, by Quest. VI, 9.5.—The value of an annuity certain for 14 years, is, (by Table II. at the beginning of the next volume) 11.296, which divided by 94.4, (twice the *complement* of the life of B, by Table IX. (a) gives .12, which, added to 9.5, gives 9.62.; and this again multiplied by 1.21, the *reserved quotient*, gives 11.64, the *present* value in *one* payment of 100*l.* payable at the death of A aged 37, to B aged 30, should A die and leave B the survivor within 14 years.

The present value for 14 years of two joint lives, one 7 and the other 30 years of age, may be found, by the help of Table XI, and the rule in the *Scholium* to Quest. VI. to be nearly 9 years purchase; and, *l.* 11.64 divided by this value with unity added, or by 10, gives 1.164, the foregoing value in *annual payments* during the joint lives for 14 years, the first payment to be made immediately, and the *last* payment at the end of 14 years, should the joint lives not fail.

SCHOLIUM.

It deserves particularly to be remembered, that in this method likewise may be calculated, what sums ought to be paid on any survivorship, within a given time, of one life

(a) This table gives the *expectations* only, but it should be remembered, that twice the *expectation* is always the *complement* of a life. See note, p. 37.

beyond

beyond another, in consideration of any given sum now advanced.—The following Example of this is a case which has offered itself in practice.

“ A person, aged 30, has in expectation
 “ an estate which is to come to him, pro-
 “ vided he survives a *minor*, aged 7, before
 “ he is out of his minority; that is, pro-
 “ vided he should be himself living at the
 “ time of the minor’s death, should that hap-
 “ pen before he is 21.—In these circum-
 “ stances, he wants to borrow 1000*l.* on his
 “ *expectation*. What *reversion* out of the
 “ estate depending on such a survivorship, is
 “ a proper equivalent for this sum now ad-
 “ vanced, interest being reckoned at 3 *per*
 “ *cent.* and the probabilities of life being
 “ supposed the same with those in Mr. *Simp-*
 “ *son’s* Table of *London* Observations?”

A N S W E R.

It appears from what has been just determined, that for *l.* 11.64 now advanced, the proper equivalent in such circumstances, is, 100*l.* to be paid, in case the survivorship should take place; or, by the *correction* in page 34, as much of the estate as 100*l.* will buy at 3 *per cent.* supposing the first rent to be received immediately; (that is, supposing the estate worth 34.33 years purchase) or *l.* 2.912 *per annum.*—By the rule of proportion, therefore, for 1000*l.* the proper equi-

equivalent will be 8591 *l.* in money, or 250 *l.* *per annum* out of the estate.

QUESTION XVI.

“ 100 *l.* will be lost to B’s heirs, should he
 “ happen to die after A, *within a given time*.
 “ What is the price of the *assurance* of it?—
 “ That is: What is the present value of
 “ 100 *l.* payable at the death of B, provided
 “ his death should happen *after* A’s death,
 “ *within a given time* ?”

ANSWER.

Divide the sum of the decrements of life in the Table of Observations from the age of B, for the given time, by the given time; and by the *quotient* divide the number of the living at the age of B; and again, by this *second quotient* (*a*), divide the given sum, reserving the *third quotient*.

Find the value of an annuity on the life A for a number of years, less by *one* year than the given time, which subtract from the value of an annuity certain for the same number of years. Multiply the *remainder* by the *reserved quotient*, and divide the *product* by the amount of 1 *l.* for one year, and let this be a *second reserved quotient*.

(*a*) Or rather, if the *London Table* is used, by the *complement* of the life of B, when his age is under 60, and the term exceeds the *difference* between it and 70.

Again,

Again. Multiply into one another the *first* reserved quotient, and the value of an annuity certain for the given time; and divide the product by twice the *complement* of A's life. This *last* quotient, added to the *second* reserved quotient, will be the *answer* in a present single payment.

E X A M P L E.

Let the age of B be 40. Of A 30. The sum 100*l*. Rate of interest 4 *per cent*. The given time 20 years. The Table of Observations, Mr. *Simpson's*, or Table VIII. in the collection of tables in the next volume.—The sum of the decrements of life, in this Table from the age of 40 for 20 years, is 127, which, divided by 20, (the given time) gives 6.38.—The number of the living at 40 is 229, which, divided by 6.38, gives 35.8; and 100*l*. (the given sum) divided by 35.8, gives 2.79, the *first* quotient to be reserved.

The value of an annuity for 19 years on a life at 30 years of age, is 10.3; which, subtracted from 13.134, (the value of an annuity *certain* for 19 years, by Table II.) and the remainder multiplied by 2.79, gives 7.89. This product divided by 1.04, (the amount of 1*l*. in one year) gives 7.60; the *second* reserved quotient.

2.79 multiplied by 13.59, (the value of an annuity certain for 20 years) gives 37.916; and this *product* divided by 94.4, (twice the *com-*

complement of A's life by Table IX.) gives .401, which, added to 7.60, gives 8*l.* the *Answer*; or, the value of 100*l.* payable at the death of B, on the contingency of his surviving A aged 30, and *both* dying in 20 years.

It is plain, that this is likewise the sum that ought to be lent to B now, on the expectation of 100*l.* at his death, provided it should happen after A's death in 20 years.

This rule gives the just solution in all cases, except when B, the expectant, is the *youngest* of the two lives, and at the same time the term of years *greater* than the complement of A's life. In this particular case the following rule must be used.

Find, by the preceding rule, the value of the assurance of the given sum for a term of years, equal to the complement of A's life, and let this value be *reserved*. Multiply by one another the given *sum*; the *value* of 1*l.* to be received at the end of a number of years equal to the complement of A's life; and the value of an *annuity certain* for as many years as the given term exceeds this complement. And the *product*, divided by the complement of B's life, and the *quotient* added to the *value reserved*, will be the true value sought.

EXAMPLE.

Let the age of B be 30; of A 40. The term 47 years; and every thing else as in the

the last Example. The complement of A's life, is, by Table IX. 39.2. The value of 100*l.* to be received at the death of B, if he survives A within 39 years, may be found by the preceding rule to be *l.*16.15; the value to be reserved—The value of 1*l.* to be received at the end of 39 years is, by Table II. .2166. The value of an annuity *certain* for 8 years, (the excess of the given term above the complement of the life of B by Table IX.) is, 6.733.

And these two values multiplied by one another, and by 100*l.* give 145.83; which, divided by 47.2, (the complement of the life of B) and 16.15, added to the quotient, make *l.*19.23, the value sought.

R E M A R K.

As after finding the present value of an estate, or annuity, it is necessary to *divide* that value by the amount of 1*l.* in one year; in order to find the present value of a *sum* equivalent to the *annuity*; so, after finding the value of a *sum*, it is necessary to *multiply* that value by the said amount, in order to find from it the value of an equivalent annuity.

In the first example, therefore, the value of an estate of 4*l. per annum*, would be *l.*8.32. In the second example 20*l.* And this is, as it ought to be, the value for the whole duration of the lives, agreeably to the Problem in the note p. 37.

In

In solving this Question, care also must be taken not to forget the *first* Remark under the foregoing Question.

In this chapter, rules have been given for finding the values of all assurances on single lives, and any two lives, or any survivorships between two lives, whether for *terms*, or their *whole duration*. In the same way rules may be investigated for finding the values of all assurances on any three lives, or any survivorships between them. But this is a work of more difficulty, and which requires great attention and skill. I can, however, with particular satisfaction acquaint the Reader, that it has been lately executed, in the compleatest manner, by Mr. Morgan, in his Treatise on the *Doctrine of Annuities and Assurances on Lives and Survivorships*.

C H A P. II.

Containing an Application of the Questions in the foregoing Chapter to the Schemes of the Societies in Great Britain, for making Assurances on Lives and Survivorships, and for granting Annuities to Widows, and to Persons in Old Age.

S E C T. I.

Of the London Annuity, and the Laudable Societies for the Benefit of Widows (a).

THE scheme mentioned in Quest. VIII. was nearly that with which the *London Annuity Society* set out in 1765. The *Laudable Society* was established in 1761, and formed on a similar plan. In both, the *annual contribution* of every member was five guineas, payable half-yearly; and for this a title was given to an annuity of 20*l.* to every widow during widowhood, if the husband after admission, lived *one* year according to the *first* scheme; or *three* years according to

(a) It must be remembered, that this section has in view the state of these societies in 1773, or at the time of the publication of the former edition of this tract.

the (a) *second*; of 30*l.* if the husband lived *seven* years, according to both schemes; and 40*l.* according to the *first* scheme, if he lived 15 years, or 13 years, according to the second. —In both schemes also, there was no other premium or fine required, than five guineas extraordinary, at admission, from every member whose age does not exceed 45. The *Laudable Society* admitted none above 45, and the *London Annuity Society* obliged every person between 45 and 55 to pay, at admission, five guineas extraordinary, for every year that he was turned of 45.

These were the main particulars in the schemes on which these Societies were formed; and, therefore, both of them, were the annuities to be enjoyed for life, received (supposing the members all under 46 at admission, and of the same ages with their wives, and money at 4 *per cent.*) but little more than three-fifths of the true value of the annuities; or about one half, supposing wives, one with another, 10 years younger than their husbands; as appears from Question VIII.

It appears further in that Question, that, supposing the annuities to be *life* annuities, and men and their wives of equal ages, the

(a) In this society a member who lived but *one* year, was entitled to no more than an annuity of 10*l.* for his widow; if he lived two years, to 15*l.* if he lived three years, to 20*l.* four years, 25*l.* seven years, 30*l.* ten years, 35*l.* thirteen years, 40*l.*

expectation to which an annual payment of five guineas beginning immediately, entitles, is nearly 14*l.* if the contributor lives a year, and 20*l.* if he lives seven years (*a*), taking the medium between the *London* and the other Tables of Observations.

It is likely, that many persons will be very unwilling to believe, that these schemes could have been so deficient as they have been now represented. I will, therefore, endeavour to prove this in a way which, though less strict, is sufficiently decisive, and may be more likely to be intelligible to persons unskilled in mathematical calculation.—I shall here confine myself to the scheme of the *London Annuity Society*. The differences between it and the scheme of the *Laudable Society* are inconsiderable, and what shall be said of the one will be fully applicable to the other.

According to this scheme, as it has been just described, all that live 15 years in the society will be entitled to annuities of 40*l.* *per annum*, for their widows. Suppose the whole society, at admission, to be men of 40 years of age, taken one with another. A person of this age has an even chance of *living* 23 years; and he has an even chance of continuing with a wife of the same age, (that

(*a*) The same annual payment will, on the same suppositions, entitle to 14*l.* if a member lives a year, and 18*l.* if he lives *three* years.

is, of continuing in the society) 13 years and $\frac{1}{2}$ (a). Not much less, therefore, than half the members will continue in the society 15 years; and, consequently, not much less than half the widows that will come upon the society will be annuitants of 40*l.* *per annum*. These widows, however, being older than the rest when they commence annuitants, will continue on the society a shorter time; and, therefore, the number constantly in life together, to which they will in a course of years increase, will be proportionably smaller. Putting every thing as favourably as possible, let us suppose, that out of 20 annuitants constantly on the society, *five* will be annuitants of 40*l.* *six* of 30*l.* and *nine* of 20*l.* To 20 annuitants then the society will pay 560*l.* *per annum*, or the 20th part of this sum, that is 28*l.* to *every* annuitant at an average. But such an annuity for a life at 40, after another equal life, provided both survive one year, is worth (by *Quest. VII. p. 24.*) in a single present payment, 85*l.* nearly, according to the *London*, and all the

(a) This is the exact truth according to Mr. *De Moivre's* hypothesis, and the *Norwich* Table. But according to Dr. *Halley's* and the *Northampton* Table, a man 40 years of age has an even chance of living no more than 22 years, and of joint continuance with a wife of the same age, 13 years.—Forty must be more than the mean age of the members of the society at admission, and on this account the number of annuitants of 40*l.* must be proportionably greater. The mean age, therefore, has been taken very moderately.

Tables of Observations, interest being all along supposed at 4 *per cent*.

It cannot appear improbable to any one, that this should be the true value of such a reversion. It is not probable, that there is any situation in which the decrements of life are such as can make it a tenth part more or less.—85*l.* in present payment is the same with 3*l.* 8*s.* *per annum* for ever.—But is an annual payment of five guineas, which must cease as soon as either of two lives each 40 fails, equal in value to such a perpetuity? Every one must see, that there is a great difference.—A set of marriages between persons all 40, will, according to the probabilities of life in Dr. *Halley's* Table, last, one with another, 15 years (*a*); and an annual payment beginning immediately, during the joint continuance of the lives of two persons of this age, is worth 10 years purchase (*b*).

(*a*) See the beginning of Essay I.

(*b*) The value of such an annual payment, by Table XI. or the *London* Observations, is 9.1; and 10.8, by Mr. *De Moivre's* hypothesis.—I have not taken into this account the five guineas *fine* paid at admission, because it is obviously of too little consequence to make any considerable difference: The allowances I have made in favour of these schemes are more than equivalent to it. In particular; it should be remembered, that the calculations suppose, that the payments required by these schemes, are yearly payments beginning immediately; (see p. 28.) and that the first payment of the annuity is not to be made till the end of the year in which the husband shall die; and also, that the annuity is to be paid yearly, and nothing to be due for any part of the year in which the annuitant shall happen to die.

The

The comparison then, in the present case, is between 3*l.* 8*s.* *per annum for ever*, and five guineas *per annum for 15 years*; or between an annuity of 3*l.* 8*s.* worth 25 years purchase, and an annuity of five guineas worth only 10 years purchase.

But to throw this subject into another light.

Let the number to which the society is kept up be supposed to be 200. It has been demonstrated in Quest. II. that at least half this number of widows will in time come to be constantly on the society; and it has also been just now shewn, that the medium of annuities, payable to them, will be at least 28*l.* After a course of years, then, the society will have a constant expence to bear of 2800*l.* *per annum*.—But what will be its income?—In order to determine this, we must consider, that there are two sources from whence its income will be derived. First, the annual payments of the members. And, secondly, the money accumulated, or the *capital* raised during the time the number of annuitants is coming to a *maximum*.—The first of these sources affords 1000 guineas, or 1050*l.* *per annum*. This wants 1750*l.* of the annual expence just mentioned; and, therefore, in order that the income of the society may be equal to the burden upon it, when the annuitants come to a *maximum*, there must be a fund raised in the mean time equal to 43,750*l.* or to an estate

in perpetuity of 1750*l. per annum*.—But 1050*l. per annum* beginning immediately, and forborn 25 years, and improved, without loss or delay, all that time at 4 *per cent.* compound interest, will but just raise such a capital (*a*). There is therefore, the fullest proof, that the scheme I am considering is extremely deficient. The truth is, that scarcely a *third* of such a capital could be raised, as will appear from the following observations.

Out of 200 persons, all 40 years of age, more than five, according to the *London Table of Observations*, and not so many by Dr. *Halley's Table*, may be expected to die in a year. Suppose then five to be the real number of members that will die in the first year of the society. In subsequent years the collective body of members will be continually growing older; and, therefore, the proportion of them that will die every year, will be continually increasing, till it gets to a *maximum*. I will, however, suppose, that during the first 20 years no more than the number just specified will die every year; and

(*a*) Every question of this kind may be easily solved by Table III. in the next volume, which shews, that 1000*l. per ann.* will, in 25 years, increase to 2666*l. per ann.*; and, therefore, 1050*l. per ann.* to 2800*l. per annum*.

that

that, consequently, no more than *five widows* will come every year on the society. The ages of all these widows, when they commence widowhood, will, it is evident, be between 40 and 60. One with another then, they may be considered as having commenced widowhood at 50 years of age. Now, five widows left every year at this age, will, in 10 years, increase to 43 constantly in life together, according to the expectations of life in Tables V, VI, and VII; and, in 20 years, to 70 (a). Suppose the true number alive together at the end of 20 years to be only 62. The greater part of these will be annuitants of 30*l.* and 40*l. per ann.* and the rest 20*l.* Were the former only equal to the latter, the medium of annuities payable to them would exceed 25*l.* Suppose then this medium to be no more than 26*l.* and it

(a) Every calculation of this kind is easily made by the rule in note (A) at the end of the next volume.—I have put the number living together at the end of 20 years at 62, not only that the reader may be better satisfied that I have kept low enough, but also to make an allowance for such widows as will be left by those members who die within a year after admission, and who, therefore, according to these schemes, will be entitled to no annuities. This allowance is too large: For, after the first year of the scheme, it will not happen above once in 4 or 5 years, that the death of a member will be so circumstanced, supposing the probability that a man at 40 will live a year, to be, as all but the *London* Tables make it, 50 to 1.

will follow, that, at the end of 20 years, the society will have an annual rent to pay of 26 *l.* multiplied by 62, and 1612 *l.* and, if then able to bear such an expence, it must, in the intermediate time, have acquired an increase of income equal to the difference between 1050 *l.* and 1612 *l. per ann.* That is; it must, with its savings, have accumulated a stock equal to 562 *l. per ann.* and worth 14,050 *l.* But as, during this time, there will be a number of annuitants constantly increasing, to whom yearly payments must be made, the savings cannot certainly be one half of what they would have been had the society been all the time free from all burdens. Suppose then the stock produced by these savings, to be equal to the stock that would arise from an income of 1050 *l. per ann.* beginning immediately, and improved perfectly at 4 *per cent.* compound interest, for half the time I have mentioned, or for 10 years, without being subject to any checks or deductions: Such an income thus improved, would in 10 years produce an additional income of 504 *l. per annum*, or a capital of 12,600 *l.*—According to these Observations, therefore, the annual income of the society at the end of 20 years, and before a third part of the highest annuitants could come upon it, would fall considerably short of its expences. About that time
then

then it would necessarily run aground; and long before the number of annuitants could rise to a 100, it would spend its whole stock, and find itself under a necessity of either doubling the annual payments of its members, or of reducing the annuities one half.

All I have now said is meant on the supposition, that the society begins with 200 members at 40 years of age, and is afterwards limited to that number, by admitting no more new members than will just supply the vacancies occasioned by the loss of old members. If it is allowed to increase, it may continue a longer time. And, for this reason, a society that wants half the income necessary to render it permanent, may very well subsist, and even prosper for 30 or 40 years.—Thus, the *Laudable Society*, was it to keep to its present number of members, might possibly feel no deficiencies for 20 years to come; but if it should continue to increase at the rate of 70 or 80 every year, it would, at the end of that time, possess a balance so much in its favour as might enable it to support itself for 15 or 20 years more (a). But bankruptcy would

(a) What has been before demonstrated in Quest. III. should be here recollected, that the number of annuitants on such a society as this, must go on to increase for more than 100 years after acquiring its greatest number of members.

The

would come at last, and with the more weight the longer it had been deferred.

The rule in the *London Annuity Society*, which obliges every person between the ages of 45 and 55, to pay at admission 5 guineas extraordinary for every year that he exceeds 45, is an advantage to it, but it is a very inadequate, and also a very unequitable advantage. For at the same time, that it obliges a person 55 years of age, to give *more* than the value of his expectation, it takes *above* two-fifths *less* than the value from a person who is 45 years of age.

If any persons remain still doubtful about what I have said, I must beg their attention to one further argument.

The *Laudable Society*, I am informed, took its rise from a calculation contained in a pamphlet entitled, *The Possibility and Probability of a SCHEME intended for the Benefit of Widows being able to support itself*. The scheme here referred to, is the same with that which this Society has since followed; and I am afraid I shall not be credited, when I say, that the calculation to prove its capacity of supporting itself, is founded on the supposition, that a hundred married men, whose common age is 36, will leave but one widow every year, though at the same time it is supposed that two of them will die every year.

This mistake has made the whole calculation one half wrong.—Nothing can be plainer than that, if the death of a married man does not leave a widow at the end of the year, the reason must be, that both himself and his wife have happened to die in the year. But it is always very improbable this should happen.

It

It must be expected that every other member of these societies, supposing them to consist of persons all of the same ages with their wives, will leave widows to whom, one with another (as already shewn), at least 28*l.* *per ann.* must be allowed, for as many years as there have been payments from each member. For every 10 guineas then received they must some time or other hereafter pay 28*l.* But let it be well considered what can enable them to do this. Did money bear no interest, for any given sum now received, they could not afford at any time hereafter to pay more than an *equal* sum. That is; (since the duration of *survivorship* is in the present case, by Quest. II. equal to the duration of *marriage*) the proper consideration for any given reversionary annuity, to be allowed to *all* the survivors of a set of marriages, would be, an equal annuity payable by each marriage during its existence; and just *half* the reversionary annuity, if it is to be allowed only to half the survivors, or to widows exclusive of widowers. The annual payment then of *five* guineas, during marriage, can entitle widows to no more than an annuity of *ten* guineas, supposing money to bear no interest. But if money does bear interest, the same payment will entitle them to more, in proportion to the degree in which it is capable of being improved, during the time be-
tween

tween that in which the annual payments begin, and the commencement of widowhood. Now, it is easy to see, that unless money bears very high interest, this improvement cannot be likely in any circumstances to produce a capital, the interest of which shall be equal to the annual payment itself. Any given annual payment perfectly improved at 4. *per cent.* compound interest, requires 17 years and a half to double itself, supposing the first payment made immediately (a). But no marriages are *likely* to last so long, except those among persons who are very young. A marriage between two persons, both 40, will not *probably* last longer than 13 years, according to the probabilities of life in Dr. *Halley's* Table. A marriage between two persons, both 50, will not probably by the same Table, last longer than *eleven* years; nor a marriage between two persons, both 30, longer than 16 years. Such marriages, it is true, may possibly last 30 or 40 years. But this circumstance is more than balanced by the fact, that no less possibly they may not last *one* year. The annual payments, then, being incapable of such an improvement as shall produce an additional income equal to themselves; it is ob-

(a) At 3 *per cent.* the period of doubling money by compound interest, is nearly 23 years and a half. At 5 *per cent.* 14 years.

vious, that no society ought to go so far as to allow to widows annuities twice as great as those which might be allowed, supposing no interest of money (*a*); so far, for instance, as to allow, instead of 10 guineas, 20 guineas for an annual payment of five guineas. In the circumstances of most of these societies three-fifths addition may be the full allowance. That is; supposing the annual payment of each member to be five guineas, time may be expected for gaining from hence a capital of 75 guineas, or that shall produce three guineas *per annum* interest; and the proper reversionary annuity will be 16 guineas; or six guineas more than the proper reversionary annuity, did money admit of no improvement (*b*).

The

(*a*) The money accumulated will not be exactly the same with that to which the annual payment would increase, if improved at compound interest for a number of years, equal to that which the joint lives have an *equal chance* of existing. Much less will the increase be the same with that which would arise from the annual payment forborn, and improved, for a number of years equal to the *expectations* of the joint lives. It will be less than either of these, for a reason explained in note (K) at the end of the next volume.

(*b*) To these accounts may be added the following short and easy method of trying the sufficiency of all schemes of this kind.

In an adequate scheme it can make no difference whether the annuities themselves are paid, or the value of them in a single payment at the time they become due.—

Suppose

The preceding observations have gone on the supposition, that the reversionary annuities are to be *for life*. What difference in favour of these societies arises from the circumstance, that the annuities are to be paid only *for widowhood*, cannot be exactly determined. Some judgment, however, may be formed of it from what has been said at the conclusion of *Quest. II.* Were even one half of the widows to marry, still the schemes I have

Suppose then a society just established, consisting of 600 members, all married men at the age of 40, each of whom, besides one payment in hand, is to make an annual payment of five guineas. Suppose the age of their wives $39\frac{1}{2}$, and every widow to be entitled, on the day her husband dies, to a life annuity of 20*l.* the first payment to be made at the end of half a year.—Suppose further, that the society is to be kept up for ever to 600 members by admitting new ones at the age of 40, as old ones drop off.—In the first year (according to Tables V, VI, and VII. in the next volume) twelve members, at least, will die, and leave twelve widows, each intitled to 20*l. per ann.* The value of such an annuity to commence at the end of half a year, the age being 40, is $14\frac{1}{2}$ years purchase by Mr. *De Moivre's* valuation of lives, (or Table I. in the last leaves of the next volume) reckoning interest at 3 *per cent.* The value, therefore, of 12 such annuities; that is, the whole amount of the sums becoming payable during the course of the first year, is 3480*l.*—The annual contribution is 600 times 5 guineas, or 3150*l.* and this, together with its interest for about half a year, or 3205*l.* is all that such a society could be possessed of to bear an annual expence of 3480*l.*—It appears, therefore, that, in order to support the expence of the supposed annuities, the annual contribution of each member ought to have been more than five guineas.

A proof

have been considering would probably be insufficient. But, in the circumstances of these societies, it cannot be expected, that above one in 10, or perhaps one in 20, will marry. The persons most likely to enter into them, are such as have not the prospect or ability of making competent provisions for their wi-

A proof of the same nature with that here given may be deduced, by considering these societies as bodies of men united for the purpose of assuring to one another, from year to year, annuities for their widows; and the way of finding the value of such an assurance is, to multiply the value of the annuity, by the probability that it will become payable in the course of the year.—For instance, Let the member's age, and also his wife's, be 40. Let the annuity be 20 *l. per ann.* for life to commence at the end of a year, or an annuity whose present value is (reckoning interest at $3\frac{1}{2}$ per cent.) 14 years purchase; that is, 280 *l.* The probability that a person at the age of 40 will die in a year, and that his wife of the same age will live a year: or, in other words, the probability, that such a member will leave a widow in the course of the year, is, by the *Breslaw* Observations, (or Table V, next volume) $\frac{9}{443}$ multiplied by $\frac{436}{443}$, or .0198. (See p. 18, and 23). That is; there will be the odds of nearly 49 to 1, against such a member leaving a widow in the course of the year. The value of the assurance, therefore, is .0198, multiplied by 280, or the 50th part of *l.* 280: that is, 5 *l.* 11s.—In the same manner the value of a like assurance for a year at any other ages may be easily calculated. At the age of 35, it is 5 *l.* 7s. At the age of 45, it is 6 *l.* 7s. The value, therefore, increases continually with age; and, if given in an annual payment constantly the same, which is the case in these societies, it ought to be greater than the annual payment due for one year at the commencement of the assurance:

Five guineas *per annum*, therefore, is, demonstrably, an insufficient payment from a married man for a life-annuity of 20 *l.* to his widow.

dows

dows in other ways. The widows left, therefore, will in general be unprovided for, and, being also left with families of children, it is quite unreasonable to expect, that any considerable proportion should marry. This is true of such as may happen to be left young; but when a society has subsisted some time, the *greater* part will not be young when left, and these, at the same time that no advantage can be expected from their marrying will be in general the *highest* annuitants, and, therefore, the *heaviest burdens*.—Moreover, the prospect of the loss of their annuities will have a particular tendency to check marriage among them.—For all these reasons it seems to me likely, that the benefit, which these societies will derive from marriages among their annuitants, will not be very considerable; or, at least not *so* considerable as to be equal to the advantages I have allowed them, by calculating on the suppositions, that the money they receive will be *always improved perfectly, without loss or delay, at the rate of 4 per cent. compound interest*; that the probabilities of life among males and females are the same, and all husbands likewise of the same ages with their wives, and that consequently the *maximum* of widows on such societies can amount to no more than half the number of marriages (a).—With respect to the

(a) Care should be taken in these societies, not to judge of the proportion of widows that will marry, from
the

the last of these suppositions, it deserves to be particularly observed, that from accounts taken annually with great care in *Scotland*, it appears, that the widows of the *ministers* and *professors* there (*a*), notwithstanding the diminution occasioned by their marrying, do exceed considerably half the number of marriages. And certainly it would be unreasonable in these societies not to reckon that the same will happen among them.—Indeed it seems certain, that notwithstanding the hazards that attend child-bearing, the probability that the woman shall survive in mar-

the proportion that may happen to marry during their first years. For most of the widows that will be left at first will be young; whereas the greater part will not be young when they commence widowhood, after a society has subsisted 30 or 40 years; and, therefore, though one in three or four should marry at first, it will not be reasonable to expect, that half so many should marry after the affairs of the society become stationary.

(*a*) The number of married ministers and professors, for 17 years, from 1750 to 1766, was at a medium 667. And from the enquiries that have been made, it appears, that from this whole body near 400 widows constantly living are derived. The medium of widows left annually has, for the last 36 years, been $19\frac{7}{10}$; and, for 10 years, ending in the year 1767, but nine of these had married.—Of the annuitants likewise (about 160 in number) on the fund established among the Dissenters in *London*, for relieving the widows of indigent ministers, it is found that few ever marry. See Chap. 2. Sect. 2. See likewise the latter end of the 4th Essay; and note (A) at the end of the next volume.—In the *Laudable Society* during 19 years from 1761, the number of widows that came upon the Society was 167; and of these only 13 had married at Lady-day 1780. Fourteen had died.

riage, and not the man, is much greater (a) than is commonly imagined. It will be shewn in the last Essay, that it is not less than the odds of 3 to 2; and had I calculated agreeably to this fact, the values of annuities for widows, would have been given near a quarter greater than they have been given on the supposition, that the chance of survivorship is equal between men and their wives.—It must be added, that I have made no account of any expences attending the execution and management of the schemes of these societies. Some such expences there must be, and some advantages should be always provided in order to compensate them.

There are in this kingdom many institutions for the benefit of widows, besides the two on which I have now remarked; and in general, as far as I have had any information concerning them, they are founded on plans equally inadequate, having being formed just as fancy has dictated, without any knowledge of the principles on which the values of re-

(a) Partly, as observed in page 8, on account of the greater mortality of males, but chiefly on account of the excess of age on the man's side.—The *Laudable Society*, for several years after its institution, paid no regard to this excess of age; and the allowance required on this account by the *London Annuity Society* was so trifling as to deserve no notice.

In March 1780, 167 husbands had died in the *Laudable Society*, and only 138 wives.

versionary

versionary annuities ought to be calculated. The motives which influence the contrivers of these institutions, may be *laudable*; but they ought, I think, to have informed themselves better. This appears sufficiently from what has been said; but I will just mention one further proof of it.

The *London Annuity Society* promises that, if in 21 years; and the *Laudable Society* that, if in 25 years, it shall appear that there has been all along an annual surplus in favour of the societies, it shall be employed in either raising the *annuities*, or in sinking the *annual payments*. Now, they may be assured, that if, at the end of these periods, they should not be possessed of a considerable surplus, the true reason will be, their having granted much higher annuities than the annual contributions are able permanently to support: For it has been demonstrated, that the number of annuitants, and consequently the amount of the annual expences, will go on increasing for a long course of years beyond these periods. The effect, therefore, of carrying into execution this regulation will be, precipitating that bankruptcy which would have come too soon had there been no such regulation.

It has been said in defence of these societies, that the deficiencies in their plans cannot be of much consequence, because their rules oblige them to preserve a constant equality

between their income and expences, by reducing the annuities as there shall be occasion. And from hence it is inferred, that they can never be in any danger of a bankruptcy.— But it has appeared, that the time when they will begin to feel deficiencies is so distant, that it will be too late to remedy past errors, without sinking the annuities so much, as to render them inconsiderable and trifling. All that is given too much to *present* annuitants is so much taken away from *future* annuitants. And if a scheme is *very* deficient, the first annuitants may, for 30 or 40 years, receive so much more than they ought to receive, as to leave little or nothing for any who come after them. Deficient schemes, therefore, are attended with particular injustice; and this injustice will be the same, if, instead of *reducing* the annuities, the annual payments should be increased; for all the difference this can make will be, to cause the injustice to fall on *future contributors*, instead of *future annuitants*.

But what requires most to be considered here is, that, after either the annuities have been for some time in a state of reduction, or the contributions in a state of increase, it will be seen that these societies have gone upon wrong plans, and, therefore, they will be deserted and avoided; the consequence of which will prove still greater deficiencies in
their

their annual income, and a more rapid desertion and decline, till a total dissolution and bankruptcy take place.—This will be the death of most of the present societies for providing for widows, if they continue to be encouraged, and do not soon alter their plans: And at that period the number of *annuitants* will be greater than ever; whose annuities, having no other support than the poor remains of a stock always insufficient, will be soon left, without the possibility of relief, to lament that ignorance and credulity which gave rise to these societies, and which had so long supported them.

In the *London Annuity Society*, there is an encouragement to *batchelors* and *widowers* to join them, arising from the additional annuities to which they will be *immediately* entitled, when they marry, in consequence of having made their payments a greater number of years; and it is imagined, that particular advantages will be derived from such members. But even these will in general pay much less than the value of their expectations.—A person who begins an annual contribution of five guineas at the age of 24, will, should he live 11 years, and marry a woman of the same age at the end of that time, entitle her immediately to 35*l.* *per ann.* during survivorship, and to 41*l.* *per annum* should he live four years after marrying,

ing, (interest being at 4 *per cent.*) (a). In this particular case, therefore, a person will pay nearly the true value of his expectation. But *all* at all ages who *marry*, in less time than 11 years after admission, will pay less than the value of their expectations.

S E C T. II.

Of the Association among the London Clergy, and the Ministers in Scotland, for providing Annuities for their Widows (b).

IN April, 1765, the clergy within the bills of mortality, and the county of *Middlesex*, at a general meeting in *Sion-College*, agreed to form themselves into a society for the support of their widows and orphans. Many in

(a) The value of five guineas *per annum* (first payment made immediately) for 11 years, subject to failure should a life now 24 fail; and, after 11 years, for the joint lives of two persons both 35, is, by the Table of *London Observations*, *l.* 69.3—By Dr. *Halley's* Table, *l.* 76.44—The present value of 35 *l.* *per annum* for life to the widow of a person now 24, should he live 11 years, and marry a woman of the same age with himself at the end of that time; and also of 6 *l.* more, or 41 *l.* *per annum* in all, should he live after marriage four years; is, by the Table of *London Observations*, *l.* 69.36.—By Dr. *Halley's* Table, *l.* 76.03.

(b) This section, as well as the former, must be considered as written in 1773. Further accounts of these institutions will be given in the supplement to this section.

this

this respectable body may be capable of doing, in a better manner, what I have attempted in this Treatise; and they are, perhaps, already sensible of the deficiencies in the plan which they have established. I shall not, however, I hope, do wrong, in taking the liberty to recite briefly this plan, in order to introduce a few observations upon it.

According to the printed articles, every clergyman possessed of any benefice, lectureship, or licensed curacy, within the bills of mortality and the county of *Middlesex*, who subscribes annually one guinea, or two guineas, or more, shall entitle his widow to an annuity; or, if he leaves no widow, he shall entitle any such children as he shall leave, to the same annuity for seven years as his widow would have had. And, in case a widow possessed of an annuity, should either *die* or *marry* before the lapse of 10 years, from the commencement of her annuity, such children of her former husband, as shall be then alive, are to be entitled to as many of the ten years payments of the annuities as she shall not have received.—The annuity is fixed to no particular sum, but instead of this, it is ordered, that a fourth part of the annual subscriptions and interest shall be divided the first three years after the establishment of the society; half only the next four years; and three fourths the next five years; provided, however, that in no one of these 12 years the dividend

shall exceed 20 *l.* to the widows and orphans of the clergy subscribing two guineas or more; and 10 *l.* to the widows and orphans of the subscribers of one guinea. And, after the expiration of 12 years, the whole amount of the subscriptions, and of the interest of the capital stock, is to be divided proportionably for ever.—It is further provided, that every clergyman who shall be married, or have children, at the time of his subscription, shall pay a fine of two guineas towards a capital stock, if a subscriber of two guineas or more, and 40 years of age or upwards. If 50 years of age or upwards, he shall pay a fine of three guineas; if 60 or upwards, five guineas. But, if not married at the time of his subscribing, and shall afterwards marry, he shall pay a fine according to the age he shall be of at the time of his marrying. The obligation laid upon all, whether married or unmarried, to become subscribers, is, an incapacity of being admitted members without the consent of a general court, unless, within two years after becoming possessed of any ecclesiastical employment, they subscribe.

Every one who has attended to the observations in this and the preceding chapter, must know what judgment to form of these regulations.

Let us suppose that all the clergy in *London* and *Middlesex* came into this association from

from the first; and that one with another they are subscribers of two guineas annually; and that there are among them as many unmarried persons as married.

In this case, it may be learnt from *Quest. XIII*, that the annuity to which widows should be entitled, (supposing no allowance to the children of any that die) ought not to exceed 10 or 11 guineas at most; and that, besides the annual subscriptions, there ought to have been a fine paid at the commencement of the scheme, by every married person, of six guineas at least, or, by the whole number of subscribers, three guineas. If the number of married members is double the unmarried, the annuity ought not to exceed eight guineas; and the fine from every member should be about four guineas.—The order, that only a fourth part of the annual subscriptions and interest shall be divided the first three years, half the next four years, and three quarters the next five, is without reason; because the number of claimants, for the first 12 years of the scheme, will be so few, that it will not be possible, during that time, that there should be occasion for dividing any proportions so large of the annual subscriptions and interest, unless they are indeed beyond all bounds too little.—After 12 years, the number of annuitants will go on increasing for near 50 years, as appears from *Quest. III*. The consequence, therefore,

therefore, of dividing, after that time, the whole amount of the annual subscriptions and interest, will be a constant yearly diminution in the dividends for near 50 years ; and making the payments to the first claimants much more considerable than they ought to be, at the expence of all subsequent claimants—For these reasons, it appears to me out of all doubt, that this scheme is by no means likely to answer the good ends proposed by it ; and that, therefore, it will be best to lay it aside. At the time it was settled it was, I find, further agreed, that the annual subscriptions of the *laity*, together with the interest of their benefactions, unless otherwise directed by the donors ; and the annual subscriptions of such of the clergy as shall so direct, shall make a *charitable fund* to be applied to the relief of the distressed widows or children of all the clergy within the limits I have mentioned, whether subscribers or not, provided that in no one year of the first twelve more than 20*l.* be given out of the fund to any one family.—This is an excellent design ; and if the money arising from all the subscriptions is thrown into this fund, an important means of relief may be provided, for such of the more indigent widows and families as will accept the help of charity.

There is one more scheme of particular consequence, which I must take notice of :

I mean,

I mean, that which is established by Act of Parliament, among the ministers and professors in *Scotland*, for making provision for their widows and orphans. The last mentioned scheme, and also several others of the same kind. (a) in this kingdom, have been formed on the model of this: and the success with which it has been hitherto attended, is one of the principle causes to which they have owed their rise. It is, therefore, proper I should give some account of it; and it will be sufficient with this view to mention, “that for an annual payment, which
“ begun immediately, of *five guineas* from
“ 1011 contributors, 667 of whom are married persons, besides a tax on weddings,
“ producing about 142 *l. per ann.* it entitles
“ every widow to an annuity of 20 *l.* during
“ widowhood, and also every family of children that shall be left by such members
“ as die without leaving widows, to 200 *l.*”
This scheme contains a variety of other particulars; but this is its substance—It commenced on the 25th of *March*, 1744; and from that time to the 22d of *November*,

(a) There is one among the Dissenting Ministers in the counties of *Chester* and *Lancaster*; and another among the Dissenting Ministers in *Cumberland*, *Northumberland*, *Westmorland*, and *Durham*.—Even the *London Annuity Society*, though its plan is totally different, professes to form itself on the principles of the *Scotch* establishment, and to derive encouragement from it.

1770 (a), or in 26 years and near 8 months, 151 ministers and professors died, and left 151 families of children without widows; that is, 5.66 such families were left annually; and the annual disbursements to them have therefore been 1132 *l.* Subtract this sum from 5450 *l.* the whole annual income; and the remainder, or 4318 *l. per ann.* will be the standing provision for bearing the expence of all the annuitants possible to be derived from 667 marriages. Such an annual payment, or 4.27 each from 1011 contributors, is the same with 6.55 each, from 667 contributors; and, consequently, it appears, that in this establishment a contribution is received equivalent to an annual payment beginning immediately, of *l.* 6.55 from every married man, in order to entitle his widow to an annuity of 20 *l.* during her widowhood.

In the societies mentioned in the last section, annuities increasing from 20 *l.* to 40 *l.* are promised to widows for an annual payment of only 5 guineas (b). And, in all the societies for the benefit of widows with which I am acquainted, there is an equal or a greater disproportion between the contributions received, and the annuities promised.—

(a) In Nov. 1779, or 35 years and 8 months, 199 ministers and professors had died, and left 199 families of children without widows; that is 5.58 annually.

(b) See page 65.

With

With what strange rashness then has the plan of this establishment been copied? And how absurdly have the societies in this kingdom pleaded it as a precedent which encourages and favours them?—It would be trifling to say more on this subject.

It may be observed that the annual income for the support of this establishment, supposing it to have only the benefit of widows in view, ought to be *l.7.19 per ann.* from every marriage, according to Quest. XIII. p. 44. and *l.7.44 per ann.* according to the calculation in Note F, at the end of the next volume.

These determinations exceed the income actually provided. But the excesses are by no means considerable enough, to afford any certain reason for concluding, that the fund of this establishment will prove insufficient. I was, however, once led to entertain some doubts on this subject. And in these doubts I thought myself confirmed by observing, that, in the calculations (a) made at the commencement of the scheme, the number

(a) See Table III. in a book printed at *Edinburgh* in 1748, entitled, *Calculations, with the Principles and Data on which they are instituted, relative to a late Act of Parliament, entitled, An Act for raising and establishing a Fund, for a provision for the widows and children of the ministers of the church, and of the heads, principals, and masters of the universities of SCOTLAND; shewing the rise and progress of the Fund.*

333 was stated, as the *maximum* of widows living at one time, likely to come upon it, or to be derived from 20 (a) widows left annually; and also, that 40 years was stated as the number of years necessary to bring on this *maximum*; whereas I was satisfied, that the *maximum* of widows would not prove much less than 400; nor the number of years necessary to bring it on, less than 60.—In the former editions of this work, I gave a distinct account of this. But I have lately received such information (b) as has convinced me that my doubts have been in a great measure groundless. I have learnt, in particular, that there have been several calculations subsequent to those I had seen; and that this establishment has enjoyed advantages and provisions for its support which I was unacquainted with, and which give reason for expecting that it will indeed be able to bear the expence of 400 annuitants, should so many come upon it. I should only tire most of my readers, were I to enter into an account of these advantages and provisions. It will be of more importance to take this opportunity to observe, that the probabilities of

(a) See the note A, among the notes at the end of the second volume.—See likewise the note in p. 81.

(b) I owe this information to the kind and very obliging candour of the reverend and ingenious Dr. WEBSTER, of Edinburgh.

life from which the determinations I have mentioned are derived, though much lower than the probabilities of life among the ministers and their wives in SCOTLAND (a), are yet such as give the values of reversion depending on survivorships among them too high.

In order to understand this, it must be considered, that the difference between the probabilities of life in different situations, takes place much more in the first and middle than in the last stages of life; and that the effect of this must be to *increase* the duration of *joint lives*, and at the same time to *lessen* the duration of *survivorship* in those situations which are most favourable to health. Or, in other words, to render the duration of marriage in such situations, greater than it would otherwise be in proportion to the duration of widowhood; and, consequently, to reduce the present value in annual payments during marriage, of any given annuity payable during widowhood. For instance. Were the probabilities of life among the ministers and their wives in SCOTLAND the same that they are in Mr. *De Moivre's* hypothesis, or in the *Breslaw* and *Northampton* Tables of Observation, the duration of marriages among them,

(a) More particular notice will be taken of this at the conclusion of the last of the following Essays.

taken one with another, could not be more than 19 years. The duration of widowhood would be 22 years, and the *maximum* of widows living at one time derived from 667 marriages constantly kept up, would be considerably more than 400.—Were the probabilities of life among them the same that they are in LONDON, the duration of marriage would be still *less*, and the duration of widowhood *greater*, and the *maximum* of widows derived from 667 marriages, could not be less than 500. But the fact is, that the duration of marriage among them is 22 years nearly; and that of widowhood about 20 years and a half (*a*). And it appears also, from accounts taken annually, that the number of widows living at one time, derived from the whole body of ministers and professors, does not exceed 400. It is, therefore, certain that a smaller income must be sufficient for the support of this scheme than would be necessary, according to the probabilities of life in the Tables just mentioned.—And upon the whole; after a careful review of all the circumstances of this establishment in its present state, I am well sa-

(*a*) See a note at the conclusion of the last Essay; and also note F, at the end of the next volume.—The *maximum* of widows (supposed 395) divided by the number left annually (or $19\frac{1}{16}$) gives $20\frac{3}{5}$, the expectation of widowhood. See page 81, and Note (A) at the end of the next volume.

tisfied,

tisfied that the success with which it has been hitherto attended, is likely to continue; and that it will indeed prove a permanent foundation of that assistance to the *widow* and *fatherless* which is intended by it.—Caution, however, and vigilance, will for some time be necessary. Many more years must pass before it can receive a decisive confirmation from experience. Events have hitherto favoured it more than could have been reasonably expected. They may perhaps hereafter try it; and deviations from probability may arise, which cannot be now foreseen.—But I ought to ask pardon for making these remarks. The venerable ministers and professors concerned will, I hope, excuse me. They are eminently distinguished by their abilities and knowledge: and can have little need of any information which I am able to give them.

S E C T. III.

Of the best Schemes for providing Annuities for Widows.

INSTITUTIONS for providing widows with annuities would, without doubt, be extremely useful, could such be contrived as would be *durable*, and at the same time *easy*

and *encouraging*. The natures of things do not admit of this in the degree that is commonly imagined. The calculations and rules, in the preceding chapter, will enable any one to determine in all cases to what reversionary annuities any given payments entitle, according to any given valuation of lives, or rate of interest. From Quest. VII. and VIII. in particular, it may be inferred that (interest being at 4 *per cent.* and the probabilities of life as in the *Breslaw*, *Norwich*, and *Northampton* Tables) for an annual payment beginning immediately of *four* guineas during marriage; and also for a guinea and half in hand, on account of each year that the age of the husband exceeds the age of the wife; every married man, under 40, might be entitled to an annuity for his widow, during *life*, of 5*l.* if he lives a year, 10*l.* if he lives *three* years, and 20*l.* if he lives *seven* years.

If such a society chuses, that those who shall happen to continue members the longest time, shall be entitled to still greater annuities, six guineas, additional to all the other payments at admission, would be the full payment for an annuity of 25*l.* and 12 guineas for an annuity of 30*l.* if a member should live 15 years.

All batchelors and widowers might be encouraged to join such a society, by admitting them on the following terms.—*Four guineas* to be paid on admission, and *three guineas* every
every

every year afterwards, during celibacy; and on marriage, the same payments with those made by persons admitted after marriage; in consideration of which, 1*l.* *per annum*, for every single payment before marriage, might be added to the annuities to which such members would have been otherwise entitled.

For example. If they have been members four years, or made five payments before marriage, instead of being entitled to life-annuities for their widows of only 5*l.* 10*l.* 20*l.* 25*l.* and 30*l.* on the conditions I have specified, they might be entitled to annuities of 10*l.* 15*l.* 25*l.* 30*l.* and 35*l.* Or, if they have been members nine years, and made 10 payments, they might, instead of the same annuities, be entitled to annuities of 15*l.* 20*l.* 30*l.* 35*l.* and 40*l.*—In this case, the contributions of such members as should happen to desert, or die in celibacy, would be so much profit to the society, tending to give it more strength and security.

This is one of the best schemes that I can think of, or would chuse to recommend. But in the following scheme there is a simplicity and fairness which seem to give it a particular preference.

Every husband, be his age what it will, for a single payment at admission, of 15*l.* with 1*l.* 10*s.* added for *every* year that his age exceeds his wife's, and an annual pay-

ment of 5*l.* during marriage, (the first to be made at the end of a year) might entitle his wife, should he leave her a widow, to an annuity of 10*l.* for her life, if he lives *one* year; 11*l.* if he lives *two* years; 12*l.* if he lives *three* years; and so on; the annuity to increase continually at the rate of 1*l.* for every year that the husband lives beyond one year.—Any addition to these payments might entitle to a proportionable addition to the annuity, and to its increase.—And should any husband under 40 wish to secure a sum for his children, *provided* he should leave no widow, he might for every annual payment of nine shillings, *during life*, entitle them to 50*l.* payable among them at his death, whenever that shall happen. Making all these payments *guineas* instead of *pounds*, might probably be sufficient, if the number of subscribers is considerable, to defray the expences of management.

There is one particular advantage which societies formed on plans of this kind would enjoy (*a*).—Persons who know themselves subject to disorders which are likely to render them short-lived, will have no great temptations to endeavour to gain admission into such societies; and, if admitted, the danger

(*a*) See another advantage mentioned under Quest. VIII.

from them will be less than on any other plan. Were it not for this danger, the following plan might be recommended.

In the plans hitherto mentioned it is implied, that, if either a member or his wife dies within any of the periods specified, the additional annuities, that would otherwise have become due, will be lost. But it would be much more agreeable to a purchaser, that they should be made certain to his wife, provided she lives to the end of these periods, though in the mean time his own life should fail. The value of such annuities may be computed by the rule in Quest. IX.

Suppose, for instance, the *scheme* to be
“ that a wife shall be intitled certainly to a
“ life-annuity of 20*l.* the first payment of
“ which shall be made at the end of 12 years,
“ provided she should be then alive, and her
“ husband dead; or at the end of any year
“ beyond this term in which she may hap-
“ pen to be left a widow.” Suppose it also
stipulated, “ that she shall be entitled to
“ 10*l.* more, or 30*l.* in all, on the same
“ terms, provided she should live 16 years.”
—The value of such an expectation (interest
being at 3 *per cent.* and the probabilities of life
as in the *Northampton* Table of Observations)
will be, in the most convenient round sums,
supposing none admitted above 50 years of
age, seven guineas in annual payments to be

continued during marriage, and to begin immediately; besides four guineas in present money for every year, as far as 15 years, that the husband's age exceeds the wife's, if he is between 40 and 50, and three guineas on the same account if he is under 40; Or, if the whole value of the expectation is given in one present payment, 70*l.* added to a guinea and half for every year that the husband's age falls short of 50, besides the payment just mentioned on account of disparity of age.

The value of this expectation at 4 *per cent.* is six guineas in annual payments; besides three guineas in present money, for every year that the husband's age exceeds the wife's, if he is between 40 and 50; and two guineas, if he is under 40: Or, if the whole value of the expectation is given in one present payment, 56*l.* added to 1*l.* 5*s.* for every year that his age falls short of 50, besides the payment last mentioned on account of inequality of age (a).

He

(a) Supposing 16 years the only term, the annuity 20*l.* and interest at 4 *per cent.* the proper payments will be nearly, in the case of equal ages and *single* payments, 46*l.*—40*l.*—29*l.* as the age of the man is 30, 40, or 50. Or, in *annual* payments, *l.* 3.80.—*l.* 3.66.—*l.* 3.13.—Supposing the woman's age 10 years less than the man's, the same values will be, in *single* payments *l.* 58.92.—*l.* 56.56.—*l.* 53.66.—in *annual* payments *l.* 4.63.—*l.* 5.—*l.* 5.41.—It appears, therefore, that a society, supposing money improved at the rate of 4 *per cent.* might entitle all married men *indiscriminately*, who are under 50 years of age,
to

He that will give himself the trouble to calculate, agreeably to the directions in the Questions to which I have referred, will find that, taking all particular cases together, the rules now given come as near the truth as there is reason to desire in an affair of this nature, the *defects* in some cases being nearly compensated by the *excesses* in others.

These determinations are agreeable to the probabilities of living in Dr. *Halley's*, as well as the *Northampton* Table of Observations, or Tables 5th and 6th in the next volume. These Tables seem to give a proper *medium* between the different values of *town* and *country* lives. In the country the probabilities of living are much higher; but in *London*, and probably in all *great* towns and some *smaller* ones, they are much lower.

It is proper to add, that, according to the values of lives deduced both from the *London* and Dr. *Halley's* Table, and taking interest as low as 3 *per cent.* all women whose husbands are under 50 years of age, might be entitled to an annuity of 24*l.* during *life* (the first payment to be made at the end of the year in which they shall be left widows) for the sum of 100*l.* supposing 3*l.*

to such an expectation as this for their wives, for either 60*l.* in *one* payment, or five guineas in *annual* payments. —But equity requires, that different payments should be made according to the different comparative ages of men and their wives.

additional given on account of every year that they are younger than their husbands.—At 4 *per cent.* an annuity of 30*l.* might be granted on the same terms.

In the year 1690, the company of *Mercers* in *London*, adopted such a scheme as that last mentioned. For 100*l.* in *one present payment*, they entitled every subscriber to a *life-annuity* for his widow of 30*l.*; and this, at that time, (when money bore 8 *per cent.* interest) was considerably less than the value of the money advanced, supposing men and their wives of equal ages. As the interest of money sunk, they sunk also the annuity, first to 25*l.* and then to 20*l.* and 15*l.* But at last, after carrying on the scheme for above 50 years, finding the burden of the annuitants too heavy, and likely to go on increasing, they were obliged to drop the scheme and to stop payment. In a little time, however, by a parliamentary aid of 3000*l.* *per ann.* they were restored to a capacity of making good all their engagements, and of paying their arrears.—Their failure, is, indeed, much to be lamented; for, in consequence of it, the public has lost the benefit of an institution, that for many years promised the happiest effects, by encouraging marriage, and affording relief to indigence. The rapid fall of the interest of money; their admitting purchasers at too advanced ages; and, particularly, their paying no regard to the
difference

difference of age between husbands and their wives, must have contributed much to hurt them. Some of the principal causes, therefore, which have rendered them unsuccessful, may be now avoided.

It must, however, be remembered, that the issue of the best schemes of this kind must be in some degree uncertain. For want of proper observations (*a*), it is not possible to determine what allowances ought to be made, on account of the higher probabilities of life among females than males. No prudence can prevent all losses in the improvement of money; nor can any care guard against the inconveniencies to such schemes, which must arise from those persons being most ready to fly to them who, by reason of concealed disorders, feel themselves most likely to want the benefit they offer.

The societies, therefore, on which I have remarked in the first section of this chapter, would have reason to take warning from what has happened to the *Mercer's Company*, were the schemes on which they are formed perfectly unexceptionable. But I have demonstrated that these schemes are very defective; and that the longer they are carried on, the more mischief they must produce. 'Tis vain (as appears from Quest. III.) to form such

(*a*) This defect will, I hope, be in some measure removed by the Observations and Tables in the next volume.

establish-

establishments with the expectation of seeing their fate determined soon by experience. If not more extravagant than any ignorance can well make them, they *will* go on prosperously for 20 or 30 years; and, if at all tolerable, they *may* support themselves for 40 or 50 years; and at last end in distress and ruin. No experiments, therefore, of this sort should be tried hastily. An unsuccessful experiment must be productive of very pernicious effects. All inadequate schemes lay the foundation of *present* relief on *future* calamity, and afford assistance to a *few* by disappointing and oppressing *multitudes*.

As the persons who conduct these schemes can mean nothing but the advantage of the public, they ought to listen to these observations. At present their plans are capable of being reformed; but they cannot continue so always; for the greater number of exorbitant payments they now make to annuitants, the more they consume the property of future annuitants, and the less practicable a retreat is rendered to a rational and equitable and permanent plan (a). They should, therefore, *immediately* (b) either reduce their

(a) See p. 83, 84. Sect. I.

(b) Thus; was the London Annuity Society to make their lowest annuity 10*l.* the next 20*l.* and the highest 30*l.* they would probably be safe. But, after proceeding on their present plan some years longer, such a reduction would by no means be sufficient.

schemes, or change them into one of those which I have proposed. But, I am afraid, this is not to be expected. The neglect with which they have received some remonstrances that have been already made to them, gives reason to fear, that what has been now said will be in vain; and that those who are to come after them, must be left to *rue* the consequences of their mistakes.

SUPPLEMENT *to the preceding Sections;*
containing a further Account of the Societies
for the Benefit of Widows.

° [Of the LONDON ANNUITY SOCIETY.

IN the first Section, the Reader has seen on what very incompetent plans the two Societies, which are the subject of it, have been formed. Some changes have taken place in them since the last edition of this Treatise, of which it is necessary I should here take notice.

The LONDON ANNUITY Society, consisting in January 1781 of 326 members, has so far reformed its plan, as to be now in little danger. Besides ordering a compensation for difference of age between husbands and wives, it determined, in 1774, not to engage
to

to pay a higher annuity than 20*l.* to widows if their husbands had lived a year after admission, for a contribution of 10 guineas in hand, and five guineas *per ann.* afterwards. At the same time, however, room was left for expecting that some additional annuities might be paid to the widows of such members as should survive 15 years in the Society; but what the additions should be, was left to be determined at the end of 15 years from the establishment of the Society. Accordingly, last year some able judges were consulted; and, if I am rightly informed, the result has been, that the Society has agreed to promise for the same contributions an addition to the 20*l.* annuity just mentioned, of 4*l.* *per ann.* to widows, if their husbands have been members 15 years or more.

He that will consider the demonstrations in the First Section of this Chapter, or compute agreeably to any table of the decrements of life by the Rule in Quest. X. may assure himself, that a contribution of ten guineas in *immediate payment* and five guineas every year after the first, is scarcely a sufficient support for an annuity of 20*l.* during life to widows, supposing husbands and wives of the same ages, and money improved at an interest of 4 *per cent.*—But money may now be improved at a higher interest. Some advantages also must be derived

derived from making the annuities payable for widowhood only; and on these accounts, such a contribution may safely enough be reckoned a proper payment for an annuity of 20*l.* as it is offered by this Society. But it cannot, without danger, offer more; particularly, as it is certain, that the lives of women in general, and more especially of women in the advanced stages of life, are more durable than the lives of males.

The additional annuity, however, not being of more value than about three guineas and a half in a single payment at entrance, the Society may possibly find itself capable of paying it, *provided* the contributions for supporting the scheme (namely ten guineas at entrance and five guineas *per ann.* from every member, besides a just compensation for the excess of his age above that of his wife) are not loaded with any of the expences of management.

Further Account of the Laudable Society for the Benefit of Widows. See the First Section.

THIS Society affords a melancholy proof of the pernicious tendency of that disposition to form annuity societies which prevailed some time ago.—In consequence of a petition to Parliament in 1774,
from

from many of the most respectable members, it *reformed* its plan; but no arguments could engage the majority of the members to consent to a reformation which was likely to be attended with any other effect than an increase of calamity by postponing it. For thirteen years from the time of its establishment, it had overlooked the differences of age between men and their wives, and gone upon the plan mentioned in page 64.—In 1774, a compensation for the wife's inferiority of age was ordered to be paid by all *new* members, and at the same time the following plan agreed to.—For an annual payment of five guineas, the first to be made immediately, every widow was entitled to an annuity during widowhood of

10 l. if	—	—	} 2 years and a day
been admitted			
15 l. if	—	—	3 years
20 l. if	—	—	6
25 l. if	—	—	8
30 l. if	—	—	11
35 l. if	—	—	13
40 l. if	—	—	15

Any one who will calculate by the rule in *Quest. VIII.* will find that the annual payment necessary to support these annuities is nearly, by *Dr. Halley's Table of Observations*, 7 l. supposing equality of age between husbands and wives, money improved at

at 4 *per cent.* and the mean age of admission *thirty-seven*.——This change, therefore, did not deserve the name of a *reformation*; and an attention to the following account will shew, that instead of doing good, it has in fact only prolonged the existence of the Society to do mischief.

From the establishment of the Society in 1761 to 1772, it had increased to 700 members; but in April 1780, it had gradually sunk, by deaths and desertions, to 550.—The whole number of widows which had come on the Society was then 168, of whom 84 had come upon it in six years, from 1774 to 1779 (*a*); that is, fourteen annually. *Thirteen* had died and *fourteen* had married, which had left 141 annuitants, the claims of 133 of whom amounted then to 3,310*l.* *per ann.* The claims of the remaining eight, reckoned at 30*l.* *per ann.* each, will make 240*l.* *per ann.*; and the expences of management (or 300*l.* *per ann.* nearly), added to these sums, will make the whole annual expence of the Society in April 1780, 3,850*l.*——Its income, consisting of the interest of 49,090*l.* three *per cent.* stock, and the subscriptions of 550 members at five guineas each, amounted at the same time to 4,360*l.* leaving a favourable balance of only 480*l.*—

(*a*) In 1780 fourteen more widows came upon the Society.

Supposing the Society to preserve its present number of members, and the number of annuitants to increase for six years to come only at the rate of ten annually, its expence at the end of the present year, or the beginning of the next, will be equal to its income; and afterwards it will find itself under the necessity of having recourse to one of the three following expedients. It will be obliged either to run into its capital, or to increase its contributions, or reduce the annuities.—The consequence of the first of these expedients will be, that the capital of the Society will be soon consumed, and the annuitants left without support.—The consequence of the *second* will be, that the contributions will be increasing every year till, in 10 or 12 years, they are *doubled*, and at last almost *tripled*.—The consequence of the *last* will be, that the annuities will sink every year till they come to be less than half the annuities promised (*a*). Such are the affairs of this infatuated Society, nor is it easy to apply any remedy to them; for in consequence of going on too long with an insufficient scheme, the Society has large payments to make in order to compensate past deficiencies; and a scheme at first adequate would now prove

(*a*) The annuitants (should the number of members continue what it is) cannot increase to much less than double their number last year.

inadequate.

inadequate.—For example. (Supposing a just allowance required for the wife's inferiority of age) an annual payment of seven guineas from every member, begun in 1772, when the number of widows was only 42, would probably have been sufficient to support the reformed scheme mentioned in the last page; but *now* an annual payment of nine guineas from the *present* members, and of seven from all *future* members, would scarcely be sufficient.

In such circumstances it seems best to break up, and to divide the present capital, as far as it will go, among the annuitants. Should this be done, the annuitants will indeed be great sufferers; for, so miserably circumstanced is the Society, that its whole stock will not pay much more than *half* (a) the value of the annuities. But this only sets in a stronger light the necessity of an *immediate* dissolution: I say *immediate*; for the annuitants are increasing fast, together with the medium of the annuities due to them; and therefore the consequence of delay must be, extending *greater* sufferings to *greater* numbers.

(a) The mean age of the widows now on the Society is not probably more than 42 or 43. The value of annuities payable half-yearly during the lives of women at this age, is not really so little as 13 years purchase, reckoning interest at 4 *per cent.* Supposing therefore the Society to break up some time or other before the end of this year (1781), and the number of widows then upon it 154, the value of their annuities, at 350*l.* each, will be 53,900*l.* of which their stock, at its present price, will not pay so much as 28,000*l.*

P O S T S C R I P T.

SINCE the preceding account was written, this Society, convinced at last of its mistakes, has resolved to reduce the annuities of such widows as became claimants before the alteration in 1774, *35 per cent.* and the annuities of all the other widows who are now claimants, *20 per cent.* It was also resolved by two general courts, that the annual payments should be increased from five to six guineas. But this resolution was revoked by the general court in July last; and a resolution substituted in its room to change the plan described in p. 110, into one which makes it necessary that a member should have been admitted 3 years to entitle his widow to an annuity of 10*l.* and 7 years to entitle her to 20*l.*; and 13 and 20 years to entitle her to 30*l.* and 40*l.*—Possibly this resolution, like the former, will be retracted by some future general court. Should it be confirmed, it will, in conjunction with the reduction just mentioned of the annuities payable to present claimants, constitute a reformation which (supposing the funds of the Society not incumbered with any expences of management) might have been *nearly* sufficient *ten or twelve years* ago to save the Society. But it is far from being sufficient *now*—The reduction of the annuities payable to present claimants is too little. It should have been extended to
the

the widows of the old members now living, and the new plan restricted to members *lately* admitted, and to *future* members; and a compensation for past over-payments to widows should have been provided.

Further Account of the Association among the London and Middlesex Clergy; and of the Establishment among the Ministers and Professors in Scotland.

THE Clergy of London and Middlesex agreed, in 1775, to new rules and orders, by which such fines were required (on account of a subscriber's exceeding the age of 40, and being older than his wife) and such reductions made in the annuities as would probably have rendered the contributions adequate to the expences of the association. But the event has been, that in consequence of this necessary reformation, the Association has dwindled, and is now sunk so low as not to be likely to subsist much longer.

On the contrary. The Establishment among the ministers and professors in Scotland has prospered to a degree which gives reason to believe that it cannot fail to answer the hopes of the venerable body interested in it. This has been owing chiefly to the great ability and faithful zeal of the Rev. Dr. Webster, its founder and conductor.—To the account already given of it in the Second

Section, I will take this opportunity to add the following particulars.

Dr. Webster (having, when the plan was first formed 38 years ago, no certain *data* to go upon) assumed 52 as the medium age at which the widows of ministers would commence annuitants. By calculating on this supposition, and taking the chances of life as they are in Dr. *Halley's* Table, he found that the number of annuitants on the scheme at Lady-day 1780 would be 310. —The fact is, that they were then 304; and that consequently there was, even in this way of calculating, a difference of *six* in favour of the funds which support the scheme. —Since the establishment of the scheme it has been discovered, that the medium age just mentioned does not probably exceed 47. Dr. Webster, therefore, some years ago, in order to put the scheme to a severer trial, instituted a new calculation, on the supposition that the medium age is no more than 44, and found that on this supposition the number of annuitants at Whitsuntide 1780 would be 328. This has made a still greater difference in favour of the Establishment, and gives a very encouraging prospect of its stability; a sufficient income having been in reality provided for bearing the expences, had the annuitants increased as in this last calculation.

Had 52 been the mean age of the widows when they commence such, the *maximum* of
widows

widows living at one time derived from 20 left annually, would be 334, according to Dr. Halley's Table; but supposing it no more than 44, this *maximum* would exceed 400; and the enquiries which have been made, give reason to expect that it will not fall much short of this number. Dr. Webster, therefore, has in his last calculations, reckoned upon the increase of the annuitants to this number; and for this reason, and to secure more certainly the Establishment, a new act of parliament was procured in 1779, by which, among other new provisions, it was ordered, that the increase of the capital (then amounting to 75,088*l.*) should not be discontinued till it rose to 100,000*l.*—This capital, joined to the annual contributions, will probably be an ample support to the Establishment, should the number of annuitants (which will go on to increase for near forty years more) become at last 400.—Circumspection and caution, however, continue to be necessary, because still unfavourable events may arise, which no human wisdom can foresee.

Having bestowed a good deal of attention on this institution, I cannot take leave of it without congratulating Dr. Webster on his happiness. By being the founder of this scheme, and by the care with which he has watched its progress, and conducted it to its present state of maturity, he has entitled himself to the blessings of many indigent

widows and orphans, and made it impossible that he should be ever remembered in the church of Scotland without gratitude and respect.

It is much to be wished that institutions of the same kind, could be established in *England*. Some efforts have been made to this purpose. The reverend and ingenious Mr. *Gandy* of Plymouth, having with much labour and ability prepared a plan of this kind, endeavoured in 1774 and 1775 to get it established in the diocese of *Exeter*. Had he succeeded, the benefits arising from it would have become in a little time very conspicuous, and an example would have been given which would perhaps have been followed in other dioceses. But he did not meet with sufficient encouragement, and the scheme was given up.—Being unwilling that the time and pains which were employed in digesting and calculating the tables for this scheme should be entirely lost; I have inserted some of the principal of them among the Tables in the next volume.—The Rev. Mr. *Grant*, of *Henly* upon *Thames*, has been also lately engaged in soliciting encouragement to a similar scheme; and I heartily wish him the success his benevolence and abilities deserve.

*Account of a Scheme established among the
East-India Commanders.*

THE *East-India Commanders* six years ago entered into an association for the purpose of providing for their *nominees*, and did me the honour to desire I would recommend a scheme to them. They approved the following, and have adopted it.—Every member is entitled to 500*l.* payable at his death to his *nominee*, in consideration of 50*l.* at his admission, and *eleven annual payments* of 25*l.* the first to be made at the beginning of the second year, and the right to every payment to depend on the continuance of the life of the subscriber. No subscribers must be admitted whose ages are not less than 50; and their mean ages are reckoned at 40.

In calculating this scheme, interest was reckoned at $3\frac{1}{2}$ *per cent.* (a); but a much

(a) The value, by Table VI. in the next volume, reckoning interest at $3\frac{1}{2}$ *per cent.* of an annuity of eleven years, on a life aged 40, first payment to be made at the end of a year, is by *Quest. VI. Chap. I.* 7.895.—The annuity, therefore, being 25*l.* its value is 197*l.* to which 50*l.* (the first payment) added makes 247*l.*; which is also, by *Quest. X.* the value, reckoning the same interest, of 500*l.* payable at the death of a person aged 40.—The same contributions, supposing money improved at 5 *per cent.* would entitle a nominee to 600*l.*

higher improvement has for some years been made of money, and the scheme has escaped the danger of being too much loaded in its infancy. I am satisfied, therefore, that, without altering the contributions, the sum payable to *nominees* may with perfect safety be increased to 550*l.* Indeed, I should be in no pain were it even advanced to 600*l.* provided only the contributions were made in *guineas*, instead of *pounds*, and all savings made to accumulate in the short annuity for 27 years from Christmas last.—The progress of such an association will be as follows.—Suppose it to consist of 46 members, kept up from year to year by admitting, as old members die off, new ones at the mean age of 40. At first, according to Mr. *De Moivre's* hypothesis, only *one* member will die annually; but, after a certain period, *two* will die annually. During this interval there will be savings which will raise a stock, the interest of which, when added to the annual contributions, will be just sufficient, when two members come to die annually, to pay two claims.—Supposing each claim 550*l.* the expence of the association, *when greatest*, will be 1100*l.* *per ann.* The contributions at that period will be first, 50*l.* each (or 100*l.* in all) from *two* new members admitted every year; and 25*l.* each
(or

(or 450*l.* in all) from 18 * *other* members who had not been admitted more than eleven years. The remainder (or 550*l.*) necessary to make up 1100*l.* *per ann.* will be the interest of the capital, which, therefore, if lodged in the *three per cents*, must be 18,333*l.* When, therefore, any such association consisting of 46 members has raised this stock; or, if it consists of any other number of members, when it has raised a stock in the same proportion to it, that the standing number of members bears to 46, it will become a reasonable object of consideration whether the increase of its stock should not be discontinued, and all subsequent savings, should any arise, be employed in either lessening the contributions or increasing the claims.

S E C T. IV.

Account of some foreign Institutions for the Benefit of Widows.

IN the Preface to the first edition of this Treatise, I took notice of an institution for the sale of annuities payable on

* Out of a body of new members derived from two admitted every year at 40 for 11 years, it may be expected that two will die before the end of the 11th year.

survivor-

survivorship, established at *Amsterdam*, which seemed to be then much encouraged, and into which, I had been informed, many had entered from different parts of *Europe*. This was so wretched a deception that it was impossible it should long stand its ground; and I am told that it now exists no more. I have, therefore, expunged the notice I took of it in that Preface; and I will not here give any further account of it.

In 1739, an institution was established in *Denmark* under the patronage and guarantyskip of the King of Denmark, which, without regarding ages, promised pensions to widows at the rate of 40 rixdollars *per ann.* for life, from the commencement of widowhood, for every present payment of 110 rixdollars. This being less than the true value of such pensions, the fate of this scheme has been the same with that of the *Mercers Company* mentioned in page 104. At the end of the year 1778, its whole fund was exhausted, and the King of Denmark found himself burdened with the support of 700 widows, and an obligation to support as many more as would be derived from 1500 marriages then remaining undissolved.

At *Bremen*, an institution was established in 1760, which promised annuities to widows for a payment at admission of a sum equal to one yearly payment of the annuity purchased, and an annual contribution during

ing marriage of 15 *per cent.* (or a little more than a 7th) of the annuity. These payments are not much more than half the proper compensation for the annuities. The conductors of the scheme have therefore been obliged to reduce the annuities 10 *per cent.*; and they will soon be obliged to reduce them much more.

The states of the dutchy of *Calenberg*, of which *Hanover* is the capital, established in 1767 a like scheme, but on terms still more deficient; for, though it differed from the two former schemes in paying a regard to the ages of married persons, yet, notwithstanding several augmentations, the contributions required by it did not two years ago come up to half the value of the annuities. Great numbers, influenced probably by the lowness of the terms and the authority of the states, have been induced to encourage this institution. In 1779, it had annuities to pay to 600 widows, and consisted of no less than 3800 members or subscribers whose widows would be entitled to annuities. In consequence of a rapid increase, its insufficiency was not then become palpable enough to force either a dissolution, or a timely and effectual reformation. It was, therefore, likely to lay the foundation of great confusion and distress.

There are probably many other foreign bubbles of this kind, of which I have no knowledge.

knowledge. The information which has enabled me to give this account I owe to Mr. OEDER of *Oldenburg*; and it is with particular satisfaction, that I can from him on this occasion add an account of one foreign institution for the benefit of widows which is founded on just principles, and likely to be productive of great good.—The plan of this institution has been formed and the calculations for settling its terms have been made by Mr. *Oeder*, who appears indeed to possess an acquaintance with this subject so extensive and correct as to be perfectly qualified for such an office.—This institution is intended only for the benefit of the inhabitants of the diocese of *Lubec* and the dutchy of *Oldenburg*; and the sovereign of this state has himself given it the sanction of a statute, and guarantied to his subjects the advantages it promises. At prices deduced by calculations at 4 *per cent.* from Mr. *Susmilch's* Table of Observations (See the Tables in the next volume), and agreeing nearly with the prices deduced by the rules in *Quest. I. and IV.* from the *Northampton* Table (or Table VI. in the next volume), it offers to a married man any annuity for his widow not exceeding 500 rixdollars, (or about 88*l.* *per ann.*) (*a*) payable for life; but with a power reserved to the husband of directing that it shall be applied to the support

(*a*) Six German rixdollars make about a guinea.

of his children in case his widow should marry.

This institution farther enables a parent to provide for his children annuities, (not exceeding 500 rixdollars) payable to them in the event of their survivorship, till they are 25 years of age.

For example. To a husband, aged 35, this institution promises a life annuity of 10*l.* payable to his widow, for either an *annual* payment of 2*l.* 15*s.*—3*l.* 1*s.*—3*l.* 7*s.* 4*d.*, &c. or a *single present* payment of 31*l.* 7*s.* 3*d.*—36*l.* 6*s.*—41*l.* 6*s.* &c. according as he is of the *same* age with his wife, or 5, 10, &c. years *older*.

And if he wishes to make a provision for any of his children, provided he should leave them orphans under age, he may purchase annuities payable to them from the time they shall happen to survive till they are 25 years of age, at the rate of an annuity of 10*l.* for every *annual* payment during the joint lives, of 1*l.* 17*s.* 4*d.*—1*l.* 14*s.*—1*l.* 7*s.* 4*d.* or a *single* payment of 19*l.* 14*s.*—18*l.* 12*s.* 9*d.*—12*l.* 7*s.* according as the child's age is two, five or ten (a) years.—

(a) It is obvious that the values in this case for every annuity of 1*l.* are, in a *single* payment, the *excess* of the value of the life of the child for as many years as his age is less than 25, above the value of the joint lives for the same time found by the *Scholium* to Quest. VI.; and, in *annual* payments beginning immediately, the *Quotient* arising from dividing the single payment by the value just mentioned of the joint lives, with unity added.

These

These values are greater or less as the age of the parent is greater or less; and all the prices of such annuities, and also of annuities for widows, are specified in Tables, for all ages and all *differences* of ages.

The sufficiency of the receipts to answer the expences in this institution, as far as it provides annuities for widows, has been proved by Mr. *Oeder* in the clearest manner from accounts which have been collected in the dutchy of *Oldenburg* of the duration of 1273 marriages, and compared with accounts of the duration of the widowhoods derived from these marriages.—One circumstance in these accounts deserves particular notice.

The ages of the men, one with another, when the marriages just mentioned commenced, was $32\frac{1}{2}$ years; of the women 28 years. The men lived after marrying $27\frac{3}{4}$ years; the women, 31 years and nine tenths. The former, according to Mr. *Susmilch's* Table (and also nearly according to the *Northampton* Table of Observations), should have lived only 27 years and one tenth; and the latter, 29 years and eight tenths. The former, therefore, having exceeded the duration of life exhibited in the Tables only *six tenths of a year*, but the latter having exceeded it above *two years*, it follows that women, notwithstanding the hazards of the critical periods and of child-bearing, live longer than men.

I will

I will add, that by examining 154 of these marriages, I find their duration to have been, one with another, 21 years and a quarter, and the duration of the survivorship of the widows derived from them, 19 years. Had Mr. *De Moivre's* hypothesis of an equal decrement of life been just, the latter would have been *longer* than the former. The reason why the contrary happens has been given in p. 95, &c.

In these marriages (if I may judge from examining only 140 of them) *four* widows were left to *three* widowers, which shews a chance of survivorship in favour of the wife in marriage, greater than could have taken place, had their been no other reason for it than inferiority of age.

At *Hamburg*, an annuity scheme has been lately established of a more comprehensive nature than any of the schemes which have been hitherto mentioned; but the account of it will be more properly given at the conclusion of the last Section of this Chapter.

S E C T. V.

Of Schemes for providing Annuities for Old Age.

A GENERAL disposition has lately shewn itself, to encourage schemes for granting *annuities* to persons in the latter stages of
of

of life ; and this has occasioned the 6th Question in the former chapter ; and, as a further and more particular direction in cases of this kind, I have thought it necessary here to give the following Table.

Values of <i>1l. per ann.</i> for life, after 50, to persons whose ages are	Values in one <i>present payment,</i> interest 4 per cent.	Interest 3 per cent.	Values in annual payments, till 50, to begin at the end of a year, interest 4 per cent.	Interest 3 per cent.
10	1.235	2.015	.0789	.113
15	1.583	2.444	.106	.146
20	2.028	2.989	.146	.193
25	2.594	3.644	.203	.259
30	3.369	4.508	.297	.366
35	4.446	5.667	.466	.559
40	5.953	7.232	.822	.950

Values of the same annuity, after 55, to ages			Values in annual payments till 55.	
30	2.114	2.937	.197	.211
35	2.722	3.632	.241	.297
40	3.732	4.708	.394	.464
45	5.088	6.115	.703	.803

Values of the same annuity, after 60, to ages			Values in annual payments till 60.	
35	1.667	2.290	.135	.168
40	2.234	2.923	.203	.245
45	3.043	3.811	.327	.384
50	4.255	5.061	.600	.679

The numbers in the 2d and 3d columns of this Table, multiplied by any annuity, will give the value of that annuity in a *single* payment,

payment, to be enjoyed for life, by the ages corresponding to those numbers in the first column, *after* the age at the head of that column.—And in the same manner; the numbers in the 4th and 5th columns will give the values in *annual* payments.—Thus: The value of 44*l. per annum*, to be enjoyed for life, after 50, by a person now 40, (interest at 4 *per cent.*) 5.95, multiplied by 44, or 1.261.9, in a *single* payment; and .822, multiplied by 44, or 1.36.16, in *annual* payments till 50, the first payment to be made at the end of a year.

In order to find the same values, partly in *annual payments*, and partly in any given *entrance* or *admission money*; say, “As the value of the *given annuity* in a *single* payment, (found in the way just mentioned) is to the given *entrance money*; so is its value in *annual* payments, to a fourth proportional; which, subtracted from the value in *annual payments*, the *remainder* will be the annual payment due, over and above the given entrance money.”

E X A M P L E.

Suppose a person now 40, to be willing to pay 200*l.* entrance-money, *besides* such an annual payment for 10 years as shall, together with his entrance-money, be sufficient to entitle him to a life-annuity of 44*l.* after 50. What ought the annual payment to be?

VOL. I.

K

ANSWER.

A N S W E R.

L. 8.55.—For, *L.* 261.9, is to 200*l.* as *L.* 36.16, to *L.* 27.61; which, subtracted from *L.* 36.16, the remainder is *L.* 8.55.

This Table has been calculated from the *probabilities* of living in Table V. at the beginning of the next volume, and Mr. *De Moivre's* valuation of lives.—The probabilities of life among the inhabitants of *London*, are (as I have often had occasion to observe) much lower than among the generality of mankind; and the values in the preceding Table, had they been given agreeably to the *London* Observations, would have been less. But, certainly, an office or society, that means to be a permanent advantage to the public, ought always to take higher rather than lower values, for the sake of rendering itself more secure, and gaining some *profits* to balance *losses* and *expences*.

There have lately been established, in *London*, several societies for granting such annuities as those now mentioned; and he that will compare their true values, as they may be learnt from the preceding Table, with the *terms* of admission into these societies, as given in their printed *Abstracts* and *Tables*, must be surpris'd and shocked. They
are

are all impositions on the public, proceeding from ignorance, and encouraged by credulity and folly.

It has been shewn; that the proper payment (allowing compound interest at 4 *per cent.*) for an annuity of 44*l.* to be enjoyed by a person now 40, for what may happen to remain of his life after 50, is 200*l.* in *admission-money*; besides 1*l.* 8*s.* 55, or 8*l.* 11*s.* in annual payments till he attains 50, the first of these payments to be made at the end of a year.—The conditions of obtaining this annuity, according to the present Tables of the *Laudable Society of Annuitants (a) for the Benefit of Age*, consisting of about 1300 members, are (a) 76*l.* 17*s.* in *admission-money*; and 6*l.* 14*s.* in *annual payments*.—According to the Tables of the society of *London Annuitants for the Benefit of Age*, the conditions of obtaining the same annuity are 30*l.* in *admission-money*, and 10*l.* in *annual payments*.—The *Equitable Society of Annuitants* requires for the same annuity 38*l.* 10*s.* in *admission-money*, and 13*l.* in *annual payments*. The true value is, over and above the *admission-money* just mentioned, an *annual payment* of 30*l.* 17*s.* (interest reckoned at 4 *per cent.*) or an *annual payment* of 36*l.* 15*s.* (interest reckoned at 3 *per cent.*)—The *London Union Society for the*

(a) The first members of this Society have paid no admission-money; and are now expecting 44*l.* *per ann.* for contributions which do not entitle them, one with another, to 10*l.* *per ann.*

comfortable support of aged members promises an annuity of no less than 50 guineas for life, after 50, to a person now 40, for 40*l.* 10*s.* admission-money, and 7*l.* in annual payments.

The *Amicable Society of Annuityants for the benefit of age*, promises an annuity of 26*l.* per annum, for life, to a person now 40, after attaining 50, for 28*l.* 16*s.* in admission-money, and 6*l.* in annual payments.—The true value of this annuity is 28*l.* 16*s.* in admission-money, and 17*l.* 8*s.* in annual payments, (interest supposed at 4 per cent.); or the same sum in admission-money, and 20*l.* 18*s.* in annual payments, interest supposed at 3 per cent.

The *Provident Society for the benefit of age*, consisting of 1280 members, promises an annuity of 25*l.* to a person now 40, after the age of 50, for 34 guineas in admission-money, and eight guineas in annual payments. The true value is, 34 guineas in admission-money, and 15*l.* 12*s.* in annual payments, interest at 4 per cent.; or, the same sum in admission-money, and 19*l.* in annual payments, interest being at 3 per cent. (a).

But I will not tire the reader, by going, in this manner, through the schemes of all these

(a) The account here given of the terms on which a person whose age is 40, is admitted into these societies, I have taken from their printed Tables as they stood at the end of the year 1770.—In the younger ages, the deficiencies are greater.

societies. The contrivers of them, it is certain, can know nothing of the principles on which the rule in Quest. VI. and the demonstration of it in Note (B) at the end of the next volume, is founded; and, therefore, if unwilling to be guided by the authority of mathematicians, it may not be possible to convince them of their mistakes. I will however, offer to them the following demonstration, which will be understood without difficulty, by every one who knows how to compute the increase of money at compound interest.

The value of a life at 50, (interest being at 4 *per cent.*) is $11\frac{1}{2}$ years purchase by the first Table in the last leaves of the next volume. For an annuity, therefore, of 44*l.* *per annum* for life, to be enjoyed by a person at this age, 498*l.* ought to be given. *Two in three* of a number of persons at the age of 32 will (by Tables V, VI, and VII, at the beginning of the next volume) live to 50; and therefore, in order to be able to pay an annuity to them of 44*l.* for life, after 50, the money now advanced by every *three*, ought to be such as will, in consequence of being laid up to be improved, increase in 18 years to double 498*l.* that is, to 996*l.*—From the preceding Table it may be learnt, that the money which ought to be advanced by every single person is 165*l.* or by *three* persons 495*l.* and this, in 18 years, will (as may be learnt from Table III. in the next volume) double

itself, or increase to just the sum that will then be the value of the annuities to be paid. —But the money required in this case by the *Laudable Society*, is 14*l.* 11*s.* 9*d.* from each member at admission, besides an *annual* payment of 4*l.* The admission-money, therefore, of two members, being 29*l.* 3*s.* 6*d.* may be increased to twice this sum, or to 58*l.* 7*s.* An annual payment of 4*l.* for 18 years will, if perfectly improved at 4 *per cent.* compound interest, increase to 102*l.*; and two such annual payments will increase to 204*l.* as may be learnt from Table IV. in the next volume.

The whole pay, therefore, of *two* members will produce at the end of 18 years 262*l.* 7*s.*—A third part, I have said, will die before 50 years of age, and these will live one with another 9 years. An annuity of 4*l.* for this time, will produce a capital of 42*l.* 6*s.* See Table IV. in the next volume; and this capital improved for nine years more will increase to 60*l.* The whole profit, therefore, from the member who will die is, his admission-money doubled and added to 60*l.* or 89*l.* 3*s.* 6*d.* And this sum added to 262*l.* 7*s.* makes 351*l.* 10*s.* 6*d.* the *whole* money with which the society can be provided, at the end of 18 years, to bear the expence of *two* life-annuities, worth together 996*l.*

By a similar computation it may be found, that the improvement of money at only 3 *per cent.*

cent. will *sink* the former sum to 324*l.* at the same time that the values of the *annuities* will be *raised* to 1100*l.*

The deficiencies in the schemes of most of the other societies, are no less considerable.—What confusion then must they produce some time or other? How barbarous is it thus to draw money from the public by promises of advantages that *cannot* be obtained? Have we not already suffered too much by *bubbles*?

I have said, that these societies are “impositions on the public, proceeding from ignorance, and supported by credulity and folly.” But this is too gentle a censure. There is reason to believe, that worse principles have contributed to their rise and support. The present members, consisting chiefly of persons in the more advanced ages, who have been admitted on the easiest terms, believe that the schemes they are supporting will last *their* time, and that *they* will be gainers. And as to the injury that may be done to their *successors*, or to *younger members*, it is at a distance, and they care little about it. Agreeably to this principle, the founders of these societies begin so low as not to require perhaps a *fourth* or a *fifth* of the values of the annuities they promise. Afterwards they advance gradually, just as if they imagined, that the value of the annuities was nothing determinate, but increased with every increase of the society. But, as no

ignorance can believe this, the true design appears to be, to form soon as large a society as possible, by leading the unwary to endeavour to be *foremost* in their applications, lest the advantage of getting in, on the easiest terms, should be lost.—It is well known, that these arts have succeeded wonderfully ; and that, in consequence of them, these societies now consist of persons who, for the *same* annuities, make higher or lower payments according to the time when they have been admitted ; and the generality of whom, therefore, must know, that either more than the values have been required of the members last admitted ; or if not, that they are themselves expecting considerable annuities, for which they have given no valuable consideration, and which, if paid them, must be *stolen* from the pockets of some of their fellow-members (a).

(a) If any person wants more information than I have given him concerning these societies, he should consult a work of great merit, published since the second edition of this treatise, and entitled, *CALCULATIONS deduced from first Principles, in the most familiar Manner, by plain Arithmetic, for the Use of the Societies instituted for the Benefit of old Age ; intended as an Introduction to the Study of the Doctrine of Annuities.* By Mr. Dale, a Member of one of the Societies. In this Treatise there is not only a very ample account given of the insufficiency and iniquity of the schemes of these Societies, but the principles on which the values of all annuities on single lives are determined, and the method of calculating them, are explained with the greatest clearness.

I do

I do not, however, mean to condemn all institutions of this kind. They may be very useful, if the full values are taken, and proper care is used in the *improvement* of money. Interest, in these cases, ought not to be reckoned higher than 3 *per cent.* and, supposing money improved at this rate, a person, for a single payment of 50*l.* before he is 40, might be entitled to a life-annuity of 10 guineas *after* 55; or, if he chuses it, to a life-annuity of 17*l.* *after* 60. But if he pays the same sum before he is 34, he might be entitled to a life-annuity of 14*l.* *after* 55, or 22*l.* *after* 60. 25*l.* would purchase for him *half* these annuities; and 100*l.* *double.*

A society or office that would go on this plan, might do great service. Persons in the lower stations of life might be brought to a habit of industry, in the beginning of life, by striving to get 25*l.* or 50*l.* beforehand in order to purchase such annuities, and thus to make provisions for themselves in the more advanced parts of life, when they will be incapable of labour (a).

It

(a) The benevolent Mr. *Howard*, in his *State of the Prisons in England and Wales*, p. 60. Octavo Edit. gives the following account of an institution at *Harlem* in *Holland*. "In this city," he says, "there is a noble hospital, " airy and spacious, called the *Proeveniers*, in which the " persons admitted are decently provided with meat, " drink, and lodging, during their lives, and a burying- " place at their decease. Persons of all ages, from 20 " to

It is proper to observe here, that institutions of this kind would furnish one of the *safest* ways of providing for widows.—A married man might, by paying 100*l.* before his wife attained 40, intitle her, after 55, or 60, to a life-annuity of 21*l.* or 34*l.* Or, by paying the same sum before she attained 34, he might intitle her, after the same ages; to a life annuity of 28*l.* or 44*l.* (a); and in this case he would have a chance of sharing himself in the benefit of the annuity.

I have called this the *safest* way of providing for widows, because attended with none of the dangers arising from disproportion of age between men and their wives, and from the admission of persons labouring under concealed distempers.

“ to the most advanced periods, are admitted, though it
 “ is not common for any under 40 to apply for admission. At entrance each person pays a greater or smaller
 “ sum, according to his age. If his age is 30, he pays
 “ 4500; if 40, he pays 3900; if 50, he pays 3300; if
 “ 60, 70, &c. he pays 2700, 2100, &c. florins; and
 “ in proportion to these sums, at all the intermediate
 “ ages.—A common table is provided for all that are not
 “ sick or infirm.—It often happens that there are not vacant places when persons desire admission. But many
 “ secure places before a vacancy, by paying 200 florins in
 “ advance of their entrance-money, in consequence of
 “ which their names are inscribed in a list, and the
 “ money deducted from the sum required when they
 “ enter.—Those who chuse to leave the house, receive,
 “ during the remainder of their lives, a certain but small
 “ interest for the sum which they paid at admission.”

(a) The same payment before 30, would entitle to an annuity of 22*l.* after 50.

I cannot

I cannot conclude this Section, without mentioning the following plan of a provision for Old Age.

Let 13 guineas be given as *entrance-money*; and let besides 1*l.* 2*l.* 3*l.* 4*l.* &c. be given at the beginning of the 1st, 2d, 3d, 4th, &c. years, as the payments for these years respectively; and let the last payment be 16*l.* at the beginning of the 16th year. All these payments put together will, according to the probabilities of life in the 5th or 6th Tables in the next volume, (interest being at 4 *per cent.*) entitle a person, whose age was 40 when he begun them, to an annuity, after 15 years, beginning with 15*l.* and increasing at the rate of 1*l.* every year, till at the end of 15 years, or (a) when he has reached to 70, it becomes a standing annuity of 30*l.* for the remainder of his life.

If the addition of three guineas is made to the *entrance-money*, for every year that any life between 30 and 40 falls short of 40, the value will be obtained nearly, of the same annuity to be enjoyed by that life, after the same number of years, and increasing in the same manner, till it becomes *stationary* and *double*.—This plan is particularly inviting, as it makes the *largest* payments

(a) According to the probabilities of life in the *London* Table, this annuity should be greater.—A *Theorem* for finding what the annuity ought to be in these cases, is given in Note (I), at the end of the next volume.

become

become due, when the *near* approach of the annuity renders the encouragement to them *greatest*; and as, likewise, the annuity is to increase continually with age, till it comes to be highest (*a*), when life is most in the decline,

(*a*) The lower part of mankind are objects of particular compassion, when rendered incapable, by accident, sickness, or age, of earning their subsistence. This has given rise to many very useful societies among them, for granting relief to one another, out of little funds supplied by *weekly* contributions: A society of this kind, formed on the following plan, would probably thrive.

Let the society, as its first establishment, consist of 100 persons, all between 30 and 40; and whose mean age may therefore be reckoned 36; and let it be supposed to be always kept up to this number, by the admission of new members, between the ages of 30 and 40, as old members die off. Let the contribution of each member be four-pence *per* week, making, from the whole body, an annual contribution of 85 *l.* 17 *s.*—Let it be further supposed, that seven of them will fall every year into disorders, that shall incapacitate them for seven weeks.—30 *l.* 12 *s.* of the annual contribution will be just sufficient to enable the society to grant to each of these 12 *s.* *per* week, during their illnesses. And the remaining 55 *l.* *per annum*, laid up and *carefully* improved at $3\frac{1}{2}$ *per cent.* will increase to a capital that shall be sufficient, according to the chances of life in Tables V, VI, and VII. in the next volume, to enable the society to pay to every member, *after* 67 years of age, or *upon* entering his 68th year, an annuity, beginning with 5 *l.* and increasing at the rate of 1 *l.* every year for seven years, till, at the age of 75, it came to be a standing annuity of 12 *l.* for the remainder of life.

Were such a society to make its contribution *seven-pence per* week, an allowance of 15 *s.* might be made, on the same suppositions, to every member during sickness; besides the payment of an annuity beginning with 5 *l.*
when

cline, and when therefore it will be most useful.—It is further a recommendation of this

when a member entered his 64th year, and encreasing for 15 years, till, at 79, it became fixed for the remainder of life at 20*l*.

If the probabilities of life are lower among the labouring poor, than among the generality of mankind, this plan will be so much the more sure of succeeding.

In 1773, a pamphlet was published, entitled, *A Proposal for establishing Life Annuities in Parishes, for the Benefit of the industrious Poor*.—"It is," says this writer, "a common (a) observation that the money annually raised for the poor, amounts to, at least, a million a year; and that yet in many places they are but indifferently provided for. To make provision for one's old age is so natural a piece of prudence, that it seems at first sight wonderful, that it should not be generally practised by the labouring poor, as it is almost universally by persons in the higher paths of industry: Nor can their negligence in this respect be accounted for, in any other way so naturally, as by ascribing it to their wanting proper opportunities of employing the money they might save, in some safe and easy method that would procure them a suitable advantage from it in the latter periods of their lives. They know, for the most part, but little of the *public funds*; and when it happens that they are acquainted with them, the smallness of the sum they would be entitled to receive, as the interest of the money they could afford to lay out in them, is no encouragement to them to dispose of it in that way. What inducement, for instance, can it be to a man who has saved ten pounds out of his year's wages, to invest it in the purchase of 3 per cent. *Bank annuities*, to consider that it will produce him six or seven

(a) The amount of the poor-rate for one year at the end of the reign of King Charles II. was 665,362*l*. See *Davenant's works*, Vol. I. p. 38.

—In 1777, it was 1,556,804*l*. according to the returns made in that year to parliament by the overseers of the poor.

this plan that less depends in it on the *improvement* of money than in most other plans.

“ seven shillings a year ? It is but the wages of three days labour.—And if they lend their money to tradesmen of their acquaintance, as they sometimes do, it happens not unfrequently that their creditor becomes a bankrupt, and the money they had trusted him with is lost for ever ; which discourages others of them from saving their money at all, and makes them resolve to spend it in the enjoyment of present pleasure. But if they saw an easy method of employing the money they could spare in such a manner as would procure them a considerable income in return for it at some future period of their lives, without any such hazard of losing it by another man’s folly or misfortune, it is probable they would frequently embrace it : And thus a diminution of the poor rate on the estates of the rich, and increase of present industry and sobriety in the poor, and a more independent and comfortable support of them in their old age, would be the happy consequences of such an establishment. Now this might be effected in the following method.”

First, “ Let the church-wardens and overseers of every parish be empowered, by act of parliament, to grant life-annuities to such of the inhabitants of the parish as shall be inclined to purchase them, to commence at the end of one, two, or three years, or such other future period of time as the purchaser shall chuse, and to be paid out of the poor rates of the parish, so that the lands and other property in the parish that is chargeable to the poor-rate, shall be answerable for the payment of these annuities.—This circumstance would give these annuities great credit with the poor inhabitants, by setting before them a solid and ample security for the payment of them.”

Secondly, “ Let the annuities, thus granted to the poor inhabitants, be such as arise from a supposition that the interest of money is 3 *per cent.* or some higher
“ rate

plans.—But I must leave these hints to be pursued by others.

The

“ rate of interest, if the church-wardens and overseers of
“ the poor think fit to make use of such higher interest.”
Thirdly, “ But at the rate of 3 *per cent.* the purchaser
“ should have a right to an annuity, and the church-
“ wardens and overseers of the poor should be compell-
“ able to grant it.”

Fourthly, “ No annuity depending on one life should
“ exceed 20 *l.* a year.”

Fifthly, “ No less sum than 5 *l.* should be allowed
“ to be employed in the purchase of an annuity.—
“ This is to avoid intricacy and multiplicity in the ac-
“ counts.”

Sixthly, “ An exact register of these grants should be
“ kept, by the church-wardens and overseers of the
“ poor, in proper books for the purpose, in which the
“ grants should be copied exactly, and the copy of each
“ grant subscribed by the person to whom it is granted.
“ And this copy, in the register-book of the parish,
“ should be good evidence of the purchaser's right to
“ the annuity, in case the original deed of grant to the
“ purchaser, which was delivered to him at the time of
“ the purchase, should be afterwards lost.”

Seventhly, “ The money thus paid to the church-
“ wardens and overseers of the poor for the purpose of
“ life-annuities, should be employed in the purchase of
“ 3 *per cent.* Bank-annuities in the joint names of all
“ the church-wardens and overseers, and by them trans-
“ ferred at the expiration of their offices to their suc-
“ cessors, and so on to the next successors for ever, so
“ as to be always the legal property of the church-
“ wardens and overseers of the poor for the time being,
“ in trust for the persons who should be entitled to the
“ several life-annuities, granted in the manner above-
“ mentioned; and the interest of this money should
“ be received every half year, and invested in the pur-
“ chase of more principal continually, so as to make a
“ perpetual fund for the payment of the annuities, &c.

&c.

The body of dissenting ministers in London had under consideration some time ago a plan of this kind; and a set of Tables were composed for them. The design was dropped; but as it is possible it may be taken up again, and the Tables may be of use, I have thought fit to preserve them by inserting them among the additional Tables and Observations in the next volume.

Additional Account of the Societies for the Benefit of old Age.

SINCE the publication, in three former editions, of the Observations in the preceding Section, almost all the societies mentioned in it, convinced of the insufficiency and pernicious tendency of their plans, have dissolved themselves, and distributed among their subscribers the money they had paid, with such interest or profit as remained after

“ &c. Deficiencies, if any should ever happen, to be made good by the poor-rates, &c. &c.”

The very able and public spirited and worthy writer of the pamphlet from which I have taken this quotation, now *Cursitor Baron of the Exchequer*, took great pains to carry into execution the design he has explained in it. With this view, a bill, with suitable Tables annexed, was brought into the House of Commons and supported by the excellent Sir *George Saville*, the late Mr. *Dowdeswell*, and many others of the most respectable members. It passed that House without much opposition, but was rejected in the House of Lords.

deducting

deducting the expences of management; and there are now left within my knowledge only two of these societies which require any particular notice. I mean, the *Amicable* and the *Laudable Societies for the Benefit of Age*. The first of these Societies, mentioned in p. 132, finding upon examination that, instead of an annuity of 26*l.* they could not in reality afford to pay a higher annuity than 8*l.* determined, with great fairness, to leave it to the option of all their members, either to continue their contributions with a view to this reduced annuity, or to take back all they had paid and withdraw. Near two hundred members having chosen the former, the Society now consists of *them* only, and therefore can scarcely be in any danger.—The other Society, mentioned p. 131, has also sunk the annuity it promises from 44*l.* to 24*l.*; but it is certain, that it cannot permanently pay to all its members a greater annuity than 15*l.* I should lose too much time were I to give an account of the calculations which prove this. He that would see it demonstrated with all possible clearness, should consult a Tract published in 1777, by Mr. Dale, entitled, *A SUPPLEMENT to Calculations deduced from first Principles, &c.* No person who understands common arithmetic can avoid being convinced by the evidence offered in this Tract, nor can any *honest* man avoid being shocked by the narrative it

contains of the obstinacy with which the majority in this Society have persevered in error, contrary to the efforts of the more respectable part of the Society; and in defiance of reason, justice, and humanity. I cannot, in short, speak more properly on this subject than in the words of Mr. *Morgan* in his *Treatise on the Doctrine of Life Annuities and Assurances*, p. 47. “ There is ONE Society
 “ for the benefit of old age still left, on
 “ which, none of the calls of justice and
 “ humanity have been able to make any proper impression. I mean the *Laudable Society* of Annuitants, whose office is held at
 “ the bottom of *Bartholomew-Lane*. In opposition to the *plainest* evidence, this Society goes on to offer *double* the annuity it
 “ can afford to pay; and the late transactions in it (as related by Mr. *Dale*
 “ in his *Supplement*) exhibit an instance
 “ of such an obstinate and wilful perseverance in imposition as has seldom
 “ been equalled.—I am sorry to add”, says Mr. *Morgan*, “ that this censure is applicable to another Society called also LAUDABLE, but in reality PERNICIOUS, as many
 “ suffering widows will some time or other
 “ experience.”

It is here said, that this Society promises *double* the annuity it can pay; that is, 24*l.* when in reality it can pay only 12*l.*; whereas I have said that it may pay 15*l.* In
 order

order to explain this difference, it is necessary to observe that Mr. *Dale* has shewn that 15*l.* nearly is the annuity which the Society can afford to pay according to the chances of living in Dr. *Halley's* or the *Breslaw* Table; but that the chances of living in the Society had for eight years before 1776 (*a*), (when the number of the Society was above 1300, and its *stock* near 96,000*l.*) been found by particular enquiry to be not much less than *double* to those at the same ages in the Table; from whence it follows, that the annuity payable by the Society being more valuable, it ought not to be so high as 15*l.* nor probably more than 12*l.*—The fact now mentioned is important, but not singular; for it has been found to take place in other similar situations (*b*); and the reason is, that Dr. *Halley's* and most other Tables of Observations give the chances of living as they exist in towns among men of all sorts taken in the gross; whereas such Societies as those for the benefit of old age, and in general all purchasers of life-annuities for *themselves*, must consist of a *selection* of the best lives.

(*a*) I know not what the rate of mortality in the Society has been since this year.

(*b*) See Additional Observations on Civil Liberty, p. 135.

P O S T S C R I P T.

Since the preceding Observations were written, I have been informed that this Society has reduced its annuity from 24% to 20%. This reduction, together with the high interest at which money may be now improved (particularly in the short annuity), will prolong considerably the duration of the Society; but, unless it is favoured by uncommon events, cannot make it permanent.

S E C T. VI.

Of the Amicable Society for a perpetual Assurance Office: And the Society for Equitable Assurances on Lives and Survivorships.

THE 10th Problem has been given, with a particular view to the corporation of the *Amicable Society*, for a perpetual Assurance-Office on single lives, kept in *Serjeant's-Inn*. This Society was established in 1706, and is the only one I am acquainted with, which has stood any considerable trial from time and experience. It is limited by its charter to the disposal of shares

shares or numbers (not to exceed 2000) held by *single* lives, and entitling to *claims* when the lives drop. For each of these shares every purchaser pays at entrance 7*l.* 10*s.* besides 1*l.* 11*s.* as the first *quarterly* payment of 6*l.* 4*s.* *per ann.* to be continued during life. An annual dividend of 1*l.* 4*s.* for each share is allowed to every purchaser out of the profits of the corporation, which reduces the *annual* payment for each share to 5*l.* The nett annual income arising from all the annual payments (making 10,000*l.* when the Society is full, and all the shares are disposed of), is equally divided among the nominees of such members as die within the year; which dividend, therefore, is more or less at the end of every year as a smaller or greater number of the members happen to die in that year. In 1757, the Society engaged that this dividend, though it might be more, should not be less on each share than 125*l.* and in 1770 (a), that it should not be less than 150*l.*—No one person is allowed to purchase more than *three* shares; nor are any admitted to be purchasers whose ages exceed 45, or fall short of 12; and all

(a) In 1757, the Society had accumulated by its savings 25,300*l.* three *per cent.* stock, which in 1770, had been increased to 33,300*l.*; and a part of this stock was in these years appropriated to the payment of claims, whenever the number of them in any year should happen to be so great as to render the annual contributions insufficient to make them up to the guarantied sums.

between these ages are admitted on the same terms.

This Society has, I doubt not, been very useful to the public; and its plan is such, that it cannot fail to *continue* to be so. It might, however, certainly have been much more useful, had it gone from the first on a different plan. It is obvious, that regulating the dividends among the *nominees* by the number of members who die every year, is not *equitable*; because it makes the benefit which a member is to receive to depend, not on the value of his contribution, but on a *contingency*; that is, the number of members that shall happen to die the same year with him. This regulation must also have been disadvantageous to the Society; as will appear from the following account of the natural progress of the affairs of such a Society, when established on a right plan.

Suppose a *thousand* persons, whose common age is 36, to form themselves into a Society for the purpose of *assuring* a particular sum at their deaths, to such persons as they shall name, in consideration of a particular annual contribution to be continued during their lives. Suppose the annual contribution to be 5*l.* and the first payment (*a*) to be made immediately. Suppose, like-

(a) Such payment, it has been shewn, Quest. VIII. p. 28, are better than any *half-yearly* or *quarterly* payments, and at the same time they save some trouble.

wife,

wife, the original number of the Society to be constantly kept up by the admission of new members, at 36 years of age, in the room of such as die.—In Quest. X. p. 31, it appears, that an annual payment, beginning immediately, of 5*l.* during a life now at the age of 36, should entitle, at the failure of such a life, to 172*l.* reckoning interest at 4*per cent.* and taking Mr. *De Moivre's* valuation of lives.—A *thousand* persons, all 36 years of age, will die off at the rate of 20 every year. The disbursements, therefore, of such a Society will be, the first year, 20 times 172*l.* or 3440*l.* and its income will be 5000*l.* It will, therefore, at the end of the year, have a surplus of 1560*l.* to put to interest.—In consequence of the yearly accessions to supply vacancies, the number dying annually will be always increasing after the first year. In 50 years (*a*) it will get to a *maximum*; and then the affairs of the Society will become *stationary*, and the number dying annually will be 40, and its annual expence will be 6,880*l.* exceeding the annual contribution, 1,880*l.* But, in the mean time, by improving its surplus monies, it will have raised a capital equal to this ex-

(*a*) This period will (by Quest. III.) be longer if the Society is any time in filling, and admits members at younger ages than 36. It will, for instance, be 84 years, if the Society is ten years in filling, and admits at all ages between 12 and 45.

cess, and, consequently, its affairs will be fixed on a firm basis for all subsequent times.

Suppose now that such a Society, at its establishment, should resolve to divide its whole yearly income among the *nominees* of deceased members. The effect of this would be, that no capital could be raised; that the dividends payable to *nominees* would diminish continually, till, at the time that the greatest number of members came to die *annually*, (or at the end of 50 years), they would be reduced to *half*; and that all claimants, after this period, would receive too little, because the first claimants had received too much (*a*).

(*a*) The reverse of this will take place, if such a Society *begins* with admitting all at all ages, and afterwards changes its plan, and *limits* the age of admission. In this case, the number of *yearly deaths* will be *greatest* at first, and the *dividends smallest*. In consequence of altering its plan, the *yearly deaths* will lessen gradually, and the *dividends* rise; but in time *both* would return again to their original state.

The following facts incline me to suspect, that this remark may be applicable to the *Amicable Corporation*.

First. In their *original charter*, as it is given in their printed abstracts, there is no limitations of age mentioned; but 31 years afterwards, I find a bye-law made against admitting any person who should be above the age of 45, or under 12.—Secondly. In their printed advertisements in 1770, it is said, that in 59 years they had paid, among 3643 claimants, 378,184*l.* from whence it follows, that though the average of their dividends, for 17 years before 1773, has been 154*l.* the average, for 59 years, has been only 104*l.*

At

At the time of the institution of the *Amicable Corporation*, the interest of money was at 6 *per cent.* and, as they admit none whose ages are not under 45, the mean age of admission cannot be much greater than 36. It appears, therefore, that had they avoided the error now mentioned, and gone from the first on the plan I have described, they might have all along paid to each *nominee* 172*l.* besides raising a capital much greater in proportion to the number of members, than that I have specified, from the premiums at admission, forfeitures, and other advantages which they have enjoyed (*a*). Indeed, I cannot doubt but that on this plan, and with these advantages, they might have found themselves always able to pay at least 200*l.* to each *nominee* (*b*).

I have already mentioned one instance in which the plan of this Society is not equit-

(*a*) A surplus from a *thousand* members of only *five shillings per annum*, duly improved at 4 *per cent.* would, in 41 years, produce a capital of 25,000*l.*

(*b*) It should be remembered, that all this was said in the former editions, on the supposition that proper care has been taken to keep out unhealthy persons; that the ages of admission have never exceeded 45; and that the probabilities of life among the members of this Society, are the same with those in the 5th, 6th, and 7th Tables, in the next volume. But I have lately found the truth to be, agreeably to the suspicion expressed in the last note but one, that for many years after the first institution of the Society, members were admitted at all ages.

able.

able. Another instance of this is, their requiring the same payments from all persons under 45, without regarding the differences of their ages; whereas, the annual payments of a person admitted at 45, ought to be double the annual payment of a person admitted at 12.

Further. The plan of this Society is so narrow, as to confine its usefulness too much. It can be of no service to any person whose age exceeds 45. It is, likewise, far from being properly adapted to the circumstances of persons, who want to make assurances on their lives, for only short terms of years.—Thus; the true value of the assurance of 150*l.* for 10 years, on the life of a person whose age is 30, is by Quest. XIV. (interest being at 3 *per cent.*) 2*l.* 13*s.* in annual payments (for 10 years), to begin at the end of the first year; and subject to failure when the life fails. But such an assurance could not be made, in this Society, without an annual payment of 5*l.*—Neither is the plan of this Society at all adapted to the circumstances of persons, who want to make assurances on particular survivorships.—For example. A person possessed of an estate, or salary, which must be lost with his life, has a person dependent upon him, for whom he desires to secure a sum of money, payable at his death. But, he desires this only as a provision against the danger of his dying
first,

first, and leaving a wife, or a parent, without support. In these circumstances, he enters himself into this Society; and by an annual payment of 5*l.* entitles his *nominee* to 150*l.* In a few years, perhaps, his *nominee* happens to die; and, having then lost the benefit he had in view, he determines to forfeit his former payments, and to withdraw from the Society. In this way, probably, this Society must have gained some advantages. But the right method would have been to have taken from such a person the true value of the sum assured, “on the “supposition of non-payment, provided he “should survive.” In this way he would have chosen to contract with the Society; and had he done this, he would have paid for the *assurance*, (supposing interest at 3 *per cent.* his age 30, the age of his *nominee* 30, and the probabilities of life as in the 5th, 6th, and 7th Tables) 3*l.* 8*s.* (a) in annual payments to begin immediately, and to be continued during the *joint* continuance of his own life, and the life of his *nominee*.

(a) The value of 150*l.* payable at the death of a person, aged 30, *provided* he survives another person of the same age, is, by *Quest.* XI. Chap. I. *l.* 45.65; and this value divided by 13.43 (the value increased by unity, of two joint lives both 30), gives *l.* 3.4, or 3*l.* 8*s.*—The value of the same reversion, according to the probabilities of life in *London*, is, *l.* 49.19, in *one* payment; and *l.* 4.16, in *annual* payments, during the joint lives, the first payment to be made immediately.

Further

Further Account of the Amicable Corporation.

THE affairs of this *Corporation* have lately taken a very favourable turn. The dividends, from the annual contributions of 5*l.* for each number, which for eight years, ended in 1769, had not been 150*l.* on each claim, have, for eight years ended at 1779, been nearly 200*l.* (a)—The subsisting shares have increased from 1120, (their number in 1769) to 1990 (their full complement nearly) in 1780; and in the same time the *stock* of the Society has been increased from 33,300*l.* to 51,300*l.* in the 3 *per cent.* annuities; in consequence of which it finds itself now possess, after discharging all expences, of a clear surplus of about 1350*l.* *per ann.*—In these circumstances a proposal has been made to the Society to discontinue the increase of its stock, in order to make use of the surplus in increasing the dividends on claims. This being an inviting proposal, it is not surprising that the Society in general has shewn itself

(a) This dividend for 1780, was 192*l.* 6*s.* 1*d.* $\frac{3}{4}$; but made up to 193*l.* in consequence of a discretionary power given the directors to take what sums they think reasonable from the savings to increase the dividends.

disposed

disposed to accede to it.—The imprudence however of such a step will be evident from the following observations.—It should be considered, that the reason of the late increase of the annual dividends, has been the late increase of the Society by the influx of young members. So great has been this increase, that the Society has been nearly *doubled* in twelve years, and *tripled* in the last 30 years.—It could not, therefore, but happen that the number of deaths should become much less in proportion to the number of members, than they were *before* the increase. The Society being now full, and admitting of no farther increase, the collective age of the members, and, together with it, the annual deaths, will be for some time increasing, till that part of the Society which consists of the late additions (a) come to

(a) An addition of near 400 to the subsisting numbers or shares of the Society was made from 1749 to 1768; and of above 900 more, from 1769 to 1779.—Even that part of the increase of the Society which consists of members to whom the first of these additions has been owing, is at present far from dying off so fast as it will. In truth, 50 or 60 years at least must elapse before this can happen. See Quest. III. in the Treatise on Reversionary Payments. I am told it has been asserted by some belonging to the Society, that a person about 40 or 45 is not less likely to die than a person 20 or 30 years older, from whence, I suppose it is inferred, that the deaths now in the Society may be as numerous as they will ever be. It would be doing an injury to the Society to suppose that
it

to die off *twice* as fast as they did at first. During this interval the dividends will be growing less and less, till at last they will fall below the dividend which the Society has guarantied (or 150*l.*), and produce a necessity of entering into the capital in order to make it up. And the reduction of the capital once begun will proceed faster and faster till it is all spent; and when spent, the Society will be thrown back into the state it was in before its increase, when frequently it could not afford a dividend of 100*l.* on each number.—Such will be the certain effect of adopting the measure I have mentioned. It is, therefore, imprudent in the highest degree; and I will add, that the *injustice* of it is equal to its imprudence; for it is benefiting the *older* part of the present members at the expence of the *younger* members, and all *present* members at the expence of *future* ones. For a few years the dividends will probably, with the

it can be influenced by such assertions. According to all observations on human mortality, a body of men at 65 or 70 will die off twice as fast as a body of men at 40; and a body of men at 75 or 80 will die off *four* or *five* times as fast as a body of men at 50. The human frame, after the age of 12 or 15, is continually wearing out and becoming less capable of combating the causes of mortality; but more or less slowly according to the degrees of firmness with which it was built, and the favourableness or unfavourableness of the situations into which it happens to fall.

help

help of the addition of the annual profits, exceed 200%. which will be paid chiefly among the claimants derived from the older members. But after a course of years they will sink to little more than half; and at that period the Society will consist of the younger part of the present members, and such *new* members as shall be hereafter admitted to fill up vacancies, who will therefore be great *losers*, because their predecessors, by neglecting to improve the estate, made themselves too great *gainers*.

In order to set this in a clearer light, I would desire it may be considered that, according to the mean probabilities of the duration of life, a body of people at the age of 36, will, one with another, live 25 years. It must, therefore, be expected that a *twenty-fifth* part will die annually of a Society which has subsisted any considerable time, and the members of which are admitted at this mean age. Supposing, therefore, 36 the mean age of admission in the *Amicable Corporation*, the time must come when (if kept (a) up to its full complement) a 25th part of the members will die annually, or when 80 numbers (the 25th of 2000) will drop, and produce 80 claims annually.—The whole

(a) Should the Society decline, more than a 25th part will die annually, and the period when this will happen will arrive sooner.

income

income of the Society (consisting of 10,000*l.* *per ann.* from the charter contributions, and 1350*l.* *per ann.* profits), when divided equally among 80 claimants, will give 142*l.* for each claim. The dividend gaurantied in 1770 being 150*l.* there will be a deficiency of 8*l.* in each claim, or of 640*l.* in the total of claims; which sum procured by selling stock in one year will leave a necessity of selling more the next year, and still more the following, and so on through every successive year, till the whole stock falls rapidly to nothing.—It is proper to observe here, that this deficiency of 640*l.* *per ann.* is a *maximum* to which the deficiencies of many preceding years had increased gradually; and that, therefore, the whole stock may be consumed even before the period arrives when the greatest deficiencies will happen.

All this reasoning supposes that no more than a 25th part of the members of this Corporation will hereafter come to die annually; or that they are admitted at the mean age of 36, and live after admission, one with another, 25 years—But these are probably too favourable suppositions. Perhaps, the mean age at which members are admitted (and particularly the purchasers of two or three numbers) may be above 40; and perhaps also, on account of the difficulty there must be in excluding from *such institutions* all bad lives,

lives, the duration of the lives of the members may be somewhat less than is common among persons at the same ages. Should this be true, their duration of life after admission will not exceed 22 years. A 22d part will hereafter die annually. The claims from 2000 numbers will be 91; and the dividend on each claim, with the addition of the savings, will be only 124*l.* 12*s.* which will produce a deficiency of 2311*l.* *per ann.*

If I may judge from what has hitherto happened in the Society, even these last suppositions favour it too much; for, I find, that before the increase (a) which begun in 1750, a *nineteenth* part of the existing numbers dropped annually, which made the di-

(a) This increase seems to have been owing to the limitation of age in 1737. In a few years after this, the greatest part of the members who had been admitted at advanced ages having died off, and the Society consisting chiefly of younger members admitted in their room; the dividends rose, which occasioned a quick *increase* of young members, raised the dividends still higher, till, in 1757, the Society thought itself capable of guarantying a dividend of 125*l.*; and, in 1770, a dividend of 150*l.* These augmentations contributed yet more to increase the Society, and consequently the dividends. It became, therefore, soon full, and now finds itself in the prosperous state described at the beginning of these Observations; a state, which the Society may render *really* and *permanently* prosperous, if they are properly attentive to its causes, and will avail themselves of the opportunity it gives them to rectify the faults in their original plan by increasing their guaranteed dividend in the manner I shall presently propose, instead of discontinuing the increase of their capital.

vidends then fall frequently below 100%. This, however, must have been owing to the admission, for some years after the establishment of the Society, of members at too advanced ages, and the neglect of proper care to exclude bad lives. Much uncertainty in this instance would be removed, and the best guide obtained in conducting the affairs of the Society, by taking an exact account of the mean age at which, for the last ten or twelve years, all members (distinguishing particularly such as have two or three numbers dependent on their lives) have been admitted; and also of the number of years which all admitted, till within the last 30 years, have lived after admission.

Upon the whole. Till new light is given by such an enquiry as this, I must think that, however prosperous the affairs of the Society seem at present, it cannot prudently act on any other expectation than that a period will come when a 22d or 23d (a) part
of

(a) If Mr. *Brand*, the clerk of this Society, is right, the members of it are uncommonly short-lived; for, according to an account which he has given of the duration of the lives of 3826 persons who have been admitted into it, and on whose deaths claims have been paid; they do not, one with another, live after admission 16 years. See *Smart's* Tables republished by Mr. *Brand*, p. 189. See likewise his Treatise on Assurances and Annuities, p. 68.

This, were it true, would be very threatening to the Society. But Mr. *Brand*, in other parts of the *little* that is his own in these Treatises, has erred so palpably, and
shewn

of its members will die annually, and when, therefore, it will want at least an *additional* income of 2000*l. per ann.* to enable it to make good its engagements. A greater additional income might be acquired by continuing to lay up all its savings after dividing the charter contributions—But were it to lay up the *whole of its income* above what may be necessary to divide 170*l.* on every claim, it might advance its *guarantied* dividend from 150*l.* to *this sum.* And this, in my opinion, is the most rational and equitable measure it can adopt.

I know, indeed, that there is a clause in the charter which limits the increase of its estate to 2000*l. per ann.* But this clause shews that the charter was framed with too little foresight; and it must be repealed, or the consequences will be that *danger* to the Society and *injustice* to its future members which I have represented.

The preceding Observations are offered very respectfully to the consideration of this Society. Should any of the members or directors think them worth their attention, I

shewn himself so wonderfully ignorant, that, however possessed in this instance of the means of information, he deserves no credit.—The truth is, in the present case, that he has included in his account such members as have been admitted and have died *lately*; and this must necessarily lead to a wrong conclusion concerning the duration of the lives of the members taken at large.

hope they will reflect, that having no interest to serve, I can mean nothing by them but the preservation of the credit and usefulness of the Society.

S E C T. VII.

Of the Society for Equitable Assurances on Lives and Survivorships.

THE Society which is to be the subject of this Section, has justly styled itself, "*A Society for Equitable Assurances on Lives and Survivorships.*" The business of it is carried on at its office, in *Chatham-Square*, near *Blackfriars Bridge*. It was founded in 1761, in consequence of lectures recommending such an institution, which had been read by *Mr. Simpson*, a name that can never be forgotten while there is any mathematical or philosophical knowledge left in the world. *Mr. Dodson* also, the author of the *Mathematical Repository*, was active in recommending the plan of this Society, and composed Tables for its use.—It assures any sums or reversionary annuities on any lives, for any number of years as well as for the whole continuance of the lives, at rates settled by particular calculation; and in any manner that may be best adapted to the views of the persons

sons assured. That is, either by making the assured sums payable *certainly* at the failure of any given lives, or on *condition* of survivorship; and also, either by taking the price of the assurance in *one present payment*, or in *annual payments* during any single or joint lives, or any terms less than the whole continuance of the lives.—In short; the plan of this Society is so extensive and so important, that I cannot satisfy my own mind, without offering to the gentlemen concerned in the direction of it, the following observations, hoping they will not think them impertinent.

First. They should consider what distress would arise from the failure of such a scheme in any future time; and what dangers there are, which ought to be carefully guarded against in order to secure success. I have already more than once observed, that those persons will be most for flying to these establishments, who have feeble constitutions, or are subject to distempers which they know render their lives particularly precarious; and it is to be feared, that no caution will be sufficient to prevent all danger from hence.

Again. In matters of chance, it is impossible to say, that an unfavourable run of events will not come, which may hurt the best contrived scheme. The calculations only determine probabilities; and, agreeably

to these, it may be depended on, that events will happen on the whole. But at particular periods, and in particular instances, great deviations will often happen; and these deviations, at the commencement of a scheme, must prove either very favourable, or very unfavourable.

But further. The calculations suppose, that all the monies received are put out immediately to accumulate at compound interest. They make no allowance for losses, or for any of the expences attending management. On these accounts, the payments to a Society of this kind, ought to be more than the calculations will warrant, and the interest of money ought to be reckoned low. Mr. *Dodson*, I find, has paid due attention to all this, by reckoning interest, in his calculations for this Society, at 3 *per cent.* and taking the lowest of all the known probabilities of life, or those deduced from the *London bills of mortality* (a). There is, besides,

(a) It ought, however, to be remembered here, that in selling life-annuities to commence either immediately, or after given terms; and also in some other cases, the values come out *less* in consequence of *lower* probabilities of life. Would it in *such* instances, be taking an unfair advantage, to estimate the values by Tables which give the *highest* rather than the *lowest* values? Thus; was the Society to sell 20*l.* *per annum*, for life, to a person now 30, after 50, the value, according to Dr. *Halley's Table*, would, reckoning interest at 3 *per cent.* be 90*l.* in a single

besides, a liberty provided of making a call on all the members, in case of any particular emergency. It is, therefore, highly probable, that this Society must be secure. The last expedient, however, would be a very disagreeable one, should there be ever any occasion for having recourse to it; and, in order to guard still more effectually against danger, it would not, I think, be amiss to charge a profit of 6 *per cent.* on all the payments.—Should the consequence of this prove, that in some future period the Society shall find itself possessed of too large a capital, the harm will be trifling, and future members will reap the advantage. But this leads me to repeat an observation of particular consequence.

As this Society is guided, in every instance, by strict calculation, it is not to be expected that it can meet with any difficulties for many years; because not till the end of many years after it has acquired its *maximum* of members, will the *maximum* of yearly claimants and annuitants come upon it. Should it, therefore, through inattention to this remark, and the encouragement arising from the possession of a large surplus, be led to check or

gle payment; but according to the *London Table*, the value would be only 70*l.*

But in reality the value, even by Dr. *Halley's Table*, is less than the Society, in such a case, ought to take, for the reason mentioned in page 147.

stop the increase of its stock too soon, the consequences might prove pernicious.

Again. I would observe, that it is of great importance to the safety of such a Society, that its affairs should be under the inspection of able mathematicians. Melancholly experience shews, that none but mathematicians are qualified for forming and conducting schemes of this kind.—In short; dangerous mistakes may sometimes be committed, if the affairs of such a Society are not managed frugally, carefully, and prudently. One instance of this I cannot avoid mentioning.

A person, who desires to assure a particular sum to be paid at the failure of his life, on condition of the survivorship of another life, may chuse to pay the value in annual contributions during the continuance of his own single life rather than during the continuance of the joint lives, because the annual contributions, in this case, ought to be much less. But a Society that would practise such a method of *assurance* would hurt itself; for as soon as the life, on whose survivorship the assurance depends, is extinct, the person assured, if then living, would have no longer any benefit in view; and, therefore, would make his payments with reluctance, and in time, perhaps, entirely withdraw them; the consequence of which would be that the Society would suffer a loss by
being

being deprived of the just value of the expectation it had granted. The plan of a Society ought always to be such, as that the losses arising from discontinuance of payment, should fall on the purchaser, and never on the Society.

I must not forget to add, that it is necessary that such a Society should be furnished with as complete a set of Tables as possible. This will render the business of the Society much more easy, and also much more capable of being conducted by persons unskilled in mathematics. It will also contribute much to its *safety*. For in all cases to which Tables can be extended, there would be no occasion for employing any calculators; and, consequently, a danger would be prevented to which, though it is not *now*, it may *hereafter* be exposed; I mean, the danger of happening to trust unskilful or careless calculators.—It is indeed furnished with Tables, by which a great part of its business is transacted; but there are some important Tables which it wants, and with which it should be supplied; and these when composed, together with all its other Tables, should be subject to the revial and examination of the best judges, and afterwards published; with a minute account of the principles assumed and the method taken in composing them. Such a publication would be a valuable addition to this part of science; and it
would

would also be the means of increasing and establishing the credit of the Society.

In Questions 4th, 6th, 10th, 11th, 14th, 15th, and 16th, I have, with a particular view to this Society, given rules by which may be formed every Table it can want, for shewing the values of assurances on the *whole duration*, or any *terms*, of any *one* or *two* lives; and nothing but care and attention can be necessary to enable any good arithmetician to calculate from them.

*Further Account of the Equitable Society,
with an Account of an Institution for the
Sale of Life-Annuities at Hamburgh.*

I Have just referred to the questions in the first Chapter of this Treatise, for the rules by which the values of assurances on any *one* life, or any *two* lives, may be computed. Since the last publication of this Treatise, investigations of this kind have been carried much farther, and this subject, as far as it respects assurances on any number of lives not exceeding *three*, has been nearly exhausted by Mr. *Morgan*, the actuary of this Society, in the Treatise referred to at the end of the First Chapter.—In this work, Mr. *Morgan* has given a distinct account of the state of this Society as he had made it out to January 1777; and to that
account,

account, and to the Observations addressed to the Society in the Introduction to Mr. Morgan's Treatise, I must refer for the fullest information that can be given of the plan and progressive increase of the Society, and of the methods employed to keep in constant view the state of its accounts. I shall here only add, that its increase has been going on ever since with rapidity; that, in the last five years (or since 1776), its *annual income* has been nearly *doubled*, and its *capital* (consisting now in part of land securities) more than *tripled*; that the rate of mortality among the persons assured has continued much below that in the Tables by which it has hitherto made its calculations (a); and that, upon the whole,

(a) The ratio of the decrements of life or rate of mortality in the Society has been, for *twelve* years, from 1768 to 1780,

		to those in London	—	to those at Breslaw
from 10 to 20	—	as 1 to $1\frac{1}{4}$	—	as 1 to 1
20 to 30	—	as 3 to 7	—	as 2 to 3
30 to 40	—	as 4 to 9	—	as 2 to 3
40 to 50	—	as 3 to 5	—	as 8 to 9
50 to 60	—	as 2 to 3	—	as 6 to 7
60 to 70	—	9 to 10	—	9 to 8

The profits of the Society by assurances only on single lives during four years, from 1775 to 1778, were, on an average, 6446 *l. per ann.* In 1779, the Society (in order better to improve a part of their capital) sold 27,000 *l.* three *per cent.* stock, yielding an interest of 810 *l. per ann.* and with the produce of the sale (amounting to 16,696 *l.* 7 *s.* in money) purchased 1334 *l.* of the short annuity for 30 years from the 5th of January 1778; by which change it added to its present income 200 *l. per ann.* after appropriating

whole, it appears at present to possess such a *surplus* of income and stock as places it (if no mismanagement takes place) above danger, except from events the most extraordinary.

In these circumstances, the Society, not willing to raise an exorbitant capital, or to take unreasonable profits, came to a resolution at the beginning of the last year, or 1781, to make such abatements in its demands as its present circumstances render safe, and to settle such new arrangements in its business as may contribute to make it as great a benefit as possible to the public.

In the preceding Section, but more particularly in the Introduction to Mr. *Morgan's* Treatise, I have expressed my wishes that the Society would order new Tables to be calculated from Observations more adapted to the general state of mortality among mankind than those given by the *London* bills of mortality. — I can now inform the public, that such an order has been given and lately

priating 324 *l. per ann.* to accumulation in this short annuity, the effect of which accumulation will be, that if the stocks continue the same, the annuity, with its increase, will always sell for more than the original purchase money; and, if never sold, will, in the two last half-yearly payments, become at least one *half* more than the purchase money. — If the stocks *rise*, the profit from a sale will be increased. If they *fall*, the accumulation will be increased. In every state, therefore, of the funds, the Society will be gainers.

carried

carried into execution—These new Tables are,

First. A Table exhibiting the values of single lives for their whole duration.

Secondly. A Table of the values of single lives for any terms of years not exceeding seven.

Thirdly. A Table of the values, in single and annual payments, of assurances on single lives for terms, and for their whole duration.

Fourthly. A Table of the values of two joint lives for all ages.

Fifthly. A Table of the values, in single and annual payments, of assurances of gross sums, and life annuities payable on the survivorship of one life beyond another.—

The most material parts of these Tables will be found among the other Tables in the next volume. They have been calculated by Mr. *Morgan* with incredible care and industry; and are correct and complete to a degree never before attempted in any Tables of this kind. They are to form the basis of the future business of the Society, and must conduce much to its growing credit and usefulness.

The second, third, and fifth Tables, have been calculated by the rules in Chap. 1st of this Treatise, Question 6th, 10th, 11th, and 14th. And the other Tables, or the 1st and 4th just mentioned, in the method described by Mr.

Mr. *Morgan* in the II^d. Section of the second Chapter of his Treatise on the Doctrine of Annuities and Assurances; a method which, at the same time that it lessens the labour of these calculations, prevents the possibility of falling into any mistakes (*a*).

They are all founded on a Table of the probabilities of the duration of human life at *Northampton*, which will be inserted among the other Tables in the second volume of this work. This Table made a part of all the former editions of this work; but it is, in the present edition, much improved, and gives, I believe, more correctly than any other, the mean probabilities of the duration of human life; and, therefore, as I shall observe again hereafter, seems to be properer than any other for general use.

I had, in the Introduction to Mr. *Morgan's* Treatise, recommended to the Society the observations on human mortality at *Chester*; and I had procured a copy of them from Dr. HAYGARTH, the ingenious founder of them. But the directors of the Society have judged very rightly, that they carry the probabilities of life too high for their business—These Observations, however, are not on this account less important. I have been enabled

(*a*) *Specimens* of the shortest and easiest method of making these calculations by *Logarithms*, and grounded on Mr. *Morgan's* method here mentioned, will be given in a Postscript to the second of the Essays at the end of this volume.

by

by them to make the *Northampton* Table of Observations more complete; and Tables of the decrements and expectations of lives, deduced from them, for both sexes, will be given in the next volume.

The interest of money in calculating the new Tables of the Society has been reckoned; as it was in the old Tables, at 3 *per cent.* This gives the Society (more especially at present when money may be improved at near double this interest) a very great advantage. It likewise possesses the two following advantages.

First. The interest of a large and fast increasing capital, the greatest part of which is a SURPLUS over and above all that is necessary to enable it to make good its engagements.

Secondly. The profits arising from higher probabilities of living among the members of the Society than are exhibited, even in the new Table of Observations, by which its demands are for the future to be governed. This Table differs but little from the *Breslaw* or Dr. *Halley's* Table of Observations, which, as may be seen in the Note, p. 171, gives probabilities of living near a *third* lower than those which have hitherto taken place in the Society.

I believe the Society might now safely trust itself to the security arising from these advantages, and take the payments for assurances

ances in strict conformity to its new Tables, without any charge upon them; and the consequence of this would be, that these payments, which four years ago were reduced a *tenth*, will be further reduced about *two tenths*, or in the whole about 30 *per cent.* (a).

But as it has been the custom of the Society (in conformity to the recommendation in p. 167) to make an addition of 6 *per cent.* to all the payments required by the old Tables, it may, I think, be excused, if, for the sake of greater safety and to provide better for the expences of management, it should make an addition of 3 or 4 *per cent.* to the payments required by the new Tables.

There still remain a few Tables, which perhaps some time or other the Society may think proper to furnish itself with.—I will mention the two following.

First. A Table of the same kind with that mentioned in p. 125, shewing the values of sums payable *at* a given age, and of annuities payable *till* a given age, to a child, should he lose his parent.

Secondly. A Table containing the values of assurances of annuities for the remainder of life after given terms.

(a) It should be remembered, that the reduction here mentioned, does not extend to the *single* payments for assurances of *life-annuities* payable on *survivorship*, the *London* Table of Observations giving these almost as low as the *Northampton* Table.

This

This would inform the public what advantages could be reasonably offered to persons who wish to be purchasers of such annuities; and the avidity with which the deceptions in this way have been encouraged renders such an information particularly proper.—The public, indeed, has been led by these deceptions to entertain such wrong ideas of the terms on which these annuities may be sold, that probably no terms which the Society can afford will appear sufficiently encouraging. There are, however, annuities of this kind which, at the same time that they have the most useful tendency, might possibly invite purchasers.—Suppose, in particular, an annuity of 1*l.* to commence at 56, and to increase at the rate of 1*l.* every year afterwards, (so as to become 15*l.* at 70, 25*l.* at 80, and 35*l.* at 90), was offered for a given sum payable at the age of 30, with proper abatements for every year that the purchase was made before this age—Would not such a proposal be likely to engage attention? And might it not be extremely useful, by holding forth an incitement to industry in the beginning of life, and providing a way of laying out small savings to the best advantage?—According to a high valuation the sum in this case would be about twenty guineas; and the proper abatement about twenty-five shillings for every year

that a purchaser's age, if not less than 20, falls short of 30.

I must not conclude these Observations on annuity schemes without taking particular notice of an excellent example in this way, which is given by a general annuity institution lately established at *Hamburg*; and to which I have referred at the end of the fourth Section, p. 127.

Having received, through Mr. Oeder at *Oldenburg*, an account of this institution, and finding that the conductors of it wish to extend its advantages beyond the limits of *Hamburg*; I embrace with pleasure this opportunity of recommending it, and reciting the following particulars in its plan.

Persons of all ages who may desire to increase their incomes by purchasing annuities for their own single lives, or for the longest of any two lives, may in this institution purchase such annuities.——A person at 50 may receive during his life $7\frac{5}{8}$ per cent. for any sum; at 60, he may receive $10\frac{1}{8}$ per cent.; at 70, 15 per cent.——Persons who depend for a subsistence on the permanency of their capacities for service or labour, may with such savings as they may be able to make in their years of vigour, purchase for themselves a competence for old age. A person at 40 may with 100*l.* purchase 18*l.* per ann. for his life after 55; or, for a payment of 17*s.* 6*d.* every half year till he is 60, he may purchase for his life after that
age

age 6*l.* 12*s.* *per ann.*—Young persons whose fortunes do not produce a sufficient income may, by sinking a part of them, procure the means of a future settlement in life. A boy, for instance, aged 10, may with 100*l.* purchase an annuity of 8*l.* $\frac{3}{4}$ dependent on his life till he is 25 years of age, when it may be supposed he will be provided with other means of supporting himself.

Persons who have friends or relations dependent upon them may purchase for them, either by a single present payment, or by half-yearly payments, any annuities to commence at the time of their survivorship, should that happen, and to be continued during the remainder of their lives. In the case of widows an abatement is made, if the purchaser chuses that the annuity should be paid only during widowhood.

Parents wishing to provide *portions* for their children, or sums for putting them out to apprenticeships when grown up to a certain age, may purchase (by either half-yearly contributions or single payments), such portions or sums to be paid them at that age, should they live to it.

The plan of this institution includes in it several other particulars; but I will only add, that the money received by the conductors of it is lodged in the chamber of *Hamburgh*; that the prices or contributions are distinctly specified for every age in

a set of Tables which have been published at *Hamburg*; and that these Tables have been calculated at an interest of 3 *per cent.* from some of the best registers of mortality, and (as far as I have examined them) with skill and correctness.

C H A P. III.

*Of PUBLIC CREDIT, and the NATIONAL
DEBT.*

THE *National Debt* is a subject in which the public is deeply interested. Some observations have occurred to me upon it, which I think important; and for this reason, though foreign to my chief purpose in this work, I shall beg leave to offer them to public attention.

The practice of raising the necessary supplies for every national service, by borrowing money on interest, to be continued till the principal is discharged, must be in the highest degree detrimental to a kingdom, unless a plan is settled for putting its debts into a regular and certain course of payment. When this is not done, a kingdom, by such a practice, obliges itself to return for every sum it borrows infinitely greater sums; and, for the sake of a present advantage, subjects itself to a burden which must be always growing heavier and heavier, till it becomes insupportable.

This seems to be now the very state of this nation. At the REVOLUTION, an æra in other respects truly glorious, the practice I have mentioned begun. Ever since, the public debt has been increasing fast, and every new war has added much more to it than was taken from it during the preceding period of peace. In the year 1700, it was 16 millions. In 1715, it was 55 millions. A peace, which continued till 1740, sunk it to 46 millions; but the succeeding war increased it to 78 millions; and the next peace sunk it no lower than 75 millions. In the *last* war it rose to 146 millions and a half. During a peace which has lasted now 10 years, it has been reduced to 138 millions: and at a sum not much less than this, it will, perhaps, be found at the commencement of another war, which may possibly raise it to 200 millions (a).—One cannot reflect on this without terror.—No resources can be sufficient to support a kingdom long in such a course. 'Tis obvious, that the consequence of accumulating debts

(a) It should be remembered that this was written in 1773.—In 1774 and 1775, two millions of the capital of the 3 per cent. annuities were paid off, which reduced the capital of the public debt to 136 millions; and at this sum, nearly, (supposing the long annuity then worth 27 years purchase) it stood at the beginning of the *American* war. The addition made to it by this war, and its amount in the present year (1790), may be learnt from the account at the end of the Second Volume of this work.

so rapidly, and of mortgaging posterity, and funding for eternity, in order to pay the interest of them, must, in the end, prove destructive. Rather than go on in this way, it is absolutely necessary, that no money should be borrowed, except on annuities, which are to terminate within a given period. Were this practised, there would be a LIMIT beyond which the national debts could not increase; and time would do that *necessarily* for the public, which, if trusted to the œconomy of the conductors of its affairs might possibly *never* be done.

This, therefore, is one of the proposals to which on this occasion, I wish I could engage attention.—I am sensible, indeed, that the *present* burdens of the state would, in this case, be increased, in consequence of the greater present interest which would be necessary to be given for money: but I do not consider this as an objection of any weight. For let the annuity be an annuity for 100 years. Such an annuity is, to the present views of men, nearly the same with an annuity for ever; and it is also nearly the same in calculation, its value at 4 *per cent.* being $24\frac{1}{2}$ years purchase, and therefore only half a year's purchase less than the value of a *perpetuity*. Supposing, therefore, the public able to borrow money at 4 *per cent.* on annuities for ever, it ought not to give above 1 s. 7 d.

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per cent. more for money borrowed on annuities for 100 years: But should it be obliged to give a *quarter*, or even an *half per cent.* more (a), the additional burdens derived from hence, would not be such as could be very sensibly felt; and the advantages, arising from the necessary annihilation of the public debts by time, would abundantly overbalance them.

These advantages would be, indeed, unspeakably great. By such a method of raising money, the expence of one war would, in time, come to be always discharged before a new war commenced; and it would be impossible that a state should ever have upon it, at any one time, the expence of many wars; or any larger debts than could be contracted within the limited period of the annuities; and, consequently, it would enjoy the invaluable privilege of being rendered, in some degree, independent of the manage-

(a) The annuities might be kept 18 years without being much diminished in value; for, supposing interest at 4 *per cent.* an annuity for 82 years, is, within a 49th part, or 2*l.* in 98*l.* worth as much as an annuity for a 100 years.

Perhaps, in this way of raising money, it might be best to offer a higher interest at first, which should fall to a lower, at the end of given intervals. Thus, though 4½ for 100 years is equal in value to 5 *per cent.* for 17 years, and after that 4 *per cent.* for 83 years, yet the latter might appear more inviting.

ment

ment of its finances by ignorant or unfaithful servants.

I must add, that it is by no means necessary, that the limited period of the annuities should be so long as I have mentioned, or 100 years; and that, at any time before the expiration of this period, the public might employ any surplus monies, in extinguishing part of the annuities, by purchasing them for itself at the market price; and thus it might aid the operations of time, and keep its debts within any bounds that its interest rendered necessary. Our government has, I know, in some instances adopted the plan now proposed; but it is to be wished that, instead of retracting (*a*) it, as was once done, it had been carried much further.

I am, however, far from intending to recommend this plan as the best a state can pursue. There is another method of gaining the same end, which is, on many accounts preferable to it. I mean, “by providing an
“annual saving, to be applied invariably,
“together with the interest of all the sums
“redeemed by it, to the purpose of discharging the public debts: Or, in other words
“by the establishment of a permanent SINK-
“ING FUND.”

(*a*) In the year 1720, the nation was put to the expence of above three millions, in order to reduce several long and short annuities then subsisting to redeemable perpetuities.

It

It is well known, that this plan has been also adopted by our government; but, though capable of producing the *greatest* effects in the *easiest* and *surest* manner, it has never been carried into execution. It will abundantly appear, from what follows, that this observation is just.

Suppose the annual saving to be 100,000*l.* This sum, applied *now* to discharge an equal debt, bearing interest at 4 *per cent.* will transfer to the public, from its creditors, an annuity of 4000*l.* The annual saving, therefore, would be increased to 104,000*l.*; and this saving would transfer to the public another annuity of 4,160*l.* and make the saving at the beginning of the 2d year, to be 108,160*l.*—Thus, the original fund would go on increasing, at the same rate with money improved at 4 *per cent.* compound interest—At the beginning of the 3d year it would be 112,486*l.* At the beginning of the 18th year, 202,581*l.* Of the 36th year, 410,393*l.* and of the 95th year, 4,151,138*l.*—In 94 years, then, the nation might be eased of above 4 millions *per annum* in taxes; and above 100 millions of its debts would be discharged, gradually and insensibly, at no greater expence than 100,000*l.* *per annum*; and without interfering with any of the resources of government; or making any other difference, than causing *funds* to be engaged
for

for a course of time to the *public*, which would have been otherwise necessarily engaged to its *creditors*, and which, therefore, must have been entirely useless to it.

It is an observation that deserves particular attention here, that on this plan, it will be of less importance to a state what interest it is obliged to give for money: For the higher the interest, the sooner will such a fund pay off the principal. Thus; a 100 millions borrowed at 8 *per cent.* and bearing an annual interest of eight millions, would be paid off by a fund, producing annually 100,000 *l.* in 56 years; that is, in 38 years less time than if the same money had been borrowed at 4 *per cent.* (a).

It

(a) What is here said, supposes the *same* fund applied to the discharge of debts bearing *different* interests. If different funds are applied, bearing to one another the same proportion with the interests of the debts which they are to discharge, the benefit derived from borrowing on lower rather than higher interests, will be reduced to almost nothing; for the disbursements of the public on account of all equal loans, will, in this case, be nearly the same.

The following example will explain and demonstrate this:

Let a million be borrowed at 3 *per cent.* and let a fund be charged with it, bringing in *six shillings per cent. per ann.* more than the interest; or 33,000 *l.* instead of 30,000 *l.* *per ann.* This *surplus, unalienably* applied, together with all the interests disengaged by it, will annihilate the *principal* in 81 years, as may be gathered from Table IVth. in the next volume. And the disbursements, on account of the loan, will be 81 multiplied by 33,000 *l.* that is, 2,673,000 *l.* Let us suppose again, a million borrowed

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It follows from hence, that reductions of interest would on this plan, be no great advantage to a state. They would, indeed, lighten its present burdens; but this advantage would be, in some measure, balanced by the addition which would be made to its *future* burdens, in consequence of the longer time during which it would be necessary to bear them.—I mean this on the supposition, that the savings produced by reductions of interest, are immediately applied to the relief of the state, by annihilating taxes equivalent to them. But if that is not the case; and if, likewise, there is either no plan established for putting the public debts into a certain course of payment, or it is not faithfully carried into execution; in these circumstances, reductions of interest may prove hurtful. For, first, They would only furnish funds for contracting further debts, and with more money for supplying the deficiencies arising from profusion and bad management. And, secondly, As, in such circumstances, they would only *retard*, and not *prevent* the increase of the burdens occasioned by the public debts, a period would come when the affairs of the state would

at 6 *per cent.* and let a fund be charged with it, producing a surplus of *twelve shillings per cent. per ann.* such a fund, besides paying the interest, will discharge the *principal* in 41 years; and the disbursements, on account of the loan, will be 66,000 *l.* multiplied by 41; that is, 2,706,000 *l.* or nearly the same with the disbursements on account of an equal loan at 3 *per cent.*

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get near to a *crisis*; and at such a period, its danger would be increased, in proportion to the reductions of interest that had been made.

In order to understand this; let us suppose that a debt, bearing an annual interest of five millions, is the whole debt which a state can bear without being so much oppressed as to be near sinking. Let it, however, be supposed to have still some last resources left, which may enable it to bear, for 23 years to come, this load, together with every additional load, which, during this time, may be necessary to be thrown upon it.—Let it further be supposed, that at this time, the state, urged by the fear of an approaching bankruptcy, resolves upon entering into some effectual measures for preserving itself.—Certain it is, that in such circumstances, no measure *so* effectual can be pursued, as the establishment of a *sinking fund*, and such a faithful application of it as I have explained. Let that then be the measure entered upon; and let the state be supposed capable of providing a fund, producing a million annually. If all the debts bear interest at 6 *per cent.* this fund would pay off three-fifths of them, within the time I have mentioned; or, in 23 years; and the state might be saved. But if, in consequence of reductions, they bear interest at no more than 3 *per cent.* the same fund would not give the same relief, in less than
than

than *double* that time; and therefore, a bankruptcy might prove unavoidable (a).

I wish I could think, that there is nothing in this representation, that can be applied to the present state of this nation. The interest of the public debts has been reduced, at different periods, from 6 to 5, from 5 to 4, and from 4 to 3 *per cent.*; but still they have grown with rapidity; and we now see ourselves overloaded, and in no way of gaining relief. Had there been no reductions of interest, we should, indeed, have been in the same condition sooner; but, we might have been relieved also sooner, and with less difficulty and danger.

In short, Reductions of interest are advantageous chiefly when made to gain additions to such a *sinking fund* as I have described.—When made with other views, they are only *palliatives*, which give *present* relief by increasing *future* danger; or *expedients* which *postpone* a public bankruptcy, by rendering it a calamity more *unavoidable* and *dreadful*. As managed therefore among us, they have been indeed the effects of too narrow a policy, and deserve none of the *encomiums* which have been bestowed upon them.—The preceding observations prove this sufficiently; but there is one farther

(a) It should not be forgotten, that the operation of such a scheme would soon raise the 3 *per cent.* debts nearly to *par*, and render the discharge of them at a discount impracticable.

proof of it which I cannot help mentioning.—Suppose 200,000*l. per ann.* to have been gained in 1716, by the reduction which was then made of the 6 *per cents.* to 5 *per cents*; or, in other words, by saving 1 *per cent. per ann.* on a capital of 20 millions. This saving, in consequence of being applied unalienably in the manner I have represented, to the payment of the public debts, would, in 37 years, have discharged a debt of 20,325,000*l.* bearing 5 *per cent.* interest. But if applied every year to current services, in order to avoid levying new taxes, the benefit derived from it in the same period, would be 37 times 200,000*l.* or 7,400,000*l.* but at the same time, a debt would have been continued of 20 millions, which must have been otherwise paid. The effect, therefore, in this case, of the reduction, would be to prevent an incumbrance on the public of 200,000*l. per ann.* by leaving upon it an incumbrance of a million *per ann.* rendered more difficult and unlikely than ever to be removed (a).

But

(a) There is an inconvenience arising from reductions of interest, which I did not attend to when this Chapter was composed, but which is greater than any here mentioned. They have given rise to the extravagant *douceurs* or *premiums* which have been annexed to our public loans, by causing the lenders of money to consider whatever interest could be offered above 3 *per cent.* as no more than a short annuity.—Had there been no reduction of interest before the war which begun in 1755, the price of the *four per*

But to return to the subject I have principally in view.

per cent. stocks would have been during the whole war either *above* par or not much *below* it, and government might have borrowed at this or a lower interest without *premiums*. But the reduction which had just taken place of 56 millions to an interest of 3 *per cent.* made it unavoidable for monied men to consider this stock as only a 3 *per cent.* stock with 1 *l. per ann.* added till peace^o was established; and this made it necessary for government, either to promise interest extravagantly high, or to offer long, or short, or life-annuities, or additional capitals, as rewards for lending money at 3 *per cent.* The last method has constantly been adopted; and in consequence of it, the public has been put to an enormous expence, which the avoiding of reductions and a little management would have rendered entirely unnecessary.

I have endeavoured to explain this in the second Tract on Civil Liberty, Part II, Sect. III, and particularly in the Supplement to that Tract, where regulations are proposed for raising the value of stocks bearing higher interest than 3 *per cent.* in order to avoid the necessity of offering high *premiums* and creating artificial debts.—The following example will shew what occasion there is for some such regulations.—The 3 *per cent.* stocks were (Oct. 25th, 1781,) at $55 \frac{1}{4}$; and the price of the 4 *per cent.* stocks, which, in order to bear a proportionable price, ought to have been $73 \frac{3}{4}$, was (deducting a fortnight's interest) $70 \frac{5}{8}$. One *per cent.* added to make a five *per cent.* stock, would have been still more undervalued; and such a stock, even with the advantage of being declared irredeemable for 15 or 20 years, would not probably have sold for more than 82 *l.* or 83 *l.* Had, therefore, government attempted to borrow by offering 5 *per cent.* it could not have succeeded without giving a *premium* of near 20 *l.* for every 100 *l.* advanced. But, by such regulations as those proposed in the Supplement to which I have just referred, it would, I imagine, have become practicable to borrow at interests corresponding nearly to the prices of the 3 *per cents.* without *premiums*.

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What I have said implies, that a state always discharges its debts, whatever interest they bear, by paying the original sum borrowed. It may, perhaps, be imagined, that when a loan is under *par*, it may be discharged at a less expence. But this is by no means so practicable as it may seem; for it should be considered, that a public loan, now under *par*, would not long keep so, after being put into a course of payment: And for this reason, as a state can never be obliged, in redeeming its debts, to pay *more* than the original sum borrowed, so neither ought it to expect, in general, to be able to redeem them by paying *less*. I have said, *in general*; for I am sensible that, at the beginning of the operations of a fund, when its produce is small, and also in a time of war, a state might derive great advantages from the low price of its debts. And I am sensible also, that considerable advantages might be derived from *lotteries*, in paying the public debts: But *lotteries* do great mischief in a state, by fostering the destructive spirit of gaming. It is wretched policy to make them familiar, by recurring to them in the ordinary course of government. There are great occasions on which they may be necessary, and for such occasions they should be reserved.

The advantages of putting the public debts into such a course of payment as I have described, are scarcely to be imagined. It would give a vigour to public credit, which would enable a state always to borrow money easily, and on the best terms. And the encouragement to lenders might be always improved, without any inconvenience, by making every loan irredeemable, during the first 20 or 30 years; for, there could seldom be any occasion for beginning to discharge any *one* loan sooner.

It might be easily shewn, that the faithful application, from the beginning of the year 1700, of only 200,000*l.* annually, would long before 1790, notwithstanding the reductions of interest, have paid off above a hundred millions of the public debts. The nation might, therefore, some years ago, have been eased of a great part of the taxes with which it is loaded. The most important relief might have been given to its trade and manufactures; and it might now have been in better circumstances than at the beginning of the last war; its credit firm; respected by foreign nations, and dreaded by its enemies. A new account might also some years ago have been begun; and another fund, not much more considerable, applied in the same way, would, in a short course of years, have paid all that would have been now unpaid
of

of our debts. And thus, without any expence that could be sensibly felt, its debts, as soon as they began to grow heavy, might have been constantly reduced to a *half*, or a *third*; and not only all *danger*, but all considerable *inconvenience* from them prevented.

All I have now said, supposes a *single* fund with a *general* appropriation to the payment of the public debts. The same ends might be answered by *particular* funds, with small surplusses, appropriated to *particular* debts. In the wars of King *William* and Queen *Anne*, an interest of six, and sometimes seven and eight *per cent.* was given for loans. It would have been easy to have annexed to each loan a *fund* producing a *surplus* of 1 *l.* *per cent.* after paying the interest; and such a *surplus* would have been sufficient to annihilate the principal of every loan in 33 years. Had this plan been followed, the disengagement of the public funds, and the relief attending it, would have begun 60 years ago; and the debts contracted, during the reigns of King *William* and Queen *Anne*, would have been all cancelled near 30 years ago, without any of that trouble, tumult, and distress, which have been occasioned by reductions of interest, and by the various schemes which have been tried for lessening the debts (a).—

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(a) The sums to be laid out would, in this case, be so small at first, that it would be proper to employ them in purchasing

A fund, yielding 1*l.* *per cent.* surplus, annexed to a loan at 5 *per cent.* would discharge the principal in 37 years (*b*). At 4 *per cent.* in 41 years. At 3 *per cent.* in 47 years.

These observations relate only to what *might* have been the state of the nation with respect to its debts, had a right plan been pursued from the first. But it will be asked, What can be done with them as they *are*?—I wish I was able to give a more satisfactory answer to this enquiry. Every one must see our prospect to be discouraging, and our state hazardous. Some have thought, that a good method might be found out of discharging the national debt, by short annuities, and life annuities. The following observations will shew how groundless an imagination this is.

purchasing part of the loan to be annihilated at the prices in the public market; and this, as far as it can be carried, is the most easy and quiet and silent way possible of extinguishing the public debts.

(*b*) I have all along supposed the produce of the public funds to come in yearly. The truth is, that it comes in *half*-yearly; but this gives no advantage in the payment of the public debts worth taking into account. 1*l.* *per annum*, together with its growing interests, at 4 *per cent.* taken yearly out of 100*l.* will reduce it to nothing in 41 years; if taken *half*-yearly, it will annihilate the same capital only four months and 12 days sooner.

Short

Short annuities and life-annuities have been always undervalued by the public; and were they offered to sale to such an amount as would be necessary to make any considerable reduction in the national debt, they would probably fall to a very low price. Let the three *per cent.* stocks be supposed at .86. 100*l.* of this stock would not be given up for a life-annuity less than 6*l.* or a short annuity for a less term than 30 years; for this is valuing them at $14\frac{1}{2}$ years purchase, and life-annuities in particular have never yet been disposed of at so high a price.—In order, therefore, to discharge in this way a capital in the 3 *per cent.* annuities of 33 millions and a third (the interest of which is a million *per ann.*) it would be necessary to provide a surplus producing another million *per ann.* for 30 years. But this surplus, employed during the same time as a sinking fund, would redeem 47 millions and a half at *par*, and 51 millions and a half at 86*l.* in *money* for every 100*l.* stock. It would, therefore, be great folly to employ such a surplus in the former way rather than the latter.

But I will beg leave to detain the reader here a little longer——The observations now made may be of use in shewing what the best method is of incurring debts as well as redeeming them.

Suppose a million raised by *annuities* on a set of lives, all at 30 years of age. The purchasers of such annuities cannot reasonably be reckoned to have an expectation of less than 30 years. That is; the duration of their lives, taking them one with another, will be 30 years; and they will be entitled, supposing interest at 4 *per cent.* to 7*l. per annum*, for every 100*l.* advanced. For a million then, the public would make 30 payments of 70,000*l.*—Let us suppose next, that a fund producing this sum annually, instead of being engaged to pay these life-annuities, is engaged for 30 years, to pay the principal and interest of a million, borrowed on *redeemable* perpetuities, at 4 *per cent.* There will, at the end of the first year, be a surplus of 30,000*l.*—In consequence of applying this to the extinction of the principal, it will be reduced to 970,000*l.* on which, at the end of the second year, the interest due will be 38,800*l.* There will, therefore, be a saving of 1200*l.* Instead of employing this saving in further sinking the *principal*, which would cause the fund to accumulate in the same manner with money at compound interest, let it be taken and employed in any other way: And let the same be done with all the subsequent savings, reserving only 30,000*l.* annually, for the purpose of sinking the principal. At the end of the second year, the principal will be 940,000*l.*;
and

and the saving of interest upon it, at the end of the third year, 2400*l.* At the end of the 30th year, the principal will be reduced to 100,000*l.* The saving of interest that year will be, 1200*l.* multiplied by 29, or 34,800*l.* and the sum of all the savings will be 522,000*l.*—Deduct from hence 100,000*l.* remaining then undischarged of the principal; and 422,000*l.* will be the loss the public would sustain, in the circumstances I have supposed, by raising money on life-annuities. But if we suppose the savings, as they arise, as well as the constant sum of 30,000*l.* to be applied to the discharge of the principal, instead of being spent on current surrices; the whole million will be annihilated in 21 years and a half; and the loss to the public by preferring life-annuities, will be $8\frac{1}{2}$ years purchase of the annuities; or 595,000*l.*—By similar deductions it may be easily found, that the loss, in *younger* lives, is greater; in *older* lives less; but never inconsiderable, except in the *oldest* lives.

It appears, therefore, that in consequence of such a way of raising money, the public must always pay much more in interest than there is any occasion for; and *waste* a sum equal to more than half the principal borrowed (a). This, however, though so wasteful,

(a) It is obvious, that the observations here made, may be applied to the common methods of raising money on life-annuities, for building churches, paving streets, making

ful, is a more frugal way of procuring money than by borrowing on *perpetuities*, without putting them into a course of redemption; for in this case the loss may be *infinite*.

But to return.

making navigations, &c. &c. And, in general, to all cases where the money received is not laid up to be improved.—For, to view this subject in another light, let us suppose 10,000*l.* borrowed for any public work, on *perpetuities*, at 4 *per cent.* And, if that will afford more encouragement, let them be made irredeemable for any number of years less than seventeen. Let us further suppose such rates, or tolls, established for the payment of the interest and principal, as shall produce *double* the interest of the sum borrowed; or 800*l.* *per annum*, instead of 400*l.* *per annum.* Let the *surplus*, as it comes in *half-yearly*, be laid up to accumulate in the public funds. In 17 years and a half, reckoning interest at 4 *per cent.* a capital will be raised, equal to the whole sum borrowed; and, therefore, at the end of that time, the whole debt may be discharged, and the whole transaction finished.—But if the same sum had been borrowed on annuities, for the lives of a set of persons 50 years of age, at 8 *per cent.* which is 1*l.* *per cent.* less than the true value of such annuities: Had this, I say, been done, *half* the annuitants would have been alive at the end of the term I have mentioned; and the whole transaction, together with the expences and trouble attending the management of it, could not have been finally closed till the extinction of all the lives; that is, not in less time, most probably, than 35, or, perhaps, 40 years.—It is a necessary observation here, that, if public credit maintains its ground, much will not depend, in the plan now proposed, on the rise and fall of Stocks. If a *war* sinks them, the money laid out, while the war lasts, will accumulate faster. If a peace raises them, the money that had been previously laid out will be proportionably increased.

The

The enquiry which has occasioned this digression, must be interesting to every person who wishes well to this country.—All schemes for discharging the public debts by life-annuities, have been shewn to be absurd and extravagant.—In general it may be observed, that it is far from probable, that any money which the nation can spare, if applied so as to bear only *simple* interest, can be capable of reducing its debts within due bounds; or of doing us, in our present circumstances, any essential service. A fund, producing a surplus of even two millions annually, would, when thus applied, pay no more than 40 millions in 20 years; and, in that time, a war might probably come, which would interrupt the application of it; and increase our debts much more than such a fund had lessened them.

Certain it is, therefore, that if our affairs are to be retrieved, it must be by a *fund* increasing itself in the manner I have explained. The smallest *fund* of this kind is, indeed, *omnipotent*, if it is allowed time to operate. But we are, I fear, got so near to the limits of the resources of the nation, that it cannot be allowed much time: And, in order to make amends for this, it is necessary that it should be *large*.

In redeeming the public debts, considerable advantages might be derived from their low price, and this advantage, in a time of war,

war, would be nearly the same in whatever manner the redemption was made: That is, whether it was always made at *par*, or at the market prices below *par*. The latter method would be the best; but supposing the former adopted, an equivalent advantage would arise from the power it would give to borrow money on lower terms—Suppose, for instance, that in a time of war, four millions must be borrowed for the service of the year; and let the produce of the fund be then increased to two millions; and the interest of money in the *stocks*, above 4 *per cent*. In these circumstances, it would be the interest of the lenders of money, to take $3\frac{1}{2}$ *per cent*. for the sums they advanced, in consideration of having their 3 *per cents*. paid off at *par*, to the amount of half these sums.—War, therefore, would accelerate the redemption of the public debts; and it would do this the more, the longer it lasted, and the higher it raised the interest of money. Or if, in consequence of paying always at *par*, this could not happen; an equivalent effect would be produced in the way just mentioned. The *stocks* would be always kept up by the operations of the fund; and, in proportion to the sums yielded by it, the public would be able to borrow money more advantageously, and less would be added to its burdens.—This seems to me an observation of particular consequence. It demonstrates,

strates, that the invariable application, in *war* as well as *peace*, of the produce of the fund I am supposing, to the payment of the national debts, rather than to any current services, would, independently of its effect in redeeming these debts, be attended with great advantages to the public. But this is a subject on which I shall have occasion to say more presently.

The *sinking fund*, in its present state, and, after supplying the deficiencies of the peace establishment, yields, I suppose, a considerable part of a million and a half *per ann.* (a). And I cannot doubt but that such savings might be made in the collection and expenditure of the national revenue, as would cause this fund to yield the *whole* of this sum, without imposing any new burdens on the public. But, were there, indeed, no way of providing any part of it, but by creating new funds, or imposing new taxes ;

(a) A more careful enquiry has shewn me that the true surplus of the public income was at the time when this was written much less than I imagined. For five years after 1763 it was no more than 30,000 *l. per ann.* For five years before 1775 it was 338,729 *l.*; and the medium for the whole duration of the peace between the two last wars was 196,000 *l. per ann.* according to the accounts in the second Tract on Civil Liberty, Part 3d, Sect. 4th. And from the commencement of the present peace to the present year 1790, the stated revenue has produced *no* surplus, as I fancy will appear from the statements at the end of this work.

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it *ought* to be done, because it *must* be done, or the nation sink.

The evils and dangers, attending an *exorbitant* public debt in this country, are so great, that they cannot be exaggerated.— Without repeating, what has been so often said, of its increasing the dependance on the crown; rendering us tributary to foreigners; and raising the price of provisions and labour; and, consequently, checking population, and loading our trade and manufactures; I will only take notice of the following evils which attend it.

First. The execrable practices of the *Alley*. These cannot be mentioned in language too strong. They are increasing every day; and the national debt, by giving occasion to them, is likely soon (with the aid of annual lotteries) to ruin all honest industry among us, and to turn us into a nation of gamblers.

Secondly. It must check the exertions of the spirit of liberty in the kingdom. The tendency of every government is to despotism; and in this it must end, if the people are not constantly jealous and watchful. Opposition, therefore, and resistance, are often necessary. But they may throw things into confusion, and occasion the ruin of the public funds. The apprehension of this must influence all who have their interest connected
with

with the preservation of the funds, - and incline them always to acquiescence and servility.

But further. It exposes us to particular danger from *foreign* enquiries by inviting insults, and rendering us incapable of meeting war, however necessary, without the hazard of bringing on terrible convulsions, by overwhelming public credit.

All these are evils which must increase with every increase of the national debt; and there is a point at which, when they arrive, the consequences must be fatal (a).

But to proceed to some further observations.

What has been said, has all along supposed a *sacred* and *inviolable* application of the fund I have described, and of all its earnings, to the purpose of sinking the national debt.

(a) "Either the nation (Mr. *Hume* says, *Essays* Vol. II. p. 145) must destroy public credit; or public credit will destroy the nation." Mr. *Gordon*, in the Preface to *Cato's Letters*, tells us, that the great and good Mr. *TRENCHARD* had two things much at heart, namely, keeping *England* free from foreign broils, and paying off the public debts. He thought that one of these depended on the other, and that the *being* of the state depended on the latter. Mr. *Gordon* adds, that he believed no one, who thought at all, could think Mr. *TRENCHARD* mistaken.

fund

The whole effect of it depends on its being allowed to operate, WITHOUT INTERRUPTION, a proper time. But it may be asked, how this can be secured? Or, by what method an object, that must be continually growing more and more tempting, can be defended against invasion and rapine?—I might here mention the superintendency and care of the representatives of the kingdom, the faithful guardians of the state, to whom ministers are responsible for the use they make of the public money. But experience has shewn, that we cannot rely on this security.—The difficulty, therefore, now mentioned, is the very greatest difficulty the nation has to struggle with in the payment of its debts.

The sinking fund was established in the year 1716, or soon after the accession of the present family, at a time when the public debts, though not much more than a third of what they are now, were thought to be so considerable as to be alarming and dangerous. It was intended as a SACRED DEPOSIT never to be touched; the law which established it declaring, that it was to be applied to the payment of the principal and interest of such national debts and incumbrances, as had been incurred before the 25th of *December 1716; and to no other use, intent, or purpose whatever.*—The faith of *parliament*, therefore,

therefore, as well as the security of the kingdom, seemed to require, that it should be preserved carefully and rigorously from alienation. But, notwithstanding this, it has been *generally* alienated; and the produce of it employed, in helping to defray such current expences as the exigences of the state rendered necessary.

In order to justify this, it has been usual to plead, that when money is wanted, it makes no difference, whether it is taken from hence, or procured by making a new loan. But in truth the difference between these two methods of procuring money is no less than *infinite*.—For, by employing the SINKING FUND in bearing current expences, rather than borrowing *new* money on new funds; the state, in order to avoid giving *simple interest* for money, is made to alienate money, that *must* have otherwise been improved at *compound interest*; and which, in time, would have *necessarily* increased to *any* sum.—Had a faithful use been made from the first, of only one THIRD of the produce of this fund, the greatest part of our present debts would now have been discharged. (a)—
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(a) The principal observations in this Chapter, I have given just as they occurred to my thoughts, without knowing that any of them had been made by other writers. Some proposals and observations of a similar nature, I have since found in an excellent pamphlet published

Can it be possible then to think, without the deepest regret, of that misapplication of this fund, which, with the consent of parliaments always complying, our ministers have practised?

S U P P L E M E N T.

THE following account of our Public Funds *in general*, and of the SINKING FUND *in particular*, having been, published in another work too incorrectly, I have thought proper to introduce it here as a *Supplement* to the preceding Chapter.

lished in 1726, entitled, *An Essay on the National Debts of this kingdom, wherein the importance of discharging them is considered, and some general mistakes about the nature and efficacy of the SINKING FUND examined and removed. In a Letter to a Member of the House of Commons. Fourth edition.*

The

The *British Funds* have been all formed into the four following classes or divisions.—The AGGREGATE FUND; the SOUTH-SEA FUND; the GENERAL FUND; and the SINKING FUND.

The *Aggregate Fund* was established by an act of *Geo. I.* cap. 12. in 1715. It had this name given it, because it consisted of a great variety of taxes and surplusses of taxes which were in that year consolidated, and given as a security for the discharge of the interest and principal of debts due to the Bank of *England*, and some other public debts; and also for the payment of 120,000*l.* per ann. to the civil list. Into this *fund* are brought the two-thirds and one-half subsidy of tonnage and poundage; half the inland duties on tea and coffee; the house-money granted by the 7th of *Will. III.*; the duty on hops; the duties on low wines, brandy, and *British* spirits; all arrears of land-taxes; all public monies not appropriated; the surplusses of the nine-penny excise, of the five sevenths of the Bank nine-penny excise, of the revenues in the annuity acts of the 4th, 5th, and 6th of *Queen Anne*, &c. and, by an act of the 1st of *Geo. III.* all the duties constituting the revenue of the civil list.

The *South-sea fund* was established, by stat. 3 *Geo. I.* cap. 9. in 1716; and is so called, because appropriated to pay the interest of the *South-sea* company's capital.

It consisted of a duty on candles, and certain imposts on wines, vinegar, tobacco, and *East-India* goods.

The *General Fund* was also established, by stat. 3 *Geo.* I. cap. 7. in 1716, and consisted of a subsidy on goods exported; a tax on hackney-coaches and chairs; duties on soap, hides, stamps, and policies of insurance; 700*l.* per week letter-money; a moiety of the inland duties on tea and coffee; and 39,855*l.* per annum out of the hereditary excise on beer for the bankers annuities. All these taxes were appropriated to the discharge of the interest of 7,808,087*l.* (originally 10,000,000*l.*) capital stock of *South-sea* annuities, together with charges of management.

All that remained of the produce of the taxes thus digested into these three *funds*, after satisfying the charges upon them, was in the same year (or 1716) carried into a fourth *fund*, to which was given the name of the *Sinking Fund*, because appropriated to the purpose of *sinking* the public debts. The words of the act of the 3d of *Geo.* I. which established this *fund*, are, “ All the
 “ monies to arise from time to time, as
 “ well of the excess and surplus of an act
 “ made this session for redeeming the *funds*
 “ of the Bank of *England*; and of the excess or surplus by virtue of one other act
 “ made likewise this session for redeeming
 “ the

“ the *funds* of the *South-sea* company ; as
“ also, of the excess or surplus of the du-
“ ties and revenues by this act appropriated
“ as aforesaid; and the overplus monies of
“ the said *General Fund* by this act esta-
“ blished, shall be appropriated to the dis-
“ charging the principal and interest of
“ such national debts as were incurred be-
“ fore the 25th of December, 1716, and
“ are declared to be national debts; and to
“ or for no other use, intent, or purpose
“ whatsoever.” The transactions with re-
spect to this fund make a very important
part of the history of *Britain*; and furnish
us with a striking instance of the depravity
and folly which often ruin kingdoms.

Before its establishment there had existed
many smaller *funds* of the same nature; that
is, such duties or taxes had been provided for
paying the interests of particular loans, as
afforded surplusses by which the principal it-
self was to be gradually redeemed. This was
the common practice in the reigns of King
William and Queen *Anne*. Most of the
public duties were given for terms of years;
and at the end of those terms they ceased of
course, unless continued for farther terms by
new acts of parliament: And, in general, it
was provided, when any money was raised,
that the principal should be cancelled either
by time, as in the case of the sale of long
and short annuities, or by the surplusses of

the duties charged with the payment of the interest. This was an excellent plan; but it was by no means carried into execution. In the year 1720, most of the long and short annuities were converted into redeemable perpetuities, at the expence of above three millions; and the surplusses of the duties charged with particular loans were often so broken into, by being either charged with new loans before they had cancelled the old, or spent on current services, as to be rendered incapable of answering the end intended by them. In consequence partly of this bad management, the public debts at the accession of the House of *Hanover* were so much increased as to be generally reckoned *insupportable*; and their reduction was made one of the first objects of parliamentary attention. This gave rise, in 1716, to the institution of the *fund* of which we are giving an account, the father of which was (as has been generally said) Sir *Robert Walpole*, but, in reality, Earl *Stanbope*. All the taxes, except the land-tax and six-pence *per bushel* malt-tax (*a*), were then made perpetual, and distributed into the three *funds* which have been described, the surplusses of which, for ever afterwards, were

(*a*) These taxes have been always voted by parliament from year to year; and are, on this account, distinguished by the name of the *annual* taxes; and wholly employed, as far as they will go, in bearing the current expences of every year.

to be held sacred, and to be applied *inviolably*, according to the words of the act just recited, to the redemption of the national debt.

A considerate person might have suspected, that the same causes which had rendered former partial appropriations ineffectual, would destroy the efficacy of this. There seemed, however, to be reason for hoping the contrary: For,

First, the future happiness and glory of the kingdom were thought to depend (*a*) on this appropriation; and the law which established it was declared to be a *fundamental* law of the realm.

Secondly, in conformity to these sentiments, the words of this law were made as strong as possible; and, in order to give additional force to it, a repetition of it, in the same words, was inserted in an act of the 5th of *Geo. I.* cap. 3. — Particular notice should be taken of these words. — They order that all the surplusses of the taxes then made perpetual, shall be applied to the discharge of the public debts, and “ to no other use or intent

(*a*) The alarm occasioned about this time by the public debts (which have been since twice doubled) and the eager expectations entertained from this fund, appear remarkably, from a fact just mentioned; or from the conversion (at the expence of *three millions*) of most of the long and short annuities then subsisting into *redeemable* perpetuities, in order to subject them to the operation of this fund.

“ whatever.” When, therefore, a debt had been paid off, the addition arising from that payment to the surplusses (or the annuity disengaged by it) became a part of the *fund*, and, together with it, was to be employed in discharging farther debts. And the same being true of every successive annuity disengaged by every payment, the *fund*, if never misapplied, must necessarily have operated in sinking the public debt, in the same manner that money accumulates, when put out to bear compound interest. And in this way this *fund* did in fact operate for a few years. While in its infancy, it was watched with great care. The improvement and the inviolable application of it was recommended in most of the speeches from the throne, and echoed back in the addresses of the House of Commons. It is particularly observable, that so well did our ministers then understand the nature and importance of this *fund*, that rather than encroach upon it, they frequently borrowed money in order to defray the necessary expences of government. From some publications in 1726 it appears, that some persons had been led to apprehend this zeal of the ministry would not be permanent, because it was not their interest to pay off the public debt, on account of the dependence and influence created by it. In answering this objection, the writers on the side of the court called such

such an apprehension an *indecent jealousy*, and took upon them to assure the public, "that in no possible exigence of affairs "could our ministers ever approve of or "recommend the alienation of the *sinking fund*." Happy would it have been for Britain had this proved true: But in a little time it appeared, that the apprehensions which had been stiled *indecent jealousies*, were too well grounded. Men in power came soon to see, that this *fund* was advancing too fast in its operations, and to change their zeal for it into a resolution to destroy it. This will abundantly appear from the following facts.

Charging the income of the *sinking fund* with the payment of the interest of new loans, is an encroachment upon it, no less subversive of its efficacy, than depriving it of gross sums; there being no difference between taking from it the annual interest of a sum, and that sum itself. Between the years 1727 and 1732 several incroachments of this kind had been made upon it; but, being of a less obvious nature, they passed without any particular opposition. The finishing blow was given in the year 1733. In that year, in order to keep the land-tax at one shilling in the pound, it was necessary either to borrow half a million for the current service, or to take half a million from the *sinking fund*. The last method was chosen; and proposed by Sir Robert Walpole

to the House of Commons. Long and warm debates ensued. A proposal to alienate, in a time of profound peace, a *fund* which the law had made sacred, and the alienation of which *no possible exigence of public affairs* could justify, only for the sake of keeping the land-tax for one year at one shilling in the pound, justly kindled the indignation of the patriotic party. They urged the prohibition of the law, the faith of parliament, and the security of the kingdom. The proposer of the alienation was reminded of his inconsistency and treachery, in endeavouring to beat down that very monument of glory which he had boasted of having erected for himself; and Sir *John Barnard* warned him, that he was drawing upon himself the curses of posterity. But all arguments were vain. The ministry pleaded that the landed interest wanted ease; that there was no occasion for being in a hurry to pay the national debt; and that the circumstances of the kingdom had altered so much since the establishment of the *sinking fund*, that the competition then among the public creditors was, not who should be *first*, but who should be *last* paid. Thus argued, among others, Sir *Robert Walpole*. His reasons prevailed; and the House of Commons consented.

The practice of alienating the *sinking fund* having been thus begun, went on of course. In the next year, or 1734, 1,200,000 *l.* was taken

taken from it. In 1735, and 1736, it was anticipated and mortgaged.

Thus expired, after an existence of a few years, the *sinking fund*; that sacred blessing (as it was once thought) and the nation's only hope. Could it have escaped, it would long before this time have eased *Britain* of all its debts, and left it safe and happy.

In order to obtain a juster sense of this, let us here compare what it *would* have done had it never been misapplied, with what it *has* done.

Though the act that established it was passed, as already said, in 1716, it did not begin its operation till 1719, when three quarters of a million in old Exchequer bills were paid off with it. The intermediate time had been employed in laying the foundations of this *fund*, and providing an income for it, by a general reduction of the public debts, from an interest of six *per cent.* and other higher interests, to five *per cent.* What made this reduction then practicable was a rapid fall of the interest of money, which begun (in consequence of an increase of trade producing an influx of money) a little before the accession (a). The means used by government for accomplishing this reduction were, first, the addition (at *Michaelmas* 1717) of the interest

(a) The *legal* interest of money was reduced in 1714, from six to five *per cent.*

of some debts bearing five *per cent.* to the principal, in order to make use of the produce of the taxes which should have paid that interest, in discharging the bankers debt and some other debts bearing six *per cent.*

Secondly, loans at five *per cent.* obtained chiefly from the Bank and the *South-sea* company, to pay off such of the public creditors as did not chuse to accept a lower interest than six *per cent.*

After this reduction, the three *funds*, before described, produced a surplus of above half a million *per annum.* In 1727 this surplus was increased to 939,103 *l.* and in 1733 (the year when the practice of alienating it begun) it had been increased so much by the redemptions made with it, and by a second reduction of interest in 1727 from five to four *per cent.* that its medium for five years had been 1,212,000 *l. per annum.* Had it, from the year 1732, been allowed no increase beyond this sum (except from the interest of debts paid by it), and been applied for the first twenty-five years to the payment of debts bearing four *per cent.* interest, and afterwards to the payment of debts bearing three *per cent.* it would (in the year 1781) have completed the redemption of more than one hundred and sixty millions of debt, leaving the public during this whole period, in possession of all
the

the surplusses which have arisen in the revenue beyond 1,212,000*l.* except those produced by redemptions. It is not possible to conceive the beneficial effects with which this would have been attended, or the vigour which would have been all along given to public credit by such a *fund*, and by the prospect it would have given of the total annihilation in a few years of all the public debts, and the disengagement of the taxes supporting them. No person who duly attends to this, and wishes well to *England*, can avoid execrating the policy which first produced, and has since continued, the alienation of the *sinking fund*, and converted an expedient for saving the kingdom, into a supply for extravagance, and a support of corruption and despotism. This, however, is a policy which it may be expected men in power will always use when they can; for few of them (except when well watched) have ever shewn themselves superior to the temptations of power, or virtuous enough to avoid using all means to extend it.

Many schemes of different kinds have been formed for paying the public debts; and certain it is, that nothing can be of more importance. But the nature of things doth not admit of any method of doing this so expeditiously and effectually as an unalienably *sinking fund*; for in such a *fund* (it has been shewn) money is improved at compound

pound interest, and therefore in the most perfect manner. The writers, therefore, who have employed themselves in contriving such schemes might have spared their labour. The best of all schemes has been long known and established, and received all the weight and efficiency which could be given it by the most solemn acts of legislature. But no legislature can give security against itself. No parliament can do any thing which it may not undo.

We have now seen what the *sinking fund* would have done, had it suited the views of the *British* ministry in 1733 to suffer it to go on with its operations. Let us next compare this with what it *has* done.

In 1737 and 1738, a million of the stock of Bank annuities and two millions of the stock of *South-sea* annuities were redeemed with it. For twelve years after 1738, it was wholly applied to the current expences of every year. In 1749, the interest of near fifty-eight millions of the public debts was reduced from four to three and a half *per cent.* interest for seven years, and afterwards to three *per cent.* for ever. But notwithstanding the great addition which this *third* reduction of interest made to the *sinking fund*, no more than three millions of the public debts were redeemed by it, during the interval of peace between the years 1748 and 1756.

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By an act of the 25th of Geo. II. 1752, a change was made in the *sinking fund*, which it is necessary to mention.

Before this act the *sinking fund* consisted only of the clear surplusses of the aggregate, the general, and the *South-sea* company's funds. By the war, which begun in 1740, there was an addition made to the public debts of near thirty-two millions. This occasioned a great increase of taxes; and the practice was, whenever any new tax produced *less* than the interest with which it was charged, to make good the deficiency out of the *sinking fund*, and afterwards to replace the sum taken from it out of the supplies for the following year. But whenever a tax produced *more* than the charge upon it, the overplus, instead of being carried to the *sinking fund*, was made a part of the supplies for the year. By the act just mentioned, all the *new* taxes, together with all the annuities to the payment of which they had been appropriated, were ordered to be carried into the *sinking fund*, and formed into one general account. Most of the new taxes having proved deficient, this *fund* at first lost more than it gained by the change. But the loss was afterwards more than made up; first, by the saving which was produced by the reduction of interest from three and a half *per cent.* to three *per cent.* in 1757; and, secondly, by the addition, in the same year,

year, of the salt-duties to this *fund*, after they had completed the redemption of a million with which they had been charged in 1745.

The war which began in 1756 (a), added seventy-one millions and a half to the public debts. This produced a new increase of taxes, which (in conformity to the consolidating act just mentioned) were brought to the general *sinking fund* account, together with the annuities or interests with the payment of which they were charged. And it had been, till the last war, the constant practice to carry every new fund or tax, imposed for paying the interest of a loan, into the *sinking fund*; in consequence of which this *fund* gained when the tax happened to produce more, but lost when it produced less than the interest which it had been given to pay. The *sinking fund*, therefore, which, before the consolidating act, consisted only of the surplusses of the aggregate, general, and *South-sea* company's *funds*, consisted afterwards of the clear surplus of all the appropriated taxes. There was only one exception; namely, the additional tax upon houses and windows, granted in 1758, towards paying the interest of four millions and a half then borrowed. This tax was never made a part of the *sinking fund*; and, having always

(a) See second Tract on Civil Liberty, p. 147.

proved deficient to the amount of about 43,000*l.* *per annum*, the deficiency was constantly made good by the *sinking fund*, and afterwards replaced from the supplies.

Before the last reduction of the interest of the public debts, the *sinking fund*, having suffered greatly from various encroachments upon it, produced little more than a million *per ann.* But after this reduction, and its increase by the addition of the salt-duties, it produced near two millions *per ann.* In 1764 it produced at *Michaelmas*, after making good deficiencies, 2,105,000*l.* nearly. For five years after 1764, its average produce, reckoned to *Christmas* in every year, was 2,234,780*l.* For five years, ended in 1774, its average produce (after making good the deficiency of the fund in 1758) was 2,610,759*l.* In 1775, it produced 2,917,869*l.* In 1776, 3,166,517*l.* In 1777 it was charged with an annuity of 100,000*l.* *per ann.* to the civil list; and, after paying three quarters of this annuity, and half a year's interest of five millions borrowed in that year, it produced from October 1776 to October 1777, 2,685,669*l.* From October 1777 to October 1778, 1779, and 1780, it produced 2,442,063*l.* 2,267,399*l.*—2,403,017*l.* after paying the said annuity, and also after making good
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the deficiency of the fund in 1758, and all the deficiencies (a) of the new taxes.

It appears from this detail, that after the peace in 1763 the income of the *sinking fund* increased considerably. The causes of this have been partly the falling in of life-annuities, and the greater productiveness of the taxes occasioned by the increase of luxury. But the principal cause was the falling in of the interest of about ten millions and a half of the public debts, which were discharged during the twelve years of peace between 1763 and 1775. This diminution of the public debts was made, not by the *sinking fund*, but by a contribution from the *East India* company of 400,000*l. per ann.* begun in 1768, and continued for five years; by the profits of ten lotteries; by the composition for maintaining *French* prisoners; sale of *French* prizes taken before the declaration of war in 1756; and other extraordinary receipts, amounting in all to above eight millions. This *fund*, therefore, did not pay off more than two millions and a half, the rest of its produce having been employed in bearing the expences of the peace establishment,

(a) These deficiencies have had two causes. First, the unproductiveness of some of the new taxes: and, secondly, the commencement of the interests of the loans before the new taxes could be collected, the payment of which interests, therefore, fell on the Sinking Fund.

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which, during that period, were not much less than double to what they had been in any former period.

To the sum just mentioned, add three millions paid off in the peace between 1748 and 1756, and three millions paid off in 1736 and 1737, and it will appear that the whole amount of the public debts paid off by the *sinking fund*, from its first alienation in 1733, to the commencement of the *American* war, is only eight millions and a half.

It has been said, that when money is wanted for defraying public expences, it makes no difference whether it is obtained by diverting the *sinking fund*, or by a new loan. I have taken notice of this fallacy in page 207. Money in a *sinking fund*, if never alienated, is improved, I have shewn, at *compound* interest; but, when procured by a loan, bears only *simple* interest. A nation, therefore, whenever it applies the income of such a *fund* to current expences rather than the redemption of its debts, chuses to lose the benefit of compound interest in order to avoid paying simple interest; and the loss in this case is equal to the difference between the increase of money at compound and simple interest. The following calculation will shew what this difference is.

One penny put out at our Saviour's birth to five *per cent.* compound interest, would,

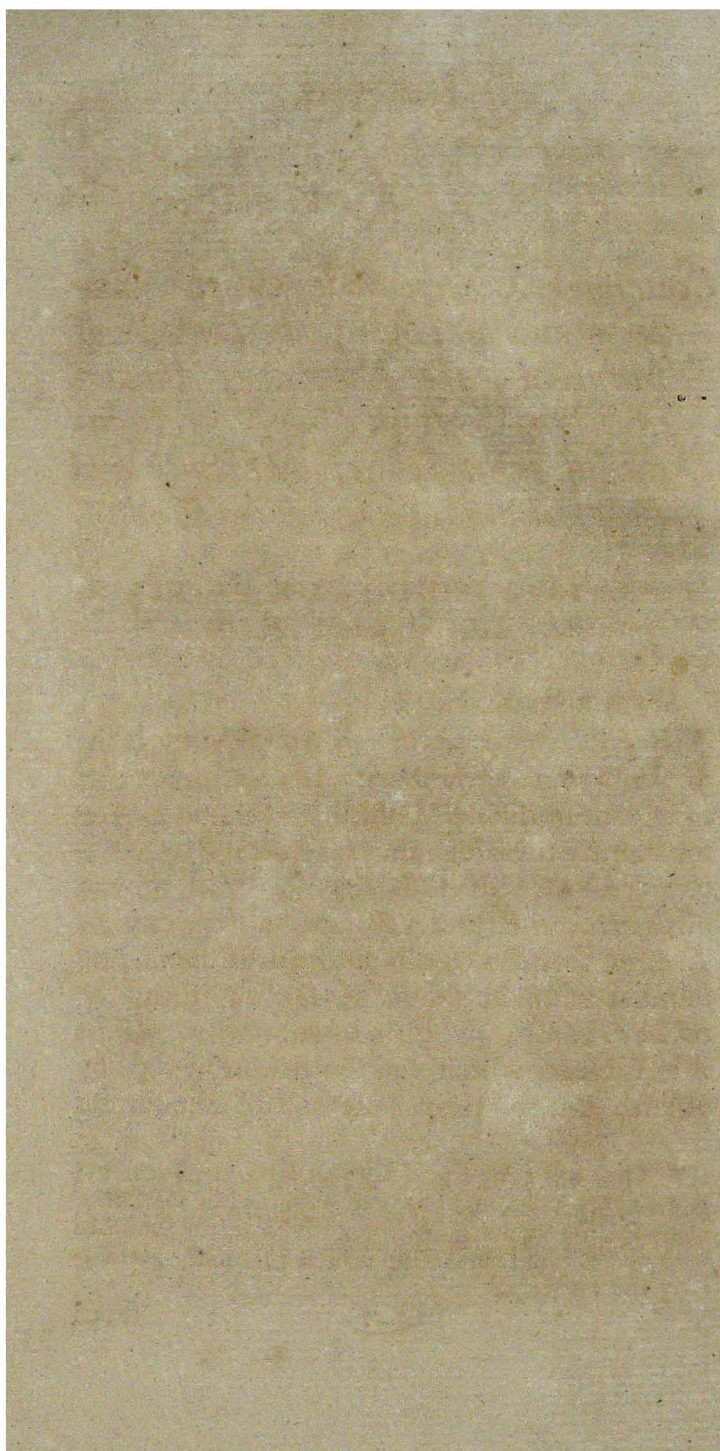
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in the year 1791, have increased to a greater sum than would be contained in THREE HUNDRED MILLIONS of earths, all solid gold. But, if put out to simple interest, it would, in the same time, have amounted to no more than SEVEN SHILLINGS AND SIX-PENCE. All governments that alienate *funds* destined for reimbursements, chuse to improve money in the *last* rather than the *first* of these ways.

N. B. The preceding account brings down the history of the Sinking Fund to the time of the last publication of this work. Since that time a new *Sinking Fund* has been established, and a new *Consolidating Act* has been passed, of which an account will be given at the end of the next volume.

FOUR ESSAYS
ON
DIFFERENT SUBJECTS
IN THE
DOCTRINE
OF
LIFE ANNUITIES
AND
POLITICAL ARITHMETIC.



E S S A Y I*.

Containing Observations on the Expectations of Lives; the Increase of Mankind; the Number of Inhabitants in LONDON; and the Influence of great Towns, on Health and Population.

In a LETTER to BENJAMIN FRANKLIN, Esq; LL. D. and F. R. S.

DEAR SIR,

I Beg leave to submit to your perusal the following observations. If you think them of any importance, I shall be obliged to you for communicating them to the Royal Society. You will find, that the chief subject of them is the present state of the city of *London*, with respect to healthfulness and number of inhabitants, as far as it can be collected from the bills of mortality. This is a subject which has been considered by others; but the proper method of calculating

* This Essay was read to the ROYAL SOCIETY, April 27th, 1769, and has been published in the Philosophical Transactions, Vol. 59. It is here republished with corrections; and with several additions, particularly the *Postscript*.

from the bills has not, I think, been sufficiently explained.

No competent judgment can be formed of the following observations, without a clear notion of what the writers on *Life-Annuities* and *Reversions* have called the *Expectation of Life*. Perhaps this is not properly understood; and Mr. *De Moivre's* manner of expressing himself about it is very liable to be mistaken.

The most obvious sense of the *expectation* of a given life is, "That particular number of years which a life of a given age has an equal chance of enjoying." This is the time that a person may reasonably *expect* to live; for the chances *against* his living longer are greater than those *for* it; and, therefore, he cannot entertain an *expectation* of living longer, consistently with probability. This period does not coincide with what the writers on Annuities call the *expectation of life*, except on the supposition of an uniform decrease in the probabilities of life, as Mr. *Simpson* has observed in his *Select Exercises*, p. 273.—It is necessary to add, that, even on this supposition, it does not coincide with what is called the *expectation of life*, in any case of joint lives. Thus, two lives of 40 have an even chance, according to Mr. *De Moivre's* hypothesis (a), of continuing together only $13\frac{1}{2}$ years. But the *expectation*

(a) See the Notes in page 2 and 23.

of two equal joint lives, being (according to the same hypothesis) always a *third* of the *common complement*; it is, in this case, $15\frac{1}{2}$ years. It is necessary, therefore, to observe, that there is another sense of this phrase, which ought to be carefully distinguished from that now mentioned. It may signify, “The *mean continuance* of any given *single*, “*joint*, or *surviving* lives, according to any “given Table of Observations:” that is, the number of years which, taking them one with another, they actually enjoy, and may be considered as sure of enjoying; those who live or survive *beyond* that period, enjoying as much *more* time in proportion to their number, as those who *fall short* of it enjoy *less*. Thus; supposing 46 persons alive, all 40 years of age; and that, according to Mr. *De Moivre’s hypothesis*, one will die every year ’till they are all dead in 46 years; half 46, or 23, will be their *expectation of life*: That is, The number of years enjoyed by them all, will be just the same as if every one of them had lived 23 years, and then died; so that, supposing no interest of money, there would be no difference in value between annuities payable for life to every single person in such a set, and equal annuities payable to another equal set of persons of the same common age, supposed to be all sure of living just 23 years and no more.

In like manner; the *third* of 46 years, or 15 years and 4 months (*a*), is the *expectation* of two joint lives both 40; and this is also the *expectation* of the survivor. That is; supposing a set of marriages between persons all 40, they will, one with another, last just this time; and the survivors will last the same time. And annuities payable during the continuance of such marriages would, supposing no interest of money, be of exactly the same value with annuities to begin at the extinction of such marriages, and to be paid, during life, to the survivors.—In adding together the years which any great number of such marriages and their survivorships have lasted, the sums would be found to be equal.

One is naturally led to understand the *expectation* of life in the first of the senses now explained, when, by Mr. Simpson and Mr. De Moivre, it is called, *the number of years which, upon an equality of chance, a person may expect to enjoy*; or, *the time which a person of a given age may justly expect to continue in being*; and, in the last sense, when it is called, *the share of life due to a person*. But, as in reality it is always used in the last of these senses, the former language should not be applied to it: And it is in this last sense that it coincides with the *sums* of the *present* probabilities, that any given single or joint lives shall attain to the end of the

(*a*) See Note (K) at the end of next volume.

1st, 2d, 3d, &c. *moments*, from this time to the end of their possible existence; or (in the case of survivorships) with the sum of the probabilities, that there shall be a survivor at the end of the 1st, 2d, 3d, &c. moments, from the present time to the end of the possible existence of survivorship. This coincidence every one conversant in these subjects must see, upon reflecting, that both these senses give the true present value of a life-annuity, secured by land, without interest of money (a).

This period in joint lives, I have observed, is never the same with the period which they have an equal chance of enjoying; and in single lives, I have observed, they are the same only on the supposition of an uniform decrease of the probabilities of life. If this decrease, instead of being always uniform, is *accelerated* in the last stages of life; the former period, in single lives, will be *less* than the latter; if *retarded*, it will be *greater*.

It is necessary to add, that the number expressing the former period, multiplied by the number of single or joint lives whose expectation it is, added annually to a society or town, gives the whole number living together, to which such an annual addition would in time grow. Thus; since 19, or the third of 57, is the *expectation* of two

(a) See Note (K) at the end of next volume.

joint lives whose common age is 29, or common *complement* 57; twenty marriages every year between persons of this age would, in 57 years, grow to 20 times 19, or 380 marriages always existing together. The number of *survivors* also arising from these marriages, and always in life together, would, in twice 57 years, increase to the same number. And, since the *expectation* of a single life is always half its *complement*; in 57 years likewise, 20 single persons aged 29, added annually to a town, would increase to 20 times 28.5 or 570; and, when arrived at this number, the deaths every year will just equal the accessions, and no further increase be possible.

It appears from hence, that the particular proportion that becomes extinct every year, out of the whole number constantly existing together of single or joint lives, must, wherever this number undergoes no variation, be exactly the same with the *expectation* of those lives, at the time when their existence commenced. Thus; was it found that a 19th part of all the marriages among any bodies of men, whose numbers do not vary, are dissolved every year by the deaths of either the husband or wife, it would appear that 19 was, at the time they were contracted, the *expectation* of these marriages. In like manner; was it found in a society, limited to a fixed number of members,

members, that a 28th part dies annually out of the whole number of members, it would appear that 28 was their common expectation of life at the time they entered. So likewise; were it found in any town or district, where the number of births and burials are equal, that a 20th or 30th part of the inhabitants die annually, it would appear, that 20 or 30 was the *expectation* of a child just born in that town or district. These *expectations*, therefore, for all single lives, are easily found by a *Table of Observations*, shewing the number that die annually at all ages, out of a given number alive at those ages; and the general rule for this purpose, is “to divide the sum of all the living in the Table, “at the age whose expectation is required, “and at all greater ages, by the sum of all “that die annually at that age, and above it; “or, which is the same, by the number (in “the Table) of the living at that age; and “half unity subtracted from the quotient will “be the required *expectation* (a).” Thus, in Dr. *Halley’s* Table, the sum of all the living at 20 and upwards, is 20,724. The number living at that age is 598; and the former

(a) This rule, and also rules for finding in all cases the expectations of joint lives and survivorships, may be deduced with great ease, by having recourse to the doctrine of fluxions. In this method, Mr. *De Moivre* says, he discovered them. See note (K), where an account will be given of these deductions, omitted by Mr. *De Moivre*.

number divided by the latter, and half unity (a) subtracted from the quotient, gives 34.15 for the *expectation* of 20. The expectation of the same life by Mr. *Simpson's* Table, formed from the bills of mortality of *London*, is 28.9 (b).

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(a) If we conceive the *recruit* necessary to supply the *waste* of every year to be made always at the *end* of the year, the *dividend* ought to be the *medium* between the numbers living at the *beginning* and the *end* of the year: That is, it ought to be taken *less* than the sum of the living in the Table at and above the given age, by *half* the number that die in the year ; the effect of which *diminution* will be the same with the *subtraction* here directed.— The reason of this subtraction will be further explained, in the beginning of the 4th Essay.

(b) It appears in p. 230 and 231, that the *expectations* of *single* and *joint* lives are the same with the values of *annuities* on these lives, supposing no interest or improvement of money.—In considering this subject, it will, probably, occur to some, that, allowing interest for money, the values of lives must be the same with the values of annuities *certain* for a number of years equal to the *expectations* of the lives. But care must be taken not to fall into this mistake. The latter values are always greater than the former : And the reason is, that, though a number of *single* or *joint* lives of given ages will, among them, enjoy a *given* number of years, yet some of them will enjoy a much *greater*, and some a much *less* number of years. Thus ; 100 marriages among persons, all 29, would, as I have said, one with another, exist 19 years ; and an office bound to pay annuities to such marriages during their continuance, might reckon upon making 19 payments for each marriage. But then, many of these payments would not be made 'till the end of 30, and some not 'till the end of 40 years. And it is apparent, that on account of the greater value of *quick* than *late* payments, when money *bears* interest, 19 payments so made cannot be worth

These observations bring me to the principal point which I have had all along in view. They suggest to us an easy method of finding the number of inhabitants in a place, from a *Table of Observations* or the *bills of mortality* for that place, supposing the yearly births and burials equal. "Find by "the Table, in the way just described, the "expectation of an infant just born; and this, "multiplied by the number of yearly births, "will be the number of inhabitants." At *Breslaw*, according to Dr. *Halley's* Table, though half die under 16, and therefore an

worth as much, as the same number of payments made regularly at the end of every year, 'till in 19 years they are all made.

This observation might be employed, to demonstrate further, the error of those who have maintained, that the value of a given life is the same with the value of an annuity certain, for as many years as the life has an equal chance of existing. Were this true, an annuity on a life, supposed to be exposed to such danger in a particular year as to create an equal chance whether it will not fail that year, would, at the beginning of the year, be worth *nothing*, though supposed to be sure of continuing for ever, if it escaped that danger: Nor, in general, would the values of annuities on a set of lives be at all affected by any alterations in the rate of mortality among them, provided these alterations were such, as did not affect the period during which they had an equal chance of existing.—But there can be no occasion for taking notice of an opinion, which has been embraced only by persons ignorant of mathematics, and plainly unacquainted with the genuine principles of calculation on this subject.—See a pamphlet on Life-Annuities by *Weyman Lee, Esq;* of the *Inner Temple*.

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infant just born has an *equal chance* of living only 16 years ; yet his *expectation*, found by the rule I have given, is near 28 years ; and this, multiplied by 1238, the number born annually, gives 34,664, the number of inhabitants. In like manner, it appears from Mr. *Simpson's* Tables, that, though an infant just born in *London* has not an *equal chance* of living 3 years, his *expectation* is 19 years and a quarter. Let us reckon it as high as 20 years. This number, multiplied by the yearly births, would give the number of inhabitants in *London*, were the births and burials equal.—The medium of the yearly births, for ten years, from 1759 to 1768, was 15,710. And 15,710 multiplied by 20, gives 314,200 ; which is the number of inhabitants that there would be in *London*, according to the Bills, were the yearly burials no more than equal to the births : that is, were it to support itself in its number of inhabitants, without any supply from the country. But for the period I have mentioned, the burials were, at an average, 22,956, and exceeded the christenings 7,246. This is, therefore, the yearly addition of people to *London* from other parts of the kingdom, by whom it is kept up. Suppose them to be all, one with another, persons who have, when they remove to *London*, an *expectation* of life equal to 30 years. That is ; suppose them to be all of the age
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of 18 or 20, a supposition certainly far beyond the truth. From hence will arise, according to what has been before observed, an addition of 30 multiplied by 7.246; that is, 217,380 inhabitants. This number, added to the former, makes 531,580; and this, I think, at most, would be the number of inhabitants in *London* were the bills perfect. But it is certain, that they give the number of births and burials too little. There are many burial places which are never brought into the bills. Many also emigrate to the navy and army and country; and these ought to be added to the number of deaths. What the deficiencies arising from hence are, cannot be determined (a). Suppose them equivalent to 6000 every year in the births, and 6000 in the burials. This would make an addition of 20 times 6000, or 120,000, to

(a) Two whole parishes are omitted in the bills, or *Marybone* and *Pancras* parishes. The former of these parishes is now one of the largest in *London*. The annual medium of burials in it for five years to 1771, was 780. In *Pancras* parish this medium for the same period was 322. From an accurate account taken in March 1772 of that part of this last parish which joins to *London*, it appeared that the number of inhabitants was then 3479, of whom 1594 were lodgers, and that the number of houses was 476, of which about 330 had been built in seven years. Mr. *Wales*, in a pamphlet of which more notice will be taken presently, gives the annual medium of *burials*, for 5 years to 1779, in *Marybone* parish 1145; of *births* 1008. In *Pancras*, he gives the *burials* for the same period, 339; the *births*, 234.

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the last number; and the whole number of inhabitants would be 651,580. If the burials are deficient only two thirds of this number, or 4000; and the births the whole of it; 20 multiplied by 6000, must be added to 314,290, on account of the defects in the births: and, since the excess of the burials above the births will then be only 5,246; 30 multiplied by 5,246 or 157,380, will be the number to be added on this account; and the sum, or number of inhabitants, will be 591,580.—But if, on the contrary, the burials are deficient 6000, and the births only 4000; 80,000 must be added to 314,200, on account of the deficiencies in the births; and 30 multiplied by 9,246, or 277,380, on account of the excess of the burials above the births; and the whole number of inhabitants will be 671,580.

Every supposition in these calculations is too high. *Emigrants* from *London* are, in particular, allowed the same *expectation* of continuance in *London* with those who are born in it, or who come to it in the firmest part of life, and never afterwards leave it; whereas it is not credible that the former *expectation* should be so much as half the latter. But I have a further reason for thinking that this calculation gives too high numbers, which has with me irresistible weight. It has been seen, that the number of inhabitants

bitants comes out less on the supposition, that the defects in the christenings are greater than those in the burials. Now it seems evident that this is really the case; and, as it is a fact not attended to, I will endeavour to explain distinctly the reason which proves it.

The proportion of the number of births in *London*, to the number who live to be 10 years of age, is, by the Bills, 16 to 5. Any one may find this to be true, by subtracting the *annual medium* of those who have died under 10 for some years past (*a*), from the *annual medium* of births for the same number of years.—Now, though without doubt *London* is very fatal to children, yet it seems incredible that it should be so fatal as this implies. The *Bills*, therefore, probably, give the number of those who die under 10 too great in proportion to the number of births; and there can be no other cause of this, than a greater deficiency in the *births* than in the *burials*. Were the deficiencies in both equal; that is, were the *burials*, in proportion to their number, just as deficient as the *births* are in proportion to *their* number, the proportion of those who reach 10 years of age to the number born, would be right in the *Bills*, let the deficiencies themselves be ever so considerable. On the contrary; were the deficiencies in the *burials* greater than in the *births*, this proportion would be given too

(*a*) That is, for some years before the date of this letter in 1769.

great; and it is only when the former are least, that this proportion can be given too little.—Thus; let the number of annual *burials* be 23,000; of *births* 15,700; and the number dying annually under 10, 10,800. Then 4,900 will reach 10, of 15,700 born annually; that is, 5 out of 16.—Were there no deficiencies in the *burials*, and were it fact that only *half* the number born die under 10; it would follow, that there was an annual deficiency equal to 4,900 subtracted from 10,800, or 5,900, in the *births*.—Were the *births* a third part too little, and the *burials* also a third too little, the true number of *births*, *burials*, and of *children dying under 10*, would be 20,933—30,666—and 14,400; and, therefore, the number that would live to 10 years of age, would be 6,533 out of 20,933, or 5 of 16 as before.—Were the *births* a third part, and the *burials* so much as two-fifths wrong, the number of *births*, *burials*, and children dying under 10 would be 20,933—32,200—and 15,120. And, therefore, the number that would live to 10 would be 5,813 out of 20,933, or five out of 18.—Were the *births* a third part wrong, and the *burials* but a sixth, the foregoing numbers would be 20,933—26,833—12,600; and therefore, the number that would live to 10 would be 8,333 out of 20,933, or 5 out of 12.56: and this proportion seems
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as low as is consistent with probability. It is somewhat less than the proportion in Mr. Simpson's Table of *London Observations*; and much less than the proportion in the Table of *Observations for Breslaw*. The deficiencies, therefore, in the register of *births*, must be greater than those in the register of *burials* (a); and the least number I have given, or 591,580 (b) is nearest to

(a) One obvious reason of this fact is, that *none* of the *births* among *Jews, Quakers, Papists*, and the *three denominations of Dissenters* are included in the Bills, whereas *many* of their *burials* are. It is further to be attended to, that the abortive and still-born, amounting to about 600 annually, are included in the burials, but never in the births. If we add these to the christenings, preserving the burials the same, the proportion of the born according to the Bills, who have reached ten, for sixteen years, from 1756 to 1771, will be very nearly one *third* instead of *five sixteenths*.

(b) Mr. *Wales*, the ingenious master of the royal mathematical school in *Christ's Hospital*, has lately, in a pamphlet entitled, *An Enquiry into the present State of the Population of England and Wales*, made several remarks on the Observations in this Essay. He objects particularly to this calculation, and expresses, p. 12, his surprise that it should have escaped my attention, that if the births are considerably more deficient than the burials, the expectation of life by which the number of these births is multiplied will be greater, particularly at this time, when the number of births approaches so much nearer than it did to the number of burials.— But Mr. *Wales* should have observed, that in order to be certain of not making the number of inhabitants in *London* less than it is, I have all along in this calculation reckoned the expectation of a child at birth in *London* so

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to the true number of inhabitants. However, should any one, after all, think that
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high as 20 years; and that this is a greater expectation than such a child could have, according to the Bills from 1759 to 1768, supposing the deficiencies in the christenings so considerable as a third, while in the *burials* they were only a sixth.——In page 15th, he says, that according to my Tables for *London*, formed on the supposition that the burials exceed the births a *fourth*, the expectation of a child just born in *London* is 20 years and three quarters.—Had Mr. *Wales* attended more to this subject, he would have found, that in reality this expectation is no more than 18; and that 20 and three quarters is the expectation, according to my Tables, not of a child just born in *London*, but of all the inhabitants of *London* at the time they enter it. See the 4th Essay towards the middle, and the Tables for *London* in the Collection of Tables.——He would also have found that even in the present improved state of *London* it is not possible, without assuming suppositions which are perfectly extravagant, to frame a table from the Bills that shall give the expectation of a child at birth in *London* much more than 20. He *intimates*, however, that it may *now* approach even to $25\frac{1}{2}$; but *concludes*, tho' he could not stop to make the calculation, that it cannot be less than 24. He will see how wrong he has been in drawing this conclusion, if he will consult the Essay, and the Tables to which I have just referred. The 16th Table, in particular, gives the probabilities of life between 8 and 16 higher than (according to Mr. *Wales's* account) they have been found to be among the children in *Christ's Hospital* for 20 years before 1781. It gives them likewise too high *after* 20; and yet even this Table makes the expectation of a child just born in *London* only $19\frac{1}{2}$.——Mr. *Wales*, in consequence of concluding without calculation this expectation to be 24, makes the inhabitants of *London* to be 625,131.——Had he taken it at 20, he would have taken it higher than it is, and
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it is not improbable that only 5 of 16 should live in *London* to be 10 years of age; or that above *two-thirds* die under this age; the consequence will still be, that the foregoing calculation has been carried too high. For it will from hence follow, that the *expectation* of a child just born in *London* must be far short of the number at which I have taken it, or of 20 years.—It is only $19\frac{1}{2}$ on the supposition that half die under 3 years of age, and that 5 of 16 live to be 29 years of age, agreeably to Mr. *Simpson's* Table. But if it is indeed true, that *half* die under 2 years of age, and 5 of 16 under 10, agreeably to the *Bills*, this expectation cannot be so much as 17; and all the numbers before given will be considerably reduced.

Upon the whole: I am forced to conclude from these observations, that the second number I have given, or 651,580, though short of the number of inhabitants commonly supposed in *London*, is, very probably, much *greater*, but cannot be *less*, than the true number. Indeed, it is in ge-

by proceeding on his own principles found that the inhabitants of *London* cannot be so many as 528,859.

I cannot conclude this Note without adding, that though it appears from hence, that Mr. *Wales* has been much too hasty in some of his remarks, yet I think myself greatly obliged to him for them. It will come in my way to take notice of more of them in the course of this work.

neral evident, that in cases of this kind numbers are very much over-rated. The ingenious Dr. *Brakenridge*, 14 years ago, when the Bills were lower than they are now, from the number of houses, and allowing six to a house, made the number of inhabitants 751,800. But he has taken the number of houses much greater than it really is; and six to a house is probably too large an allowance (*a*).

Another

(*a*) Vid. *Phil. Transactions*, Vol. xlviii. p. 788. In a paper subsequent to this, read to the Royal Society in March 1758, Dr. *Brakenridge* tells us, that in a late survey it appeared, that in all *Middlesex*, *London*, *Westminster*, and *Southwark*, there were 87,614 houses, of which 19,324 were cottages, and 4810 empty. And he acknowledges, that this, if right, proves *London* to be much less populous than he had made it. See *Phil. Trans.* Vol. L. p. 471.—Mr. *Maitland* gives two accounts of the number of houses within the Bills. One carefully taken from the books of all the parishes and precincts belonging to *London*; and another taken from a particular survey in 1737, made by himself with incredible pains. The first account makes the number of houses 85,805. The second account makes it 95,968. And the reason of the difference he observes, is, that many landlords of small places paying all taxes, they are in the parish books reckoned as so many single houses, though each of them contain several houses. See Mr. *Maitland's History of London*, 2d Book at the end.—It will be observed presently, that the number of inhabitants in *London* in 1737, was considerably greater than it is now.

From a Table which I have given at the end of this Essay, containing the results of actual surveys of the
number

Another method which Dr. *Brakenridge* took to determine the number of inhabitants in *London* was from the annual number of burials, adding 2000 to the Bills for omissions, and supposing a 30th part to die every year. In order to prove this to be a moderate supposition, he observes that, according to Dr. *Halley's* Observations, a 34th part die every year at *Breslaw*. But this obser-

number of inhabitants, houses, and families in many different places, it will appear that *six* to a house is probably too large an allowance for *LONDON*; and that certainly *five* to a house is an allowance sufficiently large for *ENGLAND* in general. And this will prove that Dr. *Brakenridge* has over-rated the number of people in *ENGLAND* as well as in *LONDON*. In a letter to *George Lewis Scott, Esq.* published in 1756 in the *Phil. Trans.* Vol. xlix. p. 877, he says, that he had been certainly informed that the number of houses rated to the window-tax was 690,000. The number of cottages not rated, he adds, could not exceed 200,000; and from these data, by allowing *six* to a house, he makes the number of inhabitants in *ENGLAND* to be 5,340,000.—Dr. *Brakenridge* was much mistaken with respect to the cottages. Their number in 1761 was (according to the returns of the surveyors of the house duties) 276,149; and the whole number of houses in *England* and *Wales* was in the same year 980,692.—In 1777, according to the same returns, the cottages were 251,261, and the whole number of houses 952,734. Let, however, the number of houses then in *England* and *Wales* be reckoned a *MILLION*; and, allowing *five* to a house, the number of people must have been *FIVE MILLIONS*.—The inhabitants of *IRELAND* may, I suppose, be stated at *TWO MILLIONS*.—The inhabitants of *SCOTLAND* consisted, in 1754.

observation was made too inadvertently. The number of annual burials there, according to Dr. *Halley's* account, was 1174, and the number of inhabitants, as deduced by him from his Table, was 34,000 ; and therefore a 29th part died every year. Besides ; any one may find, that in reality the Table is constructed on the supposition, that the whole number born, or 1238, die every year ; from whence it will follow that a 28th part died every year (a). Dr. *Brakenridge*, therefore, had he attended to this, would have stated a 24th part as the pro-

of between 16,000 and 17,000 *Papists*, and between 1,240,000 and 1,280,000 Protestants, according to an estimate that was made, I am informed, with labour and expence by the Rev. Dr. *Webster*.—It follows, therefore, that the whole number of people in *Britain* and *Ireland* may be about EIGHT MILLIONS AND A HALF, OR NINE MILLIONS. In the Supplement in the next volume I shall have occasion to say more on this subject, and to take notice of the arguments offered by Mr. *Wales* and Mr. *Howlett*, to prove that our population is increasing.

(a) Care should be taken, in considering Dr. *Halley's* Table, not to take the first number in it, or 1000, for so many just born. 1238, he tells us, was the annual medium of births, and 1000 is the number he supposes all living at one year and under. It was inattention to this that led Dr. *Brakenridge* to his mistake.

It will be shewn in the 4th Essay, that the number of the living under 20, is given too high in this table ; and from hence it will follow, that more than a 28th part of the inhabitants die at *Breslaw* annually.

portion

portion that dies in *London* every year, and this would have taken off 150,000 from the number he has given. But even this must be less than the just proportion. For let three-fourths of all who either die in *London* or migrate from it, be such as have been born in *London*; and let the rest be persons who have removed to *London* from the country, or from foreign nations. The *expectation* of the former, it has been shewn, cannot be 20 years; and 30 years have been allowed to the latter. One with another, then, they will have an *expectation* of $22\frac{1}{2}$ years. That is; one of $22\frac{1}{2}$ will die every year (*a*). And, consequently, supposing the annual

(*a*) The mean number of inhabitants in *Rome*, of all ages and conditions, for ten years ending in 1771, was 158,957. The annual medium of births for the same time was 4851; and of burials 7367. One in $21\frac{1}{2}$, therefore, died annually. See *Phil. Trans.* Vol. lxxv. p. 445. In 1752, the accurate and diligent Mr. *Struyk* took particular pains to determine the number of inhabitants in *Amsterdam*; and the result of his enquiry was, that very probably it did not amount to 200,000. The annual medium of burials for six years, from 1747 to 1752, was 8247; and for five years, from 1772 to 1776, it was 8447. One in 24, therefore, died annually.—At *Amsterdam*, there is a great number of Jews, and their burials are not included in the Bills. There must, I suppose, be other deficiencies, and an allowance for these would, I doubt not, increase the proportion of inhabitants who die annually to one in 21 or 22.—At *Dublin*, in the year 1695, the number of inhabitants was found, by an exact survey, to be 40,508. (See *Philos.*

annual recruit from the country to be
7000 (a), the number of *births* 3 times
7000

Philos. Transactions, No. 261). I find no account of the annual burials just at that time; but from 1661 to 1681, the medium had been 1613; and from 1715 to 1728 it was 2123. There can, therefore, be no material error in supposing that, in 1695, it was 1800; and this makes 1 in 22 to die annually. See Dr. *Short's Comparative History*, p 15, and *New Observations*, p. 228.—The annual medium of burials for five years ending in 1775, in *Manchester* and *Salford*, was 973. The number of inhabitants in 1773 was 27,246. About a 28th part, therefore, died annually. But it should be considered here, that *Manchester* increases fast by accessions from the country, and that the effect of such an increase must be to raise the proportion of *inhabitants* to the *deaths* and also the proportion of the *births* and *weddings* to the *burials*, higher than they would otherwise be.—The annual medium of burials at *Stockholm* in *Sweden*, from 1758 to 1763, was 3802. The number of inhabitants in 1763 was 72,979. One in 19 therefore died annually. See a memoir by M. *Vargentin*, in the 15th Vol. of the *Collection Academique*, printed at *Paris* 1772.

Mr. *Maitland*, in his *History of London*, Vol. II. page 744, by a laborious, but too unsatisfactory investigation, makes 1 in $24\frac{1}{2}$ to die in *London* annually; and on the suppositions, that this is the true proportion dying annually, at all times, in *London*, and that the deficiencies in the burials (including the burials in *Marybone* and *Pancras* parishes) amounted to 3038 annually; he determines, that the number of inhabitants within the bills was 725,903, in the year 1737.

The number of burials not brought to account in the Bills is, probably, now much greater than either Dr. *Brakenridge* or Mr. *Maitland* suppose it. I have reckoned it so high as 6000, in order to be more sure of not falling below the truth.

It will appear in the last Essay, with an evidence little short of demonstration, that, at least, 1 in $20\frac{3}{4}$ die annually

7000 or 21,000, and the *burials* and *migrations* 28,000 (which are all very high suppositions),

nually in *London*, and that, consequently, the number of inhabitants, if the burials are 26,000, cannot exceed 539,500.

(a) Mr. *Wales*, though he seems to acknowledge that formerly the number of annual recruits from the country to *London* was much greater than it is here supposed, yet reckons that when he wrote on this subject in 1782, it might be fairly stated at no more than 1779. See Mr. *Wales's* Enquiry, p. 16. It may be proper to consider here how improbable it is that such a change as this should have taken place at a time when the communication between *London* and the country has been made so easy as it is, and when also a disposition to migrate to *London* seems to be more prevalent than ever.—But it is unnecessary to insist on this, for in the 4th Essay it will be proved by decisive evidence, that these recruits could not even then be so little as double the number at which Mr. *Wales* has stated them. It is true indeed, that though the burials have been falling, the christenings have been rising, for several years. But this does not necessarily imply, that the emigrants from the country are less numerous than they were. It may, on the contrary, be owing to a greater afflux of people to *London* in the prolific stages of life occasioning an increase of the christenings, without at present occasioning such an increase of the burials as is sufficient to balance the causes that diminish them. The Lying-in Hospitals lately established in *London* increase the christenings, by drawing many into them to lye-in who reside out of the limits of the Bills; and the burials are diminished by the custom of sending infants to be nursed in the country, by the new burying grounds which have been lately opened, and particularly by an Act of Parliament which we owe to the humanity of Mr. *Hanway*, passed in 1767, and requiring all parish infants to be sent in three weeks into the country to be nursed there for six years.—The improved state of *London* with respect

positions), the number of inhabitants will be, $22\frac{1}{2}$ multiplied by 28,000, or 630,300 (a).

I will just mention here one other instance of exaggeration on the present subject.

Mr. *Corbyn Morris*, in his useful *Observations on the past growth and present state of the city of London*, published in 1651, supposes that no more than a 60th part of the inhabitants of *London*, who are above 20, die every year, and from hence he concludes that the number of inhabitants was near a million. In this supposition there was an error of at

spect to healthiness might be also here mentioned; but this has been greatly over-rated. The values of lives in *London* after the age of 20, are much the same that they were 50 years ago; and there is no evidence to prove, that they are much greater before 20. This will be shewn at the end of the 4th Essay, and in the Observations on Table 15th in the next volume.—According to Mr. *Howlett's* account, in page 91, of his Examination of my Essay on the Population of *England and Wales*, above 2000 deaths of children under two years of age have been taken out of the Bills by the Parish Act just mentioned. This probably goes much beyond the truth. Should the true number be only a *thousand*, it will follow that the state of infants in *London* is but little mended. For on this supposition a thousand must be added to the number given in the Bills as dying under two years of age, which will make it near half the number born as it was 20 years ago. But the addition of 2000 would make the mortality of infants (supposing parish infants not sent into the country) greater now in *London* than it ever was.

(a) If with Mr. *Wales* the annual recruit is taken at no more than 1779, the inhabitants on the high suppositions here made that the burials are 28,000, the expectations at birth 20, and at migration 30, will be only 577,790.

least

least one half. According to Dr. *Halley's* Table, it has been shewn, that a 34th part of all at 20 and upwards, die every year at *Breslaw*. In *London*, a 29th part, according to Mr. *Simpson's* Table, and also according to all other Tables of *London* Observations. Had, therefore, Mr. *Morris* stated a 30th part of all above 20 dying annually in *London*, he would have gone beyond the truth, and his conclusion would have been 400,000 less than it is.

Dr. *Brakenridge* observed, that the number of inhabitants, at the time he culculated, was 127,000 less than it had been. The Bills have lately advanced a little, but still they are much below what they were from 1717 to 1743. The medium of the annual *births*, for 20 years, from 1716 to 1736, was 18,000, and of *burials* 26,529; and, by calculating from hence on all the same suppositions with those which made 651,580 to be the present number of inhabitants in *London*, it will be found that the number then was 735,840, or 84,260 greater than the number in the present year 1769 (a). *London, therefore*, for the last 30 years, has been decreasing; and though now it is increasing again, yet there

(a) In the Essay on the Population of *England* and *Wales*, I have mentioned several facts which seem to shew, that even so long ago as the *Revolution*, *London* was more populous than it is at present. The chief of these facts are the following:

First;

there is reason to think that the additions lately made to the number of buildings round it, are owing, chiefly to the increase
of

First; The returns in 1777 of the surveyors of the house and window duties made the number of houses then in *Southwark, Westminster, London*, and all *Middlesex*, including cottages and uninhabited houses, to be 90,578.—Sir *William Petty*, in 1687, says, that the number of *houses* (which he expressly distinguishes from *families*) in *London* appeared by the register to be 105,315. See his *Political Arithmetic*, p. 74. His words in p. 79 are, “by certificate from the hearth-office, I find the “houses within the Bills of Mortality to be 105,315.”—Dr. *Devenant's* account agrees with this, who, from the same hearth-office, gives 111,215 as the number of houses in *London* (exclusive of *Southwark Westminster*, and all *Middlesex*, on Lady-day 1690. See Dr. *Devenant's Works*, Vol. I. p. 38. The annual average of registered burials also for five years before 1690 was considerably greater than it is at present.

This seems as direct evidence as can well be given in a point of this kind. In order to give more weight to the fact last mentioned, I have, in the Essay just referred to, observed that there are twelve parishes now included in the Bills, which were omitted formerly. But Mr. *Wales* has very properly corrected me in this instance by observing, that these parishes at the time they were added to the Bills were *new* parishes formed out of old parishes, which had been always included in the Bills. There is, therefore, no such regard due to this omission as I imagined.—It may be farther observed with respect to the excess of the burials at the Revolution, that the deficiencies in the register of burials are greater now than they were then; and there are two causes that may possibly have produced this effect. First, the opening of some burial places among the *Methodists*, where many are now buried who used to be buried in churches. And, Secondly, the interment out of the Bills of the greater part of the parish-children who die, in consequence of the Act of Parliament mentioned in the note, p. 251.—
There

of luxury, and the inhabitants requiring more room to live upon (*a*).

It

There are, however, other causes which have lessened these deficiencies; and, particularly, the decrease of the three denominations of Dissenters in *London*. My own recollection, as well as a great deal of other evidence, leave me no room to doubt of this. Mr. *Howlett*, however, in the pamphlet already quoted, asserts the contrary; and gives a list of burials among Dissenters, which makes their number more than three times greater than it was when Mr. *Maitland* published his *History of London*. But this is all a mistake. The principal burying places in his list happen to be places lately opened, to which, partly from a regard to cheapness, not Dissenters only, but people of all sorts are brought to be buried. This is particularly the case with *Coughland's ground*, *Holywell Mount*, and *Britain's ground*, *Whitechapel*.—The chief burying place of Dissenters has always been *Tindall's ground* in *Bunhill Fields*; but even this is by no means confined to Dissenters, and the number of burials in it has been for a course of years decreasing; and instead of being as Mr. *Howlett* gives it, 1400 annually, is not a *third* of this number.—In 1779 the exact number was 434, according to an account which has been extracted for me from the Register.

(*a*) The medium of annual burials in the 97 parishes within the walls was,

From 1655 to 1664,	—	3264
From 1680 to 1690,	—	3139
From 1730 to 1740,	—	2316
From 1758 to 1768,	—	1620
From 1771 to 1780,	—	1491
From 1781 to 1784,	—	1354

This account proves, that though, since 1655, *London* has doubled its inhabitants, yet *within the walls*, they have decreased; and so rapidly since 1690, as to be now reduced to less than half.—The like may be observed of the 17 parishes immediately without the walls.

Since

It should be remembered, that the number of inhabitants in *London* is now so much less as I have made it, than in was 40 years ago, on the supposition, that the proportion of the omissions in the *births* to those in the *burials*, was the same then that it is now. But it appears that this is not the fact.— From 1728, (the year when the ages of the dead were first given in the *Bills*) to 1742, near five-sixths of those who were born died under 10, according to the *Bills*. From 1742 to 1752 three quarters: and ever since 1752, this proportion has stood nearly as it is now, or at somewhat more than two-thirds. The omissions in the *births*, therefore, compared with those in the *burials*, were greater formerly; and this must render the difference between the number of inhabitants now and formerly somewhat less considerable than it may seem to be from the face of the *Bills*. One reason, why the proportion of the amounts of the *births* and *burials* in the *Bills*, comes now nearer than

Since 1730, these parishes have been decreasing so fast, that the *annual burials* in them have sunk from 8672 to near 4000, which is lower than they were before the year 1660. In *Westminster*, on the contrary, and the 23 out-parishes in *Middlesex* and *Surrey*, the *annual burials* have since 1660 advanced from about 4000 to 16,000, the medium for some years before 1769.— These facts prove, that the inhabitants of *London* are now much less crowded together than they were. It appears, in particular, that *within the walls* the inhabitants take as much room to live upon as double their number did formerly.

it did, to the true proportion, may, perhaps, be, that the number of Dissenters is lessened (a).

I will add, that it is probable that *London* is now become less fatal to children than it was; and that this is a further circumstance which must reduce the difference I have mentioned; and which is likewise necessary to be joined to the greater deficiencies in the births, in order to account for the very small proportion of children who survived 10 years of age, during the two first of the periods I have specified.—Since 1752, *London* has been thrown more open. The custom of keeping country-houses, and of sending children to be nursed in the country, has prevailed more. But, particularly, the destructive use of spirituous liquors among the poor has been checked (b).

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(a) See the end of the Note in p. 254.

(b) The enquiry in the preceding pages into the number of inhabitants in *London* was first published above twenty-one years ago. Fourteen years ago (or in 1777) the surveyors of the house and window duties were ordered to make returns to the tax office of the number of houses of all sorts in *London, Southwark, Westminster*, and the county of *Middlesex*. The number returned was 90,570. This seems to leave no room for much dispute. Allowing six to a house, the number of inhabitants within the *Bills*, with the addition of the whole county, will be 543,420. See the Note in p. 260 and 252; and a more particular account in my *Essay on the Population of England from the Revolution to the present time*.

Mr. *Wales*, in the pamphlet quoted in the Notes, p. 243 and p. 251, without taking any notice of these returns,
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I have shewn that in *London*, even in its present state, and according to the most moderate

turns, calculates the number of houses and inhabitants in *London* in the following manner.—Mr. *Maitland*, in 1737 (when the registered burials for 20 years had been near 8000 *per ann.* more than they are at present) found the number of houses in *London* to be 95,968. To this number Mr. *Wales* adds 4032, in order to make up 100,000; and by allowing $6\frac{1}{2}$ to each house, finds the number of inhabitants to be 650,000.

Leaving the reader to judge as he pleases of this calculation, I shall reckon myself more out of danger of being wrong in following the documents I have just mentioned, and in stating from them the inhabitants of *London within the Bills*, with the addition of *Pancras* and *Marybone* parishes, at *half a million*.—The annual medium of burials for the five years ending in 1780 was, according to the Bills, 26,743. Add 6000 for omissions, and the number of burials will be 26,743, or a 19th part nearly of the inhabitants, which is the proportion dying annually at *Stockholm*. See the Note, p. 250.

If the omissions are only 3038, agreeably to the result of Mr. *Maitland's* enquiries, one in 21 will die annually.—Mr. *Howlett*, in his examination already quoted, (published about eight years ago) p. 91, makes the deficiencies in the burials to be much greater than either of these estimates. He reckons that a deficiency of 2100 burials has been occasioned by the Act of Parliament, requiring parish infants to be nursed for six years in the country, which implies that so many now die annually in the country who ought to be included in the Bills. But this is not his meaning; for he says, that of 2800 infants which come annually upon parishes, and are required to be removed in three weeks into the country, only 250 die there in *six years*; whereas 450 die in the *three weeks* before their removal. The deficiency, therefore, in the Bills arising from hence, can be only 250. But this carries us to the contrary extreme, and makes the probabilities of the duration of life among infants,

derate computation, half the number born die under *three* years of age. In *Vienna* and *Stock-*

fants, committed by parishes to the care of foster-mothers, to be much greater than were ever known among infants in the best situations.—Mr. *Howlett's* meaning appears to be, that 2100 deaths are *prevented* annually by this Act of Parliament. The observation just made shews, that it is impossible this should be true; but supposing it true, it will be obvious, that a *prevention* of deaths ought not to be reckoned among *deficiencies*; for on the same ground the deaths prevented by cleansing and opening the streets, and other salutary regulations, might be so reckoned.—This Act of Parliament has undoubtedly prevented a great number of deaths. Before it was passed, almost all parish infants died in the first six years. Let us reckon that *now* of 2800 brought annually into workhouses, only a thousand die in this time, after being removed in three weeks into the country to be nursed. This would be a change unspeakably for the better; and it would imply that the probabilities of the duration of life among them is higher than is common among children in *London*. On this supposition the deficiency under consideration will be a *thousand*; and it will appear that 1100 ought to be taken from Mr. *Howlett's* total of deficiencies. But much greater deductions ought to be made on other accounts.—He gives 2000 as a deficiency occasioned by carrying out so many to be buried in the neighbouring villages, without making any allowance for the burials brought in. He gives also the burials in the *East-India* ships serving abroad; the burials in the *hospitals*, *Northampton-chapel*, *Bunhill*, as all burials of persons residing within the Bills; and thus makes the deficiencies amount to 11,273, and the total of annual burials to 31,941. He farther calculates that the kingdom in general, and *London* in particular, is improved a *tenth* in healthiness; and on this account he adds a tenth to the total just mentioned, and in *this* way makes the number of inhabitants in *London* to be about 800,000.—Such are Mr. *Howlett's* calculations.—In his list of deficiencies he sets down 1400 for the annual burials in *Bunhill*. From

Stockholm, under two. In *Manchester*, under five. In *Norwich*, under five. In *Northampton*, under (a) ten.—But it appears from *Graunt's* (b) accurate account of the births, weddings, and burials in three country parishes for 90 years; (and also, from *Dr. Short's* collection of observations in his *Comparative History*, and his Treatise, entitled, *New Observations on Town and Country Bills of Mortality*) that in country villages and parishes, the major part live to

the note in p. 261, it appears that this number is near 1000 greater than the truth. The annual burials at *Northampton-chapel, Clerkenwell*, in 1782, he makes to be 2080. The information I received from thence was, that, taking one week with another, they might be then reckoned at 30 in a week, or 1560 in a year. This, probably, *Mr. Howlett* mistook for 40 in a week, and thus has been led to make them 2080 in a year. They were, however, increasing, and every year diminishing more and more the burials in the churches, the lowness of the fees gaining for this burying ground, and the other burying grounds mentioned in the note p. 261, a particular preference among the lower ranks of people.

(a) See the Tables at the beginning of the next volume.—The whole number buried in the parish church of *Manchester* for six years, from 1773 to 1778, was 4126, of whom 2174 were children under five. But it must be considered, that in this town the births then exceeded the burials, and that consequently the Bills give the proportion dying in childhood too high.

(b) See *Natural and Political Observations on the Bills of Mortality*, by Capt. *John Graunt*, F. R. S.—Also, *Mr. Derham's Physico-Theology*, p. 174, where it appears, that in the parish of *Aynho* in *Northamptonshire*, though the births had been for 118 years to the marriages as 6 to 1; yet the burials had been to the marriages only as $3\frac{1}{2}$ to 1.
mature

mature age, and even to marry. In the parish of *Holy-Cross* (a), near *Salop*, it appears from a curious register, which has been kept by the Rev. Mr. *Gorsuch*, the vicar, that, of 655 who died there at all ages for 20 years to 1770 near

*(a) This parish contains in it a village which is a part of the suburbs of *Shrewsbury*. It consists of 1400 acres of arable and pasture land; besides 300 acres taken up by houses and gardens. It is six miles in circumference; half of which lies along the banks of the river *Severn*.—I mention these particulars to shew, that it may be reckoned a *country* parish; though, perhaps, not perfectly so, on account of its nearness to *Shrewsbury*.—The christenings in it exceed the burials in the proportion of 15 to 13; and the number of inhabitants (mostly labouring people) had for 20 years before 1771 kept nearly to 1050, without any considerable increase.—The register of this parish from 1750 to 1760, has been published in the LIId volume of the *Philosophical Transactions*, Part I. Art. 25. And a continuation of it from 1760 to 1770, in the LXIst volume, p. 57. It is kept with particular care and accuracy by Mr. *Gorsuch*; and furnishes very useful *data* for determining the value of country lives.—It deserves to be mentioned particularly, that no *foreigners* or *strangers*, who happen to die in this parish, or who may be brought into it to be buried, are entered into the register: Nor are any of the fixed inhabitants omitted, though carried out to be buried.

In Nov. 1781. Mr. *Gorsuch* was so kind as to favour me with a continuation of his Observations to 1780, which makes them complete for 30 years. An abstract of them, and a Table of the decrements of life deduced from them, which I reckon one of the most correct that has been ever published, will be found in the Collection of Tables in the next Volume. The conclusions mentioned above are confirmed by the addition of these last Observations.

one half lived to 30 years of age : And, by forming a Table of Observations from this register, in the manner which will be described in the last Essay, I find that a child just born in this parish has an expectation of 33 years ; and that, in general, under the age of 50, the *expectations* of lives there exceed those in *London*, in the proportion of about 4 to 3.—In the parish of *Ackworth, Yorkshire*, it appears, from an exact account kept by Dr. *Lee*, of the ages at which all died there for 20 years to 1767, that half the inhabitants live to the age of 46.—In the province of *Vaud, Switzerland*, consisting of 112,951 (*a*) inhabitants, half live to 41.—So great is the difference between the duration of human life in *towns* and in the *country*.—Further evidence for the truth of this observation may be deduced from the account given by Dr. *Thomas Heberden*, and published in the *Philosophical Transactions* (Vol. LVII. p. 461), of the *increase and mortality of the inhabitants of the island of Madeira*. In this island, it seems, the weddings have been to the births, for 8 years, from 1759 to 1766, as 10 to 48.8 ; and to the burials, as 10 to 27.5, or 9 to 24.75. Double these proportions, therefore, or the proportion of 20 to 46.8, and of 18 to 24.75, are the proportions of the

(.) See the Supplement in the next Volume.

number marrying annually, to the number born and the number dying. Let one marriage in three be a 2d or (a) 3d marriage on the side of either the man or the woman; or, in other words, let one in six of all that marry be *widows* and *widowers*; and 9 marriages will imply 15 persons who have grown up to maturity, and lived to marry once or oftener; and the proportion of the number marrying annually the first time, to the number dying annually, will be 15 to 24.75, or 3 to 5. It may seem to follow from hence, that in this island three-fifths of those who die have been married; and, consequently, that only two-fifths of the inhabitants die in childhood and celibacy; and this would be a just conclusion were there no increase, or had the births and burials been equal. But it must be remembered, that the general effect of an increase while it is going on in a country, is to render the proportion of persons marrying annually, to the annual deaths, *greater*, and to the annual births *less*, than the true proportion marrying out of any given number born. This proportion generally lies between the other two proportions, but always

(a) This proportion is taken nearly from fact.—In all *Pomerania*, during 9 years, from 1748 to 1756, the number of persons who married was 56,956; and of these, 10,586 were *widows* and *widowers*. *Susmilch's Works*, Vol. I. Tables, p. 98.

nearest to the first (a); and, in the present case, it cannot be so little as one half. Agreeably

(a) In a country where there is no increase or decrease of the inhabitants, and where also life, in its first periods, is so stable, and marriage so much encouraged, that half of all who are born live to be married, the *annual* births and burials must be equal, and also *quadruple* the number of weddings, after allowing for 2d and 3d marriages. Suppose in these circumstances (every thing else remaining the same) the *probabilities of life*, during its first stages, to be improved. In this case, more than *half* the born will live to be married, and an increase will take place. The births will exceed the burials, and both fall below *quadruple* the weddings; or, which is the same, below *double* the number annually married.—Suppose next (the *probabilities of life* and the *encouragement to marriage* remaining the same) the *prolifickness* only of the marriages to be improved. In this case it is plain, that an increase also will take place; but the *annual* births and burials, instead of being less, will now both rise above *quadruple* the weddings; and therefore the proportion of the born to that part of the born who marry (being by supposition two to one) will be less than the proportion of either the *annual* births or the *annual* burials, to the number marrying *annually*. Suppose again (the *encouragement to marriage* remaining the same) that the *probabilities of life* and the *prolifickness of marriages* are both improved. In this case, a more rapid increase will take place, or a greater excess of the births above the burials; but at the same time they will keep nearer to *quadruple* the weddings, than if the latter cause only had operated, and produced the same increase.—I should be too minute and tedious, were I to explain these observations at large. It follows from them, that, in every country or situation where, for a course of years, the *burials* have been either *equal to* or *less* than the *births*, and both under *quadruple* the marriages; and also that, wherever the burials are *less* than quadruple the annual marriages, and at the same time

greeably to this, it appears also from Dr. *Heberden's* account, that the *expectation* of a child just born in *Madeira* is about 39 years; or more than double the expectation of a child just born in *London*. For the number of inhabitants was found, by a survey made in the beginning of the year 1767, to be 64,614. The annual medium of *burials* had been, for eight years, 1293; of *births*

time the births *greater*, there the major part of all that are born live to marry.

I have shewn how the allowance is to be made for 2d and 3d marriages. Very wrong conclusions will be drawn if this allowance is not made. But it is, in part, compensated by the natural children which are included in the births, and which raise the proportion of the births to the weddings higher than it ought to be, and therefore bring it nearer to the true proportion of the number born *annually*, to those who marry annually, after deducting those who marry a 2d or 3d time.

In drawing conclusions from the proportion of *annual* births and burials, in different situations, some writers on the increase of mankind, have not given due attention to the difference in these proportions, arising from the different circumstances of increase or decrease among a people. One instance of this I have now mentioned; and one further instance of it is necessary to be mentioned. The proportion of *annual* births to weddings has been considered as giving the true number of children derived from each marriage, taking all marriages one with another. But this is true only when, for many years, the births and burials have kept nearly equal. Where there is an excess of the births occasioning an increase, the proportion of *annual* births to weddings must be less than the proportion of children derived from each marriage; and the contrary must take place where there is a decrease.

2201. The number of inhabitants, divided by the annual medium of *burials*, gives 49.89; or the *expectation* nearly of a child just born, supposing the *births* had been 1293, and consequently equal to the *burials*, the number of inhabitants remaining the same. And the same number, divided by the annual medium of *births*, gives 29.35; or the *expectation* of a child just born supposing the *burials* 2201, the number of *births* and of inhabitants remaining the same. And the true *expectation* of life must be somewhere near the mean between 49.89 and 29.35.

Again: A 50th part of the inhabitants of *Madeira*, it appears, die annually. In *London*, I have shewn, that above twice this proportion dies annually. In smaller towns a smaller proportion dies (*a*); and the births also

(*a*) In *London*, this proportion is, at the highest, 1 in $20\frac{3}{4}$.—In *Norwich*, 1 in $24\frac{1}{2}$.—In *Northampton*, 1 in $26\frac{2}{5}$. See the last Essay: In the parish of *Newbury*, *Berks*, consisting of 3732 persons, all *town* inhabitants, the annual medium of deaths for 19 years, or from 1747 to 1765, was 136. In this town, therefore, 1 in $27\frac{1}{2}$ die annually. The contiguous parish of *Speen* consisted, in 1757, of 1200 inhabitants, about 520 of whom were inhabitants of that part of the town of *Newbury* which is in this parish, and the rest were *country* inhabitants. For 34 years, or from 1724 to 1757, thirty-nine died here annually; or 1 in 31.—In both these parishes the births and burials are nearly equal.—I believe these facts may be depended on; and they seem to shew us very distinctly

also come nearer to the burials.—In general; there seems reason to think that in towns (allowing for particular advantages of situation, trade, police, cleanliness, and openness, which some towns may have,) the excess of the burials above the births, and the proportion of inhabitants dying annually, are more or less as the towns are greater or smaller. In *London* itself, about 160 years ago, when it was scarcely a fourth of its present bulk, the births were much nearer to the burials, than they are now. But in country parishes and villages, the births almost always exceed the burials; and I believe it never happens, except in very particular situations, that more than a 40th (a) part

distinctly the gradations in the degrees of human mortality from *great* towns to *moderate* towns, and from *moderate* towns to *small* towns, and to parishes consisting partly of town and partly of country inhabitants. The next note will shew what the degree of human mortality is in places purely country.

(a) According to *Graunt's* account of a parish in *Hampshire*, not reckoned, he says, remarkably healthful, a 50th part of the inhabitants had died annually for 90 years, *Natural and Political Observations, &c.* Chap. xii.—In the parish of *Ackworth, Yorkshire*, one of 47 die annually. See the register of this parish at the end of the first additional Essay in the next volume. In the province of *Vaud, Switzerland*, one in 45 die annually. See the first part of the *Supplement* in the next volume. In 1098 country parishes, mentioned by *Susmilch*, the annual average of deaths, for six years ending in 1749, was 5255. The number of inhabitants was 225,357.
One,

part of the inhabitants die annually. In the four provinces of *New-England* there is a very rapid increase of the inhabitants ; but, notwithstanding this, at *Boston*, the capital, the inhabitants would decrease, were there no supply from the country : For, if the account I have seen is just, from 1731 to 1762, the burials all along exceeded the

One, therefore, in 43 died annually.—In 106 other parishes, mentioned by him, this proportion was 1 in 50.

In the dukedom of *Wurtemberg*, the inhabitants, Mr. *Susmilch* says, are numbered every year ; and from the average of five years, ending in 1754, it appeared that, taking the towns and country together, 1 in 32 died annually.—In another province, which he mentions, consisting of 635,998 inhabitants, 1 in 33 died annually. From these facts he concludes, that taking a whole country in *gross*, including all cities and villages, mankind enjoy among them about 32 or 33 years each of existence. This, very probably, is below the truth ; from whence it will follow, that a child born in a country parish or village, has, at least, an expectation of 36 or 37 years ; supposing the proportion of *country* to *town* inhabitants to be as $3\frac{1}{2}$ to 1 ; which, I think, this ingenious writer's observations prove to be nearly the case in *Pomerania*, *Brandenburgh*, and some other kingdoms.

In all *Sweden*, consisting in 1763 of 2,446,394 inhabitants, the annual medium of deaths for 9 years, ending in 1763, was 69,125 ; and therefore one in 35 and two-fifths died annually. The medium of births was 90,245 ; of marriages 21,220. See the first additional Essay in the next volume.—In the kingdom of *Naples*, consisting of 4,311,503 inhabitants in 1777, the medium of deaths for 5 years was 115,412 ; and therefore one in 37 and a third died annually. The births were 166,808. See the *Essay on the Population of England*, &c. page 15.

births

births (a). So remarkably do towns, in consequence of their unfavourableness to health, and the luxury which generally prevails in them, check the increase of countries.

Healthfulness and prolificness are, probably, causes of increase seldom separated. In conformity to this observation, it appears from comparing the births and weddings, in countries and towns where registers of them have been kept, that in the former, marriages, one with another, seldom produce less than four children each; generally between four and five, and sometimes above five (b). In all *Sweden* the births and weddings

(a) See a particular account of the births and burials in this town from 1731 to 1752 in the *Gentleman's Magazine* for 1753, p. 413.

(b) Any one may see what evidence there is for this, by consulting Dr. *Sartori*'s two books already quoted, and the *Abridgment of the Philosophical Transactions*, Vol. VII. Part iv. p. 46, and *Graunt's* account, already quoted, of the births, weddings, and burials in three country parishes for 90 years; compared with similar accounts in towns. In considering these accounts, it should not be forgotten that allowances must be made for the different circumstances of increase or decrease in a place, agreeably to the observations at the end of the note in page 264.

In April 1779 the inhabitants of the parish of *Biddulph*, in *Staffordshire*, were numbered, and found to be 495 males, and 540 females, making 207 families. The annual average of births for 20 years preceding 1780 had been 21.4 males, and 17.5 females, of burials 10.85 males, and 10.3 females; of marriages 6.15.—The same averages

dings are to one another as $4\frac{1}{4}$ to 1.—In all *France* as $4\frac{2}{3}$ to 1. But in towns this proportion is generally between 3 and 4 to 1.

I have sometimes heard the great number of old people in *London* mentioned, to prove its favourableness to health and long life. But no observation can be more erroneous. There ought, in reality, to be more old people in *London*, in proportion to the number of inhabitants, than in any smaller towns; because at least one quarter of its inhabitants are persons who come into it from the country, in the most robust part of life, and with a much greater probability

averages for 60 years had been 16.9 males *born* annually and 14.7 females; 9.4 males *buried* annually, and 9.93 females; marriages 5.46.—Taking, therefore, the highest of these averages, it appears that in this parish a 46th part of the *males* die annually, but only a 52d part of the *females*; that the annual births are nearly a 26th part of the inhabitants; and that every marriage, supposing no allowance for illegitimate births, produces six children. This account I owe to an information communicated by the Rev. Mr. *Wilson*, the minister of this parish, to Dr. *Haygarth* at *Chester*.

The parish of *Swinderby*, in *Lincolnshire*, consisted in June 1771 (as I have learnt from Dr. *Disney*, then the worthy minister of that parish) of 52 families and 224 souls, 95 of whom were heads of families, 87 children, 32 servants, and 10 inmates.—The *births*, *marriages*, and *burials* for 30 years before 1771 had been 199, 47 and 154. The proportion of marriages to births therefore, was as 1 to $4\frac{1}{4}$.—A number equal to a 34th of the inhabitants had been born annually, and a 44th part died annually.—The inhabitants of *Okford*, in *Devonshire*, were in 1770, 422.
The

bility of living to old age, than if they had come into it in the weakness of infancy. But, notwithstanding this advantage, there are much fewer persons who live to great ages in *London*, than in most other places where observations have been made.—At *Breslaw* it appears, by Dr. *Halley's* Table, that 41 of 1238 born, or a 30th part, live to be 80 years of age. In the parish of *All-saints*, in *Northampton*, an account has been kept ever since 1733 of the ages at which the inhabitants die; and I find that a 22d part die there turned of 80. At *Norwich* a like account has been kept; and

The average of births for 20 years to 1769 had been 12, and of burials $7\frac{1}{2}$. A 35th part, therefore, was born annually, and a 56th part died.

From the returns of the Intendants of the different provinces of *FRANCE*, it appeared, that the annual medium of births in that kingdom for ten years to 1780, had been 940,935; of deaths 818,491; and of marriages 213,774.—See Mr. *NECKER* on the Administration of the finances of *France*, Vol. I. chap. 9.—The births and marriages were, therefore, in the proportion mentioned in the text. From the last note but one it appears that a 35th part of the inhabitants of a country may be reckoned to die annually. Multiply, therefore, 818,491 by 35, and the kingdom of *France* will appear to have consisted in 1780 of 28 millions and a half of inhabitants. Nor is there any reason to think this to be greater than the true number; for the deaths, as well as the births and marriages, are probably given too small, it being scarcely possible to avoid omissions in such returns. It appears further from the great excess of births, that the population of *France* was then increasing.—See more on this subject in the *Appendix* to my *Discourse on the Love of our Country*, p. 1.

it appears, that a 27th part of the inhabitants die turned of the same age.—According to Mr. *Kerffeboom's* Table of Observations, published at the end of Mr. *De Moivre's* Treatise on the Doctrine of Chances, a 14th part die turned of 80. And this is the very proportion that died turned of 80 in the parish of *Ackworth*, for the 20 years mentioned page 262. In the parish of *Holy-Cross*, already mentioned, p. 261, 1 in $1\frac{1}{2}$, or 2 in 22 of the inhabitants live to 80 (*a*).—But in *London*, for 30 years, ending at the year 1768, only 25 of every 1000, who had died, or a 40th part, had lived to this age (*b*); which may be easily discovered, by dividing the sum of all who have died during these years at all ages, by the sum of all who had died above 80 (*c*).

Among

(*a*) This, however, will appear itself inconsiderable, if the following account is true: “In 1761 the burials “in the district of *Christianna*, in *Norway*, amounted “to 6,924, and the christenings to 11,024. Among “those who died, 394, or 1 in 18, had lived to the “age of 90; 63 to the age of 100, and seven to the “age of 101.—In the diocese of *Bergen*, the persons “who died amounted to 2,580, of whom 18 lived “to the age of 100; one woman to the age of 104, and “another woman to the age of 108.”

See the *Annual Register* for 1761, p. 191.

(*b*) For five years to 1780 only one in 46 lived to 80.

(*c*) In the parish church of *Manchester*, of 4126 buried during six years ending in 1778, a hundred and twenty

Among the peculiar evils to which great towns are subject, I might further mention the PLAGUE. Before the year 1666, this dreadful calamity laid *London* almost waste once in every 15 or 20 years; and there is no reason to think, that it was not generally bred

twenty-nine, or a 32d part, had lived to 80 or more. This proportion would be considerably greater were there no increase of *Manchester*, and no excess of the births above the burials.—The same is true of *Warrington*, in *Lancashire*, where of 2430 buried in eight years ending in 1780, sixty-seven, or a 36th part, had lived to 80 or upwards; and also of the parish of *EC-CLES* in the same county, where of 1123 buried in four years, from 1776 to 1779, fifty-one, or a 22d part, had lived to 80.—In *CHESTER*, where the births and burials are nearly equal, of 1969 females who died in the course of 9 years, from 1772 to 1780, 149 or a 13th part, had lived to 80; but of *males*, only 72 out of 1764, or a 25th part. See the Tables in the next Volume. In all *SWEDEN*, where the births exceed the burials in the proportion of nearly 13 to 10, 710 *females* of 10,000 born (or a 14th part) and 555 *males*, of 10,000 (or an 18th part) live to 80: But in *STOCKHOLM* only one in a 100 of the *females* born there, and one in 300 of the *males*, live to this age. See the Tables in the next Volume.

These facts give a frightful view of the fatality of great towns to human life. A farther account, with answers to some objections, may be found in the next volume, in the first additional Essay on the *difference between the duration of human life in great towns and in country parishes*.

I have said above, that a 40th part of all who die in *London* live to 80. But it should be considered, that a great proportion of those who die in *London* came into

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bred within itself. A most happy alteration has taken place; which, perhaps, in part is owing to the greater advantages of cleanliness and openness which *London* has enjoyed since it was rebuilt; and which lately have been very wisely improved.

The facts I have now taken notice of are so important that, I think they deserve more attention than has been hitherto bestowed upon them. Every one knows that the strength of a state consists in the number of people. The encouragement of population, therefore, ought to be one of the first objects of policy in every state; and some of the worst enemies of population are the luxury, the licentiousness, and debility produced and propagated by great towns.

it in the firmest parts of life, and that consequently nothing can be from hence determined with respect to the proportion of the *natives* of *London* who live to 80. This must be a much smaller proportion. The corrected Table of Observations for *London* (or Table 15th in the next Volume) makes it as 25 to 1518, or as 1 to 60. But even this corrected Table certainly gives the probabilities of living in *London*, at most ages, too high; and were there such accurate *data* for forming a table for *London* as have been furnished by the Observations at *Stockholm*, the rate of mortality in the two cities would not perhaps appear to be very different. More will be said on this subject in the introduction to the Tables in the next volume.

I have

I have observed that *London* is now (a) increasing. But it appears, that, in truth, this is an event more to be dreaded than desired. The more *London* increases, the more the rest of the kingdom must be deserted; the fewer hands must be left for agriculture; and, consequently, the less must be the plenty, and the higher the price of all the means of subsistence. *Moderate* towns, being seats of refinement, emulation, and arts, may be public advantages. But *great* towns, long before they grow to half the bulk of *London*, become checks on population of too hurtful a nature, nurseries of debauchery and voluptuousness; and, in many respects, greater evils than can be compensated by any advantages (b).

Dr.

(a) If we may trust the Bills, *London* has decreased since this was written. The annual medium of burials for five years ending in 1770, 1777, and 1780, was 22,688—21,087—and 20,743. The medium for three years to 1780, was 20,445. But this decrease has probably been owing to the causes mentioned in the notes, p. 251 and 260.

(b) The mean annual *births*, *weddings*, and *burials* in the following towns, for some years before 1772, have been nearly,

	Births.	Weddings.	Burials.
At <i>Paris</i> , — —	19,100	— 4,400	— 19,400
<i>Vienna</i> , from 1757	} 5,800	—	— 6,600
to 1769			
<i>Amsterdam</i> , from	} 4,600	— 2,400	— 7,922
1761 to 1770			
<i>Copenhagen</i> , — —	2,700	— 886	— 3,300
	T 2		At

Dr. *Heberden* observes that, in *Madeira*, the inhabitants double their own number in 84 years. But this (as you, Sir, well know) is a very slow increase, compared with that which takes place among our colonies in AMERICA. In the back settlements, where the inhabitants apply themselves entirely to agriculture, and luxury is not known, they double their own number in 15 years; and all through the northern colonies, in 25 years (a). This is an instance of increase so rapid, as to have scarcely any parallel. The births in these countries must exceed the burials much more than in *Madeira*; and a greater proportion of the born must reach maturity.—In 1738, the number of inhabitants in

	Births.	Weddings.	Burials.
At <i>Berlin</i> , for 5 years, ending at 1759	3,855	— 980	— 5,054
<i>Stockholm</i> , for nine years, ending in 1763	2,535	—	— 3,781

It deserves notice, that before 1770, all that died in the hospitals at *Vienna* were omitted in the Bills.—Of the *Paris* Bills a more particular account will be given in the Postscript to this Essay.—The annual medium of burials at *Amsterdam* for 10 years to 1710, was 7,288. For 10 years to 1780, it was 8,710; but three of these last years were reckoned very sickly years.

(a) See a Discourse on *Christian Union*, by Dr. *Styles*, Boston, 1761, p. 103, 109, &c.—See also, *The Interest of Great Britain considered with regard to her Colonies, together with Observations concerning the Increase of Mankind, peopling of Countries, &c.* p. 35. 2d edit. London, 1761.

New Jersey was taken by order of the government, and found to be 47,369. Seven years afterwards, the number of inhabitants was again taken; and found to be increased, by procreation only, above 14,000; and very near one *half* of the inhabitants were found to be under (a) 16 years of age. In 22 years, therefore, they must have doubled their own number, and the births must have exceeded the burials 2000 annually. As the increase here is much quicker than in *Madeira*, we may be sure that a smaller proportion of the inhabitants must die annually. Let us, however, suppose it the same, or a 50th part. This will make the annual burials to have been, during these seven years, 1000; and annual births 3000; or an 18th part of the inhabitants.—Similar observations may be made on the much quicker increase in *Rhode Island*, as related in the preface to the *Collection of the London Bills of Mortality*; and also in the valuable pamphlet last quoted, on *the Interest of Great Britain with regard to her Colonies*, p. 36.—What a prodigious difference must there be, between the vigour and the happiness of human life in such situations, and in such a place as *London*?—The original number of persons who, in 1643,

(a) According to Dr. *Halley's* Table, the number of the living under 16, is but a *third* of all the living at all ages.

had settled in *New-England*, was 21,200. Ever since, it is reckoned, that more have left them than have gone to them (*a*). In the year 1760, they were increased to half a million. They had, therefore, all along doubled their own number in 25 years. It is not probable that they will continue to increase at the same rate; but should this happen they will, 70 years hence, in *New England* alone, be four millions; and in all the colonies (*b*), above twice the number of inhabitants in *Great Britain* (*c*).—But I am wandering

(*a*) See Dr. *Style's* pamphlet, just quoted, p. 110, &c.

(*b*) In the original letter to Dr. *Franklin*, containing these observations, and communicated by him to the Royal Society (in April 1769), the following words were here added.—“ Formerly an increasing number of “ FRIENDS, but now likely to be converted, by an unjust and fatal policy, into an increasing number of “ ENEMIES.”—This reflection was occasioned by the discontents which were then prevalent in the colonies, and which had been produced first by the Stamp Act, and after the repeal of that act, by the duties laid in *America* on tea, paper, glass, &c.—When read to the Royal Society, it was softened by the omission of the words “ unjust and fatal policy;” but, notwithstanding this, it gave offence; and was suppressed in all the former publications of these Observations.—I need not say how dreadfully the apprehensions expressed by it were afterwards verified.

(*c*) The rate of increase, supposing the procreative powers the same, depends on two causes: The “ encouragement to marriage;” and the “ expectation of “ a child just born.” When one of these is given, the
increase

dering from my purpose in this letter. The point I had chiefly in view was, the present state

increase will be always in proportion to the other.—That is; as much *greater* or *less* as the *ratio* is of the numbers who reach maturity, and of those who marry, to the number born, so much *quicker* or *slower* will be the increase.—Let us suppose the operation of these causes such, as to produce an annual excess of the *births* above the *burials*, equal to a 36th part of the whole number of inhabitants. It may seem to follow from hence, that the inhabitants would double their own number in 36 years; and thus some have calculated. But the truth is, that they would double their own number in much less time. Every addition to the number of inhabitants from the births, produces a proportionably greater number of births, and a greater excess of these above the burials; and if we suppose the excess to increase annually at the same rate with the inhabitants, or so as to preserve the *ratio* of it to the number of inhabitants always the same, and call this *ratio* $\frac{1}{r}$, the period of doubling will be the *quotient* produced by dividing the logarithm of 2 by the *difference* between the logarithms of $r + 1$ and r ; as might be easily demonstrated. In the present case, r being 36, and $r + 1$ being 37, the period of doubling comes out 25 years. If r is taken equal to 22, the period of doubling will be 15 years.—But it is certain that this ratio may, in many situations, be greater than $\frac{1}{22}$; and, instead of remaining the same, or becoming less, it may *increase*, the consequence of which will be, that the period of doubling will be shorter than this rule gives it.—According to Dr. Halley's Table, the number of persons between 20 and 42 years of age is a third part of the whole number living at all ages. The prolific part, therefore, of a country may very well be a 4th of the whole number of inhabitants; and supposing four of these, or every other marriage between persons all under 42, to produce *one*

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birth

state of *London* as to healthfulness, number of inhabitants, and its influence on population. The observations I have made may, perhaps, help to shew, how the most is to be made of the lights afforded by the *London Bills*; and serve as a specimen of the proper method of calculating from them. It is indeed extremely to be wished, that they were less imperfect than they are, and extended further. More parishes round *London* might be taken into them; and, by an easy improvement in the parish registers now kept, they might be extended through all the pa-

birth every year, the annual number of births will be a 16th part of the whole number of people. And, therefore, supposing the burials to be a 48th part, the annual excess of the births above the burials will be a 24th part, and the period of doubling 17 years.

I must not conclude this note without adding a remark to remove an objection which may occur to some in reading Dr. *Heberden's* account of *Madeira*, to which I have referred. In that account 5945 is given as the number of children under seven in the island, at the beginning of the year 1767. The medium of annual births, for eight years, had been 2201; of burials 1293. In six years, therefore, 13,206 must have been born; and if, at the end of six years, no more than 5945 of these were alive, 1210 must have died every year. That is; almost all the burials in the island, for six years, must have been burials of children under seven years of age. This is plainly incredible; and, therefore, it seems certain, that the number of children under seven years of age must, through some mistake, be given, in that account, 3000 or 4000 too little.

ishes

rishes and towns in the kingdom. The advantages arising from hence would be very considerable. It would give the precise law according to which human life wastes in its different stages; and thus supply the necessary *data* for computing accurately the values of all *life-annuities* and *reversions*. It would, likewise, shew the different degrees of healthfulness of different situations, mark the progress of population from year to year, keep always in view the number of people in the kingdom, and, in many other respects, furnish instruction of the greatest importance to the state. Mr. *De Moivre*, at the end of his book on the Doctrine of Chances, has recommended a general regulation of this kind; and observed, particularly, that at least it is to be wished, that an account was taken, at proper intervals, of all the living in the kingdom, with their ages and occupations; which would, in some degree, answer most of the purposes I have mentioned.—But, dear Sir, I am sensible it is high time to finish these remarks. I have been carried in them far beyond the limits I at first intended. I always think with pleasure and gratitude of your friendship. The world owes to you many important discoveries; and your name must live as long as there is any knowledge of philosophy among mankind. That you may ever enjoy all that
can

282 *On the Expectation of Lives;*

can make you most happy, is the sincere
wish of,

S I R,

Your much obliged,

and very humble Servant,

Newington-Green,
April 3, 1769.

RICHARD PRICE.

P O S T-

P O S T S C R I P T.

AT *Edinburgh*, bills of mortality, of the same kind with those of *London*, have been kept for many years. I have, since the foregoing letter was written, examined these Bills, and formed a Table of Observations ~~from them~~, as I found them for a period of 20 years, beginning in 1739, and ending in 1758.—As this is a town of moderate bulk, and seems to have a particular advantage of situation; I expected to find the probabilities of life in it, nearly the same with those at *Breslaw*, *Northampton*, and *Norwich*; but I have been surprized to observe, that this is not the case. During the period I have mentioned, only one in 42 of all who died at *Edinburgh*, reached 80 years of age.—In general; it appears, that the probabilities of life in this town are much the same, through all the stages of life, with those in *London*, the chief difference being, that after 30, they are rather lower at *Edinburgh*.—It is not difficult to account for this.—It affords, I think, a striking proof of the pernicious effects arising from uncleanness, and crowding together on one spot too many inhabitants. At *Edinburgh*, Mr. Maitland says, “the buildings, elsewhere called *houses*, are denominated *lands*, and the *apartments*, in other
“ places

“ places named *stories*, here called *houses*, are
 “ so many freeholds inhabited by different
 “ families; whereby the houses are so ex-
 “ cessively crowded with people, that the
 “ inhabitants of this city may be justly pre-
 “ sumed to be more numerous than those of
 “ some towns of *triple* its dimensions.” See
Maitland's History of Edinburgh, p. 140.

In the year 1748, the whole number of
apartments or *families* in the city and liberties
 of *Edinburgh*, was 9064. This Mr. *Mait-*
land mentions as the result of particular exa-
 mination, and undoubtedly right. *Ib.* p. 217,
 218.—In 1743, an accurate account was
 taken, by the desire of this writer, of the
 number of *families* and *inhabitants* in the pa-
 rish of *St. Cuthbert*. *Ib.* p. 171. The number
 of *families* was 2370, and of *inhabitants* at all
 ages, 9731. The proportion, therefore, of
inhabitants to *families*, was $4\frac{1}{10}$ to 1; and,
 supposing this the true proportion for the
 whole town, the number of inhabitants
 was $4\frac{1}{10}$ multiplied by 9064, or 37,162.
 The yearly medium of deaths in the town
 and liberties for eight years, from 1741 to
 1748, was 1783. *Ib.* p. 220 and 222. And,
 consequently, *one* in $20\frac{4}{5}$ died annually.

Mr. *Maitland*, though possessed of the *data*
 from which these conclusions necessarily fol-
 lowed, has made the number of inhabitants
 50,120, in consequence of a disposition to
 exaggerate in these matters, and of assuming,
 without

without any reason, a 28th part of the inhabitants as dying annually.

In page 220, he expresses much surprize at finding, that the number of males in this town was less than the number of females, in the proportion of 3 to 4. But this is by no means peculiar to *Edinburgh*.

“ All I have been saying must be understood of the state of *Edinburgh*, before the year 1758. The Bills, since this year, have been so irregular, and so different from the same Bills for the preceding years, and from all other Bills, that I cannot give them any credit. Either some particular incorrectness has crept into the method of keeping them; or there has been some change in the state of the town which renders them of no use.

From the note in p. 275, it appears, that the christenings and burials at *Paris*, come very near to equality. This once led me to suspect, that there must be some particular singularity in the state of *Paris*, which rendered it much less prejudicial to health and population than great towns commonly are. But better information has lately obliged me to entertain very different sentiments.—The difference between the births and burials at *Paris*, is much greater than the Bills shew.

“ Children here are baptized the instant
“ they are born; and, in a day or two af-
“ terwards, it is the custom to send them to
“ the

“ the adjacent villages to be nursed. A
“ great number, therefore, of the infants born
“ at *Paris* die in the country, and these
“ appear only in the register of christen-
“ ings.” See a book entitled the *Police*
“ of *France*, page 127. And *Buffon’s* Natural
History, Tom. II. at the end.—“ All the
“ children also received into the *Foundling-*
“ *Hospital*, are immediately sent to be nurs-
“ ed in the country, at a distance from *Paris*,
“ where they remain 5 or 6 years; at the end
“ of which time they are brought again to
“ *Paris*, the boys to be placed in the suburbs
“ of St. *Antoine*, and the girls at *Salpetriere*,
“ to be further maintained ’till they arrive at
“ the age of twelve years.” *Police of France*,
p. 81.—The following passage in the same
writer, containing a further account of this
Hospital, is important; and therefore, though
long, I cannot help transcribing it.—“ Let
“ us suppose, that out of 4000 children an-
“ nually carried into the country, two thirds
“ may die, during the five years they are
“ destined to remain at nurse; so that only
“ 1333 would constantly be the annual
“ number sent back to *Paris*; who, being
“ kept at the two Hospitals St. *Antoine* and
“ *Salpetriere* just mentioned, till they are 12,
“ and succeeded by a like number each year,
“ the total number composed of all brought
“ in the successive years, would make the
“ constant resting stock to amount to 9331.
“ But

“ But of these we will suppose a 5th part
“ to die every year. Yet even then the
“ constant resting stock of children ought to
“ be 7465. How greatly then must we be
“ surprized to find, by the authentic account
“ taken from their own books, only 640
“ boys in the college of St. *Antoine*, and not
“ more than 600 girls at the *Salpetriere*; so
“ that the resting stock of returned foundlings
“ appears to be no more than 1240, which
“ being deducted from 7465, will leave 6225.
“ What then becomes of these?—Are they
“ reclaimed by their parents?—Or do they
“ perish for want of care?—In answer to
“ which question it was explained to me;
“ that as many of the lower class of people
“ were induced to marry, in order to be ex-
“ cused from serving in the militia; so when
“ these have children, which they are un-
“ able to maintain, they usually send them to
“ this hospital; which, therefore, must be
“ looked upon, as not only a charity for the
“ care of exposed and deserted children whose
“ parents are unknown, but also as a public
“ *nursery* for the sustenance of the children
“ of poor people, who, tho’ registered at the
“ office, are often reclaimed from their coun-
“ try nurses by their parents. This accounts,
“ in some measure, for the small stock of
“ children brought back to the hospital at
“ *Paris*.—The further difference is suspected
“ to

“ to be owing to the insufficient nourishment
 “ they receive; as this particular charity, as
 “ well as the General Hospital, adopts that
 “ preposterous method of taking in an un-
 “ limited number, while there is only a li-
 “ mited income for their subsistence.” *Ib.*
 page 83.

These facts prove, that, at the same time that the register of *christenings* at *Paris* must be full, the register of *burials* must be very deficient. Let the deficiencies be reckoned at 3700; and, consequently, the annual burials at 23,100. The annual average of weddings, given in p. 275, is 4400; and, therefore, the number of persons who marry annually must be 8800. Deduct a 6th part (*a*) for *widows* and *widowers*, and 7134 will be the number of *virgins* and *batchelors* marrying annually.—The difference between the christenings and burials is 4000; which, therefore, is the number of annual recruits from the country. These, in general, must be persons in mature life. Suppose 3000 of them to marry after settling at *Paris*. Then, 7134 lessened by 3000, or 4134 will be the number of persons born at *Paris* who grow up to marry; and 14,966, or near *four-fifths* of all who are born at *Paris*, will be the number dying annually in childhood and celibacy. Nor is this at all improbable, for

(a) Vid. Note, p. 263.

it appears from the most authentic documents that *three-fifths* of all that are born at *Stockholm* die under five years of age. It has been observed in p. 264, &c. that in country parishes above half the inhabitants live to marry.

The suppositions on which I have made this computation for *Paris*, seem moderate; but if any one thinks otherwise, he may make the same calculation on any other suppositions.

The births at *Paris* are above four times the weddings; and it may seem, therefore, that here, as well as in the most healthy country situations, every wedding produces above four children. I have observed nothing like this in any other great town. Many children born in the country are, I suppose (*a*), brought to the Foundling-Hospital, and there christened. This Hospital may likewise occasion a more than common number of illegitimate births. And, besides, some who leave the country to settle at *Paris*, may come thither already married. These are circumstances that will swell the register of births, without having any effect on the weddings. I do not, however, know that

(*a*) "If the parents of a child brought to this Hospital are known, the register of its baptism must be produced. If the parents are unknown, the child must be baptised after being received." *Police of France*, page 82.

any of them take place at *Paris*; and, perhaps, it must be granted, that it is distinguished in this respect from most other towns. Nor can I wonder at this, if it be indeed true, not only, that all married men in *France* are excused serving in the militia from whence draughts are made for the army, but also, that a *fifth* of all the children born at *Paris* are sent to the *Foundling-Hospital* (a). These are encouragements to
mar-

(a) See the *Police of France*, p. 83.—This writer adds, that a *third* of all that die at *Paris* die in Hospitals. “In the *Hotel Dieu* (a great Hospital, situated in the middle of the city) we may, he says, behold a horrid scene of misery; for, the beds being too few for the number admitted, it is common to see 4, or 6, or even 8 in a bed together, lying 4 at one end and 4 at the other, ill of various distempers in several degrees; some bad, others worse; some dying, others dead.—Above a *fifth* of all admitted to this Hospital die; the annual numbers admitted being 21,823. The medium of deaths for three years from 1751 to 1753, 4650.—The medium of deaths for the same years in all the Hospitals was 6181.” *Ib.* p. 85.—In our two great city Hospitals, *St. Thomas’s* and *St. Bartholomew’s*, about 600 die annually; or one in 13 of all admitted as in-patients.—An account of the *Hotel Dieu* in *Paris*, much the same with that now given, may be found in the *Memoirs of the Year Two Thousand Five Hundred* lately published, and translated from the *French* by *W. Hooper*, M. D. “A citizen or stranger (this writer says) who falls sick, and is sent thither, is imprisoned in a noisome bed, between a corpse and a person expiring in agonies, to breathe the noxious vapours from the dead and the dying, and convert a simple indisposition into a cruel disease.—Six thousand wretches are
“crowded

marriage that no other city enjoys. It has been seen that the *Foundling-Hospital*, though attended with this effect, is, probably, in the highest degree pernicious.

At the end of the 2d vol. of *Monfieur De Buffon's Natural History*, there are Tables formed from the Observations of M. *Du Pre de S. Maur*, of the *French Academy*, containing an account of the ages at which 13,189 persons died in three parishes at *Paris*; and also, of the ages at which 10,805 persons died in 12 country parishes and villages near *Paris*.—According to these Tables, many *more* die in the beginning of life, and much *fewer* in the latter part of life, in the country than in *Paris*. But the circumstances of *Paris*, and the country round it, are such, that no argument can be drawn from hence in favour of *Paris*. Many of the children dying in the country, are children sent thither from *Paris* to be nursed; and, on the other hand, *many*,

“crowded together into this Hospital, where the air has
“no free circulation; and the arm of the river which
“flows by, receives all its filth, and is drank, abound-
“ing with the seeds of corruption, by half the city.”
The *London Hospitals*, it appears, have greatly the advantage; but indeed, with respect to hospitals in general, *as now constructed and regulated*, I cannot help fearing that they cause more distempers than they cure, and destroy more lives than they save. See *Thoughts on Hospitals*, by Mr. *Aikin*, physician, at YARMOUTH, together with a Letter to the Author, by Dr. *Percival*.

perhaps *most*, of those who die in old age at *Paris*, are persons who have removed thither from the country, some to *Hospitals*, and some to places and settlements. It is evident, therefore, that these Tables give a representation of the probabilities of life at *Paris*, which, when compared with those in the adjacent country (*a*), is just the reverse of the truth. Were the children born at *Paris*, who die in the country, to be transferred to the town register; and, on the contrary, the adults born in the country, who die at *Paris*, to be transferred to the country register, there is no reason to doubt, but that the probabilities of life at *Paris*, would be found as low, in comparison with those in the country, as the probabilities of life in *London* are; or, perhaps, much lower.—This observation is applicable, in some degree, to most other great towns; and, in general, on account of the migrations from the country to towns, navies and armies, we may be satisfied, that we err on the side of *defect*, whenever we judge of the probabilities of life in the *country*, from the numbers dying in the several stages of life; and, on the side of *excess*, whenever,

(*a*) It is for this reason that these Tables, when combined, exhibit justly the *mean* probabilities of life for town and country taken together; and that the Table of the *decrements* of life deduced from them by *M. Buffon* and *Mr. Du Pre*, agrees nearly with *Dr. Halley's* Table.

in the same way, we judge of the probabilities of life in *towns*. And this, it is obvious, has a tendency to confirm all that has been said in the preceding Essay, concerning the pernicious effects of great towns on human life.

There are several *ordonnances* and *arrets* of council which fix the boundaries of *Paris*, and prohibit all new buildings beyond those boundaries.—The reasons of this regulation, as set forth in one of these *arrets*, are remarkable; and it will not be improper to recite them.—“ By the excessive aggrandizing of the city, it is said, the air would be rendered unwholesome, and the cleaning the streets more difficult.”—“ Augmenting the number of inhabitants would augment the price of provisions, labour, and manufactures.”—“ That ground would be covered with buildings which ought to be cultivated in raising the necessary subsistence for the inhabitants; and thereby hazard a scarcity.”—“ The people in the neighbouring towns and villages would be tempted to come and fix their residence in the capital, and desert the country.”—“ And, lastly; the difficulty of governing so great a number of people, would occasion a disorder in the *Police*, and give an opportunity to rogues to commit robberies and murders (*a*).”

(*a*) Vid. *Police of France*, p. 130.

No one can think overgrown cities greater evils than I do. But, yet, I can by no means approve of this policy. The effect of it must be, crowding together too many people within the prescribed boundaries, and rendering a town more the seat of uncleanness, infection and disease.—The number of houses in *Paris* is reckoned about 28,000 (*a*), but the number of inhabitants (supposing a 20th part to die annually, and the true number of burials to be 23,000) must be 460,000; or about 16 times the number of houses.

It is happy for *LONDON*, that there have been no laws to restrain its increase. In consequence of being allowed to extend itself on all sides into the country, the inhabitants now take near twice the room to live upon that they did; and it must be rendered less the means of shortening human life.

(*a*) Vid. *Police of France*, p. 130.

I find, in a Book entitled, *Recherches sur la Population des Generalites d'Auvergne, de Lyon, de Rouen, &c.* by M. MESSANCE, and printed at *Paris* in 1766, the number of houses at *Paris* is given 23,565, from a capitation tax in 1755; and the number of families 71,114. There must, I suppose, be some deficiencies in this account; but M. Messance, by allowing most extravagantly (See the Table at the end of this Postscript) 8 to a family, infers from it that the number of inhabitants at *Paris* is 568,912.—On very unsatisfactory grounds also he makes the inhabitants of *FRANCE* to be near 24 millions. *Susmilch* calls them 16 millions. But the returns mentioned in the note, p. 271, determine them to be a much larger number, and leave little room for controversy on this subject.

In

In page 276, I have given the annual *medium* of births, weddings and burials at BERLIN, from 1755 to 1759.—In 1747, an account was taken with the utmost care, by the order of the King of PRUSSIA, of the number of inhabitants in this town; and, it was found to be 107,224.—In order to be more certain, a *second* account was taken the same year; and the number found the same within 200.—In 1755, the inhabitants were increased to 126,661. Their number, therefore, in 1758, could scarcely be less than 134,000; and must have been to the annual burials nearly as $26\frac{1}{2}$ to 1.—This proportion is higher than could be expected in a town so considerable; and also so much crowded, as to have, at an average, 16 inhabitants in every house. But an observation already made, must be here remembered.—BERLIN, for many years, had been increasing very fast, by a conflux of people from the surrounding country and provinces. About the year 1700, the medium of annual burials was no more than 1000. In 50 years, therefore, it has more than quadrupled itself.—In a city increasing with such rapidity, the *ratio* of inhabitants to the annual deaths, must be greatly above the just standard.—Were there now, such accessions to LONDON of deserters from the country, in the beginning of mature life, as would cause the number of inhabitants to increase at the rate

of 10,000 every year, it would in 50 years be doubled; and the proportion of inhabitants to deaths would rise gradually, 'till it came to be about one-third greater. BERLIN, we have seen, has, in fact, increased at *double* this rate; and, therefore, the number of inhabitants dying annually in it is in reality very high.

The ingenious *Susmilch*, to whose works I owe my information concerning BERLIN, makes the proportion of people who die annually in *great* towns, to be from $\frac{1}{24}$ to $\frac{1}{28}$; in *moderate* towns, from $\frac{1}{28}$ to $\frac{1}{31}$; and in the country from $\frac{1}{40}$ to $\frac{1}{50}$.—The observations and facts in this Essay, joined to those which will be found in the 4th Essay, and the *Supplement* in the next volume, prove, I think, that these proportions may be more truly stated as follows.—*Great* towns, from $\frac{1}{29}$ or $\frac{1}{20}$ to $\frac{1}{23}$ or $\frac{1}{24}$. *Moderate* towns, from $\frac{1}{33}$ to $\frac{1}{28}$. The *country*, from $\frac{1}{35}$ or $\frac{1}{40}$, to $\frac{1}{30}$ or $\frac{1}{60}$.—This, however, must be understood with exceptions. There may be *moderate* towns so ill situated, or whose inhabitants may be so crowded together, as to render the proportion of deaths in them greater than in the largest towns: And, of this, EDINBURGH, if it is not now, was 30 years ago an example.—There may be also *great* towns in which, from a sudden increase, this proportion may be less than in small towns: And of this I have just given an example in BERLIN.—On the
con-

contrary ; there may be *moderate* towns so advantageously circumstanced as to be equally healthy with many *country parishes* ; and of this, *Chester* seems to be a very singular instance. See the Introduction to the Tables in the next volume.—And there are some country parishes so ill situated as to be no less unhealthy than great towns ; of which a marshy parish in *Switzerland*, described in a letter to Dr. *Horsley* at the end of the next volume, is an instance.

In Nottingham, according to a survey in Sept. 1779, exclusive of 294 in hospitals and workhouses — — — — —	Families, 3,267 } 3,556 }	Inhabitants, 17,417 }	To a house, 5 $\frac{1}{2}$. To a family, 4 $\frac{1}{2}$.
Norwich, according to a survey in 1752	— Families	—	— To a house, 5.
Shrewbury, by a survey in 1750 — —	—	36,169 —	— To a family, 4 $\frac{1}{2}$.
Northampton, by a survey in 1746 — —	—	13,328 —	—
The parish of Ackworth, Yorkshire, in 1767	—	5,136 —	—
Newbury, Berkshire, in 1768 — —	—	728 —	—
Speen, adjoining to Newbury, in 1768 — —	—	3,732 —	—
Aldwinckle, Northamptonshire, in 1772	—	1,200 —	—
The parish of Holy Crofs, near Shrewsbury, in 1760 — — — — —	—	402 —	—
Altringham, Cheshire, in 1772 — —	—	1,050 —	—
The Parish of St. Michael's, Chester, in 1772 — — — — —	—	1,029 —	—
The town and parish of Bala, North-Wales, in 1774 — — — — —	—	618 —	—
Fifty-nine Dutch villages mentioned by Struyk — — — — —	—	1,723 —	—
Birmingham, in 1770 — — — — —	—	45,888 —	—
Liverpool*, in 1773, including 400 in the Poor-house, but exclusive of about 4000 absent at sea — — — — —	—	30,804 —	—
	—	34,407 —	—

3700
1391
1398

* Houses in Liverpool in 1753
Annual average of deaths for five years to 1771
of births
See an *Essay* towards the History of Liverpool, by Dr. Enfield, p. 23—34.

In Biddulph, in Staffordshire, according to an accurate enumeration in April, 1779, by the Rev. Mr. Wilton	— Families	1,935	— Inhabitants,	207	— To a house, 5.
Swinderby, in Lincolnshire, according to an enumeration by the Rev. Mr. Difney	— Families	52	—	224	— 4 $\frac{1}{2}$.
Holbee, Hunflett, and six other villages, near Leeds. See Mr. Wales's Inquiry, p. 41	— Families	2,631	—	11,468	— 4 $\frac{1}{2}$.
Manchester and Salford, in 1773	— Houses	4,338	—	27,246	— 6 $\frac{1}{2}$.
Leeds, in 1775	— Families	4,096	—	17,121	— To a family 4 $\frac{1}{2}$.
The District of Vaud in Switzerland	— Families	25,778	—	112,951	— 4 $\frac{1}{2}$.
Chester in 1774	— Families	3,428	—	14,713	— 4 $\frac{3}{8}$.
Rome, in 1770	— Families	37,449	—	158,442	— 4 $\frac{1}{2}$.
Calne, Wiltshire	— Families	776	—	3,467	— 4 $\frac{1}{2}$.
Liverpool	— Families	8,002	—	34,407	— 4 $\frac{1}{2}$.
Manchester	— Families	6,416	—	27,246	— 4 $\frac{1}{2}$.
Bolton in Lancashire, in 1773, including Little Bolton	— Houses	1,178	—	5,339	— To a house, 4 $\frac{1}{2}$.
Bury in Lancashire, in 1772	— Houses	463	—	2,090	— 4 $\frac{1}{2}$.
The parish of Bala in North-Wales, in 1774	— Houses	401	—	1,723	— 4 $\frac{1}{2}$.
Chippenham, Wilts, in 1773	— Houses	483	—	2,407	— 5.
Brenhill, near Calne, in Wiltshire	— Houses	218	—	1,206	— 5 $\frac{1}{2}$.
The Island of Sicily (see end of 2d vol. of Brydone's Travels)	— Houses	268,120	—	1,123,163	— 4 $\frac{1}{2}$.

In Fourteen market towns mentioned by Dr. Short, Comparative History, page 58	Families	20,371	Inhabitants,	97,611	To a fami	4 $\frac{1}{2}$.
					^{lv}	
Sixty-five country parishes, <i>ibid.</i>	Families	17,208		76,284		4 $\frac{3}{4}$.
	Houses	139		506	To a house,	4 $\frac{1}{2}$.
The Parish of <i>Stelton</i> , Yorkshire, in 1777	Families	500		2,461	To a family	4 $\frac{1}{2}$.
The town and parish of Wycombe, Bucks	Families	1,685		9,117		5 $\frac{3}{4}$.
Worley, Barton, Pendleton, Pendlebury, and Clifton, Lancashire, in 1778	Families	2,370		9,731		4 $\frac{1}{2}$.
Parish of St. Cuthbert, Edinburgh, in 1743 (see Maitland's History of Edinburgh, page 171)	Families	24,931		99,332		4 $\frac{1}{2}$.
In a number of small towns and parishes in the Generalities of Auvergne, Lyon, and Rouen, in France (see <i>Recherches sur la Population</i> , par M. Messance, pages 8, 26, and 62)	Families	2,525		13,786		5 $\frac{1}{2}$.
	Houses	2,412		13,786	To a house,	5 $\frac{1}{2}$.
Parish of Manchester, exclusive of the town, in 1774						
Parish in the city of London (see Phil. Transf. vol. 48, part 2d, page 796)						

In Warrington and its vicinity, by a survey in April 1781	— Houses	2,145	— Inhabitants,	9,770	— To a house, 4 $\frac{2}{3}$.
Maidstone, by a survey in 1781	{ Houses Families	1,106 1,276	—	5,650	— To a family 5 $\frac{1}{10}$.
Town and parish of Ashton under-Line, near Manchester, by a survey in 1775. See Phil. Trans. vol. 66, p. 164	{ Houses Families	1,494 1,570	—	7,956	— To a house, 5 $\frac{1}{3}$.
Tattenhall and Waverton, two parishes near Chester, by a survey in 1774.—lb. p. 165	{ Houses Families	261 292	—	7,956	— To a family 5 $\frac{1}{17}$.
Swindon, Wilts, by a survey in 1781	{ Houses Families	190 151	—	1,413	— To a house, 5 $\frac{2}{3}$.
Parish of St. Michael's, in Chester, by a survey in 1772	{ Houses Families	127 205	—	1,413	— To a family 4 $\frac{2}{3}$.
Bridgend, in Glamorganshire, in March 1783	{ Houses Families	127 205	—	1,064	— To a house, 5 $\frac{2}{3}$.
	{ Houses Families	127 205	—	618	— To a family 4 $\frac{1}{10}$.
	{ Houses Families	127 205	—	824	— To a house, 4 $\frac{2}{3}$.
	{ Houses Families	127 205	—	824	— To a family 4 $\frac{2}{3}$.

E S S A Y II.

On Mr. DE MOIVRE's Rules for calculating the Values of Joint Lives; with a Postscript, containing a Specimen of the most expeditious Method of calculating the Values of Single and Joint Lives, according to any Table of Observations.

THE calculation of the values of *single* and *joint* lives, from given Tables of Observation, being tedious and troublesome; Mr. *De Moivre* has had recourse to two *hypotheses*, which give easy rules for this purpose; and which, he thought, corresponded with sufficient exactness to Observations.—The first of these *hypotheses* is, that the probabilities of life decrease, as we advance from childhood to old age, in an *arithmetical progression*; or in such a manner, that the *difference* is always the same, between the number of persons living at the beginning of any one year, and the number living at the beginning of the next following year.—The other *hypothesis* is, that the probabilities of
life

life decrease in a *geometrical* progression ; or in such manner, that the *proportion* is always the same, between the number of persons living at the beginning of any one year, and the number living at the beginning of the next following year.—All the Tables of Observation shew, that the real law, according to which human life wastes, comes much nearer to the former *hypothesis*, than the latter.—In Tables V, VI, and VII, in the next volume, it is so near the former *hypothesis*, that the difference between them in the middle stages of life is scarcely worth regarding. According to this *hypothesis*, therefore (accommodated to the *Breslaw* Table, in the manner mentioned in the note, page 2.) Mr. *De Moivre* calculated the values of *single lives* ; and the rules founded upon it for this purpose are so easy, that an operation which would otherwise take up much time, may be performed almost immediately.

By proceeding on the same principles, the values of *joint lives* might have been calculated ; but the rules for this purpose derived from these principles, are far from being equally easy in practice. Here, therefore, Mr. *De Moivre* quitted his *first* hypothesis ; and finding, that the *second* hypothesis afforded, in the case of *joint lives*, rules that were as easy as the rules given by the other hypothesis were in the case of *single lives*, he chose to adopt this *hypothesis* ; believing
at

at the same time, that the values of *joint* lives, obtained by rules derived from it, would not deviate much from the truth. But in this he was greatly mistaken. The values of *two joint* lives obtained by these rules are so wrong, that in finding the present value, in a *single payment*, of one life after another, they generally give results which are near a *quarter* of the true value too great; and about *two-fifths* too great, when the value is sought in *annual payments* during the joint lives. These are errors so considerable, that I think it of particular importance that the public should be informed of them, in order to prevent the inconveniencies and perplexities they may occasion.

Mr. *Simpson* (in the Appendix to his Treatise on the *Doctrine of Annuities and Reversions*) has observed, that Mr. *De Moivre's* rules for finding the values of joint lives are wrong. But I don't know, that it has been ever attended to, that they are *so* wrong as I have found them. Mr. *Simpson's* remarks point out chiefly the errors in these rules, when the values of *three* or more joint lives are calculated by them; but, 'till I was forced to a particular examination of this subject by some difficulties into which I found myself brought by following Mr. *De Moivre* too implicitly, I did not at all suspect, that any such errors as I have mentioned, could arise from these rules, when
the

the values of only *two* joint lives are calculated by them. Mr. *De Moivre*, in consequence of other remarks contained in Mr. *Simpson's Appendix*, altered, in the 4th edition of his *Treatise*, some of his rules. It is surprizing he did not see reason at the same time to alter these.

That there may be no doubt about the truth of these observations, I will just mention a few examples of the difference between the values of a given reversionary annuity, according to the rules to which I have objected, and the values, according to the exact method of deducing them from Mr. *De Moivre's first hypothesis*.

Let the proposed annuity be 30*l.*, to be enjoyed for what shall happen to remain of the life of a person now 40 years of age, after the life of another person of the same age. The value of the joint lives (interest being at 4 *per cent.*) is, by the 2d hypothesis, or problem 2d of Mr. *De Moivre's Treatise* on Life-Annuities, 8.964; which subtracted from 13.196, (the value, by the first hypothesis, of a single life at 40) gives 4.23; which remainder, multiplied by 30, gives 1.126.9, or the value of the reversion in a single present payment. And 126.9, divided by the foregoing value of the joint lives, is 1.14.16; or, the value of the reversion in annual payments, during the joint lives.—But the *true* values are 1.101.1

in a *single* payment, by Quest. I. chap. I.; and *l.* 10.3, in *annual* payments, by Quest. IV.—The former values, therefore, are a *quarter* of the true value too great in the *single* payment; and near *two-fifths* too great in the *annual* payments.

The *true* value of the same annuity for a life at 66, after another life of the same age, is, (reckoning interest as before, at 4 *per cent.*) 68*l.* in a *single* payment; and *l.* 13.5 in *annual* payments.—But these values, according to the Problem just quoted, are 91*l.* and 21*l.* one of which is near a *third*, and the other above *half* the true value too great.

In *unequal* lives these errors may be no less considerable.—Thus; if the value of the proposed annuity be required for a life at 70, after a life at 30 years of age; it will, by the same Problem, be *l.* 26.5, in a *single* payment; and *l.* 5.1, in *annual* payments during the joint lives. But the *true* values are 17*l.* and *l.* 3.05.

Where 3 or more lives are concerned the errors will be still greater.

The true values of the joint lives, mentioned in these Examples, have been calculated by a rule in page 16, of Mr. *Simpson's* Treatise on the Doctrine of *Annuities* and *Reversions*, and explained in note (L) at the end of the next volume.—To save, however, a great deal of trouble hereafter, I have thought proper

proper to calculate Table II. at the end of the next volume, which gives the exact values according to Mr. *De Moivre's* first hypothesis, of two joint lives, for every five years of human life, from 10 to 70.

This *hypothesis*, I have observed, does not differ much from the Tables of Observation for *Breslaw*, *Northampton* and *Norwich*. Between the ages of 30 and 40, it gives the values of *single* lives almost the same with the *Breslaw* Table. Under 30, it gives them somewhat *less*; and above 40, somewhat *greater*. But it ought to be remembered, that wherever it does this, it gives, at the same ages, the values of the *joint* lives also too little or too great; and that, consequently, the results from it, in calculating the values of *Reversions*, and of the *longest* of given lives, come so much nearer to exactness.

The rules to which I have objected are the only ones given by Mr. *De Moivre*, in all the editions of his Treatise on Life-Annuities. But it seems, this great mathematician became at last sensible, that they were too incorrect; and therefore, at the end of the last edition of his Treatise on the *Doctrine of Chances*, page 320, (a work which gets into comparatively few hands) he had given other rules which come nearer the truth. But even these rules produce errors so great in

many cases, (particularly when combined with the errors of the hypothesis) that it will be best never to use them.

Postscript *for the* Fourth Edition.

SINCE the former editions of this work I have found reason to be dissatisfied with Mr. *De Moivre's first* as well as his *second* hypothesis. There is no situation in which, in the first and last periods of life, it corresponds to fact; and in some situations, particularly great towns and country parishes, it does not correspond sufficiently to fact in *any* periods of life. An inspection of the Tables of Observation in the next volume will prove this. However useful, therefore, this hypothesis may be in many cases, it would be best not to be under any necessity of having recourse to it; and for this reason, and also to render this work as complete as I am capable of making it, I have, while this edition has been in the press, and with the help of some friends, calculated the tables in the next volume of the values of single and joint (*a*) lives from the *Northampton* register of mortality. This

(*a*) The value of the reversionary annuity, mentioned in p. 306, is by these Tables 14.83 in annual payments, instead of 13.5, as there given from the hypothesis.

register has been chosen for this purpose, because it gives the mean values of lives between the highest and lowest, and is on this account, and also in consequence of the corrections I have made in it, better fitted for general use than any other.—I have, however, retained the Tables of these values according to Mr. *De Moivre's* Hypothesis, published in the former editions of this work, because all the examples in the preceding part of this work have been taken from them, and there are some cases in which they may still prove of use.

The computation of the values of joint lives correctly from a given table of Observations, is a business so tedious and tiresome, that it has scarcely been ever executed, except by Mr. *Simpson* from the *London* Observations; and as these give the values of lives among a body of people taken in the gross in one of the worst of all situations, they are by no means fit for common use.—I have, therefore, employed a good deal of attention to find out the most easy and expeditious method of making these calculations; and I shall here give the following *Specimen* of a method (deduced from that described by Mr. *Morgan* in his Treatise on Life-Annuities and Assurances, chap 2d, sect. 2d, p. 56) which, at the same time that it renders mistakes impossible, will ex-

pedite this work as much as the nature of it will allow, and render the computation of the values of any number of *joint* lives not more difficult or tedious than the computation of the values of an equal number of *single* lives.

Let the Table of Observations be that for *Northampton*, or Table 6th, in the next volume; and let the rate of interest be 4 *per cent*.

Write down on a paper to be always kept in sight the Logarithms of all the numbers in the column of the living without the Indices.

EXAMPLE FIRST.

Living at age 0 —	11650 —	Log ^{ms} .	.066325
1 year	8650 —		.937016
&c.	&c.		&c.
age 81 —	406 —		.608526
82 —	346 —		.539076
83 —	289 —		.460897
84 —	234 —		.369215
&c.	&c.		&c.
age 91 —	34 —		.531478
92 —	24 —		.380211
93 —	16 —		.204119
94 —	9 —		.954242
95 —	4 —		.602059
96 —	1 —		.000000
			Find

Find the Logarithm of 1*l.* increased by its interest for a year, and also the Logarithm of the value of 1*l.* payable at the end of a number of years equal to the difference between the greatest and least ages in the Table of Observations lessened by the difference of age between the joint lives whose values are to be calculated.

EXAMPLE SECOND.

Interest being at 4 *per cent.* 1*l.* increased by its interest for a year is 1.04; and the Logarithm of 1.04 is .0170333.

In the *Northampton* Table of Observations the greatest age is 96, and the least age 0. The difference, therefore, is 96; and supposing the given difference of age between the two joint lives to be 10 years, the value of 1*l.* payable at the end of a number of years equal to the difference between the greatest and least ages in the table lessened by the difference of age between the joint lives, will be the value of 1*l.* payable at the end of 86 years. Table 1st, in the next volume, shews this value to be .0342872 (reckoning interest at 4 *per cent.*) the Logarithm of which number (striking out the Index) is .535133.

N. B. The best way of finding this Logarithm is by multiplying the Logarithm of 1*l.* with its interest for a year by the difference

X 4

ference

ference between the greatest and least ages in the Table lessened by the difference of age between the joint lives, and subtracting the product from unity. The remainder will be the Logarithm sought. Thus, in the present example $.0170333$ multiplied by 86, gives (without the Index) $.464864$, which subtracted from unity leaves $.535136$.—
 Had the given difference of age between the two joint lives been 15 years, and the youngest age in the Table of Observations 3, and the oldest 94, the Logarithm $.0170333$, instead of being multiplied by 86, must have been multiplied by 76, and the product (without the Index) subtracted from unity would have been $.705469$.

Having made these preparations, the calculations must begin with the oldest joint lives, and proceed upwards according to the following specimen.

SPECIMEN of an easy and expeditious Method of calculating the Values of two Joint Lives.

Interest at 4 per Cent. — Difference of Age 10 Years. — *Northampton* Table of Observations.

Values. — — — Ages $\frac{95}{83}$ } .18739 --- Ages $\frac{94}{84}$ } .403347 --- Ages $\frac{93}{83}$ } .61455 --- Ages $\frac{92}{82}$ } .86446 --- Ages $\frac{91}{81}$ } 1.07846

.017033 (a) added continually to .535136

See *second* Example — — — A .535136 — .552169 — .569202 — .586235 — .603268

Log^{ms}. of the numbers living at 96, 95, 94, &c. years. See *first* Example — —

Log^{ms}. of the numbers living at 86, 85, 84, &c. years. See *first* Example — —

B + C — — — D .161368 — .871571 — .323457 — .665016 — .919287 — .140004

Log^{ms}. of the numbers (with unity added) of 1st, 2d, 3d, &c. Log^{ms}. in H — —

D + E — — — F .161368 — .946168 — .470619 — .873067 — .189840

.017033 (a) added to 2d, 3d, 4th, &c. Logarithms in D — —

F — G — — — G .888604 — .340490 — .682049 — .936320 — .157037

A + G — — — H .272764 — .605678 — .788570 — .936747 — .032803

H + I — — — I .423740 — .892659 — .251251 — .522555 — .760305

Numbers of the Log^{ms}. in I — — — K .696504 — .498337 — .039821 — .459302 — .793108

Numbers of the Log^{ms}. in K — — — L .265303 — .781016 — .178341 — .333087 — .575846

Numbers of the Log^{ms}. in K — — — M .49717 — .315019 — .109603 — .287941 — .621025

Proof or L + M — — — 315020 — 1096035 — .287944 — .621028

(a) This must be .0188372 --- .0149403 --- .017033 --- .021189 --- .0243059, as the rates of interest are 3, 3½, 4, 5, or 6 per cent.; and the first Logarithm in A for these several rates of interest, on which all that follow in that line depend, is to be deduced from them by the rule in the *second* Example;

OBSERVATIONS.

IN the addition of the Logarithms in this Specimen, the decimal parts only are to be retained.

In subtracting them, it is of no consequence whether a Logarithm is greater or less than that from which it is to be subtracted.

In every column, the numbers in the lines B, C, D, E, F, G, H, give the *value*. The other numbers give the *proof*.

The *first* Logarithms in the lines B and C are always the Logarithms of the numbers of the living at the oldest ages in the Table of Observations, which have the given difference of age; and the following Logarithms are the Logarithms of the numbers living at the next ages, each one year younger than the preceding.

The values of the two joint lives are the numbers of the Logarithms in H; and the proof of these values consists in the equality of the sum of the numbers in L and M in one column, to the number in M in the following column. And it should be particularly observed that this proof answers sufficiently if, in consequence of placing the numbers in L and M over one another in *any* order, a sum can be made out whose first six figures are equal within 5 or 6 units to the succeeding number

ber in M. If the proof does not answer within this limit, the calculations have been too incorrect (*a*), and it will be necessary to examine the numbers not verified by the last proof; namely, the numbers of the two Logarithms in I and H in the *preceding* column; the Logarithms in B and C and D in the *subsequent* column; and the Logarithms in A, E, F, G, H, I and K in the column where the proof is found to be deficient.

EXAMPLE.

The addition, in the foregoing specimen, of the first numbers in L and M to one another, when the *first* figure in M is placed under the *second* in L, makes 315020, which is within one unit the same with 315019, the second number in the line M; and this proves the calculations so far to be sufficiently correct.—In like manner; the addition of the *third* numbers in L and M gives the *fourth* in M within three units. But had the addition of the numbers in L and M given the subsequent number in M only within *six* units; that is, had it given the number 287947 or 287935, an incorrectness of too much consequence must have insinuated it-

(*a*) This supposes that the values of the joint lives are to be found to three places of decimals. The agreement of *five* figures within 4 or 5 units will be sufficient, if the values are required only to two places of decimals.

self,

self, and it would have been proper to examine the numbers and Logarithms just mentioned, in order to detect it.

In calculating the *last* value ; that is, the value when the youngest of the lives is the youngest in the Table (or a life just born, according to most Tables) this proof will change into a new proof, verifying all the preceding values. For the Logarithm of 1/. with its interest for a year (that is, .017033 when the interest is 4 *per cent.*) subtracted from the Logarithm in F, will leave a Logarithm, the number of which will be the sum of the numbers in L and M in the preceding column.

This Specimen will be accommodated to the calculation of the values of *single* lives (*a*), by striking out the Logarithms in C, and making those in D the same with those in B ; and also making the first Logarithm in A, from which all that follow are deduced, the Logarithm of 1/. payable at the end of a number of years, equal to the oldest age in the Table when it begins at birth, or to the difference between the oldest and youngest when it begins at any age after birth.

(*a*) In this case, and also in calculating the values of *equal* joint lives, the first Logarithm in F (when the number living at the oldest age in the Table is 1) will be nothing ; but notwithstanding this, the first Logarithm in G must be subtracted from it just as if it was unity, in order to obtain the first Logarithm in H.

This

This Specimen will be accommodated to the calculation of the values of any *three* joint lives, if to the Logarithms in B and C are added (in order to obtain D) the Logarithms of the numbers living at any other ages, the difference between which and the ages in B and C is given; and if, likewise, the series of Logarithms in A is deduced from the Logarithm of the value of 1*l.* payable at the end of a number of years equal (if the ages begin at 0) to the oldest in the Table; or (if they do not begin at 0) to the difference of age between the oldest and youngest in the Table; lessened (in both cases) by the differences of age between the oldest and youngest of the three joint lives whose values are sought.—Thus. Supposing the given differences between the ages of the joint lives whose values are to be calculated to be 5 and 10 years, and the interest 4 *per cent.* and the Table of Observations to terminate (as the *Northampton Table* does) at 96 years of age, and to begin at 0. The series of Logarithms to be added to those in B and C in order to obtain D, will be the Logarithms of the living at 91 in the first column, at 90 in the second, at 89 in the third, &c. And the Logarithms in A will be the same with those in the Specimen. But had the differences of age been 10 and 15 years, the Logarithms to be added to those in B and C would have been

been the Logarithms of the living at 81, 80, 79, &c. and the first Logarithm in A would have been the product of 81 into .017033 subtracted from unity or .620302, and the following Logarithms in A would have been .017033 added continually to this Logarithm.

It is hence evident that in this method computations of the values of any given *two* or *three* or *four* joint lives are nearly as easy as computations of the values of *single* lives; and may, after some practice, be performed almost as expeditiously as the numbers can be written.

An error in a book of Logarithms may, if not suspected, produce infinite perplexity; and therefore, when, after repeating any calculation, the source of an error cannot be discovered, it will be right to examine the Tables from whence the Logarithms have been taken. In general, the order in which the numbers follow one another will immediately discover an error of the press; but if not, a different book of Logarithms should be consulted; and if possible, Mr. GARDINER's, which is so correct as to be almost invaluable.

It may be proper to observe once more, that it is very easy to take from SHERWIN's or GARDINER's Tables the numbers of Logarithms, and the Logarithms of numbers to *six* figures; and that if this is done, the
re-

resulting values will be always accurate to at least the *third* place of decimals. But, if such a degree of accuracy is thought needless, it will be sufficient to take them to *five* figures.

The Theorems on which the method of calculation here explained (exclusive of the proof) are grounded, will be given in note N at the end of the next volume. But a more distinct investigation of these Theorems, and also an explanation of the principles on which the *proof* is founded, has been given by Mr. *Morgan* in his Treatise on Annuities and Assurances, Chap. II. d. Sect. 2d.

The rules for finding, from the values of two or three joint lives, the values of the *longest* of any *two* or *three* lives; and also a very easy rule for obtaining *nearly* the value of any *three* joint lives from the values of *two* joint lives, will be given in the next volume at the end of the Table, shewing the values of two joint lives according to the *Northampton* Observations.

E S S A Y I I I.

*Of the Method of calculating the
Values of Reversions depending on
Survivorships.*

ALL Questions relating to the values of lives and reversions, are at present of particular importance in this kingdom. Much business is continually transacted in this way; and any considerable errors in the methods of solving such questions, must in time produce very bad consequences.—The design of the following observations is to point out a particular error, into which there is danger of falling, in finding the values of such reversions as depend on survivorships. In doing this, I shall, in order to be as plain as possible, take the following case. “ A, “ aged 40, expects to come to the possession “ of an estate, should he survive B, aged “ likewise 40. In these circumstances he “ offers, in order to raise a present sum, to “ give security for 40*l. per annum*, out of “ the estate at his death, provided he should “ get

“ get into possession ; that is, provided he
 “ should survive B. What is the sum that
 “ ought now to be advanced to him, in
 “ consideration of such security, reckoning
 “ compound interest at 4 *per cent.*?”

Mr. *De Moivre*’s directions in his *Treatise on Annuities*, Problems 17th and 20th, lead us to seek the required sum in this case, by the following process.

Find first, the present sum A should receive, for the reversion of 40*l.* *per annum* for ever after his death ; supposing it *not* dependent on his surviving B. The present value of such a reversion is “ the (a) value of the
 “ life subtracted from the *perpetuity*, and
 “ the *remainder* multiplied by the annual
 “ rent.”—The value of the life is, by Mr. *De Moivre*’s Hypothesis, 13,196. This subtracted from 25, the *perpetuity*, leaves 11,80 ; which, multiplied by 40, gives *l.*472 ; the value of the supposed estate, after the life of A. But, as Mr. *De Moivre* observes, the lender having a chance to lose his money, a compensation ought to be made to him for the risk he runs, which is founded on the possibility, that a man of 40 years of age may not survive another person of the same age. This chance is an *equal* chance ; and, therefore, half the preceding sum, or

(a) By *Scholium*, p. 34, and Problem 26th, p. 293 of Mr. *Simpson*’s *Select Exercises*.

236*l.* is the money which should be advanced now on the expectation mentioned.

This solution carries a plausible appearance; and most persons will, probably, be ready to pronounce it right; nor will this be at all wonderful, as so great a master of these subjects as Mr. *De Moivre*, appears to have been misled by it.—Nothing more is necessary to prove it to be fallacious, than proceeding in the same way to solve the following similar Question.

“A, aged 40, offers to give security for
 “40*l. per annum*, to be entered upon at his
 “death, provided it should happen *before* the
 “death of B, aged likewise 40. What sum
 “should now be advanced to him for such
 “a reversion, interest being reckoned at 4
 “*per cent.*?”

In solving this problem, agreeably to the method just described, we are to find the value of 40*l. per annum*, to be entered upon *certainly* at the death of A; and then to multiply this value by the chance that A shall *not* survive B, or by $\frac{1}{2}$; and in this way the answer comes out the same with that already given.

Now it may be easily seen, that this must be wrong. The value of a reversion, to be received when a person of a given age dies, cannot be the same, whether the condition of obtaining it is, that he shall die *before*, or that he shall die *after* another person. That

is,

is, whether it is provided, that a purchaser, if he succeeds, shall get into possession *sooner* or *later*. The reversion in the latter case must, without doubt, be of less value than in the former.

The first Question here proposed, resolves itself into the following general Question.

“What is the present value of a given reversionary estate, to be entered upon after the failure of two lives, provided one *in particular* of them should be the *longest* life?”

Now, the present value of an estate to be enjoyed for ever, after the failure of the *longest* of two lives, is “the value of the *longest* of the two lives, subtracted from the *perpetuity*; and the remainder multiplied by “the annual rent of the estate.”—The value of the *longest* of two lives is (as is well known) the value of the two *joint* lives, subtracted from the sum of the (a) values of the two *single* lives. In the present case, therefore, it is 9.82, (the value of two joint lives at the age of 40 by Table II^d at the end of the notes in the next volume) subtracted from twice 13.196; (the value of a *single* life at the same age) that is, 16.57 year's purchase. And this subtracted

(a) See Mr. *De Moivre* on Annuities, Problem IV. or Mr. *Simpson's Doctrine of Annuities and Reversions*, Problem II.

from 25, (the perpetuity) gives 8.43; which, multiplied by 40, gives *l.* 337.2, the value of the given estate were it *certainly* to be enjoyed, after the extinction of the longest of two lives both 40; that is, whether *one* or *other* of them failed last. But that A's life in particular should fail last, is an even chance. The true value of the reversion, therefore, is half the last value, or *l.* 168.6.

In like manner. The second Question is the same with the Question, "What is the present value of 40*l.* *per ann.* for ever, to be entered upon after the extinction of two joint lives both 40; that is, whenever *either* of them shall fail; provided the first that fails should happen to be A's life in particular?"—And the answer is found by subtracting the present value of the *two joint* lives from the *perpetuity*, and multiplying the remainder by $\frac{1}{2}$, that is, by the chance that A in particular shall die first: And this will give the required value, *l.* 303.4 (a).

In short. It appears in *both* these cases, that, according to the first method of solution, we are to subtract from the *perpetuity* the value of *one* of the single lives, when, in the *former* case, the value of the *longest* of the two lives, and, in the *latter* case, the value

(a) I have, tho' scarcely necessary, given a demonstration of these Solutions in note M. at the end of the next volume.

of their *joint continuance*, ought, in reality, to be subtracted. I need not say what prodigious errors may often arise from hence ; and how unfit such a method of solution is for practice.

Mr. *Simpson*, in p. 322, of his *Select Exercises*, speaks on this subject in the following manner.—“ I have been very particular
“ on these kinds of Problems ; and the more
“ so, as there has been no method before
“ published, that I know of, by which they
“ can be rightly determined. ’Tis true, the
“ manner of proceeding, by first finding the
“ probability of Survivorship (which method is used in my former work, and
“ which a celebrated author has largely insisted on in three successive editions), may
“ be applied to good advantage, when the
“ given ages are nearly equal ; but then it is
“ certain, that this is not a genuine way of
“ going to work, and that the conclusions
“ hence derived are at best but near approximations.”

This excellent mathematician has here expressed himself much too favourably of the method of solution on which I have remarked.—In both the cases I have specified, the ages are equal ; and yet, in one of them the error is a good deal above a *third* of the true value, and in the other a *fifth*: And, it is obvious, that in cases where three equal lives are taken, the errors will be much greater.

—Mr. *Simpson's* Observations in this passage are true only, when applied to a *different* method used by himself, in the 28th and following Problems of his Treatise on the *Doctrine of Annuities and Reversions*. This method is exact when the lives are equal; but it gives results which are too far from the truth, when there is any considerable inequality between the lives.

It is with reluctance I have made some of these remarks. Mr. *De Moivre* has made very important improvements in this branch of science; and the highest respect is due to his name and authority. This, however, only renders these remarks more necessary.

In the first Chapter (Question 11th, 12th, 15th, and 16th) (*a*). I have given a minute account of the method of finding, in all cases, the values of the reversions which have been the subject of this Essay.—But Mr. *Morgan* has, in his Treatise on Life Assurances, carried this enquiry much farther.

(*a*) In solving questions 11th and 12th, here referred to, I have made use of Mr. *Simpson's* rules in his Select Exercises, p. 297 and p. 298. Mr. *Morgan*, in his Treatise on Life Assurances, adopted the method of solution from which Mr. *Simpson* deduced these rules; but he has lately discovered that it is an approximation which in some cases gives results that are much too distant from the truth. See more on this subject in the Preface to this Edition; and also, at the end of the next volume, where rules (investigated by Mr. *Morgan*) may be found which give in all cases the correct values of sums payable on any survivorships between any two or three lives,

E S S A Y IV.

Observations on the proper Method of constructing Tables for determining the Rate of human Mortality, the Number of Inhabitants, and the Values of Lives in any Town or District, from Bills of Mortality in which are given, the Numbers dying annually at all Ages.

IN every place that just supports itself in the number of its inhabitants, without any recruits from other places ; or where, for a course of years, there has been no increase or decrease, the number of persons dying every year at any particular age, and above it, must be equal to the number of the living at that age.—The number, for example, dying every year, at all ages, from the beginning to the utmost extremity of life, must, in such a situation, be just equal to the whole number *born* every year. And for the same reason, the number dying every year at *one* year of age and upwards ; at *two* years of age and upwards ; at *three* and upwards, and so

on; must be equal to the numbers that reach to those ages every year; or, which is the same, to the numbers of the living at those ages. It is obvious, that unless this happens, the number of inhabitants cannot remain the same. If the former number is greater than the latter, the inhabitants must *decrease*; if less, they must *increase*.—From this observation it follows, that in a town or country where there is no increase or decrease, bills of mortality which give the ages at which all die, will shew the exact number of inhabitants; and also the exact law, according to which human life wastes in that town or country.

In order to find the number of inhabitants; the mean numbers dying annually, at every particular age and upwards, must be taken as given by the bills, and placed under one another in the order of the second column of the 6th, 7th, &c. Tables at the beginning of the next volume. These numbers will, it has appeared, be the numbers of the living at 0, 1, 2, 3, &c. years of age; and, consequently, the sum, diminished by half the number living at age 0, or by half the number born annually (*a*), will be the whole

(*a*) This subtraction is necessary for the following reason.—In a Table formed in the manner here directed, it is supposed, that the numbers in the second column are all living together at the beginning of every year. Thus; the number in the *second* column opposite to 0 in the

first

whole number of inhabitants.—In such a series of numbers, the excess of each number above that which immediately follows it, will be the number dying every year, out of the particular number alive at the beginning of the year; and these excesses set down regularly, as in the third column of the Table to which I have referred, will shew the different rates at which human life wastes through all its different periods, and the different probabilities of life at all particular ages.

It must be remembered, that what has been now said goes on the supposition that the place, whose bills of mortality are given, supports itself, by procreation only, in the number of its inhabitants. In towns this very seldom happens, on account of the

first column, the Table supposes to be all just born together on the first day of the year. The number, likewise, opposite to 1, it supposes to attain to one year of age just at the same time that the former number is born. And the like is true of every number in the second column.—During the course of the year, as many will die at all ages as were born at the beginning of the year; and, consequently, there will be an excess of the number alive at the beginning of the year, above the number alive at the end of the year, equal to the whole number of the annual births; and the true number constantly alive together, is the arithmetical mean between these two numbers; or, agreeably to the rule I have given, the *sum* of the numbers in the second column of the Table, lessened by *half* the number of annual births. See Essay I. page 235, &c.

luxury

luxury and debauchery which generally prevail in them. They are, therefore, commonly kept up by a constant accession of strangers or *settlers*, who remove to them from country parishes and villages. In these circumstances, in order to find the true number of inhabitants, and probabilities of the duration of life, from bills of mortality containing an account of the ages at which all die; it is necessary that the proportion of the annual births to the annual settlers should be known; and also the period of life at which the latter remove.—Both these particulars may be discovered in the following method.

If for a course of years there has been no sensible increase or decrease in a place, the number of annual settlers will be equal to the excess of the annual burials above the annual births. If there is an *increase*, it will be *greater* than this excess. If there is a *decrease*, it will be *less*.

The period of life at which these settlers remove, will appear in the Bills by an increase in the number of deaths at that period and beyond it. Thus; in the *London* Bills, the number of deaths, between 20 and 30, is generally above double, and between 30 and 40, near triple the number of deaths between 10 and 20: And the true account of this is, that from the age of 18 or 20 to 35 or 40, there is a confluence of people every year to *London* from the country, which occasions

cations a great increase in the number of inhabitants at these ages ; and, consequently, raises the deaths for all ages *above* 20 considerably above their due proportion, when compared with the number of deaths *before* 20.—This is observable in all the bills of mortality for towns with which I am acquainted, not excepting even the *Breslaw* Bills. Dr. *Halley* takes notice, that these Bills give the number of deaths between 10 and 20, too small. This he considered as an irregularity, owing to chance ; and therefore, in forming his Table of Observations, he took the liberty so far to correct it, as to render the proportion of those who die to the living in this division of life, nearly the same with the proportion which, he says, he had been *informed* (a) die annually of the young lads in *Christ-Church Hospital*. But the truth is, that this irregularity in the Bills was derived from the cause I have just assigned.—During the five years for which the *Breslaw* Bills are given by Dr. *Halley*, the births did, indeed, a little exceed the burials ; but, it appears, that this was the effect of some peculiar causes that happened to operate just at that time ; for, during a complete century from 1633 to 1734, the annual

(a) See *Lowtherp's* Abridgment of the Philosophical Transactions, vol. III. p. 670.

medium of *births* was 1089 (a), and of burials 1256 (b). This town, therefore, must have been all along kept up by a number of yearly recruits from other places, equal to about *a seventh* part of the yearly births.

What has been now observed concerning the period of life at which people remove from the country to settle in towns, would appear sufficiently probable, were there no such evidence for it as I have mentioned; for it might be well reckoned, that these people in general, must be single persons in the beginning of mature life, who not having yet obtained settlements in the places where they were born, migrate to towns in quest of employments.

Having premised these Observations, I shall next endeavour to explain distinctly, the effect which these accessions to towns must have, on Tables of Observation formed from their bills of mortality. This is a

(a) See Dr. *Short's* Comparative History, p. 63.

(b) It appears from the account in the *Philosophical Transactions* (Abridgment, vol. VII. No. 380, p. 46, &c.) that from 1717 to 1725, the annual medium of births at *Breslaw* was 1252, of burials 1507; and also, that much the greatest part of the births died under 10 years of age.—From a Table in *Susmilch's* works, Vol. I. p. 38, it appears, that, in reality, the greater part of all that die in this town are children under five years of age.

subject proper to be insisted on, because mistakes have been committed about it; and because also, the discussion of it is necessary to shew, how near to truth the values of lives come as deduced from such Tables.

The following general rule may be given on this subject.

If a place has, for a course of years, been maintained in a state nearly stationary, as to number of inhabitants, by supplies or recruits coming in every year, to prevent the decrease that would arise from the excess of the burials above the births; a Table formed on the principle, that the number dying annually, after every particular age, is equal to the number living at that age," will give the number of inhabitants and the probabilities of life, too *great* for all ages preceding that at which the supplies cease; and after this, it will give them *right*.—If the accessions are so great as to cause an *increase* in the place, such a Table will give the number of inhabitants and the probabilities of life, too *little*, after the age at which the accessions cease (a); and too great, if there is a decrease.

(a) Agreeably to these Observations; if a place increases, not in consequence of accessions from other places, but of a constant excess of the births above the deaths; a Table, constructed on the principle I have mentioned, will give the probabilities of life too low through the whole extent of life; because, in such circumstances, the number of *deaths* in the *first* stages of life

crease. *Before* that age it will in *both* cases give them too great; but most considerably so in the former case, or when there is an increase.

For example. Let us suppose, that 244 of those born in a town, attain annually to 20 years of age; and that 250 more, all likewise 20 years of age, come into it annually from other places; in consequence of which, it has for a course of years, been just maintained in the number of its inhabitants, without any sensible increase or decrease. In these circumstances the number of the living in the town of the age of 20, will be always 244 *natives* and 250 *settlers*, or 494 in all; and, since these are supposed all to die in the town, and no more recruits are supposed to come in; 494 will be likewise the number dying annually at 20 and upwards.—In the same manner; it will appear on these suppositions, that the number of the living, at every age, subsequent to 20, will be equal to the number dying annually at that age and above it; and consequently, that the number of inhabitants and the decrements of life, for every such age, will be given exactly by the Table I

life must be too great, in comparison of the number of deaths in the *latter* stages; and more or less so, as the *increase* is more or less rapid. The contrary, in all respects, takes place where there is a decrease, arising from the excess of the *deaths* above the *births*.

have

have supposed. But for all ages *before* 20, they will be given much too great. For let 280 of all born in the town, reach 10. In this case, 280 will be the true number of the living in the town, at the age of 10; and the recruits not coming in 'till 20, the number given by the Bills, as dying between 10 and 20, will be the true number dying annually of the living in this division of life. Let this number be 36; and it will follow, that the Table ought to make the numbers of the living at the ages between 10 and 20, a series of decreasing means between 280 and (280 diminished by 36, or) 244. But in forming the Table on the principle I have mentioned, 250 (the number above 20 dying annually in the town who were not born in it) will be added to each number in this series; and, therefore, the Table will give the numbers of the living, and the probabilities of life in this division of life, almost twice as great as they really are.—This observation, it is manifest, may be applied to *all* the ages under 20.

It is necessary to add, that such a Table will give the number of inhabitants, and the probabilities of life, equally wrong before 20, whether the recruits all come in at 20, agreeably to the supposition just made, or only *begin* then to come in. In this last case, the Table will give the number of inhabitants, and probabilities of life, too
great

great throughout the whole extent of life, if the recruits come in at all ages above 20. But if they cease at any particular age, it will give them right only from that age; and before, it will err all along on the side of excess; but less considerably between 20 and that age, than before 20—For example. If, of the 250 I have supposed to come in at 20, only 150 then come in, and the rest at 30; the numbers of the living will be given 100 too high, at every age between 20 and 30; but as just shewn, they will be given 250 too high at every age before 20.—In general, therefore, the number of the living *at* any particular age, must be given by the supposed Table, as many too great as there are annual settlers *after* that age: And, if these supplies come in at all ages indiscriminately, during any certain interval of life; the number of inhabitants and the probabilities of life will be continually growing less and less wrong, the nearer any age is to the end of that interval.—These Observations prove, that Tables of Observation formed in the common way, from bills of mortality for places, where there is an excess of the burials above the births, must be erroneous, for a great part of the duration of life, in proportion to the degree of that excess. They shew likewise, at what parts of life the errors in such Tables are most considerable, and
how

how they may be in a great measure corrected.

All this I shall exemplify and illustrate in the particular case of *London*.

The number of deaths, between the ages of 10 and 20, is always so small in the *London* Bills, that it seems certain few recruits come to *London* under 20; or at least not so many as before this age are sent out for education to schools and universities. After 20, great numbers come in 'till 30, and some perhaps 'till 40 or 50.—The *London* Tables of Observation, therefore, being formed on the principle I have mentioned, cannot give the probabilities of life right 'till 40. Between 30 and 40 they must be a little too high; but more so between 20 and 30; and most of all so before 20.—It follows also, that these Tables must give the number of inhabitants in *London* much too great.

Table XIII. in the next volume, is a Table formed in the manner I have explained, from the *London* Bills for 10 years, from 1759 to 1768; and adapted to a 1000 born as a *radix*. The sum of the numbers in the second column diminished by half the number born, is 25,757. According to this Table then, for every 1000 deaths in *London*, there are 25 times and $\frac{3}{4}$ that number of inhabitants; or, in other words, the expectation of a child just born is $25\frac{3}{4}$; and

the inhabitants are to the annual burials, as $25\frac{3}{4}$ to 1.—But it has appeared, that the numbers in the second column being given on the supposition, that all who die in *London* were born there, must be too great; and we have from hence a DEMONSTRATION, that the probabilities of the duration of life are given in the common Tables of *London* Observations, too high, for, at least, the first 30 years of life; and also, that the number of inhabitants in *London* must be *less* than $25\frac{3}{4}$, multiplied by the annual *burials*.—The common Tables, therefore, of *London* Observations, undoubtedly want to be corrected (*a*); and the way of doing this, and, in general, the right method of forming genuine Tables of Observation for towns, may be learnt from the following rule.

“ From the sum of all that die annually,
 “ *after* any given age, subtract the number
 “ of annual settlers *after* that age; and the
 “ remainder will be the number of the liv-
 “ ing *at* the given age.”

This rule can want no explication or proof, after what has been already said.

(*a*) The ingenious and accurate Mr. *Simpson* saw that it was necessary to correct the *London* Tables, and he has done it with great judgment; but, I think, too imperfectly, and without going upon any fixt principles, or shewing particularly, how Tables of Observation ought to be formed, and how far in different circumstances, and at different ages, they are to be depended on.

If, therefore, the number of annual settlers in a town at every age could be ascertained; a perfect Table of Observations might be formed for that town, from Bills of mortality containing an account of the ages at which all die in it. But no more can be learnt in this instance from any Bills, than the whole number of annual *settlers*, and the general division of life in which they enter. This, however, may be sufficient to enable us to form Tables that shall be tolerably exact.—For instance. Suppose the *annual deaths* in a town which has not increased or decreased, to have been for many years, in the proportion of 4 to 3 to the *annual births*. It will hence follow, that $\frac{1}{4}$ of the persons who die in such a town are *supplies*, or *emigrants* from other places; and not *natives*: And the sudden increase in the deaths after 20, will also shew, agreeably to what was before observed, that they enter after this age. In forming therefore a Table for such a town, a *quarter* of all that die at all ages throughout the whole extent of life, must be deducted from the sum of all that die after every given age before 20; and the remainder will be the true number living at that given age. And if, at 20, and every age above it, this deduction is omitted, or the number of the living at every such age is taken the same with the sum of all that die after it, the result will

be (supposing *most* of the supplies to come in before 30, and *all* before 40) a Table exact 'till 20; too high between 20 and 30; but nearly right for some years before 40; and after 40 exact again.—Such a Table, it is evident, will be the same with the Table last described at all ages above 20; and different from it only under 20.—It is evident also that, on account of its giving the probabilities of the duration of life too great for some years, after 20, the number of inhabitants deduced from it may be depended on as greater than the truth; and more or less so as the annual recruits enter in general later or sooner after 20.

Let us now consider, what the result of these remarks will be, when applied particularly to the *London Bills*.

It must be here first observed, that, at least one *quarter* of all that die in *London* are *emigrants* from the country, and not *natives*.—The medium of annual burials for 10 years, from 1759 to 1768, was 22,956; of births 15,710. The excess is 7246; or near a *third* of the burials.—The same excess, during ten years, before 1750, was 10,500; or near *half* the burials. *London* was then *decreasing*. For 12 or 15 years before 1769 it was *increasing*. This excess, therefore, agreeably to the foregoing observations, was then *greater* than the number

ber of annual recruits ; and it is now *less*. I have chosen, however, to suppose the number of annual recruits to be now (a) no more than a *quarter* of the annual burials, in order to allow for more omissions in the births than the burials ; and also, in order to be more sure of obtaining results that shall not exceed the truth.

Of every thousand then who die in *London*, only 750 are *natives*, and 250 are *settlers* who come to it after 18 or 20 years of age : And, consequently, in order to obtain from the Bills a more correct Table than the 13th in the next volume, 250 must be subtracted from every one of the numbers in the second column 'till 20 ; and the numbers in the third column must be kept the same, the Bills always giving these right.—After 20, the Table is to be continued unaltered ; and the result will be, a Table which will give the numbers of the living at all ages in *London* much nearer the truth, but still too high.—Such is the 14th Table in the next volume.—The sum of all the numbers in the second column of this Table, diminished by 500, is 20,750. For every 1000 deaths, therefore, in *London*, there are, according to this Table, 20,750 living persons

(a) It must not be forgotten that this refers to the time of the first publication of this work ; that is, the year 1769.

in it ; or for every single death, $20\frac{3}{4}$ inhabitants. It was before shewn, that the number of inhabitants in *London* could not be so great as 25 times $\frac{3}{4}$ the deaths. It now appears, (since the numbers in the second column of this Table are too high) that the number of inhabitants in *London* cannot be so great as even 20 times $\frac{3}{4}$ the deaths. And this is a conclusion which, I believe, every one who will bestow due attention on what has been said, will find himself forced to receive. It will not be amiss, however, to confirm it by the following fact, the knowledge of which I owe to the particular enquiry and kind information of Mr. *Harris*, the ingenious master of the Royal Mathematical School in Christ-Church Hospital.--The average number of lads in this school has, for 30 years ending in 1768, been 831. They are admitted at all ages between seven and eleven; and few stay beyond 16. They are, therefore, in general, lads between the ages of eight and 16. They have better accommodations than it can be supposed children commonly have; and about 300 of them have the particular advantage of being educated in the country. In such circumstances it may be well reckoned that the proportion of children dying annually, must be less than the general proportion of children dying annually at the same ages
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in *London* (a).—The fact is, that, for the last 30 years, $11\frac{1}{5}$ have died annually; or one in $70\frac{2}{5}$.

According to Table XIV, of all who complete their 8th year in *London*, and who are living at that age and at every intermediate age till 16, one in 74 die annually. It follows, therefore, that, according to this Table, supposing the lads in *Christ-Church* School all admitted exactly at

(a) Mr. *Wales*, the present master of this school, has, in his Enquiry, &c. p. 33, confirmed this account; adding, that the number of children in it for twenty years ending in 1780, had been 851, and the average of annual deaths $10\frac{1}{5}$, or one in 83; but that the number of children for ten years (ending in 1780) had been 894, and the average of annual deaths only $8\frac{2}{5}$, or about one in $100\frac{1}{5}$.—From hence Mr. *Wales* infers a great improvement in the state of *London* with respect to healthiness. But this fact is by no means a sufficient foundation for such a conclusion. In numbers so inconsiderable, an average of ten years cannot be depended on. Were it, however, the true average, the reasons above given have a tendency to prove, notwithstanding the central situation of this school, that it must be too low for *London* in general. If so many as *three-fourths* of all that die in *London* are natives, the proportion dying annually between 8 and 16 must be as high as *one in seventy-five*; and possibly this is even now less than the true proportion. But it would be unreasonable to take it less than the first proportion mentioned by Mr. *Wales*, or one in 83. The consequence however of stating it at even one in 90, and also one in a hundred, will be shewn in the next note; from which it will appear to be impossible, without exceeding all the bounds of credibility, to make the expectation of a child just born in *London* much more than twenty years.—See 1st Essay, notes p. 243, and p. 251.

eight years of age, and none discharged before they have completed 16 years of age, or before they have resided eight years (suppositions much too favourable), only a 74th part ought to die annually. That Table, therefore, gives the decrements of life in *London* at these ages too little, and the numbers of the living too great: And if this is true of these ages, it must be true of all other ages under 20; and it follows demonstrably, in conformity to what was before shewn, that more people settle in *London* after 20, than the *quarter* I have supposed; and that from 20 to 35 or 40, the numbers of the living are given too great in proportion to the decrements of life.

In this Table the numbers in the second column are doubled at 20, agreeably to what really happens in *London*; and the sum of the numbers in this column diminished by half the whole number of deaths, gives the *expectation* of life, not of a child just born, as in other Tables, but of all the inhabitants of *London* at the time they enter it, whether that be at birth, or at 20 years of age. The *expectations*, therefore, and the *values* of *London* lives under 20, cannot be calculated from this Table. But it may be very easily fitted for this purpose by first finding the number of births which, according to the given decrements of life, will leave 494 alive at 20; and then adapting the intermediate numbers
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in such a manner to this radix, as to preserve all along the number of the living, in the same proportion to the numbers of the dead. This is done in the 15th Table in the next volume; and this Table may, I fancy, be recommended as better adapted to the present state of *London* than any other Table (a). The values of lives, however, deduced from

(a) Had I, instead of subtracting 250 from Table 13th before the age of 20 (agreeably to the directions in p. 341), subtracted only 200 (or supposed that only a fifth part of all that die annually in *London* are emigrants to it after 20), the resulting Table would have made the number dying between 8 and 16, one in 90; and the expectation at entrance into *London*, would have been 22, and at birth, $19\frac{4}{5}$.—Had 166 only been subtracted, or a 6th part of all that die in *London* supposed to be emigrants from the country, the result would have been a Table which would have made one in a 100 die between 8 and 16, and the expectations just mentioned 23 and $21\frac{1}{4}$. Nor will any difference worth regarding arise, if Table 15th in the next volume, instead of being formed after 19 from the Bills for ten years ended at 1768, had been formed from the Bills for ten years ending in 1780. Table 16th is such a Table; and the observations annexed to it will shew how wrong the ideas are which some have lately entertained of the improved state of *London*. Some alteration for the better there must be; but the correspondence between the Tables of Observation for whatever period they are formed from the Bills, demonstrates that it is not considerable. The great evils which produce the unhealthfulness of towns are the closeness and foulness of the air, and the irregular modes of living. If the former of these has been diminished in *London*, the latter may have increased. But the truth may be, that the diminution of the former of these evils has not much extended itself to the lower ranks of people in *London*, who form the body of the inhabitants.

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it, are in general nearly the same with those deduced by Mr. *Simpson*, from the *London Bills* as they stood 40 years ago. The main difference is, that after 52 and in old age, this Table gives them somewhat lower than Mr. *Simpson's* Table.

It has sufficiently appeared, what judgment we are to form of the values of lives thus deduced. During the greatest part of the interval of life, in which the annual recruits that keep up *London* come to it, these values err certainly on the side of *excess*: And it is also *probable*, that they exceed the truth in all the last stages of life (a).

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(a) When the former editions of this Treatise were published, it appeared to me probable, that, in consequence of retirements from *London* in the advanced periods of life, the Bills gave the probabilities of living in *London* after seventy years of age too *low* rather than too *high*. But I am now convinced of the contrary. Those who withdraw from *London* in advanced life are only a part of the inhabitants in the higher classes, *themselves* a small part of the whole body of inhabitants; and they withdraw, if at all, before seventy years of age, and therefore, the loss of them in the Bills can have no effect on the *proportions* of the numbers that die at all ages after seventy.—It has also occurred to me, that though the probabilities of living *before* the age of seventy, as given by the Bills, have continued remarkably the same from 1728 (when the ages were first included in the Bills) to 1780, (as will be shewn in the Observations on the *London Tables* in the next volume), yet *after* the age of 70 there has been a gradual diminution in them; so that now, of all who die at *all* ages, only one in 44 dies at a greater age than 80; whereas at the period just
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The number of inhabitants in *London* may also be learnt from what has been offered, more

mentioned, one in 32 died above this age; and of all who die above 70, only 31 in a hundred now die above 80, and 4 in a 100 above 90; whereas, at the same period, 43 in a hundred used to die above 80, and 11 in a hundred above 90.

But what has principally determined my judgment in this instance is a comparison of the probabilities of living in *Stockholm*, as deduced from the *Stockholm Bills*, with the correct probabilities as determined by an actual account taken at three different times of the number of the inhabitants living there at all ages.— This comparison shews that *Bills* of mortality for great towns give the probabilities of living too high at all ages; and particularly at the *end* as well as the *beginning* of life; for the proportion of inhabitants between 70 and 80 dying annually at *Stockholm* was, according to the *survey*, 10 out of 63; and between 80 and 90, ten out of 28; and above 90, ten out of 25; whereas, according to the *Bills*, these proportions are ten out of 100, 55, and 24 respectively.

The *London*, *Vienna*, and *Berlin Bills* give the probabilities of living between 70 and 80, and between 80 and 90, nearly the same with these, as may be learnt from the *Tables of Observations* for these towns in the next volume; and as at *Stockholm*, they are certainly too high; the reasonable conclusion is, that they are so likewise in the other towns: The truth, perhaps, may be, that more persons (invited by the conveniencies in towns) remove into them in old age, than withdraw from them.

No one, probably, will think that the change which I have mentioned in the *London Bills* can be owing to a growing unfavourableness of *London* to the health of old people. The following observations will sufficiently account for this fact.

LONDON, after the loss of a quarter of its inhabitants by the plague in 1665, and the devastation of the fire
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more nearly than by any method which has been hitherto taken. It cannot, it has been shewn, exceed 20 times $\frac{3}{4}$ the number of annual deaths. Could, therefore, the annual deaths be ascertained, we should know the number of inhabitants within pretty narrow limits. But the omissions in the Bills are such, that it is not possible to ascertain, with exactness, the annual deaths. Dr. *Brakenridge* supposed these omissions to amount to 2000 annually. The result of a very minute enquiry by Mr. *Maitland* is, that in the year 1729, they amounted to 3038. But they are probably now more considerable than they ever were (*a*). Let them be 6000; and the

in 1666, recovered so fast as in three or four years to become more populous than it had ever been; and it continued to increase till the *Revolution* in 1688; after which period, and during the reigns of King *William* and Queen *Anne*, it seems, if we may judge from the Bills, to have stagnated and declined. There must, therefore, for some years after 1666, have been a very extraordinary influx of people to it; and they must have been, for the most part, people in the beginning of mature life, who would not all die off in less than 60 or 70 years, and, therefore, would, about the year 1728, render *London* fuller of inhabitants turned of 80 and 90, than it could be at any other period.

(*a*) Vid. Preface to a Collection of the Bills of Mortality from 1657 to 1758, p. 4, &c.—Since the above was written the burials, as given in the Bills, have fallen from 22,688 (the annual average for five years to 1770) to 20,743, the same average to 1780. Adding 6000 to this

the number of inhabitants (supposing the burials 29,000) will be 601,750 at most.

All the preceding Observations are, it is plain, applicable to Bills of mortality for towns in general; and point out the way of deducing from them genuine Tables of Observations, which shall give the true probabilities and values of lives, and the true number of inhabitants, in the towns whose Bills are given.—I shall beg leave to confirm and illustrate this, in the particular case of the town of NORTHAMPTON.

In this town, containing four parishes, namely, *All-Saints*, *St. Sepulchre's*, *St. Giles*, and *St. Peter's*, an account has been kept ever since the year 1741, of the number of males and females that have been christened and buried (Dissenters included) in the whole town. And in the parish of *All-saints*, containing the greatest part of the town, an account has been kept ever since 1735, of the *ages* at which all have died there.

In 1746, an account was taken of the number of *houses*, and of *inhabitants* in the town. The number of *houses* was found to be 1083;

this last number, and multiplying the total by 20 $\frac{3}{4}$, will make the number of inhabitants in *London* in 1780 554,917. But even this computation is too high, as appears from the note in p. 257.

Since 1780, the causes mentioned in the note p. 251 and at the conclusion of the Postscript in the next volume, have sunk the registered burials in *London* to 19,494, which was the average for three years to 1790.

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and the number of *inhabitants* 5136.—In the parishes of *All-saints* and *St. Giles*, the number of *male* and *female* heads of families, *servants*, *lodgers*, and *children*, were particularly distinguished.—The *heads of families* were, 707 males; and 846 females.—*Children*, males 624; females 759.—*Servants*, males 203; females 280.—*Lodgers*, males 137; females 287.—In *St. Peter's*, males 99; females 129.—In *St. Sepulchre's*, *adults* 638; *children* 427. In the last parish the sexes were not distinguished.

The *Christenings* and *Burials* in the whole town for 40 years, from 1741 to 1780, have been as follows:

Christened	{ Males 3218 Fem. 3108 }	6326--Annual medium 158
Buried	{ Males 3757 Fem. 3823 }	7580--Annual medium 189½

In the parish of *All-Saints*, from 1735 to 1780, or 46 years,

Christened	{ Males 2152 Fem. 2068 }	4220--Annual medium 91¼
Buried	{ Males 2377 Fem. 2312 }	4689--Annual medium 102

Of these died,

Under 2 years of age	—	1529
Between 2 and 5	—	362
Between 5 and 10	—	201
Between 10 and 20	—	189

Be-

Between 20 and	30 —	373
Between 30 and	40 —	329
Between 40 and	50 —	365
Between 50 and	60 —	384
Between 60 and	70 —	378
Between 70 and	80 —	358
Between 80 and	90 —	199
Between 90 and	100 —	22

Total 4689

A Table formed from these *data* in the manner of Table XIII in the next volume ; or, on the supposition, that all who *die* in *Northampton* were *born* there, would give the expectation of a child just born 28.83 years ; and consequently the proportion of the inhabitants to the annual deaths, as 28.83 to 1. It has been shewn, that this proportion, in a place where the burials exceed the births, must be *greater* than the *true* proportion of the number of inhabitants to the annual deaths : And this appears to be the real case. For the Bills shew, that, from 1741 to 1750, or for 10 years, about the time when the number of inhabitants was 5136, the annual medium of burials was 197.5 ; which, multiplied by 28.83, gives 5693 ; or a 9th part more than the *true* number.

A Table formed in the manner of Table XIV in the next volume, would give the proportion

portion of inhabitants to the annual deaths, as 26.41 to 1; and this makes the inhabitants 5216, or very nearly the true number.

The VIth Table, in the next volume, is formed in the same manner with Table XV, for *London*: And this is the genuine Table of Observations for *Northampton* (a), from which may be calculated the true probabilities and values of lives in that town.

(a) In the fourth edition of this Treatise the following corrections were made in this Table. First. The Table printed in the three first editions having been formed from the *Northampton* Bills for 36 years, this Table was rendered a little more correct in consequence of being formed from the same Bills for 46 years.—Secondly. The Bills give the numbers dying annually between 20 and 30 greater than between 30 and 40; but this being a circumstance which does not exist in any other register of mortality, and undoubtedly owing to some accidental and local causes, the decrements were made equal between 22 and 40; preserving, however, the total of deaths between 20 and 40 the same that the Bills give them.—Thirdly. The Bills giving only the totals of deaths under two years of age, and between 2 and 5, the proportions of deaths for every particular year between 2 and 5, and for every *quarter* of a year after birth till one year of age, were made the same nearly that the *Chester* register makes them. See the Introduction to the collection of Tables in the next volume.

In consequence of these alterations, and also of increasing the *radix* from 1165 to 11650, in order to adjust the decrements with greater regularity and precision, this Table, in my opinion, gives the *mean* probabilities and values of lives at every age with more accuracy than any other Table now extant

At NORWICH; Bills of Mortality, of the same kind with those in *London* and *Northampton*, have been kept for many years. I have been favoured with a copy of these Bills for 30 years, from 1740 to 1769. The annual medium of *christenings*, during this period, has been 1057 (a), of *burials* 1206. And from hence, together with the account of the numbers dying in the several decads of life, after 10, I have formed Table VII, which shews the true probabilities of the duration of life in this town.

The following particulars seem to deserve notice here.

First. Had these Tables been formed from the NORTHAMPTON and NORWICH Bills, for no longer time than any 10 years taken together, of the periods I have men-

(a) In this register all that die before baptism, and also all that are born and die among *Quakers*, *Jews*, &c. are omitted. There are also some other omissions; and the true annual medium of births and burials must be greater than they are given in the bills. But this will have no effect on a Table of Observations, supposing the *proportions* of the births to the burials, and of the numbers dying in the different stages of life, given right. —It is proper I should mention further here, that these Bills give only the whole number of children dying under 10, without specifying the numbers dying under two years of age, between 2 and 5, and between 5 and 10, as in other Bills. I have, therefore, in forming the Table for NORWICH, supposed the proportions of these numbers the same that I have given them for NORTHAMPTON.

tioned; they would not have given the values of lives materially different. These Tables, therefore, are founded on a sufficient number of Observations; and it appears, that there is an invariable law which governs the waste of human life in these towns.—The same remark might be made concerning LONDON (a).

Secondly. An account was taken at SHREWSBURY, in 1750, of the *whole* number of inhabitants; distinguishing, particularly, the number at the age of 21 and upwards.—The former number was 8141; and the latter, 5187.—According to a Table

(a) Some have entertained a very wrong notion of the imperfections in the LONDON Bills. They do, indeed, give the *whole* number of births and deaths much too little; but the conclusions with respect to the probabilities of life in LONDON, and the proportion of inhabitants dying annually, depend only on the *proportions* of the numbers dying in the several divisions of life; and these are given *right* in the LONDON Bills.—For first, There seems nothing in this case, that can be likely to cause the deficiencies in the Bills to fall in one division of life more than in another: But what decides this point is, that these proportions, as given by the Bills for *any* ten, or even *five* years, come out nearly the same with one another; and always very different from the proportions given by other Bills.—There are no other variations, than such as must arise from the fluctuations of LONDON as to increase and decrease; and also from some improvements in its state, which have lately taken place, and particularly the law lately passed, ordering all parish infants to be nursed in the country. See the note in page 251; and the Observations on Table XVI in the next volume.

formed

formed for NORTHAMPTON, in the same manner with Table XIV for LONDON, the whole number of the living is to the number of the living at 21 and upwards, as 26,411 to 16,586; that is, as 8141 to 5113.—According to a like Table for NORWICH, these numbers are to one another, as 24,500 to 15,680; that is, as 8141 to 5210.—These Tables, therefore, give the proportion of the *whole* number of inhabitants, to the number of the living at 21 and upwards, almost exactly the same with the true proportion, as it is at SHREWSBURY (a): And this affords an additional proof of the rectitude of the principles on which these Tables have been formed.

But further.—The number of inhabitants, not reckoning children, in the parishes of *St. Giles* and *All-Saints*, NORTHAMPTON, was, in 1746, 2460; and the *whole* number of inhabitants in these two *parishes* was 3843.

(a) The annual medium of births at SHREWSBURY, for 7 years, from 1762 to 1768, was 301; of burials 329. It appears, therefore, that one in $24\frac{3}{4}$ of the inhabitants die annually. But it should be remembered, that in 1766, the small-pox and measles increased very much the mortality in this town; and I find also, that since 1750, a nursery for *foundlings* from LONDON was established here; and that in 1768 this nursery contained 660 children and servants. It seems, therefore, probable, that the true *medium* of burials about the year 1750, must have been less than 329; and that the proportion of inhabitants dying annually, may not be much greater than it is at NORTHAMPTON; or 1 in 26.41.

See p. 350.—In the account I have received, the particular age at which the limit of childhood was fixed in taking this survey, is not mentioned; but there seems reason to believe, that it was 21: And, taking this for granted, the number of inhabitants, not children, will come out (by such a Table for NORTHAMPTON as Table XIV for LONDON) 2414; or, nearly the same with the number *really* found in these parishes.—Had this number been computed, from a Table formed for NORTHAMPTON, in the manner of Table XIII, in next volume, it would have come out only 2176. This remark is applicable to the Table for *Breslaw*, formed by Dr. *Halley*, compared with the same Table, corrected for all the ages under 20 (*a*), by the rule, p. 338. The necessity, therefore, of that correction is verified by facts; and it appears, abundantly, that the Tables I

(*a*) I have given Dr. *Halley*'s Table in the next volume just as he framed it. A correction of it might be made from the proportion of births to burials, mentioned p. 332. And it would then appear, that a 25th part of the inhabitants at *Breslaw* die annually; and that half the number born die there under six, as well as at *Norwich*. This Table, as we now have it, makes half live to 16; but the account mentioned in the note, page 332, shews this not to be the truth. It likewise makes the number of inhabitants at SHREWSBURY, above the age of 21, to be 4730; and in the parishes of *All-Saints* and *St. Giles*, in NORTHAMPTON, 2230. It gives, therefore, these numbers wrong; whereas, as observed above, a corrected Table would give them true.

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have given for NORTHAMPTON and NORWICH may be depended on.

But, thirdly. In comparing these two Tables, it may be observed, that there is a difference between them in favour of NORTHAMPTON, *fewer* dying there in childhood, and *more* in old age. The same would be found to be true, were the NORTHAMPTON Table to be compared with a corrected BRESLAW Table. It appears, therefore, agreeably to what might have been expected, that NORTHAMPTON, being a small town compared with BRESLAW and NORWICH, is less unfavourable to health and longevity. The difference, however, is not considerable. After the age of 20, there is a striking conformity between all the three Tables, which gives them great weight and authority.

Further. It ought to be noted, that these Tables prove the *decrements* of life between 25 and 75, in moderate towns, to be nearly *equal*. At NORTHAMPTON it appears that, of a given number of persons alive at 20, the same number die every year 'till 78, without any considerable interruption, except between the ages of 30 and 40. — A like uniform decrease in the probabilities of the duration of life appears in the BRESLAW and NORWICH Tables; but not so remarkably. It was this circumstance in the BRESLAW Table, that led Mr. *De Moivre* to the *hypothesis*, described in p. 2, and so often

mentioned in this work.—It gives the values of lives in the middle stages nearly the same that they are by these three Tables; but it is far from being applicable with sufficient correctness to the valuation of lives before 25 and after 75 years of age (*a*); nor does it at all correspond to the law which governs the waste of human life in great towns, and in country parishes and villages. This will appear immediately from inspecting the Tables in the next volume. I will here only compare the *expectations* of life by it with the *expectations* at the same ages in *London*, and in a country parish, where the exactest observations have been made. I mean, in the parish of HOLY-CROSS near *Shrewsbury* (*b*), mentioned in the first Essay, page 261.

EXPEC-

(*a*) Having in the three first editions of this work given examples from this hypothesis, and the Tables founded upon it and printed at the end of the next volume, I have been obliged to continue them in this edition; but the truth is, that it does not in *any* part of life give such correct values, particularly of joint lives, as are necessary in some cases. And it is this, together with the other reasons mentioned here, and in the Postscript to the second Essay, that has induced me to employ a good deal of time in calculating the Tables of the values of lives from real observations, which will be found in the next volume.

(*b*) The register of this parish, with a Table of the probabilities and expectations of lives deduced from it will be given among the Tables in the next volume.
The

EXPECTATIONS of LIFE in LONDON at

		By Hypothesis.		Holy-Cross.	
Age 10	— 34.8	— 38	— 46		
20	— 28.9	— 33	— 38.66		
30	— 23.6	— 28	— 32.66		
40	— 19.6	— 23	— 26.40		
50	— 16.0	— 18	— 20.40		
60	— 12.4	— 13	— 14.86		
70	— 8.0	— 8	— 10.00		

There is one more fact which I shall here take notice of; and which deserves more attention than has been hitherto bestowed upon it.

The expectation of a child just born here is 33.9.— At NORTHAMPTON it is 25 $\frac{1}{2}$. At NORWICH, 23 $\frac{3}{4}$. In LONDON, 18.—In this parish, 1 in 11 dies at 80, and upwards. In NORTHAMPTON; 1 in 22. In NORWICH; 1 in 27. In LONDON; 1 in 60. See Essay I. p. 273.

I will add, that the probabilities of life here appear to be much the same with the probabilities of life among the ministers and professors in SCOTLAND.—This is a fact of some consequence; and, therefore, I will give a brief account of it.

The mean age at which the ministers and professors enter into benefices and professorships in Scotland is reckoned to be 27. Their number is 974. The establishment among them for providing for their widows begun on the 25th of March 1744; from which time to November 22, 1779, 1037 have died: That is, 29 annually; or 1 in 33 $\frac{3}{5}$. The expectation, therefore, of a life among them, at the age of 27, is 33.6; which is nearly the same with the expectation of a life of the same age in the

it. I mean; “the difference between the “probabilities of life among *males* and *females*, in favour of the latter.”

From the account in p. 350, it appears, that at NORTHAMPTON, though more *males*

parish of *Holy-Cross*; and $3\frac{1}{2}$ years more, than the *expectation* of the same age by Tables V, VI, and VII, in the next volume.—Now the *expectation* at a given age, being composed of all the probabilities of life from that age to the extremity of life; there arises from hence reason for concluding, that the probabilities of life among the ministers in *Scotland*, cannot differ much in any part of life from those in this parish.—But there is another fact that confirms this observation.

The annual average of weddings among the ministers and professors in SCOTLAND, for 35 years ending in 1779, has been 30. The average of married persons among them, for 17 years ending in 1757, had been 667. This number, divided by 30, gives 22, the *expectation* of marriage among them; which is above 3 years more than the expectation of marriage would be, by Dr. Halley's Table, on the supposition, that all marriages may be justly considered as commencing, one with another, so early as the age of 30.—The *expectation* of two equal *joint* lives is to the *expectation* of a *single* life of the same age, as 2 to 3, by note (K) at the end of the next volume. It follows, therefore, that among the ministers in *Scotland*, the *expectation* of a *single* life at 30 cannot be less than 33. Most probably it is more; on account of the later commencement of marriage in the situation of the *Scotch* ministers.—I reckon also that 27 must be less than the mean age at which they enter their benefices and professorships; meaning by it, not the age on each side of which equal numbers enter, but the age at which the excess of the interval of time taken to enter on one side, is just such as to compensate the greater numbers who enter on the other side. See the conclusion of note (F) in the next volume.

are born than *females*, and nearly the same number die; yet the number of living *females* is greater than the number of *males*, in the proportion of 2301 to 1770, or 39 to 30. This cannot be accounted for without supposing, that *males* are more short-lived than *females*.—One obvious reason of this fact is, that males are more subject to untimely deaths by accidents of various kinds; and also, in general, more addicted to the excesses and irregularities which shorten life. But this is by no means the *only* reason. For it should be observed that at NORTHAMPTON the number of *female* children was, in 1746, greater than the number of male children, in the proportion of 759 to 624.—The greater mortality of males, therefore, takes place among *children*.—But this, together with the greater mortality in general of males at all ages, will more particularly appear from the following recital of facts.

In the parish of *Holy-Cross, Salop*, the ingenious Vicar, Mr. *Gorsuch*, in 1760, and again in 1770, took the number of male and female inhabitants turned of 80. In 1760, the number of females turned of this age, was 13; of males, 2. In 1770, these numbers were, females, 11; males, 6. And for 10 years to 1770, eleven out of 365 had died between the
ages

ages of 85 and 102; and they were all females (a).

At BERLIN, it appeared, from the accurate account which was taken of the inhabitants in 1747, and which has been mentioned in p. 294, that the number of *female* citizens exceeded the number of *male* citizens, in the proportion of 459 to 391: And yet, out of this smaller number of males, more had died for 20 years preceding 1751, in the proportion of 19 to 17 (b).

At EDINBURGH, in 1743, the number of *females* was to the number of *males*, as 4 to 3; (See Essay I. p. 284) but the females that

(a) For 20 years, from 1760 to 1780, eleven out of 966 had died above 90 in this parish; and they were all females. See the Register of this parish to 1780, among the collection of Tables in the next volume.

From an accurate survey of the parish of *Skelton*, in *Yorkshire*, taken 1777, under the direction of Dr *Bisset*, it appeared that 39 (that is, an 18th part) were 75 and upwards, 25 of whom were females, and only 14 males.

According to an enumeration in 1762, a hundred and five parishes and villages in the generality of *Rouen* in *France*, consisted of 15,943 families, and 60,552 inhabitants, 6812 of whom were girls and 5670 boys, under the age of fourteen.

(b) Vid. *Susmilch*, *Gottliche Ordnung*, &c. where a minute account is given of the number of males and females at BERLIN in 1747; and also, of the numbers of each sex that had died from 1722 to 1750.

died

died annually, from 1749 to 1758, were to the males, in no higher proportion than $3\frac{1}{2}$ to 3. Before 1749, the Bills gave the totals of burials, without distinguishing them into the totals of males and females dying every year.

Mr. Kerseboom, in his Essay on the numbers of people in HOLLAND, informs us, that from the Tables of assignable Annuities for lives in HOLLAND, which had been kept there for 125 years, wherein the ages of the persons dying are truly entered; it appears, that females have, in all accidents of age, lived about 3 or 4 years longer than the same number of males. See *Philosophical Transactions* abridged, Vol. IX. p. 326.

In Volume the 7th of the *Philosophical Transactions* abridged, Part IV. p. 46, &c. there is an account of the numbers of *male* and *female* still-born children and chryfoms, and of boys and girls under 10, of married men and married women, and of widows and widowers, who died for a course of years at *Vienna, Breslaw, Dresden, Leipzig, Ratisbon*, and some other towns in GERMANY.

He that will take the pains to examine these accounts will find that, though in these towns the proportion of males and females born is no higher than 19 to 18, yet the
pro-

proportion of boys and girls (a) that die is 8 to 7; and that, in particular, the *still-born* and *chrysom males*, are to the *still-born* and *chrysom females*, as 3 to 2.

In these accounts it appears also, that of 7270 married persons who had died in these towns (b), 4336 were married men, and but 2934 married women; that is, *three* married men died to two married women. In all POMERANIA, during 9 years, from 1748 to 1756, there died 13,556 married men, and 10,007 married women; that is, nearly 15 to 11. *Susmilch, Gottliche Ordnung*, vol. i. Tables, p. 97. The scheme for making provision for the widows and orphans of the ministers in SCOTLAND, has obliged them to keep an account of the number of *weddings* among them, and the number of *widows* left annually: and it appears from the *reports* of the *trustees* for carrying this scheme into execution, that the annual medium of *weddings* is 30. And the annual medium of *widows*, who came upon the scheme for 35

(a) In the accounts from *Breslaw* it is particularly mentioned, that by *boys* and *girls* are meant children to 10 years of age, of whom for 8 years from 1717 to 1725, *seven* males died to *six* females, exclusively of the *still-born* and *chrysoms*.

(b) In *Breslaw* alone, for the eight years mentioned in the last note, 1891 married men died, to 1196 married women; that is 5 to 3.

years, to 1779 was $19\frac{1}{10}$. Of 30 marriages then contracted annually, $19\frac{1}{10}$ became extinct by the deaths of *husbands*; and not 11 by the deaths of *wives*. That is; among the ministers and professors in SCOTLAND, 19 married men die to 11 married women. It appears, therefore, that there is the chance of more than 7 to 4, that the *woman* shall be the survivor of a marriage, and not the *man*. In order to account for this by the difference of age between men and their wives, this difference ought to be at least 13 or 14 years. That is; supposing the mean age at which women marry to be 23, the mean age at which men marry ought to be 36 or 37. But this seems to exceed the bounds of credibility; and, therefore, very probably, the greater mortality of males must operate in this case.

It is further observable in the accounts from *Germany*, to which I have referred, that the number of *widows* dying annually, is four times the number of *widowers* (a); and,

(a) In *Dresden* alone, the number of *widows* who died, in four years, was 584. The number of *widowers*, 149. That is: 4 to 1. At WITTENBERG, during 11 years, 98 *widowers* died, and 376 *widows*.—At GOTH A, during 20 years, 210 *widowers* and 760 *widows*. *Susmilch's Got-tliche Ordnung*, Vol. II. p. 273.—In the country, on account of a less difference between the ages of husbands and wives and more early marriages, the deaths of *widowers* and *widows* come nearer to one another: for in Po-

and, as *widows* are certainly, one with another, several years younger than *widowers*; it may be concluded from hence, that the number of the former in life together could not be less than five times the latter.—This fact is likewise confirmed, by the observations which have been made among the ministers in *Scotland*. The number of *widows* in life, derived from the whole body of ministers and professors, cannot be much short of 400; but the number of *widowers* among them had, for many years before 1779, been scarcely 90; that is, not so much as a *quarter* of the number of *widows*.—It may be easily seen, and it would not be difficult to demonstrate, that neither the greater number of persons left widows, nor any probable supposition concerning the greater frequency of marriages among widowers, can completely account for this, without admitting the *greater mortality of males*.—This, therefore, appears on the whole to be a fact well established: And it follows from it, that in order to calculate the values of *Life-An-*

MERANIA, during the 9 years mentioned in p. 364, the *widowers* that died were 411, the *widows* 1553; or 2 to 5.—At CHESTER, during 9 years, from 1772 to 1779, the number of *widowers* who died was 157; of *widows* 390.—The number of *widowers* in the town in 1774 was 258; of *widows* 736.—At Warrington, during 7 years, from 1773 to 1779, seventy-nine *widowers* died, and 155 *widows*. See the Introduction to the Tables in the next volume.

nuities

muties and *Reversions* with exactness, there ought to be distinct Tables of the Probabilities of life for *males* and *females*. All that is necessary to obtain the proper *data* for forming such Tables is, that the *sexes* as well as the *ages* of the dead should be specified in the Bills; and this improvement would be rendered more complete by distinguishing the males that die under the denominations of boys, married men, widowers and batchelors; and the females under the denominations of girls, married women, widows, and virgins (a).

It has been observed, that the Author of nature has provided, that more *males* should

(a) Since the former editions of this work, Registers of mortality on the plan proposed here and in the two following pages, have been established at *Chester* under the direction of Dr. *Haygarth*; at *Warrington*, under the direction of Mr. *Aikin*; and at *Eccles* near *Manchester*, under the direction of Dr. *Percival*.—The two first of these Registers (abstracts of which will be found in the next volume) have furnished already *data* nearly sufficient for forming distinct Tables of the values of lives among males and females; and they confirm what has been here observed concerning the longer duration of human life among females. But the best information on this subject has been given by the Observations in SWEDEN, which came to my knowledge since the three first editions of this Treatise, and which have helped me not a little to improve it, as may be seen in the next volume.

For more facts relating to the longer duration of life among females, see page 126 and 127 of this volume, and the Supplement in the next volume.

be born than *females*, on account of the particular waste of *males*, occasioned by wars and other causes. Perhaps it might have been observed with more reason, that this provision had in view that particular weakness or delicacy in the constitution of males which makes them more subject to mortality; and which, consequently, renders it necessary that more of them should be produced, in order to preserve in the world a due proportion between the two sexes (*a*).

In the course of this Essay, it has often appeared, that I have been particularly indebted to an information which I have received from NORTHAMPTON.—I should be inexcusable, did I not mention, that I owe this information to Mr. *Lawton*, an ingenious gentleman in that town, who has preserved the Bills of Mortality there with much care, and been very obliging in communicating them to me.—It is much to be desired, that like accounts were kept in every town and parish. It would be extremely agreeable to learn from them the different rates of human mortality in different places, and the number of people and progress of population in the kingdom. The trouble of keeping them would be trifling; but the instruc-

(*a*) More will be said on this subject in the *Supplement* in the next volume, and the true cause of that debility in the male constitution, which shortens its duration, will be there assigned.

tion derived from them (a), would be very important.—I have already proposed one improvement of such accounts. I will add, that they would be still more useful, did they give the ages of the dead after 10, within periods of *five* instead of *ten* years.—During every period so short as *five* years, the decrements of life may, in constructing Tables, be safely taken to be *uniform*. But this cannot be equally depended on, in periods so long as ten years.

There is yet another improvement of these accounts, which I shall take this opportunity to mention. They should contain not only a list of the distempers of which all die, like that in the *London Bills*; but they should specify particularly the numbers dying of these distempers, in the several divisions of life. Accurate registers of mortality kept in this manner in all parts of the kingdom, and compared with records of the seasons, and of the weather, and with the particular circumstances which discriminate different situations, might contribute, more than can be easily imagined, to the increase of *physical* knowledge.—But to proceed no farther in these Observations; I shall now beg leave to shut up this Essay with the following general reflection.

(a) See Essay I. p. 280, 281.

I have represented particularly the great difference between the duration of human life in towns and in country parishes; and from the facts I have recited it appears, that the further we go from the artificial and irregular modes of living in great towns the fewer of mankind die in the *first* stages of life, and the more in its *last* stages. The lower animals (except such (a) as have been taken under human management) seem in general to enjoy the full period of existence allotted them, and to die chiefly of old age: And were any observations to be made among *savages*, perhaps the same would be found to be true of them.—DEATH is an evil to which the order of Providence has subjected every inhabitant of this earth; but to man it has been rendered unspeakably more an evil than

(a) Calves are the only animals taken under our peculiar care immediately after birth; and in consequence of then administering to them the same sort of physic that is given to infants, and treating them in other respects in the same manner; it is probable, that more of them die soon after being born, than of *all* the other species of animals, which we see in the same circumstances. See the *Comparative View of the State and Faculties of Man with those of the Animal World*, p. 23.—It is indeed, melancholy to think of the havock made among the human species by the unnatural *customs* as well as the *vices* which prevail in polished societies. I have no doubt, but that the custom, in particular, of committing infants, as soon as born, to the care of *foster-mothers*, destroys more lives than the sword, famine, and pestilence put together.

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it was designed to be. The greatest part of that black catalogue of diseases which ravage human life, is the off-spring of the tenderness, the luxury, and the corruptions introduced by the vices and false refinements of civil society (a). That delicacy which is injured by every breath of air, and that rottenness of constitution which is the effect of indolence, intemperance, and debauchery, were never intended by the Author of Nature; and it is impossible, that they should not lay the foundation of numberless sufferings, and terminate in premature and miserable deaths.—Let us then value more the simplicity and innocence of a life agreeable to nature; and learn to consider nothing as savageness but malevolence, ignorance, and wickedness. The order of nature is wise and kind. In a conformity to it consist health and long life; grace, honour, virtue and joy. But nature turned out of its way will always punish. *The wicked shall not*

(a) The ingenious and excellent writer quoted in the last note, observes, that the whole class of diseases which arise from catching cold, are found only among the civilized part of mankind, p. 51.—And, concerning that loss of all our higher powers which so often attends the decline of life, and which is so humiliating to human pride, he observes, that it exhibits a scene singular in nature, and that there is the greatest reason to believe, that it proceeds from adventitious causes, and would not take place among us if we led natural lives, p. 62.

live.

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live out half their days. Criminal excesses embitter and cut short our *present* lives; and the highest authority has taught us to expect, that they will not only kill the *body*, but the *soul*; and deprive of ETERNAL LIFE.

END OF VOL. I.